2. Steady State and Non-Steady State Technological Processes

2.1 Definitions

In this section the meaning of some technical terms, which are necessary for the understanding of the following parts, will be explained. Besides the terms "fibre formation" and "fibre processing" characterising the *textile technological process* the title of the book contains the terms of *dynamics* and *modelling*. The latter are to be defined first.

2.1.1 The Technological Process

Before the consideration of some specific points, the technological process itself is defined:

The technological process is an organised series of scientifically determined changes of the treated product in order to fulfill a certain manufacturing task. Those changes are initiated by the employment of production instruments and machines.

The succession of each technological process is coupled with the appearance of certain energetic processes:

Each technological process is a meaningful coupling of different energetic processes, which are applied to the treated product. Their combination serves the realisation of a goal-directed procedure.

The example of the drawing process, a basic process in the production of chemical fibres, shall clarify the given definitions.

Figure 2.1 shows a series of important parameters of the manufacturing task of drawing. During the process they are connected by varied cause-effect relations. The drawing process is put into reality by input and output godet pairs driven with different velocities. The parameters can be divided into two groups:

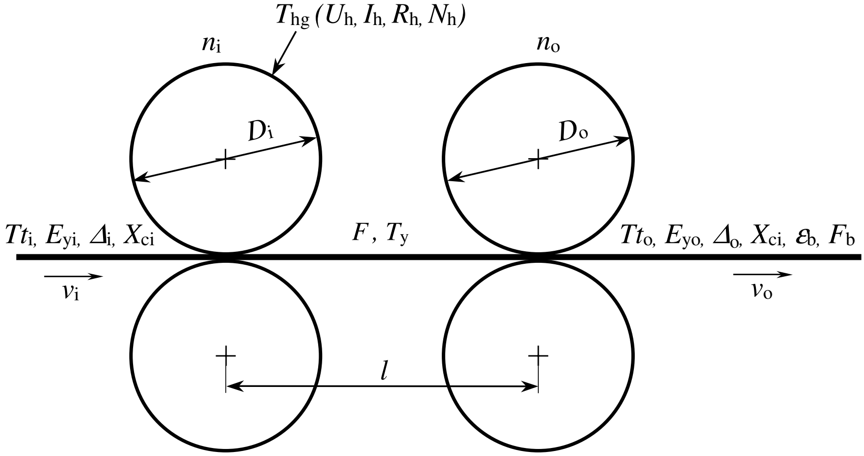

Fig. 2.1. Technological scheme of the drawing process, essential process control and product variables

1. the *process characteristics*, which define the mechanical, geometric and energetic, in other words, the technical-physical set point values characterising a certain technological operating point;
2. the *product characteristics*, which define the quality or state of the input, the processed and the output materials of the process. Thereby the input materials can either be raw materials or semifinished products, whereas the processed materials are semifinished or finished products.

According to Fig. 2.1 the group of the process characteristics contains the input velocity v_i and the output velocity v_o of the drawing godets, which are characterised by the diameters D_i and D_o and the motive revolutions per minute n_i and n_o. Furthermore, the length of the drawing zone l, as well as the godet temperature T_{hg} (only if a hot drawing process, for instance for the polyester production, is considered), which is given by the heating voltage U_h, the heating current I_h, the heating resistance R_h and the heating power N_h, belong to this group.

The product characteristics group comprises for instance the finesses Tt_i and Tt_o, the elastic moduli E_{yi} and E_{yo}, the birefringences Δ_i and Δ_o and the crystallinities X_{ci} and X_{co} of the incoming undrawn and the outcoming drawn yarns, the breaking elongation ε_b of the drawn yarn, the breaking force F_b of the drawn yarn, the tensile force F of the yarn in the drawing zone and the temperature T_y of the yarn at a certain point of the drawing zone.

As the above definitions show, the modification of the properties of the involved materials is characteristic for each technological process. Those modifications can either be intentional or disturbing. The materials' properties, which partly serve as a measure of the quality of the raw materials and the semifinished or finished products, are the product characteristics.

Just as well, the process characteristics are not constant during a certain period of the process. Compared with those of the product characteristics the modifications of the process characteristics are quite small but in general undesirable. In order to emphasise the dynamic character it seems to be meaningful to employ the terms *product variable* and *process variable* instead. As already mentioned, the product and process variables are only constant in some very special cases. In the quality characterisation and classification it is therefore usual to permit tolerances around a desired mean value of a product property.

This fact allows a further conclusion. As the process variables vary around their operating point, the product variables of interest are oscillating around the defined set point value, as well. In other words, the quantities fluctuate with the dynamics caused by the cause-effect relations of the single process lines. Thus, the process dynamics are an inherent component of the process itself.

2.1.2 Dynamics, Process Dynamics

In technical mechanics, dynamics mean the theory of motions caused by forces. In connection with the technological process the term *dynamics* has to be modified and adjusted. It must be restricted to:

Dynamics relating to the technological process, which will be called process dynamics in the following sections, mean the behaviour of a process run during a transition (transition behaviour) from one technologically adjusted operating point to another one. This transition is initiated by a determined or a stochastic disturbance and can be described in its quantitative and temporary progress.

The above definition can be easily explained for the drawing process. Let us take a yarn, which is, according to its macromolecular structure, relatively unoriented.

As shown in Fig. 2.2 the yarn is classically spun between the drawing godets and consequently continuously formed by the elongation $\varepsilon(t)$. The elongation results in a higher oriented form of the yarn which is connected with some desired physical properties. The realised elongation $\varepsilon(t)$ depends on the input and output velocities of the drawing godets $v_\mathrm{i}(t)$ and $v_\mathrm{o}(t)$ in the following way:

$$\varepsilon(t) = \frac{v_\mathrm{o}(t) - v_\mathrm{i}(t)}{v_\mathrm{i}(t)} \tag{2.1}$$

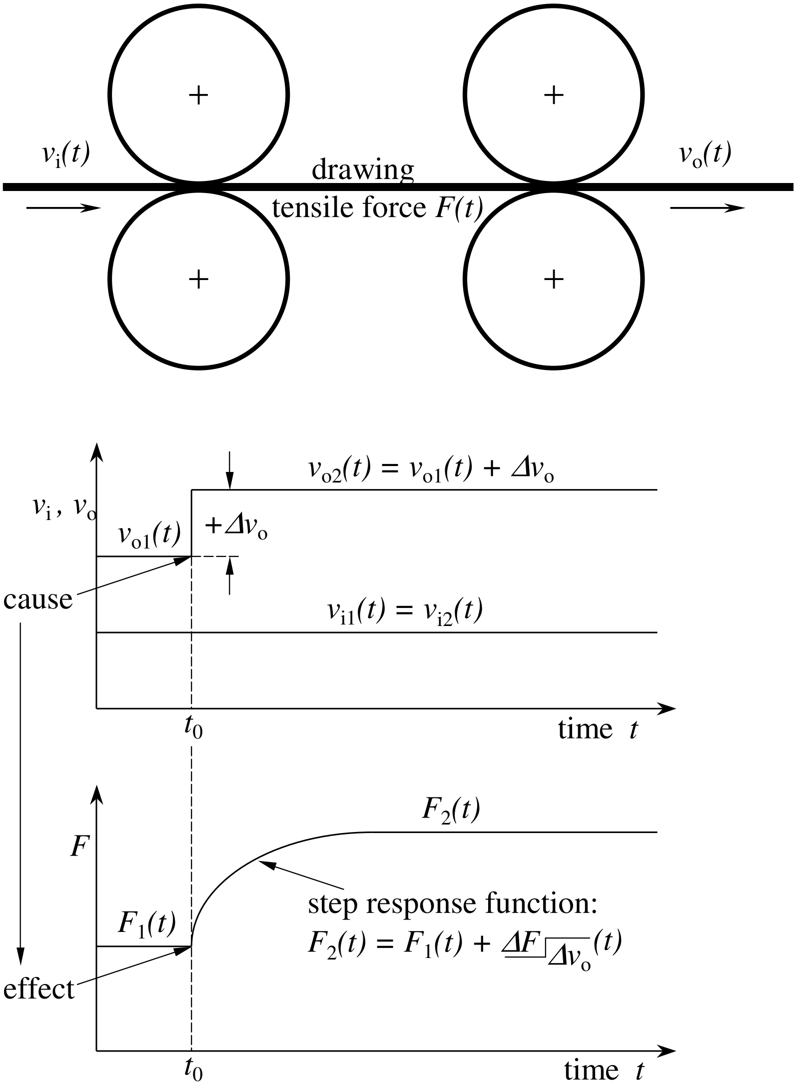

Fig. 2.2. Technological scheme of the drawing process, step-like disturbance of the output velocity Δv_o (symbol ⌐, see Sect. 2.4.2 too), effect $\Delta F(t)$

The raised elongation causes a reaction force in the drawn yarn, the drawing yarn tensile force $F(t)$. Let the process run at the technological operating point 1 characterised by the input velocity $v_{i1}(t) = const.$ and the drawing yarn tensile force $F_1(t) = const.$ At the moment $t = 0$ the output velocity $v_o(t)$ of the drawing godets is step-like increased by Δv_o and consequently reaches the new level $v_{o2}(t) = v_{o1}(t) + \Delta v_o$, which shall characterise the operating point 2.[1] Let level 2 be constant for $t > t_0$. The input velocity shall remain constant. Caused by the step-like disturbance $+\Delta v$, which can be classified as determined aperiodic disturbance (Fig. 2.2), the drawing tensile force (as effect quantity) will leave the level $F_1(t)$ for $t > t_0$. This transition is continuous and not step-like! Thus, the tensile drawing force reaches a level $F_2(t)$ after a transition period. Then, $F_2(t)$ remains constant if there is no further change of the new godet velocities $v_{\mathrm{i}2}(t)$ and $v_{\mathrm{o}2}(t)$.

The quantitative chronological description of the transition behaviour of the effect quantity drawing yarn tensile force from $F_1(t)$ to $F_2(t)$ caused by a step-like disturbance of a cause quantity (here: step-like increase of $v_{\mathrm{o}1}(t)$ to $v_{\mathrm{o}2}(t)$) is called step response function or simply *step response*. The response function describes the transitory behaviour of an effect caused by a step-like disturbance and is thus an expression for the *dynamics* of the process under consideration. Already the evaluation of such a simple dynamic standard function allows some fundamental statements regarding the characterisation and the assessment of the dynamic properties of technological processes or specific parts of them. This will be explained in detail in further sections, general facts are given in Sect. 2.4.2.

Let us assume we have a stable technological operating point. Now let the output velocity $v_{\mathrm{o}}(t)$ periodically (for instance sinusoidal) oscillate with the amplitude $\Delta v_{\mathrm{o}1}$ and the circular frequency ω_1 around its mean value v_{om} (see Fig. 2.3).

This periodic disturbance makes the drawing tensile force $F(t)$ oscillate with the amplitude ΔF_1 and the same circular frequency ω_1 around its mean value F_{m}. The initial oscillation (v_{o}-disturbance) and the response oscillation (F-disturbance) are not synchronous for the effect runs behind the cause. This behaviour is reflected in the phase shift angle φ_1. A change of the circular frequency of the periodic disturbance from ω_1 to ω_2 at a constant amplitude $\Delta v_{\mathrm{o}1}$ results in a modification of the response oscillation. Then, $F(t)$ oscillates at ω_2 with the modified amplitude ΔF_2 and a changed phase shift angle φ_2. The magnitude of the ratio $\Delta F/\Delta v_{\mathrm{o}}$ and the change of the phase shift angle φ dependent upon the disturbing frequency are, similarly to the step response, an expression of the dynamic characteristics of the drawing process. They lead to the terms *transfer function, frequency response, amplitude frequency response* (simply *amplitude response*) and *phase frequency response* (simply *phase response*). Explanations will follow later in Sect. 2.4.3.

[1] A step-like change in mass effected technical systems is normally not possible to realise. Nevertheless, this model image will be used here and in the following, because this affords an insight in the dynamic process behaviour on the basis of a well developed mathematical signal and system theory.

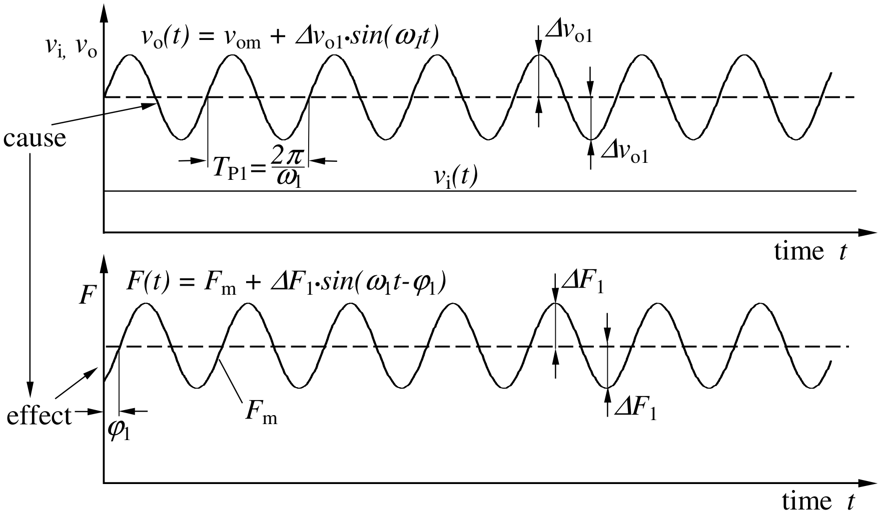

Fig. 2.3. Drawing process, periodic disturbances of the output velocity Δv_{o1}, effect ΔF_1, circular frequency of the disturbances ω_1, phase shift angle φ_1

2.2 Modelling of the Steady State Melt Spinning Process

2.2.1 Goal of Modelling

The goal of each modelling procedure is to obtain a mathematical description of the technological process for better understanding of the main relations between process parameters, material behaviour and product properties. An entire theory of melt spinning should also take into account the history of the polymer, thermal and deformational (rheological) behaviour, and the non-equilibrium conditions for the transfer processes. Such a theory would be quite complicated and is only marginally realised with respect to rheology. But, stationarity in melt spinning means that there are stable conditions in time, that there are no dependences of the process variables, no changes, disturbances or drifts in time. The values describing the fibre formation process change only with respect to space and describe therefore a *steady state* process, but it is noticeable that the process in reality is not one of equilibrium. A reasonable model for this process involves the dynamics of melt spinning and the resulting fibre properties. The best model then is one which is simple enough for handling but good enough for answering the questions which the fibre producers and developers are interested in. Early investigations to the melt spinning process, and based on this knowledge the development of the fundamentals of the model of fibre formation were done by Ziabicki [14–23], Andrews [24], Kase and Matsuo [25–28], Hamana [29,30], Han [31–37], George [38–40], Shimizu [41–53, 56–60], Yasuda [61–67], and many other authors [78–99, 176–183, 256].

The research on this interesting topic is still actively persued, there are many efforts made by scientists, engineers and producers to get a deeper understanding of the process. Recently, a multitude of contributions to the theory of fibre formation and melt spinning have been published. A major review about the literature is given in two books by Ziabicki [184, 274].

The following two sections only give a short and very simplified introduction to the model of fibre formation, more details to the steady state model will be discussed in Chap. 3 later. The goal of this section is only to give an impression of what the model analysis is capable of doing, therefore, as an example, a first and simple estimation to the fibre cooling process is made.

2.2.2 Balance Equations

To form filaments, the molten polymer is extruded through capillaries and is drawn down by a take-up unit which applies the necessary force. The take-up unit is often realised by godets but in principle it is also possible to wind up the fibres directly onto bobbins. Another common procedure for taking up the filaments is by using special air suction devices (for example in the spunbonded nonwoven process). At their path from the spinneret to the take-up unit, the filaments cool down, become accelerated to their final take-up speed, solidify and at last they can be partly oriented and crystallised. The engineering analysis of this process is made by the application of the physical balance equations of mass, energy and momentum to the fibre forming process in combination with material behaviour. In the following, the three basic equations are briefly summarised.

The most important (and simplest) relation is the continuity equation, it describes the *mass balance* in melt spinning:

$$Q = Tt \cdot v \ . \tag{2.2}$$

The filament cooling, i. e. the heat loss by heat transfer from the fibre surface to the surrounding air, is described by the *energy balance*. If we only regard the convective heat transfer (see later, Eq. 3.10) it is given in the following form:

$$\frac{\mathrm{d}T}{\mathrm{d}x} = -(T - T_{\mathrm{air}}) \cdot \frac{1}{L_{\mathrm{c}}} \ . \tag{2.3}$$

Then, the *momentum balance* describes the forces acting at the fibres:

$$F = F_0 + F_{\mathrm{surf}} + F_{\mathrm{inert}} + F_{\mathrm{drag}} - F_{\mathrm{grav}} \ . \tag{2.4}$$

Using Eq. 2.2 above, the fineness (titre) Tt_{L} of the as-spun filament with take-up velocity v_{L} then is given by

$$Tt_{\mathrm{L}} = \frac{Q}{v_{\mathrm{L}}} \ . \tag{2.5}$$

2.2.3 Example: Heat Transfer

The differential equation 2.3 can easily be integrated for constant parameter L_c and constant temperature T_{air} of surrounding air. The solution is a simple exponential function

$$T(x) = T_{air} + (T_0 - T_{air}) e^{-x/L_c} \; . \tag{2.6}$$

with the cooling length L_c proportional to the primary variable mass throughput: $L_c \propto Q$ (for more details see Sect. 3.1.2). The formula (2.6) can be used as a rough estimation for how fast the filaments will cool down. For poly(ethylene terephthalate) (PET) as typical melt spinning polymer the cooling length L_c per mass throughput becomes $L_c/Q \approx 0.2\,\mathrm{m}/(\mathrm{g}\cdot\mathrm{min}^{-1})$ (see Sect. 3.1.2). If the exponential course of the filament temperature is taken into consideration it means for PET, to cool down a filament with a mass throughput of about 1 g/min from an initial melt temperature of 290 ℃ to a temperature of 50 ℃ where solidification is surely reached, one needs about 0.5 m cooling length, for 2 g/min about 1.0 m, etc. (see Fig. 2.4).

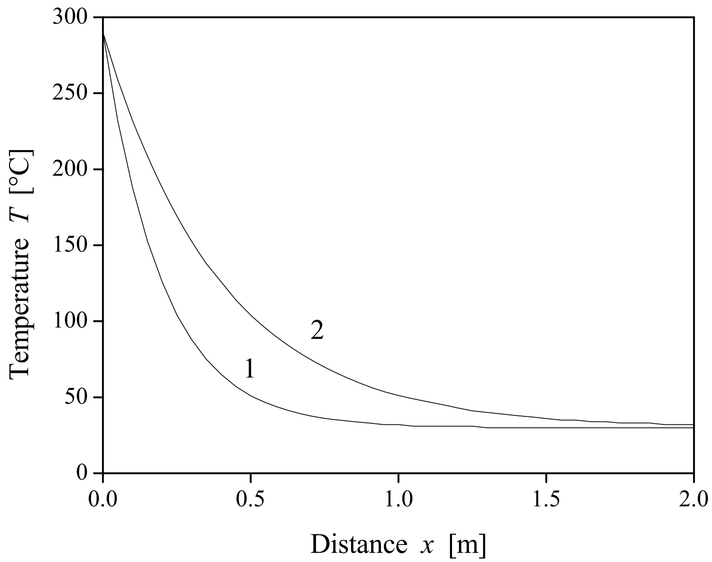

Fig. 2.4. Estimated course of PET filament temperature vs. distance, 1 – mass throughput $Q = 1$ g/min, 2 – $Q = 2$ g/min

2.3 Modelling of Non-Steady State Dynamic Process

2.3.1 System and Signal

In view of the closing remarks of Sect. 2.1.2 a second definition of the technological process is necessary. The definition can be derived from the following way of thinking, which is the norm in the fields of automatic control engineering, system engineering or information technology:
The realisation of each technological process requires an arrangement of certain mechanical, electrical, electronic, pneumatic or hydraulic devices and instruments, which are assembled in a machine or a part of a machine. They are the material basis for the fulfillment of the manufacturing task for the materials passing the process.

The machine and the passing and processed materials together are called the *system*.

As soon as the system, which is assumed to be at standstill and is not performing any material or energy transfer at first, begins to run, one can speak about a *process*. Only then the previously defined process and product variables begin to interact according to their varied cause-effect relations. Those interactions imply that all process and product variables are reflected in their (desired) mean values *and* their (in most cases undesired) fluctuations around the mean values. Those lapses of time are called *time functions* of the process and product variables, for instance $Tt_{\mathrm{o}}(t)$, $F(t)$, $\varepsilon_{\mathrm{b}}(t)$, $v_{\mathrm{i}}(t)$, $T_{\mathrm{hg}}(t)$ and so on. The information content of a time function, in other words the mean value and the fluctuations, are called their *signal*. If those signals are missing (for instance process interruption after a fibre break) the interaction between the process and product variables will not exist any longer. The process will fall back into its static (unproductive) state. Thus, the following process definition, which is the most contensed one, seen under the system technical viewpoint, is possible:

$$\text{process} = \text{system} + \text{signal(s)}$$

Consequently, the scientific analysis of a process with respect to its dynamics includes both, at least one signal analysis and one system analysis; in other words:

$$\text{process analysis} = \text{system analysis} + \text{signal analysis}$$

This situation is once more illustrated in Fig. 2.5 (after [4]), some terms appearing in this figure will be explained in the following section within the presentation of specific examples.

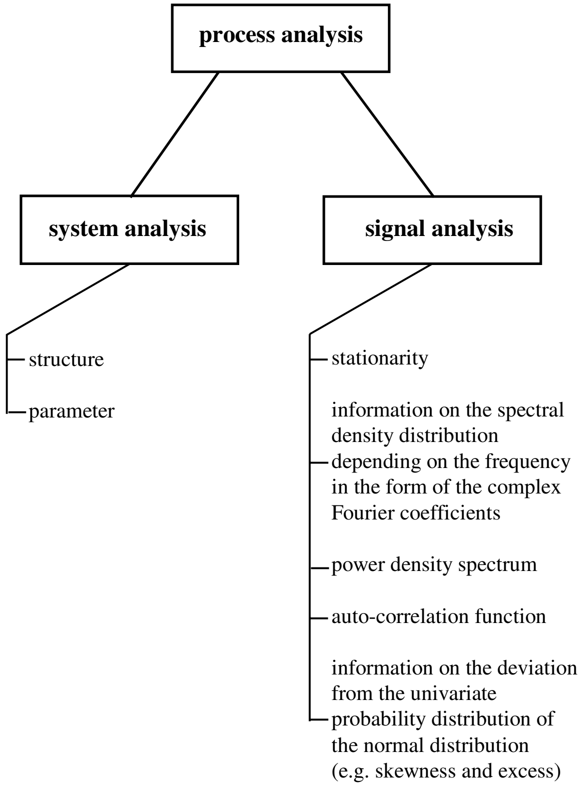

Fig. 2.5. Elements of a process analysis under system-technical aspect according to [4]

2.3.2 Model

The term *model* is another important expression being introduced now. Dealing with a given process it always has a certain reason, for instance:

- The process runs too instable. The prescribed tolerances of the product qualities cannot be maintained. The causes have to be investigated and eliminated.
- There are too many disturbances leading to process interruptions (= interruptions of the signal exchange). The causes must be determined and removed, too.
- A better machine shall be designed (= construction and design task).

Each of those exemplary mentioned tasks means that, at first, an analysis of the given state has to be made. This fact leads to the theoretical and/or experimental attempt to investigate the quantitative relations between the cause-effect relations of the process and product variables and their time-dependent behaviour. In most cases the results of this work have the shape of an image of the real constellation. This can be a formula, a graph, a regression equation, a DEq., a machine on a reduced scale, which is more transparent in its working, or an electronically aided simulation of the process or its parts. Those representations are designated as a more or less complete and sufficient *model*. Thus, relating to the technological process,

the model is an image of a process in its most significant parts, which describes the essential aspects of and the relations between the process and product variables with reference to a certain question.

Within the scope of this book the following classification is chosen:
The term *model* is divided into the *steady state* model on the one hand and the *dynamic* model on the other. The first describes the relations between the constant mean values of the process and product variables whereas the latter includes the relations between the changes and fluctuations of the process and product variables. Thus, the dynamic model represents the time dependence of the cause-effect relations.

As both models will be presented mathematically, it has to be mentioned that in most cases the steady state model does not require DEqs. in time whereas the dynamic model *always* leads to such DEqs.. This must be explained with the fact that the time-dependent behaviour can only be described with differentials of time.

As in the former section, the above definitions shall be clarified by an easy example. Again, the drawing process according to Fig. 2.1 is employed. With some fundamental technological knowledge we can write for the fineness of the drawn yarn Tt_o:

$$Tt_\mathrm{o} = Tt_\mathrm{i} \cdot \frac{v_\mathrm{i}}{v_\mathrm{o}} \tag{2.7}$$

Here, Tt_o, Tt_i, v_o and v_i are the mean values of the corresponding process and product variables. In this case, Eq. 2.7 would be the quite simple steady state mathematical model expressing the relations between the mean values of the target quantity Tt_o and its determining variables Tt_i, v_o and v_i.

The dynamic mathematical model describing the modification behaviour of the same target quantity Tt_o at the same process level is given by the mass balance equation. In our case, the balance equation is realised by the following DEq.:

$$v_\mathrm{o} \cdot Tt_\mathrm{o} + l \cdot \frac{\mathrm{d}Tt_\mathrm{o}}{\mathrm{d}t} - v_\mathrm{i} \cdot Tt_\mathrm{i} = 0 \tag{2.8}$$

The meaning of the differential element $l \cdot \dfrac{\mathrm{d}Tt_\mathrm{o}}{\mathrm{d}t}$ in Eq. 2.8 will be explained below the Eq. 2.23 in Sect. 2.5.2.

For a step-like disturbance of the input velocity v_i of the magnitude Δv_i the solution of the DEq. 2.8 would be the following step response function:

$$\underline{\Delta Tt_\mathrm{o}}\overline{|\Delta v_\mathrm{i}} = \Delta v_\mathrm{i} \cdot \frac{Tt_\mathrm{i}}{v_\mathrm{o}} \left[1 - \exp\left(-\frac{v_\mathrm{o}}{l} \cdot t\right)\right] \quad , \tag{2.9}$$

which describes the lapse of time of the effect of this disturbance on the modification of the output fineness ΔTt_o.

The DEq. 2.8 and its solutions (for the case of an aperiodic step-like disturbance such a solution is given by the Eq. 2.9) are a *dynamic mathematical model* of the drawing process.

2.4 Characterisation of the Dynamic Process Behaviour

2.4.1 Differential Equation

Restricting the dynamic mathematical modelling of processes or process steps on the clear representation in the time range and/or the frequency range, further terms, which accompany the methodical approach, have to be introduced.

For the determination of the dynamics of technological processes the DEq., describing the time behaviour of the system, plays a fundamental role. The DEq. is the mathematical reproduction of the cause-effect relations of those processes and product variables which are taken into account within the scope of the model. The DEqs. formation for specific technical systems is based on the dynamic mass-, energy- and momentum balance relations. Hereby, not only the steady state, but the dynamic (related to the general case of a running process) balances (processing mode) must be employed. Different a-priori knowledge of the process of interest, mathematical-methodical knowledge as well as some basic scientific laws are required for the DEqs. development.

2.4.2 Description in the Time Range; Step Response

If the DEq. shall supply facts about the dynamic properties of the system forming its basis, it must be solved first. According to the type of the time function of the independent variables of the DEq., which correspond to the causes for the system modifications, several solutions for the dependent variable, corresponding to the effect, are possible.

If the time function of the independent (cause-)variable, which disturbs the system, initiates a single (aperiodic) change of the previous (steady state) mean value or technological operating point, the arising new time function of the dependent (effect-)variable will be called *response function.* For special time functions of the independent (disturbance- or cause-) variables the *response time-* (or simple only *time-*) *function* can be designated more exactly. If the cause-time function is a single step (symbol _|‾, see also Fig. 2.2), the response time function will be called *step response*. If the cause time function is a single impulse (symbol $\perp$), which means that the cause variable leaves its steady state value only for an infinitely short time $t = t_0$, say impulse-like, and comes back to this value instantaneously, the response time function will be called *impulse response function*. Additionally, it has to be mentioned that, besides those two standard types of a cause time function, arbitrary signal types can appear as disturbances. The mathematical algorithm for the solution of the DEq. for step-like and impulse-like disturbances is well-known and ready for application. But at technological process steps the impulse-like disturbance has almost no practical significance, for in an experiment this type cannot be realised with sufficient exactness. Therefore, all common descriptions are based on the *step response* or simply *response function.* As the response functions are solutions of the DEq. reflecting the time behaviour of the dependent effect variables, we also speak about a representation in the time range. The following equation shows the general form:

$$\underline{\Delta y}\overline{|\Delta x} = f(t) \tag{2.10}$$

with Δx as cause, Δy as effect and t as running time.

2.4.3 Description in the Frequency Range

Dynamic Transfer Function; Complex Frequency Response

The cause variable can also be assumed to be a periodic function of time. In other words, the variable is disturbed frequently and not only at one time as considered in the above section. Now, in contrast to the case of a single disturbance, the solution of the DEq. does not reveal the exact time behaviour of the effect variable after a defined cause disturbance, but the solution specifies:

a) how the ratio of the amplitudes of the dependent and the periodic independent variable, and
b) how the phase shift angle φ between the cause-and the effect disturbance

will depend on the frequency ω of the periodic disturbance.

In principle, similar to the case of aperiodic disturbances, arbitrary types of disturbance signals are possible. In most practical cases the sine function is used because its mathematical treatment is quite easy.

However, the amplitude- and phase shift angle dependence on the disturbing frequency are the solution of the DEq., which models the dynamics of the system under investigation, in the *frequency range*. This solution is called *dynamic transfer function* or *complex frequency response*. In the common literature of system control or automatic control engineering the dynamic transfer function is given as the following complex function:

$$G(\mathrm{j}\omega) = \frac{\widetilde{\Delta y}(\omega)}{\widetilde{\Delta x}(\omega)} \cdot \mathrm{e}^{\mathrm{j}\varphi(\omega)} \tag{2.11}$$

with the following symbols[2]:

- $\widetilde{\Delta x}(\omega)$ vector of the sinusoidal disturbance of the independent cause variable $\Delta x \sin(\omega t)$
- $\widetilde{\Delta y}(\omega)$ vector of the sinusoidal response of the dependent response (effect) variable $\Delta y \sin(\omega t + \varphi)$
- $\mathrm{e}^{\mathrm{j}\varphi(\omega)}$ factor, which gives the phase shift between cause- and effect oscillation in the complex plane
- j imaginary unit, $\mathrm{j}^2 = -1$

As detailed examples later in the text will show, the dynamic transfer function or complex frequency response are directly obtained as the solution of the DEq. for periodic sinusoidal disturbances of the independent variable.

The complex frequency response Eq. 2.11 can, as common with complex quantities, be separated into its real and imaginary part:

$$G(\mathrm{j}\omega) = \mathrm{Re}(\omega) + \mathrm{j} \cdot \mathrm{Im}(\omega) \quad , \tag{2.12}$$

whereas $\mathrm{Re}(\omega)$ represents the real part and $\mathrm{Im}(\omega)$ corresponds to the imaginary part of the complex frequency response.

The representation of the complex frequency response Eq. 2.11 or Eq. 2.12 in the complex plane (x axis = real axis; y axis = imaginary axis) marks the end points of all those vectors, which can be drawn from the origin of the coordinate system dependent upon the excitation frequency ω. This curve is called *transfer locus* of the complex frequency response (Fig. 2.6).

Equivalent to the response function in the time range, the transfer locus describes the dynamic behaviour of a system in the frequency range, because the amplitude of the dependent effect variable is now plotted against the frequency.

[2] In mathematics the imaginary unit is usually designated with i. As in technical context the symbol i is often used for electric currents, the symbol j is used for the imaginary unit here.

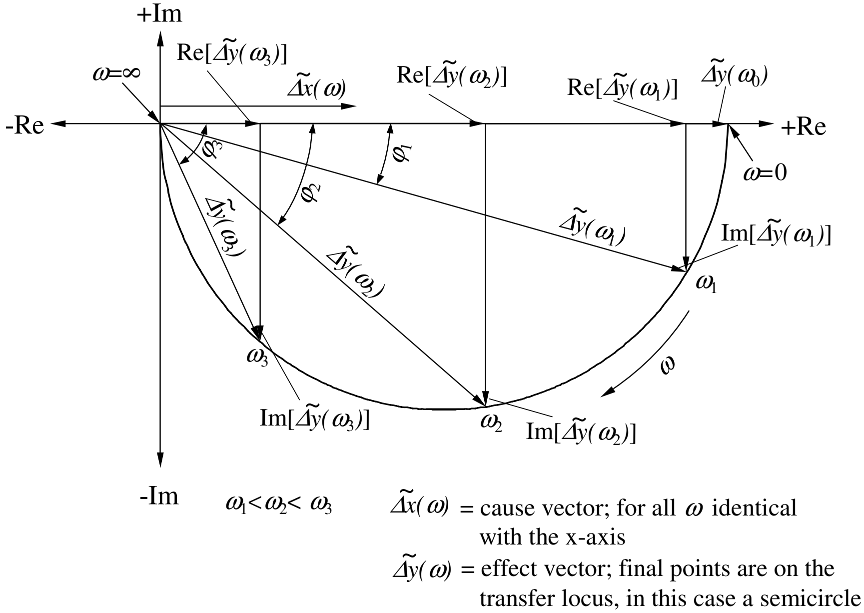

Fig. 2.6. Transfer locus of a complex frequency response; general example

It can be easily seen that for the excitation frequency $\omega = 0$, the amplitude vector of the independent cause variable (which always remains on the real axis) and the amplitude vector of the dependent variable have the same direction and melt together on the real axis. This represents the initial point of the transfer locus defining a kind of steady state excitation for the very special case of $\omega = 0$. For this exemplary situation there is no phase shift between cause and effect oscillation: $\varphi = 0$.

Amplitude Frequency Response

Proceeding from the complex frequency response Eq. 2.12 the ratio of the amplitudes of the cause- and the effect oscillation, $\widetilde{\Delta y}(\omega)/\widetilde{\Delta x}(\omega)$, can be plotted against the excitation frequency ω. This amplitude ratio corresponds to the absolute value of the single complex number of the frequency response $G(\mathrm{j}\omega)$, that is $|G(\mathrm{j}\omega)|$, or to the ratio of the mentioned vectors of the transfer locus of the complex frequency response. Those absolute values are calculated as follows:

$$|G(\mathrm{j}\omega)| = \sqrt{\mathrm{Re}(\omega)^2 + \mathrm{Im}(\omega)^2} \tag{2.13}$$

Equation 2.13 gives the *amplitude frequency response*, which is one part of the complex frequency response. Consequently it is just another form of

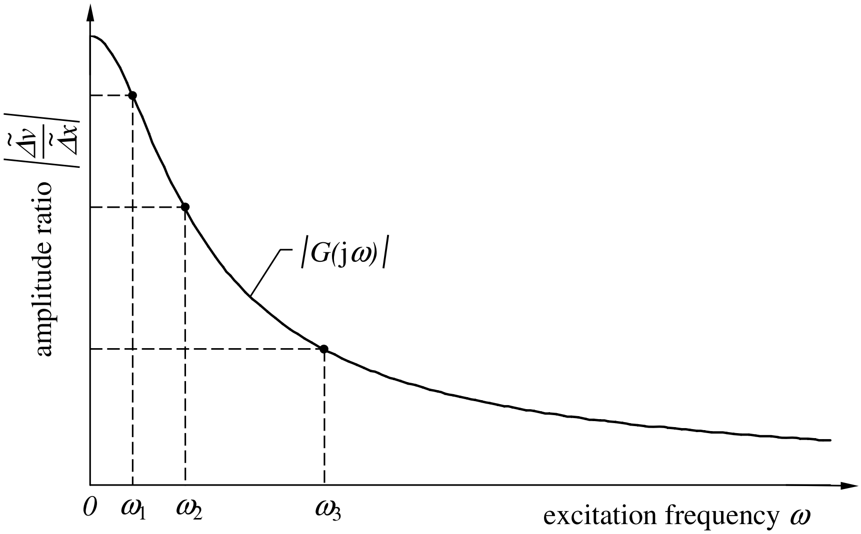

Fig. 2.7. Amplitude frequency response of the transfer locus according to Fig. 2.6

representation in the frequency range, as the solution is plotted against the frequency ω, again (Fig. 2.7).

Phase Frequency Response

The second part which can be extracted from the complex frequency response, is the *phase frequency response.* This function describes how the phase shift angle φ between cause- and effect oscillation depends on the excitation frequency ω. In the transfer response representation it is exactly that angle included by the real axis and each vector beginning at the origin of the coordinate system and ending on the transfer locus curve. The tangent of the phase shift angle φ is equal to the ratio of the imaginary and the real part of the complex frequency response 2.12, namely:

$$\tan[\varphi(\omega)] = \frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)} \tag{2.14}$$

According to Eq. 2.14, the phase shift angle φ can be calculated directly:

$$\varphi(\omega) = \arctan\left[\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)}\right] \tag{2.15}$$

For the general case the phase frequency response is depicted in Fig. 2.8.

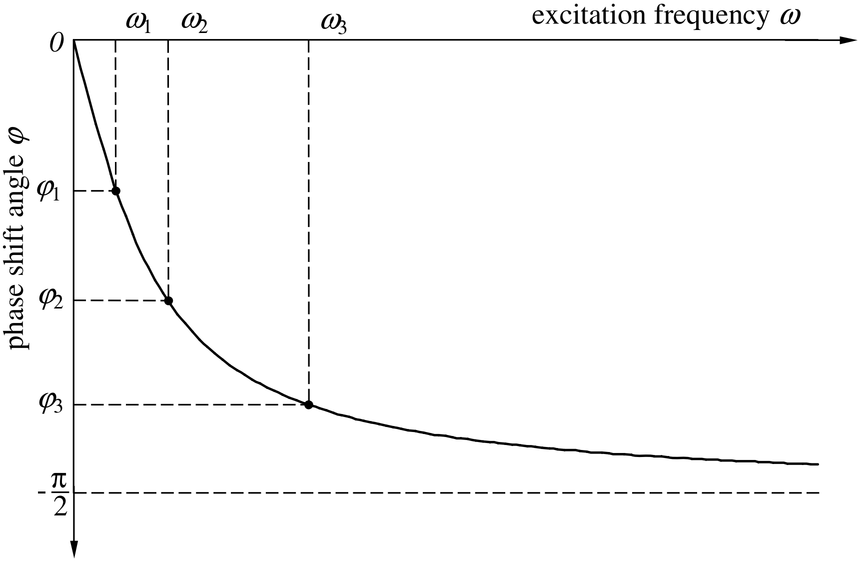

Fig. 2.8. Phase frequency response of the transfer locus according to Fig. 2.6

While the complex frequency response or its transfer locus, respectively, contain the information from the DEqs. solution for a certain sinusoidal disturbance of the independent cause variable, i.e. the dependence of the amplitude ratio *and* the phase shift angle on ω, the amplitude frequency response and the phase frequency response provide only the dependence of *either* the amplitude ratio *or* the phase shift angle on ω.

For more detailed information, didactically reasoned descriptions and analytical proofs see for instance [5], [6] or [7]. Within the scope of this book, only the basic knowledge which seems to be inherently necessary for the understanding of the following sections, has been sketched. A very detailed example, which is thought to clarify the above mathematical terms and connections, is presented in Sect. 2.5.2.

2.4.4 Correlation and Power Density Spectrum Functions

In the previous sections only such functions characterising the dynamic system behaviour have been considered, which are based on the transfer description of *determined* disturbances or changes of the system. But in principle it is also possible to develop a mathematical description for the transfer of *non-determined* or *stochastic* disturbances of the cause variables. This becomes necessary as soon as it is experimentally impossible to produce determined disturbances or if the process of interest should not be faced with higher disturbance amplitudes, which are in most cases needed for determined disturbances, for reasons of safety or any other reasons. In those cases the arbitrary (stochastic) fluctuations of the cause and effect variables around their mean values, which are generally always present and which determine their technological operating point, can be used for the dynamic system analysis. Consequently, this method of analysis implies the possibility to gain information during the normal process run. In doing so, some very strict conditions and standards have to be fulfilled. Those conditions and standards specifically concern the parts of system engineering and system analysis of such an investigation. This methodical procedure for the system analysis will not be shown in this book, the approach to this specific theoretical and experimental discipline can be studied for instance in [5–8]. Within the scope of stochastic system investigations statistic characteristic functions of the time functions of the process and product variables are used as they allow a so far hardly practised approach to the system analysis with a higher gain of information in the field of advanced, process related disturbance analysis of textile products. Therefore the fundamental equations with some short explications are now given and will be applied in later sections. These equations are the *correlation function* and the *power density spectrum function.*

The *correlation function* $K(\tau)$ of *one* time function $x(t)$ is defined in the following way.

Integral representation:

$$K(\tau) = \lim_{\mathrm{t}\to\infty} \frac{1}{T} \int_0^t [x(t) - \bar{x}][x(t+\tau) - \bar{x}]\mathrm{d}t \tag{2.16}$$

Sum representation:

$$K(k \cdot \Delta t) = \frac{1}{n-k} \sum_{\mathrm{i}=0}^{n-k} (x_{\mathrm{i}} - \bar{x})(x_{\mathrm{i+k}} - \bar{x}) \tag{2.17}$$

Boundary condition: $k_{\max} \leq \dfrac{n}{5}$, with $k = 0, 1, 2, 3, \ldots k_{\max}$.

$x(t)$	time function
$\bar{x}$	mean value of the time function in the evaluation range $0 \leq t \leq T$ or $0 \leq k \cdot \Delta t \leq n-k$, respectively
x_{i}	discrete value of time function $x(t)$, taken in steps of Δt
T	time period of the integration range
$\tau, k \cdot \Delta t$	time shift
k, n, m	running (sequence) indices
$K(\tau), K(k \cdot \Delta t)$	single values of the correlation function for τ or $k \cdot \Delta t$, respectively

As can be seen from Eqs. 2.16 and 2.17 all values of the time function are multiplied continuously or at certain points with the values of the same time function after a time shift of τ or $k \cdot \Delta t$, namely $x(t+\tau)$ or $x_{\mathrm{i+k}}$, and finally added up to get $K(\tau)$ or $K(k \cdot \Delta t)$ (see Fig. 2.9 too).

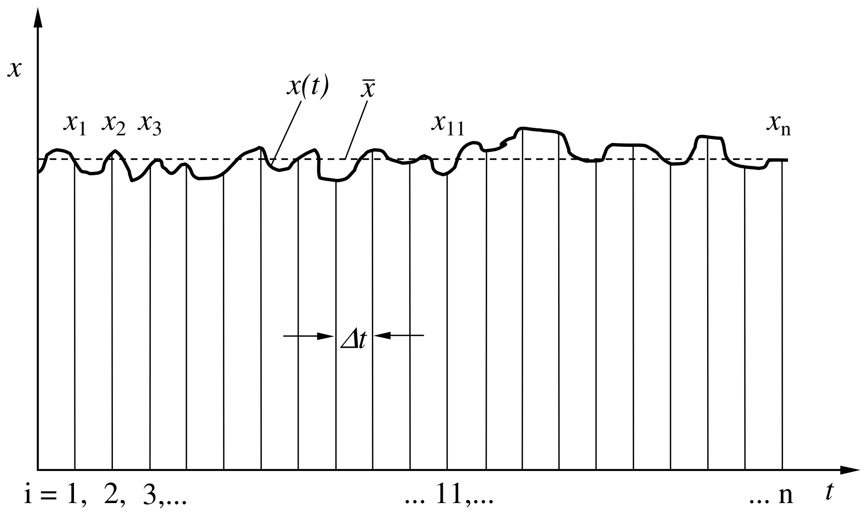

Fig. 2.9. Time function $x(t)$, distributed into n single values for the design of the auto-correlation function

The result of this summation divided by the length of the addition- or interpretation interval T represents the single value of the correlation function $K(\tau)$ or $K(k \cdot \Delta t)$. The repetition of this calculation for different τ or $k \cdot \Delta t$ yields the complete correlation function.

As $x(t)$ is correlated with itself according to Eqs. 2.16 and 2.17, the corresponding correlation function is called *auto-correlation function.*

Without an extended discussion of the efficiency and the interpretation possibilities (more about this topic in Sect. 6.4.2), it should be mentioned that the initial value of the auto-correlation function $K(0)$ is equal to the square spread of the time function in the interpretation- and integration range.

The algorithm for the calculation of the correlation function can also be applied to *two* different time functions $x(t)$ and $y(t)$, which could be for instance coupled by certain cause-effect relations. Analogous to Eqs. 2.16 and 2.17 the corresponding equations are:

$$K(\tau) = \lim_{t\to\infty} \frac{1}{T} \int_0^t [x(t) - \bar{x}][y(t+\tau) - \bar{y}]\mathrm{d}t \quad , \tag{2.18}$$

$$K(k \cdot \Delta t) = \frac{1}{n-k} \sum_{\mathrm{i}=1}^{n-k} (x_{\mathrm{i}} - \bar{x})(y_{\mathrm{i+k}} - \bar{y}) \tag{2.19}$$

Boundary condition: $k_{\max} \leq \frac{n}{5}$, with $k = 0, 1, 2, 3, \ldots k_{\max}$.

As the partial products of two cross-wisely analyzed time functions are summed up, the resulting correlation function is called *cross-correlation function.* To support a better understanding the situation of a cross-correlation is visualised in Fig. 2.10.

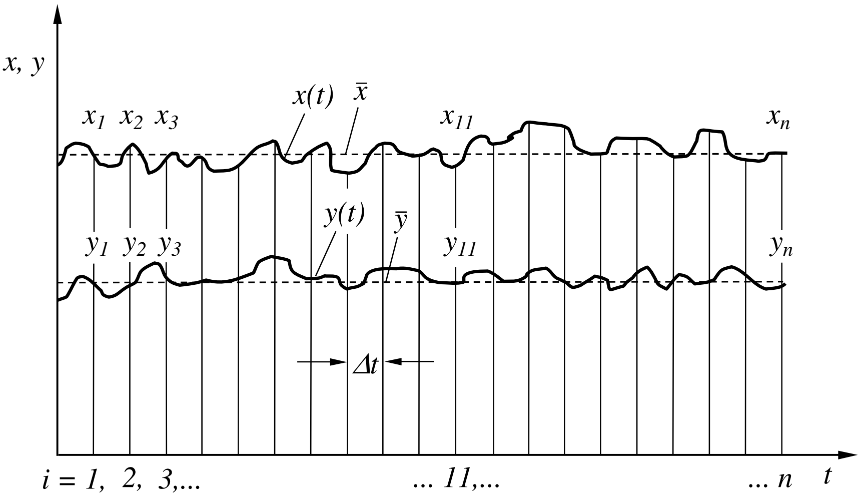

Fig. 2.10. Time functions $x(t)$ and $y(t)$, distributed into n single values for the design of the cross-correlation function

The independent variable of the auto- or cross-correlation function is the time shift τ or $k \cdot \Delta t$. Thus, this form of representation is a representation in the time range (see also Sect. 2.4.2).

The statistic characteristic function, which supplies the equivalent information about the signal shape of the time function in the frequency range is the *power density spectrum function.* This function can also be written as auto- (APSF) or cross-power density spectrum function (CPSF).

Following, the calculation rule for the power density spectrum function is given, as Sect. 6.4.2 will exclusively refer to it. Furthermore, the principle is commercially available in the form of modules for yarn and fibre uniformity testing devices or spectrometers. These instruments allow the automatised calculation of the amplitude spectrum, which is similar to the power density spectrum function, of measured signal shapes of the fibre fineness.[3]

Integral representation to the auto-power density spectrum function:

$$S(\omega) = \lim_{\mathrm{T}\to\infty} \frac{1}{T} \left\{ \left[\int_0^T (x(t) - \bar{x}) \cos(\omega t) \mathrm{d}t \right]^2 + \left[\int_0^T (x(t) - \bar{x}) \sin(\omega t) \mathrm{d}t \right]^2 \right\} \tag{2.20}$$

Sum representation:

$$S(\omega) = \frac{1}{n} \left\{ \left[\sum_{\mathrm{i}=1}^{n} (x(t) - \bar{x}) \cos(\omega \cdot i \cdot \Delta t) \right]^2 + \left[\sum_{\mathrm{i}=1}^{n} (x(t) - \bar{x}) \sin(\omega \cdot i \cdot \Delta t) \right]^2 \right\} \tag{2.21}$$

Boundary condition: $\dfrac{10\pi}{n \cdot \Delta t} \leq \omega \leq \dfrac{\pi}{\Delta t}$

If the auto-correlation function, in its sum definition according to Eq. 2.17, has already been calculated the single function values of $K(k \cdot \Delta t)$ can be applied directly for the determination of the power density spectrum function without a repeated access to the values for the x_{i} of the basic time function. The conversion formula is:

$$S(\omega) = 2\Delta t \sum_{\mathrm{k}=1}^{m} K(k \cdot \Delta t) \cdot \cos(\omega \cdot k \cdot \Delta t) \tag{2.22}$$

Boundary condition: $\dfrac{2\pi}{m \cdot \Delta t} \leq \omega \leq \dfrac{\pi}{\Delta t}$

Further explications are given in [8,9].

[3] The *spectrograph* of the ZELLWEGER CO., Uster, Switzerland, which is delivered as a module for the *Uster uniformity tester*, has been popular for decades.

2.5 Dynamic Process Analysis and Modelling

2.5.1 Methodical Procedure

Since the most essential terms have been introduced and explained quite independently in the previous sections, now some fundamental rules with respect to the methodical procedure for dealing with process-analytic tasks, which are well-tried, are given. These rules will help to get a general idea of the connections and interactions between those essential terms.

There are two stages in the system analysis, which are classified according to the qualitative representation of their result:

a) the stage of the steady state system analysis aiming on the development of a steady state model of the process, and
b) the stage of the dynamic system analysis aiming on the setting up of a dynamic model of the process, which can include the steady state model as a special case.

According to Table 2.1 (taken from [1]) the mathematical-analytical approach to the modelling of technological flow-processes (which are, in contrast to piece-processes, such processes where the process variables are exposed to continuous changes and not to generally step-like changes) always leads to at least one DEq. or usually to a system of DEqs.

Table 2.1. Fundamental mathematical-analytical approaches to the modelling of technological flow-processes; $x, y, z \ldots$ space coordinates, $t \ldots$ time

	Processes with concentrated parameters	Processes with scattered parameters
Steady state model	Systems of algebraic equations	Systems of ordinary DEqs. with derivatives $\frac{\mathrm{d}}{\mathrm{d}x}$
	Systems of transcendental equations	Systems of partial DEqs. with derivatives $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$
Dynamic model	Systems of ordinary DEqs. with derivatives $\frac{\mathrm{d}}{\mathrm{d}t}$	Systems of partial DEqs. with derivatives $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$

Considering the principied type of the chosen methodical approach, the system analysis can be arranged in three classes:

a) the theoretical,
b) the experimental, and
c) the combined theoretical-experimental system analysis.

The latter is based on a continuous interaction between theoretically established relations and experimental investigations, see Fig. 2.12. Practically, this type is the most common one.

The specific steps for the dynamic system analysis or the modelling are:

1. All process and product variables, being involved in the process and probably being connected with each other via cause-effect relations, are to be collected and sorted. For that purpose the establishment of a *cause-effect-scheme*, which inherently goes along with an exact physical-analytical way of thinking and the inclusion of all available a-priori knowledge, is inevitable.
2. If possible, all DEqs. of the system, following from mass-, energy- and momentum balances, have to be set up.
3. The DEqs. coefficients have to be determined theoretically and experimentally. The DEq. is to be solved (step response function and/or frequency-, phase- and amplitude response).
4. If it is impossible to set up a DEq., the cause-effect-scheme has to be split into meaningful smaller cause-effect-blocks. The structure and the parameters of those partial systems must be investigated by actively performed experiments.
5. The model is tested and improved.
6. Mathematical simulations are performed in order to answer the technological questions. The model is applied.

The realisation of the experiments mentioned in 4., aiming on the investigation of the cause-effect relations between the process and product variables, requires the modification of the concerned cause variable under a certain law and the subsequent measurement of the system's response (of the effect variable). The evaluation of the time functions of the varied input variable (test signal) *and* of the output variable allows the determination of the dynamic behaviour of the system. Figure 2.11 (after [4]) summarises the possible test signals for the input variable (cause variable).

The transfer functions are the response time functions for aperiodic determined test signals. From their characteristics the dynamic parameters and the structure of the system can be derived. If periodic determined test signals, for instance sinusoidal signals of tunable frequency, are used, the dynamic transfer function or the complex frequency response of the system can be investigated experimentally. Analogously, the dynamic system characteristics can be determined from them. Correlation and spectral analysis methods belong to the group of methods which employ stochastic signals for the determination of the system parameters and the system's structure. However, their application is coupled with a considerable expenditure. Nevertheless, in several cases it is the only possibility to carry out any dynamic process investigation at all.

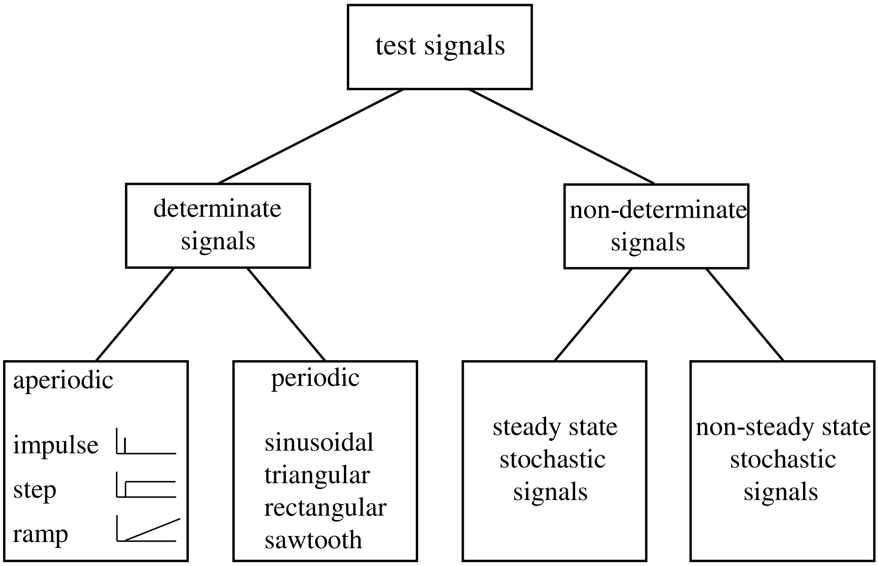

Fig. 2.11. Classification of test signals

Figure 2.12 schematically shows the different methods of dynamic process analysis.

As can be seen here, the problem can be solved theoretically as well as experimentally. That model which starts from the setting up of the DEq. taking all qualitative physical-analytical relationships into consideration and in which only the quantitative fixing of the DEqs. parameters is tested experimentally, must be seen as the most valuable model. All dynamic characteristic functions, for instance the step response function or the complex frequency response, can be calculated if the DEq. of the process or process part behaviour is known.

As soon as the setting up of the DEq. becomes impossible, the questions of interest have to be exclusively solved by experiments. In principle, all experimental methods are equally good, but practically there are big differences with respect to the expense of time and aid and the achievable accuracy. The step or impulse test quickly supplies characteristics in the time range, which can be converted into the qualitatively more valuable characteristic in the frequency range (the complex frequency response or the dynamic transfer function) only with a limited accuracy. Investigations applying a periodic test signal permit a direct access to the complex frequency response – however, with a higher expenditure of time but also with a good accuracy. Those methods which utilise stochastic test signals take a lot of time, as well. Their accuracy decisively depends on the quality of the measuring instruments.

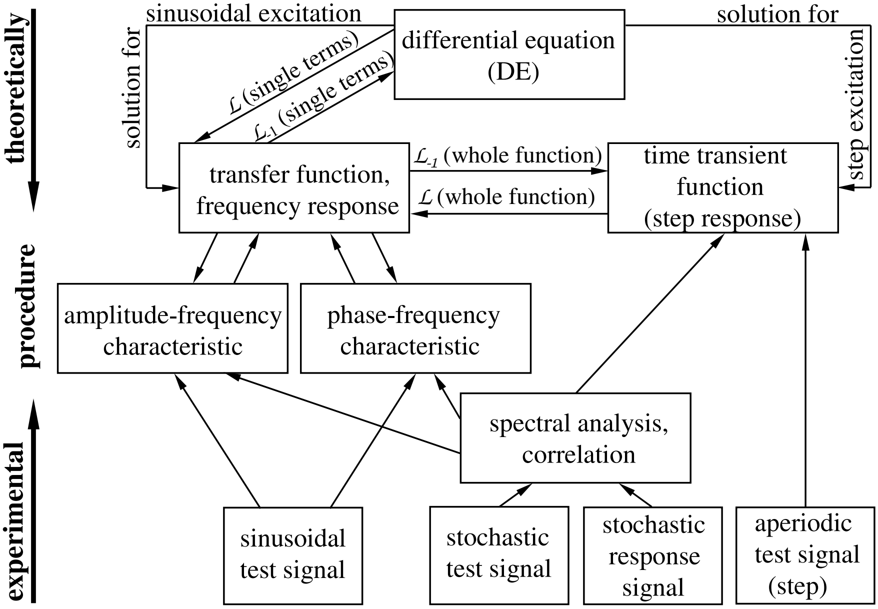

Fig. 2.12. Methods of the dynamic process analysis: L means LAPLACE-transformation from the time to the frequency range, L_{-1} means retransformation of the LAPLACE-transformation from the frequency to the time range

Here, an important problem relating to the purely experimental procedure (often of necessity) shall be mentioned. Of course the experimental procedure leads to a quantitatively correct model of the process under consideration. But such a model cannot contribute to an understanding of the fundamental physical-analytical relations of the process and product variables. The experimentally gained model is usually limited to that process that was employed for its acquisition. It is quite good for the process control but not for a general explication of the connections between several input and output variables. Furthermore, such a model should not be transferred to a similar but larger process (up scaling) without testing its validity again.

Finally it is pointed out that the conversion laws between the dynamic transfer function and the step response, formally given in Fig. 2.12, are theoretically based on the LAPLACE-transformation (abbreviation symbols L and L_{-1}), which will be used in some later examples. Some literature hints will also be given to this later.

2.5.2 Detailed Example (Drawing Process)

Deviation of the Specified Differential Equation and its Solutions

Here, the methods introduced during the previous sections are demonstrated by the setting up, the evaluation and interpretation of a simple dynamic model for the drawing process. For that purpose the following question has to be answered: Which dynamic transfer properties has the process step of drawing relating to changes or fluctuations of the input velocity v_{i} in its effect on the fineness Tt_{o} of the yarn leaving the drawing zone?

In this special case the first step of the analytical procedure recommended in Sect. 2.5.1, namely the qualitative coverage of the process and product variables of interest, can be omitted. With some technological basic knowledge the fundamental statistic equation for the output yarn fineness Tt_{o}, see Eq. 2.7, can be given at once. As already mentioned, the output yarn fineness depends on the input velocity v_{i} as well as on the output velocity v_{o} and the fineness Tt_{i} of the incoming yarn.

The setting up of the DEq. requires the dynamic mass balance equation for the draw field in which the fibre mass is exchanged continuously (the undrawn fibre goes in, the drawn fibre comes out).

As the draw field and, by the way, a lot of other process steps of textile processing processes are systems where a certain mass is stored, the general balance equation (continuity equation) taking the dynamic *processing mode* into consideration can be utilised as the basis for the formulation of the DEq.:

$$\frac{\text{mass inflow}}{\text{time}} = \frac{\text{mass discharge}}{\text{time}} + \text{change of stored mass} \quad (2.23)$$

Specifically for the drawing zone the terms are:

$$\frac{\text{mass inflow}}{\text{time}} = Tt_{\mathrm{i}} \cdot v_{\mathrm{i}} \quad , \quad \frac{\text{mass discharge}}{\text{time}} = Tt_{\mathrm{o}} \cdot v_{\mathrm{o}}$$

The term *change of stored mass* needs, relating to the modelling of the filament drawing process, a short explanation leading to a simplification. The yarn mass stored in the drawing zone is $Tt_{\mathrm{o}} \cdot l$, see Fig.2.1. We assume now that all changes of the fibre fineness take place homogeneously, quasi rubber like, over the whole drawing zone. This assumption is sufficiently exact, if only small changes of process variables around the operating point (<10%) are in the view.

For the situation, that immediately after the inflow of the undrawn fibre the full draw ratio of some 100% would be realised the fibre would reach its output fineness Tt_{o} shortly after the inflow (neck like deformation). The fibre had to be transported only over the length l of the drawing zone with the output velocity v_o. For the last case (which is not shown here) the model description would be valid for a dead time thread line, which is explained more in detail in Sect. 5.1.3.

The term *change of stored mass* means the mass per time interval which is *additionally* flowing in or being discharged from the drawing zone after a perturbation. Mathematically, this term is the first derivative of the stored mass after the time:

$$\frac{d(Tt_o \cdot l)}{\mathrm{d}t} = \frac{\mathrm{d}Tt_o}{\mathrm{d}t} \cdot l = \dot{T}t_o \cdot l \tag{2.24}$$

By inserting this equation in Eq. 2.23 we get the following DEq.:

$$Tt_i \cdot v_i = Tt_o \cdot v_o + \dot{T}t_o \cdot l \tag{2.25}$$

Equation 2.25, which is equal to Eq. 2.8, is the DEq. describing the drawing process under the simplified conditions explained above. The cause variable was v_i and the effect variable was Tt_o. The values of Tt_i, v_o and l should be constant.

Before we continue with the solution of the DEq., an agreement concerning the symbols must be reached. Constants characterising the technological operating point are written with the index m, standing for *mean* or *mean value*. Variables are split into their constant mean value, also marked by the index m, and their fluctuating part, which is symbolised by a Δ in front.

In our example we have to introduce $Tt_o = Tt_{om} + \Delta Tt_o$ and $v_i = v_{im} + \Delta v_i$ for the variables and Tt_{im}, v_{om} and l_m for the constants.

Then, Eq. 2.25 appears as:

$$Tt_{im} \cdot (v_{im} + \Delta v_i) = (Tt_{om} + \Delta Tt_o) \cdot v_{om} + (\dot{T}t_{om} + \Delta\dot{T}t_o) \cdot l_m$$

After multiplication, with $Tt_{im} \cdot v_{im} = Tt_{om} \cdot v_{om}$, which is equivalent to the steady state balance equation, and with the knowledge that the derivative of a constant is equal to zero the DEq. gets the following form:

$$Tt_{im} \cdot \Delta v_i = v_{om} \cdot \Delta Tt_o + l_m \cdot \Delta\dot{T}t_o \tag{2.26}$$

In contrast to Eq. 2.25 this DEq. contains only those variables which have real signal character and not the constant mean values. Consequently, Eq. 2.26 fits better with the definition of the dynamic model, which mainly investigates the fluctuation behaviour of the process and product variables. Thus, Eq. 2.26, represents a view point which is typical for the comparison between actual value and rated value in the automatic control technique. There, only fluctuations around the rated value are also considered [10].

For the calculation of the transfer function the common approach being well-known in the field of DEq. analysis

$$\Delta Tt_\mathrm{o} = C_1 \cdot \mathrm{e}^{C_2 \cdot t} \tag{2.27}$$

is inserted in the homogeneous DEq. 2.26. Setting the independent cause variable Δv_i equal to zero, we get the so-called **volatile** solution:

$$v_\mathrm{om} \cdot C_1 \cdot \mathrm{e}^{C_2 \cdot t} + l_\mathrm{m} \cdot C_1 \cdot C_2 \cdot \mathrm{e}^{C_2 \cdot t} = 0 \quad ,$$

which means:

$$C_2 = -\frac{v_\mathrm{om}}{l_\mathrm{m}} \tag{2.28}$$

We insert Eq. 2.28 in Eq. 2.27, extend the approach by the so-called steady state solution and get the modified, complete approach:

$$Tt_\mathrm{o} = C_1 \cdot \exp\left(-\frac{v_\mathrm{om}}{l_\mathrm{m}} \cdot t\right) + C_3 \tag{2.29}$$

Aiming on the determination of the coefficients C_1 and C_3 the approach of Eq. 2.29 is inserted into the complete DEq. 2.26:

$$Tt_\mathrm{im} \cdot \Delta v_\mathrm{i} = v_\mathrm{om}\left[C_1 \cdot \exp\left(-\frac{v_\mathrm{om}}{l_\mathrm{m}} \cdot t\right) + C_3\right] - l_\mathrm{m} \cdot \frac{v_\mathrm{om}}{l_\mathrm{m}} \cdot C_1 \cdot \exp\left(-\frac{v_\mathrm{om}}{l_\mathrm{m}} \cdot t\right)$$

For $t \rightarrow \infty$ we get:

$$C_3 = \frac{\Delta v_\mathrm{i}}{v_\mathrm{om}} \cdot Tt_\mathrm{im} \tag{2.30}$$

Inserting Eq. 2.30 into the approach of Eq. 2.29 leads to:

$$\Delta Tt_\mathrm{o} = C_1 \cdot \exp\left(-\frac{v_\mathrm{om}}{l_\mathrm{m}} \cdot t\right) + \frac{\Delta v_\mathrm{i}}{v_\mathrm{om}} \cdot Tt_\mathrm{im} \tag{2.31}$$

For the determination of the constant C_1 we investigate the solution of Eq. 2.31 for the case of $t = 0$, which corresponds to the beginning of the step-like excitation (Δv_i just applied). ΔTt_o is still equal to zero for $t = 0$, that means:

$$0 = C_1 + \frac{\Delta v_\mathrm{i}}{v_\mathrm{om}} \cdot Tt_\mathrm{im} \quad \Longrightarrow \quad C_1 = -\frac{\Delta v_\mathrm{i}}{v_\mathrm{om}} \cdot Tt_\mathrm{im} \tag{2.32}$$

Equation 2.32 inserted in Eq. 2.31 results in the complete time transient function or *step response function*:

$$\underline{\Delta Tt_\mathrm{o}}\overline{|\Delta v_\mathrm{i}} = \Delta v_\mathrm{i} \cdot \frac{Tt_\mathrm{im}}{v_\mathrm{om}}\left[1 - \exp\left(-\frac{v_\mathrm{om}}{l_\mathrm{m}} \cdot t\right)\right] \tag{2.33}$$

According to Fig. 2.12 the transfer function Eq. 2.33 represents the solution of the DEq. 2.26 for step-like excitations of the system by the independent cause variable. Equation 2.33 is identical with Eq. 2.9 of Sect. 2.3.2 but includes the more detailed symbols for the several mean values and fluctuations. Before the interpretation of Eq. 2.33 it is shown, how the *complex frequency response* can be derived from the DEq. 2.26. As explained in one of the former sections the complex frequency response represents the DEqs. steady state solution for sinusoidal excitations.

For the cause variable we write:

$$\Delta v_{\mathrm{i}} = \widetilde{\Delta v_{\mathrm{i}}} \cdot \mathrm{e}^{\mathrm{j}\omega t} \quad , \tag{2.34}$$

and for the effect variable:

$$\Delta Tt_{\mathrm{o}} = \widetilde{\Delta Tt_{\mathrm{o}}} \cdot \mathrm{e}^{\mathrm{j}(\omega t+\varphi)} \quad , \tag{2.35}$$

where ω is the excitation frequency and φ is the phase shift angle. The Eqs. 2.34 and 2.35 correspond to an oscillating behaviour of the two variables and are written in the complex form, which will directly lead to the questioned complex frequency response (see for instance [10] for further information). When we use Eqs. 2.34 and 2.35 in Eq. 2.26 we gain:

$$Tt_{\mathrm{im}} \cdot \widetilde{\Delta v_{\mathrm{i}}} \cdot \mathrm{e}^{\mathrm{j}\omega t} = v_{\mathrm{om}} \cdot \widetilde{\Delta Tt_{\mathrm{o}}} \cdot \mathrm{e}^{\mathrm{j}(\omega t+\varphi)} + l_{\mathrm{m}} \cdot \widetilde{\Delta Tt_{\mathrm{o}}} \cdot \mathrm{j}\omega \cdot \mathrm{e}^{\mathrm{j}(\omega t+\varphi)}$$

or

$$Tt_{\mathrm{im}} \cdot \widetilde{\Delta v_{\mathrm{i}}} = \widetilde{\Delta Tt_{\mathrm{o}}} \cdot \mathrm{e}^{\mathrm{j}\varphi}(v_{\mathrm{om}} + \mathrm{j}\omega \cdot l_{\mathrm{m}})$$

Calculating the cause/effect-ratio finally leads to:

$$G(\mathrm{j}\omega) = \frac{\widetilde{\Delta Tt_{\mathrm{o}}}}{\widetilde{\Delta v_{\mathrm{i}}}} \cdot \mathrm{e}^{\mathrm{j}\varphi} = \frac{Tt_{\mathrm{im}}}{v_{\mathrm{om}}} \cdot \frac{1}{1 + \mathrm{j}\omega \cdot \frac{l_{\mathrm{m}}}{v_{\mathrm{om}}}} \tag{2.36}$$

Eqation 2.36 represents the *complex frequency response* of the system under investigation and corresponds to Eq. 2.11 in Sect. 2.4.3.
According to Eq. 2.13 the *amplitude frequency response* is equal to the absolute value of the complex frequency response. For its calculation we have to seperate the real and the imaginary part of Eq. 2.36. This can be reached by **extension** of the fraction by the factor $\left(1 - \mathrm{j}\omega \cdot \dfrac{l_{\mathrm{m}}}{v_{\mathrm{om}}}\right)$:

$$G(\mathrm{j}\omega) = \frac{\widetilde{\Delta Tt_{\mathrm{o}}}}{\widetilde{\Delta v_{\mathrm{i}}}} \cdot \mathrm{e}^{\mathrm{j}\varphi} = \frac{Tt_{\mathrm{im}}}{v_{\mathrm{om}}} \cdot \frac{1 - \mathrm{j}\omega \cdot \dfrac{l_{\mathrm{m}}}{v_{\mathrm{om}}}}{\left(1 + \mathrm{j}\omega \cdot \dfrac{l_{\mathrm{m}}}{v_{\mathrm{om}}}\right)\left(1 - \mathrm{j}\omega \cdot \dfrac{l_{\mathrm{m}}}{v_{\mathrm{om}}}\right)}$$

$$G(\mathrm{j}\omega) = \frac{\widetilde{\Delta Tt_{\mathrm{o}}}}{\widetilde{\Delta v_{\mathrm{i}}}} \cdot \mathrm{e}^{\mathrm{j}\varphi} = \frac{Tt_{\mathrm{im}}}{v_{\mathrm{om}}} \cdot \frac{1 - \mathrm{j}\omega \cdot \dfrac{l_{\mathrm{m}}}{v_{\mathrm{om}}}}{1 + \left(\omega \cdot \dfrac{l_{\mathrm{m}}}{v_{\mathrm{om}}}\right)^2} \tag{2.37}$$

The absolute value of Eq. 2.37 leads to the *amplitude frequency response* due to Eq. 2.13:

$$|G(\mathrm{j}\omega)| = \left|\frac{\widetilde{\Delta Tt_{\mathrm{o}}}}{\widetilde{\Delta v_{\mathrm{i}}}}\right| = \frac{Tt_{\mathrm{im}}}{v_{\mathrm{om}}} \cdot \frac{1}{\sqrt{1 + \left(\omega \cdot \dfrac{l_{\mathrm{m}}}{v_{\mathrm{om}}}\right)^2}} \tag{2.38}$$

Employing Eq. 2.15 the *phase frequency response* is:

$$\varphi(\omega) = \arctan\left[-\frac{\omega \cdot l_{\mathrm{m}}}{v_{\mathrm{om}}}\right]. \tag{2.39}$$

A more elegant and, mainly for the solution of DEqs. of higher order, faster technique for the calculation of the complex frequency response is the LAPLACE-transformation. Cutting out any details, the method is now explained for the (quite simple) DEq. 2.25.

The derivatives with respect to the time $\frac{\mathrm{d}^u}{\mathrm{d}t^u}$ are replaced by the so-called LAPLACIAN p^u. Because only first derivatives occur ($\Delta\dot{T}t_{\mathrm{o}}$) results here $u = 1$:

$$Tt_{\mathrm{im}} \cdot \Delta v_{\mathrm{i}} = v_{\mathrm{om}} \cdot \Delta Tt_{\mathrm{o}} + p \cdot l_{\mathrm{m}} \cdot \Delta Tt_{\mathrm{o}} \quad ,$$

where $p \cdot \Delta Tt_{\mathrm{o}}$ stands for $\Delta\dot{T}t_{\mathrm{o}}$ or $\frac{d(\Delta Tt_{\mathrm{o}})}{\mathrm{d}t}$.

It follows:

$$G(p) = \frac{\Delta Tt_{\mathrm{o}}}{\Delta v_{\mathrm{i}}} = \frac{Tt_{\mathrm{im}}}{v_{\mathrm{om}}} \cdot \frac{1}{1 + p} \cdot \frac{l_{\mathrm{m}}}{v_{\mathrm{om}}} \tag{2.40}$$

It can be easily seen that Eq. 2.40, which is called dynamic transfer function $G(p)$ in a narrower sense, proceeds to the complex frequency response $G(\mathrm{j}\omega)$ (Eq. 2.36) when the LAPLACIAN p^u is replaced by the complex frequency $(\mathrm{j}\omega)^u$. The advantage of this direct method, which indirectly includes all boundary conditions, is obvious.

If we look for the step response we will need a transformation from the frequency range back to the time range. For that purpose the retransformation integral of the LAPLACE-transformation must be applied. In our special case that integral is:

$$\underline{\Delta Tt_{\mathrm{o}}}\overline{|\Delta v_{\mathrm{i}}} = \frac{\Delta v_{\mathrm{i}}}{2\pi \mathrm{j}} \int\limits_{-\mathrm{j}\infty}^{+\mathrm{j}\infty} \frac{G(p)}{p} \cdot \mathrm{e}^{pt} \cdot \mathrm{d}p \quad , \tag{2.41}$$

or with Eq. 2.40:

$$\underline{\Delta Tt_{\mathrm{o}}}\overline{|\Delta v_{\mathrm{i}}} = \frac{\Delta v_{\mathrm{i}} \cdot Tt_{\mathrm{im}}}{2\pi \mathrm{j} \cdot v_{\mathrm{om}}} \int\limits_{-\mathrm{j}\infty}^{+\mathrm{j}\infty} \frac{\mathrm{e}^{pt}}{p \cdot \left(1 + p \cdot \dfrac{l_{\mathrm{m}}}{v_{\mathrm{om}}}\right)} \cdot \mathrm{d}p \tag{2.42}$$

The value of the integral Eq. 2.42 can either be extracted from the appropriate literature [11], [12] or solved with the *residue theorem*of the LAPLACE-transformation [13]. The residue theorem states that the value of the integral is equal to the sum of all residues multiplied by the factor 2πj.

This means for the given example:

$$\underline{\Delta Tt_{\mathrm{o}}}\overline{|\Delta v_{\mathrm{i}}} = \Delta v_{\mathrm{i}} \cdot \frac{Tt_{\mathrm{im}}}{v_{\mathrm{om}}} \cdot \sum_{\nu=1}^{2} \operatorname*{Res}_{p_\nu}[S(p)] \quad , \tag{2.43}$$

where p_ν are the zero values of the integrand of Eq. 2.42. Those are:

$$p_1 = 0 \quad \text{and} \quad p_2 = -\frac{v_{\mathrm{om}}}{l_{\mathrm{m}}}$$

The general equation for the calculation of the residues is:

$$\operatorname*{Res}_{p_\nu}[S(p_\nu)] = \lim_{p \to p_\nu} (p - p_\nu) \cdot S(p)$$

Here $S(p)$ represents the whole integrand of Eq. 2.42. Now, the two residues can be calculated:

$$\operatorname*{Res}_{p_1}[S(p_1)] = \lim_{p \to 0} \frac{(p-0) \cdot \mathrm{e}^{pt}}{p \cdot \left(1 + p \cdot \dfrac{l_{\mathrm{m}}}{v_{\mathrm{om}}}\right)} = 1 \quad , \tag{2.44}$$

$$\operatorname*{Res}_{p_2}[S(p_2)] = \lim_{p \to -\frac{v_{\mathrm{om}}}{l_{\mathrm{m}}}} \frac{\left(p + \dfrac{v_{\mathrm{om}}}{l_{\mathrm{m}}}\right) \cdot \mathrm{e}^{pt}}{p \cdot \left(1 + p \cdot \dfrac{l_{\mathrm{m}}}{v_{\mathrm{om}}}\right)} = -\exp\left(-\frac{v_{\mathrm{om}}}{l_{\mathrm{m}}} \cdot t\right) \tag{2.45}$$

The input of Eqs. 2.44 and 2.45 into the Eq. 2.43 leads to the transfer function

$$\underline{\Delta Tt_{\mathrm{o}}}\overline{|\Delta v_{\mathrm{i}}} = \Delta v_{\mathrm{i}} \cdot \frac{Tt_{\mathrm{im}}}{v_{\mathrm{om}}} \left[1 - \exp\left(-\frac{v_{\mathrm{om}}}{l_{\mathrm{m}}} \cdot t\right)\right] \tag{2.46}$$

Obviously, Eq. 2.46 is the same as Eq. 2.43, which has been acquired by the classical approach with the DEq. 2.25.

Evaluation of Results

We are now ready to deal with the sixth step (application of the model, simulations with respect to technological questions) of rules for the methodical procedure for the dynamic modelling given in Sect. 2.5.1. For that purpose the potential of the mathematical solutions found in the previous section is demonstrated by means of a concrete example. Let us assume a drawing process of a synthetic yarn characterised by the following process and product variables:

mean input yarn fineness	$Tt_{\mathrm{im}} = 30$ tex
mean output yarn fineness	$Tt_{\mathrm{om}} = 10$ tex
($\Rightarrow$ mean draw ratio = 3)	
mean input velocity	$v_{\mathrm{im}} = 300$ m/min
mean output velocity	$v_{\mathrm{om}} = 900$ m/min
assumed step-like or sinusoidal	
shift of the input velocity	Δv_{i} or $\widetilde{\Delta v_{\mathrm{i}}} = 15$ m/min.

Figure 2.13 shows a transfer function calculated with Eq. 2.33 by means of the above values for $l_{\mathrm{m}} = 0.3$ m.

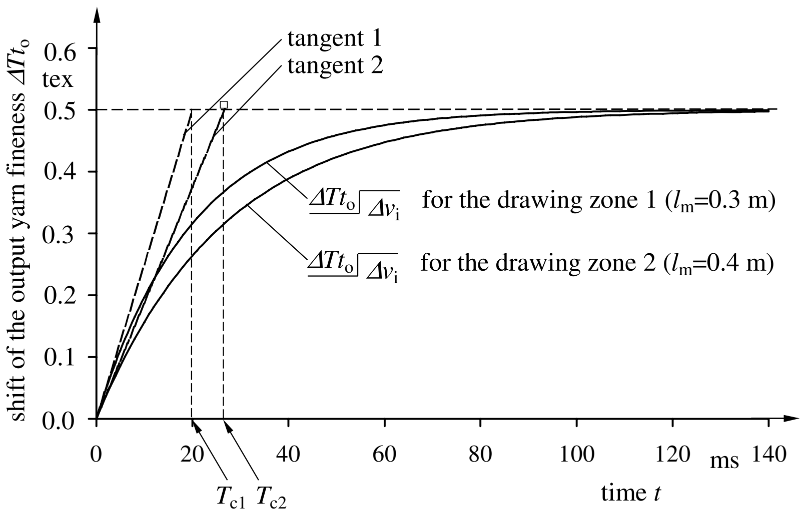

Fig. 2.13. Drawing process; time transient function of the fineness $\Delta Tt_{\mathrm{o}}(t)$ by means of an input velocity step $\Delta v_{\mathrm{i}} = 15$ m/min:
$Tt_{\mathrm{im}} = 30$ tex; $Tt_{\mathrm{om}} = 10$ tex; $v_{\mathrm{im}} = 300$ m/min; $v_{\mathrm{om}} = 900$ m/min

The following information can be extracted from the curves:

a) The change of the output yarn fineness ΔTt_{o} shows an exponential behaviour and reaches a steady state value of 0.5 tex. Consequently, if the disturbance of the input velocity is 15 m/min the outgoing yarn will have a constant fineness of 10.5 tex after a certain transition time.

b) The transition period between leaving the old technological operating point ($Tt_o = 10$ tex) and reaching the new disturbed one ($Tt_o = 10.5$ tex) lasts 100 ms.
c) The proportional change of the fineness after the end of the transition period is equal to the proportional change of the input velocity (5%).

There are a number of further considerations to be made. First, the transition time of 100 ms seems to be quite short. But we have to bear in mind that already 1.5 m of the fibre with fineness values differing from the desired value have left the drawing zone during that period. If the input velocity jumped back from the disturbed value of 315 m/min to the initial value of 300 m/min after those 100 ms we would have to wait for further 100 ms until the output yarn fineness had reached its original value of 10 tex again. Thus, an only 100 ms lasting change of the input velocity to a 5% higher value would have caused a 3 m fibre segment with undefined fineness and – which is even worse for synthetic silks – with different orientation and structure characteristics, which possibly show negative effects on the staining homogeneity. Here, the terms of skitteriness and barre suggest themselves and do not have to be interpreted in detail. As for the rest, the changes of the input velocity cause slippage effects of the fibre on the input godet roll (in diminishing direction), which are often barely recognised, appear for extremely short periods and can be hardly measured.

The parameter in the transfer function which finally determines how fast the effect variable can follow the step-like cause variable, is in our case (and in many other similar cases) the exponent of the exponential function. The reciprocal value of the factor $v_{\mathrm{om}}/l_{\mathrm{m}}$ is called the *time constant* T_{c} of the system:

$$T_{\mathrm{c}} = \frac{l_{\mathrm{m}}}{v_{\mathrm{om}}} \tag{2.47}$$

This means for our example:

$$T_{\mathrm{c}} = \frac{0.3\ \mathrm{m}}{900\ \mathrm{m/min}} = 3.33 \cdot 10^{-4}\ \mathrm{min} = 0.02\ \mathrm{s} = 20\ \mathrm{ms}.$$

As can be checked easily there is the following correlation between the time since the beginning of a disturbance and the percentage of the alteration range being passed by the effect variable:

$$\begin{aligned} 1 \cdot T_{\mathrm{c}} &\iff 63\%, \\ 3 \cdot T_{\mathrm{c}} &\iff 95\%, \\ 5 \cdot T_{\mathrm{c}} &\iff 99\%. \end{aligned}$$

Therefore it seems to be justified to consider the transition process as finished after a period of $5{\cdot}T_{\rm c}$ (here: 100 ms). For less exact approximations (error: 5%) even $3{\cdot}T_{\rm c}$ can be assumed as long enough. Consequently the time constant can be extracted from a purely experimentally acquired transfer function with sufficient exactness (in case of an exponential behaviour, which has to be tested before): One determines the point of the curve, where 63% of the whole alteration range has been passed, and extracts the corresponding time value from the abscissa (Fig. 2.13). Another (less exact) method is the construction of the tangent on the exponential function in the zero value of time. That tangent also meets the parallel to the abscissa which corresponds to the steady state final value of the alteration range of the effect variable at the distance $T_{\rm c}$.

Eqation 2.47 shows how the dynamic behaviour of the system (here: the drawing process) can be changed. A faster responding system reacts with a diminishing of the time constant which is equivalent to a shortening of the drawing zone and/or an enlargement of the output velocity, whereas a slower responding system reacts with an enlargement of the time constant which is equivalent to a longer drawing zone and/or a smaller output velocity. If, for a certain reason, the fibre length, being influenced by disturbances, has to be changed, for instance minimised, this will only be possible by an adequate diminishing of $l_{\rm m}$, as the disturbance fibre length is proportional to the product $T_{\rm c} \cdot v_{\rm om}$. The right curve of Fig. 2.13 illustrates how the transition process changes when the drawing zone is enlarged to 0.4 m. For this special case $T_{\rm c}$ is 26.7 ms and the transition process is practically finished after 133 ms.

Figures 2.14 to 2.16 show the solutions in the frequency range: the frequency response in its transfer locus representation (Fig. 2.14), the amplitude frequency response (Fig. 2.15) and the phase frequency response (Fig. 2.16).

The transfer locus as a summarising representation teaches us that

a) the oscillation $\widetilde{\Delta Tt_{\rm o}}$, which is caused by sinusoidal exciting oscillation with constant amplitude $\widetilde{\Delta v_{\rm i}}$ (vector on the real axis), gets smaller for growing excitation frequencies ω and finally vanishes for $\omega \rightarrow \infty$,
b) the phase shift angle φ, which is a measure for the delay of the effect oscillation behind the (exciting) cause oscillation, grows from initially zero (for $\omega = 0$, quasi steady state excitation) to a value of $-\dfrac{\pi}{2}$ (for $\omega \rightarrow \infty$).

Amplitude and phase frequency response show those statements separately. The ordinate values of the amplitude frequency response are explicitly given as the amplitude ratio $\widetilde{\Delta Tt_{\rm o}}/\widetilde{\Delta v_{\rm i}}$ for our example. The maximum value for this ratio, appearing at $\omega = 0$, is $\dfrac{0.5}{15}\dfrac{\rm tex}{\rm m/min} = 0.033\dfrac{\rm tex}{\rm m/min}$. This means that at the beginning of the amplitude frequency response, therefore at small

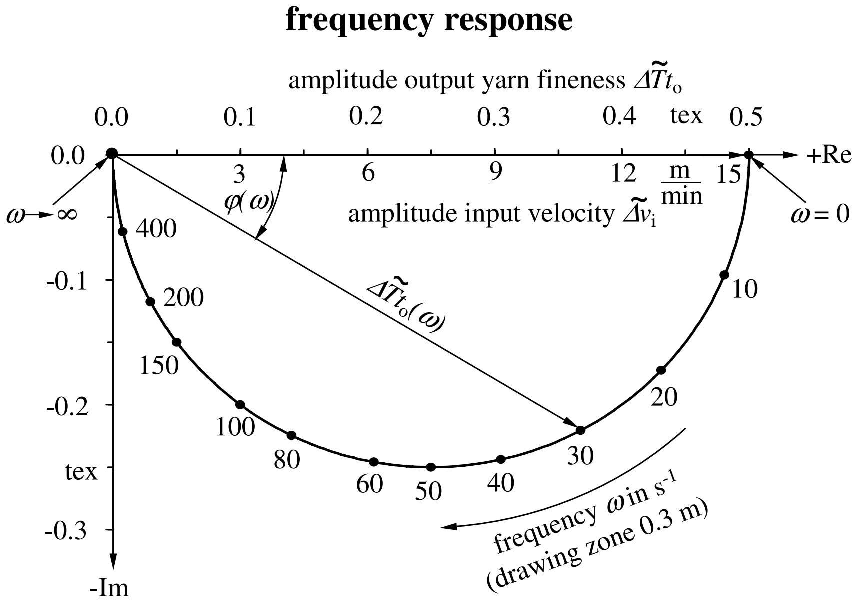

Fig. 2.14. Drawing process; transfer locus of the fineness $\widetilde{\Delta Tt}_o$ by means of periodic input velocity disturbances $\widetilde{\Delta v}_i = 15$ m/min:
$Tt_{im} = 30$ tex; $Tt_{om} = 10$ tex; $v_{im} = 300$ m/min; $v_{om} = 900$ m/min

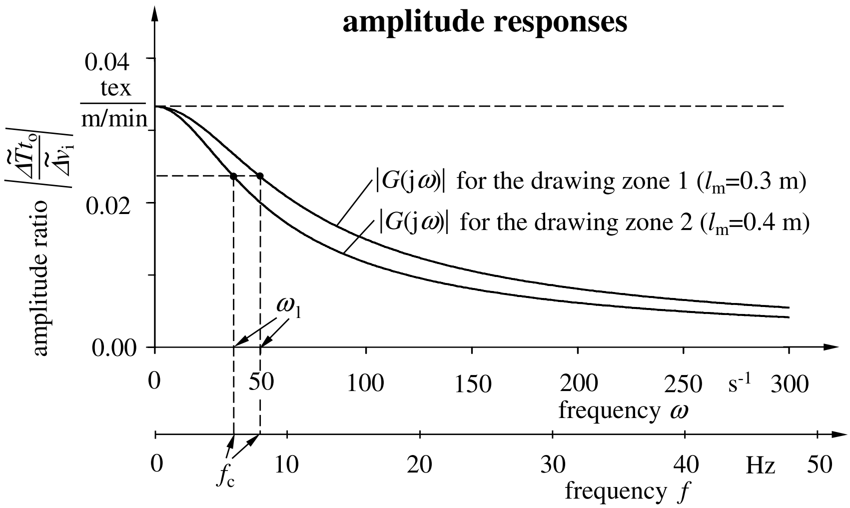

Fig. 2.15. Drawing process; amplitude frequency responses of the fineness $\widetilde{\Delta Tt}_o$ by means of periodic input velocity disturbances $\widetilde{\Delta v}_i = 15$ m/min:
$Tt_{im} = 30$ tex; $Tt_{om} = 10$ tex; $v_{im} = 300$ m/min; $v_{om} = 900$ m/min

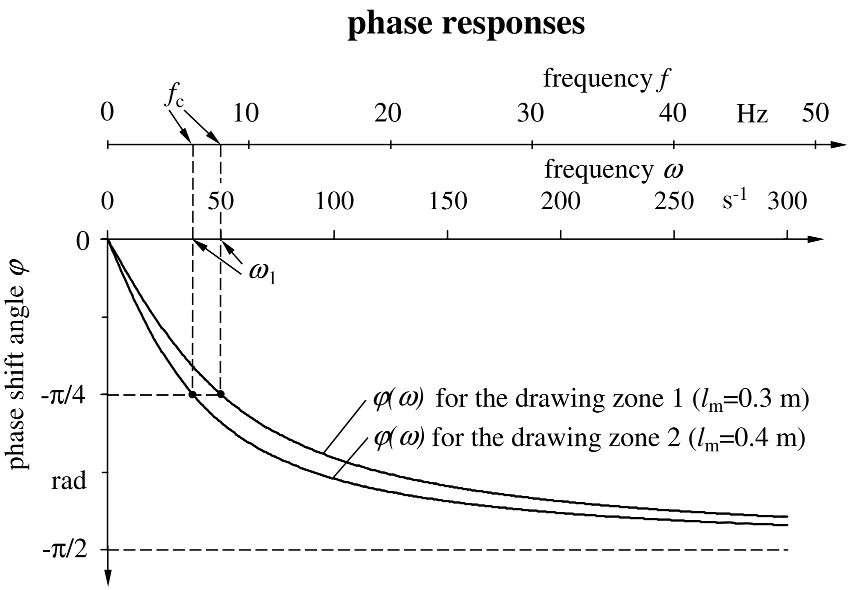

Fig. 2.16. Drawing process; phase frequency responses of the fineness $\widetilde{\Delta Tt_o}$ by means of periodic input velocity disturbances $\widetilde{\Delta v_i} = 15$ m/min:
$Tt_{im} = 30$ tex; $Tt_{om} = 10$ tex; $v_{im} = 300$ m/min; $v_{om} = 900$ m/min

disturbance frequencies, a fluctuation amplitude $\widetilde{\Delta v_i}$ =1 m/min results in a fluctuation amplitude $\widetilde{\Delta Tt_o}$ =0.033 tex.

The so-called critical (circuit) frequency ω_c can be taken as a characteristic value for the estimation of the dynamic system properties in the frequency range representation. It is defined as the excitation frequency, at which the effect oscillation amplitude has fallen to $1/\sqrt{2}$ of the value being valid for the case of steady state excitation (for $\omega = 0$). In the transfer locus (Fig. 2.14) the imaginary part of the vector for the effect oscillations is equal to its real part at this point. This is the case for Eq. 2.37 with

$$\omega \cdot \frac{l_m}{v_{om}} = 1 \Longrightarrow \omega = \omega_c = \frac{v_{om}}{l_m}$$

or, for Eq. 2.47 with:

$$\omega_c = \frac{v_{om}}{l_m} = \frac{1}{T_c} \tag{2.48}$$

There, the phase shift angle is $\varphi_l = -\dfrac{\pi}{4} = -45^\circ$. For our technological example we get: $\omega_c = \dfrac{900}{0.3}\dfrac{\text{m/min}}{\text{m}} = 3000 \text{ min}^{-1} \cong 50 \text{ s}^{-1}$. It is clearer to give the values for the circular frequency ω in the more familiar dimension

Hertz corresponding to the number of oscillations per time unit f. The well-known relation $\omega = 2\pi f$ lets Eq. 2.48 appear as

$$f_c = \frac{\omega_c}{2\pi} = \frac{v_{om}}{2\pi \cdot l_m} \tag{2.49}$$

For the example we get $f_c = \frac{50}{2\pi}\text{Hz} = 8$ Hz. The frequency abscissas of Figs. 2.15 and 2.16 contain both values: ω and f. Practically, this means that the drawing zone transfers periodic sinusoidal fluctuations of the input velocity $\widetilde{\Delta v_i} \cdot \sin(\omega t)$ or $\widetilde{\Delta v_i} \cdot \sin(2\pi f \cdot t)$ at frequencies of 0 Hz $\leqq f \leqq$ 8 Hz to the fineness of the outgoing fibre with a transfer factor of at least 0.7 related to the maximum amplitude ratio for $f = 0$ Hz. The input velocity would fluctuate between 285 and 315 m/min. The fineness of the outgoing fibre would fluctuate for a disturbance frequency of

$\approx$ 0 Hz between 9.5 tex and 10.5 tex,
8 Hz between 9.65 tex and 10.35 tex.

If the disturbance frequency exceeds f_c the cause oscillation will be transferred less to the effect variable ΔTt_o. Consequently the drawing zone with its stored fibre mass will dampen the disturbance the better as the disturbance frequency reaches higher values. This behaviour corresponds to the part of the amplitude frequency response which approaches zero for $f > f_c$. As in most practical cases such a system behaviour is formally desired for the following appropriate measure, which directly follows from the explained relations, can be recommended: The *critical frequency* of the technological system for the critical cause-effect relations should be as *small* as possible because then the desired dampening for the dynamic disturbance transfer already begins at lower disturbance frequencies.

After the Eqs. 2.47 or 2.48 this means for the drawing zone: diminishing of v_{om} and/or enlargement of l_m. As the first one lowers the productivity, an enlargement of l_m should be the aim. It is quite clear that other aspects, for instance the technical conditions must be considered as well and consequently compromises have to be made. This point is not further discussed at this place. For comparison, Fig. 2.15 includes an amplitude frequency response which is valid for a drawing process with a drawing zone being enlarged to $l_m = 0.4$ m. As can be calculated with Eq. 2.48 the critical frequency drops to $f_c = 6$ Hz for this case, so an effective dampening of disturbances can be expected already for that frequency.

In a similar way as the time constant of the transfer function does, the phase frequency response $\varphi(\omega)$ (Eq. 2.39) allows statements about the thread length, which leaves the drawing zone before that point of the thread, which corresponds to a disturbance, appears at the end of the drawing zone. This thread length is called **delay thread length** L_d and is calculated as follows:

$$L_{\mathrm{d}} = \frac{|\varphi(\omega)|}{2\pi \cdot \omega} \cdot v_{\mathrm{om}} = \frac{|\varphi(\omega)|}{4\pi^2 \cdot f} \cdot v_{\mathrm{om}} \tag{2.50}$$

For our example we get:

$L_{\mathrm{d}} = 37$ mm for $f_{\mathrm{c}} = 8$ Hz (drawing zone 0.3 m),
$L_{\mathrm{d}} = 50$ mm for $f_{\mathrm{c}} = 6$ Hz (drawing zone 0.4 m).

In Sect. 5.1.5 other problems for which the phase frequency response plays an important role will be discussed.

The next sections cover important aspects of the dynamics of main process steps of fibre/yarn/thread formation and processing technologies. There, the mathematical techniques explained so far will be employed again.