CONTENTS

CHAPTER

10

Performance of Transmission Lines

Introduction

The important considerations in the design and operation of a transmission line are the determination of voltage drop, line losses and efficiency of transmission. These values are greatly influenced by the line constants *R*, *L* and *C* of the transmission line. For instance, the voltage drop in the line depends upon the values of above three line constants. Similarly, the resistance of transmission line conductors is the most important cause of power loss in the line and determines the transmission efficiency. In this chapter, we shall develop formulas by which we can calculate voltage regulation, line losses and efficiency of transmission lines. These formulas are important for two principal reasons. Firstly, they provide an opportunity to understand the effects of the parameters of the line on bus voltages and the flow of power. Secondly, they help in developing an overall understanding of what is occuring on electric power system.

CONTENTS

10.1 Classification of Overhead Transmission Lines

A transmission line has *three constants R, L and C distributed uniformly along the whole length of the line. The resistance and inductance form the series impedance. The capacitance existing between conductors for 1-phase line or from a conductor to neutral for a 3-phase line forms a shunt path throughout the length of the line. Therefore, capacitance effects introduce complications in transmission line calculations. Depending upon the manner in which capacitance is taken into account, the overhead transmission lines are classified as :

(*i*) *Short transmission lines.* When the length of an overhead transmission line is upto about 50 km and the line voltage is comparatively low (< 20 kV), it is usually considered as a short transmission line. Due to smaller length and lower voltage, the capacitance effects are small and hence can be neglected. Therefore, while studying the performance of a short transmisison line, only resistance and inductance of the line are taken into account.

(*ii*) *Medium transmission lines.* When the length of an overhead transmission line is about 50-150 km and the line voltage is moderatly high (>20 kV < 100 kV), it is considered as a medium transmission line. Due to sufficient length and voltage of the line, the capacitance effects are taken into account. For purposes of calculations, the distributed capacitance of the line is divided and lumped in the form of condensers shunted across the line at one or more points.

(*iii*) *Long transmission lines.* When the length of an overhead transmission line is more than 150 km and line voltage is very high (> 100 kV), it is considered as a long transmission line. For the treatment of such a line, the line constants are considered uniformly distributed over the whole length of the line and rigorous methods are employed for solution.

It may be emphasised here that exact solution of any tranmission line must consider the fact that the constants of the line are not lumped but are distributed unfiormly throughout the length of the line. However, reasonable accuracy can be obtained by considering these constants as lumped for short and medium transmission lines.

10.2 Important Terms

While studying the performance of a transmission line, it is desirable to determine its voltage regulation and transmission efficiency. We shall explain these two terms in turn.

(*i*) **Voltage regulation.** When a transmission line is carrying current, there is a voltage drop in the line due to resistance and inductance of the line. The result is that receiving end voltage (V_R) of the line is generally less than the sending end voltage (V_S). This voltage drop ($V_S - V_R$) in the line is expressed as a percentage of receiving end voltage V_R and is called voltage regulation.

*The difference in voltage at the receiving end of a transmission line **between conditions of no load and full load is called* **voltage regulation** and is expressed as a percentage of the receiving end voltage.

---

* There is also a fourth constant *i.e.*, shunt conductance. It represents the conductance between conductors or between conductor and ground and accounts for the leakage current at the insulators. It is very small in case of overhead lines and may be assumed zero.

** At no load, there is no drop in the line so that at no load, $V_R = V_S$. However, at full load, there is a voltage drop in the line so that receiving end voltage is V_R.

∴ Difference in voltage at receiving end between no load and full load

$$= V_S - V_R$$

Mathematically,

$$\%\text{ age Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

Obviously, it is desirable that the voltage regulation of a transmission line should be low *i.e.*, the increase in load current should make very little difference in the receiving end voltage.

(*ii*) Transmission efficiency. The power obtained at the receiving end of a transmission line is generally less than the sending end power due to losses in the line resistance.

The ratio of receiving end power to the sending end power of a transmission line is known as the **transmission efficiency** *of the line i.e.*

$$\%\text{ age Transmission efficiency, } \eta_T = \frac{\text{Receiving end power}}{\text{Sending end power}} \times 100$$

$$= \frac{V_R I_R \cos\phi_R}{V_S I_S \cos\phi_S} \times 100$$

where V_R, I_R and $\cos\phi_R$ are the receiving end voltage, current and power factor while V_S, I_S and $\cos\phi_S$ are the corresponding values at the sending end.

10.3 Performance of Single Phase Short Transmission Lines

As stated earlier, the effects of line capacitance are neglected for a short transmission line. Therefore, while studying the performance of such a line, only resistance and inductance of the line are taken into account. The equivalent circuit of a single phase short transmission line is shown in Fig. 10.1 (*i*). Here, the total line resistance and inductance are shown as concentrated or lumped instead of being distributed. The circuit is a simple a.c. series circuit.

Let I = load current

R = loop resistance *i.e.*, resistance of both conductors

X_L = loop reactance

V_R = receiving end voltage

$\cos\phi_R$ = receiving end power factor (lagging)

V_S = sending end voltage

$\cos\phi_S$ = sending end power factor

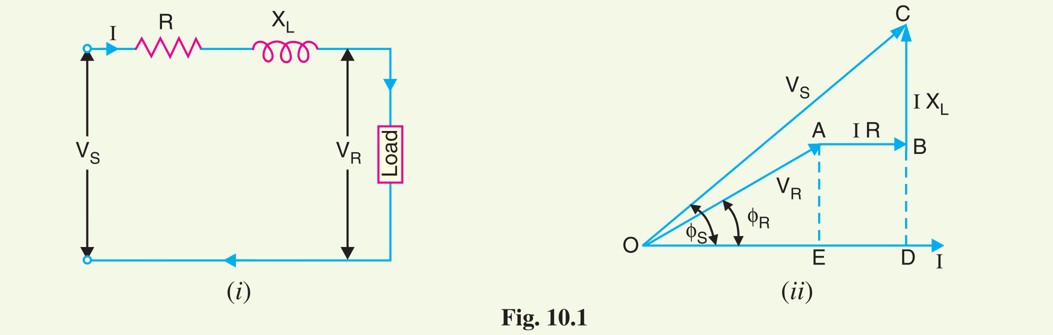


Fig. 10.1

The *phasor diagram of the line for lagging load power factor is shown in Fig. 10.1 (*ii*). From the right angled traingle *ODC*, we get,

* **Phasor diagram.** Current I is taken as the reference phasor. OA represents the receiving end voltage V_R leading I by ϕ_R. AB represents the drop IR in phase with I. BC represents the inductive drop IX_L and leads I by 90°. OC represents the sending end voltage V_S and leads I by ϕ_S.

$$(OC)^2 = (OD)^2 + (DC)^2$$

or
$$V_S^2 = (OE + ED)^2 + (DB + BC)^2$$
$$= (V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2$$

$$\therefore \quad V_S = \sqrt{(V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2}$$

(i) %age Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100$

(ii) Sending end *p.f.*, $\cos \phi_S = \dfrac{OD}{OC} = \dfrac{V_R \cos \phi_R + IR}{V_S}$

(iii) Power delivered $= V_R I_R \cos \phi_R$

Line losses $= I^2 R$

Power sent out $= V_R I_R \cos \phi_R + I^2 R$

%age Transmission efficiency $= \dfrac{\text{Power delivered}}{\text{Power sent out}} \times 100$

$$= \frac{V_R\ I_R \cos \phi_R}{V_R\ I_R \cos \phi_R + I^2 R} \times 100$$

An approximate expression for the sending end voltage V_S can be obtained as follows. Draw perpendicular from *B* and *C* on *OA* produced as shown in Fig. 10.2. Then *OC* is *nearly* equal to *OF i.e.*,

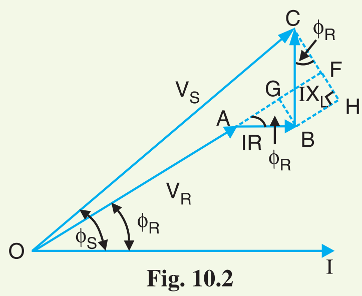


Fig. 10.2

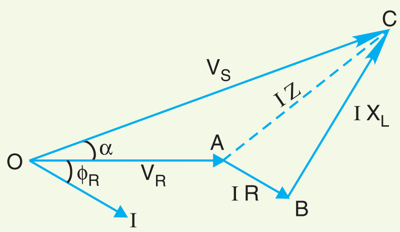


Fig. 10.3

$$OC = OF = OA + AF = OA + AG + GF$$
$$= OA + AG + BH$$
$$\therefore \quad V_S = V_R + I R \cos \phi_R + I X_L \sin \phi_R$$

Solution in complex notation. It is often convenient and profitable to make the line calculations in complex notation.

Taking $\vec{V_R}$ as the reference phasor, draw the phasor diagram as shown in Fig 10.3. It is clear that $\vec{V_S}$ is the phasor sum of $\vec{V_R}$ and $\vec{I}\,\vec{Z}$.

$$*\vec{V_R} = V_R + j\,0$$
$$\vec{I} = \vec{I} \angle -\phi_R = I(\cos \phi_R - j \sin \phi_R)$$
$$\vec{Z} = R + jX_L$$
$$\therefore \quad \vec{V_S} = \vec{V_R} + \vec{I}\,\vec{Z}$$
$$= (V_R + j\,0) + I(\cos \phi_R - j \sin \phi_R)(R + j\,X_L)$$

* Phasors are shown by arrows and their magnitudes without arrow. Thus $\vec{V_R}$ is the receiving end voltage phasor, whereas V_R is its magnitude.

$$= (V_R + I\,R\cos\phi_R + I\,X_L\sin\phi_R) + j\,(IX_L\cos\phi_R - I\,R\sin\phi_R)$$

$$\therefore \qquad V_S = \sqrt{(V_R + I\,R\cos\phi_R + I\,X_L\sin\phi_R)^2 + (I\,X_L\cos\phi_R - I\,R\sin\phi_R)^2}$$

The second term under the root is quite small and can be neglected with reasonable accuracy. Therefore, approximate expression for V_S becomes :

$$V_S = V_R + I\,R\cos\phi_R + I\,X_L\sin\phi_R$$

The following poins may be noted :

(*i*) The approximate formula for V_S ($= V_R + I\,R\cos\phi_R + I\,X_L\sin\phi_R$) gives fairly correct results for lagging power factors. However, appreciable error is caused for leading power factors. Therefore, approximate expression for V_S should be used for lagging p.f. only.

(*ii*) The solution in complex notation is in more presentable form.

10.4 Three-Phase Short Transmission Lines

For reasons associated with economy, transmission of electric power is done by 3-phase system. This system may be regarded as consisting of three single phase units, each wire transmitting one-third of the total power. As a matter of convenience, we generally analyse 3-phase system by considering

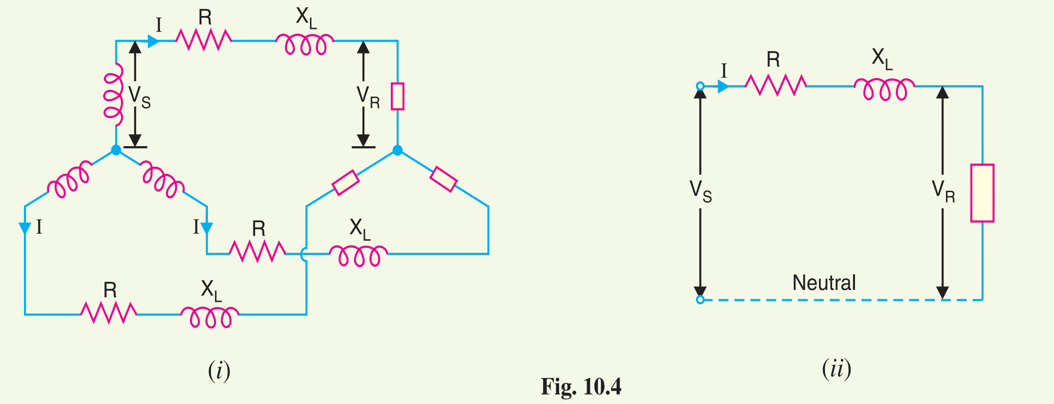


Fig. 10.4

*one phase only. Therefore, expression for regulation, efficiency etc. derived for a single phase line can also be applied to a 3-phase system. Since only one phase is considered, phase values of 3-phase system should be taken. Thus, V_S and V_R are the phase voltages, whereas R and X_L are the resistance and inductive reactance per phase respectively.

Fig. 10.4 (*i*) shows a *Y*-connected generator supplying a balanced *Y*-connected load through a transmission line. Each conductor has a resistance of $R\,\Omega$ and inductive reactance of $X_L\,\Omega$. Fig. 10.4 (*ii*) shows one phase separately. The calculations can now be made in the same way as for a single phase line.

10.5 Effect of Load p.f. on Regulation and Efficiency

The regulation and efficiency of a transmission line depend to a considerable extent upon the power factor of the load.

1. Effect on regulation. The expression for voltage regulation of a short transmission line is given by :

$$\text{\%age Voltage regulation} = \frac{I\,R\cos\phi_R + I\,X_L\sin\phi_R}{V_R} \times 100 \qquad \textit{(for lagging p.f.)}$$

* As similar conditions prevail in the three phases.

$$\%\text{age Voltage regulation} = \frac{I\,R\cos\phi_R - I\,X_L\sin\phi_R}{V_R}\times 100 \qquad (\textit{for leading p.f.})$$

The following conclusions can be drawn from the above expressions :

(*i*) When the load p.f. is lagging or unity or such leading that $I\,R\cos\phi_R > I\,X_L\sin\phi_R$, then voltage regulation is positive *i.e.*, receiving end voltage V_R will be less than the sending end voltage V_S.

(*ii*) For a given V_R and I, the voltage regulation of the line increases with the decrease in p.f. for lagging loads.

(*iii*) When the load p.f. is leading to this extent that $I\,X_L\sin\phi_R > I\,R\cos\phi_R$, then voltage regulation is negative *i.e.* the receiving end voltage V_R is more than the sending end voltage V_S.

(*iv*) For a given V_R and I, the voltage regulation of the line decreases with the decrease in p.f. for leading loads.

2. Effect on transmission efficiency. The power delivered to the load depends upon the power factor.

$$P = V_R\,{}^{*}I\cos\phi_R \qquad \text{(For 1-phase line)}$$

$$\therefore \quad I = \frac{P}{V_R\cos\phi_R}$$

$$P = 3\,V_R\,I\cos\phi_R \qquad \text{(For 3-phase line)}$$

$$\therefore \quad I = \frac{P}{3V_R\cos\phi_R}$$

Power Factor Meter

It is clear that in each case, for a given amount of power to be transmitted (P) and receiving end voltage (V_R), the load current I is inversely proportional to the load p.f. $\cos\phi_R$. Consequently, with the decrease in load p.f., the load current and hence the line losses are increased. This leads to the conclusion that transmission efficiency of a line decreases with the decrease in load p.f. and *vice-versa*,


Power Factor Regulator

Example 10.1. *A single phase overhead transmission line delivers 1100 kW at 33 kV at 0·8 p.f. lagging. The total resistance and inductive reactance of the line are 10 Ω and 15 Ω respectively. Determine : (i) sending end voltage (ii) sending end power factor and (iii) transmission efficiency.*

Solution.

Load power factor, $\cos\phi_R = 0{\cdot}8$ lagging

Total line impedance, $\vec{Z} = R + j\,X_L = 10 + j\,15$

---

* $I_R = I_S = I$

Receiving end voltage, $V_R = 33$ kV $= 33{,}000$ V

$$\therefore \quad \text{Line current, } I = \frac{kW \times 10^3}{V_R \cos \phi_R} = \frac{1100 \times 10^3}{33{,}000 \times 0{\cdot}8} = 41{\cdot}67 \text{ A}$$

$$\text{As } \cos \phi_R = 0{\cdot}8 \quad \therefore \quad \sin \phi_R = 0{\cdot}6$$

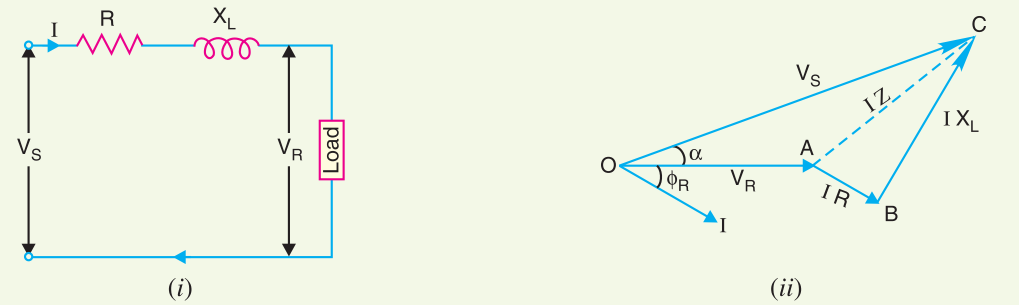


Fig. 10.5

The equivalent circuit and phasor diagram of the line are shown in Figs. 10.5 (*i*) and 10.5 (*ii*) respectively. Taking receiving end voltage $\vec{V_R}$ as the reference phasor,

$$\vec{V_R} = V_R + j\,0 = 33000 \text{ V}$$

$$\vec{I} = I(\cos \phi_R - j \sin \phi_R)$$

$$= 41{\cdot}67\,(0{\cdot}8 - j\,0{\cdot}6) = 33{\cdot}33 - j\,25$$

(*i*) Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I}\,\vec{Z}$

$$= 33{,}000 + (33{\cdot}33 - j\,25{\cdot}0)\,(10 + j\,15)$$

$$= 33{,}000 + 333{\cdot}3 - j250 + j500 + 375$$

$$= 33{,}708{\cdot}3 + j\,250$$

$$\therefore \quad \text{Magnitude of } V_S = \sqrt{(33{,}708{\cdot}3)^2 + (250)^2} = \mathbf{33{,}709 \text{ V}}$$

(*ii*) Angle between $\vec{V_S}$ and $\vec{V_R}$ is

$$\alpha = \tan^{-1} \frac{250}{33{,}708{\cdot}3} = \tan^{-1} 0{\cdot}0074 = 0{\cdot}42^\circ$$

∴ Sending end power factor angle is

$$\phi_S = \phi_R + \alpha = 36{\cdot}87^\circ + 0{\cdot}42^\circ = 37{\cdot}29^\circ$$

$$\therefore \quad \text{Sending end p.f., } \cos \phi_S = \cos 37{\cdot}29^\circ = \mathbf{0{\cdot}7956 \text{ lagging}}$$

(*iii*) Line losses $= I^2R = (41{\cdot}67)^2 \times 10 = 17{,}364$ W $= 17{\cdot}364$ kW

Output delivered $= 1100$ kW

Power sent $= 1100 + 17{\cdot}364 = 1117{\cdot}364$ kW

$$\therefore \quad \text{Transmission efficiency} = \frac{\text{Power delivered}}{\text{Power sent}} \times 100 = \frac{1100}{1117{\cdot}364} \times 100 = \mathbf{98{\cdot}44\%}$$

Note. V_S and ϕ_S can also be calculated as follows :

$$V_S = V_R + I\,R \cos \phi_R + I\,X_L \sin \phi_R \text{ (approximately)}$$

$$= 33{,}000 + 41{\cdot}67 \times 10 \times 0{\cdot}8 + 41{\cdot}67 \times 15 \times 0{\cdot}6$$

$$= 33{,}000 + 333{\cdot}36 + 375{\cdot}03$$

$$= 33708{\cdot}39 \text{ V which is approximately the same as above}$$

$$\cos \phi_S = \frac{V_R \cos \phi_R + I\,R}{V_S} = \frac{33{,}000 \times 0{\cdot}8 + 41{\cdot}67 \times 10}{33{,}708{\cdot}39} = \frac{26{,}816{\cdot}7}{33{,}708{\cdot}39}$$

$$= 0{\cdot}7958$$

As stated earlier, this method gives fairly correct results for lagging p.f. The reader will find that this method is used in the solution of some numericals.

Example 10.2. *What is the maximum length in km for a 1-phase transmission line having copper conductor of 0·775 cm² cross-section over which 200 kW at unity power factor and at 3300V are to be delivered ? The efficiecny of transmission is 90%. Take specific resistance as 1.725 μ Ω cm.*

Solution.

$$\text{Receiving end power} = 200 \text{ kW} = 2{,}00{,}000 \text{ W}$$

$$\text{Transmission efficiency} = 0{\cdot}9$$

$$\therefore \quad \text{Sending end power} = \frac{2{,}00{,}000}{0\cdot 9} = 2{,}22{,}222 \text{ W}$$

$$\therefore \quad \text{Line losses} = 2{,}22{,}222 - 2{,}00{,}000 = 22{,}222 \text{ W}$$

$$\text{Line current, } I = \frac{200\times 10^3}{3{,}300 \times 1} = 60{\cdot}6 \text{ A}$$

Let R Ω be the resistance of one conductor.

$$\text{Line losses} = 2\, I^2\, R$$

or $$22{,}222 = 2\,(60{\cdot}6)^2 \times R$$

$$\therefore \quad R = \frac{22{,}222}{2\times(60\cdot 6)^2} = 3{\cdot}025\,\Omega$$

Now, $$R = \rho\, l/a$$

$$\therefore \quad l = \frac{Ra}{\rho} = \frac{3\cdot 025\times 0\cdot 775}{1\cdot 725\times 10^{-6}} = 1{\cdot}36\times 10^6 \text{ cm} = \mathbf{13{\cdot}6\ km}$$

Example 10.3. *An overhead 3-phase transmission line delivers 5000 kW at 22 kV at 0·8 p.f. lagging. The resistance and reactance of each conductor is 4 Ω and 6 Ω respectively. Determine :* (i) *sending end voltage* (ii) *percentage regulation* (iii) *transmission efficiency.*

Solution.

$$\text{Load power factor, } \cos\phi_R = 0{\cdot}8 \text{ lagging}$$

$$\text{Receiving end voltage/phase,}^{*}V_R = 22{,}000/\sqrt{3} = 12{,}700 \text{ V}$$

$$\text{Impedance/phase, } \quad \vec{Z} = 4 + j\,6$$

$$\text{Line current, } \quad I = \frac{5000\times 10^3}{3\times 12700\times 0\cdot 8} = 164 \text{ A}$$

As $\cos\phi_R = 0{\cdot}8 \quad \therefore \sin\phi_R = 0{\cdot}6$

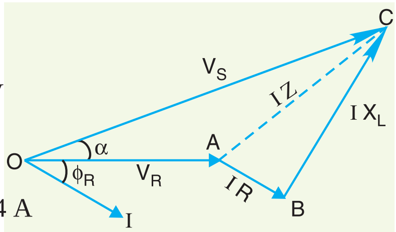


Fig. 10.6

Taking $\vec{V_R}$ as the reference phasor (see Fig. 10.6),

$$\vec{V_R} = V_R + j\,0 = 12700 \text{ V}$$

$$\vec{I} = I\,(\cos\phi_R - j\sin\phi_R) = 164\,(0{\cdot}8 - j\,0{\cdot}6) = 131{\cdot}2 - j\,98{\cdot}4$$

(*i*) Sending end voltage per phase is

$$\vec{V_S} = \vec{V_R} + \vec{I}\,\vec{Z} = 12700 + (131{\cdot}2 - j\,98{\cdot}4)\,(4 + j\,6)$$
$$= 12700 + 524{\cdot}8 + j\,787{\cdot}2 - j\,393{\cdot}6 + 590{\cdot}4$$
$$= 13815{\cdot}2 + j\,393{\cdot}6$$

$$\text{Magnitude of } V_S = \sqrt{(13815\cdot 2)^2 + (393\cdot 6)^2} = 13820{\cdot}8 \text{ V}$$

* If not mentioned in the problem, star-connection is understood.

Line value of V_S = $\sqrt{3} \times 13820\cdot8 = 23938$ V = **23·938 kV**

(ii) % age Regulation = $\dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{13820\cdot8 - 12700}{12700} \times 100$ = **8·825%**

(iii) Line losses = $3I^2R = 3 \times (164)^2 \times 4 = 3{,}22{,}752$ W = 322·752 kW

∴ Transmission efficiency = $\dfrac{5000}{5000 + 322\cdot752} \times 100$ = **93·94%**

Example 10.4. *Estimate the distance over which a load of 15000 kW at a p.f. 0·8 lagging can be delivered by a 3-phase transmission line having conductors each of resistance 1 Ω per kilometre. The voltage at the receiving end is to be 132 kV and the loss in the transmission is to be 5%.*

Solution.

Line current, $I = \dfrac{\text{Power delivered}}{\sqrt{3} \times \text{line voltage} \times \text{power factor}} = \dfrac{15000 \times 10^3}{\sqrt{3} \times 132 \times 10^3 \times 0\cdot8} = 82$ A

Line losses = 5% of power delivered = $0\cdot05 \times 15000 = 750$ kW

Let R Ω be the resistance of one conductor.

Line losses = $3\,I^2R$

or $750 \times 10^3 = 3 \times (82)^2 \times R$

∴ $R = \dfrac{750 \times 10^3}{3 \times (82)^2} = 37\cdot18\,\Omega$

Resistance of each conductor per km is 1 Ω (*given*).

∴ Length of line = **37·18 km**

Example 10.5. *A 3-phase line delivers 3600 kW at a p.f. 0·8 lagging to a load. If the sending end voltage is 33 kV, determine* (i) *the receiving end voltage* (ii) *line current* (iii) *transmission efficiency. The resistance and reactance of each conductor are 5·31 Ω and 5·54 Ω respectively.*

Solution.

Resistance of each conductor, $R = 5\cdot31\,\Omega$

Reactance of each conductor, $X_L = 5\cdot54\,\Omega$

Load power factor, $\cos\phi_R = 0\cdot8$ (*lagging*)

Sending end voltage/phase, $V_S = 33{,}000/\sqrt{3} = 19{,}052$ V

Let V_R be the phase voltage at the receiving end.

Line current, $I = \dfrac{\text{Power delivered / phase}}{V_R \times \cos\phi_R} = \dfrac{1200 \times 10^3}{V_R \times 0\cdot8}$

$$= \frac{150 \times 10^5}{V_R} \qquad \ldots(i)$$

(i) Using approximate expression for V_S, we get,

$$V_S = V_R + I\,R\cos\phi_R + I\,X_L\sin\phi_R$$

or $19{,}052 = V_R + \dfrac{15 \times 10^5}{V_R} \times 5\cdot31 \times 0\cdot8 + \dfrac{15 \times 10^5}{V_R} \times 5\cdot54 \times 0\cdot6$

or $V_R^2 - 19{,}052\,V_R + 1{,}13{,}58{,}000 = 0$

Solving this equation, we get, $V_R = 18{,}435$ V

∴ Line voltage at the receiving end = $\sqrt{3} \times 18{,}435 = 31{,}930$ V = **31·93 kV**

(ii) Line current, $I = \dfrac{15 \times 10^5}{V_R} = \dfrac{15 \times 10^5}{18{,}435} =$ **81·36 A**

(iii) Line losses, $= 3\,I^2 R = 3 \times (81{\cdot}36)^2 \times 5{\cdot}31 = 1{,}05{,}447$ W $= 105{\cdot}447$ kW

$\therefore$ Transmission efficiency $= \dfrac{3600}{3600 + 105{\cdot}447} \times 100 =$ **97·15%**

Example 10.6. *A short 3-φ transmission line with an impedance of* (6 + *j* 8) Ω *per phase has sending and receiving end voltages of 120 kV and 110 kV respectively for some receiving end load at a p.f. of 0·9 lagging. Determine* (*i*) *power output and* (*ii*) *sending end power factor.*

Solution.

Resistance of each conductor, $R = 6\ \Omega$

Reactance of each conductor, $X_L = 8\ \Omega$

Load power factor, $\cos \phi_R = 0{\cdot}9$ lagging

Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63508$ V

Sending end voltage/phase, $V_S = 120 \times 10^3/\sqrt{3} = 69282$ V

Let I be the load current. Using approximate expression for V_S, we get,

$$V_S = V_R + I\,R \cos \phi_R + I\,X_L \sin \phi_R$$

or $69282 = 63508 + I \times 6 \times 0{\cdot}9 + I \times 8 \times 0{\cdot}435$

or $8{\cdot}88\,I = 5774$

or $I = 5774/8{\cdot}88 = 650{\cdot}2$ A

(i) Power output $= \dfrac{3\,V_R\,I \cos \phi_R}{1000}$ kW $= \dfrac{3 \times 63508 \times 650{\cdot}2 \times 0{\cdot}9}{1000}$

$=$ **1,11,490 kW**

(ii) Sending end p.f., $\cos \phi_S = \dfrac{V_R \cos \phi_R + I\,R}{V_S} = \dfrac{63508 \times 0{\cdot}9 + 650{\cdot}2 \times 6}{69282} =$ **0·88 lag**

Example 10.7. *An 11 kV, 3-phase transmission line has a resistance of 1·5* Ω *and reactance of 4* Ω *per phase. Calculate the percentage regulation and efficiency of the line when a total load of 5000 kVA at 0.8 lagging power factor is supplied at 11 kV at the distant end.*

Solution.

Resistance of each conductor, $R = 1{\cdot}5\ \Omega$

Reactance of each conductor, $X_L = 4\ \Omega$

Receiving end voltage/phase, $V_R = \dfrac{11 \times 10^3}{\sqrt{3}} = 6351$ V

Load power factor, $\cos \phi_R = 0{\cdot}8$ lagging

Load current, $I = \dfrac{\text{Power delivered in kVA} \times 1000}{3 \times V_R}$

$= \dfrac{5000 \times 1000}{3 \times 6351} = 262{\cdot}43$A

Using the approximate expression for V_S (sending end voltage per phase), we get,

$$V_S = V_R + I\,R \cos \phi_R + I\,X_L \sin \phi_R$$

$= 6351 + 262{\cdot}43 \times 1{\cdot}5 \times 0{\cdot}8 + 262{\cdot}43 \times 4 \times 0{\cdot}6 = 7295{\cdot}8$ V

% regulation $= \dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{7295{\cdot}8 - 6351}{6351} \times 100 =$ **14·88%**

Line losses $= 3\,I^2 R = 3 \times (262{\cdot}43)^2 \times 1{\cdot}5 = 310 \times 10^3$ W $= 310$ kW

$$\text{Output power} = 5000 \times 0{\cdot}8 = 4000 \text{ kW}$$

$$\text{Input power} = \text{Ouput power + line losses} = 4000 + 310 = 4310 \text{ kW}$$

$$\text{Transmission efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100 = \frac{4000}{4310} \times 100 = \mathbf{92{\cdot}8\%}$$

Example 10.8. *A 3-phase, 50 Hz, 16 km long overhead line supplies 1000 kW at 11 kV, 0·8 p.f. lagging. The line resistance is 0·03 Ω per phase per km and line inductance is 0·7 mH per phase per km. Calculate the sending end voltage, voltage regulation and efficiency of transmission.*

Solution.

Resistance of each conductor, $R = 0{\cdot}03 \times 16 = 0{\cdot}48\,\Omega$

Reactance of each conductor, $X_L = 2\pi f L \times 16 = 2\pi \times 50 \times 0{\cdot}7 \times 10^{-3} \times 16 = 3{\cdot}52\,\Omega$

Receiving end voltage/phase, $V_R = \dfrac{11 \times 10^3}{\sqrt{3}} = 6351 \text{ V}$

Load power factor, $\cos\phi_R = 0{\cdot}8$ lagging

$$\text{Line current, } I = \frac{1000 \times 10^3}{3 \times V_R \times \cos\phi} = \frac{1000 \times 10^3}{3 \times 6351 \times 0{\cdot}8} = 65{\cdot}6\text{A}$$

$$\text{Sending end voltage/phase, } V_S = V_R + I R \cos\phi_R + I X_L \sin\phi_R$$

$$= 6351 + 65{\cdot}6 \times 0{\cdot}48 \times 0{\cdot}8 + 65{\cdot}6 \times 3{\cdot}52 \times 0{\cdot}6 = 6515 \text{ V}$$

$$\therefore \quad \text{\%age Voltage regulation} = \frac{V_S - V_R}{V_R} \times 100 = \frac{6515 - 6351}{6351} \times 100 = \mathbf{2{\cdot}58\%}$$

$$\text{Line losses} = 3 I^2 R = 3 \times (65{\cdot}6)^2 \times 0{\cdot}48 = 6{\cdot}2 \times 10^3 \text{ W} = 6{\cdot}2 \text{ kW}$$

$$\text{Input power} = \text{Output power + Line losses} = 1000 + 6{\cdot}2 = 1006{\cdot}2 \text{ kW}$$

$$\therefore \quad \text{Transmission efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100 = \frac{1000}{1006{\cdot}2} \times 100 = \mathbf{99{\cdot}38\%}$$

Example 10.9. *A 3-phase load of 2000 kVA, 0·8 p.f. is supplied at 6·6 kV, 50 Hz by means of a 33 kV transmission line 20 km long and 33/6·6 kV step-down transfomer. The resistance and reactance of each conductor are 0·4 Ω and 0·5 Ω per km respectively. The resistance and reactance of transformer primary are 7.5 Ω and 13.2 Ω, while those of secondary are 0.35 Ω and 0.65 Ω respectively. Find the voltage necessary at the sending end of transmission line when 6.6 kV is maintained at the receiving end. Determine also the sending end power factor and transmission efficiency.*

Solution. Fig. 10.7 shows the single diagram of the transmission system. Here, the voltage drop will be due to the impedance of transmission line and also due to the impedance of transformer.

Resistance of each conductor $= 20 \times 0{\cdot}4 = 8\,\Omega$

Reactance of each conductor $= 20 \times 0{\cdot}5 = 10\,\Omega$

Let us transfer the impedance of transformer secondary to high tension side *i.e.*, 33 kV side.

Equivalent resistance of transformer referred to 33 kV side

$$= \text{Primary resistance} + 0{\cdot}35\,(33/6{\cdot}6)^2$$

$$= 7{\cdot}5 + 8{\cdot}75 = 16{\cdot}25\Omega$$

Equivalent reactance of transformer referred to 33 kV side

$$= \text{Primary reactance} + 0{\cdot}65\,(33/6{\cdot}6)^2$$

$$= 13{\cdot}2 + 16{\cdot}25 = 29{\cdot}45\Omega$$

Total resistance of line and transformer is

$$R = 8 + 16{\cdot}25 = 24{\cdot}25\Omega$$

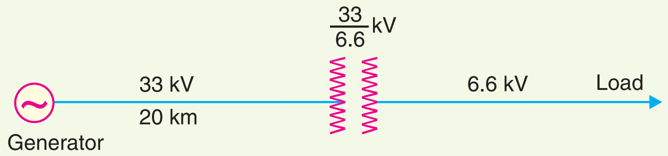


Fig. 10.7

Total reactance of line and transformer is

$$X_L = 10 + 29{\cdot}45 = 39{\cdot}45\,\Omega$$

Receiving end voltage per phase is

$$V_R = 33{,}000/\sqrt{3} = 19052 \text{ V}$$

Line current, $$I = \frac{2000 \times 10^3}{\sqrt{3} \times 33000} = 35 \text{ A}$$

Using the approximate expression for sending end voltage V_S per phase,

$$\begin{aligned} V_S &= V_R + I\,R\cos\phi_R + I\,X_L\sin\phi_R \\ &= 19052 + 35 \times 24{\cdot}25 \times 0{\cdot}8 + 35 \times 39{\cdot}45 \times 0{\cdot}6 \\ &= 19052 + 679 + 828 = 20559 \text{ V} = 20{\cdot}559 \text{ kV} \end{aligned}$$

Sending end line voltage $= \sqrt{3} \times 20{\cdot}559 \text{ kV} = \mathbf{35{\cdot}6\ kV}$

Sending end p.f., $$\cos\phi_S = \frac{V_R\cos\phi_R + I\,R}{V_S} = \frac{19052 \times 0{\cdot}8 + 35 \times 24{\cdot}25}{20559} = \mathbf{0{\cdot}7826\ lag}$$

Line losses $$= \frac{3\,I^2 R}{1000}\text{ kW} = \frac{3 \times (35)^2 \times 24{\cdot}25}{1000} = 89{\cdot}12 \text{ kW}$$

Output power $= 2000 \text{ kVA} \times 0{\cdot}8 = 1600 \text{ kW}$

∴ Transmission efficiency $$= \frac{1600}{1600 + 89{\cdot}12} \times 100 = \mathbf{94{\cdot}72\%}$$

TUTORIAL PROBLEMS

1. A single phase overhead transmission line delivers 4000 kW at 11 kV at 0·8 p.f. lagging. If resistance and reactance per conductor are 0·15 Ω and 0·02 Ω respectively, calculate :

(*i*) percentage regulation (*ii*) sending end power factor

(*iii*) line losses **[(*i*) 19·83% (*ii*) 0·77 lag (*iii*) 620 kW]**

2. A single phase 11 kV line with a length of 15 km is to transmit 500 kVA. The inductive reactance of the line is 0·5 Ω/km and the resistance is 0·3 Ω/km. Calculate the efficiency and regulation of the line for 0·8 lagging power factor. **[97·74%, 3·34%]**

3. A load of 1000 kW at 0·8 p.f. lagging is received at the end of a 3-phase line 20 km long. The resistance and reactance of each conductor are 0·25 Ω and 0·28 Ω per km. If the receiving end line voltage is maintained at 11 kV, calculate :

(*i*) sending end voltage (line-to-line) (*ii*) percentage regulation

(*iii*) transmission efficiency **[(*i*) 11·84 kV (*ii*) 7·61% (*iii*) 94·32%]**

4. Estimate the distance over which a load of 15000 kW at 0·85 p.f. can be delivered by a 3-phase transmission line having conductors of steel-cored aluminium each of resistance 0·905 Ω/phase per kilometre. The voltage at the receiving end is to be 132 kV and the loss in transmission is to be 7·5% of the load. **[69·55 km]**

5. A 3-phase line 3 km long delivers 3000 kW at a p.f. 0·8 lagging to a load. The resistance and reactance per km of each conductor are 0·4 Ω and 0·3 Ω respectively. If the voltage at the supply end is maintained at 11 kV, calculate :

(i) receiving end voltage (line-to-line) (ii) line current

(iii) transmission efficiency. **[(i) 10.46 kV (ii) 207 A (iii) 95%]**

6. A short 3-ϕ transmission line with an impedance of $(5 + j\,20)\ \Omega$ per phase has sending end and receiving end voltages of 46·85 kV and 33 kV respectively for some receiving end load at a p.f.of 0·8 lagging. Determine :

(i) power output

(ii) sending end power factor **[(i) 22·86 kW (ii) 0·657 lag]**

7. A substation receives 6000 kVA at 6 kV, 0.8 p.f. lagging on low voltage side of a transformer from a generating station through a 3-phase cable system having resistance of 7 Ω and reactance of 2 Ω per phase. Identical 6600/33000 V transformers are installed at each end, 6600 V side being delta connected and 33000 V side star connected. The resistance and reactance of each transformer are 1 Ω and 9 Ω respectively, referred to *h.v.* side. Calculate the voltage at the generating station bus bars **[6778 V]**

8. A short 3-phase transmission line connected to a 33kV, 50 Hz generating station at the sending end is required to supply a load of 10 MW at 0·8 lagging power factor at 30 kV at the receiving end. If the minimum transmission efficiency is to be limited to 96%, estimate the per phase value of resistance and inductance of the line. **[2·4 Ω; 0·028 H]**

9. A single phase transmission line is delivering 500 kVA load at 2 kV. Its resistance is 0·2 Ω and inductive reactance is 0·4 Ω. Determine the voltage regulation if the load power factor is (i) 0·707 lagging (ii) 0·707 leading. **[(i) 5·3% (ii) –1·65%]**

10.6 Medium Transmission Lines

In short transmission line calculations, the effects of the line capacitance are neglected because such lines have smaller lengths and transmit power at relatively low voltages (< 20 kV). However, as the length and voltage of the line increase, the capacitance gradually becomes of greater importance. Since medium transmission lines have sufficient length (50-150 km) and usually operate at voltages greater than 20 kV, the effects of capacitance cannot be neglected. Therefore, in order to obtain reasonable accuracy in medium transmission line calculations, the line capacitance must be taken into consideration.

The capacitance is uniformly distributed over the entire length of the line. However, in order to make the calculations simple, the line capacitance is assumed to be lumped or concentrated in the form of capacitors shunted across the line at one or more points. Such a treatment of localising the line capacitance gives reasonably accurate results. The most commonly used methods (known as *localised capacitance methods*) for the solution of medium transmissions lines are :

(i) End condenser method *(ii)* Nominal *T* method *(iii)* Nominal π method.

Although the above methods are used for obtaining the performance calculations of medium lines, they can also be used for short lines if their line capacitance is given in a particular problem.

10.7 End Condenser Method

In this method, the capacitance of the line is lumped or concentrated at the receiving or load end as shown in Fig. 10.8. This method of localising the line capacitance at the load end overestimates the effects of capacitance. In Fig. 10.8, one phase of the 3-phase transmission line is shown as it is more convenient to work in phase instead of line-to-line values.

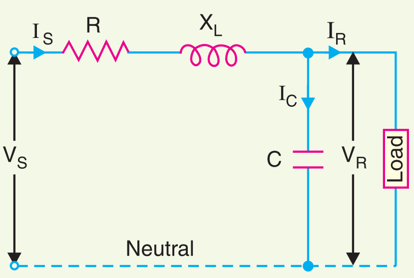


Fig. 10.8

Let I_R = load current per phase

R = resistance per phase

X_L = inductive reactance per phase

C = capacitance per phase

$\cos\phi_R$ = receiving end power factor (*lagging*)

V_S = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig 10.9. Taking the receiving end voltage $\vec{V_R}$ as the reference phasor, we have, $\vec{V_R} = V_R + j\,0$

Load current, $\vec{I_R} = I_R\,(\cos\phi_R - j\sin\phi_R)$

Capacitive current, $\vec{I_C} = j\,\vec{V_R}\,\omega C = j\,2\pi f C\,\vec{V_R}$

The sending end current $\vec{I_S}$ is the phasor sum of load current $\vec{I_R}$ and capacitive current $\vec{I_C}$ *i.e.*,

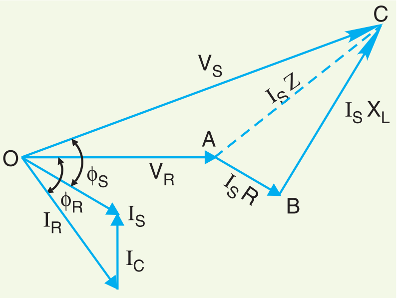


Fig. 10.9

$$\vec{I_S} = \vec{I_R} + \vec{I_C}$$
$$= I_R\,(\cos\phi_R - j\sin\phi_R) + j\,2\pi f C\,V_R$$
$$= I_R\cos\phi_R + j\,(-I_R\sin\phi_R + 2\pi f C V_R)$$

Voltage drop/phase $= \vec{I_S}\,\vec{Z} = \vec{I_S}\,(R + j\,X_L)$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\,\vec{Z} = \vec{V_R} + \vec{I_S}\,(R + j\,X_L)$

Thus, the magnitude of sending end voltage V_S can be calculated.

$$\%\text{ Voltage regulation} = \frac{V_S - V_R}{V_R}\times 100$$

$$\%\text{ Voltage transmission efficiency} = \frac{\text{Power delivered / phase}}{\text{Power delivered / phase + losses / phase}}\times 100$$

$$= \frac{V_R\,I_R\cos\phi_R}{V_R\,I_R\cos\phi_R + I_S^2\,R}\times 100$$

Limitations. Although end condenser method for the solution of medium lines is simple to work out calculations, yet it has the following drawbacks :

(i) There is a considerable error (about 10%) in calculations because the distributed capacitance has been assumed to be lumped or concentrated.

(ii) This method overestimates the effects of line capacitance.

Example 10.10. *A (medium) single phase transmission line 100 km long has the following constants :*

Resistance/km = 0·25 Ω ; *Reactance/km = 0·8 Ω*

Susceptance/km = 14 × 10^{-6} siemen ; *Receiving end line voltage = 66,000 V*

Assuming that the total capacitance of the line is localised at the receiving end alone, determine (i) the sending end current (ii) the sending end voltage (iii) regulation and (iv) supply power factor. The line is delivering 15,000 kW at 0.8 power factor lagging. Draw the phasor diagram to illustrate your calculations.

Solution. Figs. 10.10 (*i*) and (*ii*) show the circuit diagram and phasor diagram of the line respectively.

* Note the construction of phasor diagram. The load current $\vec{I_R}$ lags behind $\vec{V_R}$ by ϕ_R. The capacitive current $\vec{I_C}$ leads $\vec{V_R}$ by 90º as shown. The phasor sum of $\vec{I_C}$ and $\vec{I_R}$ is the sending end current $\vec{I_S}$. The drop in the line resistance is $\vec{I_S}R$ (*AB*) in phase with $\vec{I_S}$ whereas inductive drop $\vec{I_S}X_L$ (*BC*) leads I_S by 90º. Therefore, *OC* represents the sending end voltage $\vec{V_S}$. The angle ϕ_S between the sending end voltage $\vec{V_S}$ and sending end current $\vec{I_S}$ determines the sending end power factor cos ϕ_S.

Total resistance, $R = 0{\cdot}25 \times 100 = 25\,\Omega$

Total reactance, $X_L = 0{\cdot}8 \times 100 = 80\,\Omega$

Total susceptance, $Y = 14 \times 10^{-6} \times 100 = 14 \times 10^{-4}\,S$

Receiving end voltage, $V_R = 66{,}000$ V

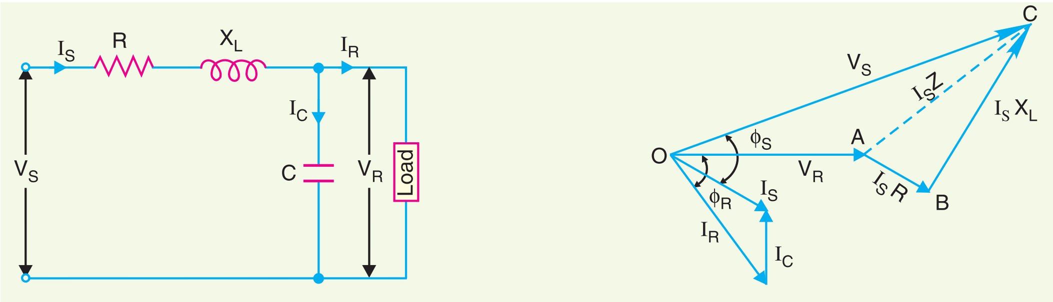
$\therefore$ Load current, $I_R = \dfrac{15{,}000 \times 10^3}{66{,}000 \times 0{\cdot}8} = 284$ A

$\cos\phi_R = 0{\cdot}8\,;\quad \sin\phi_R = 0{\cdot}6$

Taking receiving end voltage as the reference phasor [see Fig.10.10 (*ii*)], we have,

$$\vec{V_R} = V_R + j\,0 = 66{,}000\,V$$

Load current, $\vec{I_R} = I_R\,(\cos\phi_R - j\sin\phi_R) = 284\,(0{\cdot}8 - j\,0{\cdot}6) = 227 - j\,170$



(*i*) (*ii*)

Fig. 10.10

Capacitive current, $\vec{I_C} = j\,Y \times V_R = j\,14 \times 10^{-4} \times 66000 = j\,92$

(*i*) Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (227 - j\,170) + j\,92$

$= 227 - j\,78$ ... (*i*)

Magnitude of $I_S = \sqrt{(227)^2 + (78)^2} =$ **240 A**

(*ii*) Voltage drop $= \vec{I_S}\,\vec{Z} = \vec{I_S}(R + j\,X_L) = (227 - j\,78)\,(25 + j\,80)$

$= 5{,}675 + j\,18,160 - j\,1950 + 6240$

$= 11{,}915 + j\,16{,}210$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\,\vec{Z} = 66{,}000 + 11{,}915 + j\,16{,}210$

$= 77{,}915 + j\,16{,}210$ ...(*ii*)

Magnitude of $V_S = \sqrt{(77915)^2 + (16210)^2} =$ **79583V**

(*iii*) % Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{79{,}583 - 66{,}000}{66{,}000} \times 100 =$ **20·58%**

(*iv*) Referring to exp. (*i*), phase angle between $\vec{V_R}$ and $\vec{I_R}$ is :

$$\theta_1 = \tan^{-1} - 78/227 = \tan^{-1}(-0{\cdot}3436) = -18{\cdot}96^\circ$$

Referring to exp. (*ii*), phase angle between $\vec{V_R}$ and $\vec{V_S}$ is :

$$\theta_2 = \tan^{-1}\frac{16210}{77915} = \tan^{-1}(0{\cdot}2036) = 11{\cdot}50^\circ$$

$\therefore$ Supply power factor angle, $\phi_S = 18{\cdot}96^\circ + 11{\cdot}50^\circ = 30{\cdot}46^\circ$

$\therefore$ Supply p.f. $= \cos\phi_S = \cos 30{\cdot}46^\circ =$ **0·86 lag**

10.8 Nominal T Method

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on its either side as shown in Fig. 10.11. Therefore, in this arrangement, full charging current flows over half the line. In Fig. 10.11, one phase of 3-phase transmission line is shown as it is advantageous to work in phase instead of line-to-line values.

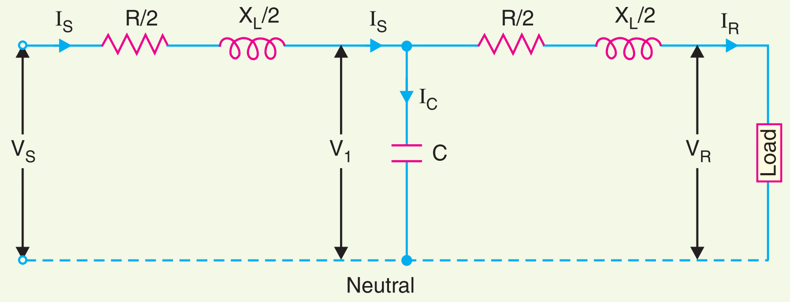


Fig. 10.11

Let I_R = load current per phase ; R = resistance per phase

X_L = inductive reactance per phase ; C = capacitance per phase

$\cos \phi_R$ = receiving end power factor (*lagging*) ; V_S = sending end voltage/phase

V_1 = voltage across capacitor C

The *phasor diagram for the circuit is shown in Fig. 10.12. Taking the receiving end voltage $\vec{V_R}$ as the reference phasor, we have,

Receiving end voltage, $\vec{V_R} = V_R + j\,0$

Load current, $\vec{I_R} = I_R (\cos \phi_R - j \sin \phi_R)$

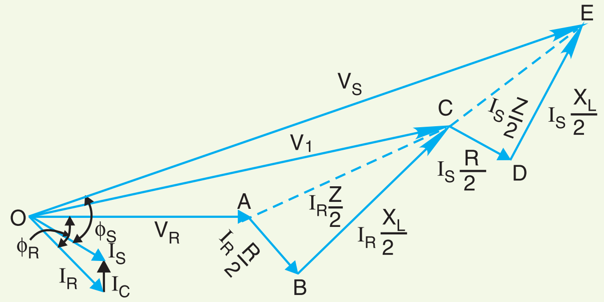


Fig. 10.12

* Note the construction of phasor diagram. $\vec{V_R}$ is taken as the reference phasor represented by OA. The load current $\vec{I_R}$ lags behind $\vec{V_R}$ by ϕ_R. The drop $AB = I_R\, R/2$ is in phase with $\vec{I_R}$ and $BC = I_R \cdot X_L/2$ leads $\vec{I_R}$ by 90º. The phasor OC represents the voltage $\vec{V_1}$ across condenser C. The capacitor current $\vec{I_C}$ leads $\vec{V_1}$ by 90º as shown. The phasor sum of $\vec{I_R}$ and $\vec{I_C}$ gives $\vec{I_S}$. Now $CD = I_S\, R/2$ is in phase with $\vec{I_S}$ while $DE = I_S\, X_L/2$ leads $\vec{I_S}$ by 90º. Then, OE represents the sending end voltage $\vec{V_S}$.

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2$

$$= V_R + I_R(\cos\phi_R - j\sin\phi_R)\left(\frac{R}{2} + j\frac{X_L}{2}\right)$$

Capacitive current, $\vec{I_C} = j\,\omega\,C\,\vec{V_1} = j\,2\pi\,f\,C\,\vec{V_1}$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C}$

Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\dfrac{\vec{Z}}{2} = \vec{V_1} + \vec{I_S}\left(\dfrac{R}{2} + j\,\dfrac{X_L}{2}\right)$

Example 10.11. *A 3-phase, 50-Hz overhead transmission line 100 km long has the following constants :*

Resistance/km/phase $= 0.1\ \Omega$

Inductive reactance/km/phase $= 0{\cdot}2\ \Omega$

Capacitive susceptance/km/phase $= 0{\cdot}04 \times 10^{-4}$ *siemen*

Determine (i) the sending end current (ii) sending end voltage (iii) sending end power factor and (iv) transmission efficiency when supplying a balanced load of 10,000 kW at 66 kV, p.f. 0·8 lagging. Use nominal T method.

Solution. Figs. 10.13 (*i*) and 10.13 (*ii*) show the circuit diagram and phasor diagram of the line respectively.

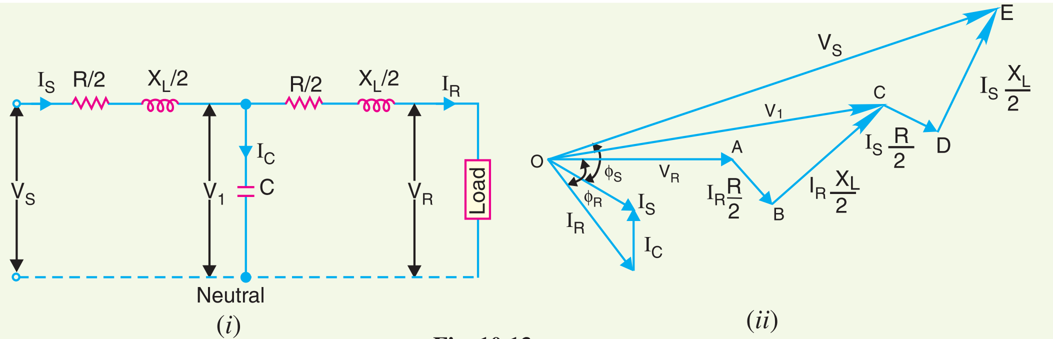


Fig. 10.13

Total resistance/phase, $R = 0{\cdot}1 \times 100 = 10\ \Omega$

Total reactance/phase. $X_L = 0{\cdot}2 \times 100 = 20\ \Omega$

Capacitive susceptance, $Y = 0{\cdot}04 \times 10^{-4} \times 100 = 4 \times 10^{-4}$ S

Receiving end voltage/phase, $V_R = 66{,}000/\sqrt{3} = 38105\ V$

Load current, $$I_R = \frac{10{,}000 \times 10^3}{\sqrt{3} \times 66 \times 10^3 \times 0{\cdot}8} = 109 \text{ A}$$

$$\cos\phi_R = 0{\cdot}8\ ;\quad \sin\phi_R = 0{\cdot}6$$

Impedance per phase, $\vec{Z} = R + j\,X_L = 10 + j\,20$

(*i*) Taking receiving end voltage as the reference phasor [see Fig. 10.13 (*ii*)], we have,

Receiving end voltage, $\vec{V_R} = V_R + j\,0 = 38{,}105\ V$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 109\,(0{\cdot}8 - j\,0{\cdot}6) = 87{\cdot}2 - j\,65{\cdot}4$

Voltage across C, $\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2 = 38,105 + (87{\cdot}2 - j\,65{\cdot}4)\,(5 + j\,10)$

$$= 38{,}105 + 436 + j\,872 - j\,327 + 654 = 39{,}195 + j\,545$$

Charging current, $\vec{I_C} = j\,Y\,\vec{V_1} = j\,4\times10^{-4}(39{,}195 + j\,545) = -\,0\cdot218 + j\,15\cdot6$

Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (87\cdot2 - j\,65\cdot4) + (-\,0\cdot218 + j\,15\cdot6)$

$= 87\cdot0 - j\,49\cdot8 = 100\,\angle - 29°47'$ A

∴ Sending end current = **100 A**

(ii) Sending end voltage, $\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2 = (39{,}195 + j\,545) + (87\cdot0 - j\,49\cdot8)\,(5 + j\,10)$

$= 39{,}195 + j\,545 + 434\cdot9 + j\,870 - j\,249 + 498$

$= 40128 + j\,1170 = 40145\,\angle 1°40'$ V

∴ Line value of sending end voltage

$= 40145\times\sqrt{3} = 69\,533$ V = **69·533 kV**

(iii) Referring to phasor diagram in Fig. 10.14,

θ_1 = angle between $\vec{V_R}$ and $\vec{V_S}$ = 1°40′

θ_2 = angle between $\vec{V_R}$ and $\vec{I_S}$ = 29° 47′

∴ ϕ_S = angle between $\vec{V_S}$ and $\vec{I_S}$

$= \theta_1 + \theta_2 = 1°40' + 29°47' = 31°27'$

∴ Sending end power factor, $\cos\phi_S = \cos 31°27'$ = **0·853 lag**

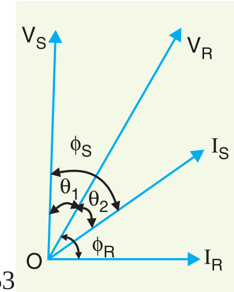


Fig. 10.14

(iv) Sending end power $= 3\,V_S I_S \cos\phi_S = 3\times40{,}145\times100\times0\cdot853$

$= 10273105$ W $= 10273\cdot105$ kW

Power delivered = 10,000 kW

∴ Transmission efficiency $= \dfrac{10{,}000}{10273\cdot105}\times100$ = **97·34%**

Example 10.12. *A 3-phase, 50 Hz transmission line 100 km long delivers 20 MW at 0·9 p.f. lagging and at 110 kV. The resistance and reactance of the line per phase per km are 0·2 Ω and 0·4 Ω respectively, while capacitance admittance is $2\cdot5\times10^{-6}$ siemen/km/phase. Calculate : (i) the current and voltage at the sending end (ii) efficiency of transmission. Use nominal T method.*

Solution. Figs. 10.15 (*i*) and 10.15 (*ii*) show the circuit diagram and phasor diagram respectively.

Total resistance/phase, $R = 0\cdot2\times100 = 20\ \Omega$

Total reactance/phase, $X_L = 0\cdot4\times100 = 40\ \Omega$

Total capacitance admittance/phase, $Y = 2\cdot5\times10^{-6}\times100 = 2\cdot5\times10^{-4}$ S

Phase impedance, $\vec{Z} = 20 + j40$

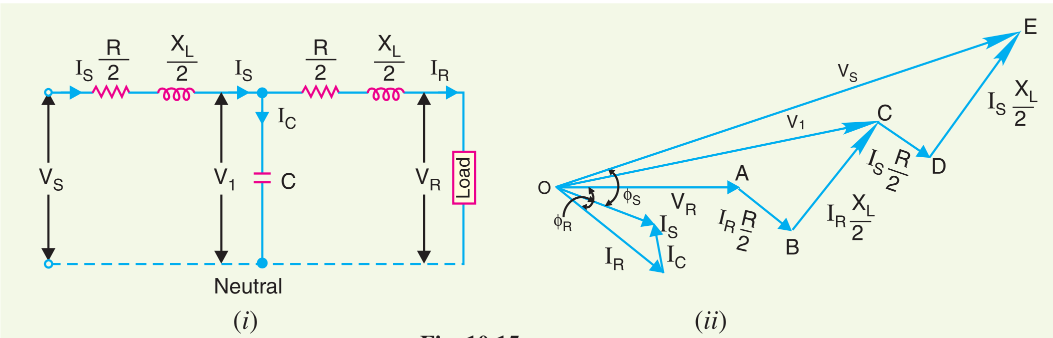


Fig. 10.15

Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63508$ V

Load current, $I_R = \dfrac{20 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0 \cdot 9} = 116.6$ A

$\cos \phi_R = 0 \cdot 9 \text{ ; } \sin \phi_R = 0 \cdot 435$

(*i*) Taking receiving end voltage as the reference phasor [see phasor diagram 10.15 (*ii*)], we have,

$$\vec{V}_R = V_R + j0 = 63508 \text{ V}$$

Load current, $\vec{I}_R = I_R (\cos \phi_R - j \sin \phi_R) = 116 \cdot 6\,(0 \cdot 9 - j\,0 \cdot 435) = 105 - j50 \cdot 7$

Voltage across C, $\vec{V}_1 = \vec{V}_R + \vec{I}_R\,\vec{Z}/2 = 63508 + (105 - j\,50 \cdot 7)\,(10 + j\,20)$

$= 63508 + (2064 + j1593) = 65572 + j1593$

Charging current, $\vec{I}_C = j\,Y\vec{V}_1 = j\,2 \cdot 5 \times 10^{-4}\,(65572 + j1593) = -0 \cdot 4 + j\,16 \cdot 4$

Sending end current, $\vec{I}_S = \vec{I}_R + \vec{I}_C = (105 - j\,50 \cdot 7) + (-0 \cdot 4 + j16 \cdot 4)$

$= (104 \cdot 6 - j\,34 \cdot 3) = 110 \angle -18°9'$ A

$\therefore$ Sending end current $=$ **110 A**

Sending end voltage, $\vec{V}_S = \vec{V}_1 + \vec{I}_S\,\vec{Z}/2$

$= (65572 + j1593) + (104 \cdot 6 - j34 \cdot 3)\,(10 + j20)$

$= 67304 + j\,3342$

$\therefore$ Magnitude of $V_S = \sqrt{(67304)^2 + (3342)^2} = 67387\ V$

$\therefore$ Line value of sending end voltage

$= 67387 \times \sqrt{3} = 116717 \text{ V} =$ **116·717 kV**

(*ii*) Total line losses for the three phases

$= 3\,I_S^2\,R/2 + 3I_R^2\,R/2$

$= 3 \times (110)^2 \times 10 + 3 \times (116 \cdot 6)^2 \times 10$

$= 0 \cdot 770 \times 10^6 \text{ W} = 0 \cdot 770 \text{ MW}$

$\therefore$ Transmission efficiency $= \dfrac{20}{20 + 0 \cdot 770} \times 100 =$ **96·29%**

10.9 Nominal π Method

In this method, capacitance of each conductor (*i.e.*, line to neutral) is divided into two halves; one half being lumped at the sending end and the other half at the receiving end as shown in Fig. 10.16. It is obvious that capacitance at the sending end has no effect on the line drop. However, its charging current must be added to line current in order to obtain the total sending end current.

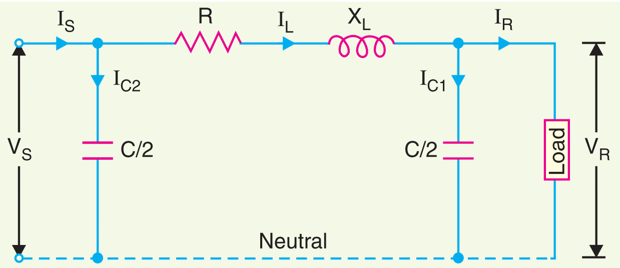


Fig. 10.16

Let I_R = load current per phase

R = resistance per phase

X_L = inductive reactance per phase

C = capacitance per phase

$\cos \phi_R$ = receiving end power factor (*lagging*)

V_S = sending end voltage per phase

The *phasor diagram for the circuit is shown in Fig. 10.17. Taking the receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j\,0$$

Load current, $\vec{I_R} = I_R (\cos \phi_R - j \sin \phi_R)$

Charging current at load end is

$$\vec{I_{C1}} = j\,\omega\,(C/2)\,\vec{V_R} = j\,\pi\,f\,C\,\vec{V_R}$$

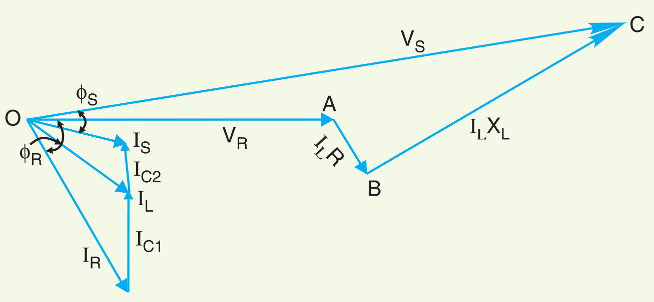


Fig. 10.17

Line current, $\vec{I_L} = \vec{I_R} + \vec{I_{C1}}$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + jX_L)$

Charging current at the sending end is

$$\vec{I_{C2}} = j\,\omega\,(C/2)\,\vec{V_S} = j\,\pi\,f\,C\,\vec{V_S}$$

∴ Sending end current, $\vec{I_S} = \vec{I_L} + \vec{I_{C2}}$

Example 10.13 *A 3-phase, 50Hz, 150 km line has a resistance, inductive reactance and capacitive shunt admittance of 0·1 Ω, 0·5 Ω and 3×10^{-6} S per km per phase. If the line delivers 50 MW at 110 kV and 0·8 p.f. lagging, determine the sending end voltage and current. Assume a nominal π circuit for the line.*

---

* Note the construction of phasor diagram. $\vec{V_R}$ is taken as the reference phasor represented by OA. The current $\vec{I_R}$ lags behind $\vec{V_R}$ by ϕ_R. The charging current $\vec{I_{C1}}$ leads $\vec{V_R}$ by 90º. The line current $\vec{I_L}$ is the phasor sum of $\vec{I_R}$ and $\vec{I_{C1}}$. The drop $AB = I_L R$ is in phase with $\vec{I_L}$ whereas drop $BC = I_L X_L$ leads $\vec{I_L}$ by 90º. Then OC represents the sending end voltage $\vec{V_S}$. The charging current $\vec{I_{C2}}$ leads $\vec{V_S}$ by 90º. Therefore, sending end current $\vec{I_S}$ is the phasor sum of the $\vec{I_{C2}}$ and $\vec{I_L}$. The angle ϕ_S between sending end voltage V_S and sending end current I_S determines the sending end p.f. $\cos \phi_S$.

Solution. Fig. 10.18 shows the circuit diagram for the line.

Total resistance/phase, $R = 0{\cdot}1 \times 150 = 15\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}5 \times 150 = 75\ \Omega$

Capacitive admittance/phase, $Y = 3 \times 10^{-6} \times 150 = 45 \times 10^{-5}$ S

Receiving end voltage/phase, $V_R = 110 \times 10^3/\sqrt{3} = 63{,}508$ V

Load current, $I_R = \dfrac{50 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0{\cdot}8} = 328$ A

$\cos \phi_R = 0{\cdot}8\ ;\quad \sin \phi_R = 0{\cdot}6$

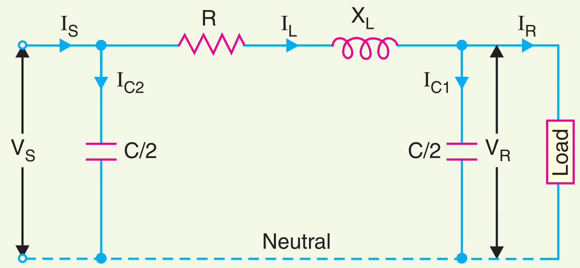


Fig. 10.18

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j\,0 = 63{,}508 \text{ V}$$

Load current, $\vec{I_R} = I_R(\cos \phi_R - j \sin \phi_R) = 328\,(0{\cdot}8 - j0{\cdot}6) = 262{\cdot}4 - j196{\cdot}8$

Charging current at the load end is

$$\vec{I_{C1}} = \vec{V_R}\, j\,\frac{Y}{2} = 63{,}508 \times j\,\frac{45 \times 10^{-5}}{2} = j\,14{\cdot}3$$

Line current, $\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = (262{\cdot}4 - j\,196{\cdot}8) + j\,14{\cdot}3 = 262{\cdot}4 - j\,182{\cdot}5$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + j\,X_L)$

$= 63,\,508 + (262{\cdot}4 - j\,182{\cdot}5)\,(15 + j\,75)$

$= 63{,}508 + 3936 + j\,19{,}680 - j\,2737{\cdot}5 + 13{,}687$

$= 81{,}131 + j\,16{,}942{\cdot}5 = 82{,}881\,\angle\,11°47'$ V

∴ Line to line sending end voltage $= 82{,}881 \times \sqrt{3} = 1{,}43{,}550$ V = **143·55 kV**

Charging current at the sending end is

$$I_{C2} = j\vec{V_S}\,Y/2 = (81{,}131 + j\,16{,}942{\cdot}5)\,j\,\frac{45 \times 10^{-5}}{2}$$

$= -\,3{\cdot}81 + j\,18{\cdot}25$

Sending end current, $\vec{I_S} = \vec{I_L} + \vec{I_{C_2}} = (262{\cdot}4 - j\,182{\cdot}5) + (-\,3{\cdot}81 + j\,18{\cdot}25)$

$= 258{\cdot}6 - j\,164{\cdot}25 = 306{\cdot}4\,\angle -32{\cdot}4°$ A

∴ Sending end current = **306·4 A**

Example 10·14. *A 100-km long, 3-phase, 50-Hz transmission line has following line constants:*

Resistance/phase/km = 0·1 Ω

Reactance/phase/km = 0·5 Ω

Susceptance/phase/km = 10×10^{-6} S

If the line supplies load of 20 MW at 0·9 p.f. lagging at 66 kV at the receiving end, calculate by nominal π method :

(i) sending end power factor *(ii) regulation*

(iii) transmission efficiency

Solution. Fig. 10·19 shows the circuit diagram for the line.

Total resistance/phase, $R = 0{\cdot}1 \times 100 = 10\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}5 \times 100 = 50\ \Omega$

Susceptance/phase, $Y = 10 \times 10^{-6} \times 100 = 10 \times 10^{-4}$ S

Receiving end voltage/phase, $V_R = 66 \times 10^3/\sqrt{3} = 38105$ V

Load current, $$I_R = \frac{20 \times 10^6}{\sqrt{3} \times 66 \times 10^3 \times 0{\cdot}9} = 195 \text{ A}$$

$$\cos \phi_R = 0{\cdot}9 \quad ; \quad \sin \phi_R = 0{\cdot}435$$

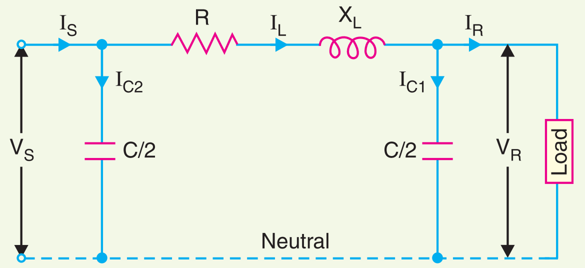


Fig. 10.19

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j0 = 38105 \text{ V}$$

Load current, $\vec{I_R} = I_R (\cos \phi_R - j \sin \phi_R) = 195\,(0{\cdot}9 - j\,0{\cdot}435) = 176 - j\,85$

Charging current at the receiving end is

$$\vec{I_{C1}} = \vec{V_R}\, j\, \frac{Y}{2} = 38105 \times j\, \frac{10 \times 10^{-4}}{2} = j\,19$$

Line current, $\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = (176 - j\,85) + j\,19 = 176 - j\,66$

Sending end voltage , $\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \vec{I_L}\,(R + j\,X_L)$

$= 38{,}105 + (176 - j\,66)\,(10 + j\,50)$

$= 38{,}105 + (5060 + j\,8140)$

$= 43{,}165 + j\,8140 = 43{,}925 \angle 10{\cdot}65^\circ$ V

Sending end line to line voltage $= 43,925 \times \sqrt{3} = 76 \times 10^3$ V $= 76$ kV

Charging current at the sending end is

$$\vec{I_{C2}} = \vec{V_S}\, jY/2 = (43{,}165 + j\,8140)\, j\, \frac{10 \times 10^{-4}}{2}$$

$= -4{\cdot}0 + j\,21{\cdot}6$

$\therefore$ Sending end current, $\vec{I_S} = \vec{I_L} + \vec{I_{C2}} = (176 - j\,66) + (-4{\cdot}0 + j\,21{\cdot}6)$

$= 172 - j\,44{\cdot}4 = 177{\cdot}6 \angle -14{\cdot}5^\circ$ A

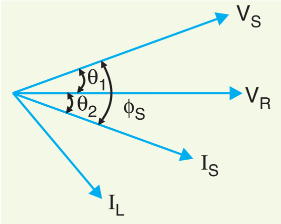


Fig. 10.20

(*i*) Referring to phasor diagram in Fig. 10·20,

θ_1 = angle between $\vec{V_R}$ and $\vec{V_S}$ = 10·65º

θ_2 = angle between $\vec{V_R}$ and $\vec{I_S}$ = – 14·5º

∴ ϕ_S = angle between $\vec{V_S}$ and $\vec{I_S}$ = $\theta_2 + \theta_1$

= 14·5º + 10·65º = 25·15º

∴ Sending end p.f., cos ϕ_S = cos 25·15º = **0·905 lag**

(*ii*) % Voltage regulation $= \dfrac{V_S - V_R}{V_R} \times 100 = \dfrac{43925 - 38105}{38105} \times 100$ = **15·27 %**

(*iii*) Sending end power $= 3\, V_S\, I_S \cos \phi_S = 3 \times 43925 \times 177{\cdot}6 \times 0{\cdot}905$

$= 21{\cdot}18 \times 10^6$ W = 21·18 MW

Transmission efficiency = (20/21·18) × 100 = **94 %**

TUTORIAL PROBLEMS

1. A (medium) single phase transmission line 100 km long has the following constants :

Resistance/km/phase = 0·15Ω
Inductive reactance/km/phase = 0·377 Ω
Capacitive reactance/km/phase = 31·87 Ω
Receiving end line voltage = 132 kV

Assuming that the total capacitance of the line is localised at the receiving end alone, determine :

(*i*) sending end current (*ii*) line value of sending end voltage
(*iii*) regulation (*iv*) sending end power factor

The line is delivering 72 MW at 0·8 p.f. lagging.

[(*i*) 377·3 A (*ii*) 155·7 kV (*iii*) 17·9% (*iv*) 0·774 lag]

2. A 3-phase, 50 Hz overhead transmission line has the following constants :

Resistance/phase = 9·6 Ω
Inductance/phase = 0·097 mH
Capacitance/phase = 0·765 μF

If the line is supplying a balanced load of 24,000 kVA 0·8 p.f. lagging at 66 kV, calculate :

(*i*) sending end current (*ii*) line value of sending end voltage
(*iii*) sending end power factor (*iv*) percentage regulation
(*v*) transmission efficiency **[(*i*) 204 A (*ii*) 75 kV (*iii*) 0·814 lag (*iv*) 13·63 % (*v*) 93·7%]**

3. A 3-phase, 50 Hz, overhead transmission line delivers 10 MW at 0·8 p.f. lagging and at 66 kV. The resistance and inductive reactance of the line per phase are 10 Ω and 20 Ω respectively while capacitance admittance is 4×10^{-4} siemen. Calculate :

(*i*) the sending end current (*ii*) sending end voltage (line-to-line)
(*iii*) sending end power factor (*iv*) transmission efficiency

Use nominal *T* method. **[(*i*) 100 A (*ii*) 69·8 kV (*iii*) 0·852 (*iv*) 97·5%]**

4. A 3-phase, 50 Hz, 100 km transmission line has the following constants ;

Resistance/phase/km = 0·1 Ω
Reactance/phase/km = 0·5 Ω
Susceptance/phase/km = 10^{-5} siemen

If the line supplies a load of 20 MW at 0·9 p.f. lagging at 66 kV at the receiving end, calculate by using nominal π method :

(*i*) sending end current (*ii*) line value of sending end voltage (*iii*) sending end power factor (*iv*) regulation

[(*i*) 177·6 A (*ii*) 76kV (*iii*) 0·905 lag (*iv*) 15·15%]

5. A 3-phase overhead transmission line has the following constants :

Resistance/phase = 10 Ω

Inductive reactance/phase = 35 Ω

Capacitive admittance/phase = 3×10^{-4} siemen

If the line supplied a balanced load of 40,000 kVA at 110 kV and 0·8 p.f. lagging, calculate :

(*i*) sending end power factor (*ii*) percentage regulation

(*iii*) transmission efficiency **[(*i*) 0·798 lag (*ii*) 10% (*iii*) 96·38%]**

6. A 3-phase, 50 Hz overhead transmission line, 100 km long, 110 kV between the lines at the receiving end has the following constants :

Resistance per km per phase = 0·153 Ω

Inductance per km per phase = 1·21 mH

Capacitance per km per phase = 0·00958 μF

The line supplies a load of 20,000 kW at 0·9 power factor lagging. Calculate using nominal π representation, the sending end voltage, current, power factor, regulation and the efficiency of the line. Neglect leakage. **[115·645 kV (line voltage) : 109 ∠ – 16·68º A ; 0·923 lag ; 5·13 %; 97·21 %]**

10.10 Long Transmission Lines

It is well known that line constants of the transmission line are uniformly distributed over the entire length of the line. However, reasonable accuracy can be obtained in line calculations for short and medium lines by considering these constants as lumped. If such an assumption of lumped constants is applied to long transmission lines (having length excess of about 150 km), it is found that serious errors are introduced in the performance calculations. Therefore, in order to obtain fair degree of accuracy in the performance calculations of long lines, the line constants are considered as uniformly distributed throughout the length of the line. Rigorous mathematical treatment is required for the solution of such lines.

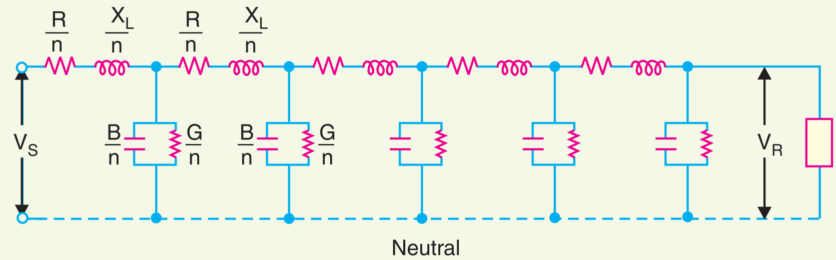


Fig. 10.21

Fig. 10.21 shows the equivalent circuit of a 3-phase long transmission line on a phase-neutral basis. The whole line length is divided into n sections, each section having line constants $\frac{1}{n}th$ of those for the whole line. The following points may by noted :

(*i*) The line constants are uniformly distributed over the entire length of line as is actually the case.

(*ii*) The resistance and inductive reactance are the series elements.

(*iii*) The leakage susceptance (B) and leakage conductance (G) are shunt elements. The leakage susceptance is due to the fact that capacitance exists between line and neutral. The leakage conductance takes into account the energy losses occurring through leakage over the insulators or due to corona effect between conductors. Admittance = $\sqrt{G^2 + B^2}$.

(*iv*) The leakage current through shunt admittance is maximum at the sending end of the line and decreases continuously as the receiving end of the circuit is approached at which point its value is zero.

10.11 Analysis of Long Transmission Line (*Rigorous method*)

Fig. 10.22 shows one phase and neutral connection of a 3-phase line with impedance and shunt admittance of the line uniformly distributed.

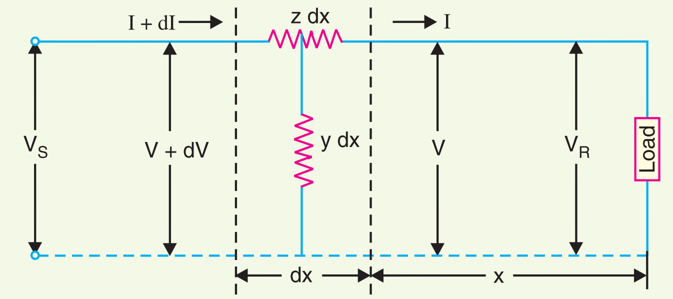


Fig. 10.22

Consider a small element in the line of length *dx* situated at a distance *x* from the receiving end.

Let
z = series impedance of the line per unit length
y = shunt admittance of the line per unit length
V = voltage at the end of element towards receiving end
$V + dV$ = voltage at the end of element towards sending end
$I + dI$ = current entering the element dx
I = current leaving the element dx

Then for the small element *dx*,

$z\,dx$ = series impedance
$y\,dx$ = shunt admittance

Obviously, $dV = I\,z\,dx$

or $$\frac{dV}{dx} = I\,z \qquad ...(i)$$

Now, the current entering the element is $I + dI$ whereas the current leaving the element is I. The difference in the currents flows through shunt admittance of the element *i.e.*,

dI = Current through shunt admittance of element $= V\,y\,dx$

or $$\frac{dI}{dx} = V\,y \qquad ...(ii)$$

Differentiating eq. (*i*) w.r.t. *x*, we get,

$$\frac{d^2V}{dx^2} = z\,\frac{dI}{dx} = z\,(V\,y) \qquad \left[\because \frac{dI}{dx} = V\,y \text{ from exp. } (ii)\right]$$

or $$\frac{d^2V}{dx^2} = y\,z\,V \qquad ...(iii)$$

The solution of this differential equation is

$$V = k_1 \cosh\left(x\sqrt{y\,z}\right) + k_2 \sinh\left(x\sqrt{y\,z}\right) \qquad ...(iv)$$

Differentiating exp. (*iv*) *w.r.t. x*, we have,

$$\frac{dV}{dx} = k_1 \sqrt{y\,z} \sinh (x \sqrt{y\,z}) + k_2 \sqrt{y\,z} \cosh (x \sqrt{y\,z})$$

But $$\frac{dV}{dx} = I\,z \qquad \text{[from exp. } (i)\text{]}$$

$\therefore$ $$I\,z = k_1 \sqrt{y\,z} \sinh (x\sqrt{y\,z}) + k_2 \sqrt{z\,y} \cosh (x \sqrt{y\,z})$$

or $$I = \sqrt{\frac{y}{z}} \left[k_1 \sinh\ (x\sqrt{y\,z}) + k_2 \cosh \left(x\sqrt{y\,z}\right)\right] \qquad ...(v)$$

Equations (*iv*) and (*v*) give the expressions for V and I in the form of unknown constants k_1 and k_2. The values of k_1 and k_2 can be found by applying end conditions as under :

At $x = 0$, $V = V_R$ and $I = I_R$

Putting these values in eq. (*iv*), we have,

$$V_R = k_1 \cosh 0 + k_2 \sinh 0 = k_1 + 0$$

$\therefore$ $$V_R = k_1$$

Similarly, putting $x = 0$, $V = V_R$ and $I = I_R$ in eq. (*v*), we have,

$$I_R = \sqrt{\frac{y}{z}} \left[k_1 \sinh 0 + k_2 \cosh 0\right] = \sqrt{\frac{y}{z}} \left[0 + k_2\right]$$

$\therefore$ $$k_2 = \sqrt{\frac{z}{y}}\, I_R$$

Substituting the values of k_1 and k_2 in eqs. (*iv*) and (*v*), we get,

$$V = V_R \cosh (x\sqrt{y\,z}) + \sqrt{\frac{z}{y}}\, I_R \sinh (x\sqrt{y\,z})$$

and $$I = \sqrt{\frac{y}{z}} V_R \sinh (x\sqrt{y\,z}) + I_R \cosh (x\sqrt{y\,z})$$

The sending end voltage (V_S) and sending end current (I_S) are obtained by putting $x = l$ in the above equations *i.e.*,

$$V_S = V_R \cosh (l \sqrt{y\,z}) + \sqrt{\frac{z}{y}} I_R \sinh (l\sqrt{y\,z})$$

$$I_S = \sqrt{\frac{y}{z}}\, V_R \sinh (l\sqrt{y\,z}) + I_R \cosh (l\sqrt{y\,z})$$

Now, $$l\sqrt{y\,z} = \sqrt{l\,y \cdot l\,z} = \sqrt{Y\,Z}$$

and $$\sqrt{\frac{y}{z}} = \sqrt{\frac{y\,l}{z\,l}} = \sqrt{\frac{Y}{Z}}$$

where Y = total shunt admittance of the line

Z = total series impedance of the line

Therefore, expressions for V_S and I_S become :

$$\mathbf{V_S = V_R \cosh \sqrt{YZ} + I_R \sqrt{\frac{Z}{Y}} \sinh \sqrt{YZ}}$$

$$\mathbf{I_S = V_R \sqrt{\frac{Y}{Z}} \sinh \sqrt{YZ} + I_R \cosh \sqrt{YZ}}$$

It is helpful to expand hyperbolic sine and cosine in terms of their power series.

$$\cosh \sqrt{Y\,Z} = \left(1 + \frac{Z\,Y}{2} + \frac{Z^2Y^2}{24} + \ldots\ldots\right)$$

$$\sinh\sqrt{YZ} = \left(\sqrt{YZ} + \frac{(YZ)^{3/2}}{6} + \ldots\ldots\right)$$

Example 10.15. *A 3-φ transmission line 200 km long has the following constants :*

Resistance/phase/km = *0·16 Ω*

Reactance/phase/km = *0·25 Ω*

Shunt admittance/phase/km = $1{\cdot}5 \times 10^{-6}$ *S*

Calculate by rigorous method the sending end voltage and current when the line is delivering a load of 20 MW at 0·8 p.f. lagging. The receiving end voltage is kept constant at 110 kV.

Solution :

Total resistance/phase, $R = 0{\cdot}16 \times 200 = 32\ \Omega$

Total reactance/phase, $X_L = 0{\cdot}25 \times 200 = 50\ \Omega$

Total shunt admittance/phase, $Y = j\,1{\cdot}5 \times 10^{-6} \times 200 = 0{\cdot}0003 \angle 90^\circ$

Series Impedance/phase, $Z = R + j\,X_L = 32 + j\,50 = 59{\cdot}4 \angle 58^\circ$

The sending end voltage V_S per phase is given by :

$$V_S = V_R \cosh\sqrt{YZ} + I_R\sqrt{\frac{Z}{Y}}\sinh\sqrt{ZY} \qquad \ldots(i)$$

Now $$\sqrt{ZY} = \sqrt{59{\cdot}4 \angle 58^\circ \times 0{\cdot}0003 \angle 90^\circ} = 0{\cdot}133 \angle 74^\circ$$

$$ZY = 0{\cdot}0178 \angle 148^\circ$$

$$Z^2Y^2 = 0{\cdot}00032 \angle 296^\circ$$

$$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{59{\cdot}4 \angle 58^\circ}{0{\cdot}0003 \angle 90^\circ}} = 445 \angle -16^\circ$$

$$\sqrt{\frac{Y}{Z}} = \sqrt{\frac{0{\cdot}0003 \angle 90^\circ}{59{\cdot}4 \angle 58^\circ}} = 0{\cdot}00224 \angle 16^\circ$$

$$\therefore \quad \cosh\sqrt{YZ} = 1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} \text{ approximately}$$

$$= 1 + \frac{0{\cdot}0178}{2}\angle 148^\circ + \frac{0{\cdot}00032}{24}\angle 296^\circ$$

$$= 1 + 0{\cdot}0089 \angle 148^\circ + 0{\cdot}0000133 \angle 296^\circ$$

$$= 1 + 0{\cdot}0089\,(-0{\cdot}848 + j\,0{\cdot}529) + 0{\cdot}0000133\,(0{\cdot}438 - j\,0{\cdot}9)$$

$$= 0{\cdot}992 + j\,0{\cdot}00469 = 0{\cdot}992 \angle 0{\cdot}26^\circ$$

$$\sinh\sqrt{YZ} = \sqrt{YZ} + \frac{(YZ)^{3/2}}{6} \text{ approximately}$$

$$= 0{\cdot}133 \angle 74^\circ + \frac{0{\cdot}0024 \angle 222^\circ}{6}$$

$$= 0{\cdot}133 \angle 74^\circ + 0{\cdot}0004 \angle 222^\circ$$

$$= 0{\cdot}133\,(0{\cdot}275 + j\,0{\cdot}961) + 0{\cdot}0004\,(-0{\cdot}743 - j\,0{\cdot}67)$$

$$= 0{\cdot}0362 + j\,0{\cdot}1275 = 0{\cdot}1325 \angle 74^\circ 6'$$

Receiving end voltage per phase is

$$V_R = 110 \times 10^3/\sqrt{3} = 63508 \text{ V}$$

Receiving end current, $$I_R = \frac{20 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0{\cdot}8} = 131\ A$$

Putting the various values in exp (*i*), we get,

$$\begin{aligned} V_S &= 63508 \times 0{\cdot}992 \angle 0{\cdot}26^\circ + 131 \times 445 \angle -16^\circ 0' \times 0{\cdot}1325 \angle 74^\circ 6' \\ &= 63000 \angle 0{\cdot}26^\circ + 7724 \angle 58^\circ 6' \\ &= 63000\,(0{\cdot}999 + j\,0{\cdot}0045) + 7724\,(0{\cdot}5284 + j\,0{\cdot}8489) \\ &= 67018 + j\,6840 = 67366 \angle 5^\circ 50' \text{ V} \end{aligned}$$

Sending end line-to-line voltage $= 67366 \times \sqrt{3} = 116{\cdot}67 \times 10^3$ V = **116·67 kV**

The sending end current I_S is given by :

$$I_S = V_R \sqrt{\frac{Y}{Z}} \sinh \sqrt{Y\,Z} + I_R \cosh \sqrt{Y\,Z}$$

Putting the various values, we get,

$$\begin{aligned} I_S &= 63508 \times 0{\cdot}00224 \angle 16^\circ \times 0{\cdot}1325 \angle 74^\circ 6' + 131 \times 0{\cdot}992 \angle 0{\cdot}26^\circ \\ &= 18{\cdot}85 \angle 90^\circ 6' + 130 \angle 0{\cdot}26^\circ \\ &= 18{\cdot}85\,(-0{\cdot}0017 + j\,0{\cdot}999) + 130\,(0{\cdot}999 + j\,0{\cdot}0045) \\ &= 129{\cdot}83 + j\,19{\cdot}42 = 131{\cdot}1 \angle 8^\circ \text{ A} \end{aligned}$$

∴ Sending end current = **131·1 A**

TUTORIAL PROBLEMS

1. A 3-phase overhead transmission line has a total series impedance per phase of 200 ∠80° ohms and a total shunt admittance of 0·0013∠90° siemen per phase. The line delivers a load of 80 MW at 0·8 p.f. lagging and 220 kV between the lines. Determine the sending end line voltage and current by rigorous method. **[263·574 kV ; 187·5 A]**
2. A 3-phase transmission line, 160 km long, has the following constants :

   Resistance/phase/km = 0·2 Ω

   Reactance/phase/km = 0·3127 Ω

   Shunt admittance/phase/km = $1{\cdot}875 \times 10^{-6}$ S

   Determine the sending end voltage and current by rigorous method when the line is delivering a load of 25 MVA at 0·8 p.f. lagging. The receiving end voltage is kept constant at 110 kV. **[116·67 kV ; 131·1 A]**

10.12 Generalised Circuit Constants of a Transmission Line

In any four terminal *network, the input voltage and input current can be expressed in terms of output voltage and output current. Incidentally, a transmission line is a 4-terminal network ; two input terminals where power enters the network and two output terminals where power leaves the network.

Therefore, the input voltage $(\vec{V}_S)$ and input current $(\vec{I}_S)$ of a 3-phase transmission line can be expressed as :

$$\vec{V}_S = \vec{A}\,\vec{V}_R + \vec{B}\,\vec{I}_R$$

$$\vec{I}_S = \vec{C}\,\vec{V}_R + \vec{D}\,\vec{I}_R$$

where $\vec{V}_S$ = sending end voltage per phase

$\vec{I}_S$ = sending end current

$\vec{V}_R$ = receiving end voltage per phase

$\vec{I}_R$ = receiving end current

* The network should be *passive* (containing no source of e.m.f.), *linear* (impedances independent of current flowing) and *bilateral* (impedances independent of direction of current flowing). This condition is fully met in transmission lines.

and $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ (generally complex numbers) are the constants known as *generalised circuit constants* of the transmission line. The values of these constants depned upon the particular method adopted for solving a transmission line. Once the values of these constants are known, performance calculations of the line can be easily worked out. The following points may be kept in mind :

(*i*) The constants $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ are generally complex numbers.

(*ii*) The constants $\vec{A}$ and $\vec{D}$ are dimensionless whereas the dimensions of $\vec{B}$ and $\vec{C}$ are ohms and siemen respectively.

(*iii*) For a given transmisson line,

$$\vec{A} = \vec{D}$$

(*iv*) For a given transmission line,

$$\vec{A}\,\vec{D} - \vec{B}\,\vec{C} = 1$$

We shall establish the correctness of above characteristics of generalised circuit constants in the following discussion.

10.13 Determination of Generalised Constants for Transmission Lines

As stated previously, the sending end voltage $(\vec{V_S})$ and sending end current $(\vec{I_S})$ of a transmission line can be expressed as :

$$\vec{V_S} = \vec{A}\,\vec{V_R} + \vec{B}\,\vec{I_R} \quad ...(i)$$

$$\vec{I_S} = \vec{C}\,\vec{V_R} + \vec{D}\,\vec{I_R} \quad ...(ii)$$

We shall now determine the values of these constants for different types of transmission lines.

(*i*) **Short lines.** In short transmission lines, the effect of line capacitance is neglected. Therefore, the line is considered to have series impedance. Fig. 10.23 shows the circuit of a 3-phase transmission line on a single phase basis.

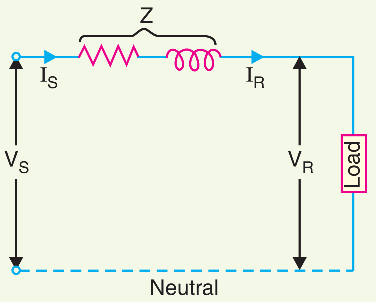


Fig. 10.23

Here, $$\vec{I_S} = \vec{I_R} \quad ...(iii)$$

and $$\vec{V_S} = \vec{V_R} + \vec{I_R}\,\vec{Z} \quad ...(iv)$$

Comparing these with eqs. (*i*) and (*ii*), we have,

$$\vec{A} = 1\,; \quad \vec{B} = \vec{Z}, \quad \vec{C} = 0 \quad \text{and} \quad \vec{D} = 1$$

Incidentally ; $\vec{A} = \vec{D}$

and $$\vec{A}\,\vec{D} - \vec{B}\,\vec{C} = 1 \times 1 - \vec{Z} \times 0 = 1$$

(*ii*) **Medium lines – Nominal T method.** In this method, the whole line to neutral capacitance is assumed to be concentrated at the middle point of the line and half the line resistance and reactance are lumped on either side as shown in Fig. 10.24.

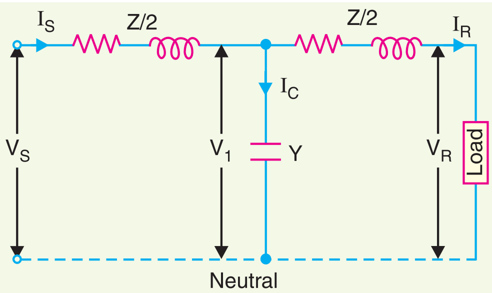


Fig. 10.24

Here, $$\vec{V_S} = \vec{V_1} + \vec{I_S}\,\vec{Z}/2 \quad ...(v)$$

and $$\vec{V_1} = \vec{V_R} + \vec{I_R}\,\vec{Z}/2$$

Now, $$\vec{I_C} = \vec{I_S} - \vec{I_R}$$

$$= \vec{V_1}\,\vec{Y} \quad \text{where } Y = \text{shunt admittance}$$

$$= \vec{Y}\left(\vec{V_R} + \frac{\vec{I_R}\,\vec{Z}}{2}\right)$$

$$\therefore \qquad \vec{I_S} = \vec{I_R} + \vec{Y}\,\vec{V_R} + \vec{Y}\,\frac{\vec{I_R}\,\vec{Z}}{2}$$

$$= \vec{Y}\,\vec{V_R} + \vec{I_R}\left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right) \qquad \text{...}(vi)$$

Substituting the value of V_1 in eq. (v), we get,

$$\vec{V_S} = \vec{V_R} + \frac{\vec{I_R}\,\vec{Z}}{2} + \frac{\vec{I_S}\,\vec{Z}}{2}$$

Substituing the value of I_S, we get,

$$\vec{V_S} = \left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right)\vec{V_R} + \left(\vec{Z} + \frac{\vec{Y}\,\vec{Z}^2}{4}\right)\vec{I_R} \qquad \text{...}(vii)$$

Comparing eqs. (vii) and (vi) with those of (i) and (ii), we have,

$$\vec{A} = \vec{D} = 1 + \frac{\vec{Y}\,\vec{Z}}{2}\ ; \quad \vec{B} = \vec{Z}\left(1 + \frac{\vec{Y}\,\vec{Z}}{4}\right)\ ; \ \vec{C} = \vec{Y}$$

Incidentally :

$$\vec{A}\,\vec{D} - \vec{B}\,\vec{C} = \left(1 + \frac{Y\,Z}{2}\right)^2 - Z\left(1 + \frac{Y\,Z}{4}\right)Y$$

$$= 1 + \frac{Y^2 Z^2}{4} + Y\,Z - Z\,Y - \frac{Z^2 Y^2}{4} = 1$$

(iii) Medium lines—Nominal π method. In this method, line-to-neutral capacitance is divided into two halves ; one half being concentrated at the load end and the other half at the sending end as shown in Fig. 10.25.

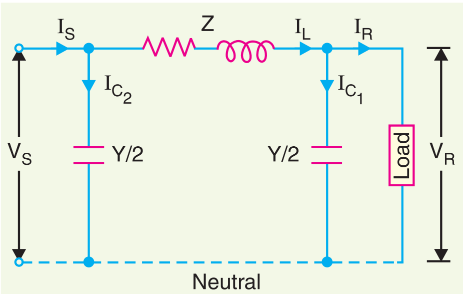


Fig. 10.25

Here, $\vec{Z} = R + jX_L$ = series impedenace/phase

$\vec{Y} = j\,\omega\,C$ = shunt admittance

$$\vec{I_S} = \vec{I_L} + \vec{I_{C2}}$$

or

$$\vec{I_S} = \vec{I_L} + \vec{V_S}\,\vec{Y}/2 \qquad \text{...}(viii)$$

Also

$$\vec{I_L} = \vec{I_R} + \vec{I_{C1}} = \vec{I_R} + \vec{V_R}\,\vec{Y}/2 \qquad \text{...}(ix)$$

Now

$$\vec{V_S} = \vec{V_R} + \vec{I_L}\,\vec{Z} = \vec{V_R} + \left(\vec{I_R} + \vec{V_R}\,\vec{Y}/2\right)\vec{Z} \quad \text{(Putting the value of } \vec{I_L}\text{)}$$

$$\therefore \qquad \vec{V_S} = \vec{V_R}\left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right) + \vec{I_R}\,\vec{Z} \qquad \text{...}(x)$$

Also

$$\vec{I_S} = \vec{I_L} + \vec{V_S}\,\vec{Y}/2 = \left(\vec{I_R} + \vec{V_R}\,\vec{Y}/2\right) + \vec{V_S}\,\vec{Y}/2$$

(Putting the value of $\vec{I_L}$)

Putting the value of $\vec{V_S}$ from eq. (x), we get,

$$\vec{I_S} = \vec{I_R} + \vec{V_R}\,\frac{\vec{Y}}{2} + \frac{\vec{Y}}{2}\left\{\vec{V_R}\left(1 + \frac{\vec{Y}\,\vec{Z}}{2}\right) + \vec{I_R}\,\vec{Z}\right\}$$

$$= \vec{I_R} + \vec{V_R}\frac{\vec{Y}}{2} + \frac{\vec{V_R}\,\vec{Y}}{2} + \frac{\vec{V_R}\,\vec{Y}^2\,\vec{Z}}{4} + \frac{\vec{Y}\,\vec{I_R}\,\vec{Z}}{2}$$

$$= \vec{I_R}\left(1+\frac{\vec{Y}\,\vec{Z}}{2}\right) + \vec{V_R}\,\vec{Y}\left(1+\frac{\vec{Y}\,\vec{Z}}{4}\right) \qquad ...(xi)$$

Comparing equations (*x*) and (*xi*) with those of (*i*) and (*ii*), we get,

$$\vec{A} = \vec{D} = \left(1+\frac{\vec{Y}\,\vec{Z}}{2}\right)\ ;\ \vec{B} = \vec{Z}\,;\ \vec{C} = \vec{Y}\left(1+\frac{\vec{Y}\,\vec{Z}}{4}\right)$$

Also

$$\vec{A}\,\vec{D} - \vec{B}\,\vec{C} = \left(1+\frac{Y\,Z}{2}\right)^2 - Z\,Y\left(1+\frac{Y\,Z}{4}\right)$$

$$= 1+\frac{Y^2 Z^2}{4} + Y\,Z - Z\,Y - \frac{Z^2 Y^2}{4} = 1$$

(*iv*) Long lines—Rigorous method. By rigorous method, the sending end voltage and current of a long transmission line are given by :

$$V_S = V_R \cosh\sqrt{Y\,Z} + I_R\sqrt{\frac{Z}{Y}}\sinh\sqrt{Y\,Z}$$

$$I_S = V_R\sqrt{\frac{Y}{Z}}\sinh\sqrt{Y\,Z} + I_R\cosh\sqrt{Y\,Z}$$

Comparing these equations with those of (*i*) and (*ii*), we get,

$$\vec{A} = \vec{D} = \cosh\sqrt{Y\,Z}\ ;\quad \vec{B} = \sqrt{\frac{Z}{Y}}\sinh\sqrt{Y\,Z}\ ;\quad \vec{C} = \sqrt{\frac{Y}{Z}}\sinh\sqrt{Y\,Z}$$

Incidentally

$$\vec{A}\,\vec{D} - \vec{B}\,\vec{C} = \cosh\sqrt{Y\,Z}\times\cosh\sqrt{Y\,Z} - \sqrt{\frac{Z}{Y}}\sinh\sqrt{Y\,Z}\times\sqrt{\frac{Y}{Z}}\sinh\sqrt{Y\,Z}$$

$$= \cosh^2\sqrt{Y\,Z} - \sinh^2\sqrt{Y\,Z} = 1$$

Example 10.16. *A balanced 3-phase load of 30 MW is supplied at 132 kV, 50 Hz and 0·85 p.f. lagging by means of a transmission line. The series impedance of a single conductor is (20 + j52) ohms and the total phase-neutral admittance is 315 $\times$ 10^{-6} siemen. Using nominal T method, determine: (i) the A, B, C and D constants of the line (ii) sending end voltage (iii) regulation of the line.*

Solution. Fig. 10.26 shows the representation of 3-phase line on the single phase basis.

Series line impedance/phase, $\vec{Z} = (20 + j\,52)\ \Omega$

Shunt admittance/phase, $\vec{Y} = j\,315\times 10^{-6}$ S

(*i*) Generalised constants of line. For nominal *T* method, various constants have the values as under :

$$\vec{A} = \vec{D} = 1 + \vec{Z}\,\vec{Y}/2 = 1 + \frac{20 + j\,52}{2}\times j\,315\times 10^{-6}$$

$$= 0{\cdot}992 + j\,0{\cdot}00315 = \mathbf{0{\cdot}992\ \angle 0{\cdot}18^\circ}$$

$$\vec{B} = \vec{Z}\left(1+\frac{\vec{Z}\,\vec{Y}}{4}\right) = (20 + j\,52)\left[1+\frac{(20 + j\,52)\,j\,315\times 10^{-6}}{4}\right]$$

$$= 19{\cdot}84 + j\,51{\cdot}82 = \mathbf{55{\cdot}5\ \angle 69^\circ}$$

$$\vec{C} = \vec{Y} = \mathbf{0{\cdot}000315\ \angle 90^\circ}$$

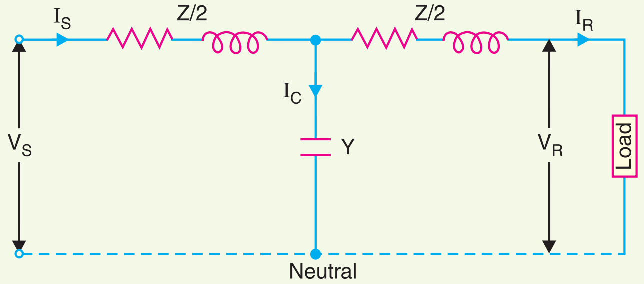


Fig. 10.26

***(ii)* Sending end voltage.**

Receiving end voltage/phase, $V_R = 132 \times 10^3/\sqrt{3} = 76210$ V

Receiving end current, $I_R = \dfrac{30 \times 10^6}{\sqrt{3} \times 132 \times 10^3 \times 0{\cdot}85} = 154$ A

$\cos \phi_R = 0{\cdot}85\,; \quad \sin \phi_R = 0{\cdot}53$

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j0 = 76210 \text{ V}$$

$$\vec{I_R} = I_R (\cos \phi_R - j \sin \phi_R) = 154\,(0{\cdot}85 - j\,0{\cdot}53) = 131 - j\,81{\cdot}62$$

Sending end voltage per phase is

$$\vec{V_S} = \vec{A}\,\vec{V_R} + \vec{B}\,\vec{I_R}$$
$$= (0{\cdot}992 + j\,0{\cdot}0032)\,76210 + (19{\cdot}84 + j\,51{\cdot}82)\,(131 - j\,81{\cdot}62)$$
$$= 82{,}428 + j\,5413$$

$\therefore$ Magnitude of sending end voltage is

$$V_S = \sqrt{(82{,}428)^2 + (5413)^2} = 82{\cdot}6 \times 10^3 \text{V} = 82{\cdot}6 \text{ kV}$$

$\therefore$ Sending end line-to-line voltage

$$= 82{\cdot}6 \times \sqrt{3} = \mathbf{143\ kV}$$

***(iii)* Regulation.** Regulation is defined as the change in voltage at the receiving end when full-load is thrown off.

Now, $\vec{V_S} = \vec{A}\,\vec{V_R} + \vec{B}\,\vec{I_R}$

At no load, $\vec{I_R} = 0$

$\therefore$ $\vec{V_S} = \vec{A}\,\vec{V_{R0}}$

where $\vec{V_{R0}}$ = voltage at receiving end at no load

or $\vec{V_{R0}} = \vec{V_S}/\vec{A}$

or $V_{R0} = V_S/A$ (in magnitude)

$\therefore$ % Regulation $= \dfrac{(V_S/A - V_R)}{V_R} \times 100 = \dfrac{(82{\cdot}6/0{\cdot}992) - 76{\cdot}21}{76{\cdot}21} \times 100 = \mathbf{9{\cdot}25\%}$

Example 10.17. *A 132 kV, 50 Hx, 3-phase transmission line delivers a load of 50 MW at 0·8 p.f. lagging at the receiving end. The generalised constants of the transmission line are :*

$A = D = 0{\cdot}95\ \angle 1{\cdot}4^o\,; \ B = 96\ \angle 78^o\,; \ C = 0{\cdot}0015\ \angle 90^o$

Find the regulation of the line and charging current. Use Nominal-T method.

Solution.

Receiving end voltage/phase, $V_R = 132 \times 10^3/\sqrt{3} = 76210$ V

Receiving end current, $I_R = \dfrac{50 \times 10^6}{\sqrt{3} \times 132 \times 10^3 \times 0{\cdot}8} = 273$ A

$\cos \phi_R = 0{\cdot}8; \quad \sin \phi_R = 0{\cdot}6$

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = V_R + j\,0 = 76210 \angle 0^\circ$$

$$\vec{I_R} = I_R \angle -\phi_R = 273 \angle -36{\cdot}9^\circ$$

Sending end voltage per phase is

$$\begin{aligned}
\vec{V_S} &= \vec{A}\ \vec{V_R} + \vec{B}\ \vec{I_R} \\
&= 0{\cdot}95 \angle 1{\cdot}4^\circ \times 76210 \angle 0^\circ + 96 \angle 78^\circ \times 273 \angle -36{\cdot}9^\circ \\
&= 72400 \angle 1{\cdot}4^\circ + 26208 \angle 41{\cdot}1^\circ \\
&= 72400 (\cos 1{\cdot}4^\circ + j \sin 1{\cdot}4^\circ) + 26208 (\cos 41{\cdot}1^\circ + j \sin 41{\cdot}1^\circ) \\
&= 72400 (0{\cdot}9997 + j\,0{\cdot}0244) + 26208 (0{\cdot}7536 + j\,0{\cdot}6574) \\
&= (72378 + j\,1767) + (19750 + j\,17229) \\
&= 92128 + j\,18996 = 94066 \angle 11{\cdot}65^\circ \text{ V}
\end{aligned}$$

Sending end current,

$$\begin{aligned}
\vec{I_S} &= \vec{C}\ \vec{V_R} + \vec{D}\ \vec{I_R} \\
&= 0{\cdot}0015 \angle 90^\circ \times 76210 \angle 0^\circ + 0{\cdot}95 \angle 1{\cdot}4^\circ \times 273 \angle -36{\cdot}9^\circ \\
&= 114 \angle 90^\circ + 260 \angle -35{\cdot}5^\circ \\
&= 114 (\cos 90^\circ + j \sin 90^\circ) + 260 (\cos 35{\cdot}5^\circ - j \sin 35{\cdot}5^\circ) \\
&= 114 (0 + j) + 260 (0{\cdot}814 - j\,0{\cdot}58) \\
&= j\,114 + 211 - j\,150 = 211 - j\,36
\end{aligned}$$

Charging current,

$$\begin{aligned}
\vec{I_C} &= \vec{I_S} - \vec{I_R} = (211 - j36) - 273 \angle -36{\cdot}9^\circ \\
&= (211 - j\,36) - (218 - j\,164) = -7 + j\,128 = \mathbf{128{\cdot}2 \angle 93{\cdot}1^\circ \ A}
\end{aligned}$$

$$\% \text{ Regulation} = \frac{(V_S/A) - V_R}{V_R} \times 100 = \frac{94066/0{\cdot}95 - 76210}{76210} \times 100 = \mathbf{30\%}$$

Example 10.18. *Find the following for a single circuit transmission line delivering a load of 50 M VA at 110 kV and p.f. 0·8 lagging :*

(i) sending end voltage (ii) sending end current (iii) sending end power (iv) efficiency of transmission. Given A = D = 0·98 ∠3°; B = 110 ∠75° ohm ; C = 0·0005 ∠80° siemen.

Solution.

Receiving end voltage/phase,

$$V_R = \frac{110}{\sqrt{3}} = 63{\cdot}5 \text{ kV}$$

Receiving end current, $I_R = \dfrac{50 \times 10^6}{\sqrt{3} \times 110 \times 10^3} = 262{\cdot}4$ A

Taking receiving end voltage as the reference phasor, we have,

$$\vec{V_R} = (63500 + j0)$$

$$\vec{I_R} = 262{\cdot}4 \angle -\cos^{-1} 0{\cdot}8 = 262{\cdot}4\,(0{\cdot}8 - j\,0{\cdot}6) = (210 - j\,157{\cdot}5)\text{ A}$$

(i) Now sending-end voltage per phase is

$$\vec{V_S} = \vec{A}\ \vec{V_R} + \vec{B}\ \vec{I_R}$$

Here $\vec{A}\,\vec{V_R} = 0{\cdot}98\,\angle 3^\circ \times 63500\,\angle 0^\circ = 62230\,\angle 3^\circ = (62145 + j\,3260)\text{ V}$

and $\vec{B}\,\vec{I_R} = 110\,\angle 75^\circ \times 262{\cdot}4\,\angle -36{\cdot}86^\circ$

$= 28865\,\angle 38{\cdot}14^\circ = (22702 + j\,17826)\text{ V}$

$\therefore$ $\vec{V_S} = (62145 + j\,3260) + (22702 + j\,17826)$

$= 84847 + j\,21086 = 87427\,\angle 14^\circ\text{ V}$

$\therefore$ Magnitude of sending-end voltage/phase = **87427 V**

(ii) Sending-end current is given by ;

$$\vec{I_S} = \vec{C}\ \vec{V_R} + \vec{D}\ \vec{I_R}$$

Here $\vec{C}\,\vec{V_R} = 0{\cdot}0005\,\angle 80^\circ \times 63500\,\angle 0^\circ = 31{\cdot}75\,\angle 80^\circ = (5{\cdot}5 + j\,31{\cdot}3)\text{ A}$

and $\vec{D}\,\vec{I_R} = 0{\cdot}98\,\angle 3^\circ \times 262{\cdot}4\,\angle -36{\cdot}86^\circ$

$= 257{\cdot}15\,\angle -33{\cdot}8^\circ = (213{\cdot}5 - j\,143{\cdot}3)\text{ A}$

$\therefore$ $\vec{I_S} = (5{\cdot}5 + j\,31{\cdot}3) + (213{\cdot}5 - j\,143{\cdot}3)$

$= 219 - j\,112 = 246\,\angle -27^\circ\text{ A}$

$\therefore$ Magnitude of sending-end current = **246 A**

(iii) Sending-end power $= 3\,V_S\,I_S \cos\phi_S$

Here $V_S = 87427$ V ; $I_S = 246$ A ; $\cos\phi_S = \cos(-27^\circ - 14^\circ)$

$\therefore$ Sending-end power $= 3 \times 87427 \times 246 \times \cos(-27^\circ - 14^\circ)$

$= 48{\cdot}6 \times 10^6\text{ W} =$ **48·6 MW**

(iv) Receiving end power $= 50 \times 0{\cdot}8 = 40$ MW

Transmission efficiency, $\eta = \dfrac{40}{48{\cdot}6} \times 100 =$ **82·3%**

TUTORIAL PROBLEMS

1. A 150 km, 3-ϕ, 110 kV, 50 Hz transmission line transmits a load of 40,000 kW at 0·8 p.f. lagging at the receiving end. Resistance/km/phase = 0·15Ω ; reactance/km/phase = 0·6 Ω; susceptance/km/phase = 10^{-5} S. Determine (*i*) the *A*, *B*, *C* and *D* constants of the line (*ii*) regulation of the line.

[(*i*) A = D = 0·968 ∠1° ; B = 92·8 ∠7·5°Ω ; C = 0·00145 ∠90·5° S (*ii*) 33·5%]

2. A balanced load of 30 MW is supplied at 132 kV, 50 Hz and 0·85 p.f. lagging by means of a transmission line. The series impedance of a single conductor is (20 + j52) ohms and the total phases-neutral admittance is 315 microsiemens. Shunt leakage may be neglected. Using the nominal *T* approximation, calculate the line voltage at the sending end of the line. If the load is removed and the sending end voltage remains constant, find the percentage rise in voltage at the receiving end.

[143 kV; 9%]

3. Calculate *A*, *B*, *C* and *D* constants of a 3-phase, 50 Hz transmission line 160 km long having the following distributed parameters :

$R = 0{\cdot}15\ \Omega/\text{km}$; $L = 1{\cdot}20 \times 10^{-3}$ H/km ; $C = 8 \times 10^{-9}$ F/km ; $G = 0$

[A = D = 0·988 ∠0·3° ; B = 64·2 ∠68·3° Ω ; C = 0·4 × 10^{-3} ∠90·2° S]

SELF-TEST

1. Fill in the blanks by inserting appropriate words/figures.

(i) In short transmission lines, the effects of .... are neglected.

(ii) .... of transmission lines, is the most important cause of power loss in the line.

(iii) In the analysis of 3-phase transmission line, only .... is considered.

(iv) For a given V_R and I, the regulation of the line .... with the decrease in p.f. for lagging loads.

(v) If the p.f. of the load decreases, the line losses ....

(vi) In medium transmission lines, effects of .... are taken into account.

(vii) The rigorous solution of transmission lines takes into account the .... nature of line constants.

(viii) In any transmission line, $AD - BC$ = ...

(ix) In a transmission line, generalised constants .... and .... are equal.

(x) The dimensions of constants B and C are respectively .... and ....

2. Pick up the correct words/figures from the brackets and fill in the blanks.

(i) The line constants of a transmission line are .... [*uniformly distributed, lumped*]

(ii) The length of a short transmission line is upto about .... [*50 km, 120 km, 200 km*]

(iii) The capacitance of a transmission line is a .... element. [*series, shunt*]

(iv) It is desirable that voltage regulation of a transmission line should be .... [*low, high*]

(v) When the regulation is positive, then receiving and voltage (V_R) is .... than sending and voltage (V_S). [*more, less*]

(vi) The shunt admittance of a transmission line is 3 microsiemens. Its complex notation will be .... siemen. [$3 \times 10^{-6} \angle 90^o$, $3 \times 10^{-6} \angle 0^o$]

(vii) The exact solution of any transmission line must consider the fact that line constants are .... [*uniformly distributed, lumped*]

(viii) The generalised constants A and D of the transmission line have .... [*no dimensions, dimensions of ohm*]

(ix) $30 \angle 10^o \times 60 \angle 20^o$ = ... [$2 \angle 2^o$, $1800 \angle 30^o$, $1800 \angle 2^o$]

(x) $\sqrt{9 \angle 90^o \times 4 \ \angle 10^o}$ = ... [$6 \angle 50^o$, $6 \angle 80^o$, $6 \angle 10^o$]

ANSWERS TO SELF-TEST

1. (i) capacitance (ii) resistance (iii) one phase (iv) increases (v) increase (vi) capacitance (vii) distributed (viii) 1 (ix) A and D (x) ohm, siemen

2. (i) uniformly distributed (ii) 50 km (iii) shunt (iv) low (v) less (vi) $3 \times 10^{-6} \angle 90^o$ (vii) uniformly distributed (viii) no dimensions (ix) $1800 \angle 30^o$ (x) $6 \angle 50^o$

CHAPTER REVIEW TOPICS

1. What is the purpose of an overhead transmission line ? How are these lines classified ?
2. Discuss the terms voltage regulation and transmission efficiency as applied to transmission line.
3. Deduce an expression for voltage regulation of a short transmission line, giving the vector diagram.
4. What is the effect of load power factor on regulation and efficiency of a transmission line ?
5. What do you understand by medium transmission lines ? How capacitance effects are taken into account in such lines ?
6. Show how regulation and transmission efficiency are determined for medium lines using
   (i) end condensor method
   (ii) nominal T method
   (iii) nominal π method
   Illustrate your answer with suitable vector diagrams.

7. What do you understand by long transmission lines ? How capacitance effects are taken into account in such lines ?
8. Using rigorous method, derive expressions for sending end voltage and current for a long transmission 1ine.
9. What do you understand by generalised circuit constants of a transmission line ? What is their importance?
10. Evaluate the generalised circuit constants for
    (*i*) short transmission line
    (*ii*) medium line — nominal T method
    (*iii*) medium line — nominal π method

DISCUSSION QUESTIONS

1. What is the justification in neglecting line capacitance in short transmission lines ?
2. What are the drawbacks of localised capacitance methods ?
3. A long transmission line is open circuited at the receiving end. Will there be any current in the line at the sending end ? Explain your answer.
4. Why is leakage conductance negligible in overhead lines ? What about underground system ?
5. Why do we analyse a 3-phase transmission line on single phase basis ?

GO To FIRST