– 5 –

Magnetics

Before describing the magnetic properties of a chemical substance we will briefly discuss some of the properties of a magnetic field itself. Whereas the sources of an electrostatic field are motionless electric charges, the sources of a magnetic field are moving charges, i.e., an electric current. Let us consider some characteristics of a permanent electric current and the conditions of its maintenance.

5.1 GENERAL CHARACTERISTICS OF THE MAGNETIC FIELD

5.1.1 A permanent (direct) electric current

An electric current can run in a substance in which free charges (*current carriers*) are present. They can be either electrons (in metals, for example) or ions (in liquids or solid electrolytes). Such substances are referred to as the *conductors* of an electric current. However, if a conductor is brought into an external electrostatic field there will be only an instant displacement of charges according to electrostatic laws; this will lead to the creation of an internal electrostatic field in a conductor directed opposite to the external electrostatic field and numerically equal to it; the current instantly stops. Therefore, inside a conductor the electric field is always zero. This means that additional conditions to support the current flow are necessary. We shall consider these a little later, but first we shall introduce some ideas about the electric current.

The ordered motion of electric charges is referred to as an electric current (a current of conductivity). The scalar value I determined by a total charge dQ having run through a cross-section of the conductor in a unit time interval dt, i.e.,

$$I = \frac{dQ}{dt}, \tag{5.1.1}$$

is referred to as a current strength or simply current. The motion of positive charges is accepted as the current direction, the current flow generally being opposite to the electron movement. If the current magnitude does not change with time it is referred to as a *permanent (constant, direct) current.* If the current strength changes with time it is referred to as an *alternating current.*

The current I is a macroscopic characteristic of a particular conductor. For the distribution of an electric current throughout the conductor section a vector *of current density* **j** is introduced. The vector of current density **j** is directed along the carrier's motion and is numerically equal to the electric charge current, which in the unit time crosses a unit conductor area perpendicular to the carrier velocity (see Figures 2.19 and 2.20). Thus, the modulus of a **j**-vector is equal to the ratio of the current strength dI through an elementary area dS located in a given conductor point perpendicular to the direction of the ordered carrier motion: $j = dI/dS_{\perp}$. To attach to the current density a vector character, we can multiply j to the unit vector of a direction of the carrier motion $\boldsymbol{\upsilon}/\upsilon$; therefore,

$$\mathbf{j} = \frac{dI}{dS_{\perp}} \frac{\boldsymbol{\upsilon}}{\upsilon}. \tag{5.1.2}$$

The integral current strength is therefore

$$I = \int_{S} \mathbf{j} \cdot d\mathbf{S}, \tag{5.1.3}$$

where S is the conductor cross-section.

Let us establish correlation between the microscopic current characteristics, the density of current **j**, the concentration of current carriers n and the average velocity of carrier motion $\langle \boldsymbol{\upsilon} \rangle$ (current speed of drifts). Let charge ΔQ be transferred in time Δt through the cross-section S, perpendicular to the ordered carrier movement (Figure 5.1). By definition, the density of a current **j** is numerically equal to the ratio I/S. The charge ΔQ is equal to the product of a single carrier charge and their full number in the volume $\Delta V = S_{\perp}\, l$. Therefore $j = q\, n \Delta l \Delta S_{\perp} / \Delta t \Delta S_{\perp} = q\, n \langle \boldsymbol{\upsilon} \rangle$. Because current density vector **j** and the velocity of positive curriers $\langle \boldsymbol{\upsilon} \rangle$ are codirectional, hence

$$\mathbf{j} = qn\langle \boldsymbol{\upsilon} \rangle. \tag{5.1.4}$$

Taking into account the possible movement of both positive and negative charges (for instance in an electrolyte) the total current density is

$$j = q_{+} n_{+} \langle \upsilon_{+} \rangle + q_{-} n_{-} \langle \upsilon_{-} \rangle.$$

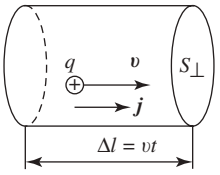


Figure 5.1 Relationship of a current density j, carrier concentration n and drift velocity υ.

Now derive Ohm's law in differential form. At a fragment of a uniform conductor in an electric circuit, the current I is proportional to voltage loss U and inversely proportional to its resistance R: $I = U/R$. Consider an elementary cylindrical volume in the vicinity of an arbitrary point inside the conductor (Figure 5.2) with the cylinder generatrix parallel to current density vector $\mathbf{j}$. A current jdS flows through the cross-section of the cylinder. The voltage loss U on the cylinder ends is equal to Edl, where E is the strength of the electric field in the vicinity of the given point. This means that

$$dI = jdS = \frac{Edl}{R} = \frac{Edl}{\rho dl} dS,$$

where ρ is the specific resistance and $R = \rho(dl/dS)$. Since the charge carriers at every point move parallel to vector **E,** the Ohm's law in differential form acquires the form $\mathbf{j} = \mathbf{E}/\rho = \sigma\mathbf{E}$, where $\sigma = 1/\rho$ is the specific electroconductivity. Therefore, the current density at any current point $\mathbf{j}(\mathbf{r})$ is equal to the product of specific electroconductivity and the electric field strength $\mathbf{E}(\mathbf{r})$:

$$\mathbf{j}(\mathbf{r}) = \sigma\mathbf{E}(\mathbf{r}). \tag{5.1.5}$$

This is Ohm's law in differential form.

Another characteristic of a current is the mobility of the current carrier b, which is the average speed acquired by the current carrier in a field of unit electric field strength. If charges have average speed $\langle \upsilon \rangle$ in a field E then, by definition, their mobility is $b = \langle \upsilon \rangle / E$. The mobility b can be expressed through the specific electroconductivity ρ and carrier concentration n. As the current density is $j = nq\langle \upsilon \rangle$ and $j = \sigma E$, therefore, $\sigma E = nq\langle \upsilon \rangle$. Having divided both parts of equality by E we shall obtain: $\sigma = nqb$ or

$$b = \frac{\sigma}{nq}. \tag{5.1.6}$$

Let us now consider the conditions that can maintain the electric current in a closed electric circuit. In fact, the current in a circuit can exist only if external forces maintain at the conductor ends a constant voltage difference (to say nothing about superconductivity). Therefore, in the closed circuit, along with the parts in which positive charges move along the decreasing potential φ, there should be parts where the positive charges move against

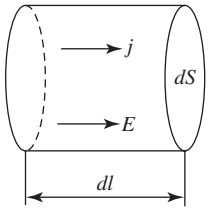


Figure 5.2 Derivation of the Ohm law in differential form.

the potential loss, i.e., against the forces of an electrostatic field. Figure 5.3 shows a closed circuit with a part where the electrostatic field is acting (1-*a*-2) and a part (2-*b*-1), where the so-called *extraneous (outside) forces* operate. The extraneous forces are of a different, nonelectrostatic nature; they can be of chemical (galvanic cells), induction (electrogenerators), or of thermal origin, etc. These extraneous forces are capable of maintaining the ordered movement of carriers against Coulomb forces. There exists *a field of extraneous forces*, characterized by the extraneous force $\mathbf{F}_{\text{ext}}$ having field strength $\mathbf{E}_{\text{ext}}$. By analogy with eq. (4.1.2) we can write

$$\mathbf{E}_{\text{ext}} = \frac{\mathbf{F}_{\text{ext}}}{Q}. \tag{5.1.7}$$

The extraneous forces produce the work on charges moving along a circuit. The physical value equal to the work of the extraneous forces produced in moving a positive unit charge Q from point 1 to point 2 is referred to as *electromotive force* (EMF) ε. Thus,

$$\varepsilon_{12} = \frac{A_{\text{ext}}}{Q} = \int_1^2 \mathbf{E}_{\text{ext}} d\boldsymbol{l}.$$

Thus, for the closed circuit

$$A = Q \oint \mathrm{E}_{\ell,\text{ext}} d\boldsymbol{l}.$$

Therefore, the work of the EMF along the closed circuit is $\varepsilon = Q \oint \mathbf{E}_l d\boldsymbol{l}$.

Both extraneous and Coulomb forces act on the charge moving along the closed circuit. The work produced is $\boldsymbol{A} = Q \oint \mathbf{E}\, d\boldsymbol{l}$, where $\mathbf{E}$ symbolizes the sum of the extraneous $\mathbf{E}_{\text{ext}}$ and the Coulomb $\mathbf{E}_{\text{col}}$ field strengths, i.e., $A = Q \oint (\mathbf{E}_{\text{ext}} + \mathbf{E}_{\text{cou}}) d\boldsymbol{l}$. Since $\oint \mathbf{E}_{\text{cou}} d\boldsymbol{l} = 0$, hence $A = Q \oint \mathbf{E}_{\text{ext}} d\boldsymbol{l} = A_{\text{ext}}$. Therefore $\varepsilon = A/Q$, i.e., the work on the unit positive charge along the closed circuit, i.e., the EMF force ε is

$$\varepsilon = \oint \mathbf{E} \cdot d\boldsymbol{l}, \tag{5.1.8}$$

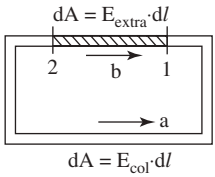


Figure 5.3 A closed circuit with extraneous forces.

where **E** is the combined strength of both fields. If we consider the charge motion in a limit from point 1 to point 2, then $A_{12}=Q\int_1^2 \mathbf{E}_{ext}\cdot d\boldsymbol{l}+Q\int_1^2 \mathbf{E}_{cou}\cdot d\boldsymbol{l}$, and consequently

$$A_{12} = Q\varepsilon_{12} + Q(\varphi_1 - \varphi_2). \tag{5.1.9}$$

The value U_{12} is numerically equal to the work of both fields, electrostatic and extraneous, at the displacement of a unit positive charge from point 1 to point 2, referred to as voltage loss (voltage) at a given section of circuit

$$U_{12} = IR_{12} = \varepsilon_{12} + (\varphi_1 - \varphi_2). \tag{5.1.10}$$

If the circuit is open ended ($I = 0$, $U_{12} = 0$) then $\varepsilon_{12} = (\varphi_2 - \varphi_1)$, i.e., the EMF is equal to the potential difference on the clumps of a current source.

5.1.2 A magnetic field induction

The electric current (moving charges) creates a magnetic field in the surrounding space. This field affects the charges (currents) moving in it. Thus, the interaction of two currents has an electromagnetic character. Ampere's experiments showed that two parallel infinite currents I_1 and I_2 running in one direction attract each other, whereas currents directed oppositely repelled (Figure 5.4). The interaction force falling at a unit conductor length, *f,* is inversely proportional to distance *b* between them, i.e., $f\sim(I_1I_2/b)$. The given statement is the essence of Ampere's law. In SI this law takes the form

$$f = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{b}, \tag{5.1.11}$$

where μ_0 is the permeability constant (refer to Appendix 1).

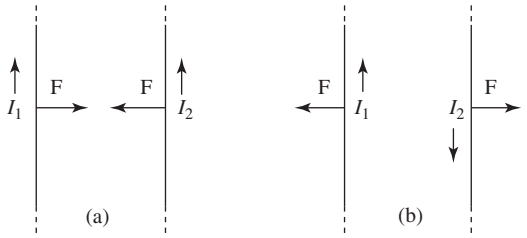


Figure 5.4 Ampère's law: interactions of currents, (a*)* currents are codirectional and (b) antidirectional.

It can be imagined that one conductor creates the magnetic field and the other in this field acquires its action.

Consider first how an electric current creates a magnetic field and then how the magnetic field acts on the conductor with a current. When we analyzed an electrostatic field it was convenient to use a unit point positive charge (probe charge), which we imagined to be placed in each point of the field and measured the force acting on it; this was the electric field strength **E**. It is impossible to apply such a procedure in the case of a magnetic field because there are no magnetic charges (monopoles) in nature. It is necessary to use a multipole of the next order, i.e., a dipole, placing it at different points in the magnetic field and measuring the torque acting on it (refer to Appendix 3).

In order to carry out such a magnetic field analysis, we need to use a model. Such a model is a magnetic dipole moment, which is a small (ideally, a point) flat contour with a current (a probe contour). Orientation of the probe contour in space is determined by a vector **n** normal to the contour (Figure 5.5). Placing the probe contour into every point one can see that in a given point it accepts a definite position: the normal vector **n** is oriented in a strictly determined direction. Experience also shows that the action of the magnetic field on the contour is associated with the value IS (where I is the current flowing around an area S). This contour $\mathcal{M} = IS$ is called *the magnetic moment* of a contour. The magnetic moment of a contour is the vector value directionally coinciding with the normal **n** to the contour and determined by the contour current by a right-hand system rule, so

$$\mathcal{M}=\mathcal{M}\mathrm{n} = IS\mathrm{n}. \tag{5.1.12}$$

Accept the direction of the probe contour normal **n** freely oriented in a given point of the magnetic field as a direction of the force line. The force characteristic of the magnetic field is a vector **B** in the direction of the normal vector **n**; its absolute value is determined by the maximum torque acting on a contour in this point. Then,

$$\mathbf{B} = \frac{M_{\max}}{\mathcal{M}}. \tag{5.1.13}$$

Vector $\mathbf{B} = B\mathbf{n}$ is referred to as a magnetic field induction vector.

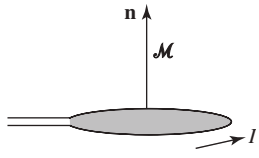


Figure 5.5 An elementary magnetic moment.

The magnetic field is schematically presented in full analogy to the electric field: the **B** vectors at any point are parallel to the tangent to force lines and their density is proportional to the $|\mathbf{B}|$ value.

For the description of the magnetic field we require one more characteristic—field strength **H**. In vacuum, the two characteristics differ only by a constant, namely, $\mathbf{B} = \mu_0\mathbf{H}$; however inside of a body magnetic characteristics mentioned differ noticeably (refer to Section 5.4). Remember that the two characteristics appear also in the description of the electric field properties (see Section 4.2.2).

The magnetic field obeys the general principle of superposition (see Section 2.9.1). With reference to the case given it can be formulated as follows: the magnetic field created at a given point of space by any current does not depend on whether there are other sources of a field (other currents) in this space or not. Owing to the vector character of the magnetic field, the total induction of a system of currents is equal to the vector sum of particular field inductions, which are created by each current separately:

$$\mathbf{B} = \sum_{i=1}^{N} \mathbf{B}_i, \tag{5.1.14}$$

and when current is distributed continuously

$$\mathbf{B} = \int_L d\mathbf{B}, \tag{5.1.15}$$

where $d\mathbf{B}$ is the induction created by the elementary current Idl.

An empirical method of calculating the induction of a magnetic field at some point in space if the distribution of currents is known was suggested by Biot and Savart; a corresponding law relates a current element $Id\boldsymbol{l}$ (I is a scalar current running in the conductor element $d\boldsymbol{l}$) to an induction $d\mathbf{B}$ at a point A, the latter being assigned by the radius vector **r** drawn from the element $d\boldsymbol{l}$ to point A (Figure 5.6):

$$d\mathbf{B} = \frac{\mu_0 I\,[d\boldsymbol{l} \cdot \mathbf{r}]}{4\pi r^3}. \tag{5.1.16}$$

A multiplier $\mu_0/4\pi$ entered into the law formula in the SI.

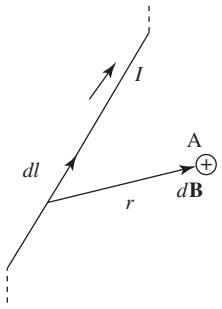


Figure 5.6 Biot–Savart law.

We can illustrate the application of the Biot–Savart law with two important examples. Calculate a field that creates a circular current (Figure 5.7) in its center point O. The magnetic induction $d\mathbf{B}$ at the point O is created by an element of current $Id\boldsymbol{l}$ accordingly. The vector product in (5.1.16) assign elementary vector $d\mathbf{B}$: it is directed perpendicular to a plane of a drawing and is related to the current element by the right-handed system rule. The vector $d\mathbf{B}$ is directed along the same axis (a symmetry axis of a ring) independently of where on the circuit the current element is chosen; therefore, integration (5.1.15) can be executed in a scalar form. Proceeding from the general formula (5.1.16), keeping in mind that the angle between vectors $d\boldsymbol{l}$ and $\mathbf{r}$ remains $\pi/2$, we arrive at $d\mathbf{B}=(\mu_0/4\pi)(Idl/R^2)$, where $r = R$ is the ring radius. The circular current integration gives

$$\mathbf{B}_0 = \int_{\mathrm{L}} d\mathbf{B} = \frac{\mu_0 I}{4\pi R}\int_0^{2\pi R} dl = \frac{\mu_0 I}{2R}. \tag{5.1.17}$$

Next, calculate a field created by the current I running along a rectilinear section of the conductor. In Figure 5.8 conductor MN and an observation point A at a distance b from it are presented alongside the current element $d\boldsymbol{l}$ and a corresponding vector $\mathbf{r}$. Wherever the element $d\boldsymbol{l}$ is chosen, the vector $d\mathbf{B}$ is directed along the same direction (perpendicular to the plane of the drawing). Therefore, integration (5.1.15) can be executed in a scalar form. Applying (5.1.16), we obtain

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Idl\sin\alpha}{r^2}.$$

There are several variables in this expression; they should be expressed by a single variable. We shall choose for the integration variable an angle α (see Figure 5.8). Expressing r and dl through the distance b and an angle α:

$$r = \frac{b}{\sin\alpha}, \quad dl = \frac{rd\alpha}{\sin\alpha} = \frac{bd\alpha}{\sin^2\alpha}.$$

Having substituted these expressions in the formula for $d\mathbf{B}$, we obtain $d\mathbf{B}=(rd\alpha/4\pi)(I/b)\times$ $\sin\alpha d\alpha$, and consequently

$$|\mathbf{B}| = \frac{\mu_0 I}{4\pi b}\int_{\alpha_1}^{\alpha_2}\sin\alpha d\alpha = \frac{\mu_0 I}{4\pi b}(\cos\alpha_2 - \cos\alpha_1). \tag{5.1.18}$$

Angles α_1 and α_2 are defined by the extreme points M and N.

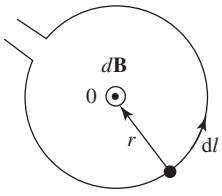


Figure 5.7 Application of Biot–Savart law to the calculation of the magnetic field of a circular current.

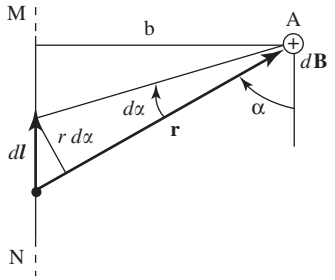


Figure 5.8 Application of Biot–Savart law to the calculation of the magnetic field of a direct current.

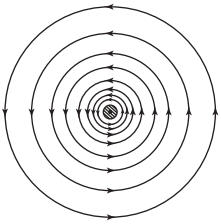


Figure 5.9 Magnetic force lines of a direct current in planes perpendicular to the current.

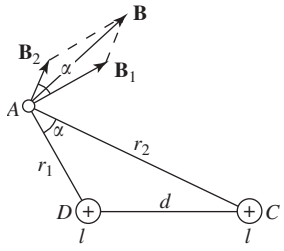
For the infinite direct conductor wire (where $\alpha_1 = 0$ and $\alpha_2 = \pi$ and the difference $\cos 0 - \cos \pi$ is 2):

$$B = \frac{\mu_0 I}{4\pi b} \times 2 = \frac{\mu_0 I}{2\pi b}. \tag{5.1.19}$$

The magnetic field of a direct current in a plane, perpendicular to a conductor, is depicted in Figure 5.9 by the force lines. They appear as concentric circles; their density decreases as the distance increases.

EXAMPLE E5.1

Along two parallel indefinitely long wires identical currents $I = 60$ A flow in the same direction. The wires are located at distance $d = 10$ cm from each other. Define the magnetic induction **B** at a point A (Figure E5.1) at distance of $r_1 = 5$ cm from one conductor and $r_2 = 12$ cm from another.



Solution: In order to find the magnetic induction at the specified point A we should define the directions of induction vectors $\mathbf{B}_1$ and $\mathbf{B}_2$ created by each conductor separately and then combine two vectors $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2$. We can find the module of total induction according to the cosine theorem

$$B = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos\alpha^*}.$$

Induction values B_1 and B_2 are expressed accordingly by the current strength I and distances r_1 and r_2: $B_1=(\mu_0 I/2\pi r_1)$ and $B_2=(\mu_0 I/2\pi r_2)$. Substituting these values in * we obtain

$$B = \frac{\mu_0 I}{2\pi}\sqrt{\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{2}{r_1 r_2}\cos\alpha}.$$

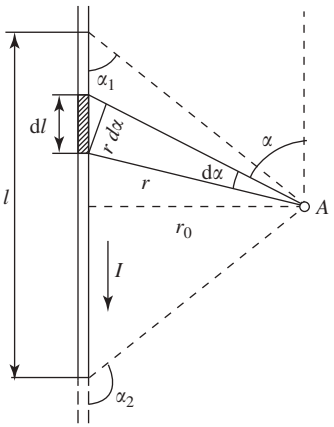
Find $\cos\alpha$ using the cosine theorem

$$\cos\alpha = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2} = 0.576.$$

Substituting all constants and given values we arrive at 286 μT.

EXAMPLE E5.2

Determine the magnetic field induction **B** produced by a section of infinitely long wire at point A at a distance $r_0 = 20$ cm from the center of the wire segment (Figure E5.2). The flowing current $I = 30$ A, the segment length $l = 60$ cm.



Solution: The magnetic field induction **B** can be calculated according to eq. (5.1.18). The scalar form of this equation is $dB=(\mu_0 I \sin \alpha d\ell/4\pi r^2)$. There are three variables in this equation: α, *r* and *l*. It is more convenient to integrate over an angle α provided all the variables are expressed through this angle. So $d\ell = rd\alpha/\sin\alpha$. Substituting this ratio into the first equation we obtain

$$dB=\frac{\mu_0 \ell \sin \alpha r d\alpha}{4\pi r^2 \sin \alpha} = \frac{\mu_0 I d\alpha}{4\pi r};$$

besides *r* is also a variable and should be expressed through α $r = (r_0/\sin\alpha)$ and therefore $dB=\mu_0 I \sin \alpha d\alpha/4\pi r_0$. This expression should be integrated over variable α.

$$dB = \frac{\mu_0 I}{4\pi r_0}\int_{\alpha 1}^{\alpha 2} \sin\alpha\, d\alpha = \frac{\mu_0 I}{4\pi r_0}(\cos\alpha_1 - \cos\alpha_2)^*.$$

Note that in the case of the symmetrical position of wire $\cos\alpha_1 = -\cos\alpha_2$, therefore the formula * is $(\mu_0 I/2\pi r_0)\cos\alpha_1$. We need to define angle α. It can be seen from Figure E5.2 that

$$\cos\alpha = \frac{\frac{\ell}{2}}{\sqrt{r_0^2 + \left(\frac{\ell}{2}\right)^2}} = \frac{\ell}{\sqrt{4r_0^2 + \ell^2}}.$$

Therefore,

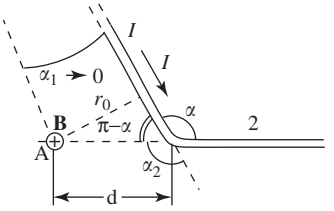
$$B = \frac{\mu_0 I}{2\pi r_0}\cos\alpha_1 = \frac{\mu_0 I}{2\pi r_0}\frac{\ell}{\sqrt{4r_0^2 + \ell^2}}.$$

Substituting all data into the formula obtained we arrive at

$$B = 2.49\times 10^{-5}\ \text{N/A m} = 24.9\,\mu\text{T}.$$

EXAMPLE E5.3

A long wire with a current $I = 50$ A is bent at an angle $\alpha = 2\pi/3$. Determine induction **B** at a point A (Figure E5.3). The distance *d* is equal to $d = 5$ cm.



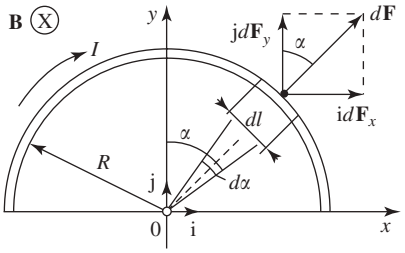
Solution: A bent wire can be considered as consisting of two semiinfinite pieces. According to the principle of magnetic fields superposition, the magnetic induction **B** at a point A is equal to the geometrical sum of the magnetic inductions $\mathbf{B}_1$ and $\mathbf{B}_2$, i.e., the fields created by two wire pieces 1 and 2, $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2$. The field magnetic induction $\mathbf{B}_2$ is zero: it follows from the Biot–Savart law, according to which in points lying on an axis of a conductor, $d\mathbf{B} = 0$ ($[d\boldsymbol{l}\mathbf{r}] = 0$). Therefore, we need to find only $\mathbf{B}_1$. In Section 5.1.2 this problem was considered in detail and eq. (5.1.18), i.e., $\mathbf{B}=(\mu_0 I/4\pi r_0)$ $(\cos\alpha_1 - \cos\alpha_2)$ was derived, where r_0 is the shortest distance from the wire to point A (the length of the perpendicular descended from point A on the wire). In the case considered $\alpha_1 \to 0$ (the wire is infinite, $\cos\alpha_1 = 1$), $\alpha_2 = \alpha = 2\pi/3$ ($\cos\alpha_2 = 2\pi/3 = -1/2$). The distance is $r_0 = d\sin(\pi - \alpha) = d\sin(\pi/3) = d(\sqrt{3}/2)$. Correspondingly,

$$B_1 = \frac{2\mu_0 I}{4\pi d\sqrt{3}}\left(1+\frac{1}{2}\right) = \frac{\sqrt{3}\mu_0 I}{4\pi d}.$$

Executing calculations we obtain $B = 34.6$ μT. Vector **B** is directed perpendicular to the drawing in downward direction.

EXAMPLE E5.4

A wire in the form of a thin half ring of radius $R = 10$ cm is in a uniform magnetic field ($B = 50$ mT) perpendicular to magnetic force lines. A current $I = 10$ A flows along the wire. Find the force acting on the half ring.



Solution: (See first the Section 5.1.4). (Let us arrange the wire in a plane of the drawing and direct the coordinate axes as is represented in Figure E5.4. On the wire allocate an elementary section with a current $Id\boldsymbol{l}$. On this area an Ampere force $d\mathbf{F} = I[d\boldsymbol{l}\ \mathbf{B}]$ (5.1.24) operates. Let us divide the elementary force into two components $d\mathbf{F} = \mathbf{i}\,dF_x + \mathbf{j}\,dF_y$. The force acting on the whole conductor can be found by the integration

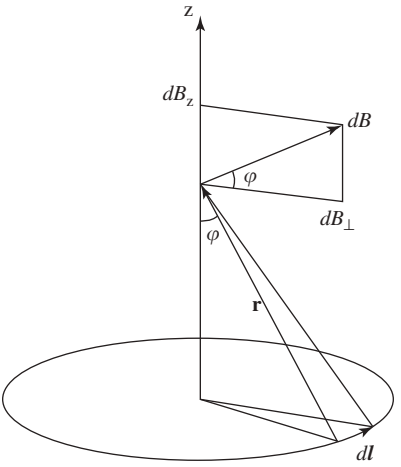
$$\mathbf{F} = \mathbf{i}\int_{\mathrm{L}} dF_x + \mathbf{j}\int_{\mathrm{L}} dF_y,$$

where the symbol L indicates that the integration is taken over the whole half ring length.

Using symmetry of the problem we can *a priory* write $\int_L dF_x = 0$. Therefore the whole force depends only on *y*-component $dF_y = dF \cos \alpha$, where dF is the modulus of the force $d\mathbf{F}$, $dF = IBdl$. Expressing dl through R and α ($dl = Rd\alpha$) we can write $dF_y = IBR \cos \alpha d\alpha$. Integration over a quarter of ring (multiplied by 2) gives $\mathbf{F} = \mathbf{j} \times IRB \times 2\int_0^{\pi/2} \cos \alpha d\alpha$ and $|\mathbf{F}| = 2IBR$. It can be seen that the force is directed along the *y*-axis. Executing calculations gives $F = 0.1$ N.

EXAMPLE E5.5

A current $I = 80$ A is flowing along a thin conducting ring of radius $R = 10$ cm. Find the magnetic induction **B** at the *z*-axis perpendicular to the circle crossing the ring center at an arbitrary point of the axis. Then find the magnetic induction at a point $r = 20$ cm (Figure E5.5).



Solution: Since the *z*-axis is perpendicular to the ring plane and passes the center of the ring, it is a symmetry axis L_∞. This means that the induction vector certainly must be codirectional to the *z*-axis and only the B_z component gives contribution to the field induction **B**. First, derive the general expression for $B(z)$.

Use the Biot–Savart law $d\mathbf{B} = (\mu_0/4\pi)(I[d\boldsymbol{l} \cdot \mathbf{r}]/r^3)$ to determine the dB_z component: $dB_z = |d\mathbf{B}| \sin\varphi = dB\,(R/\sqrt{R^2+Z^2})$. The vector product $[d\boldsymbol{l} \cdot \mathbf{r}]$ is perpendicular to the plane fixed by vectors $d\boldsymbol{l}$ and **r**. Then

$$dB_z = \frac{\mu_0 I}{4\pi} \frac{\mathrm{d}\boldsymbol{l}}{R^2 + z^2} \frac{R}{(R^2 + z^2)^{1/2}},$$

where $\sin \varphi = R/(R^2+z^2)^{1/2}$. The only variable is $d\boldsymbol{l}$. The integration over the whole circle gives

$$B_z = \frac{\mu_0 I}{4\pi} \frac{2\pi R \cdot R}{(z^2 + R^2)^{3/2}} = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}.$$

This is the general result. To make sure of the result we can use the border conditions $B(0)$ and $B(\infty)$. We know that $B(0)$ is equal to $(\mu_0 I/2R)$ (see eq. (5.1.17)); by assuming $z = 0$ we arrive at the correct result. The magnetic field diminishes at $z \to \infty$, which is also in agreement with our result.

In order to find the field at an equidistant point r we can use the result obtained, accept $r^2 = x^2 + R^2$ and substitute it into the general result; therefore $B=(\mu_0 IR^2/2r^3)$. Executing calculations, we arrive at 62.8×10^{-5} T or 628 μT.

5.1.3 The law of a total current (Ampere law)

The sign of the potential character of a force field is the equality to zero of the circulation of the field intensity vector along any closed contour. Let us see whether the magnetic field is potential, i.e., whether the integral $\oint_L \mathbf{B}\cdot d\boldsymbol{l}$ is equal to zero or not.

Consider the simplest case when a magnetic field is created by a linear conductor with current I. The magnetic force lines in this case are the concentric circles lying in parallel planes, perpendicular to the conductor, with their centers on the linear conductor (Figure 5.9). Choose for simplicity contour L coinciding with one circular force line of any radius R (Figure 5.10). Then the circulation of the vector **B** along the contour L will be equal to

$$\oint_L \mathbf{B}\cdot d\boldsymbol{l} = \frac{\mu_0 I}{2\pi R}\oint_L R d\varphi = \frac{\mu_0 I}{2\pi R}\int_0^{2\pi R} d\boldsymbol{l}.$$

Therefore, by using eq. (5.1.19):

$$\oint_L \mathbf{B}\cdot d\boldsymbol{l} = \mu_0 I. \tag{5.1.20}$$

Since circulation of the magnetic field induction is not zero *the magnetic field is not potential*. (Notice that in the above integral dl is an element of a contour L but not current.) To obtain this ratio for a noncircular contour of any form is not a difficult task.

Expression (5.1.20) is the essence of Ampere's law: *circulation of the induction vector along a closed contour L is equal to the current multiplied by μ_0 comprised by this contour.*

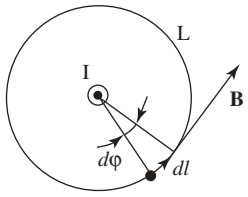


Figure 5.10 An Ampere law consideration.

(This means that if the current passes outside the contour chosen, this particular current does not contribute to the total current.) A field satisfying the condition (5.1.20) is referred to as a *nonpotential field*. Expression (5.1.20) is also referred to as Ampere's law to emphasize the unity of the phenomena of interaction of currents with each other and with the magnetic fields.

If contour L comprised N currents then, according to the principle of superposition, the circulation of a vector $\mathbf{B}$ is equal to their algebraic sum

$$\oint_{\mathrm{L}} \mathbf{B} \cdot d\boldsymbol{l} = \mu_0 \sum_{i=1}^{\mathrm{N}} I_i, \tag{5.1.21}$$

the current is considered positive if it corresponds to the clockwise rule, otherwise it is considered negative.

If the current is distributed nonuniformly across the conductor this law can be rewritten as

$$\oint_{\mathrm{L}} \mathbf{B} \cdot d\boldsymbol{l} = \mu_0 \int_S \mathbf{j}(\mathbf{r}) dS, \tag{5.1.22}$$

where a surface S is resting on contour L (Figure 5.11).

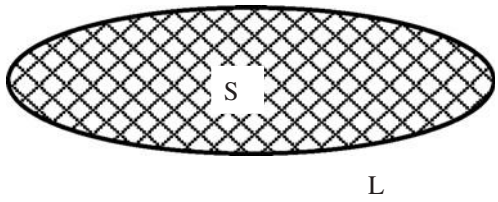


Figure 5.11 A surface rested on a contour loop.

Let us apply Ampere's law to calculate the induction of a magnetic field created by a solenoid. Remember that a coil that has been reeled up by thin wires without misses on the cylinder (Figure 5.12) is referred to as a solenoid. We shall choose a rectangular contour 1–2–3–4, depicted in the figure. Then circulation along the whole contour can be divided into four integrals:

$$\oint_{\mathrm{L}} \mathbf{B} \cdot d\boldsymbol{l} = \oint_{\mathrm{L}} B_l dl = \int_{1-2} B_l dl + \int_{2-3} B_l dl + \int_{3-4} B_l dl + \int_{4-1} B_l dl.$$

It can be seen that the integrals along segments $2-3$ and $4-1$ are zero since the angle between $\mathbf{B}$ and $d\boldsymbol{l}$ is $\pi/2$. The integral on the segment $3-4$ is also zero because this segment can be chosen far enough from the solenoid where $B = 0$. Therefore,

$$\oint_{\mathrm{L}} B_l dl = \int_{1-2} B_l dl = \mu_0 \sum_{1-2} I_i = \mu_0 nlI,$$

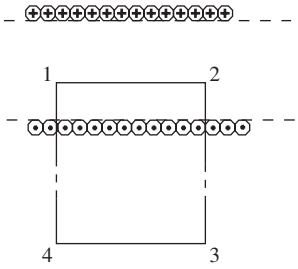


Figure 5.12 Calculation of a solenoid magnetic field strength.

where l is the length of segment 1–2, n is a number of turns over a unit solenoid length, I is the current in the solenoid. Therefore,

$$B = \mu_0 nI, \tag{5.1.23}$$

i.e., the induction inside an infinitely long solenoid is proportional to the overall current running onto the length unit.

The magnetic field inside the solenoid is uniform. In this respect the solenoid plays the same role as a plate condenser plays in electrostatics.

5.1.4 Action of the magnetic field on the current, on the moving charge

Comparing the expression of the Ampere law (5.1.11) with the expression for a field created by an infinite conductor (5.1.19), one can imagine that current I_1 (Figure 5.4) is creating the magnetic field B_1 which in turn acts on a neighboring conductor with current I_2; then $f_{12} = B_1 I_2$. Accordingly, $f_{21} = B_2 I_1$. If the conductor is not rectilinear, we should consider a conductor element dl and then $f=(dF/dl)=IB$ or, finally, in the vector form

$$d\mathbf{F} = I[d\boldsymbol{l}\,\mathbf{B}]. \tag{5.1.24}$$

Figure 5.13 shows the arrangement of the vectors describing the field **B**, the conductor element $d\boldsymbol{l}$ and force acting on the conductor $d\mathbf{F}$. Since the differential expression (5.1.24) is obtained starting from the Ampere law (5.1.11) the force $d\mathbf{F}$ is accordingly referred to as Ampere force.

Another manifestation of the Ampere force is the action of a magnetic field on a moving charge. Moreover, from the point of view of electromagnetic dynamics all macroscopic (ponderomotive) forces can be reduced finally to the forces applied to the electric charges included in this body. We shall reduce force $\mathbf{F}_A$, which operates on the whole conductor, to a force that

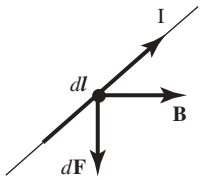


Figure 5.13 Vector disposition according an Ampere law in the differential form.

operates on each single charge moving inside the conductor. For this purpose allocate the elementary cylinder of a volume $dV = dlS$ inside a conductor so that its generatrix is parallel to the direction of the carriers' motion (Figure 5.14). According to eq. (5.1.24) force $d\mathbf{F}_A$, acting on this cylinder, is equal to $d\mathbf{F}_A = I\,[d\boldsymbol{l}\cdot\mathbf{B}] = \mathbf{j}S\,[d\boldsymbol{l}\cdot\mathbf{B}]$. On the other hand, the current density $\mathbf{j}$ can be submitted according to eq. (5.1.4). Taking into account that $\mathbf{j}$ is codirected with $\mathbf{u}$ and, accordingly, with $d\boldsymbol{l}$, the force $d\mathbf{F}_A$ can be rewritten as $dF_A = qnSdl[vB] = qdN[vB]$, where dN is the number of electric carriers in the allocated cylinder. If the overall force is divided by the number of carriers (dF_A/dN), force acting on a single charge is obtained

$$\mathbf{F}_{lor} = \frac{d\mathbf{F}_A}{dN} = q[\boldsymbol{\upsilon}\cdot\mathbf{B}]. \tag{5.1.25}$$

The force acting on a single charge moving with a speed $\boldsymbol{\upsilon}$ in a magnetic field $\mathbf{B}$ is referred to as a Lorentz force. The sign (direction) of the force depends on the sign of the moving charge, i.e., from sign q in eq. (5.1.25).

From the expression defined for the Lorentz force $\mathbf{F}_{lor}$ it can be seen that it is always perpendicular to the particle velocity. Therefore, a Lorentz force does not produce work. It follows that it is impossible to accelerate the particles by means of a Lorentz force, i.e., an electric field is required to do so. (Nevertheless, the Lorentz force is used in accelerators to make a motion cyclic.)

The Lorentz force defines the movement of charged particles in a magnetic field. If the particles enter the magnetic field in a plane perpendicular to the induction $\mathbf{B}$, the Lorentz force will act perpendicular to both vectors $\boldsymbol{\upsilon}$ and $\mathbf{B}$. In the absence of any other force, the Lorentz force is centripetal, and a circular movement will occur (Figure 5.15). Write the equation of Newton's second law for this case $q\upsilon B = ma_n = m(\upsilon^2/R)$. The radius of a circle R can be derived from this expression

$$R = \frac{\upsilon}{B}\frac{1}{(q/m)}, \tag{5.1.26}$$

where q/m is the specific particle charge. The period T of the circular particle motion turns out to be

$$T = \frac{2\pi R}{\upsilon} = 2\pi\frac{1}{B(q/m)}. \tag{5.1.27}$$

The period does not depend either on the particle radius R or on the speed.

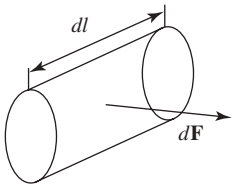


Figure 5.14 To a Lorentz force derivation.

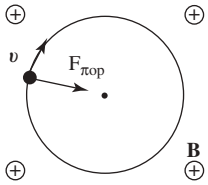


Figure 5.15 Movement of a charge in a plane perpendicular to a magnetic field induction.

If the particle velocity is directed at an angle α to a vector **B** (Figure 5.16, where **B** is directed along the z-axis), vector υ has to be projected in two directions: perpendicular $\upsilon_{\perp}$ and parallel $\upsilon_{||}$ to the induction vector **B**. Accordingly, the component $\upsilon_{\perp}$ defines the circular motion of the particle and another component $\upsilon_{||}$ determines its uniform motion along axis z since the Lorentz force for this component is zero. This results in the particle's spiral movement. The basic spiral increment h is defined as

$$h = \upsilon_{||} T = 2\pi \frac{\upsilon \cos\alpha}{B(q/m)}, \tag{5.1.28}$$

whereas the circus radius R depending upon the perpendicular constituent $\upsilon_{\perp} = \upsilon \sin\alpha$ is

$$R = \frac{\upsilon \sin\alpha}{B(q/m)}. \tag{5.1.29}$$

Therefore, the particle entering the magnetic field moves with a winding-up movement on the magnetic force lines.

The charged particle movement described above permitted the development of a powerful instrument – the mass spectrometer: a device for "sorting" ions on their specific charge q/m. Such an opportunity is extremely tempting for modern chemistry for the analysis and synthesis of new substances and for many other problems.

The basic scheme of a mass spectrometer is shown in Figure 5.17. The gas to be analyzed enters a vacuum chamber at a point S and is ionized (by any method, for example, by an electron beam impact). Between points S and A the potential difference $\Delta\varphi$ is applied and ions are accelerated by an electric field. Passing an aperture (at point A) all ions possess identical energy, but not speed. To select ions with identical speed from a beam a speed filter is used in which both forces, Coulomb' ($\mathbf{F}_{cou} = qE$) and Lorentz' ($F_{lor} = qB_1\upsilon$), operate perpendicular to

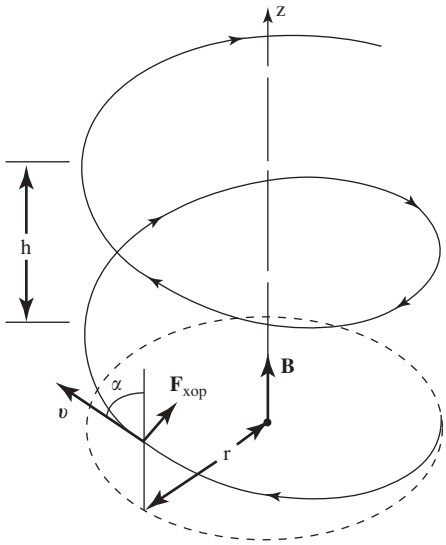


Figure 5.16 Movement of a charge at an arbitrary orientation of a field induction **B** and a charge velocity vector.

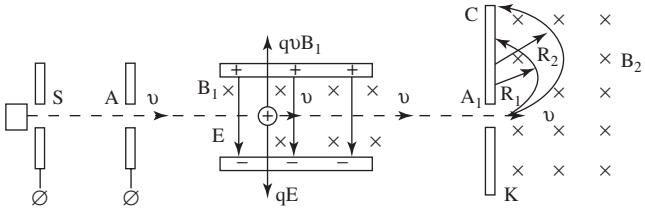


Figure 5.17 Scheme of a mass spectrometer.

each other; the filter allows the singly ionized particles with speed $\upsilon = E/B_1$ to pass through. This filtered beam goes to another point A_1 in the chamber where the ions start to move in a circular trajectory with a radius depending on the q/m ratio. In a collector (or a film, or special detector device) located on line A_1C ions with different specific charges q/m fall in different points. In Figure 5.18 the mass spectrum of the air is shown, where along abscissa axis values A/q (A is a mass number, q is a particle charge) are plotted, whereas along the ordinate a relative number of the given molecules in the object under investigation are shown.

The construction of the modern mass spectrometer differs significantly from the one described above, although it is based on the same principle.

EXAMPLE E5.6

An electron is projected into a uniform field of induction **B** ($B = 30$ mT) with its velocity vector $\boldsymbol{\upsilon}$ ($\upsilon = 2 \times 10^6$ m/sec) making an angle of $\alpha = 30^o$ with **B.** It begins

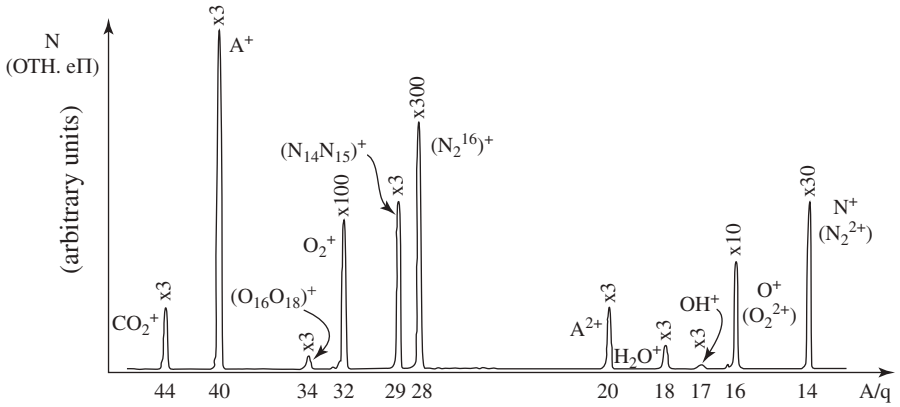


Figure 5.18 Air mass spectrum (every peak relates to a definite ion; numbers over intense peaks show how many times its height was curtailed in order to present all peaks in one spectrogram).

to move along the helix (refer to Figure 5.16). Find the radius R of the helix and its pitch h.

Solution: A Lorentz force is acting on the electron changing the direction of travel of the electron. Let us divide electron's velocity into two components $\upsilon_{\parallel}$ and $\upsilon_{\perp}$ relative to vector **B**: $\upsilon_{||} = \upsilon \sin \alpha$ and $\upsilon_{\perp} = \upsilon \cos \alpha$. The helix radius can be found from the second Newton law: $F = ma_n$, therefore $F = |e|\, \upsilon_{\perp} B$ and $a_n = \upsilon_{\perp}^2/R$. Then $|e|\upsilon_{\perp} B = m(\upsilon_{\perp}^2/R)$ and further $R = (m\upsilon_{\perp}/|e|B) = (m\upsilon \sin \alpha/|e|B)$. Substituting the given values we obtain $R = 0.19$ mm.

The helix pitch is $h = \upsilon_{||}\, T$, where $T = 2\pi R/\upsilon_{\perp}$. Substituting this value into an expression for h we obtain

$$h = \left(\frac{2\pi R \upsilon_{\parallel}}{\upsilon_{\perp}}\right) \text{or } h = \left(\frac{2\pi R \upsilon \cos\alpha}{\upsilon \sin\alpha}\right) = 2\pi R \tan\alpha.$$

Calculation shows that the pitch is $h = 2.06$ mm.

EXAMPLE E5.7

An electron enters a uniform magnetic field of induction $B = 0.03$ T and begins to move along a circle of radius $R = 10$ cm. Determine the electron's speed υ.

Solution: The second Newton law can be applied to the movement of the electron along a circle $(m\upsilon^2/r) = |e|B\upsilon$; its momentum can be found from the expression $p = m\upsilon = |e|Br$. However, relativistic laws should be used in this case (as we will

see in the end). In this case $p=(m_0c\beta/\sqrt{1-\beta^2})$ (the electron's velocity is included in the value β). Solve the last expression relative to β:

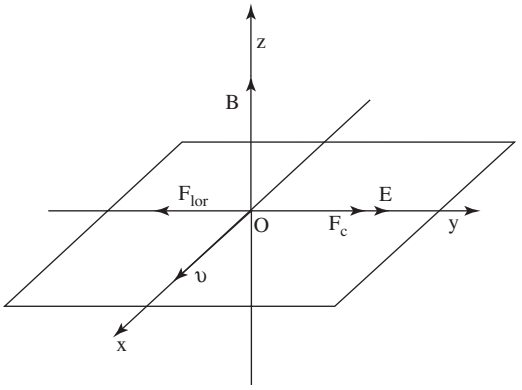
$$\beta = \frac{p}{m_0c^2\sqrt{1+\left(\frac{p}{m_0c}\right)^2}} = \frac{\frac{|e|Br}{m_0c}}{\sqrt{1+\left(\frac{|eBr|}{m_0c}\right)^2}}.$$

Our solution can be simplified; calculating separately the value appears equal to $(|e|Br/m_0c) = 1.76$. Therefore, the β value can be calculated ($\beta = 0.871$) and then the velocity found $\upsilon = c\beta = 2.61 \times 10^8$ m/sec. The electron moving with such a velocity is the relativistic one.

EXAMPLE E5.8

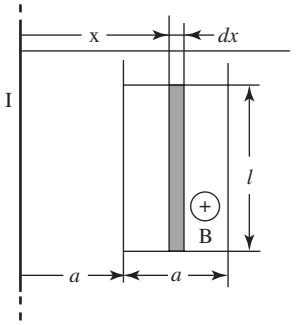
An alpha particle runs through the accelerating potential difference $\Delta\varphi = 104$ V and projects into two fields crossing at right angles – magnetic $B = 0.1$ T and electrostatic $E = 10$ kV/m. Find the charge/mass ratio of this particle if, when traveling in these fields perpendicular to both the fields, it does not diverge from a rectangular motion.

Solution: In order to find the ratio it is useful to apply the relationship between the electrostatic forces work qU and the change of its kinetic energy $m\upsilon^2/2$. From this equality it follows that $(q/m)=(\upsilon^2/2\Delta\varphi)$*. The special arrangement of the Coulomb and Lorentz fields provides the equality of their action and straight-line motion (Figure E5.8). Therefore, $qE = q\upsilon B$. This equation permits us to find particle speed υ as $\upsilon = E/B$. Inserting this equality into the * fraction, we arrive at $(q/m)=(E^2/2\Delta\varphi B^2)$. Executing calculations, we obtain $(q/m)=4.81\times10^7$ C/kg. Let us check the dimension of the result: $(E^2/\Delta\varphi B^2)=((1B/m)^2/1BT^2)=(1B\ A^2/1B\ N^2)=1J\ C/(1N\ sec)^2=1C\ m/(1N\ sec)^2=1C/kg$. Dimension is just the specific charge.



EXAMPLE E5.9

In one plane with an infinite direct wire is a frame with sizes as shown in Figure E5.9. Along the wire flows a current $I = 50$ A. Find the magnetic induction flux $d\Phi$ through the frame in square $S = al$.

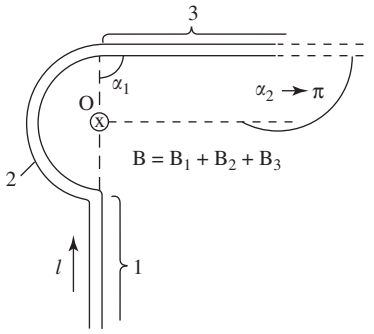


Solution: In full analogy with the electrostatic strength flux the induction of the magnetic field is $d\Phi = \mathbf{B}\, d\mathbf{S}$. In our problem induction of the magnetic field depends on the distance from the wire x, therefore the flux is

$d\Phi = \int_a^{2a} B(x)dx$. Since $B(x)=(\mu_0 I/2\pi x)$ the flux is equal to $\Phi=(\mu_0 l I/2\pi)\ln 2$. Executing the calculation we arrive at $\Phi = 4.5$ μWb.

EXAMPLE E5.10

An infinite wire is bent as is shown in Figure E5.10. The circle radius is $R = 10$ cm and the current flowing along it is $I = 80$ A. Determine the magnetic induction **B** at a point O.



Solution: Divide the current wire into three pieces: 1, 2 and 3. The magnetic field induction is the vector sum $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3$. The first segment does not produce a magnetic field at point O since $d\mathbf{B} = 0$ ($[d\boldsymbol{l}\ \mathbf{r}] = 0$), according to Biot–Savart law along the whole segment piece. Two components remain both giving codirectional

induction perpendicular to the drawing plane; they can be summed as scalar values. The half-ring induction B_2 can be found according to eq. (5.1.17) taking only half of it: $B_2=(\mu_0 I/4R)$. Induction B_3 can be found according to eq. (5.1.18) taking into account that $\alpha_1 = \pi/2$ and $\alpha_2 = \pi$; therefore $B_3 = (\mu_0 I / 4\,\pi\, R)$. Then

$$B = B_2 + B_3 = \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4\pi R}(\pi + 1). \text{ Therefore } B = 3.31\times 10^{-4}\ \text{T} = 331\,\mu\text{T}.$$

5.1.5 A magnetic dipole moment in a magnetic field

Just as an electrostatic field acts on an electric dipole moment (refer to Section 4.1.5), a magnetic field acts on a magnetic dipole moment. At a significant difference of working forces the results are very similar.

Let us first consider a homogeneous magnetic field; we accept a magnetic dipole moment as a rectangular hoop (Figure 5.19). If the contour is oriented so that vector **B** is parallel to its plane, the sides having length b will not fill any action of the Ampere force because the vector product in eq. (5.1.24) is zero. The forces acting on the side a are $F = IaB$ (refer to eq. (5.1.24)), the force couple renders the torque rotating moment of the contour

$$M_{\text{F}} = Fb = IaBb = ISB = \mathcal{M}B$$

or in vector form:

$$\mathbf{M}_{\text{F}} = [\boldsymbol{\mathcal{M}}\,\mathbf{B}]. \tag{5.1.30}$$

This expression corresponds to eq. (4.1.32). The torque $\mathbf{M}_{\text{F}}$ aspires to turn the contour so that the magnetic moment $\boldsymbol{\mathcal{M}}$ as a vector turns along the field. On two sides having length b the Ampere forces act oppositely and will stretch (or compress) the contour but not rotate it. It is also possible to show that formula (5.1.30) is valid for a contour of any form and, hence, can be used regardless of the form of the magnetic moment $\boldsymbol{\mathcal{M}}$.

The potential energy of a magnetic moment in a magnetic field can be calculated according to the recipe given in Section 1.4.5. Taking into account that in this case M_{F} is the moment of external forces, we obtain

$$dU = M_F d\alpha = \mathcal{M}B\sin\alpha d\alpha$$

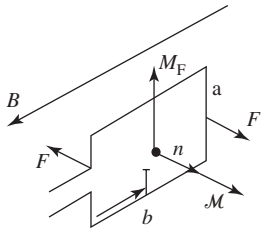


Figure 5.19 A frame with a current in a uniform magnetic field.

and after integration

$$U = \int M_F d\alpha = -\mathcal{M}B\cos\alpha + C.$$

Accept C to be zero at $\alpha = 0$. Thus, finally in vector form

$$\mathbf{U} = -(\boldsymbol{\mathcal{M}}\,\mathbf{B}). \tag{5.1.31}$$

The graph is just the same as for an electric analog (Figure 4.18).

In a nonuniform field besides the torque, a force F will act on the magnetic dipole moment (providing the moment is turned into a stable position with $\alpha = 0$ and $\cos\alpha = \pm 1$). To calculate the force we can take the advantage of eq. (1.4.32) describing it as dependent on potential energy. Then (for a one-dimensional case, $B = B(x)$)

$$F_x = -\frac{\partial U}{\partial x} = \mathcal{M}\frac{dB}{dx}\cos\alpha. \tag{5.1.32}$$

It follows from this equation that the magnetic moment $\boldsymbol{\mathcal{M}}$ can be dragged in or pushed out of the magnetic field depending on the angle α ($\cos\alpha = \pm 1$) and the sign of magnetic field gradient.

5.1.6. Electromagnetic induction

The concept of a vector flux $d\Phi$ through a surface dS has been given in Sections (2.8.3) and (4.1.3). Being a particular case of a more general theory of a vector field, the same concepts can be applied to a magnetic field as well. An elementary flux $d\Phi$ of a magnetic field induction vector $\mathbf{B}$ through the surface $d\mathbf{S}$ is equal to a scalar product of $\mathbf{B}$ and $d\mathbf{S}$

$$d\Phi = (\mathbf{B}\,d\mathbf{S}). \tag{5.1.33}$$

Depending on the angle between a normal $\mathbf{n}$ to the surface dS (Figure 2.20) and the induction vector $\mathbf{B}$, a flux $d\Phi$ can vary in limits $\pm BdS$. In general, flux Φ through surface S is defined by the integration

$$\Phi = \int_S (\mathbf{B}\,dS) = \int_S B_n\,dS. \tag{5.1.34}$$

In 1812 the English physicist Michael Faraday made a discovery, which has significantly influenced the development of all mankind. Having made a real conducting contour, confined to a surface S, he established that by changing the flux (5.1.34) an electric current appeared in the circuit. By numerous experiments, Faraday established that current value does not depend on the way the flux changes but on the speed of this change. The mathematical form of Faraday's law is extraordinary simple:

$$\varepsilon_i = -\frac{d\Phi}{dt}. \tag{5.1.35}$$

The minus sign in this expression corresponds to the general physical law of inertia: the induction current in a contour is always directed in a way that opposes the reason of its appearance. This statement is referred to as the Lenz rule.

To exemplify Faraday's law, let us imagine a simple experiment. We shall create a closed electric contour with motionless rails and a metal axis with wheels moving on them (Figure 5.20). On this "construction," impose a magnetic field perpendicular to the plane of the image. Move the axis at a uniform speed υ. At the same time and with the same speed begin to move electric carriers creating a current in the metal axis and in the circuit in general. The force $\mathbf{F}'$ directed along the axis and equal to $q[\upsilon \cdot \mathbf{B}]$ will operate on the carriers. The action of this force is equivalent to the action of the electric field $\mathbf{E}' = [\upsilon \cdot \mathbf{B}]$.

This field is not electrostatic because it has been created in a different way – the movement of charges in a magnetic field. The circulation of the electric field strength $\mathbf{E}'$ along the contour will produce EMF in the contour (see eq. (5.1.8)):

$$\varepsilon_i = \oint_{\mathrm{L}} \mathbf{E}' d\boldsymbol{l} = \oint_{\mathrm{L}} [\upsilon \cdot \mathbf{B}] d\boldsymbol{l}. \tag{5.1.36}$$

Only the movable part of the contour creates EMF, therefore

$$\varepsilon_i = \oint_{\mathrm{L}} \mathbf{E}' d\boldsymbol{l} = \int_1^2 [\upsilon \cdot \mathbf{B}] d\boldsymbol{l}.$$

Supposing that both the axis movement and the magnetics field are uniform, we can obtain

$$\varepsilon_i = \upsilon B \int_1^2 d\ell = \upsilon B \ell.$$

Multiplying and dividing this intermediate expression by dt we derive

$$\varepsilon_i = \frac{B \ell \upsilon \, dt}{dt}.$$

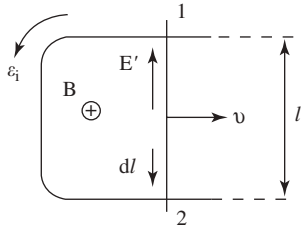


Figure 5.20 A modeling of a Faraday magnetic induction law.

Taking into account that $\upsilon l dt = dS$ and $BdS = d\Phi$, we obtain $\varepsilon_i = (d\Phi/dt)$, which coincides with eq. (5.1.35). The sign appeared in this expression after analysis of the vector disposition and accounting for the negative sign of electron.

The integral $\oint_L \mathbf{E}' d\boldsymbol{l}$ is not zero since $\mathbf{E}'$ is not an electrostatic field. It represents quite different, solenoidal (or vertex or curl) electric field. The acting force $\mathbf{F}'$ is also non-Coulomb in origin; in contrast, it can be related to the extraneous one. Therefore, we can proceed by adding to the nonzero integral $\oint_L \mathbf{E}' d\boldsymbol{l}$ the zero's addition $\oint_L E dl$ and write the expression

$$\varepsilon_i = \oint_L E^* d\ell. \tag{5.1.37}$$

Here, however, E^* is the strength of both the conservative and (potential and nonpotential) electric fields.

Furthermore, if the left-hand part of eq. (5.1.35) is written in the form (5.1.37) and using eq. (5.1.34), we can arrive at an important expression which is referred to as a Maxwellian equation

$$\oint_L E_\ell^* d\ell = -\frac{d}{dt}\int_S B_n dS = -\int_S \left(\frac{\partial B}{\partial t}\right)_n dS. \tag{5.1.38}$$

At the left-hand side the integral is taken on any contour L, whereas the surface S is fixed by the already chosen contour L: the surface S rests on the contour L (see Figure 5.11). From this Maxwellian equation it follows that *any change of a magnetic field* (the right-hand side of the equation) *generates an electric field* (the left-hand side of the equation). If a conducting wire is drawn along the contour L an electric current would occur in it. If contour L is in vacuum, along it the electric field would be excited.

Once again we would like to emphasize that an induction electric field is not electrostatic. In fact, the source of an electrostatic field is motionless electric charge whereas in producing a nonpotential electric field the source of the field is an alternating magnetic field. This field, induction by origin, is solenoidal (i.e., is not potential) and certainly possesses other than electrostatic field properties.

Faraday's law is one of the general laws of electrodynamics.

In an alternating magnetic field, induction leads to the excitement of EMF. It defines the mutual induction of one conductor onto another. However, even if there is only one conductor with an alternative current a force appears that renders back the current value state. A magnetic flux that *penetrates its own contour with current I and generates current variation, is referred to as its own, intrinsic magnetic flux* and is designated as Φ_S. This flux is not influenced by a change of counter orientation; it is firmly connected with a contour. Since, according to Biot–Savart law, B is proportional to I, therefore it is also proportional to Φ

$$\Phi_c = LI. \tag{5.1.39}$$

The coefficient L is called the self-inductance of the counter. It describes the relation between an alternating current and the intrinsic magnetic field produced by it.

If the counter consisted of N windings and the magnetic flux penetrated all of them without omission, then the total magnetic flux linkage Ψ is equal to $\Psi = N\Phi_c$ or

$$\Psi = LIN = L'I. \tag{5.1.40}$$

Excitation of an induction ε in a closed counter when a current change is taking place is referred to as self-induction. It is equal to the speed of flux linkage Ψ change taken with the sign minus:

$$\varepsilon_S = -\frac{d\Psi}{dt}$$

or for solenoids without a ferromagnetic core:

$$\varepsilon_S = -L'\frac{dI}{dt}. \tag{5.1.41}$$

In order to calculate the inductance of a long solenoid, we can use an expression for the total magnetic flux (Ampere law) $\Psi = LI$ and $\Psi = N\Phi_c = nl\mathbf{B}S$ (where n is the number of solenoid windings on a unit length and l is the total solenoid length; the stroke at L is omitted). Since the solenoid magnetic induction is $B = \mu_0 nI$ (see eq. (5.1.22)), hence

$$\Psi = \mu_0 n^2 lSI.$$

Therefore,

$$L = \mu_0 n^2 V. \tag{5.1.42}$$

It can be seen that the solenoid inductance depends quadratically upon the number of windings on a unit length and is proportional to the solenoid volume. In the absence of a ferromagnetic core the inductance is constant and is not dependent on the current strength.

5.2 MAGNETIC PROPERTIES OF CHEMICAL SUBSTANCES

From the point of view of their reaction to an external magnetic field, all substances are referred to as magnetic. Its chemical structure defines the magnetic properties of a substance. All magnetic materials can be divided mainly into three main classes: diamagnetic, paramagnetic and magnetically ordered substances.

Diamagnetics are pushed out of an external nonuniform magnetic field; they consist of those molecules that do not possess their own (i.e., “intrinsic”) magnetic dipole moments. Paramagnetics are drawn into the external nonuniform magnetic field; they consist of molecules possessing inherent magnetic dipole moments in the absence of an internal magnetic field. Among the magnetically ordered substances are ferromagnetics and ferrimagnetics, highly reacting on an external magnetic field. There is also a class of antiferromagnetic substances, weakly reacting on an external field. Further we shall describe in more detail the nature of all these substances and consider those characteristics that describe macro- and microproperties of magnetics.

Moreover, we shall follow approximately the same logic we used for the description of dielectrics. In particular, we should establish where the magnetic properties originated from. It is logical to connect magnetic properties with magnetic dipole moments of atoms and molecules.

5.2.1 Atomic magnetism

In semiclassical Bohr theory an atom is represented as consisting of a nucleus and electrons traveling along circular stationary orbits. This motion can be characterized by an orbital angular momentum: the orbital angular (mechanical) moment $\mathbf{L}_l$ (see Section 1.3.9). An intrinsic electron angular momentum is also considered; sometimes it is described by electron rotation around its own axis (see Chapter 6.7 and Section 7.5.5); it is referred to as electron spin with its own angular momentum $\mathbf{L}_S$. Orbital and spin states are sometimes imagined as circular electric currents. These currents create orbital and spin magnetic dipole moments. More advanced notions have been developed in quantum mechanics, but these semiclassical representations are very distinct and useful at this point.

There are three sources of the magnetic properties of substances: (1) electron spin; (2) orbital electron motion; (3) change of the electron orbital angular momentum at the imposition of an external magnetic field. The first two can explain paramagnetism, and the third can be used in considering diamagnetism.

A nuclear magnetic moment is very weak in comparison with orbital and spin electron magnetic moments; thus it can be temporarily neglected here. However, nuclear magnetism will be closely considered in Chapter 8 because the nuclei participate strongly in resonant methods of investigations in chemistry.

We shall take advantage of the semiclassical Bohr theory as it provides an elementary model for understanding the physical essence of the phenomenon. We will begin by calculating a inasmuch as the Bohr theory permits us to do it very easily. The gyromagnetic ratio is the ratio of the magnetic and mechanical moments is referred to as giromagnetic ratio (Figure 5.21).

Remember that the angular momentum $\mathbf{L}$ of an MP (an electron, in our case) relative to an origin (nucleus) is defined as a vector product $\mathbf{L} = [\mathbf{r}\cdot\mathbf{p}]$, where $\mathbf{r}$ is the electron radius vector relative to the nucleus, and $\mathbf{p}$ the linear electron momentum ($\mathbf{p} = m\boldsymbol{\upsilon}$). On traveling along a circular orbit the linear velocity is perpendicular to $\mathbf{r}$ ($\boldsymbol{\upsilon} \perp \mathbf{r}$), then $L_l = |\mathbf{L}_l| = m\upsilon r$. The direction of the vector $\mathbf{L}_l$ is defined by the rule of vector product (the right-hand screw rule).

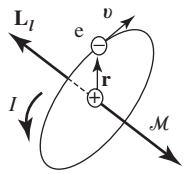


Figure 5.21 A gyromagnetic ratio for orbital electron movement in an atom.

The circular current caused by the electron traveling along the orbit produces an orbital magnetic moment $\mathcal{M}_l$, the direction of which is also defined by the right-hand screw rule (Figure 5.21). The module of the orbital magnetic moment is $\mathcal{M}_l = IS = (|e|\pi r^2/T)$, where T is the period of revolution. Thus, the gyromagnetic ratio is

$$\frac{\mathcal{M}_l}{L_l} = \frac{|e|\pi r^2}{T m v r}$$

Canceling r and taking into account that $v = (2\pi r/T)$ we arrive at

$$\frac{\mathcal{M}_l}{L_l} = \tfrac{1}{2}\frac{|e|}{m}. \tag{5.2.1}$$

Eq. (5.2.1) can be rewritten as

$$\frac{\mathcal{M}_l}{L_l} = g\,\tfrac{1}{2}\frac{|e|}{m}, \tag{5.2.2}$$

where g is the gyromagnetic ratio in the unit $\frac{1}{2}(|e|/m)$. In this unit the orbital gyromagnetic ratio is equal to 1 ($g_{\text{orb}} = 1$).

In their outstanding experiments, Einstein and de Haas showed that for spin the gyromagnetic ratio is equal to 2 ($g_{\text{sp}} = 2$) and consequently $(\mathcal{M}_s/L_s)=2 \times \frac{1}{2}(|e|/m)$.

This gyromagnetic ratio anomaly is a source of some of the most interesting and important phenomena and is used, in particular, in many physical methods of chemical substances' research.

5.2.2 Macroscopic properties of magnetics

The magnetization of a substance is quantitatively characterized *by magnetization* $\mathfrak{I}$, which is numerically equal to the magnetic moment of a volume unit

$$\mathfrak{I} = \frac{\sum \mathcal{M}_i}{\Delta V}. \tag{5.2.3}$$

Alongside with magnetization of a unit volume, a specific and mole magnetization are considered as well. The specific magnetization (magnetization of a mass unit) is equal to:

$$\mathfrak{I}_{\text{sp}} = \frac{1}{\Delta m}\sum \mathcal{M}_i, \tag{5.2.4}$$

where Δm is a mass of the physically infinitesimal volume ΔV (refer to Section 4.2.1). Having replaced Δm on $\rho \Delta V$ where ρ is a substance density we shall obtain

$$\mathfrak{I}_{\text{sp}} = \tfrac{1}{\rho}\mathfrak{I}. \tag{5.2.5}$$

Mole magnetization (magnetization of one mole) is

$$\mathfrak{J}_{\mathrm{M}} = \frac{1}{v}\sum_{i=1}^{N} \boldsymbol{\mathcal{M}}_i, \tag{5.2.6}$$

where v is the number of moles in a physically infinitesimal volume $v = \Delta m/M$, M is a molar mass.

Alongside with a magnetic induction **B** one more value characterizes the magnetic field **H,** a strength of the magnetic field, is used; in isotropic magnetic the magnetization $\mathfrak{J}$ is proportional to magnetic field strength **H**, that is

$$\mathfrak{J} = \chi \mathbf{H} \tag{5.2.7}$$

where χ is the scalar value referred to as *a magnetic susceptibility*. The magnetic susceptibility characterizes ability of substance to be magnetized in a magnetic field. As $\mathfrak{J}$ and **H** have identical dimension χ is dimensionless value.

For diamagnetic materials $\chi < 0$, its value is $\sim -10^{-5} \div 10^{-7}$, for paramagnetic materials $\chi > 0$, its value is $\sim 10^{-3} \div 10^{-6}$; for ferromagnetic $\chi \sim 10^3 - 10^5$. One can see that diamagnetic weakly and oppositely magnetized in an inner magnetic field and therefore pushed out from it. Paramagnetic weakly magnetized too but positively. Ferromagnetic magnetized very strong and intensively drawn in magnetic field.

Alongside with magnetic susceptibility χ of the unit volume a specific magnetic susceptibility χ_{sp} is often used in practice $\chi_{\mathrm{sp}} = (\mathfrak{J}_{\mathrm{sp}}/H)$ the relation being exist $\chi_{\mathrm{sp}} = (1/\rho)\chi$. The same is for molar magnetic susceptibility χ_{M} which is equal to

$$\chi_{\mathrm{M}} = \frac{\mathfrak{J}_{\mathrm{M}}}{H} \tag{5.2.8}$$

and

$$\chi_{\mathrm{M}} = \frac{M}{\rho}\chi.$$

5.2.3 An internal magnetic field in magnetics

One more magnetic characteristic, a magnetic permeability μ, is usually introduced; it shows how much the magnetic induction in a magnetic B is larger then that of the external magnetic field B_0:

$$\mu = B/B_0. \tag{5.2.9}$$

This means that this value should be substituted into the Bio-Savare law (5.1.16)

$$d\mathbf{B} = \frac{\mu\mu_0}{4\pi}\frac{I[\mathrm{d}\boldsymbol{l}\cdot\mathbf{r}]}{r^3} \tag{5.2.10}$$

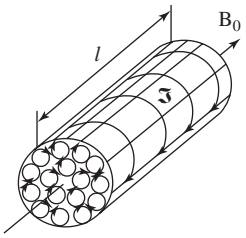


Figure 5.22 A magnetic in an external magnetic field; a circular micro-molecular current and surface currents can be seen.

Consider now a sample of a cylinder form placed in an external magnetic field with induction B_0. Let the cylinder by a cross section S and length l is oriented along the external field force lines (Figure 5.22). Under an action of a field all of molecular currents (vectors of the magnetic dipole moments) will be ordered in the field (along or opposite, it does not matter at the moment). At averaging, inside of the magnetic cylinder molecular currents will be mutually canceled. Not compensated micro-currents will be left on the cylinder's surface. The picture remained a solenoid and can be considered as a certain total macroscopic current, flowing the cylinder over (with a magnetization current J_m). We can introduce a value of current linear density $\wp$ being equal to $\wp = J_m/l$. The situation is really just like the solenoid with the magnetic field induction B_1 inside. The field inside the solenoid is defined by eqn (5.1.23). Considering the magnetized cylinder as the solenoid we can calculated the induction B_1 superimposed on the external field

$$B_1 = \mu_0 nI = \mu_0 \wp \tag{5.2.11}$$

Find the relation of the magnetic magnetization $\mathfrak{J}$ and the surface density of the current $\wp$. According to eqn (5.2.3), the magnetization is the magnetic moment of the unit volume. Therefore for this case

$$\mathfrak{J} = \sum \mathcal{M}_i / \Delta V = I\mathrm{S}/(\mathrm{S}l) = \wp, \tag{5.2.12}$$

i.e. the magnetization of a piece of the magnetic is numerically equal to the linear density of a surface current.

The expressions obtained allow one to find a ratio outside and inside of the magnetic and to establish the ratio between a magnetic susceptibility and magnetic permeability. The macroscopic field in a substance is characterized by a magnetic induction which is the geometrical sum of magnetic inductions of the external $\mathbf{B}_0$ and internal $\mathbf{B}_1$ fields, i.e.

$$B = B_0 + B_1. \tag{5.2.13}$$

Having replaced in this expression $\mathbf{B}_0$ and $\mathbf{B}_1$ according to (5.2.9) and (5.2.11), and taking also into account a collinear arrangement of all three vectors, we obtain

$$B = \mu_0 H + \mu_0 \wp = \mu_0 H + \mu_0 \mathfrak{J}. \tag{5.2.14}$$

It shows that the strength of the magnetic field inside of the magnetic differs from that of an external field on the value of magnetization. Writing further the magnetization according to (5.2.7) one can obtain

$$B = \mu_0 H + \mu_0 \chi H = \mu_0 (1+\chi) H. \tag{5.2.15}$$

On the other hand, according to (5.2.9), B = μB_0. Therefore

$$\mu = 1 + \chi \tag{5.2.16}$$

and the induction inside of isotropic magnetic is

$$\mathbf{B} = \mu_0 \mu \mathbf{H}. \tag{5.2.17}$$

This expression corresponds to the definition of the magnetic susceptibility (5.2.9). It follows that

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathfrak{J}$$

Let's note here a distinction of the induction and the strength notions. The induction of a magnetic field according to (5.2.9) depend on substance property, however the strength of the field outside and that of inside of the magnetic is the same. Moreover, the strength doesn't depend at all on the sample magnetic properties (i.e. on μ) ($H = B/\mu_0\mu = \mu_0\mu H/\mu_0\mu = H$). At the same time the induction varies at transmitting from one magnetic to another. Therefore at calculations of magnetic circuits to use the strength **H** is more convenient.

5.2.4 Microscopic mechanism of magnetization

It has already been mentioned that from the point of view of their magnetic properties, we can distinguish three main classes of substance: diamagnetic, paramagnetic and magnetically ordered substances. We shall now consider the same question from a microscopic point of view, i.e., which processes cause magnetic properties and how these properties are related to their chemical structure. We shall start with a diamagnetic.

Diamagnetics magnetize opposite to an external magnetic field and are pushed out of it. The magnetic susceptibility of a diamagnetic is thus negative and depends neither on temperature, nor on the strength of the magnetic field.

Diamagnetic properties are defined by the electron atomic orbit. It is easier to begin with one-electron atom. In a magnetic field the electron orbit precesses in the same way as a spinning top in a gravitational field (refer to Appendix 2). This precession arises because an atom possesses both magnetic and angular (mechanic) momentums (refer to 1.3.57 and Figure 1.19). Find the angular velocity ω of orbit precession.

Let the atom possess an angular momentum $\mathbf{L}$ and magnetic moment $\boldsymbol{\mu}$, directed opposite to each other (see Figure 5.23). In an external magnetic field $\mathbf{B}$, excited along an axis z, a torque $\mathbf{M}_F$ will operate $\mathbf{M}_F = [\boldsymbol{\mu}\cdot\mathbf{B}]$, directed perpendicular to vectors $\boldsymbol{\mu}$ and $\mathbf{B}$ (Figure 5.24). Under the action of this torque, vector $\mathbf{L}$ in time dt will acquire an increment $d\mathbf{L} = \mathbf{M}_F dt$ and, accordingly, $\mathbf{L}'(t + dt) = \mathbf{L}(t) + d\mathbf{L}$. The vector $d\mathbf{L}$ is perpendicular to vector $\mathbf{L}$ and therefore $|\mathbf{L}'| = |\mathbf{L}|$. Thus, the action of torque $\mathbf{M}_F$ changes the direction of vector $\mathbf{L}$, but not its length. Thereof, the plane in which the axis z and vector $\mathbf{L}$ lie, will turn by an angle $d\varphi$

$$d\varphi = \frac{dL}{L\sin\theta} = \frac{Mdt}{L\sin\theta}.$$

Since $M = |\mathbf{M}_F| = \mathcal{M}B \sin\theta$, hence

$$d\varphi = \frac{\mathcal{M}B\sin\theta}{L\sin\theta}dt = \frac{\mathcal{M}}{L}Bdt.$$

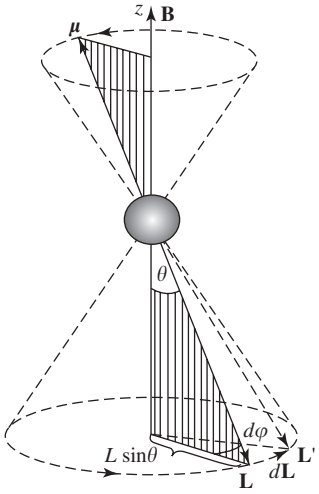


Figure 5.23 An electron orbit precession.

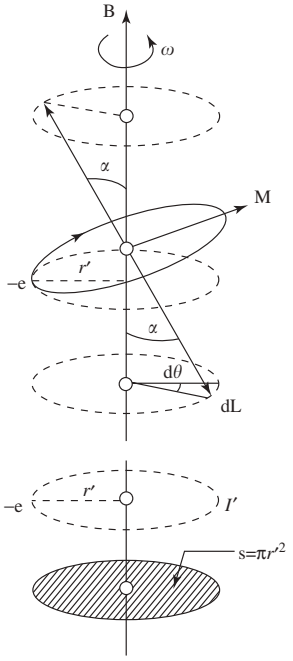


Figure 5.24 Origin of a diamagnetic moment.

Dividing both sides by *dt*, we obtain the angular velocity of the electron orbit precession

$$\omega_{\mathrm{L}} = \frac{d\varphi}{dt} = \frac{\mathcal{M}}{L}\mathrm{B}$$

Using the gyromagnetic ratio the orbit angular velocity can be found:

$$\omega_{\mathrm{L}} = g\frac{1}{2}\frac{|e|}{m}\mathrm{B} \tag{5.2.18}$$

This expression can be applied both to orbit and spin precession taking the *g*-factor into account. The angular velocity is named after J. Larmor and the magnetic precession is also referred to as Larmor precession. In particular, for orbit precession, the angular velocity is

$$\omega_{\mathrm{L}} = \frac{1}{2}\frac{|e|}{m}\mathrm{B}. \tag{5.2.19}$$

Additional electron movement caused by orbit precession leads to the excitation of an equivalent circular current I' (Figure 5.24). This current induces the magnetic moment $\mathcal{M}$, which is the diamagnetic moment. Irrespective of the direction of the

torque vector $\mathbf{M}_F$ or of the direction of induction of an external magnetic field $\mathbf{B}$, the induced diamagnetic moment $\mathcal{M}$ is always directed against $\mathbf{B}$. This is the origin of the diamagnetic effect. Therefore, the minus sign is present in the expressions for the diamagnetic moment.

The additional electron movement occurs along a circle smaller than r (the electron radius designated in Figure 5.24 as r'). A circular electric current corresponds to this movement

$$I' = \frac{e\omega_L}{2\pi} = \frac{e^2}{4\pi m} B.$$

Strictly speaking, the radius (r') is not the radius of an electron orbit; besides, it depends on the inclination angle α. As an electron orbit can be inclined to the induction direction (to an external field), averaging over all these parameters allows us to obtain

$$\langle (r')^2 \rangle = \frac{2}{3} \langle (r)^2 \rangle,$$

where $\langle (r)^2 \rangle$ is the average value of the square of the electron distance from the z-axis. Having substituted this value in the expression by $\mathcal{M}$, we shall obtain for one-electron atom

$$\mathcal{M} = \frac{e^2}{6m} \langle r^2 \rangle B. \tag{5.2.20}$$

Summing up the expression obtained for all electrons in a multielectron atom, we shall find its induced moment as

$$\mathcal{M} = -\frac{e^2}{6m} B \sum_{k=1}^{k=Z} \langle r_k^2 \rangle, \tag{5.2.21}$$

where Z is the number of electrons in an atom. If we now increase this value by Avogadro number N_A we shall obtain the value of the mole magnetizations

$$\mathfrak{I} = \mathcal{M} N_A = -\frac{N_A e^2}{6m} B \sum_{k=1}^{k=Z} \langle r_k^2 \rangle.$$

Multiplying numerator and denominator by μ_0 and comparing the expression obtained with eq. (5.2.7), we shall find the molar magnetic susceptibility of multielectron atoms

$$\chi_M = -\mu_0 \frac{N_A e^2}{6m} B \sum_{k=1}^{k=Z} \langle r_k^2 \rangle. \tag{5.2.22}$$

It can be seen that the more electrons in an atom and the larger the radius of electron orbits, the greater is the diamagnetic susceptibility. Substituting here values of fundamental physical values and accepting the radius of atoms $\sim 10^{-10}$ m, we obtain $\chi_M \sim 10^{-7} \div 10^{-8}$ m^3/mol which corresponds well to experiment for molar susceptibilities of diamagnetics.

The molar susceptibility of a number of compounds is presented in Table 5.1. Notice that diamagnetism is inherent in all substances without exception.

Paramagnetics magnetize in the direction of an external magnetic field and are drawn into it. The magnetic susceptibility of a paramagnetic is positive, depending on the temperature and value of the magnetic field strength. Paramagnetism is inherent in substances whose molecules possess permanent magnetic moments regardless of magnetic field. We shall connect all these experimental facts with the microscopic properties of paramagnetics.

In the absence of an external magnetic field, the magnetic moments of paramagnetic molecules are disordered in space by the action of chaotic thermal movement (Figure 5.25a). This means that the vector sum for magnetization (5.2.3) is zero; hence, the magnetization $\mathfrak{I}$ is also zero.

When a paramagnetic substance is brought into the magnetic field, each magnetic moment aspires to be guided in the field's direction; however, the molecule's thermal movement prevents it from doing so. A balance is established (Figure 5.25b); as a result the vector sum in eq. (5.2.3) becomes distinct from zero, the substance is magnetized.

A factor which should be taken into account when describing the competition of the ordering action of a magnetic field (whose energy is $U(\alpha)$) and the disordering tendency due to chaotic movement (with averaged energy κT) is the Boltzmann factor: $\exp(-U(\alpha)/\kappa T)$. It is necessary to take all these circumstances into account when considering the magnetization process.

Table 5.1

Values of molar susceptibilities of some diamagnetic compounds

Substance	χ_M, 10^{-11} m^3/mol	Substance	χ_M, 10^{-11}m^3/mol
Helium (He)	−2.4	Silver (Ag)	−27
Neon (Ne)	−8.2	Bismuth (Bi)	−350
Argon (Ar)	−25	Glass (SiO_2)	−50
Krypton	−40	Methane (CH_4)	−76
Xenon (Xe)	−84	Naphthalene ($C_{10}H_8$)	−240 (perpendicular to the molecule plane)

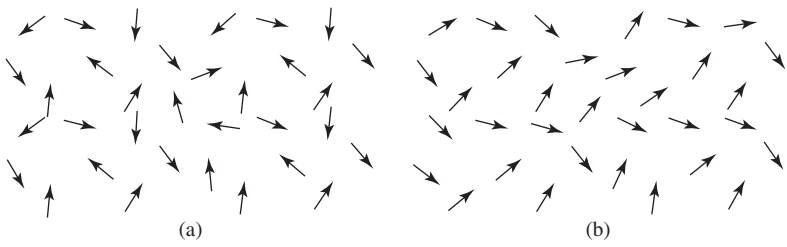


Figure 5.25 A paramagnetic (a) outside a magnetic field ($\Sigma \boldsymbol{\mathcal{M}}_i = 0$) and (b) inside a magnetic field ($\Sigma \boldsymbol{\mathcal{M}}_i \neq 0$).

An analysis of all these processes was carried out by the French physicist P. Langevin. He considered the behavior of a system of magnetic dipole moments in an external magnetic field. (An identical problem exists in dielectric physics when describing the orientation polarization, see Section 4.2.4.) The essence of both phenomena consists of the competition between the two processes: the aspiration of a field to direct the dipole moments along the field direction and the action of chaotic thermal movement interfering with it. At each temperature a compromise is achieved. Finding this compromise is the essence of the Langevin theorem. (The Langevin theorem is presented in full in Appendix 4.)

The result of this theorem is an expression for mole magnetic susceptibility of a paramagnetic substance. At $(\mu B/\kappa T) \ll 1$ the mole paramagnetic susceptibility has been found to be

$$\chi_{\mathrm{M}} = \frac{\mu_0 N_{\mathrm{A}} \mu^2}{3\kappa T} \tag{5.2.23}$$

(see eq. (A4.10) in Appendix 4). It can be seen from this formula that χ_{M} is inversely proportional to temperature:

$$\chi_{\mathrm{M}} = \frac{C}{T} \tag{5.2.24}$$

This dependence is referred to as Curie's law. Comparing eqs. (5.2.23) and (5.2.24), we can find a constant C, which is also named after Curie:

$$C = \frac{\mu_0 N_{\mathrm{A}} \mathcal{M}^2}{3\kappa}. \tag{5.2.25}$$

Substituting into formula (5.2.23) the values of fundamental physical constants and the value of the spin magnetic moment, we can obtain $\chi_{\mathrm{M}} \approx 10^{-8}$ m^3/mol, which is compatible with most experimental data (see Table 5.2 below).

In very highly magnetic fields and/or at very low temperatures ($\mu B \gg \kappa T$) the field can orient all the magnetic moments of all molecules in parallel; this results in saturation, i.e., further increase of the field intensity cannot appreciably change the magnetization of the sample since all moments are already parallel along the field

$$\mathfrak{I}_{\mathrm{M}} = N_{\mathrm{A}} \mathcal{M}. \tag{5.2.26}$$

Table 5.2

The molar magnetic susceptibility of some paramagnetics

Substance	χ_{M}, 10^{-11} m^3/mol	Substance	χ_{M}, 10^{-11} m^3/mol
Sodium (^{11}Na)	2.0	$MnSO_4$	1.7
Aluminum (^{13}Al)	2.1	Fe_2O_3	4.8
Lithium (^{3}Li)	3.1	$NiSO_4$	5.0
Vanadium (^{23}V)	37	$FeCl_2$	16
Oxygen (O_2)	430	Dysprosium (^{66}Dy)	150

In an intermediate area where $\mu B \approx \kappa T$ one dependence $\mathfrak{I}_m(H)$ smoothly passes to another (Figure 5.26). The curve repeats for orientation polarization (Figure 4.28).

Consider now a number of examples that will allow us to see how the structure of chemical substances influences their magnetic properties.

In the creation of diamagnetic properties the main role is played by the electron orbit. This means that diamagnetism is inherent to all substances without any exception. All atoms possess diamagnetic properties but they cannot always be measured because, as a rule, diamagnetism is masked by a larger paramagnetic effect. Accordingly, diamagnetism becomes apparent in substances consisting of atoms with completely compensated magnetic moments.

This takes place in noble gases (He, Ne, Ar, Xe and Rn). As an example consider a neon atom: it has 10 electrons in its electron shell ($1s^2 2s^2 2p^6$). Figure 5.27 shows the electron distribution among quantum cells. Notice that for s-electrons the orbital magnetic moment is zero ($l = 0$). The total magnetic moment of s-shells is also zero because all quantum cell are occupied by pairs; the p-shell is also filled completely. So the total magnetic moment of the neon atom is zero. In this case diamagnetic properties can be exhibited and measured.

Ions Na^+ and Cl^- whose electron configurations coincide with Ne and Ar will also be diamagnetic. On the other hand, the neutral atoms Na and Cl possess magnetic moments as there is one noncoupled 3s electron in the Na atom and a 3p electron in the Cl electron shell. While forming a chemical compound, the Na atom's 3s electron passes to the Cl atom and a NaCl molecule with ionic bond is formed. Since Na^+ and Cl^- magnetic moments do not possess noncoupled spins, the molecule NaCl is diamagnetic.

Note that, in general, the majority of chemical compounds are diamagnetic. In particular, this is true for the ionic compounds of the type considered and covalence compounds with nonsaturated bonds. By way of example, we can consider a CCl_4 molecule. Upon formation of this molecule, the carbon atom, having two noncoupled 2p electrons, is excited (Figure 5.28); 2s electrons are dicoupled and, as a result, sp^3-hybridizations are generated; four equivalent sp^3-hybrid orbits arise, with two electrons on each bond. These orbits form chemical bonds with the Cl atom. Thus, in the molecule CCl_4 there are no free noncoupled electrons and consequently it is diamagnetic.

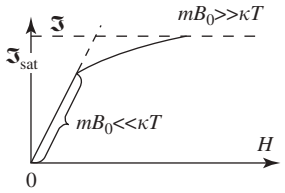


Figure 5.26 Paramagnetic magnetization versus magnetic field strength H.

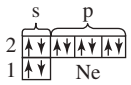


Figure 5.27 Neon atom's electron configuration.

In addition to the substances already listed, metals Pb, Zn, Hg, Sb, Ge, elementary sulfur, water and others also exhibit diamagnetic properties. (See Table 5.1 above for values of the molar susceptibilities for some of diamagnetic substances.)

The origin of paramagnetism is mainly due to the electron spins. Therefore, an analysis of paramagnetism should begin with a consideration of quantum cell filling by spins. A lithium atom, for example, has two coupled electrons in 1s-state and one 2s electron (Figure 5.29). The spin magnetic moments of the coupled electrons are zero. Therefore, the paramagnetic moment of a lithium atom is defined exclusively by the single 2s-electron spin.

A free carbon atom is also paramagnetic as it possesses a significant magnetic moment due to the two noncoupled $2p^2$ electrons. However, diamond and graphite are diamagnetic; the diamond's diamagnetism is caused by four equivalent saturated sp^3-hybrid covalence bonds participating in the crystal formation. The diamagnetism of graphite with layered structure is of the other type. The layer-forming carbon atoms are bonded by sp^2-hybrid bonds; whereas between the layers, the forces are of the van der Waals type. Within each layer probably free electron's movement is possible along the closed orbits of the large ring radius; this leads to strong diamagnetic effect.

Oxygen paramagnetism is defined by the presence of a noncoupled electron pair on the antibonding molecular orbits.

Many rare earth elements together with their alloys and compounds clearly exhibit paramagnetic properties. The atoms of these elements have incompletely filled deeply laying 4f-shells; they are shielded by outer electrons. So, for example, a dysprosium atom ^{66}Dy has four noncoupled (according to the Hund rule) 4f electrons (Figure 5.30). They define

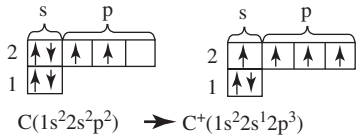


Figure 5.28 A carbon atomic configuration in sp^3-hybridization state.

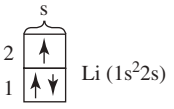


Figure 5.29 Lithium atom's electron configuration.

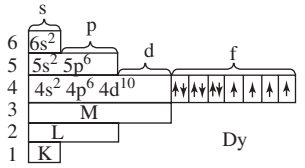


Figure 5.30 Dysprosium atom's electron configuration (K, L and M are closed and omitted).

the magnetic properties of this atom. At the same time, the outer electron shell ($5s^2 5p^6 6s^2$) takes part in bond formation and very poorly influences the magnetism. Therefore, the magnetic molar susceptibility of the rare earth element compounds depends only insignificantly on particular compounds and on what state the rare earth atom is in.

Compounds of elements with incompletely filled d-shells exhibit strong magnetic properties, especially 3d-metals. However, in this case some of the 3d electrons take part both in magnetism and in chemical bonding. Therefore, the magnetic properties of compounds of the same d-element can behave differently in different chemical compounds.

Analysis of the participation of orbital state in the formation of atomic magnetic moments shows that, for a variety of reasons, its contribution can frequently be ignored; it is either too small in comparison with spin, or is suppressed ("frozen") by a crystal field. In Table 5.2 values of molar susceptibilities of some paramagnetic substances are given.

There are, however, exceptions to this general law (for example, ^{29}Cu, ^{47}Ag, ^{83}Bi with odd numbers of electrons), exhibiting diamagnetic properties. Due to the large number of electrons (Z in eq. (5.2.22)) diamagnetism is great in these substances and suppresses paramagnetic effects. If the number of electrons is even the orbital and spin magnetic moments can be compensated in pairs.

5.3 MAGNETICALLY ORDERED STATE

A magnetic state is magnetically ordered if the atomic magnetic moments in the absence of an external magnetic field in a macroscopic crystal volume are orderly directed to each other. From this definition it is clear that a magnetically ordered state is mainly typical for a crystalline state.

There are two basic magnetically ordered structures, which can be distinguished from each other by the presence or absence of permanent macroscopic magnetization, namely ferromagnetic and antiferrimagnetic.

In the ferromagnetic-ordered state the atomic magnetic moments are spontaneously oriented parallel to each other (Figure 5.31a), and in antiferromagnetics they are oriented mutually antiparallel (Figure 5.31b). A ferrimagnetic state is referred to as an ordered state and arises when the antiferromagnetic structure consists of atoms (ions) with different magnetic moments. The magnetic moments in this case do not completely compensate each other (the moments in positions A in Figure 5.31c differ from that in position B). Macroscopically such a state appears to be ferromagnetic; however, the magnetic ordering is nearer to an incompletely compensated antiferromagnetic state.

5.3.1 Ferromagnetism

Ferromagnetics are intensively macroscopically magnetized in an external field and are strongly drawn into it. They have the greatest application techniques. Ferromagnetic

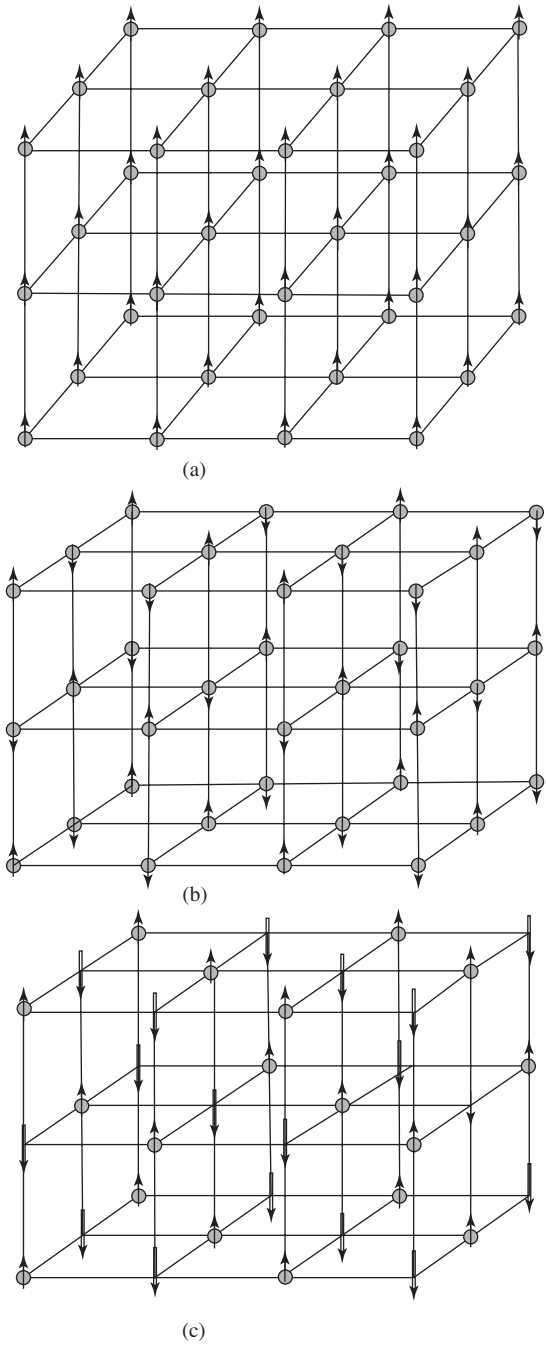


Figure 5.31 Magnetic ordering in (a) ferro, (b) antiferro and (c) ferrimagnetic crystals.

susceptibility χ is $\chi \gg 1$ and, consequently, magnetic permeability is nearly equal to its susceptibility ($\mu \approx \chi$). The ferromagnetic state is also characterized by the following features:

- the existence of spontaneous magnetization in macroscopic volumes that are referred to as domains; magnetic hysteresis, i.e., ambiguous dependence of the magnetization on the strength of an external magnetic field;
- pronounced dependence of ferromagnetic susceptibility on temperature and on the intensity of an external magnetic field;
- the presence of a Curie point, i.e., temperatures above which the ferromagnetic loses its specific properties and transforms into a paramagnetic state (for example, the Curie point for Fe is 1043 K, for Ni is 631 K, etc.).

Einstein and de Haas carried out an experiment to determine the gyromagnetic ratio (see Section 5.2.1) for ferromagnetic materials; later their results were repeatedly confirmed. Experience has inevitably led to the gyromagnetic ratio $g = 2$ instead of the expected $g = 1$ (as for orbital magnetic moments). Goudsmit and Uleneck explained this result suggesting an inherent electron magnetic moment (spin). Thus, it was established that ferromagnetism is defined by electron spin.

A typical example of a ferromagnetic is iron. Figure 5.32 shows a scheme of the electron distribution on the quantum cells in an iron atom. The d-shell has four noncoupled electrons. They define the atomic magnetic moment. In an isolated iron atom an orbital electron movement creates some orbital moment. In crystals, however, the orbital moments are "frozen" by the action of intracrystalline fields and do not contribute to the atomic magnetic moment. The reason for the "freezing" of the orbital moments is not yet completely understood. However, experience clearly shows that the orbital moments do not participate in the ferromagnetism of transition d-elements. Moreover, in the transition metals (iron, for instance) the distribution of the valence electrons on the quantum cells is influenced by many factors; so it is impossible to predict, *a priori,* in which state outer 3d and 4s electrons are. From saturation magnetization measurements (see formula (5.2.35)), it is known only that the atomic magnetic moment of iron is equal to 2.86 μ_B (Bohr *magnetons*, see Section 7.5.5).

In iron compounds the iron atom can be either in a di- or a trivalent state; in this case the electron distribution on quantum cells is certainly known (Figure 5.33). In an Fe^{2+} ion there are four noncoupled spins and in Fe^{3+} there are five such electrons. Accordingly, the magnetic moment of ion Fe^{2+} is 4 μ_B and ion Fe^{3+} is 5 μ_B.

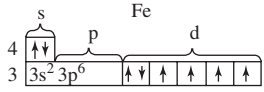


Figure 5.32 Electronic configuration of an iron atom.

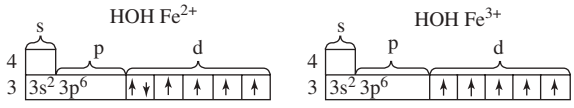


Figure 5.33 Iron's ions Fe^{2+} and Fe^{3+} electron configuration.

In addition to transition d-elements (Fe, Co, Ni), magnetic ordering (including ferromagnetic) is found in rare earth metals: gadolinium, terbium, dysprosium, etc., as well as other elements of the yttrium group, their compounds and alloys. In spite of the fact that the number of magnetically active atoms (ions) is limited, the number of their combinations in alloys and chemical compounds is very high.

The presence of noncoupled d- and f-electrons in atoms does not yet completely provide the conditions necessary for ferromagnetism to occur. For example, such elements as Cr, Mn, Pt, etc., also have noncoupled electrons, but they are not ferromagnetic. In some cases they possess antiferromagnetic behavior.

The phenomenon of magnetic ordering is *collective*; magnetic atoms themselves, being isolated from each other, do not show any ferromagnetic properties. The nature of magnetic ordering has a quantum mechanical origin, a description of which is beyond the scope this book.

5.3.2 Domains: magnetization of ferromagnetics

At temperatures lower than Curie point T_c the magnetic moments of a ferromagnetic are ordered, i.e., they build in parallel to each others. If such ordering is established in the whole sample (as in Figure 5.31a) it would be saturated even in the absence of an external magnetic field. However this is not the case. Below T_c the ferromagnetic sample splits up into small volumes referred as domains. There is nearly perfect magnetic ordering in each domain however all of them are oriented disorderly, i.e., a sample exhibits no magnetic moment. A scheme of the ferromagnetic sample splitting into domains is shown in Figure 5.34a-d; in Figure 5.34e the final splitting is depicted.

The sample splitting into domains is due to the fact that a single-domain sample would posses too big energy because of a presence on a sample edges the similar poles (Figure 5.34a). The sample splitting into domains with opposite moments directions leads to reduction of energy (Figure 5.34b-d). Partitioning the sample body into domains is accomplished by creation of the large number of domain walls to what the energy is spent. Splitting comes to the end when the gain as a result of splitting into domains is equal to expense of the domain walls production. In Figure 5.35 the scheme of such domain wall between two neighboring domains is shown.

In Figure 5.36 a curve of a preliminary ferromagnetic magnetization, i.e., dependence of a magnetization $\mathfrak{J}$ on the magnetic field strength $\mathbf{H}$ is given. Figure 5.37 describes graphically a respective alteration of the sample domain structure.

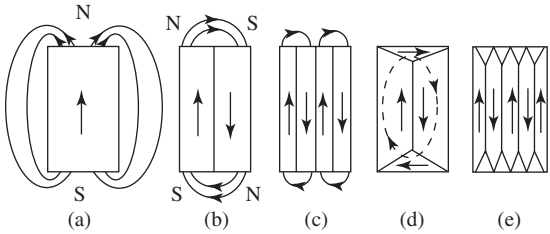


Figure 5.34 Scheme of the ferromagnetic sample splitting into domains.

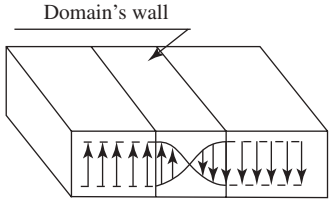


Figure 5.35 A domain wall.

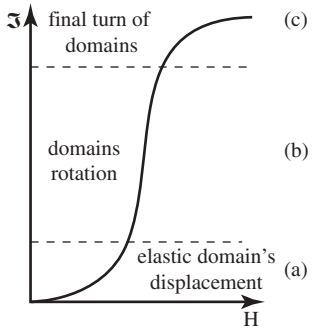


Figure 5.36 At stages of a primary ferromagnetic magnetization process: (a) elastic domain walls displacement, (b) turn of the domain magnetization directions, (c) final turn of domains directions.

The magnetization process can be conditionally divided into some stages. In the beginning (at low **H**) volume of those domains which are energetically more favorable (from the point of view of their orientation in the magnetic field) increases. This increase is accomplished by the displacement of the domain walls. The volume of "unfavorable" domain decreases in their total amount and vise versa. When process of domain walls comes to the end an additional turn of an already single domain to the direction of the external field takes place. As a result almost all atomic magnetic moments of the whole sample drew up in the direction of the external field; the sample comes nearly to saturation. There is only a small part of the moments disoriented due to the thermal vibration. Finally, all magnetic moments are aligned, there comes the saturation.

The subsequent reduction of intensity of magnetic field strength does not occur in the same way as discussed earlier. The magnetization decrease follows a curve 1–2 (Figure 5.38). When the external field disappears, magnetization is kept at a nonzero level $\mathfrak{I}_r$, referred to as *residual magnetization*. In fact, this is the magnetization that we all experienced working with permanent magnets. To demagnetize the sample, an opposite field (a part of a curve 2–3) must be applied.

The field strength corresponding to the sample demagnetization H_c is referred to as the *coercive force*. To continue increasing an opposite field, the sample magnetization will follow along curve 3–4; to then change the field direction the magnetization process will follow curve 4–5–6–1. The closed curve of the ferromagnetic magnetization (Figure 5.38) is called a magnetic *hysteresis loop*.

Modern techniques have certain requirements of various magnetic materials in respect of their saturation magnetization value, residual magnetization, coercive force, etc. Thin

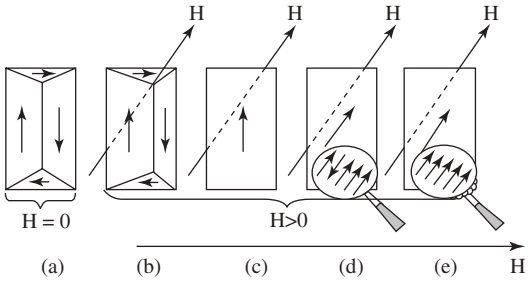


Figure 5.37 Change of a ferromagnetic domain structure at magnetization: (a) the domain structure of a sample out of the magnetic field, (b) the growth of the "favorable" domain in the external magnetic field, (c) the single domain crystal, (d) the turn of the whole domain along the external magnetic field to say nothing about the single magnetic moments, (e) saturation.

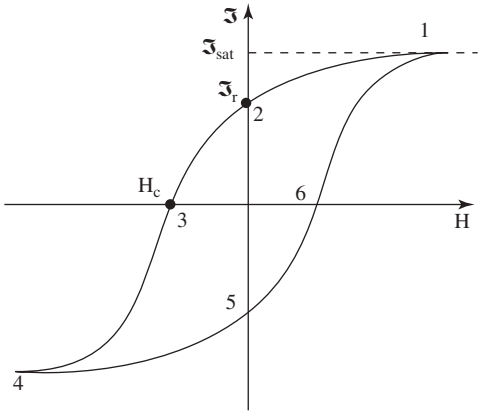


Figure 5.38 A hysteresis loop.

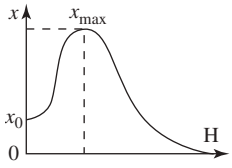


Figure 5.39 A ferromagnetic susceptibility χ in external magnetic field H.

and thick, high- and flat-loop materials are required and these can be achieved by the synthesis of new magnetic compounds and alloys. Differentiation of the primary magnetization curve (Figure 5.36) shows that the magnetic susceptibility $\chi(H)$ of a ferromagnetic depends on the strength of the external field (Figure 5.39).

5.3.3 Antiferro- and ferrimagnetics

The magnetic structure of antiferromagnetics can be imagined as two identical ferromagnetic sublattices inserted into each other with opposite moment directions. As an

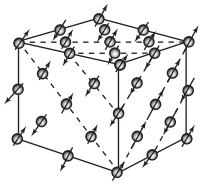


Figure 5.40 An antiferromagnetic structure of MnO.

example, Figure 5.40 shows the magnetic structure of MnO (diamagnetic oxygen atoms are not shown). It can also be visualized by ferromagnetic atomic layers with antiparallel orientation of the magnetic moments in every following layer. Naturally, such crystal does not show ferromagnetic properties, all the magnetic moments are mutually compensated. The study of antiferromagnetics is nevertheless of interest exclusively from the point of view of the theory of magnetic interactions in magnetic materials.

Also, antiferromagnetism exists in a limited range of temperature. The transition of an antiferromagnetic (as well as a ferrimagnetic) to a paramagnetic state occurs at certain temperature points, referred to as Neel points.

There is technical interest in ferrimagnetics. The majority of them are antiferromagnetics, the structure of which includes two (or more) ions whose magnetic moments differ, and therefore sublattices do not completely compensate each other in the crystal. The macroscopic magnetic moment arises. In many respects, a ferrimagnetic crystal behaves as a ferromagnetic one. The domain structure and, consequently, a hysteresis loop, also exhibit.

Magnetite Fe_3O_4, having the structure of a noble spinel ($CaAl_2O_4$) is a good example of a ferrimagnetic. The magnetite's chemical formula does not, however, reflect a valence state of atoms in this compound. It would be more correct to present the chemical magnetite formula as $Fe_2^{3+}Fe^{2+}O_4$. As we have already mentioned, the ionic magnetic moments of iron Fe^{3+} and Fe^{2+} are different (Figure 5.33). They can occupy (in ordered or disordered manner) different crystallographic positions in the structure. In every case crystals can possess different properties. Moreover, a lot of isomorphic substitutions are possible in the crystal structure. They are all referred to as ferrites. Each particular compound can have different magnetic properties (saturation magnetization, coercive force, residual magnetization, etc.). They are dielectrics and do not lose energy for induction currents and Joule heat release, which is why they are widely used in portable radio sets.

The creation of special materials with preset properties is one of the tasks of the chemist.

5.4 DISPLACEMENT CURRENT: MAXWELL'S EQUATIONS

To end this chapter it is useful to generalize all the aspects discussed; they compose a system of equations of classical Maxwellian electrodynamics from which all the laws of electricity and magnetism can be derived, including electromagnetic radiation. Nearly all the equations are already known to the readers, so we can concentrate mainly on the physical conclusions.

Let us start with those equations that describe stationary phenomena. One of the equations is the Gauss law (Section 4.1.3). Its physical sense concerns the statement: *the sources*

of an electrostatic field are motionless (in the given reference frames) *electric charges*. In fact, if there are no electric charges in a system the right-hand parts of all specified equations are equal to zero; this also leads to the absence of an electrostatic strength flux. The significance of Coulomb and Gauss laws has already been mentioned in Section 4.1.3.

The Ampere law from which the nonpotential character of the magnetic field follows is the next Maxwell equation. It follows from this law that *the source of the magnetic field is the electric current and that it is of a nonpotential character*:

$$\int_{\mathrm{L}} \mathbf{H}_{\ell}\, dl = \int_{S} \mathbf{j}\, d\mathbf{S} \quad \mathbf{H}, dl, d\mathbf{S}, \mathbf{j} \tag{5.4.1}$$

The potential character of an electrostatic field follows from equality to zero of circulation of the strength vector **E** of the electrostatic field.

Since there are no magnetic charges (monopoles) in nature, the flux of the magnetic induction through the closed surface is zero ($\oint_S \mathbf{B}\cdot d\mathbf{S} \neq 0$).

The next equation in the Maxwellian series is the Faraday's law (5.1.38)

$$\oint_{\mathrm{L}} E_{\ell} dl = -\int_{S} \left(\frac{\partial B}{\partial t}\right)_n dS, \tag{5.4.2}$$

a change of the magnetic field causes the appearance of an electric vortex field (being not of an electrostatic nature).

Comparison of eqs. (5.4.1) and (5.4.2) allows us to find an infringement of symmetry in the magnetic and electric laws. In fact, if the source of a vertex electric field is an alternating magnetic field, it can be expected that the alternating electric field should cause the occurrence of the magnetic field. From the equations presented above, this does not follow. So Maxwell came analytically to the symmetry brake phenomenon, which later originated the new notion of displacement current.

Let us consider an electric contour with a condenser as its component (Figure 5.41). If the contour is connected to a permanent electric source, the current in the circuit does not flow. If, however, the source generates an alternating voltage, there appears an alternating current in the circuit, which depends on the frequency of the electric generator and on condenser capacity. Charges will periodically appear on the condenser clamps creating a variable electric field in the condenser. At the same time lines of electric current density seem to terminate on the surface of the condenser plates. Moreover, the alternative current in the circuit creates a magnetic field around it, though such a field is unlikely to be present around the condenser. Maxwell solved this uncertainty by introducing a current inside the condenser (Figure 5.41), which closed the electric current lines. This current is referred to as a displacement current.

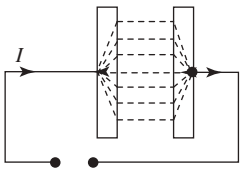


Figure 5.41 Lines of a displacement current.

Consider these phenomena in more detail. According to the definition the conductivity current I in an electric circuit is dq/dt. It can also be written as $j_{\text{cond}}\, S = dq/dt$, from which follows $j_{\text{cond}} = dq/(Sdt) = d\sigma/dt$ (since dq/S is a surface charge density $d\sigma$). On the other hand, the electric strength of the field inside the condenser is $\sigma/\varepsilon_0\varepsilon$. Therefore,

$$\left(\frac{d\sigma}{dt}\right) = \varepsilon_0\varepsilon\left(\frac{dE}{dt}\right) = \left(\frac{dD}{dt}\right) = \dot{D}. \tag{5.4.3}$$

The value $(d\boldsymbol{D}/dt)=\dot{\boldsymbol{D}}$ is the displacement current. Denote it as j_{disp}. Notice that this value is of a vector nature as far as the $\mathbf{j}_{\text{cond}}$ is concerned; then we can write $(d\boldsymbol{D}/dt)=\dot{\boldsymbol{D}}$. The displacement current force lines enclose the conduction current forces lines, besides, the displacement current produces a magnetic field around.

Therefore the current density is the sum

$$j = j_{\text{cond}} + j_{\text{disp}} = j_{\text{cond}} + \dot{D}. \tag{5.4.4}$$

Eq. (5.4.1) can be rewritten as

$$\oint_{\text{L}} \mathbf{H}_\ell \cdot d\mathbf{l} = \int_S (\mathbf{j}_{\text{cond}} + \dot{\mathbf{D}}) d\mathbf{S}. \tag{5.4.5}$$

Two more equations are included in the Maxwellian system, which connect the fields in a vacuum to the fields inside a medium (refer to Sections 4.2 and 5.2).

$$\mathbf{B} = \mu_0\mu\,\mathbf{H}, \tag{5.4.6}$$

$$\mathbf{D} = \varepsilon_0\varepsilon\mathbf{E}. \tag{5.4.7}$$

And the last equation

$$\mathbf{j}(\mathbf{r}) = \sigma\mathbf{E}(\mathbf{r}) \tag{5.4.8}$$

relates to the local characteristics of the medium. Remember that this is the Ohm's equation in differential form (refer to eq. (5.1.5)).

Let us come back to eqs. (5.4.2) and (5.4.5). There are no conductivity currents in a vacuum; therefore the last equation can, for this particular case, be rewritten as

$$\oint_{\text{L}} \mathbf{E} \cdot d\mathbf{l} = -\int_S \left(\frac{\partial \mathbf{B}}{\partial t}\right) d\mathbf{S}, \tag{5.4.9}$$

$$\oint_{\text{L}} \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\frac{\partial \mathbf{D}}{\partial t}\right) d\mathbf{S}. \tag{5.4.10}$$

The symmetry between the two is clearly seen.

It follows from them that the change in time of a magnetic field is the source of an electric field, whereas a time-varying electric field produces a magnetic field. Together they explain the existence of an electromagnetic wave. Notice that the excitation of electromagnetic waves needs only one field: the variable electric field is easier to create.

Consider in brief the principle of the creation of an electromagnetic wave. This can be produced by a dipole antenna. It is well known that in an electric contour containing inductance L and capacitance C (insertion in Figure 5.42), oscillations can be excited with a period T depending on the values of L and C. Turn the condenser plates to align them as a simple dipole antenna (Figure 5.42). Certainly, the value of the condenser's capacitance will change, but this does not interest us at the moment. Suppose, in a given instant of time, the left side of the antenna is charged positive and the right side is charged negative. In a plane xy there will be an electric field, and an excitement created immediately begins to move away from the antenna along the x-axis with a high but finite speed. This speed c in vacuum is equal to $c \sim 3 \times 10^{10}$ m/sec (see Appendix 5). After a quarter of the period, the charge on the sides of the antenna becomes zero, the force lines of the already created electric field become closed, but excitement continues the movement. After another quarter of the period, the antenna will have an opposite charge to the initial one. Again, there will be an electric field with oppositely directed force lines. After three quarters of the period, the antenna again becomes uncharged and the force lines of the second "ringlet" become isolated (closed), but in the opposite direction. All these "ringlets" will follow each other and never be caught because they move at the same speed. In a period of time, all the events will be repeated.

According to eq. (5.4.10) the change in time of an electric field generates a variable magnetic field in a plane perpendicular to the "ringlets" of an electric field. There will be similar "ringlets" of a magnetic field moving together with the first. Sometimes this picture is represented as "couplings" as shown in Figure 5.43a. However, a more realistic picture is given in Figure 5.43b: in the Cartesian coordinates two mutually perpendicular planes result in which oscillations of the vectors of the strengths of electric and magnetic fields take place. The wave is propagated with a speed c.

It was shown in Section 2.8.3 that the energy transferred by an electromagnetic wave is proportional to the square of the wave amplitude **E**. Without proof, we state that the energy density carried by a wave is defined by the vector product $[\mathbf{E}\cdot\mathbf{B}] = \mathbf{S}$; vector **S** being referred to as the Poynting vector.

The mechanism of electromagnetic radiation emission in electronic and nuclear subsystems of atoms differs significantly from that discussed above. However, all characteristics

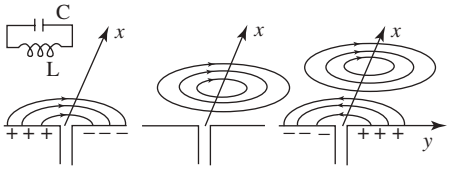


Figure 5.42 A work of a dipole antenna; inset (*top left*): an oscillation circuit.

of the radiation remain the same. A more detailed consideration of the Maxwellian equation system is given in Appendix 5.

Table 5.3 gives the range of known electromagnetic waves.

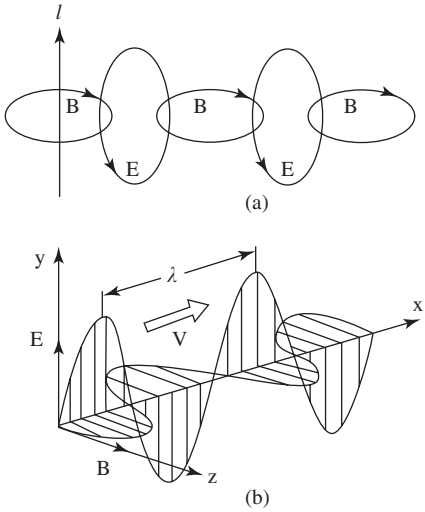


Figure 5.43 Electromagnetic wave: (a) schematic representation of electromagnetic bundles and (b) oscillation of the field inductions vectors **E** and **B** in an electromagnetic wave.

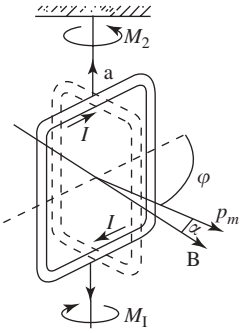
Table 5.3

Electromagnetic wave scales and their peculiarities

Energy		Frequency	Wavelength	Radiation type	Emitter	Detector
lg(E/1J)	lg(E/1eV)	(s^{-1})	(γ/1m)			
−30	−11	3	6	Long / Medium / Short / Radio waves	Oscillatory circuit	Radio devices; NMR, NQR
−29	−10	4	5			
−28	−9	5	4			
−27	−8	6	3			
−26	−7	7	2			
−25	−6	8	1			
−24	−5	9	0			
−23	−4	10	−1	Micro waves		
−22	−3	11	−2			
−21	−2	12	−3		Klystron, magnetron	Crystal diodes
−20	−1	13	−4	Infra red	Heated body	
−19	0	14	−5			
−18	+1	15	−6	Visible light.		Eye, photo
−17	+2	16	−7	Ultra violet	Exited molecules	Counter of nuclear particles
−16	+3	17	−8			
−15	+4	18	−9			
−14	+5	19	−10	x – rays	Valence electrons	
−13	+6	20	−11	gamma rays	nuclei	

EXAMPLE E5.11

A square frame with a side length of $a = 2$ cm consisting of $N = 100$ winds of a thin wire is suspended by an elastic thread. The thread elasticity C is equal to $C = 10$ μN/m grad. The frame plane coincides with an external magnetic field induction. Determine the field induction if at a current $I = 1$ A the frame plane is turned at angle $\alpha = 60°$ (Figure E5.11).



Solution: The frame is in equilibrium if two angular momentums, the action of the magnetic field $M_1 = p_m B \sin\alpha$ and the resistance of the elastic thread $M_2 = C\varphi$, are equal: $M_1 = M_2$ and, consequently, $p_m B \sin\alpha = C\varphi$. We know that $p_m = ISN = Ia^2N$ (where $S = a^2$); we can rewrite the previous equation as $NIa^2B\sin\alpha = C\varphi$. Therefore, $B = (C\varphi/Na^2 \sin\alpha)$.

In this particular case $\alpha = (\pi/2) - \varphi$ (refer to Figure E5.8); therefore $\sin\alpha = \cos\varphi$. Finally we can find $B = (C\varphi/Na^2 \cos\varphi)$ and substituting all known values arrive at $B = 0.03$ T $= 30$ mT.

EXAMPLE E5.12

A frame of area $S = 150$ cm^2 consisting of $N = 1000$ winds is rotating in a magnetic field of induction $B = 0.1$ T with frequency $n = 10$ sec^{-1}. Find the instant EMF ε_i in the magnetic field with induction $B = 0.1$ T which corresponds to the turn angle $\alpha = 30^o$.

Solution: An instant value of ε_i can be determined according to eq. (5.1.35): $\varepsilon_i = (d\Psi/dt)$, where $\Psi = N\Phi$ is a magnetic linkage. Therefore, $\varepsilon_i = -N(d\Phi/dt)$. At revolution Φ changes according to the induction law $\Phi = BS\cos\omega t$, where $\omega = 2\pi n$ is the angular frequency of rotation. Substituting the last equation into the previous equation we obtain $\varepsilon_i = 2\pi nNBS \sin\omega t$. Executing the calculations we arrive at $\varepsilon_i = 47.1$ V.

EXAMPLE E5.13

Determine the bismuth magnetic susceptibility χ and its molar susceptibility χ_M if specific bismuth susceptibility $\chi_{sp} = -1.3 \times 10^{-9}$ m^3/kg. The bismuth density is $\rho = 9.8 \times 10^3$ kg/m^3.

Solution: All the equations in use are given in Section 5.2.2. In order to solve the problem we will use the equation $\chi = \chi_{sp}\rho$: $\chi = -1.3 \times 10^{-5}$ (dimensionless quantity). To determine the molar susceptibility χ_M we have to multiply χ_{sp} by the molar mass M: $\chi_M = M\chi_{sp}$. Executing the calculations we arrive at $\chi_M \approx -2.7 \times 10^{-10}$ m^3/mol.

EXAMPLE E5.14

A bismuth ball of radius $R = 1$ cm is in a uniform magnetic field with $B = 0.5$ T. Determine the magnetic moment of the ball p_m acquired by it if the bismuth magnetic susceptibility χ is -1.5×10^{-4}.

Solution: Use the expression for magnetization $\mathfrak{J}$ (5.2.3) $\mathfrak{J} = \frac{1}{V}\Sigma_{i=1}^{N}\boldsymbol{\mu}_i$, where $\boldsymbol{\mu}_i$ is the magnetic moment of an ith bismuth atom and N is the number of atoms in the volume V. The vector qualities stay under the sum sign; however, bismuth is a very strong diamagnetic. Therefore all induced atomic moments are codirectional and equal on modulus. Therefore, we can rewrite the expression in scalar form as $\mathfrak{J} = (1/V)\ \mu_J N$. The product $\mu_J N$ is the total magnetic moment of the unit volume; therefore the ball magnetic moment is $p_m = \mathfrak{J}V$. Since the magnetization $\mathfrak{J}$ relates to the magnetic field strength H as $\mathfrak{J} = \chi H = \chi(B/\mu_0)$ and the ball volume is $V = (4/3)\pi R^3$, hence $p_m = \frac{4}{3}\pi\chi(B/\mu_0)R^3$. Express all values in SI and execute calculations:

$$p_m = \frac{4}{3}\pi(-1.5\times10^{-4})\frac{0.5}{4\pi\times10^{-7}}(0.01)^3 = -2.5\times10^{-4}\text{Am}^2 = -250\ \mu\text{A m}^2.$$

The minus sign shows that the acquired ball magnetic moment is directed opposite to the external magnetic field.

EXAMPLE E5.15

A square frame with a side $b = 10$ cm is made of a thin wire. It consists of $N = 255$ winds. The frame is in a uniform magnetic field with induction $B = 0.25$ T; the frame can rotate around an axis which passes through the middle of its opposite sides and is perpendicular to the magnetic induction force lines. Determine the maximum amplitude value of EMF induction $E_{i,max}$ arising in the frame winding with frequency $n = 1800$ min^{-1}.

Solution: The change of a magnetic flux crossing the frame occurs at the rotation of the frame. Instant value of magnetic flux $\Phi(t)$ is defined by the expression $\Phi(t) = BS\cos\varphi$, where S is the frame area ($S = b^2$); φ is the angle between the normal vectors **n** to the frame plane and that of a magnetic induction **B**. At uniform rotation this angle linearly changes with t, i.e., $\varphi = \omega t$ where ω is the angular velocity of the frame rotation. According to law of Faraday electromagnetic induction in which E_i is defined by the time derivative from the flux $E_i = -(d\Phi/dt)$ and using flux linkage notion E_i is $E_i = -(d\Psi/dt)$ or $E_i = -N(d\Phi/dt)$. Executing differentiation we obtain $E_i = NBb^2\omega \sin\omega t$. The maximum instant EMF is at maximum sin ωt, i.e., sin $\omega t = 1$.

The $E_{i,max} = NBb^2\omega$ or $E_{i,max} = 2\pi NBb^2 n$. Express all values in the SI: $N = 255$; $B = 0.25$ T; $b = 10$ cm $= 0.1$ m; $n = 1800$ min$^{-1} = 30$ sec^{-1}. Substitute them in the expression obtained: $E_{i,max} = 2\pi \times 255 \times 0.25(0.1)^2 \times 30 = 120$ V.

EXAMPLE E5.16

Two identical codirectional point magnetic moments $p_m = 5$ μAm^2 are at a distance $r = 1$ m from each other. Determine: (1) the potential energy U of their interaction; (2) the force F of their interaction.

Solution: In order to solve this problem it is convenient to use the result of Example E5.5, where an expression for the magnetic field induction on the perpendicular axes $B(z)$ was obtained. In the case of point dipole $r \gg R$ and the expression takes the form

$$B = \frac{\mu_0 IR^2}{2r^3} *.$$

The magnetic moment $\mathbf{p}_m$ of the ring with electric current I is $\mathbf{p}_m = \mathbf{n}IS$, where $\mathbf{n}$ is a normal unit vector to the plane of the ring and S is an area of the ring $S = \pi R^2$. It is easy to obtain from the equation *: $B=(\mu_0 I(\pi R^2)/2\pi r^3)$ and further $\mathbf{B} = (\mu_0 \mathbf{p}_m/2\pi r^3)$**.

(1) Assume that one dipole creates a field and the other is experiencing its action. The potential energy of the magnetic dipole in the magnetic field is defined by an expression $U = -p_m B\cos\theta$ (see Section 5.1.5), where θ is the angle between vectors $\mathbf{p}_m$ and **B.** In our case $\theta = 0$ and $\cos\theta = 1$. Therefore, $U = -p_m B$. Substituting B by the expression ** we obtain $U=-(\mu_0 p_m^2/2\pi r^3)$. Expressing all values in SI units and executing calculations we obtain

$$U = -\frac{4\pi\times10^{-7}(5\times10^{-3})^2}{2\pi\times2^3} = -6.25\times10^{-13}\,\text{J} = 0.625\,\text{pJ}.$$

(2) In order to determine the interaction force we can use the relation (1.4.28)

$$F = -\left(\frac{\partial U}{\partial r}\right).$$

Using the potential energy expression obtained after differentiation one has

$$F = -\left(\frac{3\mu_0 p_m^2}{2\pi r^4}\right).$$

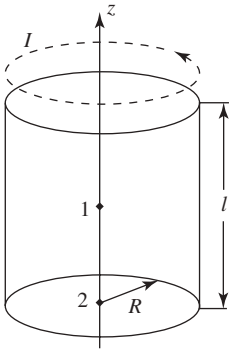
Substituting all the values into this expression (in SI) we arrive at

$$F = -\frac{3\times4\pi\times10^{-7}(5\times10^{-3})^2}{2\pi\times2^4}\,\text{N} = -9.38\times10^{-13}\,\text{N} = -0.938\,\text{nN}.$$

The minus sign in both final expressions shows that the force is attractive.

PROBLEMS/TASKS

5.1. A thin tape $l = 40$ cm in width is folded into a cylinder of radius $R = 30$ cm (Figures T5.1). An electric current $I = 200$ A flows, uniformly distributed, across the tape. Define the magnetic induction B on the axis of the cylinder at two points: at a central point of the cylinder B_1 and at a point at the end of the cylinder B_2.



5.2. Entering the uniform magnetic field with $B = 0.1$ T an electron began to move along a circle of radius $R = 5$ cm. Find the equivalent current I produced by this movement and associated magnetic moment μ.

5.3. Assume that in metallic state an iron atom contains four noncoupled 3d electrons. Determine the theoretical value of the corresponding saturation magnetization $\mathfrak{I}_{sat}$ of the metallic iron.

5.4. An infinitely long straight wire is bent at a right angle at point O. An electric current flows through the wire ($I = 100$ A). Calculate the magnetic field induction B at the points on the line of bisection of the right angle at distances $a = 10$ cm from point O to both sides of it.

5.5. An electric current $I = 60$ A flows through a thin wire bent as a rectangle. The dimensions of the rectangle are 30 and 40 cm. Calculate the magnetic field induction (B) at the point of intersection of the diagonals.

5.6. An electric current $I = 1$ kA flows through a straight wire passing through a uniform magnetic field that is perpendicular to the lines of the field induction. With what force F/l does the magnetic field act on a $l = 1$ m long wire. The magnetic field induction is equal to $B = 1$ T.

5.7. A square wire frame is placed in the same plane as a long straight wire; the wire is parallel to the two opposite sides of the square frame. The same current ($I = 1$ kA) flows along all wires. Determine the force F applied to the whole wire frame if the distance of the nearest frame side from the direct wire is equal to the length of the frame side.

5.8. A current of $I = 10$ A flows along two similar wire rings with a radius $R = 10$ cm. Determine the force F interaction between the rings if they are on the same plane parallel to each other; the distance (d) between the rings is 1 mm.

5.9. A thin ring with a radius $R = 10$ cm is carrying a charge $Q = 10$ nC. The ring is rotating at $n = 10\ \text{sec}^{-1}$ around its symmetry axis, i.e., perpendicular to the ring plane. Find: (1) the magnetic moment p_m of the ring; (2) the relationship between the magnetic moment and the angular momentum (p_m/L), if the weight of the ring is 10 g.

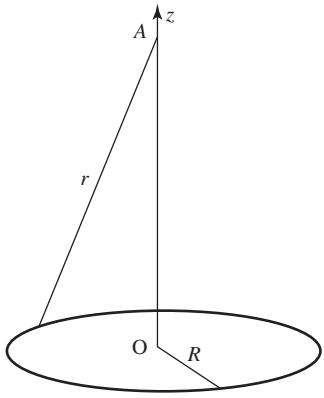
5.10. Two ions with the same charge but different masses move into a uniform magnetic field. The first ion moves along a circular path with a radius $R_1 = 5$ cm and the second ion with a radius $R_2 = 2.5$ cm. Find the ratio between the masses (m_1/m_2) of the two ions if they initially pass through the same accelerating potential difference.

5.11. An electron enters a uniform magnetic field of strength $H = 16$ kA/m with a velocity $\upsilon = 8 \times 10^6$ m/sec. The velocity vector is oriented at an angle $\alpha = 60°$ relative to the force field lines. Determine the spiral radius R and pitch h.

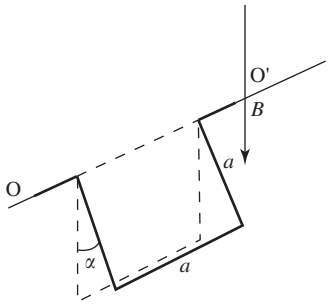
5.12. A rod of length $l = 10$ cm is spinning in a uniform magnetic field with an induction $B = 0.4$ T in the plane perpendicular to the induction force lines. The axis of rotation passes through one end of the rod. Determine the potential difference $\Delta\varphi$ inducted at ends of the rod if the rod rotates with frequency $n = 16\ \text{sec}^{-1}$.

5.13. Determine the nonuniformity of the magnetic field (dB/dx) if the maximum force acting on the point magnetic dipole is $F_{max} = 1$ mN. The dipole moment of the point dipole is $\mu_m = 2\ \text{mA m}^2$.

5.14. Find the electric current I flowing along the thin ring of a radius $R = 0.2$ m if the magnetic field induction in the equidistant point at $r = 0.3$ m on the z-axis from the ring is $B = 20\ \mu\text{T}$ (Figure T5.14).



5.15. A rectangular copper bail (Figure T5.15) of a wire section $S = a^2 = 2\ \text{mm}^2$ is in a uniform magnetic field of $B = 10$ mT directed vertically downwards. The bail can freely rotate around a line OO′. A current $I = 20$ A is flowing along the wire. Find the angle α on which the bail will turn as a result of Ampere interaction. The copper density is $\rho = 8.9 \times 10^3\ \text{kg/m}^3$.



5.16. A long solenoid is wound closely with wire $d = 5$ mm in diameter. What is the strength H of the magnetic field inside the solenoid at electric current $I = 4$ A?

ANSWERS

5.1. $B_1 = 349\ \mu\text{T}$, $B_2 = 251\,\mu\text{T}$.
5.2. $I_{ef} = 1.1$ mA, H = 10 MA/m.
$\mu = Be^2R^2/(2m) = 14.08\,\text{pA m}^2$.
5.3. $\mathfrak{J}_{sat} = 3.13\,\text{MA/m}$.
5.4. $B_1 = \dfrac{\mu_0 I}{2\pi a}(\sqrt{2}+1) = 482\,\mu\text{T}$; $B_2 = \dfrac{\mu_0 I}{2\pi a}(\sqrt{2}-1) = 82.8\,\mu\text{T}$.
5.5. $B = \dfrac{2\mu_0 I\sqrt{a^2+b^2}}{\pi ab} = 200\ \mu\text{T}$.
5.6. $(F/l) = 1$ kN/m.
5.7. $F = \dfrac{\mu_0 I^2}{4\pi} = 0.1\,\text{N}$.
5.8. $F = \dfrac{\mu_0 I^2 r}{d} = 12.6\,\text{mN}$.
5.9. (1) $\mu_m = \pi qnR^2 = 3.14\,\text{nA m}$; (2) $(\mu_m/L) = 500\,\text{nC/kg}$.
5.10. $(m_1/m_2) = 4$.
5.11. $R = 1.96$ mm, $h = 7.1$ mm.
5.12. $\Delta\varphi = \pi l^2 Bn = 201\,\text{mV}$.
5.13. $(dB/dx) = (F_{max}/\mu_m) = 0.5$ T/m.
5.14. $I = \dfrac{2Br^3}{\mu_0 R^2} = 21.5\,\text{A}$.
5.15. $\alpha = \arctan\dfrac{IB}{2\rho gS} = 29.8°$.
5.16. $H = 8$ kA/m.