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CHAPTER

7

Supply Systems

Introduction

In early days, there was a little demand for electrical energy so that small power stations were built to supply lighting and heating loads. However, the widespread use of electrical energy by modern civilisation has necessitated to produce bulk electrical energy economically and efficiently. The increased demand of electrical energy can be met by building big power stations at favourable places where fuel (coal or gas) or water energy is available in abundance. This has shifted the site of power stations to places quite away from the consumers. The electrical energy produced at the power stations has to be supplied to the consumers. There is a large network of conductors between the power station and the consumers. This network can be broadly divided into two parts *viz*., transmission and distribution. The purpose of this chapter is to focus attention on the various aspects of transmission of electric power.

7.1 Electric Supply System

The conveyance of electric power from a power station to consumers' premises is known as **electric supply system**.

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An electric supply system consists of three principal components *viz.*, the power station, the transmission lines and the distribution system. Electric power is produced at the power stations which are located at favourable places, generally quite away from the consumers. It is then transmitted over large distances to load centres with the help of conductors known as transmission lines. Finally, it is distributed to a large number of small and big consumers through a distribution network.

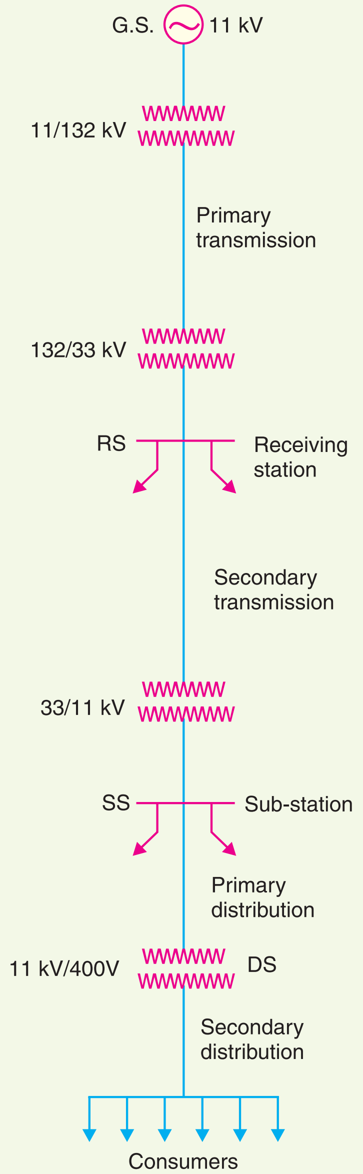

Fig. 7.1

The electric supply system can be broadly classified into (*i*) d.c. or a.c. system (*ii*) overhead or underground system. Now-a-days, 3-phase, 3-wire a.c. system is universally adopted for generation and transmission of electric power as an economical proposition. However, distribution of electric power is done by 3-phase, 4-wire a.c. system. The underground system is more expensive than the overhead system. Therefore, in our country, overhead system is *mostly adopted for transmission and distribution of electric power.

7.2 Typical a.c. Power Supply Scheme

The large network of conductors between the power station and the consumers can be broadly divided into two parts *viz.*, transmission system and distribution system. Each part can be further sub-divided into two—primary transmission and secondary transmission and primary distribution and secondary distribution. Fig. 7.1. shows the layout of a typical a.c. power supply scheme by a single line diagram. It may be noted that it is not necessary that all power schemes include all the stages shown in the figure. For example, in a certain power scheme, there may be no secondary transmission and in another case, the scheme may be so small that there is only distribution and no transmission.

(*i*) *Generating station :* In Fig 7.1, G.S. represents the generating station where electric power is produced by 3-phase alternators operating in parallel. The usual generation voltage is †11 kV. For economy in the transmission of electric power, the generation voltage (*i.e.*, 11 kV) is stepped upto 132 kV (or **more) at the generating station with the help of 3-phase transformers. The transmission of electric power at high voltages has several advantages including the saving of conductor material and high transmission efficiency. It may appear advisable to use the highest possible voltage for transmission of electric power to save conductor material and have other advantages. But there is a limit to which this voltage can be increased. It is because increase in transmission voltage introduces insulation problems as

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* In certain densely populated cities, the underground system is being employed for distribution. This is to eliminate the danger to human life which would be present with overhead system and to avoid ugly appearance and inconvenience of pole lines running down the main thorough fares.

† It may be 6·6 kV or even 33 kV in certain cases.

** Depending upon the length of transmission line and the amount of power to be transmitted.

well as the cost of switchgear and transformer equipment is increased. Therefore, the choice of proper transmission voltage is essentially a question of economics. Generally the primary transmission is carried at 66 kV, 132 kV, 220 kV or 400 kV.

(ii) *Primary transmission.* The electric power at 132 kV is transmitted by 3-phase, 3-wire overhead system to the outskirts of the city. This forms the primary transmission.

(iii) *Secondary transmission.* The primary transmission line terminates at the receiving station (*RS*) which usually lies at the outskirts of the city. At the receiving station, the voltage is reduced to 33kV by step-down transformers. From this station, electric power is transmitted at 33kV by 3-phase, 3-wire overhead system to various sub-stations (*SS*) located at the strategic points in the city. This forms the secondary transmission.

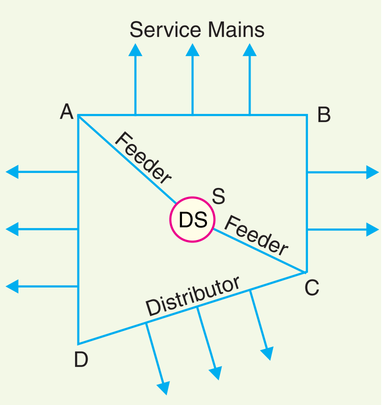
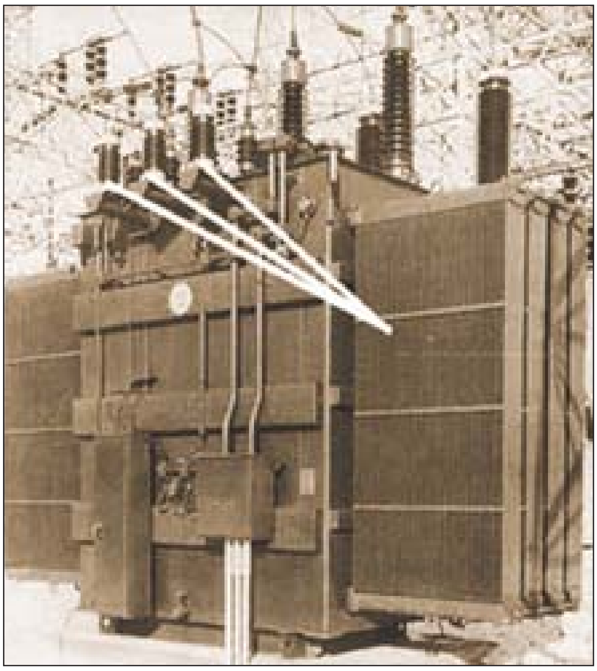


Fig. 7.2

(iv) *Primary distribution.* The secondary transmission line terminates at the sub-station (*SS*) where voltage is reduced from 33 kV to 11kV, 3-phase, 3-wire. The 11 kV lines run along the important road sides of the city. This forms the primary distribution. It may be noted that big consumers (having demand more than 50 kW) are generally supplied power at 11 kV for further handling with their own sub-stations.

(v) *Secondary distribution.* The electric power from primary distribution line (11 kV) is delivered to distribution sub-stations (*DS*). These sub-stations are located near the consumers' localities and step down the voltage to 400 V, 3-phase, 4-wire for secondary distribution. The voltage between any two phases is 400 V and between any phase and neutral is 230 V. The single-phase residential lighting load is connected between any one phase and neutral, whereas 3-phase, 400 V motor load is connected across 3-phase lines directly.

It may be worthwhile to mention here that secondary distribution system consists of *feeders, distributors and service mains*. Fig. 7.2 shows the elements of low voltage distribution system. Feeders (*SC* or *SA*) radiating from the distribution sub-station (*DS*) supply power to the distributors (*AB*, *BC*, *CD* and *AD*). No consumer is given direct connection from the feeders. Instead, the consumers are connected to the distributors through their service mains.



Power Transformer

Note. A practical power system has a large number of auxiliary equipments (*e.g.*, fuses, circuit breakers, voltage control devices etc.). However, such equipments are not shown in Fig. 7.1. It is because the amount of information included in the diagram depends on the purpose for which the diagram is intended. Here our purpose is to display general lay out of the power system. Therefore, the location of circuit breakers, relays etc., is unimportant.

Further, the structure of power system is shown by a single line diagram. The complete 3-phase circuit is seldom necessary to convey even the most detailed information about the system. In fact, the complete diagram is more likely to hide than to clarify the information we are seeking from the system viewpoint.

7.3 Comparison of D.C. and A.C. Transmission

The electric power can be transmitted either by means of d.c. or a.c. Each system has its own merits and demerits. It is, therefore, desirable to discuss the technical advantages and disadvantages of the two systems for transmission of electric power.

1. D.C. transmission. For some years past, the transmission of electric power by d.c. has been receiving the active consideration of engineers due to its numerous advantages.

Advantages. The high voltage d.c. transmission has the following advantages over high voltage a.c. transmission :

(*i*) It requires only two conductors as compared to three for a.c. transmission.

(*ii*) There is no inductance, capacitance, phase displacement and surge problems in d.c. transmission.

(*iii*) Due to the absence of inductance, the voltage drop in a d.c. transmission line is less than the a.c. line for the same load and sending end voltage. For this reason, a d.c. transmission line has better voltage regulation.

(*iv*) There is no skin effect in a d.c. system. Therefore, entire cross-section of the line conductor is utilised.

(*v*) For the same working voltage, the potential stress on the insulation is less in case of d.c. system than that in a.c. system. Therefore, a d.c. line requires less insulation.

(*vi*) A d.c. line has less corona loss and reduced interference with communication circuits.

(*vii*) The high voltage d.c. transmission is free from the dielectric losses, particularly in the case of cables.

(*viii*) In d.c. transmission, there are no stability problems and synchronising difficulties.

Disadvantages

(*i*) Electric power cannot be generated at high d.c. voltage due to commutation problems.

(*ii*) The d.c. voltage cannot be stepped up for transmission of power at high voltages.

(*iii*) The d.c. switches and circuit breakers have their own limitations.

2. A.C. transmission. Now-a-days, electrical energy is almost exclusively generated, transmitted and distributed in the form of a.c.

Advantages

(*i*) The power can be generated at high voltages.

(*ii*) The maintenance of a.c. sub-stations is easy and cheaper.

(*iii*) The a.c. voltage can be stepped up or stepped down by transformers with ease and efficiency. This permits to transmit power at high voltages and distribute it at safe potentials.

Disadvantages

(*i*) An a.c. line requires more copper than a d.c. line.

(*ii*) The construction of a.c. transmission line is more complicated than a d.c. transmission line.

(*iii*) Due to skin effect in the a.c. system, the effective resistance of the line is increased.

(*iv*) An a.c. line has capacitance. Therefore, there is a continuous loss of power due to charging current even when the line is open.

Conclusion. From the above comparison, it is clear that high voltage d.c. transmission is superior to high voltage a.c. transmission. Although at present, transmission of electric power is carried by a.c., there is an increasing interest in d.c. transmission. The introduction of mercury arc rectifiers and thyratrons have made it possible to convert a.c. into d.c. and *vice-versa* easily and efficiently. Such devices can operate upto 30 MW at 400 kV in single units. The present day trend is towards a.c. for generation and distribution and high voltage d.c. for transmission.

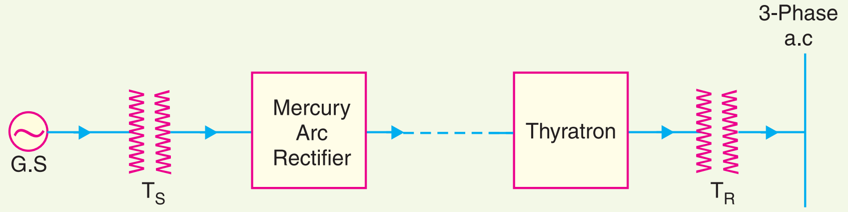


Fig. 7.3

Fig. 7.3 shows the single line diagram of high voltage d.c. transmission. The electric power is generated as a.c. and is stepped up to high voltage by the sending end transformer T_S. The a.c. power at high voltage is fed to the mercury arc rectifiers which convert a.c. into d.c. The transmission of electric power is carried at high d.c. voltage. At the receiving end, d.c. is converted into a.c. with the help of thyratrons. The a.c. supply is stepped down to low voltage by receiving end transformer T_R for distribution.

7.4 Advantages of High Transmission Voltage

The transmission of electric power is carried at high voltages due to the following reasons :

(*i*) Reduces volume of conductor material. Consider the transmission of electric power by a three-phase line.

Let P = power transmitted in watts

V = line voltage in volts

$\cos\phi$ = power factor of the load

l = length of the line in metres

R = resistance per conductor in ohms

ρ = resistivity of conductor material

a = area of X-section of conductor

$$\text{Load current, } I = \frac{P}{\sqrt{3}\, V \cos\phi}$$

$$\text{Resistance/conductor, } R = \rho l/a$$

$$\text{Total power loss, } W = 3 I^2 R = 3\left(\frac{P}{\sqrt{3}\, V \cos\phi}\right)^2 \times \frac{\rho l}{a}$$

$$= \frac{P^2 \rho l}{V^2 \cos^2\phi\, a}$$

$$\therefore \quad \text{Area of X-section, } a = \frac{P^2 \rho l}{W V^2 \cos^2\phi}$$

Total volume of conductor material required

$$= 3\, a\, l = 3\left(\frac{P^2 \rho\, l}{W V^2 \cos^2\phi}\right) l$$

$$= \frac{3P^2 \rho l^2}{W V^2 \cos^2 \phi} \quad ...(i)$$

It is clear from exp. (*i*) that for given values of *P*, *l*, ρ and *W*, the volume of conductor material required is inversely proportional to the square of transmission voltage and power factor. In other words, the greater the transmission voltage, the lesser is the conductor material required.

(*ii*) Increases transmission efficiency

$$\text{Input power} = P + \text{Total losses}$$

$$= P + \frac{P^2 \rho l}{V^2 \cos^2 \phi \, a}$$

Assuming *J* to be the current density of the conductor, then,

$$a = I/J$$

$$\therefore \quad \text{Input power} = P + \frac{P^2 \rho l J}{V^2 \cos^2 \phi \, I} = P + \frac{P^2 \rho l J}{V^2 \cos^2 \phi} \times \frac{1}{I}$$

$$= P + \frac{P^2 \rho l J}{V^2 \cos^2 \phi} \times \frac{{}^{*}\sqrt{3}\, V \cos \phi}{P}$$

$$= P + \frac{\sqrt{3}\, P J \rho l}{V \cos \phi} = P \left[1 + \frac{\sqrt{3}\, J \rho l}{V \cos \phi} \right]$$

$$\text{Transmission efficiency} = \frac{\text{Output power}}{\text{Input power}} = \frac{P}{P \left[1 + \frac{\sqrt{3}\, J \rho l}{V \cos \phi} \right]} = \frac{1}{\left[1 + \frac{\sqrt{3}\, J \rho l}{V \cos \phi} \right]}$$

$$= {}^{**}\left[1 - \frac{\sqrt{3}\, J \rho l}{V \cos \phi} \right] \text{ approx.} \quad ...(ii)$$

As *J*, ρ and *l* are constants, therefore, transmission efficiency increases when the line voltage is increased.

(*iii*) Decreases percentage line drop

$$\text{Line drop} = I R = I \times \frac{\rho l}{a}$$

$$= I \times \rho l \times J/I = \rho l J \quad [\because a = I/J]$$

$$\text{\%age line drop} = \frac{J \rho l}{V} \times 100 \quad ...(iii)$$

As *J*, ρ and *l* are constants, therefore, percentage line drop decreases when the transmission voltage increases.

Limitations of high transmission voltage. From the above discussion, it might appear advisable to use the highest possible voltage for transmission of power in a bid to save conductor material. However, it must be realised that high transmission voltage results in

(*i*) the increased cost of insulating the conductors

(*ii*) the increased cost of transformers, switchgear and other terminal apparatus.

Therefore, there is a limit to the higher transmission voltage which can be economically employed in a particular case. This limit is reached when the saving in cost of conductor material due to

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* $I = \frac{P}{\sqrt{3} V \cos \phi}$

** Binomial theorem.

higher voltage is offset by the increased cost of insulation, transformer, switchgear etc. Hence, the choice of proper transmission voltage is essentially a question of economics. The reader may find further discussion on this topic later in this chapter.

7.5 Various Systems of Power Transmission

It has already been pointed out that for transmission of electric power, 3-phase, 3-wire a.c. system is universally adopted. However, other systems can also be used for transmission under special circumstances. The different possible systems of transmission are :

1. **D.C. system**
   (*i*) D.C. two-wire.
   (*ii*) D.C. two-wire with mid-point earthed.
   (*iii*) D.C. three-wire.
2. **Single-phase A.C. system**
   (*i*) Single-phase two-wire.
   (*ii*) Single-phase two-wire with mid-point earthed.
   (*iii*) Single-phase three-wire.
3. **Two-phase A.C. system**
   (*i*) Two-phase four-wire.
   (*ii*) Two-phase three wire.
4. **Three-phase A.C. system**
   (*i*) Three-phase three-wire.
   (*ii*) Three-phase four-wire.

From the above possible systems of power transmission, it is difficult to say which is the best system unless and until some method of comparison is adopted. Now, the cost of conductor material is one of the most important charges in a system. Obviously, the best system for transmission of power is that for which the volume of conductor material required is minimum. Therefore, the volume of conductor material required forms the basis of comparison between different systems.

While comparing the amount of conductor material required in various systems, the proper comparison shall be on the basis of equal maximum stress on the *dielectric. There are two cases :

(*i*) *When transmission is by overhead system.* In the overhead system, the maximum disruptive stress** exists between the conductor and the earth. Therefore, the comparison of the system in this case has to be made on the basis of maximum voltage between conductor and earth.

(*ii*) *When transmission is by underground system.* In the underground system, the chief stress on the insulation is between conductors. Therefore, the comparison of the systems in this case should be made on the basis of maximum potential difference between conductors.

7.6 Comparison of Conductor Material in Overhead System

In comparing the relative amounts of conductor material necessary for different systems of transmission, similar conditions will be assumed in each case *viz.*,

* In long transmission lines, the voltage is only limited by the problem of insulating the conductors against disruptive discharge. Therefore, comparison should be on the basis of equality of maximum potential difference *i.e.*, equal maximum stress on the dielectric.

** In overhead system, insulation between conductors whether at the supports or intermediate points is always provided by suitably spacing the conductors. Therefore, electric dischrage cannot occur between conductors. However, the insulation has to be provided between the conductor and supporting structure. Therefore, maximum stress is between conductor and earth.

(*i*) same power (*P watts*) transmitted by each system.
(*ii*) the distance (*l metres*) over which power is transmitted remains the same.
(*iii*) the line losses (*W watts*) are the same in each case.
(*iv*) the maximum voltage between any conductor and earth (V_m) is the same in each case.

1. Two-wire d.c. system with one conductor earthed

In the 2-wire d.c. system, one is the outgoing or positive wire and the other is the return or negative wire as shown in Fig. 7.4. The load is connected between the two wires.

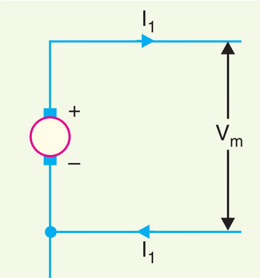


Fig. 7.4

Max. voltage between conductors = V_m

Power to be transmitted = P

∴ Load current, $I_1 = P/V_m$

If R_1 is the resistance of each line conductor, then,

$$R_1 = \rho\, l/a_1$$

where a_1 is the area of X-section of the conductor.

$$\text{Line losses,} \quad W = 2I_1^2\, R_1 = 2\left(\frac{P}{V_m}\right)^2 \rho\, \frac{l}{a_1}$$

$$\therefore \quad \text{Area of X-section, } a_1 = \frac{2\, P^2 \rho\, l}{W\, V_m^2}$$

Volume of conductor material required

$$= 2\, a_1\, l = 2\left(\frac{2\, P^2 \rho\, l}{W\, V_m^2}\right) l = \frac{4\, P^2 \rho\, l^2}{W\, V_m^2}$$

It is a usual practice to make this system as the basis for comparison with other systems. Therefore, volume of conductor material required in this system shall be taken as the basic quantity *i.e.*

$$\frac{4\, P^2 \rho\, l^2}{W\, V_m^2} = K \text{ (say)}$$

2. Two-wire d.c. system with mid-point earthed. Fig. 7.5 shows the two-wire d.c. system with mid-point earthed. The maximum voltage between any conductor and earth is V_m so that maximum voltage between conductors is $2V_m$.

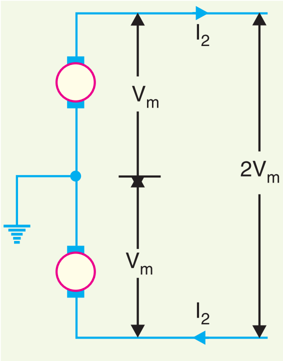


Fig. 7.5

Load current, $I_2 = P/2V_m$

Let a_2 be the area of X-section of the conductor.

$$\text{Line losses, } W = 2I_2^2\, R_2 = 2\left(\frac{P}{2\, V_m}\right)^2 \times \frac{\rho l}{a_2}$$

$$[\because R_2 = \rho\, l/a_2]$$

$$\therefore \quad W = \frac{P^2 \rho\, l}{2 a_2\, V_m^2}$$

$$\therefore \quad \text{Area of X-section, } a_2 = \frac{P^2 \rho\, l}{2 W\, V_m^2}$$

∴ Volume of conductor material required

$$= 2\, a_2\, l = 2\left(\frac{P^2 \rho\, l}{2W\, V_m^2}\right) l = \frac{P^2 \rho\, l^2}{W\, V_m^2}$$

$$= \frac{K}{4} \qquad \left[\because K = \frac{4P^2 \rho l^2}{W V_m^2}\right]$$

Hence, the volume of conductor material required in this system is *one-fourth* of that required in a two-wire d.c. system with one conductor earthed.

3. Three-wire d.c. system. In a 3-wire d.c. system, there are two outers and a middle or neutral wire which is earthed at the generator end as shown in Fig. 7.6. If the load is balanced, the current in the neutral wire is zero. Assuming balanced loads,

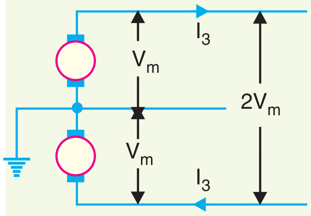


Fig. 7.6

Load current, $I_3 = P/2V_m$

Let a_3 be the area of X-section of each outer wire.

$$\text{Line losses, } W = 2I_3^2 R_3 = 2\left(\frac{P}{2V_m}\right)^2 \times \rho \frac{l}{a_3} = \frac{P^2 \rho l}{2 V_m^2 a_3}$$

$$\therefore \quad \text{Area of X-section, } a_3 = \frac{P^2 \rho l}{2W V_m^2}$$

Assuming the area of X-section of neutral wire to be half that of the outer wire,

Volume of conductor material required

$$= 2{\cdot}5\, a_3\, l = 2{\cdot}5\left(\frac{P^2 \rho l}{2 W V_m^2}\right) l = \frac{2{\cdot}5}{2}\left(\frac{P^2 \rho l^2}{W V_m^2}\right)$$

$$= \frac{5}{16} K \qquad \left[\because K = \frac{4P^2 \rho l^2}{W V_m^2}\right]$$

Hence the volume of conductor material required in this system is 5/16th of what is required for a 2-wire d.c. system with one conductor earthed.

4. Single phase 2-wire a.c. system with one conductor earthed. Fig. 7.7 shows a single phase 2-wire a.c. system with one conductor earthed. The maximum voltage between conductors is V_m so that r.m.s. value of voltage between them is $V_m / \sqrt{2}$. Assuming the load power factor to be cos ϕ,

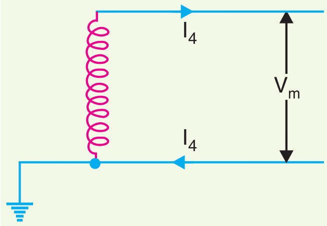


Fig. 7.7

$$\text{Load current, } I_4 = \frac{P}{(V_m / \sqrt{2}) \cos \phi} = \frac{\sqrt{2}\, P}{V_m \cos \phi}$$

Let a_4 be the area of X-section of the conductor.

$$\therefore \quad \text{Line losses, } W = 2 I_4^2 R_4 = 2\left(\frac{\sqrt{2}\, P}{V_m \cos \phi}\right)^2 \times \frac{\rho l}{a_4} = \frac{4 P^2 \rho l}{\cos^2 \phi\, V_m^2\, a_4}$$

$$\therefore \quad \text{Area of X-section, } a_4 = \frac{4 P^2 \rho l}{\cos^2 \phi\, W V_m^2}$$

Volume of conductor material required

$$= 2\, a_4\, l = 2\left(\frac{4 P^2 \rho l}{V_m^2\, W \cos^2 \phi}\right) l$$

$$= \frac{2}{\cos^2 \phi} \times \frac{4 P^2 \rho l^2}{W V_m^2}$$

$$= \frac{2K}{\cos^2 \phi} \qquad \left[\because K = \frac{4 P^2 \rho l^2}{W V_m^2}\right]$$

Hence, the volume of conductor material required in this system is $2/\cos^2\phi$ times that of 2-wire d.c. system with the one conductor earthed.

5. Single phase 2-wire system with mid-point earthed. Fig. 7.8 shows a single phase a.c. system with mid-point earthed. The two wires possess equal and opposite voltages to earth (*i.e.*, V_m). Therefore, the maximum voltage between the two wires is $2V_m$. The r.m.s. value of voltage between conductors is $= 2V_m/\sqrt{2} = \sqrt{2}V_m$. Assuming the power factor of the load to be cos φ,

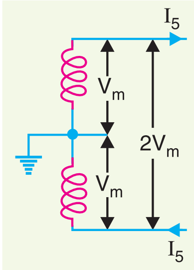


Fig. 7.8

$$\text{Load current, } I_5 = \frac{P}{\sqrt{2}\, V_m \cos\phi}$$

Let a_5 be the area of X-section of the conductor.

$$\text{Line losses, } W = 2\, I_5^2\, R_5 = 2\left(\frac{P}{\sqrt{2}\, V_m \cos\phi}\right)^2 R_5$$

$$\therefore \quad W = \frac{P^2 \rho\, l}{a_5\, V_m^2 \cos^2\phi}$$

$$\therefore \quad \text{Area of X-section, } a_5 = \frac{P^2 \rho\, l}{W\, V_m^2 \cos^2\phi}$$

Volume of conductor material required

$$= 2\, a_5\, l = 2\left(\frac{P^2 \rho\, l}{W\, V_m^2 \cos^2\phi}\right) l = \frac{2\, P^2 \rho\, l^2}{W\, V_m^2 \cos^2\phi}$$

$$= \frac{2}{\cos^2\phi} \times \frac{P^2 \rho l^2}{W\, V_m^2}$$

$$= \frac{K}{2\cos^2\phi} \qquad \left[\because K = \frac{4\, P^2 \rho\, l^2}{W\, V_m^2}\right]$$

Hence the volume of conductor material required in this system is $1/2\cos^2\phi$ times that of 2-wire d.c. system with one conductor earthed.

6. Single phase, 3-wire system. The single phase 3-wire system is identical in principle with 3-wire d.c. system. The system consists of two outers and neutral wire taken from the mid-point of the phase winding as shown in Fig. 7.9. If the load is balanced, the current through the neutral wire is zero. Assuming balanced load,

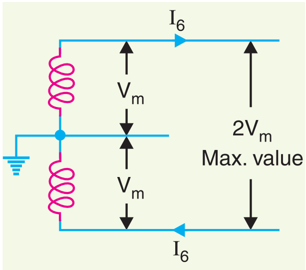


Fig. 7.9

Max. voltage between conductors $= 2\, V_m$

R.M.S.value of voltage between conductors $= 2V_m/\sqrt{2} = \sqrt{2}V_m$

If the p.f of the load is cos φ, then,

$$\text{Load current, } I_6 = \frac{P}{\sqrt{2}\, V_m \cos\phi}$$

Let a_6 be the area of X-section of each outer conductor.

$$\text{Line losses, } W = 2\, I_6^2\, R_6 = 2\left(\frac{P}{\sqrt{2}\, V_m \cos\phi}\right)^2 \times \frac{\rho\, l}{a_6}$$

$$= \frac{P^2 \rho\, l}{a_6\, V_m^2 \cos^2\phi}$$

$$\therefore \quad \text{Area of X-section, } a_6 = \frac{P^2\, \rho\, l}{W\, V_m^2 \cos^2\phi}$$

Assuming the area of X-section of neutral wire to be half that of the outer wire,

Volume of conductor material required

$$= 2{\cdot}5\, a_6\, l = 2{\cdot}5\left(\frac{P^2 \rho\, l}{W\, V_m^2 \cos^2\phi}\right) l = \frac{2\cdot 5\, P^2 \rho\, l^2}{W\, V_m^2 \cos^2\phi}$$

$$= \frac{2\cdot 5}{\cos^2\phi} \times \frac{P^2 \rho\, l^2}{W\, V_m^2}$$

$$= \frac{5K}{8\cos^2\phi} \qquad \left[\because K = \frac{4P^2 \rho\, l^2}{W\, V_m^2}\right]$$

Hence, the volume of conductor material required in this system is 5/8 $\cos^2\phi$ times that required in a 2-wire d.c. system with one conductor earthed.

7. Two phase, 4-wire a.c. system. As shown in Fig. 7.10, the four wires are taken from the ends of the two-phase windings and the mid-points of the two windings are connected together. This system can be considered as two independent single phase systems, each transmitting one half of the total power.

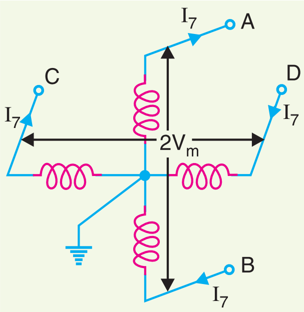


Fig. 7.10

Max. voltage between outers A and $B = 2V_m$

R.M.S. value of voltage $= 2V_m/\sqrt{2} = \sqrt{2}\, V_m$

Power supplied per phase (*i.e.*, by outers A and B) $= P/2$

Assuming p.f. of the load to be $\cos\phi$,

$$\text{Load current, } I_7 = \frac{P/2}{\sqrt{2}\, V_m \cos\phi} = \frac{P}{2\sqrt{2}\, V_m \cos\phi}$$

Let a_7 be the area of X-section of one conductor.

$$\text{Line losses, } W = 4\, I_7^2\, R_7 = 4\left(\frac{P}{2\sqrt{2}\, V_m \cos\phi}\right)^2 \times \frac{\rho\, l}{a_7}$$

$$\therefore \quad W = \frac{P^2 \rho\, l}{2a_7\, V_m^2 \cos^2\phi}$$

$$\therefore \quad \text{Area of X-section, } a_7 = \frac{P^2 \rho\, l}{2\, W\, V_m^2 \cos^2\phi}$$

$\therefore$ Volume of conductor material required

$$= 4\, a_7\, l$$

$$= 4\left(\frac{P^2 \rho\, l}{2\, W\, V_m^2 \cos^2\phi}\right) l = \frac{4P^2 \rho\, l^2}{2\, W\, V_m^2 \cos^2\phi}$$

$$= \frac{1}{2\cos^2\phi} \times \frac{4P^2 \rho\, l^2}{W\, V_m^2}$$

$$= \frac{K}{2\cos^2\phi} \qquad \left[\because K = \frac{4P^2 \rho\, l^2}{W\, V_m^2}\right]$$

Hence, the volume of conductor material required for this system is $1/2\cos^2\phi$ times that of 2-wire d.c. system with one conductor earthed.

8. Two-phase, 3-wire system. Fig. 7.11 shows two-phase, 3-wire a.c. system. The third or neutral wire is taken from the junction of two-phase windings whose voltages are in quadrature with each other. Obviously, each phase transmits one half of the total power. The R.M.S. voltage between outgoing conductor and neutral

$$= V_m/\sqrt{2}$$

$\therefore$ Current in each outer, $I_8 = \dfrac{P/2}{\dfrac{V_m}{\sqrt{2}}\cos\phi} = \dfrac{P}{\sqrt{2}\,V_m\cos\phi}$

Current in neutral* wire $= \sqrt{I_8^2 + I_8^2} = \sqrt{2}\,I_8$

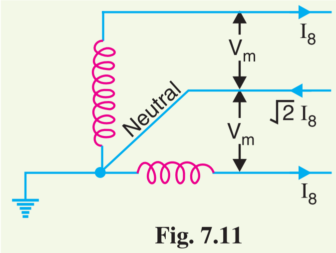


Fig. 7.11

Assuming the current density to be constant, the area of X-section of the neutral wire will be $\sqrt{2}$ ** times that of either of the outers.

$\therefore$ Resistance of neutral wire $= \dfrac{R_8}{\sqrt{2}} = \dfrac{\rho\, l}{\sqrt{2}\, a_8}$

Line losses, $W = 2\,I_8^2\,R_8 + \left(\sqrt{2}\,I_8\right)^2 \dfrac{R_8}{\sqrt{2}} = I_8^2\,R_8\left(2+\sqrt{2}\right)$

$$= \left(\frac{P}{\sqrt{2}\,V_m\cos\phi}\right)^2 \times \frac{\rho\, l}{a_8}\left(2+\sqrt{2}\right)$$

$\therefore$ $W = \dfrac{P^2 \rho\, l}{2\,a_8\,V_m^2\cos^2\phi}\left(2+\sqrt{2}\right)$

$\therefore$ Area of X-section, $a_8 = \dfrac{P^2\,\rho\, l}{2W\,V_m^2\cos^2\phi}\left(2+\sqrt{2}\right)$

Volume of conductor material required

$$= 2\,a_8\,l + \sqrt{2}\,a_8\,l = a_8\,l\left(2+\sqrt{2}\right)$$

$$= \frac{P^2\,\rho\, l^2}{2W\,V_m^2\cos^2\phi}\left(2+\sqrt{2}\right)^2$$

$$= \frac{1\cdot 457}{\cos^2\phi}K \qquad \left[\because K = \frac{4P^2\,\rho\, l^2}{W\,V_m^2}\right]$$

Hence, the volume of conductor material required for this system is $1{\cdot}457/\cos^2\phi$ times that of 2-wire d.c. system with one conductor earthed.

9. 3-Phase, 3-wire system. This system is almost universally adopted for transmission of electric power. The 3-phase, 3-wire system may be star connected or delta connected. Fig. 7.12 shows 3-phase, 3-wire star† connected system. The neutral point N is earthed.

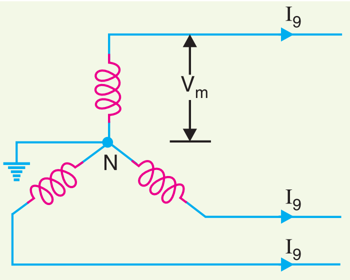


Fig. 7.12

R.M.S. voltage per phase $= V_m/\sqrt{2}$

Power transmitted per phase $= P/3$

* Current in the neutral wire is the phasor sum of currents in the outer wires. Now, the currents in the outers are in quadrature (*i.e.*, 90° apart) with each other.

** Since the neutral wire carries $\sqrt{2}$ times the current in each of the outers, its X-section must be increased in the same ratio to maintain the same current density.

† The same result will be obtained if Δ-connected system is considered.

Load current per phase, $I_9 = \dfrac{P/3}{\left(V_m/\sqrt{2}\cos\phi\right)} = \dfrac{\sqrt{2}\,P}{3\,V_m\cos\phi}$

Let a_9 be the area of X-section of each conductor.

$$\text{Line losses, } W = 3\,I_9^2\,R_9 = 3\left(\frac{\sqrt{2}\,P}{3\,V_m\cos\phi}\right)^2\frac{\rho\,l}{a_9} = \frac{2\,P^2\rho\,l}{3\,a_9\,V_m^2\cos^2\phi}$$

$$\therefore \quad \text{Area of X-section, } a_9 = \frac{2\,P^2\rho\,l}{3\,W\,V_m^2\cos^2\phi}$$

Volume of conductor material required

$$= 3\,a_9\,l = 3\left(\frac{2\,P^2\rho\,l}{3\,W\,V_m^2\cos^2\phi}\right)l = \frac{2\,P^2\rho\,l^2}{W\,V_m^2\cos^2\phi}$$

$$= \frac{0{\cdot}5\,K}{\cos^2\phi} \qquad \left[\because K = \frac{4P^2\rho\,l^2}{W\,V_m^2}\right]$$

Hence, the volume of conductor material required for this system is $0{\cdot}5/\cos^2\phi$ times that required for 2-wire d.c. system with one conductor earthed.

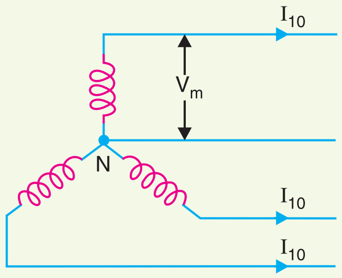


Fig. 7.13

10. 3-phase, 4-wire system. In this case, 4th or neutral wire is taken from the neutral point as shown in Fig. 7.13. The area of X-section of the neutral wire is generally one-half that of the line conductor. If the loads are balanced, then current through the neutral wire is zero. Assuming balanced loads and p.f. of the load as $\cos\phi$,

Line losses, W = Same as in 3 phase, 3-wire

$$= \frac{2\,P^2\rho\,l}{3\,a_{10}\,V_m^2\cos^2\phi}$$

$$\therefore \quad \text{Area of X-section, } a_{10} = \frac{2\,P^2\rho\,l}{3\,W\,V_m^2\cos^2\phi}$$

As the area of X-section of neutral wire is one-half that of any line conductor,

$\therefore$ Volume of conductor material required

$$= 3{\cdot}5\,a_{10}\,l = 3{\cdot}5\left(\frac{2\,P^2\rho\,l}{3\,W\,V_m^2\cos^2\phi}\right)\times l$$

$$= \frac{7\,P^2\rho\,l^2}{3\,W\,V_m^2\cos^2\phi} = \frac{7}{3\cos^2\phi}\times\frac{P^2\rho\,l^2}{W\,V_m^2}$$

$$= \frac{7\,K}{12\cos^2\phi} \qquad \left[\because K = \frac{4P^2\rho\,l^2}{W\,V_m^2}\right]$$

Hence, the volume of conductor material required for this system is $7/12\cos^2\phi$ times that required for 2-wire d.c. system with one conductor earthed.

7.7 Comparison of Conductor Material in Underground System

In an underground transmission using multi-core* belted type cables, the chief stress on the insulation is usually between conductors. Under such situations, comparison is made on the basis of maximum voltage between conductors. Again assumptions made are :

(i) The same power (*P* watts) is transmitted by each system.

(ii) The distance (*l* metres) over which power is transmitted remains the same.

(iii) The line losses (*W* watts) are the same in each case.

(iv) The maximum voltage between conductors (V_m) is the same in each case.

1. Two-wire d.c. system. If V_m denotes the maximum potential difference between the conductors, it will also be the working voltage in this case.

Load current, $I_1 = P/V_m$

Line losses, $W = 2I_1^2 R_1 = 2\left(\frac{P}{V_m}\right)^2 \frac{\rho l}{a_1}$

$\therefore \quad W = \frac{2P^2 \rho l}{a_1 V_m^2}$

$\therefore$ Area of X-section, $a_1 = \frac{2P^2 \rho l}{W V_m^2}$

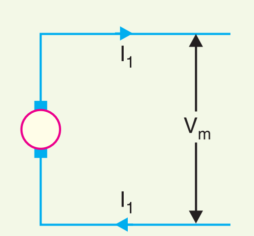


Fig. 7.14

$\therefore$ Volume of conductor material required

$$= 2\, a_1\, l = 2\left(\frac{2P^2 \rho l}{W V_m^2}\right) l = \frac{4 P^2 \rho l^2}{W V_m^2} = K\ (say)$$

This volume will be taken as the basic quantity and comparison shall be made for other systems *i.e.*,

$$\frac{4 P^2 \rho l^2}{W V_m^2} = K$$

2. Two-wire d.c. system with mid point earthed

Load current, $I_2 = P/V_m$

Line losses, $W = 2I_2^2 R_2 = 2\left(\frac{P}{V_m}\right)^2 \rho \frac{l}{a_2}$

$\therefore \quad W = \frac{2P^2 \rho l}{V_m^2 a_2}$

$\therefore$ Area of X-section, $a_2 = \frac{2P^2 \rho l}{W V_m^2}$

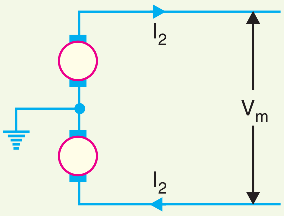


Fig. 7.15

Volume of conductor material required

$$= 2\, a_2\, l = 2\left(\frac{2P^2 \rho l}{W V_m^2}\right) l = \frac{4P^2 \rho l^2}{W V_m^2} = K$$

---

* When the underground transmission is by single-core or multi-core cables of S.L. type, the stress is from conductor to earth. Under such conditions, maximum voltage between conductor and earth forms basis of comparison of various systems.

Hence, the volume of conductor material required in this system is the same as that for 2-wire d.c. system.

3. Three wire d.c. system. The maximum voltage between the outers is V_m. Assuming balanced load, the current through the neutral wire will be zero.

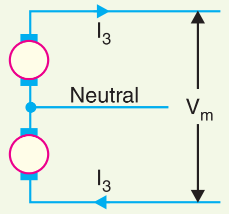


Fig. 7.16

$$\text{Load current, } I_3 = P/V_m$$

$$\text{Line losses, } W = 2\,I_3^2\,R_3 = 2\left(\frac{P}{V_m}\right)^2 \rho\,\frac{l}{a_3}$$

$$\therefore \quad W = \frac{2P^2\,\rho\,l}{V_m^2\,a_3}$$

$$\therefore \quad \text{Area of X-section, } a_3 = \frac{2P^2\,\rho\,l}{W\,V_m^2}$$

Assuming the area of X-section of neutral wire to be half of that of either outers,

Volume of conductor material required

$$= 2{\cdot}5\,a_3 l = 2{\cdot}5\left(\frac{2P^2\,\rho\,l}{W\,V_m^2}\right) l = \frac{5P^2\,\rho\,l^2}{W\,V_m^2}$$

$$= 1{\cdot}25\,K \qquad \left[\because K = \frac{4P^2\,\rho\,l^2}{W\,V_m^2}\right]$$

Hence, the volume of conductor material required in the system is 1·25 times that required for 2-wire d.c. system.

4. Single phase, 2-wire a.c. system. As the maximum voltage between conductors is V_m, therefore, r.m.s. value of voltage is $V_m/\sqrt{2}$. Assuming the p.f. of the load to be cos ϕ,

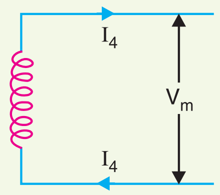


Fig. 7.17

$$\text{Load current, } I_4 = \frac{P}{V_m/\sqrt{2}\,\cos\phi} = \frac{\sqrt{2}P}{V_m\cos\phi}$$

$$\text{Line losses, } W = 2I_4^2 R_4$$

$$= 2\left(\frac{\sqrt{2}P}{V_m\cos\phi}\right)^2 \rho\,\frac{l}{a_4} = \frac{4P^2\,\rho\,l}{a_4\,V_m^2\cos^2\phi}$$

$$\therefore \quad \text{Area of X-section, } a_4 = \frac{4P^2\,\rho\,l}{W\,V_m^2\cos^2\phi}$$

Volume of conductor material required

$$= 2a_4\,l = 2\left(\frac{4P^2\,\rho\,l}{W\,V_m^2\cos^2\phi}\right) l = \frac{8P^2\,\rho\,l^2}{W\,V_m^2\cos^2\phi}$$

$$= \frac{2}{\cos^2\phi} \times \frac{4P^2\,\rho\,l^2}{W\,V_m^2}$$

$$= \frac{2\,K}{\cos^2\phi} \qquad \left[\because K = \frac{4P^2\,\rho\,l^2}{W\,V_m^2}\right]$$

Hence, the volume of conductor material required for this system is $2/\cos^2\phi$ times that required in a 2-wire d.c. system.

5. Single phase, 2-wire system with mid-point earthed. The maximum value of voltage between the outers is V_m. Therefore, the r.m.s. value of voltage is $V_m/\sqrt{2}$. If the p.f.of the load is cos ϕ, Load current, $I_5 = \dfrac{\sqrt{2}P}{V_m \cos\phi}$

Line losses, $W = 2\,I_5^2\,R_5 = 2\left(\dfrac{\sqrt{2}P}{V_m\cos\phi}\right)^2 \rho\,\dfrac{l}{a_5}$

$\therefore \quad W = \dfrac{4P^2\,\rho\,l}{a_5\,V_m^2\cos^2\phi}$

$\therefore$ Area of X-section, $a_5 = \dfrac{4P^2\,\rho\,l}{W\,V_m^2\cos^2\phi}$

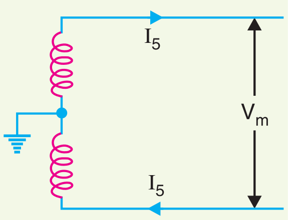


Fig. 7.18

Volume of conductor material required

$$= 2\,a_5\,l = 2\left(\frac{4P^2\,\rho\,l}{W\,V_m^2\cos^2\phi}\right)l = \frac{8P^2\,\rho\,l^2}{W\,V_m^2\cos^2\phi}$$

$$= \frac{2}{\cos^2\phi}\times\frac{4P^2\,\rho\,l^2}{W\,V_m^2}$$

$$= \frac{2\,K}{\cos^2\phi} \qquad \left[\because K = \frac{4P^2\,\rho\,l^2}{W\,V_m^2}\right]$$

Hence, the volume of conductor material required in this system is $2/\cos^2\phi$ times that required in 2-wire d.c. system.

6. Single phase, 3-wire system. If the load is considered balanced, the system reduces to a single phase 2-wire except that a neutral wire is provided in addition. Assuming the area of X-section of the neutral wire to be half of either of the outers,

Volume of conductor material required

$$= 2{\cdot}5\,{}^*a_4\,l$$

$$= 2{\cdot}5\left(\frac{4\,P^2\,\rho\,l}{W\,V_m^2\cos^2\phi}\right)l$$

$$= \frac{10\,P^2\,\rho\,l^2}{W\,V_m^2\cos^2\phi}$$

$$= \frac{2{\cdot}5}{\cos^2\phi}\times\frac{4P^2\,\rho\,l^2}{W\,V_m^2}$$

$$= \frac{2{\cdot}5\,K}{\cos^2\phi} \qquad \left[\because K = \frac{4P^2\,\rho\,l^2}{W\,V_m^2}\right]$$

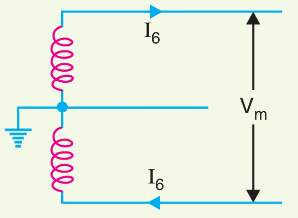


Fig. 7.19

Hence, the volume of conductor material required in this system is $2{\cdot}5/\cos^2\phi$ times that required in 2-wire d.c. system.

7. Two-phase, 4-wire system. This system can be considered as two independent single phase systems, each transmitting one-half of the total power. It is clear form Fig. 7.20 that voltage across outers (AB or CD) is twice that of single phase 2-wire (refer back to Fig. 7.17). Therefore, current (I_7) in each conductor will be half that in single-phase 2-wire system. Consequently, area of X-section of each conductor is also half but as there are four wires, so volume of conductor

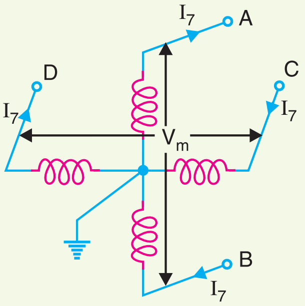


Fig. 7.20

* Area of X-section of conductor for single phase 2-wire system.

material used is the same as in a single phase, 2-wire system *i.e.*

Volume of conductor material required.

$$= \frac{2\,K}{\cos^2 \phi}$$

Hence, volume of conductor material required in this system is $2/\cos^2 \phi$ times that required in 2-wire d.c. system.

8. Two-phase, 3-wire system. Fig. 7.21 shows two phase, 3-wire a.c. system. Let us suppose that maximum voltage between the outers is V_m. Then maximum voltage between either outer and neutral wire is $V_m/\sqrt{2}$ *.

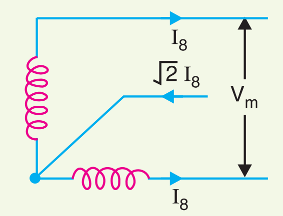


Fig. 7.21

R.M.S. voltage between outer and neutral wire

$$= \frac{V_m/\sqrt{2}}{\sqrt{2}} = \frac{V_m}{2}$$

Current in each outer, $$I_8 = \frac{P/2}{V_m/2 \cos \phi} = \frac{P}{V_m \cos \phi}$$

Current in neutral wire $$= \sqrt{I_8^2 + I_8^2} = \sqrt{2} I_8$$

Assuming the current density to be constant, the area of X-section of neutral wire will be $\sqrt{2}$ times that of either of the outers.

$\therefore$ Resistance of neutral wire $= \dfrac{R_8}{\sqrt{2}} = \dfrac{\rho\, l}{\sqrt{2}\, a_8}$

$$\text{Line losses, } W = 2 I_8^2 R_8 + (\sqrt{2}\, I_8)^2 \frac{R_8}{\sqrt{2}} = I_8^2 R_8 (2 + \sqrt{2})$$

$$= \left(\frac{P}{V_m \cos \phi}\right)^2 \times \rho \frac{l}{a_8} \left(2 + \sqrt{2}\right)$$

$$\therefore \quad W = \frac{P^2 \rho l}{a_8 V_m^2 \cos^2 \phi} \left(2 + \sqrt{2}\right)$$

$$\therefore \quad \text{Area of X-section, } a_8 = \frac{P^2 \rho l}{W V_m^2 \cos^2 \phi} \left(2 + \sqrt{2}\right)$$

Volume of conductor material required

$$= 2\, a_8\, l + \sqrt{2}\, a_8\, l = a_8\, l\, (2 + \sqrt{2})$$

$$= \frac{P^2 \rho\, l^2}{W V_m^2 \cos^2 \phi} \left(2 + \sqrt{2}\right)^2$$

$$= \frac{2 \cdot 194\, K}{\cos^2 \phi} \qquad \left[\because K = \frac{4 P^2 \rho\, l^2}{W V_m^2}\right]$$

Hence, the volume of conductor material required in this system is $2{\cdot}914/\cos^2 \phi$ times that required in 2-wire d.c. system.

9. 3-Phase, 3-wire system. Suppose that the maximum value of voltage between the conductors is V_m. Then maximum voltage between each phase and neutral is $V_m/\sqrt{3}$. Therefore, r.m.s. value of voltage per phase

* The voltages in two phase windings are 90º out of phase.

$$= \frac{V_m}{\sqrt{3}} \times \frac{1}{\sqrt{2}} = \frac{V_m}{\sqrt{6}}$$

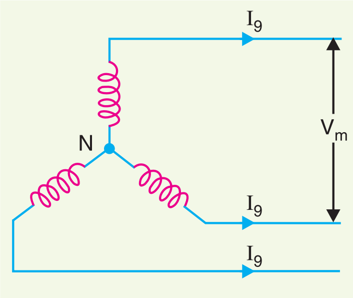


Fig. 7.22

Power transmitted per phase $= P/3$

$\therefore$ Load current per phase, $I_9 = \dfrac{P/3}{V_m/\sqrt{6}\cos\phi} = \dfrac{\sqrt{6}\,P}{3\,V_m\cos\phi}$

Line losses, $W = 3\,I_9^{\,2}\,R_9$

$$= 3\left(\frac{\sqrt{6}\,P}{3V_m\cos\phi}\right)^2 \times \rho\frac{l}{a_9}$$

$$\therefore \quad W = \frac{2P^2\rho\, l}{a_9 V_m^2\cos^2\phi}$$

$\therefore$ Area of X-section, $a_9 = \dfrac{2P^2\rho l}{W\,V_m^2\cos^2\phi}$

Volume of conductor material required

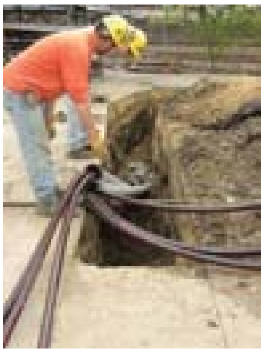
$$= 3\,a_9\,l = 3\left(\frac{2P^2\rho\, l}{W\,V_m^2\cos^2\phi}\right) l = \frac{6P^2\rho\, l^2}{W\,V_m^2\cos^2\phi}$$

$$= \frac{1\cdot 5}{\cos^2\phi} \times \frac{4P^2\rho\, l^2}{W\,V_m^2}$$

$$= \frac{1\cdot 5\,K}{\cos^2\phi} \qquad \left[\because K = \frac{4\,P^2\rho\, l^2}{W\,V_m^2}\right]$$

Hence, the volume of conductor material required in this system is $1\cdot 5/\cos^2\phi$ times that required in 2-wire d.c. system.

10. 3-phase, 4-wire system. Fig. 7.23 shows the 3-phase, 4-wire system. If the loads are balanced, then neutral wire carries no current. Consequently, the system reduces to a 3-phase, 3-wire system except that there is additional neutral wire. Assuming the area of X-section of the neutral wire to be half that of line conductor,


Underground transmission system.

Volume of conductor material required

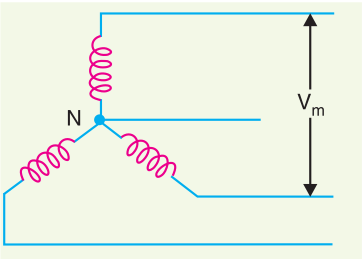


Fig. 7.23

$$= 3\cdot 5\,a_9\,l$$

$$= 3\cdot 5\left(\frac{2P^2\rho\, l}{W\,V_m^2\cos^2\phi}\right) l$$

$$= \frac{7P^2\rho\, l^2}{W\,V_m^2\cos^2\phi} = \frac{7}{\cos^2\phi} \times \frac{P^2\rho\, l^2}{W\,V_m^2}$$

$$= \frac{1\cdot 75\,K}{\cos^2\phi} \qquad \left[\because K = \frac{4\,P^2\rho\, l^2}{W\,V_m^2}\right]$$

Hence the volume of conductor material required in this system is $1\cdot 75/\cos^2\phi$ times that required in 2-wire d.c. system.

7.8 Comparison of Various Systems of Transmission

Below is given the table which shows the ratio of conductor-material in any system compared with that in the corresponding 2-wire d.c. system. Cos ϕ is the power factor in an a.c. system.

System	Same maximum voltage to earth	Same maximum voltage between conductors
1. D.C. system		
(i) Two-wire	1	1
(ii) Two-wire mid-point earthed	0·25	1
(iii) 3-wire	0·3125	1·25
2. Single phase system		
(i) 2 wire	$2/\cos^2\phi$	$2/\cos^2\phi$
(ii) 2-wire with mid-point earthed	$\frac{0\cdot5}{\cos^2\phi}$	$\frac{2}{\cos^2\phi}$
(iii) 3-wire	$\frac{0\cdot625}{\cos^2\phi}$	$\frac{2\cdot5}{\cos^2\phi}$
3. Two-phase system		
(i) 2-phase, 4-wire	$\frac{0\cdot5}{\cos^2\phi}$	$\frac{2}{\cos^2\phi}$
(ii) 2-phase, 3-wire	$\frac{1\cdot457}{\cos^2\phi}$	$\frac{2\cdot914}{\cos^2\phi}$
4. Three-phase system		
(i) 3-phase, 3-wire	$\frac{0\cdot5}{\cos^2\phi}$	$\frac{1\cdot5}{\cos^2\phi}$
(ii) 3-phase, 4-wire	$\frac{0\cdot583}{\cos^2\phi}$	$\frac{1\cdot75}{\cos^2\phi}$

The following points may be noted from the above table :

(i) There is a great saving in conductor material if d.c. system is adopted for transmission of electric power. However, due to technical difficulties, d.c. system is not used for transmission.

(ii) Considering the a.c. system, the 3-phase a.c. system is most suitable for transmission due to two reasons. Firstly, there is considerable saving in conductor material. Secondly, this system is convenient and efficient.

Example 7.1 *What is the percentage saving in feeder copper if the line voltage in a 2-wire d.c. system is raised from 200 volts to 400 volts for the same power transmitted over the same distance and having the same power loss ?*

Solution. Fig. 7.24 (*i*) shows 200 volts system, whereas Fig. 7.24 (*ii*) shows 400 volts system. Let *P* be the power delivered and *W* be power loss in both cases. Let v_1 and a_1 be the volume and area of X-section for 200 *V* system and v_2 and a_2 for that of 400 *V* system.

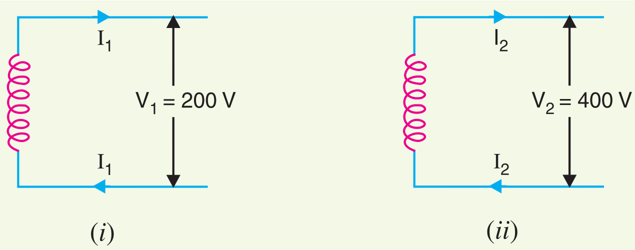


Fig. 7.24

Now, $P = V_1 I_1 = 200\, I_1$ ...(*i*)

And $P = V_2 I_2 = 400\, I_2$ ...(*ii*)

As same power is delivered in both cases,

$\therefore \quad 200 I_1 = 400\, I_2$ or $I_2 = (200/400)\, I_1 = 0{\cdot}5 I_1$

Power loss in 200 *V* system, $W_1 = 2 I_1^{\,2} R_1$

Power loss in 400 *V* system, $W_2 = 2 I_2^{\,2} R_2 = 2(0{\cdot}5\, I_1)^2 R_2 = 0{\cdot}5 I_1^{\,2} R_2$

As power loss in the two cases is the same,

$$\therefore \quad W_1 = W_2$$

or $2\, I_1^2 R_1 = 0{\cdot}5\, I_1^2 R_2$

or $R_2/R_1 = 2/0{\cdot}5 = 4$

or $a_1/a_2 = 4$

or $v_1/v_2 = 4$

or $v_2/v_1 = 1/4 = 0{\cdot}25$

$\therefore$ % age saving in feeder copper $= \dfrac{v_1 - v_2}{v_1} \times 100 = \left(\dfrac{v_1}{v_1} - \dfrac{v_2}{v_1}\right) \times 100$

$= (1 - 0{\cdot}25) \times 100 =$ **75%**

Example 7.2 *A d.c. 2-wire system is to be converted into a.c. 3-phase, 3-wire system by the addition of a third conductor of the same cross-section as the two existing conductors. Calculate the percentage additional load which can now be supplied if the voltage between wires and the percentage loss in the line remain unchanged. Assume a balanced load of unity power factor.*

Solution. Fig. 7.25 (*i*) shows the 2-wire d.c. system, whereas Fig. 7.25 (*ii*) shows the 3-phase, 3-wire system. Suppose *V* is the voltage between conductors for the two cases. Let *R* be the resistance per conductor in each case.

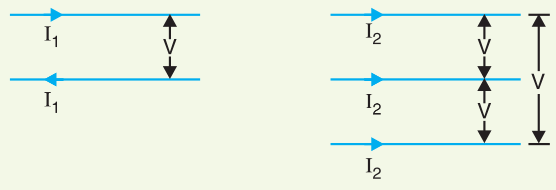


Fig. 7.25

Two-wire d.c. system. Referring to Fig. 7.25 (*i*),

Power supplied, $P_1 = V I_1$

Power loss, $W_1 = 2\, I_1^2 R$

Percentage power loss $= \dfrac{2\, I_1^2\, R}{V\, I_1} \times 100$ ...(*i*)

3-phase, 3-wire a.c. system. Referring to Fig. 7.25 (*ii*),

Power supplied, $P_2 = \sqrt{3}\, V I_2$

Power loss, $W_2 = 3\, I_2^2 R$

Percentage power loss $= \dfrac{3\, I_2^2\, R}{\sqrt{3}\, V\, I_2} \times 100$ ...(*ii*)

As the percentage power loss in the two cases is the same,

$$\therefore \quad \text{exp. }(i) = \text{exp. }(ii)$$

or $$\frac{2\, I_1^2\, R}{V\, I_1} \times 100 = \frac{3\, I_2^2\, R}{\sqrt{3}\, V\, I_2} \times 100$$

or $$2\, I_1 = \sqrt{3}\, I_2$$

or $$I_2 = \frac{2}{\sqrt{3}}\, I_1$$

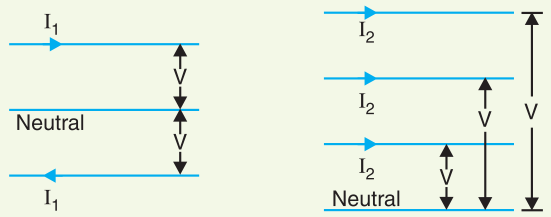
Now, $$\frac{P_2}{P_1} = \frac{\sqrt{3}\, V\, I_2}{V\, I_1} = \frac{\sqrt{3}\, V \times \frac{2}{\sqrt{3}}\, I_1}{V\, I_1} = 2$$

$$\therefore \quad P_2 = 2\, P_1$$

i.e. additional power which can be supplied at unity p.f. by 3-phase, 3-wire a.c. system = **100%**.

Example 7.3. *A d.c. 3-wire system is to be converted into a 3-phase, 4-wire system by adding a fourth wire equal in X-section to each outer of the d.c. system. If the percentage power loss and voltage at the consumer's terminals are to be the same in the two cases, find the extra power at unity power factor that can be supplied by the a.c. system. Assume loads to be balanced.*

Solution. Fig. 7.26 (*i*) shows the 3-wire d.c. system, whereas Fig. 7.26 (*ii*) shows 3-phase, 4-wire system. Suppose that V is consumer's terminal voltage (*i.e.*, between conductor and neutral) in the two cases. Let R be the resistance per conductor in each case.



(*i*) (*ii*)

Fig. 7.26

3-wire d.c. system. Refer to Fig. 7.26 (*i*). As the loads are balanced, therefore, neutral wire carries no current. Consequently, there is no power loss in the neutral wire.

$$\text{Power supplied, } P_1 = 2\, V\, I_1$$

$$\text{Power loss, } W_1 = 2\, I_1^{\,2}\, R$$

$$\text{Percentage power loss} = \frac{2\, I_1^2\, R}{2\, V\, I_1} \times 100 \quad ...(i)$$

3-phase, 4-wire a.c. system. Refer to Fig. 7.26 (*ii*). Since the loads are balanced, the neutral wire carries no current and hence there is no power loss in it.

$$\text{Power supplied, } P_2 = {}^*3\, V\, I_2 \quad [\because \cos\phi = 1]$$

$$\text{Power loss, } W_2 = 3\, I_2^{\,2}\, R$$

$$\text{Percentage power loss} = \frac{3\, I_2^2\, R}{3\, V\, I_2} \times 100 \quad ...(ii)$$

As the percentage power loss in the two cases is the same, therefore, exp. (*i*) is equal to exp. (*ii*) *i.e.*,

* Power per phase = $V\, I_2$

∴ Power in 3-phases = $3V\, I_2$

$$\frac{2\,I_1^2\,R}{2\,V\,I_1} \times 100 = \frac{3\,I_2^2\,R}{3\,V\,I_2} \times 100$$

or $$I_1 = I_2$$

Now $$\frac{P_2}{P_1} = \frac{3\,V\,I_2}{2\,V\,I_1} = \frac{3\,V\,I_1}{2\,V\,I_1} = 1{\cdot}5$$

$\therefore$ $$P_2 = 1{\cdot}5\,P_1$$

i.e., extra power that can be supplied at unity power factor by 3-phase, 4-wire a.c. system = **50%.**

Example 7.4. *A single phase a.c. system supplies a load of 200 kW and if this system is converted to 3-phase, 3-wire a.c. system by running a third similar conductor, calculate the 3-phase load that can now be supplied if the voltage between the conductors is the same. Assume the power factor and transmission efficiency to be the same in the two cases.*

Solution. Fig. 7.27 (*i*) shows the single phase 2-wire a.c. system, whereas Fig. 7.27 (*ii*) shows 3-phase, 3-wire system. Suppose that V is the voltage between the conductors in the two cases. Let R be the resistance per conductor and cos ϕ the power factor in each case.

Single phase 2-wire system. Referring to Fig. 7.27 (*i*),

$$\text{Power supplied, } P_1 = V\,I_1 \cos\phi$$

$$\text{Power loss, } W_1 = 2\,I_1^{\,2}\,R$$

$$\%\text{ age power loss} = \frac{2\,I_1^2\,R}{V\,I_1 \cos\phi} \times 100 \quad \ldots(i)$$

3-phase, 3-wire a.c. system. Referring to Fig. 7.27 (*ii*),

$$\text{Power supplied, } P_2 = \sqrt{3}\,V\,I_2 \cos\phi$$

$$\text{Power loss, } W_2 = 3\,I_2^{\,2}\,R$$

$$\%\text{ age power loss} = \frac{3\,I_2^2\,R}{\sqrt{3}\,V\,I_2 \cos\phi} \times 100 \quad \ldots(ii)$$

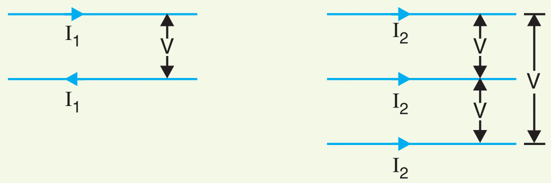


Fig. 7.27

As the transmission efficiency in the two cases is the same, therefore, percentage power loss will also be the same *i.e.*,

$$\text{exp. } (i) = \text{exp. } (ii)$$

or $$\frac{2\,I_1^2\,R}{V\,I_1 \cos\phi} \times 100 = \frac{3\,I_2^2\,R}{\sqrt{3}\,V\,I_2 \cos\phi} \times 100$$

or $$2\,I_1 = \sqrt{3}\,I_2$$

or $$I_2 = \frac{2}{\sqrt{3}}\,I_1$$

Now, $$\frac{P_2}{P_1} = \frac{\sqrt{3}\,V\,I_2 \cos\phi}{V\,I_1 \cos\phi} = \frac{\sqrt{3}\,V\,\frac{2}{\sqrt{3}}\,I_1 \cos\phi}{V\,I_1 \cos\phi} = 2$$

As the power supplied by single phase, 2-wire (*i.e.*, P_1) is 200 kW,

∴ Power supplied by 3-phase, 3-wire a.c. system is

$$P_2 = 2P_1 = 2 \times 200 = \textbf{400 kW}$$

It may be seen that 3-phase, 3-wire system can supply 100% additional load.

Example 7.5. *A 50 km long transmission line supplies a load of 5 MVA at 0·8 p.f. lagging at 33 kV. The efficiency of transmission is 90%. Calculate the volume of aluminium conductor required for the line when (i) single phase, 2-wire system is used (ii) 3-phase, 3-wire system is used. The specific resistance of aluminium is* $2{\cdot}85 \times 10^{-8}\ \Omega$ *m.*

Solution.

$$\text{Power transmitted} = \text{MVA} \times \cos\phi = 5 \times 0{\cdot}8 = 4 \text{ MW} = 4 \times 10^6 \text{ W}$$

$$\text{Line loss, } W = 10\% \text{ of power transmitted} = (10/100) \times 4 \times 10^6 = 4 \times 10^5 \text{ W}$$

$$\text{Length of line, } l = 50 \text{ km} = 50 \times 10^3 \text{ m}$$

(*i*) Single phase, 2-wire system

$$\text{Apparent power} = V I_1$$

$$\therefore \quad I_1 = \frac{\text{Apparent power}}{V} = \frac{5 \times 10^6}{33 \times 10^3} = 151{\cdot}5 \text{ A}$$

Suppose a_1 is the area of cross-section of aluminium conductor.

$$\text{Line loss, } W = 2\, I_1^2 R_1 = 2 I_1^2 \left(\rho \frac{l}{a_1}\right)$$

$$\therefore \quad \text{Area of X-section, } a_1 = \frac{2\, I_1^2\, \rho\, l}{W} = \frac{2 \times (151 \cdot 5)^2 \times \left(2 \cdot 85 \times 10^{-8}\right) 50 \times 10^3}{4 \times 10^5}$$

$$= 1{\cdot}635 \times 10^{-4} \text{ m}^2$$

$$\text{Volume of conductor required} = 2\, a_1\, l = 2 \times (1{\cdot}635 \times 10^{-4}) \times 50 \times 10^3 = \textbf{16·35 m}^3$$

(*ii*) 3-phase, 3-wire system

$$\text{Line current, } I_2 = \frac{\text{Apparent power}}{\sqrt{3}\, V} = \frac{5 \times 10^6}{\sqrt{3} \times 33 \times 10^3} = 87{\cdot}5 \text{ A}$$

Suppose a_2 is the area of cross-section of the conductor in this case.

$$\text{Line loss, } W = 3\, I_2^2 R_2 = 3\, I_2^2 \left(\rho \frac{l}{a_2}\right)$$

$$\therefore \quad \text{Area of X-section, } a_2 = \frac{3\, I_2^2\, \rho\, l}{W} = \frac{3 \times (87 \cdot 5)^2 \times \left(2 \cdot 85 \times 10^{-8}\right) \times 50 \times 10^3}{4 \times 10^5}$$

$$= 0{\cdot}818 \times 10^{-4} \text{ m}^2$$

$$\text{Volume of conductor required} = 3\, a_2\, l = 3 \times (0{\cdot}818 \times 10^{-4}) \times 50 \times 10^3 = \textbf{12·27 m}^3$$

Note that volume of conductor (and hence weight) required is less in case of 3-phase, 3-wire system.

Example 7.6. *A sub-station supplies power at 11 kV, 0·8 p.f. lagging to a consumer through a single phase transmission line having total resistance (both go and return) of 0.15 Ω. The voltage drop in the line is 15%. If the same power is to be supplied to the same consumer by two wire d.c. system by a new line having a total resistance of 0·05 Ω and if the allowable voltage drop is 25%, calculate the d.c. supply voltage.*

Solution :

Single phase system

Let I_1 be the load current. Then,

$$\text{Voltage drop} = I_1 R_1 = I_1 \times 0{\cdot}15 \text{ volts} \quad ...(i)$$

$$\text{Also voltage drop} = \frac{15}{100} \times 11000 = 1650 \text{ volts} \quad ...(ii)$$

From eqs. (*i*) and (*ii*), $I_1 \times 0{\cdot}15 = 1650$

$$\therefore \quad I_1 = \frac{1650}{0{\cdot}15} = 11000 \text{ A}$$

$$\text{Power received by consumer} = \text{Apparent power} \times \cos\phi$$

$$= \frac{(11{,}000 \times 11{,}000) \times 0{\cdot}8}{1000} \text{ kW} = 9{\cdot}68 \times 10^4 \text{ kW}$$

D.C. two-wire system. The power to be supplied by d.c. 2-wire system is $9{\cdot}68 \times 10^4$ kW = $9{\cdot}68 \times 10^7$ W. Let V volts be the supply voltage.

$$\therefore \quad \text{Load current, } I_2 = \frac{9{\cdot}68 \times 10^7}{V}$$

$$\text{Voltage drop} = I_2 R_2 = \left(\frac{9{\cdot}68 \times 10^7}{V}\right) \times 0{\cdot}05 \quad ...(iii)$$

$$\text{Allowable voltage drop} = \frac{25}{100} \times \text{V} = 0{\cdot}25 \text{ V} \quad ...(iv)$$

$$\therefore \quad \frac{9{\cdot}68 \times 10^7 \times 0{\cdot}05}{V} = 0{\cdot}25 \text{ V}$$

$$\text{or} \quad V^2 = \frac{9{\cdot}68 \times 10^7 \times 0{\cdot}05}{0{\cdot}25} = 1936 \times 10^4$$

$$\therefore \quad V = \sqrt{1936 \times 10^4} = \mathbf{4400\ V}$$

TUTORIAL PROBLEMS

1. What is the percentage saving in copper feeder if the line voltage in a 2-wire d.c. system is raised from 220 V to 500 V for the same power transmitted over the same distance and having the same power loss? **[80·64%]**
2. A single phase load of 5 MW is transmitted by a pair of overhead conductors. If a third conductor of the same cross-section and material be added and 3-phase supply be thus substituted for the original single phase, calculate the 3-phase load which can now be transmitted if the voltage between the conductors and the percentage loss in the lines remains unchanged. **[10 MW]**
3. Electric power of 50 MW is to be transmitted over a 132 KV, 3-phase, 3-wire transmission line. The length of the line is 300 km and the efficiency of transmission is 85%. Aluminium is used for conductor material which has resistivity of 3×10^{-9} Ωm. Calculate the volume of conductor material required for a power factor of 0·8 lagging. **[242 m^3]**
4. A 3-phase, 4-wire system is used for lighting. Compare the amount of copper required with that needed for a 2-wire d.c. system with the same lamp voltage. Assume the same losses and balanced load. The neutral is one-half the cross-section of one of the respective outers. **[Copper for 3ϕ, 4 wire : Copper for 2-wire d.c. = 0·292 : 1]**
5. 30,000 kW at power factor 0·8 lagging is being transmitted over a 220 kV, three-phase transmission line. The length of the line is 275 km and the efficiency of transmission is 90%. Calculate the weight of copper required. Also calculate the weight of copper had the power been transmitted over a single-phase transmission line for the same line voltage and losses. Assume that the resistance of 1 km long conductor and 1 sq. cm is 0·173 Ω and specific gravity of copper is 8·9. **[338 × 10^3 kg : 450·67 × 10^3 kg]**

7.9 Elements of a Transmission Line

For reasons associated with economy, transmission of electric power is done at high voltage by 3-phase, 3-wire overhead system. The principal elements of a high-voltage transmission line are :

(*i*) *Conductors,* usually three for a single-circuit line and six for a double-circuit line. The usual material is aluminium reinforced with steel.

(*ii*) *Step-up and step-down transformers,* at the sending and receiving ends respectively. The use of transformers permits power to be transmitted at high efficiency.

(*iii*) *Line insulators,* which mechanically support the line conductors and isolate them electrically from the ground.

(*iv*) *Support,* which are generally steel towers and provide support to the conductors.

(*v*) *Protective devices,* such as ground wires, lightning arrestors, circuit breakers, relays etc. They ensure the satisfactory service of the transmission line.

(*vi*) *Voltage regulating devices,* which maintain the voltage at the receiving end within permissible limits.

All these elements will be discussed in detail in the subsequent chapters.

7.10 Economics of Power Transmission

While designing any scheme of power transmission, the engineer must have before him the commercial aspect of the work entrusted to him. He must design the various parts of transmission scheme in a way that maximum economy is achieved. The economic design and layout of a complete power transmission scheme is outside the scope of this book. However, the following two fundamental economic principles which closely influence the electrical design of a transmission line will be discussed :

(*i*) Economic choice of conductor size

(*ii*) Economic choice of transmission voltage

7.11 Economic Choice of Conductor Size

The cost of conductor material is generally a very considerable part of the total cost of a transmission line. Therefore, the determination of proper size of conductor for the line is of vital importance. The most economical area of conductor is that for which the total annual cost of transmission line is minimum*. This is known as *Kelvin's Law* after Lord Kelvin who first stated it in 1881. The total annual cost of transmission line can be divided broadly into two parts *viz.*, annual charge on capital outlay and annual cost of energy wasted in the conductor.

(*i*) *Annual charge on capital outlay.* This is on account of interest and depreciation on the capital cost of complete installation of transmission line. In case of overhead system, it will be the annual interest and depreciation on the capital cost of conductors, supports and insulators and the cost of their erection. Now, for an overhead line, insulator cost is constant, the conductor cost is proportional to the area of X-section and the cost of supports and their erection is partly constant and partly proportional to area of X-section of the conductor. Therefore, annual charge on an overhead† transmission line can be expressed as :

$$\text{Annual charge} = P_1 + P_2\, a \qquad \text{...}(i)$$

* The question of voltage regulation is unimportant in a transmission line. Generally, the X-sectional area of the conductor is decided on the basis of minimum annual cost.

† **Underground system.** A similar relationship exists for underground system. In this system, the annual charge is on account of interest and depreciation on the cost of conductors, insulation and the cost of laying the cables. Now, the cost of insulation is constant and the cost of conductor is proportional to area of X-section of conductor.

$$\therefore \qquad \text{Annual charge} = P_1 + P_2\, a$$

where P_1 and P_2 are constants and a is the area of X-section of the conductor.

(ii) Annual cost of energy wasted. This is on account of energy lost mainly‡ in the conductor due to I^2R losses. Assuming a constant current in the conductor throughout the year, the energy lost in the conductor is proportional to resistance. As resistance is inversely proportional to the area of X-section of the conductor, therefore, the energy lost in the conductor is inversely proportional to area of X-section. Thus, the annual cost of energy wasted in an overhead transmission line can be expressed as :

$$\text{Annual cost of energy wasted} = P_3/a \quad \ldots(ii)$$

where P_3 is a constant.

$$\text{Total annual cost, } C = \text{exp. } (i) + \text{exp. } (ii)$$
$$= (P_1 + P_2\,a) + P_3/a$$
$$\therefore \quad C = P_1 + P_2\,a + P_3/a \quad \ldots(iii)$$

In exp. (*iii*), only area of X-section a is variable. Therefore, the total annual cost of transmission line will be minimum if differentiation of C w.r.t. a is zero *i.e.*

$$\frac{d}{da}(C) = 0$$

$$\text{or} \quad \frac{d}{da}(P_1 + P_2\,a + P_3/a) = 0$$

$$\text{or} \quad P_2 - \frac{P_3}{a^2} = 0$$

$$\text{or} \quad P_2 = P_3/a^2$$

$$\text{or} \quad P_2\,a = \frac{P_3}{a}$$

i.e. Variable part of annual charge = Annual cost of energy wasted

Therefore Kelvin's Law can also be stated in an another way *i.e. the most economical area of conductor is that for which the variable part* of annual charge is equal to the cost of energy losses per year.*

Graphical illustration of Kelvin's law. Kelvin's law can also be illustrated graphically by plotting annual cost against X-sectional area 'a' of the conductor as shown in Fig. 7.28. In the diagram, the straight line (1) shows the relation between the annual charge (*i.e.*, $P_1 + P_2a$) and the area of X-section a of the conductor. Similarly, the rectangular hyperbola (2) gives the relation between annual cost of energy wasted and X-sectional area a. By adding the ordinates of curves (1) and (2), the curve (3) is obtained. This latter curve shows the relation between total annual cost ($P_1 + P_2a + P_3/a$) of transmission line and area of X-section a. The lowest point on the curve (*i.e.*, point P) represents the most economical area of X-section.

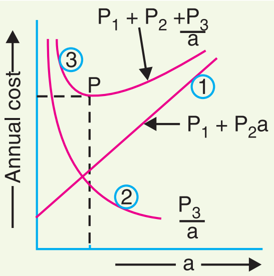


Fig. 7.28

Limitations of Kelvin's law. Although theoretically Kelvin's law holds good, there is often considerable difficulty in applying it to a proposed scheme of power transmission. In practice, the limitations of this law are :

(*i*) It is not easy to estimate the energy loss in the line without actual load curves, which are not available at the time of estimation.

(*ii*) The assumption that annual cost on account of interest and depreciation on the capital outlay is in the form $P_1 + P_2a$ is strictly speaking not true. For instance, in cables neither the cost of cable dielectric and sheath nor the cost of laying vary in this manner.

‡ There may be some losses in the insulating material but these are quite small and may be neglected.

* That part of annual charge (*i.e.*, $P_2\,a$) which is proportional to the area of X-section of conductor.

(*iii*) This law does not take into account several physical factors like safe current density, mechanical strength, corona loss etc.

(*iv*) The conductor size determined by this law may not always be practicable one because it may be too small for the safe carrying of necessary current.

(*v*) Interest and depreciation on the capital outlay cannot be determined accurately.

Example 7.7. *A 2-conductor cable 1 km long is required to supply a constant current of 200 A throughout the year. The cost of cable including installation is Rs. (20 a + 20) per metre where 'a' is the area of X-section of the conductor in cm^2. The cost of energy is 5P per kWh and interest and depreciation charges amount to 10%. Calculate the most economical conductor size. Assume resistivity of conductor material to be 1·73 μ Ω cm.*

Solution.

$$\text{Resistance of one conductor} = \frac{\rho\, l}{a} = \frac{1\cdot73\times10^{-6}\times10^{5}}{a} = \frac{0\cdot173}{a}\ \Omega$$

$$\text{Energy lost per annum} = \frac{2I^2\ Rt}{1000}\text{kWh}$$

$$= \frac{2\times(200)^2\times0\cdot173\times8760}{1000\times a} = \frac{1{,}21{,}238\cdot4}{a}\ \text{kWh}$$

$$\text{Annual cost of energy lost} = \text{Cost per kWh}\times\text{Annual energy loss}$$

$$= \text{Rs}\ \frac{5}{100}\times\frac{1{,}21{,}238\cdot4}{a}$$

$$= \text{Rs}\ 6062/a \qquad ...(i)$$

The capital cost (variable) of the cable is given to be Rs 20 *a* per metre. Therefore, for 1 km length of the cable, the capital cost (variable) is Rs. 20 $a\times1000$ = Rs. 20,000 *a*.

Variable annual charge = Annual interest and depreciation on capital cost (variable) of cable

$= \text{Rs}\ 0\cdot1\times20{,}000\ a$

$= \text{Rs}\ 2000\ a \qquad ...(ii)$

According to Kelvin's law, for most economical X-section of the conductor,

Variable annual charge = Annual cost of energy lost

or $2000\ a = 6062/a$

$$\therefore \quad a = \sqrt{\frac{6062}{2000}} = \mathbf{1\cdot74\ cm^2}$$

Example 7.8 *The cost of a 3-phase overhead transmission line is Rs (25000 a + 2500) per km where 'a' is the area of X-section of each conductor in cm^2. The line is supplying a load of 5 MW at 33kV and 0·8 p.f. lagging assumed to be constant throughout the year. Energy costs 4P per kWh and interest and depreciation total 10% per annum. Find the most economical size of the conductor. Given that specific resistance of conductor material is 10^{-6} Ω cm.*

Solution.

$$\text{Resistance of each conductor, } R = \frac{\rho\, l}{a} = \frac{10^{-6}\times10^{5}}{a} = \frac{0\cdot1}{a}\ \Omega$$

$$\text{Line current, } I = \frac{P}{\sqrt{3}V\cos\phi} = \frac{5\times10^6}{\sqrt{3}\times33\times10^3\times0\cdot8} = 109\cdot35\ \text{A}$$

$$\text{Energy lost per annum} = \frac{3I^2Rt}{1000}\ \text{kWh} = \frac{3\times(109\cdot35)^2\times0\cdot1\times8760}{1000\times a} = \frac{31{,}424}{a}\ \text{kWh}$$

$$\text{Annual cost of energy lost} = \text{Rs } 0{\cdot}04 \times 31{,}424/a = \text{Rs } \frac{1256{\cdot}96}{a}$$

The capital cost (variable) of the cable is given to be Rs 25000 a per km length of the line.

$\therefore$ Variable annual charge = 10% of capital cost (variable) of line

$$= \text{Rs } 0{\cdot}1 \times 25{,}000a = \text{Rs } 2{,}500\ a$$

According to Kelvin's law, for most economical X-section of the conductor,

Variable annual charge = Annual cost of energy lost

$$2500\ a = \frac{1256{\cdot}96}{a}$$

or
$$a = \sqrt{\frac{1256{\cdot}96}{2500}} = \mathbf{0{\cdot}71\ cm^2}$$

Example 7.9. *A 2-wire feeder carries a constant current of 250 A throughout the year. The portion of capital cost which is proportional to area of X-section is Rs 5 per kg of copper conductor. The interest and depreciation total 10% per annum and the cost of energy is 5P per kWh. Find the most economical area of X-section of the conductor. Given that the density of copper is 8·93gm/cm*^{3} *and its specific resistance is* $1{\cdot}73 \times 10^{-8}\ \Omega$ *m.*

Solution. Consider 1 metre length of the feeder. Let a be the most economical area of X-section of each conductor in m^2.

$$\text{Resistance of each conductor, } R = \frac{\rho\, l}{a} = \frac{1{\cdot}73 \times 10^{-8} \times 1}{a} = \frac{1{\cdot}73 \times 10^{-8}}{a}\ \Omega$$

$$\text{Energy lost per annum} = \frac{2I^2 R\, t}{1000}\ \text{kWh} = \frac{2 \times (250)^2 \times 1{\cdot}73 \times 10^{-8} \times 8760}{1000 \times a}$$

$$= \frac{18{,}94{,}350}{a} \times 10^{-8}\ \text{kWh}$$

$$\text{Annual cost of energy lost} = \text{Rs } \frac{5}{100} \times \frac{18{,}94{,}350 \times 10^{-8}}{a} = \text{Rs } \frac{94{,}717{\cdot}5}{a} \times 10^{-8}$$

$$\text{Mass of 1 metre feeder} = 2\ (\text{Volume} \times \text{density}) = 2 \times a \times 1 \times 8{\cdot}93 \times 10^3\ \text{kg}$$

$$= 17{\cdot}86 \times 10^3 a\ \text{kg}$$

$$\text{Capital cost (variable)} = \text{Rs } 5 \times 17{\cdot}86 \times 10^3\ a = \text{Rs } 89{\cdot}3 \times 10^3 a$$

Variable Annual charge = 10% of capital cost (variable)

$$= 0{\cdot}1 \times 89{\cdot}3 \times 10^3 a = \text{Rs } 8930\ a$$

For most economical area of X-section,

Variable annual charge = Annual cost of energy lost

or
$$\text{Rs } 8930\ a = \frac{94{,}717{\cdot}5}{a} \times 10^{-8}$$

$\therefore$
$$a = \sqrt{\frac{94{,}717{\cdot}5 \times 10^{-8}}{8930}} = 3{\cdot}25 \times 10^{-4} \text{m}^2 = \mathbf{3{\cdot}25\ cm^2}$$

Example 7.10. *Determine the most economical cross-section for a 3-phase transmission line, 1 km long to supply at a constant voltage of 110 kV for the following daily load cycle :*

6 hours	*20 MW*	*at p.f. 0·8 lagging*
12 hours	*5 MW*	*at p.f. 0·8 lagging*
6 hours	*6 MW*	*at p.f. 0·8 lagging*

The line is used for 365 days yearly. The cost per km of line including erection is Rs (9000 + 6000 a) where 'a' is the area of X-section of conductor in cm^2*. The annual rate of interest and* *depreciation is 10% and the energy costs 6P per kWh. The resistance per km of each conductor is 0·176/a.*

Solution.

Resistance per km of each conductor is

$$R = \frac{0 \cdot 176}{a}\ \Omega$$

$$\text{Line voltage, } V = 110 \text{ kV} = 110 \times 10^3 \text{ V}$$

The load currents at various loads are :

$$\text{At 20 } MW,\ I_1 = \frac{20 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0 \cdot 8} = 131 \cdot 2 \text{ A}$$

$$\text{At 5 } MW,\ I_2 = \frac{5 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0 \cdot 8} = 32 \cdot 8 \text{ A}$$

$$\text{At 6 } MW,\ I_3 = \frac{6 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0 \cdot 8} = 39 \cdot 36 \text{ A}$$

Energy loss per day in 3-phase line

$$= 3 \times \frac{0 \cdot 176}{a} \times \frac{1}{1000} [(131 \cdot 2)^2 \times 6 + (32 \cdot 8)^2 \times 12 + (39 \cdot 36)^2 \times 6]$$

$$= \frac{0 \cdot 528}{1000\ a} [1{,}03{,}280 \cdot 64 + 12{,}910 \cdot 08 + 9{,}295 \cdot 26] \text{ kWh}$$

$$= \frac{66 \cdot 26}{a} \text{ kWh}$$

$$\text{Energy lost per annum} = \frac{66 \cdot 26}{a} \times 365 = \frac{24{,}184 \cdot 9}{a} \text{ kWh}$$

$$\text{Annual cost of energy} = \text{Rs } \frac{6}{100} \times \frac{24{,}184 \cdot 9}{a} = \frac{1451 \cdot 09}{a}$$

Variable annual charge = 10% of capital cost (variable) of line

$$= \text{Rs } 0 \cdot 1 \times 6000\ a = \text{Rs } 600\ a$$

According to Kelvin's law, for most economical X-section of the conductor,

Variable annual charge = Annual cost of energy

or $$600\ a = \frac{1451 \cdot 09}{a}$$

$$\therefore \quad a = \sqrt{\frac{1451 \cdot 09}{600}} = \mathbf{1 \cdot 56\ cm^2}$$

TUTORIAL PROBLEMS

1. Determine the best current density in A/mm^2 for *a* 3-ϕ overhead line if the line is in use for 2000 hours per year and if the conductor costing Rs 3·0 per kg has a specific resistance of 1·73Ω m and weighs 6200 kg/m^3. Cost of energy is 10 P/unit. Interest and depreciation is 12% of conductor cost. **[0·705 A/mm^2]**

2. Determine the most economical size of a 3-phase line which supplies the following loads at 10 kV :

(*i*) 100 kW at 0·8 p.f. (lag) for 10 hours

(*ii*) 500 kW at 0·9 p.f. (lag) for 8 hours

(*iii*) 100 kW at unity p.f. for 6 hours.

The above gives the daily load cycle. The cost per km of the completely erected line is Rs (8000 *a* + 1500) where *a* is the area of cross-section of each conductor. The combined interest and depreciation is 10% per annum of capital cost. Cost of energy losses is 5 paise per kWh. Resistivity of conductor material = 1·72 × 10^{-6} Ω cm. **[0·844 cm^2]**

3. If the cost of an overhead line is Rs 2000 A (where A is the cross-section in cm^2) and if the interest and depreciation charges of the line are 8%, estimate the most economical current density to use for a transmission requiring full load current for 60% of the year. The cost of generating electrical energy is 5 paise/kWh. The resistance of the conductor one km long and 1 cm^2 X-section is 0·18 Ω. **[41·12 A/cm²]**

7.12 Economic Choice of Transmission Voltage

It has been shown earlier in the chapter that if transmission voltage is increased, the volume of conductor material required is reduced. This decreases the expenditure on the conductor material. It may appear advisable to use the highest possible transmission voltage in order to reduce the expenditure on conductors to a minimum. However, it may be remembered that as the transmission voltage is increased, the cost of insulating the conductors, cost of transformers, switchgear and other terminal apparatus also increases. Therefore, for every transmission line, there is optimum transmission voltage, beyond which there is nothing to be gained in the matter of economy. The transmission voltage for which the cost of conductors, cost of insulators, transformers, switchgear and other terminal apparatus is minimum is called *economical transmission voltage.*

The method of finding the economical transmission voltage is as follows. Power to be transmitted, generation voltage and length of transmission line are assumed to be known. We choose some standard transmission voltage and work out the following costs :

(*i*) *Transformers,* at the generating and receiving ends of transmission line. For a given power, this cost increases slowly with the increase in transmission voltage.

(*ii*) *Switchgear.* This cost also increases with the increase in transmission voltage.

(*iii*) *Lightning arrestor.* This cost increases rapidly with the increase in transmission voltage.

(*iv*) *Insulation and supports.* This cost increases sharply with the increase in transmission voltage.

(*v*) *Conductor.* This cost decreases with the increase in transmission voltage.

The sum of all above costs gives the total cost of transmission for the voltage considered. Similar calculations are made for other transmission voltages. Then, a curve is drawn for total cost of transmission against voltage as shown in Fig. 7.29. The lowest point (P) on the curve gives the economical transmission voltage. Thus, in the present case, OA is the optimum transmission voltage. This method of finding the economical transmission voltage is rarely used in practice as different costs cannot be determined with a fair degree of accuracy.

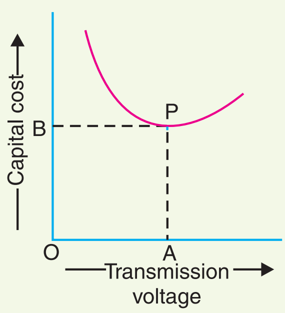


Fig. 7.29

The present day trend is to follow certain empirical formulae for finding the economical transmission voltage. Thus, according to American practice, the economic voltage between lines in a 3-phase a.c. system is

$$V = 5 \cdot 5 \sqrt{0 \cdot 62\, l + \frac{3P}{150}}$$

where V = line voltage in kV

P = maximum kW per phase to be delivered to single circuit

l = distance of transmission line in km

It may be noted here that in the above formula, power to be transmitted and distance of transmission line have been taken into account. It is because both these factors influence the economic voltage of a transmission line. This can be easily explained. If the distance of transmission line is increased, the cost of terminal apparatus is decreased, resulting in higher economic transmission voltage. Also if power to be transmitted is large, large generating and transforming units can be employed. This reduces the cost per kW of the terminal station equipment.

7.13 Requirements of Satisfactory Electric Supply

The electric power system in India is 3-phase a.c. operating at a frequency of 50 Hz. The power station delivers power to consumers through its transmission and distribution systems. The power delivered must be characterised by constant or nearly constant voltage, dependability of service, balanced voltage, efficiency so as to give minimum annual cost, sinusoidal waveform and freedom from inductive interference with telephone lines.

(*i*) Voltage regulation. A voltage variation has a large effect upon the operation of both power machinery and lights. A motor is designed to have its best characteristics at the rated voltage and consequently a voltage that is too high or too low will result in a decrease in efficiency. If the fluctuations in the voltage are sudden, these may cause the tripping of circuit breakers and consequent interruptions to service. Usually the voltage at the generator terminals, where this is done, in some cases the voltage variations at the load may be made sufficiently small by keeping the resistance and reactance of the lines and feeders low.

(*ii*) Dependability. One important requirement of electric supply is to furnish uninterrupted service. The losses which an industrial consumer sustains due to the failure of electric power supply are usually vastly greater than the actual value of the power that he would use during this period. It is on account of the expense of idle workmen and machines and other overhead charges. Interruptions to service cause irritation and are sometimes positively dangerous to life and property. For example, failure of power in hospitals, in crowded theatres and stores may lead to very grave consequences. Therefore, it is the duty of electric supply company to keep the power system going and to furnish uninterrupted service.

(*iii*) Balanced voltage. It is very important that the polyphase voltage should be balanced. If an unbalanced polyphase voltage is supplied to a consumer operating synchronous or induction motors, it will result in a decrease in the efficiency of his machinery and also a decrease in its maximum power output. Motors called upon to deliver full load when their terminal voltages are unbalanced are liable to considerable damage due to overheating. One method of maintaining balance of voltage is by having balanced loads connected to the circuit.

(*iv*) Efficiency. The efficiency of a transmission system is not of much importance in itself. The important economic feature of the design being the layout of the system as a whole so as to perform the requisite function of generating and delivering power with a minimum overall annual cost. The annual cost can be minimised to a considerable extent by taking care of power factor of the system. It is because losses in the lines and machinery are largely determined by power factor. Therefore, it is important that consumers having loads of low power factor should be penalised by being charged at a higher rate per kWh than those who take power at high power factors. Loads of low power factor also require greater generator capacity than those of high power factor (for the same amount of power) and produce larger voltage drops in the lines and transformers.

(*v*) Frequency. The frequency of the supply system must be maintained constant. It is because a change in frequency would change the motor speed, thus interfering with the manufacturing operations.

(*vi*) Sinusoidal waveform. The alternating voltage supplied to the consumers should have a sine waveform. It is because any harmonics which might be present would have detrimental effect upon the efficiency and maximum power output of the connected machinery. Harmonics may be avoided by using generators of good design and by avoidance of high flux densities in transformers.

(*vii*) Freedom from inductive interference. Power lines running parallel to telephone lines produce electrostatic and electromagnetic field disturbances. These fields tend to cause objectionable noises and hums in the apparatus connected to communication circuits. Inductive interference with telephone lines may be avoided by limiting as much as possible the amount of zero-sequence and harmonic current and by the proper transposition of both power lines and telephone lines.

SELF - TEST

1. Fill in the blanks by inserting appropriate words/figures.

(*i*) In India .... system is adopted for transmission of electric power.
(*ii*) .... voltage is used for power transmission as a matter of economy.
(*iii*) The distribution system comprises of three elements *viz.*, ... and ....
(*iv*) D.C. transmission is .... to a.c. transmission.
(*v*) The higher the transmission voltage, the .... is the conductor material required.
(*vi*) The choice of proper transmission voltage is essentially a question of ....
(*vii*) In overhead system, the comparison of various systems is made on the basis of maximum voltage between ....
(*viii*) The economic size of conductor is determined by ....
(*ix*) In a transmission system, the cost of conductor is proportional to .... of conductor.
(*x*) The economic transmission voltage is one for which the transmission cost is ....

2. Pick up the correct words/figures from brackets and fill in the blanks.

(*i*) Primary transmission is done by 3-phase ....wire a.c. system. (*3, 4*)
(*ii*) The .... distribution is done by 3-phase, 4-wire a.c. system. (*primary, secondary*)
(*iii*) The greater the power to be transmitted, the .... is the economic transmission voltage. (*smaller, larger*)
(*iv*) The annual charge of a transmission line can be expressed as $P_1 + P_2$ .... (a, $1/a$)
(*v*) The economic transmission voltage .... the distance of transmission. (*depends upon, does not depend upon*)

ANSWERS TO SELF-TEST

1. (*i*) 3-phase, 3-wire (*ii*) high (*iii*) feeders, distributors, service mains (*iv*) superior (*v*) lesser (*vi*) economics (*vii*) conductor and earth (*viii*) Kelvin's law (*ix*) area (*x*) minimum.

2. (*i*) 3 (*ii*) secondary (*iii*) larger (*iv*) *a* (*v*) depends upon.

CHAPTER REVIEW TOPICS

1. What is electric power supply system ? Draw a single line diagram of a typical a.c power supply scheme.
2. What are the advantages and disadvantages of d.c. transmission over a.c. transmission ?
3. Discuss the advantages of high transmission voltage.
4. Compare the volume of conductor material required for a d.c. 3-wire system and 3-phase, 3-wire system on the basis of equal maximum potential difference between one conductor and earth. Make suitable assumptions.
5. Compare the volume of conductor materiel required in d.c. single phase and three-phase a.c. system.
6. State and prove Kelvin's law for size of conductor for transmission. Discuss its limitations.
7. How will you determine the economic transmission voltage ?

DISCUSSION QUESTIONS

1. What is the need of primary distribution in an electric supply scheme ?
2. The present trend is towards a.c for generation and distribution and d.c. for transmission. Discuss the reasons for it.
3. In an overhead system, the basis of comparison is the maximum voltage between conductor and ground. Why ?
4. Kelvin's law does not give the exact economical size of conductor. Give reasons in support of your answer.

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