

## Underground Cables

11.1 Underground Cables
11.2 Construction of Cables
11.3 Insulating Materials for Cables
11.4 Classification of Cables
11.5 Cables for 3-Phase Service
11.6 Laying of Underground Cables
11.7 Insulation Resistance of a Single-Core Cable
11.8 Capacitance of a Single-Core Cable
11.9 Dielectric Stress in a Single-Core Cable
11.10 Most Economical Conductor Size in aCable
11.11 Grading of Cables
11.12 Capacitance Grading
11.13 Intersheath Grading
11.14 Capacitance of 3-Core Cables
11.15 Measurements of $C_{e}$ and $C_{c}$
11.16 Current-Carrying Capacity of Under-ground Cables
11.17 Thermal Resistance
11.18 Thermal Resistance of Dielectric of aSingle-Core Cable
11.19 Permissible Current Loading
11.20 Types of Cable Faults
11.21 Loop Tests for Location of Faults inUnderground Cables
11.22 Murray Loop Test

## Introduction

Electric power can be transmitted or dis tributed either by overhead system or by underground cables. The underground cables have serveral advantages such as less liable to damage through storms or lightning, low maintenance cost, less chances of faults, smaller voltage drop and better general appearance. However, their major drawback is that they have greater installation cost and introduce insulation problems at high voltages compared with the equivalent overhead system. For this reason, underground cables are employed where it is impracticable to use overhead lines. Such locations may be thickly populated areas where municipal authorities prohibit overhead lines for reasons of safety, or around plants and substations or where maintenance conditions do not permit the use of overhead construction.

The chief use of underground cables for many years has been for distribution of electric power in congested urban areas at comparatively low or moderate voltages. However, recent improvements in the design and manufacture have led to the development of cables suitable for use at high voltages. This has made it possible to employ underground cables for transmission of electric
power for short or moderate distances. In this chapter, we shall focus our attention on the various aspects of underground cables and their increasing use in power system.

### 11.1 Underground Cables

An underground cable essentially consists of one or more conductors covered with suitable insulation and surrounded by a protecting cover.

Although several types of cables are available, the type of cable to be used will depend upon the working voltage and service requirements. In general, a cable must fulfil the following necessary requirements:
(i) The conductor used in cables should be tinned stranded copper or aluminium of high conductivity. Stranding is done so that conductor may become flexible and carry more current.
(ii) The conductor size should be such that the cable carries the desired load current without overheating and causes voltage drop within permissible limits.
(iii) The cable must have proper thickness of insulation in order to give high degree of safety and reliability at the voltage for which it is designed.
(iv) The cable must be provided with suitable mechanical protection so that it may withstand the rough use in laying it.
(v) The materials used in the manufacture of cables should be such that there is complete chemical and physical stability throughout.

### 11.2 Construction of Cables

Fig. 11.1 shows the general construction of a 3-conductor cable. The various parts are :
(i) Cores or Conductors. A cable may have one or more than one core (conductor) depending upon the type of service for which it is intended. For instance, the 3-conductor cable shown in Fig. 11.1 is used for 3-phase service. The conductors are made of tinned copper or aluminium and are usually stranded in order to provide flexibility to the cable.
(ii) Insulatian. Each core or conductor is provided with a suitable thickness of insulation, the thickness of layer depending upon the voltage to be withstood by the cable. The commonly used materials for insulation are impregnated paper, varnished cambric or rubber mineral compound.
(iii) Metallic sheath. In order to protect the cable from moisture, gases or other damaging liquids (acids or alkalies) in the soil and atmosphere, a metallic sheath of lead or aluminium is provided over the insulation as shown in Fig. 11.1
(iv) Bedding. Over the metallic


Fig. 11.1 Construction of a Cable sheath is applied a layer of bedding which consists of a fibrous material like jute or hessian tape. The purpose of bedding is to protect the metallic sheath against corrosion and from mechanical injury due to armouring.
(v) Armouring. Over the bedding, armouring is provided which consists of one or two layers of galvanised steel wire or steel tape. Its purpose is to protect the cable from mechanical injury while laying it and during the course of handling. Armouring may not be done in the case of some cables.
(vi) Serving. In order to protect armouring from atmospheric conditions, a layer of fibrous
material (like jute) similar to bedding is provided over the armouring. This is known as serving.
It may not be out of place to mention here that bedding, armouring and serving are only applied to the cables for the protection of conductor insulation and to protect the metallic sheath from mechanical injury.

### 11.3 Insulating Materials for Cables

The satisfactory operation of a cable depends to a great extent upon the characteristics of insulation used. Therefore, the proper choice of insulating material for cables is of considerable importance. In general, the insulating materials used in cables should have the following properties :
(i) High insulation resistance to avoid leakage current.
(ii) High dielectric strength to avoid electrical breakdown of the cable.
(iii) High mechanical strength to withstand the mechanical handling of cables.
(iv) Non-hygroscopic i.e., it should not absorb moisture from air or soil. The moisture tends to decrease the insulation resistance and hastens the breakdown of the cable. In case the insulating material is hygroscopic, it must be enclosed in a waterproof covering like lead sheath.
(v) Non-inflammable.
(vi) Low cost so as to make the underground system a viable proposition.
(vii) Unaffected by acids and alkalies to avoid any chemical action.

No one insulating material possesses all the above mentioned properties. Therefore, the type of insulating material to be used depends upon the purpose for which the cable is required and the quality of insulation to be aimed at. The principal insulating materials used in cables are rubber, vulcanised India rubber,
 impregnated paper, varnished cambric and polyvinyl chloride.

1. Rubber. Rubber may be obtained from milky sap of tropical trees or it may be produced from oil products. It has relative permittivity varying between 2 and 3, dielectric strength is about $30 \mathrm{kV} / \mathrm{mm}$ and resistivity of insulation is $10^{17} \Omega \mathrm{~cm}$. Although pure rubber has reasonably high insulating properties, it suffers form some major drawbacks viz., readily absorbs moisture, maximum safe temperature is low (about $38^{\circ} \mathrm{C}$ ), soft and liable to damage due to rough handling and ages when exposed to light. Therefore, pure rubber cannot be used as an insulating material.
2. Vulcanised India Rubber (V.I.R.). It is prepared by mixing pure rubber with mineral matter such as zine oxide, red lead etc., and 3 to $5 \%$ of sulphur. The compound so formed is rolled into thin sheets and cut into strips. The rubber compound is then applied to the conductor and is heated to a temperature of about $150^{\circ} \mathrm{C}$. The whole process is called vulcanisation and the product obtained is known as vulcanised India rubber.

Vulcanised India rubber has greater mechanical strength, durability and wear resistant property than pure rubber. Its main drawback is that sulphur reacts very quickly with copper and for this reason, cables using VIR insulation have tinned copper conductor. The VIR insulation is generally used for low and moderate voltage cables.
3. Impregnated paper. It consists of chemically pulped paper made from wood chippings and impregnated with some compound such as paraffinic or napthenic material. This type of insulation has almost superseded the rubber insulation. It is because it has the advantages of low cost, low capacitance, high dielectric strength and high insulation resistance. The only disadvantage is that paper is hygroscopic and even if it is impregnated with suitable compound, it absorbs moisture and thus lowers the insulation resistance of the cable. For this reason, paper insulated cables are always
provided with some protective covering and are never left unsealed. If it is required to be left unused on the site during laying, its ends are temporarily covered with wax or tar.

Since the paper insulated cables have the tendency to absorb moisture, they are used where the cable route has a *few joints. For instance, they can be profitably used for distribution at low voltages in congested areas where the joints are generally provided only at the terminal apparatus. However, for smaller installations, where the lenghts are small and joints are required at a number of places, $V I R$ cables will be cheaper and durable than paper insulated cables.
4. Varnished cambric. It is a cotton cloth impregnated and coated with varnish. This type of insulation is also known as empire tape. The cambric is lapped on to the conductor in the form of a tape and its surfaces are coated with petroleum jelly compound to allow for the sliding of one turn over another as the cable is bent. As the varnished cambric is hygroscopic, therefore, such cables are always provided with metallic sheath. Its dielectric strength is about $4 \mathrm{kV} / \mathrm{mm}$ and permittivity is 2.5 to 3.8.
5. Polyvinyl chloride ( $P V C$ ). This insulating material is a synthetic compound. It is obtained from the polymerisation of acetylene and is in the form of white powder. For obtaining this material as a cable insulation, it is compounded with certain materials known as plasticizers which are liquids with high boiling point. The plasticizer forms a gell and renders the material plastic over the desired range of temperature.

Polyvinyl chloride has high insulation resistance, good dielectric strength and mechanical toughness over a wide range of temperatures. It is inert to oxygen and almost inert to many alkalies and acids. Therefore, this type of insulation is preferred over VIR in extreme enviormental conditions such as in cement factory or chemical factory. As the mechanical properties (i.e., elasticity etc.) of $P V C$ are not so good as those of rubber, therefore, $P V C$ insulated cables are generally used for low and medium domestic lights and power installations.

### 11.4 Classification of Cables

Cables for underground service may be classified in two ways according to (i) the type of insulating material used in their manufacture (ii) the voltage for which they are manufactured. However, the latter method of classification is generally preferred, according to which cables can be divided into the following groups :
(i) Low-tension (L.T.) cables - upto 1000 V
(ii) High-tension (H.T.) cables - upto $11,000 \mathrm{~V}$
(iii) Super-tension (S.T.) cables - from 22 kV to 33 kV
(iv) Extra high-tension (E.H.T.) cables - from 33 kV to 66 kV
(v) Extra super voltage cables - beyond 132 kV

A cable may have one or more than one core depending upon the type of service for which it is intended. It may be (i) single-core (ii) two-core (iii) three-core (iv) four-core etc. For a 3-phase service, either 3-single-core cables or three-core cable can be used depending upon the operating voltage and load demand.

Fig. 11.2 shows the constructional details of a single-core low tension cable. The cable has ordinary construction because the stresses developed in the cable for low voltages (upto Stranded 6600 V ) are generally small. It consists of one circular core of conductor tinned stranded copper (or aluminium) insulated by layers of


Fig. 11.2

[^0]impregnated paper. The insulation is surrounded by a lead sheath which prevents the entry of moisture into the inner parts. In order to protect the lead sheath from corrosion, an overall serving of compounded fibrous material (jute etc.) is provided. Single-core cables are not usually armoured in order to avoid excessive sheath losses. The principal advantages of single-core cables are simple construction and availability of larger copper section.

### 11.5 Cables for 3-Phase Service

In practice, underground cables are generally required to deliver 3-phase power. For the purpose, either three-core cable or *three single core cables may be used. For voltages upto $66 \mathrm{kV}, 3$-core cable (i.e., multi-core construction) is preferred due to economic reasons. However, for voltages beyond 66 kV , 3-core-cables become too large and unwieldy and, therefore, single-core cables are used. The following types of cables are generally used for 3-phase service :

1. Belted cables - upto 11 kV
2. Screened cables - from 22 kV to 66 kV
3. Pressure cables - beyond 66 kV .
4. Belted cables. These cables are used for voltages upto 11 kV but in extraordinary cases, their use may be extended upto 22 kV . Fig. 11.3 shows the constructional details of a 3 -core belted cable. The cores are insulated from each other by layers of impregnated paper. Another layer of impregnated paper tape, called paper belt is wound round the grouped insulated cores. The gap between the insulated cores is filled with fibrous insulating material (jute etc.) so as to give circular cross-section to the cable. The cores are generally stranded and may be of noncircular shape to make better use of available space. The belt is covered with lead sheath to protect the cable against ingress of moisture and mechanical injury. The lead sheath is covered with one or more layers of armouring with an outer serving (not shown in the fig-


Fig. 11.3 ure).

The belted type construction is suitable only for low and medium voltages as the electrostatic stresses developed in the cables for these voltages are more or less radial i.e., across the insulation. However, for high voltages (beyond 22 kV ), the tangential stresses also become important. These stresses act along the layers of paper insulation. As the insulation resistance of paper is quite small along the layers, therefore, tangential stresses set up **leakage current along the layers of paper insulation. The leakage current causes local heating, resulting in the risk of breakdown of insulation at any moment. In order to overcome this difficulty, screened cables are used where leakage currents are conducted to earth through metallic screens.
2. Screened cables. These cables are meant for use upto 33 kV , but in particular cases their use may be extended to operating voltages upto 66 kV . Two principal types of screened cables are $\mathrm{H}-$ type cables and S.L. type cables.
(i) H-type cables. This type of cable was first designed by H. Hochstadter and hence the name. Fig. 11.4 shows the constructional details of a typical 3-core, $H$-type cable. Each core is insulated by layers of impregnated paper. The insulation on each core is covered with a metallic screen which usually consists of a perforated aluminium foil. The cores are laid in such a way that metallic screens

[^1]make contact with one another. An additional conducting belt (copper woven fabric tape) is wrapped round the three cores. The cable has no insulating belt but lead sheath, bedding, armouring and serving follow as usual. It is easy to see that each core screen is in electrical contact with the conducting belt and the lead sheath. As all the four screens ( 3 core screens and one conducting belt) and the lead sheath are at $\dagger$ earth potential, therefore, the electrical stresses are purely radial and consequently dielectric losses


Fig. 11.4 are reduced.

Two principal advantages are claimed for $H$-type cables. Firstly, the perforations in the metallic screens assist in the complete impregnation of the cable with the compound and thus the possibility of air pockets or voids (vacuous spaces) in the dielectric is eliminated. The voids if present tend to reduce the breakdown strength of the cable and may cause considerable damage to the paper insulation. Secondly, the metallic screens increase the heat dissipating power of the cable.


H-Type Cables
(ii) S.L. type cables. Fig. 11.5 shows the constructional details of a 3-core *S.L. (separate lead) type cable. It is basically $H$-type cable but the screen round each core insulation is covered by its own lead sheath. There is no overall lead sheath but only armouring and serving are provided. The S.L. type cables have two main advantages over $H$-type cables. Firstly, the separate sheaths minimise the possibility of core-to-core breakdown. Secondly, bending of cables becomes easy due to the elimination of overall lead sheath. However, the disadvantage is that the three lead sheaths of S.L. cable are much thinner than the single sheath of $H$-cable and, therefore, call for greater care in manufacture.


Fig. 11.5

Limitations of solid type cables. All the cables of above construction are referred to as solid type cables because solid insulation is used and no gas or oil circulates in the cable sheath. The voltage limit for solid type cables is 66 kV due to the following reasons:
(a) As a solid cable carries the load, its conductor temperature increases and the cable com-

[^2]pound (i.e., insulating compound over paper) expands. This action stretches the lead sheath which may be damaged.
(b) When the load on the cable decreases, the conductor cools and a partial vacuum is formed within the cable sheath. If the pinholes are present in the lead sheath, moist air may be drawn into the cable. The moisture reduces the dielectric strength of insulation and may eventually cause the breakdown of the cable.
(c) In practice, $\dagger$ voids are always present in the insulation of a cable. Modern techniques of manufacturing have resulted in void free cables. However, under operating conditions, the voids are formed as a result of the differential expansion and contraction of the sheath and impregnated compound. The breakdown strength of voids is considerably less than that of the insulation. If the void is small enough, the electrostatic stress across it may cause its breakdown. The voids nearest to the conductor are the first to break down, the chemical and thermal effects of ionisation causing permanent damage to the paper insulation.
3. Pressure cables For voltages beyond 66 kV , solid type cables are unreliable because there is a danger of breakdown of insulation due to the presence of voids. When the operating voltages are greater than 66 kV , pressure cables are used. In such cables, voids are eliminated by increasing the pressure of compound and for this reason they are called pressure cables. Two types of pressure cables viz oil-filled cables and gas pressure cables are commonly used.
(i) Oil-filled cables. In such types of cables, channels or ducts are provided in the cable for oil circulation. The oil under pressure (it is the same oil used for impregnation) is kept constantly supplied to the channel by means of external reservoirs placed at suitable distances (say 500 m ) along the route of the cable. Oil under pressure compresses the layers of paper insulation and is forced into any voids that may have formed between the layers. Due to the elimination of voids, oil-filled cables can be used for higher voltages, the range being from 66 kV upto 230 kV . Oil-filled cables are of three types viz., single-core conductor channel, single-core sheath channel and three-core filler-space channels.

Fig. 11.6 shows the constructional details of a single-core conductor channel, oil filled cable. The oil channel is formed at the centre by stranding the conductor wire around a hollow cylindrical steel spiral tape. The oil under pressure is supplied to the channel by means of external reservoir. As the channel is made of spiral steel tape, it allows the oil to percolate between copper strands to the wrapped insulation. The oil pressure compresses the layers of paper insulation and prevents the possibility of void formation. The system is so designed that when the oil gets expanded due to increase in cable temperature, the extra oil collects in the reservoir. However, when the cable temperature falls during light load conditions, the oil from the reservoir flows to the channel. The disadvantage of this type of cable is that the channel is at the middle of the cable and is at full voltage w.r.t. earth, so that a very complicated system of joints is necessary.

Fig. 11.7 shows the constructional details of a singlecore sheath channel oil-filled cable. In this type of cable, the conductor is solid similar to that of solid cable and is paper insulated. However, oil ducts are provided in the


Fig. 11.6 Single-core conductor channel, oil-filled cable metallic sheath as shown. In the 3-core oil-filler cable shown in Fig. 11.8, the oil ducts are located in the filler spaces. These channels are composed of perforated metal-ribbon tubing and are at earth potential.

[^3]

Fig. 11.7


Fig. 11.8

The oil-filled cables have three principal advantages. Firstly, formation of voids and ionisation are avoided. Secondly, allowable temperature range and dielectric strength are increased. Thirdly, if there is leakage, the defect in the lead sheath is at once indicated and the possibility of earth faults is decreased. However, their major disadvantages are the high initial cost and complicated system of laying.
(ii) Gas pressure cables. The voltage required to set up ionisation inside a void increases as the pressure is increased. Therefore, if ordinary cable is subjected to a sufficiently high pressure, the ionisation can be altogether eliminated. At the same time, the increased pressure produces radial compression which tends to close any voids. This is the underlying principle of gas pressure cables.

Fig. 11.9 shows the section of external pressure cable designed by Hochstadter, Vogal and Bowden. The construction of the cable is similar to that of an ordinary solid type except that it is of triangular shape and thickness of lead sheath is $75 \%$ that of solid cable. The triangular section reduces the weight and gives low thermal resistance but the main reason for triangular shape is that the lead sheath acts as a pressure membrane. The sheath is protected by a thin metal tape. The cable is laid in a gas-tight steel pipe. The pipe is filled with dry nitrogen gas at 12 to 15 atmospheres. The gas pressure produces radial compression and closes the voids that may have formed between the layers of paper insulation. Such cables can carry more load current and operate at higher voltages than a normal cable. Moreover,


Fig. 11.9 maintenance cost is small and the nitrogen gas helps in quenching any flame. However, it has the disadvantage that the overall cost is very high.

### 11.6 Laying of Underground Cables

The reliability of underground cable network depends to a considerable extent upon the proper laying and attachment of fittings i.e., cable end boxes, joints, branch connectors etc. There are three main methods of laying underground cables viz., direct laying, draw-in system and the solid system.

1. Direct laying. This method of laying underground cables is simple and cheap and is much favoured in modern practice. In this method, a trench of about 1.5 metres deep and 45 cm wide is dug. The trench is covered with a layer of fine sand (of about 10 cm thickness) and the cable is laid over this sand bed. The sand prevents the entry of moisture from the ground and thus protects the cable from decay. After the cable has been laid in the trench, it is covered with another layer of sand of about 10 cm thickness.


Fig. 11.10

The trench is then covered with bricks and other materials in order to protect the cable from mechanical injury. When more than one cable is to be laid in the same trench, a horizontal or vertical interaxial spacing of atleast 30 cm is provided in order to reduce the effect of mutual heating and also to ensure that a fault occurring on one cable does not damage the adjacent cable. Cables to be laid in this way must have serving of bituminised paper and hessian tape so as to provide protection against corrosion and electorlysis.

## Advantages

(i) It is a simple and less costly method.
(ii) It gives the best conditions for dissipating the heat generated in the cables.
(iii) It is a clean and safe method as the cable is invisible and free from external disturbances.

## Disadvantages

(i) The extension of load is possible only by a completely new excavation which may cost as much as the original work.
(ii) The alterations in the cable netwok cannot be made easily.
(iii) The maintenance cost is very high.
(iv) Localisation of fault is difficult.
(v) It cannot be used in congested areas where excavation is expensive and inconvenient.

This method of laying cables is used in open areas where excavation can be done conveniently and at low cost.
2. Draw-in system. In this method, conduit or duct of glazed stone or cast iron or concrete are laid in the ground with manholes at suitable positions along the cable route. The cables are then pulled into position from manholes. Fig. 11.11 shows section through four-way underground duct line. Three of the ducts carry transmission cables and the fourth duct carries relay protection connection, pilot wires. Care must be taken that where the duct line changes direction ; depths, dips and offsets be made with a very long radius or it will be difficult to pull a large cable between the manholes. The distance between the manholes should not be too long so as to simplify the pull-


Fig. 11.11 ing in of the cables. The cables to be laid in this way need not be armoured but must be provided with serving of hessian and jute in order to protect them when being pulled into the ducts.

## Advantages

(i) Repairs, alterations or additions to the cable network can be made without opening the ground.
(ii) As the cables are not armoured, therefore, joints become simpler and maintenance cost is reduced considerably.
(iii) There are very less chances of fault occurrence due to strong mechanical protection provided by the system.

## Disadvantages

(i) The initial cost is very high.
(ii) The current carrying capacity of the cables is reduced due to the close grouping of cables and unfavourable conditions for dissipation of heat.
This method of cable laying is suitable for congested areas where excavation is expensive and inconvenient, for once the conduits have been laid, repairs or alterations can be made without open-
ing the ground. This method is generally used for short length cable routes such as in workshops, road crossings where frequent digging is costlier or impossible.
3. Solid system. In this method of laying, the cable is laid in open pipes or troughs dug out in earth along the cable route. The troughing is of cast iron, stoneware, asphalt or treated wood. After the cable is laid in position, the troughing is filled with a bituminous or asphaltic compound and covered over. Cables laid in this manner are usually plain lead covered because troughing affords good mechanical protection.

## Disadvantages

(i) It is more expensive than direct laid system.
(ii) It requires skilled labour and favourable weather conditions.
(iii) Due to poor heat dissipation facilities, the current carrying capacity of the cable is reduced.

In view of these disadvantages, this method of laying underground cables is rarely used now-adays.

### 11.7 Insulation Resistance of a Single-Core Cable

The cable conductor is provided with a suitable thickness of insulating material in order to prevent leakage current. The path for leakage current is radial through the insulation. The opposition offered by insulation to leakage current is known as insulation resistance of the cable. For satisfactory operation, the insulation resistance of the cable should be very high.

Consider a single-core cable of conductor radius $r_{1}$ and internal sheath radius $r_{2}$ as shown in Fig. 11.12. Let $l$ be the length of the cable and $\rho$ be the resistivity of the insulation.

Consider a very small layer of insulation of thickness $d x$ at a radius $x$.


Fig. 11.12 The length through which leakage current tends to flow is $d x$ and the area of X-section offered to this flow is $2 \pi x l$.
$\therefore$ Insulation resistance of considered layer

$$
=\rho \frac{d x}{2 \pi x l}
$$

Insulation resistance of the whole cable is

$$
\begin{aligned}
R & =\int_{r_{1}}^{r_{2}} \rho \frac{d x}{2 \pi x l}=\frac{\rho}{2 \pi l} \int_{r_{1}}^{r_{2}} \frac{1}{x} d x \\
\therefore \quad R & =\frac{\rho}{2 \pi l} \log _{e} \frac{r_{2}}{r_{1}}
\end{aligned}
$$

This shows that insulation resistance of a cable is inversely proportional to its length. In other words, if the cable length increases, its insulation resistance decreases and vice-versa.

Example 11.1. A single-core cable has a conductor diameter of 1 cm and insulation thickness of 0.4 cm . If the specific resistance of insulation is $5 \times 10^{14} \Omega-\mathrm{cm}$, calculate the insulation resistance for a 2 km length of the cable.

## Solution

Conductor radius, $\quad r_{1}=1 / 2=0.5 \mathrm{~cm}$
Length of cable, $\quad l=2 \mathrm{~km}=2000 \mathrm{~m}$
Resistivity of insulation, $\quad \rho=5 \times 10^{14} \Omega-\mathrm{cm}=5 \times 10^{12} \Omega-\mathrm{m}$
Internal sheath radius, $\quad r_{2}=0.5+0.4=0.9 \mathrm{~cm}$
$\therefore \quad$ Insulation resistance of cable is

$$
\begin{aligned}
R & =\frac{\rho}{2 \pi l} \log _{e} \frac{r_{2}}{r_{1}}=\frac{5 \times 10^{12}}{2 \pi \times 2000} \log _{e} \frac{0 \cdot 9}{0 \cdot 5} \\
& =0.234 \times 10^{9} \Omega=234 \mathrm{M} \Omega
\end{aligned}
$$

Example 11.2. The insulation resistance of a single-core cable is $495 \mathrm{M} \Omega$ per km . If the core diameter is 2.5 cm and resistivity of insulation is $4.5 \times 10^{14} \Omega-\mathrm{cm}$, find the insulation thickness.

## Solution.

Length of cable,
$l=1 \mathrm{~km}=1000 \mathrm{~m}$
Cable insulation resistance, $R=495 \mathrm{M} \Omega=495 \times 10^{6} \Omega$
Conductor radius, $\quad r_{1}=2.5 / 2=1.25 \mathrm{~cm}$
Resistivity of insulation, $\quad \rho=4.5 \times 10^{14} \Omega-\mathrm{cm}=4.5 \times 10^{12} \Omega \mathrm{~m}$
Let $r_{2} \mathrm{~cm}$ be the internal sheath radius.
Now

$$
R=\frac{\rho}{2 \pi l} \log _{e} \frac{r_{2}}{r_{1}}
$$

or

$$
\log _{e} \frac{r_{2}}{r_{1}}=\frac{2 \pi l R}{\rho}=\frac{2 \pi \times 1000 \times 495 \times 10^{6}}{4.5 \times 10^{12}}=0.69
$$

or
$2.3 \log _{10} r_{2} / r_{1}=0.69$
$r_{2} / r_{1}=$ Antilog 0.69/2.3 $=2$
$r_{2}=2 r_{1}=2 \times 1.25=2.5 \mathrm{~cm}$
or
$\therefore \quad$ Insulation thickness $=r_{2}-r_{1}=2.5-1.25=\mathbf{1 . 2 5} \mathbf{c m}$
Example 11.3. A single core cable 5 km long has an insulation resistance of $0.4 \mathrm{M} \Omega$. The core diameter is 20 mm and the diameter of the cable over the insulation is 50 mm . Calculate the resistivity of the insulating material.

## Solution.

Length of cable,

$$
l=5 \mathrm{~km}=5000 \mathrm{~m}
$$

Cable insulation resistance, $R=0.4 \mathrm{M} \Omega=0.4 \times 10^{6} \Omega$
Conductor radius,

$$
r_{1}=20 / 2=10 \mathrm{~mm}
$$

Internal sheath radius, $\quad r_{2}=50 / 2=25 \mathrm{~mm}$
$\therefore \quad$ Insulation resistance of the cables is
or

$$
R=\frac{\rho}{2 \pi l} \log _{e} \frac{r_{2}}{r_{1}}
$$

$$
0.4 \times 10^{6}=\frac{\rho}{2 \pi \times 5000} \times \log _{e} \frac{25}{10}
$$

$\therefore \quad \rho=13.72 \times 10^{9} \Omega \mathrm{~m}$

## TUTORIAL PROBLEMS

1. A single-core cable has a conductor diameter of 2.5 cm and insulation thickness of 1.2 cm . If the specific resistance of insulation is $4.5 \times 10^{14} \Omega \mathrm{~cm}$, calculate the insulation resistance per kilometre length of the cable.
[ $305.5 \mathrm{M} \Omega$ ]
2. A single core cable 3 km long has an insulation resistance of $1820 \mathrm{M} \Omega$. If the conductor diameter is 1.5 cm and sheath diameter is 5 cm , calculate the resistivity of the dielectric in the cable.

$$
\left[28.57 \times 10^{12} \Omega \mathrm{~m}\right]
$$

3. Determine the insulation resistance of a single-core cable of length 3 km and having conductor radius 12.5 mm , insulation thickness 10 mm and specific resistance of insulation of $5 \times 10^{12} \Omega \mathrm{~m}$. [ $156 \mathrm{M} \Omega$ ]

### 11.8 Capacitance of a Single-Core Cable

A single-core cable can be considered to be equivalent to two long co-axial cylinders. The conductor (or core) of the cable is the inner cylinder while the outer cylinder is represented by lead sheath which is at earth potential. Consider a single core cable with conductor diameter $d$ and inner sheath diameter $D$ (Fig. 11.13). Let the charge per metre axial length of the cable be $Q$ coulombs and $\varepsilon$ be the permittivity of the insulation material between core and lead sheath. Obviously ${ }^{*} \varepsilon=\varepsilon_{0} \varepsilon_{r}$ where $\varepsilon_{r}$ is the relative permittivity of the insulation.

Consider a cylinder of radius $x$ metres and axial length 1 metre. The


Fig. 11.13 surface area of this cylinder is $=2 \pi x \times 1=2 \pi x \mathrm{~m}^{2}$
$\therefore \quad$ Electric flux density at any point $P$ on the considered cylinder is

$$
D_{x}=\frac{Q}{2 \pi x} \mathrm{C} / \mathrm{m}^{2}
$$

Electric intensity at point $P, E_{x}=\frac{D_{x}}{\varepsilon}=\frac{Q}{2 \pi x \varepsilon}=\frac{Q}{2 \pi x \varepsilon_{0} \varepsilon_{r}}$ volts $/ \mathrm{m}$
The work done in moving a unit positive charge from point $P$ through a distance $d x$ in the direction of electric field is $E_{x} d x$. Hence, the work done in moving a unit positive charge from conductor to sheath, which is the potential difference $V$ between conductor and sheath, is given by :

$$
V=\int_{d / 2}^{D / 2} E_{x} d x=\int_{d / 2}^{D / 2} \frac{Q}{2 \pi x \varepsilon_{0} \varepsilon_{r}} d x=\frac{Q}{2 \pi \varepsilon_{0} \varepsilon_{r}} \log _{e} \frac{D}{d}
$$

Capacitance of the cable is

$$
\begin{aligned}
C & =\frac{Q}{V}=\frac{Q}{\frac{Q}{2 \pi \varepsilon_{0} \varepsilon_{r}} \log _{e} \frac{D}{d}} \mathrm{~F} / \mathrm{m} \\
& =\frac{2 \pi \varepsilon_{o} \varepsilon_{r}}{\log _{e}(D / d)} \mathrm{F} / \mathrm{m} \\
& =\frac{2 \pi \times 8 \cdot 854 \times 10^{-12} \times \varepsilon_{r}}{2 \cdot 303 \log _{10}(D / d)} \mathrm{F} / \mathrm{m} \\
& =\frac{\varepsilon_{r}}{41 \cdot 4 \log _{10}(D / d)} \times 10^{-9} \mathrm{~F} / \mathrm{m}
\end{aligned}
$$

If the cable has a length of $l$ metres, then capacitance of the cable is

$$
C=\frac{\varepsilon_{r} l}{41 \cdot 4 \log _{10} \frac{D}{d}} \times 10^{-9} \mathrm{~F}
$$

Example 11.4. A single core cable has a conductor diameter of 1 cm and internal sheath diameter of 1.8 cm . If impregnated paper of relative permittivity 4 is used as the insulation, calculate the capacitance for 1 km length of the cable.

## Solution.

Capacitance of cable,

$$
C=\frac{\varepsilon_{r} l}{41 \cdot 4 \log _{10}(D / d)} \times 10^{-9} \mathrm{~F}
$$

[^4]Here | $\varepsilon_{r}=4 ;$ |  | $l=1000 \mathrm{~m}$ |
| :--- | :--- | :--- |
| $D$ | $=1.8 \mathrm{~cm} ;$ |  |
|  | $d=1 \mathrm{~cm}$ |  |

Substituting these values in the above expression, we get,

$$
C=\frac{4 \times 1000}{41 \cdot 4 \log _{10}(1 \cdot 8 / 1)} \times 10^{-9} \mathrm{~F}=0.378 \times 10^{-6} \mathrm{~F}=0.378 \mu \mathrm{~F}
$$

Example 11.5. Calculate the capacitance and charging current of a single core cable used on a 3-phase, 66 kV system. The cable is 1 km long having a core diameter of 10 cm and an impregnated paper insulation of thickness 7 cm . The relative permittivity of the insulation may be taken as 4 and the supply at 50 Hz .

## Solution.

Capacitance of cable, $\quad C=\frac{\varepsilon_{r} l}{41 \cdot 4 \log _{10}(D / d)} \times 10^{-9} \mathrm{~F}$
Here, $\quad \varepsilon_{r}=4 ; \quad l=1000 \mathrm{~m}$

$$
d=10 \mathrm{~cm} ; \quad D=10+2 \times 7=24 \mathrm{~cm}
$$

Substituting these values in the above expression,

$$
C=\frac{4 \times 1000}{41.4 \times \log _{10}(24 / 10)} \times 10^{-9} \mathrm{~F}=0.254 \times 10^{-6} \mathrm{~F}=0.254 \mu \mathrm{~F}
$$

Voltage between core and sheath is

$$
\begin{aligned}
V_{p h} & =66 / \sqrt{3}=38 \cdot 1 \mathrm{kV}=38.1 \times 10^{3} \mathrm{~V} \\
\text { Charging current } & =V_{p h} / X_{C}=2 \pi f C V_{p h} \\
& =2 \pi \times 50 \times 0.254 \times 10^{-6} \times 38.1 \times 10^{3} \mathrm{~A}=3.04 \mathrm{~A}
\end{aligned}
$$

Example 11.6. A $33 \mathrm{kV}, 50 \mathrm{~Hz}$, 3-phase underground cable, 4 km long uses three single core cables. Each of the conductor has a diameter of 2.5 cm and the radial thickness of insulation is 0.5 cm. Determine (i) capacitance of the cable/phase (ii) charging current/phase (iii) total charging $k V A R$. The relative permittivity of insulation is 3 .

## Solution.

(i) Capacitance of cable/phase, $\quad C=\frac{\varepsilon_{r} l}{41 \cdot 4 \log _{10}(D / d)} \times 10^{-9} \mathrm{~F}$

Here $\quad \varepsilon_{r}=3 \quad ; \quad l=4 \mathrm{~km}=4000 \mathrm{~m}$

$$
d=2.5 \mathrm{~cm} \quad ; \quad D=2.5+2 \times 0.5=3.5 \mathrm{~cm}
$$

Putting these values in the above expression, we get,
(ii) $\quad$ Voltage/phase, $V_{p h}=\frac{33 \times 10^{3}}{\sqrt{3}}=19.05 \times 10^{3} \mathrm{~V}$

Charging current/phase, $I_{C}=\frac{V_{p h}}{X_{C}}=2 \pi f C V_{p h}$

$$
=2 \pi \times 50 \times 1984 \times 10^{-9} \times 19.05 \times 10^{3}=11.87 \mathrm{~A}
$$

(iii) Total charging kVAR $=3 V_{p h} I_{C}=3 \times 19.05 \times 10^{3} \times 11.87=\mathbf{6 7 8 . 5} \times \mathbf{1 0}^{\mathbf{3}} \mathrm{kVAR}$

## TUTORIAL PROBLEMS

1. A single core cable has a conductor diameter of 1 cm and internal sheath diameter of 1.8 cm . If the impregnated paper of relative permittivity 3 is used as insulation, calculate the capacitance for 1 km length of the cable.
[0.282 $\mu \mathrm{F}]$
2. Calculate the capacitance and charging current of a single core cable used on 3-phase, 66 kV system. The cable is 1 km long having a core diameter of 15 cm and impregnated paper insulation of thickness 22.5 cm . The relative permittivity of the insulation may be taken as 3.5 and supply at 50 Hz .
[0.144 $\mu \mathrm{F} ; 1.74 \mathrm{~A}]$
3. An $11 \mathrm{kV}, 50 \mathrm{~Hz}$, single phase cable 2.5 km long, has a diameter of 20 mm and internal sheath radius of 15 mm . If the dielectric has a relative permittivity of 2.4 , determine (i) capacitance (ii) charging current (iii) total charging kVAR.
[(i) $0.822 \mu \mathrm{~F}$ (ii) 2.84 A (iii) 31.24 kVAR$]$

### 11.9 Dielectric Stress in a Single-Core Cable

Under operating conditions, the insulation of a cable is subjected to electrostatic forces. This is known as dielectric stress. The dielectric stress at any point in a cable is infact the potential gradient (or *electric intensity) at that point.

Consider a single core cable with core diameter $d$ and internal sheath diameter $D$. As proved in Art 11.8, the electric intensity at a point $x$ metres from the centre of the cable is

$$
E_{x}=\frac{Q}{2 \pi \varepsilon_{o} \varepsilon_{r} x} \text { volts } / \mathrm{m}
$$

By definition, electric intensity is equal to potential gradient. Therefore, potential gradient $g$ at a point $x$ metres from the centre of cable is

$$
g=E_{x}
$$

or

$$
\begin{equation*}
g=\frac{Q}{2 \pi \varepsilon_{o} \varepsilon_{r} x} \text { volts } / \mathrm{m} \tag{i}
\end{equation*}
$$

As proved in Art. 11.8, potential difference $V$ between conductor and sheath is


Fig. 11.14
or

$$
\begin{align*}
V & =\frac{Q}{2 \pi \varepsilon_{o} \varepsilon_{r}} \log _{e} \frac{D}{d} \text { volts } \\
Q & =\frac{2 \pi \varepsilon_{o} \varepsilon_{r} V}{\log _{e} \frac{D}{d}} \tag{ii}
\end{align*}
$$

Substituting the value of $Q$ from exp. (ii) in exp. (i), we get,

$$
\begin{equation*}
g=\frac{2 \pi \varepsilon_{o} \varepsilon_{r} V}{\frac{\log _{e} D / d}{2 \pi \varepsilon_{o} \varepsilon_{r} x}}=\frac{V}{x \log _{e} \frac{D}{d}} \text { volts/m } \tag{iii}
\end{equation*}
$$

It is clear from exp. (iii) that potential gradient varies inversely as the distance $x$. Therefore, potential gradient will be maximum when $x$ is minimum i.e., when $x=d / 2$ or at the surface of the conductor. On the other hand, potential gradient will be minimum at $x=D / 2$ or at sheath surface.
$\therefore \quad$ Maximum potential gradient is

$$
g_{\max }=\frac{2 V}{d \log _{e} \frac{D}{d}} \text { volts } / \mathrm{m}
$$

[Putting $x=d / 2$ in $\exp$. (iii)]

Minimum potential gradient is

$$
g_{\text {min }}=\frac{2 V}{D \log _{e} \frac{D}{d}} \text { volts } / \mathrm{m}
$$

[Putting $x=D / 2$ in exp. (iii)]

[^5]$$
\therefore \quad \frac{g_{\text {max }}}{g_{\text {min }}}=\frac{\frac{2 V}{d \log _{e} D / d}}{\frac{2 V}{D \log _{e} D / d}}=\frac{D}{d}
$$

The variation of stress in the dielectric is shown in Fig. 11.14. It is clear that dielectric stress is maximum at the conductor surface and its value goes on decreasing as we move away from the conductor. It may be noted that maximum stress is an important consideration in the design of a cable. For instance, if a cable is to be operated at such a voltage that *maximum stress is $5 \mathrm{kV} / \mathrm{mm}$, then the insulation used must have a dielectric strength of atleast $5 \mathrm{kV} / \mathrm{mm}$, otherwise breakdown of the cable will become inevitable.

Example 11.7. A 33 kV single core cable has a conductor diameter of 1 cm and a sheath of inside diameter 4 cm . Find the maximum and minimum stress in the insulation.

Solution.
The maximum stress occurs at the conductor surface and its value is given by;

$$
g_{\max }=\frac{2 V}{d \log _{e} \frac{D}{d}}
$$

Here,

$$
V=33 \mathrm{kV}(\mathrm{r} . \mathrm{m} . \mathrm{s}) ; d=1 \mathrm{~cm} ; D=4 \mathrm{~cm}
$$

Substituting the values in the above expression, we get,

$$
g_{\max }=\frac{2 \times 33}{1 \times \log _{e} 4} \mathrm{kV} \dagger / \mathrm{cm}=47.61 \mathrm{kV} / \mathrm{cm} \mathrm{r} . \mathrm{ms} .
$$

The minimum stress occurs at the sheath and its value is give by ;

$$
g_{\min }=\frac{2 V}{D \log _{e} \frac{D}{d}}=\frac{2 \times 33}{4 \times \log _{e} 4} \mathrm{kV} / \mathrm{cm}=11.9 \mathbf{k V} / \mathrm{cm} \mathrm{r} . \mathrm{m} . \mathrm{s}
$$

Alternatively;

$$
g_{\min }=\mathrm{g}_{\max } \times \frac{d}{D}=47.61 \times 1 / 4=11.9 \mathrm{kV} / \mathrm{cm} \text { r.m.s. }
$$

Example 11.8. The maximum and minimum stresses in the dielectric of a single core cable are $40 \mathrm{kV} / \mathrm{cm}$ (r.m.s.) and $10 \mathrm{kV} / \mathrm{cm}$ (r.m.s.) respectively. If the conductor diameter is 2 cm , find :
(i) thickness of insulation
(ii) operating voltage

## Solution.

Here,

$$
g_{\text {max }}=40 \mathrm{kV} / \mathrm{cm} ; \quad g_{\text {min }}=10 \mathrm{kV} / \mathrm{cm} ; d=2 \mathrm{~cm} ; D=?
$$

(i) As proved in Art. 11.9,
or

$$
\frac{g_{\max }}{g_{\min }}=\frac{D}{d}
$$

$$
D=\frac{g_{\max }}{g_{\min }} \times d=\frac{40}{10} \times 2=8 \mathrm{~cm}
$$

$\therefore \quad$ Insulation thickness $=\frac{D-d}{2}=\frac{8-2}{2}=3 \mathrm{~cm}$
(ii)

$$
g_{\max }=\frac{2 V}{d \log _{e} \frac{D}{d}}
$$

[^6]$$
\therefore \quad V=\frac{g_{\max } d \log _{e} \frac{D}{d}}{2}=\frac{40 \times 2 \log _{e} 4}{2} \mathrm{kV}=\mathbf{5 5 . 4 5} \mathrm{kV} \text { r.m.s. }
$$

Example 11.9. A single core cable for use on $11 \mathrm{kV}, 50 \mathrm{~Hz}$ system has conductor area of 0.645 $\mathrm{cm}^{2}$ and internal diameter of sheath is 2.18 cm . The permittivity of the dielectric used in the cable is 3.5. Find (i) the maximum electrostatic stress in the cable (ii) minimum electrostatic stress in the cable (iii) capacitance of the cable per km length (iv) charging current.

## Solution.

Area of cross-section of conductor, $a=0.645 \mathrm{~cm}^{2}$
Diameter of the conductor, $\quad d=\sqrt{\frac{4 a}{\pi}}=\sqrt{\frac{4 \times 0 \cdot 645}{\pi}}=0.906 \mathrm{~cm}$
Internal diameter of sheath, $\quad D=2.18 \mathrm{~cm}$
(i) Maximum electrostatic stress in the cable is

$$
g_{\max }=\frac{2 V}{d \log _{e} \frac{D}{d}}=\frac{2 \times 11}{0.906 \log _{e} \frac{2 \cdot 18}{0.906}} \mathrm{kV} / \mathrm{cm}=27.65 \mathrm{kV} / \mathrm{cm} \mathrm{r.m.s.}
$$

(ii) Minimum electrostatic stress in the cable is
(iii) Capacitance of cable, $\quad C=\frac{\varepsilon_{r} l}{41 \cdot 4 \log _{10} \frac{D}{d}} \times 10^{-9} \mathrm{~F}$

Here $\quad \varepsilon_{r}=3.5 ; l=1 \mathrm{~km}=1000 \mathrm{~m}$

$$
\therefore \quad C=\frac{3.5 \times 1000}{41.4 \log _{10} \frac{2 \cdot 18}{0.906}} \times 10^{-9}=0.22 \times 10^{-6} \mathbf{F}
$$

(iv) Charging current,

$$
I_{C}=\frac{V}{X_{C}}=2 \pi f C \quad V=2 \pi \times 50 \times 0.22 \times 10^{-6} \times 11000=0.76 \mathrm{~A}
$$

### 11.10 Most Economical Conductor Size in a Cable

It has already been shown that maximum stress in a cable occurs at the surface of the conductor. For safe working of the cable, dielectric strength of the insulation should be more than the maximum stress. Rewriting the expression for maximum stress, we get,

$$
\begin{equation*}
g_{\max }=\frac{2 V}{d \log _{e} \frac{D}{d}} \text { volts } / \mathrm{m} \tag{i}
\end{equation*}
$$

The values of working voltage $V$ and internal sheath diameter $D$ have to be kept fixed at certain values due to design considerations. This leaves conductor diameter $d$ to be the only variable in exp. (i). For given values of $V$ and $D$, the most economical conductor diameter will be one for which $g_{\max }$ has a minimum value. The value of $g_{\max }$ will be minimum when $d \log _{e} D / d$ is maximum i.e.
or
or $\quad \log _{e}(D / d)-1=0$
or $\quad \log _{e}(D / d)=1$
or

$$
(D / d)=e=2.718
$$

$\therefore$ Most economical conductor diameter is

$$
d=\frac{D}{2.718}
$$

and the value of $g_{\max }$ under this condition is

$$
g_{\max }=\frac{2 V}{d} \text { volts } / \mathrm{m}
$$

[Putting $\log _{e} D / d=1$ in exp. (i)]
For low and medium voltage cables, the value of conductor diameter arrived at by this method (i.e., $d=2 \mathrm{~V} / g_{\max }$ ) is often too small from the point of view of current density. Therefore, the conductor diameter of such cables is determined from the consideration of safe current density. For high voltage cables, designs based on this theory give a very high value of $d$, much too large from the point of view of current carrying capacity and it is, therefore, advantageous to increase the conductor diameter to this value. There are three ways of doing this without using excessive copper :
(i) Using aluminium instead of copper because for the same current, diameter of aluminium will be more than that of copper.
(ii) Using copper wires stranded round a central core of hemp.
(iii) Using a central lead tube instead of hemp.

Example 11.10. Find the most economical value of diameter of a single-core cable to be used on 50 kV , single-phase system. The maximum permissible stress in the dielectric is not to exceed 40 $\mathrm{kV} / \mathrm{cm}$.

Solution.
Peak value of cable voltage, $\quad V=50 \times \sqrt{2}=70.7 \mathrm{kV}$
Maximum permissible stress, $g_{\max }=40 \mathrm{kV} / \mathrm{cm} \quad$ (assumed peak)
$\therefore$ Most economical conductor diameter is

$$
d=\frac{2 \mathrm{~V}}{g_{\max }}=\frac{2 \times 70 \cdot 7}{40}=3.53 \mathrm{~cm}
$$

Example 11.11 Find the most economical size of a single-core cable working on a 132 kV , 3phase system, if a dielectric stress of $60 \mathrm{kV} / \mathrm{cm}$ can be allowed.

## Solution

$$
\text { Phase voltage of cable }=132 / \sqrt{3}=76 \cdot 21 \mathrm{kV}
$$

Peak value of phase voltage, $V=76.21 \times \sqrt{2}=107.78 \mathrm{kV}$
Max. permissible stress, $g_{\max }=60 \mathrm{kV} / \mathrm{cm}$
$\therefore$ Most economical conductor diameter is

$$
d=\frac{2 V}{g_{\max }}=\frac{2 \times 107.78}{60}=3.6 \mathrm{~cm}
$$

Internal diameter of sheath, $D=2.718 d=2.718 \times 3.6=9.78 \mathrm{~cm}$
Therefore, the cable should have a conductor diameter of 3.6 cm and internal sheath diameter of 9.78 cm .

### 11.11 Grading of Cables

The process of achieving uniform electrostatic stress in the dielectric of cables is known as grading of cables.

It has already been shown that electrostatic stress in a single core cable has a maximum value $\left(g_{\max }\right)$ at the conductor surface and goes on decreasing as we move towards the sheath. The maximum voltage that can be safely applied to a cable depends upon $g_{\max }$ i.e., electrostatic stress at the conductor surface. For safe working of a cable having homogeneous dielectric, the strength of di-
electric must be more than $g_{\max }$. If a dielectric of high strength is used for a cable, it is useful only near the conductor where stress is maximum. But as we move away from the conductor, the electrostatic stress decreases, so the dielectric will be unnecessarily overstrong.

The unequal stress distribution in a cable is undesirable for two reasons. Firstly, insulation of greater thickness is required which increases the cable size. Secondly, it may lead to the breakdown of insulation. In order to overcome above disadvantages, it is necessary to have a uniform stress distribution in cables. This can be achieved by distributing the stress in such a way that its value is increased in the outer layers of dielectric. This is known as grading of cables. The following are the two main methods of grading of cables :
(i) Capacitance grading (ii) Intersheath grading

### 11.12 Capacitance Grading

The process of achieving uniformity in the dielectric stress by using layers of different dielectrics is known as capacitance grading.

In capacitance grading, the homogeneous dielectric is replaced by a composite dielectric. The composite dielectric consists of various layers of different dielectrics in such a manner that relative permittivity $\varepsilon_{r}$ of any layer is inversely proportional to its distance from the centre. Under such conditions, the value of potential gradient at any point in the dieletric is *constant and is independent of its distance from the centre. In other words, the dielectric stress in the cable is same everywhere and the grading is ideal one. How ever, ideal grading requires the use of an infinite number of dielectrics which is an impossible task. In practice, two or three dielectrics are used in the decreasing order of permittivity ; the dielectric of highest permittivity being used near the core.

The capacitance grading can be explained beautifully by referring to Fig. 11.15. There are three dielectrics of outer diameter $d_{1}, d_{2}$ and $D$ and of relative permittivity $\varepsilon_{1}, \varepsilon_{2}$ and $\varepsilon_{3}$ respectively. If the permittivities are such that $\varepsilon_{1}>\varepsilon_{2}>\varepsilon_{3}$ and the three dielectrics are worked at the same maximum stress, then,


Fig. 11.15
or

$$
\begin{aligned}
\frac{1}{\varepsilon_{1} d} & =\frac{1}{\varepsilon_{2} d_{1}}=\frac{1 \dagger}{\varepsilon_{3} d_{2}} \\
\varepsilon_{1} d & =\varepsilon_{2} d_{1}=\varepsilon_{3} d_{2}
\end{aligned}
$$

Potential difference across the inner layer is

* As $\varepsilon_{r} \propto \frac{1}{x} \quad \therefore \quad \varepsilon_{r}=k / x$ where $k$ is a constant.

Potential gradient at a distance $x$ from the centre

$$
=\frac{Q}{2 \pi \varepsilon_{0} \varepsilon_{r} x}=\frac{Q}{2 \pi \varepsilon_{0}(k / x) x}=\frac{Q}{2 \pi \varepsilon_{0} k}=\text { Constant }
$$

This shows that if the condition $\varepsilon_{r} \propto 1 / x$ is fulfilled, potential gradient will be constant throughout the dielectric of the cable.

$$
\begin{aligned}
& \dagger \quad g_{1 \max }=\frac{Q}{\pi \varepsilon_{0} \varepsilon_{1} d} ; \quad g_{2 \max }=\frac{Q}{\pi \varepsilon_{0} \varepsilon_{2} d_{1}} ; \quad g_{3 \max }=\frac{Q}{\pi \varepsilon_{0} \varepsilon_{3} d_{2}} \\
& \text { If } g_{1 \max }=g_{2 \max }=g_{3 \max }=g_{\max }(\text { say }), \text { then, } \\
& \frac{1}{\varepsilon_{1} d}=\frac{1}{\varepsilon_{2} d_{1}}=\frac{1}{\varepsilon_{3} d_{2}}
\end{aligned}
$$

$$
\begin{aligned}
V_{1} & =\int_{d / 2}^{d_{1} / 2} g d x=\int_{d / 2}^{d_{1} / 2} \frac{Q}{2 \pi \varepsilon_{0} \varepsilon_{1} x} d x \\
& =\frac{Q}{2 \pi \varepsilon_{0} \varepsilon_{1}} \log _{e} \frac{d_{1}}{d}=\frac{g_{\max }}{2} d \log _{e} \frac{d_{1}}{d}\left[\because \frac{Q}{2 \pi \varepsilon_{0} \varepsilon_{1}}=\frac{* g_{\max }}{2} d\right]
\end{aligned}
$$

Similarly, potential across second layer $\left(V_{2}\right)$ and third layer $\left(V_{3}\right)$ is given by ;

$$
\begin{aligned}
& V_{2}=\frac{g_{\max }}{2} d_{1} \log _{e} \frac{d_{2}}{d_{1}} \\
& V_{3}=\frac{g_{\max }}{2} d_{2} \log _{e} \frac{D}{d_{2}}
\end{aligned}
$$

Total p.d. between core and earthed sheath is

$$
\begin{aligned}
V & =V_{1}+V_{2}+V_{3} \\
& =\frac{g_{\max }}{2}\left[d \log _{e} \frac{d_{1}}{d}+d_{1} \log _{e} \frac{d_{2}}{d_{1}}+d_{2} \log _{e} \frac{D}{d_{2}}\right]
\end{aligned}
$$

If the cable had homogeneous dielectric, then, for the same values of $d, D$ and $g_{\max }$, the permissible potential difference between core and earthed sheath would have been

$$
V^{\prime}=\frac{g_{\max }}{2} d \log _{e} \frac{D}{d}
$$

Obviously, $V>V^{\prime}$ i.e., for given dimensions of the cable, a graded cable can be worked at a greater potential than non-graded cable. Alternatively, for the same safe potential, the size of graded cable will be less than that of non-graded cable. The following points may be noted :
(i) As the permissible values of $g_{\max }$ are peak values, therefore, all the voltages in above expressions should be taken as peak values and not the r.m.s. values.
(ii) If the maximum stress in the three dielectrics is not the same, then,

$$
V=\frac{g_{1 \max }}{2} d \log _{e} \frac{d_{1}}{d}+\frac{g_{2 \max }}{2} d_{1} \log _{e} \frac{d_{2}}{d_{1}}+\frac{g_{3 \max }}{2} d_{2} \log _{e} \frac{D}{d_{2}}
$$

The principal disadvantage of this method is that there are a few high grade dielectrics of reasonable cost whose permittivities vary over the required range.

Example 11.12. A single-core lead sheathed cable is graded by using three dielectrics of relative permittivity 5, 4 and 3 respectively. The conductor diameter is 2 cm and overall diameter is 8 cm . If the three dielectrics are worked at the same maximum stress of $40 \mathrm{kV} / \mathrm{cm}$, find the safe working voltage of the cable.

What will be the value of safe working voltage for an ungraded cable, assuming the same conductor and overall diameter and the maximum dielectric stress?

## Solution.



Graded cable. As the maximum stress in the three dielectrics is the same,

$$
\therefore \quad \varepsilon_{1} d=\varepsilon_{2} d_{1}=\varepsilon_{3} d_{2}
$$

or $\quad 5 \times 2=4 \times d_{1}=3 \times d_{2}$

$$
\therefore \quad d_{1}=2.5 \mathrm{~cm} \text { and } d_{2}=3.34 \mathrm{~cm}
$$

* $\quad g_{\max }=\frac{Q}{\pi \varepsilon_{0} \varepsilon_{1} d} \quad \therefore \quad g_{\max } d=\frac{Q}{\pi \varepsilon_{0} \varepsilon_{1}} \quad$ or $\quad \frac{g_{\max }}{2} d=\frac{Q}{2 \pi \varepsilon_{0} \varepsilon_{1}}$

Permissible peak voltage for the cable

$$
\begin{aligned}
& =\frac{g_{\max }}{2}\left[d \log _{e} \frac{d_{1}}{d}+d_{1} \log _{e} \frac{d_{2}}{d_{1}}+d_{2} \log _{e} \frac{D}{d_{2}}\right] \\
& =\frac{40}{2}\left[2 \log _{e} \frac{2 \cdot 5}{2}+2 \cdot 5 \log _{e} \frac{3 \cdot 34}{2 \cdot 5}+3 \cdot 34 \log _{e} \frac{8}{3 \cdot 34}\right] \\
& =20[0.4462+0.7242+2 \cdot 92] \mathrm{kV} \\
& =20 \times 4.0904=81 \cdot 808 \mathrm{kV}
\end{aligned}
$$

$\therefore \quad$ Safe working voltage (r.m.s.) for cable

$$
=\frac{81 \cdot 808}{\sqrt{2}}=57 \cdot 84 \mathbf{k V}
$$

Ungraded cable. Permissible peak voltage for the cable

$$
=\frac{g_{\max }}{2} d \log _{e} \frac{D}{d}=\frac{40}{2} \times 2 \log _{e} \frac{8}{2} \mathrm{kV}=55.44 \mathrm{kV}
$$

$\therefore \quad$ Safe working voltage (r.m.s.) for the cable

$$
=\frac{55 \cdot 44}{\sqrt{2}}=39 \cdot 2 \mathrm{kV}
$$

This example shows the utility of grading the cable. Thus for the same conductor diameter (d) and the same overall dimension $(D)$, the graded cable can be operated at a voltage $(57.84-39.20)=$ 18.64 kV (r.m.s.) higher than the homogeneous cable - an increase of about $47 \%$.

Example 11.13. A single core lead sheathed cable has a conductor diameter of 3 cm ; the diameter of the cable being 9 cm . The cable is graded by using two dielectrics of relative permittivity 5 and 4 respectively with corresponding safe working stresses of $30 \mathrm{kV} / \mathrm{cm}$ and $20 \mathrm{kV} / \mathrm{cm}$. Calculate the radial thickness of each insulation and the safe working voltage of the cable.

## Solution.

Here,

$$
\begin{array}{rlrlrl}
\text { Here, } \begin{aligned}
d & =3 \mathrm{~cm} & ; & d_{1}
\end{aligned}=? ; \quad D=9 \mathrm{~cm} \\
\varepsilon_{1} & =5 ; & \varepsilon_{2} & =4 \\
g_{1 \max } & =30 \mathrm{kV} / \mathrm{cm} ; & g_{2 \max } & =20 \mathrm{kV} / \mathrm{cm} \\
g_{1_{\max }} & \propto \frac{1}{\varepsilon_{1} d} ; & & g_{2 \max } & \propto \frac{1}{\varepsilon_{2} d_{1}} \\
& & \frac{g_{1 \max }}{g_{2 \max }} & =\frac{\varepsilon_{2} d_{1}}{\varepsilon_{1} d} \\
\therefore & & d_{1} & =\frac{g_{1 \max }}{g_{2 \max }} \times \frac{\varepsilon_{1} d}{\varepsilon_{2}}=\frac{30}{20} \times \frac{5 \times 3}{4}=5.625 \mathrm{~cm} \\
& & & & &
\end{array}
$$

or
$\therefore \quad$ Radial thickness of inner dielectric

$$
=\frac{d_{1}-d}{2}=\frac{5 \cdot 625-3}{2}=1.312 \mathrm{~cm}
$$

Radial thickness of outer dielectric

$$
=\frac{D-d_{1}}{2}=\frac{9-5 \cdot 625}{2}=1.68 \mathrm{~cm}
$$

Permissible peak voltage for the cable

$$
\begin{aligned}
& =\frac{g_{1_{\max }}}{2} d \log _{e} \frac{d_{1}}{d}+\frac{g_{2 \max }}{2} d_{1} \log _{e} \frac{D}{d_{1}} \\
& =\frac{30}{2} \times 3 \log _{e} \frac{5 \cdot 625}{3}+\frac{20}{2} \times 5.625 \log _{e} \frac{9}{5.625} \\
& =28.28+26.43=54.71 \mathrm{kV}
\end{aligned}
$$

$\therefore \quad$ Safe working voltage (r.m.s.) for the cable

$$
=54.71 / \sqrt{2}=38.68 \mathrm{kV}
$$

Example 11.14. A $66-\mathrm{kV}$ single-core lead sheathed cable is graded by using two dielectrics of relative permittivity 5 and 3 respectively; thickness of each being 1 cm . The core diameter is 2 cm . Determine the maximum stress in the two dielectrics.

Solution. Fig. 11.16 shows the composite dielectric of a capacitance graded cable. The potential difference $V$ between conductor and earthed sheath is given by ;

$$
\begin{align*}
V & =\int_{d / 2}^{d_{1} / 2} g_{1} d x+\int_{d_{1} / 2}^{D / 2} g_{2} d x \\
& =\int_{d / 2}^{d_{1} / 2} \frac{Q}{2 \pi \varepsilon_{0} \varepsilon_{1} x} d x+\int_{d_{1} / 2}^{D / 2} \frac{Q}{2 \pi \varepsilon_{0} \varepsilon_{2} x} d x \\
& =\frac{Q}{2 \pi \varepsilon_{0}}\left[\frac{1}{\varepsilon_{1}} \log _{e} \frac{d_{1}}{d}+\frac{1}{\varepsilon_{2}} \log _{e} \frac{D}{d_{1}}\right]  \tag{i}\\
g_{1 \text { max }} & =\frac{Q}{\pi \varepsilon_{0} \varepsilon_{1} d} \tag{ii}
\end{align*}
$$



Fig. 11.16

Putting the value of $Q=g_{1 \text { max }} \pi \varepsilon_{0} \varepsilon_{1} d$ from exp. (ii) in exp. (i), we get,
or

$$
V=\frac{g_{1_{\max }} \varepsilon_{1} d}{2}\left[\frac{1}{\varepsilon_{1}} \log _{e} \frac{d_{1}}{d}+\frac{1}{\varepsilon_{2}} \log _{e} \frac{D}{d_{1}}\right]
$$

$$
g_{1 \text { max }}=\frac{2 V}{d\left[\log _{e} \frac{d_{1}}{d}+\frac{\varepsilon_{1}}{\varepsilon_{2}} \log _{e} \frac{D}{d_{1}}\right]}
$$

Here, $d=2 \mathrm{~cm}, \quad d_{1}=4 \mathrm{~cm}, \quad D=6 \mathrm{~cm} ; V=\frac{66}{\sqrt{3}} \times \sqrt{2}=53.9 \mathrm{kV}, \varepsilon_{1}=5, \quad \varepsilon_{2}=3$
Substituting the values, we get,

$$
\begin{aligned}
g_{1 \max } & =\frac{2 \times 53 \cdot 9}{2\left[\log _{e} 4 / 2+5 / 3 \log _{e} 6 / 4\right]} \mathrm{kV} / \mathrm{cm} \\
& =\frac{2 \times 53.9}{2[0.6931+0.6757]}=39.38 \mathrm{kV} / \mathrm{cm}
\end{aligned}
$$

Similarly, it can be *proved that :

$$
\begin{align*}
g_{2 \max } & =\frac{2 V}{d_{1}\left[\frac{\varepsilon_{2}}{\varepsilon_{1}} \log _{e} \frac{d_{1}}{d}+\log _{e} \frac{D}{d_{1}}\right]}  \tag{iii}\\
& =\frac{2 \times 53.9}{4\left[3 / 5 \log _{e} 4 / 2+\log _{e} 6 / 4\right]} \mathrm{kV} / \mathrm{cm} \\
& =\frac{2 \times 53.9}{4[0.4158+0 \cdot 4054]}=32.81 \mathrm{kV} / \mathrm{cm}
\end{align*}
$$

### 11.13 Intersheath Grading

In this method of cable grading, a homogeneous dielectric is used, but it is divided into various layers by placing metallic intersheaths between the core and lead sheath. The intersheaths are held at suitable potentials which are inbetween the core potential and earth potential. This arrangement im-

[^7]
## Underground Cables

proves voltage distribution in the dielectric of the cable and consequently more uniform potential gradient is obtained.

Consider a cable of core diameter $d$ and outer lead sheath of diameter $D$. Suppose that two intersheaths of diameters $d_{1}$ and $d_{2}$ are inserted into the homogeneous dielectric and maintained at some fixed potentials. Let $V_{1}$, $V_{2}$ and $V_{3}$ respectively be the voltage between core and intersheath 1 , between intersheath 1 and 2 and between intersheath 2 and outer lead sheath. As there is a definite potential difference between the inner and outer layers of each intersheath, therefore, each sheath can be treated like ${ }^{\text {s }}$ a homogeneous single core cable. As proved in Art. 11.9,

Maximum stress between core and intersheath 1 is

$$
\text { Similarly, } \begin{aligned}
g_{1 \max } & =\frac{V_{1}}{\frac{d}{2} \log _{e} \frac{d_{1}}{d}} \\
g_{2 \max } & =\frac{V_{2}}{\frac{d_{1}}{2} \log _{e} \frac{d_{2}}{d_{1}}} \\
g_{3 \max } & =\frac{V_{3}}{\frac{d_{2}}{2} \log _{e} \frac{D}{d_{2}}}
\end{aligned}
$$

Since the dielectric is homogeneous, the maximum


Fig. 11.17 stress in each layer is the same i.e.,

$$
\begin{array}{rlrl}
g_{1 \max } & =g_{2 \max }=g_{3 \max }=g_{\max }(\text { say }) \\
\therefore & \frac{V_{1}}{\frac{d}{2} \log _{e} \frac{d_{1}}{d}} & =\frac{V_{2}}{\frac{d_{1}}{2} \log _{e} \frac{d_{2}}{d_{1}}}=\frac{V_{3}}{\frac{d_{2}}{2} \log _{e} \frac{D}{d_{2}}}
\end{array}
$$

As the cable behaves like three capacitors in series, therefore, all the potentials are in phase i.e. Voltage between conductor and earthed lead sheath is

$$
V=V_{1}+V_{2}+V_{3}
$$

Intersheath grading has three principal disadvantages. Firstly, there are complications in fixing the sheath potentials. Secondly, the intersheaths are likely to be damaged during transportation and installation which might result in local concentrations of potential gradient. Thirdly, there are considerable losses in the intersheaths due to charging currents. For these reasons, intersheath grading is rarely used.

Example 11.15. A single core cable of conductor diameter 2 cm and lead sheath of diameter 5.3 cm is to be used on a 66 kV , 3-phase system. Two intersheaths of diameter 3.1 cm and 4.2 cm are introduced between the core and lead sheath. If the maximum stress in the layers is the same, find the voltages on the intersheaths.

## Solution.

Here,

$$
\begin{aligned}
d= & 2 \mathrm{~cm} ; \quad d_{1}=3.1 \mathrm{~cm} ; d_{2}=4.2 \mathrm{~cm} \\
D= & 5.3 \mathrm{~cm} ; V=\frac{66 \times \sqrt{2}}{\sqrt{3}}=53.9 \mathrm{kV} \\
& g_{1 \max }=\frac{V_{1}}{\frac{d}{2} \log _{e} \frac{d_{1}}{d}}=\frac{V_{1}}{1 \times \log _{e} \frac{3 \cdot 1}{2}}=2.28 V_{1}
\end{aligned}
$$

$$
\begin{aligned}
& g_{2 \max }=\frac{V_{2}}{\frac{d_{1}}{2} \log _{e} \frac{d_{2}}{d_{1}}}=\frac{V_{2}}{1 \cdot 55 \log _{e} \frac{4.2}{3.1}}=2 \cdot 12 V_{2} \\
& g_{3 \max }=\frac{V_{3}}{\frac{d_{2}}{2} \log _{e} \frac{D}{d_{2}}}=\frac{V_{3}}{2 \cdot 1 \log _{e} \frac{5 \cdot 3}{4 \cdot 2}}=2 \cdot 04 V_{3}
\end{aligned}
$$

As the maximum stress in the layers is the same,

$$
\begin{array}{lrl}
\therefore & g_{1 \max } & =g_{2 \max }=g_{3 \max } \\
\text { or } & 2.28 V_{1} & =2.12 V_{2}=2.04 V_{3} \\
\therefore & V_{2} & =(2.28 / 2.12) V_{1}=1.075 V_{1} \\
\text { and } & V_{3} & =(2.28 / 2.04) V_{1}=1.117 V_{1} \\
\text { Now } & V_{1}+V_{2}+V_{3} & =V \\
\text { or } & V_{1}+1.075 V_{1}+1.117 V_{1} & =53.9 \\
\text { or } & V_{1} & =53.9 / 3.192=16.88 \mathrm{kV} \\
\text { and } & V_{2} & =1.075 V_{1}=1.075 \times 16.88=18.14 \mathrm{kV}
\end{array}
$$

$\therefore \quad$ Voltage on first intersheath (i.e., near to the core)

$$
=V-V_{1}=53.9-16.88=37.02 \mathrm{kV}
$$

Voltage on second intersheath $=V-V_{1}-V_{2}=53.9-16.88-18.14=18.88 \mathbf{k V}$
Example 11.16. A single-core 66 kV cable working on 3-phase system has a conductor diameter of 2 cm and a sheath of inside diameter 5.3 cm . If two intersheaths are introduced in such a way that the stress varies between the same maximum and minimum in the three layers, find:
(i) positions of intersheaths
(ii) voltage on the intersheaths
(iii) maximum and minimum stress

## Solution.

Here, $d=2 \mathrm{~cm} ; D=5.3 \mathrm{~cm} ; V=\frac{66 \times \sqrt{2}}{\sqrt{3}}=53.9 \mathrm{kV}$
(i) Positions of intersheaths. Suppose that diameters of intersheaths are $d_{1}$ and $d_{2} \mathrm{~cm}$ respectively. Let $V_{1}, V_{2}$ and $V_{3}$ respectively be the voltage between conductor and intersheath 1, between intersheath 1 and 2 and between intersheath 2 and outer lead sheath.

$$
g_{1 \max }=\frac{V_{1}}{\frac{d}{2} \log _{e} \frac{d_{1}}{d}} \quad ; \quad g_{2 \max }=\frac{V_{2}}{\frac{d_{1}}{2} \log _{e} \frac{d_{2}}{d_{1}}} \quad ; \quad g_{3 \max }=\frac{V_{3}}{\frac{d_{2}}{2} \log _{e} \frac{D}{d_{2}}}
$$

As the maximum stress in the three layers is the same,

$$
\begin{equation*}
\therefore \quad \frac{V_{1}}{\frac{d}{2} \log _{e} \frac{d_{1}}{d}}=\frac{V_{2}}{\frac{d_{1}}{2} \log _{e} \frac{d_{2}}{d_{1}}}=\frac{V_{3}}{\frac{d_{2}}{2} \log _{e} \frac{D}{d_{2}}} \tag{i}
\end{equation*}
$$

In order that stress may vary between the same maximum and minimum in the three layers, we have,

$$
\begin{array}{ll} 
& d_{1} / d=d_{2} / d_{1}=D / d_{2} \\
\therefore & \frac{V_{1}}{d}=\frac{V_{2}}{d_{1}}=\frac{V_{3}}{d_{2}} \tag{iii}
\end{array}
$$

[^8]From exp. (ii), we get,

$$
[\because d=2 \mathrm{~cm}]
$$

$$
d_{1}^{2}=d \times d_{2}=2 d_{2}
$$

and

$$
d_{1} d_{2}=D \times d=5.3 \times 2=10.6 \mathrm{~cm}
$$

or $\quad d_{1} \times d_{1}^{2} / 2=10.6$
or

$$
d_{1}=(21.2)^{1 / 3}=2.76 \mathrm{~cm}
$$

$\therefore \quad d_{2}=d_{1}^{2} / 2=(2.76)^{2} / 2=3.8 \mathrm{~cm}$
Hence intersheaths of diameters 2.76 cm and 3.8 cm are required to be used.
(ii) Voltage on intersheaths
or

$$
V=V_{1}+V_{2}+V_{3}
$$

$$
53.9=V_{1}+\frac{d_{1}}{d} V_{1}+\frac{d_{2}}{d} V_{1}
$$

[From eq. (iii)]

$$
\therefore \quad V_{1}=53.9 / 4 \cdot 28=12.6 \mathrm{kV}
$$

and

$$
V_{2}=\frac{d_{1}}{d} \times V_{1}=\frac{2.76}{2} \times 12.6=17.39 \mathrm{kV}
$$

Voltage on first intersheath $=V-V_{1}=53.9-12.6=41.3 \mathrm{kV}$ max
Voltage on second intersheath $=V-V_{1}-V_{2}=53.9-12.6-17.39=23.91 \mathrm{kV}$ max
(iii) Stresses in dielectrics

$$
\begin{aligned}
\text { Maximum stress } & =\frac{V_{1}}{\frac{d}{2} \log _{e} \frac{d_{1}}{d}}=\frac{12 \cdot 6}{1 \times \log _{e} \frac{2 \cdot 76}{2}} \mathrm{kV} / \mathrm{cm}=39 \mathrm{kV} / \mathrm{cm} \\
\text { Minimum stress } & =\frac{V_{1}}{\frac{d_{1}}{2} \log _{e} \frac{d_{1}}{d}}=\frac{12 \cdot 6}{1 \cdot 38 \log _{e} \frac{2 \cdot 76}{2}} \mathrm{kV} / \mathrm{cm}=28.35 \mathrm{kV} / \mathrm{cm}
\end{aligned}
$$

## TUTORIAL PROBLEMS

1. A 33 kV , single-core cable has a conductor diameter of 1 cm and insulation of 1.5 cm . Find the maximum and minimum stress in the insulation.
[ $47.62 \mathrm{kV} / \mathrm{cm}$ (r.m.s.), $11.9 \mathrm{kV} / \mathrm{cm}$ (r.m.s.)]
2. Find the economic size of a single-core cable working on 220 kV , 3-phase system. The maximum permissible stress in the dielectric is not to exceed $250 \mathrm{kV} / \mathrm{cm}$. $[d=1.43 \mathrm{~cm}, \mathrm{D}=3.88 \mathrm{~cm}]$
3. The inner conductor of a concentric cable has a diameter of 3 cm with insulation of diameter 8.5 cm . The cable is insulated with two materials having relative permittivities of 5 and 3 with corresponding safe working stresses of $38 \mathrm{kV} / \mathrm{cm}$ and $26 \mathrm{kV} / \mathrm{cm}$. Calculate the radial thickness of insulating layers and the safe working voltage of the cable.
[ $2 \cdot 15 \mathrm{~cm}, \mathbf{0 . 6} \mathrm{~cm}, 46 \cdot 1 \mathrm{kV}$ (r.m.s.)]
4. A single-core lead covered cable is to be designed for 66 kV to earth. Its conductor diameter is 2 cm and its three insulating materials have permittivities of 5, 4 and 3 respectively with the corresponding maximum safe working stress of $38 \mathrm{kV} / \mathrm{cm}$ (r.m.s.), $26 \mathrm{kV} / \mathrm{cm}$. (r.m.s.) and $20 \mathrm{kV} / \mathrm{cm}$ (r.m.s.) respectively. Find the minimum diameter of lead sheath.
[ 8.3 cm ]
5. A single-core 66 kV cable has a conductor diameter of 2 cm and a sheath of inside diameter 5.3 cm . The cable has an inner layer of 1 cm thick of rubber of dielectric constant 4.5 and the rest impregnated paper of dielectric constant $3 \cdot 6$. Find the maximum stress in the rubber and in the paper.
[ $63 \mathrm{kV} / \mathrm{cm}, 39.5 \mathrm{kV} / \mathrm{cm}$ ]
6. A single-core cable working on 66 kV on 3 -phase system has a conductor diameter of 2 cm and a sheath of inside diameter 5.3 cm . If two intersheaths are used, find the best positions, maximum stress and the voltage on the intersheaths. $\quad\left[d_{1}=2.77 \mathrm{~cm} ; d_{2}=3.84 \mathrm{~cm} ; 38.7 \mathrm{kV} / \mathrm{cm} ; \mathrm{V}_{1}=41.1 \mathrm{kV}, \mathrm{V}_{2}=23.9 \mathrm{kV}\right]$

### 11.14 Capacitance of 3-Core Cables

The capacitance of a cable system is much more important than that of overhead line because in cables (i) conductors are nearer to each other and to the earthed sheath (ii) they are separated by a dielectric of permittivity much greater than that of air. Fig. 11.18 shows a system of capacitances in a 3-core belted cable used for 3-phase system. Since potential difference exists between pairs of conductors and between each conductor and the sheath, electrostatic fields are set up in the cable as shown in Fig. 11.18 (i). These electrostatic fields give rise to core-core capacitances $C_{c}$ and conduc-tor-earth capacitances $C_{e}$ as shown in Fig. 11.18 (ii). The three $C_{c}$ are delta connected whereas the three $C_{e}$ are star connected, the sheath forming the star point [See Fig. 11.18 (iii)].

(i)

(ii)

(iii)

Fig. 11.18
They lay of a belted cable makes it reasonable to assume equality of each $C_{c}$ and each $C_{e}$. The three delta connected capacitances $C_{c}$ [See Fig. 11.19 (i)] can be converted into equivalent star connected capacitances as shown in Fig. 11.19 (ii). It can be easily *shown that equivalent starcapacitance $C_{e q}$ is equal to three times the deltacapacitance $C_{c}$ i.e. $C_{e q}=3 C_{c}$.

(i)

(ii)

Fig. 11.19
The system of capacitances shown in Fig.
11.18 (iii) reduces to the equivalent circuit shown in Fig. 11.20 (i). Therefore, the whole cable is equivalent to three star-connected capacitors each of capacitance [See Fig. 11.20 (ii)],

(i)

(ii)

Fig. 11.20

[^9]\[

$$
\begin{aligned}
C_{N} & =C_{e}+C_{e q} \\
& =C_{e}+3 C_{c}
\end{aligned}
$$
\]

If $V_{p h}$ is the phase voltage, then charging current $I_{C}$ is given by ;

$$
\begin{aligned}
I_{C} & =\frac{V_{p h}}{\text { Capacitive reactance per phase }} \\
& =2 \pi f V_{p h} C_{N} \\
& =2 \pi f V_{p h}\left(C_{e}+3 C_{c}\right)
\end{aligned}
$$

### 11.15 Measurements of $C_{e}$ and $C_{c}$

Although core-core capacitance $C_{c}$ and core-earth capacitance $C_{e}$ can be obtained from the empirical formulas for belted cables, their values can also be determined by measurements. For this purpose, the following two measurements are required :
(i) In the first measurement, the three cores are bunched together (i.e. commoned) and the capacitance is measured between the bunched cores and the sheath. The bunching eliminates all the three capacitors $C_{c}$, leaving the three capacitors $C_{e}$ in parallel. Therefore, if $C_{1}$ is the measured capacitance, this test yields :

$$
\begin{aligned}
C_{1} & =3 C_{e} \\
C_{e} & =\frac{C_{1}}{3}
\end{aligned}
$$

Knowing the value of $C_{1}$, the value of $C_{e}$ can be determined.
(ii) In the second measurement, two cores are bunched with the sheath and capacitance is measured between them and the third core. This test yields $2 C_{c}+C_{e}$. If $C_{2}$ is the measured capacitance, then,

$$
C_{2}=2 C_{c}+C_{e}
$$

As the value of $C_{e}$ is known from first test and $C_{2}$ is found experminentally, therefore, value of $C_{c}$ can be determined.
It may be noted here that if value of $C_{N}\left(=C_{e}+3 C_{c}\right)$ is desired, it can be found directly by another test. In this test, the capacitance between two cores or lines is measured with the third core free or connected to the sheath. This eliminates one of the capacitors $C_{e}$ so that if $C_{3}$ is the measured capacitance, then,

$$
\begin{aligned}
C_{3} & =C_{c}+\frac{C_{c}}{2}+\frac{C_{e}}{2} \\
& =\frac{1}{2}\left(C_{e}+3 C_{c}\right) \\
& =\frac{1}{2} C_{N}
\end{aligned}
$$

Example 11.17. The capacitance per kilometre of a 3-phase belted cable is $0.3 \mu \mathrm{~F}$ between the two cores with the third core connected to the lead sheath. Calculate the charging current taken by five kilometres of this cable when connected to a 3-phase, $50 \mathrm{~Hz}, 11 \mathrm{kV}$ supply.

Solution. The capacitance between a pair of cores with third core earthed for a length of 5 km is

$$
\begin{aligned}
C_{3} & =0.3 \times 5=1.5 \mu F \\
V_{p h} & =\frac{11 \times 10^{3}}{\sqrt{3}}=6351 \mathrm{~V} ; f=50 \mathrm{~Hz}
\end{aligned}
$$

As proved in Art 11.15, core to neutral capacitance $C_{N}$ of this cable is given by :

$$
C_{N}=2 C_{3}=2 \times 1.5=3 \mu F
$$

$$
\therefore \quad \text { Charging current, } \quad \begin{aligned}
I_{C} & =2 \pi f V_{p h} C_{N} \\
& =2 \pi \times 50 \times 6351 \times 3 \times 10^{-6} \mathrm{~A}=\mathbf{5 . 9 8} \mathbf{A}
\end{aligned}
$$

Example 11.18. The capacitances of a 3-phase belted cable are $12.6 \mu F$ between the three cores bunched together and the lead sheath and $7.4 \mu F$ between one core and the other two connected to sheath. Find the charging current drawn by the cable when connected to $66 \mathrm{kV}, 50 \mathrm{~Hz}$ supply.

## Solution.

Here,

$$
V_{p h}=\frac{66 \times 10^{3}}{\sqrt{3}}=38105 V ; f=50 \mathrm{~Hz} ; C_{1}=12.6 \mu F ; C_{2}=7.4 \mu F
$$

Let core-core and core-earth capacitances of the cable be $C_{c}$ and $C_{e}$ respectively. As proved in Art. 11.15,

$$
C_{1}=3 C_{e}
$$

$\therefore \quad C_{e}=C_{1} / 3=12.6 / 3=4.2 \mu \mathrm{~F}$
and

$$
C_{2}=2 C_{c}+C_{e}
$$

$\therefore \quad C_{c}=\frac{C_{2}-C_{e}}{2}=\frac{7.4-4.2}{2}=1.6 \mu \mathrm{~F}$
$\therefore$ Core to neutral capacitance is

$$
\text { Charging current, } \quad \begin{aligned}
C_{N} & =C_{e}+3 C_{c}=4.2+3 \times 1.6=9 \mu F \\
I_{C} & =2 \pi f V_{p h} C_{N} \\
& =2 \pi \times 50 \times 38105 \times 9 \times 10^{-6} \mathrm{~A}=\mathbf{1 0 7 . 7 4} \mathbf{A}
\end{aligned}
$$

Example 11.19. The capacitance per kilometre of a 3-phase belted cable is $0 \cdot 18 \mu F$ between two cores with the third core connected to sheath. Calculate the kVA taken by 20 km long cable when connected to 3-phase, $50 \mathrm{~Hz}, 3300 \mathrm{~V}$ supply.

Solution. The capacitance between a pair of cores with third core earthed for a length of 20 km is

$$
C_{3}=0.18 \times 20=3.6 \mu F, V_{p h}=3300 / \sqrt{3}=1905 \mathrm{~V} ; f=50 \mathrm{~Hz}
$$

Core to neutral capacitance, $C_{N}=2 C_{3}=2 \times 3.6=7.2 \mu \mathrm{~F}$
Charging current, $\quad I_{C}=2 \pi f V_{p h} C_{N}$

$$
=2 \pi \times 50 \times 1905 \times 7.2 \times 10^{-6} \mathrm{~A}=4.3 \mathrm{~A}
$$

kVA taken by the cable

$$
=3 V_{p h} I_{C}=3 \times 1905 \times 4.3 \times 10^{-3} \mathrm{kVA}=\mathbf{2 4 . 5 7} \mathbf{~ k V A}
$$

## TUTORIAL PROBLEMS

1. The capacitances per kilometre of a 3-phase cable are $0.63 \mu \mathrm{~F}$ between the three cores bunched together and the sheath and $0.37 \mu \mathrm{~F}$ between one core and the other two connected to the sheath. Calculate the charging current taken by eight kilometres of this cable when connected to a 3-phase, $50 \mathrm{~Hz}, 6600 \mathrm{~V}$ supply.
[4.31 A]
2. The capacitances of a 3-core belted type cable are measured as detailed below :
(i) Between three cores bunched together and sheath is $8 \mu \mathrm{~F}$
(ii) Between a conductor and the other two connected to the sheath together is $6 \mu \mathrm{~F}$. Calculate the capacitance per phase.
$[23 / 3 \mu \mathrm{~F}]$
3. A 3-core, 3-phase belted cable tested for capacitance between a pair of cores on single phase, with the third core earthed, gave a capacitance of $0.4 \mu \mathrm{~F}$ per km . Calculate the charging current for 15 km length of this cable when connected to $22 \mathrm{kV}, 3$-phase, 50 Hz supply.

### 11.16 Current-Carying Capacity of Underground Cables

The safe current-carrying capacity of an underground cable is determined by the maximum permissible temperature rise. The cause of temperature rise is the losses that occur in a cable which appear
as heat. These losses are :
(i) Copper losses in the conductors
(ii) Hysteresis losses in the dielectric
(iii) Eddy current losses in the sheath

The safe working conductor temperature is $65^{\circ} \mathrm{C}$ for armoured cables and $50^{\circ} \mathrm{C}$ for lead-sheathed cables laid in ducts. The maximum steady temperature conditions prevail when the heat generated in the cable is equal to the heat dissipated. The heat dissipation of the conductor losses is by conduction through the insulation to the sheath from which the total losses (including dielectric and sheath losses) may be conducted to the earth. Therefore, in order to find permissible current loading, the thermal resistivities of the insulation, the protective covering and the soil must be known.

### 11.17 Thermal Resistance

The thermal resistance between two points in a medium (e.g. insulation) is equal to temperature difference between these points divided by the heat flowing between them in a unit time i.e.

$$
\text { Thermal resistance, } S=\frac{\text { Temperature difference }}{\text { Heat flowing in a unit time }}
$$

In SI units, heat flowing in a unit time is measured in watts.

$$
\begin{aligned}
\therefore & \text { Thermal resistance, } S & =\frac{\text { Temperature rise }(t)}{\text { Watts dissipated }(P)} \\
\text { or } & S & =\frac{t}{P}
\end{aligned}
$$

Clearly, the SI unit of thermal resistance is ${ }^{\circ} \mathrm{C}$ per watt. This is also called thermal ohm.
Like electrical resistance, thermal resistance is directly proportional to length $l$ in the direction of transmission of heat and inversely proportional to the cross-section area $a$ at right angles to that direction.

$$
\begin{array}{ll}
\therefore & S \propto \frac{l}{a} \\
\text { or } & S=k \frac{l}{a}
\end{array}
$$

where $k$ is the constant of proportionality and is known as thermal resistivity.

$$
\begin{aligned}
k & =\frac{S a}{l} \\
\therefore \quad \text { Unit of } k & =\frac{\text { Thermal ohm } \times m^{2}}{m}=\text { thermal ohm-metre }
\end{aligned}
$$

### 11.18 Thermal Resistance of Dielectric of a Single-Core Cable

Let us now find the thermal resistance of the dielectric of a single-core cable.
Let

$$
\begin{aligned}
r & =\text { radius of the core in metre } \\
r_{1} & =\text { inside radius of the sheath in metre } \\
k & =\text { thermal resistivity of the insulation (i.e. dielectric) }
\end{aligned}
$$

Consider 1 m length of the cable. The thermal resistance of small element of thickness $d x$ at radius $x$ is (See Fig. 11.21)

$$
d S=k \times \frac{d x}{2 \pi x}
$$

$\therefore$ Thermal resistance of the dielectric is


Fig. 11.21

$$
\begin{aligned}
S & =\int_{r}^{r_{1}} k \times \frac{d x}{2 \pi x} \\
& =\frac{k}{2 \pi} \int_{r}^{r_{1}} \frac{1}{x} d x \\
\therefore \quad S & =\frac{k}{2 \pi} \log _{e} \frac{r_{1}}{r} \text { thermal ohms per metre length of the cable }
\end{aligned}
$$

The thermal resistance of lead sheath is small and is generally neglected in calculations.

### 11.19 Permissible Current Loading

When considering heat dissipation in underground cables, the various thermal resistances providing a heat dissipation path are in series. Therefore, they add up like electrical resistances in series. Consider a cable laid in soil.

Let $I=$ permissible current per conductor
$n=$ number of conductors
$R \quad=$ electrical resistance per metre length of the conductor at the working temperature
$S=$ total thermal resistance (i.e. sum of thermal resistances of dielectric and soil) per metre length
$t \quad=$ temperature difference (rise) between the conductor and the soil
Neglecting the dielectric and sheath losses, we have,

$$
\text { Power dissipated }=n I^{2} R
$$

Now $\quad$ Power dissipated $=\frac{\text { Temperature rise }}{\text { Thermal resistance }}$
or

$$
n I^{2} R=\frac{t}{S}
$$

$\therefore$ Permissible current per conductor in given by;

$$
I=\sqrt{\frac{t}{n R S}}
$$

It should be noted that when cables are laid in proximity to each other, the permissible current is reduced further on account of mutual heating.

Example 11.20. A single-core cable is laid in the ground, the core diameter being 30 mm and the dielectric thickness 40 mm . The thermal resistivity of the dielectric is 5 thermal ohm-metres and the thermal resistance between the sheath and the ground surface is 0.45 thermal ohm per metre length of the cable. Neglecting dielectric and sheath losses, estimate the maximum permissible current loading if the temperature difference between the conductor and the ground surface is not to exceed $55^{\circ} \mathrm{C}$. The electrical resistance of the cable is $110 \mu \Omega$ per metre length.

Solution. Thermal resistance of the dielectric of the cable is

$$
S_{1}=\frac{k}{2 \pi} \log _{e} \frac{r_{1}}{r} \text { thermal-ohms per metre length of cable }
$$

Here $k=5$ thermal ohm-metres; $r=30 / 2=15 \mathrm{~mm} ; r_{1}=15+40=55 \mathrm{~mm}$
$\therefore \quad S_{1}=\frac{5}{2 \pi} \log _{e} \frac{55}{15}=1.03$ thermal ohms per metre length
Thermal resistance of soil, $S_{2}=0.45$ thermal ohm per metre length (given)
$\therefore$ Total thermal resistance, $S=S_{1}+S_{2}=1.03+0.45=1.48$ thermal ohm per metre length

$$
\begin{aligned}
& \text { Now } I=\sqrt{\frac{t}{n R S}} \\
& \text { Here } t=55^{\circ} \mathrm{C} ; n=1, R=110 \times 10^{-6} \Omega ; S=1 \cdot 48 \\
& \therefore \quad I=\sqrt{\frac{55}{1 \times 110 \times 10^{-6} \times 1 \cdot 48}}=\mathbf{5 8 1} \mathbf{A}
\end{aligned}
$$

### 11.20 Typesof Cable Faults

Cables are generally laid directly in the ground or in ducts in the underground distribution system. For this reason, there are little chances of faults in underground cables. However, if a fault does occur, it is difficult to locate and repair the fault because conductors are not visible. Nevertheless, the following are the faults most likely to occur in underground cables :
(i) Open-circuit fault
(ii) Short-circuit fault
(iii) Earth fault.
(i) Open-circuit fault. When there is a break in the conductor of a cable, it is called opencircuit fault. The open-circuit fault can be checked by a megger. For this purpose, the three conductors of the 3 -core cable at the far end are shorted and earthed. Then resistance between each conductor and earth is measured by a megger. The megger will indicate zero resistance in the circuit of the conductor that is not broken. However, if the conductor is broken, the megger will indicate infinite resistance in its circuit.
(ii) Short-circuit fault. When two conductors of a multi-core cable come in electrical contact with each other due to insulation failure, it is called a short-circuit fault. Again, we can seek the help of a megger to check this fault. For this purpose, the two terminals of the megger are connected to any two conductors. If the megger gives zero reading, it indicates shortcircuit fault between these conductors. The same step is repeated for other conductors taking two at a time.
(iii) Earth fault. When the conductor of a cable comes in contact with earth, it is called earth fault or ground fault. To identify this fault, one terminal of the megger is connected to the conductor and the other terminal connected to earth. If the megger indicates zero reading, it means the conductor is earthed. The same procedure is repeated for other conductors of the cable.

### 11.21 Loop TestsForLocation of Faults in Underground Cables

There are several methods for locating the faults in underground cables. However, two popular methods known as loop tests are :
(i) Murray loop test
(ii) Varley loop test

These simple tests can be used to locate the earth fault or short-circuit fault in underground cables provided that a sound cable runs along the faulty cable. Both these tests employ the principle of Wheatstone bridge for fault location.

### 11.22 Murray Loop Test

The Murray loop test is the most common and accurate method of locating earth fault or short-circuit fault in underground cables.
(i) Earth fault : Fig. 11.22 shows the circuit diagram for locating the earth fault by Murray loop test. Here $A B$ is the sound cable and $C D$ is the faulty cable; the earth fault occuring at point $F$. The far end $D$ of the faulty cable is joined to the far end $B$ of the sound cable through a low resistance
link. Two variable resistances $P$ and $Q$ are joined to ends $A$ and $C$ (See Fig. 11.22) respectively and serve as the ratio arms of the Wheatstone bridge.

Let $R=$ resistance of the conductor loop upto the fault from the test end
$X=$ resistance of the other length of the loop


Note that $P, Q, R$ and $X$ are the four arms of the Wheatstone bridge. The resistances $P$ and $Q$ are varied till the galvanometer indicates zero deflection.

In the balanced position of the bridge, we have,
or

$$
\begin{aligned}
\frac{P}{Q} & =\frac{R}{X} \\
\frac{P}{Q}+1 & =\frac{R}{X}+1 \\
\frac{P+Q}{Q} & =\frac{R+X}{X}
\end{aligned}
$$

or

If $r$ is the resistance of each cable, then $R+X=2 r$.
$\therefore \quad \frac{P+Q}{Q}=\frac{2 r}{X}$
or

$$
X=\frac{Q}{P+Q} \times 2 r
$$

If $l$ is the length of each cable in metres, then resistance per metre length of cable $=\frac{r}{l}$.
$\therefore$ Distance of fault point from test end is
or

$$
\begin{aligned}
d & =\frac{X}{r / l}=\frac{Q}{P+Q} \times 2 r \times \frac{l}{r}=\frac{Q}{P+Q} \times 2 l \\
d & =\frac{Q}{P+Q} \times(\text { loop length }) * \text { metres }
\end{aligned}
$$

Thus the position of the fault is located. Note that resistance of the fault is in the battery circuit and not in the bridge circuit. Therefore, fault resistance does not affect the balancing of the bridge. However, if the fault resistance is high, the sensitivity of the bridge is reduced.

[^10](ii) Short-circuit fault : Fig. 11.23 shows the circuit diagram for locating the short-circuit fault by Murray loop test. Again $P, Q, R$ and $X$ are the four arms of the bridge. Note that fault resistance is in the battery circuit and not in the bridge circuit. The bridge in balanced by adjusting the resistances $P$ and $Q$. In the balanced position of the bridge :
\[

$$
\begin{array}{rlrl} 
& \frac{P}{Q} & =\frac{R}{X} \\
& \text { or } & \frac{P+Q}{Q} & =\frac{R+X}{X}=\frac{2 r}{X} \\
\therefore & X & =\frac{Q}{P+Q} \times 2 r \\
& \text { or } & X & \left.=\frac{Q}{P+Q} \times \text { (loop length }\right) \text { metres }
\end{array}
$$
\]

or
or


Thus the position of the fault is located.

### 11.23 Va rey Loop Test

The Varley loop test is also used to locate earth fault or short-circuit fault in underground cables. This test also employs Wheatstone bridge principle. It differs from Murray loop test in that here the ratio arms $P$ and $Q$ are fixed resistances. Balance is obtained by adjusting the variable resistance $S$


Fig. 11.24 Varley Loop Test (Earth Fault)
connected to the test end of the faulty cable. The connection diagrams for locating the earth fault and short-circuit fault by Varley loop test are shown in Figs. 11.24 and 11.25 respectively.


Fig. 11.25 Varley Loop Test (Short Circuit Test)
For earth fault or short-circuit fault, the key $K_{2}$ is first thrown to position 1. The variable resistance $S$ is varied till the bridge is balanced for resistance value of $S_{1}$. Then,

$$
\frac{P}{Q}=\frac{R}{X+S_{1}}
$$

or

$$
\frac{P+Q}{Q}=\frac{R+X+S_{1}}{X+S_{1}}
$$

or

$$
\begin{equation*}
X=\frac{Q(R+X)-P S_{1}}{P+Q} \tag{i}
\end{equation*}
$$

Now key $K_{2}$ is thrown to position 2 (for earth fault or short-circuit fault) and bridge is balanced with new value of resistance $S_{2}$. Then,

$$
\frac{P}{Q}=\frac{R+X}{S_{2}}
$$

$$
\begin{equation*}
(R+X) Q=P S_{2} \tag{ii}
\end{equation*}
$$

From eqs. (i) and (ii), we get,

$$
X=\frac{P\left(S_{2}-S_{1}\right)}{P+Q}
$$

Since the values of $P, Q, S_{1}$ and $S_{2}$ are known, the value of $X$ can be determined.

$$
\text { Loop resistance }=R+X=\frac{P}{Q} S_{2}
$$

If $r$ is the resistance of the cable per metre length, then,
Distance of fault from the test end is

$$
d=\frac{X}{r} \text { metres }
$$

Example 11.21. Murray loop test is performed on a faulty cable 300 m long. At balance, the resistance connected to the faulty core was set at $15 \Omega$ and the resistance of the resistor connected to the sound core was $45 \Omega$. Calculate the distance of the fault point from the test end.

## Solution.

Distance of the fault point from test end is

$$
d=\frac{Q}{P+Q} \times \text { loop length }
$$

Here $Q=15 \Omega ; P=45 \Omega ;$ loop length $=2 \times 300=600 \mathrm{~m}$

$$
\therefore \quad d=\frac{15}{45+15} \times 600=\mathbf{1 5 0} \mathbf{~ m}
$$

Example 11.22. In a test by Murray loop for ground fault on 500 m of cable having a resistance of $1.6 \Omega / \mathrm{km}$, the faulty cable is looped with a sound cable of the same length and area of crosssection. If the ratio of the other two arms of the testing network at balance is $3: 1$, find the distance of the fault from the testing end of cables.

Solution.

$$
\frac{P}{Q}=3 \quad \text { or } \quad \frac{P+Q}{Q}=4
$$

Distance of fault from test end is

$$
d=\frac{Q}{P+Q} \times \text { loop length }=\frac{1}{4} \times(2 \times 500)=\mathbf{2 5 0} \mathbf{~ m}
$$

Example 11.23. In a test for a fault to earth on a 500 m length of cable having a resistance of $1 \Omega$ per 1000 m , the faulty cable is looped with a sound cable of the same length but having a resistance of $2.25 \Omega$ per 1000 m . The resistance of the other two arms of the testing network at balance are in the ratio 2.75:1. Calculate the distance of the fault from the testing end of the cable.

Solution.

$$
\frac{P}{Q}=2.75 \quad \text { or } \quad \frac{P+Q}{Q}=2.75+1=3.75
$$

$$
\text { Resistance of loop }=\frac{1}{1000} \times 500+\frac{2 \cdot 25}{1000} \times 500=1.625 \Omega
$$

Resistance of faulty cable from test end upto fault point is

$$
X=\frac{Q}{P+Q} \times(\text { loop resistance })=\frac{1}{3.75} \times 1.625=0.433 \Omega
$$

Distance of fault point from the testing end is

$$
d=\frac{X}{1 / 1000}=0.433 \times 1000=433 \mathrm{~m}
$$

Example 11.24. Varley loop test is performed to locate an earth fault on a 20 km long cable. The resistance per km of the single conductor is $20 \Omega$. The loop is completed with a similar healthy conductor. At balance, the variable resistance connected to the faulty conductor is $200 \Omega$. The fixed resistors have equal values. Calcualte the distance of the fault from the test end.

## Solution.

Resistance of faulty cable from test end to fault point is

$$
X=\frac{Q(R+X)-P S}{P+Q}
$$

Here $P=Q ; S=200 \Omega ; R+X=20(20+20)=800 \Omega$

$$
\therefore \quad X=\frac{Q(800)-Q \times 200}{Q+Q}=300 \Omega
$$

The resistance per km $=20 \Omega$
$\therefore$ Distance of fault from test end is

$$
d=\frac{X}{20}=\frac{300}{20}=15 \mathrm{~km}
$$

## TUTORIAL PROBLEMS

1. The Murray loop test is used to locate an earth fault on one core of a two-core cable. The other core is used to complete the loop. When the network is balanced, the resistance connected to the faulty core has a value of $3.2 \Omega$. The other resistance arm has a value of $11.8 \Omega$. The fault is 42.7 m from the test end. Find the length of the cable.
[ 100 m ]
2. Murray loop test is performed to locate an earth fault on one core of a 2 -core cable 100 m long. The other core is healthy and used to form the loop. At balance, the resistance connected to the faulty core was $4 \Omega$. The other resistance arm has a value of $16 \Omega$. Calculate the distance of the fault from the test end.
[40 m]
3. The Varley loop test is used to find the position of an earth fault on a line of length 40 km . The resistance $/ \mathrm{km}$ of a single line is $28 \Omega$. The fixed resistors have resistances of $250 \Omega$ each. The fault is calculated to be 7 km from the test end. To what value of resistance was the variable resistor set?
[1848 $\Omega$ ]

## SELF - TEST

1. Fill in the blanks by inserting appropriate words/figures.
(i) The underground system is $\qquad$ costly than the equivalent overhead line system.
(ii) Voltage drop in cable system is less than that of equivalent overhead line because of . $\qquad$ of conductors in a cable.
(iii) A metallic sheath is provided over the insulation to protect the cable from $\qquad$
$\qquad$
(iv) In single-core cables, armouring is not done in order to avoid $\qquad$
(v) The most commonly used insulation in high-voltage cables is $\qquad$
(vi) Belted cables are generally used upto $\qquad$ kV .
(vii) The working voltage level of belted cable is limited to 22 kV because of the $\qquad$ set up in the dielectric.
(viii) For voltages beyond 66 kV , solid type cables are unreliable because there is a danger of breakdown of insulation due to the $\qquad$
(ix) If the length of a cable increases, its insulation resistance $\qquad$ .. .
$(x)$ Under operating conditions, the maximum stress in a cable is at $\qquad$
2. Fill in the blanks by picking up correct words/figures from brackets.
(i) For voltages less than 66 kV , a 3-phase cable usually consists of.
[3-core cable, 3 single-core cables]
(ii) If the length of a cable is doubled, its capacitance is $\qquad$ [doubled, halved, quadrupled]
(iii) A certain cable has an insulation of relative permittivity 2 . If the insulation is replaced by one of relative permittivity 4 , then capacitance of cable is $\qquad$ [doubled, halved]
(iv) The minimum dielectric stress in a cable is at .................. [conductor surface, lead sheath]
(v) If a cable of homogeneous insulation has maximum stress of $5 \mathrm{kV} / \mathrm{mm}$, then the dielectric strength of insulation should be. $\qquad$ [ $5-\mathrm{kV} / \mathrm{mm}, 2 \cdot 5 \mathrm{kV} / \mathrm{mm}, 3 \mathrm{kV} / \mathrm{mm}$ ]
(vi) In capacitance grading of cables, we use a $\qquad$ dielectric.
[homogeneous, composite]
(vii) For the same safe potential, the size of a graded cable will be $\qquad$ [less, more]
(viii) For operating voltages beyond 66 kV , $\qquad$ cables are used.
[Belted, S.L. type, oil-filled]
(ix) Voids in the layers of impregnated paper insualtion $\qquad$ the breakdown voltage of the cable. [increase, decrease]
( $x$ ) For voltages beyond 66 kV , 3-phase system usually employs. $\qquad$

# ANSWERS TO SELF-TEST <br> 1. (i) more (ii) closer spacing (iii) moisture (iv) excessive sheath losses (v) impregnated paper (vi) 11 (vii) tangential stresses (viii) presence of voids (ix) decreases ( $x$ ) conductor surface <br> 2. (i) 3-core cable (ii) doubled (iii) doubled (iv) lead sheath (v) $5 \mathrm{kV} / \mathrm{mm}$ (vi) composite (vii) less (viii) oil-filled (ix) decrease <br> (x) 3 single-core cables 

## CHAPTER REVIEW TOPICS

1. Compare the merits and demerits of underground system versus overhead system.
2. With a neat diagram, show the various parts of a high voltage single-core cable.
3. What should be the desirable characteristics of insulating materials used in cables ?
4. Describe briefly some commonly used insulating materials for cables.
5. What is the most general criterion for the classification of cables ? Draw the sketch of a single-core low tension cable and label the various parts.
6. Draw a neat sketch of the cross-section of the following :
(i) 3-core belted cable
(ii) H-type cable
(iii) S.L. type cable
7. What are the limitations of solid type cables ? How are these overcome in pressure cables ?
8. Write a brief note on oil-filled cables.
9. Describe the various methods of laying underground cables. What are the relative advantages and disadvantages of each method?
10. Derive an expression for the insulation resistance of a single-core cable.
11. Deduce an expression for the capacitance of a single-core cable.
12. Show that maximum stress in a single-core cable is

$$
\frac{2 V}{d \log _{e} D / d}
$$

where $V$ is the operating voltage and $d$ and $D$ are the conductor and sheath diameter.
13. Prove that $g_{\max } / g_{\min }$ in a single-core cable is equal to $D / d$.
14. Find an expression for the most economical conductor size of a single core cable.
15. Explain the following methods of cable grading :
(i) Capacitance grading
(ii) Intersheath grading
16. Write short notes on the following :
(i) Laying of 11 kV underground power cable
(ii) Capacitance grading in cables
(iii) Capacitance of 3-core belted cables
17. Derive an expression for the thermal resistance of dielectric of a single-core cable.
18. What do you mean by permissible current loading of an underground cable ?
19. With a neat diagram, describe Murray loop test for the location of (i) earth fault (ii) short-circuit fault in an underground cable.
20. Describe Varley loop test for the location of earth fault and short-circuit fault in an underground cable.

## DISCUSSION QUESTIONS

1. Overhead system can be operated at 400 kV or above but underground system offers problems at such voltages. Why?
2. Why are VIR cables preferred to paper insulated cables for smaller installations ?
3. Why do we use 3 single-core cables and not 3-core cables for voltages beyond 66 kV ?
4. What is the mechanism of breakdown of an underground cable ?
5. How do voids in the insulation cause breakdown of the cable ?

[^0]:    * Special precautions have to be taken to preclude moisture at joints. If the number of joints is more, the installation cost increases rapidly and prohibits the use of paper insulated cables.

[^1]:    * Separate single-core cable for each phase.
    ** It is infact a leakage current but should not be confused with the capacitance current.

[^2]:    $\dagger$ The four screens and lead sheath are in electrical contact and lead sheath is at earth potential.

    * In this arrangement, each core is separately lead sheathed and hence the name S.L. cable.

[^3]:    $\dagger$ Voids are unintentional spaces in the insulation of cable filled with air or gas, usually at low pressure.

[^4]:    * It may be recalled $\varepsilon_{r}=\varepsilon / \varepsilon_{0}$ where $\varepsilon_{0}$ is the permittivity of free space. In the SI units, $\varepsilon_{0}=8.854 \times 10^{-12} \mathrm{~F} / \mathrm{m}$.

[^5]:    * It may be recalled that potential gradient at any point is equal to the electric intensity at that point.

[^6]:    * Of course, it will occur at the conductor surface.
    $\dagger$ Note that unit will be $\mathrm{kV} / \mathrm{cm}$. It is because $V$ in the numerator is in kV and $d$ in the denominator is in cm .

[^7]:    * $g_{2 \max }=\frac{Q}{\pi \varepsilon_{0} \varepsilon_{2} d_{1}}$. Putting the value of $Q=g_{2 \max } \pi \varepsilon_{0} \varepsilon_{2} d_{1}$ in exp. (i), we get the exp. (iii).

[^8]:    * This equation is obtained if we put the values of eq. (ii) in eq. (i).

[^9]:    * Refer to Fig. 11.19. The capacitance between any two conductors of star and delta connected system must be the same.
    $\therefore$

    $$
    \begin{aligned}
    C_{c}+\frac{1}{2} C_{c} & =\frac{1}{2} C_{e q} \\
    C_{e q} & =3 C_{c}
    \end{aligned}
    $$

[^10]:    * Note that the term $Q / P+Q$ is dimensionless ; being the ratio of resistances.

