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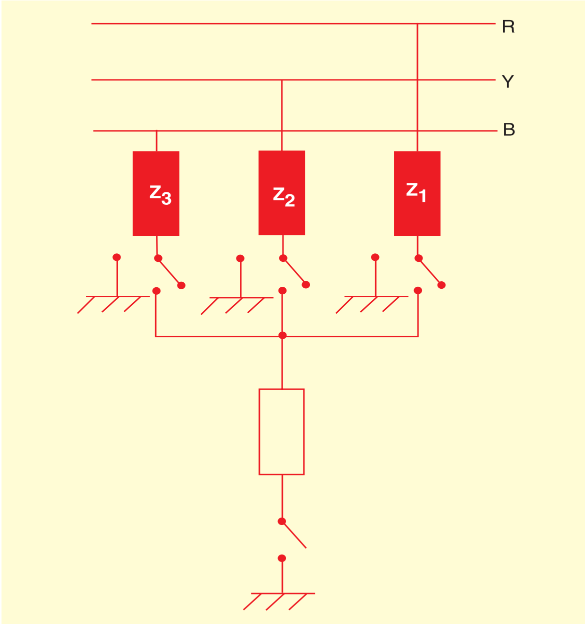

CHAPTER

18

Unsymmetrical Fault Calculations

Introduction

In the previous chapter, attention was confined to the analysis of symmetrical faults *e.g.* all three lines short-circuited (*L — L — L*) *or* all three lines short-circuited with an earth connection at the fault (*L —L — L — G*). When such a fault occurs, it gives rise to symmetrical fault currents *i.e.* fault currents in the three lines are equal in magnitude and displaced 120º electrical from one another. Although symmetrical faults are the most severe and impose heavy duty on the circuit breakers, yet the analysis of such faults can be made with a fair degree of ease. It is because the balanced nature of fault permits to consider only one phase in calculations ; the conditions in the other two phases being similar.

The great majority of faults on the power system are of unsymmetrical nature; the most common type being a short-circuit from one line to ground. When such a fault occurs, it gives rise to unsymmetrical currents *i.e.* the magnitude of fault currents in the three lines are different having unequal phase displacement. The calculation procedure known as *method of symmetrical components* is used to determine the currents and voltages on the occurrence of an unsymmetrical fault. In this chapter, we shall focus our attention on the analysis of unsymmetrical faults.

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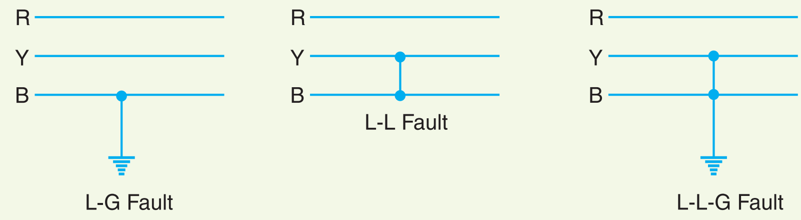
18.1 Unsymmetrical Faults on 3-Phase System

Those faults on the power system which give rise to unsymmetrical fault currents (i.e. unequal fault currents in the lines with unequal phase displacement) are known as **unsymmetrical faults.**

On the occurrence of an unsymmetrical fault, the currents in the three lines become unequal and so is the phase displacement among them. It may be noted that the term 'unsymmetry' applies only to the fault itself and the resulting line currents. However, the system impedances and the source voltages are always symmetrical* through its main elements *viz.* generators, transmission lines, synchoronous reactors *etc.* There are three ways in which unsymmetrical faults may occur in a power system (see Fig. 18.1).

(*i*) Single line-to-ground fault ($L — G$)
(*ii*) Line-to-line fault ($L — L$)
(*iii*) Doube line-to-ground fault ($L — L — G$)



(*i*) (*ii*) (*iii*)

Fig. 18.1

The solution of unsymmetrical fault problems can be obtained by either (*a*) Kirchhoff's laws or (*b*) Symmetrical components method. The latter method is preferred because of the following reasons :

(*i*) It is a simple method and gives more generality to be given to fault performance studies.
(*ii*) It provides a useful tool for the protection engineers, particularly in connection with tracing out of fault currents.

Electronic earth fault indicator

18.2 Symmetrical Components Method

In 1918, Dr. C.L. Fortescue, an American scientist, showed that any unbalanced system of 3-phase currents (or voltages) may be regarded as being composed** of three separate sets of balanced vectors *viz.*

* In other words, no piece of equipment ever has a red phase impedance which differs from a yellow phase impedance.

** This has come to be known as *symmetrical component theory.* This is a general theory and is applicable to any three vector system whose resultant is zero.

(*i*) a balanced *system of 3-phase currents having positive† (or normal) phase sequence. These are called *positive phase sequence components.*

(*ii*) a balanced system of 3-phase currents having the opposite or negative phase sequence. These are called *negative phase sequence components.*

(*iii*) a system of three currents equal in magnitude and having zero phase displacement. These are called *zero phase sequence components.*

The positive, negative and zero phase sequence components are called the *symmetrical components* of the original unbalanced system. The term 'symmetrical' is appropriate because the unbalanced 3-phase system has been resolved into three sets of balanced (or symmetrical) components. The subscripts 1, 2 and 0 are generally used to indicate positive, negative and zero phase sequence components respectively. For instance, $\overrightarrow{I_{R0}}$ indicates the zero phase sequence component of the current in the red phase. Similarly, $\overrightarrow{I_{Y1}}$ implies the positive phase sequence component of current in the yellow phase.

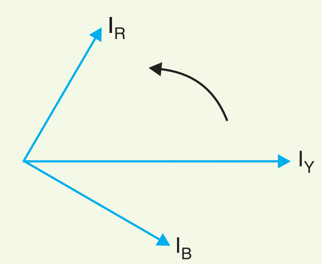


Fig. 18.2

Illustration. Let us now apply the symmetrical components theory to an unbalanced 3-phase system. Suppose an unsymmetrical fault occurs on a 3-phase system having phase sequence *RYB*. According to symmetrical components theory, the resulting unbalanced currentes $\overrightarrow{I_R}$, $\overrightarrow{I_Y}$ and $\overrightarrow{I_B}$ (see Fig. 18.2) can be resolved into :

(*i*) a balanced system of 3-phase currents, $\overrightarrow{I_{R1}}$, $\overrightarrow{I_{Y1}}$ and $\overrightarrow{I_{B1}}$ having positive phase sequence (*i.e. RYB*) as shown in Fig. 18.3 (*i*). These are the positive phase sequence components.

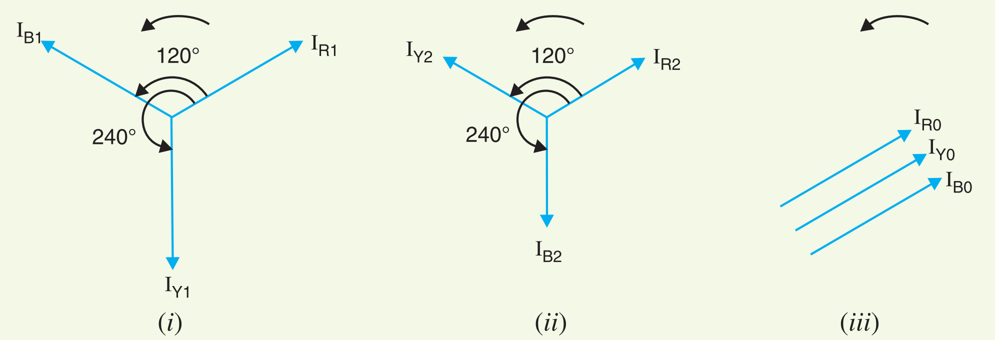


Fig. 18.3

(*ii*) a balanced system of 3-phase currents $\overrightarrow{I_{R2}}$, $\overrightarrow{I_{Y2}}$ and $\overrightarrow{I_{B2}}$ having negative phase sequence (*i.e. RBY*) as shown in Fig. 18.3 (*ii*). These are the negative phase sequence components.

(*iii*) a system of three currents $\overrightarrow{I_{R0}}$, $\overrightarrow{I_{Y0}}$ and $\overrightarrow{I_{B0}}$ equal in magnitude with zero phase displacement from each other as shown in Fig. 18.3 (*iii*). These are the zero phase sequence components.

* A balanced system of 3-phase currents implies that three currents are equal in magnitude having 120° displacement from each other.

† Positive phase sequence means that phase sequence is the same as that of the original 3-phase system.

The current in any phase is equal to the vector sum of positive, negative and zero phase sequence currents in that *phase as shown in Fig. 18.4.

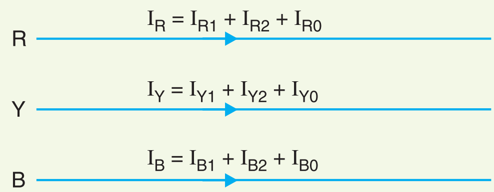


Fig. 18.4

$$\vec{I_R} = \vec{I_{R1}} + \vec{I_{R2}} + \vec{I_{R0}}$$
$$\vec{I_Y} = \vec{I_{Y1}} + \vec{I_{Y2}} + \vec{I_{Y0}}$$
$$\vec{I_B} = \vec{I_{B1}} + \vec{I_{B2}} + \vec{I_{B0}}$$

The following points may be noted :

(*i*) The positive phase sequence currents ($\vec{I_{R1}}$, $\vec{I_{Y1}}$ and $\vec{I_{B1}}$), negative phase sequence currents ($\vec{I_{R2}}$, $\vec{I_{Y2}}$ and $\vec{I_{B2}}$) and zero phase sequence currents ($\vec{I_{R0}}$, $\vec{I_{Y0}}$ and $\vec{I_{B0}}$) separately form balanced system of currents. Hence, they are called symmetrical components of the unbalanced system.

(*ii*) The symmetrical component theory applies equally to 3-phase currents and voltages both phase and line values.

(*iii*) The symmetrical components do not have separate existence. They are only mathematical components of unbalanced currents (or voltages) which actually flow in the system.

(*iv*) In a balanced 3-phase system, negative and zero phase sequence currents are zero. This is demonstrated in example 18.7.

18.3 Operator 'a'

As the symmetrical component theory involves the concept of 120º displacement in the positive sequence set and negative sequence set, therefore, it is desirable to evolve some operator which should cause 120º rotation. For this purpose, operator '*a*' (symbols *h* or λ are sometimes used instead of '*a*') is used. It is defined as under :

The ****operator 'a'*** *is one, which when multiplied to a vector rotates the vector through 120º in the anticlockwise direction.*

Consider a vector *I* represented by *OA* as shown in Fig. 18.5. If this vector is multiplied by operator '*a*', the vector is rotated through 120º in the anticlockwise direction and assumes the position *OB*.

$$\therefore \quad a\,I = I \angle 120^\circ$$
$$= I\,(\cos 120^\circ + j \sin 120^\circ)$$

* Star connected system being considered in Fig. 18.4.

** Just as the operator *j* rotates a vector through 90º in the anticlockwise direction.

$$= I(-0{\cdot}5 + j\,0{\cdot}866)$$

$$\therefore \quad a = -0{\cdot}5 + j\,0{\cdot}866 \qquad \ldots (i)$$

If the vector assuming position *OB* is multiplied by operator '*a*', the vector is further rotated through 120° in the anticlockwise direction and assumes the position *OC*.

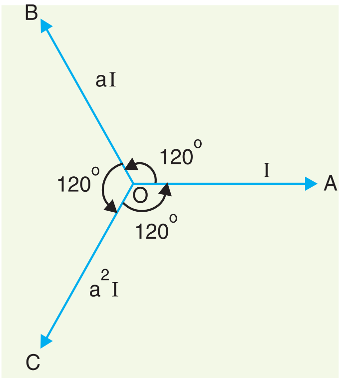


Fig. 18.5

$$\therefore \quad a^2 I = I\angle 240°$$

$$= I(\cos 240° + j \sin 240°)$$

$$= I(-0{\cdot}5 - j\,0{\cdot}866)$$

$$\therefore \quad a^2 = -0{\cdot}5 - j\,0{\cdot}866 \qquad \ldots (ii)$$

Thus the operator 'a^2' will turn the vector through 240° in the anticlockwise direction. This is the same as turning the vector through 120° in clockwise direction.

$$\therefore \quad a^2 I = I\angle -120°$$

$$\text{Similarly,} \quad a^3 I = I\angle 360°$$

$$= I(\cos 360° + j \sin 360°)$$

$$\therefore \quad a^3 = 1 \qquad \ldots (iii)$$

Properties of Operator '*a*'

(*i*) Adding exps. (*i*) and (*ii*), we get,

$$a + a^2 = (-0{\cdot}5 + j\,0{\cdot}866) + (-0{\cdot}5 - j\,0{\cdot}866) = -1$$

$$\therefore \quad 1 + a + a^2 = 0$$

(*ii*) Subtracting exp. (*ii*) from exp. (*i*), we get,

$$a - a^2 = (-0{\cdot}5 + j\,0{\cdot}866) - (-0{\cdot}5 - j\,0{\cdot}866) = j\,1{\cdot}732$$

$$\therefore \quad a - a^2 = j\sqrt{3}$$

18.4 Symmetrical Components in Terms of Phase Currents

The unbalanced phase currents in a 3-phase system can be expressed in terms of symmetrical components as under :

$$\overrightarrow{I_R} = \overrightarrow{I_{R1}} + \overrightarrow{I_{R2}} + \overrightarrow{I_{R0}}$$

$$\overrightarrow{I_Y} = \overrightarrow{I_{Y1}} + \overrightarrow{I_{Y2}} + \overrightarrow{I_{Y0}}$$

$$\overrightarrow{I_B} = \overrightarrow{I_{B1}} + \overrightarrow{I_{B2}} + \overrightarrow{I_{B0}}$$

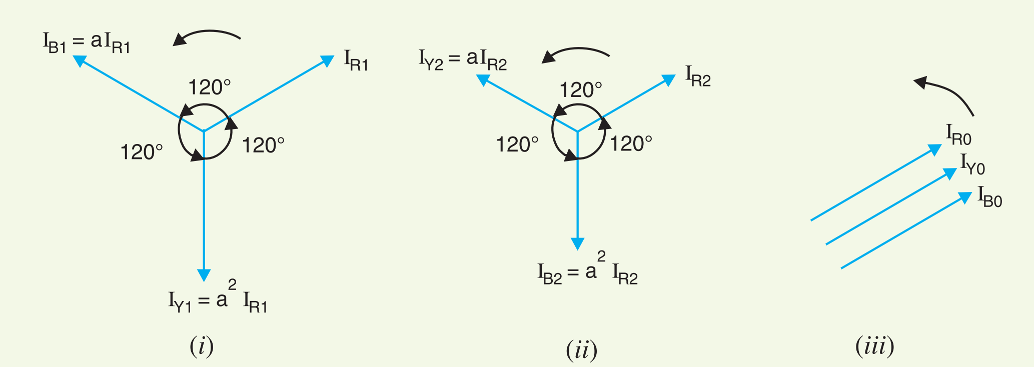


Fig. 18.6

Fig. 18.6 shows the vector representation of symmetrical components. It is usually profitable in calculations to express the symmetrical components in terms of unbalanced phase currents. Let us express the symmetrical components of *R*-phase in terms of phase currents $\vec{I_R}$, $\vec{I_Y}$ and $\vec{I_B}$. For this purpose, express all symmetrical components of *Y* and *B* phases in terms of the symmetrical components of R-phase by means of operator '*a*' as shown in Fig. 18.6.

Note that the positive sequence set shown in Fig. 18.6 (*i*) can be expressed in terms of $\vec{I_{R1}}$ by means of operator *a*. Thus positive sequence current $\vec{I_{B1}}$ in phase *B* leads $\vec{I_{R1}}$ by 120º and, therefore, $\vec{I_{B1}} = a\,\vec{I_{R1}}$. Similarly, positive sequence current in phase *Y* is 240º ahead of $\vec{I_{R1}}$ so that $\vec{I_{Y1}} = a^2\,\vec{I_{R1}}$. In an exactly similar manner, the negative sequence set can be expressed in terms of $\vec{I_{R2}}$ by means of operator '*a*' as shown in Fig. 18.6(*ii*). It is clear from Fig. 18.6 that :

$$\vec{I_R} = \vec{I_{R1}} + \vec{I_{R2}} + \vec{I_{R0}} \quad ...(i)$$

$$\vec{I_Y} = \vec{I_{Y1}} + \vec{I_{Y2}} + \vec{I_{Y0}}$$

$$= a^2\,\vec{I_{R1}} + a\,\vec{I_{R2}} + \vec{I_{R0}} \quad ...(ii)$$

$$\vec{I_B} = \vec{I_{B1}} + \vec{I_{B2}} + \vec{I_{B0}}$$

$$= a\,\vec{I_{R1}} + a^2\,\vec{I_{R2}} + \vec{I_{R0}} \quad ...(iii)$$

(*i*) Zero sequence current. By adding exps. (*i*), (*ii*) and (*iii*), we get,

$$\vec{I_R} + \vec{I_Y} + \vec{I_B} = \vec{I_{R1}}\,(1 + a^2 + a) + \vec{I_{R2}}\,(1 + a + a^2) + 3\vec{I_{R0}}$$

$$= \vec{I_{R1}}\,(0) + \vec{I_{R2}}\,(0) + 3\,\vec{I_{R0}} = 3\,\vec{I_{R0}} \qquad (\because\ 1 + a + a^2 = 0)$$

$$\therefore \quad \vec{I_{R0}} = \frac{1}{3}\left(\vec{I_R} + \vec{I_Y} + \vec{I_B}\right)$$

As the red phase is always taken as the reference phase, therefore, subscript *R* is usually omitted.

$$\therefore \quad \vec{I_0} = \frac{1}{3}\left(\vec{I_R} + \vec{I_Y} + \vec{I_B}\right)$$

(*ii*) Positive sequence current. Multiply exp.(*ii*) by '*a*' and exp. (*iii*) by 'a^2' and then adding these exps. to exp. (*i*), we get,

$$\vec{I_R} + a\,\vec{I_Y} + a^2\,\vec{I_B} = \vec{I_{R1}}\,(1 + a^3 + a^3) + \vec{I_{R2}}\,(1 + a^2 + a^4) + \vec{I_{R0}}\,(1 + a + a^2)$$

$$= 3\,\vec{I_{R1}} + \vec{I_{R2}}\,(0\,{*}) + \vec{I_{R0}}\,(0) = 3\,\vec{I_{R1}}$$

$$\therefore \quad \vec{I_{R1}} = \frac{1}{3}\left(\vec{I_R} + a\,\vec{I_Y} + a^2\vec{I_B}\right)$$

Omitting the subscript *R*, we have,

$$\vec{I_1} = \frac{1}{3}\left(\vec{I_R} + a\,\vec{I_Y} + a^2\vec{I_B}\right)$$

(*iii*) Negative sequence current. Multiply exp. (*ii*) by 'a^2' and exp. (*iii*) by '*a*' and then adding these exps. to (*i*), we get,

$$\vec{I_R} + a^2\vec{I_Y} + a\,\vec{I_B} = \vec{I_{R1}}\,(1 + a^4 + a^2) + \vec{I_{R2}}\,(1 + a^3 + a^3) + \vec{I_{R0}}\,(1 + a^2 + a)$$

$$= \vec{I_{R1}}\,(0) + \vec{I_{R2}}\,(3) + \vec{I_{R0}}\,(0) = 3\,\vec{I_{R2}}$$

$$\therefore \quad \vec{I_{R2}} = \frac{1}{3}\left(\vec{I_R} + a^2\,\vec{I_Y} + a\,\vec{I_B}\right)$$

$$\text{or} \quad \vec{I_2} = \frac{1}{3}\left(\vec{I_R} + a^2\,\vec{I_Y} + a\,\vec{I_B}\right)$$

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* $a^4 = a^3 \times a = 1 \times a = a$

$\therefore \quad 1 + a^2 + a^4 = 1 + a^2 + a = 0$

The following points may be noted carefully :

(*i*) The currents $\vec{I_1}$, $\vec{I_2}$ and $\vec{I_0}$ are the symmetrical components of *R*-phase. Because of the symmetry of each set, the symmetrical components of yellow and blue phases can be easily known.

(*ii*) Although the treatment has been made considering currents, the method applies equally to voltages. Thus the symmetrical voltage components of *R*-phase in terms of phase voltages shall be :

$$\vec{E_0} = \frac{1}{3}\left(\vec{E_R} + \vec{E_Y} + \vec{E_B}\right)$$

$$\vec{E_1} = \frac{1}{3}\left(\vec{E_R} + a\,\vec{E_Y} + a^2\,\vec{E_B}\right)$$

$$\vec{E_2} = \frac{1}{3}\left(\vec{E_R} + a^2\,\vec{E_Y} + a\,\vec{E_B}\right)$$

18.5 Some Facts about Sequence Currents

It is now desirable to get the readers acquainted with the following facts about positive, negative and zero phase sequence currents :

(*i*) A balanced 3-phase system consists of positive sequence components only; the negative and zero sequence components being zero.

(*ii*) The presence of negative or zero sequence currents in a 3-phase system introduces unsymmetry and is indicative of an abnormal condition of the circuit in which these components are found.

(*iii*) The vector sum of the positive and negative sequence currents of an unbalanced 3-phase system is zero. The resultant solely consists of three zero sequence currents *i.e.*

Vector sum of all sequence currents in 3-phase unbalanced system

$$= \vec{I_{R0}} + \vec{I_{Y0}} + \vec{I_{B0}}$$

(*iv*) In a 3-phase, 4 wire unbalanced system, the magnitude of zero sequence components is one-third of the current in the neutral wire *i.e.*

$$\text{Zero sequence current} = \frac{1}{3}\,[\text{Current in neutral wire}]$$

In the absence of path through the neutral of a 3-phase system, the neutral current is zero and the line currents contain no zero -sequence components. *A delta-connected load provides no path to the neutral and the line currents flowing to delta-connected load can contain no zero-sequence components.*

(*v*) In a 3-phase unbalanced system, the magnitude of negative sequence components cannot exceed that of the positive sequence components. If the negative sequence components were the greater, the phase sequence of the resultant system would be reversed.

(*vi*) The current of a single phase load drawn from a 3-phase system comprises equal positive, negative and zero sequence components.

Example 18.1. *Prove that :*

(*i*) $\dfrac{1-a^2}{a-a^2} = -a$ (*ii*) $\dfrac{1-a}{1+a^2} = 1-a^2$

Solution.

(*i*) $\dfrac{1-a^2}{a-a^2} = \dfrac{(1+a)(1-a)}{a(1-a)} = \dfrac{1+a}{a} = -\dfrac{a^2}{a} = -a$ $(\because\ 1 + a + a^2 = 0)$

(ii) $\dfrac{1-a}{1+a^2} = \dfrac{1-a}{-a} = \dfrac{(1-a)(-a^2)}{(-a)(-a^2)} = \dfrac{-a^2+a^3}{a^3} = 1-a^2$

Example 18.2. *In a 3-phase, 4-wire system, the currents in R, Y and B lines under abnormal conditions of loading are as under :*

$$\vec{I_R} = 100\ \angle 30^o\ A \quad ; \quad \vec{I_Y} = 50\ \angle 300^o\ A \quad ; \quad \vec{I_B} = 30\ \angle 180^o\ A$$

Calculate the positive, negative and zero sequence currents in the R-line and return current in the neutral wire.

Solution. Let $\vec{I_0}$, $\vec{I_1}$ and $\vec{I_2}$ be the zero, positive and negative sequence currents respectively of the line current in red line.

$\therefore$
$$\begin{aligned}
\vec{I_0} &= \tfrac{1}{3}\left[\vec{I_R}+\vec{I_Y}+\vec{I_B}\right]\\
&= \tfrac{1}{3}[100\ \angle 30^o + 50\ \angle 300^o + 30\ \angle 180^o]\\
&= \tfrac{1}{3}\ {}^*[(86{\cdot}60\ + j\ 50) + (25 - j\ 43{\cdot}3) + (-30 + j0)]\\
&= \tfrac{1}{3}\ [\ 81{\cdot}6 + j\ 6{\cdot}7]\\
&= (27{\cdot}2 + j\ 2{\cdot}23) = \mathbf{27{\cdot}29\ \angle 4{\cdot}68^o\ A}
\end{aligned}$$

$$\begin{aligned}
\vec{I_1} &= \tfrac{1}{3}\left[\vec{I_R} + a\ \vec{I_Y} + a^2\ \vec{I_B}\right]\\
&= \tfrac{1}{3}\ [\ 100\ \angle 30^o + 1\ \angle 120^o \times 50\ \angle 300^o + 1\ \angle{-120^o} \times 30\ \angle 180^o]\\
&= \tfrac{1}{3}\ [100\ \angle 30^o + 50\ \angle 60^o + 30\ \angle 60^o]\\
&= \tfrac{1}{3}[(86{\cdot}6 + j\ 50) + (25 + j\ 43{\cdot}3) + (15 + j\ 25{\cdot}98)]\\
&= \tfrac{1}{3}\ [126{\cdot}6 + j\ 119{\cdot}28]\\
&= (42{\cdot}2 + j\ 39{\cdot}76) = \mathbf{57{\cdot}98\ \angle 43{\cdot}3^o\ A}
\end{aligned}$$

$$\begin{aligned}
\vec{I_2} &= \tfrac{1}{3}[\vec{I_R} + a^2\ \vec{I_Y} + a\ \vec{I_B}]\\
&= \tfrac{1}{3}[100\ \angle 30^o + 1\ \angle{-120^o} \times 50\ \angle 300^o + 1\ \angle 120^o \times 30\ \angle 180^o]\\
&= \tfrac{1}{3}[100\ \angle 30^o + 50\ \angle 180^o + 30\ \angle 300^o]\\
&= \tfrac{1}{3}[(86{\cdot}6 + j\ 50) + (-50 + j\ 0) + (15 - j\ 25{\cdot}98)]\\
&= \tfrac{1}{3}[51{\cdot}6 + j\ 24{\cdot}02]\\
&= (17{\cdot}2 + j\ 8) = \mathbf{18{\cdot}96\ \angle 24{\cdot}9^o\ A}
\end{aligned}$$

Current in the neutral wire $= \vec{I_R} + \vec{I_Y} + \vec{I_B} = (81{\cdot}6 + j\ 6{\cdot}7) = \mathbf{81{\cdot}87\ \angle 4{\cdot}7^o\ A}$

Example 18.3. *The currents in a 3-phase unbalanced system are :*

$$\vec{I_R} = (12 + j\ 6)\ A \quad ; \quad \vec{I_Y} = (12 - j\ 12)\ A \quad ; \quad \vec{I_B} = (-15 + j\ 10)\ A$$

The phase sequence in RYB. Calculate the zero, positive and negative sequence components of the currents.

Solution.

Red phase

Zero phase sequence component,

$$\vec{I_{R0}} = \frac{1}{3}\left[\vec{I_R} + \vec{I_Y} + \vec{I_B}\right]$$

* With the help of scientific calculator, *polar form* can be directly changed to *rectangular form* and *vice-versa*.

$$= \frac{1}{3}[(12 + j\,6) + (12 - j\,12) + (-15 + j\,10)]$$

$$= \frac{1}{3}[9 + j\,4] = \mathbf{(3 + j\,1{\cdot}33)}\,A$$

Positive phase sequence component is

$$\vec{I_{R1}} = \frac{1}{3}\left[\vec{I_R} + a\,\vec{I_Y} + a^2\,\vec{I_B}\right]$$

$$= \frac{1}{3}[(12 + j\,6) + (-0{\cdot}5 + j\,0{\cdot}866)(12 - j\,12) + {}^*(-0{\cdot}5 - j\,0{\cdot}866)(-15 + j\,10)]$$

$$= \frac{1}{3}[32{\cdot}55 + j\,30{\cdot}39] = \mathbf{(10{\cdot}85 + j\,10{\cdot}13)\,A}$$

Negative phase sequence component is

$$\vec{I_{R2}} = \frac{1}{3}\left[\vec{I_R} + a^2\,\vec{I_Y} + a\,\vec{I_B}\right]$$

$$= \frac{1}{3}[(12 + j\,6) + (-0{\cdot}5 - j\,0{\cdot}866)(12 - j\,12) + (-0{\cdot}5 + j\,0{\cdot}866)(-15 + j\,10)]$$

$$= \frac{1}{3}[-5{\cdot}55 - j\,16{\cdot}41] = \mathbf{(-1{\cdot}85 - j\,5{\cdot}47)}\,A$$

Yellow phase

Zero phase sequence component is

$$\vec{I_{Y0}} = \vec{I_{R0}} = \mathbf{(3 + j\,1{\cdot}33)}\,A$$

Positive phase sequence component is

$$\vec{I_{Y1}} = a^2\,\vec{I_{R1}} = (-0{\cdot}5 - j\,0{\cdot}866)(10{\cdot}85 + j\,10{\cdot}13)$$
$$= \mathbf{(3{\cdot}35 - j\,14{\cdot}4)}\,A$$

Negative phase sequence component is

$$\vec{I_{Y2}} = a\,\vec{I_{R2}} = (-0{\cdot}5 + j\,0{\cdot}866)(-1{\cdot}85 - j\,5{\cdot}47) = \mathbf{(5{\cdot}7 + j\,1{\cdot}13)}\,A$$

Blue phase

Zero phase sequence component is

$$\vec{I_{B0}} = \vec{I_{Y0}} = \vec{I_{R0}} = \mathbf{(3 + j\,1{\cdot}33)}\,A$$

Positive phase sequence component is

$$\vec{I_{B1}} = a^2\,\vec{I_{R1}} = (-0{\cdot}5 + j\,0{\cdot}866)(10{\cdot}85 + j\,10{\cdot}13) = \mathbf{(-14{\cdot}2 + j\,4{\cdot}31)}\,A$$

Negative phase sequence component is

$$\vec{I_{B2}} = a^2\,\vec{I_{R2}} = (-0{\cdot}5 - j\,0{\cdot}866)(-1{\cdot}85 - j\,5{\cdot}47) = \mathbf{(-3{\cdot}82 + j\,4{\cdot}34)}\,A$$

Example 18.4. *The sequence voltages in the red phase are as under :*

$\vec{E_{R0}} = 100$ V ; $\quad \vec{E_{R1}} = (200 - j\,100)$ V ; $\quad \vec{E_{R2}} = -100$ V

Find the phase voltages $\vec{E_R}$, $\vec{E_Y}$ and $\vec{E_B}$.

Solution. In the polar form, we have,

$\vec{E_{R0}} = 100\,\angle 0°$ V; $\quad \vec{E_{R1}} = 223{\cdot}6\,\angle{-26{\cdot}56°}$ V; $\quad \vec{E_{R2}} = 100\,\angle 180°$ V

$$\vec{E_R} = \vec{E_{R0}} + \vec{E_{R1}} + \vec{E_{R2}}$$
$$= 100 + (200 - j\,100) + (-100)$$
$$= 200 - j\,100 = \mathbf{223{\cdot}6\,\angle{-26{\cdot}56°}\ volts}$$

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* $a = -0{\cdot}5 + j\,0{\cdot}866$ and $a^2 = -0{\cdot}5 - j\,0{\cdot}866$

$$\vec{E_Y} = \vec{E_{R0}} + a^2\vec{E_{R1}} + a\vec{E_{R2}}$$
$$= 100\angle 0^\circ + 1\angle 240^\circ \times 223{\cdot}6\angle{-26{\cdot}56^\circ} + 1\angle 120^\circ \times 100\angle 180^\circ$$
$$= 100\angle 0^\circ + 223{\cdot}6\angle 213{\cdot}44^\circ + 100\angle 300^\circ$$
$$= (100 + j\,0) + (-186{\cdot}58 - j\,123{\cdot}2) + (50 - j\,86{\cdot}6)$$
$$= -36{\cdot}58 - j\,209{\cdot}8 = \mathbf{213\angle{-99{\cdot}89^\circ}\ volts}$$

$$\vec{E_B} = \vec{E_{R0}} + a\vec{E_{R1}} + a^2\vec{E_{R2}}$$
$$= 100\angle 0^\circ + 1\angle 120^\circ \times 223{\cdot}6\angle{-26{\cdot}56^\circ} + 1\angle 240^\circ \times 100\angle 180^\circ$$
$$= 100\angle 0^\circ + 223{\cdot}6\angle 93{\cdot}44^\circ + 100\angle 420^\circ$$
$$= (100 + j\,0) + (-13{\cdot}4 + j\,223{\cdot}2) + (50 + j\,86{\cdot}6)$$
$$= 136{\cdot}6 + j\,309{\cdot}8 = \mathbf{338{\cdot}57\angle 66{\cdot}2^\circ\ volts}$$

Example 18.5. *The zero and positive sequence components of red phase are as under :*

$\vec{E_{R0}} = (0{\cdot}5 - j\,0{\cdot}866)\ V$; $\vec{E_{R1}} = 2\angle 0^\circ\ V$

If the phase voltage $\vec{E_R} = 3\angle 0^\circ$ V, *find the negative sequence component of red phase and the phase voltages* $\vec{E_Y}$ *and* $\vec{E_B}$.

Solution.

$$\vec{E_R} = \vec{E_{R0}} + \vec{E_{R1}} + \vec{E_{R2}}$$

or $$3 = (0{\cdot}5 - j\,0{\cdot}866) + 2 + \vec{E_{R2}}$$

∴ Negative sequence component in *R*-phase is

$$\vec{E_{R2}} = 0{\cdot}5 + j\,0{\cdot}866 = \mathbf{1\angle 60^\circ\ volts}$$

In polar form, $$\vec{E_{R0}} = 0{\cdot}5 - j\,0{\cdot}866 = 1\angle{-60^\circ}$$

Now $$\vec{E_Y} = \vec{E_{R0}} + a^2\vec{E_{R1}} + a\vec{E_{R2}}$$
$$= [1\angle{-60^\circ}] + [1\angle 240^\circ \times 2\angle 0^\circ] + [1\angle 120^\circ \times 1\angle 60^\circ]$$
$$= 1\angle{-60^\circ} + 2\angle 240^\circ + 1\angle 180^\circ$$
$$= (0{\cdot}5 - j\,0{\cdot}866) + (-1 - j\,1{\cdot}732) + (-1 + j\,0)$$
$$= -1{\cdot}5 - j\,2{\cdot}598$$
$$= \mathbf{3\angle{-120^\circ}\ volts}$$

$$\vec{E_B} = \vec{E_{R0}} + a\vec{E_{R1}} + a^2\vec{E_{R2}}$$
$$= [1\angle{-60^\circ}] + [1\angle 120^\circ \times 2\angle 0^\circ] + [1\angle 240^\circ \times 1\angle 60^\circ]$$
$$= 1\angle{-60^\circ} + 2\angle 120^\circ + 1\angle 300^\circ$$
$$= (0{\cdot}5 - j\,0{\cdot}866) + (-1 + j\,1{\cdot}732) + (0{\cdot}5 - j\,0{\cdot}866)$$
$$= \mathbf{0\ volt}$$

Example 18.6. *The current from neutral to ground connection is 12 A. Calculate the zero phase sequence components in phases.*

Solution. We know that zero sequence components in all phases have the same value and that each component is equal to one-third the current in the neutral wire.

∴ Zero sequence current in each phase

$$= \frac{1}{3} \times 12 = \mathbf{4\ A}$$

Example 18.7. *A balanced star connected load takes 90 A from a balanced 3-phase, 4-wire supply. If the fuses in the Y and B phases are removed, find the symmetrical components of the line currents*

(*i*) *before the fuses are removed* (*ii*) *after the fuses are removed*

Solution. Fig. 18.7. shows the star-connected system with fuses in phases *B* and *Y*.

(*i*) Before removal of fuses. Before fuses are removed from *Y* and *B* lines, the system is balanced and current in each line is 90 A.

$\therefore \quad \vec{I_R} = 90\angle 0°\text{A}\ ; \qquad \vec{I_Y} = 90\angle 240°\ \text{A}\ ; \qquad \vec{I_B} = 90\angle 120°\text{A}$

Since the system is balanced, it will have only positive sequence currents *i.e.*, negative sequence and zero sequence components will be zero in the three lines. This can be readily established.

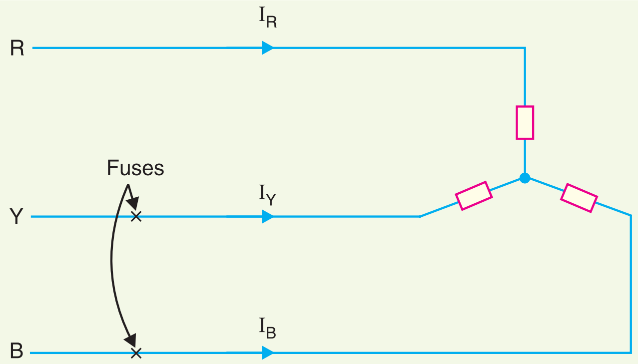

Fig. 18.7

$$\vec{I_{R0}} = \vec{I_{Y0}} = \vec{I_{B0}}$$
$$= \frac{1}{3}\left[\vec{I_R} + \vec{I_Y} + \vec{I_B}\right] = \frac{1}{3}\,[90\angle 0° + 90\angle 240° + 90\angle 120°]$$
$$= \frac{1}{3}\,[90\angle 0° + 90\angle -120° + 90\angle 120°] = 0\ \text{A}$$

Hence zero sequence components in three lines are zero.

$$\vec{I_{R2}} = \frac{1}{3}\left[\vec{I_R} + a^2\,\vec{I_Y} + a\,\vec{I_B}\right]$$
$$= \frac{1}{3}\,[\,90\angle 0° + 1\angle -120° \times 90\angle 240° + 1\angle 120° \times 90\angle 120°]$$
$$= \frac{1}{3}\,[90\angle 0° + 90\angle 120° + 90\angle 240°]$$
$$= \frac{1}{3}\,[\,90\angle 0° + 90\angle 120° + 90\angle -120°] = 0\ \text{A}$$

Also $\quad \vec{I_{Y2}} = a\,\vec{I_{R2}} = 1\angle 120° \times 0 = 0\ \text{A}$

and $\quad \vec{I_{B2}} = a^2\,\vec{I_{R2}} = 1\angle 240° \times 0 = 0\ \text{A}$

Hence negative sequence components in the three lines are also zero. It can be easily shown that three positive sequence components will have the following values :

$\vec{I_{R1}} = \vec{I_R} = 90\angle 0°\ \text{A}\ ; \qquad \vec{I_{Y1}} = \vec{I_Y} = 90\angle 240°\ \text{A}; \qquad \vec{I_{B1}} = \vec{I_B} = 90\angle 120°\ \text{A}$

***(ii)* After removal of fuses.** When the fuses are removed in Y and B phases, the system becomes unbalanced with line currents as under :

$$\vec{I_R} = 90\angle 0^\circ A \quad ; \quad \vec{I_Y} = \vec{I_B} = 0\ A$$

The sequence currents in the three lines can be found out as under :

$$\vec{I_{R0}} = \vec{I_{Y0}} = \vec{I_{B0}}$$
$$= \frac{1}{3}\left[\vec{I_R} + \vec{I_Y} + \vec{I_B}\right]$$
$$= \frac{1}{3}[90\angle 0^\circ + 0 + 0] = \mathbf{30\ \angle 0^\circ\ A}$$

i.e. zero sequence current in each line is 30 $\angle 0^\circ$ A.

$$\vec{I_{R1}} = \frac{1}{3}\left[\vec{I_R} + a\,\vec{I_Y} + a^2\,\vec{I_B}\right]$$
$$= \frac{1}{3}[90\angle 0^\circ + 0 + 0] = \mathbf{30\ \angle 0^\circ A}$$
$$\vec{I_{Y1}} = a^2\,\vec{I_{R1}} = 1\ \angle 240^\circ \times 30\ \angle 0^\circ = \mathbf{30\ \angle 240^\circ A}$$
$$\vec{I_{B1}} = a\,\vec{I_{R1}} = 1\ \angle 120^\circ \times 30\ \angle 0^\circ = \mathbf{30\ \angle 120^\circ A}$$
$$\vec{I_{R2}} = \frac{1}{3}\left[\vec{I_R} + a^2\,\vec{I_Y} + a\,\vec{I_B}\right]$$
$$= \frac{1}{3}[90\ \angle 0^\circ + 0 + 0] = \mathbf{30\ \angle 0^\circ A}$$
$$\vec{I_{Y2}} = a\,\vec{I_{R2}} = 1\ \angle 120^\circ \times 30\angle 0^\circ = \mathbf{30\ \angle 120^\circ\ A}$$
$$\vec{I_{B2}} = a^2\,\vec{I_{R2}} = 1\ \angle 240^\circ \times 30\angle 0^\circ = \mathbf{30\ \angle 240^\circ\ A}$$

The reader may wonder how sequence currents can flow in the yellow and blue lines when fuses are removed in them. The answer is that these components do not have separate existence. They are only the mathematical components of the current which does exist. Thus the current in the yellow line is zero and this can be readily established from its sequence components :

$$\vec{I_Y} = \vec{I_{Y0}} + \vec{I_{Y1}} + \vec{I_{Y2}}$$
$$= 30\ \angle 0^\circ + 30\ \angle 240^\circ + 30\ \angle 120^\circ$$
$$= 30\ \angle 0^\circ + 30\ \angle -120^\circ + 30\ \angle 120^\circ = 0\ A$$

Similary, it can be proved that sum of sequence currents in the blue line is zero and that is what the circuit reveals.

Example 18.8. *A 3-ϕ, 4-wire-system supplies loads which are unequally distributed in the three phases. An analysis of the current flowing in R, Y and B lines shows that in R line, positive phase sequence component is 200 $\angle 0^\circ$ A and the negative phase sequence component is 100 $\angle 60^\circ$A. The total observed current flowing back to the supply in the neutral conductor is 300 $\angle 300^\circ$ A. Calculate the currents in the three lines.*

Solution.

Zero phase sequence current in R-line is

$$\vec{I_{R0}} = \frac{1}{3} \times \text{Current in neutral wire}$$
$$= \frac{1}{3} \times 300\ \angle 300^\circ = 100\ \angle 300^\circ A$$

Positive phase sequence current in R-line is

$$\vec{I_{R1}} = 200\ \angle 0^\circ A$$

Negative phase sequence current in *R*-line is

$$\vec{I}_{R2} = 100 \angle 60^\circ \text{A}$$

$\therefore$ Current in the *R*-line, $\vec{I}_R = \vec{I}_{R0} + \vec{I}_{R1} + \vec{I}_{R2} = 100 \angle 300^\circ + 200 \angle 0^\circ + 100 \angle 60^\circ$

$= (50 - j\,86{\cdot}6) + (200 + j\,0) + (50 + j\,86{\cdot}6) =$ **$300 \angle 0^\circ$ A**

Current in the *Y*-line, $\vec{I}_Y = \vec{I}_{R0} + a^2\,\vec{I}_{R1} + a\,\vec{I}_{R2}$

$= 100 \angle 300^\circ + 1 \angle 240^\circ \times 200 \angle 0^\circ + 1\angle 120^\circ \times 100\angle 60^\circ$

$= 100 \angle 300^\circ + 200 \angle 240^\circ + 100 \angle 180^\circ$

$= (50 - j\,86{\cdot}6) + (-100 - j\,173{\cdot}2) + (-100 + j\,0)$

$= -150 - j\,259{\cdot}8 =$ **$300 \angle -120^\circ$ A**

Current in *B* line, $\vec{I}_B = \vec{I}_{R0} + a\,\vec{I}_{R1} + a^2\,\vec{I}_{R2}$

$= 100 \angle 300^\circ + 1 \angle 120^\circ \times 200 \angle 0^\circ + 1 \angle 240^\circ \times 100 \angle 60^\circ$

$= 100 \angle 300^\circ + 200 \angle 120^\circ + 100 \angle 300^\circ$

$= (50 - j\,86{\cdot}6) + (-100 + j\,173{\cdot}2) + (50 - j\,86{\cdot}6) =$ **0 A**

Example 18.9. *One conductor of a 3-phase line is open. The current flowing to the Δ-connected load through the line R is 10 A. With the current in line R [See Fig. 18.8] as reference and assuming that line B is open, find the symmetrical components of the line currents.*

Solution. The line currents are :

$\vec{I}_R = 10\angle 0^\circ$ A ; $\vec{I}_Y = 10\angle 180^\circ$ A ; $\vec{I}_B = 0$ A

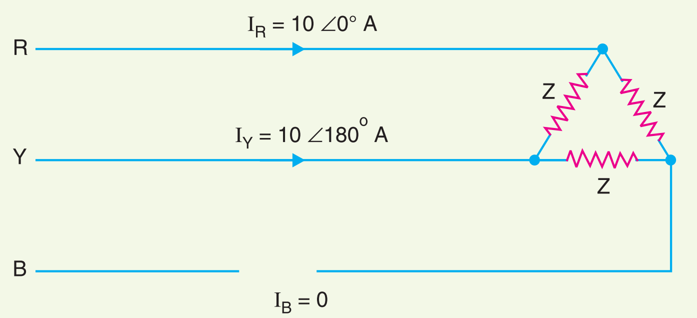

Fig. 18.8

***R*-line**

$$\vec{I}_{R0} = \frac{1}{3}\left[\vec{I}_R + \vec{I}_Y + \vec{I}_B\right] = \frac{1}{3}\,[\,10\angle 0^\circ + 10\angle 180^\circ + 0] = \textbf{0 A}$$

$$\vec{I}_{R1} = \frac{1}{3}\left[\vec{I}_R + a\,\vec{I}_Y + a^2\,\vec{I}_B\right] = \frac{1}{3}\,[\,10\angle 0^\circ + 1\angle 120^\circ \times 10\angle 180^\circ + 0]$$

$$= 5 - j\,2{\cdot}89 = \textbf{5·78}\ \angle\textbf{–30°A}$$

$$\vec{I}_{R2} = \frac{1}{3}\left[\vec{I}_R + a^2\,\vec{I}_Y + a\,\vec{I}_B\right] = \frac{1}{3}\,[\,10\angle 0^\circ + 1\angle 240^\circ \times 10\angle 180^\circ + 0]$$

$$= 5 + j\,2{\cdot}89 = \textbf{5·78}\ \angle\textbf{30°A}$$

***Y*-line**

$$\vec{I}_{Y0} = \vec{I}_{R0} = \textbf{0 A}$$

$$\vec{I}_{Y1} = a^2\,\vec{I}_{R1} = 1\angle 240^\circ \times 5{\cdot}78\angle -30^\circ = \textbf{5·78}\ \angle\textbf{–150° A}$$

$$\vec{I}_{Y2} = a\,\vec{I}_{R2} = 1\angle 120^\circ \times 5{\cdot}78\angle 30^\circ = \mathbf{5{\cdot}78\angle 150^\circ A}$$

***B*-line**

$$\vec{I}_{B0} = \vec{I}_{R0} = \mathbf{0\ A}$$

$$\vec{I}_{B1} = a\,\vec{I}_{R1} = 1\angle 120^\circ \times 5{\cdot}78\angle -30^\circ = \mathbf{5{\cdot}78\angle 90^\circ\ A}$$

$$\vec{I}_{B2} = a^2\,\vec{I}_{R2} = 1\angle 240^\circ \times 5{\cdot}78\angle 30^\circ = \mathbf{5{\cdot}78\angle -90^\circ A}$$

Note that components I_{B1} and I_{B2} have finite values although the line B is open and can carry no net current. As expected, the sum of I_{B1} and I_{B2} is zero. However, the sum of components in line R is $10\angle 0^\circ$A and the sum of components in line Y is $10\angle 180^\circ$A.

Example 18.10. *Three resistors of 5Ω, 10Ω and 20Ω are connected in delta across the three phases of a balanced 100 volts supply. What are the sequence components in the resistors and in supply lines ?*

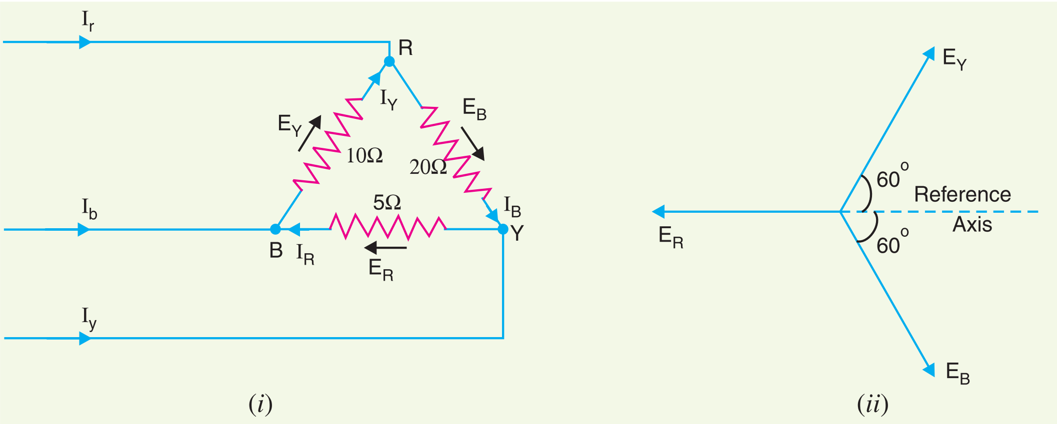

Fig. 18.9

Solution. Let the voltages across 5Ω, 10Ω and 20Ω be $\vec{E}_R$, $\vec{E}_Y$ and $\vec{E}_B$ respectively and the corresponding currents in the resistors be $\vec{I}_R$, $\vec{I}_Y$ and $\vec{I}_B$. These voltages can be represented by the vector diagram shown in Fig. 18.8 (*ii*).

$$\vec{E}_R = -100\angle 0^\circ \text{ V} \quad ; \quad \vec{E}_Y = 100\angle 60^\circ \text{ V} \quad ; \quad \vec{E}_B = 100\angle -60^\circ\text{V}$$

Current in 5Ω, $\quad \vec{I}_R = \vec{E}_R/5 = \dfrac{-100\angle 0^\circ}{5} = -20\angle 0^\circ$ A

Current in 10Ω, $\quad \vec{I}_Y = \vec{E}_Y/10 = \dfrac{100\angle 60^\circ}{10} = 10\angle 60^\circ$A

Current in 20Ω, $\quad \vec{I}_B = \vec{E}_B/20 = \dfrac{100\angle -60^\circ}{20} = 5\angle -60^\circ$A

Sequence currents in resistors

Zero sequence component of $\vec{I}_R$ is

$$\vec{I}_{R0} = \frac{1}{3}\left[\vec{I}_R + \vec{I}_Y + \vec{I}_B\right]$$

$$= \frac{1}{3}\,[-20\angle 0^\circ + 10\angle 60^\circ + 5\angle -60^\circ]$$

$$= \frac{1}{3}\,[(-20 + j\,0) + (5 + j\,8{\cdot}66) + (2{\cdot}5 - j\,4{\cdot}33)]$$

$$= \frac{1}{3}\,[-12{\cdot}5 + j\,4{\cdot}33]$$

$$= -\,4{\cdot}17 + j\,1{\cdot}44 = \mathbf{4{\cdot}41\,\angle 160{\cdot}9^\circ A}$$

Positive sequence component of $\vec{I_R}$ is

$$\vec{I_{R1}} = \frac{1}{3}\left[\vec{I_R} + a\,\vec{I_Y} + a^2\,\vec{I_B}\right]$$

$$= \frac{1}{3}\,[-20\,\angle 0^\circ + 1\,\angle 120^\circ \times 10\,\angle 60^\circ + 1\,\angle 240^\circ \times 5\,\angle -60^\circ]$$

$$= \frac{1}{3}\,[-20\,\angle 0^\circ + 10\,\angle 180^\circ + 5\,\angle 180^\circ]$$

$$= \frac{1}{3}\,[(-20 + j\,\,0) + (-10 + j\,0) + (-5 + j\,0)]$$

$$= \frac{1}{3}\,[-35 + j\,0]$$

$$= -11{\cdot}66 + j\,0 = \mathbf{11{\cdot}66\,\angle 180^\circ A}$$

Negative sequence component of $\vec{I_R}$ is

$$\vec{I_{R2}} = \frac{1}{3}\left[\vec{I_R} + a^2\,\vec{I_Y} + a\,\vec{I_B}\right]$$

$$= \frac{1}{3}\,[-20\angle 0^\circ + 1\,\angle 240^\circ \times 10\,\angle 60^\circ + 1\,\angle 120^\circ \times 5\,\angle -60^\circ]$$

$$= \frac{1}{3}\,[-20\,\angle 0^\circ + 10\,\angle 300^\circ\;\; + 5\,\angle 60^\circ]$$

$$= \frac{1}{3}\,[(-20 + j\,0) + (5 - j\,8{\cdot}66) + (2{\cdot}5 + j\,4{\cdot}33)]$$

$$= \frac{1}{3}\,[-\,12{\cdot}5 - j\,4{\cdot}33] = -\,4{\cdot}17 - j\,1{\cdot}44 = \mathbf{4{\cdot}4\,\angle -160{\cdot}9^\circ\;A}$$

The sequence components of $\vec{I_Y}$ and $\vec{I_B}$ can be found as under :

$$\vec{I_{Y0}} = \vec{I_{R0}} = \mathbf{4{\cdot}41\,\angle 160{\cdot}9^\circ\;A}$$

$$\vec{I_{Y1}} = a^2\,\vec{I_{R1}} = 1\,\angle 240^\circ \times 11{\cdot}66\,\angle 180^\circ = \mathbf{11{\cdot}66\,\angle 60^\circ\;A}$$

$$\vec{I_{Y2}} = a\,\vec{I_{R2}} = 1\,\angle 120^\circ \times 4{\cdot}4\,\angle -160{\cdot}9^\circ = \mathbf{4{\cdot}4\,\angle -40{\cdot}9^\circ\;A}$$

$$\vec{I_{B0}} = \vec{I_{R0}} = \mathbf{4{\cdot}41\,\angle 160{\cdot}9^\circ\;A}$$

$$\vec{I_{B1}} = a\,\vec{I_{R1}} = 1\,\angle 120^\circ \times 11{\cdot}66\,\angle 180^\circ = \mathbf{11{\cdot}66\,\angle 300^\circ\;A}$$

$$\vec{I_{B2}} = a^2\,\vec{I_{R2}} = 1\,\angle 240^\circ \times 4{\cdot}4\,\angle -160{\cdot}9^\circ = \mathbf{4{\cdot}4\,\angle 79{\cdot}1^\circ\;A}$$

Sequence currents in supply lines

Line current in R-line, $\vec{I_r} = \vec{I_B} - \vec{I_Y} = 5\,\angle -60^\circ - 10\,\angle 60^\circ$

$$= (2{\cdot}5 - j\,4{\cdot}33) - (5 + j\,8{\cdot}66)$$

$$= -\,2{\cdot}5 - j\,12{\cdot}99 = 13{\cdot}22\angle -100{\cdot}9^\circ\;A$$

Line current in Y-line, $\vec{I_y} = \vec{I_R} - \vec{I_B} = -\,20\angle 0^\circ - 5\,\angle -60^\circ$

$$= (-20 + j\,0) - (2{\cdot}5 - j\,4{\cdot}33)$$

$$= -22{\cdot}5 + j\,4{\cdot}33 = 22{\cdot}91\angle 169^\circ\;A$$

Line current in B-line, $\vec{I_b} = \vec{I_Y} - \vec{I_R} = 10\angle 60^\circ - (-20\angle 0^\circ)$

$$= (5 + j\,8{\cdot}66) - (-20 + j\,0)$$

$$= 25 + j\,8{\cdot}66 = 26{\cdot}45\angle 19{\cdot}1^\circ A$$

Zero sequence component of $\vec{I_r}$ is

$$\vec{I_{r0}}\,* = \frac{1}{3}\left(\vec{I_r}+\vec{I_y}+\vec{I_b}\right) = \frac{1}{3}[0] = \mathbf{0A}^{**}$$

Positive sequence component of $\vec{I_r}$ is

$$\vec{I_{r1}} = \frac{1}{3}\left(\vec{I_r}+a\,\vec{I_y}+a^2\,\vec{I_b}\right)$$

$$= \frac{1}{3}\left[\left(\vec{I_B}-\vec{I_Y}\right)+a\left(\vec{I_R}-\vec{I_B}\right)+a^2\left(\vec{I_Y}-\vec{I_R}\right)\right]$$

$$= \frac{1}{3}\left[a\left(\vec{I_R}+a\,\vec{I_Y}+a^2\,\vec{I_B}\right)-a^2\left(\vec{I_R}+a\,\vec{I_Y}+a^2\,\vec{I_B}\right)\right]$$

$[\because\ a^3 = 1 \text{ and } a^4 = a]$

$$= \frac{1}{3}\left[\left(a-a^2\right)\left(\vec{I_R}+a\,\vec{I_Y}+a^2\,\vec{I_B}\right)\right]$$

Now $\quad a-a^2 = j\sqrt{3}$ and $\vec{I_R}+a\,\vec{I_Y}+a^2\,\vec{I_B} = 3\,\vec{I_{R1}}$

$\therefore$

$$\vec{I_{r1}} = \frac{1}{3}\left[\left(j\sqrt{3}\right)\left(3\,\vec{I_{R1}}\right)\right]$$

$$= j\sqrt{3}\,\vec{I_{R1}} = j\sqrt{3}\,(-11{\cdot}66+j\,0)$$

$$= -j\,20{\cdot}2 = \mathbf{20{\cdot}2\,\angle{-90^\circ}\ A}$$

Negative sequence component of $\vec{I_r}$ is

$$\vec{I_{r2}} = \frac{1}{3}\left[\vec{I_r}+a^2\,\vec{I_y}+a\,\vec{I_b}\right]$$

$$= \frac{1}{3}\left[\left(\vec{I_B}-\vec{I_Y}\right)+a^2\left(\vec{I_R}-\vec{I_B}\right)+a\left(\vec{I_Y}-\vec{I_R}\right)\right]$$

$$= \frac{1}{3}\left[a^2\left(\vec{I_R}+a^2\,\vec{I_Y}+a\,\vec{I_B}\right)-a\left(\vec{I_R}+a^2\,\vec{I_Y}+a\,\vec{I_B}\right)\right]$$

$$= \frac{1}{3}\left[(a^2-a)\left(\vec{I_R}+a^2\,\vec{I_Y}+a\,\vec{I_B}\right)\right]$$

Now $\quad a^2-a = -j\sqrt{3}$ and $\vec{I_R}+a^2\,\vec{I_Y}+a\,\vec{I_B} = 3\,\vec{I_{R2}}$

$\therefore$

$$\vec{I_{r2}} = \frac{1}{3}\left(-j\sqrt{3}\times 3\,\vec{I_{R2}}\right) = -j\sqrt{3}\times\vec{I_{R2}}$$

$$= -j\sqrt{3}\times(-4{\cdot}17-j\,1{\cdot}44)$$

$$= -2{\cdot}5+j\,7{\cdot}2 = \mathbf{7{\cdot}62\ \angle 109{\cdot}1^\circ A}$$

Note. Incidentally, we have the formulas for relation among sequence components in the phases and lines.

$$\vec{I_{r1}} = j\sqrt{3}\,\vec{I_{R1}}\quad ;\quad \vec{I_{r2}} = -j\sqrt{3}\,\vec{I_{R2}}$$

Example 18.11. *A delta connected load is supplied from a 3-phase supply. The fuse in the B line is removed and current in the other two lines is 20 A. Find the symmetrical components of line currents.*

Solution. Let *R*, *Y* and *B* be the supply lines. When fuse in the line *B* is removed, the various line currents are :

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* Since vector sum of $\vec{I_r}+\vec{I_y}+\vec{I_b} = 0,\ \ \vec{I_{r0}} = 0$

** This shows that in delta formation, the zero sequence currents are present in phases but they disappear in line currents. As line current is the difference of two phase currents, therefore, the zero sequence components cancel out.

$$\vec{I_r} = 20 \angle 0^\circ \text{ A} ; \; \vec{I_y} = 20 \angle 180^\circ \text{ A} ; \; \vec{I_b} = 0 \text{ A}$$

***R*-line**

$$\vec{I_{r0}} = \frac{1}{3}\left[\vec{I_r} + \vec{I_y} + \vec{I_b}\right] = \frac{1}{3}[20 \angle 0^\circ + 20 \angle 180^\circ + 0]$$

$$= \frac{1}{3}[(20 + j0) + (-20 + j0) + 0] = \frac{1}{3}[0] = \mathbf{0A}$$

$$\vec{I_{r1}} = \frac{1}{3}\left[\vec{I_r} + a\,\vec{I_y} + a^2\,\vec{I_b}\right]$$

$$= \frac{1}{3}[20 \angle 0^\circ + 1 \angle 120^\circ \times 20 \angle 180^\circ + 0]$$

$$= \frac{1}{3}[20 \angle 0^\circ + 20 \angle 300^\circ] = \frac{1}{3}[(20 + j0) + (10 - j\,17{\cdot}32)]$$

$$= \frac{1}{3}[30 - j\,17{\cdot}32] = 10 - j\,5{\cdot}77 = \mathbf{11{\cdot}54 \angle{-30}^\circ A}$$

$$\vec{I_{r2}} = \frac{1}{3}\left[\vec{I_r} + a^2\,\vec{I_y} + a\,\vec{I_b}\right]$$

$$= \frac{1}{3}[20 \angle 0^\circ + 1 \angle 240^\circ \times 20 \angle 180^\circ + 0]$$

$$= \frac{1}{3}[20 \angle 0^\circ + 20 \angle 60^\circ] = \frac{1}{3}[(20 + j0) + (10 + j\,17{\cdot}32)]$$

$$= \frac{1}{3}[30 + j\,17{\cdot}32] = 10 + j\,5{\cdot}77 = \mathbf{11{\cdot}54 \angle 30^\circ A}$$

***Y*-line**

$$\vec{I_{y0}} = \vec{I_{r0}} = \mathbf{0A}$$

$$\vec{I_{y1}} = a^2\,\vec{I_{r1}} = 1 \angle 240^\circ \times 11{\cdot}54 \angle{-30}^\circ = \mathbf{11{\cdot}54 \angle 210^\circ\ A}$$

$$\vec{I_{y2}} = a\,\vec{I_{r2}} = 1 \angle 120^\circ \times 11{\cdot}54 \angle 30^\circ = \mathbf{11{\cdot}54 \angle 150^\circ\ A}$$

***B*-line**

$$\vec{I_{bo}} = \vec{I_{ro}} = \mathbf{0A}$$

$$\vec{I_{b1}} = a\,\vec{I_{r1}} = 1 \angle 120^\circ \times 11{\cdot}54 \angle{-30}^\circ = \mathbf{11{\cdot}54 \angle 90^\circ A}$$

$$\vec{I_{b2}} = a^2\,\vec{I_{r2}} = 1 \angle 240^\circ \times 11{\cdot}54 \angle 30^\circ = \mathbf{11{\cdot}54 \angle 270^\circ\ A}$$

Example 18.12. *Three impedances of 5—j10, 6 + j 5 and 3 + j15 ohms are connected in star to red, yellow and blue lines of a 3300 V, 3-phase, 3-wire supply. The phase sequence is RYB. Calculate the line current I_R.*

Solution. This is a case of unbalanced 3-phase star connected load supplied from a balanced 3-phase supply. Since the phase sequence is *RY B*,

$$\therefore \quad \vec{V_{RY}} = 3300 \angle 0^\circ \text{ V} \quad ; \quad \vec{V_{YB}} = a^2\,\vec{V_{RY}} = 3300 \angle 240^\circ \text{ V}$$

Let $\vec{V_R}$, $\vec{V_Y}$ and $\vec{V_B}$ be the voltages across impedances in *R*, *Y* and *B* phases respectively and $\vec{I_R}$, $\vec{I_Y}$ and $\vec{I_B}$ the resulting line currents.

$$\therefore \quad \vec{V_R} - \vec{V_Y} = \vec{V_{RY}} = 3300 + j\,0$$

$$\text{and} \quad \vec{V_Y} - \vec{V_B} = \vec{V_{YB}} = 3300(-0{\cdot}5 - j\,0{\cdot}866)$$

$$\text{Since} \quad \vec{I_R} + \vec{I_Y} + \vec{I_B} = 0\text{A}$$

$$\therefore \qquad \vec{I_{R0}} = \vec{I_{Y0}} = \vec{I_{B0}} = 0\text{A}$$

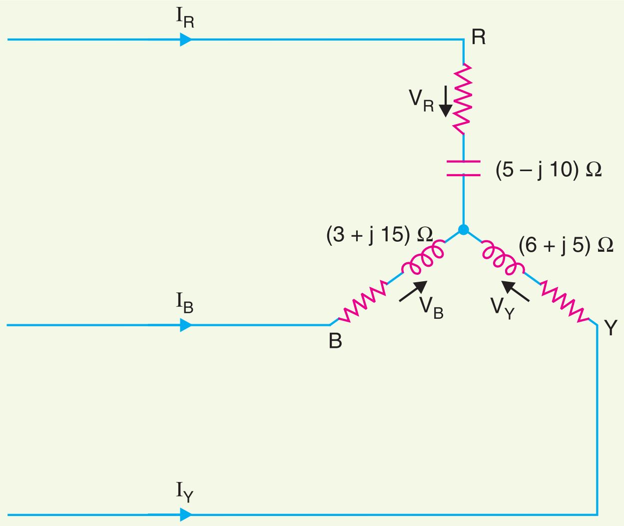


Fig. 18.10

$$\vec{V_R} = (5 - j10)\,\vec{I_R} = (5-j10)\left(\vec{I_{R0}} + \vec{I_{R1}} + \vec{I_{R2}}\right)$$

$$= (5-j10)\left(\vec{I_{R1}} + \vec{I_{R2}}\right) \qquad \text{...}(i)$$

$$\vec{V_Y} = (6 + j5)\,\vec{I_Y} = (6+j5)\left(\vec{I_{Y0}} + \vec{I_{Y1}} + \vec{I_{Y2}}\right)$$

$$= (6+j5)\left(\vec{I_{Y1}} + \vec{I_{Y2}}\right)$$

$$= (6+j5)\left(a^2\,\vec{I_{R1}} + a\,\vec{I_{R2}}\right) \qquad \text{...}(ii)$$

$$\vec{V_B} = (3 + j15)\,\vec{I_B} = (3+j15)\left(\vec{I_{B0}} + \vec{I_{B1}} + \vec{I_{B2}}\right)$$

$$= (3+j15)\left(\vec{I_{B1}} + \vec{I_{B2}}\right)$$

$$= (3+j15)\left(a\,\vec{I_{R1}} + a^2\,\vec{I_{R2}}\right) \qquad \text{...}(iii)$$

Subtracting exp. (*ii*) from exp. (*i*), we get,

$$\vec{V_R} - \vec{V_Y} = \left(\vec{I_{R1}} + \vec{I_{R2}}\right)(5 - j10) - \left(a^2\,\vec{I_{R1}} + a\,\vec{I_{R2}}\right)(6 + j5)$$

or $$3300 = (3\cdot 67 - j\,2\cdot 3)\vec{I_{R1}} + (12\cdot 33 - j\,12\cdot 7)\,\vec{I_{R2}} \qquad \text{...}(iv)$$

Subtracting exp. (*iii*) from exp. (*ii*), we get,

$$\vec{V_Y} - \vec{V_B} = \left(a^2\,\vec{I_{R1}} + a\,\vec{I_{R2}}\right)(6 + j5) - \left(a\,\vec{I_{R1}} + a^2\,\vec{I_{R2}}\right)(3 + j15)$$

or $$3300(-0\cdot 5 - j\,0\cdot 866) = (15\cdot 8 - j\,2\cdot 8)\vec{I_{R1}} - (18\cdot 84 - j\,12\cdot 8)\vec{I_{R2}}$$

or $$-1650 - j\,2858 = (15\cdot 8 - j\,2\cdot 8)\,\vec{I_{R1}} - (18\cdot 84 - j\,12\cdot 8)\vec{I_{R2}} \qquad \text{...}(v)$$

Solving exps. (*iv*) and (*v*), we get,

$$\vec{I_{R1}} = 134 - j\,65$$

and $$\vec{I_{R2}} = 95 + j\,141$$

$$\therefore \quad \vec{I_R} = \vec{I_{R1}} + \vec{I_{R2}} = (134 - j\,65) + (95 + j\,141)$$
$$= 229 + j\,76 = \mathbf{241 \angle{-18 \cdot 4^\circ}\ A}$$

Example 18.13. *A star connected load consists of three equal resistors of 1 Ω resistance. The load is assumed to be connected to an unsymmetrical 3-phase supply, the line voltages are 200 V, 346 V and 400 V. Find the magnitude of current in any phase by the method of symmetrical components.*

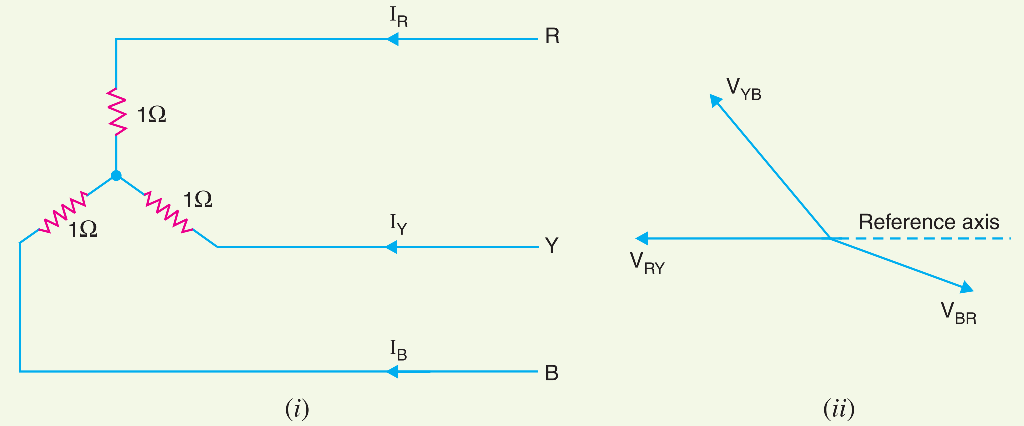


Fig. 18.11

Solution. This is a case of a balanced star-connected load supplied from an unbalanced 3-phase supply. Fig. 18.11 (*i*) shows the balanced star-connected load receiving unbalanced supply. Fig. 18.11 (*ii*) shows the vector diagram. Since the vector sum of three voltages is zero, these can be represented by the three sides of a triangle as shown in Fig. 18.12. Referring to Fig. 18.12, it is clear that :

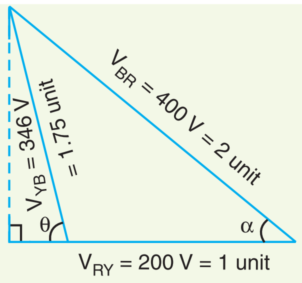


Fig. 18.12

$$(2)^2 = (1 + 1 \cdot 75 \cos\theta)^2 + (1 \cdot 75 \sin\theta)^2$$

or $$4 = 1 + (1 \cdot 75)^2 (\cos^2\theta + \sin^2\theta) + 2 \times 1 \cdot 75 \cos\theta$$

or $$4 = 1 + 3 \times 1 + 3 \cdot 5 \cos\theta$$

$$\therefore \quad \cos\theta = (4 - 4)/3 \cdot 5 = 0$$

$$\therefore \quad \theta = 90^\circ$$

and $$\cos\alpha = \frac{1 + 1 \cdot 75 \cos\theta}{2} = \frac{1+0}{2} = 0 \cdot 5$$

$$\therefore \quad \alpha = 60^\circ$$

As the phase sequence is *RYB*, therefore, various line voltages are :

$$\vec{V_{RY}} = 200 \angle 180^\circ = (-200 + j0)\ \text{V}$$
$$\vec{V_{YB}} = 346 \angle 180^\circ - 90^\circ = 346 \angle 90^\circ = (0 + j\,346)\ \text{V}$$
$$\vec{V_{BR}} = 400 \angle{-60^\circ} = (200 - j\,346)\ \text{V}$$

The current in any phase (or line) is equal to phase voltage divided by resistance in that phase.

$$\therefore \quad \text{Line current, } \vec{I_R} = \frac{200 \angle 180^\circ}{1 \times \sqrt{3}} = 115 \cdot 47 \angle 180^\circ\ \text{A}$$

$$\text{Line current, } \vec{I_Y} = \frac{346 \angle 90^\circ}{1 \times \sqrt{3}} = 199 \cdot 77 \angle 90^\circ\ \text{A}$$

Line current, $\vec{I_B} = \dfrac{400 \angle -60^\circ}{1 \times \sqrt{3}} = 230{\cdot}94 \angle -60^\circ A$

Sequence components in red phase are :

$$\vec{I_{R0}} = \frac{1}{3}\left[\vec{I_R} + \vec{I_Y} + \vec{I_B}\right]$$
$$= \frac{1}{3}\left[115{\cdot}47 \angle 180^\circ + 199{\cdot}77 \angle 90^\circ + 230{\cdot}94 \angle -60^\circ\right]$$
$$= \frac{1}{3}\left[(-115{\cdot}47 + j\,0) + (0 + j\,199{\cdot}77) + (115{\cdot}47 - j\,199{\cdot}99)\right]$$
$$= \frac{1}{3}[0] = \mathbf{0A}$$
$$\vec{I_{R1}} = \frac{1}{3}\left[\vec{I_R} + a\,\vec{I_Y} + a^2\,\vec{I_B}\right]$$
$$= \frac{1}{3}\left[115{\cdot}47 \angle 180^\circ + 1 \angle 120^\circ \times 199{\cdot}99 \angle 90^\circ + 1 \angle 240^\circ \times 230{\cdot}94 \angle -60^\circ\right]$$
$$= \frac{1}{3}\left[115{\cdot}47 \angle 180^\circ + 199{\cdot}99 \angle 210^\circ + 230{\cdot}94 \angle 180^\circ\right]$$
$$= \frac{1}{3}\left[(-115{\cdot}47 + j0) + (-173 - j\,99{\cdot}99) + (-230{\cdot}94 + j0)\right]$$
$$= \frac{1}{3}\left[-519{\cdot}4 - j\,99{\cdot}99\right] = -173{\cdot}13 - j\,33{\cdot}3 = \mathbf{176{\cdot}3 \angle -169^\circ A}$$
$$\vec{I_{R2}} = \frac{1}{3}\left[\vec{I_R} + a^2\,\vec{I_Y} + a\,\vec{I_B}\right]$$
$$= \frac{1}{3}\left[115{\cdot}47 \angle 180^\circ + 1 \angle 240^\circ \times 199{\cdot}99 \angle 90^\circ + 1 \angle 120^\circ \times 230{\cdot}94 \angle -60^\circ\right]$$
$$= \frac{1}{3}\left[115{\cdot}47 \angle 180^\circ + 199{\cdot}99 \angle 330^\circ + 230{\cdot}94 \angle 60^\circ\right]$$
$$= \frac{1}{3}\left[(-115{\cdot}47 + j0) + (173 - j\,99{\cdot}99) + (115{\cdot}47 + j\,199{\cdot}99)\right]$$
$$= \frac{1}{3}\left[173 + j\,100\right] = 57{\cdot}66 + j\,33{\cdot}3 = \mathbf{66{\cdot}58 \angle 30^\circ\ A}$$

TUTORIAL PROBLEMS

1. In a 3-phase, 4-wire system, currents in *R*, *Y* and *B* lines under abnormal conditions of loading are:

$I_R = 150 \angle 45^\circ$ A ; $I_Y = 250 \angle 150^\circ$ A ; $I_B = 100 \angle 300^\circ$ A

Calculate the zero, positive and negative phase sequence currents in the *R*-line and return current in the neutral connection.

[I_{R0} = 52·2 ∠112·7° A ; I_{R1} = 48·02 ∠–87·6° A; I_{R2} = 163·21 ∠40·45° A; I_N = 156·6 ∠112·7° A]

2. In a 3-phase system, the phase voltages are as under :

$E_R = 1 \angle 0^\circ$ V ; $E_B = 1 \angle -120^\circ$ V ; $E_Y = 0$ V

Find the zero, positive and negative phase sequence components in the *R*-phase.

[E_{R0} = –0·33 ∠120°V; E_{R2} = – 0·33 ∠240°V; E_{R1} = 0·66 ∠0° V]

3. The currents in a 3-phase unbalanced system are :

$I_R = (80 + j\,0)$ A ; $I_Y = (-10 - j\,60)$ A ; $I_B = (70 + j\,60)$ A

The phase sequence is *RYB*. Calculate the zero, positive and negative sequence components of the red line current and determine the current in the neutral wire.

[I_{R0} = 0A; I_{R1} = 76·58 ∠13°A ; I_{R2} = 18·12 ∠–72·6°A; I_N = 0A]

4. A 3-phase, 4-wire system supplies loads which are unequally distributed in the three phases. An analysis of the circuit shows that positive and negative phase sequence components of the current in the red line are as under :

$I_{R1} = (7{\cdot}89 + j\,0{\cdot}732)$ A ; $I_{R2} = (2{\cdot}11 - j\,2{\cdot}732)$ A

The total observed current flowing back to supply in the neutral conductor is zero. Calculate the current in the three lines. **[$I_R = (10 - j\,2)$A; $I_Y = (-2 - j\,4)$ A; $I_B = (-8 + j\,6)$ A]**

18.6 Sequence Impedances

Each element of power system will offer impedance to different phase sequence components of current which may not be the same. For example, the impedance which any piece of equipment offers to positive sequence current will not necessarily be the same as offered to negative sequence current or zero sequence current. Therefore, in unsymmetrical fault calculations, each piece of equipment will have three values of impedance—one corresponding to each sequence current *viz.*

(*i*) Positive sequence impedance (Z_1)

(*ii*) Negative sequence impedance (Z_2)

(*iii*) Zero sequence impedance (Z_0)

The impedance offered by an equipment or circuit to positive sequence current is called positive sequence impedance and is represented by Z_1. Similarly, impedances offered by any circuit or equipment to negative and zero sequence currents are respectively called negative sequence impedance (Z_2) and zero sequence impedance (Z_0).

The following points may be noted :

(*a*) In a 3-phase balanced system, each piece of equipment or circuit offers only one impedance– the one offered to positive or normal sequence current. This is expected because of the absence of negative and zero sequence currents in the 3-phase balanced system.

(*b*) In a 3-phase unbalanced system, each piece of equipment or circuit will have three values of impedance *viz.* positive sequence impedance, negative sequence impedance and zero sequence impedance.

(*c*) The positive and negative sequence impedances of linear, symmetrical and static circuits (*e.g.* transmission lines, cables, transformers and static loads) are equal and are the same as those used in the analysis of balanced conditions. This is due to the fact that impedance of such circuits is independent of the phase order, provided the applied voltages are balanced. It may be noted that positive and negative sequence impedances of rotating machines (*e.g.* synchronous and induction motors) are normally different.

(*d*) The zero sequence impedance depends upon the path taken by the zero sequence current. As this path is generally different from the path taken by the positive and negative sequence currents, therefore, zero sequence impedance is usually different from positive or negative sequence impedance.

18.7 Sequence Impedances of Power System Elements

The concept of impedances of various elements of power system (*e.g.* generators, transformers, transmission lines *etc.*) to positive, negative and zero sequence currents is of considerable importance in determining the fault currents in a 3-phase unbalanced system. A complete consideration of this topic does not fall within the scope of this book, but a short preliminary explanation may be of interest here. The following three main pieces of equipment will be considered :

(*i*) Synchronous generators

(*ii*) Transformers

(*iii*) Transmission lines

(*i*) **Synchronous generators.** The positive, negative and zero sequence impedances of rotating machines are generally different. The positive sequence impedance of a synchronous generator is equal to the synchronous impedance of the machine. The negative sequence impedance is much less than the positive sequence impedance. The zero sequence impedance is a variable item and if its value is not given, it may be assumed to be equal to the positive sequence impedance. In short :

Negative sequence impedance	<	Positive sequence impedance
Zero sequence impedance	=	Variable item
	=	may be taken equal to +ve sequence impedance if its value is not given

It may be worthwhile to mention here that any impedance Z_e in the earth connection of a star-connected system has the effect to introduce an impedance of 3 Z_e per phase. It is because the three equal zero-sequence currents, being in phase, do not sum to zero at the star point, but they flow back along the neutral earth connection.

(*ii*) Transformers. Since transformers have the same impedance with reversed phase rotation, their positive and negative sequence impedances are equal; this value being equal to the impedance of the transformer. However, the zero sequence impedance depends upon earth connection. If there is a through circuit for earth current, zero sequence impedance will be equal to positive sequence impedance otherwise it will be infinite. In short,

Positive sequence impedance	=	Negative sequence impedance
	=	Impedance of Transformer
Zero sequence impedance	=	Positive sequence impedance, if there is circuit for earth current
	=	Infinite, if there is no through circuit for earth current.

(*iii*) Transmission lines. The positive sequence and negative sequence impedance of a line are the same; this value being equal to the normal impedance of the line. This is expected because the phase rotation of the currents does not make any difference in the constants of the line. However, the zero sequence impedance is usually much greater than the positive or negative sequence impedance. In short :

Positive sequence impedance	=	Negative sequence impedance
	=	Impedance of the line
Zero sequence impedance	=	Variable item
	=	may be taken as three times the +ve sequence impedance if its value is not given

18.8 Analysis of Unsymmetrical Faults

In the analysis of unsymmetrical faults, the following assumptions will be made :

(*i*) The generated e.m.f. system is of positive sequence only.

(*ii*) No current flows in the network other than due to fault *i.e.* load currents are neglected.

(*iii*) The impedance of the fault is zero.

(*iv*) Phase R shall be taken as the reference phase.

In each case of unsymmetrical fault, e.m.f.s' per phase are denoted by E_R, E_Y and E_B and the terminal p.d. per phase by V_R, V_Y and V_B.

18.9 Single Line-to-Ground Fault

Consider a 3-phase system with an earthed neutral. Let a single line-to-ground fault occur on the red phase as shown in Fig. 18.13. It is clear from this figure that :

$$^*\vec{V_R} = 0 \quad \text{and} \quad \vec{I_B} = \vec{I_Y} = 0$$

* Note that V_R is the terminal potential of phase R *i.e.* p.d. between N and R. Under line-to-ground fault, it will obviously be zero.

The sequence currents in the red phase in terms of line currents shall be :

$$\vec{I_0} = \frac{1}{3}\left(\vec{I_R} + \vec{I_Y} + \vec{I_B}\right) = \frac{1}{3}\vec{I_R}$$

$$\vec{I_1} = \frac{1}{3}\left(\vec{I_R} + a\,\vec{I_Y} + a^2\,\vec{I_B}\right) = \frac{1}{3}\vec{I_R}$$

$$\vec{I_2} = \frac{1}{3}\left(\vec{I_R} + a^2\,\vec{I_Y} + a\,\vec{I_B}\right) = \frac{1}{3}\vec{I_R}$$

$$\therefore \qquad \vec{I_0} = \vec{I_1} = \vec{I_2} = \frac{1}{3}\vec{I_R}$$

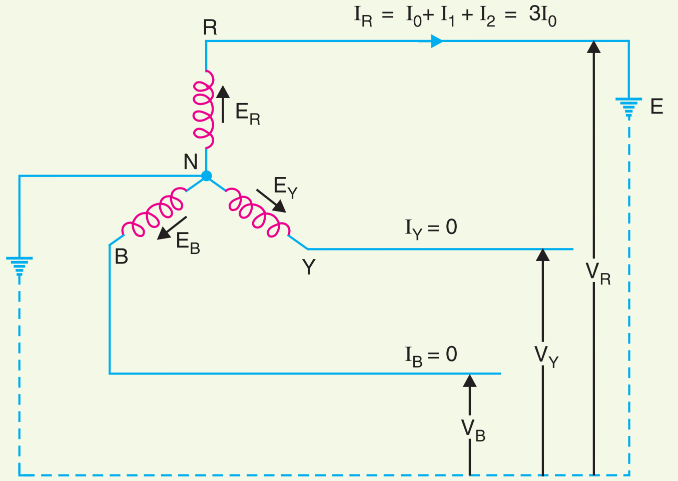


Fig. 18.13

Fault current. First of all expression for fault current $\vec{I_R}$ will be derived. Let $\vec{Z_1}$, $\vec{Z_2}$ and $\vec{Z_0}$ be the positive, negative and zero sequence impedances of the generator respectively. Consider the closed loop *NREN*. As the sequence currents produce voltage drops due only to their respective sequence impedances, therefore, we have,

$$\vec{E_R} = \vec{I_1}\,\vec{Z_1} + \vec{I_2}\,\vec{Z_2} + \vec{I_0}\,\vec{Z_0} + \vec{V_R}$$

As $\qquad \vec{V_R} = 0$ and $\vec{I_1} = \vec{I_2} = \vec{I_0}$

$$\therefore \qquad \vec{E_R} = \vec{I_0}\left(\vec{Z_1} + \vec{Z_2} + \vec{Z_0}\right)$$

or

$$\vec{I_0} = \frac{\vec{E_R}}{\vec{Z_1} + \vec{Z_2} + \vec{Z_0}}$$

$\therefore$ Fault current,

$$\vec{I_R} = 3\,\vec{I_0} = \frac{3\,\vec{E_R}}{\vec{Z_1} + \vec{Z_2} + \vec{Z_0}} \qquad ...(i)$$

Examination of exp. (*i*) shows that the equivalent circuit from which fault current may be calculated is as given in Fig. 18.14. It is clear that fault current is obtained by connecting the phase sequence impedances in series across an imaginary generator of voltage 3 E_R. This is a wonderful part of the method of symmetrical components and makes the analysis easy and interesting. In fact, this method permits to bring any unsymmetrical fault into a simple circuit of

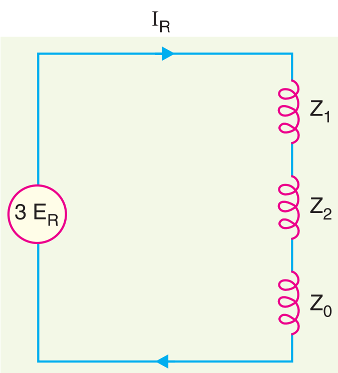


Fig. 18.14

interconnection of sequence impedances appropriate to the fault condition prevailing.

The assumption made in arriving at exp. (*i*) is that the fault impedance is zero. However, if the fault impedance is Z_e, then expression for fault current becomes :

$$\vec{I_R} = \frac{3\,\vec{E_R}}{\vec{Z_1} + \vec{Z_2} + \vec{Z_0} + 3\,\vec{Z_e}}$$

It may be added here that if the neutral is not grounded, then zero sequence impedance will be infinite and the fault current is zero. This is expected because now no path exists for the flow of fault current.

Phase voltages at fault. Now let us calculate the phase voltages at fault (*i.e.* voltage between each line and fault). Since the generated e.m.f. system is of positive sequence only, the sequence components of e.m.f. in *R*-phase are :

$$\vec{E_0} = 0\ ;\ \vec{E_2} = 0 \text{ and } \vec{E_1} = \vec{E_R}$$

The sequence voltages at the fault for *R*-phase are :

$$\vec{V_1} = \vec{E_R} - \vec{I_1}\,\vec{Z_1} = \vec{E_R} - \frac{\vec{E_R}\,\vec{Z_1}}{\vec{Z_1} + \vec{Z_2} + \vec{Z_0}}$$

$$\therefore \quad \vec{V_1} = \frac{\vec{Z_2} + \vec{Z_0}}{\vec{Z_1} + \vec{Z_2} + \vec{Z_0}}\,\vec{E_R}$$

$$\vec{V_2} = 0 - \vec{Z_2}\,\vec{I_2} = \frac{-\vec{Z_2}}{\vec{Z_1} + \vec{Z_2} + \vec{Z_0}}\,\vec{E_R}$$

$$\vec{V_0} = 0 - \vec{I_0}\,\vec{Z_0} = \frac{-\vec{Z_0}}{\vec{Z_1} + \vec{Z_2} + \vec{Z_0}}\,\vec{E_R}$$

It can be readily seen that $\vec{V_1} + \vec{V_2} + \vec{V_0} = 0$. This is expected because *R*-phase is shorted to ground.

$\therefore$ The phase voltages at fault are :

$$\vec{V_R} = \vec{V_0} + \vec{V_1} + \vec{V_2} = 0$$

$$\vec{V_Y} = \vec{V_0} + a^2\,\vec{V_1} + a\,\vec{V_2}$$

$$\vec{V_B} = \vec{V_0} + a\,\vec{V_1} + a^2\,\vec{V_2}$$

Summary of Results. For line (*R*-phase)-to-ground fault :

(*i*) $$\vec{I_R} = \text{Fault current} = \frac{3\,\vec{E_R}}{\vec{Z_1} + \vec{Z_2} + \vec{Z_0}}\ ;\ \vec{I_Y} = 0\ ;\ \vec{I_B} = 0$$

(*ii*) $$\vec{V_R} = 0$$

$$\vec{V_Y} = \vec{V_0} + a^2\,\vec{V_1} + a\,\vec{V_2}$$

$$\vec{V_B} = \vec{V_0} + a\,\vec{V_1} + a^2\,\vec{V_2}$$

18.10 Line–to–Line Fault

Consider a line-to-line fault between the blue (*B*) and yellow (*Y*) lines as shown in Fig. 18.15. The conditions created by this fault lead to :

$$\vec{V_Y} = \vec{V_B}\ ;\ \vec{I_R} = 0 \text{ and } \vec{I_Y} + \vec{I_B} = 0$$

Again taking *R*-phase as the reference, we have,

$$\vec{I_0} = \frac{1}{3}\left(\vec{I_R} + \vec{I_Y} + \vec{I_B}\right) = 0$$

Now $\quad \vec{V_Y} = \vec{V_B}$

Expressing in terms of sequence components of red line, we have,

$$\vec{V_0} + a^2\,\vec{V_1} + a\,\vec{V_2} = \vec{V_0} + a\,\vec{V_1} + a^2\,\vec{V_2}$$

or $\quad \vec{V_1}\,(a^2 - a) = \vec{V_2}\,(a^2 - a)$

$\therefore \quad \vec{V_1} = \vec{V_2} \quad$ ...(*i*)

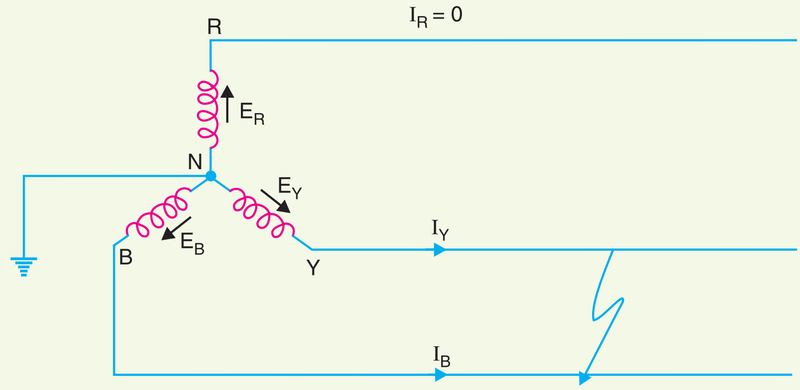


Fig. 18.15

Also $\quad \vec{I_Y} + \vec{I_B} = 0$

or $\quad \left(\vec{I_0} + a^2\,\vec{I_1} + a\,\vec{I_2}\right) + \left(\vec{I_0} + a\,\vec{I_1} + a^2\,\vec{I_2}\right) = 0$

or $\quad (a^2 + a)\left(\vec{I_1} + \vec{I_2}\right) + 2\vec{I_0} = 0$

or $\quad \vec{I_1} + \vec{I_2} = 0 \quad [\because I_0 = 0] \quad$ ...(*ii*)

Fault current. Examination of exp. (*i*) and exp (*ii*) reveals that sequence impedances should be connected as shown in Fig. 18.16. It is clear from the figure that :

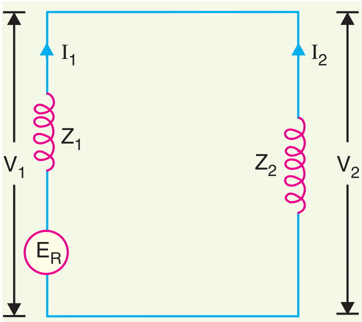


Fig. 18.16

$$\vec{I_1} = -\vec{I_2} = \frac{\vec{E_R}}{\vec{Z_1} + \vec{Z_2}}$$

Fault current, $\quad \vec{I_Y} = \vec{I_0} + a^2\,\vec{I_1} + a\,\vec{I_2}$

$$= 0 + a^2\left(\frac{\vec{E_R}}{\vec{Z_1} + \vec{Z_2}}\right) + a\left(\frac{-\vec{E_R}}{\vec{Z_1} + \vec{Z_2}}\right)$$

$$= (a^2 - a)\,\frac{\vec{E_R}}{\vec{Z_1} + \vec{Z_2}}$$

$$= \frac{-j\sqrt{3}\,\vec{E_R}}{\vec{Z_1} + \vec{Z_2}} = -\,\vec{I_B}$$

Phase voltages. Since the generated e.m.f. system is of positive phase sequence only, the sequence components of e.m.f. in *R*-phase are :

$$\vec{E_0} = 0\ ;\quad \vec{E_2} = 0 \quad \text{and } \vec{E_1} = \vec{E_R}$$

The sequence voltages at the fault for R-phase are :

$$\vec{V}_1 = \vec{E}_R - \vec{I}_1 \vec{Z}_1 = \vec{E}_R - \left(\frac{\vec{E}_R}{\vec{Z}_1 + \vec{Z}_2} \right) \vec{Z}_1$$

$$\therefore \quad \vec{V}_1 = \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R$$

$$\vec{V}_2 = 0 - \vec{I}_2 \vec{Z}_2 = \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R$$

$$\vec{V}_0 = 0 - \vec{I}_0 \vec{Z}_0 = 0$$

The phase voltages at fault are :

$$\vec{V}_R = \vec{V}_0 + \vec{V}_1 + \vec{V}_2$$

$$= 0 + \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R + \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R$$

$$\therefore \quad \vec{V}_R = \frac{2\,\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R$$

$$\vec{V}_Y = \vec{V}_0 + a^2 \vec{V}_1 + a\,\vec{V}_2$$

$$= 0 + a^2 \left(\frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R \right) + a \left(\frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R \right)$$

$$= (a^2 + a) \left(\frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R \right)$$

$$\therefore \quad \vec{V}_Y = - \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R \qquad (\because\ a^2 + a = -1)$$

$$\vec{V}_B = \vec{V}_0 + a\,\vec{V}_1 + a^2 \vec{V}_2$$

$$= 0 + a \left(\frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R \right) + a^2 \left(\frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R \right)$$

$$= (a^2 + a) \left(\frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R \right)$$

$$\therefore \quad \vec{V}_B = - \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R$$

Summary of Results. For line-to-line fault (Blue and Yellow lines) :

(*i*) $\vec{I}_R = 0 \quad ; \quad \vec{I}_Y = -\vec{I}_B = \dfrac{-j\sqrt{3}\,\vec{E}_R}{\vec{Z}_1 + \vec{Z}_2}$

(*ii*) $\vec{V}_Y = \vec{V}_B = -\dfrac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R$ and $\vec{V}_R = \dfrac{2\,\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2} \vec{E}_R$

18.11 Double Line-to-Ground Fault

Consider the double line-to-ground fault involving Y–B lines and earth as shown in Fig. 18.17. The conditions created by this fault lead to :

$$\vec{I}_R = 0 \quad ; \quad \vec{V}_Y = \vec{V}_B = 0$$

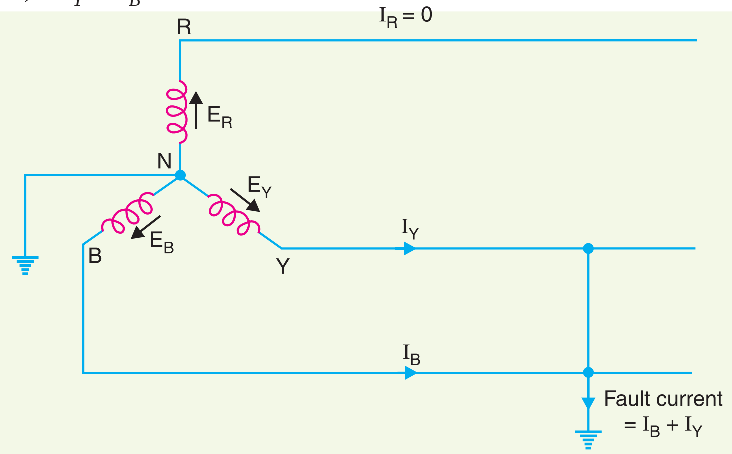


Fig. 18.17

Since $\vec{V}_Y = \vec{V}_B = 0$, it is implied that :

$$\vec{V}_1 = \vec{V}_2 = \vec{V}_0 = \frac{1}{3}\vec{V}_R \qquad ...(i)$$

Also $\vec{I}_R = \vec{I}_1 + \vec{I}_2 + \vec{I}_0 = 0$ *(given)* ...(*ii*)

Fault current. Examination of exp. (*i*) and exp. (*ii*) reveals that sequence impedances should be *connected as shown in Fig. 18.18. It is clear that :

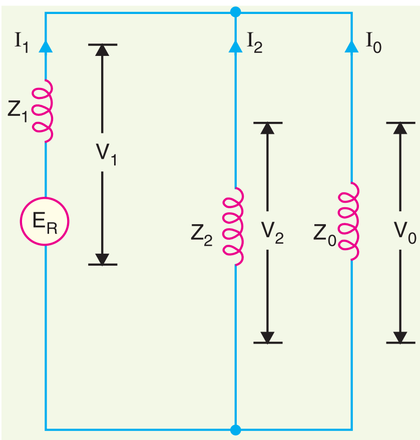


Fig. 18.18

$$\vec{I}_1 = \frac{\vec{E}_R}{\vec{Z}_1 + \dfrac{\vec{Z}_2\,\vec{Z}_0}{\vec{Z}_2 + \vec{Z}_0}}$$

$$\vec{I}_2 = -\vec{I}_1\,\frac{\vec{Z}_0}{\vec{Z}_2 + \vec{Z}_0}$$

$$\vec{I}_0 = -\vec{I}_1\,\frac{\vec{Z}_2}{\vec{Z}_2 + \vec{Z}_0}$$

Fault current, $\vec{I}_F = \vec{I}_Y + \vec{I}_B = 3\,\vec{I}_0$ ** $= 3\left(-\vec{I}_1\,\dfrac{\vec{Z}_2}{\vec{Z}_2 + \vec{Z}_0}\right)$

$$= -\frac{3\,\vec{Z}_2}{\vec{Z}_2 + \vec{Z}_0} \times \frac{\vec{E}_R}{\vec{Z}_1 + \dfrac{\vec{Z}_2\vec{Z}_0}{\vec{Z}_2 + \vec{Z}_0}}$$

$$= -\frac{3\,\vec{Z}_2\,\vec{E}_R}{\vec{Z}_0\,\vec{Z}_1 + \vec{Z}_0\,\vec{Z}_2 + \vec{Z}_1\,\vec{Z}_2}$$

---

* Since $\vec{V}_1 = \vec{V}_2 = \vec{V}_0 = \frac{1}{3}\vec{V}_R$, sequence impedances must be in parallel.

** $\vec{I}_0 = \frac{1}{3}\left(\vec{I}_R + \vec{I}_Y + \vec{I}_B\right) = \frac{1}{3}$ (0 + Fault Current)

$\therefore$ Fault current $= 3\,\vec{I}_0$

Phase Voltages. The sequence voltages for phase R are :

$$\vec{V_1} = \vec{E_R} - \vec{I_1}\,\vec{Z_1}\,; \quad \vec{V_2} = 0 - \vec{I_2}\,\vec{Z_2}\,; \quad \vec{V_0} = 0 - \vec{I_0}\,\vec{Z_0}$$

Now $$\vec{V_1} = \vec{V_2} = \vec{V_0} = \frac{1}{3}\vec{V_R}$$

$\therefore$ $$\vec{V_R} = \vec{V_1} + \vec{V_2} + \vec{V_0} = 3\,\vec{V_2}$$

$$\vec{V_Y} = a^2\,\vec{V_1} + a\,\vec{V_2} + \vec{V_0} = (a^2 + a + 1)\,\vec{V_2} \qquad (\because \vec{V_1} = \vec{V_2} = \vec{V_0})$$

$$= 0 \times \vec{V_2} = 0 \qquad (\because a^2 + a + 1 = 0)$$

$$\vec{V_B} = a\,\vec{V_1} + a^2\,\vec{V_2} + \vec{V_0} = (a + a^2 + 1)\,\vec{V_2} = 0$$

Example 18.14. *A 3-phase, 10 MVA, 11 kV generator with a solidly earthed neutral point supplies a feeder. The relevant impedances of the generator and feeder in ohms are as under :*

	Generator	*feeder*
Positive sequence impedance	$j\,1{\cdot}2$	$j\,1{\cdot}0$
Negative sequence impedance	$j\,0{\cdot}9$	$j\,1{\cdot}0$
Zero sequence impedance	$j\,0{\cdot}4$	$j\,3{\cdot}0$

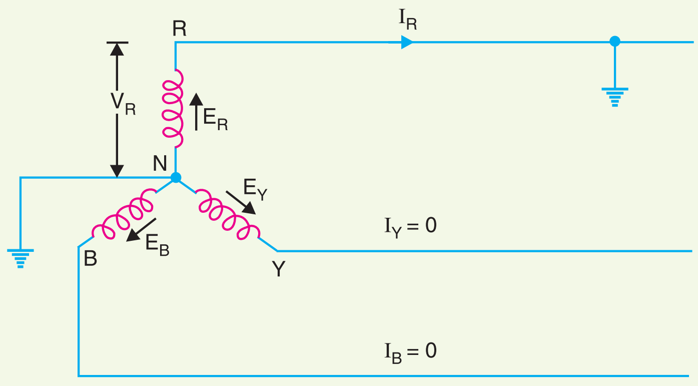


Fig. 18.19

If a fault from one phase to earth occurs on the far end of the feeder, calculate

(i) *the magnitude of fault current*

(ii) *line to neutral voltage at the generator terminal*

Solution. The circuit diagram is shown in Fig. 18.19. The fault is assumed to occur on the red phase. Taking red phase as the reference,

Phase e.m.f. of R-phase, $\vec{E_R} = 11 \times 10^3/\sqrt{3} = 6350$ V

(i) The total impedance to any sequence current is the sum of generator and feeder impedances to that sequence current.

$\therefore$ $$\text{Total } \vec{Z_1} = j\,1{\cdot}2 + j\,1{\cdot}0 = j\,2{\cdot}2\,\Omega$$

$$\text{Total } \vec{Z_2} = j\,0{\cdot}9 + j\,1{\cdot}0 = j\,1{\cdot}9\,\Omega$$

$$\text{Total } \vec{Z_0} = j\,0{\cdot}4 + j\,3{\cdot}0 = j\,3{\cdot}4\,\Omega$$

For a line-to-ground fault, we have,

$$\vec{I_1} = \vec{I_2} = \vec{I_0} = \frac{\vec{E_R}}{\vec{Z_1} + \vec{Z_2} + \vec{Z_0}} = \frac{6350}{j\,2{\cdot}2 + j\,1{\cdot}9 + j\,3{\cdot}4}$$

$$= \frac{6350}{j\,7 \cdot 5} = -j\,846 \text{ A}$$

$\therefore$ Fault current, $\quad \vec{I_R} = 3\,\vec{I_0} = 3 \times (-j\,846) =$ **$-j$ 2538 A**

(ii) Line-to-neutral voltage of R-phase,

$$\vec{V_R} = \vec{E_R} - \vec{I_1}\,\vec{Z_1} - \vec{I_2}\,\vec{Z_2} - \vec{I_0}\,\vec{Z_0}$$

where $\vec{Z_0}$, $\vec{Z_1}$ and $\vec{Z_2}$ are the sequence impedances of generator.

$$= \vec{E_R} - \vec{I_0}\left(\vec{Z_1} + \vec{Z_2} + \vec{Z_0}\right)$$

$$= 6350 - (-j\,846)\,(j\,1 \cdot 2 + j\,0 \cdot 9 + j\,0 \cdot 4)$$

$$= 6350 + j\,846\,(j\,2 \cdot 5)$$

$$= 6350 - 2115 = \mathbf{4235\ V}$$

Example 18.15. *A 3-phase, 11 kV, 10 MVA alternator has sequence reactances of $X_0 = 0 \cdot 05$ p.u., $X_1 = 0 \cdot 15$ p.u. and $X_2 = 0 \cdot 15$ p.u. If the generator is on no load, find the ratio of fault currents for L-G fault to that when all the 3-phases are dead short-circuited.*

Solution. Taking red phase as the reference, let its phase e.m.f. be $\vec{E_R} = 1$ p.u.

Line-to-ground fault. Suppose the fault occurs on the red phase. Then,

$$\vec{I_1} = \vec{I_2} = \vec{I_0} = \frac{\vec{E_R}}{\vec{X_1} + \vec{X_2} + \vec{X_0}}$$

$$\therefore \quad \vec{I_0} = \frac{1}{j\,0 \cdot 15 + j\,0 \cdot 15 + j\,0 \cdot 05} = \frac{1}{j\,0 \cdot 35} = -j\,2 \cdot 85$$

$\therefore$ Fault current, $\quad \vec{I_R} = 3\,\vec{I_0} = 3 \times (-j\,2 \cdot 85) = -j\,8 \cdot 55$ A

Three phase fault. When a dead short circuit occurs on all the three phases, it gives rise to symmetrical fault currents. Therefore, the fault current (say I_{sh}) is limited by the positive sequence reactance (*i.e.* X_1) only.

$$\therefore \text{ Fault current, } \quad \vec{I_{sh}} = \frac{\vec{E_R}}{\vec{X_1}} = \frac{1}{j\,0 \cdot 15} = -j\,6 \cdot 66$$

$$\text{Ratio of two fault currents} = \frac{\vec{I_R}}{\vec{I_{sh}}} = \frac{-j\,8 \cdot 55}{-j\,6 \cdot 66} = \mathbf{1 \cdot 284}$$

i.e. single line-to-ground fault current is 1·284 times that due to dead short circuit on the 3-phases.

Example 18.16. *A 3-phase, 11 kV, 25 MVA generator with $X_0 = 0 \cdot 05$ p.u., $X_1 = 0 \cdot 2$ p.u. and $X_2 = 0 \cdot 2$ p.u. is grounded through a reactance of 0·3 Ω. Calculate the fault current for a single line to ground fault.*

Solution. Fig. 18.20 shows the circuit diagram. The fault is assumed to occur on the red phase. Taking red phase as the reference, let its phase e.m.f. be $\vec{E_R} = 1$ p.u.

First of all, convert the reactance X_n into p.u. value from the following relation :

$$^{*}\text{p.u. value of } X_n = X_n \text{ in ohms} \times \frac{\text{kVA rating}}{(\text{kV})^2 \times 1000}$$

* $\% X_n = X_n$ in ohms $\times \dfrac{\text{kVA rating}}{(\text{kV})^2 \times 10}$. If this value is divided by 100, we get p.u. value.

$$= 0{\cdot}3 \times \frac{25{,}000}{(11)^2 \times 1000} = 0{\cdot}062 \text{ p.u.}$$

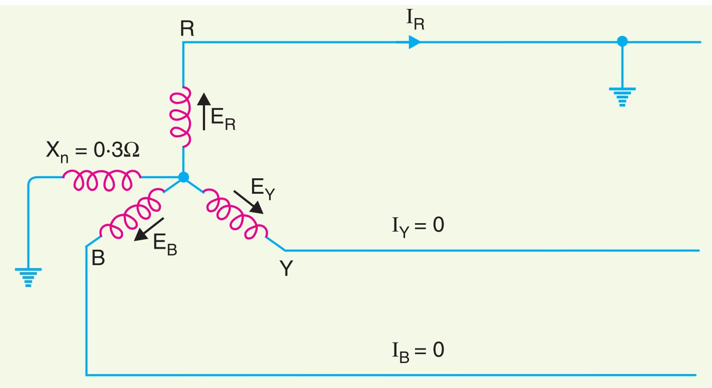


Fig. 18.20

For a line-to-ground fault, we have,

$$\vec{I_1} = \vec{I_2} = \vec{I_0} = \frac{\vec{E_R}}{\vec{X_1} + \vec{X_2} + (\vec{X_0} + 3\,\vec{X_n})}$$

$$= \frac{1}{j\,0{\cdot}2 + j\,0{\cdot}2 + j(0{\cdot}05 + 3 \times 0{\cdot}062)}$$

$$= \frac{1}{j\,0{\cdot}636} = -j\,1{\cdot}572 \text{ p.u.}$$

Fault current, $\vec{I_R} = 3\,\vec{I_0} = 3 \times (-j\,1{\cdot}572) = -j\,4{\cdot}716$ p.u.

$\therefore$ Fault current in amperes = Rated current $\times$ p.u. value

$$= \frac{25 \times 10^6}{\sqrt{3} \times 11 \times 10^3} \times 4{\cdot}716 = \textbf{6188 A}$$

Example 18.17. *A 3-phase, 3-wire system has a normal voltage of 10·4 kV between the lines. It is supplied by a generator having positive, negative and zero sequence reactances of 0·6, 0·5 and 0·2 Ω per phase respectively. Calculate the fault current which flows when a line-to-line fault occurs at the generator terminals.*

Solution. Suppose the short circuit fault occurs between yellow and blue phases. Taking red phase as the reference, its phase e.m.f. is :

Phase e.m.f. of R-phase, $\vec{E_R} = 10{\cdot}4 \times 10^3/\sqrt{3} = 6000$ V

Now $\vec{X_1} = j\,0{\cdot}6\,\Omega$; $\vec{X_2} = j\,0{\cdot}5\,\Omega$; $\vec{X_0} = j\,0{\cdot}2\,\Omega$

For line-to-line fault, we have,

Fault current, $$I_F = \frac{\sqrt{3}\,E_R}{X_1 + X_2}$$ *(in magnitude)*

$$= \frac{\sqrt{3} \times 6000}{(0{\cdot}6 + 0{\cdot}5)} = \textbf{9447·5 A}$$

Example 18.18. *The per unit values of positive, negative and zero sequence reactances of a network at fault are 0·08, 0·07 and 0·05. Determine the fault current if the fault is double line-to-ground.*

Solution. Suppose the fault involves yellow and blue phases and the ground. Taking red phase as the reference, let its phase e.m.f. be $\vec{E}_R = 1$ p.u.

Now, $\vec{X}_1 = j\,0{\cdot}08$ p.u. ; $\vec{X}_2 = j\,0{\cdot}07$ p.u. ; $\vec{X}_0 = j\,0{\cdot}05$ p.u.

For a double line-to-ground fault, we have,

$$\text{Fault current,} \quad \vec{I}_F = \vec{I}_Y + \vec{I}_B = \frac{-3\vec{X}_2\,\vec{E}_R}{\vec{X}_1\,\vec{X}_2 + \vec{X}_1\,\vec{X}_0 + \vec{X}_2\,\vec{X}_0}$$

$$= \frac{-3 \times j\,0{\cdot}07 \times 1}{j\,0{\cdot}08 \times j\,0{\cdot}07 + j\,0{\cdot}08 \times j\,0{\cdot}05 + j\,0{\cdot}07 \times j\,0{\cdot}05}$$

$$= \frac{-j\,0{\cdot}21}{-(56 + 40 + 35) \times 10^{-4}}$$

$$= \frac{j\,0{\cdot}21 \times 10^4}{131} = \mathbf{j\;16\;p.u.}$$

Example 18.19. *A 20 MVA, 11 kV, 3-phase, 50 Hz generator has its neutral earthed through a 5% reactor. It is in parallel with another identical generator having isolated neutral. Each generator has a positive sequence reactance of 20%, negative sequence reactance of 10% and zero sequence reactance of 15%. If a line to ground short circuit occurs in the common bus-bar, determine the fault current.*

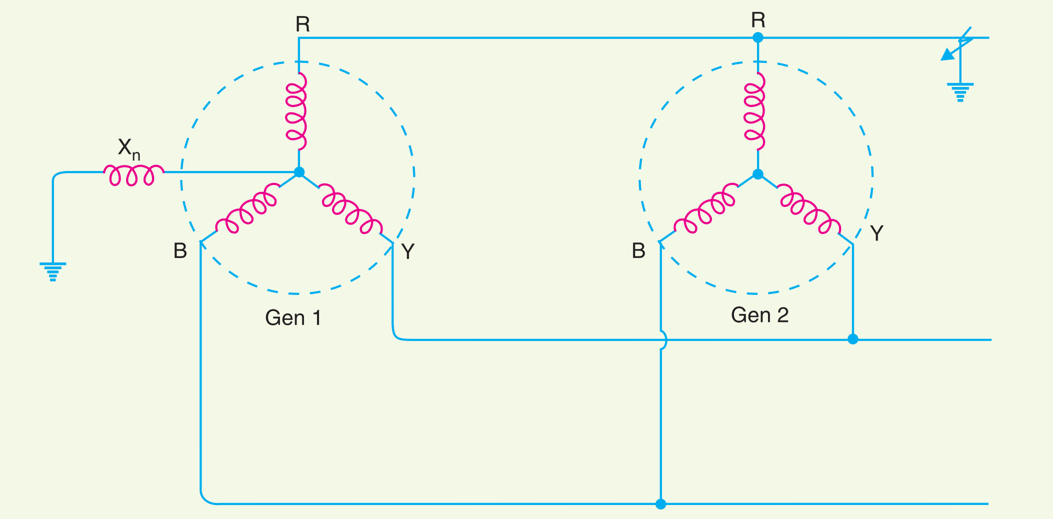


Fig. 18.21

Solution. Fig. 18.21 shows the two generators in parallel. The generator 1 has its neutral earthed through a reactance (= 5%) whereas generator 2 has ungrounded neutral. The earth fault is assumed to occur on the red phase. Taking red phase as the reference, its phase e.m.f. $E_R = 11 \times 10^3/\sqrt{3} = 6351$ V. For a line to ground fault, the *equivalent circuit will be as shown in Fig. 18.22 (*i*) which further reduces to the circuit shown in Fig. 18.22 (*ii*).

* Note the equivalent circuit diagram. The positive sequence reactances (20%) of two generators are in parallel and so are their negative sequence reactances (10%). The zero sequence reactance of generator 2 is zero because its neutral is ungrounded. However, the zero sequence reactance of generator 1 = 15% + 3 × 5% = 30%.

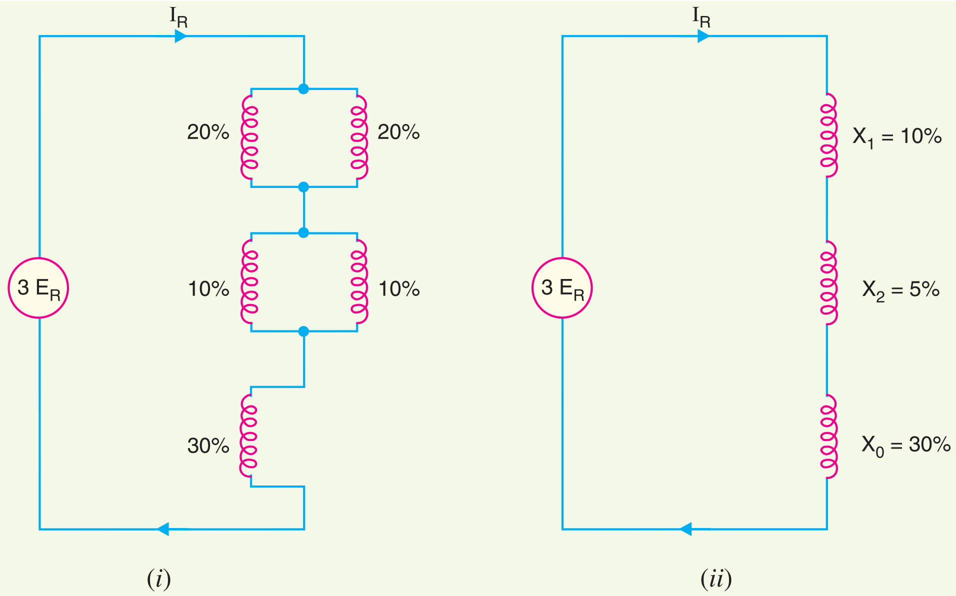


Fig. 18.22

The percentage reactances in Fig. 18.22 (*ii*) can be converted into ohmic values as under :

$$X_1 = \% \text{ reactance} \times \frac{(\text{Voltage in kV})^2 \times 10}{\text{Base kVA}}$$

$$= 10 \times \frac{(11)^2 \times 10}{10^3 \times 20} = 0{\cdot}605\ \Omega$$

$$X_2 = 5 \times \frac{(11)^2 \times 10}{10^3 \times 20} = 0{\cdot}3025\ \Omega$$

$$X_0 = 30 \times \frac{(11)^2 \times 10}{10^3 \times 20} = 1{\cdot}815\ \Omega$$

$$\therefore \quad \text{Fault current, } \vec{I}_R = \frac{3\,\vec{E}_R}{\vec{X}_1 + \vec{X}_2 + \vec{X}_0} = \frac{3 \times 6351}{j\,0{\cdot}605 + j\,0{\cdot}3025 + j\,1{\cdot}815}$$

$$= \frac{19053}{j\,2{\cdot}7225} = -\,j\ 6998\ \text{A}$$

Example 18.20. *A 50 MVA, 11 kV three-phase alternator was subjected to different types of faults. The fault currents are as under :*

3-phase fault = 2000 A ; Line-to-Line fault = 2600 A ; Line-to-ground fault = 4200 A

The generator neutral is solidly grounded. Find the values of the three sequence reactances of the alternator. Ignore resistances.

Solution. Let X_1, X_2 and X_0 be the positive, negative and zero sequence reactances respectively of the alternaor.

$$\text{For 3-phase fault, Fault current} = \frac{E_{ph}}{X_1} \qquad (magnitude)$$

or

$$2000 = \frac{11000/\sqrt{3}}{X_1}$$

$$\therefore \quad X_1 = \frac{11000}{\sqrt{3} \times 2000} = \mathbf{3{\cdot}175\ \Omega}$$

For line-to-line fault, we have,

$$\text{Fault current} = \frac{\sqrt{3}\ E_{ph}}{X_1 + X_2} \qquad (\textit{magnitude})$$

or
$$2600 = \frac{\sqrt{3} \times 11000/\sqrt{3}}{X_1 + X_2}$$

or
$$X_1 + X_2 = \frac{11000}{2600} = 4{\cdot}231\ \Omega$$

$$\therefore \quad X_2 = 4{\cdot}231 - X_1 = 4{\cdot}231 - 3{\cdot}175 = \mathbf{1{\cdot}056\ \Omega}$$

For line-to-ground fault, we have,

$$\text{Fault current} = \frac{3\ E_{ph}}{X_1 + X_2 + X_0} \qquad (\textit{magnitude})$$

or
$$4200 = \frac{3 \times 11000/\sqrt{3}}{X_1 + X_2 + X_0}$$

or
$$X_1 + X_2 + X_0 = \frac{3 \times 11000}{\sqrt{3} \times 4200} = 4{\cdot}536\ \Omega$$

$$\therefore \quad X_0 = 4{\cdot}536 - X_1 - X_2 = 4{\cdot}536 - 3{\cdot}175 - 1{\cdot}056 = \mathbf{0{\cdot}305\ \Omega}$$

TUTORIAL PROBLEMS

1. A 3-phase, 75 MVA, 0·8 p.f. (lagging), 11·8 kV star-connected alternator having its star point solidly earthed supplies a feeder. The relevant per-unit (p.u.) impedances, based on the rated phase voltage and phase current of the alternator are as follows :

	Generator	Feeder
Positive sequence impedance (p.u.)	j 1·7	j 0·1
Negative sequence impedance (p.u.)	j 0·18	j 0·1
Zero sequence impedance (p.u.)	j 0·12	j 0·3

Determine the fault current for a one line-to-earth fault occuring at the far end of the feeder. The generated e.m.f. per phase is of positive sequence only and is equal to the rated voltage per phase. **[4400 A]**

2. A 3-phase, 75 MVA, 11·8 kV star-connected alternator with a solidly earthed neutral point has the following p.u. impedances based on rated phase voltage and rated phase current :

Positive phase sequence impedance	=	j 2 p.u.
Negative phase sequence impedance	=	j 0·16 p.u.
Zero phase sequence impedance	=	j 0·08 p.u.

Determine the steady-state fault current for the following : (*i*) 3-phase symmetrical short-circuit (*ii*) one line-to-earth fault (*iii*) two line-to-earth fault. The generated e.m.f. per phase is equal to the rated voltage. **[(*i*)1840 A (*ii*) 4920 A (*iii*) 3580 A]**

3. The per unit values of positive, negative and zero sequence reactances of a network at fault are 0·08, 0·07 and 0·05 respectively. Determine the fault current if fault is line-to-line-to-ground. **[*j* 16 p.u.]**

18.12 Sequence Networks

The analysis of an unsymmetrical fault by symmetrical components method can be conveniently done by drawing sequence networks. A sequence network of a particular sequence current in a given power system is the path for the flow of that sequence current in the system. It is composed of impedances offered to that sequence current in the system. Since there are three sequence currents (*viz*. positive sequence current, negative sequence current and zero sequence current), there will be three sequence networks for a given power system, namely ;

1. Positive sequence network
2. Negative sequence network
3. Zero sequence network

1. Positive sequence network. The positive sequence network for a given power system shows all the paths for the flow of positive sequence currents in the system. It is represented by one-line diagram and is composed of impedances offered to the positive sequence currents. While drawing the positive sequence network of a given power system, the following points may be kept in view:

(*i*) Each generator in the system is represented by the generated voltage in series with appropriate reactance and resistance.

(*ii*) Current limiting impedances between the generator's neutral and ground pass no positive sequence current and hence are not included in the positive sequence network.

(*iii*) All resistances and magnetising currents for each transformer are neglected as a matter of simplicity.

(*iv*) For transmission lines, the shunt capacitances and resistances are generally neglected.

(*v*) Motor loads are included in the network as generated e.m.f. in series with appropriate reactance.

2. Negative sequence network. The negative sequence network for a given power system shows all the paths for the flow of negative sequence currents in the system. It is also represented by one-line diagram and is composed of impedances offered to the negative sequence currents. The negative sequence network can be readily obtained from positive sequence network with the following modifications :

(*i*) Omit the e.m.fs. of 3-phase generators and motors in the positive sequence network. It is because these devices have only positive sequence-generated voltages.

(*ii*) Change, if necessary, the impedances that represent rotating machinery in the positive sequence network. It is because negative sequence impedance of rotating machinery is generally different from that of positive sequence impedance.

(*iii*) Current limiting impedances between generator's neutral and ground pass no negative sequence current and hence are not included in the negative sequence network.

(*iv*) For static devices such as transmission lines and transformers, the negative sequence impedances have the same value as the corresponding positive sequence impedances.

3. Zero sequence network. The zero sequence network for a given power system shows all the paths for the flow of zero sequence currents. The zero sequence network of a system depends upon the nature of connections of the 3-phase windings of the components in the system. The following points may be noted about zero sequence network :

(*i*) The zero sequence currents will flow only if there is a return path *i.e.* path from neutral to ground or to another neutral point in the circuit.

(*ii*) In the case of a system with no return path for zero sequence currents, these currents cannot exist.

18.13 Reference Bus for Sequence Networks

While drawing the sequence networks, it is necessary to specify the reference potential *w.r.t.* which all sequence voltage drops are to be taken. For this purpose, the reader may keep in mind the following points :

(*i*) For positive or negative sequence networks, the neutral of the generator is taken as the reference bus. This is logical because positive or negative sequence components represent balanced sets and hence all the neutral points must be at the same potential for either positive or negative sequence currents.

(*ii*) For zero sequence network, the reference bus is the ground at the generator.

Example 18.21. *An unloaded generator is grounded through a reactance Z_n as shown in Fig. 18.23. If a single line-to-ground fault occurs, draw (i) the positive sequence network (ii) negative sequence network and (iii) zero sequence network.*

Solution. Fig. 18.23 shows the unloaded generator with single line-to-ground fault. We shall now draw the sequence networks for this system.

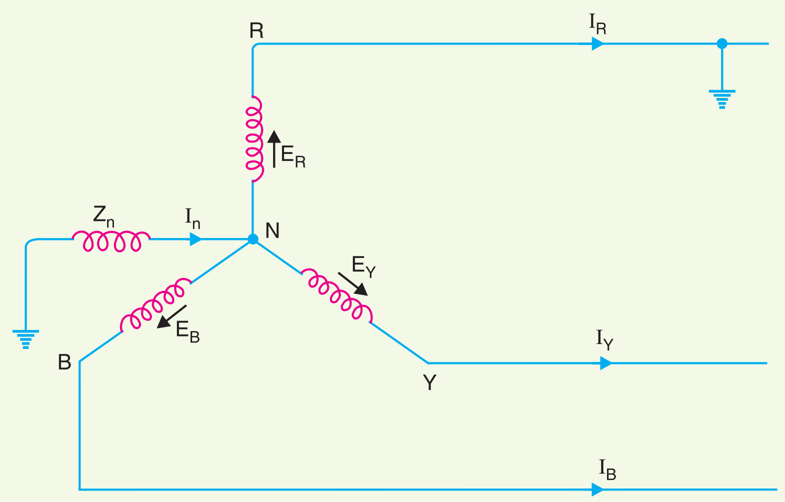


Fig. 18.23

(*i*) **Positive sequence network.** The generated voltages are of positive sequence only becasue the generator is designed to supply 3-phase balanced voltages. Therefore, the positive sequence network is composed of phase e.m.fs. in series with positive sequence impedance of the generator. Fig. 18.24 (*i*) shows the positive sequence current paths whereas Fig. 18.24 (*ii*) shows the single-phase positive sequence network.

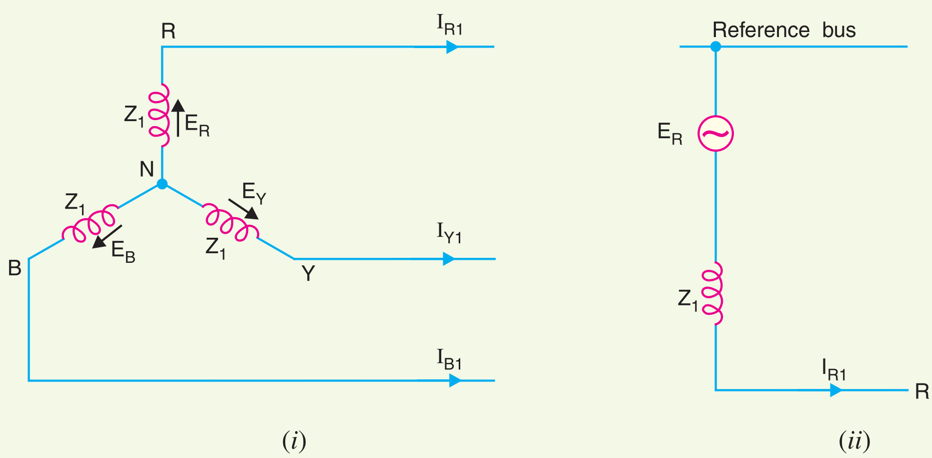


Fig. 18.24

***(ii)* Negative sequence network.** A negative sequence network contains no e.m.f. but includes the impedances of the generator to negative sequence currents. Thus negative sequence network is readily obtained by omitting e.m.fs. in the positive sequence network. Fig. 18.25 (*i*) shows the negative sequence current paths whereas Fig. 18.25 (*ii*) shows the single-phase negative sequence network.

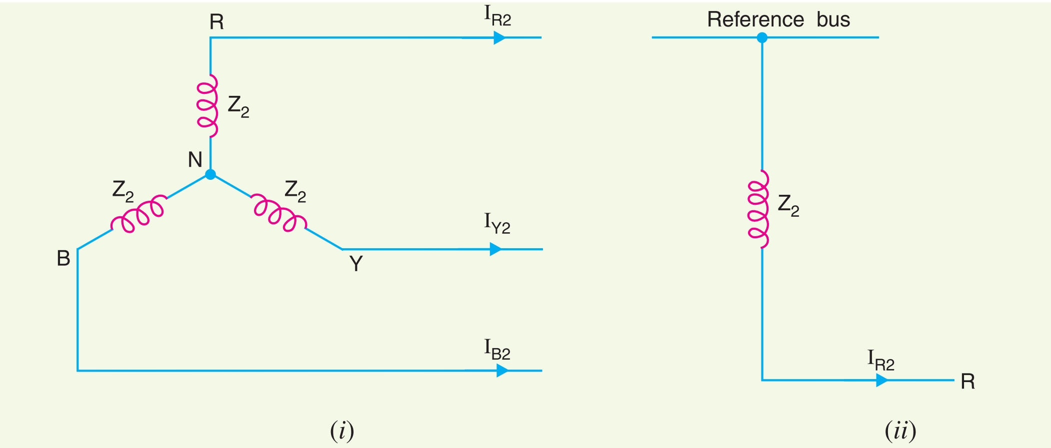


Fig. 18.25

***(iii)* Zero sequence network.** The zero sequence currents flow through phases as well as through the reactance Z_n as shown in Fig. 18.26 (*i*). It is clear that current flowing in impedance Z_n is $3I_{R0}$. It is because $I_{R0} = I_{B0} = I_{Y0}$.

∴ Voltage drop of zero sequence current from R to ground

$$= -3I_{R0}Z_n - I_{R0}Z_0$$

$$= -I_0(3\,Z_n + Z_0)$$

Therefore, the per phase impedance to zero sequence current is $3\,Z_n + Z_0$. Fig. 18.26 (*ii*) shows the zero sequence network.

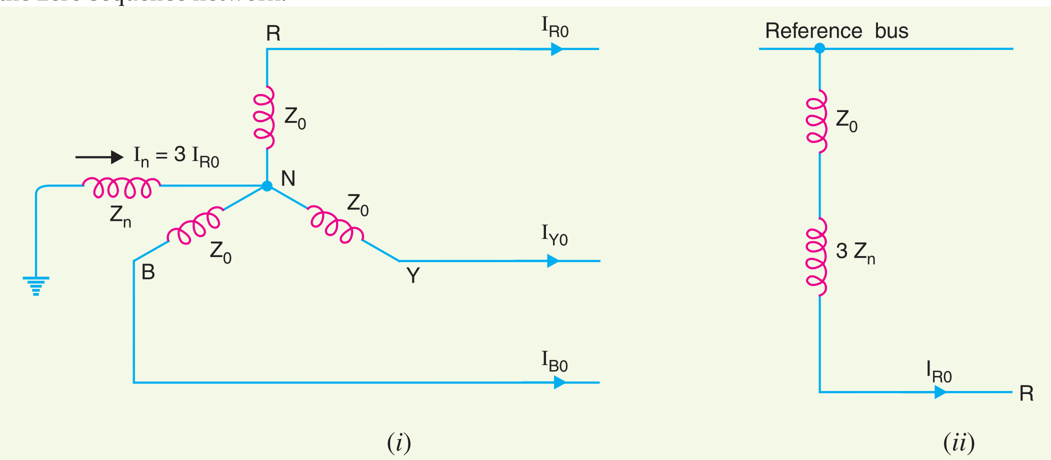
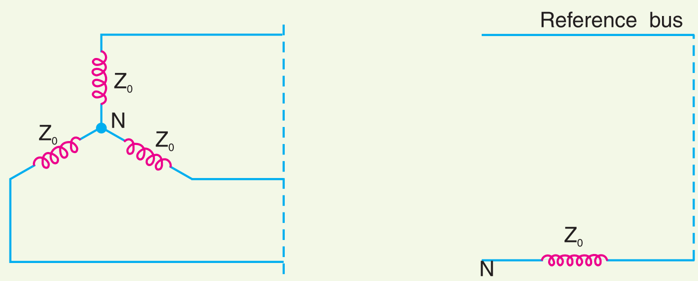


Fig. 18.26

Example 18.22. *Draw the zero sequence network for (i) star-connected load with no earth connection (ii) star-connected load with Z_n from neutral to ground (iii) delta-connected load.*

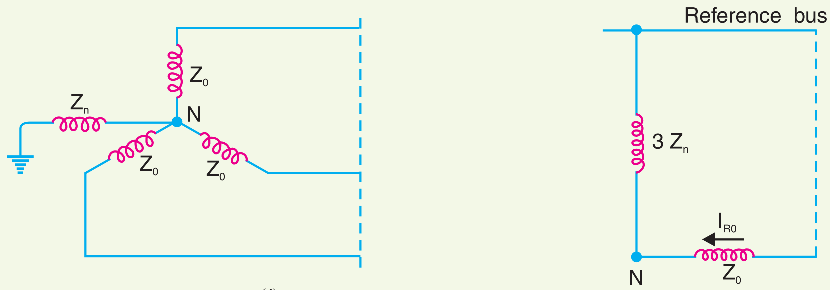
Solution. *(i)* Fig. 18.27 (*i*) shows the star connected load with no earth connection. In this case, neutral current is zero and no zero sequence current can exist. Fig. 18.27 (*ii*) shows the zero sequence network.



(i) (ii)

Fig. 18.27

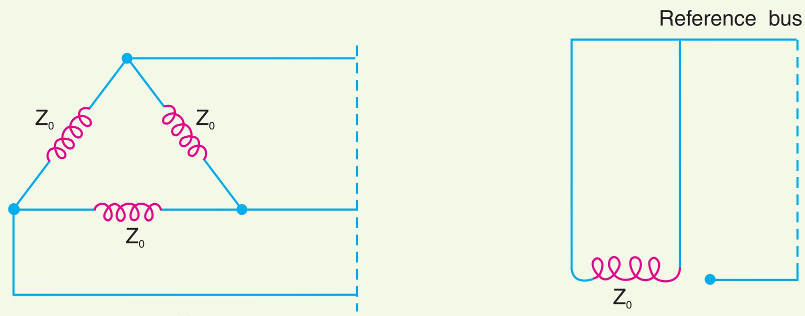
(ii) Fig. 18.28 (*i*) shows a star connected load with an impedance Z_n between neutral and ground. Fig. 18.28 (*ii*) shows the zero sequence network. Note that if impedance Z_n is placed between neutral and ground, then an impedance of 3 Z_n must be placed between the neutral and reference bus of zero sequence network.



(i) (ii)

Fig. 18.28

(iii) Since a delta connected load provides no return path, zero sequence currents cannot exist in the phase windings. In other words, a delta connected circuit provides infinite impedance to zero sequence line currents. The zero sequence network is open at the delta connected circuit. Fig. 18.29(*ii*) shows the zero sequence network for a delta connected circuit.



(i) (ii)

Fig. 18.29

SELF-TEST

1. Fill in the blanks by appropriate words/figures.

(i) The most common type of 3ϕ unsymmetrical fault is ....

(ii) In a balanced 3–ϕ system, negative and zero phase sequence currents are ....

(*iii*) In a 3-phase, 4-wire unbalanced system, the magnitude of zero sequence current is .... of the current in the neutral wire.

(*iv*) The positive sequence impedance of a transmission line is .... to the negative sequence impedance.

(*v*) The zero sequence impedance of different elements of power system is generally ....

2. Pick up the correct words/figures from the brackets and fill in the blanks.

(*i*) A symmetrical fault on a power system is .... severe than an unsymmetrical fault. (*more, less*)

(*ii*) The operator 'a' rotates the vector through .... in the anticlockwise direction. (*90º, 120º, 180º*)

(*iii*) $a - a^2 =$ .... $(j\sqrt{3}, -j\sqrt{3}, 1)$

(*iv*) On the occurrence of an unsymmetrical fault, the positive sequence component is always ...... than that of negative sequence component. (*more, less*)

(*v*) The zero sequence impedance of an element in a power system is generally .... the positive or negative sequence impedance. (*the same as, different from*)

ANSWERS TO SELF-TEST

1. (*i*) Single line-to-ground (*ii*) zero (*iii*) one-third (*iv*) equal (*v*) different
2. (*i*) more (*ii*) 120º (*iii*) $j\sqrt{3}$ (*iv*) more (*v*) different from

CHAPTER REVIEW TOPICS

1. What is a 3-ϕ unsymmetrical fault ? Discuss the different types of unsymmetrical faults that can occur on a 3-ϕ system.
2. Discuss the 'symmetrical components method' to analyse an unbalanced 3-ϕ system.
3. What is operator '*a*' ? Show that :
   (*i*) $a^2 = -0{\cdot}5 - j\,0{\cdot}866$ (*ii*) $a^3 = 1$
   (*iii*) $1 + a + a^2 = 0$ (*iv*) $a - a^2 = j\sqrt{3}$
4. Express unbalanced phase currents in a 3-ϕ system in terms of symmetrical components.
5. What do you understand by positive, negative and zero sequence impedances ? Discuss them with reference to synchronous generators, transformers and transmission lines.
6. Derive an expression for fault current for single line-to-ground fault by symmetrical components method.
7. Derive an expression for fault current for line-to-line fault by symmetrical components method.
8. Derive an expression for fault current for doube line-to-ground fault by symmetrical components method.
9. What do you understand by sequence networks ? What is their importance in unsymmetrical fault calculations ?
10. Write short notes on the following :
    (*i*) Positive sequence network
    (*ii*) Negative sequence network
    (*iii*) Zero sequence network

DISCUSSION QUESTIONS

1. Why is 3-ϕ symmetrical fault more severe than a 3-ϕ unsymmetrical fault ?
2. In a 3-ϕ system, it has been found that negative sequence components and zero sequence components are absent. What do you conclude from it ?
3. Do the sequence components physically exist in a 3-ϕ system ?
4. Why do we prefer to analyse unsymmetrical faults by symmetrical components method ?
5. The positive sequence network of a power system is similar to the negative sequence network. What do you infer from it ?

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