5
The bending properties of woven fabrics

5.1 General bending behaviour of woven fabrics

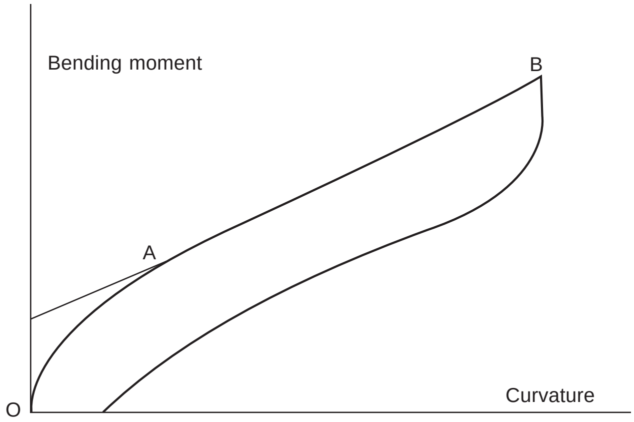
5.1.1 Introduction

The bending properties of fabrics govern many aspects of fabric performance, such as hand and drape, and they are an essential part of the complex fabric deformation analysis. Thus, the bending of woven fabrics has received considerable attention in the literature. Computational models for solving large-deflection elastic problems from theoretical models have been applied to specific fabric engineering and apparel industry problems, for example, the prediction of the robotic path for controlling the laying of fabric onto a work surface (Brown *et al.*, 1990, Clapp and Peng, 1991).

The most detailed analyses of the bending behaviour of plain-weave fabrics were given by Abbott *et al*. (1973), de Jong and Postle (1977), Ghosh *et al.* (1990a,b,c), Lloyd *et al*. (1978) and Hu *et al*. (1999, 2000). Modelling the bending of a woven fabric requires knowledge of the relationship between fabric bending rigidity, the structural features of the fabric, and the tensile/bending properties of the constituent yarns, measured empirically or determined through the properties of its constituent fibres and the yarn structure. It requires a large number of parameters and is very difficult to express in a closed form. Thus, the applicability of such models is very limited. Konopasek (1980a) proposed a cubic-spline-interpolation technique to represent the fabric moment–curvature relationship.

5.1.2 Moment–curvature curve of bending behaviour

Fabrics are very easy to bend. Their rigidity is usually less than 1/10 000 that of metal materials and about 1/100 that of tensile deformation. Bending properties of a fabric are determined by the yarn-bending behaviour, the weave of the fabric and the finishing treatments applied. Yarn-bending behaviour, in turn, is determined by the mechanical properties of the constituent



5.1 Typical bending curve of woven fabrics.

fibres and the structure of the yarn. The relationships among them are highly complex. Figure 5.1 illustrates a typical bending curve of woven fabrics.

For this curve, it is normally thought that there is a two-stage behaviour with a hysteresis loop within low-stress deformation: (a) an initial higher stiffness non-linear region, OA; within this region the curve shows that the effective stiffness of the fabric decreases with increasing curvature from the zero-motion position, as more and more of the constituent fibres are set in motion at the contact points; (b) a close-to-linear region, AB; since all the contact points are set in motion, the stiffness of the fabric seems to be close-to-constant.

It should be noted that when a woven fabric is bent in the warp or weft direction, the curvature imposed on the individual fibres in the fabric is almost the same as the curvature imposed on the fabric as a whole. As high curvatures meet when fabrics are wrinkled, the coercive couple or hysteresis is affected by viscoelastic decay of stress in the fibre during the bending cycle (Postle *et al.*, 1988). However, in applications where the fabric is subjected to low-curvature bending, such as in drapes, the frictional component dominates the hysteresis. Thus, if the strain in the individual fibres is sufficiently small that viscoelastic deformation within the fibres can be neglected, the hysteresis in Fig. 5.1 is attributed to non-recoverable work done in overcoming the frictional forces. The effect of the fibre's viscoelasticity in this section will not be considered because the bending of fabrics on the KES tester is within low-stress regions.

5.1.3 Bending stiffness

The primary concern with the conventional research in fabric bending is the bending stiffness. Bending stiffness is one of the main properties that control fabric bending. It should be defined as the first derivative of the moment–curvature (M–ρ) curve. If the structure of the bending curve is linear, M is directly proportional to the curvature produced. Some studies have been conducted to predict fabric bending stiffness. It has proved very difficult to calculate bending stiffness explicitly, due to the numerous factors that affect its value if the stiffness of the whole bending process is considered. In reality, the bending stiffness of fabrics is usually approximated to a constant which can be considered as steady-state-average-stiffness and the initial non-linear region is ignored. This is a low-order approximation to the actual non-linear bending properties present in most fabrics. Clapp and Peng (1991) have shown that the approximation to a constant stiffness may yield inaccurate values when calculating the fabric-buckling force in the initial buckling stage (Brown, 1998). As we can see in Fig. 5.1, the actual experimental M–ρ curves are non-linear, at least in the initial region in which the slope of the M–ρ curve for small values of ρ is greater than that for larger values of ρ. Thus, the bending-stiffness, B, should be a non-linear, continuous function of curvature.

5.1.4 Relationship between bending stiffness and bending hysteresis

The effect of friction on the steady-state-stiffness, known as 'elastic stiffness' in the literature, of fabric bending is well known to us and has been studied by a number of workers, including Peirce, Platt, Kleine and Hamburger, and Cooper before Liversey and Owen. But different researchers have different views on the manner and extent of this effect. Peirce suggested that a theoretical minimum warpway or weftway stiffness for a fabric might be calculated by summing the bending stiffness of the yarns; this was examined more fully by Cooper who found that friction or binding between the fibres causes the observed stiffness to exceed this minimum. The contribution of inter-fibre friction to the stiffness of a fabric has usually been studied by subjecting the specimen to a bending cycle and examining the resulting hysteresis curves. Liversey and Owen (1964) derived a mathematical formula for the minimum fabric bending stiffness, neglecting interactions between the fibres; this formula took account of the twist and crimp in the yarns. An instrument was described in their classical paper titled 'Cloth stiffness and hysteresis in bending' to assist in determining the nature of the interactions between fibres which cause the observed fabric bending stiffness to exceed the theoretical minimum.

In Grosberg's conclusion (1980), however, there is no friction present in the region of the close-to-linear portion of the bending curves; friction only affects the coercive moment. Postle *el al*. (1988) also thought that the internal friction has no effect on elastic bending or shear stiffness but did not mention whether friction exists during this period of deformation. Skelton (1974 and

1976) thought internal friction is always present during deformation but is independent of elastic stiffness. They all agreed that hysteresis is a measure of internal friction.

5.2 Modelling the bending behaviour of woven fabrics

5.2.1 Modelling the bending curves using non-linear regression

The modelling of the bending (moment–curvature) curve of woven fabrics started with the work of Peirce (1930). The theoretical modelling can be divided into three categories: predictive modelling, descriptive modelling and numerical modelling. The majority of the existing research work has been in the area of predictive modelling, in which the analytical relationship between fabric bending properties, yarn-bending behaviour and constituent-fibre behaviour, on the assumption of a given geometrical disposition of fibres or yarns in the fabric, is obtained. This kind of model was very difficult to solve in a closed form and thus very difficult to apply. A review of the research in this field was carried out by Ghosh *et al.* (1990a,b,c). It is not intended to re-review here due to its limited relevance.

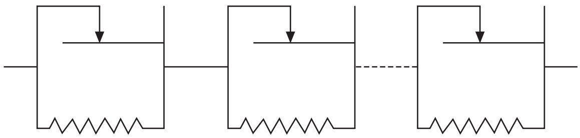
Many numerical modelling methods are used in mechanical engineering, and they are useful for the stress–strain analysis of a structure. Konopasek (1980) proposed the use of the cubic-spline-interpolation technique to represent the stress–strain relationship of fabric bending. The cubic-spline-interpolation technique is useful when the mathematical relationship between moment and curvature is not available, but it is rather cumbersome in computation and application. When the relationship of moment–curvature of fabric bending is available, a non-linear regression method may be used to estimate constants in the equation. The following introduces the descriptive model established by Oloffson (1967). It is expected that this model can be fitted using the non-linear regression technique.

There are examples scattered through the literature of rheological studies, or descriptive modelling, including sliding elements that are in accordance with Oloffson's study, in which a simple non-viscous combination consists of a sliding element (f_N) in parallel with an elastic element (E_N) in Fig. 5.2 or a block connected by a spring to a wall.

If the initial strain is equal to zero and $\sigma \geq \sigma_N$, the conditions exist for the displacement $\varepsilon_N|$ as a function of the external stress σ. If a series of coupled elements of the type is considered arranged in the sequence:

$$\sigma_{f1} < \sigma_{f2} < \sigma_{f3} < \ldots < \sigma_{fN} < \ldots \quad [5.1]$$

the force on all the elements is then the same:



5.2 Assembly of frictional and elastic elements.

$$\sigma_1 = \sigma_2 = \sigma_3 \ldots \sigma_N = \sigma \quad [5.2]$$

and the total deformation can be found by summing:

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \ldots \varepsilon_N + \ldots = \varepsilon \quad [5.3]$$

$$\varepsilon = \sum_{N=1}^{N-1} \varepsilon_n = \sigma \sum_{N=1}^{N-1} \frac{1}{E_N} - \sum_{N=1}^{N-1} \frac{\sigma_{fN}}{E_N} \quad [5.4]$$

If a continuous model considered by changing the step function

$$\sigma_{f1} < \sigma_{f2}\sigma_{f3} < \ldots < \sigma_{fN} < \ldots \quad [5.5]$$

corresponding to finite elements of Fig. 5.2 into a continuous function σ which increase with F (differential elements), then a continuous function for E_N can we expressed as a function of σ:

$$\frac{1}{E_N} = k\sigma^m \mathrm{d}\sigma \quad [5.6]$$

where the infinitesimal range dσ is introduced and β is the curvature of the fabric. The equation can thus be obtained:

$$\varepsilon = \sum_{N=1}^{N-1} \frac{(\sigma - \sigma_{fN})}{E_N} = \int_0^{\sigma} (\sigma - \sigma_f)\,\varphi(\sigma_f)\,\mathrm{d}\sigma_f \quad [5.7]$$

$$\varphi(\sigma_f) = k\sigma_f^m \quad [5.8]$$

and

$$\varepsilon = k\int_0^{\sigma} (\sigma - \sigma_f)\sigma^m \mathrm{d}\sigma = \frac{k}{(m+1)(m+2)}\,\sigma^{m+2} \quad [5.9]$$

where m is the conditional coefficient.

For an assembly of identical or nearly identical elements $m = 0$, hence a stress–strain relationship of the form:

$$\varepsilon = A\sigma^2 \quad [5.10]$$

or

$$\sigma = B\varepsilon^{\frac{1}{2}} \quad [5.11]$$

where A and B are two arbitrary constants. Equation (5.11) has been used in several cases for bending and shear initial behaviour. From the derivation conditions, this equation could be valid for the whole range of the deformation. But in practice, we can see that only the initial part was thought to obey this law. The principal range of m for fabric bending was reported to be $-0.1 > m > -0.9$.

In conventional studies, the Oloffson's model has only been applied when $m = 0$ and been used in the initial region of the moment–curvature curve; the latter stage has been considered as a linear relationship and even independent of the frictional element. The present work makes an attempt to modify equation 5.11 into a two-parameter function and to extend it to fit to the whole curve of experimental results using a non-linear regression method. The modified function including two constants α and β is as follows:

$$M = \alpha \rho^{\beta} \qquad [5.12]$$

where M is the bending moment and ρ the curvature.

5.2.2 Bending stiffness

Considering bending stiffness as a constant, the bending curve of fabrics can be described using equation 5.12. If the B–K (bending stiffness, B, versus curvature, K) curve is defined as the first derivative of the M–K curve,

$$B = \alpha\beta\rho^{(\beta-1)} \qquad [5.13]$$

the simulated bending stiffness now is a continuous, non-linear function of the curvature.

5.2.3 Estimation of two constants

Similar to the methods in Chapter 4, there are several ways to estimate the two constants α and β, but the most reliable one should be the non-linear regression method. The second choice may be the application of a general least squares method using more than two points. Suppose there are n sets of data from a bending curve of a woven fabric $(\rho_1, M_1), (\rho_2, M_2), \ldots, (\rho_n, M_n)$, then we have:

$$M_i = \alpha \rho_i^{\beta} \qquad [5.14]$$

So the sum of the squares of deviation from the true line is

$$S = \sum_{i=1}^{n} (M_i - \alpha \rho_i^{\beta})^2 \qquad [5.15]$$

By mathematical operation using the least squares principle, the following two equations can be obtained:

$$\alpha = \frac{\sum_{i=1}^{n} M_i \rho_i^{\beta-1}}{\sum_{i=1}^{n} \rho_i^{\beta}}, \quad \alpha = \frac{\sum_{i=1}^{n} M_i \rho_i^{\beta}}{\sum_{i=1}^{n} \rho_i^{\beta}} \qquad [5.16]$$

5.3 Modelling the bending properties of woven fabrics using viscoelasticity

5.3.1 Introduction

The bending performance of fabrics is characterised through parameters such as bending rigidity and hysteresis. However, the problem of how to separate the viscoelastic and frictional components in hysteresis remains unsolved. A detailed investigation of the bending of woven fabrics that determines the frictional couple through the cyclic bending curve of the fabric is needed. Hence, a theoretical model composed of a standard-solid model in parallel with a sliding element is proposed. The bending properties of woven fabrics are quantitatively studied.

Linear viscoelasticity is in fact applicable to many viscoelastic materials like wool, polyester, nylon and so on. In the study of fabric rheology from the phenomenological viewpoint, two simple rheological models consisting of linearly elastic and frictional elements, proposed by Oloffson (1967), are most popular in the textile literature (Grosberg 1966; Hamilton and Postle, 1974; Gibson and Postle, 1978; Hu, 1996). These models do not account for fibre viscoelastic processes which occur during fabric deformation and recovery. Chapman proposed a theoretical model in which the material is termed as 'generalized linear viscoelastic' and showed that the result fits single wool and nylon fibres at low strains (1 %) under changing temperature and relative humidity (Chapman, 1973; 1974a, 1975). The fabric has been shown to behave as a GLVE sheet in bending with an internal frictional moment (Chapman, 1974b). The frictional couple associated with each fibre in bending is principally considered as a function of strain and absolute time (Chapman, 1974c, 1980; Grey and Leaf, 1975, 1985; Ly, 1985). One of the fundamental ways to characterise the rheology of viscoelastic material is to bend the sample to a designated curvature and observe its transient behaviour. The recovery of fabrics from bending (Chapman, 1976), shear (Asvadi and Postle, 1994), creasing (Chapman, 1974d; Shi *et al*., 2000a,b,c) and wrinkling (Denby, 1974a,b; Denby, 1980; Postle *et al*., 1988) can be calculated through the knowledge of stress relaxation.

5.3.2 The linear viscoelasticity theory in the modelling of bending behaviour

Deformation, stress relaxation and subsequent recovery of fabrics can be studied quantitatively using the rheological model of linear viscoelasticity. Linear viscoelasticity is applicable for many viscoelastic materials when they are deformed to low strain (Postle *et al*., 1988). Modelling the viscoelastic behaviour of materials may involve using simple multiple-element models or generalised integrated forms.

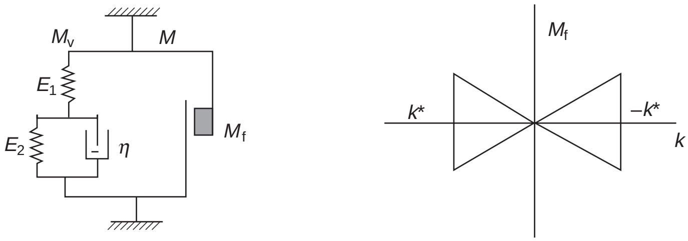
In order to simplify the calculation, the fibre is assumed to be linearly viscoelastic and its bending behaviour can be described by the standard solid model. The fabric is considered to be a viscoelastic sheet with internal frictional constraint. Its bending behaviour can be described by a three-element linear viscoelastic model in parallel with a frictional element, as shown in Fig. 5.2. The model is governed by the following equation (Chapman, 1974a):

$$M(k) = M_v(k) + \dot{k}/|\dot{k}| \times M_f \qquad [5.17]$$

In equation (5.17), $M(k)$ is the bending moment of the fabric, k is the curvature of the fabric at time t, M_f is the frictional constraint and m_v is the viscoelastic bending moment of the fabric. $\dot{k}$ is the rate of change of curvature (cm/s). The factor $\dot{k}/|\dot{k}|$ is the sign of the curvature change, which means that any curvature change of the fabric is opposed by the frictional constraint M_f. The frictional constraint interacts with the viscoelastic behaviour of single fibres to impose a limit on the recovery a fabric may eventually attain.

Frictional constraint restricts free movement of the fibres in fabric during bending. It is supposed that the fabric in bending acts like a linear spring in parallel with a frictional element and the frictional constraint is assumed to be a constant M_0 (Grosberg, 1966; Oloffson, 1967). The couple of the frictional sliding element is termed the 'coercive couple'. The coercive couple for fabrics in bending is half the distance between the cut-offs on the vertical or moment axis of the cyclic bending curve.

The intercept has been interpreted as being entirely due to the frictional moment and equal to $2M_0$ in the past (Grosberg, 1966). However, the frictional moment, in fact, only accounts for a portion of this intercept. Another portion of the intercept will be due to viscoelastic effects because the fibres are viscoelastic in nature (Konopasek, 1980b). In fact, the frictional constant varies with the maximum curvature imposed on the fabric (Ly, 1985). Since constant frictional constraint will lead to greater error and reduce the applicability of the model and the intercept on the bending moment axis made by the hysteresis loop is smaller than the 2*HB* from the Pure Bending Tester in Kawabata's Evaluation System, we assume that the frictional constraint is proportional to the curvature imposed on the fabric, as depicted in Fig. 5.3.



5.3 A three-element-plus-frictional viscoelastic model for bending of fabric.

If a fabric is bent at a constant rate of change of curvature ρ, the viscoelastic bending moment of the fabric of unit length can be expressed as

$$M_v(t) = \rho \int_0^t B(\tau)\,d\tau \qquad [5.18]$$

where $B(\tau)$ is relaxation modulus of the fabric. For a standard solid model, $B(\tau)$ is given by

$$B(\tau) = E_1 e^{-t/T} + \frac{E_1 E_2}{E_1 + E_2}(1 - e^{-t/T}) \qquad [5.19]$$

where the constant $T = \eta/(E_1 + E_2)$ is the relaxation time of the model, E_1 and E_2 are elasticity moduli of the springs, η is the viscosity coefficient of the damper. Substituting equation (5.19) into equation (5.18), the viscoelastic bending moment of the fabric can be written as follows:

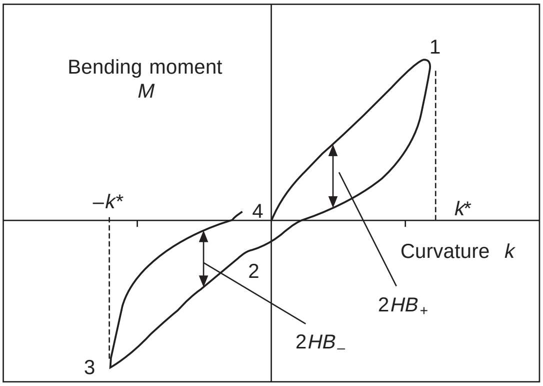
$$M_v(t) = \frac{E_1 E_2}{E_1 + E_2}\rho t + \frac{E_1^2}{(E_1 + E_2)^2}\rho\eta(1 - e^{-t/T}) \qquad [5.20]$$

$$= at + b(1 - e^{-t/T})$$

In equation (5.20),

$$a = \frac{E_1 E_2}{E_1 + E_2}\rho, \quad b = \frac{E_1^2}{(E_1 + E_2)^2}\rho\eta$$

When the fabric is cycled between curvature k^* and $-k^*$, a typical hysteresis curve for bending deformation is as shown in Fig. 5.4. The cyclic bending curve can be separated into regions where alternate positive and negative rates of change of curvature are inserted. By applying equation (5.20) the complete bending hysteresis cycle due to the viscoelasticity of the sample can be calculated. Using the Boltzman superposition principle to add the effects caused by the component strain rate for each portion of the hysteresis curve of the viscoelastic component, we can calculate the moment at points 1, 2, 3 and 4 in Fig. 5.4. For bending at a constant rate of ρ and limiting curvature k^*, $k = \rho t$, $t^* = k^*/\rho$, the viscoelastic bending moment at time t^*, $2t^*$, $3t^*$ and $4t^*$, is respectively obtained as



5.4 An idealised hysteresis loop for fabric bending.

$$M_{v1} = M_v(t^*) \qquad (t = t^*) \qquad [5.21]$$

$$M_{v2} = M_v(2t^*) - 2M_v(t^*) \qquad (t = 2t^*) \qquad [5.22]$$

$$M_{v3} = M_v(3t^*) - 2M_v(2t^*) \qquad (t = 3t^*) \qquad [5.23]$$

$$M_{v4} = M_v(4t^*) - 2M_v(3t^*) + 2M_v(t^*) \qquad (t = 4t^*) \qquad [5.24]$$

where M_{v1}, M_{v2}, M_{v3} and M_{v4} are viscoelastic components of the bending moment at points 1, 2, 3 and 4 in Fig. 5.4. Substituting equation (5.22) into equation (5.23), the viscoelastic moments at time t^*, $2t^*$, $3t^*$ and $4t^*$ can be expressed as, respectively

$$M_{v1} = at^* + b(1 - \gamma) \qquad (t = t^*) \qquad [5.25]$$

$$M_{v2} = -b(1 - \gamma)^2 \qquad (t = 2t^*) \qquad [5.26]$$

$$M_{v3} = -at^* - b(1 - 2\gamma^2 + \gamma^3) = -M_{v1} + \gamma M_{v2} \qquad (t = 3t^*) \qquad [5.27]$$

$$M_{v4} = b(1 - \gamma^2)(1 - \gamma)^2 \qquad (t = 4t^*) \qquad [5.28]$$

where

$$\gamma = e^{-t^*/T} = e^{-(E_1+E_2)t^*/\eta} \qquad [5.29]$$

For cyclic bending between curvature k^* and $-k^*$, as depicted in Fig. 5.4, the frictional constraint at points 1, 2, 3 and 4 varies and the total moments at each point can be defined in the following manner:

$$M_1 = M_v + \mu k^* = at^* + b(1 - \gamma) + \mu k^* \qquad (t = t^*) \qquad [5.30]$$

$$M_2 = M_{v2} = -b(1 - \gamma)^2 \qquad (t = 2t^*) \qquad [5.31]$$

$$M_3 = M_{v3} - \mu k^* = -M_{v1} + \gamma M_{v2} - \mu k^* \qquad (t = 3t^*) \qquad [5.32]$$

$$M_4 = M_{v4} = b(1 - \gamma^2)(1 - \gamma)^2 \qquad (t = 4t^*) \qquad [5.33]$$

However, there are only three independent equations in equations (5.30–5.33). Another equation must be established in order to find the solution to the other two unknown variables. One of the parameters used to characterise the bending properties of the fabric in the KES-FB-2 Bending Tester is 2*HB*, as depicted in Fig. 5.4, which is independent of equation (5.33) and is given by:

$$2HB_+ = M_+(k) - M_-(k) = M_{v+}(k) - M_{v-}(k) + 2M_f(k)$$
$$= b\left(2 - e^{\frac{k}{\rho T}} - 2e^{\frac{k^*-k}{\rho T}} + e^{\frac{2k^*-k}{\rho T}}\right) + 2\mu k \qquad [5.34a]$$

and

$$2HB_- = M_+(-k) - M_-(-k) = M_{v+}(-k) - M_{v-}(-k) + 2M_f(-k)$$
$$= b\left(2 - e^{-\frac{4k^*-k}{\rho T}} + 2e^{-\frac{3k^*-k}{\rho T}} + e^{-\frac{2k^*+k}{\rho T}} - 2e^{-\frac{k^*-k}{\rho T}} - 2e^{-\frac{k^*+k}{\rho T}}\right)$$
$$+ 2\mu k \qquad [5.34b]$$

where, the subscript $_+$ means the fabric is bent forward and the subscript $_-$ means the fabric is bent backwards. $2HB_+$ and $2HB_-$ are the width of the hysteresis loop at a specific curvature $\pm k$. In the KES-FB Pure Bending Tester, it is defined at curvature $\pm 1\ \text{cm}^{-1}$. Their average can be obtained as

$$2HB = (2HB_+ + 2HB_-)/2 = bQ + 2\mu k \qquad [5.35a]$$

where

$$Q = 0.5\left(4 - e^{-\frac{4k^*-k}{\rho T}} + 2e^{-\frac{3k^*-k}{\rho T}} + e^{-\frac{2k^*+k}{\rho T}} - 2e^{-\frac{k^*-k}{\rho T}} + e^{-\frac{2k^*-k}{\rho T}}\right.$$
$$\left. - 2e^{-\frac{k^*+k}{\rho T}} - 2e^{-\frac{k^*-k}{\rho T}} - e^{-\frac{k}{\rho T}}\right) + 2\mu k \qquad [5.35b]$$

Equation (5.33) can be merged as

$$\left.\begin{aligned} M_1 &= M_{v1} + \mu k^* = at^* + b(1-\gamma) + \mu k^* \\ M_2 &= -b(1-\gamma)^2 \\ M_1 + M_3 &= -b\gamma(1-\gamma)^2 \end{aligned}\right\} \qquad [5.36]$$

Solving simultaneous equations (5.35) and (5.36), the parameters are given by

$$\left.\begin{aligned} \gamma &= \frac{M_1 + M_3}{M_2} \\ b &= -\frac{M_1 + M_3}{\gamma(1-\gamma)^2} \\ \mu &= \frac{2HB - bQ}{2k} \\ a &= \frac{M_1 - b(1-\gamma) - \mu k^*}{t^*} \\ T &= -t^*/\ln\gamma \end{aligned}\right\} \qquad [5.37]$$

Then, three parameters of the standard solid model can be obtained as follows:

$$\left.\begin{aligned} E_1 &= \frac{aT + b}{T\rho} \\ E_2 &= \frac{a(aT + b)}{b\rho} \\ \eta &= -T(E_1 + E_2) = -\frac{(aT + b)^2}{b\rho} \end{aligned}\right\} \quad [5.38]$$

Thus, the proposed bending model for a fabric can be established through three points in the moment–curvature curve and a hysteresis parameter.

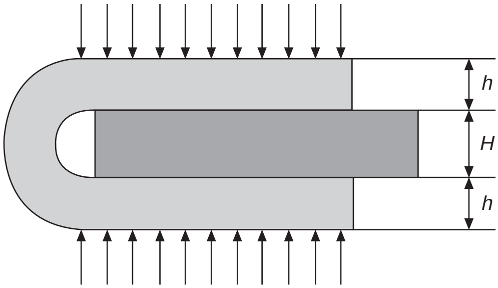
5.4 Modelling the wrinkling properties with viscoelasticity theory

5.4.1 Introduction

When a fabric is creased and then released, the residual forces in the fibres enable the fabric to unfold or recover. Wrinkle recovery is thus defined as the property of a fabric that enables it to recover from folding deformations. The most common method of testing crease recovery (ISO 2313, IWTO Drift TM 42) and wrinkle recovery (AATCC 66-1990) is to bend a strip of fabric by heavy loading at controlled time and air conditions and measure the angle of recovery after releasing the load.

During wrinkling deformation, all fabrics show a varying degree of inelasticity, such as viscoelasticity and inter-fibre friction, because of the viscoelastic nature of the constituent fibres and the rearrangement within the fibre assembly. Their responses to applied loads are rate- or time-dependent. At any time, the state of stress within a fabric depends on the entire loading history. The viscoelastic nature of the constituent fibre is responsible for the phenomenon of stress relaxation, and the inter-fibre friction provides the fabric frictional stress during deformation and is responsible for the irreversible deformation. Studying these inelastic effects in fabrics enables us to understand and eventually predict important performance characteristics.

In this section the modelling of wrinkling, wrinkle recovery and set of fabrics are established using the rheological model of linear viscoelasticity based on the bending model developed in Section 5.3. The recovery of the fabrics after release from wrinkling is analysed and the wrinkle recovery angle of the fabrics is calculated using the model parameters derived from pure bending test.



5.5 Wrinkling of a fabric for testing of wrinkle recovery angle.

5.4.2 Modelling the wrinkle recovery angle of woven fabrics

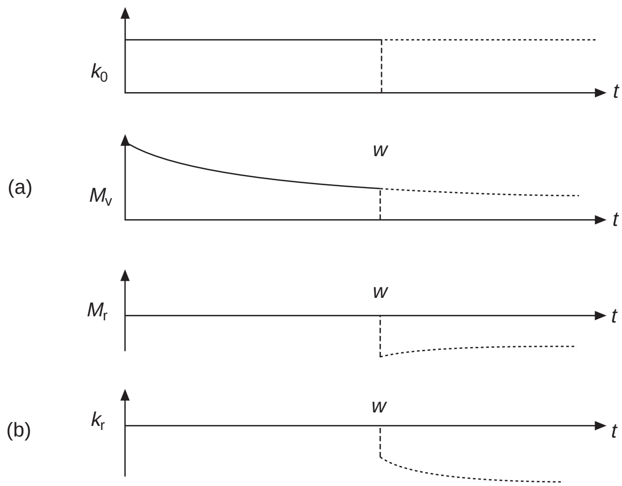
Assume that a woven fabric is simply folded in the warp or weft direction and pressed together by a uniform pressure normal to the surface of the fabric, as shown diagrammatically in Fig. 5.5.

When a fabric is held at a fixed curvature k_0 for a period of time t, and if the fabric is considered as viscoelastic sheets with internal constraints, which follow the three-element model in parallel with a sliding element and the frictional constraint is considered to be proportional to the curvature of the fabric as shown in Figs 5.1 and 5.2, the relaxation stress for the standard solid model may expressed as (Creus, 1986; Yan, 1990)

$$M_v(t) = E_1 k_0 e^{-t/T} + \frac{E_1 E_2}{E_1 + E_2} k_0 (1 - e^{-t/T}) \quad [5.39]$$

It can be found that the relaxation moment decreases progressively when the fabric is held at a constant curvature. That is to say, the residual moment in the fibre drops with time or the moment needed to maintain the fabric at a constant curvature reduces gradually as indicated in Fig. 5.6(a).

The fabric is creased for a length of time w and then released against a restraining couple M_f. Based on the Boltzmann superimposition principle, removing the applied force that maintains constant curvature k_0 is equivalent to a $-M_r$ being exerted in the opposite direction on the fabric, that is, $M_r(t)$ $(t > w)$ equal to $M_v(t)$ $(t > w)$ in magnitude, but opposite in direction, as shown in Fig. 5.6(b). M_r acts on the fabric and makes it recover from wrinkling or creasing deformation. M_r can be divided into two portions. One portion, M_{rv} acts on the standard solid element. Another portion, M_{rf} is assumed in the frictional element. The frictional constraining couple is directly proportional to the curvature of the fabric according to the assumption above. If the fabric has a curvature k_r from curvature k_0 under the action of M_r, then the frictional constraining couple is equal to μk_r.



5.6 Stress and strain relation of the model during insertion of wrinkles and wrinkle recovery (a) step curvature applied during insertion of wrinkles and stress relaxation; (b) residual stress and curvature recovery of the fabric after releasing.

At instant t after the fabric is released, the moment can be expressed as

$$M_r(w + \tau) = M'_v(\tau) + \mu k_r \quad [5.40]$$

where k_r is the curvature of the fabric produced by M_r. To calculate wrinkle recovery of the fabric after release from a fixed curvature, we consider now the curvature change of the fabric under a stress $-M_r$. The constitutive equations for the standard solid element can be established as follows:

$$M'_{rv} + \frac{\eta}{E_1 + E_2}\dot{M}'_{rv} = \frac{E_1 E_2}{E_1 + E_2}k_r + \frac{E_1}{E_1 + E_2}\eta\dot{k}_r \quad [5.41]$$

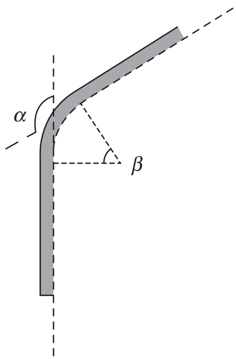
Substituting equation (5.40) into and rearranging equation (5.41) gives

$$\dot{k}_r + \left(\frac{E_1 + E_2}{E_1\eta}\mu + \frac{E_2}{\eta}\right)k_r = \frac{E_2}{\eta}k_0 - \frac{\mu}{E_1} \quad [5.42]$$

Solving equation (5.42), the recovery deformation of the fabric is given by

$$k_r(\tau) = \frac{E_1 k_0}{(E_1 + E_2)(\mu + E_1)}(E_1 e^{-w/T} + E_2)e^{-\left(\frac{E_1+E_2}{E_1\eta} + \frac{E_2}{\eta}\right)\tau} + \frac{E_1 E_2 k_0 - \mu\eta}{(E_1 + E_2)\mu + E_1 E_2}\left[1 - e^{-\left(\frac{E_1+E_2}{E_1\eta} + \frac{E_2}{\eta}\right)\tau}\right] \quad [5.43]$$

The remnant curvature of the fabric at moment τ after the applied force is removed can be expressed as



5.7 The proposed model for the wrinkle recovery angle of a fabric.

$$k(\tau) = k_0 - k_r(\tau) \tag{5.44}$$

We assume that the bent portion of the fabric takes a semi-circular profile during the insertion of wrinkles and a circular arc profile during recovery from wrinkles, as shown in Fig. 5.7. If the length of the circular arc is constant and equal to that of the semi-circle, that is

$$\frac{\pi}{k_0} = \frac{\pi - \alpha}{k} \tag{5.45}$$

then, the wrinkle recovery angle of the fabric can be expressed as

$$\alpha = \pi\left(1 - \frac{k}{k_0}\right) = \pi\frac{k_r}{k_0} \tag{5.46}$$

The instantaneous wrinkle recovery angle α_0 and the maximum wrinkle recovery angle α_∞ at time $\tau = 0$ and $\tau = \infty$ can be derived respectively as follows:

$$\alpha_0 = \frac{E_1}{(E_1 + E_2)(\mu + E_1)}(E_1 e^{-w/T} + E_2) \times 180° \tag{5.47}$$

$$\alpha_\infty = \frac{E_1 E_2 k_0 - \mu\eta}{(E_1 + E_2)\mu + E_1 E_2}\,\frac{180°}{k_0} \tag{5.48}$$

It can be seen that the wrinkle recovery angle is completely determined once we know the values of k_0, w, τ and the parameters of the elements in the model. Thus, the wrinkle recovery angle of the fabric can be predicted using the model parameters derived from the pure bending test.

5.5 Anisotropy of woven fabric bending properties

5.5.1 Introduction

Bending behaviour of a woven fabric can be characterised by bending rigidity (B) and bending hysteresis ($2HB$). Bending rigidity is the resistance of a fabric to bending, which can be defined as the first derivative of the moment–curvature curve. Bending hysteresis is the energy loss within a bending cycle when a fabric is deformed and allowed to recover, denoting the difference in bending moment between the loading and the unloading curves when the bending curvature is fixed.

Postle *et al.* and Hu have proved the close relationship between bending rigidity and bending hysteresis. In particular, Postle *et al*. reported very good correlation between the bending and the hysteresis parameters measured from fabric bending deformation recovery curves (1988). Moreover, the research done by Chung and co-workers (Chung *et al*., 1990; Chung and Hu, 2000) indicates that the correlation coefficient of bending stiffness and bending hysteresis is quite high, 0.9333 for cotton fabric. For worsted and Schengen woven fabrics, B and $2HB$ are also very high, 0.7872 and 0.7596 respectively. This implies that bending stiffness and bending hysteresis are not independent, but have a linear relationship (Hu, 1994).

There may be some differences in the mechanism operating in bending rigidity and bending hysteresis of woven fabrics but, based on the above findings, it is assumed that they have similar mechanisms. Thus this section discusses an attempt to apply the existing models for bending rigidity to bending hysteresis of plain woven fabrics. Also presented is an attempt to examine which of the existing models is the best for predicting bending hysteresis.

5.5.2 Directionality of fabric bending rigidity

Peirce (1930) produced a formula for calculating the stiffness of a fabric in any direction in terms of the stiffness in the warp and weft direction. This was derived from the theory for homogenous elastic material and it was found to be empirically satisfactory. It is suggested that the reason for this is that most of the fabrics which Peirce tested were made from cotton. In addition, he also reported a formula to predict the bending stiffness in various directions, in which the values in the warp and weft directions were known.

Go *et al.* (1958) measured the bending stiffness of fabrics using the heart loop method. They indicated that the bending stiffness of the fabric is dependent on the bending model of the test piece. The bending stiffness of fabric having long floats on its surface was smaller in face-to-face bending than back-to-back. The effect of the crimp of the component yarn of fabric on the fabric bending stiffness was generally small. Later, Go and Shinohara (1962) reported that on the polar diagram of bending stiffness there was minimum presented at 45° to the warp when the fabric was bent. Their formula neglected the restriction at the interaction of the warp and weft directions. They concluded that the stiffness of textile fabrics depended upon their bending directions and that, in general, the stiffness in bias directions was relatively small.

Cooper (1960) used cantilever methods to determine fabric stiffness and stated that there was no evidence to suggest that there was any appreciable shearing of the fabric caused by its own weight. He concluded that the stiffness of a fabric may vary with direction of bending in different ways, but for most practical purposes measurement along warp, weft and one other direction was sufficient to describe it.

Cooper conducted a detailed study of the stiffness of fabrics in various directions and has produced polar diagrams of bending stiffness. He found that some fabrics had a distinct minimum value at an angle between the warp and weft direction while others had similar values between the warp and weft. In general, viscose rayon fabrics provided an example of the former and cotton fabrics an example of the latter.

These effect were explained in terms of the fabric bending stiffness in the warp and weft direction and the resistance offered by the yarns to the torsional effects which are inseparable from bending at an angle to warp and weft (Cooper, 1960). He concluded that the resistance offered by the yarns to the torsional deformation is low when the interaction between the yarns is low and vice versa.

Shinohara *et al*. (1980) derived an equation empirically which is similar to the equation introduced by Peirce and analysed the problems using three-dimensional elasticas. They assumed the constituent yarns of woven fabrics to be perfectly elastic, isotropic, uncrimped and circular in cross-section, and to behave in a manner free from inter-fibre friction. In addition, they also presented another equation containing a parameter n which was related to V introduced by Cooper (1960) in order to predict the shape of a polar diagram.

5.5.3 Theoretical study of fabric bending properties

Peirce first introduced the bending rigidity of a fabric by applying an equation in his classical paper as follows:

$$B = wc^3 \qquad [5.49]$$

where B is the bending rigidity, w is the weight of the fabric in grams per square cm and c is the bending length. He also introduced another equation for bending rigidity in various directions. This formula enabled the value for any direction to be obtained when the values in the warp and weft directions were known:

$$B_\theta = \left[\frac{\cos^2\theta}{\sqrt{B_1}} + \frac{\sin^2\theta}{\sqrt{B_2}}\right]^{-2} \qquad [5.50]$$

where B_1, B_2 and B_θ are bending rigidities in warp, weft and θ directions, respectively.

A similar equation could also be considered empirically by Shinohara *et al.* (1980):

$$B_\theta = (\sqrt{B_1}\cos^2\theta + \sqrt{B_2}\sin^2\theta)^2 \qquad [5.51]$$

Go *et al.* also reported an equation which was theoretically derived by neglecting twist and frictional effects from equation (5.50):

$$B_\theta = B_1\cos^4\theta + B_2\sin^4\theta \qquad [5.52]$$

(Go *et al.* 1958; Go and Shinohara 1962).

Later, Cooper (1960) presented an equation including twist effect. The results of the twisting effect were found to be valuable in practical applications and so equation (5.53) was derived:

$$B_\theta = B_1\cos^4\theta + B_2\sin^4\theta + (J_1 + J_2)\cos^2\theta\sin^2\theta \qquad [5.53]$$

where J_1 and J_2 are constants due to torsional moment.

Chapman and Hearle (1972) also derived a similar equation by energy analysis of helical yarns as follows:

$$\begin{aligned} B_T &= n_1 v_1 \sin^2\theta(B\sin^2\theta + J_y\cos^2\theta) \\ &\quad + n_2 v_2\cos^2\theta(B\cos^2\theta + J_y\sin^2\theta) \qquad [5.54] \\ B_T &= n_1\theta + \eta\cos^2\theta + n_2 v_2\cos^2\theta(B\cos^2\theta + J) \end{aligned}$$

where B_T is an expression for the bending rigidity per unit width of a thin fibre web of linearly elastic fibres and there are n_1 yarns per unit length in the warp direction, each containing v_1 number of fibres, and n_2 yarns per unit length in the weft direction, each containing v_2 number of fibres. They assume that they have a two-dimensional assembly of very long straight fibres of the same type, with bending rigidity B and torsional rigidity J_y. Their approach utilises energy considerations instead of the 'force method'. Chapman and Hearle's model involves many variables which will complicate the mathematical calculation. Their approach is, in fact, very similar to Cooper's so Cooper's model is chosen for the study.

From equation (5.53), B_1 and B_2 may be obtained directly by experimental work while J_1 and J_2 cannot. The theoretical treatment suggests that measurements of stiffness in two directions are insufficient to define a fabric's bending properties, since different types of variation with direction are still possible for fabrics with similar B_1 and B_2. An investigation into a third direction is therefore necessary, and the most convenient in practice is at bias direction (45°). In this direction, twisting effects are small provided that B_1 and B_2 are similar in magnitude. Nevertheless, the sum $(J_1 + J_2)$ may be deduced from measurements in three different directions by considering specimens cut along the warp, weft and 45° directions. Therefore, when considering $\theta = 45°$,

$$B_{45} = B_1 \cos^4 45° + B_2 \sin^4 45° + (J_1 + J_2) \cos^2 45° \sin^2 45°$$

$$= B_1\left(\frac{1}{\sqrt{2}}\right)^4 + B_2\left(\frac{1}{\sqrt{2}}\right)^4 + (J_1 + J_2)\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2 \quad [5.55]$$

$$= \frac{1}{4}(B_1 + B_2 + J_1 + J_2)$$

where

$$J_1 + J_2 = 4B_{45} - (B_1 + B_2)$$

The term $(J_1 + J_2)$ is replaced by the stiffness value at the warp, weft and 45° directions. We may use this result to calculate other bending rigidities over all possible directions as in equation (5.56):

$$B_\theta = B_1 \cos^4 \theta + B_2 \sin^4 \theta + [4B_{45} - (B_1 + B_2)] \cos^2 \theta \sin^2 \theta \quad [5.56]$$

In Cooper's paper, he argued that the shape of polar diagrams of bending rigidity B may show three types of variation between fabrics. The ratio $(J_1 + J_2)/(B_1 + B_2) = V$ is introduced to predict the trends in polar diagrams. When the term $(J_1 + J_2)$ is replaced by the stiffness values of warp, weft and 45° directions, the equation for the ratio V will change as follows:

$$V = \frac{4B_{45} - (B_1 + B_2)}{B_1 + B_2} \quad [5.57]$$

Cooper's model for calculation of ratio V is dependent on bending rigidity (B_1 and B_2) and torsional rigidity (J_1 and J_2). This leads to different shaped polar diagrams. Furthermore, different ratios of bending rigidity in warp and weft directions can also contribute different shapes of polar diagrams. When the torsional rigidity is replaced by the bending rigidity of warp, weft and 45° directions, the calculation of ratio V is simplified.

From the results provided by Cooper (1960), it may be seen that the range of ratio V is between 0 and 1. He found that some fabrics with very open structure had a distinct minimum value at an angle between warp and weft direction when $V = 0$. In this case, the model is identical to that derived by Go *et al*. (1958). When $V = 1$, these minima are absent and the model is qualitatively similar to that described by Peirce (1937) and Shinohara *et al*. (1980).

In Cooper's model (1960), the coefficient of $\cos^2\theta \sin^2\theta$ was related to the torsional rigidities of the yarn. It was found that the polar diagram of fabric bending rigidity fitted reasonably with other models. However, there is a limitation, which relates to ratio V (equation 5.57) introduced by Cooper (1960). Since fibres display marked non-linear viscoelasticity, and this is superimposed on a complicated yarn and fabric geometry, this also gives rise to frictional restraints between fibres and between yarns. If the fabric is bent in the bias direction, high inter-yarn friction arises due to the relative movement of the yarns (Chapman *et al.*, 1972); it is, therefore, impossible to obtain $V = 0$.

On Cooper's theoretical polar diagram (1960), distinct minima are presented in the polar diagram of fabric bending rigidity between the two principal directions when a very open plain fabric is examined. Go *et al.*'s (1958) model may be applicable to a very open structure fabric as the twist and frictional effect in this type of fabric is small. However, their model cannot be applied in the prediction of fabric bending rigidity of other types of fabrics.

Since observed values do not always agree with the theoretical model (equation 5.51) derived by Shinohara *et al.* (1972), they presented another model containing a parameter n which relates to V introduced by Cooper (1960) as follows:

$$B_\theta = B_1 \cos^4 \theta + B_2 \sin^4 \theta + 2n\sqrt{B_1 B_2} \cos^2 \theta \sin^2 \theta \qquad [5.58]$$

and

$$\frac{V}{n} = \frac{2\sqrt{B_1 B_2}}{B_1 + B_2} \qquad [5.59]$$

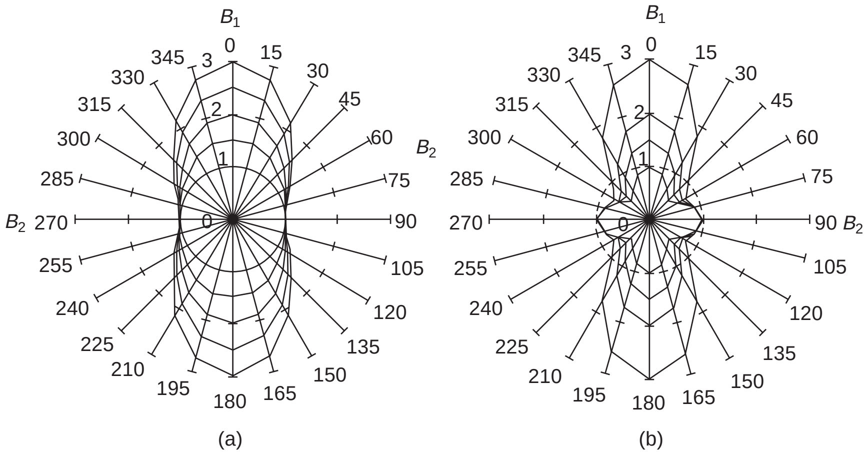
where V/n is a ratio of geometrical mean to arithmetical mean of B_1 and B_2. From experimental results on commercially available fabrics, Shinohara *et al.* found that the values of n are in the range from 0 to 1 and minimum values exist in 45° directions for certain types of fabrics. The term n presented by Shinohara *et al*. (1980) is also used to predict the trends in polar diagrams, and similar trends are observed in Cooper's ratio V. They reported that tight fabrics generally have larger values of n, and sleazy fabrics have a smaller value of n.

5.5.4 Polar diagrams of the bending model

5.5.4.1 General features of the polar diagrams

Similar polar diagrams are observed from three of the existing models (Peirce's model, Shinohara *et al.*'s model, and Cooper's model). These polar diagrams and the diagram produced from Go *et al.*'s model can be classified generally into two types according to their shape. The polar diagrams of various values of B_1/B_2 in Types 1 and 2 models are shown in Fig. 5.8, which demonstrates the theoretical polar diagrams of fabric bending rigidity in various directions.

It can easily be observed from Fig. 5.8 that the anisotropy of Type 2 omits the resistance at the intersection of warp and weft. Therefore, distinct minima are present between the warp and weft directions. However, a circular shaped polar diagram is obtained when B_1 equals B_2 in anisotropy of Type 1. If the



5.8 Theoretical curves of bending rigidity of fabric anisotropy; (a) type 1: by Peirce, Shinohara *et al*. and Cooper; (b) type 2: by *Go et al.*

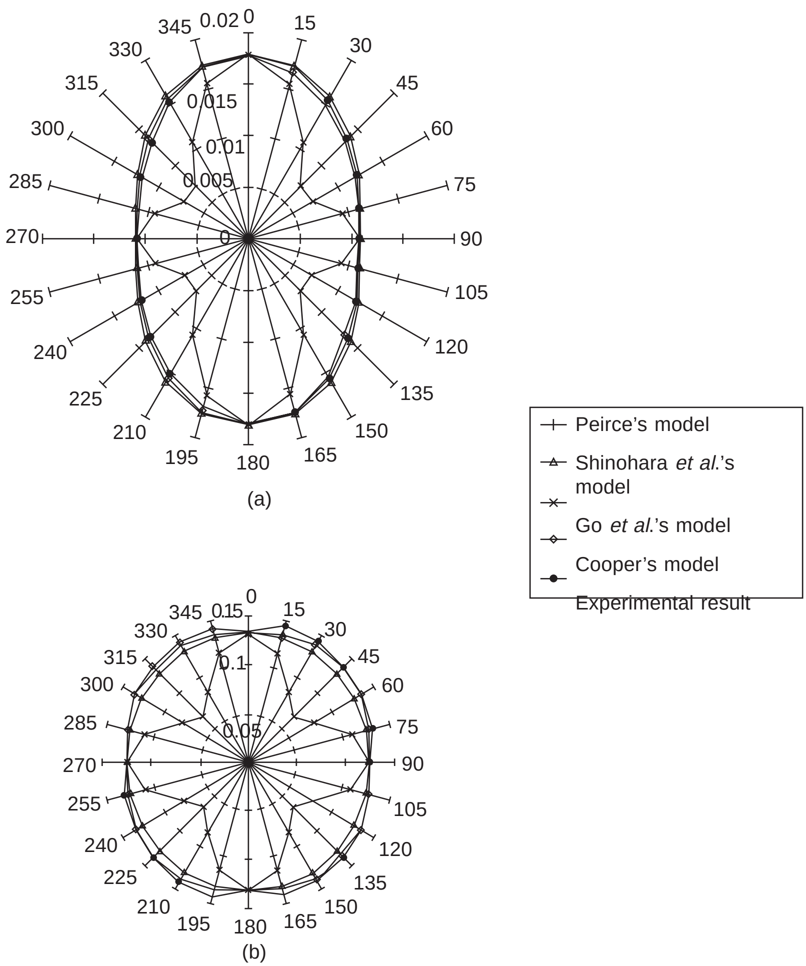
difference between the bending rigidity B_1 and B_2 becomes larger, an ellipse or gourd shape is illustrated in the polar diagram of fabric bending rigidity.

5.5.4.2 Comparison of four models

In this section, the predictability of the four models (Go *et al*., Peirce, Shinohara *et al.* and Cooper) discussed above will be compared. Additionally, the effect of ratio V on plain woven fabrics in different weave densities will be investigated as will the shapes of the polar diagrams of bending hysteresis from different values of ratio V on cotton plain woven fabrics.

For ease of comparison it is convenient to fix the bending hysteresis in the warp direction so that changes occurring along any other directions can be easily observed from the polar diagrams. This fixed bending hysteresis can be obtained by averaging all results recorded in the warp direction, and then multiplying or dividing the bending hysteresis of each fabric by this average value. In this way, the points in the warp direction can be fixed and any differences other than warp direction can be seen in the polar diagrams of different fabrics. Trends in the ratio V can also be observed with different types of plain woven fabrics.

Figure 5.9 illustrates the bending hysteresis of lighter (loose) and heavier (tight) plain fabrics produced from the outputs of four models against the experimental result. As Go *et al.*'s model neglects the twist and frictional effect, the polar diagram of this model exhibits a cross shape with minima around the 45° direction. Peirce's, Shinohara *et al.*'s and Cooper's models show elliptic shapes.



5.9 Comparison of four models with the experimental result of bending hysteresis: (a) light plain woven fabric; (b) heavy plain woven fabric.

All experimental results on plain woven fabrics are close to values calculated from Peirce's, Shinohara *et al.*'s and Cooper's models. It is also found that the average deviation between Go *et al.*'s model and the experimental result is the largest when compared with other models, which indicates that Go *et al.*'s model cannot be applied to the prediction of polar diagrams of bending hysteresis. Therefore, the twisting and frictional effects play significant roles in the calculation of bending properties.

From Cooper's theoretical polar diagram (Cooper, 1960), there are distinct minima in the polar diagram of bending rigidity between warp and weft directions when a very open plain light woven fabric is examined. Go *et al.*'s model may also be applicable to loose fabrics as the twist and frictional effect in a very open plain fabric is small. In contrast, these minima are absent in the polar diagrams of bending hysteresis. Bending hysteresis is a measurement of inter-yarn friction. When the fabric is bent on the bias, relative movement of the yarns occurs and is maintained by high inter-yarn friction. Therefore, there are no minima present on the bias directions. In Fig. 5.9(a), it is found that Peirce's, Shinohara *et al.*'s and Cooper's models are well fitted to the polar diagrams of bending hysteresis of loose fabrics. However, Go *et al.*'s model should not be applied to the prediction of the shape of a polar diagram of bending hysteresis in loose fabric.

Another fact is that each model produces larger deviation on heavy fabrics than on light fabrics. Besides, it is not difficult to see from Fig. 5.9(b) that the highest value of the bending hysteresis of heavy fabrics is observed around 15° to the warp. It also reveals that the component around this angle contributes the highest bending hysteresis. However, beyond 15°, the bending hysteresis of these fabrics decreases with the increase in angle.

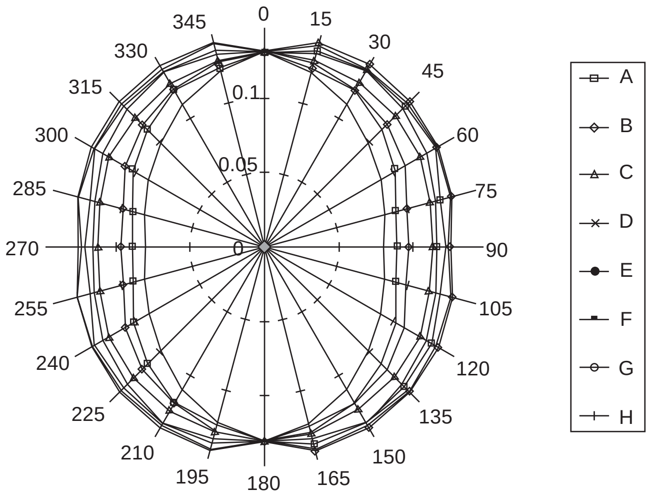
In addition, although Peirce's, Shinohara *et al.*'s and Cooper's models can all be applied to the prediction of polar diagrams of bending hysteresis for loose to tight plain woven fabrics, of the three, Cooper's model presents the lowest deviation from the experimental results. Therefore, it can be seen that the twist and frictional effects in Cooper's model play an important role in the prediction of bending hysteresis on either loose or tight plain woven fabrics. Moreover, when comparing the bending hysteresis of loose and tight plain woven fabrics, the deviation in loose plain fabric is smaller than that in tight plain fabric.

From the above analysis, we may conclude that Cooper's model is the most reliable in the prediction of bending hysteresis in both loose and tight plain woven fabrics.

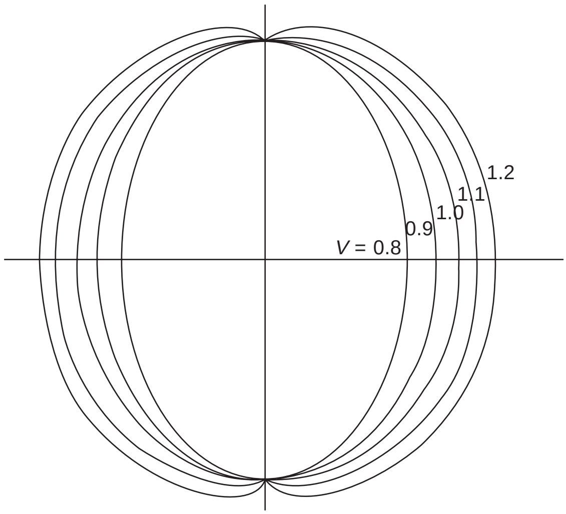
5.5.4.3 Effect of ratio V on bending hysteresis

Figure 5.10 illustrates the *HB* polar diagrams of eight different types of plain cotton fabrics, from which we can see that the trends are spreading outwards along the weft direction with the increase in ratio V. If the ratio V is larger than 1.10, the maximum value of bending hysteresis will be observed at around 15° to the warp.

The loose structure allows the movement of yarns along the warp and weft directions. The floating yarns present in this structure may lead to lower inter-yarn friction. As a result, the lowest bending hysteresis is obtained from the loose fabrics. On the contrary, the tight structure avoids yarn movement and this will increase the bending hysteresis of the fabrics. Therefore, larger bending hysteresis leads to the expansion of the polar diagram along the weft direction. The predicted shapes of the polar diagrams of bending hysteresis from ratio V of cotton plain woven fabrics are given in Fig. 5.11.



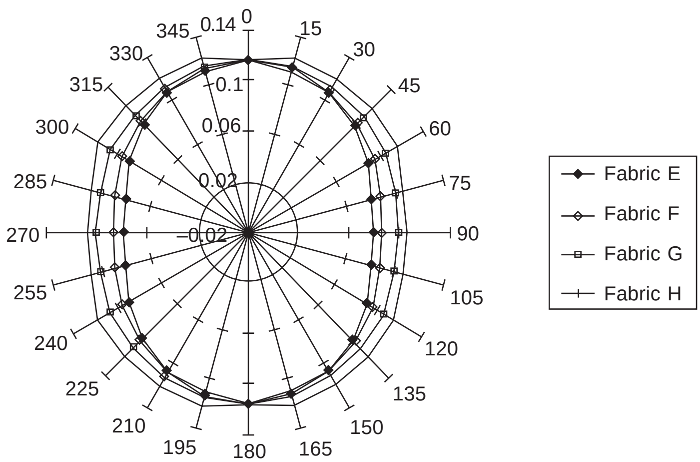
5.10 Effect of ratio *V* on bending hysteresis of plain woven fabrics.



5.11 The relationship between ratio *V* and bending hysteresis of cotton plain woven fabrics.

5.5.4.4 Effect of weft density on polar diagram

Similar to the effect of ratio *V*, with the increase in weft density, the *HB* polar diagrams also exhibit a tendency to spread outwards, as shown in Fig. 5.12.



5.12 Effect of weft density of plain woven fabrics. Ranking of weft density: *E* < *F* < *G* < *H*.

5.6 Summary

This chapter presents a comprehensive study of the bending properties of woven fabrics. After an explanation of the general concept of bending properties, two different methods, non-linear regression technique and viscoelasticity theory, are introduced to model the bending behaviour of woven fabrics. Finally, as in Chapter 4, a study of the anisotropy of the bending properties of woven fabrics is also provided. We may draw the following conclusions from this chapter:

(1) Non-linear regression techniques can be successfully used to model the bending curves of woven fabrics based on Oloffson's rheological model. The estimates obtained using non-linear regression seem to be different from those used in the existing research. In addition, bending stiffness is thought to be continuous and can be obtained by the differentiation of the moment–curvature curve with respect to curvature.

(2) The inelastic bending behaviour of woven fabrics can be quantitatively analysed using linear viscoelasticity theory. With the rheological model developed, it is found that the bending properties of the fabric under low curvature can be characterised using a standard solid element in parallel with a frictional element. Element parameters of the model can be determined through three points in the bending curve and the bending hysterisis. The difference between 2*HB*+ and 2*HB*– and the intercept of the curve at moment axis should be attributed to friction between the fibres.

(3) Based on Chapman's assumption of a semi-circular form for a fabric

bend, a simple rheological model consisting of a linearly elastic element and a frictional element is successfully used to study creasing of the fabric strips and the compression of the fabric loops. The relationship between the creasing behaviour and deformation is established and solutions are given for a linearly elastic material with constant internal frictional constraint. In addition, the formula deduced by the authors can provide a better fit than the formula given by Chapman.

(4) An extensive study of the existing bending hysteresis models reveals that Peirce's, Shinohara *et al.*'s and Cooper's models can be applied to predict the bending hysteresis anisotropy of various apparel woven fabrics, but Go *et al.*'s model can be applied only to fabrics with very open structure. This finding confirms the statement that the twisting and frictional effects have a significant role in bending properties. Moreover, all the four models are better used to predict light fabrics. In particular, Cooper's model is found to be the best one to predict the anisotropy of bending rigidity and it is also the only one that can be extended to industrial woven fabrics.

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