Chapter 2

Historical Development of Color Order Systems

2.1 COLOR AND COLOR ORDER SYSTEMS

One of the definitions in the Oxford English Dictionary, not without its difficulties, of the word *color* is "The quality or attribute in virtue of which objects present different appearances to the eye, when considered with regard only to the kind of light reflected from their surfaces." The definition is symptomatic of the age-old problems of clearly describing this psychological quality. But we all (those of us with normal color vision) know colors when we encounter them. In 1912 the philosopher B. Russell expressed himself on the subject as follows ". . . truths about the colour do not make me know the colour itself better than I did before . . . I know the color perfectly and completely when I see it. . . ."

The conventional view of colors is that they are the result of standard stimuli associated with them. I. Newton's seven primary hues (R O Y G B I V) are the result of spectral stimuli of particular wavelengths (as seen in a dark surround). In color technology certain pigment or dye combinations result in certain color perceptions when viewed under a standard light in a light booth. That the appearance may change significantly if we change the light source or the surround is described in terms of color inconstancy. The idea that there is a standard stimulus for a given color perception has often been questioned (independent of the issue of metamerism). In the 1960s the inventor of the Polaroid photographic process, E. Land, sharply delineated the limitations of such views. In natural scenes the colors assigned to a given spectral power distribution entering the eye depend on the spectral structure of the total visual field according to rules that are not yet understood. It does not

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seem appropriate to talk about color illusions when the implied stimulus/perception association is violated. Colors perceived in any given situation are the *real* colors. Our mind constructs them on the basis of its interpretation of the total visual field as well as of items in this field that appear to belong together. This view causes severe problems for the idea of a color order system in which the specimens are defined by their reflectances. On the other hand, there appears to be a finite limit in number of color perceptions that the color normal observer can experience when seen as colors of objects. There is a legitimate question as to how one can systematically demonstrate these perceptions. The answer is that we have to select a standard set of conditions in which to view the objects in order to be able to more closely relate stimuli and perceptions. This is the approach taken implicitly or explicitly by all developers of physical color order systems. After the materials and the observation situation have been fixed the variability of color normal observers comes into focus. As a result more recent efforts in development of color order systems always involved average data from many observers. From this perspective, extensive relativization of the viewing circumstances to tightly controlled conditions, relating color perceptions to particular spectral stimuli has a degree of validity. Any claim to general validity is erroneous, however.

There are two situations in which it is meaningful to associate a color perception with a specific stimulus:

1. A specific set of conditions fixes the association, but it is understood that the ensuing color experience can be the result of many other sets of stimuli and surround conditions. All the associations are taken to represent the universe of possible object color perceptions.
2. The surround conditions represent a reasonable and practical limitation for the purposes of controlling the industrial production of materials with reflectance/transmittance properties making the objects seen as colored.

The latter situation deals with two issues:

1. The initial colorant formulation for objects needs to meet criteria of closeness to the reference sample and, perhaps, degree of color constancy (lack of perceived change of color of formulation under different light sources).
2. The degree of perceived difference between the approved formulation sample and subsequent repeated manufacture of the material should be minimal.

In both cases it appears justified to use simplified surround conditions for visual tests. Objective test methods must reflect these conditions.

Color order systems that, as used here, include color appearance systems are logical arrangements of color chips and/or geometrical and numerical arrangements of symbols for such chips. One system (NCS) is based on mental references defining the color appearance. The chips themselves are symbols for color perceptions that can be created when the average color normal person views them under specified conditions of lighting and surround. Color appearance systems, specifically, are atlases of collections of color chips or prints.[1]

The remainder of this chapter presents the historical development of color order systems in the Western world.

2.2 FROM ANCIENT GREECE TO THE MIDDLE AGES

Pre-Platonists

Among the oldest extant uses of color words are those by the poet, theologian, and natural philosopher Xenophanes, active in the sixth century BC. He explained all things to have come from water and earth and commented on the rainbow: "And she whom they call Iris, she too is actually a cloud, purple and flame-red and yellow to behold."[2]

Pythagoras (ca. 582–507 BC) philosopher and founder of a religious brotherhood is credited by his followers as having discovered that the relationship between musical notes could be expressed with numbers, as could any other relationship. Of considerable importance to the Pythagoreans was the *tetractys*, their name for the sum of the first four numbers, regarded as the source of all things. According to Philolaus, a disciple of Pythagoras, he equated colors with the number 5. Plutarch remarks on the views of the Pythagoreans on color: "[They] called the surface of a solid *chroma*, that means color. Additionally, they named the species of color, white, black, red and yellow. They looked for the cause of the differences in color in various mixtures of the elements, the manifold colors of animals, however, in their nutrients as well as the climatic regions."[3]

Empedocles (ca. 492–432 BC), a Sicilian philosopher and politician, is credited with the view that everything in existence is composed of four indestructible elements: earth, fire, water, and air. On color Empedocles is quoted as follows: (after Simplicius, 28) "As when painters decorate temple-offerings with colors—men who, following their intelligence, are well-skilled in their craft—these, when they take many-colored pigments in their hands, and have mixed them in harmony, taking more of some, less of another, create from them forms like all things. . . ." (after Aetios, 132) "Empedokles explained color as that fitting into the pores of the visual organ. The multiplicity of colors, so he said, derive from the fixed mixtures of the elements. That there are four colors, exactly as many as elements: white [*leukhyn*], black [*melan*], red [*erydron*], yellow-green [*ochron*]."[4]

Democritus (ca. 460–? BC) expanded the atomistic theory of his predecessor Leucippus. Among many other works he is reported to have written a text

On colors, not extant. Grammarians credited him as the inventor of then unusual words, among them one with the meaning "change of color." He is reported by Galen to be the author of "By convention there is color, by convention sweetness and bitterness, but in reality only atoms and void." On primary colors and color mixture Democritus is quoted by Theophrastus (97): "According to Democritus there are four primary colors. White [*leukhyn*], he explained, is the smooth, polished. . . . The black color [*melan*], on the other hand, consists of (forms) of an opposite nature: rough, nodulated and uneven. . . . Red [*erydron*] he said to consist of similar but larger (forms), as warmth. . . . The green color [*khloron*], on the other hand, consists of hardness and emptiness, as a mixture of both. . . . The primary colors he said to be based on these forms. Each of them being purer the less it is composed of mixed forms. The other colors he said to derive from mixtures of the (four primary colors)."[5]

Plato

Plato (427?–347 BC), philosopher and mathematician, friend and pupil of Socrates, founder of the Academy, was one of the preeminent figures in Greek philosophy. His influence remains active today. Plato made several statements about colors in his dialogues. Concerning primary colors, and their mixtures his key statement is found in *Timaeus* (67, 68):

> . . . we ought to term white that which dilates the visual ray, and the opposite of this black. . . . in (the eye) the fire, mingling with the ray of the moisture, produces a color like blood, to which we give the name red. A bright hue mingled with red and white gives the color auburn [*xandon*]. The law of proportion, however, according to which the several colors are formed, even if a man knew he would be foolish in telling, for he could not give any necessary reason, nor indeed any tolerable or probable explanation of them. Again, red when mingled with black and white, becomes purple, but it becomes umber [*orphninon*] when the colors are burned as well as mingled and the black more thoroughly mixed with them. Flame color [pyrron] is produced by a union of auburn and dun [*phaion*], dun by an admixture of black and white; and pale yellow [*ochron*] by an admixture of white and auburn. White and bright meeting and falling upon a full black, become dark blue [*kyanoyn*], and when the dark blue mingles with white a light blue [*glaykon*] color is formed as flame color with black makes leek-green [*prasion*]. There will be no difficulty in seeing how and by what mixtures the colors derived from these are made according to the rules of probability.

Plato's color mixture scheme is shown graphically in Fig. 2-1. Note that the classification into primary, and so on, colors is by the author.

Aristotle

Aristotle (384–322 BC) was a student of Plato at the Academy. After teaching Alexander the Great he founded a school in the Lyceum in Athens. Together with Plato, Aristotle had an enormous influence on philosophical thinking

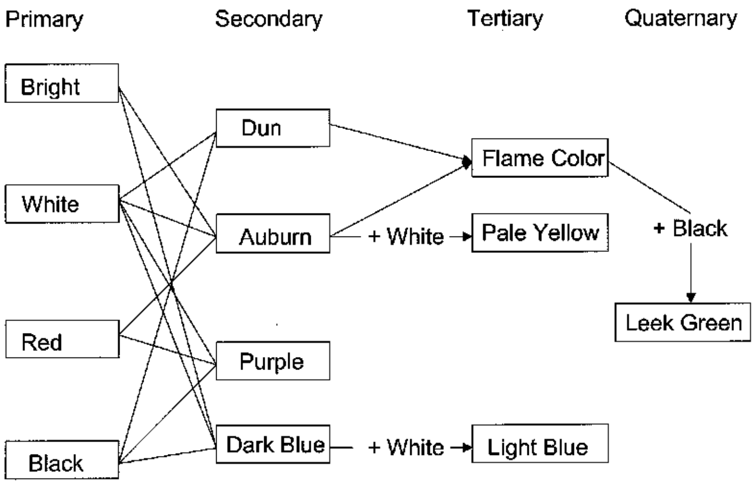

Fig. 2-1 *Plato's color mixture scheme. The four primary experiences are bright, white, red and black. Additions of these in various combinations form the secondary and later stage mixture colors.*

during the next 2000 years. In regard to basic colors and color scales Aristotle expressed himself on more than one occasion and not always in the same way.

In *Sense and Sensibilia* (442a20 ff): "Savors and colors contain respectively about the same number of species. For there are seven species of each, if, as is reasonable, we regard gray as a variety of black [*melanon*] (for the alternative is that yellow [*xandon*] should be classed with white [*leukhon*], as rich with sweet); while crimson [*phoinikoyn*], violet [*aloyrgon*], leek-green [*prasinon*] and deep blue [*kyanoyn*], come between white and black, and from these all others are derived by mixture."

Earlier he stated: (439b20 ff):

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On Colors is a text attributed to Aristotle, but by some writers also to Theophrastus, Aristotle's successor at the Lyceum, and believed to represent Aristotle's final thinking on the subject. Here he stated (1–3):

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Ptolemy

This Greco-Egyptian mathematician and astronomer (second century AD) is reported to have mentioned in the lost first book of his *Optics* a list of twelve simply seen colors. Eleven of these are listed in a biography of Ptolemy found in the library of the ninth-century Byzantine scholar and patriarch of Constantinople Photius.[6] They are, in no particular order: black, white, orange [*xandos*], dun, gray [*phaion*], yellow [*ochros*], red [*erythros*], blue [*kyanos*], purple [*halurgos*], shining or bright [*lampron*], and dark brown [*orphinon*].

Ancient Greek Thinking on Color Species and Color Order

The idea that there are four material elements and correspondingly four fundamental colors belongs to Empedocles. It proved to be influential for many centuries. The quote about painters attributed to him indicates that naturalistic painting and the mixture of pigments to achieve certain colors were well established in Greece at his time. Democritus who explained the world in atomistic terms is reported by Aristotle to have said that color does not have existence because it is generated by the turning of atoms. Democritus changed the name of the fourth, yellow/green elemental color from Empedocles's *ochron* to *khloron*.

Several other pre-Socratic philosophers are reported to have written texts on natural philosophy and are likely to have discussed senses. But their works are lost. Among them are Heraclitus, Alcmaeon, Anaxagoras, Diogenes, and Leucippus.

From Plato's statements it is evident that his elemental colors bright, white, red and black are not colorants but ideas of color fundamentals. From these in secondary, tertiary, and quaternary mixtures the next eight colors are created (see Fig. 2-1). Additional colors are generated from further mixture. There is clearly an awareness of lightness from the sequences "auburn plus white makes light yellow" and "dark blue plus white makes light blue", as well as "purple burned makes umber". But there is no indication of a systematic arrangement by lightness.

It is apparent that Aristotle was aware of a fundamental difference between mixing lights and mixing pigments. In *Sense and Sensibilia* he offers seven fundamental colors seemingly ordered by brightness. In *On Colors* they appear to be reduced to two, white of air, water and earth, and yellow of fire. Black is the result of transmutation. Here he also showed his awareness of differences in intensity and brightness that can be present in colors of a given hue. Influenced by Pythagoras and his followers, he believed that colors appearing harmonious together are formed by simple ratios. Like Plato he was overwhelmed by the multitude of colors and could not see a way of organizing them systematically.

Ptolemy's list appears derived from Plato's except that orange [*xandos*] is added, as is gray [*phaion*]. Missing is leek green [*prasinon*] and flame color [*pyrrhon*], and there is only one bluish color. It is a simple list without any apparent order.

Pliny (AD 23–79), in a history of Greek art indicates that the painter Polygnotus, active in the fifth-century BC, added light ochre and other pigments to the standard early colors of painting, black, white, and red. He is reported to have painted women in light robes and their jewelry in various bright colors. However, the idea of light and shadow was not yet understood. Pliny puts the invention of cinnabar pigment into the same time period. Light and shade appear to have become common in Greek painting one or two generations later. Invention of further new pigments in the fifth to second centuries BC made more and more naturalistic painting possible and required more extensive practical knowledge in pigment mixture.

Of considerable and ongoing controversy is Pliny's statement that the most refined classical Greek painters only used four colors in their work: white, black, yellow, and red.[7] Blue pigments must have been known to the Greeks through trade with Egypt and other nations in the region. Traces of blue pigments have been discovered on statuary and on temple friezes. Works of the celebrated classical painters, like Apelles, have not survived. What is reputed to be a copy in mosaic of a painting by Philoxenos (4th c. BC) was found in the ruins of the House of the Faun (estimated to have been made ca. 100 BC) in Pompeii. It represents Alexander the Great confronting Darius at the battle of Issus. The colors of its tesserae are muted, beiges, browns, and grays, without any blues or greens, thus being in agreement with a four-color painting theory.

The sacral colors of the ancient Greeks are white, the color of festivities, black, the color of mourning, and various reddish shades from scarlet to violet.[8] White bulls or lambs were often used in sacrifices. Red, as the color of blood, invoked both death and life.

E. Veckenstedt has shown that the classical Greek epic writers used some 140 color words, while the philosophers of the same time period only used approximately 50 words. He also showed that of the words used by the epic writers and the philosophers nearly twice as many tended toward white than toward black (Veckenstedt, 1888). Clearly, in the practical world many color experiences were distinguished by name even though philosophers mentioned only relatively few. At the same time there was considerable confusion as to the exact meaning of given color names, well understandable in the absence of color standards. B. Berlin and P. Kay (1969) have classified Homeric Greek as representing stage IIIb in their scheme of development of basic color terms, namely having the basic terms of black, white yellow, red, and blue. By the time of the later classical epic writers the color palette had become quite extensive, with many terms indicating mixed hues and toned colors. There is no record, however, of a comprehensive effort to sort and order the multitude of colors.

A perennial problem is that of interpreting the meaning of color names. Without having specific examples for the meaning of various names, later interpretations vary considerably. It is evident that throughout the classical history of Greece what developed into color names had originally more general meanings. The situation became more complicated in Roman times as different translators of Greek texts sometimes used different Latin words for a given Greek color word. A particularly confusing example is the word *glaukos*, variously translated as light blue, grayish green, or gray, in one case as flashy. As seen later, in the late Middle Ages it briefly assumed also the meaning of yellow.

Color Order in the Middle Ages

According to the record Romans made no fundamental contributions to the subject of color order. Lucretius when discussing vision in *De rerum natura* (The way things are), his poetic account of Epicurean philosophy, essentially followed the atomic theory of Leucippus and Democritus as commented upon by Epicurus.

In the forth-century AD the translator and commentator Chalcidius (life dates not known) produced the translation of Plato's *Timaeus* dialogue most widely known in the Middle Ages. His text includes commentary. In the commentary on section 67ff of the *Timaeus*, Chalcidius described a five-term scale of basic colors as follows: white–yellow–red–blue–black, apparently the first written record of a reordering of the classical Greek fundamental colors into a single tonal scale with red in the middle.

Avicenna and Averroës

The step to multiple tonal scales is found in documentary evidence related to the Persian philosopher and physician Avicenna (Abu Ali al-Husayn ibn Sina, 980–1037). Avicenna was knowledgeable in Aristotle's writings. In his speculations on the human soul, influenced by Aristotle, he also addressed issues of color vision. In his manuscript, translated into Latin in the twelfth century, *Liber de anima seu sextus de animalibus* (Book of the soul) he may have been the first to describe tonal scales of (more or less) individual hues. Changes along Avicenna's tonal scales[9] involve, in modern terminology, most likely both lightness and chroma. He described three such sequences:

1. From *subpallidum* to *pallidum* (a gray or perhaps yellow scale, the generally accepted translation of *pallidus* is pale, wan, and its meaning as a hue term is in dispute)
2. From *subrubreum* to *rubreum* (a red scale)
3. From green to indigo (a scale involving hue as well as lightness and chroma changes)

The prefix "sub-" has the meaning of below, and the first two scales refer to perhaps grays or browns to yellow, respectively wine red to full red. The third scale travels across an expanse of hue as well as lightness, from a medium green to a dark reddish blue. These appear to be the first instance of a color arrangement that requires at least two dimensions to represent it.

The Spanish-Arabic philosopher and commentator ibn-Rushd (1126?–1198?), known under the Latin name of Averroës, wrote extensive commentaries on Aristotle, among many other texts. In his commentary on *Sense and Sensibility* he introduced terms that were translated into Latin as *remittere*, with a meaning to yield, to abate, and *intendere*, with a meaning to spread, direct. They were applied to describe an interpretation of Aristotle's thesis on colors. Accordingly, as Bacon later expressed it: ". . . as Aristotle means, the causes generating colors are elementary qualities that are augmented and abated in their generations through the power of brightness" (Parkhurst 1990). Applied to chromatic colors, it may mean tonal scales, and to black and white, a gray scale.

Eraclius

Eraclius is believed to have been an Italian monk. Nothing is known about his life dates or his activities. There are several manuscripts in Latin in existence that are copies of two books in rhymes and one in prose, less certainly by him, describing the manufacture of colorants for artistic purposes. Stylistic and content comparisons have resulted in estimates that he lived in the tenth century. In section 50 of the prose manuscript Eraclius[10] discusses the "various kinds and names of the principle and intermediate colors . . .":

> Of colours, some are white and some are black. . . . The intermediate colours are red, green, yellow, purple, prasinus [leek green], azure and indicus [indigo], which are each of them, in themselves, beautiful; but are more so when mixed, because, by their variety, they give beauty to one another. And then, in composition, they have a different hue, . . . colours of different kinds are mixed together, in order that they may partake of the nature of the others as well as their own, and make as many, and beautiful, and pleasing, varieties as possible. In this mixture, and in the order in which one is laid over another in painting, great skill is excercised.

In section 58 he describes how shading and highlights are best achieved for various colors: "And note that you must shade azure with black; and lay on the lights with white lead. . . . If you wish to make a colour like lily green, mix azure with white lead; shade it with azure; lay on the lights with white lead; and when it is dry, cover it over with clear saffron."

It is evident from this that Eraclius was well acquainted with color mixing and painting technology. Mixing of pigments, lasing, and toning in black and white directions were standard procedures. However, his color sequence does not indicate a concern with systematic color arrangement.

Theophilus

Explicit tonal scales going from the full color of a bright pigment in both directions toward white and black were first described by the German Benedictine monk Theophilus (1080?–1125?), also known under the name Rugerus. In approximately 1122 he wrote a treatise, *De diversis artibus* (The various arts) in which he describes for the benefit of brother monks at some length technical details for painting, glass making, and metalworking as practiced in his time. It seems likely that Theophilus was acquainted with the works of Eraclius. In Book I of his manuscript he describes how to mix colorants for painting flesh tones with their shadows and highlights as well as facial and head hair. He also describes how to mix colorants for the painting of draperies in wall and ceiling painting and how to imitate the rainbow in painting. For flesh color he begins with heated lead white, having turned yellowish, to which unheated lead white and vermilion (mercuric sulfide, red pigment) is added until flesh color is attained. More white is added for light faces, for pallid ones green earth instead of vermilion. Theophilus proceeds with first and second shadow colors, progressively darker, with first and second rose colors and a dark red, first and second highlight colors as well as a dark gray used for painting eyes. Thus Theophilus describes scales of colors centered on average flesh color and going into lighter, darker, redder, and greener directions. For example, the second shadow color for flesh is described as follows:

> Afterwards take the (first) shadow color for flesh which has been referred to above, and mix with it more green earth and burnt ochre so that it is a darker shade of the former color. Then fill the middle space between the eyebrows and eyes, under the middle of the eyes, near the nose, between the mouth and chin, on the down or beards of young men, on the half-palms toward the thumb, on the feet above the smaller areas of relief, and on the faces of children and women from the chin right up to the temples.

Similarly, for painting drapery various tonal scales are described, for example, for a greenish yellow hue:

> Mix pure viridian (green) with yellow ochre so that the yellow ochre predominates, and fills the drapery. Add to this color a little sap green and a little burnt ochre and make the drawing. Mix white with the ground-color and paint the first light areas. Add more white, and paint the lighter areas on top. Mix with the above shadow-color more sap green and burnt ochre and a little viridian and make the shadow on the outside. . . . Mix dark blue with white in the above way. Similarly mix black with white. In the same way mix yellow ochre with white and for its shadow add a little burnt ochre.

The colors of the rainbow he describes as follows:

> The band which looks like a rainbow is composed of various colors: namely vermilion and viridian, also vermilion and dark blue, viridian and yellow ochre, and also vermilion and folium [a vegetable red lake]. . . . Then mix from vermilion and white whatever tones you please so that the first contains a little vermilion, the second more, the third still more, the fourth yet more, until you reach pure vermilion. Then mix with this a little burnt ochre, then burnt ochre mixed with black and finally black. . . . You can never have more than twelve of these strokes in each color range. And if you want these many so arrange your combinations that you place a plain color in the seventh row.

Here Theophilus clearly describes twelve-tone tonal scales for various pure color pigments that represent the most intense color available for a given hue. Presumably, the scale colors are to be mixed so that the steps appear approximately even. In this fashion he extends Avicenna's scales in a systematic manner, however, without proposing a formal arrangement to place these in.

Learned discussions concerning color scales continued without adding substantially to what Avicenna, Eraclius, and Theophilus had provided. In the later twelfth century the Sicilian physician Urso de Salerno[11] commented in a discussion about the four element-related basic colors that there were many and often unnamed intermediate colors and rather than name them a good painter could produce them by mixture from the four elemental colors.

Albertus Magnus (ca. 1200–1280) Dominican monk, philosopher, teacher, and saint had considerable interest in optics and the sense of vision. He added to Avicenna's scales a number of colors he believed were missing: *fuscum* (dark colored) to the *pallidum* scale; *croceum* (saffron colored, golden), *purpureum* (purple), and *indicum* (here considered to be a red color) to the red scale; and *viride clarum* (bright green) and *viriditas intensa* (intense greenness) to the green scale.[12]

Grossteste and Bacon

Robert Grosseteste (ca. 1168–1253) was an English Franciscan monk, later to become chancellor of Oxford University and bishop of Lincoln. His extensive writings include works on optics and color. By postulating a direct path from the material world to its essential nature through optics, he legitimized extensive studies of optical phenomena. In *De colore* (On color, ca. 1233), one of four books on optics, he described a seven-color scale (colors unnamed) credited to Aristotle and Averroës. These colors [presumably including black and white] are related to bright light [*lux clara*] by remission and, implicitly, to darkness [*lux obscura*] by intention (Parkhurst, 1990).

Roger Bacon (1214/20–1292), also a Franciscan, Aristotelian philosopher and scientist, lived a generation after Grossteste. He wrote three different, if related, texts with sections on color: *Liber de sensu et sensato* (Book of the senses and sensibilities, ca. 1255, attributed to Bacon), *Opus majus* (part of a projected encyclopedia), and *De multiplicatione specierum* (On the multiplication of species).[13] Bacon attempted to apply the second-century Greek philosopher Porphyry's system of predicables, a logical sequence of attributes—genus, species, difference, property, and accident—to the problem of color. He applied the term genus to a Chalcidian list of five fundamental color properties: whiteness [*albedo*], yellowness [*glaucitas*], redness [*rubedo*], bluegreeness [*viriditas*], and blackness [*nigredo*]. Each of these has a range of color species attached to them: for example, *glaucitas* is found in *lividus, flavus, glaucus, ceruleus, pallidus*, and *citrinus* (see below for translation). Bacon discussed at length Grosseteste's views on color and their relation to lightness and darkness. He used the specific Latin term *gradus* (step, gradation) to describe color variations representing difference in the Porphyry sequence.

In *De sensu et sensato* Bacon included a list of twenty colors, sorted in approximately tonal sequence, he considered important and explained their meaning as derived from older sources (interpretations of Bacon's meaning are in parentheses).

candidus	shining white
albus	white, dead white
lividus	incomplete white (ivory, oatmeal)
flavus	like *lividus* (pale yellow)
glaucus	yellow
ceruleus	between *glaucus* and *citrinus* (color of beeswax)
pallidus	between *citrinus* and *ceruleus* (pale)
citrinus	orange, between *glaucus* and *puniceus*
puniceus	(reddest of the yellows)
rufus	(golden, scarlet)
croceus	saffron colored, blood red
rubeus	true red, the median color between white and black
rubicundus	darker, more bluish than *rubeus*
purpureus	(purple, violet)
viridis	(blue-green)
venetius	between true blue and black (dark blue)
lividus	(dark gray?)
lazulus	(dark blue)
fuscus	(dark colored, no specific hue)
nigrum	black

Bacon was on the threshold of a more complete understanding of color phenomena, without being able to establish a clear system.

Dietrich von Freiberg

The German Dominican monk Freiberg wrote circa 1310 a manuscript *De iride* (About the rainbow) in which he provided the first accurate explanation for the rainbow phenomenon. He argued (against Aristotle) that chromatic colors could not be produced from mixture of white and black: ". . . a mixture of white and black does not produce anything except a remission of the white and black from their perfections" (various grays, as well known by artists; Wuerschmitt 1914). He also used the term *glaucus* to denote yellow, as did Bacon and the English encyclopedist Bartholomew Anglicus in 1250 (Parkhurst, 1990).

Medieval Progress toward a Systematic View of Colors

Chalcidius, the translator and commentator of Plato, introduced a tonally ordered scale of five fundamental colors that was influential until the late

Renaissance (see d'Aguilon and Kircher below). Avicenna's attempt at ordering colors, influenced by Aristotle, resulted for the first time in scales that modulate a given color along a more or less tonal path. Albertus Magnus commented on the need to add additional steps to Avicenna's scales. Roger Bacon's sustained efforts at color order did not result in significant progress. It was the practical Theophilus who, based on needs of painters and artisans in enameling to create life-like images, proposed the toning of a given pigment color in twelve systematic steps toward white and black. If he saw in this approach the possibility of a systematic description of the world of colors, he did not say so in his writings.

2.3 COLOR ORDER IN THE RENAISSANCE

Despite the flowering of the Renaissance on both sides of the Alps and the writing of books on painting (Leon Battista Alberti, 1335, and Cennino Cennini, ca. 1400), there was only modest progress in thinking on the subject of color scales until the sixteenth century. Cennini mentions a list of seven colors (but not as a scale) that appear to derive from Aristotle: "Know that there are seven natural colors, or rather four actually mineral in character, namely, black, red, yellow and green; there are natural colors but need to be helped artificially, as lime-white, blue-ultramarine, azzurite, giallorino" (Cennini, ch. 36).[14] Cennini described and recommended a style of painting in which the full pigment colors represent the darkest colors in a picture and gradations are made exclusively with the addition of white (known as the Cennini style; Hall, 1992). Alberti offered a painter's view on colors. While he discussed (in the Latin version of his manuscript) various views on color of philosophers and experts, in the Italian version he said: "I speak here as a painter. . . . Through the mixing of colors infinite other colors are born, but there are only four true colors—as there are four elements—from which more and more other kinds of colors may be thus created. Red is the color of fire, blue of the air, green of water, and of the earth gray and ash [*bigio et cenericio*]. . . . Therefore, there are four genera of colors, and these make their species according to the addition of dark and light, black or white. They are thus almost innumerable. Therefore the mixing of white does not change the genus of colors but forms the species. Black contains a similar force in its mixing to make almost infinite species of color." The interpretation of Alberti's choice of the earth-related fourth primary chromatic color is difficult. It has been described as a dull yellowish gray (Gavel, 1979). It appears that the association of his primary chromatic colors with the four elements was for Alberti more important than a system that would recognize yellow as primary. However, he clearly envisaged the systematic toning of the primary colors toward black and white.

The painter and scientist Leonardo da Vinci (1452–1519) proposed a sort of tonal scale of six colors: "The simple colors are six, of which the first is white, although some philosophers do not accept white or black in the number of colors, because one is the origin of all colors and the other their absence. But as painters cannot do without them, we include them in the number of the others, and say that in this order white is the first among the simple, yellow is the second, green is third, blue is fourth, red is fifth, and black is sixth."[15] Leonardo's hue sequence clearly considers the hue circle. It is obvious that Leonardo, like Alberti, was also well versed in tonal expansions achievable with the addition of white and/or black pigments to the pigments representing the four simple chromatic colors and their mixtures.

A curious twelve-step scale based on brightness and without regard to a systematic arrangement of hues was proposed by the Italian neoplatonic philosopher Marsilio Ficino (1433–1499) in a letter to a friend.[16] "In light there are many ideas of colors as there are colors in objects. At the lowest degree where it is communicable, there is the idea of black, at the second the idea of brown, at the third dark yellow, at the fourth dark blue and green, at the fifth sky blue and sea green, at the sixth full red, at the seventh light red, at the eighth saffron yellow, at the ninth white, at the tenth the transparent or the shining, at the eleventh the brilliant, and finally, there is the idea of splendor."

In 1528 the Italian philologist and poet Antonio Telesio (1482–1534) published in Venice a small book *Antonii Thylesii de coloribus libellus* (Antonio Telesio's little book on colors). It is reprinted in its entirety in the original Latin in Goethe's *Geschichte der Farbenlehre* (1810). Telesio described twelve basic colors, not in any particular order. Aside from the differently named black only three colors are identical to colors in Bacon's list. For each of these basic colors Telesio offered a historical commentary and several related color names. The list[17] and some related color names are as follows:

CATEGORY NAME	RELATED COLOR NAMES
1. *Coeruleus* (blue, sky blue)	*indicum, cyaneum, venetus, blavus*
2. *Caesius* (bluish gray, gray)	*glaucus, baios, charopon*
3. *Ater* (black, dead black)	*niger, anthracinus, fuscus*
4. *Albus* (white, dead white)	*pallidus, candidus, leucophaeus*
5. *Pullus* (dark colored, grayish black)	*impluviatus*
6. *Ferugineus* (rust colored, dusky)	*hyacinthus*
7. *Rufus* (red, ruddy)	*rutilum, russum, sanguinatus*
8. *Ruber* (red, ruddy)	*purpureus, xerampelinus*
9. *Roseus* (rose colored)	*incarnatus*
10. *Puniceus* (purple)	*spadiceus*
11. *Fulvus* (yellowish brown, tawny)	*croceus, luteus, flammeus*
12. *Viridis* (green)	*prasinus*

The list is remarkable for its focus on desaturated, natural colors and the absence, perhaps influenced by Alberti, of saturated yellow. It is apparent that

Telesio understood *rufus* to be a yellowish red and *ruber* a bluish one. It also shows that Telesio's meaning of *glaucus* has returned to bluish gray.

Hieronymus Cardanus (1501–1576) Italian physician and mathematician offered a further step in a color value scale.[18] The subject of color appears in more than one of his works, primarily in *De gemmis et coloribus* (On gems and colors, 1563). Here Cardanus described an Aristotelian seven-step color scale as follows: "white, yellow, red, green, wine color, blue, and black." White and black are primordial colors, the other five "intermediate." Additional mixed colors fall between these primary colors. What is new in Cardanus's list is that he assigns numbers of brightness to these colors, thus: "we assume that white contains a hundred parts of light, scarlet fifty, black nothing." These colors fix the beginning, middle, and end of the scale. Yellow is described as containing 65 to 78 parts of light, green 62, deep green 40, wine color 30, blue 25, and blackish gray 20. This scale is not influenced by concerns of painters or by issues of color mixing. It is the first-known "quantitative" assessment of the intrinsic brightness/lightness of object colors.

Sigfrid Aronus Forsius

Significant progress was made by the Finnish mathematician, astronomer and clergyman Sigfrid Forsius (1560–1624). In 1611, while in Stockholm, he wrote a manuscript on physics, chapter VII of which is titled "On Vision." There he presented two figures in form of circles. The first has sixteen colors ranging in one direction from white via red to black and in the other direction from white via blue to black. The red scale has the steps white gold, gold, burnt gold, red, purple, brown and violet brown; the blue scale dapple, gray, sky blue, blue, blue green, green, blackish green. Forsius discussed this scale as follows: "Among the colors there are two prime colors, white and black, from which all others have their origin. . . . In the middle between these colors, red since ancient times has been placed on one side and blue on the other one. . . . Gold between white and red, . . . brown between red and black. . . . Then on the other side between white and blue is gray. . . . And on the lower part green between blue and black" (Fig. 2-2*a*).

With this circle Forsius put classical and medieval ideas on tonal color scales into graphical form.[19] However, he was not satisfied with the result and continued:

> But if you want right to consider the origin and relations of the colors, you should start from the five principle middle colors which are red, blue, green, gold, and gray of white and black. And their gradings, they rise either closer to white by their paleness or to black by their darkness; albeit they are (as above has been made known) related to one another as previously shown. Because red rises to white through pale red (pink) and skin color; to black through purple, brown, violet brown and black brown. Similarly gold relates toward white through pale gold, wooden and wheat color; to black through burnt gold and blackish brown.

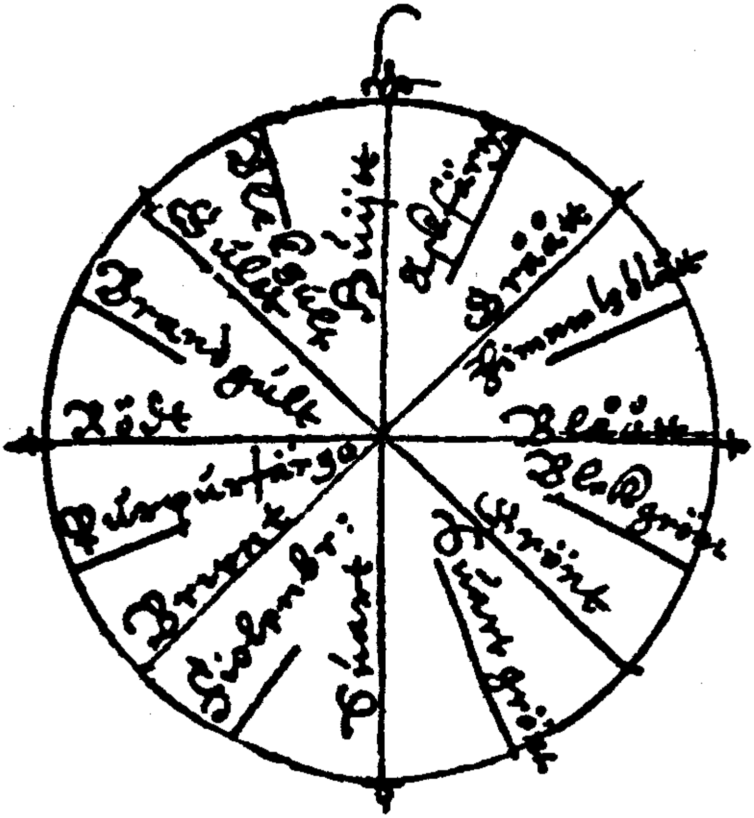


Fig. 2-2a *Classical color order is interpreted 67 Forsius.*

> Equally blue rises to white through sky blue and pale blue, like Dutch cloth; and to black through dark blue like indigo color that has some brownish to it. So rises also green toward white through verdigris and pale green; to black through blackish green. Gray approaches white by the color of light gray, dapple gray and lime: to black by mouse gray, black gray and pale black. And this is the correct relationship of colors that in their number agree with that of the planets as do the lower colors with the five membranes of the eye, and with the five senses. All this can be seen from the accompanying figure. (Fig. 2-2*b*)

Even though some commentators have interpreted the second of Forsius' circles as a color sphere, with white and black at the poles and four primary hues: gold (yellow) and blue, as well as red and green, opposing each other, there is no indication in the text that Forsius had a three-dimensional arrangement in mind. To be interpreted as a sphere with the primary hues in the proper sequence of a hue circle requires an unusual interpretation of Forsius' figure: the left semicircle would be paired with the first right curved line to

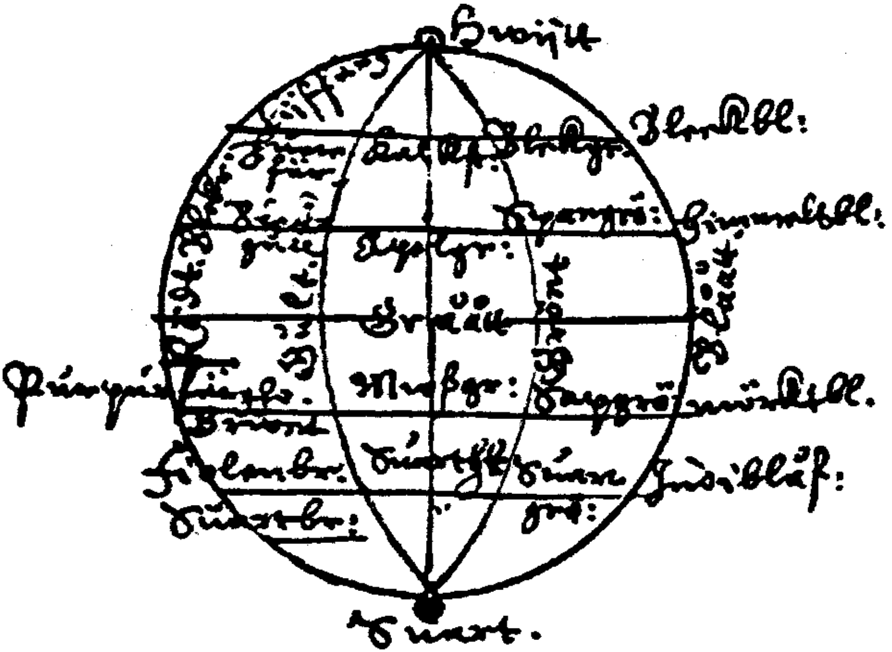


Fig. 2-2b *Forsius' proposal of tonal scales ending in white and black.*

form a circle, and vice versa. How to properly draw a transparent sphere was well known in the seventeenth century from several sixteenth-century and earlier books on perspective. On the other hand, Forsius certainly knew that there are many intermediate hues between his primary hues and with them many more curved lines. How he would have arranged these is not clear. It is evident that Forsius' second figure does not represent a color sphere but is rather a figure that illustrates four chromatic and an achromatic value scale beginning and ending at the same points. His manuscript was not published in book form and thus did not become known outside Sweden.

The Pythagorean ideas on musical harmony as expressed in the tetractys were expanded by the Roman philosopher and statesman Boethius (ca. 475–525). Boethius wrote an extensive treatise on ancient music *De institutione musica*, influential into the sixteenth century. Figure 2-3, from a twelfth-century manuscript of Boethius' work on music, is the representation of an octave, with the basic ratio of 6 and 12.[20] Additional ratios show fifths (*diapente*), fourths (*diatesseron*), and tone. A diagram of a similar style was used by the fourteenth-century French mathematician Nicole Oresme (1323–1382) to show the relationship between various basic polygons.[21] The same type of diagram was used in the 1542 edition of Argyropolus' commentary on Porphyry's *Isagoge* to show

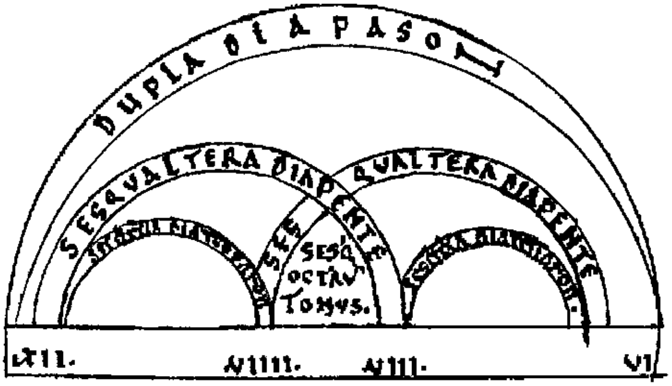


Fig. 2-3 *Graphical interpretation of a musical octave (or double diapason) split into dual-tone cords, after Boethius (fifth c.). From a twelfth-century manuscript of Boethius'* De institutione musica. *This type of diagram was used to represent several kinds of connections in the first half of the second millennium.*

the relationship between the five types of classification of things (see Parkhurst, 1990). The use of such diagrams was thought to show the simplicity of natural laws as expressed in the classical four areas of study: arithmetic, geometry, music, and astronomy. It is not surprising that classically schooled thinkers also used diagrams of this kind to show relationships between colors.

D'Aguilon, Fludd, and Kircher

François d'Aguilon (Aguilonius, 1567–1617) of the Jesuit order in Brussels is the author of a six volume work on optics *Opticorum libri sex* (1613), famous because of its illustrations by Peter Paul Rubens. In regard to colors d'Aguilon expresses himself incapable of dealing exhaustively with the complexities of color mixing as practiced by artists. He included a relational diagram, in the style of Boethius's diagram, in which he expressed the classical ideas of all colors generated from white and black. The top portion of the figure can be interpreted as a semiquantitative tonal description of the three primary hue colors: yellow close to white and far from black, with blue the opposite and red halfway between white and black. In the bottom area three important mixed hues are shown: gold as a mixture of yellow and red, purple of red and blue, and green of yellow and blue. D'Aguilon's diagram follows Chalcidius's sequence of colors restricting the chromatic primary colors to yellow, red, and blue, the painter's primaries (Fig. 2-4).

Robert Fludd (1574–1637), an English mystic, philosopher, and physician, is known for a re-interpretation of an earlier scale of the colors of urine and as the first creator of a printed color circle (Fig. 2-5).[22] This is a curious, perhaps mystically influenced, construction. It simply connects the white and black ends of a seven-color linear arrangement to form a circle. The contents of these

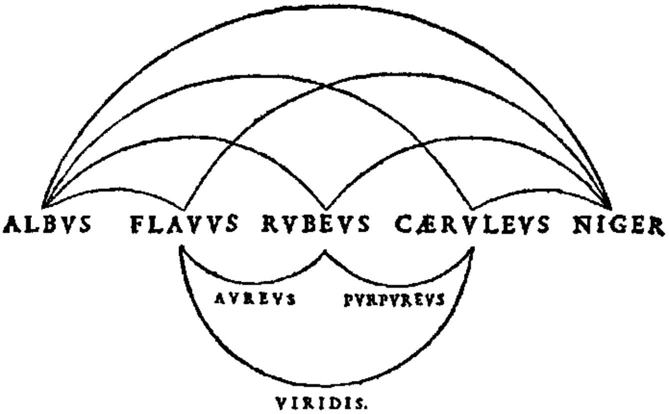


Fig. 2-4 *François d'Aguilon's basic color scale and mixture diagram of 1613. In the upper portion the tonal scales of white, the full colors, and black are shown, and in the lower section important secondary colors resulting from mixtures of the primary chromatic colors yellow, red, and blue.*

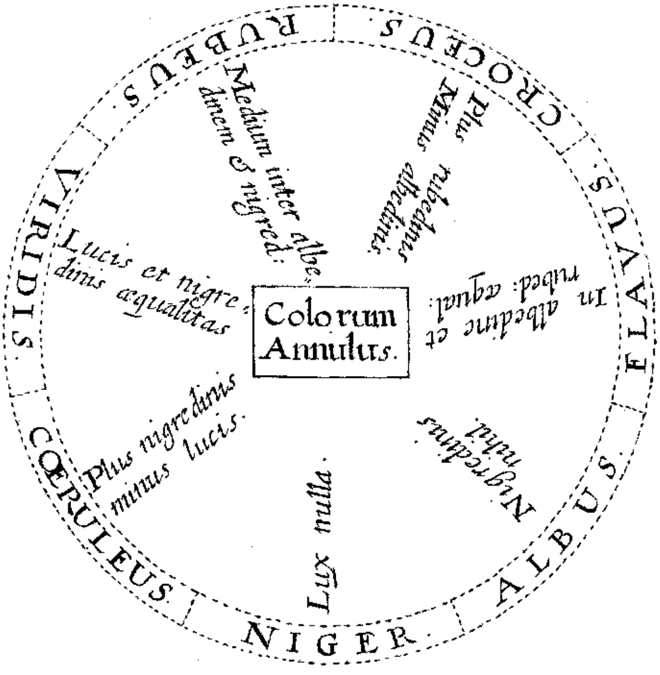
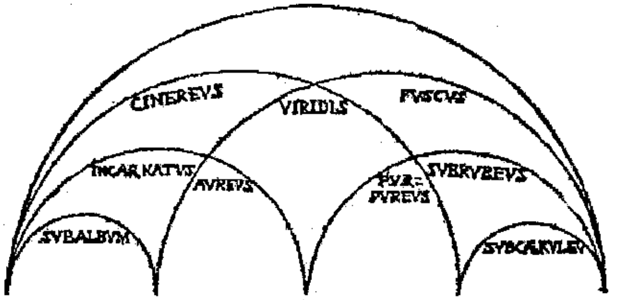


Fig. 2-5 *Color circle of Robert Fludd of 1629, perhaps the first in print.*

colors is described as follows: black: no light; blue: more blackness, less light; green: equality of light and blackness; red: middle between whiteness and blackness; orange: more redness, less whiteness; yellow: equality of whiteness and redness; white: no blackness. The Aristotelian number of seven colors and the designation of red as halfway between black and white are conventional. What is unusual is the description of colors toward black in terms of light and toward white in terms of whiteness.

Athanasius Kircher (ca. 1601–1680), German Jesuit with wide interests in the sciences, author of 44 volumes of writing, professor at the University of Würzburg and later at the College of Rome, wrote a text on light and color: *Ars magna lucis et umbrae* (All there is to know about light and shadow), published in 1646. He also considered, in the Greek tradition, colors to be the result of activity of light and darkness: "Since color is the property of a dark body or, as some say, a shadowed light, the true offspring of light and shadow, we must treat thereof. . . ." In the second chapter, *On the multitudinous variety of colors*, he used a modified version of d'Aguilon's color diagram to illustrate the arrangement of colors (Fig. 2-6). In the cusps formed by semicircles the mixture colors of the two chromatic colors involved are placed: *aureus*, *viridis*, and *purpureus*. Below the peaks of the semicircles are located important tonal



Analogia rerum cum coloribus.

Albus	Flauus	Rubeus	Cæruleus	Niger
Lux pura	Lux tincta	Lux colorata	Vmbra	Tenebræ
Lux	Vmbra tenuissima	Vmbra moderata	Vmbra densa	Tenebræ
Dulce	Dulce temperatum	[illegible]	Acidum	Amarum
Ignis	Aër vel æther	Auroræ medium	Aqua	Terra
Pueritia	Adolescentia	Iuuentus	Virilitas	Senectus
Intellectus	Opinio	Error	Pertinacia	Ignorantia
Deus	Angelus	Homo	Brutum	Plantæ
Nete	Parenete	Mese	Paramese	Hypathe.

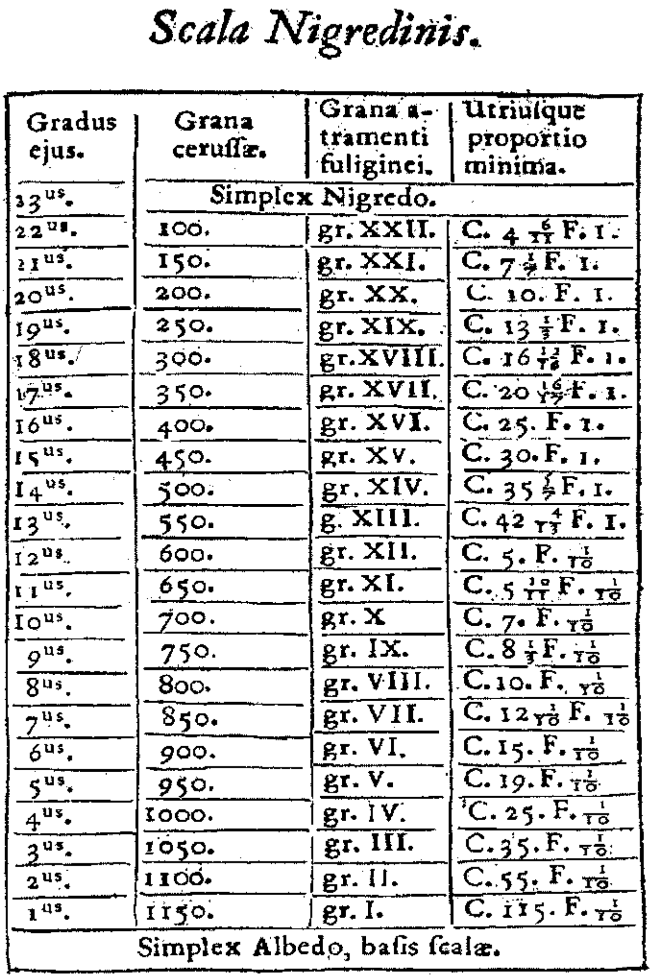
Fig. 2-6 Athanasius Kircher's color diagram in the Boethius style, 1646. Below are his analogues in other qualities and ideas. Tonal scales are shown as by d'Aguilon, with intermediate tonal colors identified (e.g., cinereus, ash colored, between white and blue). The secondary mixtures are shown at the intersections of the corresponding tonal arches.

colors resulting from mixture of the chromatic color with white or black. Thus *subalbum* is a mixture of yellow and white while *fuscus* is a mixture of yellow and black. The diagram is interesting because of the attached analogies between colors and various other qualities, properties, and things: light and shade, taste, the four classical elements, human age, intellect, a scale from God to plants, and finally, the strings of the Greek *lyra*.

Francis Glisson's Color Specification System

Glisson (1597?–1677), an English physician and Regius professor at Cambridge, wrote several books on medicine, among them *Tractatus de ventriculo et intestinis* (Treatise of ventricle and intestines), published in London in 1677. Its chapter IX is surprisingly titled *De coloribus pilorum* (On the colors of hair). The main purpose of this chapter seems to have been to present Glisson's ideas about a color specification system that could not only be used to specify the color of hair but of any colored object. Glisson was a supporter of the idea of five fundamental colors: white, black, yellow, red, and blue. Using four visually equally spaced scales, a gray scale, and scales from white to full color of yellow, red, and blue, all object colors could be specified. Glisson used a novel approach in his scale development. He determined the "equivalent strength" of his white and black pigments (lead white, carbon black) by making a mixture that visually fell halfway between the two extremes, that is, a middle gray. To achieve this, he found that he needed to use a weight ratio of 50:1. Since he wanted to have a scale of 24 steps and the middle gray was to be step 12, he multiplied the weights of his two pigments by 12 to arrive at 600 grains of white and 12 grains of black. For the next darker step he used 1 grain of black more and 50 grains of white less, that is, 13 and 550. In this fashion he completed his scale in both directions (Fig. 2-7).

Glisson also provided a schematical figure of the arrangement of the scales (Fig. 2-8). He described the gray scale (or, in his terms, blackness scale) as being straight and the three chromatic scales as being rounded and sideways from or oblique to (*tres . . . scalae obliquae sunt*) the gray scale. He also was aware of intermediate hues but did not indicate how they should be placed. The midpoints, indicated by marks, are the locations of his pure chromatic pigments (orpiment, vermilion, azurite or bice) extending in one direction to white and in the other to black. But Glisson believed that it was not necessary and a waste of effort to make complete chromatic scales: the half scales from white to the full pigment color together with the gray scale would suffice for color specification. Glisson described as an example of a chromatic scale the redness scale, with 12 steps. He had determined the strength of vermilion against lead white to be 1:20 and used the same methodology as in the gray scale. Glisson provided an example of color specification for the golden yellow blossoms of a flower. This color he judged to be equivalent to grade 11 of the yellowness scale, grade 3 of the redness scale, and grade 2 of the gray scale. If necessary, he said, colors could be specified to half steps between his grades.



Scala Nigredinis.

Gradus ejus.	Grana cerußæ.	Grana atramenti fuliginei.	Utriuſque proportio minima.
23^{us}.	Simplex Nigredo.		
22^{us}.	100.	gr. XXII.	C. 4 $\frac{6}{11}$ F. 1.
21^{us}.	150.	gr. XXI.	C. 7 $\frac{1}{7}$ F. 1.
20^{us}.	200.	gr. XX.	C. 10. F. 1.
19^{us}.	250.	gr. XIX.	C. 13 $\frac{1}{3}$ F. 1.
18^{us}.	300.	gr. XVIII.	C. 16 $\frac{12}{18}$ F. 1.
17^{us}.	350.	gr. XVII.	C. 20 $\frac{16}{17}$ F. 1.
16^{us}.	400.	gr. XVI.	C. 25. F. 1.
15^{us}.	450.	gr. XV.	C. 30. F. 1.
14^{us}.	500.	gr. XIV.	C. 35 $\frac{5}{7}$ F. 1.
13^{us}.	550.	g. XIII.	C. 42 $\frac{4}{13}$ F. 1.
12^{us}.	600.	gr. XII.	C. 5. F. $\frac{1}{10}$
11^{us}.	650.	gr. XI.	C. 5 $\frac{10}{11}$ F. $\frac{1}{10}$
10^{us}.	700.	gr. X	C. 7. F. $\frac{1}{10}$
9^{us}.	750.	gr. IX.	C. 8 $\frac{1}{3}$ F. $\frac{1}{10}$
8^{us}.	800.	gr. VIII.	C. 10. F. $\frac{1}{10}$
7^{us}.	850.	gr. VII.	C. 12 $\frac{1}{10}$ F. $\frac{1}{10}$
6^{us}.	900.	gr. VI.	C. 15. F. $\frac{1}{10}$
5^{us}.	950.	gr. V.	C. 19. F. $\frac{1}{10}$
4^{us}.	1000.	gr. IV.	C. 25. F. $\frac{1}{10}$
3^{us}.	1050.	gr. III.	C. 35. F. $\frac{1}{10}$
2^{us}.	1100.	gr. II.	C. 55. F. $\frac{1}{10}$
1^{us}.	1150.	gr. I.	C. 115. F. $\frac{1}{10}$
Simplex Albedo, baſis ſcalæ.			

Fig. 2-7 *Pigment mixture chart for Francis Glisson's gray scale. First column: grade of scale; second column: weight of lead white (in grains); third column: weight of carbon black; fourth column: reduced pigment ratio.*

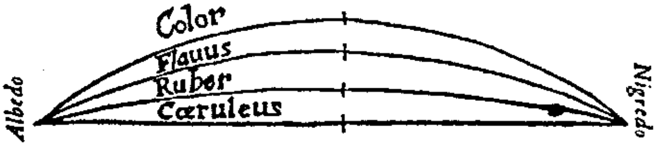


Fig. 2-8 *Glisson's sketch of the arrangement of tonal scales. The blue, red, and yellow scales arch over the horizontal gray scale according to their lightness. The vertical dashes denote the location of the pure pigment.*

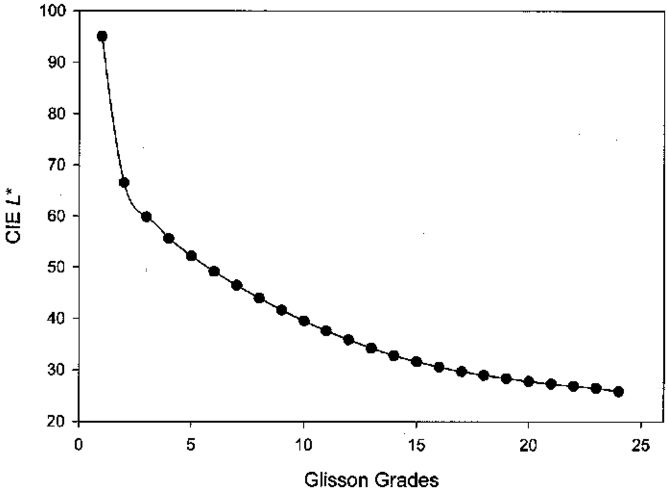


Fig. 2-9 CIE L lightness values of reconstructed Glisson gray scale, beginning with white on the left and ending with black on the right. The scale is only approximately uniform in terms of CIE lightness. There is a very large step between white and the first mixture.*

Glisson's gray and red scales have been reconstructed using the specified pigments and weights (Kuehni and Stanziola, 2002). Reflectance curves were measured and tristimulus values, as well as CIE lightness values L^*, calculated. In Fig. 2-9 the differences in CIELAB (see Chapter 6) L^* between the steps of the gray scale are shown. Near both ends the step sizes increase strongly. In between, they gradually decline in size toward black. Only extensive visual scaling would have disclosed this fact. Similar results were obtained for the red scale with, again, the first and last steps being too large and toward full red the steps gradually becoming smaller.

Brenner and Waller

On a different front progress was made toward a color atlas. In 1680 the Swedish painter and archeologist Elias Brenner (1647–1717) published in Stockholm his *Nomenclatura et species colorum* (Nomenclature and species of colors) containing 31 color samples in six groups: white, yellow, red, green, blue, and black, to assist miniature painters. Six years later, improving on Brenner to whom he referred, the member of the Royal Society of London Robert Waller published *A Catalogue of simple and mixt Colours, with a Specimen of each Colour prefixt to its proper Name* (Waller, 1686). It contains in a rectangular chart 112 mixed colors (many named in Latin, Greek, French, and English). The composition of the mixtures is identified by the colorant names of the column and row headings. As Waller explained: "... I have mixt each of the *Simple Yellows* and *Reds* with each of the *simple Blews*, and these *Mixtures* give most of the *mean* Colours, *viz. Greens*, *Purples*, &c." (italics in the original). The column header colorants are Spanish white, azurite, ultramarine, smalt, litmus, indigo, ink black. The row header yellow colorants are (lead white), Naples yellow, gamboge, ochre, orpiment, and umbra, the red colorants minium, burnt ochre, vermilion, carmine, red lake, dragon's blood, red ochre (carbon black). One-to-one mixtures were made to fill in the intersecting 98 fields. Waller's expressed idea was to have a scientific systematic arrangement of colors. He defended mixtures in single weight ratios by indicating that the possible number of mixture ratios was infinite and therefore not practically doable. In some copies of the printed paper the colorant mixtures were dabbed in.

2.4 NEWTON'S COLOR DIAGRAM

It is remarkable that the first explicit, if incomplete, hue circle was the work of Newton. Isaac Newton (1642–1727), celebrated mathematician and physicist, clarified the composition of white light as a mixture of lights of different wavelengths. When these are viewed individually they create various hue experiences, the spectral colors. In his early lectures on optics, deposited as a matter of his job responsibilities as Lucasian professor of mathematics in the library of Cambridge University (*Optica*, 1670–1672) Newton was primarily concerned with the refrangeability of light rays and had little to say about color mixture and logical arrangements of colors. Proposition 3 states: "The colors white and black together with intermediate ashens or grays are generated from rays of every sort confusedly mixed." In proposition 2 Newton described a scale of eleven hues that he considered "prominent primitives." He experimented with overlaid mixtures of prismatic lights by appropriately arranging three prisms and experienced the well-known difficulties in obtaining white from mixture of two prismatic colors. He found, and expressed in proposition 4, that "[p]rimitive colors can be exhibited by the composition of the neighboring colors on each side of them."

In his mature reflections on colors, *Opticks* (1704), Newton introduced a partly scientifically based color circle and color mixture diagram (Fig. 11 of Plate II, Part II, Book I) (Fig. 2-10). He described it (in part) as follows:

> With the Center O and Radius OD describe a Circle ADF and distinguish its circumference into seven parts . . . proportional to the seven musical Tones or Intervals of the eight Sounds, contained in an Eight. . . . Let the first part DE represent a red Colour, the second EF orange, the third FG yellow, the fourth GH green, the fifth AB blue, the sixth BC indico, and the seventh CD violet, And conceive that these are all the Colours of uncompounded Light gradually passing into one another, as they do when made by Prisms. . . . Let p be the center of gravity of the Arch DE [comparably for q, r, s, t, v, x] and about those centers of gravity let

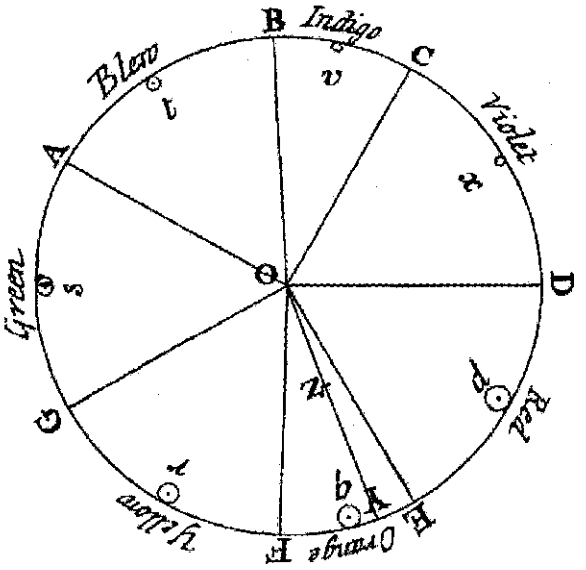


Fig. 2-10 *Newton's color circle, 1704.*

> Circles proportional to the number of rays of each Colour in the given mixture be described. . . . Find the common center of gravity of all those Circles p, q, r, s, t, v, x. Let that center be Z; and from the center of the Circle ADF, through Z to the circumference, drawing the right line OY, the place of the point Y in the circumference shall shew the Colour arising from the composition of all the Colours in the given mixture, and the line OZ shall be proportional to the fullness or intenseness of the Colour, that is, to its distance from whiteness. As if Y fall in the middle between F and G, the compounded Colour shall be the best yellow; if Y verge from the middle toward F or G, the compounded Colour shall accordingly be a yellow, verging toward orange or green. If Z fall upon the circumference the Colour shall be intense and florid in the highest degree; if it fall in the mid way between the circumference and center it shall be but half so intense, that is, it shall be such a Colour as would be made by diluting the intensest yellow with an equal quantity of whiteness; and if it fall upon the center O, the Colour shall have lost all its intenseness and become a white. . . . if the point Z fall in or near the line OD, the main ingredient being the red and violet, the Colour compounded shall not be any of the prismatic Colours, but a purple, inclining to red or violet. . . .

Newton describes with his figure a diagram that is a complete spectral hue circle as well as an additive mixture diagram. At the same time he uses the figure to give a specific example of additive color mixture:

> . . . suppose a Colour is compounded of these homogeneal Colours, of violet 1 part, of indico 1 part, of blue 2 parts, of green 3 parts, of yellow 5 parts, of orange

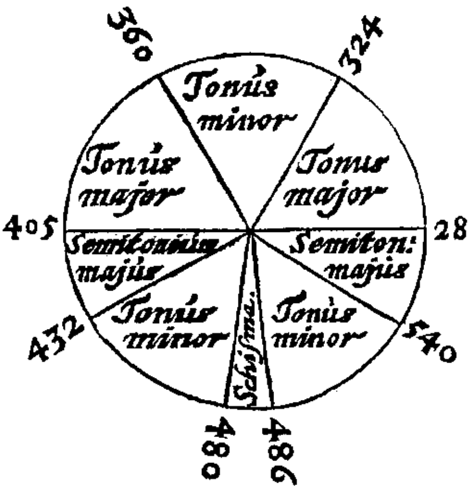


Fig. 2-11 Descartes's circular diapason, 1650. It may have served as a model for Newton's color circle.

> 6 parts, and of red 10 parts. Proportional to these parts I describe the Circles x, v, t, s, r, q, p respectively, that is, so that if the Circle x be 1, the Circle s 3. . . . Then I find Z, the common center of gravity of these Circles, and through Z drawing the line OY the point Y falls upon the circumference between E and F, . . . and thence I conclude, that the Colour compounded of these ingredients will be an orange, verging a little more to red than to yellow. Also I find that OZ is a little less than one half of OY, and thence I conclude, that this orange hath a little less than half the fullness or intenseness of an uncompounded orange . . . this proportion being not of the quantities of mixed orange and white powders, but of the quantities of the lights reflected from them.[23]

The form of Newton's diagram, reflecting his belief in a correspondence between colors and musical tones, may have been influenced by an illustration of the relationship between musical ratios in form of a circle (Fig. 2-11) by the French philosopher and scientist René Descartes (1596–1650).[24] In agreement with the musical scale Newton used the classical number of seven for the basic colors of the spectrum (the Aristotelian seven colors, however, include white and black). He was intrigued by the possibility of parallels between musical harmony and color harmony and pointed out the common reddidshness of shortest and longest wave colors of the spectrum and compared it with the similarity of the tones at the beginning and end of an octave (as reported by Diderot).[25] Newton's work, as is well known, represented a sea change in thinking about colors, opening furious and extended discussions only settled some 200 years later.

2.5 DEVELOPMENT OF THE COLOR CIRCLE

C.B.'s Color Circle

Only four years after the publication of Newton's *Opticks* with its spectral color circle, an anonymous, probably French, author published an image of a hue circle in color. The Dutch printer van Dole had issued several editions of a book on painting miniatures, beginning in 1673. The author was only identified with the initials C.B.[26] To date the identity of the author remains unclear. In 1708 a new edition was published under a new title *Traité de la Peinture en Mignature* (Treatise on miniature painting). It contained, for the first time, images of a seven-hue circle (Fig. 2-12) and of a twelve-hue circle. The author described the circles as follows: "Here are two figures by which one will be able to see how the primitive colors, yellow, red, crimson and blue generate the other colors and which one might call the *Encyclopedia of Colors*. The first figure includes the four primitive colors and three composed from them, and the second includes those same colors with five others which are produced as much from the primitives as of their composites." It is evident that the figures

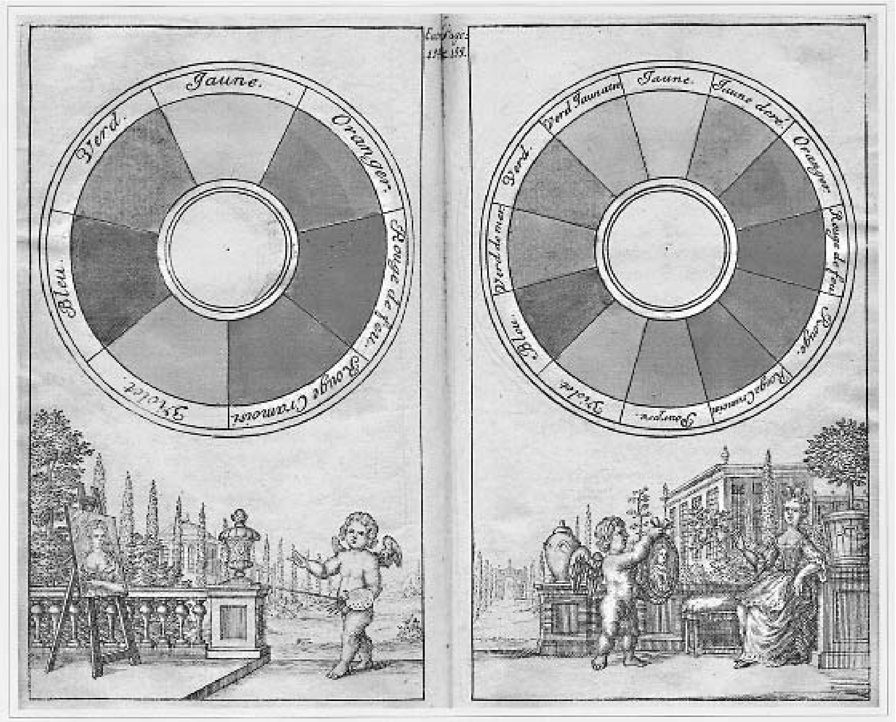


Fig. 2-12 *C.B.'s hand-painted color circles, 1708. Left: The seven-color circle; right: the twelve-color circle. Note that the pigments used for some of the colors have deteriorated. (See color plate.)*

represent pigment-mixing diagrams. In Gage's view (1993) the author used a yellowish and a bluish red primary pigment because only with those could he produce a mix of true red (shown in the twelve-hue circle).

Identity of number of colors between Newton's circle and that of the first circle in the *Traité* may be suggestive, but the anonymous author's selection of the seven colors, related to pigments, is noticeably different from Newton's. The colors of the seven-hue circle are yellow, orange, fire red, crimson, violet, blue, and green. Newton's indigo is missing. A neutral red only makes an entrance in the twelve-color circle. Here the colors are yellow, golden yellow, orange, fire red, red, crimson, purple, violet, blue, sea green, green, yellowish green. We notice that the circle is heavily red-directed: there are seven colors containing redness and only three colors containing greenness. Yellowish and bluish colors are balanced with five each. C.B.'s color circle is the first known explicit and complete (considering also extra spectral purple) circular hue arrangement. By the year 1800 this book had seen 33 editions and thus was widely known, at least in artistic circles.

Castel and Schiffermüller

Louis-Bertrand Castel (1688–1757), a French mathematician, Jesuitical priest, opponent of Newton in matters of color, was deeply interested in a connection between color and music and hoped to construct a color organ.[27] As described in *Projet d'une nouvelle optique* (Project of a new optics) of 1739, by mixing the three basic colors yellow, red, and blue, he constructed a twelve hue color spiral which he correlated with the tones and half tones of musical octaves. Surprisingly, and apparently, as Lambert later reported, as a result of French national preferences, his primary colors were a yellowish red (fire red), a neutral to reddish yellow (*stil de grain*, yellow vegetable dye lake), and "true sky blue." He derived the twelve colors directly from work with the prism in a refutation of Newton's findings of seven basic colors: crimson, red, orange, golden yellow, yellow, olive, green, sea green, blue, "violant," "agathe," and violet. "Higher-octave" color spiral segments where obtained by adding white to his original spiral segment, "lower-octave" colors by adding black. He constructed scales of twelve steps each of additions of white and black for the twelve hues of a hue spiral, ending up with a system of 144 colors. Castel hoped to expand the gradations in all directions and to display the results in a color room that he envisaged to be covered with colored woven bands or wallpaper specimens representing his colors. He attempted to make the gradations uniform in terms of perceived differences and, on the hue spiral, ended up with a different number of steps between his three primary colors. Spirals imply three dimensions and Castel's color room seems the first implicit idea of a color space.

Castel's work was continued and expanded by the Viennese entomologist and brother Jesuit Ignaz Schiffermüller (1727–1806/9). Schiffermüller did not share Castel's enthusiasm for a possible connection between music and color

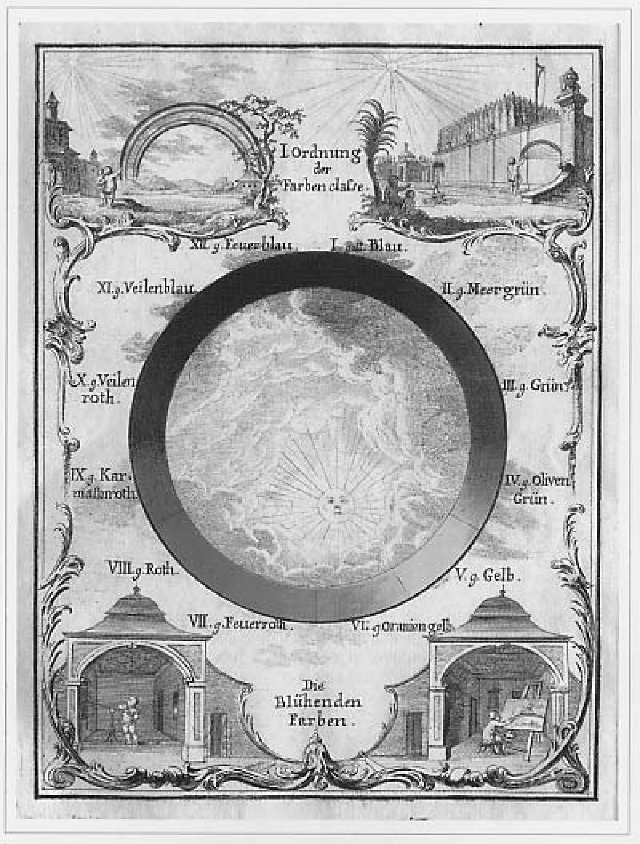


Fig. 2-13 Continuous color circle of Ignaz Schiffermüller, with twelve classes of colors, 1771. (See color plate.)

but was interested in a system that could be used, among other things, to deduce rules of color harmony. This required a physical expression of the system in considerable detail. Castel's central color spiral segment was flattened into a color circle. To specify the twelve hues Schiffermüller used more descriptive hue terms. Again, they were arranged in attempted perceptually equal steps. Between yellow and red there are two intermediate steps: orange and fire red; between red and blue there are four steps: crimson red, violet red, violet blue, and fire blue; between blue and yellow there are three steps: sea green, green, and olive green. Each of the twelve hues is the key representative of a "color class," and Schiffermüller used Roman numerals to identify them, starting with blue (Fig. 2-13). Next Schiffermüller began to create steps toward white and black but did not get beyond three examples in the blue region (Fig. 2-14). His book *Versuch eines Farbensystems* (Attempt to construct a color system) was published in Vienna in 1771, thirteen years after Tobias

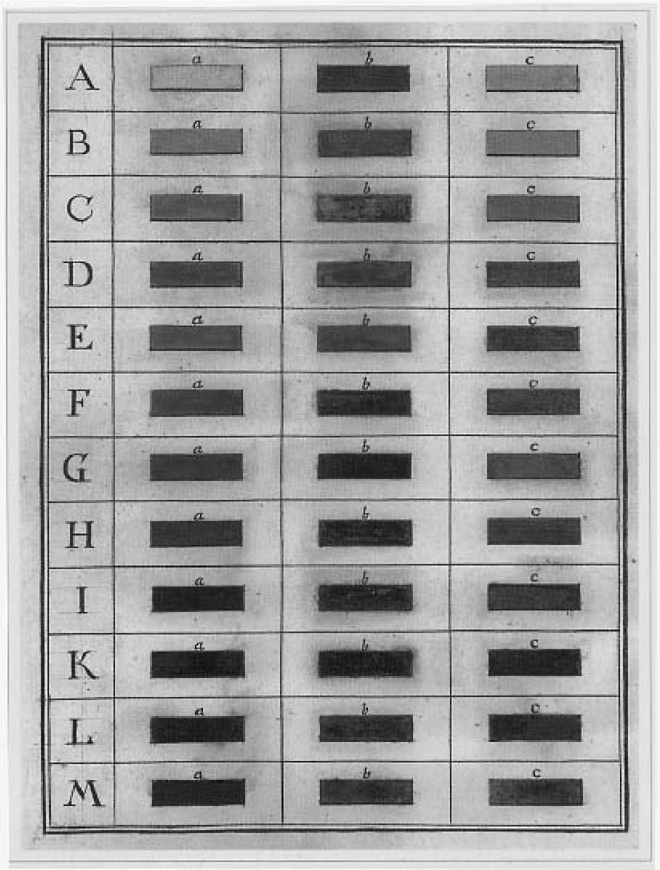


Fig. 2-14 *Schiffermüller's tonal scales of three blues, from his 1771 work. Some of the colorations have deteriorated. (See color plate.)*

Mayer presented his three-dimensional system in Göttingen and one year before Lambert published his book on the subject of a color pyramid (see below).

Why Color Circles?

One of the questions arising is why, given the linear nature of the visible spectrum, hues began to be uniformly represented in a hue circle. The first extant circular representations of colors appeared in medieval (continuing into the fifteenth century) diagnostical diagrams of the color of urine.[28] Obviously these were not complete hue circles but may have been influential for circular depictions of colors of different hues. The circles by Forsius and Fludd do not represent hue circles. Both of Forsius' two circles have a central vertical axis of the colors of a gray scale, and he placed tonal scales of four primary hues, red, yellow, green, and blue, on circular segments on either side of the gray scale. It is not obvious that the circle has a clear meaning. In case of Fludd it is not unlikely that mystical, perhaps alchemical reasons produced his circle, which begins with white and ends in black next to it.

Newton's choice of a circle for his spectral colors may have been influenced by Descartes's circular diapason, but it was also based on his knowledge of desaturation of spectral light by white light, common for all spectral colors. Thus the radius length is an indicator of the saturation of a color. Newton's placing of the spectral hues on the periphery of the circle in proportion to a musical scale meant that opposite colors were not exactly complementary colors (if close to them). For this reason, and the varying chromatic strength of different hues, opposite colors "when mixed in an equal proportion . . . the Colour compounded of those two shall not be perfectly white, but some faint anonymous Colour." Newton also was aware of compounded purple colors that belong near line D–O in his circle and that, thus, spectral hues and compounded purples can form a closed series. Newton chose a distribution of hues according to a musical scale on his color circle that, with its common white center, also represents geometrical parsimony.

C.B.'s color circle, as shown, is a pigment-mixing diagram based on the idea of yellow, red, and blue as primaries. The circular form, unlikely to have been influenced by Newton, may have been the result of independent realization by a painter that, perceptually, a series of hues derived from three primary pigments and produced by mixing neighboring pigments can return upon itself.

As an entomologist Schiffermüller had interest in systematic color arrangement as a means for classifying butterflies and other colored insects. He also had contact with many artists and was aware of issues in finding harmonious color combinations, devoting a chapter of his book to this question. In addition he thought that the time had arrived to create a complete system of colors. His circle was a first attempt in this direction. Its form was likely derived from the spiral arrangement of hues of different tonal values of Castel and his presumed knowledge of Newton's work and C.B.'s book. By this time the hue circle had become a convention that made intuitive sense. Many future systems would be based on it.

The arguments offered so far for the color circle consist of rationalizations based on likely reasoning and insight available in the seventeenth century. There is a further and perhaps stronger argument. As will be shown later and in Chapter 6, a form of data analysis called multidimensional scaling (see Chapter 3), when applied to spectral colors, results in a geometrical distribution of data points best fit with a circle. In addition mathematical analysis of large collections of spectra of colored objects by various methods, but without consideration of human color vision properties, locates the spectra in spaces where a series of hues approximately forms a circle. A hue circle therefore is a pattern that may have become subconsciously apparent and is the result of one of the strategies of our visual system.

A full-fledged color order system based on a hue circle was delayed because of the growing importance of the idea of three primary chromatic colors, presented as a triangle, that could be used to create all other hues by mixture. The source of the idea of yellow, red, and blue as three primary colors is unknown, and it might have developed from dyeing and painting technology. We have seen these three chromatic primary colors in d'Aguilon's diagram of 1613. The same primary chromatic colors are mentioned by Robert Boyle in his *Experiments and Considerations Touching Colours* (1664). Using "pigments" as generic word for colorants he wrote: ". . . there are but few Simple and Primary Colours (if I may so call them) from whose Various Compositions all the rest do as it were Result. . . . I have not yet found, that to exhibit this strange Variety they [painters] need imploy any more than *White*, and *Black*, and *Red*, and *Blew*, and *Yellow*; these *five*, Variously Compounded, . . . being sufficient to exhibit a Variety and Number of Colours, such as those that are altogether Strangers to the Painters Pallets, can hardly imagine. . . . by these simple compositions again Compounded among themselves, the Skilfull Painter can produce what kind of Colour he pleases, and a great many more than we have yet Names for." Yellow, red, and blue had been celebrated as key colors in several works of the contemporary French painter Nicholas Poussin (1594–1665). Francis Glisson was a supporter of these colors as primary chromatic colors. The inventor of four-color printing (yellow, red, blue, and black), Johann Christoffel Le Blon (1667–1741), published his book on this subject, *Coloritto*, in 1725. As a result, in the first explicit steps toward a three-dimensional color system, the form of a triangular pyramid (tetrahedron) was used with the primary colors yellow, red, blue, and white at the vertices.

2.6 MAYER AND LAMBERT'S COLOR SOLIDS

Tobias Mayer (1723–1762), self-educated German geographer, astronomer, and physicist developed, perhaps through his work with map printers, an interest in color and presented in 1758, in Göttingen, a public lecture on the relationship of colors. A report about the lecture appeared in a local journal. The lecture was published posthumously in 1775 in a collection of works, *Opera inedita Tobiae Mayeri* (Unpublished Works of T. M.) with the title *De affinitate colorum commentatio* (Commentary on the relationship of colors).[29] Here Mayer, for the first time, presented a plan for a systematic color solid, based on three primary chromatic colors. In principle, the samples representing the solid were to be spaced to be visually equidistant. The chosen geometric form of the solid is that of a double tetrahedron. Even though using pigments as a basis for his work, he also had colored lights in mind: "There are three simple or basic colors and no more than that, from mixture of which all others can be generated, which themselves from others, in whatever ratio they might be mixed, cannot be generated in any way: red, yellow and blue. We see them in rainbows, but even more distinctly in rays of the sun captured by a glass prism,

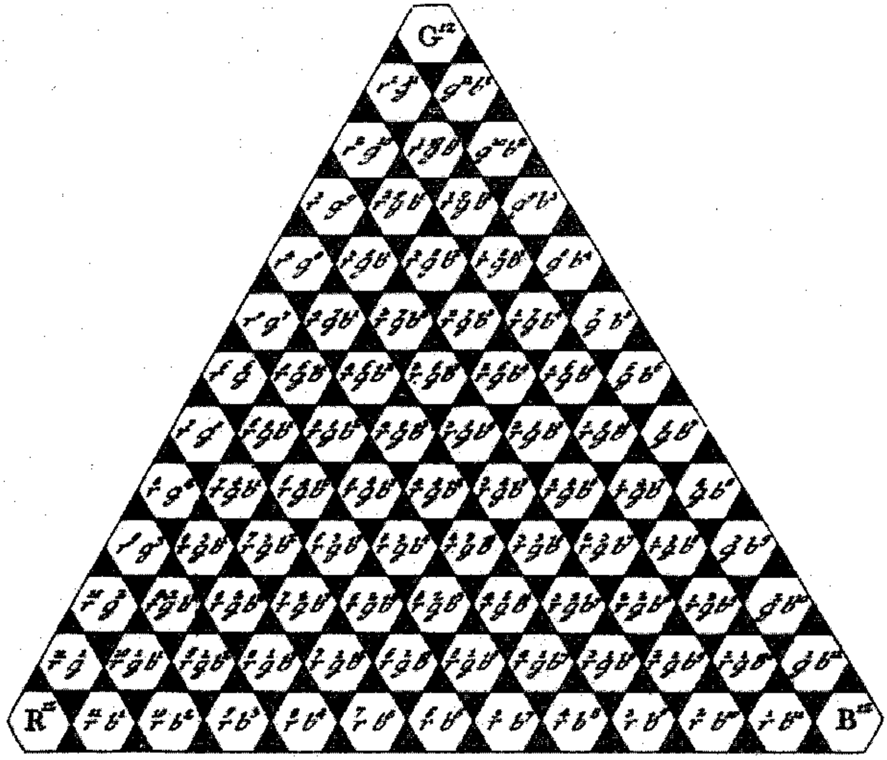


Fig. 2-15 *Arrangement of 91 colors of the central plane of Tobias Mayer's double triangular pyramid, with the color designation scheme based on the three primary colors R, G, and B, 1758.*

though there there are accompanied and surrounded by secondary colors." Based on experiments with pigments Mayer concluded that there should be twelve steps between each of the three primary colors as well as (just as Glisson) twelve steps toward white from the central plane and twelve steps toward black: "One has to employ such a ratio between colors to be mixed so that it can be expressed with numbers which are not very large. . . . Neither in architecture nor music are proportions greater than twelve generally accepted since such ratios could barely be apprehended by unaided senses." Mayer recognized the principle of threshold differences (he may have been the first to express it relatively unambiguously).

Mayer designated his three primaries with **r**, **g** (for *gelb*, yellow), and **b** and indicated their value in the ratio with superscript numbers. In this manner an equilateral triangle is generated in which there are 91 designated colors (Fig. 2-15). He maintained a constant sum of colorant parts in mixtures with white and black and on the next level where colors were mixed with one part white

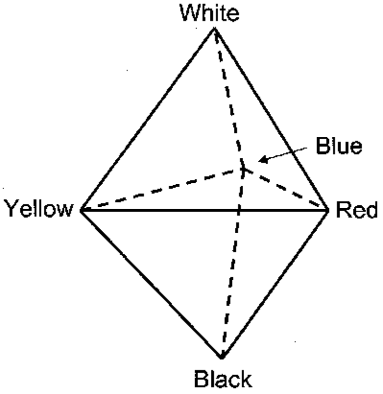


Fig. 2-16 *Schematic representation of Tobias Mayer's double triangular pyramid color solid.*

each, and with eleven divisions per side, obtained 78 colors. Pursuing this approach he ended up with 364 colors above the middle plane and the same number below, a total of 819 color samples for his double tetrahedron (Fig. 2-16), each with its own designation. Mayer did not envisage an explicit central gray scale made from mixtures of white and black but expected that mixture of identical parts of the three primary colors (colorants) in the central plane would produce neutral grays. In the published paper Mayer did not assign specific pigments to his primary colors. However, in the newspaper report of his presentation in Göttingen the "Mayer coordinates" of several pigments were identified, among them, with the identifier 12 (i.e., primary colors), orpiment (king's yellow), vermilion, and azurite (*Bergblau*). (Note that these are the same pigments as those selected by Glisson.)

It is apparent that Mayer's proposal was one of theory, applicable also to mixture of lights. It is known that he made some experiments with pigment mixture but not to what extent he investigated visually equally spaced scales. There is no indication that he was fully aware of the nonlinear relationship between pigment mixture ratio and the resulting visual experiences, as already Johann Wolfgang von Goethe in his comments indicated (Goethe, 1810). Mayer's publisher, Georg Christoph Lichtenberg, added a commentary where he applied Newton's center of gravity principle to Mayer's proposal. He also printed a reproduction of a color triangle hand painted by Mayer and commented extensively on the difficulties in obtaining a good result. In 1772, three years before Mayer's previously unpublished works appeared in print, the Alsatian mathematician, physicist, and astronomer Johann Heinrich Lambert (1728–1777) issued a small book under the title *Beschreibung einer mit dem Calauischen Wachse ausgemalten Farbenpyramide wo die Mischung jeder Farbe aus Weiss und den drey Grundfarben angeordnet, dargelegt und derselben Berechnung und vielfachen Gebrauch gewiesen wird* (Description of a color pyramid painted with Caulau wax where the mixture of each color from white and the three basic colors is arranged, explained, and its calculation and various uses indicated). He had read the report about Mayer's lecture and cited it in his book *Photometria* (Lambert, 1760). Using Mayer's proposal as a basis Lambert developed his own system. He also employed colorant mixtures and the same identification scheme. Lambert justified the triangular scheme from a presumably visually equidistant color circle in which yellow occupies the 12 o'clock position, the border of red against violet the 4 o'clock and blue the 8 o'clock positions. He searched among the available pigments for a yellow that was neither reddish nor greenish and, correspondingly, for a neutral red and blue and on that basis selected gamboge, carmine (cochineal), and *Berlinerblau* (prussian blue, potassium ferric ferrocyanide).[30] A condition Lambert required, beside neutrality of hue, was that his colorants should approach as nearly as possible the intensity of spectral colors. He commented: "I leave it undecided if in the future colorants will be found which approach the spectral colors even closer than the mentioned carmine, gamboge and Prussian blue." Realizing from practical experience that the coloristic strengths of his three colorants was significantly different, he established them by finding (as Glisson did) middle colors between the three primaries and reported them as two parts carmine to three parts prussian blue to twelve parts gamboge. Mixing his primary colorants in a corresponding ratio (12 parts prussian blue, 12 parts gamboge, and 2 parts carmine) produced a near black when applied to white paper and Lambert did not see a need for mixtures of his colorants with black and discarded the lower half of Mayer's pyramid. Lambert expressed dissatisfaction with Mayer's uniform twelve steps. He had the Prussian court painter Calau experiment to develop originally a six-step, later a nine-step visually equidistant triangle. Mixing the colorants with a near water-soluble wax of Calau's invention and gum resulted in colorations of high chroma and good stability. Since all three colorants had a high degree of transparency, Lambert believed he could avoid adding a white pigment and instead used the white of the paper together with increasing dilutions of the colorant mixtures to achieve the tonal declines toward white. Nevertheless, Lambert found that he had to produce a total of 67 mixtures to color his pyramid of 108 colors so that the result met his visual criteria. Interestingly, near black colors are pushed close to basic blue (colors 11, 12, 19, 20; see Fig. 2-17) at the lowest level rather than at 21, 27, 28 where, based on the gravimetric rule, one would expect them.

Lambert proposed for the grades along the sides of the triangle a color naming system that is only moderately intuitive. He used a system borrowed from the methodology for designating directions around the compass, for example: 1, blue; 10, blue toward red; 18, bluish reddish blue; 25, bluish red toward blue; 31, blue red or red blue; 36, red blue toward red; 40, reddish bluish red; 43, red toward blue; 45, red. For the interior of the triangle he used primarily descriptive color terms such as "chestnut red brown."

Lambert saw his pyramid as a general color atlas of use to merchants, for example, to determine if they had fabrics in stock in the desirable colors. Consumers could use the atlas to decide what color clothing to buy and what

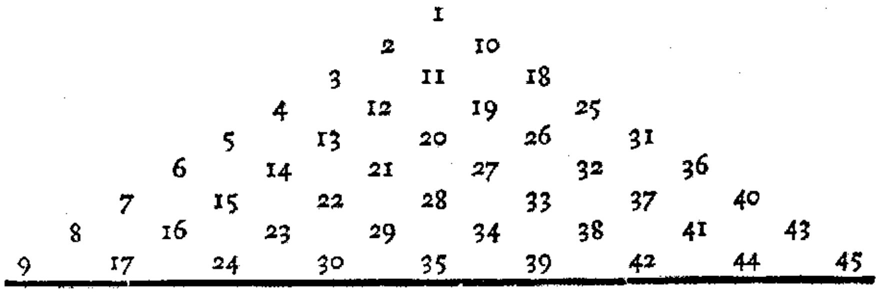


Fig. 2-17 *Identification scheme of primary colors (1, 9, and 45) and color mixes used in the basis plane of Lambert's triangular pyramid color solid, 1772.*

colors to combine: "Caroline wants to have a dress like Selinda's. She memorizes the color number from the pyramid and will be sure to have the same color. Should the color need to be darker or go more in the direction of another color, this will not pose a problem." Lambert believed the pyramid to be of particular use for dyers. If they could find three dyes approximating his three primary colorants, and after having determined their coloristic strength relative to Lambert's primaries, dyers could calculate how much of each dye to use to achieve a given shade in the pyramid. Other potential users were artists who, after sketching, for example, flowers in pencil and writing their color numbers next to them, could reproduce the natural colors in the studio by referring to the corresponding colors in the pyramid.

Lambert built a wooden pyramidal structure, and included an image of it in his book (Fig. 2-18). In this display the 108 selected colors could be properly displayed. For comparison purposes he had Calau paint chips with twelve common artist's colorants along the bottom border: naples yellow, king's yellow, orpiment, azurite, smalt, indigo, lamp black, sap green, chrysocolla, verdigris, vermilion, and florentine lake.

Lambert went about the task with a considerable amount of scientific zeal. Aside from developing an arithmetic of colorant mixture, he explained the result of black from a mixture of his three primary colorants logically as the simultaneous prevention of the activity of each of the three colorants by the other two. His color pyramid is the first attempt to create a geometrical, physical model of object color experiences achievable with his primary colorants, a color solid. The shortcomings of Mayer and Lambert's proposals will become apparent as we proceed.

2.7 COLOR CIRCLES FROM HARRIS TO HENRY

Moses Harris

Color circles continued to be proposed in various formats. In 1766 Moses Harris (1731–1785), an English entomologist and illustrator of insects, dedi-

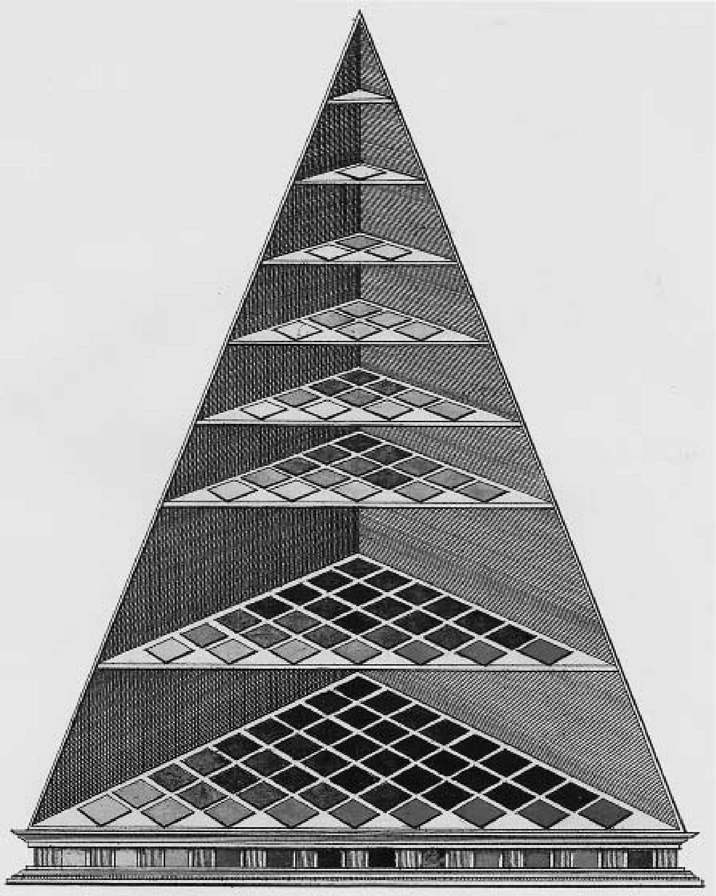


Fig. 2-18 *Depiction of Lambert's color pyramid, 1772. The lowest level contains the 45 colors identified in Fig. 2-17. The higher levels contain reduced sets at higher lightness, ending in white on top of the pyramid. Black is located on the lowest level. The colors displayed on the front of the model represent well-known artist's pigments of the time. (See color plate.)*

cated a small volume titled *The Natural System of Colours* to the then president of the Royal Academy, the painter Joshua Reynolds. It contains copperplates illustrating two color circles, one based on "prismatic" primaries yellow, red, and blue (whose mixture is illustrated as black, however; see Fig. 2-19) and one based on "compound" primaries orange, green, and purple. There are eighteen hues per circle and each hue is illustrated in twenty gradations. The first circle is said to contain only those colors "shewn by the prism." The second one contains "all other colours in nature, not found in the prismatic part." Harris provides specific examples of what he means when using color names, for example: red–vermilion, wild poppy; blue–ultramarine, cornbottle-flower; green–sap-green, leaves of the lime-tree.

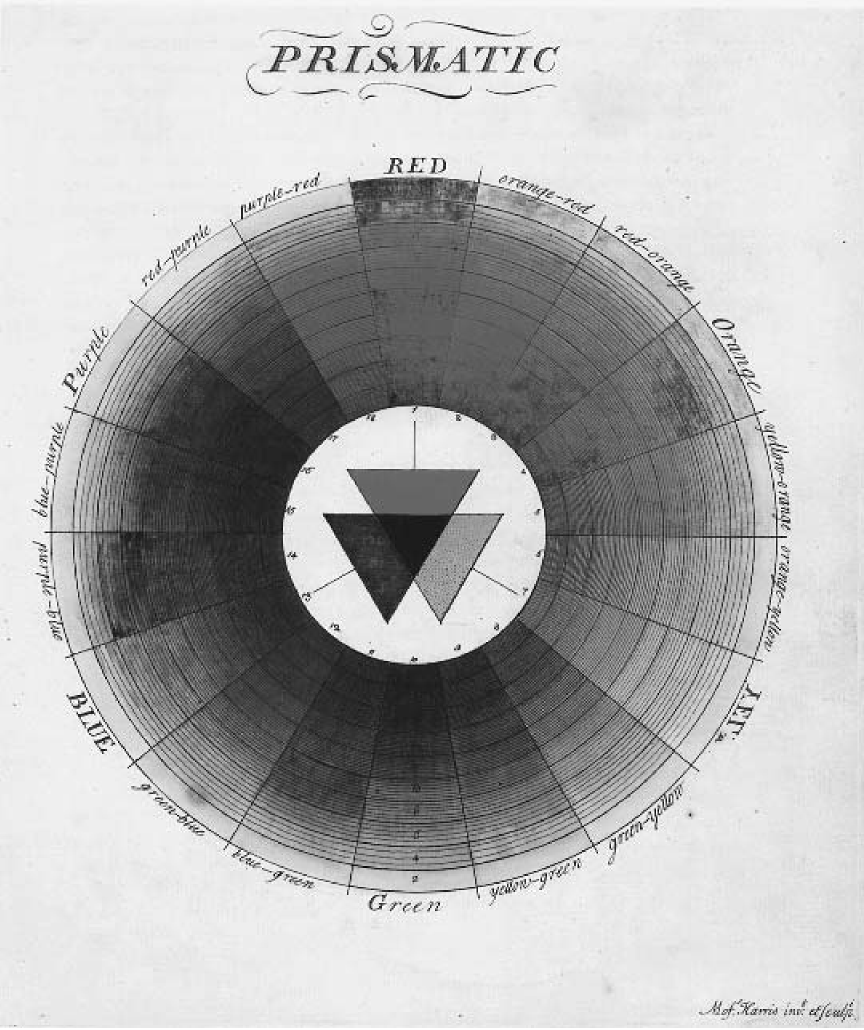


Fig. 2-19 *Prismatic version of Moses Harris's color circle of 1786. Some deterioration of colorants is evident. (See color plate.)*

Concerning the twenty gradations of each hue Harris explained: "The number of colours in this circle are supposed to be eighteen, each of these being divided into twenty parts or degrees of power, from the deepest or strongest to the weakest, . . . so that each of the colours in the innermost or smallest circle contains 20 degrees of power, but each of the outermost but one." They then represent tonal scales from the maximal color near the center of the circle to whitish colors at the periphery. Harris' circle is the first in which maximally contrasting colors are systematically opposed: "Suppose it is required what colour is most opposite or contrary in hue to red, look directly opposite to that colour in the system and it will be found to be green . . . of every colour or teint in the system no one of them contain in their compositions any of the colours of which those on the opposite side are formed. . . .” Harris remarked on the problems of coloring his system properly. Certain colorants (indigo, gamboge, carmine, sap green) are said by him to contain twenty degrees of power while others do not.

Frisch

A color circle with tonal scales in direction of both black and white was offered in 1788 by the Prussian painter Johann Christoph Frisch (1738–1815).[31] Frisch’s immediate purpose was to demonstrate tonal colors in much detail as an aid for painters at a time when subdued coloration was fashionable. Frisch’s color circle, presented only in outline, consists of 40 concentric rings, beginning with black in the center. Only two blackish colors are found on the second ring: blackish red and blue. On ring 3 the range is expanded to eight hues with reduced blackness: yellow, orange, red, purple, violet, blue, sea green, and leaf green. On ring 4 the hues swell to sixteen, with further reduced blackess and on ring 5 to 32, the maximum number. Here the explicit description ends and rings 6 to 10 were to contain tonal values toward white. Rings 11 to 20 are even less well described but apparently were to contain further tonal values ending up in black again at the periphery. Frisch expressed the opinion that there are more equal-sized hue steps between yellow and blue and between red and blue than between yellow and red, and he selected the size of his hue segments accordingly.

Goethe

Because of its importance in aesthetics the color circle of the German poet and natural philosopher Johann Wolfgang von Goethe (1749–1832), introduced in his *Zur Farbenlehre* (On the doctrine of colors) of 1810, should be mentioned here. Goethe’s six-color circle is based on two triangles: the fundamental three colors with red on top, and the intermediary colors. The position of red results from a process Goethe called *Steigerung* (intensification) in which red represents the peak. At the same time it reflects successive contrast: “To recognize quickly which colors are called forth by this contrast one can use the illuminated color circle of our tables that is altogether organized according to natural principles and offers here good service because its opposing colors are those that are demanding each other in the eye. In this manner yellow demands violet, orange blue, purple green and vice versa.” Goethe interpreted his color circle also in regard to principles of color harmony.

Color circles for the primary purpose of demonstrating rules of color harmony have also been developed by the German painter Matthias Klotz (1748–1821) in 1816 (see Section 2.10), the English colorant producer and dealer George

Field (1777–1854) in 1817, the French chemist Michel-Eugène Chevreul (1786–1889) in 1839 (see Section 2.12), the German art historian Friedrich Wilhelm Unger (1810–1876) in 1858, the Austrian physiologist and son of an artist Ernst Brücke (1819–1892) in 1866, the French physiologist Charles Henry (1859–1926) in 1888, as well as others.

2.8 THREE PRIMARY COLOR THEORIES

The idea of three fundamental colors received an important boost in 1801 from the English physicist and physician Thomas Young (1773–1829). He restated an hypothesis first mentioned by the English glassmaker George Palmer in his *Theory of Colours and Vision* (Palmer, 1777).[32] Palmer believed that "The surface of the retina is compounded of particles of three different kinds, analogous to the three rays of light [necessary to mix all colors]; and each of these particles is moved by its own ray." In Young's mature version the eye is provided with distinct sets of nervous fibers: ". . . if we seek for the simplest arrangement, which would enable [the eye] to receive and discriminate the impressions of the different parts of the spectrum, we may suppose three distinct sensations only to be excited by the rays of the three principal pure colours, falling on any given point of the retina, the red, the green, and the violet, while the rays occupying the intermediate spaces are capable of producing mixed sensations, the yellow those which belong to the red and the green" (Young, 1824). However, Young's three primary colors were not those of Boyle, Le Blon, or the dyers and painters, a situation that continued to cause confusion until resolved by Maxwell and Helmholtz (see glossary entry *primary colors*).

In 1809 the English painter and conchologist James Sowerby (1757–1822) published *A new elucidation of colours, original, prismatic, and material: showing their concordance in three primitives, yellow, red and blue, and the means of producing, measuring, and mixing them: with some observations on the accuracy of Sir Isaac Newton*. In this treatise he attempted a synthesis of ideas about colors from lights and from colorants. Sowerby experimented with additive color mixture by placing narrow strips of paintings of his three primary colorants gamboge, carmine, and prussian blue next to each other and viewing them from a distance. As a painter he was interested in the effects of adding white and black pigments to his primary colorants. In this work a two-dimensional color order system progresses from white in the center to the most intense chromatic colors and from there toward black on the periphery.

2.9 RUNGE'S COLOR SPHERE

Phillip Otto Runge (1777–1810), German romantic painter and acquaintance of Goethe, published in his last year *Die Farben-Kugel oder Construction des*

Verhältnisses aller Mischungen der Farben zueinander, und ihrer vollständigen Affinität, mit angehängtem Versuch einer Ableitung der Harmonie in den Zusammenstellungen der Farben (Color sphere or construction of the relationship of all mixtures of colors, and their complete affinity, with an attached essay on the derivation of harmony in color compositions), 1810. Runge made first mention of his color sphere in a letter to Goethe in 1807. The book contains also an essay by Runge's friend Henrik Steffens *Über die Bedeutung der Farben in der Natur* (On the significance of colors in nature).[33]

Runge distinguished between transparent and opaque colors. The color sphere was constructed to contain the totality of opaque colors created from the five elemental colors red, yellow, and blue as well as black and white. The central color circle has blue on top (in agreement with Castell and Schiffermüller) green at 4 o'clock and red at 8 o'clock. Of the twelve segments reddish colors occupy seven, as do yellowish. Bluish colors occupy five and greenish colors only three segments. The circle is described as a rounding out of two opposing equilateral triangles (in agreement with Goethe). The first contains the fundamental colors yellow, red, and blue. The other triangle contains the mixture colors seen as half way between the fundamental colors: orange, violet and green. When all three colors of a triangle are mixed in equal amounts, neutral gray results. A gray axis is erected in the center of the circle so that gray is obtained by mixture of black and white, by mixture of the three primary chromatic colors, or by mixture of two opposing colors in the hue circle. The gray axis and the color circle form a double cone that is extended to the ideal geometric figure of the sphere (Figs. 2-20, 2-21).

> Since all three colors, blue, yellow and red stand at the same distance from white and black, therefore, the center of the color disk in which those three have lost their individuality through equal activity must be in the same relationship and

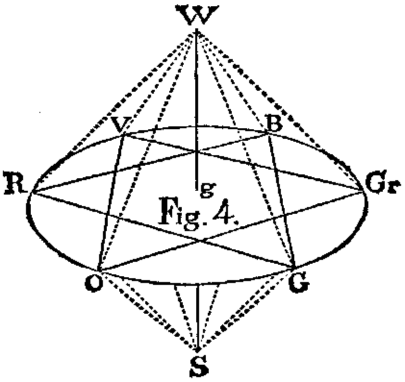


Fig. 2-20 Runge's basic color triangle (R for red, G for yellow, B for blue) combined with the triangle of secondary colors (orange, violet, green), extended horizontally to a circle and vertically in cone shape toward white and black, 1810.

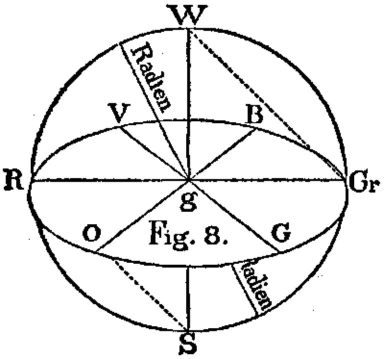


Fig. 2-21 *Runge's double-cone color solid rounded to final spherical form, with white at the top and black at the bottom of the sphere, 1810.*

> in the same distance to white as to black as those three. Since both of these points (the center point between white and black and that of the triangle Blue, Yellow, Red) mathematically coincide it follows that both must be one and the same . . . and that from the identical difference a complete indifference results into which all individual qualities have dissolved. . . . This point, since it is in equal distance to all five elements, is therefore to be seen as the general central point of them all. . . . All mixtures that result from the inclination of a point on the complete color circle toward white or black (a tendency which is common to all these points) will slowly loose themselves toward white and toward black. . . . as the differences of all points of the inclination toward white or black from the central point are radii the points form nothing but circle segments ending in the poles white and black. . . . Thereby, the complete relationship of all five elements, through its differences and inclinations, forms a perfect sphere. Its surface contains all five elements and those of their mixtures which are generated in friendly inclination of their qualities and toward its center all colors of the surface dissolve in equal steps into a balanced gray. . . . Every color is placed in its proper relationship to all pure elements as well as all mixtures and in this manner the sphere is to be seen as a general table by which he who requires various tables in his business, can always find the relationship of the totality of all colors. It now must be evident to the attentive reader that it is not possible to find a plane figure that is a complete table of all mixtures; the relationship can only be presented as a solid.

In a hand-colored copperplate figure views of the sphere from the white and the black poles, a horizontal cross section along the equator and a vertical cross section through the two poles are given (Fig. 2-22). Unlike Lambert's, Runge's idea was not to offer just a color atlas but to present an idealized theoretical construct that not only was meant to represent color relations for the painter but also presumed deeper psychological and mystical relationships gleaned in part from Goethe. As indicated in the title of the publication, Runge used the sphere geometry to also develop his ideas about color harmony.

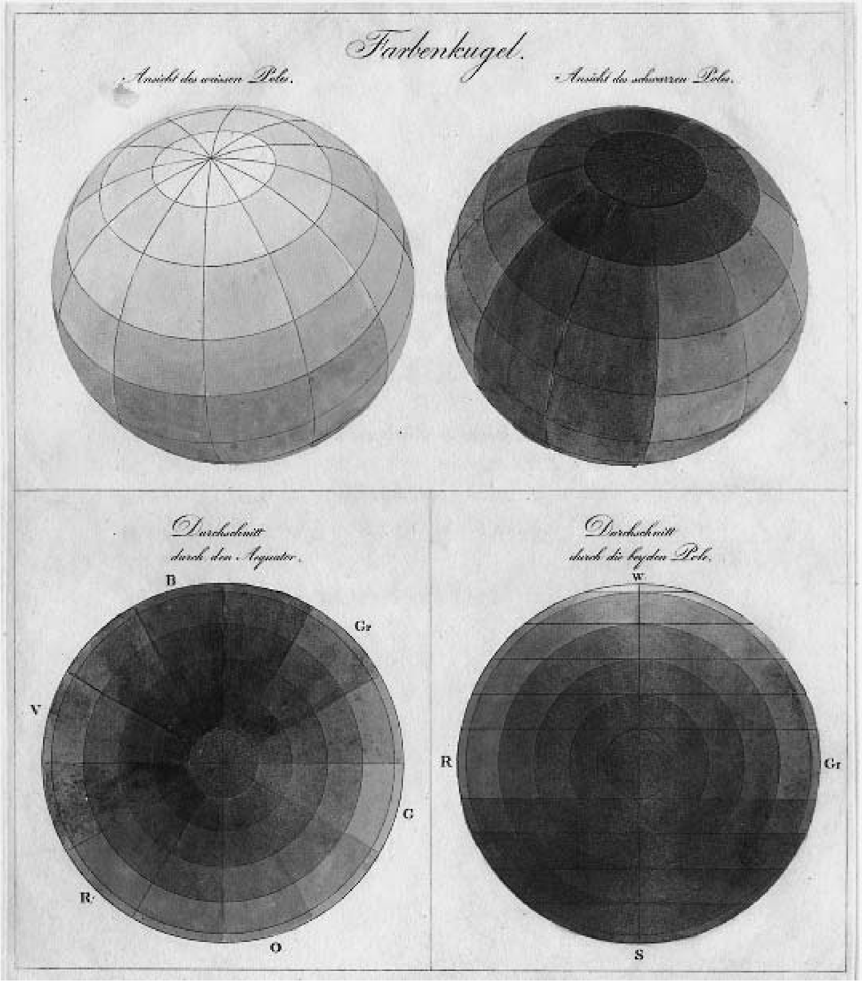


Fig. 2-22 *Hand-colored copperplate of Runge's Farben-Kugel (1810). Views toward the white and black poles are on top. The equatorial cross section is on bottom left and the polar cross section on the right. There are four saturation steps between the full color on the surface and the middle gray in the center of the sphere. (See color plate.)*

While representing the most complete color order system so far, Runge's proposal still suffers from unresolved issues. The central axis from pole to pole represents a gray scale, but the full colors of the equatorial circle are of various lightnesses so that the meaning of the vertical axis is not explicitly defined. Colorants that in equal mixture produce a neutral gray do not exist, but Runge did not address the problems materializing when mixing colorants. These and other issues have only been resolved in the twentieth century.

2.10 THE CYLINDRICAL SYSTEM OF MATTHIAS KLOTZ

Runge was an admirer of Goethe's *Farbenlehre*, but another of his countrymen, the Bavarian court and theater painter M. Klotz (1748–1816), was highly critical of Goethe's efforts and paid in his country for his views with a lack of recognition of his own efforts in this field.[34] In 1797 he announced in a journal *Aussicht auf eine Farbenlehre* (Prospect for a color doctrine), followed in 1806 with Notification concerning a color doctrine, in 1810 with Explanatory notification, and finally, in 1816 with *Gründliche Farbenlehre* (a thorough doctrine of color, published in 300 copies at the expense of the author). Klotz' book consists of two sections, one called *Chromatik* and the other *Prismatik*, the former concerned with object colors, the latter with lights. As a painter Klotz had extensive experience in pigment mixture and believed in the primacy of yellow, red, and blue. Before Runge, Klotz defined his primary colors in the sense of unique hues, as later used by Hering. Comparable to Lambert, Klotz used white paper as his source of whiteness by diluting the pigment dispersions appropriately. In this manner he prepared nine visually equidistant grades (including white) of the three primary pigments, as well as of a gray scale. The highest degree of saturation was obtained at grade 4 of his scale. The darker grades were termed "oversaturated" by Klotz. He next mixed seven intermediary combinations between each of the primaries at grade level 4, resulting in a total of 24 equidistant hues. Klotz believed that in this color circle complementary colors were placed opposite. Klotz placed neutral midgray in the center of the circle and placed between each of the 24 full hues and the central gray three steps of reduced saturation. In this manner he obtained a color chart of 97 colors, with saturated chromatic colors at the periphery and neutral gray at the center. Lightness varies considerably in this chart. Klotz recognized three modification potentials in his system: *Buntmodifikation* (hue modification), *Brechungsmodifikation* (saturation modification) and *Hell-Dunkelmodifikation* (lightness/darkness modification). Klotz understood the structure of the cylindrical model resulting from these modifications but refrained from explicitly describing a cylinder model because he could not see clearly how to bring object colors and light colors in such a system into agreement.

2.11 THE EARLY DEVELOPMENT OF PSYCHOPHYSICS

Uniformity of spacing of colors in color scales, an atlas, or color solid had been a clear but difficult to implement goal for Glisson, Castel, Mayer, Lambert, Runge, and the just mentioned Klotz. It began to be approached from a different angle as part of general psychological investigations. The idea of a differential threshold for illumination was introduced by the French mathematician Pierre Bouguer (1698–1758) in the posthumous *Traité d'optique sur la gradation de la lumière* (Optical treatise on the gradation of light, 1760). He had experimentally determined the intensity a light must have to make a weaker one disappear. In a particular experiment involving the light of candles he found that an increment of 1/64 of the intensity of the candlelight had to be added to it so that the difference was perceptible. Bouguer concluded that just perceptible differences in brightness were caused by changes in illumination and represent a nearly constant fraction.[35]

The term limen was introduced by the German philosopher Johann Friedrich Herbart (1776–1841). He defined a limen or threshold of consciousness, by arguing that ideas could only emerge above a threshold of activity of consciousness.

Weber and Fechner

Thresholds of human senses soon began to be investigated quantitatively by Ernst Heinrich Weber (1795–1878), German anatomist and physiologist. Weber began with investigating the sense of touch. Later he extended his inquiries to temperature discrimination and visual discrimination of the length of lines. Regarding the perception of weight differences Weber stated in 1834: "For experience has taught us that expert and practiced men feel a disparity of weight if it is not less than 1/30 of the heavier weight, and perceive the disparity to be the same if, in place of ounces, we put drams." Regarding length of lines he found: "The length, therefore, in which the disparity lies, even though it is two times less on the former instance, is, however, recognized just as easily, because in both cases the difference between the compared lines is 1/100 of the longer line." In 1846 he contributed the chapter *Der Tastsinn und das Gemeingefühl* (Sense of touch and common sensibility) to *Wagner's Handwörterbuch III*, part of a multivolume encyclopedia of psychology, which made his findings widely known. Weber did not state his findings in an explicit law.

The German physicist and mystic Gustav Theodor Fechner (1801–1887) began to think about issues of psychophysics before he knew of Weber. In his mystical work *Zend-Avesta* (Fechner, 1851) he stated: "If the strength of the physical activity actually underlying some mental activity at some point in space and time is measured by its energy β (energy understood in the sense of mechanics) and if its change, assuming an infinitely small part of time and space, is named $d\beta$, then the accompanying change in the intensity of the mental activity, to be estimated by feeling or in consciousness, is not proportional to the energy change $d\beta$, but to the relative change $d\beta/\beta$. . . ." In his *Elemente der Psychophysik* (Elements of psychophysics; Fechner, 1860) Fechner introduced the term *psychophysics* and argued that sensation cannot be measured. All that can be measured are stimuli and the amount of stimuli that result in a particular sensation or the difference between two sensations. The smallest difference that results in a noticeable difference is the threshold difference. Fechner saw the just noticeable difference (JND), expressed in terms of stimulus, to be the unit of sensation, with the magnitude of sensation being the sum of JNDs that lead from the absolute threshold to a given sensation. Fechner expressed Weber's findings, meanwhile known to him, for the JND increments as follows:

$$d\gamma = K\frac{d\beta}{\beta}, \tag{2-1}$$

where $d\gamma$ is the difference in sensation, K is a constant, and $d\beta$ is the difference in stimulus β. Fechner assumed that if this equation is valid for the JND, it is also valid for small increments of β, the stimulus: "In fact, if one multiplies $d\beta$ *and* β by any number, so long as it is the same number for both, the proportion remains constant and with it also the sensation difference $d\gamma$. This is Weber's law. If one doubles or triples the value of the variation $d\beta$ without changing the initial value β, $d\beta$ is also doubled or tripled." If equation (2-1) applies, Fechner reasoned, the relation between sensation and stimulus can be expressed in the simplest form as

$$\gamma = \log \beta. \tag{2-2}$$

Fechner saw this formula as a differential formula and integrated it with the result

$$\gamma = K(\log \beta - \log b), \tag{2-3}$$

where b represents a threshold; or expressed differently,

$$\gamma = K \log\left(\frac{\beta}{b}\right). \tag{2-4}$$

Fechner termed the value β/b the fundamental stimulus value. "The magnitude of the sensation γ is . . . proportional to the logarithm of the fundamental stimulus value." Fechner named this formula the *Maasformel* (measurement formula). It expresses the number of JNDs the stimulus is above threshold. "In the measurement formula one has a general dependent relation between the size of the fundamental stimulus and the size of the corresponding sensation. . . . This permits the amount of sensation to be calculated from the relative amounts of the fundamental stimulus and thus we have a measurement of sensation."

Fechner's work resulted in three types of experimental methods applied in psychology: just noticeable differences or the method of limits, the method of right and wrong cases or method of constant stimuli, and the method of average error. Each of these methods found champions in succeeding years and is still employed today.

Fechner introduced the idea of "inner" and "outer" psychophysics. Outer psychophysics considers the relationship between the measurable physical stimulus and the reported psychological response. Fechner understood inner psychophysics to mean the relationship between the excitation of nerve fibers and the mind. Thus he saw the entire process as consisting of physical stimulus–excitation–sensation–reported response. He was particularly interested in where in this chain the cause of the logarithmic relation is found.

Plateau and Delboeuf

In 1872 the Belgian physicist Joseph Antoine Ferdinand Plateau (1801–1883) read a paper before the Belgian Royal Society: *Sur la mesure des sensations physiques, et sur la loi qui lie l'intensité de ces sensations à l'intensité de la cause excitante* (On the measurement of physical sensations and on the law that connects the intensity of sensations to the intensity of the cause of the excitation). Plateau asked several painters to halve the perceptual distance between white and black oil-painted squares and found that that the results were in close agreement. He also concluded from viewing etchings in different lights that luminance ratios, and not differences as Fechner required, were applicable and that the formula connecting luminance with perceived brightness was a power function.

Objections were raised against Fechner that humans have no intrinsic sense of the magnitude of a sensation. This objection was countered by the Belgian physicist Joseph Rémi Léopold Delboeuf (1831–1896) who played an important role in early psychophysics. He showed that observers can judge the size of the interval between two sensations immediately and directly (they can effectively compare perceptual distance between two different gray samples against the perceptual distance between two different, say, red samples). Delboeuf demonstrated that for sensations to be measured, they only have to be arranged on a measurable scale, and that absolute magnitude was not required. This idea of sense-distance was further developed by the American psychologist Edward Bradford Titchener (see below). Perceptual scaling of color space since is usually based on Delboeuf's idea. Delboeuf was a pupil of the Belgian inventor of statistics Adolphe Quetelet (1796–1874) who was the first to apply the Laplace and Gauss normal law of error to human data. Thus Delboeuf was sensitive to the effects of judgment error on the determination of perceptual increments. Delboeuf also made an elaborate experiment in which he had observers perceptually halve various distances between gray samples. His analysis of the results confirmed the applicability of the Weber-Fechner law. As a result Plateau's power law fell into obscurity until the 1930s.

Investigation of a visually equidistant eight-step gray scale by the German psychologist Hermann Ebbinghaus (whom we will encounter again later) in 1887 resulted in measured light intensities not quite representative of the Weber-Fechner law.

2.12 CHEVREUL'S HEMISPHERIC SYSTEM

Design of a hemispheric color order system was offered in 1839 in his book *De la loi du contrast simultané des couleurs* by Michel-Eugène Chevreul

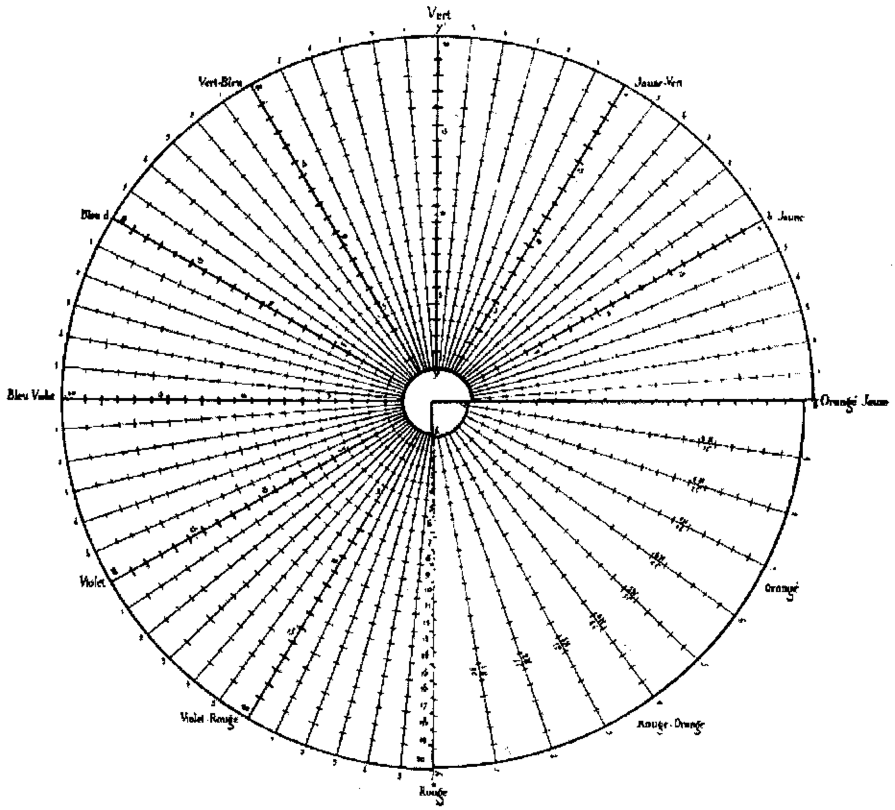


Fig. 2-23 *Chevreul's concept for a 72-hue circle with twenty grades each. White is in the center and black at the circumference of the circle, 1839.*

(1786–1889), a French chemist and director of the *Manufacture Impériale des Gobelins*, the famous tapestry manufacturer. The base plane of Chevreul's system consists of a 72-hue color circle based on the primary colors yellow, red, and blue, separated by equal segments. Twenty grades of lightness of the corresponding hue are located on radial lines from the white center, ending in black on the periphery of the circle (Fig. 2-23). In this ladder the full color of each hue is located at its appropriate level of lightness:[36]

> In each of the scales . . . there is one tone which, when pure, represents in its purity the colour of the scale to which it belongs: therefore I name it the normal tone of this scale. . . . If the tone 15 of the Red scale is the normal tone, the normal tone of the Yellow scale will be a lower number, while the normal tone of the Blue scale will be of a higher number. This depends on the unequal degree of brilliancy and luminousness of the colours.

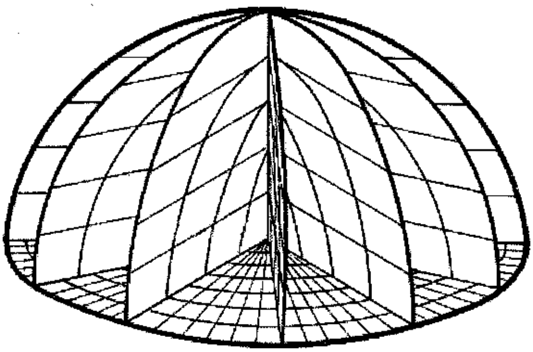


Fig. 2-24 *Schematic depiction of the tonal hemisphere raised above Chevreul's color circle. Black covers the hemisphere, with white at its origin.*

However, later Chevreul placed all full colors on the same grade, a fact that resulted in considerable confusion about his system by readers of different book versions. Colors toward the center from the "normal tone" are mixed with white, and those toward the periphery with black in appropriate amounts to make the constant hue ladder visually equidistant. Chevreul realized that there are many other colors of a given hue that he had not represented in his plane (the plane being in essence a flat representation of the surface of a color solid). To represent the missing colors Chevreul chose to erect a hemisphere above the plane (Fig. 2-24). The apex of the hemisphere is occupied by black, the line between white at the center of the plane and the vertical axis is formed by a 20-step gray scale. Chevreul then proposed to mix each of the colors of the base plane in ten steps with increasing amounts of black but comments: "It is understood that these proportions relate to the effect of the mixtures upon the eye and not to material quantities of the Red and Black substances." In this manner as the angle of the radial lines increases toward 90° the colors become increasingly blackish. In such a system there would be large visual steps between the ninth line of colors, nearing the vertical center line, and the gray scale of the center line, particularly near the core of the hemisphere. This can be avoided, as Schwarz has pointed out (Schwarz, 1997), if one assumes that Chevreul had in mind mixtures of the base plane colors with the appropriate gray rather than with black so that a smooth transition to the gray scale would result. Calculations show that in neither case explicit redundancy of colors occurs, thus negating later criticism of Ostwald and others in this respect. Assuming the mixtures to be with gray would have the advantage that all colors in a hemispherical layer would have the same lightness throughout, a result that Chevreul likely intended. Chevreul's system was never fully illustrated. Ten 72-hue circles, from the full colors with increasing amounts of black, and twelve constant hue ladders representing the key colors of the base plane were produced under Chevreul's supervision using paper printing techniques (Fig. 2-25; Chevreul, 1864).

2.13 DOPPLER'S SPHERE OCTANT

The Austrian mathematician Christian Doppler (1803–1853), discoverer of the Doppler effect, the change in frequency of energy as a result of relative motion of the source and the observer, developed an interest in color in connection with astronomy. He explained changes in the apparent color of stars, often described by astronomers, as due to the effect that became named after him. To be able to explain the effect systematically, Doppler required a systematic color order system.[37] He described such a system in a paper with the title *Versuch einer systematischen Classification der Farben* (Essay of a systematical classification of colors) published in the Proceedings of the Royal Bohemian Society of Sciences in 1847. Placing his three primary prismatic colors yellow, red, and blue on orthogonal axes with the common origin of black, he erected an achromatic axis separated by identical angles from the three chromatic axes. He believed mixture colors of equal intensity between two chromatic primary colors to be located on circular segments. The resulting space has the form of a sphere octant (Fig. 2-26).

> At the points R, B, G, M are located pure red, blue, yellow and gray (white) of intensity $AB = AR = AG = AM$, the quarter circles BR, BG and GR contain all binary mixtures of violet, green and orange, therefore not yet mixed with gray (white). At the points α, β and γ neutral violet, orange and green are located.—Colors tending toward B, R and G are more bluish, reddish, or greenish. . . . The circle segments MR, MB and MG are the loci for all mixtures of red with gray, blue with gray and yellow with gray. . . . There is an infinite number of concentric spherical sections [$M\alpha$, $M\beta$ and $M\gamma$] or at least as many as there are gradations of white light from black via gray to the most intense white.

Doppler was the first to develop a three-dimensional color system not based on object colors. As a mathematician he was able to describe the relationships between colors in correct mathematical terms (see also Schwarz, 1991).

2.14 YELLOW, RED, AND BLUE, FOR A TIME FIRMLY ESTABLISHED AS PRIMARY COLORS

The speculations of Palmer, the work of Le Blon and his competitors, and the choices of Mayer, Lambert, Runge, and others, helped to establish yellow, red, and blue as the generally accepted primary colors. Young, possibly influenced by Palmer, also had concluded that there are three primary colors but his informed considerations made him eventually select red, green, and violet as the additive primaries.

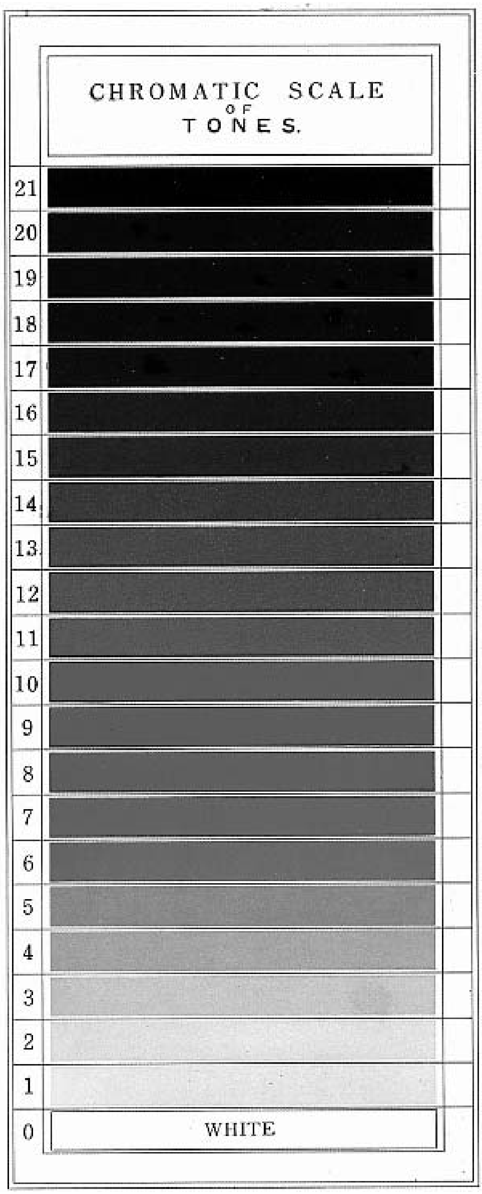


Fig. 2-25 *One of twelve chromatic scales by Chevreul, with local discoloration. The colors range from white through the full color (grade 11) to black. These scales are located on the base plane of the hemisphere. (See color plate.)*

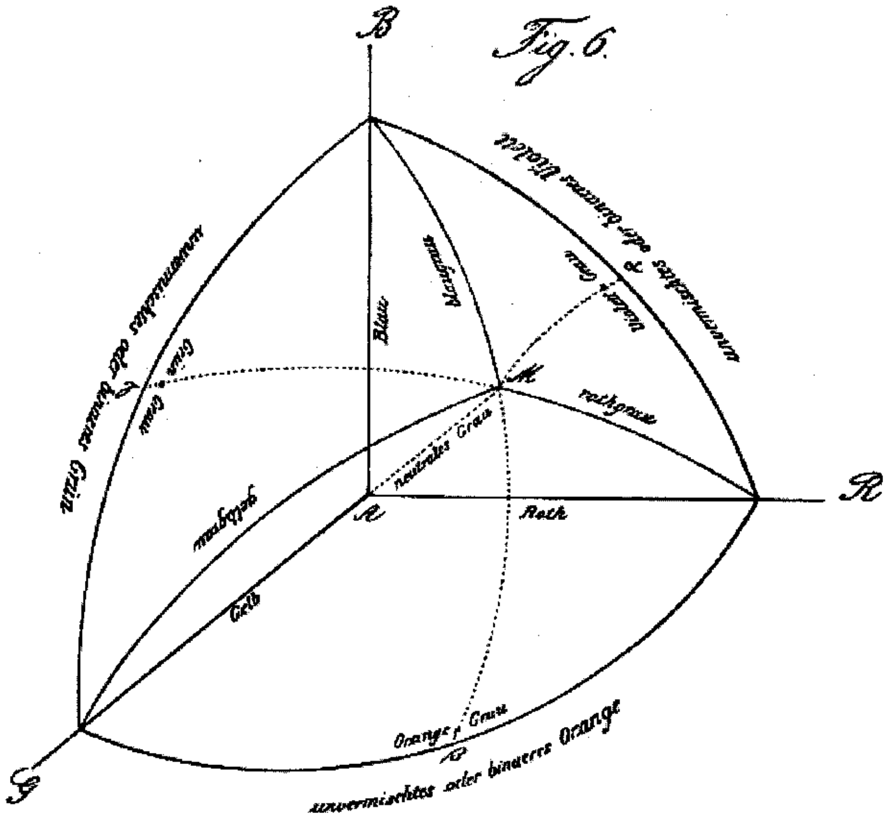


Fig. 2-26 *Sphere octant color space of Christian Doppler, 1847. The axes represent the three primary colors, yellow (G), red (R), and blue (B).*

Hayter

In 1826 the English architect and painter Charles Hayter (1761–1835) self-published a book titled *A new practical treatise on the three primitive colours assumed as a perfect system of rudimentary information.* Hayter's choice of primary colors were yellow, red, and blue, and he referred to Leonardo, Newton, and Young as his predecessors. At the same time Hayter understood that there is a difference "... between the properties of such materials as give their colours to substances suitable to the purposes of art, and the transient effects of light, which must not be considered as belonging to a system of mixing colours for the purpose of painting." Hayter offered six proofs for the primacy of yellow, red, and blue:

> First—That Yellow, Red, and Blue, are entire colours of themselves, and cannot be produced by the mixture of any other colours. ... Secondly—Yellow, Red, and

> Blue, contain the sole properties of producing all other colours whatsoever, as to colour . . . ; Thirdly—Because, by mixing proper portions of the Three Primitives together, Black is obtained, providing for every possible degree of shadow. Fourthly—And every practical degree of light is obtained by diluting any of the colours . . . by the mixture of white paint. Fifthly—All transient or prismatic effects can be imitated with the Three Primitive Colours, as permanently considered, but only to the same degree of compensation as white bears to light. Sixthly—There are no other materials, in which colour is found, that are possessed of any of the foregoing perfections."

Yellow, red, and blue as universal primaries received strong support for a time from the Scottish physicist David Brewster (1781–1868), best known for his work on light polarization. In his 1831 book *A treatise on optics*, Brewster stated: "Red, yellow, and blue light exist at every point of the solar spectrum. As a certain position of red, yellow, and blue constitute white light, the colour of every point of the spectrum may be considered as consisting of the predominating colour at any point mixed with white light. . . ." He offered spectral curves representing the content of red, yellow, and blue in spectral colors. Brewster's theory of three universal primary colors was widely accepted for some thirty years until Helmholtz and Maxwell demonstrated its errors.

2.15 HELMHOLTZ, GRASSMANN, AND MAXWELL

In 1852 the German physicist Hermann Ludwig Ferdinand von Helmholtz (1821–1894) clarified the difference between additive and subtractive color mixture. In the same paper he published a table of mixtures of pairs of five spectral lights. Only the yellow and blue lights he selected resulted, when appropriately mixed, in white light. Reading Helmholtz's account, the German mathematician Hermann Günther Grassmann (1809–1877), purely on the basis of logical thinking, demonstrated in 1853 that if Newton's theory of compound colors is true it must be possible to match any perceived color with three properly selected color stimuli. These colors must be related to the sensitivities of Palmer's and Young's postulated three sensors. Grassmann then showed that as a result any color of the spectrum must have a complementary color with the mixture of the two in appropriate ratio adding up to white light. Grassmann offered a color circle (Fig. 2-27) demonstrating this fact. The color arrangement in the circle uses the identification of key Fraunhofer lines in the spectrum (capital letters) for orientation and in this fashion making the circle for the first time semiquantitative. Grassmann's circle is an advanced version of Newtons's circle. Less than a year after their publication Helmholtz (1854) experimentally validated Grassmann's theoretical considerations, thus confirming Young's view.

In 1857 the Scottish physicist James Clerk Maxwell (1831–1879) published a paper on his color disk mixture experiments in which he described the results of disk mixtures in an equilateral triangle. Maxwell stated: "From [various]

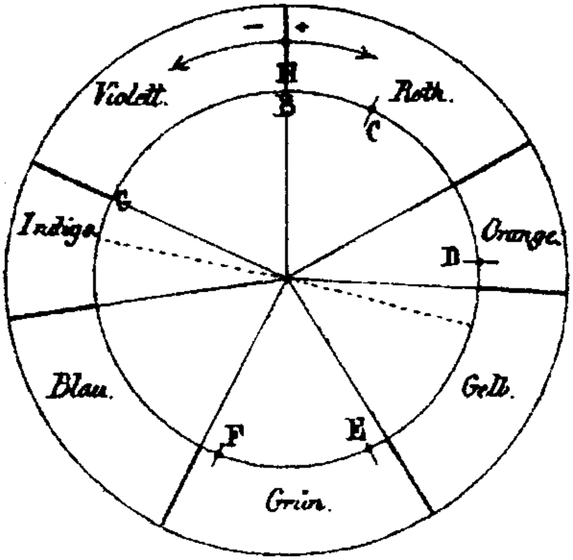


Fig. 2-27 *Grassmann's color circle, derived from Newton's circle, 1853. The beginning and end of the spectrum have been moved to the 12 o'clock position. The sector widths are identical to Newton's.*

facts I would conclude that every ray of the spectrum is capable of stimulating all three pure sensations [related to Young's three sensors], though in different degrees. The curve, therefore, which we have supposed to represent the spectrum will be entirely within the triangle of colour. All natural or artificial colours, being compounded of the colours of the spectrum, must lie within this curve. . . ." In 1855 he had produced a schematic sketch of such an arrangement. The triangle has at its vertices the three primary colors red, violet, and green. One to one mixtures of these produce intermediate hues carmine, blue, and yellow. The spectral colors are shown on the circle, with white at its center. All real colors must fall on or within the circle (Fig. 2-28).

In the first edition of his *Handbuch der physiologischen Optik* (Treatise on physiological optics) of 1867 Helmholtz offered estimated spectral sensitivity curves for the three Youngian sensors (see Fig. 5-3) and a Maxwell-type triangle with the resulting trace of spectral colors (Fig. 2-29). These excitation curves were later measured first by Maxwell (Fig. 5-4) and later by Helmholtz's assistant Artur König (Fig. 5-5). Here Helmholtz also presented a view of the basis plane, with opposing colors being complementary colors, as well as a view from the top of his color cone which he compared to Lambert's tetrahedron. (In Lambert's "subtractive" pyramid, black is in the center of the basis plane and white is at the top, while in Helmholtz's "additive" cone, white is in the center of the basis plane and black on top; Figs. 2-30 and 2-31.)

In an 1872 paper on color vision Maxwell stated:

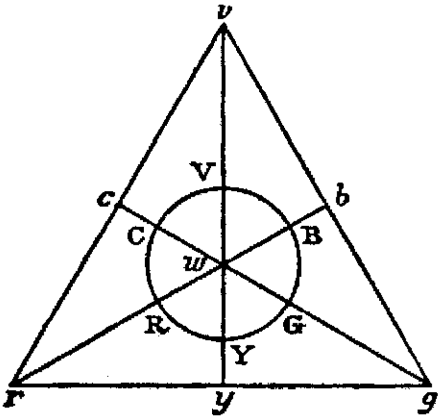


Fig. 2-28 *Maxwell's sketch of the fundamental color triangle based on the primaries red, green, and violet at the apices, and white in the center. The prismatic and extraspectral purple colors are located on a circle, with object colors falling within the circle, 1856.*

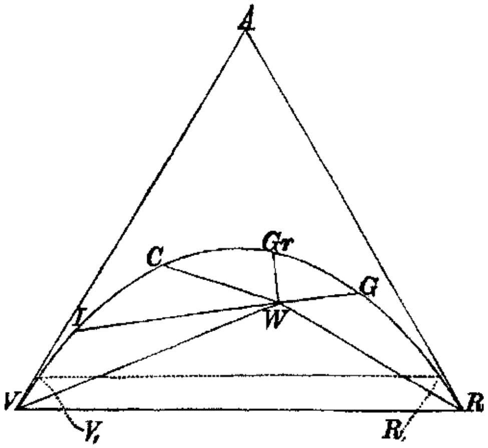


Fig. 2-29 *Helmholtz's estimate of the location of the spectral curve (from the right: red, yellow (G), green, cyan, indigo, violet, with white at the intersection point) in Maxwell's primary triangle, 1867.*

> Let us . . . suppose the colour sensations measured on some scale of intensity, and a point found for which the three distances, or coordinates, contain the same number of feet as the sensations contain degrees of intensity. Then we may say, by a useful geometrical convention, that the colour is represented, to our mathematical imagination, by the point so found in the room; and if there are several colours, represented by several points the chromatic relations of the colours will be represented by the geometrical relations of the points. This method of expressing the relations of colors is of great help to the imagination.

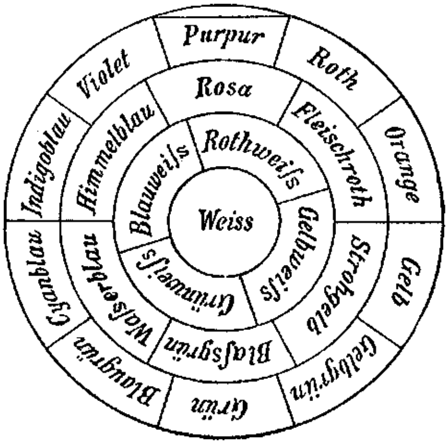


Fig. 2-30 *Basis plane of Helmholtz's color cone with white in the center and the saturated colors on the periphery, 1867.*

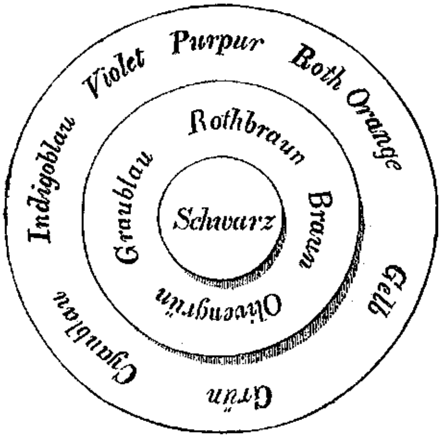


Fig. 2-31 *View from the top of Helmholtz's cone, with black on top and tonal colors from black to the saturated colors.*

With these statements Maxwell provided the basis for all future mathematical/geometrical models of psychological color space.

2.16 HERING

Helmholtz and his theory of color vision was opposed, at times bitterly, by the German physician and physiologist Karl Ewald Konstantin Hering (1834–1918) (e.g., see Turner, 1994). In the mid-1870s Hering proposed a seemingly much different idea of color vision based not on three but on four primary hues: yellow, red, blue, and green, which he called the natural color system (Hering, 1920). Hering referred to physiologist Hermann Aubert's (1826–1892) statement in his 1865 book *Physiologie der Netzhaut* (Physiology of the retina): "If we want to be clear about color sensations, then the words black, white, red, yellow, green, and blue suffice as main designations and I may therefore treat them as principal sensations or principal colors. . . ." Hering developed the concept of antagonistic assimilation-dissimilation processes to explain brightness perception from black to white. At equilibrium the two processes produce midgray. Two similar antagonistic or opponent processes produce four fundamental colors (*Urfarben*), phenomenologically simple colors: red versus green, and yellow versus blue. Hering declared the hues of the four fundamental colors to be the salient features in a color circle. He used a now classical psychological argument in support:

> All hues can be arranged in the circle so that these primary hues divide it into its four quadrants. All colors in (one) half of this circle are clearly yellowish . . . all those (in the other) have more or less blueness in common. . . . If one imagines this color circle halved so that the dividing line passes . . . through two intermediate colors that are opposite each other, for example, through a violet and one of the greenish yellows, and if the hues in either half are compared, then one finds no one chromatic quality common to all these hues. . . . If the dividing line does not pass through two primary colors we always encounter colors that have no chromatic property in common with certain other colors of the same half and thus they do not have the slightest similarity of hue. In this way we recognize . . . that a rational division of the color circle or grouping of color hues in terms of their internal relations is possible only by using the four specified primary colors. (Fig. 2-32)

Adjacent primary hues (but not those opposed) can form hue mixtures. The hues in their most intense form (*C*) can be "nuanced" by admixture of whiteness (*W*) and/or blackness (*S*). Hering presented as an example an equilateral triangle with the full color, white and black at the corners (Fig. 2-33). He developed arithmetic principles that guide the composition of mixed colors. They are composed of relative proportions of *C*, *W*, and *S*. *Reinheit* (purity) of color can be expressed by $C/(C + W + S)$. Hering was fully aware of the intrinsic lightness of colors:

> If one has a primary blue that is as clear as possible and finds a primary yellow that one cannot say is either lighter or darker than the blue, then anyone with good color vision who has even a little practice in color analysis will also observe that the yellow is less clear than the blue or that it is more or less grayish or blackish. On the other hand, if he has next to the clearest possible yellow a blue that does not look decidedly darker than the yellow, then he will see that the blue is whitish. . . . Moreover, I find a good primary red, that is, the clearest possible, lighter than the clearest primary green available.

Hering did not construct an explicit color order system, and it is not clear how he would have done so, that is, how he would have resolved the issue of the different "brightness" of his primary colors.

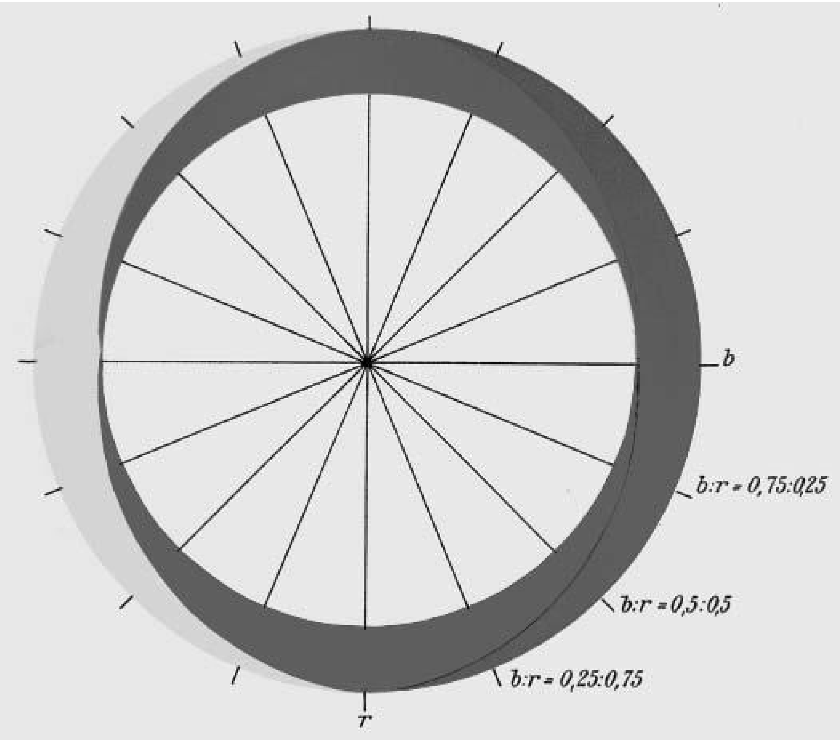


Fig. 2-32 Hering's diagram illustrating the composition of mixed hue perceptions from the unique hues located on the main axes. The fractions of blue and red of three mixed hues are shown bottom right, 1905–1911. (See color plate.)

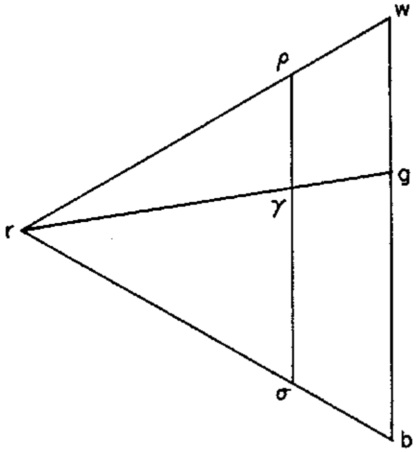


Fig. 2-33 Schematical view of Hering's constant hue triangle, with full color r, white w, and black s on the vertices. Colors along the line parallel to that connecting w and s have constant chromaticness. On the line r to g are located all colors derived from r by veiling with various amounts of g, 1905–1911.

Hering was not satisfied with the Weber-Fechner law, which he found not to apply in his investigations into gray scales. Instead, he proposed a relationship between stimulus intensity and lightness response that equals a hyperbolic function. When the luminous reflectance is plotted on a log scale, this relationship results in an S-shaped function, later found to also apply to cone response kinetics.

Aubert was the first to recognize that Hering's opponent color theory does not have to be contradictory to that of Young and Helmholtz: ". . . if one strictly distinguishes between the process of excitation and the process of sensation."[38] This idea was pursued by Helmholtz and further developed by Schrödinger and Luther (see below). It was the Dutch physiologist Franciscus Donders (1818–1889) and the German physiologist Johannes von Kries (1853–1928) who proposed a combination of the two theories in a "zone" theory, assigning the Young-Helmholtz theory to the cone level and the Hering theory to a later zone on the visual pathway.[39] Despite Donders's and Kries's proposals, the Young-Helmholtz theory of color vision became the leading paradigm until Hering's opponent-color theory was resuscitated in the second half of the twentieth century.

2.17 GEOMETRICAL SYSTEMS OF THE NINETEENTH CENTURY

Cubic Systems

In 1868 William Benson, an English architect, was, in his book *Principles of the science of colors, concisely stated to aid and promote their useful application in the decorative arts,* the first to propose a cubic system. He placed white and black on two opposed corners of the tilted cube with yellow, pink, and sea green on the upper three intermediate corners and red, blue, and green on the lower three (Fig. 2-34). The center of the cube is occupied by a medium gray. Variations of a cubic system were proposed by E. A. Hickethier (1940), and others.

Pyramidal and Cone Systems

A color solid in form of a decagonal pyramid was offered in 1874 by the German physicist Wilhelm von Bezold (1837–1907), the discoverer of the Bezold-Brücke color effect. The hues of his decagon are red, orange, yellow, yellow-green, green, blue-green, blue, violet, and purple. White is located in the center of the base plane and black at the pyramid's apex (Fig. 2-35), that is, it is based on light mixture.

Influenced by Hering, Mayer's triangular double pyramid, was modified in 1897 by the Austrian psychologist Alois Höfler (1853–1928) into a double square pyramid. However, the Hering primaries occupy the corners of the central square of Höfler's construction (Fig. 2-36).

The issue raised by Hering, but not resolved by him, as to how to reconcile the lightness of full colors with his arrangement in an equilateral triangle con-

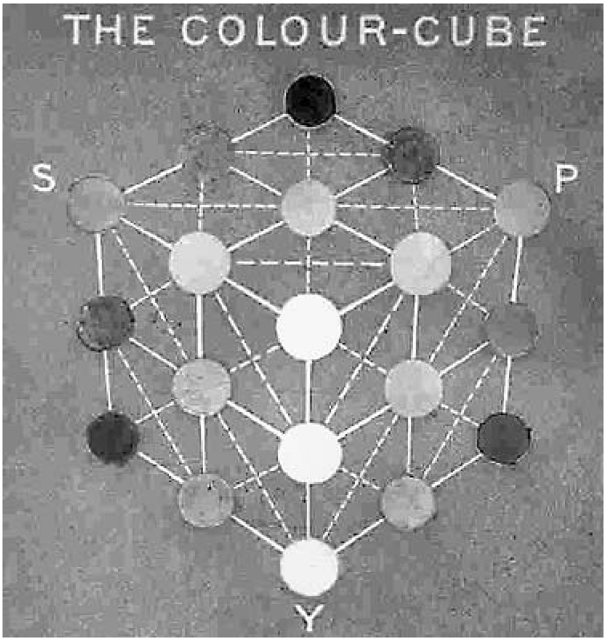


Fig. 2-34 View toward the top, white corner of Benson's tilted color cube, 1868. The gray scale is hidden behind the white sphere. (See color plate.)

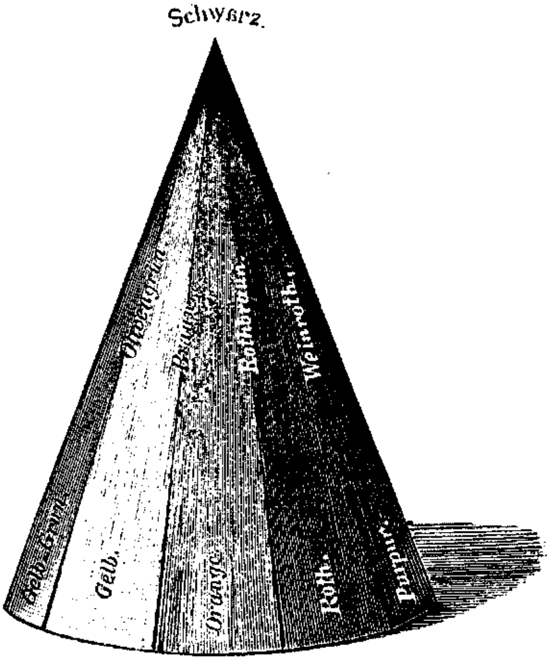


Fig. 2-35 *Bezold's decagonal color pyramid, with black on top and white at the origin, 1874.*

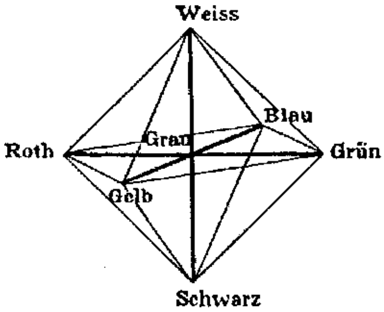


Fig. 2-36 *Höfler's double pyramid based on Hering's unique hues, with the gray scale on the vertical axis, 1897.*

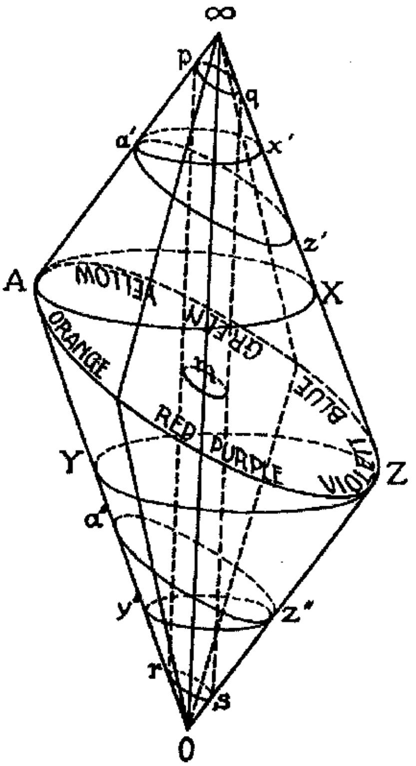


Fig. 2-37 *Tilted double cone of Kirschmann, 1895. Zero and infinite brightness are at the apices. Colors of constant brightness are on horizontal circular planes.*

tinued to be of concern to psychologists. The first attempt to solve it in form of a tilted double cone was offered in 1895 by the German psychologist and student of Wilhelm Wundt (see below) August Kirschmann (1860–1932) (Fig. 2-37). Kirschmann was also concerned with the difference between lights and

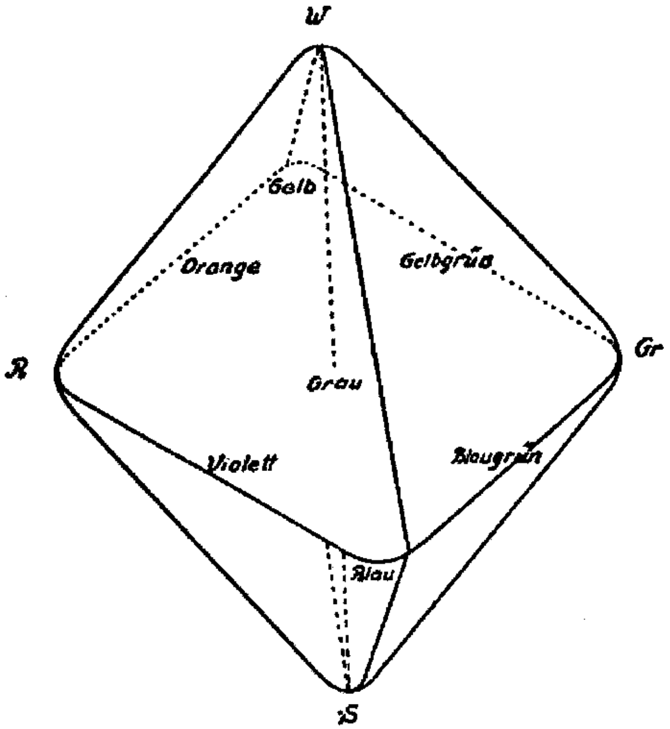


Fig. 2-38 *Ebbinghaus's tilted double pyramid with rounded corners, 1902. The gray scale is located on the vertical axis.*

object colors. His system was to be applicable to both, and he did not put "white" and "black" at the two apices of his tilted double cone but the signs 0 and ∞, explaining: "It is an error to set "white" or "black" at the ends of the axis of a color sphere or a double cone because these expressions do not correctly designate the extremes of the achromatic series of light sensations but instead are ideas of a rather complex nature."

The German psychologist Hermann Ebbinghaus (1850–1909) offered in 1902 a color system in form of a tilted double pyramid. The neutral axis of Ebbinghaus's system is vertical, but the central opponent color plane is tilted along the red–green axis with yellow being closer to white. All surface edges of the solid are rounded because, as Ebbinghaus argued, the transitions between colors are fluid (Fig. 2-38). In 1909 the English-American psychologist Edward Bradford Titchener (1867–1927) offered detailed instructions for building a square tilted double-pyramid model (Fig. 2-39). Neither Höfler, Kirschmann, Ebbinghaus, nor Titchener provided quantitative details of the color arrangements (a painted version of Titchener's pyramid, colored by the

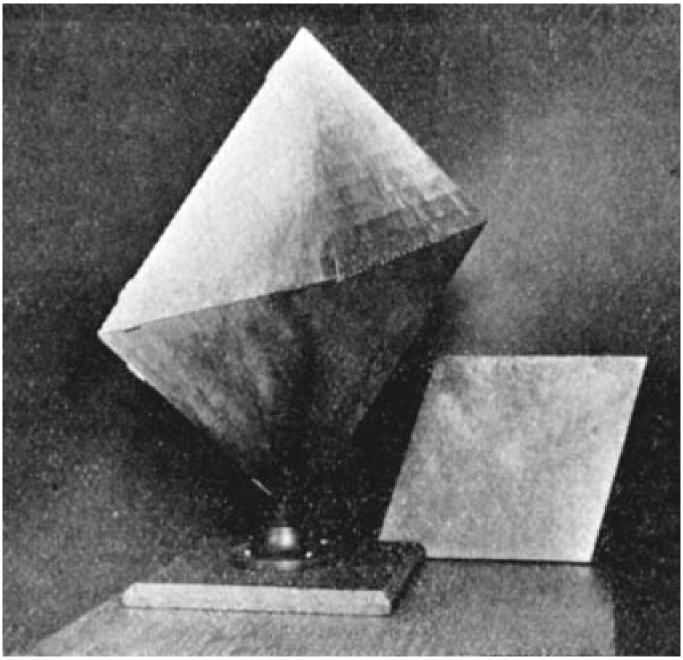


Fig. 2-39 *Titchener's model of the tilted double pyramid. The rhombic central cross section is shown on the right, 1909.*

bird painter Fuentes, is said to be in existence at Cornell University). Their aim was to provide conceptual models of the psychological color solid.

2.18 THE NINETEENTH-CENTURY EXPERIMENTAL PSYCHOLOGISTS

In the footsteps of Fechner and contemporaneously with Maxwell and Helmholtz experimental psychology began to become a science, with particular strength in Germany. The sense of vision was of considerable interest. Experimental psychology claimed Fechner and Helmholtz as its immediate ancestors. The great man of experimental psychology was Wilhelm Max Wundt (1832–1920), a prolific researcher and teacher with many well-known pupils. Under Wundt's supervision considerable research was conducted involving the sense of vision. Wundt discussed color order at several occasions. In 1874, in his *Grundzüge der physiologischen Psychologie*, he proposed a color sphere, not unlike Runge's. But based on the then newly and explicitly investigated color spectrum, yellowish colors assumed a much smaller portion of his hue circle. The eight identified main hues are red, yellow, yellow-green, green, blue-green, blue, violet, and purple. Of the eight sectors reddish and greenish colors

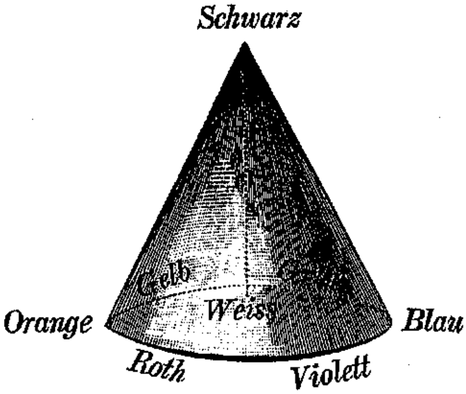


Fig. 2-40 *Wundt's cone with black on top and white on the origin, 1893.*

occupy four, bluish five, and yellowish colors three. In 1892, in the second edition of *Vorlesungen über die Menschen- und Thierseele* (Lectures on the human and animal soul), Wundt offered a Helmholtzian cone with six basic hues: red, orange, yellow, green, blue, and violet (Fig. 2-40). White was located in the center of the base circle and black at the apex of the cone. In *Grundriss der Psychologie* (1896), Wundt offered a double cone.

Further Development of Psychophysics

After Weber and Fechner and their immediate followers, major research activity in psychophysics began to shift to the United States.[40] The Polish-American psychologist Joseph Jastrow worked on the methods of determining the differential limen and recommended that the general limen be replaced by the probable error of visual determination (Jastrow, 1888). Before Jastrow the limen had been defined as the point where 50% of the observers were above and the other 50% below in their evaluation. Jastrow proposed to move the point up based on the statistical error of evaluation, specifically to 75% (the concept of standard deviation was developed only later).

In the same time period the American psychologist James McKeen Cattell (1860–1944) also worked on psychophysical problems. He experimentally pursued the idea that general difference thresholds could be determined by measuring reaction times of observers in psychological tasks. He strengthened the use of statistical methods for the analysis of psychological measurements and became the champion of this new approach that stressed the relationship between the individual and the average observer. In his *On the perception of small differences* (together with his colleague, the philosopher G. S. Fullerton) Cattell criticized Fechner's law on just noticeable differences as being too much dependent on introspection and indicated his preference for Fechner's method of constant stimuli, making use of the error law (Fullerton and Cattell, 1892).

Psychophysics faced fundamental criticism by Kries who argued that numbers applied to sensations are not quantities but merely convenient labels, an issue not yet fully resolved. In the midtwentieth century the American psychologist S. S. Stevens (1906–1973) developed a theory of sensory magnitude based on extensive experimental data. He found that such data could usually be modeled well with power functions. Chapter 3 offers a more detailed view of psychophysics.

2.19 THE MUNSELL SYSTEM

A decisive step forward toward a systematic arrangement of object colors was taken by the American artist Albert H. Munsell (1858–1918).[41] At the age of twenty-one he read Ogden N. Rood's *Modern Chromatics* (then just published), a book that influenced many artists of the period (Rood, 1879). It offered him a solid foundation in color theory and psychophysics of the time. After art studies in Paris and Rome Munsell became lecturer in color composition and artistic anatomy at the Massachusetts Normal Art School in Boston. There he began to concern himself with how best to teach systematic arrangement of colors to his students and how to systematically express pleasing color combinations for artwork. Plotting the colors of one of his pictures in form of a double spiral in 1898 suggested a sphere to Munsell. He began to construct small spheres with color fields on the surfac that, when spun, demonstrated color balance (Fig. 2-41). Early in 1899 Munsell initiated a patent application for his color sphere (granted in the year 1900). In the same year he decided to use the decimal system as a basis of his systematic color arrangement and chose five primary colors: yellow, red, purple, blue, and green. The central vertical axis of the sphere was to be formed by a 100-step lightness scale (named "value scale") and Munsell conceived the then new idea of the chroma attribute for the colors of equal lightness, extending radially from the central axis. By applying these and the hue attribute systematically, Munsell moved away from the tradition of placing the most intense colors of a given hue on the same central plane. This resulted in the vertical and horizontal dimensions of his color solid having unambiguous definitions in terms of the attributes. A portable visual photometer of Munsell's design provided semi-quantitative luminous reflectance data for the colors he selected. As early as 1900 Munsell realized that the form of his color solid could not be a sphere because of the varying intensity of different pigments but would need to be a spheroid. By 1904 Munsell had made visual uniformity by attribute a key concept of his systematic color arrangement and had settled on his color terminology. In 1905 he published *A Color Notation*, a description of the system, with print reproductions of color charts (Munsell, 1905). In the same year

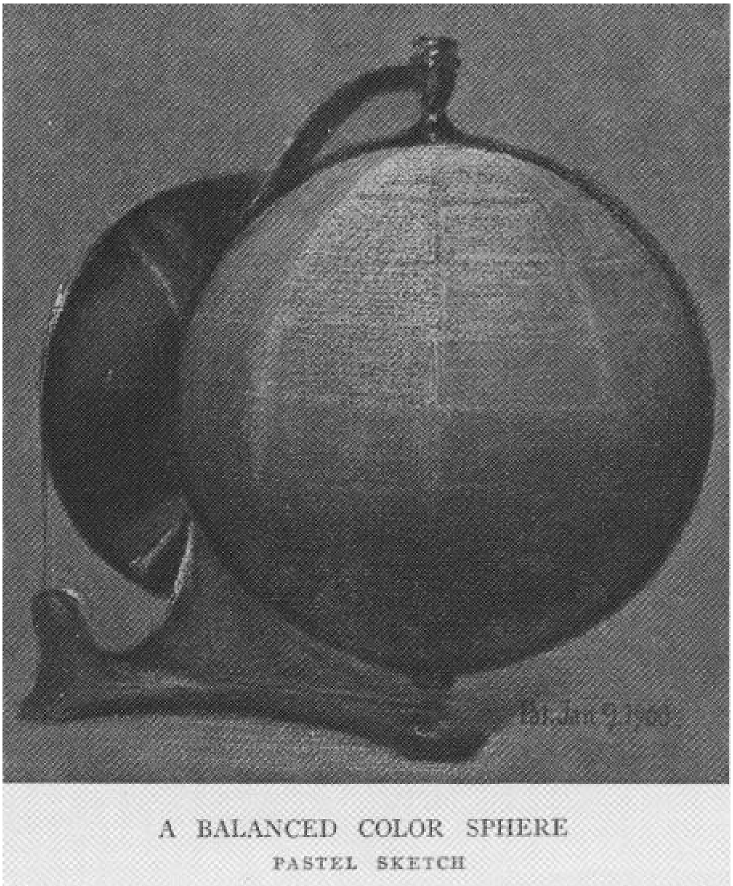


Fig. 2-41 *Artist's rendition of Munsell's balanced color sphere, patented in 1900. The sphere was rotatable to achieve additive color mixture to gray and thereby show the "balance" of the colors on the sphere. The mirror in the back discloses the blue region of the sphere. (See color plate.)*

Munsell met with Ostwald (see below) and exchanged views on color order, but both were to pursue their own views on what such a system was to look like. In 1907 the first version of his *Atlas of the Munsell Color System*, a portfolio of eight plates with painted paper samples, was published, followed in 1915 by an extended version with fifteen plates and a total of 880 color samples (Munsell, 1907, 1915). The two versions offered for the first time detailed internal views of a color solid.

In 1918, shortly before Munsell's death, the Munsell Color Company was formed by Munsell's son and friends. It produced painted paper chips of Munsell colors and in 1929 issued the first version of the *Munsell Book of*

Color, a collection of forty pages, each containing chips of various value and chroma of one given hue. The system is open ended in that the chips represent what is possible with available pigments. Identification of the samples is by hue identifier, value level, and chroma level, for example, 2.5BG5/10 meaning a sample of hue 2.5 blue-green, value 5, and chroma 10. The units of the three visual scales are not identical in perceived magnitude.

Modifications to the system (the so-called Munsell Renotations) were proposed in 1943 by the Optical Society of America Subcommittee on the Spacing of Munsell Colors (Newhall, Nickerson, and Judd, 1943) in form of colorimetric values of the aim colors. They are the basis of the modern system (Fig. 2-42). Expanded editions, based on the Renotations, have been issued in Japan (*Chroma Cosmos 5000* and a condensed version thereof: *Chromaton 707*), and the system has also been implemented in the form of textile samples. For more details and an analysis of the Munsell system, see Chapter 7.

2.20 RIDGWAY'S COLOR ATLAS

In 1912, before the publication of Munsell's 15 plate color *Atlas*, the American ornithologist and botanist Robert Ridgway (1850–1925) published *Color standards and color nomenclature*, an extensive proposal for what amounts to a double-cone color order system. It consists of fifty-three plates each with twenty-seven pigment painted color chips, a total of 1115 chips (excluding multiple whites and blacks). In several respects Ridgway's system is a forerunner of Ostwald's. It is based on Maxwellian disk mixture. His primary disks were produced using the brightest then available synthetic-organic dyes. His central color plane was divided into 71 hue steps, taken in double steps, resulting in thirty-six full color hues. Ridgway considered six hues to be fundamental: red, orange, yellow, green, blue and violet. His fundamental red was located at 644 nm (as he had been assured it should be by the then associate physicist of the U.S. National Bureau of Standards, P. G. Nutting), therefore being a quite yellowish red. His yellow, green, and blue are located near the corresponding generally accepted unique hues. There are 22 steps containing redness, 17 with yellowness, 13 with greenness, and 19 with blueness. The hue steps were meant to be visually equidistant. There are 13 steps between unique red and blue but only six between unique yellow and unique green. Disk mixtures toward white and black for all hues were made in three grades each in constant increments (Fig. 2-43). As Lambert already did Ridgway remarked that the admixtures to yellow have a much different lightness scaling than those to blue. From the central neutral gray scale of nine grades to the full color there are grades of decreasing but always identical admixture of gray. Recent measurements of an atlas have shown the gray scale to be essentially a Fechner type scale and the color circle to be less than uniform. (Aside from possible colorant deterioration it is also necessary to keep in mind the limited color measurement capabilities when Ridgway developed his *Atlas*.)

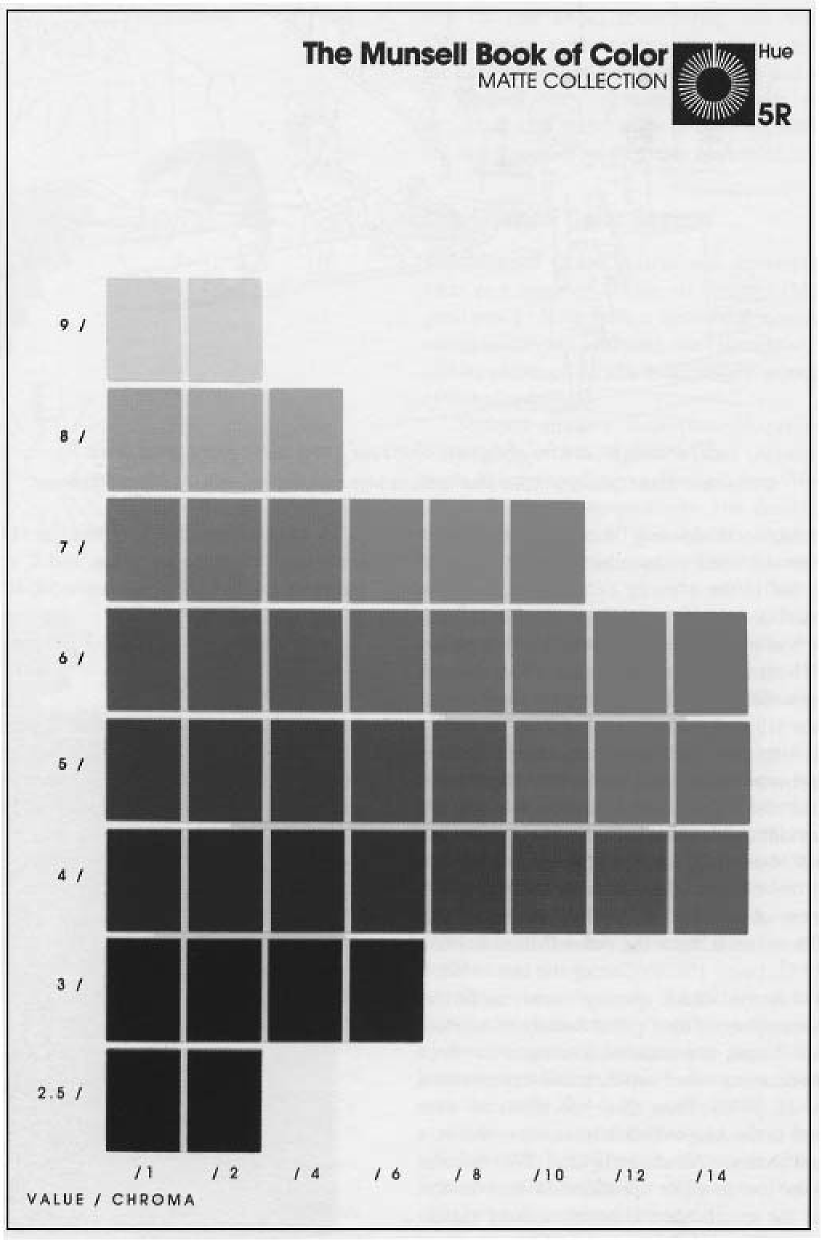


Fig. 2-42 *Constant hue page from a modern version of the Munsell Book of Color. The gray scale is not shown. The chroma scale begins at 1 and continues from 2 at two-grade intervals to chroma 14. Value grades are shown from 2.5 to 9. Courtesy Gretag-Macbeth Company. (See color plate.)*

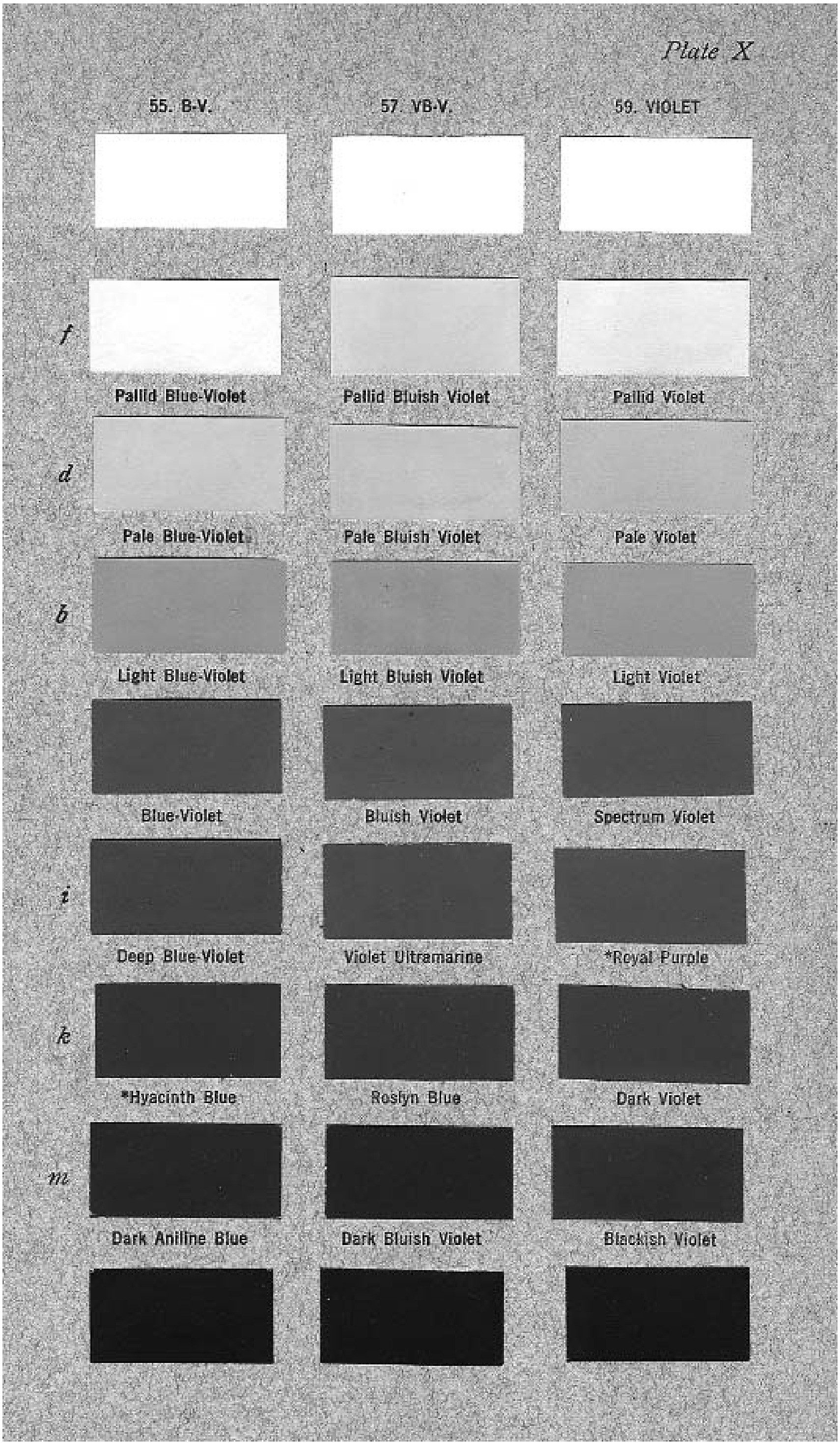


Plate X

	55. B-V.	57. VB-V.	59. VIOLET
f	Pallid Blue-Violet	Pallid Bluish Violet	Pallid Violet
d	Pale Blue-Violet	Pale Bluish Violet	Pale Violet
b	Light Blue-Violet	Light Bluish Violet	Light Violet
	Blue-Violet	Bluish Violet	Spectrum Violet
i	Deep Blue-Violet	Violet Ultramarine	*Royal Purple
k	*Hyacinth Blue	Roslyn Blue	Dark Violet
m	Dark Aniline Blue	Dark Bluish Violet	Blackish Violet

Fig. 2-43 *Page from Ridgway's color atlas showing three reddish blue hues lightened from the central color in three steps toward white and darkened in four steps toward black, 1912. (See color plate.)*

2.21 OSTWALD'S *FARBKÖRPER* (COLOR SOLID)

The German chemist and Nobel prize winner Wilhelm Ostwald (1853–1932) began in 1917 to publish a series of texts on color science, the first of which is called *Mathetische Farbenlehre* (Theory of logical ordering of colors). Here Ostwald described a double-cone color solid of related colors based on additive disk mixture. He scaled the resulting pychophysical solid using the Weber-Fechner law to approximate the implied psychological color order. Ostwald used Hering's color equation: $r + s + w = 1$, where r is the amount of the *Vollfarbe* (full color, colors of highest intensity), s that of black, and w that of white. He interpreted the equation in terms of reflectance data in the visible portion of the spectrum. However, the real colors he used in the disk mixture had, of necessity, in all cases reflectances different from those idealized. Ostwald based his system on three fundamental hues: yellow, red, and blue. The hue circle begins with yellow occupying the 12 o'clock position, proceeding clockwise to red, blue, and green (unlike Munsell who placed red at 12 o'clock and proceeded clockwise toward yellow). Opposing hues are complementary; that is, when optically mixed they result in an achromatic color. Full colors are located conventionally on the periphery of the central plane. All colors of a given hue are placed on an equilateral triangle that is half of a vertical section through the center of the double cone (Fig. 2-44). The central vertical axis is a gray scale. As in Hering's constant hue triangles, lines parallel to the line connecting the full color and white are lines of equal blackness, lines parallel to the line connecting full color and black are lines of equal whiteness. Lines parallel to the line connecting black and white are lines of equal purity. Tonal colors (tints) toward white are located in the upper half of the double cone and tonal colors toward black (shades) in the bottom half.

To bring his psychophysical solid into agreement with psychological scaling, Ostwald applied logarithmic scaling so that 16 grades from black to white and black to full color, as well as white to full color, resulted in 15 visually equidistant steps according to the Weber-Fechner law. In such an arrangement there are 120 chromatic samples in a triangle and Ostwald applied a double-letter system to identify them, in addition to the hue number. The system reveals slices through the color solid in four different directions: equal hue, equal whiteness and blackness, and equal purity. In Europe, the abridged *Farbkörper* illustrating on 12 plates 680 samples (24 hues and 8-step gray scale), when first published in 1919 was a revelation of the multitude of colors, presented systematically (see Fig. 2-45).

Ostwald's system was analyzed extensively by Foss, Nickerson, and Granville in 1944. They demonstrated that, unsurprisingly, it represents a series of compromises and that it fills only a portion of the available gamut of the Rösch-MacAdam limits (see Section 2.23). Perceptual differences between grades in different areas of the solid were found to be of varying size.

Ostwald developed several sets of colorations as atlases of his system, the most elaborate published as *Der Farbenatlas*, with 2500 samples (Ostwald,

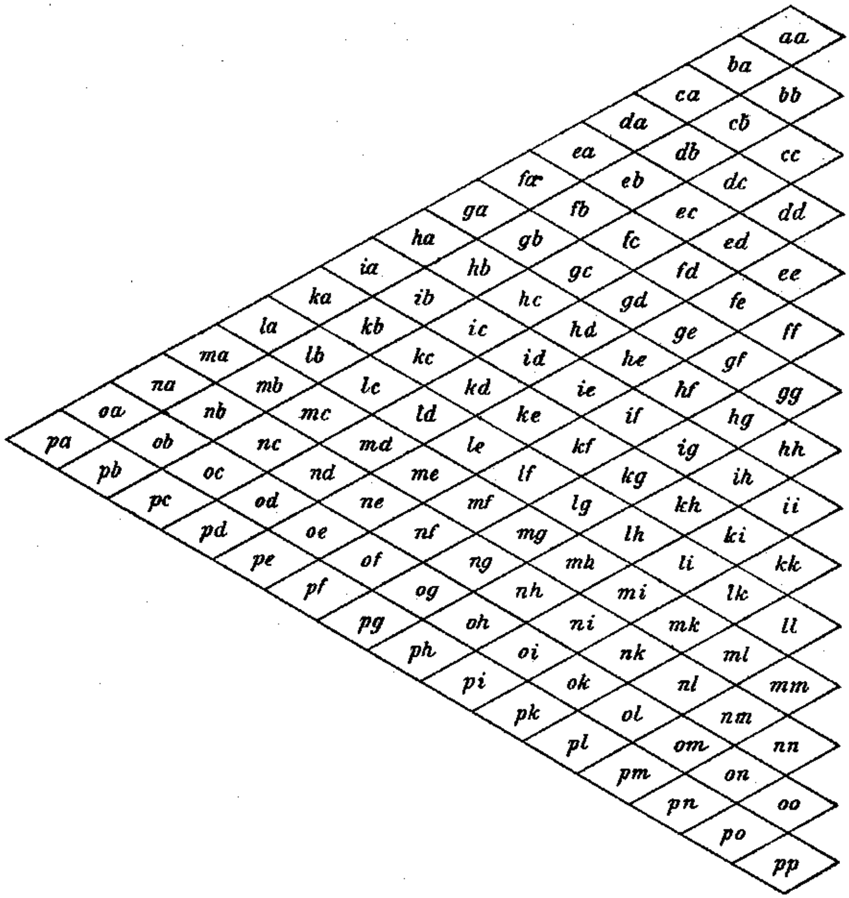


Fig. 2-44 *Color identification scheme for colors in a constant hue plane of Ostwald's double-cone color solid (1917). The grey scale is represented on the extreme right by colors* aa *to* pp. *The full color is represented by* pa.

1917). An atlas in two different sizes implemented with dyed wool samples was also offered (*Der Wollatlas*). Ostwald used his color solid to demonstrate his rules of color harmony (Ostwald, 1918). Implementations of the Ostwald system after the Second World War are those by Aemilius Müller, who gave up complementarity for improved uniform hue spacing (Müller, 1953) and the *Color Harmony Manual* of the Container Corp. of America (Jacobson, 1942; Granville, 1994), issued in three editions.

2.22 GEOMETRICAL SYSTEMS OF THE TWENTIETH CENTURY

After Munsell and Ostwald several new color order systems were proposed, all built on the ideas of double cones, double pyramids, or cubes. All of these

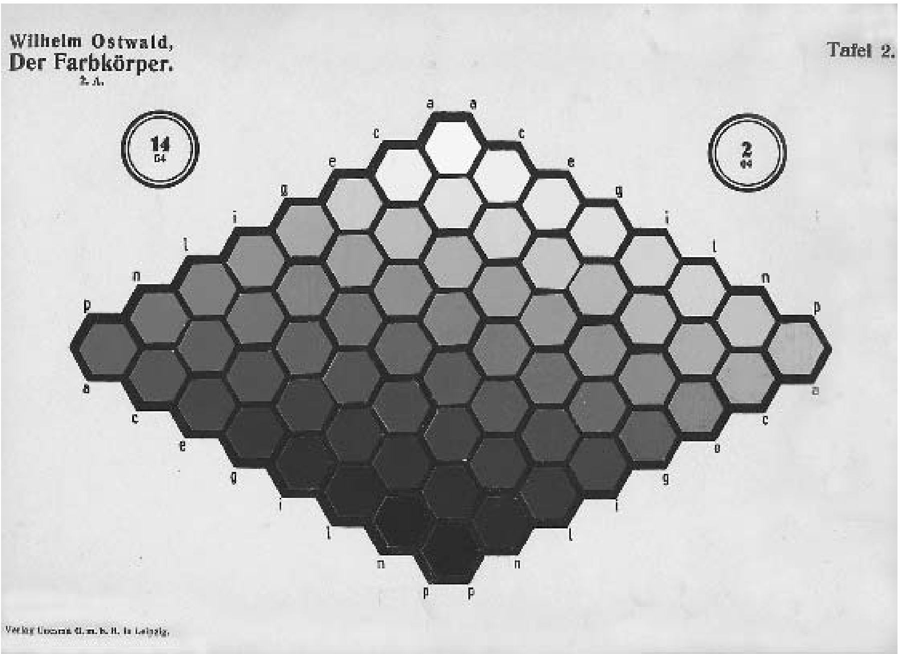


Fig. 2-45 *Vertical cross section through Ostwald's double-cone color solid illustrating constant hue colors 1 and 13, with veiling toward white and black. The achromatic scale is at the center. From Farbkörper, undated. (See color plate.)*

systems formed more or less idealized geometrical solids and did not offer any fundamentally new ideas. Among them are:

- Rounded double cone with tilted central plane from 1920 by the American art educator Arthur Pope (1880–1974). Similar systems were subsequently developed in Sweden by Tryggve Johansson (1939) and Sven Hesselgren (1953), and by Frans Gerritsen (1975).
- Modified double cone by C. Villalobos-Dominguez and J. Villalobos in 1947, developed in Argentina and with an atlas consisting of 7000 samples produced by halftone printing.
- Double pyramid of 1929 by the American psychologist Edwin G. Boring.
- Tilted cube systems by E. Alfred Hickethier, 1940, issued with 1000 printed samples, and Harald Küppers, 1972, issued with 1400 printed samples.

2.23 RÖSCH-MACADAM COLOR SOLID

In the early twentieth-century color measurement and specification began to make significant progress. In 1931 the *Commission Internationale de*

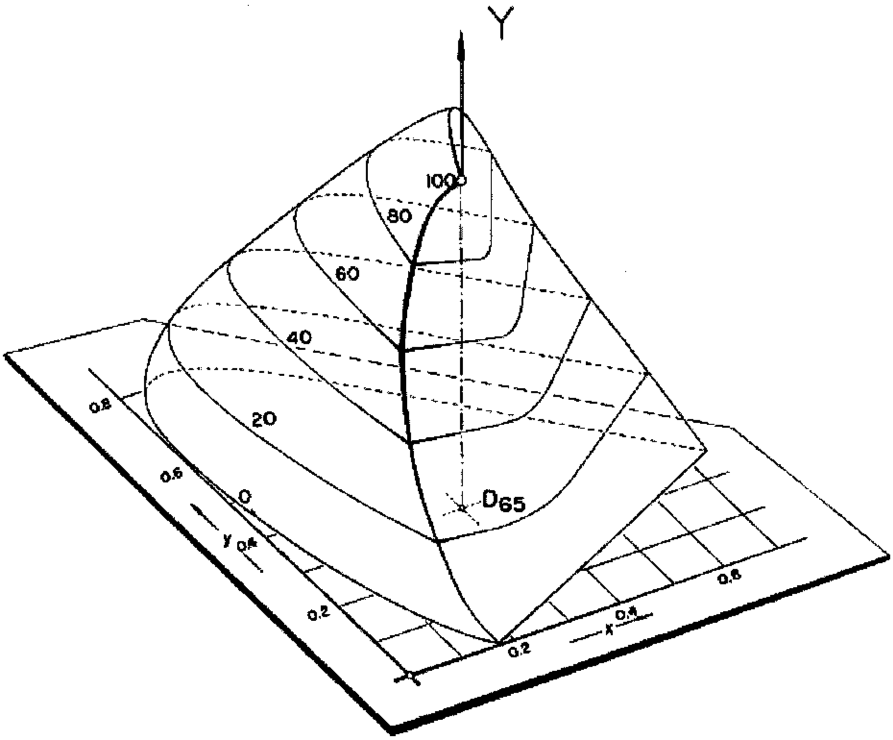


Fig. 2-46 Projective view of the Rösch-MacAdam color solid containing all object colors as viewed by the CIE standard observer in standard daylight D65. The axes represent CIE chromaticity coordinates x and y and luminous reflectance Y. From Wyszecki and Stiles (1982).

l'Éclairage (CIE, International Commission on Illumination) promulgated the 2° standard observer and standard illuminants A, B, and C, making possible the specification of colors as seen by the standard observer under a given illuminant by three numbers, the tristimulus values X, Y, and Z. Alternately, colors can be specified in terms of their chromaticity coordinates x and y in the CIE chromaticity diagram and the luminous reflectance value Y (see Chapter 5). The Austrian physicist Erwin Schrödinger (1887–1961) developed in 1920 a theory of optimal color stimuli. It was further developed by Sigfried Rösch in 1928 and by David L. MacAdam in 1935. MacAdam calculated for the 2° standard observer and illuminants C and A the chromaticity loci of optimal colors as a function of luminous reflectance Y. The resulting solid represents an object color space meant to encompass all possible object colors, specifiable in terms of measured values (Fig. 2-46). Investigations have indicated that this space is not visually uniform. The CIE later recommended more uniform color spaces based on transformations of the tristimulus space.

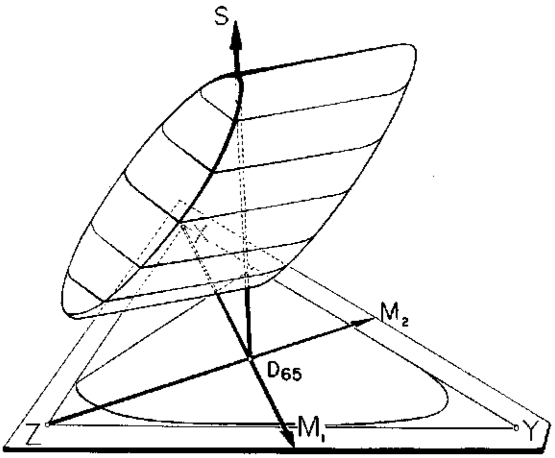


Fig. 2-47 Luther-Nyberg color solid for object colors viewed in standard daylight D65, based on the color moments M1 and M2 and the color weight S. From Wyszecki and Stiles (1982).

2.24 THE LUTHER-NYBERG COLOR SOLID

At about the same time, and employing a different interpretation of a colorimetric system, the Austrian physiologist Robert Luther (1868–1945) and the Russian mathematician N. D. Nyberg independently developed a vector color solid based on what Luther called color moments (Luther, 1927; Nyberg, 1929). The two color moments M_1 and M_2 are located at right angle and are defined as $M_1 = Y - X$ and $M_2 = Y - Z$. The color weight S, defined as $S = X + Y + Z$, is located perpendicular to the crossing point of the two color moments. Within the resulting solid all possible object colors are located (Fig. 2-47). The Luther-Nyberg solid is an early linear expression of an opponent color space where the two color moments are interpretable as greenness-redness respectively yellowness-blueness system axes. In more modern systems the color weight is replaced by the luminous reflectance Y. The Luther-Nyberg solid is located in a psychophysical vector space that is not visually equidistant.

2.25 THE GERMAN DIN6164 SYSTEM

A comprehensive proposal for a color order system, based on ideas of Manfred Richter, was made beginning in 1955 in Germany and promulgated as German standard DIN6164. By 1962 a collection of 590 painted paper chips was available. In 1984 a glossy set of 1004 color chips was added. The color solid forms a modified double cone with the distance from the full hue plane

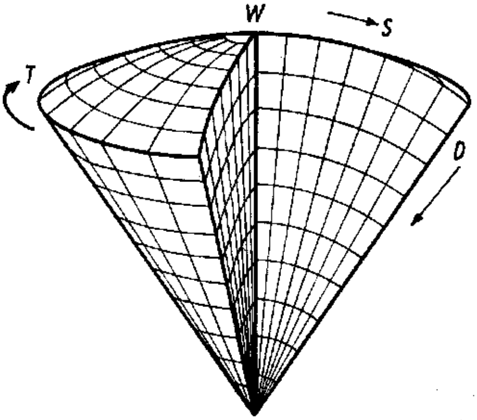


Fig. 2-48 *Schematic depiction of the DIN6164 color solid based on attributes hue (T), saturation (S), and darkness degree (D). W denotes the white point. From Richter (1976).*

to white much shorter than to black. A schematical view is shown in Fig. 2-48. The three color attributes are hue number T, saturation degree S, and darkness degree D. Colors of constant hue as defined in the system have constant dominant wavelength regardless of degree of saturation, and the perceived hue therefore generally varies slightly along these lines. The 24 simple hues of the system, modified from the Ostwald system, begin with greenish yellow at 12 o'clock and proceed clockwise toward red. They are intended to be visually equidistant. Between unique yellow (approximately DIN color 2) and unique red (DIN 9) there are seven steps, between red and blue (DIN 17) eight steps, between blue and green (DIN 21) four steps, and from green to yellow five steps. Six saturation steps were visually scaled at one lightness level only, and the results were extrapolated to other levels. Colors of equal degree of saturation are located on roughly elliptical contours in the chromaticity diagram, not unlike the equal chroma contours of the Munsell Renotations. The darkness degree D is on a scale of 0 to 10, with twenty steps illustrated. It is based on a formula recommended by Delboeuf in 1872, with experimentally determined values for the constants:

$$D = 10 - 6.1723 \log(40.7h + 1), \tag{2-5}$$

where h is the relative lightness A/A_0, with A_0 representing the lightness of the optimal color of the same hue (at the Rösch-MacAdam limit). White has the value $D = 0$ and black $D = 10$.

The DIN system is based on limited visual data (scaling of hue at one level of saturation, scaling of saturation for eight hues at one level of lightness, scaling of darkness degree to find the applicable constants in the Delboeuf equation) and inter- and extrapolation to other areas. Three compromises between psychological results and ease of expression in a psychophysical system have been made:

1. Expression of hues as constant dominant wavelengths, regardless of saturation.
2. Straight-line extrapolations of points of equal saturation intervals at one level of lightness to other levels.
3. Definition of lightness scale based on optimal colors rather than on psychological experiment.

The system was described by K. Witt (1981), K. Richter and Witt (1986), and G. Derefeldt (1991).

2.26 OPTICAL SOCIETY OF AMERICA UNIFORM COLOR SCALES

Intensive study of uniform color scales during and after the Second World War resulted in new insights about difficulties in representing hue, chroma, and lightness in a euclidean system. The Munsell Renotations were completed in 1943, but there was an obvious, considerable discrepancy between those data and the MacAdam color-matching error data of 1942 (see Chapter 6). In 1947 the Optical Society of America formed a research committee to develop as uniform an atlas with samples of a uniform color solid as possible and a formula to express the implicit space, usable for calculation of color differences from colorimetric data. It was headed from 1947 to 1972 by Dean B. Judd and then by MacAdam. The committee very early on decided to present its results in form of a crystalline internal structure of color space, thus to abandon the Munsellian attributes. In the chosen system twelve equal distances around a center color form a cubo-octahedron (Fig. 2-49). The result is the Optical Society of America Uniform Color Scales (MacAdam, 1974). Overlapping cubo-octahedra make it possible to fill color space with this crystalline infrastructure. The cubo-octahedra are arranged in a manner that results in a grid of squares in a given lightness plane, with all differences along the horizontal and vertical directions of the grid implicitly of equal perceptual size (Fig. 2-50). Grids at the next higher and next lower lightness increment level are offset. This arrangement does not result in explicit lines of equal hue or circles of equal chroma but allows seven different kinds of cleavage planes that provide new vistas within the color solid (Fig. 2-51). In 1974 an atlas with 558 painted samples was made commercially available and the colorimetric aim values for these samples have been published (MacAdam, 1978). The system is open ended, and samples are identified by three rectangular coordinates L, j, g, where L denotes lightness, j approximately yellowness or blueness, and g approximately greenness or redness. Formulas have been

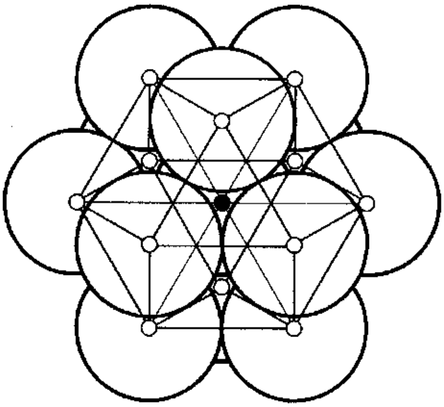


Fig. 2-49 A central color (black dot) surrounded by twelve visually equidistant colors located in the center of spheres and forming a cubo-octahedral structure. From Gerstner (1986).

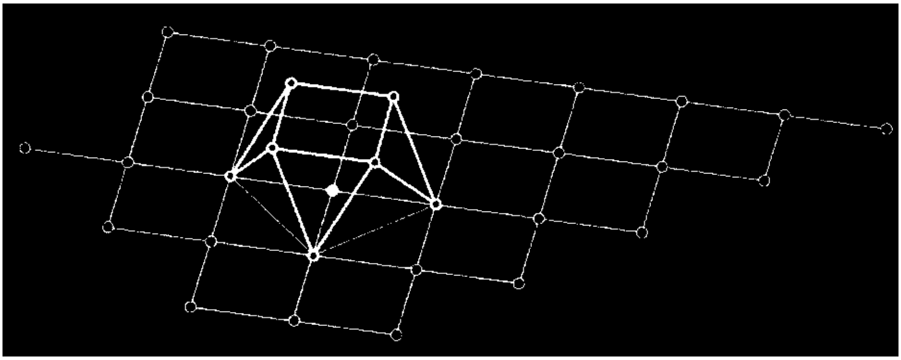


Fig. 2-50 Square grid constant lightness plane of the L, g, j space with the rotated half cubo-octahedron placed over it. Grids at the next higher and lower lightness levels are offset by one half chromatic step, as illustrated by the portion of the cubo-octahedron. From Gerstner (1986).

developed that attempt to represent an accurate psychophysical model of the psychological space (see Chapter 6). They include adjustments of the lightness scale for the crispening effect and for the Helmholtz-Kohlrausch effect (see Sections 5.8 and 8.2 for discussions of lightness and chromatic crispening and Section 5.8 for the Helmholtz-Kohlrausch effect). For more details and an analysis of these scales, see Chapter 7.

2.27 SWEDISH NATURAL COLOR SYSTEM

A system called, after Hering, Natural Color System was developed in Sweden in the 1970s. It traces its sources to Hering and Ostwald, and more recently to

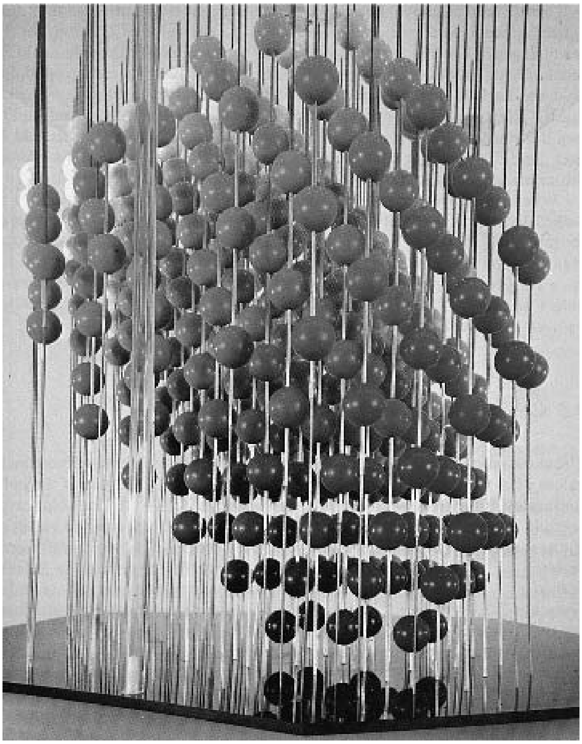


Fig. 2-51 *View of MacAdam's model of the OSA-UCS color solid illustrating the existence of several cleavage planes. See text for more detail. Slide courtesy D. L. MacAdam. (See color plate.)*

efforts by Johansson (1949) and Hesselgren (1954). The fundamental idea was that any color normal observer is able to determine in any object color the "content" of one or two fundamental full colors and of black and white. The three, respectively four, percentages always add up to 100%. In a manner comparable to Ostwald's, the Natural Color System is represented by a double cone. A forty-hue circle of full colors based on experimentally determined unique hues is located at the edge of the double cone (Fig. 2-52). The central axis is represented by a gray scale with ten steps, with white on top. Mixtures of full colors with white or black are located on the surface of the double cone. Mixtures of full colors with white and black are located in the interior. Colors of equal hue are located on triangles bounded by white, black, and the full color (Fig. 2-53). Colors are identified by hue number, NCS chromaticness c, and NCS blackness s. An atlas representing the system has been developed, containing 1750 matt color chips arranged in triangles on forty equal hue planes. Every other plane illustrates colors from $c = 10$ while those between

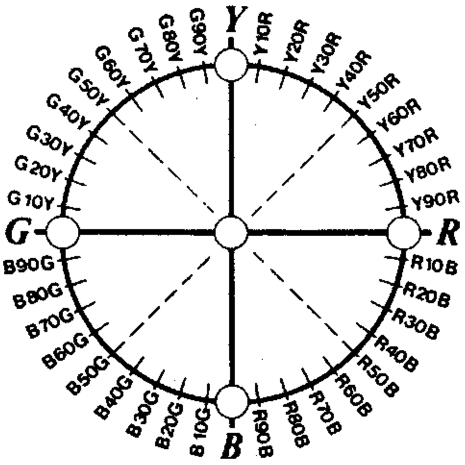


Fig. 2-52 Hue circle of the NCS color solid with hue designation scheme, identifying ten hue steps between unique hues. Courtesy NCS.

begin at $c = 40$. Colorimetric values based on the 2° standard observer of the aim colors and typical color chips have been provided in a table (Swedish Standard, 1982). For more details and an analysis of the NCS system, see Chapter 7.

2.28 UNIVERSAL COLOR LANGUAGE

In 1931 the then just formed (American) Inter-Society Color Council (ISCC) began work under Godlove on a standardized method (in English) to designate colors. An initial list was issued in 1939, and Kelly and Judd issued in 1955 NBS Circular 553 *The ISCC-NBS Method of Designating Colors and a Dictionary of Color Names*. In 1976 the NBS Special Publication 440 *Color: Universal Language and Dictionary of Names* was issued (Kelly and Judd, 1976). The Munsell color solid was subdivided into regions that can be designated by common names. The system is applicable to opaque, translucent, and transparent colored materials. Twenty-nine major color regions contain a total of 267 subregions around centroid colors. Among the twenty-six major regions are those of the primary colors black, white, yellow, red, blue, and green, as well as secondary and tertiary colors such as orange, brown, olive, purplish pink, and violet. The subregions are identified by modifiers, such as vivid, strong, deep, dark, light, and grayish. A supplement contains on eighteen pages a sample of the centroid color of the 267 subregions, with its ISCC-NBS name and Munsell Renotation.

In the 1976 publication Kelly describes six levels of precision of designat-

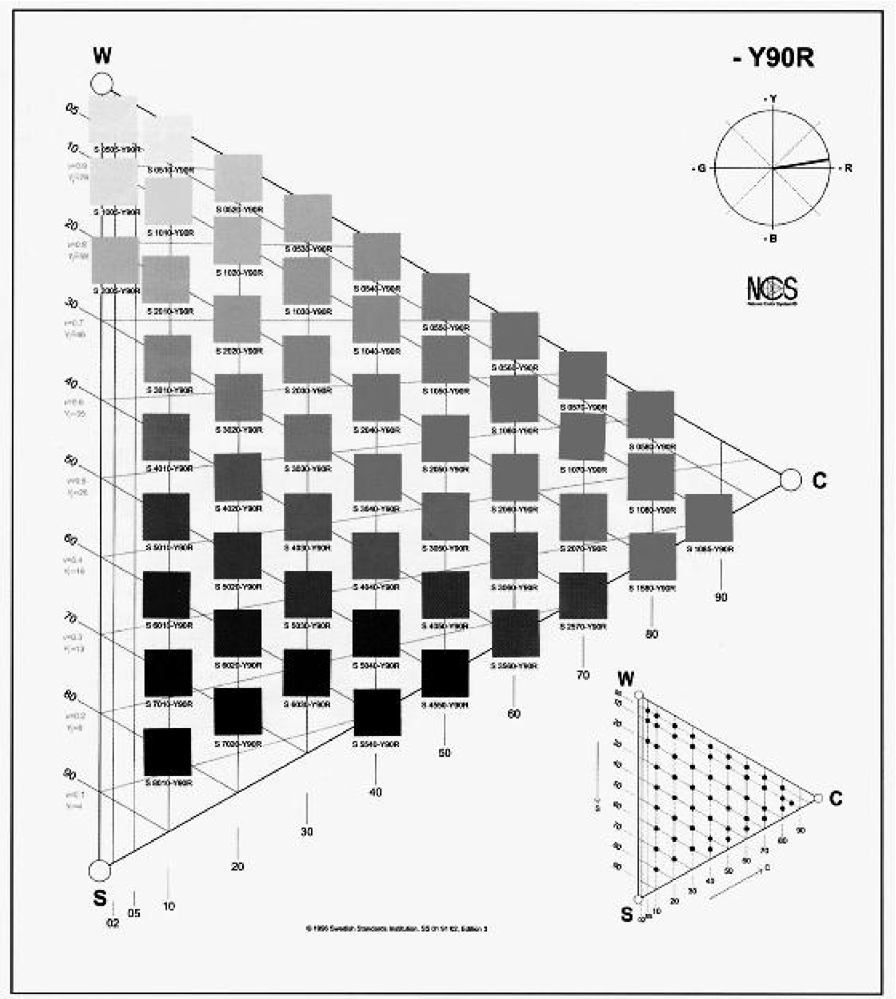


Fig. 2-53 NCS constant hue triangle of hue Y90R with full color C, white W, and black S. Colors of constant blackness s are located on lines parallel to W–C, colors of constant chromaticness c fall on lines parallel to W–S. Courtesy NCS. (See color plate.)

ing colors. Level 1 consists of thirteen generic color names: white, gray and black, yellow, orange, brown, red, pink, purple, blue, green, olive, yellow green. On level 2 sixteen intermediate hue names are added, such as reddish brown, bluish green, and purplish pink. Level 3 contains the 267 subregions. On level 4 the space is further subdivided into more than 1000 colors, as in the *Munsell Book of Color*. Some 100,000 colors can be specified on level 5 by visually interpolating between Munsell chips, such as a color specified as 7.8PB

2.2/12.5. On the final level colors are specified by standard colorimetric data, such as CIE x, y, Y values. Here the number of definable colors is in the millions.

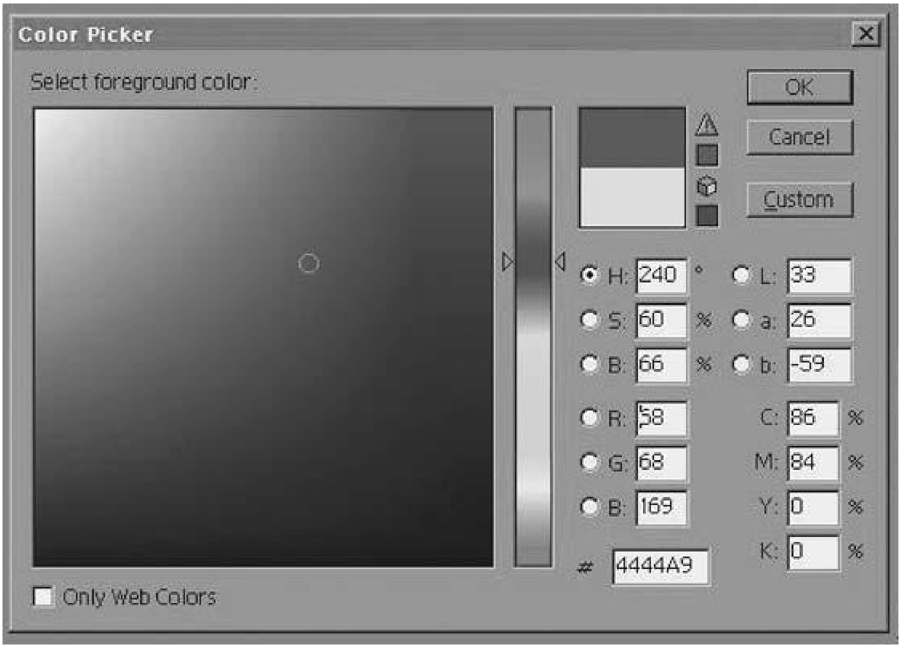
2.29 COLOR MIXING SPACES

In computer and color reproduction technology of the late twentieth century several types of color space are in routine use. In cathode ray tube (CRT) displays, color television, and most computer video displays, color stimuli are generated with three different types of phosphors that can be activated with electron beams. The colors resulting from such activation typically are orange-red, leaf green, and violet, additive color primaries. A large variety of color experiences can be created as a result of their mixture. These stimuli are represented in the RGB color cube. The abbreviations R, G, B designate red, green, and blue, loose verbal designations for additive color primaries used. The cube resembles the Benson cube (see Section 2.15). Colors can be identified in terms of R, G, B units of activation. In the RGB version the standard values of the three components range from 0 to 255. As the cube is rotated so that white and black fall on the vertical axis, a version of a polar coordinate system is imitated and the system is termed HSB (for hue, saturation, brightness) space. In this model hue is expressed as hue angle in degrees, saturation as a percentage from 0% at the achromatic point (gray) to 100% at full saturation, and brightness as a percentage from 0% at black to 100% at white. Achromatic colors have identical values of the three components, chromatic colors have varying values. The phosphors used define the gamut (the maximum chromatic range) of colors that can be achieved. The HSB and RGB spaces depend on the exact definitions of the primary colors used in their creation. They are regular spaces but not uniform as defined in Chapter 1.

Software programs such as Adobe Photoshop® display slices through the HSB space with a continuous gray scale on the left-hand side and the full colors at different lightness on the right. The plane is filled with intermediate colors (Fig. 2-54). Because of the additive mixture the hue of such a plane is not uniform but changes slightly as a function of saturation and brightness.

Another space used in design software is the CMYK space. The letters stand for the subtractive primaries of four-color printing: cyan, magenta, yellow, and black (K to avoid confusion with B). In printing processes the primary colors are defined in terms of standard printing inks. For CRT simulation (e.g., in Adobe's Illustrator® software) CMYK is defined in terms of individual percentages of the four components. A white color has zero values for all components. The gray scale differs in percentage of K. A chromatic color may have percentage values in all four categories. The gamut of CMYK is usually smaller than the gamut for RGB because of the limited chroma of printing primaries.

In programs like Illustrator® and Photoshop® colors can be defined simul-



Select foreground color:

● H: 240 °
○ S: 60 %
○ B: 66 %
○ R: 58
○ G: 68
○ B: 169
4444A9
○ L: 33
○ a: 26
○ b: -59
C: 86 %
M: 84 %
Y: 0 %
K: 0 %

☐ Only Web Colors

Fig. 2-54 *Color Picker screen from Adobe® Photoshop showing specification of a given reddish blue color in four systems: HSB, Lab (=L*, a*, b*), RGB, and CMYK. (See color plate.)*

taneously in terms of HSB, RGB, CMYK, and CIELAB $L^*a^*b^*$, and they allow input values from any of these systems to display (with a degree of accuracy that depends on many factors) the corresponding color. There are complex issues of color management in reproducibility of such CRT colors in different media (color transparency or opaque color photos, printing on different types of printers, from computer printers to commercial printers).

2.30 SPECTRAL SPACES

Spectral functions can be placed in regular spaces without consideration of human observers. Such spaces have use in machine vision, scanners, and copiers and other applications. Several methods have been applied for this purpose. Among these is principal component analysis (PCA), first used in connection with colors for a subset of Munsell colors by J. Cohen (1964). Another is through training of computerized neural networks. In PCA basis functions are calculated describing the spectral functions with a degree of accuracy that depends on the number of functions. If the number of basis functions is three, they form an orthogonal space in which each spectrum plots as a point. For an extensive set of samples, such as those of the Munsell system or of images of natural scenes, about 90% of the variation in the spectral functions is explained by three basis functions (Lenz et al., 1996; Wachtler et al., 2001). There is an all-positive nearly horizontal function that can be seen as being the spectral equivalent of a lightness function. The next two functions have positive and negative lobes and can be seen as rough approximations of opponent color functions (see Section 6.18). They extract information in spectral functions more efficiently than functions describing the spectral sensitivities of human cones. On the other hand, while both kinds of functions place the reflectances of a Munsell hue circle into ordinal order, (with some exceptions) only the cone and color-matching functions place them approximately in interval order in regard both to hue difference and chroma. There are several mathematical treatments that can be used for PCA and similar operations. Some of these have found use in color constancy and color appearance modeling (Maloney, 1999). However, for the reasons given and others it is not justified to call spectral spaces based on reflectance functions color spaces.

This review of the historical development of thought on color order and of color-ordering systems has concentrated, with few exceptions, on psychological systems that have often found a physical expression in form of atlases. When comparing such atlases, the samples are found to differ. There is a considerable literature on the results of comparisons among the Munsell, NCS, DIN, and OSA-UCS systems (e.g., see Derefeldt, 1991). The results have been summed up by the philosopher of color C. L. Hardin (1988): ". . . the issue is not that there cannot be a consistent scheme of representing phenomenal color, for there can. And, indeed, there is reason to suppose that the various representations can be mapped into each other (though the mapping relations will often be quite complex)." When doing such mapping, one should distinguish between systems that aim to be perceptually uniform (e.g., Munsell and OSA-UCS) and other systems that are regular in some way (e.g., NCS). Accordingly we may expect common principles behind the first group but significant discrepancies between the two groups, as will be seen in Chapter 7.

In discussion of the Munsell system in Chapter 1 we have seen early scaling experiments that resulted in significantly different ratios of hue differences to value and chroma differences depending on the size of the difference. This points to an apparent dependence of color space on the size of the differences selected to represent the space, a matter discussed in detail in Chapter 6.

As this chapter indicates, development of a uniform color space has been a struggle over more than two centuries, a struggle that has not yet been successful. A modern view for the reason is that our color perceptions are empirically determined and thus not in a relationship to stimuli that is expressible by simple logical rules (Purves and Lotto, 2002) or that there are no explicit neural correlates for what we perceive (Dennett, 1991; O'Regan and Noë, 2001). Under conditions of extensive relativity (average observer pool, constant defined lighting, and surround conditions) it may be possible to reduce the visual situation to one in which no ambiguities are present, and empiricism may be limited to a constant set for the observer pool. The modest degree of agreement in different attempts at a visually uniform space in the last century perhaps gives an indication that this is possible. Additional experimental work is required to establish what is feasible. The value of such a space would be limited to the specific conditions under which it was determined. For many applications this can nevertheless be useful.