

Chapter 6

Historical Development of Color Space and Color Difference Formulas

A color space formula is a set of mathematical equations describing a color space model in terms of cone sensitivities or tristimulus values and spectral power distributions. A color difference formula is a mathematical expression resulting in numbers proportional to perceived difference between points in the space. Historically development of color space and color difference formulas has taken a standard association between a color stimulus (as represented by a family of metamers) and a color perception as a given. Recognition of the need for exact specification of the conditions of viewing the stimuli is a relatively recent phenomenon. Most of the formulas discussed below betray no such recognition. As such they need to be understood as lacking fundamentality and being limited to implicit or explicit assumptions/representations of viewing conditions.

6.1 LINE ELEMENTS

The line element is the so-called first fundamental form of a regular surface.¹ For small color differences it is given explicitly by the Riemannian metric

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which determines the arc length of a curve on a surface. Connected with the line element is the geodesic: the curve that locally minimizes the length of a curve. It depends on the geometry of the space to which it applies: in plane geometry the geodesic is a straight line, on a sphere a segment of a great circle (e.g., the equator). In connection with color Wyszecki and Stiles (1982) defined the line element as “a measure of distance in a postulated space in which perceived colors are represented by points or vectors.” Historically the spaces related to given line elements have been based on color fundamentals R , G , and B or cone sensitivities. The differences between colors in these spaces have been defined with increments, originally based on the Weber-Fechner law and later on more complex relationships.

Helmholtz and Schrödinger

Interest in a stimulus intensity based model of uniform color space, a model of outer psychophysics in Fechner’s sense, grew since the end of the nineteenth century. In 1886 König wrote that it should be easy to construct a color mixture diagram in a manner that equal distances in all directions would be proportional to equal numbers of discrimination steps. What he had in mind is a model that would partition the diagram in agreement with the Weber-Fechner law. The first quantitative version was introduced by Helmholtz in 1896 based on ideas discussed in 1891. He assumed three fundamental color vision processes R , G , and B and the uniform applicability of the Weber-Fechner law:

$$\frac{dR}{R} = \frac{dG}{G} = \frac{dB}{B} = \text{constant}, \quad (6-1)$$

where dR is the increment/decrement in magnitude of the stimulus described by the fundamental process R , and comparable for the other two stimuli. Assuming in addition a euclidean space, we can calculate the threshold differences as

$$ds = \left[\left(\frac{dR}{R} \right)^2 + \left(\frac{dG}{G} \right)^2 + \left(\frac{dB}{B} \right)^2 \right]^{0.5}, \quad (6-2)$$

where ds is the distance between two neighboring points in the R , G , B space representing a threshold difference. Helmholtz attempted to make his model fit experimental data of König and Dieterici by appropriately shaping his three visual processes with linear transformations from color-matching functions. All of the resulting functions had two maxima, and none was in agreement with the experimental luminous efficiency function. Helmholtz was unaware of and did not consider contributing factors important for an accurate line element.

A more complex line element was proposed in 1920 by Schrödinger in an attempt to correct perceived shortcomings of the Helmholtz model. Schrödinger recognized that Helmholtz’s form of the line element results in a luminance function that is much different from experimental functions. His proposal has the following form:

$$(ds)^2 = \frac{1}{l_R R + l_G G + l_B B} \left[\frac{l_R (dR)^2}{R} + \frac{l_G (dG)^2}{G} + \frac{l_B (dB)^2}{B} \right], \quad (6-3)$$

where l_R , l_G , and l_B are constants corresponding approximately to brightnesses of the three fundamental processes derived from the luminous efficiency function, R , G , B are the fundamental processes as experimentally determined by König and Dieterici. The line element obeys the Weber-Fechner law while being additive. Schrödinger's proposal did not receive much practical attention, perhaps because of the difficulties of executing such calculations on a routine basis.

Stiles

In 1946 Stiles offered a version of a line element based on extensive investigation of two-color thresholds. Importantly Stiles found that different Weber fractions apply to the three visual processes rather than the common fraction assumed by Helmholtz and Schrödinger. Stiles's line element has the following form:

$$(ds)^2 = \left[\frac{\zeta(R)}{\rho} dR \right]^2 + \left[\frac{\zeta(G)}{\gamma} dG \right]^2 + \left[\frac{\zeta(B)}{\beta} dB \right]^2, \quad (6-4)$$

where $\zeta(R) = 9/(9+9R)$ and comparably for G and B ; ρ , γ , β are proportional to the Weber fractions, with values of 1.28 for ρ , 1.65 for γ , and 7.25 for β , indicating that the "blue" process is much less sensitive than the "red" and "green" processes. The three processes were defined by linear transformation from the 1931 CIE color-matching functions. Stiles's line element was tested extensively in succeeding years and was found to provide good approximations to several sets of visual data, including the luminous efficiency function, wavelength discrimination, and chromatic thresholds.

Luneburg's Line Element for Visual Space

Mention should be made here of a line element for the human visual space developed in 1947 by the mathematician R. K. Luneburg. Visual space refers to the space segment covered by our eyes when looking ahead. Visual space and color space are two different aspects of our visual sense, and there is no immediate reason for color space to be in agreement with visual space. The fact that a similar geometry appears to apply to both situations may be coincidental. Based on comparisons between physical measurements and related judgments of distance, Luneburg determined visual space to be Riemannian (i.e., elliptical) with constant curvature. He proposed the following line element:

$$ds^2 = \frac{d\alpha^2 + d\beta^2 + d\gamma^2}{\left[1 + \frac{1}{4K(\alpha^2 + \beta^2 + \gamma^2)}\right]^2} \tag{6-5}$$

where α, β, γ are orthogonal sensory coordinates and K denotes Gaussian total curvature. If $K = 0$, the space is euclidean; if negative, the space is hyperbolic, and if positive, it is elliptic.

Walraven, Bouman, Vos

In 1966 P. L. Walraven and M. A. Bouman offered a new proposal for a line element of color employing photon noise methodology (De Vries-Rose behavior; De Vries, 1948) in place of the empirical Weber-Fechner law. With the exception of the photon noise methodology replacing the Weber-Fechner law the proposal was identical to that of Schrödinger. Testing against new wavelength discrimination and other data did not provide fully satisfactory results.

Walraven continued work with a new co-worker, H. Vos, and they developed the Vos-Walraven line element (Vos and Walraven, 1972, 1991). The model underwent several modifications. In its original form it was based on a model of retinal color processing which included a Helmholtz type cone absorption step taking input of the R, G, B cones at a ratio of 32:16:1, with cone response compression. In a summation zone the compressed R and G signals form the yellow signal, and all three together the luminance signal. Separately, R and G signals are balanced to form an antagonistic Hering type red-green signal. Similarly the yellow signal is balanced against the B signal, forming the yellow-blue output signal. Finally the luminance signal is balanced against a surround luminance signal, resulting in a brightness contrast signal. Vos and Walraven concluded that opponent processing does not play a role at the threshold level and their line element is essentially of the Helmholtz type. It is defined as follows (Vos, 1979):

$$(ds)^2 = \left\{ \frac{dR}{\left[R \left(1 + \frac{R}{R_0} + \frac{R^2}{R_1^2} \right) \right]^{0.5}} \right\}^2 + \left\{ \frac{dG}{\left[G \left(1 + \frac{G}{G_0} + \frac{G^2}{G_1^2} \right) \right]^{0.5}} \right\}^2 + \left\{ \frac{dB}{\left[B \left(1 + \frac{B}{B_0} + \frac{B^2}{B_1^2} \right) \right]^{0.5}} \right\}^2 \tag{6-6}$$

where R, G, B are the cone signals at the ratio of 32:16:1, subscript 0 indicates the number of quanta for which saturation occurs, and subscript 1 the

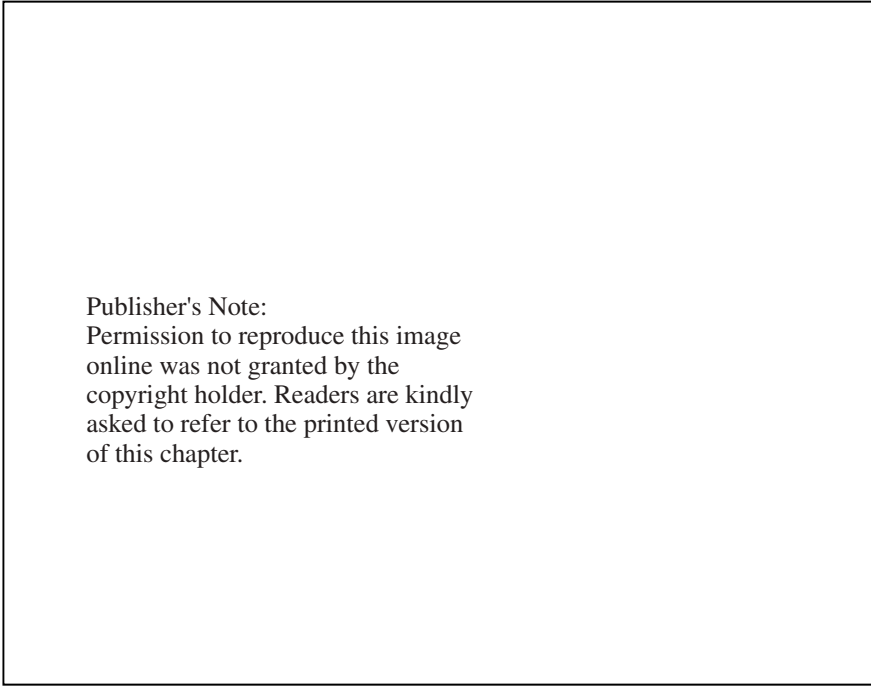
number of quanta for which supersaturation occurs. The model thus can account for various levels of saturation of the cone system and in this manner can predict, in addition to color discrimination at the threshold level, the Bezold-Brücke and the Stiles-Crawford effects. While the model is a modified version of the Helmholtz line element, it considers the ratio of cones that gives rise to different Weber fractions for the three cone types as well as saturation and supersaturation effects, making it not just applicable to a middle range of luminance but to the complete range.

The Helmholtz, Schrödinger, Stiles, and Vos-Walraven line elements were believed to be more or less uniform in terms of thresholds. However, as will be shown in Chapter 8, they are regular color spaces. In his paper Schrödinger suggested that geodesics, the lines indicating the paths of smallest numbers of threshold differences, are lines of constant hue. MacAdam originally determined geodesics based on his color-matching error data mechanically, by stretching threads across a model representing his ellipses (MacAdam, 1981). Geodesics can be calculated, for example, by using linear programming (Jain, 1972). For an example of constant hue and saturation geodesic lines in the CIE chromaticity diagram calculated from MacAdam's 1965 geodesic chromaticity diagram (MacAdam, 1981), see Fig. 6-1. With known hue, chroma, and lightness geodesics, it is possible to develop formulas transforming nonuniform cone sensitivity or tristimulus spaces into a space uniform in terms of the assumptions of the model involved.

6.2 PROJECTIVE TRANSFORMATIONS

Another approach to a uniform color space was based on the idea that the CIE 1931 x, y chromaticity diagram could be linearly transformed to result in a modified diagram in which distances were proportional to visual distances. In 1932 D. B. Judd offered an early version representing his own threshold and other published data (Fig. 6-2) and based on color-matching functions recommended by the Optical Society of America in 1922. The diagram contains radial lines of constant dominant wavelength and ovoids of constant colorimetric purity. In 1935 Judd published a modified version of a uniform chromaticity diagram in the form of a Maxwell triangle (Fig. 6-3). In a different reference frame the diagram became in 1960 the basis for the CIE u, v diagram. In 1935 Judd also introduced the symbol ΔE to denote a color difference. In the following year Judd published a graph of the CIE 1931 chromaticity diagram with ellipses that represent circles of equal size in his 1935 diagram (Fig. 6-4). These ellipses are intended to represent uniform threshold color differences, enlarged 100 times. They illustrated for the first time explicitly the perceptual nonuniformity of the CIE chromaticity diagram.

In 1937 MacAdam modified Judd's 1935 diagram with simplified coefficients resulting in a rectangular coordinate diagram. In 1939 F. C. Breckenridge and W. R. Schaub developed the rectangular uniform chromaticity scale diagram (RUCS), a transformed version of Judd's 1935 diagram.



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Fig. 6-1 Geodesic lines of constant hue and chroma in the CIE chromaticity diagram, calculated from MacAdam's 1965 geodesic chromaticity diagram (Fig. 6-9). Solid lines: constant hue; dashed lines: constant saturation. From MacAdam (1981).

In the same year Judd, Scofield, and Hunter proposed the α , β diagram and a color space related to it. In 1942 this linear transformation of the CIE chromaticity diagram became the basis of the National Bureau of Standards (NBS) formula, with the difference units designated as NBS units or *judds* (after D. B. Judd).² The formula is as follows:

$$\Delta E = f_g \left\{ \left[221Y^{0.25}(\Delta\alpha^2 + \Delta\beta^2)^{0.5} \right]^2 + [k\Delta(Y^{0.5})]^2 \right\}^{0.5}, \quad (6-7)$$

where f_g is a factor adjusting for glossiness and is defined as $f_g = Y/(Y + K)$, with K usually taken as 2.5; k , having normally a value of 10, adjusts the lightness difference to the chromatic difference

$$\alpha = \frac{2.4266x - 1.3631y - 0.3214}{1.0000x + 2.2633y + 1.1054},$$

$$\beta = \frac{0.5710x + 1.2447y - 0.5708}{1.0000x + 2.2633y + 1.1054}. \quad (6-8)$$

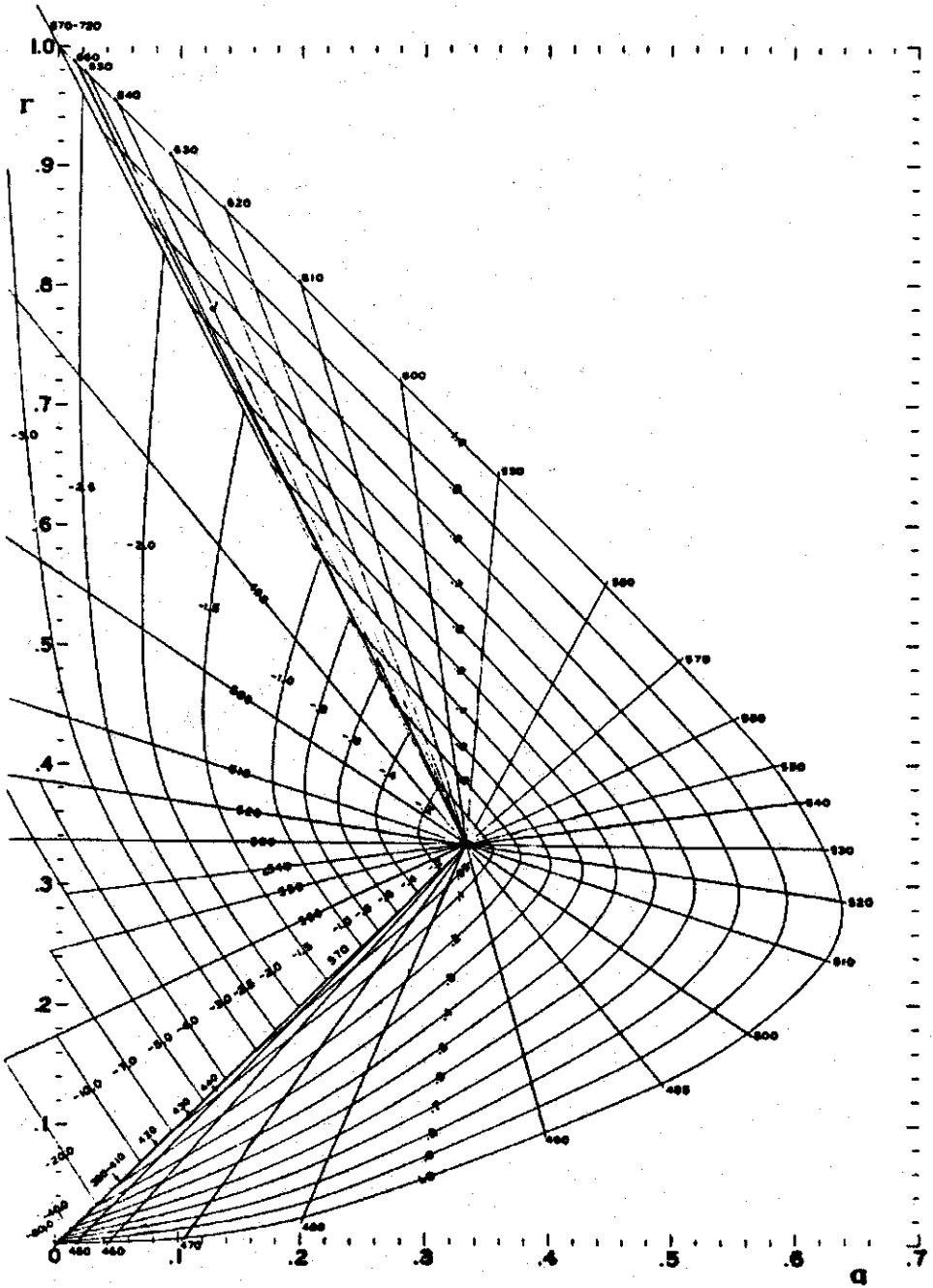


Fig. 6-2 Judd's projective transformation of the CIE chromaticity diagram of 1932 meant to result in proportionality of distances with visual distances. The numbers on the spectral trace represent wavelength in nanometers.

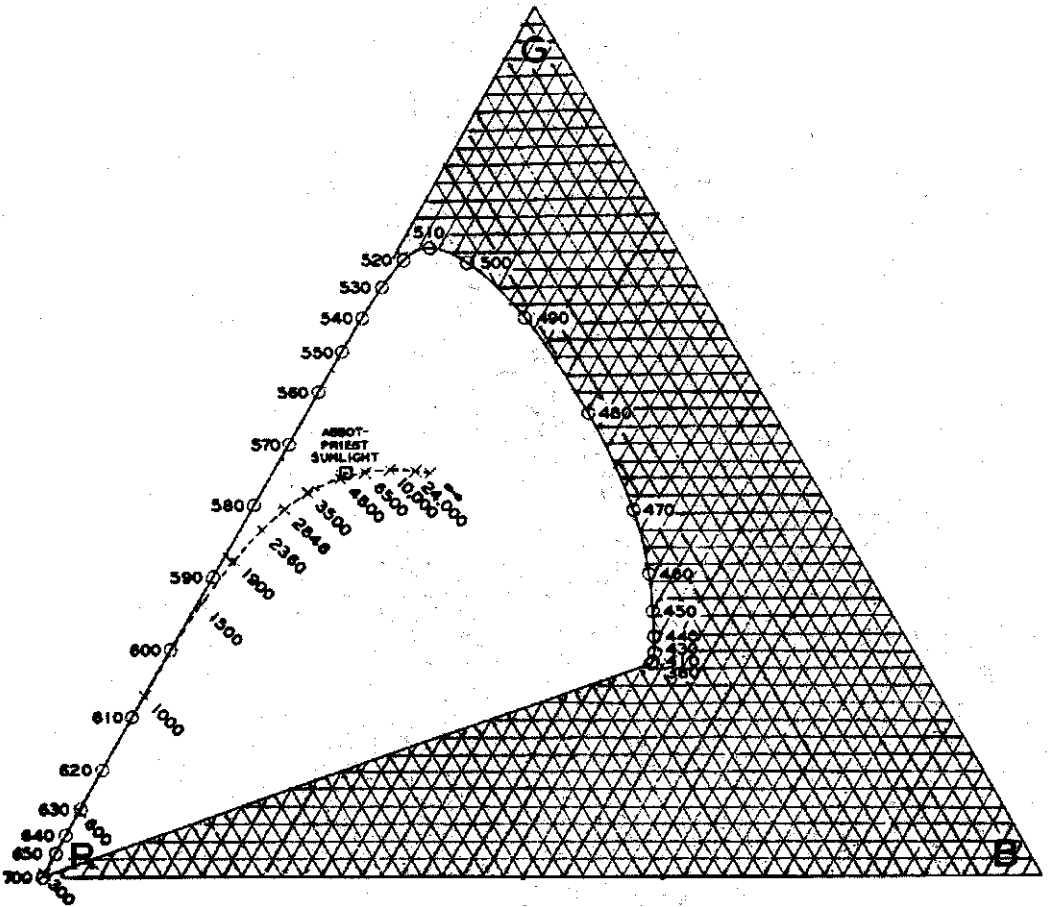


Fig. 6-3 Judd's 1935 uniform chromaticity diagram embedded in the Maxwell triangle. On the dashed curve are colors of a black body at the indicated absolute temperatures in Kelvin. Compare with Fig. 2-29.

In 1944 D. Farnsworth proposed a simplified transformation. It provides for minimum deviation from circularity of Munsell hues at chroma 10 and value 5, centered on illuminant C. In 1959 Y. Sugiyama and T. Fukuda optimized a transformation such that both the Munsell system and the MacAdam ellipses were reasonably well represented. In 1963 Wyszecki proposed a transformation formula that became the basis of the CIE 1964 ($U^*V^*W^*$) system:

$$\begin{aligned}
 U^* &= 13 W^* (u - u_0), \\
 V^* &= 13 W^* (v - v_0), \\
 W^* &= 25 Y^{1/3} - 17,
 \end{aligned}
 \tag{6-9}$$

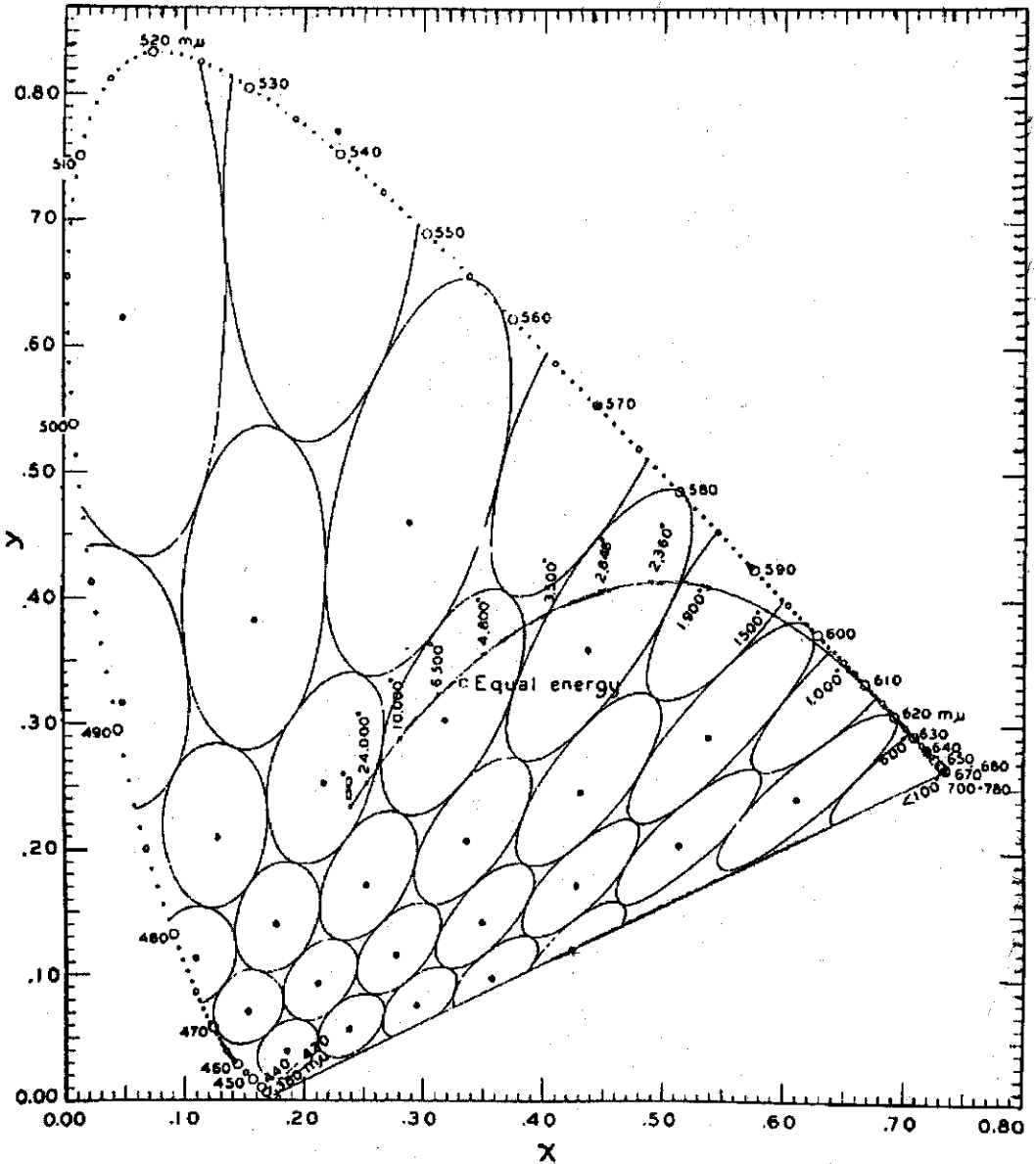


Fig. 6-4 CIE chromaticity diagram with ellipses that represent circles of equal size in Judd's 1935 diagram. From Judd (1936).

where $u = 4X / (X + 15Y + 3Z)$, $v = 6Y / (X + 15Y + 3Z)$, X, Y, Z are CIE tristimulus values, u_0, v_0 are the values of the variables u and v for the achromatic color at the origin of the U^*, V^* diagram. The total color difference was calculated as the square root of the sum of the squares of the differences in the three dimensions. This formula has been superseded by the CIELUV formula.

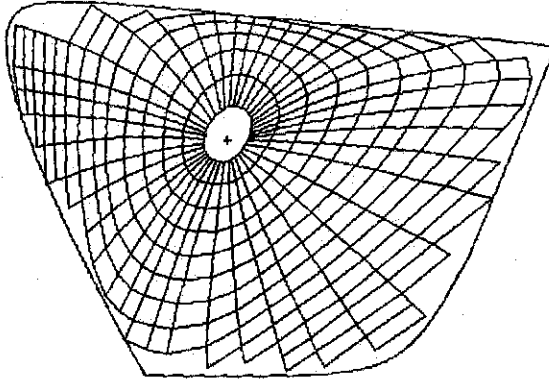


Fig. 6-5 Munsell Renotation colors at value 5 as represented in the CIELUV projective transformation formula. From Mahy et al. (1994).

The development of projective transformation came to a conclusion with the recommendation in 1976 by the CIE of the 1976 $L^*u^*v^*$ transformation (CIELUV). This is a minor modification of MacAdam's 1937 diagram (expansion of v by a factor 1.5). A cross section of the Munsell space at value 6 as represented by this formula is shown in Fig. 6-5.

Projective transformation employs the following basic formulas:

$$x' = \left(\frac{a_{11}x + a_{12}y + a_{13}}{a_{31}x + a_{32}y + a_{33}} \right), \quad y' = \left(\frac{a_{21}x + a_{22}y + a_{23}}{a_{31}x + a_{32}y + a_{33}} \right), \quad (6-10)$$

where x', y' are the transformed chromaticity coordinates and x, y are the CIE chromaticity coordinates; a_{xx} are transformation coefficients. Several of the formulas are in a simplified form requiring only five coefficients. The coefficients of the above and other linear transformation formulas are given in Table I (6.4) of Wyszecki and Stiles (1982). Linear transformations have the advantage, and are thus of interest to optical and lighting engineers, that the system remains additive and mixtures of two lights fall on a straight line connecting their locations in the diagram. The CIELUV formula will be further discussed below.

6.3 FITTING MODELS TO THE MUNSELL SYSTEM

Another avenue pursued involved attempts to develop a uniform color space model from analysis of the structure of the Munsell system. This system had been extensively investigated by the Subcommittee on Uniform Spacing of Munsell Colors of the Optical Society of America. It published a final report in 1943, containing colorimetric specifications of the smoothed Munsell Reno-

tation colors. In this form the Munsell system was taken to represent a good approximation of a visually uniform psychological color space.

In 1936 Nickerson proposed the first formula with the goal of predicting perceptually equal color differences, the Nickerson Index of Fading (see equation 4-1). The formula has a purely psychological basis, since no physical measurements are involved in its application. Its immediate purpose was to provide a measure of fading by light of dyed textiles. Component differences were combined by simple addition. The formula recognized the dependence in a polar system of the hue difference on chroma and also indicated that in the Munsell system unit differences in the three attributes are of different perceptual magnitude. As a result such a system is not uniform, as it doesn't represent distances in all directions proportionally to differences in its euclidean space (see Chapter 7).

In 1942 Adams plotted Munsell colors at several values in what he called a "chromance" diagram, representing a linear opponent color diagram normalized to the equal energy point. The two dimensions were defined as $X - Y$ and $Z - Y$. By applying the Munsell-Sloan-Godlove lightness formula (a power 0.5 modulation; see Chapter 5) not only to the luminous reflectance value Y but also to the other two tristimulus values, Adams converted the chromance diagram to a chromatic value diagram. Adams interpreted the subtractions as representations of his 1923 theory of color vision that took the color-matching functions to be cone responses. In this theory Adams had assumed an inhibitory effect of the output of the postulated Y cone on the outputs of the X , respectively Z cones. He represented this inhibitory effect with subtractions. He called the outputs V_X , V_Y , and V_Z . Adams calculated chromatic values of a high chroma Munsell hue circle and found those related to Z to be larger than those related to X and Y . To have scales of maximum value 10 in all three dimensions he proposed to multiply the V_X values by 1.90 and the V_Z values by 0.72. The result of these operations showed reasonably circular contours in the chromatic value diagram for Munsell colors of the 1929 *Book of Color* and the preliminary smoothed values of the committee (see Fig. 5-12). Adams did not propose a color difference formula based on his V_X, V_Y, V_Z space, but assuming that it is a euclidean space, such a formula is obvious. Adams's chromatic value system proved to be influential in coming years.

Aside from the Munsell data, in flux until the release of the Renotations in 1943, there were few object color sample sets with statistically supported visual data. In 1941 Balinkin reported on a set of five pale green tiles. One pair of tiles was designated as the standard difference, and sixty observers estimated the differences of all possible pairs against the reference pair. The data were used in several studies of color difference formulas. In 1944 Nickerson and her co-worker Stultz investigated the usefulness of color difference calculation for color quality control work. The visual data consisted of category judgments (five categories) by twelve observers of painted textile samples, primarily in two color regions: yellowish brown and olive green. In this study they used, among others, a formula based on Adams's chromatic value space as follows:

$$\Delta E_A = \left[(\Delta V_Y)^2 + \{ \Delta(V_X - V_Y) \}^2 + \{ 0.4 \Delta(V_Z - V_Y) \}^2 \right]^{0.5}, \quad (6-11)$$

where V_X, V_Y, V_Z are the Adams chromatic values of the samples, now interpreted in terms of the revised Munsell value function. This formula became known as the Adams-Nickerson color difference formula and proved of enduring value. In their evaluations Nickerson and Stultz found the formula only marginally better than formulas derived by Judd but considerably easier to calculate. The experiments also indicated considerable individual variability in judgment.

In 1946 J. L. Saunderson and B. L. Milner proposed a modification of the Adams chromatic value system to obtain closer agreement with the Munsell system. Contours of constant chroma are somewhat eccentric in the Adams chromatic diagram, and the Saunderson-Milner solution corrected for the eccentricity using an empirical trigonometric method. The Saunderson-Milner color space model was described as the “Zeta” space, based on their use of the Greek letter. It is defined as

$$\begin{aligned} \zeta_1 &= (V_X - V_Y)(9.37 + 0.79 \cos \Theta), \\ \zeta_2 &= kV_Y, \\ \zeta_3 &= (V_Z - V_Y)(3.33 + 0.87 \sin \Theta), \end{aligned} \quad (6-12)$$

where Θ is the angle calculated from $\tan \Theta = 0.4(V_Z - V_Y)/(V_Z - V_Y)$, and k is a constant depending on the observation conditions. Color differences are calculated as the square root of the sum of the squares of the differences in the three ζ values. A somewhat different procedure with a comparable effect was proposed in 1952 by Godlove.

Assuming that two concentric circles of five equally spaced Munsell hues are a good representation of psychologically uniform space, Burnham in 1949 investigated the performance of ten formulas (including the CIE x, y and x, z diagrams) and found the Saunderson-Milner Zeta space to perform best. However, all of the formulas resulted in deviations that were in visual terms statistically significant.

6.4 JUDD'S MODEL OF MÜLLER'S THEORY OF COLOR VISION

In 1949 Judd began to develop a model of G. E. Müller's theory of color vision (to be discussed in more detail later in this chapter). Development continued into the 1960s and the model was influential in Friele's treatment of MacAdam's data resulting in the FMC I and FCM II formulas to be discussed below. The Müller-Judd space is further discussed in Section 6.17.

6.5 COLOR DIFFERENCE THRESHOLDS AND MATCHING ERROR

Threshold differences were first investigated along the spectrum because exact setting of spectral differences was technically relatively easy. However, spectral differences are complex in visual terms since every difference is composed of hue, saturation, and brightness components. Wavelength discrimination began to be investigated at the turn of the twentieth century. A classical investigation is that by W. D. Wright and F. H. G. Pitt (1934). Wright subsequently also investigated threshold differences along straight lines in the CIE chromaticity diagram. The results of both investigations are illustrated in Fig. 6-6 (Wright, 1941). Wright's important data were overshadowed by MacAdam's extensive color matching error data of a single observer, published a year later (1942). As discussed in Chapter 3, in MacAdam's work two hemi-fields were displayed against a dark surround in a specially constructed colorimeter. In one-half a standard color was displayed. The color of the second half could be adjusted by the observer along straight lines passing through the standard color in the CIE chromaticity diagram, in a constant luminance plane. Using a single knob, the observer adjusted the color of the test field by the method of adjustment until a visual match between the two hemi-fields was achieved. The match was approached along a given line from both sides. Color matches along several lines were repeatedly set for each of 25 standard colors. From the visual data (some 20,000 observations) MacAdam calculated the standard error of color matching for his single observer. The result, fitted with ellipses, is illustrated in Fig. 6-7. MacAdam also determined that the threshold differences around his standard colors were approximately twice the standard deviation of the color-matching errors. Subsequently comparable but three-dimensional contours involving also brightness differences were determined for additional observers, and the results indicated that observers vary significantly in this task (Brown and MacAdam, 1949; Brown, 1957). The MacAdam ellipses rapidly became a key set of data used as test data for line elements and color difference formulas. Color-matching error was explained in 1949 in terms of cone activity by Y. LeGrand (see Chapter 8). The ellipses obtained good confirmation in an experiment by R. M. Boynton and N. Kambe (1980). Additional determinations of color-matching error were made by G. Wyszecki and G. Fielder (1971). Determinations of achromatic and chromatic thresholds using industrially relevant conditions were performed by K. Richter (1985) and by Witt (1987, 1990). For a comparison of such data, see Chapter 8. As mentioned in the previous chapter, thresholds and color-matching error data of limited groups of colors and using various methodologies have been performed in recent years by several researchers.

MacAdam's Empirical Line Element

Using a method suitable for describing line elements independent of the form of the implied space (i.e., applicable to euclidean and non-euclidean space),

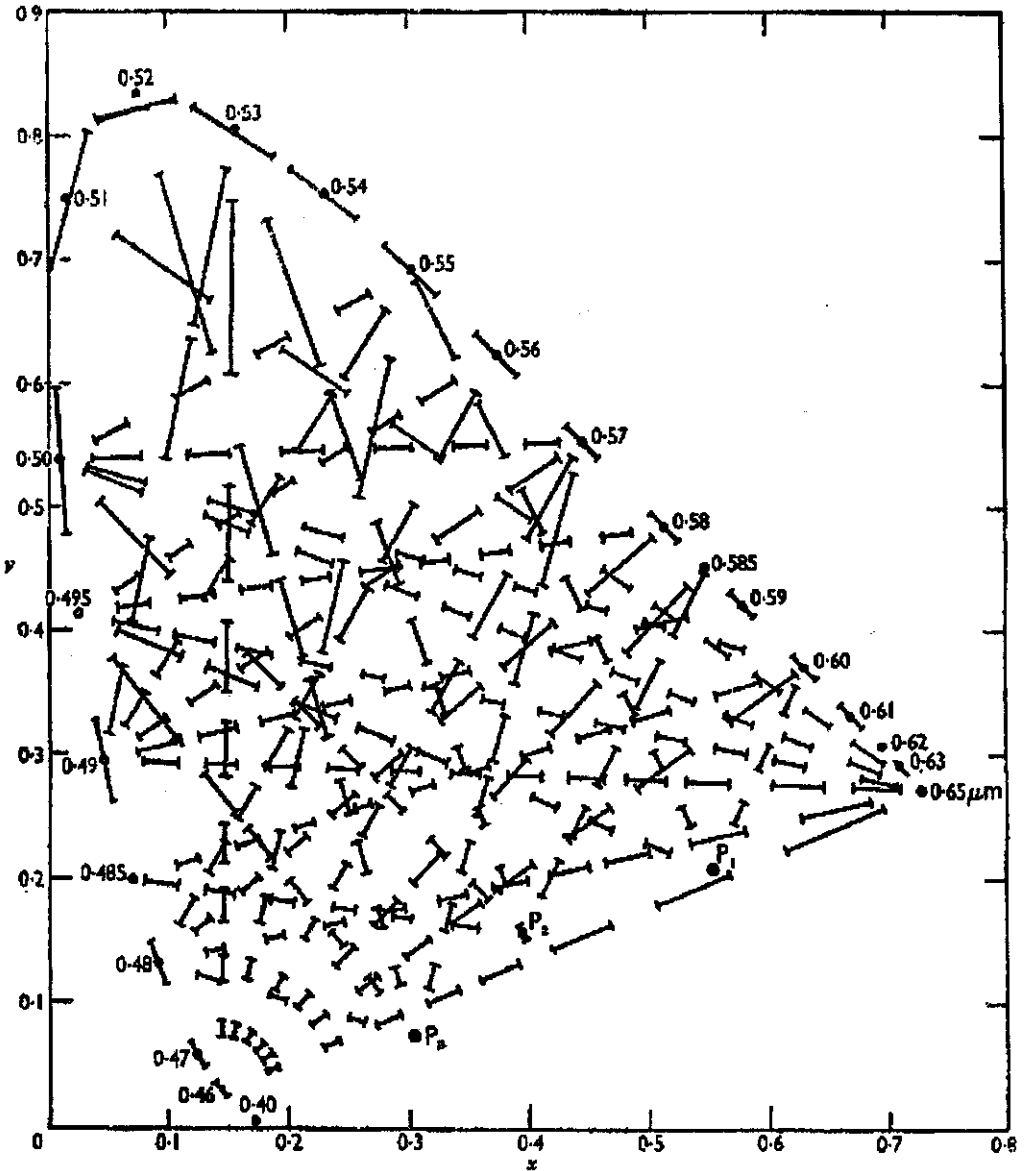


Fig. 6-6 Perceptually equal steps between spectral colors as well as colors in the interior of the chromaticity diagram, as determined by Wright (1969).

MacAdam calculated g_{ik} values from the following equation that describes his ellipses in geometrical terms:

$$(ds)^2 = g_{11}(dx)^2 + 2g_{12}dxdy + g_{22}(dy)^2 = 1, \tag{6-13}$$

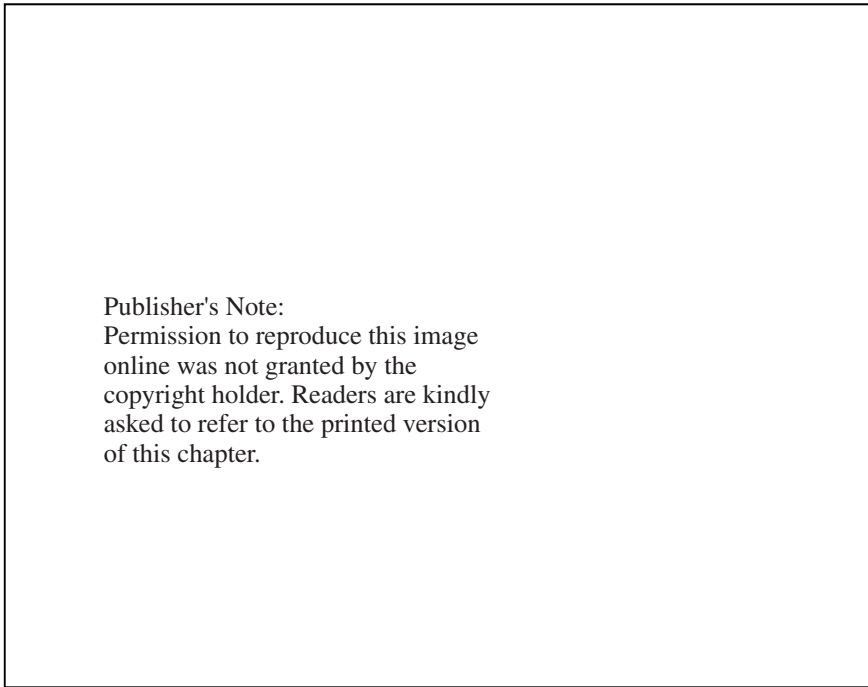
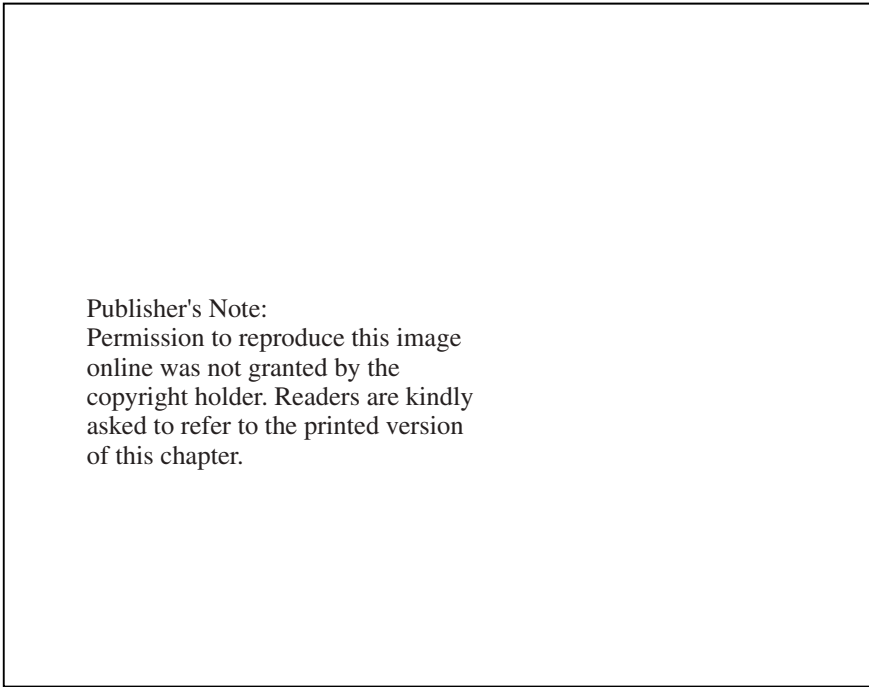


Fig. 6-7 Color matching error ellipses of observer PGN for 25 colors at the center of the ellipses, enlarged 10 times, in the CIE 2° chromaticity diagram. From MacAdam (1942).

where ds is the distance between the center of an ellipse and a point on its contour and x, y are CIE chromaticity coordinates. MacAdam drew interpolating lines connecting his data points. Results are shown in Fig. 6-8a-c. Knowing the g_{ik} values for a particular location in the chromaticity diagram by reading them from tables or graphs with interpolation between neighboring values makes possible the calculation of color differences using eq. 6-13 and applying the square root. Alternately, the chromaticity diagram could be modified locally to convert the ellipses to circles of equal size. Sets of charts that simplified the calculation of color differences by this method have been available in the 1950s and 1960s. From the results of such calculation it was apparent that linear transformation of the CIE chromaticity diagram does not yield a uniform chromaticity diagram in which the MacAdam ellipses form circles of equal size.

By 1950 the situation presented itself as follows: On the one hand, there were line elements, either derived from theoretical considerations using best esti-



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Fig. 6-8a-c Contours of the g_{11} , $2g_{12}$, and g_{22} ellipse parameter functions in the CIE chromaticity diagram for the ellipses of Fig. 6-7.

mates of cone sensitivity functions, or empirically as in the case of the MacAdam ellipses. On the other hand, there were linear transforms of the CIE chromaticity diagram as well as the Adams chromatic value diagram and modifications thereof in attempts to predict the Munsell data. In terms of psychophysics there were thus color space models based on accumulation of threshold differences (local psychophysics extended to global psychophysics) and models based on global scaling data.

Color Difference Formulas Derived from the MacAdam Data

In 1961 L. F. C. Friele began devising a formula with which MacAdam's ellipses could be described with good accuracy (Friele, 1961, 1965). The formula was slightly modified and simplified by MacAdam and later by K. D. Chickering and became known as the Friele-MacAdam-Chickering or FMC I formula.

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Fig. 6-8a-c (Continued)

The Roman numeral distinguished it from a later version, FMC II, with a different treatment of lightness. (For an informative discussion of the development of these formulas see MacAdam 1981.) Friele based his approach on the three-stage color vision theory by G. E. Müller (1930). The CIE tristimulus values were converted to cone sensitivity functions P , Q , and S . Opponent color differences and lightness differences were calculated from these in two steps.

FMC II formula

$$P = 0.724X + 0.382Y - 0.098Z,$$

$$Q = -0.48X + 1.37Y + 0.1276Z,$$

$$S = 0.686Z,$$

$$\Delta C_{rg} = \frac{(Q\Delta P - P\Delta Q)}{(P^2 + Q^2)^{0.5}},$$

$$\Delta C_{yb} = \frac{S\Delta L_1}{(P^2 + Q^2)^{0.5}} - \Delta S,$$

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Fig. 6-8a-c (Continued)

$$\Delta L_1 = \frac{(P\Delta P + Q\Delta Q)}{(P^2 + Q^2)^{0.5}},$$

$$\Delta L_2 = \frac{0.279\Delta L_1}{a},$$

$$\Delta C_1 = \left[\left(\frac{\Delta C_{rg}}{a} \right)^2 + \left(\frac{\Delta C_{yb}}{b} \right)^2 \right]^{0.5},$$

$$a^2 = \frac{0.0000173(P^2 + Q^2)}{[1 + 2.73P^2Q^2/(P^4Q^4)]},$$

$$b^2 = 0.0003098(S^2 + 0.2015Y^2),$$

$$\Delta C = K_1\Delta C_1,$$

$$\Delta L = K_2\Delta L_2,$$

$$\begin{aligned}
 K_1 &= 0.054 + 0.46Y^{1/3}, \\
 K_2 &= 0.465K_1 - 0.062.
 \end{aligned}
 \tag{6-14}$$

The total color difference is calculated from the chromatic and the lightness difference as

$$\Delta E = [(\Delta C)^2 + (\Delta L^2)]^{0.5}.
 \tag{6-15}$$

The two K functions have the purpose of adjusting the size of the implied ellipse as a function of luminous reflectance and to adjust the lightness value to be in reasonable agreement with Munsell lightness. They are shown above in the simplified form proposed by MacAdam.

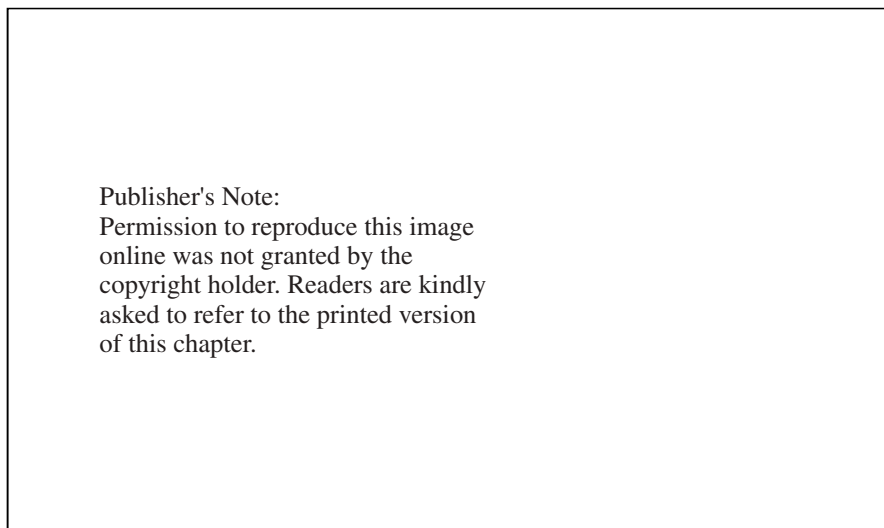
A uniform (in terms of color-matching error) color space model is implicit in this difference formula, but it has not been stated explicitly. The formula is an elaborate fitting of the MacAdam ellipses in a cone-based opponent color framework. It represents a compromise in regard to size of the ellipse as a function of luminous reflectance. MacAdam had found that the color-matching error ellipses were affected only to a small extent (less than 20%) by changes within reasonable levels in luminosity. The K_l function made them change in size in line with cube root compression. In his work with Brown, MacAdam had determined that ellipsoids generated from threshold determinations including luminance differences had one of their three axes parallel to the luminance axis. In the Adams chromatic value space, on the other hand, corresponding ellipsoids are tilted toward the neutral point of the chromatic diagram. One of the advantages of the FMC formula is that it implicitly adjusts for the Helmholtz-Kohlrausch effect.

Nonlinear Transformation of the CIE Chromaticity Diagram

MacAdam continued efforts to represent his ellipses in a chromaticity diagram in which they would appear as circles of equal size. He abandoned a cone sensitivity based solution and concentrated on mathematical methods. Using at first a paper and scissor method and then stepwise linear regression he developed a geodesic chromaticity diagram, illustrated in Fig. 6-9.

Using four linear transforms of the CIE chromaticity diagram, MacAdam, 1965 proposed the following chromaticity coordinates for this diagram:

$$\begin{aligned}
 \xi &= 3751a_1^2 - 10a_1^4 - 520b_1^2 + 13295b_1^3 + 32327a_1b_1 - 25492b_1^2 \\
 &\quad - 41672a_1b_1^2 + 10a_1^3b_1 - 5227a_1^{0.5} + 2952a_1^{0.25} \\
 \eta &= 404b_2 - 185b_2^2 + 52b_2^3 + 69a_2(1 - b_2^2) - 3a_2^2b_2 + 30a_2b_2^3,
 \end{aligned}
 \tag{6-16}$$



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Fig. 6-9 MacAdam's geodesic uniform chromaticity diagram, 1965. Lines of constant value of chromaticity coordinates x and y are drawn in the interior.

where

$$a_1 = 10x / (2.4x + 34y + 1),$$

$$a_2 = 10x / (4.2y - x + 1),$$

$$b_1 = 10y / (2.4x + 34y + 1),$$

$$b_2 = 10y / (4.2y - x + 1).$$

In evaluations (to be discussed below) against a growing number of sets of visual color difference data involving object color samples, formulas based on MacAdam's ellipses performed relatively poorly, and the development of color space and color difference model formulas generally proceeded along different routes.

More recently a nonlinear transformation was proposed by C. Oleari (2001). It uses an angular transformation of the CIE chromaticity diagram, with dilatation, to form the δ, v chromaticity diagram in which the MacAdam ellipses approach circles of equal size. When this diagram is applied to the CIE 10° observer and the OSA-UCS colors are plotted in it, they are aligned along slanted lines which Oleari believes are due to the difference in the two and ten degree observers, which he corrected with an additional dilatation.

6.6 FURTHER DEVELOPMENT OF FORMULAS BASED ON OPPONENT COLOR SYSTEMS

Realizing the opportunities arising from instrumental quality control of colored materials, R. S. Hunter began to build filter tristimulus colorimeters in 1948. These were relatively inexpensive instruments that could determine CIE tristimulus values directly and with good accuracy from four measurements per sample. Hunter, in addition, provided for output from the instruments directly in terms of color difference values. Hunter proposed the following formulas for this purpose:

$$\begin{aligned} L &= 10Y^{0.5}, \\ a_L &= \frac{17.5(1.02X - Y)}{Y^{0.5}}, \\ b_L &= \frac{7.0(Y - 0.847Z)}{Y^{0.5}}, \end{aligned} \quad (6-17)$$

where X , Y , Z are the CIE tristimulus values for illuminant C taken as percentages. The total color difference was given as the square root of the sum of the squares of the differences in L , a_L , and b_L . These instruments were quite popular and continued to be used into the 1980s. Kuehni showed that unit chromatic contours derived from the formula were not in good agreement with ellipses fitted to small color difference data (1982).

The calculation of the Munsell value function from luminous reflectance with the quintic formula was troublesome, and efforts were made to find a simpler solution. It was found that properly scaled cube roots of luminous reflectance resulted in good agreement with the quintic formula. In 1958 L. G. Glasser, A. H. McKinney, C. D. Reilley, and P. D. Schnelle proposed a cube root model of color space:

$$\begin{aligned} L^* &= 25.29G^{1/3} - 18.38, \\ a^* &= K_a(R^{1/3} - G^{1/3}), \\ b^* &= K_b(G^{1/3} - B^{1/3}), \end{aligned} \quad (6-18)$$

where $R = 1.02X$, $G = Y$, $B = 0.847Z$, $K_a = 106.0$, and $K_b = 42.34$. For closer agreement with the Munsell data the K_a and K_b constants were adjusted by quadrant. The total color difference was calculated as the square root of the sum of the squares of the differences in L^* , a^* , and b^* . With square and cube root tables and/or then modern electronic calculators, color differences from these formulas could be calculated comparatively rapidly.

In the 1960s, then, there were several competing formulas that were used

in industry to assess quality of coloration. Among these were the FMC II, the Adams-Nickerson or the cube root, Saunderson-Milner, CIE 1964, and the Hunter L, a, b formulas.

6.7 NEW SMALL COLOR DIFFERENCE DATA

The unsatisfactory situation in regard to industrial color quality control using formulas available by 1960 resulted in several new sets of visual data and considerable activity in testing these data and developing formula modifications. The following sets of data were developed from the 1950s until 1980:

1. Davidson and Friede (1953). Data consisted of 287 pairs of wool textile samples around 19 standards, evaluated by 8 observers in terms of acceptability as a production match to the standards. The results were expressed as acceptability percentages.
2. Robinson (1969). Data consisted of a single blue paint standard with 31 samples around it evaluated by 132 observers with the results expressed as acceptability percentages.
3. Thurner and Walther (1969). The data consisted of 500 textile samples around 27 standards, assessed for acceptability by 16 textile colorists.
4. Kuehni (1971a). 113 samples around three standards, pigments on polyester/cotton fabric, were assessed for acceptability by 10 observers.
5. Kuehni/Metropolitan Section (1971b). Data consisted of 180 textile samples around 10 standards, assessed for acceptability by 16 observers.
6. HATRA (Jaeckel 1975). 854 textile samples around 12 standards were assessed for acceptability by 24 to 32 observers.
7. MMB (Morley, Munn and Billmeyer 1975). 555 pairs of painted samples around 19 standards were assessed by 20 observers according to four categories from no difference to very large difference. Psychometric scales were constructed from the visual data.
8. VVVR (Friele 1978). 20 glossy painted samples each around 10 standards were assessed by 14 observers using a pair-comparison method and by 15 to 25 observers using the acceptability method. Psychometric scales were constructed.
9. ISCC (Kuehni and Marcus 1979). 180 samples around six standards were assessed, four consisting of matte paint on cardboard and two of dyed textiles. They were assessed by 26 to 37 observers using two assessment methods: (a) subjective estimates on a scale with a maximum value of 10; (b) acceptability as a commercial match. Psychometric scales were constructed from the subjective estimates.

Most of these data sets were established by industrial specialists interested in a color difference formula for quality control purposes with a reliability at least equal to that of an average observer.

In 1970 the Society of Dyers and Colourists in England recommended the use of the Adams-Nickerson formula (called ANLAB) for application in the textile industry based on extensive evaluations of formulas against various sets of visual data. It had been shown to be equal or marginally better than a dozen other formulas under consideration. (In 1973 Kuehni found that 13 different formulas were in use in U.S. color-related industries.) Levels of correlation between visual and calculated data for all of these, including ANLAB, were unsatisfactory. Formulas performed poorer than the average observer. D. Strocka showed in 1971 that a simple formula based on circles of equal size in the CIE chromaticity diagram and a simple lightness formula resulted in levels of correlation comparable to that obtained with the best formulas.

Looking for causes of the low level of correlation between visual data and formula, K. MacLaren (1971) used multiple linear regression in the directions of lightness, metric chroma, and hue angle. He was able to show that the magnitude of perceptually equal color differences, when calculated with formulas such as Adams-Nickerson, increased with increasing metric chroma and lightness.

6.8 ELLIPSE AND ELLIPSOID FITTING

Strocka's results with the circular formula were puzzling but indicated that the actual unit difference contours in the CIE chromaticity diagram could not be circles, since the correlation with the circular formula would have been greater. The implication was that the ellipsoidal contours implicit in a formula such as Adams-Nickerson did not match the ellipsoidal contours implicit in the visual data well, perhaps in direction as well as in size. In 1971 Kuehni graphically fitted unit ellipses in the CIE chromaticity diagram to various sets of visual data (Fig. 6-10; Kuehni 1971a). The results indicated that the major axes of the resulting ellipses were usually tilted 20° to 30° clockwise compared to the MacAdam ellipses and that ellipses tended to increase in size as chroma increased (as McLaren had found).

Kuehni asked MacAdam to fit the parameters of his xi-eta equation (see above) to fitted ellipses. The resulting, then unpublished, equations are as follows:

$$\xi = -4067a^2 - 133a^4 - 1675b^2 - 38280b^3 + 46479ab - 12064a^2b - 2806ab^2 - 37a^3b + 843a^{0.5} - 10254a^{0.25},$$

where $a = 10x/(2.5x + 34y + 1)$ and $b = 10y/(2.5x + 34y + 1)$;

$$\eta = 343b - 125b^2 - 41b^3 + 31a + 41a^3 + 4a^4 - 4ab^2 - 75a^2b^2 - 85a^2b + 137ab^3, \tag{6-19}$$

where $a = 10x/(4.3y - x + 1)$ and $b = 10y/(4.3y - x + 1)$.

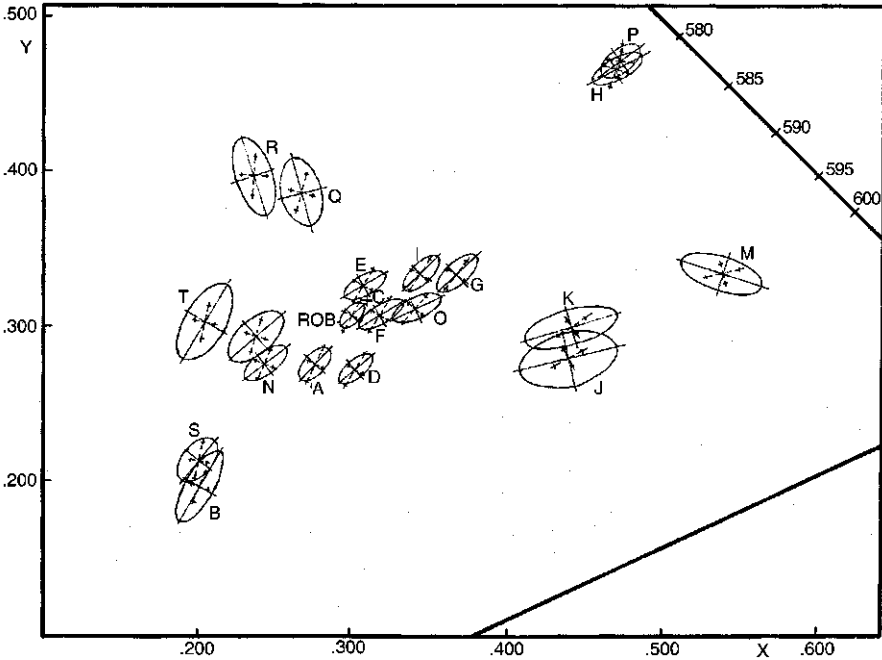


Fig. 6-10 Ellipses graphically fitted to small suprathreshold color difference data by Robinson 1962 (Rob), and by Davidson and Friede 1953 (A–T). From Kuehni (1971a).

The lightness scale used together with the formulas above was the cube root scale. The combined formulas resulted in a correlation with visual data significantly higher than the cube root and ANLAB or the circle formula.

In 1972 Kuehni refined the ellipse fitting method by systematically varying ellipse parameters until highest correlation for the samples around a standard were obtained. In 1975 R. M. Rich, F. W. Billmeyer, and W. G. Howe employed a computer algorithm using the maximum likelihood function to fit ellipses to visual data with greater statistical validity. Mathematical ellipse or ellipsoid fitting has since become a standard tool of investigation of color differences.

6.9 CONTROVERSIES OF DETAIL

A number of controversies abounded in the late 1960s to early 1970s. One of these involved the question of change in ellipse size as a function of lightness. MacAdam’s findings using a visual spectrophotometer had, as mentioned earlier, indicated only small changes in the ellipse’s size with changes in luminance. Formulas, such as Adams-Nickerson, on the other hand, implied significant changes in unit contours as a function of lightness.

As shown in Section 8.7 the final answer to this question has not been found yet.

Similarly Brown and MacAdam had found that the third axis of color-matching error ellipsoids was aligned parallel with the luminance axis. Formulas such as Adams-Nickerson have implied third axes that are tilted toward the white point of the color space. Work based on small suprathreshold differences reported by W. Schultze and L. Gall in 1971 indicated no tilt of the ellipsoids. The comparative success of power modulation-based formulas indicated that tilt is appropriate.

The dependence of the ellipse's size in a constant luminous reflectance plane on metric chroma has already been mentioned. In addition there was the controversy of perceptibility judgments versus acceptability judgments. While in perceptibility experiments presumably purely psychophysical judgments are obtained (but see Chapter 3), acceptability experiments can, and sometimes do, include additional cognitive overlays. Perceptibility experiments are usually difference magnitude estimations against a reference pair. Acceptability experiments also involve difference magnitude estimation, however, against an internal standard of acceptability in a commercial situation. Biases in the latter case are possible based on specific situations; that is, in a chromatic diagram the limit contour of acceptable color differences may not be positioned symmetrically around the standard. E. Allen and B. Yuhas (1984) and, more recently, Berns (1996) have shown how this situation can be mathematically treated. Another issue is that acceptability tolerances for identically colored materials may vary significantly depending on the context. However, experiments have shown that tolerance contours from acceptability judgments and unit difference contours from perceptibility judgments of the same sample pairs, when determined in the absence of a specific context, are symmetrical; that is to say, acceptability judgment in that situation is guided by perceptibility (Kuehni, 1975; McLaren, 1976; Mahy, Van Eycken, and Oosterlinck, 1994).

6.10 DEPENDENCE OF CALCULATED COLOR DIFFERENCE ON METRIC CHROMA

Multiple linear regression as well as fitting of ellipses and ellipsoids had clearly indicated a dependence of the chromatic differences (in the equal luminance plane) on metric chroma. In 1972 Kuehni proposed a modification to the Glasser et al. (see above) cube root color difference formula that adjusted the size of the calculated total color difference as a function of the radial difference of the standard from the illuminant point in the CIE chromaticity diagram:

$$\Delta E = [(\Delta C)^2 + (\Delta L)^2]^{0.5}, \quad (6-20)$$

where

$$\begin{aligned}\Delta C &= [(\Delta a)^2 + (\Delta b)^2]^{0.5} / F, \\ F &= 1.0 + 6S, \\ S &= [(x - 0.3100)^2 + (y - 0.3162)^2]^{0.5}.\end{aligned}$$

S is applicable for illuminant C . The need for such an adjustment was ascribed to Takasaki's chromatic crispening effect. For a combined set of data the formula provided modest improvement over the FMC II or the unmodified Glasser formula.

McLaren's work with multiple linear regression had shown that it was useful to consider separately three components of the total color difference, assumed to be related in a euclidean manner: metric lightness, metric chroma, and metric hue differences. Metric chroma was calculated as the square root of the sum of the squares of the cartesian coordinates in the Adams-Nickerson chromatic diagram and metric lightness and metric chroma were subtracted in the euclidean manner from total color difference to result in metric hue difference:

$$\Delta H = [(\Delta E)^2 - (\Delta L)^2 - (\Delta C)^2]^{0.5}. \quad (6-21)$$

While the L, a, b view of the system is the cartesian coordinate view of the euclidean space, that of L, C, H is the polar coordinate view.

Using new experimental data, and expanding on McLaren's multiple linear regression, R. McDonald in 1974 found that he could obtain significant improvement in correlation between visual and calculated data by adjusting the total color difference as a function of metric chroma as follows:

$$DE_a = \frac{DE}{1 + 0.022C}, \quad (6-22)$$

where DE_a is the equivalent color difference at the achromatic point and C is the chroma value.

6.11 THE CIE 1976 $L^*a^*b^*$ AND $L^*u^*v^*$ SPACES

Because of the mentioned large number of different formulas used in industry, the CIE organization in the early 1970s became increasingly aware of the need to promote uniformity of practice in industrial color control. However, the emerging picture was less than clear, and superiority had been claimed for various formulas and in respect to frequently changing sets of visual data. After much discussion a compromise was found, and in 1976 two formulas were recommended for study and "in the interest of uniformity of

usage” (CIE, 1976). One formula is a simplified version of the Adams-Nickerson formula, abbreviated CIELAB, the other the earlier mentioned modified version of MacAdam’s 1937 linear transformation of the CIE chromaticity diagram, abbreviated CIELUV. The two formulas are defined as follows:

CIELUV

$$\begin{aligned}
 L^* &= 116\left(\frac{Y}{Y_n}\right)^{0.333} - 16, \\
 u^* &= 13L^*(u' - u'_n), \\
 v^* &= 13L^*(v' - v'_n), \\
 \Delta E_{uv}^* &= [(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2]^{0.5}, \\
 u' &= \frac{4X}{X + 15Y + 3Z}, \\
 v' &= \frac{9Y}{X + 15Y + 3Z},
 \end{aligned} \tag{6-23}$$

where

$$\begin{aligned}
 u'_n &= \frac{4X_n}{X_n + 15Y_n + 3Z_n}, \\
 v'_n &= \frac{9Y_n}{X_n + 15Y_n + 3Z_n},
 \end{aligned}$$

and X_n, Y_n, Z_n are the tristimulus values of the nominally white object color stimulus. The optimal object color solid and the spectral trace derived from this formula are illustrated in Fig. 6-11.

CIELAB

$$\begin{aligned}
 L^* &= 116\left(\frac{Y}{Y_n}\right)^{0.333} - 16, \\
 a^* &= 500\left[\left(\frac{X}{X_n}\right)^{0.333} - \left(\frac{Y}{Y_n}\right)^{0.333}\right], \\
 b^* &= 200\left[\left(\frac{Y}{Y_n}\right)^{0.333} - \left(\frac{Z}{Z_n}\right)^{0.333}\right], \\
 \Delta E_{ab}^* &= [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{0.5},
 \end{aligned} \tag{6-24}$$

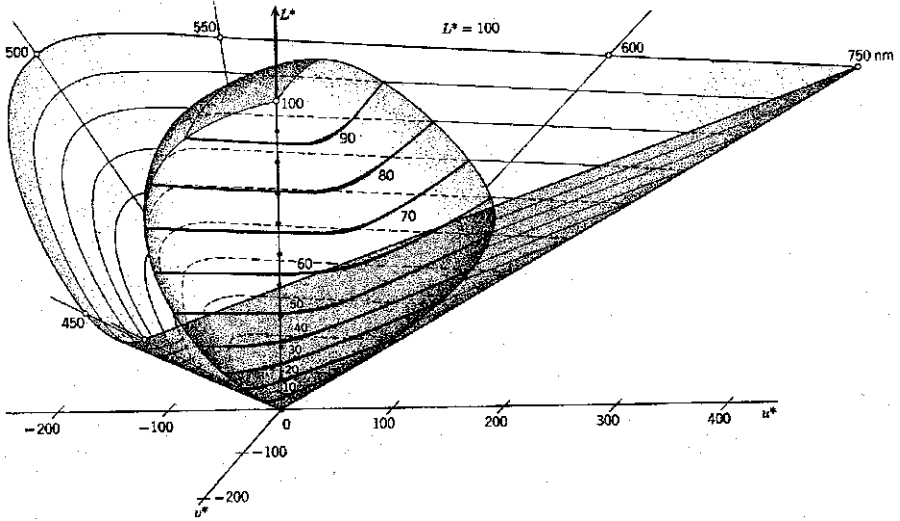


Fig. 6-11 Projective view of the L*, u*, v* object color space for the CIE 10° standard observer and illuminant D65 (inner contour) and the spectrum locus. From Judd and Wyszecki (1975).

where X_n, Y_n, Z_n are the tristimulus values of the nominally white object color stimulus. A slightly different calculation for $L^*, a^*,$ and b^* applies for low tristimulus value ratios. For $X/X_n, Y/Y_n,$ and $Z/Z_n > 0.01$, instead of cube roots, factors f are applied to the ratios, determined as follows: $f(Y/Y_n) = 7.787 (Y/Y_n) + 16/116$ and comparably for Y and Z . This adjustment is valid for the $L^*, a^*,$ and b^* scales.

Differences can also be calculated in a polar coordinate version, where

Metric chroma	$C^* = [(a^*)^2 + (b^*)^2]^{0.5}$
Hue angle	$h_{ab} = \arctan (b^*/a^*)$

From lightness and metric chroma differences, the total color difference ΔE is calculated as follows:

$$\Delta E = [(\Delta L^*)^2 + (\Delta C^*)^2 + (\Delta H^*)^2]^{0.5}, \tag{6-25}$$

where $\Delta H^* = [(\Delta E)^2 - (\Delta L^*)^2 - (\Delta C^*)^2]^{0.5}$ and $\Delta C^* = [(\Delta a^*)^2 + (\Delta b^*)^2]^{0.5}$. The optimal object color solid and the spectral trace derived from this formula are illustrated in Fig. 6-12.

As mentioned before, the former formula is primarily of interest to lighting engineers as it provides for additivity of light mixtures. The latter was recommended for use with object colors. CIELAB is a simplification of, but no advancement over, the Adams-Nickerson formula, and it was apparent at the

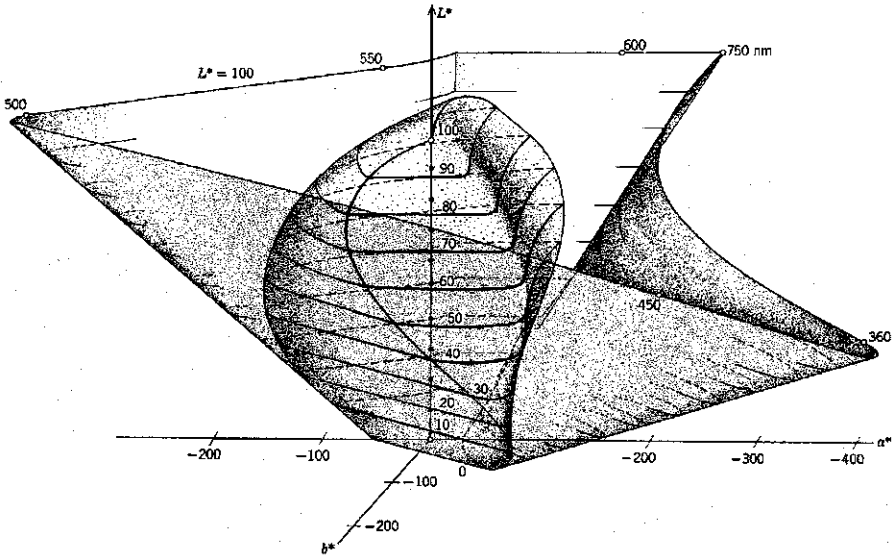


Fig. 6-12 Projective view of the L^* , a^* , b^* object color space for the CIE 10° standard observer and illuminant D65 (inner contour) and the spectrum locus. From Judd and Wyszecki (1975).

time of its recommendation that further modification was necessary to improve the correlation of calculated with average visual color differences.

6.12 FRIELE'S FCM FORMULA

In 1978 Friele published a new formula optimized against various sets of visual data. It was a modification of his earlier effort resulting in the FMC metric. Friele had created psychometric scales for the various sets of acceptability data that he used in addition to perceptibility data. The formula provided significant improvements in correlation for these data, compared to CIELAB.

FCM

$$\begin{aligned}
 R &= 0.760X + 0.401Y - 0.124Z, \\
 G &= -0.484X + 1.381Y + 0.079Z, \\
 B &= 0.847Z.
 \end{aligned}
 \tag{6-26}$$

For $R > G$,

$$T = 125Y^{1/3} \left(\frac{1 - Y^{1/3}}{R^{1/3}} \right).$$

For $R < G$,

$$T = \frac{0.760}{-0.484} 125Y^{1/3} \left(\frac{1 - Y^{1/3}}{G^{1/3}} \right).$$

$$D = \int_B^Y \left[\left(\frac{0.085x^{4/3}}{Y^{2/3}} \right)^2 + (0.055Y^{2/3})^2 \right]^{-1/2} dx. \quad (6-27)$$

$$L = 18Y^{1/3},$$

$$a = T - 0.3D,$$

$$b = D - 0.3|T|,$$

$$\text{Chroma} = (a^2 + b^2)^{1/2},$$

$$\tau = \frac{0.024R^{4/3}}{Y^{2/3}},$$

$$\delta = \left[\left(\frac{0.085B^{4/3}}{Y^{2/3}} \right)^2 + (0.055Y^{2/3})^2 \right]^{1/2}.$$

$$c^2 = T^2 + D^2,$$

$$\alpha = \arctan \left(\frac{D^2}{T^2} \right),$$

$$f = 1.6[1 - \exp(-0.0015c^2)] \sin 2\alpha + \exp(-0.0015c^2),$$

$$\Delta L = 6Y^{-2/3} \Delta Y$$

$$\tau \Delta T = 0.760 \left(\Delta X - \frac{X}{Y} \Delta Y \right) - 0.124 \left(\Delta Z - \frac{Z}{Y} \Delta Y \right),$$

$$\tau \Delta D = -0.847 \left(\Delta Z - \frac{Z}{Y} \Delta Y \right),$$

$$\Delta E = \frac{2.5}{1 + 0.01Y} \left[(f\Delta L_1)^2 + (\Delta T^2) + (\Delta D)^2 - f\Delta T\Delta D \right]^{0.5}. \quad (6-28)$$

Parameter f_1 determines the weight of the lightness difference relative to the chromatic difference. It ranges from 0.4 to 1.0 for different data sets. The model represents an implementation of the Müller three-stage theory of color vision mentioned earlier. R , G , and B represent the first, photopigment stage, and L , T , and D the second stage opponent-response functions at the receptor level. To get to the third, opponent stage at the “optic nerve” level, T and D are

modified, resulting in the rectangular coordinate system L, a, b . In this system, somewhat comparable to the CIELAB space, the unit difference contours are ellipsoids, rather than spheres, and the contours have to be normalized for the effect of chromatic crispening and the ellipse shape of the unit contour with the parameters τ and δ . Friele also implemented the controversial findings by Schultze and Gall (1971) that unit small color difference ellipsoids are not tilted toward the achromatic axis of color space.

The FCM formula, despite the improved correlation with visual data, did not receive any significant practical use, perhaps because of its complexity and the fact that its results were soon equaled by further optimization of the CIELAB formula.

6.13 RICHTER'S LABHNU2 FORMULA

In the late 1970s K. Richter proposed a series of formulas as good models for the Munsell and the OSA-UCS systems (Richter, 1980). Of these only the non-linear LABHNU2 formula will be mentioned:

$$\begin{aligned} a' &= \frac{[(x/y) + (1/6)]^{2/3}}{15}, \\ b' &= \frac{[(z/y) + (1/6)]^{1/3}}{12}, \end{aligned} \quad (6-29)$$

where x, y, z are CIE chromaticity coordinates. The CIELAB lightness formula is used for lightness difference calculation

6.14 WEIGHTING OF METRIC LIGHTNESS, CHROMA, AND HUE DIFFERENCES

JPC79 Formula

As mentioned above, in 1976 McLaren optimized weights for the three color difference components based on the Adams-Nickerson formula. He found that the optimum weights varied by set of visual data. McDonald continued to pursue individual adjustment of the three color difference components and developed new industrial visual data. In 1980 he proposed a formula employing continuous weight adjustment for all three metric components. It was based on an extensive set of 640 samples around 55 standards distributed over a significant portion of the object color solid. The samples consisted of dyed spun polyester sewing thread and the visual evaluations (acceptability judgments) were performed by eight industrial color matchers. An additional set of some 8500 judgments against 600 color standards by a single observer in an

industrial dyehouse was also included. McDonald's color difference formula, called the JPC79 formula, is based on the ANLAB formula as follows:

$$\Delta E_{\text{JPC79}} = \left[\left(\frac{\Delta L}{S_L} \right)^2 + \left(\frac{\Delta C}{S_C} \right)^2 + \left(\frac{\Delta H}{S_H} \right)^2 \right]^{0.5}, \quad (6-30)$$

where ΔL , ΔC , and ΔH are, respectively, the lightness, chroma, and hue differences calculated from ANLAB,

$$S_L = 0.08195L / (1 + 0.01765L),$$

$$S_C = [0.0638C / (1 + 0.0131C)] + 0.638,$$

$$S_H = S_C T,$$

$$T = 1 \text{ if } C = 0.38, \text{ otherwise}$$

$$T = 0.38 + |0.4 \cos(h + 35)|, \text{ unless } h \text{ is between } 164^\circ \text{ and } 345^\circ, \text{ then}$$

$$T = 0.56 + |0.2 \cos(h + 168)|.$$

The JPC79 formula represents the prototype of several further enhancements fitting formulas with the help of various functions to ellipsoids optimized to average visual data. It represents a practical, empirical approach to dealing with the non-euclidean nature of a uniform color space.

CMC (1:c) Formula

This formula was slightly modified from JPC79 and based on CIELAB component differences by the Color Measurement Committee of the Society of Dyers and Colourists in England (Clark, 1984). It has been standardized in England and in the United States, and is recommended by the International Standards Organization (ISO). It is defined as follows:

$$\Delta E_{\text{CMC}} = \left[\left(\frac{\Delta L^*}{L S_L} \right)^2 + \left(\frac{\Delta C^*}{C S_C} \right)^2 + \left(\frac{\Delta H^*}{S_H} \right)^2 \right]^{0.5}, \quad (6-31)$$

where

$$S_L = 0.04097L^* / (1 + 0.01765L^*) \text{ unless } L^* < 16, \text{ then } S_L = 0.511,$$

$$S_C = [0.0638C^* / (1 + 0.0131C^*)] + 0.638,$$

$$S_H = S_C (TF + 1 - F),$$

$$F = \left\{ (C^*)^4 / [(C^*)^4 + 1900] \right\}^{0.5},$$

$$T = 0.38 + |0.4 \cos(h + 35)|,$$

and l and c are additional weights adjusting the relative weight of lightness and chroma differences. Analysis has shown that for flat surface materials (paints, plastics) viewed in sharp juxtaposition values of 1 in both cases are appropriate, while for textile materials, the value $l = 2$ was found to improve correlation. CMC found wide international usage in industry and continues to be used in many firms.

6.15 NEW SETS OF VISUAL DATA

In 1978 the CIE issued a set of guidelines for coordinated research on color difference equations (Robertson, 1978). In these guidelines particular emphasis was placed on five color centers: gray, yellow, red, blue, and green. As a result several experimenters provided data for these centers but new data for many other centers were also established.

Luo and Rigg

As a prelude to their formula fitting (see below) R. Luo and B. Rigg assembled 13 previously published sets of perceptibility and acceptability data (Luo and Rigg, 1986). They fitted separate ellipsoids in x, y, Y space to each subset and compared these. The main difference for ellipsoids in the same neighborhood of global color space appeared to be a size factor (see Fig. 6-13 showing chromatic ellipses). This is understandable as the implicit acceptability tolerances of different observer groups are likely to be different and different scaling techniques had been used in the perceptibility judgments. The relative sizes were adjusted by judging 400 pairs of samples representing 70 color centers against a gray scale. Luo and Rigg also deleted certain subsets that they found to be internally inconsistent and, as a result of additional experiments, modified certain ellipses. No consideration was given to differences in surround in various data sets. The result was 132 ellipses that were considered to be reliable and formed a reasonably regular pattern in the CIE chromaticity diagram. Luo and Rigg found little difference between perceptibility and acceptability ellipses. The data are available on the University of Derby Web site (Derby, 1999).

Cheung and Rigg

In support of the CIE effort M. Cheung and Rigg established in 1986 suprathreshold small color difference data for the suggested five centers. The samples consisted of dyed wool fabric, with 59 to 82 pairs per center, assessed by 20 observers against a standard difference pair under artificial daylight and tungsten light against a neutral gray background of $Y = 13$. The samples had CIELAB color differences from 1 to 9 units from the respective standards.

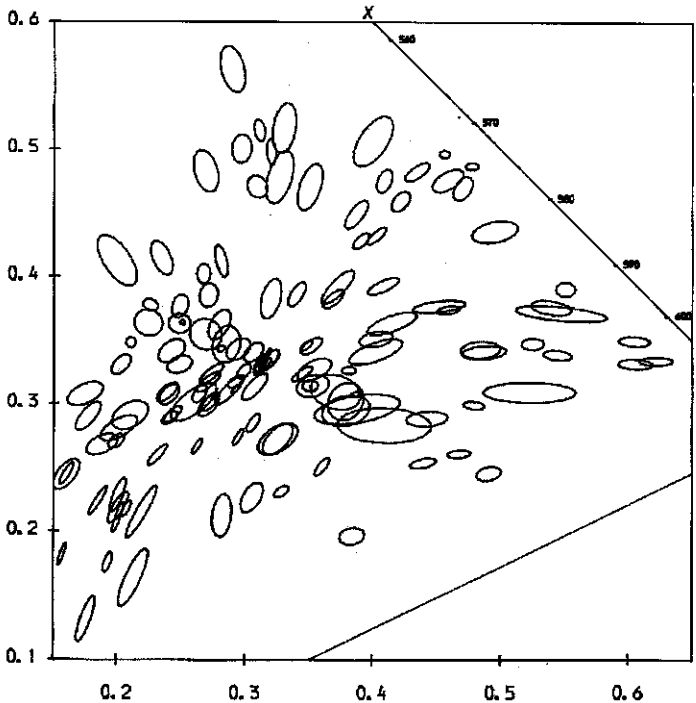


Fig. 6-13 Ellipses fitted to unadjusted data of the Luo and Rigg data set, enlarged 1.5 times, in a portion of the CIE chromaticity diagram. From Luo and Rigg (1986).

Badu and Rigg Large Difference Data

In connection with a Ph.D. thesis, A. Badu prepared in 1986 a set of data with relatively large differences. Samples were prepared to form a grid of approximately equilateral triangles on the a^* , b^* diagram. Some samples also involved lightness differences. There were 408 sample pairs, 238 consisting of nylon dyeings and 170 of glossy paint samples. They were visually evaluated by a group of 20 observers using a gray scale as standard.

Witt (1987–1990)

In 1987 K. Witt reported on the results of a threshold experiment using 50 to 64 sample pairs in four of the five CIE color regions prepared with high gloss acrylic paint. Some 24 observers determined at least four times (some observers 10 times) under simulated daylight and against a surround of $Y = 20$ if they could perceive a difference between the pairs. Ellipses were calculated from the resulting thresholds for individual observers and for sets of 4 to 22 observers with from 22 to 118 repetitions. Considerable intra- and inter-

observer variability was noted. Average color differences of the thresholds were approximately 0.4 CMC (2:1) units.

In 1990 Witt reported a further experiment involving the five CIE color centers. In addition to determining thresholds, observers were asked to assess perceptible differences in terms of hue, chroma, and lightness. Another purpose of the experiment was to determine the effects on the thresholds of changing the surround lightness and of a gap between the samples. Interestingly better agreement was found between the chromatic ellipse axis and the hue judgment axis in the CIE chromaticity diagram than in the a^* , b^* diagram. A small surround lightness effect and a larger gap effect were found.

RIT-DuPont Data

In an effort to develop a set of high-reliability small color difference perceptibility data D. Alman of DuPont and Berns of RIT (Rochester Institute of Technology) collaborated in 1990 and published the RIT-DuPont data (Berns et al., 1991). They are based on 156 acrylic lacquer spray-painted sample pairs on primed aluminum panels representing 19 color centers. The sample pairs did not have a common reference and were arranged so as to represent specific vector directions in CIELAB color space. The sample pairs, presented edge-to-edge on unprimed aluminum panels, were evaluated by 50 observers against a near gray standard pair presented identically. Probit analysis was used to establish psychometric scales, and the colorimetric values were set so that visual differences equal to that caused by a 1.0 ΔE CIELAB lightness difference resulted. Ellipsoids in CIELAB space have later been optimized to the visual data. The ellipses in the a^* , b^* plane show fair agreement with those of Luo and Rigg (see Fig. 5-29). Both show ellipses more or less pointed toward the origin of the diagram, except for blue colors. The data have been published and are also available on the University of Derby Web site.

Leeds Data

Two sets of data were established by D. H. Kim and J. H. Nobbs at Leeds University in 1997. They consist of matte painted samples. Sample set PC consists of 152 pairs evaluated juxtaposed by 15 observers against a near neutral reference pair. Sample set GS consists of 204 sample pairs evaluated against a gray scale by 12 observers. Color differences ranged from 0.4 to 3.7 CIELAB units in the two sets. The data are available on the University of Derby Web site.

Pointer and Attridge Large Difference Data

In 1997 Pointer and Attridge published the results of evaluation using a set of samples with relatively large differences (from approximately 2 to 20 CIELAB units). The samples were prepared by varying red, green, and blue exposures

on photographic paper. Samples were mounted in gray, commercial 35 mm slide mounts, and displayed against an identically mounted near neutral standard pair. There are four subsets (value 5, saturated, light, and dark) each with six to eight colors in the same categories (red, green blue, cyan, magenta, yellow, skin, and neutral). The total set consists of 384 pairs. Comparison of pairs of samples and standard were made by nine observers against a neutral pair with a designated visual color difference of 10 units. Sixty percent of the sample pairs have less than 6 units of CIE94 color differences; that is, the set contains a large component of small color differences. The data are available from the authors.

Witt (1999)

In 1999 Witt published a set of data involving the five CIE standard color regions. For each region there are approximately 30 painted samples. These have been evaluated by from 10 to 13 observers in approximately 85 different combinations of two each, thus providing a detailed evaluation in each region of color difference vectors in two and three dimensions. Comparisons were made against a specially prepared gray scale (near logarithmic) found to be visually uniform, with the result expressed in steps and third or quarter fraction steps of the reference scale. Color differences ranged from approximately 0.3 to 8 CIELAB units. Means and standard deviations of the visual judgments were calculated. The coefficient of variation of the visual results ranged from 15% to 60%. The data have been published in Witt's paper and are available on the University of Derby Web site.

Guan and Luo Large Difference Data

These authors published in 1999 results of evaluations of a set of relatively large differences (6 to 21 CIELAB units). The samples form a grid of approximate squares in the a^* , b^* diagram at three different levels of L^* (40, 50, and 60). Samples were compared by ten observers not only in the a^* , b^* directions but also on diagonals, involving changes in both a^* and b^* . The samples consist of wool dyeings and they were evaluated in 292 pairs against an eight-step gray scale, also consisting of wool dyeings.

6.16 NEW FORMULAS

BFD (l:c)

Using the above-described composite data set, Luo and Rigg (1987) optimized a formula within the general CMC framework as follows:

$$\Delta E_{\text{BFD}} = \left[\left(\frac{\Delta L}{l} \right)^2 + \left(\frac{\Delta C^*}{cD_C} \right)^2 + \left(\frac{\Delta H^*}{D_H} \right)^2 + R_T \left(\frac{\Delta C^*}{D_C} \right) \left(\frac{\Delta H^*}{D_H} \right) \right]^{0.5}, \quad (6-32)$$

where

$$D_C = 0.035\bar{C}^*/(1 + 0.0365\bar{C}^*) + 0.521,$$

$$D_H = D_C(GT' + 1 - G),$$

$$G = \{(\bar{C}^*)^4 / [(\bar{C})^4 + 14000]\}^{0.5},$$

$$T' = 0.627 + 0.055 \cos(\bar{h} - 254) - 0.040 \cos(2\bar{h} - 136^\circ) + 0.070 \cos(3\bar{h} - 32^\circ), \\ + 0.049 \cos(4\bar{h} + 114^\circ) - 0.015 \cos(5\bar{h} - 103^\circ),$$

$$R_T = R_H R_C,$$

$$R_H = -0.260 \cos(\bar{h} - 308^\circ) - 0.379 \cos(2\bar{h} - 160^\circ) - 0.636 \cos(3\bar{h} + 254^\circ), \\ + 0.226 \cos(4\bar{h} + 140^\circ) - 0.194 \cos(5\bar{h} - 280^\circ),$$

$$R_C = \{(\bar{C}^*)^6 / [(\bar{C})^6 + 70000]\}^{0.5},$$

$$L = 54.6 \log_{10}(Y + 1.5) - 9.6.$$

ΔC^* and ΔH^* are CIELAB chroma and hue differences. The overbar indicates the mean value between standard and sample. Constants l and c are comparable to the corresponding constants in the CMC equation.

While D_C is a slight modification of S_C in CMC D_H is a complex function that corrects for experimental variation in the relationship between hue angle difference and visual hue difference around the hue circle. An additional term is added to accommodate ellipses of bluish colors not directed toward the origin.

SVF Formula

Also in 1986, a different approach was taken by T. Seim and A. Valberg. They fitted a formula to the Munsell Renotation data based on cone sensitivities. In the first step tristimulus values are converted to cone sensitivities that are then centered on white (S_1 comparable to L , S_2 comparable to M and S_3 comparable to S). The lightness value V is obtained with a hyperbolic formula that represents a close fit of the polynomial Munsell value formula:

$$V_Y = 40v_1(Y), \quad (6-33)$$

where

$$v_1(Y) = \frac{(Y - 0.43)^{0.51}}{(Y - 0.43)^{0.51} + 31.75}.$$

Following Adams, the same model is applied to the two chromatic dimensions. Opponent color signals are calculated in two steps:

$$\begin{aligned} p_1 &= v_1(S_1) - v_1(Y), \\ p_2 &= v_1(Y) - v_1(S_3) \quad \text{if } S_3 \leq Y, \\ p_2 &= v_2(Y) - v_2(S_3) \quad \text{if } S_3 > Y \end{aligned} \quad (6-34)$$

where

$$v_2(Y) = \frac{[(Y/k(V_Y)) - 0.1]^{0.86}}{[(Y/k(V_Y)) - 0.1]^{0.86} + 103.2},$$

$$k(V_Y) = 0.140 + 0.175V_Y.$$

The final opponent-color functions are calculated as

$$\begin{aligned} F_1 &= 700 p_1 - 54 p_2, \\ F_2 &= 96.5 p_2. \end{aligned}$$

Color differences are calculated as

$$\Delta E_{SVF} = [(\Delta F_1)^2 + (\Delta F_2)^2 + 2.3(\Delta V_Y)^2]^{0.5}.$$

Unlike in the Müller-Judd approach the p level does not involve the subtraction of cone responses but the subtraction of Y from the response of L in one case and the response of S in the other. As a result the yellow-blue axis is established in the final form (except for scaling) at the p level. The green-red axis needs to be rotated; thus semicolon the subtractive form of F_1 . A different hyperbolic function is applied to $S > Y$ to account for the apparent change in the modulation in the Munsell system of the S function of yellowish and bluish colors, which we will encounter again later.

The formula provides a reasonably good fit to the Munsell system even though the yellowish and bluish constant hue lines are significantly curved. The formula was also applied to the OSA-UCS data where it is less successful because the implicit modulations in this system are different from those implicit in the Munsell system. Seim and Valberg applied the formula also to small suprathreshold color difference data with reportedly good results.

CIE94

In the early 1990s CIE technical committee 1–29 investigated three data sets considered reliable in regard to the relationship between ΔL^* and L^* , ΔC^* , and C^* , and ΔH^* and hue angle h (Berns, 1993). Considerable scatter was found in all three cases. As a result the committee optimized simple weights for CIELAB lightness, chroma, and hue differences and issued the formula known as CIE94:

$$\Delta E_{94}^* = \left[\left(\frac{\Delta L^*}{k_L S_L} \right)^2 + \left(\frac{\Delta C^*}{k_C S_C} \right)^2 + \left(\frac{\Delta H^*}{k_h S_h} \right)^2 \right]^{0.5}, \quad (6-35)$$

where

$$S_L = 1,$$

$$S_C = 1 + 0.045C^*,$$

$$S_H = 1 + 0.015C^*.$$

The k constants are additional weights on the lightness, chroma, and hue differences. Under reference conditions they all equal 1. For textile applications k_L typically has a value of 2. No weighting of lightness differences was made because the scatter in the combined experimental data (without consideration of surround lightness) was too large for a statistically meaningful weight.

Kim and Nobbs Weights for CIELAB

In 1997 Kim and Nobbs proposed new lightness, chroma, and hue difference weights for the CIELAB formula based on an analysis of five data sets including the Luo and Rigg, RIT-DuPont, and Leeds data:

$$\Delta E_{LCD} = \left[\frac{(\Delta L^*/S_L)^2}{K_L^2} + \frac{(\Delta C^*/S_C)^2 + (\Delta H^*/S_H)^2 + S_R \Delta C^* \Delta H^*}{K_{CH}^2} \right]^{0.5}, \quad (6-36)$$

where

$$S_L = 1 - 0.01L^* + 0.0002(L^*)^2, \text{ if } L^* < 50, \text{ then } S_L = 1.0,$$

$$S_C = (1 + 0.045C^*)S_{CH},$$

$$S_H = (1 + 0.015C^*)S_{HH},$$

$$S_R = [-C^*/(2 + 0.07C^3)] \sin(2\Delta\Theta),$$

$$\Delta\Theta = 30 \exp\{-[h - 275/25]^2\},$$

$$K_L = K_{CH} = 1.0 \text{ for nontextile samples, } K_L = 1.5 \text{ for textile samples.}$$

Complex sinusoidal S_{CH} and S_{HH} functions have been developed that improved the correlation between calculated and visual data for some data sets but not others:

$$S_{CH} = 1 + 0.07 \sin(h) - 0.16 \cos(2h + 250) - 0.05 \cos(3h) - 0.03 \cos(4h),$$

$$S_{HH} = 1 + 0.03 \cos(h + 60) + 0.12 \cos(2h) + 0.12 \cos(3h) - 0.07 \cos(4h - 45).$$

Integrating Weights

Proposals to integrate the weights on chroma and hue differences in CIE94 directly into the calculation of modified a^* and b^* values were made by E. Rohner and D. C. Rich in 1996, by H. G. Völz in 1998 and 1999, and by K. Thomsen in 2000 (see also DIN99 below). In Thomsen's version,

$$\begin{aligned} a^{*'} &= a^* f(C^*), \\ b^{*'} &= b^* f(C^*), \end{aligned} \quad (6-37)$$

where

$$f(C^*) = \frac{\ln(1 + 0.0531301C^*)}{0.0500951C^*}.$$

The two numerical factors have been optimized against CIE94 from 200,000 positions at constant L^* in the a^* , b^* diagram. The maximum discrepancy found by this method, compared to conventional calculation using CIE94, is 10.5%. Given the variability of the visual data behind the S_X weights, this is not problematical. The only direct advantage of such an approach is that euclidean relations are maintained. However, the additional optimization parameters added in CIEDE2000 (see below) would require new, more complex integration efforts to maintain euclidean relationship with questionable ultimate value, as all are based on the uncertain fundament of CIELAB. In addition CIEDE2000 is only applicable from threshold to 6 to 8 units of total difference.

Guan and Luo Large Difference Formula

In 1999 Guan and Luo fitted a formula to their large difference data mentioned above and to other large difference data sets. The formula, named GLAB by the authors, follows the CIE94 format with the following weights:

$$S_L = 0.76, \quad S_C = 1 + 0.016 C_{sb}, \quad S_H = 1.0,$$

where $C_{sb} = (C_{Std}^* C_{Spl}^*)^{0.5}$; that is, the chroma value used is the average of those of the standard and the sample being compared. The factors K_L , K_C , K_H were taken as 1. From this proposal it is evident that different formulas and/or different modification functions are required for small and large differences.

Lübbe Adjustment for Surround Lightness

In 1999 Lübbe proposed a formula for adjusting the L^* scale for the effects of surround

$$L^{*'} = L^* - f(L_u^*, L^*)(L_u^* - L^*)$$

where $L^{*'}$ is the adjusted lightness value, L^* the original value, and L_u^* the original L^* value of the surround. f is an experimentally determined factor

depending on the lightness of the test and the surround fields. It results in S-shaped functions of the relationship between L^* and $L^{*'}.$

DIN99 Formula

A new color space and difference formula was developed in 1999 in Germany (DIN 6176 2000). Its purpose is to apply the integration method proposed by Rohner and Rich (1996) to produce an euclidean space and difference formula with good performance against various sets of small color difference data. The formula went through several stages of development, and presently there are four versions in existence. Only the latest (and best performing) formula DIN99d is given here. It is based on the CIELAB formula and the X tristimulus values of reference, and test colors have been adjusted by the procedure proposed by Kuehni (1999).

$$\begin{aligned}
 X' &= 1.12X - 0.12Z, \\
 L_{99d} &= 325.22 \ln(1 + 0.0036L^*), \\
 e &= a^* \cos(50^\circ) + b^* \sin(50^\circ), \\
 f &= 1.14[-a^* \sin(50^\circ) + b^* \cos(50^\circ)], \\
 G &= (e^2 + f^2)^{0.5}, \\
 C_{99d} &= 22.5 \ln(1 + 0.06G), \\
 h_{99d} &= \arctan\left(\frac{f}{e}\right) + 50^\circ, \\
 a_{99d} &= C_{99d} \cos(h_{99d}), \\
 b_{99d} &= C_{99d} \sin(h_{99d}), \\
 \Delta E_{99d} &= \frac{1}{k_E} (\Delta L_{99d}^2 + \Delta a_{99d}^2 + \Delta b_{99d}^2)^{0.5}. \quad (6-38)
 \end{aligned}$$

The formula has been tested against the same data used in testing CIEDE2000 (see below) and performs somewhat better than CMC and CIE94 but slightly inferior to CIEDE2000. Its error against the combined test data set is 35% (compared to 38% for CIE94 and 33% for CIEDE2000) (Cui et al., 2002).

Kuehni Optimization of the CIE94 Formula

In 2001 Kuehni published a report on his analysis of the RIT-DuPont, Witt, Leeds GS and PC, as well as the Pointer-Attridge large color-difference data (Kuehni, 2001b). As a result of the analysis, and limiting changes to those applicable to all visual data sets individually, he recommended a modified CIE94 formula. The first step consists of an adjustment of the \bar{x} color-matching function as follows:

$$\bar{x}_{\text{mod}} = 1.1\bar{x} - 0.1\bar{z}. \quad (6-39)$$

The factor 1.1 is applicable to the CIE 10° observer data. It has a value of 1.06 for the 2° observer data. This adjustment results in a rotation of the b^* axis that better aligns the unit ellipses of blue colors with lines of constant hue angle.

The lightness scale is revised to make it dependent on surround lightness, resulting in a true opponent color scale. An S_C type adjustment for the lightness crispening effect is introduced:

$$L^{\wedge} = 116 \left[\left(\frac{Y}{Y_0} \right)^{0.333} - \left(\frac{Y_s}{Y_0} \right)^{0.333} \right],$$

$$\Delta L = \frac{\Delta L^{\wedge}}{S_L}, \quad (6-40)$$

where Y_s is the luminous reflectance of the surround and Y_0 that of the illuminant,

$$S_L = 1 + 0.010L^{\wedge}.$$

These changes resulted in relative improvements in prediction from 5% to 20% depending on the data set. Other changes were found to result in improved correlation with visual data for individual sets but not for all sets. Surprisingly, when used as a last step in the optimization, hue angle dependent functions to adjust the size of the hue difference component such as investigated by Kim and Nobbs (1997) or the function used in CIEDE2000 (see below), were found to have no meaningful positive effect on the correlation for any of the data sets investigated.

Kuehni found that the introduction of nonsystematic size adjustment factors for individual color groups, ranging from approximately 0.75 to 1.5, improved correlation significantly in all cases. This indicates considerable variation among and within data sets in how observer groups judge the size of color differences in specific locations in color space, compared to the prediction by the formula. The source of this variation has not yet been systematically investigated.

CIEDE2000

In 2001 Luo, Cui, and Rigg, based on extensive analysis of several sets of perceptual color difference data, proposed a modified BFD formula they initially called M2b. It is based on CIELAB with the following analytically arrived modifications:

$$\begin{aligned}
 L' &= L^*, \\
 a' &= (1+G)a^*, \\
 b' &= b^*, \\
 C'_{ab} &= [(a')^2 + (b')^2]^{0.5}, \\
 h'_{ab} &= \tan^{-1}\left(\frac{b'}{a'}\right), \\
 G &= 0.5 \left\{ 1 - \left[\frac{(\bar{C}^*_{ab})^7}{(\bar{C}^*_{ab})^7 + (25)^7} \right]^{0.5} \right\}, \tag{6-41}
 \end{aligned}$$

where \bar{C}^*_{ab} is the arithmetic mean of the C^*_{ab} values for a pair of samples.

$$\begin{aligned}
 \Delta L' &= L'_b - L'_s, \\
 \Delta C'_{ab} &= C'_{ab,b} - C'_{ab,s}, \\
 \Delta H'_{ab} &= \left[2(C'_{ab,b}C'_{ab,s})^{0.5} \sin\left(\frac{\Delta h'_{ab}}{2}\right) \right], \\
 \text{where } \Delta h'_{ab} &= h'_{ab,b} - h'_{ab,s}.
 \end{aligned}$$

The subscripts b and s refer to the comparison sample and the standard sample, respectively.

$$\Delta E = \left[\left(\frac{\Delta L'}{k_L S_L} \right)^2 + \left(\frac{\Delta C'_{ab}}{k_C S_C} \right)^2 + \left(\frac{\Delta H'_{ab}}{k_H S_H} \right)^2 + R_T \left(\frac{\Delta C'_{ab}}{k_C S_C} \right) \left(\frac{\Delta H'_{ab}}{k_H S_H} \right) \right]^{0.5}, \tag{6-42}$$

where

$$\begin{aligned}
 S_L &= 1 + \frac{0.015(\bar{L}' - 50)^2}{[20 + (\bar{L}' - 50)^2]^{0.5}}, \\
 S_C &= 1 + 0.045\bar{C}'_{ab}, \\
 S_H &= 1 + 0.015\bar{C}'_{ab}T,
 \end{aligned}$$

with $T = 1 - 0.17 \cos(\bar{h}'_{ab} - 30^\circ) + 0.24 \cos(2h'_{ab}) + 0.32 \cos(3\bar{h}'_{ab} + 6^\circ) - 0.20 \cos(4\bar{h}'_{ab} - 63^\circ)$. $R_T = -\sin(2\Delta\Theta)R_C$ and

$$\Delta\Theta = 30 \exp \left\{ - \left[\frac{(\bar{h}'_{ab} - 275^\circ)^2}{25} \right] \right\},$$

$$R_C = 2 \left(\frac{\bar{C}'_{ab}{}^7}{\bar{C}'_{ab}{}^7 + 25^7} \right)^{0.5}.$$

\bar{L}' , \bar{C}'_{ab} , and \bar{h}'_{ab} are the arithmetic means for a pair of samples of the respective individual values.

This formulation corrects for the slanted ellipses near the negative b^* axis, contracts the a^* axis near the neutral point so that ellipses located in that area can be treated as such, introduces a new lightness weighting function, and uses the hue difference weighting function T proposed by Berns at the Warsaw CIE meeting in 1999 (Berns, 2000). The new lightness difference weight is adjusted, without explicitly stating so, for a surround with $L^* = 50$. For the combined set of all data the new formula has an error of prediction of 33% compared to 38% for CIE 94 and CMC. The formula performs about equally well as CIE 94 for the RIT-DuPont data set and inferior to BFD for the Luo-Rigg (now BFD) set. It is marginally better with the Witt set and distinctly better with the Leeds and a new set of data, BIT, involving CRT display colors. The formula was adopted as CIEDE2000. Unit difference ellipses in the a^* , b^* diagram generated by this formula are illustrated in Fig. 6-14.

Sections 6.5 to 6.16 have focused on the development of industrially important color difference formulas for use in color quality control. This development has proceeded largely independent of development of models of color vision and their implications for color spaces. Such efforts in a few cases go much beyond those of industrial color difference formulas in that they attempt to encompass all aspects of color vision while color difference formulas tend to only reflect one set of test conditions: daylight illumination and a single, achromatic surround. Visual data sets involving physical samples are expensive to prepare, and visual evaluations of existing ones require different formula optimizations, for reasons that are not clear. Less expensive color difference evaluation using monitor colors is in its infancy, and the monitor conditions that produce identical results by the same observers to those obtained with comparable physical samples in simplified conditions are not yet known. In the area of formula fitting the CIE recommendation of the CIELAB formula had a strong effect in that since then most formulas, for better or worse, have been based on the foundation of CIELAB. As will be discussed in Chapter 9, progress over what is available now with CIEDE2000 depends on new systematic visual data and on a color space model more in agreement with facts than CIELAB.

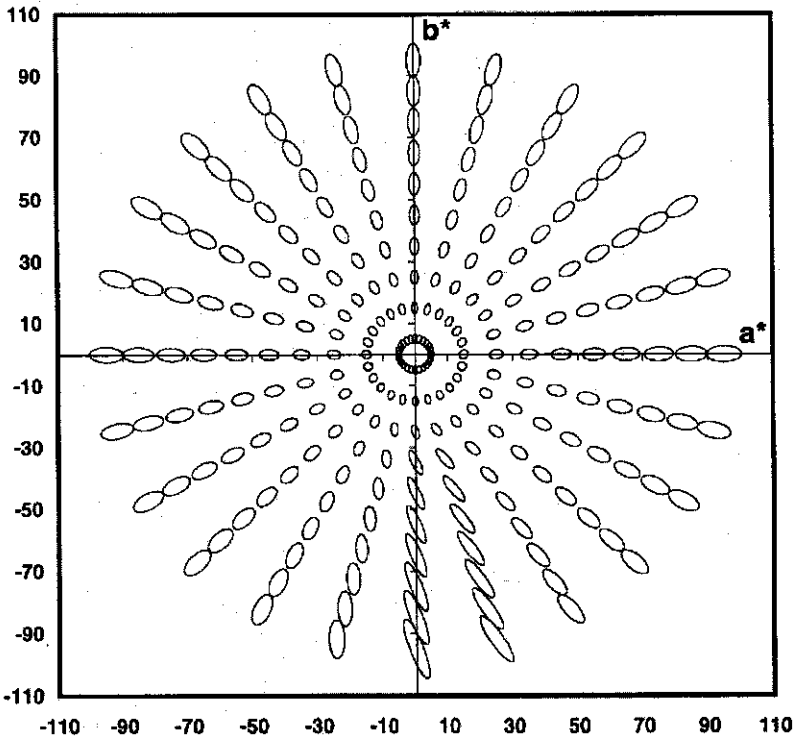


Fig. 6-14 Unit ellipses in the CIELAB a^* , b^* diagram as calculated from the CIEDE2000 formula. From Luo, Cui, and Rigg (2001).

6.17 COLOR SPACE FORMULAS AND COMPREHENSIVE MODELS OF COLOR VISION

Since the 1950s several comprehensive models of color vision have been proposed, some of which have had an impact on mathematical formulations of color space and color differences. A detailed discussion of this subject is outside the scope of this text and only a brief overview will be presented.

Müller-Judd

G. E. Müller (1850–1934) was a German psychophysicist much interested in color vision. He adopted Hering's theory of three reversible photochemical substances and attempted to bring an opponent color model in line with psychological facts. Müller developed the concept of cortical gray as representing the neutral state of color vision. In a series of articles published in 1896–97 titled *Zur Psychophysik der Gesichtsempfindungen* (On the psychophysics of the visual sense) he described a three-stage process of color vision and later expanded on the subject in a book-length treatise *Über die*

Farbenempfindungen (On color sensations, 1930). The first stage represents cone absorptions, followed by an intermediate differential second stage of coding and final neural coding in the third stage, resulting in chromatic and achromatic opponent color signals. The first two stages of the process are today loosely supported by neurophysiology, while the third stage continues to remain a hypothesis.

Beginning in 1949 and continuing until the end of his life, Judd worked on a model of the Müller theory, attempting to express it in terms of CIE tristimulus values. Three cone responses are derived from tristimulus values as follows:

$$\begin{aligned}
 P_1 &= 3.1956X + 2.4478Y - 0.6434Z, \\
 P_2 &= -2.5455X + 7.0492Y + 0.4963Z, \\
 P_3 &= 5.0000Z.
 \end{aligned}
 \tag{6-43}$$

In the second stage the cone signals are converted to intermediate opponent color signals as follows:

$$\begin{aligned}
 \alpha_1 &= P_1 - P_2, \\
 \alpha_2 &= 0.015P_1 + 0.3849P_2 - 0.4000P_3.
 \end{aligned}
 \tag{6-44}$$

In the third stage these signals are converted to the final chromatic opponent color signals as follows:

$$\begin{aligned}
 \beta_1 &= \alpha_1 - 0.6265\alpha_2, \\
 \beta_2 &= \alpha_2 + 0.1622\alpha_1.
 \end{aligned}
 \tag{6-45}$$

The achromatic final signal is equal to the CIE tristimulus value Y . The chromatic signal calculation can be reduced to expressions of tristimulus values with the following result:

$$\begin{aligned}
 \beta_1 &= 6.325(X - Y), \\
 \beta_2 &= 2.004(Y - Z), \\
 \beta_3 &= Y.
 \end{aligned}
 \tag{6-46}$$

At the third stage this model bears similarity to the Adams zone theory model and the Jameson-Hurvich model to be presented below. Judd used his implementation of the Müller theory to predict results of color vision impairment and good agreement with wavelength discrimination data of protanopes, tritanopes and normal dichromats was obtained (Judd and Yonemura, 1970). The efforts by Friele to use the Müller framework to develop color difference formulas were discussed above. The Müller framework has also been used by S. L. Guth (see section below).

Adams

As mentioned earlier, Adams offered a zone theory of color vision in 1923 and a chromatic value diagram based on it in 1942. He equated cone sensitivity with tristimulus values and applied the Munsell value function, representing the modulation of the output of the “cones,” to all three tristimulus values. The chromatic signals were calculated as follows:

$$\begin{aligned} a &= 1.9(V_X - V_Y), \\ b &= 0.72(V_Z - V_Y), \end{aligned} \quad (6-47)$$

where V_X , V_Y , and V_Z are square root Munsell value type functions of the CIE tristimulus values X , Y , and Z that Adams equated with cone responses. The constants were arrived at by comparing the ranges of chromatic value resulting from the color-matching functions and by bringing them into agreement with the Munsell value scale. The ratio is 2.64, larger than the theoretical value for a balanced linear system and larger than the value of 2.5 of the CIELAB space. As seen earlier, this model was used in modified form as a basis for the Adams-Nickerson color space and difference formula and the various formulas derived from it.

Hurvich and Jameson

In the early 1950s, as mentioned, Hurvich and Jameson developed an interest in the Hering theory by then widely disregarded by color science (except in Germany). They experimentally determined chromatic response functions by evaluating the amounts of certain chromatic stimuli required to cancel the hue of other stimuli. A typical result is shown in Fig. 6-15 and can be seen as in agreement with the Hering theory. The two spectral functions can be interpreted to represent a greenness-redness system (filled circles) and a yellowness-blueness system (open circles). Hurvich and Jameson took these functions to be linearly related to CIE color-matching functions, and their functions for white adaptation for the 1931 standard observer were shown in the previous chapter (Fig. 5-13). The general validity of such functions rests on the assumption that they are additive. Tests by Larimer and co-workers (1974-5) indicated the greenness-redness system to be additive and the yellowness-blueness system approximately so. The chromatic functions of Fig. 5-13 are approximated by the equations

$$\begin{aligned} a &= \bar{x} - \bar{y}, \\ b &= 0.4\bar{y} - 0.4\bar{z}, \end{aligned} \quad (6-48)$$

where \bar{x} , \bar{y} , \bar{z} , are the CIE 1931 color-matching functions. These functions can be expressed in relative terms as hue coefficients (Fig. 6-16), and the latter

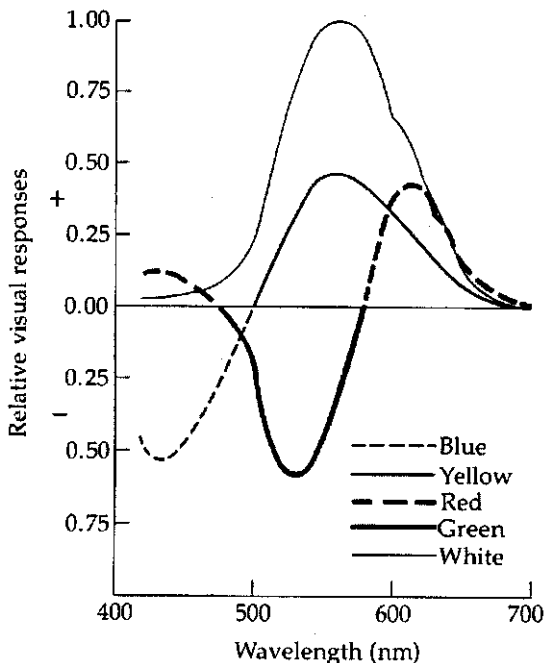


Fig. 6-15 Spectral opponent color (and lightness) functions determined from hue cancellation experiments, of a single observer. From Hurvich (1981).

were found to be in good agreement with the results of spectral hue naming experiments. Except for the weights, they are identical to the final Müller-Judd functions and in basic agreement with the Adams functions. Jameson also expressed the chromatic functions in terms of König type cone fundamentals (1972). The Hurvich-Jameson functions provide good support, derived through the paradigm of hue cancellation, for the basic ideas of Hering, Müller-Judd, and Adams.

Hurvich and Jameson proposed a polar chromatic diagram they termed a psychological diagram. It is derived by multiplying a spectral power distribution with their two chromatic functions a and b , based on their own experimental data. The two functions form the axes of a polar diagram (see Fig. 6-17) where saturation is calculated as the euclidean sum of the two chromatic responses. Conceptually unique hues fall on the axes of this diagram. However, their chromatic functions resemble the functions of equation (6-48), and average experimental unique red and green have significant positive b values. The diagram is essentially identical to a linear opponent diagram in polar coordinate form based on CIE color-matching functions. For a description of the work of Jameson and Hurvich related to the Hering system, see Hurvich (1981).

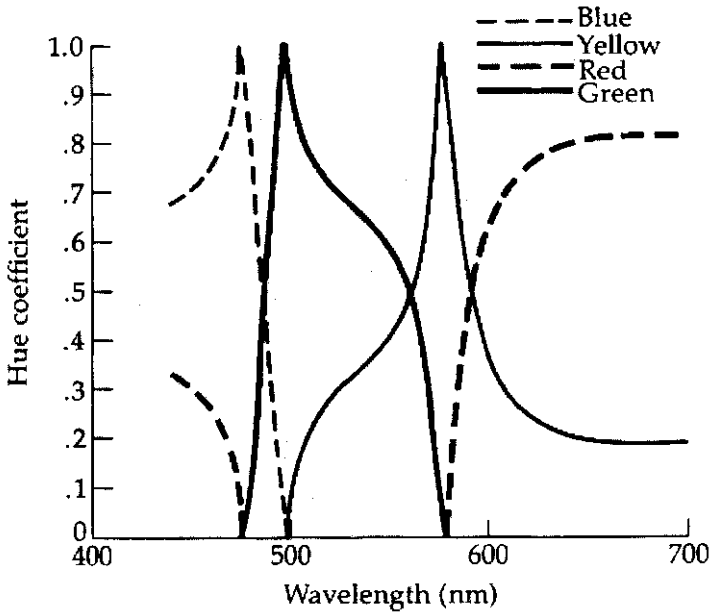


Fig. 6-16 Spectral hue coefficient functions for an average observer (Hurvich, 1981). Compare with Figs. 4-8 and 4-9.

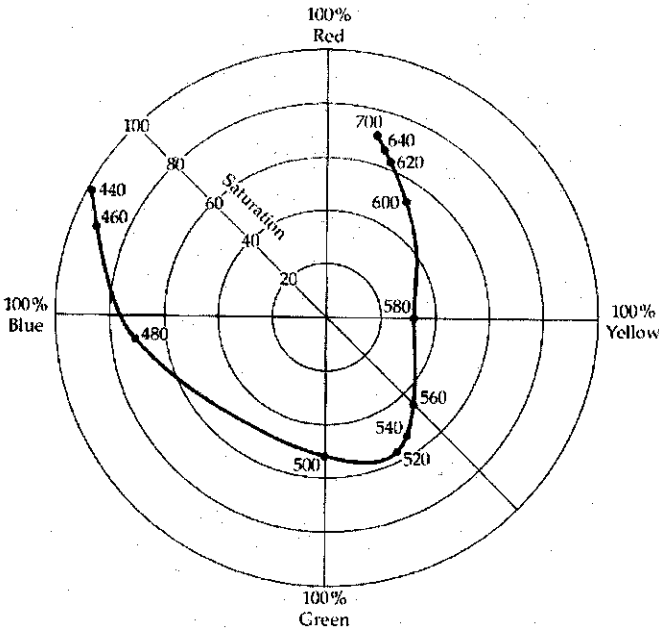


Fig. 6-17 The polar hue/saturation diagram of Hurvich and Jameson (Hurvich, 1981). Spectral colors fall on the heavy line in the interior.

Guth ATD Color Vision Model

A comprehensive color vision model has been developed by Guth and co-workers since the early 1970s (Guth, 1972, 1991). It is described as a modernized form of the Müller model. It was revised several times since the original proposal, and the most recent version is known as ATD 01 (Guth, 2001). Since different sets of visual data require different parameters and parameter functions for best fit, there is no single version applicable to all aspects of color vision. Guth does not consider ATD to be a model but “a quantitative theory of color vision.”

CIE tristimulus values of the test and the surround are converted to modulated cone responses according to

$$\begin{aligned}
 L &= (0.16X + 0.56Y - 0.034Z)^{0.70}, \\
 M &= (-0.40X + 1.16Y + 0.084Z)^{0.70}, \\
 Z &= (0.017Y + 0.27Z)^{0.70}.
 \end{aligned}
 \tag{6-49}$$

Output from the three cone types is subjected to gain control by multiplying them with their attenuation factors:

$$\begin{aligned}
 &\frac{\sigma}{\sigma + L + k(L_a - L)}, \\
 &\frac{\sigma}{\sigma + M + k(M_a - M)}, \\
 &\frac{\sigma}{\sigma + S + k(S_a - S)},
 \end{aligned}
 \tag{6-50}$$

where $\sigma = 200$ and $k = 5.5$, and subscript a denotes the surround (adapting field). The equations apply only if the cone response of the surround is larger than that of the test field; otherwise, the difference between the test and surround cone responses in the calculation of the attenuation factors is taken as zero. The resulting cone responses after gain control (subscript g) are used to calculate uncompressed responses for stage 1 and stage 2 mechanisms as follows:

$$\begin{aligned}
 A_{1i} &= 3.75L_g + 2.64M_g, & A_{2i} &= 0.09A_{1i}, \\
 T_{1i} &= 6.90L_g - 6.90M_g, & T_{2i} &= 0.60T_{1i} + 0.75D_{1i}, \\
 D_{1i} &= -0.83L_g + 1.00S_g, & D_{2i} &= -D_{1i},
 \end{aligned}
 \tag{6-51}$$

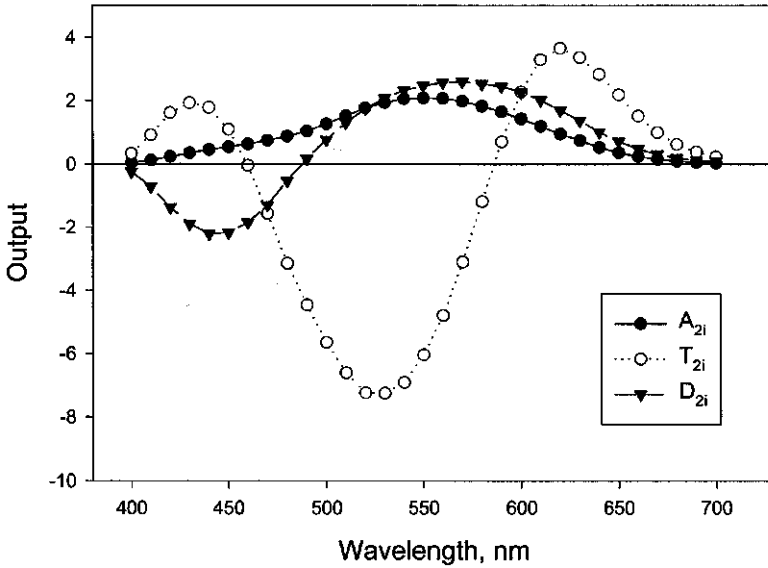


Fig. 6-18 Spectral functions of Guth's A, T, D color space.

where subscripts 1 and 2 refer to the mechanism stages and subscript *i* to the initial uncompressed response. The uncompressed stage 2 opponent color responses are illustrated in Fig. 6-18 for comparison to the Jameson-Hurvich functions and other functions below.

In the next step the final compression for the second-stage *ATD* values are calculated as follows (the compressed values for the stage 1 *ATD* values are calculated in the same manner):

$$\begin{aligned}
 A_2 &= A_{2i}(200 + |A_{2i}|), \\
 T_2 &= T_{2i}(200 + |T_{2i}|), \\
 D_2 &= D_{2i}(200 + |D_{2i}|).
 \end{aligned}
 \tag{6-52}$$

At stage 1 the brightness signal is equal to the vector sum of the compressed achromatic and two chromatic components. Lightness is calculated as the brightness compared to the brightness of the reference white (scale 100). Colorfulness, chroma, and saturation are all calculated according to

$$Co = C = Sa = (T_2^2 + D_2^2)^{0.50} A_2.$$

Hue *H* is expressed as hue angle and calculated as $H = \arctan (D_2/T_2)$.

Color differences are calculated differently, according to their magnitude. Small color differences (MacAdam ellipses, small suprathreshold differences)

are calculated as the square root of the sum of the squares of the three component vector differences at the compressed stage 1 level, while large color differences are calculated in the same manner at the compressed stage 2 level.

Many kinds of data have been predicted, with varying success, with different versions of the model. It has not found industrial use as a uniform color space formula. As a color appearance model the ATD95 version has received faint praise by Fairchild (1998).

OSA-UCS Space Formula

In connection with the development of the OSA-UCS system (see Chapters 2 and 8) the committee also developed a formula to describe the system in terms of the CIE 10° observer data and illuminant D65 (MacAdam, 1974). Tristimulus values are converted to cone sensitivity values that are much different from those of Smith and Pokorny:

$$\begin{aligned}
 R_{10} &= 0.799X_{10} + 0.4194Y_{10} - 0.1648Z_{10}, \\
 G_{10} &= -0.4493X_{10} + 1.3265Y_{10} + 0.0927X_{10}, \\
 B_{10} &= -0.1149X_{10} + 0.3394Y_{10} + 0.717Z_{10}.
 \end{aligned}
 \tag{6-54}$$

Two chromaticness and a lightness coordinate are calculated as follows:

$$\begin{aligned}
 g &= C(-13.7R^{1/3} + 17.7G^{1/3} - 4B^{1/3}), \\
 j &= C(1.7R^{1/3} + 8G^{1/3} - 9.7B^{1/3}), \\
 L &= 5.9 \left[\frac{Y_0^{1/3} - 2}{3 + 0.042(Y_0 - 30)^{1/3}} \right],
 \end{aligned}$$

where

$$C = \frac{1 + 0.042(Y_0 - 30)^{1/3}}{Y_0^{1/3} - 2/3},$$

$$Y_0 = Y(4.4934x^2 + 4.3034y^2 - 4.276xy - 1.3744x - 2.5643y + 1.8103).$$

Variable *C* adjusts chromaticness for the lightness crispening effect by a modified Semmelroth (1970) formula. Variable *Y*₀ adjusts lightness and chromaticness for the Helmholtz-Kohlrausch effect with a formula modified from that of Sanders and Wyszecki (1958). (Both factors are discussed in Chapter 5.) The committee explicitly stated that the formula is not to be used for color difference calculation of small color differences. The spectral *R*, *G*, *B* and *g*, *j* functions of the space are illustrated in Fig. 7-17*a* and *b*.

Cohen's Fundamental Color Space

Beginning in 1982 J. B. Cohen and W. E. Kappauf began to describe what Cohen later termed fundamental color space (FCS; Cohen and Kappauf, 1982; Cohen, 2001). They found that all metamers of a particular spectral power distribution (SPD) can be shown to consist of a common component (the fundamental) and a variable component (metameric black), as predicted by Wyszecki in 1953. Mathematically the fundamentals can be extracted from an SPD with the help of an orthogonal projector Cohen called matrix R . Matrix R projects spectral power distributions into the FCS, the mathematical space of all possible fundamentals. The axes of FCS can be arbitrarily selected, but Cohen proposed two preferred reference frames, the canonical frame where one of the orthonormal functions is the equal energy function and the frame where the three axes are orthogonal spectral vectors (455, 513, and 584 nm). Since the CIE luminous reflectance function is different from the equal energy and any of the orthogonal vector functions, neither configuration is comparable to the CIE X, Y, Z space.

There is no reason why FCS should be perceptually uniform, and it is not. Burns and co-workers (1990) projected the spectral power distributions of the color chips of the 1929 Munsell *Book of Color* as viewed under illuminant C into two different versions of FCS. Only when using the luminosity function as one of the three axes of FCS did the samples arrange themselves into layers that correspond to Munsell value (Fig. 6-19). When the space is viewed from the top (along the luminosity axis), they do not form concentric circles as one might expect but a somewhat elliptical cloud. Data plotting confirms the elliptical nature of the constant chroma contours. The spectral orthonormal F functions for this situation are illustrated in Fig. 6-20. The two chromatic functions are significantly different from opponent color functions such as shown in Fig. 6-15.

The Cardinal Planes Space of Derrington, Krauskopf, and Lennie

As mentioned in Chapter 5, in 1984 Derrington and co-workers reported on their findings regarding chromatic mechanisms in the lateral geniculate nuclei (LGN) of macaque monkeys. Given the close genetic relationship to humans results from macaques are considered relevant to human color vision. Measuring parvocellular activity in the LGN of their test objects the authors identified two types of cells with opponent chromatic activity: $R - G$ and $B - (R + G)$ (or the reverse). Taking cone cells of the macaques to be comparable to those of humans, they analyzed the results in terms of Smith and Pokorny's fundamental cell responses and identified three cardinal planes in a space taken to be euclidean. The constant luminance plane has a constant $R + G$ (cone response) axis and perpendicular to it a constant B axis. Perpendicular to the resulting plane is the luminance axis (Fig. 6-21). Colors are defined by their azimuth Φ and elevation Θ in the space. Derrington et al. determined the

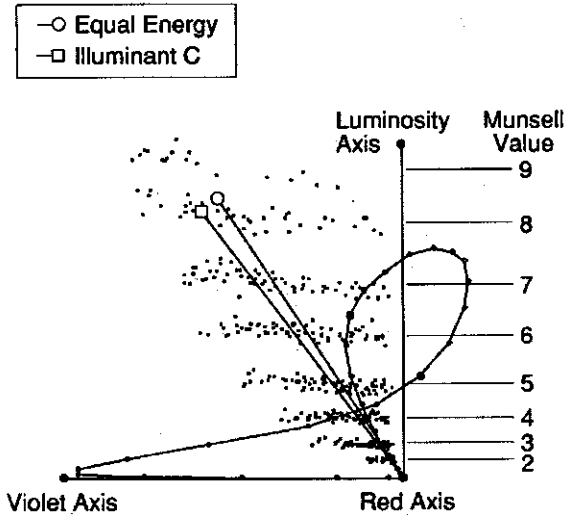


Fig. 6-19 Colors of the 1929 Munsell Book of Color as viewed under CIE illuminant C in Cohen's RLV fundamental color space (Burns et al., 1990). The spectral trace is also shown as are the vectors of the equal energy illuminant and illuminant C.

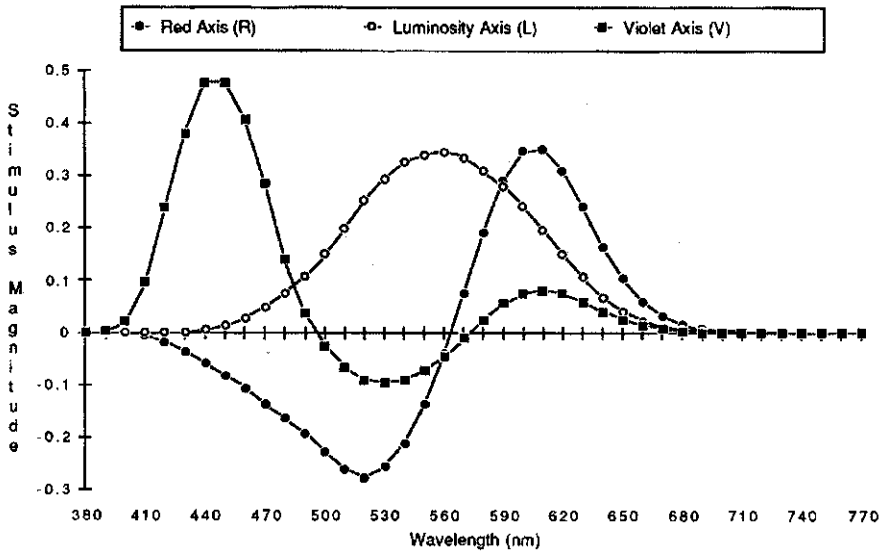


Fig. 6-20 Spectral functions representing the red, violet and luminosity axes of the RLV version of Cohen's fundamental color space (Burns et al., 1990).

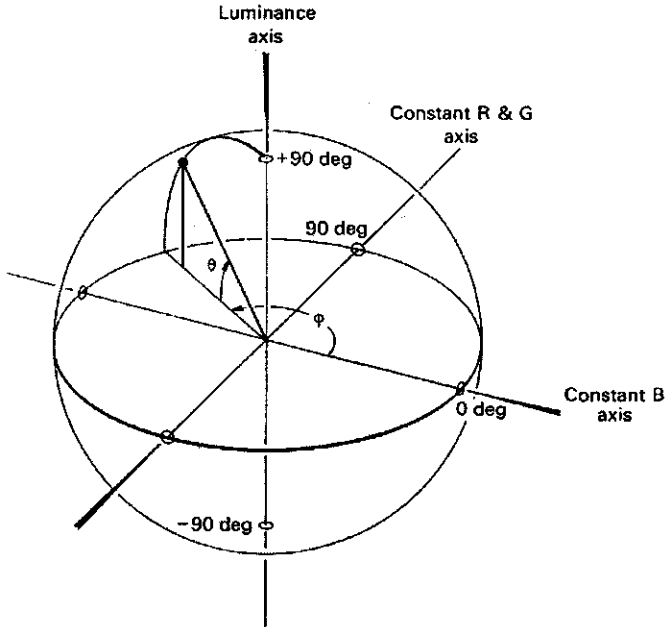


Fig. 6-21 Schematic view of the DKL color space based on cone sensitivity data. A color (black dot) is represented by its azimuth, Φ , and its elevation, Θ (Derrington et al., 1984.)

location of the chromatic axes in the CIE chromaticity diagram (Fig. 6-22). This space has direct neurophysiological support from measured cell activity in the LGN. The axes are neither in agreement with average unique hues nor with those of the CIE-based opponent color chromatic diagram.

Color vision models that imply a color space have also been developed in connection with color appearance models. An inclusion of two of these (Hunt and Nayatani) in the Mahy et al. (1994) evaluation of uniform color spaces indicates that they represent the Munsell value 5 plane and the OSA-UCS systems with significant deviations. Such systems, as mentioned earlier, are considered outside the scope of this text.

De Valois and De Valois

In 1993 R. L. and K. K. De Valois presented a paper titled *A multi-stage color model*. They had been involved in some of the earliest experiments identifying opponent color type cells in the LGN of macaques in the 1960s. They were cognizant of the discrepancy between the output of opponent cells in the LGN and psychological scaling of color space touched on above. The multi-stage model was developed to account for such discrepancies. The De Valois model consists of four stages. In the first stage, three cone types provide responses to light striking them, and the authors used the Smith-Pokorny functions to

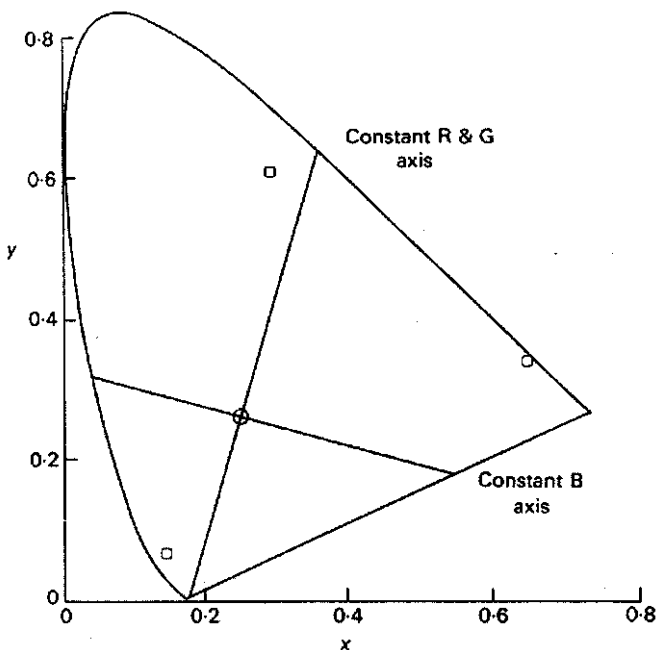


Fig. 6-22 Axes of constant R&G and constant B of the DKL space in the CIE chromaticity diagram. The squares represent the chromaticities of the phosphors used in the experiment.

describe cone sensitivity. They assumed proportions in the retina of *L*, *M*, and *S* cones in a ratio of 10:5:1. In the second stage, cone opponency signals are generated. Two kinds of surround are considered here: an indiscriminate surround based on the output of horizontal cells in the retina and the other a cone type specific surround. The response functions of three cone opponent cell types, illustrated in Fig. 6-23, are derived as follows:

$$\begin{aligned}
 L_0 &= L - (L + M + S), \\
 M_0 &= M - (L + M + S), \\
 S_0 &= S - (L + M + S).
 \end{aligned}
 \tag{6-55}$$

These cells also have mirror image copies. The cells are considered to carry both luminance and color information at different spatial frequencies.

In the third stage, in the parvocellular pathway, the authors' proposal posits combinations of signals in a way that separates luminance and color information. Accordingly, for example, $L_0 - M_0$ sums color and cancels luminance, while $L_0 + M_0$ sums luminance and cancels color. The response functions of the third stage are illustrated in Fig. 6-24. They resemble somewhat the Hurvich-Jameson opponent color functions but are not balanced. They are calculated, in the indiscriminate version, according to

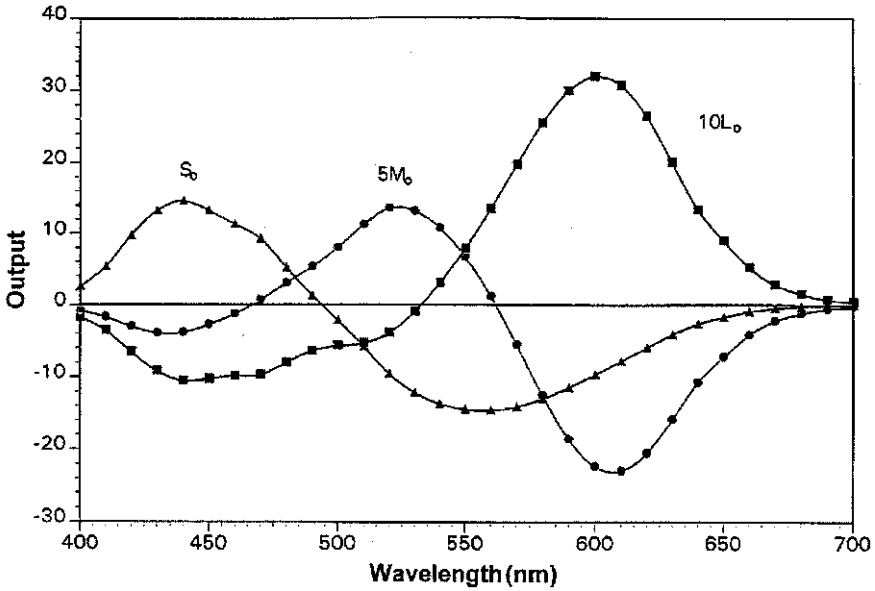


Fig. 6-23 Three cone opponency signals of the second stage of the De Valois and De Valois color model (indiscriminate surround). From De Valois and De Valois (1996).

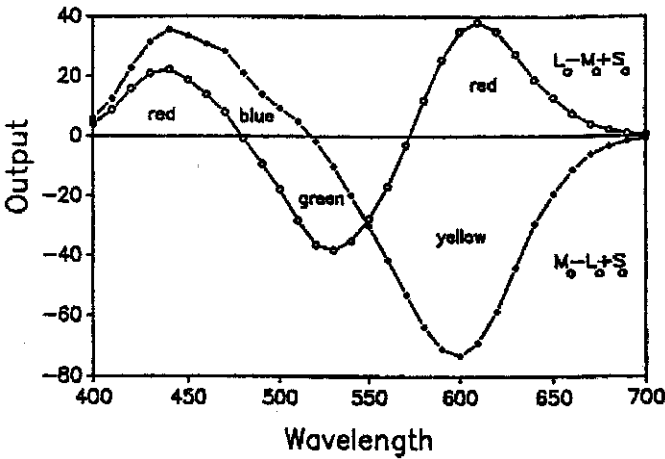


Fig. 6-24 Third-stage chromatic functions of the De Valois and De Valois color model (indiscriminate surround). From De Valois and De Valois (1993).

$$\begin{aligned}
 RG &= 18L - 23M + 5S, \\
 YB &= -26L + 19M + 7S.
 \end{aligned}
 \tag{6-56}$$

For the fourth stage, the De Valois proposal has complex color selective cells resulting from the summing of the responses of simpler cells earlier in the

pathway. The authors believe that simple opponency such as posited by Hering and Hurvich and Jameson ends at the LGN level and that already in area V1 of the brain's visual system there are cells that fire to stimulation from specific spectral regions and not to those of others. "The chromatic opponency at this stage is between, not within, individual cells." At this stage two cell types each, such as "red" and "blue" or "red" and "yellow," can fire at the same time to the same stimulus, resulting in mixed hues. In the form presented here, the De Valois model does not result in a close representation of the Munsell system.

The De Valois model is of particular interest because it considers a wealth of neurophysiological data accumulated over the last fifty years. It represents informed assumptions in regard to how the visual system operates (up until area V4) and is in this respect different from Guth's early models that optimized the Müller structure to psychophysical data, as the authors point out in a commentary in 1996.

In 1997 De Valois et al. reported on a hue-naming experiment whose results they explained in terms of a modified version of their model where in the third stage they subtracted modified amounts of second stage S_0 from L_0 and M_0 (roughly comparable to the change from the α, β diagram to the a, b diagram; see Section 5.7) They also found the red and the green systems to likely receive non-symmetrical inputs from cones or LGN cells. The De Valois proposal is a recent example of a color vision model that assumes a relatively simple relationship between proposed outputs of cells in the visual area of the cortex and color perceptions.

The Opponent Color System: Asymmetrical?

In 1999 E. J. Chichilnisky and B. A. Wandell added data to the thesis that the opponent color system might be asymmetrical. They conducted an experiment in which square color stimuli of 2.5° visual angle on a CRT were briefly flashed against various backgrounds. Observers were asked to identify if the stimuli were greenish or reddish, yellowish or bluish. One type of their presentation of the results is in cone contrast space as differences in L , M , and S as opposed to the corresponding values of the surround. Significant differences were found between the three observers in the experiment, but all showed classification boundaries that were bent, usually in a shallow bowl shape. The bowl shape did not in all cases pass through the origin; that is, "opponent classifications are not based exclusively on the difference between test stimulus and background light." An example of the results of the redness-greenness classification boundary against a greenish appearing surround is illustrated in Fig. 6-25. The shape of the bowl usually has various bulges, indicating not just global but also local nonlinearities of the boundaries between unique hues. The results were fitted with what the authors call an increment-decrement opponency model, indicating that increments and decrements in cone absorptions from those of the surround were treated differently. (Nonsymmetrical effects of increments and decrements are known from other experiments, as mentioned in Chapter 5.) A

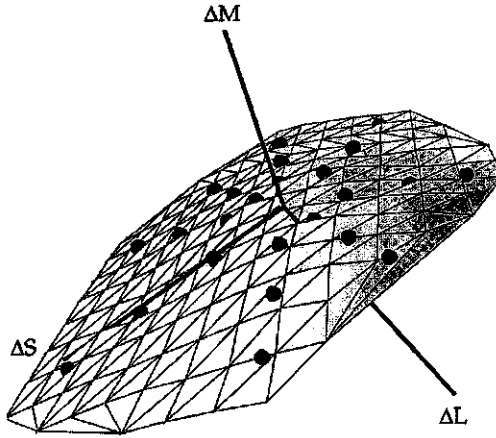


Fig. 6-25 Redness/greenness classification boundary against a greenish surround in the L, M, S cone contrast space. From Chichilnisky and Wandell (1999).

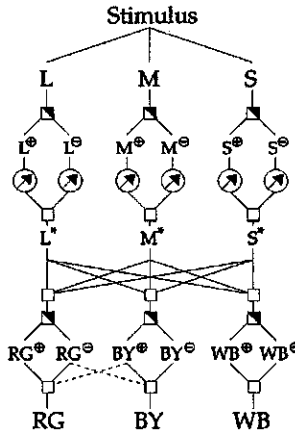


Fig. 6-26 Schematic view of the color vision model of Chichilnisky and Wandell to account for their hue classification results. Output from cones L, M, S is split into increment/decrement portions that are separately modulated to form the intermediate cone signals L^*, M^*, S^* . These are linearly combined to form incremental and decremental pre-opponent signals. They, in turn, are again linearly combined to form the opponent signals $RG, BY,$ and WB . From Chichilnisky and Wandell (1999).

schematic view of the model is found in Fig. 6-26. Linear cone signals L, M, S are treated separately as increments or decrements with respect to a neutral point. These are scaled through gain control depending only on the surround. The resulting intermediate cone signals $L^*, M^*,$ and S^* are combined linearly and then separated into pairs of increment and decrement pre-opponent

signals. These are combined linearly to create the final opponent color signals, whose signs determine the opponent color invoked. Concerning the neural location of opponent color null surfaces the authors comment: “The non-planarity of opponent color classification boundaries raise the possibility that either individual parvocellular LGN neurons are not the neural substrate of perceptual opponency, or that significant nonlinearities in neural responses were overlooked.” Increments and decrements have also been found to be treated differently in spatial patterns (Bäumel, 2002).

The relevance of these findings for a model of a global object color space viewed under conditions much different from those of the described experiment remains to be determined. There is the likelihood that all models using simple subtractions of cone absorptions or tristimulus values are much simplified approximations of the real mechanisms of color vision. The issue of the strategy pursued by evolution in the development of the primate visual system assumes key importance as well as if perception ultimately can be modeled from implicit cone responses.

6.18 IS THE OPPONENT COLOR SYSTEM “SOFT-WIRED”?

It is an accepted supposition of color psychology that reddish-greenish colors and yellowish-bluish colors are not possible. However, this has been questioned since 1983 when H. D. Crane and T. P. Piantanida published a paper “On seeing reddish green and yellowish blue.” More recently Billock and co-workers (2001) reported on experiments where observers viewed fields of opposing colors at personal equiluminance for each observer under image stabilization (the image impacted the same retinal region at all times). Four out of seven observers experienced under these conditions homogeneous mixtures of red and green or yellow and blue (some only after several trials). The authors proposed a “winner take all” multi-stage model of vision in which opponency is not hardwired into certain types of cells but is the result of the combined output of many cells. As a result one or the other of the unique hues (the winner) takes over. This soft assignment of opponent hues can be defeated in certain observation conditions resulting in mixed opponent hue perceptions.

Models of human color vision must be able to explain vast amounts of different experimental data. All models are empirical because the translation of chemical signals in area V4 to perceptual experiences is unknown. There is a likelihood that the strategies generally pursued so far will not succeed in explaining all visual data because the visual system may pursue empirical strategies that do not directly follow from the identified mechanisms in the retina, the LGN and the early visual areas but are the statistical results of past experiences, possibly elaborated beyond the classical visual areas.

6.19 SPECTRAL SPACES

As briefly mentioned in Chapter 2, regular spaces of reflectance or spectral power data can also be formed by content analysis and dimensionality reduction different from that implied in the color matching functions. When comparing the reflectance functions of Munsell chips of constant chroma and value, regularly spaced around the hue circuit, they are found to change in a regular fashion. In Fig. 6-27*a* through *d* the reflectance functions of twenty (every second) Munsell chips at constant value and chroma representing a hue circle are shown. Given the possibility of many metamers for each chip, this fairly regular change is in part due to the limited number of pigments with their own specific spectral function shape and in part is a reflection of the (relativized) hue change between chips. Lenz and co-workers (1996) have calculated principal components of the reflectance functions of 1269 Munsell chips (including those of Fig. 6-27) by forming a correlation matrix and determining its eigenvectors. The first three eigenvectors are illustrated in Fig. 6-28. It is evident that the first vector, accounting for over 80% of the correlation between the functions in the total set, has all positive values and represents in a general way the average height of the reflectance function, roughly indicative of lightness. The second and third eigenvectors have both positive and negative values and have a crude resemblance to opponent color curves (compare to Fig. 5-13). The functions are orthogonal to each other. When the twenty reflectances of Fig. 6-27 are plotted, they do not form a circle on a flat plane in the resulting space but an irregular three-dimensional contour as shown in Fig. 6-29*a* and *b*. The drop lines in Fig. 6-29*a* provide an indication of the variation in terms of the first eigenvector. It is evident that this space orders Munsell colors in an ordinal manner only in that it places the chips in a sequence that is in agreement with the perceptual sequence. Colors varying in chroma or value (not shown) are also placed relative to each other in proper ordinal order. However, the space is far from uniform in terms of the Munsell system. Ordinal order can be also obtained with many arbitrary continuous functions and does not in itself indicate a useful color space. Other dimensionality reduction methods do not place such color series always in ordinal order (Ramanat et al., 2003). Eigenvector spaces depend on the color stimuli and the sample distribution. Eigenvectors determined from different collections of color samples will be different. In addition metameric color samples do not plot in the same location in eigenvector spaces. For these reasons it is not appropriate to call such spaces color spaces.

6.20 PERFORMANCE COMPARISON OF VARIOUS FORMULAS

Over the last hundred years color discrimination data have been established at four different levels of difference:

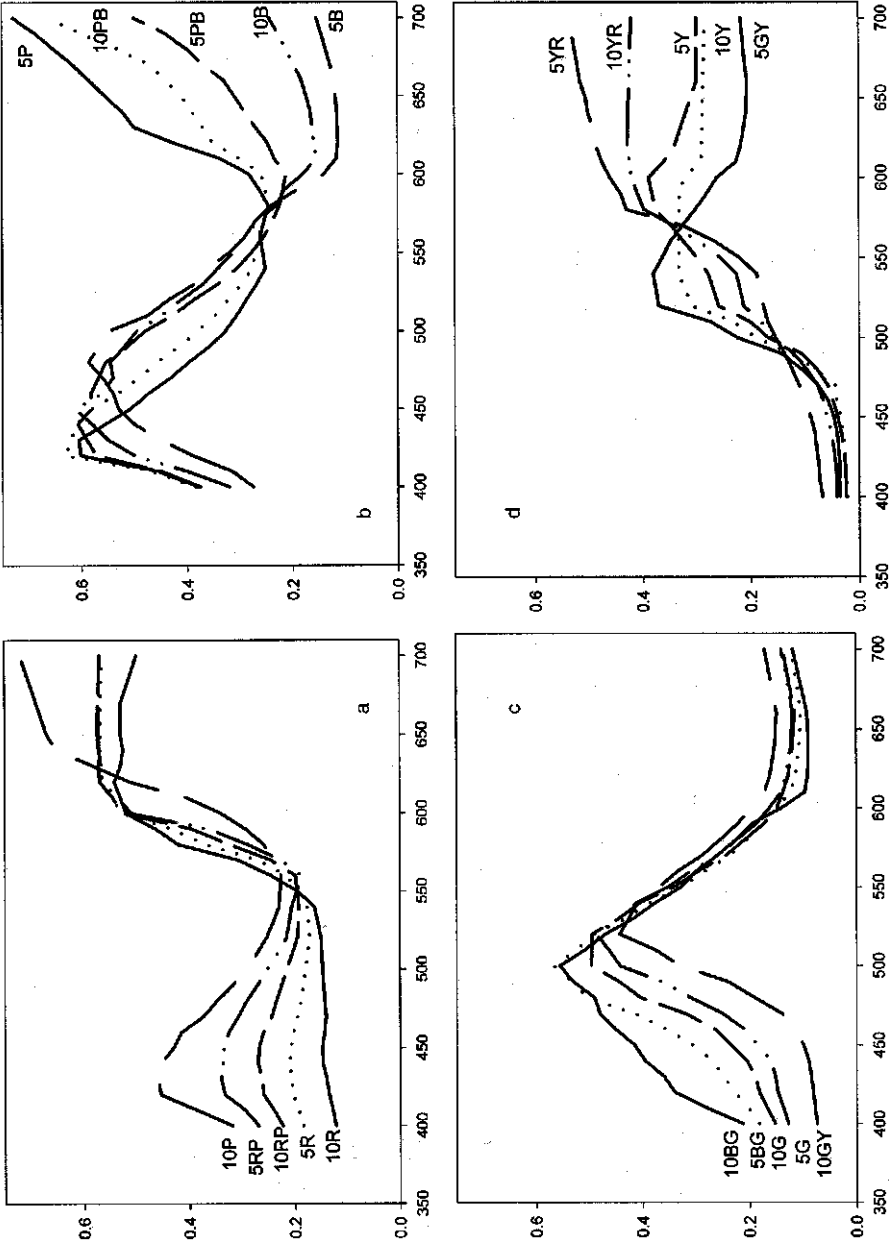


Fig. 6-27a-d Sequential plot of spectral reflectance functions of every second Munsell chip around the hue circle at value 6 and chroma 8, indicating the comparatively regular changes in reflectance. Note the common crossovers between 550 and 600 nm.

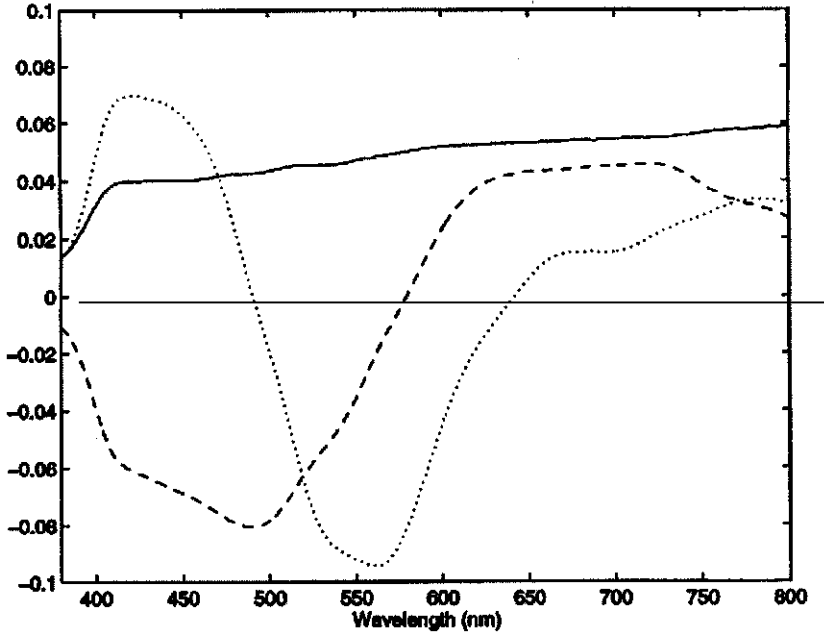
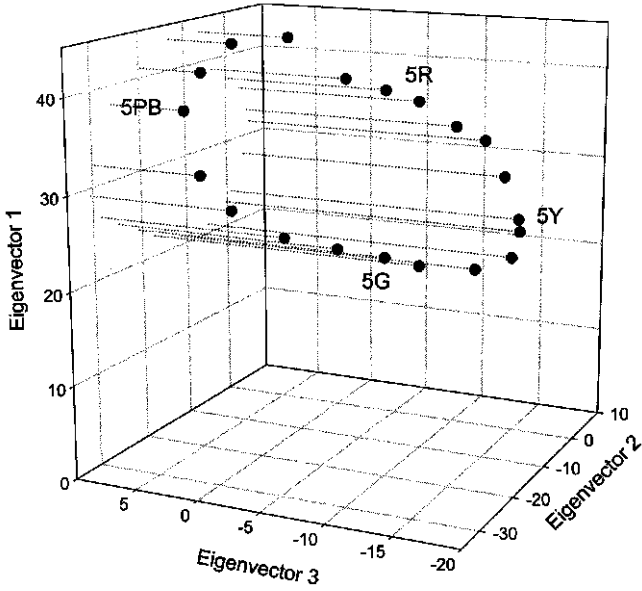


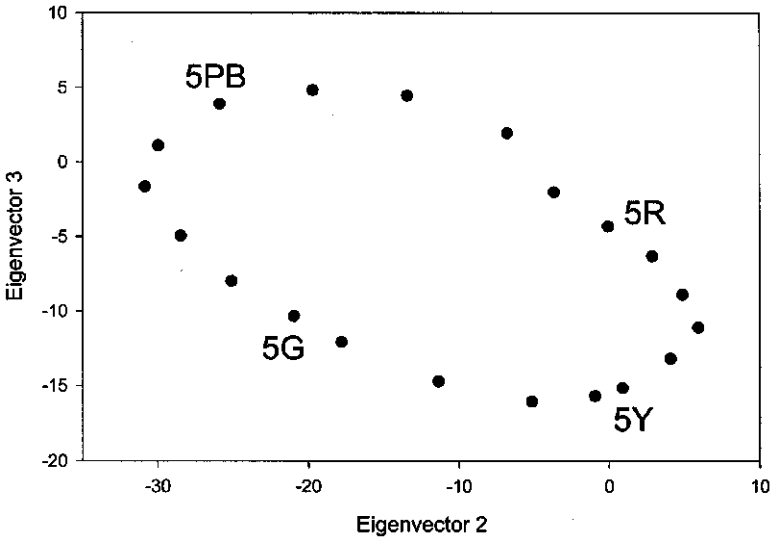
Fig. 6-28 First three eigenvectors representative of the first three principal components, fitted to a database of reflectance functions of 1269 Munsell chips. Solid line: eigenvector 1; dashed line: eigenvector 2; dotted line: eigenvector 3. From Lenz et al. (1996).

1. Subthreshold. These are color-matching error data, as determined by MacAdam and others.
2. Threshold or just noticeable difference data. Examples are luminance difference thresholds, wavelength discrimination data, purity threshold data, and the Wright, Richter, and Witt threshold data.
3. Suprathreshold small differences. These include the RIT-DuPont, Witt data, and others.
4. Large color difference data. Examples are the Munsell and OSA-UCS data, and the Guan and Luo data.

Ideally all these data sets would be describable with high accuracy using a single model/formula. It is obvious from past discussions that this is impossible. When comparing various models of color spacing with typical data sets, significant variation in results is obtained. In a few cases formulas have been optimized against a particular data set, and not surprisingly, these formulas tend to perform better than others against these data. Typical examples are the FMC formulas optimized against the MacAdam data, the OSA-UCS formula optimized against the corresponding large difference data, and the BFD formula optimized mainly against the Luo and Rigg modified data set.



(a)



(b)

Fig. 6-29a and b Plots of the twenty Munsell colors of Fig. 6-27 in the space formed by the three eigenvectors of Fig. 6-28 and in the diagram formed by the second and third eigenvectors. Hue designations of four colors are shown for identification. It is apparent that the space orders the Munsell colors regularly in an ordinal sense, but it is far from uniform.

TABLE 6-1 Prediction accuracy of three color difference formulas for various sets of color difference data

Data set	Coefficient of Variation, %		
	CIELAB	CMC	BFD
MacAdam, 24 ellipses	34.5	32.2	27.7
Wyszecki-Fielder/GW, 28 ellipses	26.9	20.2	18.8
Witt threshold, 5 ellipses	39.3	39.1	42.6
Luo-Rigg, 131 ellipses	25.1	16.2	13.6
Cheung-Rigg, 5 ellipses	31.6	39.1	42.6
Munsell value, 232 differences	7.3	29.0	21.5
Munsell hue, 365 differences	64.6	50.7	38.4
Munsell chroma, 356 differences	20.5	35.5	41.8

Source: Adapted from Melgosa et al. (1992).

In 1992 Melgosa and co-workers compared a number of data sets using the CIELAB, CIELUV, CMC, and BFD formulas. From their extensive data only a few examples are shown in Table 4-1. Results, in form of coefficients of variation for the difference between visual and calculated differences, are bewildering. Some conclusions can be drawn from the table:

1. There are systematic differences between the MacAdam data, on the one hand, and the Wyszecki-Fielder/observer GW data, on the other.
2. Witt threshold data are poorly predicted by any of the three formulas.
3. CMC and BFD are significant improvements over CIELAB for the Luo-Rigg data but not for the Cheung-Rigg data.
4. CIELAB is a good predictor of Munsell value but a poor predictor of Munsell hue differences, while CMC and BFD are poorer in predicting value differences but better (if still poor) in predicting hue differences. CIELAB is a better predictor of Munsell chroma differences than the other two formulas (in agreement with findings in Chapter 8 of absence of chromatic crispening in color differences of the size of Munsell system differences).

Another view of the progress of color difference formulas in predicting average visually judged small color differences and of color space formulas in modeling color solid data is offered in Table 6-2 and in Figs. 6-30 and 6-31. Typical data sets have been used for this purpose. The small color difference data set selected is the RIT-DuPont set with 156 difference pairs in 18 color centers. When the data were established, considerable care was taken to control significant aspects of the experiment. The nature of the experiment resulted in all color differences in this data set having a visual difference of one unit and therefore should have calculated differences of one unit. An appropriate statistic of the performance of color difference formulas for this data set is, again, the coefficient of variation. In Table 6-2 coefficients of

TABLE 6-2 Comparison of performance of some color-difference formulas against the RIT-DuPont data set

Formula	Coefficient of Variation, %
CIELAB	35
CIELUV	39
SVF	38
BFD	28
CIE94	21
CIEDE2000	20
Kuehni optimization of CIE94	18

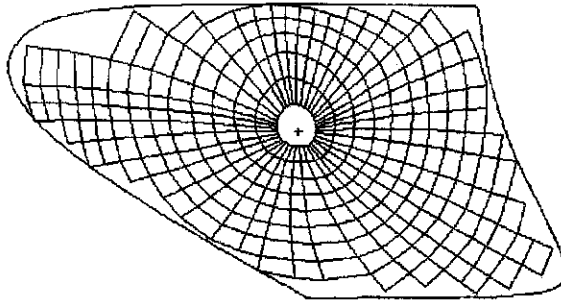


Fig. 6-30 Munsell Renotation colors at value 5 plotted in the chromatic diagram of Richter's LABHNU formula. From Mahy et al. (1994).

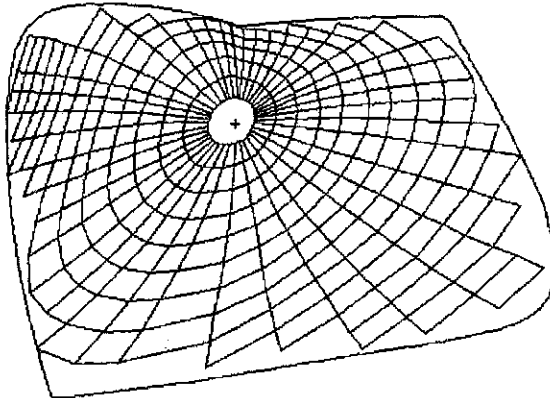


Fig. 6-31 Munsell Renotation colors at value 5 plotted in the chromatic diagram of Hunt's 1991 color appearance model. From Mahy et al. (1994).

variation for color differences calculated by various formulas for the set are listed.

The variation in calculated differences for the data set selected is now approximately half of that for the 1976 CIE formulas. The largest improvement was achieved, as indicated earlier, by applying variable weights to the implicit lightness, chroma, and hue differences to deal with chromatic crispening and the non-euclidean nature of uniform color space (in the absence of euclidization procedure).

The Munsell system was selected for the purpose of comparison of the performance of color space formulas. Constant value, respectively metric lightness, planes are illustrated. To be in perfect agreement with the perceptual results, a color space formula should represent an equal value plane of the Munsell system as a series of concentric circles with equal radial spacing and radial lines that are equidistant from each other in terms of hue angle. In the evaluation of Mahy et al. (1994) the formulas (then known) most accurate in modeling the Munsell system were the LABHNU formula of Richter (a somewhat different version from the one documented above, but found in the same reference), SVF, and the CIELAB formula (for OSA-UCS it is the OSA formula and SVF). A significant further improvement will be shown in Chapter 7 (compare Fig. 7-6). Formulas such as CIE94 cannot be used as color space formulas because of the variable weights on the difference components, unless the weights are integrated such as in the Rohner and Rich, Völz, Thomsen, or DIN99 proposals (see above). It is evident, however, that they would perform poorer than the formulas above against the Munsell and OSA-UCS systems because most of the integrated weight of S_C relates to chromatic crispening, absent in the Munsell system and OSA-UCS.

We have seen a rich tapestry of efforts to develop psychophysical models of color space and color differences applicable to differences of various size. But as discussed in Chapter 4, we do not have extensive reliable, replicated data of which we can be confident that they accurately describe color spacing for a given situation of illumination, surround, size of differences, and the truly average observer. Various data sets developed at different times vary considerably for generally unknown reasons and are described optimally by different formulas. It is important to develop data sets that can be reproduced in different locations by observer groups of comparable and known relation to a truly average standard observer (that may still need to be developed). If this proves to be impossible, it may be due to irreproducible cognitive input into the judgments. Truly reliable data sets and further knowledge of the human color vision system may make it possible to develop a psychophysical basis model, perhaps non-euclidean and certainly nonlinear, that can, with various parameters, accurately describe the tiling of color space uniform under closely defined observation conditions. There is a considerable way to get there and it is evident that CIEDE2000 is a mere milestone on that path.