4
The tensile properties of woven fabrics

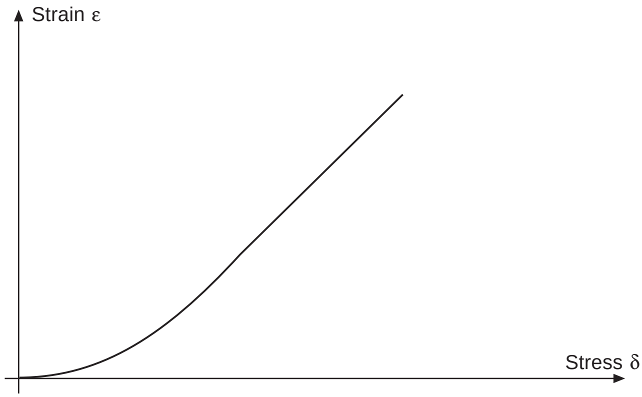
4.1 General tensile behaviour of woven fabrics

4.1.1 Introduction

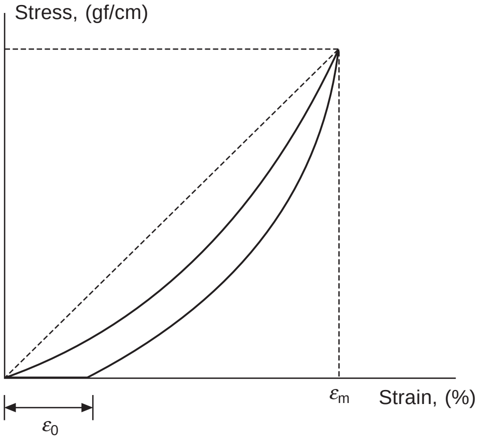
Tensile properties are one of the most important characteristics governing fabric performance in use. Their study involves many difficulties due to the great degree of bulkiness in fabric structure and the strain variation during deformation. In particular, each piece of fabric consists of a large quantity of constituent fibres and yarns and hence any slight deformation of the fabric will give rise subsequently to a chain of complex movements of these. This makes the situation more complicated since both fibres and yarns behave in a non-Hookean way during deformation and present hysteresis with time effect (Konopasek, 1970).

4.1.2 Tensile stress–strain curve of woven fabrics

Figure 4.1 illustrates a typical tensile stress–strain curve of a woven fabric derived on the KES-F apparatus. For this curve, the initial region demonstrates



4.1 Tensile stress–strain curves.



4.2 Loading and unloading cycle in the tensile stress–strain curve.

a low slope due to decrimping and crimp-interchange. After that, the slope of the stress–strain curve rises steeply until its summit is reached, an effect which can be assumed to stem from the induced fibre extension. In addition, the magnitude of the summit of the stress–strain curve is governed by the level of yarn crimp and the relative ease of distortion of the yarn *per se*.

If what the fabric undergoes is a cyclic loading process, i.e. the fabric was first stretched from zero stress to a maximum and then the stress was fully released, then an unloading process will follow the loading process. As a result, a residual strain, ε_0, will be observed since textile materials are viscoelastic in nature. Due to the existence of residual strain, the recovery curve will never return to the origin, as shown in Fig. 4.2. This is the hysteresis effect, which denotes the energy lost during the loading and unloading cycle. Due to the existence of hysteresis, a deformed fabric cannot resume its original geometrical state. In Fig. 4.2, the shift to the right from the origin of the unloading curve depicts the magnitude of the hysteresis effect and indicates the amount of permanent set resulting from the loading history.

4.1.3 Extension in the principal directions

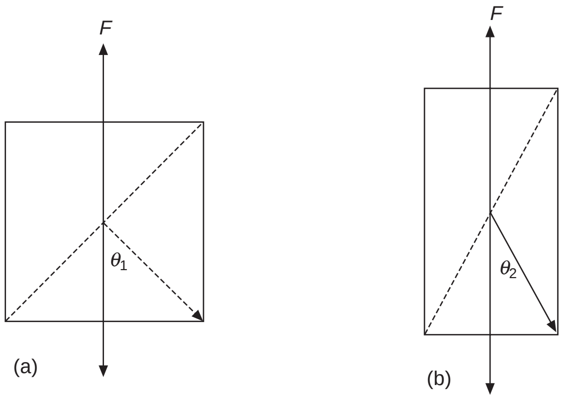
Usually, when a plain woven fabric is extended in either of the principal directions, a straightening of the crimped yarns will also occur in the direction of force. The interaction between the two sets of yarns must therefore be considered. A drop in the yarn amplitude and the weave angle will thus be found when the contact of two sets of yarns in the warp and weft directions grows. During tensioning, these yarns appear to become less flattened due to their consolidation into a rounder or more circular cross-section (Hearle *et al*., 1969). In addition, the crimp-interchange will take place at the crossover point, i.e. one set of yarns increases in crimp level, while the other decreases.

When the fabric is further extended by an applied force, yarn and fibre extensions will occur, but the yarn extension only accounts for a small portion of the total extension as compared with the effect of decrimping. Individual fibre movement within the yarns also occurs at the contact point of two sets of yarns. This movement allows the fibre to avoid the high strains that might be induced by extension. Energy loss will also take place during tensioning due to the restriction of fibre movement posed by inter-fibre strains.

Regarding the uniaxial tensile properties of plain woven fabrics, De Jong and Postle (1977a,b) stated that there are six independent dimensionless parameters to be considered in the case of a balanced woven fabric (produced from identical warp and weft yarns). The six parameters include: (a) the ratio of warp to weft yarn length per crossing yarn; (b) the ratio of yarn diameter to yarn modular length; (c) the ratio of yarn compression rigidity to bending rigidity; (d) the yarn compression index; (e) the ratio of yarn extension rigidity to bending rigidity; and (f) the degree of set. They also stated that the effect of the ratio of yarn compression rigidity to bending rigidity on the relative fabric extension can be accounted for by the yarn extension. Thus, a large part of the fabric extension can be explained by yarn extension when this ratio is lower. In addition, the average of Poisson's ratios can be explained in the selected range where the inter-yarn distance can increase to allow the yarns to turn into a rounder or more circular cross-section during tensioning.

4.1.4 Extension in bias directions

When a plain woven fabric is extended to its final state in bias direction, it can be seen from Fig. 4.3 that the warp yarn will rotate which brings the maximum elongation close to the direction of force (*F*). There is a deviation



4.3 Fabric extension in bias directions (arrow within square indicating the warp direction): (a) initial position before extension; (b) final position after extension.

in the direction ($\theta_1 > \theta_2$) at the final position. If a force is applied, the fabric will suffer a strain to reach its final position. The magnitude of the strain is governed by the deviation of the angle to the warp direction, and the diagonal direction (± 45°) presents the maximum changes in length. As a result, the maximum elongation will occur at one diagonal and the other diagonal direction demonstrates the maximum contraction. Thus the characteristics of the Poisson effect should be identified by the changes in length parallel and perpendicular to the direction of force.

If stress is applied at an angle to the warp or weft direction, the mode of deformation will become rather complicated, presented as a combination of extension and shear. If the unit cell of the plain woven fabric is regarded as a trellis, it will be extended by rotating the numbers of the trellis relative to each other in bias directions (Weissenberg, 1949).

During KES-F tensile and shear testing, if a fabric is prepared in bias directions, for some yarns, only one or neither of their ends will be clamped. As a result, three cases need to be considered in this event:

(1) both ends of the yarns are held between two clamps for the extension of the warp or weft direction;
(2) only one end of the yarns is held for the extension from ± 15° to ± 75° (bias directions) to the warp direction;
(3) both ends of yarns are free for the extension of 0° or 90° to the warp direction corresponding to the warp or weft direction.

When deformation takes place in bias directions, little tension will exist in the yarns. However, due the existence of the frictional restraint between the interlaced yarns at their contact points, there is still some tension in the yarns between two neighbouring crossovers and a bending couple in deforming the crossed yarns (Spivak and Treloar, 1968). In addition, shear properties will lead to an increase in the tension of individual yarns between the contact points.

Due to the above reasons, a higher magnitude of tensile properties will be recorded in the bias directions than in the warp and weft directions. As higher crimp is usually obtained in the warp yarns, the values of tensile properties in the warp direction are comparatively higher than those in the weft direction. All these observations are discussed with reference to the experimental findings in the following sections.

4.2 Modelling of tensile behaviour of woven fabrics

4.2.1 Introduction

The pioneer in the investigation of tensile deformation of woven fabrics is Peirce (1937). His model assumed that the cross-section of the yarns in the fabrics is circular, but this assumption of circular yarn cross-section in the fabric is highly theoretical. Therefore, many researchers modified his geometrical model to analyse tensile behaviour. Based on Peirce's rigid thread model (1937), Grosberg and Kedia (1966) analysed the small strain within the initial load–extension curve. Three approaches are reported by Leaf (1980) in order to analyse the tensile behaviour of plain woven fabrics. His first approach is based on Gastigliano's theorem for small deformation only. A force equilibrium method and an energy approach are used for the analysis of large deformation.

In an attempt to define theoretically the planar stress–strain relationships of woven fabric, Weissenberg (1949) introduced the trellis model. He treated the yarns as rods which are inextensible and inflexible. The yarns are pinpointed at their nodal points with lines of zero elongation in the pattern by offering no resistance to the allowed changes in the orientation of the lines. With these assumptions, the theory of strain, stress and the relationship between them is clearly defined. He also stated that the Poisson effect and evading action are observed when external forces are applied. Textile material in its nature and applied state is anisotropic, and the orientation of the framework of the trellis to the direction of the pull would vary with the direction of pull in the material. The Poisson's ratio could be found from a given lengthwise extension and a given widthwise contraction. He indicated that the experiments made on the model would predict a modified Poisson effect with maximum elongations and contractions occurring not parallel and perpendicular to the direction of the pull but along the bisectrices. In addition, there would be a rotation of the directions of maximum elongation and contraction, which would bring, in the terminal position, the direction of maximum elongation nearer to that of the pull. He described the material as taking an 'evading action' by rotating round and having its maximum extension in a direction different from that of the pull.

Chadwick *et al.* (1949) investigated the bias deformation of a woven fabric with the application of the trellis model under a simple pull. They showed that the warp and weft yarns underwent changes not only in length and spacing under bias extension, but also in their orientation to one another and to the direction of pull. In their experimental work, rectangular specimens are cut out in various directions. Each specimen is subjected to a series of simple pulls of increasing amount in a direction lengthwise to the rectangle. In the experiments, the behaviour of the model showed directly the various characteristics of the strains from the initial to the terminal position. They found good agreement between the mode of deformation of the trellis model and the fabrics in bias directions.

Cooper (1963) investigated the relationship between bias extension and bias shear. Kilby (1963) examined the planar stress–strain relationships of a simple trellis which is different from that discussed by Weissenberg (1949).

He suggested that a fabric might be regarded as being equivalent to an anisotropic lamina which shows the Poisson effect and has two planes of symmetry at right angles to one another. Kilby was the first to derive the fabric tensile modulus in any direction other than the warp and weft directions. He indicated that the bias extension involved the shear modulus.

Later, an analysis of the geometrically similar tests of bias extension and simple shear for plain woven fabrics was carried out by Spivak and Treloar (1968). They showed that the bias extension test involved a reduction in area and hence did not correspond to pure shear; on the contrary, simple shear had a constant-area deformation. They concluded that it is impossible to predict the complete stress–strain properties of a fabric in simple shear from measurements of bias extension.

De Jong and Postle applied an energy analysis to the woven fabric structure to investigate deformation (De Jong and Postle, 1977a,b). The independence of the fabric construction is advantageous in using energy analysis of fabric behaviour. They observed that there are some difficulties encountered when applying a generalised force analysis to fabric structure. Thus, when using force analysis, it is found necessary to divide the unit cell of the structure into segments, at the ends of which forces and/or couples might act. The length of each segment had to be varied because the point of action of the internal forces is not fixed. In their study, the fabric load–extension curves and yarn-decrimping curves for the plain-weave construction are computed for a realistic range of input parameters. They tested the tensile properties of plain woven fabrics in both the grey and the finished state, and the computed results are employed to explain the behaviour of yarns during fabric extension.

Skelton (1971) compared the mechanical properties of triaxial and orthogonal fabrics. He found that the tensile strength of the triaxial fabric is dependent upon the amount of shear distortion sustained by the fabric at rupture, but it seemed probable that the variation of strength with direction would be less than the variation found in orthogonal fabrics. More recently, Anandjiwala and Leaf (1991a,b) studied large-scale tensile deformation of plain woven fabrics. Their investigation used the approximation of non-linear yarn bending behaviour for both the undeformed fabric state and the stress analysis. They found that the agreement between experiment and theory is sometimes reasonable, but it is better during extension than during recovery.

Anandijawala and Leaf mainly concentrated on the tensile and shear modulus of plain woven fabrics. No numerical models are found to predict the anisotropy of fabric tensile properties, such as tensile work (*WT*), tensile elongation (*EMT*), tensile linearity (*LT*) and tensile resilience (*RT*), measured using Kawabata's system (KES-F).

The majority of previous research into the tensile behaviour of woven fabrics concentrated on predictive modelling, which always involved very complicated mathematical relations between stresses and strains (Grosberg and Kedia, 1966; Shanahan *et al*., 1978; Anandjiwala and Leaf, 1991a,b). In addition, the predictability of these models is not always satisfactory.

In particular, Kawabata and Bassett used numerical modelling methods for fitting the tested tensile curves. Kawabata *et al*. reported a linearisation method to model the biaxial tensile stress–strain relation of fabrics, and verified its validity in a paper by Kageyama *et al*. (1988).

In Bassett's Ph.D thesis (1988), the determination of constitutive laws of fabrics and the use of these properties in calculating stress–strain in fabric in garment-like systems were studied. For this purpose, the least squares method was used to fit multivariate polynomial curves to experimental data from woven worsted fabrics.

From the work of both Kawabata and Bassett it is clear that very complicated procedures were still involved for obtaining the constants and strain transformation. Thus, to date, there exists no practical explicit function between stress and strain for tensile deformation of woven fabrics. Therefore, better models for tensile stress–strain relations are needed in terms of both accuracy and practicality.

What is presented here is an attempt to establish an equation for the tensile stress–strain relationship for woven fabrics. An exponential function with two parameters was selected to describe tensile stress–strain curves. A non-linear regression technique was first used to estimate the unknown parameters in the proposed function. Using the proposed function, the predicted results of tensile stress–strain relationships show excellent agreement with experimental data. In addition, several methods which may be used to estimate the unknown parameters in the proposed function are suggested.

4.2.2 Modelling of tensile loading curves

To obtain a satisfactory model for the tensile stress–strain relationship of a fabric, some principles must be followed:

(1) The proposed function should belong to the correct function group, for example exponential or power function.
(2) It should have a format which is easy to compute or interpret. Usually a function with more than four parameters can rarely be evaluated satisfactorily using the non-linear regression method.
(3) It should satisfy the initial conditions of a physical process, e.g. when force equals zero, strain equals zero.
(4) Hopefully, the parameters in the selected function will be related, especially to the yarn physical properties and fabric structure for the intended purpose.

An exponential function with two parameters is chosen to depict the tensile curve of a woven fabric:

$$f = \frac{e^{\alpha\varepsilon} - 1}{\beta} + e_r \qquad [4.1]$$

where f is stress and ε strain, α and β are unknown parameters, e_r the error term.

The SPSS non-linear regression programme can be used to fit the tested tensile curve with the proposed model, in which f and ε can be read off tensile stress–strain curves tested on the KES system while α and β are unknowns to be estimated.

4.2.3 Estimates of the two constants

The two unknown parameters in the chosen function can be estimated in a non-linear regression technique using data read off the tested curves of woven fabrics on the KES tensile tester. The determined values are very useful for the estimation of initial values. Another three methods of estimating the two parameters follow.

4.2.3.1 Estimation of α and β using WT and EMT

From tensile testing of a fabric on the KES tensile tester, there are two parameters extracted from the chart which are *WT* – work done during tension – and *EMT* – strain when stress is equal to 500 gf/cm. We can use these to construct two equations which can be solved for α and β:

$$500 = \frac{e^{\alpha \cdot EMT} - 1}{\beta} \qquad [4.2]$$

and

$$WT = \frac{1}{100}\int_0^{EMT} f\,d\varepsilon = \frac{e^{\alpha \cdot EMT} - 1 - \alpha \cdot EMT}{100\alpha\beta} \qquad [4.3]$$

4.2.3.2 Estimation of α and β taking two points

If the error term in equation (4.1) is ignored and two sets of test data can be obtained, say (ε_1, f_1) and (ε_2, f_2), the following simultaneous system of equations can be solved for α and β:

$$f_1 = \frac{e^{\alpha\varepsilon_1} - 1}{\beta} \qquad [4.4]$$

and

$$f_2 = \frac{e^{\alpha\varepsilon_2} - 1}{\beta} \qquad [4.5]$$

4.2.3.3 *Estimation of α and β by the least squares method (more points method)*

Another method for the estimation of α and β is the more points method. This is in fact the least squares as in linear regression. Suppose we have n sets of data from a tensile curve of a fabric (ε_1, f_1), (ε_2, f_2), . . . (ε_n, f_n), then we can write

$$f_i = \frac{e^{\alpha\varepsilon_i} - 1}{\beta} + e_{ri} \quad [4.6]$$

so that the sum of squares of deviation from the true line is

$$s = \sum_{i=1}^{n} e_{ri} = \sum_{i=1}^{n} \left(f_i - \frac{e^{\alpha\varepsilon_i} - 1}{\beta} \right)^2 \quad [4.7]$$

We can differentiate the above equation first with respect to α and then β, setting the results equal to zero, and hence we get two normal equations:

$$\frac{\partial S}{\partial \alpha} = \frac{2\alpha}{\beta} \sum_{i=1}^{n} \left(f_i - \frac{e^{\alpha\varepsilon_i} - 1}{\beta} \right) e^{\alpha\varepsilon_i} = 0 \quad [4.8]$$

and

$$\frac{\partial S}{\partial \beta} = \frac{2}{\beta^2} \sum_{i=1}^{n} \left(f_i - \frac{e^{\alpha\varepsilon_i} - 1}{\beta} \right) (e^{\alpha\varepsilon_i} - 1) = 0 \quad [4.9]$$

The solution of α and β can thus be given by:

$$\alpha = \frac{\sum_{i=1}^{n} (e^{\alpha\varepsilon_i} - 1)^2}{\sum_{i=1}^{n} (f_i (e^{\alpha\varepsilon_i} - 1))} \quad [4.10]$$

and

$$\beta = \frac{\sum_{i=1}^{n} (e^{\alpha\varepsilon_i} (e^{\alpha\varepsilon_i} - 1))^2}{\sum_{i=1}^{n} (f_i \cdot e^{\alpha\varepsilon_i})} \quad [4.11]$$

If $n = 2$, the results are equal to the two equations used in the two-point method. Theoretically, more data would provide more accurate estimates. However, this is not always true because it may cause a big residual due to the difficulty of reading data off a curve, in which case more data can result in a more biased subjective measurement. Therefore, n is not necessarily very big. It is found that $n = 13$ is big enough for good results; $n = 4$ or 5 may result in accurate estimates of parameters. When n is not very big, say 5, it can be calculated on a package like TK Solver as long as initial guesses are not far beyond the expected results.

4.2.4 Interpretation of the selected function

4.2.4.1 Young's modulus

A particularly important part of the stress–strain curve is the initial portion; starting at zero stress in most cases it could be seen that the first portion of the curve is fairly straight, indicating a linear relationship between the stress and the strain. The tangent of the angle between the initial part of the curve and the horizontal axis is the stress/strain ratio, which is termed initial Young's modulus E_0. It describes the initial resistance to extension of a textile material. From equation 4.1, the derivative of f with respect to ε yields the modulus curve of tensile deformation:

$$E = \frac{\mathrm{d}f}{\mathrm{d}\varepsilon} = \frac{\alpha}{\beta}\,\mathrm{e}^{\alpha\varepsilon} \tag{4.12}$$

when $\varepsilon = 0$, $E = E_0 = \alpha/\beta$. Thus α/β is the initial Young's modulus. The above equation can be written as follows:

$$E = E_0 \cdot \mathrm{e}^{\alpha\varepsilon} \tag{4.13}$$

Because the tangent of the angle between the initial part of the curve and the horizontal axis is the stress/strain ratio, E carries a unit of gf/cm in the case of the KES system. Again from equation 4.1, β carries a unit of cm/gf while α is a dimensionless quantity. We can use the Maclaurin expansion formula:

$$f(\varepsilon) = \frac{1}{\beta}\left(\frac{\alpha\varepsilon}{1!} + \frac{\alpha^2 X^2}{2!} + \ldots + \frac{\alpha^n X^n}{n!} + R_n(\varepsilon)\right) \tag{4.14}$$

where $R_n\varepsilon$ is the remainder.

If ε is infinitesimal,

$$f(\varepsilon) = \frac{\alpha}{\beta}\varepsilon \tag{4.15}$$

which is a straight line with residual of

$$R_1(\varepsilon) = \alpha^2 \mathrm{e}^{\alpha\theta\varepsilon}(1 - \theta)\varepsilon^2 \quad (0 < \theta < 1) \tag{4.16}$$

This indicates that the initial portion of the stress–strain curve starting at zero stress is close to a linear line. Besides, if we further differentiate E with respect to ε, the following equation can be obtained:

$$\frac{\mathrm{d}E}{\mathrm{d}\varepsilon} = \alpha E \quad \text{or} \quad \frac{\mathrm{d}E}{E} = \alpha\,\mathrm{d}\varepsilon \tag{4.17}$$

From this equation, it appears that α is a reinforcing factor for the increase of Young's modulus E. This relation is frequently found in many physical phenomena.

The initial region of very low slope for the fabric tensile stress–strain curves represents the region of decrimping and crimp-interchange in woven fabrics for which only very small fibre stresses are developed within the fabric when the weave crimp has been fully extended. This means that further extension of the fabric is possible by extension of fibres with the interlaced yarns.

4.2.4.2 Relationship of α with crimp

According to equation 4.1, it may be assumed that there exist two different groups of factors which affect the stress–strain relation. One group affects α while the other influences β.

First it was hypothesised that the larger the yarn crimp the higher the value of β since larger β denotes larger stress induced for a given strain. The analysis, however, reveals that parameter α is correlated with crimp reversely and β has no obvious relation with crimp c. In fact, it is found that an obvious linear relationship exists between α and crimp c. This linear relationship between α and c agrees with the existing recognition that the larger the crimp, the more extensible a fabric. Moreover, this relationship is also consistent with the above interpretation of the selected function, in which α is a dimensionless factor.

In addition, it is also demonstrated that the tangents of stress–strain curves differ with the direction even for the same woven fabric. This fact indicates that there exists an anisotropy for the tensile properties of woven fabrics, as reported in the next section. Another focus of this chapter, the strain hardening effect, can also provide a strong explanation.

4.3 Anisotropy of woven fabric tensile properties

4.3.1 Introduction

One of the difficulties in analysing the tensile behaviour of woven fabrics lies in the fact that any extension occurring at an angle to the warp or weft direction usually involves a different mechanism of deformation. For example, in the 45° direction to the warp and weft, the modulus is almost completely determined by the shear behaviour of the fabric, while if it is extended in the warp or weft direction, the shear behaviour has no part to play (Hearle *et al.*, 1969). Therefore, the tensile performance of a fabric is apparently an integration of a multi-directional effect. We term this phenomenon the 'anisotropy' of tensile properties of woven fabrics and it becomes the subject of the following section. This topic is very meaningful in that little literature about the anisotropy phenomenon can be found despite the fact that there is already a sea of publications dealing with fabric tensile properties; but most of these have concentrated on what happens to the warp and weft directions.

As the term indicates, the word 'anisotropy' means that there is great variation in fabric tensile properties with changes in direction. Firstly, this is because a woven fabric is highly anisotropic in nature. Secondly, most fabric structures are asymmetrical, like twill and satin woven fabrics, and thus the force needed to stretch fabrics in different directions will vary a lot from one to another. This is basically a reflection of the different underlying deformation mechanism. For example, when a woven fabric is under bias extension, shear deformation will occur and thus shear property comes into play to influence the tensile behaviour of a fabric. In this case, the tensile behaviour of a fabric will apparently differ from what occurs when the extension happens merely in two principal directions (Hearle and Amirbayat, 1986a,b).

The work covers all the four parameters measured on Kawabata's system (KES-F): tensile work (*WT*), tensile elongation (*EMT*), tensile linearity (*LT*) and tensile resilience (*RT*) based on Kilby's Young's Modulus model (1963).

4.3.2 Modelling the anisotropy of tensile properties

4.3.2.1 Tensile work (WT)

Kilby (1963) firstly introduced the Young's modulus in any direction other than warp and weft as follows:

$$\frac{1}{E_\theta} = \frac{\cos^4\theta}{E_1} + \left[\frac{1}{G} - \frac{2\sigma_{pt}}{E}\right]\sin^2\theta\cos^2\theta + \frac{\sin^4\theta}{E_2} \qquad [4.18]$$

where E_1, E_2 and E_θ are the Young's moduli to the warp, weft and θ directions respectively, G denotes the shear modulus, and σ_{pt} indicates the Poisson's ratio relating the contraction in the weft direction to the strain in the warp direction. In order to simplify the calculation, he rearranged the above equation into:

$$\frac{1}{E_\theta} = \frac{\cos^4\theta}{E_1} + \left[\frac{3}{E_{45}} - \frac{1}{E_1} - \frac{1}{E_2}\right]\cos^2\theta\sin^2\theta + \frac{\sin^4\theta}{E_2}$$

or

$$\frac{1}{E_\theta} = \frac{\cos^4\theta}{E_1} + \frac{\cos^2\theta\sin^2\theta}{G'} + \frac{\sin^4\theta}{E_2} \qquad [4.19]$$

where

$$\frac{1}{G'} = \frac{4}{E_{45}} - \frac{1}{E_1} - \frac{1}{E_2}$$

Equation (4.19) is very useful for predicting the full form of the polar diagram of modulus against angle, when values of the parameters in only warp, weft and θ directions are known. Rearranging equation 4.1, the following relation is obtained:

$$\varepsilon = \frac{\ln(\beta F + 1)}{\alpha} \quad \Rightarrow \quad E = \frac{dF}{d\varepsilon} = \frac{\alpha}{\beta} e^{\alpha\varepsilon} \qquad [4.20]$$

The tensile work, *WT*, is thus

$$WT = \int_0^{\varepsilon_m} F(\varepsilon)\, d\varepsilon \qquad [4.21]$$

where ε_m is the strain at the upper-limit load F_m = 500 gf/cm, and F denotes the tensile load, a function of strain. Combining equations (4.20) and (4.21), we have

$$WT = \frac{E}{\alpha^2} - \left[\frac{\ln(\beta F_m + 1) + 1}{\alpha\beta}\right] \qquad [4.22]$$

F_m is a constant (= 500 gf/cm) which is directly obtained from the stress–strain curve on the KES-F apparatus. α and β are variables and will have different values in different directions. To simplify the procedure, the term $[\ln(\beta F + 1) + 1]/\alpha\beta$ is set to be K. Then *WT* may be written, and if E varies with angle θ, *WT* will also vary with θ and the above equation becomes

$$WT = \frac{E}{\alpha^2} - K \quad \Rightarrow \quad E_\theta = (WT_\theta + K)\alpha^2 \qquad [4.23]$$

Putting $E_\theta = (WT_\theta + K)\alpha^2$ into equation (4.19) gives

$$\frac{1}{\alpha^2(WT_\theta + K)} = \frac{\cos^4\theta}{\alpha^2(WT_1 + K)} + \frac{\cos^2\theta\sin^2\theta}{G'} + \frac{\sin^4\theta}{\alpha^2(WT_2 + K)} \qquad [4.24]$$

where

$$\frac{1}{G'} = \frac{4}{\alpha^2(WT_{45} + K)} - \frac{1}{\alpha^2(WT_1 + K)} - \frac{1}{\alpha^2(WT_2 + K)}$$

$$= \frac{1}{\alpha^2}\left[\frac{4}{(WT_{45} + K)} - \frac{1}{(WT_1 + K)} - \frac{1}{(WT_2 + K)}\right]$$

$$= \frac{1}{\alpha^2}\left(\frac{1}{G''}\right)$$

Substituting $1/G' = 1/\alpha^2 (1/G'')$ into equation (4.24), the tensile energy (*WT*) of the tensile parameters is derived as follows:

$$\frac{1}{(WT_\theta + K)} = \frac{\cos^4\theta}{(WT_1 + K)} + \frac{\cos^2\theta\sin^2\theta}{G''} + \frac{\sin^4\theta}{(WT_2 + K)} \qquad [4.25]$$

Principally, K varies with α and β which are different values in various directions. K is taken as a constant, due to the linear relationship found in its numerator and the denominator in various directions. In order to simplify the calculation of tensile energy (WT), α and β can be recorded directly from the warp direction. Thus, the tensile work in any direction can be obtained using equation (4.25) when values of the tensile work at the warp, weft and 45° directions are known.

4.3.2.2 Tensile elongation (EMT)

EMT reflects the extensibility of a fabric. It is a measure of a fabric's ability to be stretched under tensile load. The larger the *EMT* value, the more extensible the fabric. A similar approach to that of tensile work (WT) is adopted in the derivation of a tensile elongation (EMT) model. The model for the prediction of Young's modulus in any direction other than the warp and weft directions is derived by Kilby (1963), and a mathematical rearrangement is made to form equation (4.26):

$$\frac{1}{E_\theta} = \frac{\cos^4\theta}{E_1} + \left[\frac{1}{G} - \frac{2\sigma_{pt}}{E}\right]\cos^2\theta\sin^2\theta + \frac{\sin^4\theta}{E_2}$$

$$\Rightarrow \quad \frac{1}{E_\theta} = \frac{\cos^4\theta}{E_1} + \left[\frac{4}{E_{45}} - \frac{1}{E_1} - \frac{1}{E_2}\right]\cos^2\theta\sin^2\theta + \frac{\sin^4\theta}{E_2} \qquad [4.26]$$

where E_1, E_2 and E_θ are the Young's moduli to the warp, weft and θ directions respectively, G is the shear modulus, σ_{pt} the Poisson ratio. Equation (4.26) is useful for predicting the full form of the polar diagram of modulus against angle, when values of the parameters in the warp, weft and 45° directions are known.

In general, the tensile stress–strain relationship for textile materials is non-linear and characterised by a simple concave shape. However, in order to simplify the analysis, the author assumed that a tensile curve is linear to derive a model for *EMT* in different directions relative to its warp or weft direction. With this assumption, *EMT* of a woven fabric may be derived very conveniently from the simple relation as $E = F/\varepsilon$.

Since F is kept constant at 500 gf/cm during experiments, tensile modulus is inversely proportional to extension when the tensile curve is assumed to be linear. Then we can write $E_\theta = F_\theta/\varepsilon_\theta$ in terms of θ and substitute into equation (4.26). Thus:

$$\frac{\varepsilon_\theta}{F} = \frac{\varepsilon_1\cos^4\theta}{F} + \left[\frac{4\varepsilon_{45}}{F} - \frac{\varepsilon_1}{F} - \frac{\varepsilon_2}{F}\right]\cos^2\theta\sin^2\theta + \frac{\varepsilon_2\sin^4\theta}{F} \qquad [4.27]$$

where

$$G' = \frac{1}{F}\ (4\varepsilon_{45} - \varepsilon_1 - \varepsilon_2)$$

ε_{45} is the mean value of ε_θ of $\pm$ 45° and G' is a constant. After rewriting equation (4.27),

$$\varepsilon_\theta = \varepsilon_1 \cos^4\theta + \frac{\cos^2\theta \sin^2\theta}{G'} + \varepsilon_2 \sin^4\theta \qquad [4.28]$$

Replacing ε by EMT, the tensile strain of the tensile parameters is derived as follows:

$$EMT_\theta = EMT_1 \cos^4\theta + \frac{\cos^2\theta \sin^2\theta}{G'} + EMT_2 \sin^4\theta \qquad [4.29]$$

where

$$G' = \frac{4}{EMT_{45}} - \frac{1}{EMT_1} - \frac{1}{EMT_2}$$

In the previous case, E is considered to vary linearly with ε and EMT_θ is derived based on this linear relationship. In this case, however, a more precise model for EMT is derived based on the non-linear relationship from Hu (1994). Her model described the tensile stress–strain curve with an exponential function with two parameters shown as follows.

$$F = \frac{e^{\alpha\varepsilon} - 1}{\beta} \qquad [4.30]$$

where F is stress, ε is strain, α and β are unknown parameters. The unknown parameters α and β can be solved by using the SPSS non-linear regression method or TK Solver. F and ε are read off from tensile stress–strain curves tested on a particular fabric on the KES-F apparatus. α and β are obtained from the warp direction. Now, E is treated as the derivative of F with respect to ε:

$$\frac{1}{E} = \frac{\beta}{\alpha}\,\frac{1}{e^{\alpha\varepsilon}} \qquad [4.31]$$

α and β are taken as constant regardless of the different directions so that they are obtained from the warp direction. When E changes with angle, and this is the case for ε, then $1/E_\theta = (\beta/\alpha)(1/e^{\alpha\varepsilon_\theta})$ and substituting it into equation (4.26) yields equation (4.32).

$$\frac{\beta}{\alpha}\left(\frac{1}{e^{\alpha\varepsilon_\theta}}\right) = \frac{\beta}{\alpha}\left(\frac{\cos^4\theta}{e^{\alpha\varepsilon_p}}\right) + \frac{\cos^2\theta \sin^2\theta}{G} + \frac{\beta}{\alpha}\left(\frac{\sin^4\theta}{e^{\alpha\varepsilon_t}}\right) \qquad [4.32]$$

Hence

$$\frac{1}{G} = \frac{\beta}{\alpha}\left(\frac{4}{e^{\alpha\varepsilon_{45}}} - \frac{1}{e^{\alpha\varepsilon_p}} - \frac{1}{e^{\alpha\varepsilon_t}}\right) = \frac{\beta}{\alpha}\,\frac{1}{G''}$$

Substituting $1/G = (\beta/\alpha)\,(1/G'')$ into equation (4.26) gives

$$\frac{1}{e^{\alpha\varepsilon_\theta}} = \left(\frac{\cos^4\theta}{e^{\alpha\varepsilon_p}}\right) + \frac{\cos^2\theta\sin^2\theta}{G''} + \left(\frac{\sin^4\theta}{e^{\alpha\varepsilon_t}}\right)$$

$$e^{-\alpha\varepsilon_\theta} = e^{-\alpha\varepsilon_p}\cos^4\theta + \frac{\cos^2\theta\sin^2\theta}{G''} + e^{-\alpha\varepsilon_t}\sin^4\theta$$

and

$$\varepsilon_\theta = -\frac{1}{\alpha}\left(e^{-\alpha\varepsilon_p}\cos^4\theta + \frac{\cos^2\theta\sin^2\theta}{G''} + e^{-\alpha\varepsilon_t}\sin^4\theta\right) \qquad [4.33]$$

Replacing ε by *EMT*,

$$EMT_\theta = -\frac{1}{\alpha}\left(e^{-\alpha EMT_p}\cos^4\theta + \frac{\cos^2\theta\sin^2\theta}{G''} + e^{-\alpha EMT_t}\sin^4\theta\right) \qquad [4.34]$$

The tensile strain (*EMT*) in any direction can be obtained by using the above models (equations 4.22 and 4.24) when values of the tensile strains at warp, weft and 45° directions are known. The polar diagram of *EMT* of different woven fabrics can also be predicted by this model (Lo and Tsang, 1999).

4.3.2.3 Linearity (LT)

Linearity is a measure of the extent of non-linearity of the tensile stress–strain curve. It depends on the ratio between tensile work (*WT*) and tensile elongation (*EMT*) from the stress–strain curve. The model of tensile linearity (*LT*) can thus be denoted as follows:

$$LT_\theta = \frac{WT_\theta}{cons\tan t^* \, EMT_\theta} \qquad [4.35]$$

4.3.2.4 Tensile resilience (RT)

Tensile resilience (*RT*), which is the ratio of work recovered to the work done in tensile deformation, is expressed as a percentage ($RT = WT'/WT$). Work recovery (WT') is the tensile force at the recovery process while tensile energy (*WT*) in tensile deformation is represented by the area under the stress–strain curve in the loading process. And thus the existing *WT* model and the proposed WT' model for the loading and unloading processes respectively can be used to predict the tensile resilience (*RT*) of woven fabrics.

For the loading process of the tensile stress–strain curve, a model is derived by Hu and Newton (1993) for the loading stress–strain curve. Their approach is to establish a model with an exponential function, in which two parameters are derived by using a non-linear regression method.

As the tensile recovery curve is followed by a very rapid decrease in the fabric stress in the unloading process, function 4.36 is thus established:

$$F' = ae^{b\varepsilon} - c \qquad [4.36]$$

where F' is stress, ε is strain, a, b and c are unknown parameters.

Work recovery model (WT')

The work recovery (WT') of the tensile stress–strain curve could also be described with the proposed exponential function with two unknown parameters. The unknown parameters a, b and c in function 4.36 can be solved using the SPSS non-linear regression program and F' and ε can be directly recorded from tensile stress–strain curves tested on the KES system. The derivative of F' with respect to ε yields tensile modulus,

$$E' = \frac{dF'}{d\varepsilon} = abe^{b\varepsilon} \quad \Rightarrow \quad \varepsilon = \frac{\ln(E'/ab)}{b} \qquad [4.37]$$

Tensile recovery (WT') as defined in the KES system is work recovery in tensile deformation represented by the area under the stress–strain curve. Combining equations 4.36 and 4.37, we get

$$\begin{aligned} WT' &= \int_{\varepsilon_0}^{\varepsilon_m} F'(\varepsilon)d\varepsilon \\ &= \int_{\varepsilon_0}^{\varepsilon_m} (ae^{b\varepsilon} - c)d\varepsilon \\ &= \left| \frac{ae^{b\varepsilon}}{b} - c\varepsilon \right|_{\varepsilon_0}^{\varepsilon_m} \\ &= \frac{E'_m - E'_0}{b^2} - \frac{c}{b}\left(\ln \frac{E'_m}{E'_0} \right) \end{aligned} \qquad [4.38]$$

If E' varies with an angle θ, WT' will also vary with θ and the above equation becomes

$$WT'_\theta = \frac{(E'_m)_\theta - (E'_0)_\theta}{b^2} - \frac{c}{b}\left[\ln \frac{(E'_m)_0}{(E'_0)_\theta} \right] \qquad [4.39]$$

where $(E'_m)_\theta$ and $(E'_0)_\theta$ are the Young's moduli of the tensile recovery curve in various directions. All can be calculated from the Kilby Young's modulus model (equation 4.18). As the two parameters b and c vary with angle θ, they will give different values in various directions. To simplify the calculation of WT' by statistical mean of least squares analysis, b and c take their average in the warp, weft and ± 45° directions rather than the value of their corresponding individual direction (Lo *et al.*, 1999a,b).

Tensile resilience model (*RT*)
With the work recovery model (WT'), tensile resilience (*RT*) of woven fabrics could be easily predicted from the work recovery (WT') and tensile work (*WT*) models. Tensile resilience (*RT*) is the ratio of work recovered to work done in tensile deformation expressed as a percentage:

$$RT = \frac{WT'}{WT} \times 100\ \% \qquad [4.40]$$

If *RT* varies with angle θ, equation 4.40 becomes

$$RT_\theta = \frac{WT'_\theta}{WT_\theta} \times 100\ \% \qquad [4.41]$$

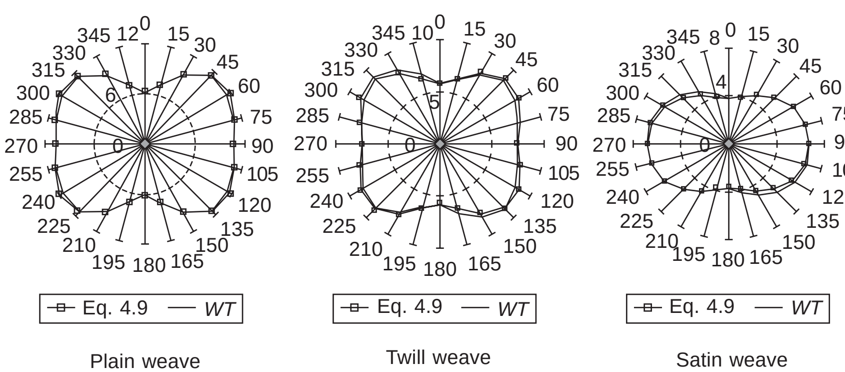
4.3.3 Polar diagrams of the tensile model

4.3.3.1 General features

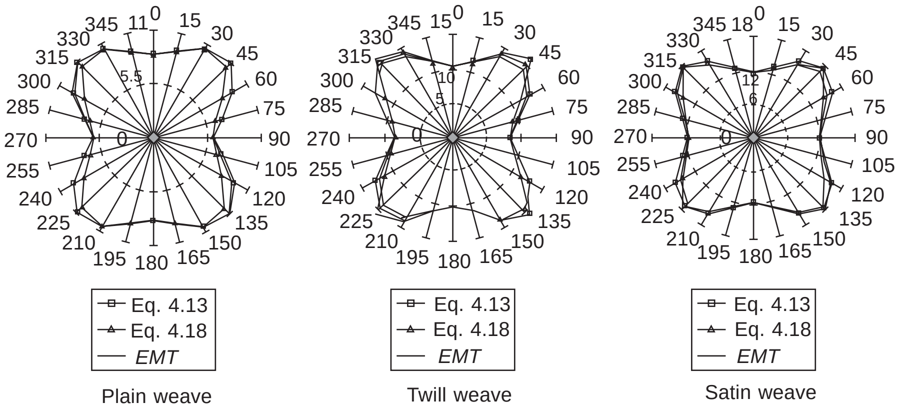
The values of tensile parameters (*WT*, *EMT*, *LT* and *RT*) usually differ in direction; thus some points in the polar diagram have to be normalised in the warp direction to simplify the comparison and analysis. When the value of these parameters in the warp direction is fixed, changes in the bias directions can be easily observed from the polar diagram. The normalised tensile parameters can be obtained by dividing each parameter by their averaged value.

As shown by Figs 4.4–4.7, many similarities can be found in the polar diagrams of different tensile parameters:

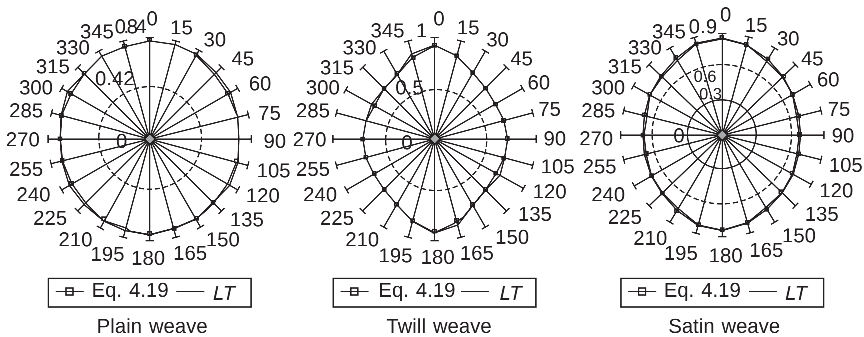
(1) The pattern is symmetrical to the warp and weft directions.
(2) The value of each parameter differs with the angle and the maximum happens exactly at either the warp (*WT* of satin, *LT*) or weft directions



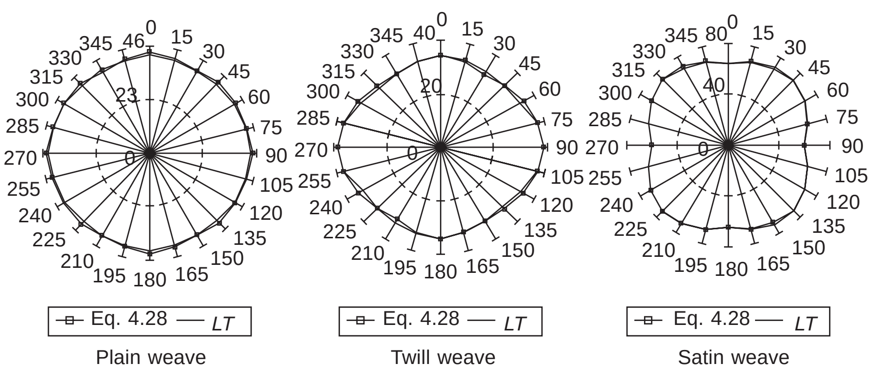
4.4 Typical polar diagram of tensile work (*WT*).



4.5 Typical polar diagram of tensile extension (*EMT*).



4.6 Typical polar diagram of tensile linearity (*LT*).



4.7 Typical polar diagram of the tensile resilience model (*RT*).

Table 4.1 Classification of polar diagrams of tensile parameters

Types	Conditions	Shape of polar diagram
1	$WT_1 \approx WT_2 \approx WT_\theta$	Circular shape (isotropic)
	$(WT_1 \approx WT_2) > WT_\theta$	Butterfly or Hexagonal Shape
	$(WT_1 \approx WT_2) < WT_\theta$	Rhombus shape
2	$WT_1 > WT_\theta > WT_2$	Gourd or elliptic shape (maximum value in warp direction)
	$WT_1 < WT_\theta < WT_2$	Gourd or elliptic shape (maximum value in weft direction)

or at ± 45° angle (*WT*, *EMT*) corresponding to the warp and weft directions.

(3) For some tensile parameters, the polar diagram generated from satin woven fabrics might differ somewhat from that of plain and twill woven fabrics due to its long floats, examples of which include the *WT* polar diagram.

(4) The polar diagrams of each parameter can be classified into two types on the basis of their general shapes. If the value of the parameter between the warp and weft directions is similar, the polar diagram will display a circular, rhombus or butterfly shape. We name this pattern Type 1. Type 2 refers to those featuring a gourd shape with their maximum value either in the warp or in the weft direction, as shown in Table 4.1.

4.3.3.2 Factors influencing tensile parameters

Influence of fabric weaves on *WT*

Twill and satin woven fabrics usually demonstrate lower tensile work (*WT*) as compared with plain woven fabrics, due to the presence of floats, but no apparent difference can be observed in the *WT* values of plain, 2/2 twill and 3/3 twill woven fabrics in the warp direction if their warp densities are kept constant. In addition, the value of *WT* will increase with the rise in weft densities, indicating that more work is needed to extend the fabric with high weft density.

Due to the frictional force between the contact points of the warp and weft yarns, several factors will contribute to the amount of energy loss, including the ratio of the yarn counts in the warp and weft yarns, the ratio of the yarn spacing, the average yarn spacing, the type of weaves, the direction of force applied to the fabrics, etc. The experimental data of *WT* indicates that the tensile work of plain woven fabrics is generally higher than that of the twill and satin woven fabrics. As the ratio of yarn spacing and the average yarn spacing of plain woven fabrics are comparatively smaller than those of twill and satin woven fabrics, greater energy is needed to overcome the frictional restraint existing at the interlacing points of the warp and weft yarns. As neighbouring yarns cross over interlacing points, the yarns are prevented from moving, especially when extension occurs in bias directions. In such a case, the yarns are extended by rotating the unit cell in the direction of the pull. Thus, larger energy consumption results in the extension of plain woven fabric in bias directions.

In contrast with this, lower yarn crimp is exhibited in the looser weave construction of twill and satin woven fabrics, especially in the weft direction. Consequently, yarn extension is no longer the main factor affecting fabric deformation, viz. fibre extension also comes into play. As a result, an increase in the work in bias directions occurs due to the addition of fibre extension. This also explains why the tensile work in the weft direction is found to be higher than that in the warp direction.

Influence of weft density on *EMT*

Weft density is an important factor governing *EMT* values. With the increase in weft density for any fabric type, a rise in the magnitude of *EMT* in all directions will be observed. A direct image of this is an outward spreading along any direction for all woven fabrics. As the width of a fabric is usually fixed, the yarns will jam and come into contact when the weft density has reached its limit. Hence, an increase in the inter-yarn friction will be found when the weft yarns are closely packed together.

For a unit cell of a plain woven fabric, the warp and weft yarns interlace with each other in a format of one up and the other down. Thus, when a fabric is under tension, the yarn bending rigidity in this lattice structure will restrict yarn movement by producing frictional force. Generally, this restriction will increase with the rise in the weave density. In addition, the yarn crimp will also grow with the increase in weave density and, in the meantime, a reduction in the modular length will be found. Therefore, more energy is needed to extend a fabric with high density.

A twill woven fabric usually exhibits larger elongation than plain woven fabrics due to its loose structure, despite its low yarn crimp. In addition, the extension of a loose-structured fabric usually involves yarn slippage or even fibre extension when large tension is applied. Thus, a broadening effect can also be found in the contour of its polar diagram when the weft density is increased.

Influence of Poisson's ratio on *EMT*

When a fabric is extended lengthwise in one direction, widthwise contraction will be found in the other as revealed by our experiments, made on the KES-F apparatus using all specimens. Poisson's ratio is such a measure of the relative changes in length in the directions of the pull to that in a direction perpendicular to it (Chadwick *et al.*, 1949). Therefore, it can be predicted that a relationship exists between the Poisson effect and the anisotropy of tensile elongation (*EMT*) and tensile work (*WT*) of woven fabrics.

For a woven fabric, it seems that its Poisson's ratio differs considerably from one direction to another. In addition, the maximum elongation can be experimentally proven at ± 45° directions as discussed in the previous section. It is also evident that maximum widthwise contraction and evading action happen at ± 45° directions. These maxima at ± 45° might be attributable to the increased internal friction in the perpendicular direction and maximum pressure existing at the warp and weft interlacing points. Also observed in tensile testing is a necking effect, which might stem from yarn migration towards both ends of the clamps.

For a clear picture of the Poisson effect on widthwise contraction, the lengthwise extension is kept constant. Consequently, in common with the situation with *EMT* and *WT*, a similar effect is found in the Poisson's ratio of woven fabrics. For woven fabrics, their Poisson's ratios in the two principal directions are very close to each other. However, an apparent difference exists in the Poisson's ratios of twill and satin woven fabrics. This indicates that a lower crimp interchange effect appears in loose fabric structures.

During uniaxial extension, only one set of yarns is firmly held at both ends, while both ends of the other set of yarn are free of tension; this leads to the great similarity in its Poisson's ratios in the two principal directions. However, in the bias directions, only one end of the yarns will be clipped during extension. The yarns can thus easily move along each other, especially in a loose structure such as in twill and satin weaves. These yarns also present a tendency to move towards the clamps due to the lack of pinpointing effect. Also proved is a higher ratio of lengthwise elongation to widthwise contraction in bias directions.

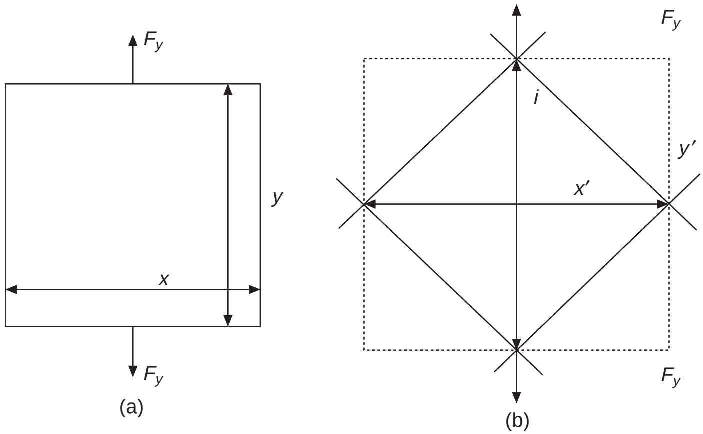
Figure 4.8 shows the unit cell of a uniaxially stretched plain woven fabric. It is quite clear that the lengthwise extension in ± 45° direction (length y' in Fig. 4.8b) is larger than that in either of the two principal directions (length y in Fig. 4.8a). If the lengthwise extension is kept constant, a higher widthwise contraction of woven fabric will also be observed in the ± 45° direction as indicated by x'.

Although the above results were developed on apparel fabrics, it has been experimentally confirmed that their validity can also be extended to industrial woven fabrics.

4.4 Strain-hardening of warp yarns in woven fabrics

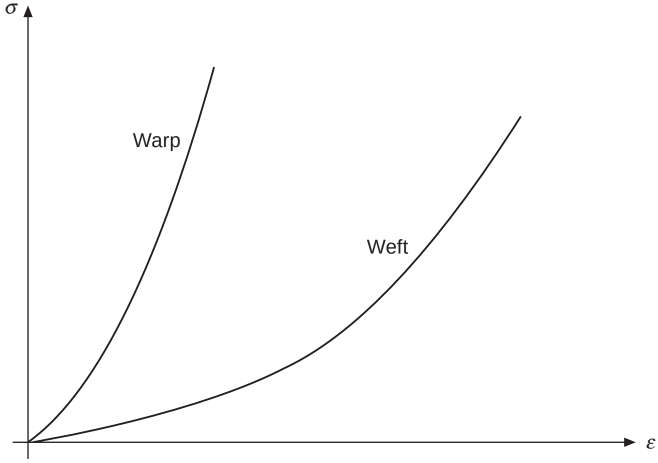
4.4.1 Introduction

From intuition, it would be expected that a square plain woven fabric should exhibit similar extensibility in its two principal directions. For poplin fabrics,

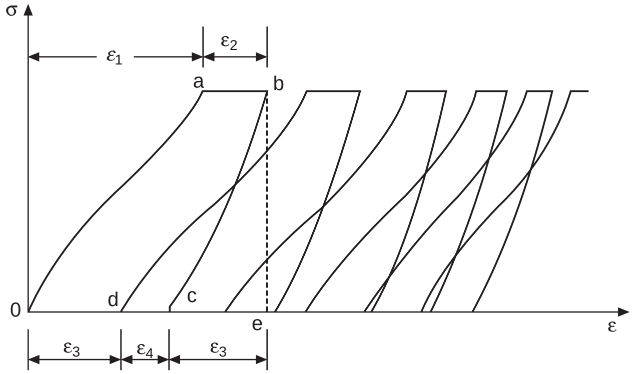


4.8 Unit cell of plain woven fabric after uniaxial tension: (a) Extension in the warp or weft direction; (b) Extension in 45° direction.

a fabric type with prominent larger crimp level in warp yarns, the extensibility of warp yarn should be far superior to that of weft yarns. However, KES testing reveals that against our expectation, the square fabric possesses greater variation in extension in its two principal directions than the poplin. In addition, tensile resilience, *RT*, seems to have a larger value in the warp direction than that in the weft for many fabrics while *WT*, the tensile energy, always exhibits a lower value in the warp. These facts reveal that a woven fabric is more extensible in the weft direction. A direct image of this phenomenon is that the shape of the tensile stress–strain curve of a woven fabric is usually steeper in the warp direction than that in the weft, as shown in Fig. 4.9. This phenomenon is apparently due to the repeated loading and unloading a woven fabric experiences during manufacturing and processing.



4.9 Comparison of the tensile stress–strain curves in principle directions of woven fabrics.



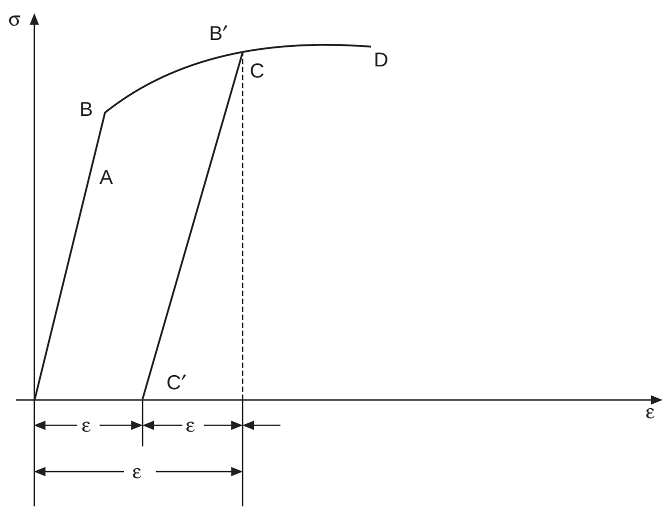
4.10 Cyclic tensile stress–strain curves of textile materials.

4.4.2 Theory of plasticity

4.4.2.1 Work-hardening

Figure 4.10 shows the cyclic tensile stress–strain curves of textile materials. It is obvious that when an inelastic material like a yarn or a fabric is subjected to cyclic loading, the loops will get narrower and narrower as cyclic loading and unloading goes on. As a result, the energy needed to stretch the material gets less and less while the strain becomes smaller and smaller at the same maximum strain.

In Fig. 4.11, the tensile curve of a general engineering material is presented. If at any point between the elastic limit B and the maximum load point C the



4.11 Stress–strain curve for a conventional engineering material.

tensile stress in the material is removed, unloading will take place along a line parallel to the elastic line, as shown in the figure by B′C′. Part of the strain is thus recovered and part remains permanently. Upon reloading, the unloading line B′C′ is retracted with very minor deviations. Actually a very thin hysteresis loop is formed, which is usually neglected. Plastic flow does not start again until the point B′ is reached. With further loading, the stress–strain curve is continued along B′C′ as if no unloading had occurred. Point B′ can thus be considered as a new yield point for this material. From this. fact, it appears that the Young's modulus in this second cycle would be equal to or larger than that of the first cycle. Three conclusions follow from this, Firstly we can infer that the modulus at any point is the smallest in the first loading cycle. The modulus of a later cycle is generally larger than that of the immediately previous cycle, or the extension of the material in the first cycle is the largest among all cycles for a given cyclic stress. For example, the Young's modulus of the third cycle is larger than that of the second cycle; in turn, the modulus of the fifth cycle may be larger than that of the fourth cycle … Secondly, when a constant stress is given for two cycles, for example at B′, we can see that the energy to extend the material during the first cycle is much larger than that in the second cycle, which is only a part of that in the first cycle; in turn, energy in the first cycle will be larger than that in the second one. Thirdly, if we release the loading at B′ it is obvious that the tensile resilience of the second cycle is 100 %, but the first cycle has only a fraction of it; that is, the tensile resilience of the second cycle will be larger than that of the first cycle and consequently the energy resilience in the later cycle may be larger than in all the previous ones. This is caused by plastic strain in the previous steps. The effect of this strain is called work–hardening or strain-hardening.

In plasticity theory, when a real material is deformed plastically, it 'work-hardens'. That is, as the material deforms, its resistance to further deformation increases. The degree of hardening is a function of the total plastic work and is otherwise independent of the strain path. This is sometimes known as the equivalence of plastic work. In other words, the resistance to further distortion depends on the amount of the work. The effect of different discontinuous processing procedures of woven fabrics, namely the extension stresses in the warp direction, can be simulated at irregular cyclic loading. They produce accumulated plastic strains in yarns of woven fabrics. The effect of plasticity is a permanent deformation. Even though a fabric is fully relaxed, the deformation caused by processing cannot be removed entirely.

Thus when a final product, a fabric, is tested on a tensile tester in a laboratory, this causes the *EMT* difference between warp direction and weft direction, even though the other conditions are the same for warp and weft yarns; *WT* in the weft direction is larger than that in the warp direction; the recovered energy is larger in the warp direction; and it can be observed that the warp yarns in a woven fabric are harder to extend than those in the weft direction.

4.4.2.2 Plastic strains

From the above section, plastic strain in woven fabrics before tensile testing is recognised by the comparison of the warp and weft tensile properties of a woven fabric and it is suggested by comparing cross-sectional areas of warp and weft yarns.

If it is assumed that yarn density remains unchanged before and after fabric manufacturing and processing, the difference in the yarn areas between warp and weft directions in a fabric can be attributed as a warp yarn extension along its axis direction.

The value of warp yarn extension may be calculated using a plastic deformation principle, assuming that plastic strain involves no volume change, thus:

$$e_{xp} + e_{yp} + e_{zp} = 0 \qquad [4.42]$$

where e_{xp}, e_{yp}, e_{zp} are logarithmic plastic strain changes in the x, y, z direction respectively.

The logarithmic strain is defined as:

$$e = \ln \frac{l}{l_0} \qquad [4.43]$$

In the case of the KES system, we use engineering strain $\varepsilon \times 100$ %:

$$\varepsilon = \frac{l - l_0}{l_0} \qquad [4.44]$$

These two strains have the following relation:

$$e = \ln (1 + \varepsilon) \qquad [4.45]$$

The definition of logarithmic strain was suggested by Ludwik. For small extensions, the engineering strain, ε (first defined by Cauchy), is approximately equal to the logarithmic strain e.

Under the condition of volume constancy, the relationship of three principal engineering strains can be expressed:

$$(1 + \varepsilon_x)(1 + \varepsilon_y)(1 + \varepsilon_z) = 1 \qquad [4.46]$$

For infinitesimal strains, we may neglect the products of the strains and equation 4.46 reduces to

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = 0 \qquad [4.47]$$

In the case of yarns in woven fabrics, for elliptic yarn cross-section, the z

axis represents yarn axial direction, the x axis the major diameter direction and the y axis the minor diameter direction. Then equation 4.42 can be written as:

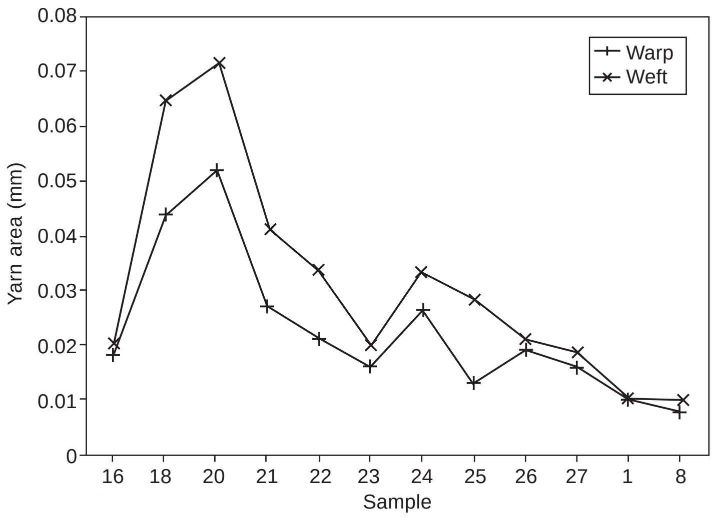
$$\ln\left(\frac{a}{d_0}\right) + \ln\left(\frac{b}{d_0}\right) + e_{zp} = 0 \quad [4.48]$$

where a and b are major and minor diameters of yarn in a woven fabric, d_0 is the original diameter of the yarn.

If the cross-sections of yarn are regarded as circular and the equivalent diameter, d, is calculated using measured yarn area, the relation can be expressed:

$$2\ln\left(\frac{d}{d_0}\right) + e_{zp} = 0 \quad [4.49]$$

One piece of evidence of the effects of the plastic strain on different directions of a woven fabric is that the cross-sectional areas of warp yarns are generally smaller than those of weft when made of the same yarns, as shown in Fig. 4.12. Statistical calculations of the plastic strains in the longitudinal direction of yarns in a fabric show that the warp yarns in a woven fabric have a positive or extended plastic strain before testing and the weft yarns a negative one, which makes warp yarns in a woven fabric harder to stretch than weft yarns. The difference of plastic strains in warp and weft directions is shown to have linear relationships with *EMT*, *WT* and *RT*.

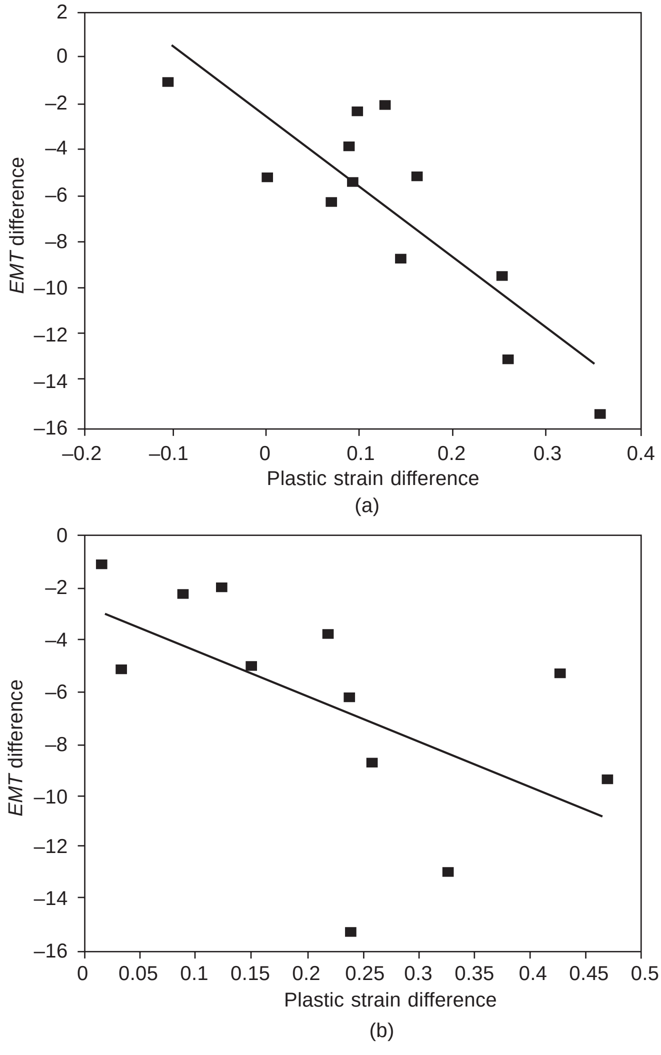


4.12 Comparison of yarn cross-section areas of warp and weft yarns.

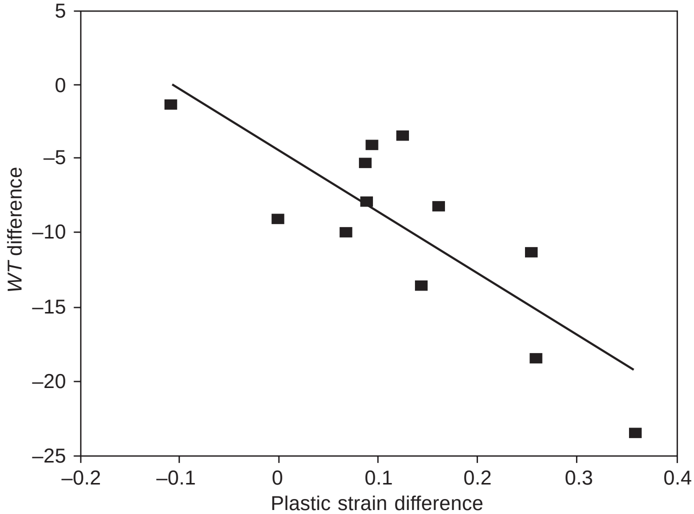
4.4.3 Relationship between plastic strain and tensile properties

It has been shown qualitatively that the tensile properties of a woven fabric are closely related to the strain-hardening effect. In this section, the quantitative relations between them will be provided.

Figure 4.13 shows the relationship between the *EMT* difference and the plastic strain difference of warp and weft yarns in woven fabrics. From Fig.



4.13 Relationship between ΔEMT and Δe for (a) circular cross-section and (b) elliptic cross-section.



4.14 Relationship between ΔWT and Δe.

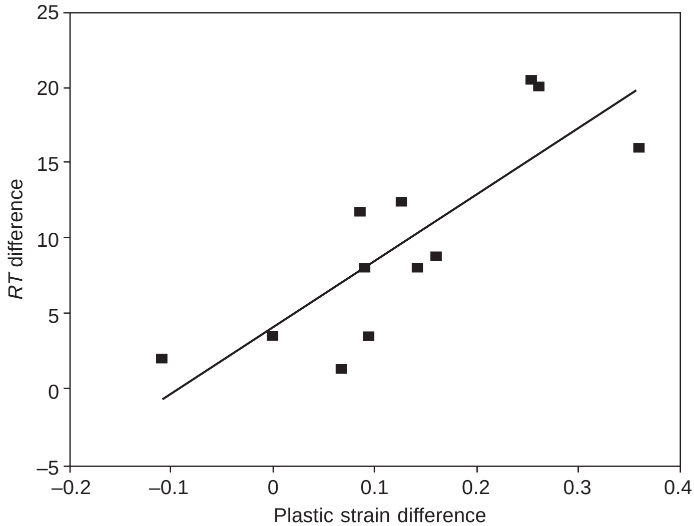
4.13a, whose plastic strain is calculated using equation 4.49 for the equivalent circular cross-sectional diameter of measured yarn area, it appears that a good linear relationship exists. Figure 4.13b, whose calculation is based on elliptic cross-section, shows similar trends.

Figure 4.14 plots the relationships between *WT* difference and plastic strain difference of warp and weft yarns in woven fabrics. It is clear that a good linear relationship also holds. Figure 4.15 depicts the relationships between *RT* difference and plastic strain difference of warp and weft yarns in woven fabrics. It is apparent that the larger *e* produces the larger *RT*.

4.5 Summary

This chapter introduces the author's contribution to the study of the tensile properties of woven fabrics. It starts with an introduction of the general concept of tensile properties with the focus placed on the features of tensile stress–strain curves of woven fabrics as well as the complexity of the deformation of woven fabrics under tensile load. This is followed by modelling the tensile behaviour of woven fabrics. Also presented is a study of the anisotropy of the tensile properties of woven fabrics together with an in-depth investigation of the strain-hardening effect observed from tensile stress–strain curves. The conclusions reached include:

(1) A woven fabric's tensile property is very difficult to study due to the great bulkiness in fabric structure in addition to the complexity in the structure and strain distribution of its constituent fibres and yarns and of the fabric itself as well as the strain variation during deformation. A



4.15 Relationship between ΔRT and Δe.

deformed fabric cannot resume its original geometrical state due to the existence of hysteresis. This is inherent since textile material is viscoelastic.

(2) The tensile stress–strain relationship of a woven fabric can be successfully described by an exponential function, $f = [(e^{\alpha\varepsilon} - 1)/\beta] + e_r$, in which α is a reinforcing factor for the increase of Young's modulus E. There is also found an obvious linear relationship between α with the crimp level c of the fabric.

(3) Regarding the various tensile parameters (*WT*, *EMT*, *LT*, *RT*), a great deal of similarity is found in their polar diagrams: their shapes are all symmetrical to the warp and weft directions; the value of each parameter differs with the angle; and the maximum happens exactly at either the warp (*WT* of satin, *LT*) or weft directions or at ± 45° angle (*WT*, *EMT*) corresponding to the warp and weft directions. The polar diagram of each parameter can be classified into two similar groups depending on the relationship of parameter values between the warp and weft directions.

(4) The strain-hardening phenomenon is found in woven fabrics. This phenomenon has a significant effect on the tensile properties of a woven fabric, as reflected by the variation in the Young's modulus value between warp and weft directions. It is believed that this phenomenon could be associated with the repeated loading and unloading a woven fabric experiences during manufacturing and processing.

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