1 Introduction

1.1 Role of woven fabric mechanics

The science and engineering of textiles and clothing have played an important role in one of the major technological transformations known to mankind: the computer revolution. For example, the Jacquard principle of weaving shares its basis with the binary system in the computer. Textile manufacture, particularly the woven fabric computer-aided design (CAD) system, is one of the earliest success stories in the development of CAD. Therefore, today's textile and clothing plant is significantly different from that of the past. The integration of the principal functions carried out in the production of textile materials and end products (fibres/yarns/fabrics/garments), namely product design, production planning and scheduling, manufacturing, material handling, and distribution, into a single entity is giving rise to the computer-integrated textile enterprise. The implementation of management philosophies, such as quick response and just-in-time, in the textile and apparel industries requires increased flexibility, higher quality and faster response times in new manufacturing systems. Automation and the linking of processes are two ways to reduce labour, improve quality and increase productivity. This trend towards automation and computerisation in textile and clothing manufacturing is not only inevitable but also beneficial.

However, there are still many problems preventing automation and the integration of processes for the textile and clothing industries. For example, automation of the handling and transport of apparel fabrics is of vital interest to researchers and industrialists, where the cost of labour is a significant portion of the total product cost. However, automated handling of textile materials is a difficult task because of their unique engineering properties and the variability of these properties in diverse product applications. Knowledge-based systems are required to control highly flexible automated devices for handling limp materials. These computer systems must be able to take fabric property information and predict the fabric bending behaviour or other mode deformation properties during the handling process. The computer algorithm must be based on numerical models for predicting the deformations of typical fabrics.

In addition, as consumers have become increasingly sophisticated in their demand for quality textile products, this has led to a requirement for automatic and objective evaluation of fabric appearance with respect to such characteristics as pilling, hairiness, wrinkling, etc. All these issues add up to a need for greater knowledge and more thorough understanding together with mathematical models of fabric structure and mechanics, especially in low-stress mechanical responses and their relationship with fabric structure.

Indeed, woven fabrics are the end products of spinning and weaving, but they are also the raw materials for clothing and other industries such as composites and medical textiles. The study of fabric mechanics under the low-stress conditions which exist in ordinary manufacturing and wear/application processes should be applicable to different sectors, namely apparel manufacturing, wear performance and fabric formation, as well as technical textiles.

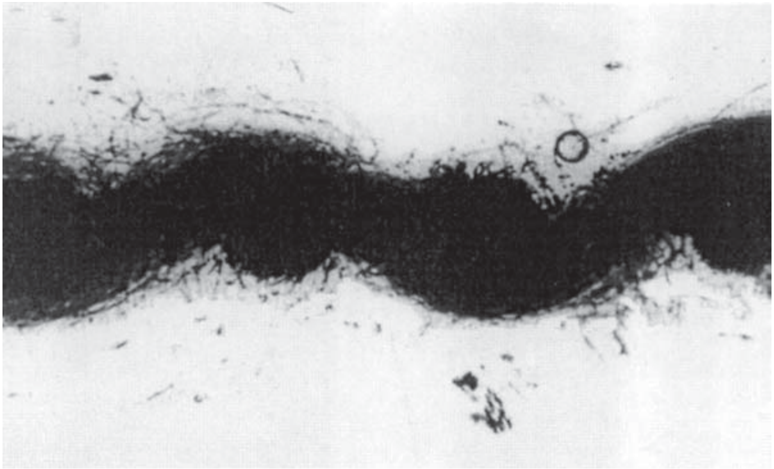
An understanding of the formation mechanisms of fabrics is useful for fabric design and process control, and includes investigation of the relationships between fibre properties, yarn structure, fabric construction and fabric physical properties. The constitutive laws of fabrics and other properties will be indispensable to the investigation of clothing construction, automation of clothing manufacturing, and computer-aided clothing design. In addition, low-stress mechanical responses are related to fabric hand, quality and performance; therefore, low-stress structural mechanics can be applied to quality control, process control, product development, process optimisation and product specification, clothing construction, automation of clothing manufacturing, and computer-aided clothing design.

1.2 General features of woven fabric mechanical behaviour

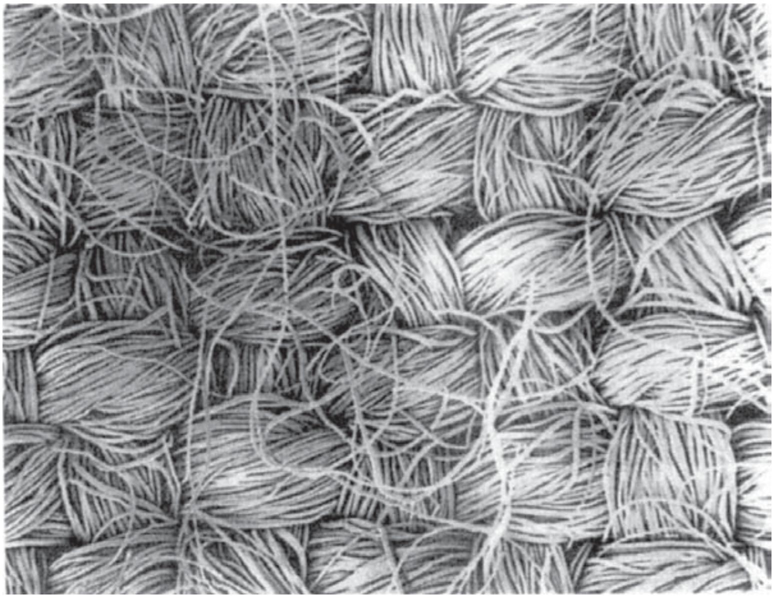
Textile materials differ considerably from conventional engineering materials in many ways. They are inhomogeneous, lack continuity and are highly anisotropic; they are easily deformed, suffering large strains and displacements even at low stress, under ordinary conditions or in normal use; they are non-linear and plastic even at low stress and at room temperature; they often achieve success rather than failure through buckling into shapes with double curvature without forming the sharp corners which appear in the case of paper when it is folded (Amirbayat and Hearle, 1989; Amirbayat, 1991). Thus they possess unique characteristics suitable especially for the human being’s body movement, for the satisfaction of the human being’s eyes and other physiological and psychological requirements.

1.2.1 Complicated geometric structure

The geometric structure of a fabric is extremely complicated. Figures 1.1 and 1.2 show photos of cross-sectional and surface images of a woven fabric. It is clear that each yarn in the fabric is crimped. The yarn cross-sectional



1.1 Cross-section image of a woven fabric.



1.2 Surface image of a woven fabric.

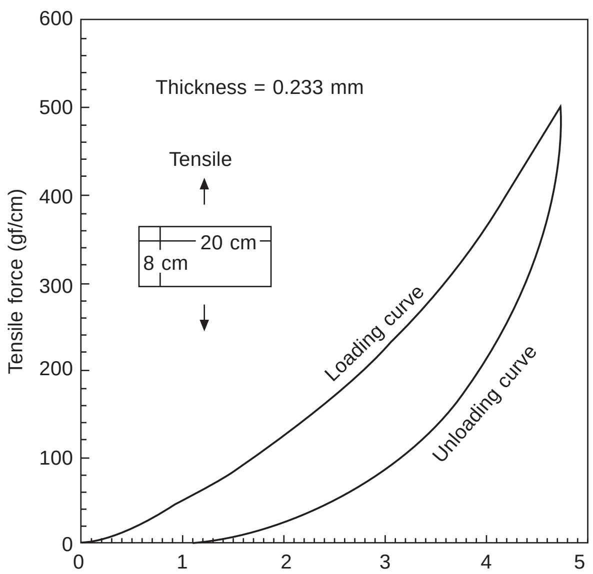
shape is rather irregular. Moreover, there are also many fibres which protrude from the yarn surfaces.

Every piece of woven fabric is an integration of warp yarns and weft yarns through intersection. The extent of this intersection is largely dependent on the friction between fibres and yarns together with fibre entanglement, while the distance between two parallel adjacent yarns determines the porosity of a fabric structure. The existence of such a discrete porous structure is what differentiates a fabric from a continuum engineering structure such as a metal sheet.

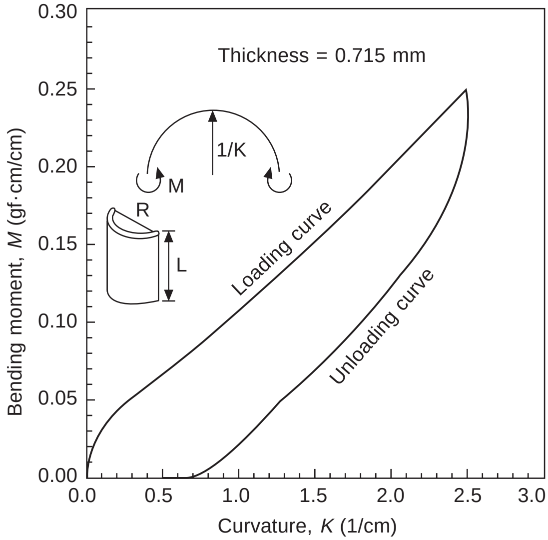
The simplest theoretical model of yarn configuration is that developed by Peirce (1937). This contrasts with reality because in the theoretical study, the cross-sectional shape and physical properties of a yarn are always simplified and idealised. However, even for this simple model, the calculations required by the geometrical parameters still involve transcendental functions (see Chapter 3).

1.2.2 Large deformability

Figure 1.3 is a typical tensile stress–strain curve of woven fabrics, where the applied tensile force per unit length is plotted against tensile strain. Because fabric sheet is very thin, the usual practice of textile researchers is to use force and moment per unit length rather than stresses in plotting stress–strain curves. This figure shows that the membrane strain is quite large even at



1.3 Tensile stress–strain curve of a woven fabric.

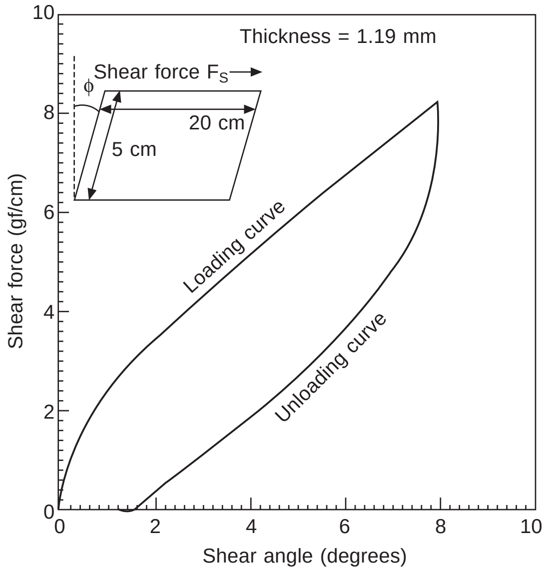


1.4 Moment–curvature curve of a woven fabric.

small forces, due to the straightening of the crimped configuration of the yarns within the fabric. The initial tensile modulus of a typical fabric is of the order of 10 MPa, compared to steel which has an elastic modulus of 2×10^5 MPa.

Compared to tensile deformations, fabrics are even more susceptible to bending deformations when under transverse loading, as shown by Fig. 1.4. Assuming that slippage between fibres is not constrained, we can easily work out the ratio of a yarn's bending stiffness to that of a solid rod of the same cross-section, i.e. $\alpha(\gamma/R)^2$, where α is the porosity ratio (the ratio of the summed area of fibres to that of the yarn cross-sectional area, and is always smaller than 1), R and γ are the radii of the yarn and its constituent fibres, respectively. For a typical yarn which contains 100 fibres, this ratio is ~1:10 000. This makes it possible to produce a thick yarn with great flexibility. In addition, due to the low thickness of fabric sheet, the ratio between bending stiffness and membrane stiffness is small. These factors contribute to the generation of a very low bending stiffness of fabrics, much lower even than their corresponding membrane (stretching) stiffness.

While large deformations can often be neglected in the engineering design of structures using stiff materials, at least in the service stage, they are required in the engineering of fabrics. Fabric under its own weight and/or external forces tends to move through these large deformations and buckle at very small in-plane compressive stresses in order to approach a state of membrane tension which it is better able to resist.



1.5 Shear behaviour of a woven fabric.

1.2.3 Non-linear stress–strain behaviour and inelasticity

Figures 1.3–1.5 show the typical stress–strain curves of woven fabrics (produced from different fabrics). For conventional engineering materials, low-stress deformations usually cause small strains which are related to stresses in a linear manner. By contrast, stress–strain curves of fabrics are complicated and generally non-linear in the low-stress range, becoming almost linear beyond a certain critical stress level. This critical value varies for different deformation modes. It is comparatively high for tension, but is very low and can be near zero for bending and shear.

This unique stress–strain behaviour of fabrics can be attributed to the porous, crimped and loosely connected structure of woven fabrics. Under tension, straightening of crimped yarns occurs at low stresses, and this is why the initial tensile stiffness is small. At high stresses when decrimping is nearly complete and inter-fibre friction is increased, the fabric structure becomes consolidated and the fibres better oriented. This leads to a stress–strain relationship close to linear, which is similar to a solid. In the intermediate range, the stress–strain curve is non-linear, reflecting the consolidation and yarn reorienting process. This behaviour makes an interesting comparison with the tensile behaviour of conventional engineering materials. For the latter, the microstructure of the material changes from order to disorder as the stresses increase. For fabrics, the applied stresses bring about order in the microstructure.

For bending and shear, when the applied stresses are low, inter-fibre friction provides a high initial resistance. However, inter-fibre slippage gradually dominates the behaviour once the frictional resistance between fibres is overcome by applied stresses, and this leads to a reduction in the stiffness as shown in Figs 1.4 and 1.5. Another interesting phenomenon observed from Figs 1.3–1.5 is that loops between loading and unloading curves exist even for low stresses, implying that irrecoverable deformations (inelasticity) occur for fabrics at small stresses. This differs from the situation for most conventional engineering materials for which inelastic deformations are usually associated with stresses which are so high that failure of the material may be imminent.

However, it is by no means the case that textile materials differ from conventional engineering materials in every way. For example, all terms such as inhomogeneous, anisotropical and non-linear come directly from conventional mechanics rather than being invented by the textile scientist. This suggests that such characteristics as non-linear, viscoelastic and inhomogeneous are problems of engineering materials. The view can be justified that the main difference between textile materials and conventional engineering materials is that the former show very complicated mechanical responses to external loads, even under ordinary conditions of low stress and at room temperature, while this happens to the latter usually under large stress, high temperature or other specific conditions. After recognition of this double identity of textile materials, it is reasonable to import conventional mechanical treatments into the study of textile in some circumstances.

1.3 Study of woven fabric mechanics

1.3.1 Summary of previous study

The study of woven fabric mechanics dates from very early work reported by Haas in the German aerodynamic literature in 1912 at a time of worldwide interest in the development of airships. In the English literature, the paper by Peirce (1937) presented a geometrical and a mathematical force model of the plain-weave structure, both of which have been used extensively and modified by subsequent workers in the field.

Considerable progress has been made over the last century in the development of the theory of geometrical structure and mechanical properties of fabrics. Responding to demands from industry, the investigation of the geometry and mechanical behaviour of fabrics has moved successively through observation, explanation and prediction. The main advances were included in the two books (Hearle *et al.*, 1969; 1980) edited by the leading figures: Hearle, Grosberg, Backer, Thwaites, Amirbayat, Postle, and Lloyd. The maturity of textile mechanics, and thus of fabric mechanics, was highlighted at the workshop at the NATO Advanced Study Institute held in 1979 (Hearle *et al.*, 1980). One of the major achievements in this process has been the development of the Kawabata Evaluation System (KES) for fabric testing, which proved to be beneficial for the objective measurement of fabric and clothing manufacturing control as well as the development of new materials for apparel fabrics. Since the 1980s the focus for research has been empirical investigations examining the relationship between the parameters obtained from the KES (Kawabata, 1980; Kawabata *et al.*, 1982; Postle *et al.*, 1983; Barker *et al.*, 1985, 1986, 1987) and characteristics such as fabric handle and tailorability. The KES system can provide five modes of tests under low-stress conditions, 17 parameters with 29 values in warp and weft and five charts consisting of nine curves for one fabric. This large amount of data was intended to provide a full description of the fabric. As a whole, it can suit a wide range of purposes in research and applications.

Research in this field in terms of methods and emphasis has taken three directions. These are:

(1) **Component-oriented:** this direction was led by Hearle, Grosberg and Postle and starts from physical concepts and assumptions which are used to facilitate further deductions and for which the theoretical basis is Newton's third law, minimum energy principles and mathematical analysis of construction. The aim is to predict the mechanical responses of fabrics by combining yarn properties, inter-yarn interactions and fabric structures with these assumptions. Many pages of mathematics and personalised programs are involved.

(2) **Phenomena-oriented:** responses of fabrics to applied loads involve elastic, viscoelastic, frictional and plastic parts. Therefore, rheological models consisting of different combination of components, such as the spring that represents the elastic part or the dashpot which represents the frictional part, simulate combined responses of fabrics to applied forces. From these, general relationships of stress–strain could be deduced.

(3) **Results-oriented:** this can be contrasted with the component-oriented direction in that it starts not from assumptions and concepts but from a hypothesis – a function or a statement to describe the experimental results. It then goes back to find the relationship of this function with fabric components such as spacing, dash pot and simulated combined responses before finally subjecting it to further analysis. The theoretical background of this approach is more concerned with pure mathematics, especially numerical methods and statistics. This type of theory is helpful in the ordering of observations. It allows estimates to be made of purely mathematical operations, thus avoiding many subjective assumptions that may be misleading. As the analysis develops, further and more complex phenomena may be revealed and an effective and realistic approach may be developed from this.

There are several questionable features which have been noticed in previous analysis of woven fabric mechanics:

(1) In general, along with the well-established exchange of ideas and the qualitative consideration of experimental results, there is a perceptible worsening of the mutual communications and practical applications as mathematical models become more and more complicated and implicit. This can lead to misunderstandings and redundancy in theoretical research.
(2) There exist few specific investigations of the explicit mathematical expression of the stress–strain relationships (constitutive laws) of fabrics.
(3) In particular, although the KES system has received wide attention for fabric objective measurement in which the investigation and application of the system are confined to the parameters extracted from the test equipment (Kawabata, 1980, Barker *et al.*, 1985, 1986, 1987), the interpretation of the charts recorded from each tester is strictly ignored. This apparent neglect of an area of important technological interest stems from the difficulties inherent in the complexity of curves themselves which are intrinsically non-linear.

Additionally, in practice, the information from the KES system is so comprehensive and extensive that it is too complicated to handle or to interpret. A technique of extracting information from massive amounts of data of this type is needed to explain the main features of the relationship hidden or implied in the data and charts.

1.3.2 Constitutive laws of fabric as a sheet

Fabric is a type of textile material and it shares the complexity characteristic of other textile materials. In order to reduce the complexity of fabric behaviour to manageable proportions, deformation must be separated into different modes.

To the first approximation, a fabric may be simulated as a sheet. In some cases, a fabric is approximated to an elastica – this was discussed by Lloyd *et al*. (1978). In engineering treatments, a simplified sheet can be subjected to four different modes of deformations which can be superposed by simple addition to give any more complicated form of deformation at a point on a sheet. In addition to two independent in-plane strains, i.e. tensile and shear strains, there are two out-of-plane deformations generated by bending and twist. In an orthogonally woven fabric, it is convenient to make use of structural axes. The desirable features of textile materials, such as double curvature, may be synthesised from the above mentioned modes of deformations. No matter how complex a fabric deformation is, constitutive laws always apply. A stress–strain relationship is usually called a constitutive equation, or constitutive law.

1.3.2.1 Basic framework

One of the simplest constitutive equations is the linear equation from the infinitesimal-elasticity theory that is applicable to the Hookean elastic body under the assumption of infinitesimal strain. Woven fabrics, however, as pointed out above, are not Hookean bodies but accord typical non-linear stress–strain relationships. Nevertheless, based on the basic frame of the infinitesimal elastic theory of a sheet, the complicated mechanical behaviour of fabric can be explored.

In the most general case, the stress–strain relationships, or constitutive laws, of a linearly elastic plate (an initially flat) sheet are as follows:

$$\begin{matrix} \text{Tensile stress} \rightarrow \\ \text{Tensile stress} \rightarrow \\ \text{Shear stress} \rightarrow \\ \text{Bending stress} \rightarrow \\ \text{Bending stress} \rightarrow \\ \text{Twist strain} \rightarrow \end{matrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_{12} \\ M_1 \\ M_2 \\ M_{12} \end{Bmatrix}$$

$$= \underset{\substack{\uparrow \\ \text{21 elements}}}{\begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{14} & B_{15} & B_{16} \\ & A_{22} & A_{23} & B_{24} & B_{25} & B_{26} \\ & & A_{33} & B_{34} & B_{35} & B_{36} \\ & & & D_{44} & D_{45} & D_{46} \\ & & & & D_{55} & D_{56} \\ & & & & & D_{66} \end{bmatrix}} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_{12} \\ K_1 \\ K_2 \\ K_{12} \end{Bmatrix} \begin{matrix} \leftarrow \text{Tensile strain} \\ \leftarrow \text{Tensile strain} \\ \leftarrow \text{Shear strain} \\ \leftarrow \text{Bending curvature} \\ \leftarrow \text{Bending curvature} \\ \leftarrow \text{Twist strain} \end{matrix} \qquad [1.1]$$

or

$$\text{Stress matrix} \rightarrow [\sigma] = \underset{\substack{\uparrow \\ \text{Stiffness matrix}}}{[S]}[\varepsilon] \leftarrow \text{Strain matrix} \qquad [1.2]$$

In equation 1.1, where T_1, T_2, ε_1 and ε_2 are the tensile stresses and strains respectively in the plane of the fabric, and T_{12} and ε_{12} are the shear stress and shear strain in the fabric plane, M_1, M_2, K_1 and K_2 are the bending stresses and curvatures, M_{12} and K_{12} are twisting stress and strain, and the submatrices A_{ij} and D_{ij} represent the membrane and bending (and twisting) stiffness respectively. The B_{ij} is coupling stiffness that connects the membrane and bending modes of deformation. In short, as in equation 1.2, $[\sigma]$ is the stress matrix, $[S]$ the stiffness matrix and $[\varepsilon]$ the strain matrix. Thus, in the general case 21 stiffnesses are required to specify the elastic behaviour of an originally flat sheet: six for membrane deformations, six for bending and twisting, and nine for coupling between the two modes.

Fabrics are usually assumed to be orthotropic, i.e. they have lines of symmetry along their two constructional directions, and the stiffness matrix [S] for linear elastic situation becomes

$$[S] = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{14} & B_{15} & 0 \\ & A_{22} & 0 & B_{24} & B_{25} & 0 \\ & & A_{33} & 0 & 0 & B_{36} \\ & & & D_{44} & D_{45} & 0 \\ & & & & D_{55} & 0 \\ & & & & & D_{66} \end{bmatrix} \quad [1.3]$$

where directions 1 and 2 are assumed to coincide with the principal directions of orthotropy, i.e. the warp and weft directions in a woven fabric. As summarised by Lloyd, this has 13 independent stiffnesses, reducing to 12 if the coupling matrix is symmetric; to eight if the fabric is symmetric; to eight if the fabric is symmetrical about its central plane so that the B_{ij} disappears; to 6 for a square fabric such as a plain-weave with the same yarns in each direction; to four for an isotropic sheet with bending behaviour unrelated to planar behaviour; and to two plus the thickness for an isotropic solid sheet. However, if the relationship were non-linear, many of the interaction terms would reappear. The interpretation of the parameters is made by Lloyd (1980) using the special case of an orthotropic fabric, initially flat, with no elastic coupling between membrane strain and bending/twisting modes.

1.3.2.2 Extensions to basic framework

The treatment of low-strain, linear elastic deformations is unrealistic in relation to textile materials. However, the framework outlined above opens up more realistic possibilities. Lloyd (1980) discussed various modifications to deal with the non-linearities common in fabric deformations: non-linear material properties, large strains and large displacements. Particularly for non-linear material properties, if the form of non-linear stress–strain laws is already known, the tangential elasticity matrix [S_T]

$$[S_T] = \frac{d[\sigma]}{d[\varepsilon]} \quad [1.4]$$

can be used in the continuum analysis. Alternatively, if [S] is kept constant, the resulting linear elastic solution will require corrections to the stresses calculated from the previous step. If the initial stresses are zero at zero displacement, then the non-linearities can be contained in [σ_0] and used to apply the necessary corrections. This is known as the initial stress method in such analysis as finite element methods.

1.3.2.3 Mathematical modelling of fabric constitutive laws

As can be seen in the above treatment of non-linear fabric properties, finding non-linear stress–strain relationships of any single deformation mode is necessary for the general continuum analysis. The widespread use of computers and the development of numerical techniques such as the finite element method opens up new possibilities: attempting problems such as fitting woven fabrics to a three-dimensional surface becomes feasible; other complex fabric deformations can be predicted; and clothing CAD systems can be developed. All these need the relationships between the constitutive laws governing fabric extension, shear and bending. However, the mathematical modelling of fabric stress–strain relationships is a very tough topic. During the last 60 years, many outstanding textile scientists, including F.T. Peirce, J.W.W. Hearle, P. Grosberg and R. Postle, have devoted their talents to this field. However, their theories are self-contained, that is it is difficult to apply the results of one piece of research work to another. For example, even though there are many papers and books on fabrics, it is well known that fabrics are non-linear and elasto-plastic in nature. In the investigation of fabric complex deformations, like drape (Collier *et al*., 1991) or ballistic penetration (Lloyd, 1980), one also assumes that basic deformation behaviour, like tensile, obeys the Hookean law of solid materials. The reasons for this stem from the complex procedures of prediction or, basically, the fact that the development of mathematical models for woven fabrics is an extremely complicated and difficult task due to the large numbers of factors on which the behaviour of the fabric depends. Usually, a mathematical model is based on a large number of assumptions, covering missing knowledge or inability to express some of the relevant factors. It is not surprising that, out of the huge bulk of works published in the area, a considerable amount appears to be of theoretical interest only and largely inadequate to cope with real fabrics. Therefore, it is necessary to introduce a different approach for the mathematical modelling of fabric constitutive equations.

With fabric, fundamental distinctions may be made between three kinds of modelling, namely: predictive, descriptive and fitting or numerical models. The predictive models, as developed by Hearle *et al*. (1969) and Postle *et al*. (1988), which form most of the existing research into fabric mechanics, are based on the consideration of at least the most important of the relevant factors, while the effect of the remaining ones is covered by suitable assumptions, defining the limits of validity and the accuracy of the resulting theories. Under these restrictions, the predictive models are directly characteristic of the physics of the fabric and permit the evaluation of the effects of the various parameters involved and the development of design procedures. Models of this form may provide a basis for evaluation of the internal state of the fabric at a microscopic level, for example, the state of stress developed between warp and weft yarns under strictly determined fabric geometries and loading conditions.

The transition from the microscopic level to the macroscopic one is usually obtained through the concept of the representative unit cell. In this way, it is possible to derive a stress–strain curve for the fabric in any of these modes of deformation and to evaluate the build-up in the level of internal forces or lateral pressures acting within the fabric as it is deformed. A detailed study of the mechanisms of fabric deformation is therefore possible, yielding relationships between the structural parameters of a woven fabric and its important mechanical properties. The number of assumptions, for models of this kind, required for an exact theory is obviously high. It is necessary to include a number of initial assumptions relating to the nature of yarn contacts and yarn cross-sectional shape within the unit cell of the fabric. Such assumptions are usually based on a great degree of simplification and they are liable to introduce large errors in any analysis of fabric mechanical or rheological properties.

However, the treatment of this relationship is usually too complicated either to understand or to apply. The increased mathematical complexity of the better solutions has made them less accessible to those who might use them, or even to other specialists. These approaches all require several pages of mathematics. Some of it is interesting, but a good deal of messy algebra has made them difficult or impossible to apply to more realistic situations.

The descriptive models (Paipetis, 1981), on the other hand, are largely empirical and reflect the need for simple mathematical relations, expressing the phenomenological behaviour of a fabric from the point of view of a particular property. For example, linear viscoelastic materials can be modelled by means of properly connected spring-and-dashpot elements. However, such models completely ignore the physics of the material, need adjustment to reality through a number of experimental values and operate within a specific range of the relevant parameters only. Still, they are undoubtedly useful, if no rigorous models are available.

In contrast to the complexity of the predictive models and the subjectivity of the descriptive models, some sort of simple mathematical equation may be used to relate stress–strain. Even if no sensible physical relationship exists between variables when introducing the function and even although the equation might be meaningless, it may nevertheless be extremely valuable for predicting the values of fabric complex deformation from the knowledge of stress or strain. Furthermore, by examining such a function we may be able to learn more about the underlying relationship and to appreciate the separate and joint effects produced by changes in certain important parameters.

These are fitting or numerical models. The modelling of this group, at the first stage, may ignore the exact mechanism taking place within the structure but emphasise the numerical relations of two variables such as stress–strain relations. This method is based on statistical considerations; it needs fewer assumptions and provides, perhaps, an approach more relevant to real situations.

There exist various methods for fitting a curve in many industrial or science fields. Constitutive laws are often estimated by using a polynomial, which contains the appropriate variables and approximates to the true function over some limited range of the variables involved. Spline, especially the cubic spline interpolation method, is also widely used for this purpose. The research work in this field, which has received comparatively little attention, can be seen in Kageyama *et al.* (1988).

1.3.3 Computational fabric mechanics

Section 1.3.2 has in fact touched on the content of computational fabric mechanics. In this section, a more specific introduction to this technique is given. Since the workshop at the NATO Advanced Study Institute (Hearle *et al.*, 1980), progress in fabric mechanics has begun to slow down. The hindrance to further development of fabric mechanics stems from the complexity of the mathematical equations used to describe the complex behaviour of fabrics. The very limited solvability of these equations by traditional analytical techniques has caused much frustration among the research community, which is increasingly losing confidence in the significance of fabric mechanics in practical applications. As the mathematics becomes more complicated and less transparent, there is also a perceptible worsening of communication between theoreticians and experimentalists, leading to misunderstandings on both sides and redundancy of theoretical research. Even Hearle, who has worked in textile mechanics for about 50 years, advocated the application of advanced computational techniques as the way forward (Hearle, 1992).

Computational fabric mechanics presents a unique opportunity where cooperation between researchers with different backgrounds will be most effective. The many challenging numerical problems will be of interest to the computational mechanics community, while the participation of textile material scientists will ensure a balanced and practically useful approach. The final product should be an intelligent CAD system, the development of which relies heavily on the contribution of computer graphics experts.

1.3.3.1 General

The application of computational techniques in fabric mechanics first appeared in the late 1960s. Konopasek, Hearle and Newton at the University of

Manchester Institute of Science and Technology (UMIST) first launched a project to use computer programs to approach textile mechanics problems including fabric behaviour (Hearle *et al.*, 1972). Computational techniques have in fact gained wide application in many engineering areas: airplane designing, machine manufacturing, civil engineering, etc. One key algorithm used in computational techniques is the numerical method, particularly the finite element method, which enables the possibility of accurately predicting the behaviour of an engineering structure under a certain loading condition. Therefore, in this section, particular emphasis is put on the finite element method as well as on fabric deformation analysis.

Continuum models

As the name says, in these models, the fabric is treated as a continuum without explicit reference to its discrete microstructure. Established mathematical methods in continuum mechanics can then be applied to the analysis of fabric deformations. In the first attempt at using computers to obtain continuum solutions to fabric deformation problems, numerical solutions were adopted after differential equations had been set up. However, this approach was difficult to apply to complex non-linear deformations of fabrics as specific equations needed to be established and a computer program needed to be written for a given problem. Representative work may be found in Konopasek (1972), Lloyd *et al*., (1978), Shanahan *et al.*, (1978), Brown *et al.* (1990), Clapp and Peng (1991).

A more versatile and powerful approach is the finite element method which can be applied to predict fabric behaviour under complex conditions. The finite element method was initially developed for engineering structures made of steel and other stiff materials. It has been developed since the 1950s and is now an essential analysis tool in many engineering fields (Zienkiewicz and Taylor, 1989, 1991). In this approach, the cloth is divided into many small patches which are called the finite elements. The cloth needs to be modelled using flat or curved shell elements, as both bending and stretching are involved.

Several researchers have attempted the finite element approach with varying degrees of success. The earliest attempt was made by Lloyd (1980) who achieved some success in dealing with in-plane deformations. Collier *et al*. (1991) developed a large-deflection/small-strain analysis using a 4-noded shell element and treated the fabrics as orthotropic sheets with properties determined from KES testers. They analysed the draping of a circular piece of fabric over a pedestal as in a traditional drape test. Their numerical draping coefficients agreed reasonably well with experimentally determined values. Gan *et al.* (1991) produced a similar analysis employing a curved shell element which belongs to the degenerated isoparametric family (Surana, 1983). They presented numerical results for the draping of a circular piece of cloth over a circular surface and a square piece over a square surface. No comparisons with results from other sources were presented. Kim (1991) also treated fabrics as orthotropic sheets in his large-deflection analysis using shell elements and presented several examples of fabric draping. He was also the only researcher to provide quantitative comparisons which demonstrated that the deformed positions of the draped fabric predicted by his analysis differ from those from physical tests by about 10 %. Another similar study is described briefly by Yu *et al.* (1993) and Kang *et al.* (1994).

The above facts show that it is possible to simulate fabric drape by non-linear finite element analysis treating fabrics as two-dimensional orthotropic sheets with both bending and membrane stiffnesses. These studies have only been able to analyse simple draping tests. Analysis of deformations is more difficult for fabrics than for other conventional engineering materials. Much work needs to be done before an accurate, reliable and efficient analysis can be developed to model all possible deformation modes in fabrics. In the immediate future, more work should be carried out to produce more precise comparisons between numerical results and physical experiments for a variety of draping cases. This will further establish the validity of the continuum approach in modelling fabric deformations.

Another area that has not been touched upon is the effect of non-linear stress–strain relationships on fabric deformations. This is partly due to the lack of explicit non-linear constitutive equations of woven fabrics in the past. Recently, Hu and Newton (1993) and Hu (1994) described a comprehensive study of the structures and mechanical properties of woven fabrics in which they established a whole set of non-linear constitutive equations for woven fabrics in tension, bending, shear and lateral compression. The inclusion of these equations in finite element simulation is expected to improve prediction accuracy in many cases and shed light on the effect of non-linear properties of fabrics on garment appearance and performance.

Discontinuum models

In contrast to the continuum model, fabrics may be modelled as an assemblage of their constituent yarns. Grosberg and his co-workers (Grosberg and Kedia, 1966; Nordy, 1968; Leaf, 1980), Hearle and Shananhan (1978), Postle *et al.* (1988) and Ghosh *et al.* (1990) adopted discrete models to predict mechanical responses of fabrics by combining yarn properties, inter-yarn interactions and fabric structures. Their work is analytical, rather than numerical, involving many pages of mathematics with the aid of personalised programs in the solution phase. In the textile literature, this work is usually referred to as structural mechanics of fabrics (Hearle *et al.*, 1969).

Viewing the yarns as curved or straight rod elements with frictional connections at the crossing points between the warp and weft yarns, the finite element method can be extended to study fabrics using discontinuum models. Torbe (1975) defined a cruciform element with arms in the directions of the threads in woven fabrics. In the same paper, the element stiffness matrix was derived, but no example of its actual application was given. Leech and Abood (1991) dealt with the dynamic response of fabric subject to tensile and tearing loads.

A discontinuum model by itself has limited value in predicting complex fabric deformations due to the prohibitive number of yarns present, but may be useful in predicting fabric mechanical properties from yarn properties, because only a small patch of cloth needs to be modelled. The problem is thus computationally feasible. Realistic constitutive laws required for fabric deformation analysis at present are only obtainable in laboratory tests. However, such laboratory tests are not possible before a particular fabric is actually manufactured. The discontinuum method may enable the accurate modelling of fabric deformations before they are manufactured.

1.3.3.2 Other approaches

Researchers in the computer graphics community are interested in producing cloth-like behaviour for computer animation. They have produced various models based on a geometric process and/or a simplified physical model, but their purpose is not to produce accurate deformation predictions of a particular deformable material. Geometric processes, together with simple physical constraints, have also been applied successfully in the composites manufacturing field.

Breen *et al.* (1994) proposed a particle-based model to simulate the draping behaviour of woven cloth. In their physical model, the cloth is treated not as a continuous sheet but as a collection of particles that conceptually represent the crossing points of warp and weft threads in a plain weave. The various constraints and interactions between particles are represented by energy functions which are defined using KES test data. Some promising results have been obtained. This kind of model has now become almost standard for various systems of cloth simulation.

1.3.3.3 Future of computational fabric mechanics

Dictated by fashion trends, textile and clothing products move through fast cycles of renovation. Just-in-time and quick response systems are becoming increasingly important in the textiles and clothing industries. Consequently, new technologies such as automation of production processes for textiles and clothing are attracting much attention. Computational fabric mechanics and understanding of fabric structure have much to offer in realising these new technologies. This section provides a brief examination of some of these areas, particularly those related to fabric deformations and clothing CAD, where application of computational fabric mechanics should be fruitful.

Complex fabric deformation and clothing CAD

In practical use, textile fabrics are subject to a wide range of complex deformations such as drape, handle and wrinkling or buckling. If textile technologists and clothing designers are to be able to make a rational engineering design of a new fabric or garment, then these complex deformations of fabrics must first be understood. With improved understanding of the deformation characteristics of various fabrics, it is then possible to design new fabrics targeted to the needs of specific end uses.

The ultimate aim is to enable a future garment designer to carry out the whole design and simulate the final product using a computer. The computer will automatically produce completed patterns based on a vivid picture drawn freehand by the designer and a few comments on the requirements of fabrics and clothing styles. The designer can then see the garment dressed up on a body simulated using computational fabric mechanics and computer graphics. In this way, a designer or customer can survey the scene as if it were a fashion show (a virtual reality fashion show!).

Automation of clothing industry

Automation and the linking of processes are two ways to reduce labour, improve quality and increase productivity in a modern enterprise. For example, automation of the handling and transport of apparel fabrics is of vital interest to industrialised nations, where the cost of labour is a significant portion of the total product cost. However, automated handling of textile materials is a difficult task because of their unique engineering properties and the variability of these properties in diverse product applications. To automate the handling process, computer software must be developed which can predict fabric bending behaviour and other modes of deformation during the handling process based on fabric property information. Such computer software will only come with developments in computational fabric mechanics.

Other applications

Computational fabric mechanics may be interpreted to include many other aspects apart from fabric deformations, although they are the most important in developing clothing CAD systems. It may be expected that computational fabric mechanics will be equally useful in studying thermal behaviour, fatigue and wear behaviour, and air and water permeability, and dynamic problems such as the ballistic penetration behaviour of fabrics for military garments.

1.4 References

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