

Production Inventory Model for Uniform Demand Considering Recycle of Defective Items

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Abstract— In this paper, a deterministic production inventory model is developed considering recycle of defective items usable for the production of same product in the next cycle when demand is uniform throughout the cycle time. Due to process deterioration or other factors, generation of defective items with finished goods is inevitable in almost every production process. Raw material from these defective items can be recovered and reused for the same product after being recycled. Proposed model describe how recycle of defective items reduces the procurement of raw material in order to minimize the total cost of the inventory system. Results are explained numerically and sensitivity of the model parameters is also carried out to justify the effectiveness by Mathematica 9.0.

Keywords—Recycle, Defective Items, Production cycle, Cycle Time, Total cost.

1 INTRODUCTION

Inventory of goods is essential in every production process. Inventory exist primarily to make goods available to the customers or producers without delay and to increase sales and profit. By the character of demand, inventory models are classified in two types. When a demand and lead time are known, the model is called deterministic. In the present paper, we considered deterministic inventory models.

In reality, every production system produces some defective items either in a small or in a large amount. It is general phenomenon that the defective items are disposed that may harm the environment remarkably or may increase the total inventory cost. We also can classify defective items into two general types: Recyclable & Non-recyclable. Recyclable items can be transformed into raw material again but all recyclable defective items are not reusable for the same product. But there are some raw materials that can be recovered and reused as raw material for the same product again for the next production cycle.

For a long time, researchers have avoided defective items in EPQ models. But during last three decades a lot of research works have been published in the

area of EPQ of imperfect quality items. Recently Sivashankari and Panayappan discussed a number of research papers on deterministic inventory system of defective items. In their paper, they did not consider recycling concept. We consider here that all the defective items can be recycled for the next production cycle. It is commonly believed that recycling is a service that costs huge money. But we are trying to show that recycling of defective product is good for both economy and environment because recycling maximizes the use of resources and minimizes wastage.

Moreover, reworked defective items may reduce customer's satisfaction and that may create a poor branding image for the company. This is another reason for considering recycle of defective items. Therefore, it is important to develop production inventory models giving due consideration to the cost associated with recycle so that total cost will be minimized. These models focused on recycle of defective items with 100% recovery of raw material usable for the same factory for the same product in the next production cycle. As a result, recycled raw material can be a reliable and alternative source of raw material in case of any uncertainty or obligation.

Inventory models with defective items are studied by researchers in past three decades. **Ghare and Schrader** (1963) first pointed out the effect of decay in inventory analysis. However, they studied a simple Economic Order Quantity (EOQ) model with exponential decay. **Salameh and Jaber** (2000)

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assumed that the defective items could be sold at a discount price and found that the economic lot size quantity tends to increase as the average percentage of imperfect quality items increase. **Cardenas-Barron (2009)** develops an EPQ type inventory model with planned backorders for determining the economic production quantity for a single product, which is manufactured in a single-stage manufacturing system that generates imperfect quality products and all these defective products are reworked in the same cycle and also establish the range of real values of the proportion of defective products for which there is an optimal solution and the closed form for the total cost of inventory system. **Hung-Chi Chang and Chia-Huei Ho (2010)** developed an inventory problem for items with imperfect quality and shortage backordering and adopts the renewal reward theorem to derive the expected net profit per unit time and this study applied algebraic methods to derive the exact closed-form solutions for optimal lot size, backordering quantity and maximum expected profit. **Chiu et al. (2011)** developed an EPQ inventory model with multiple shipments, rework and scrap. They have used a different solution method to obtain the optimal values for lot size and number of shipments instead of the classical optimization technique. **Roy et al. (2011)** developed an EOQ model for imperfect items where a portion of demand partially backlogged. **Jia-Tzer Hsu and Lie-Fern Hsu (2013)** developed inventory model for vendor-buyer coordination under the imperfect production process and the proportion of defective items in each production lot is assumed to be stochastic and follows a known probability density function. **Brojeswar Pal et al. (2013)** developed an EPQ model in an imperfect production system may undergo in 'Out-of-Control' state from 'in-control' state after a certain time that follows a probability density function. The defective items produced in 'out-of-control' state are reworked at a cost just after the regular production time. **Krishnamoorth and Panayappan (2013)** developed a single stage production process where defective items produced are reworked and two models of rework process are considered, an EPQ model without shortage and with shortages. **Chiu et al. (2013)** developed an EPQ inventory model with multiple shipments, rework and scrap. They have used a different solution method to obtain the optimal values for lot size and number of shipments

instead of the classical optimization technique. **Sivashankari and Panayappan (2014a-d)** studied and developed several production inventory models of defective items. In their research they introduced several concepts such as: production Inventory Model for Two levels of production with deteriorating items and shortage, production inventory model with reworking of imperfect production, scrap and shortages, production Inventory model for two levels of production with defective items and incorporating multi-delivery policy, production inventory model for three level of production with defective items and integrates cost reduction delivery policy.

In this paper, a manufacturing process of a single product is under consideration and a single machine manufactures periodically. Raw material is needed in the form of lot size 'Q'. Incorporating with the production cycle of the model of defective items proposed by Sivashankari and Panayappan, we consider the defective items can be sent to the recycling process and recycled raw material can be used in the next production cycle to minimize the total cost and to minimize the wastage of inventory. Assumptions & notations of proposed models are explicitly provided in forthcoming sections.

2 ASSUMPTIONS

- I. Production process is not perfect and it Produces only single product.
- II. Production rate of perfect product is finite & constant and is greater than the sum of demand rate & defective rate.
- III. Items are added to the inventory as soon as it is produced.
- IV. There are sufficient space capacity and capital to stock and produce as well as recycle the desired quantities.
- V. Demand rate is known & constant.
- VI. Rate of Defective item is constant.
- VII. The inventory reaches zero level at the end of each cycle. Production of next cycle starts as soon as the inventory level is zero.
- VIII. To avoid lost-sales and loss of goodwill, shortage are ignored.
- IX. Defective items are recyclable with 100% recovery of raw material usable for the same factory for the same product.
- X. Holding cost for defective items is considered during regular production period.

- XI. Inspection or screening cost for defective items are not considered; because, all such kind of cost is included in the production cost.
- XII. Set up cost, raw material cost, production cost, holding cost and recycle cost are known and fixed. They are separately calculated.

3 NOTATIONS

- P** : Production rate in units per unit time.
- D** : Demand rate of perfect product per unit time.
- d** : Defective rate in units per unit time.
- t_1 : Production time.
- t_2 : Non-production time.
- T** : Cycle time.
- W** : Amount of defective items per cycle.
- Q** : Production lot size per cycle.
- Q_D : Maximum amount of items in the inventory.
- Q_d : Maximum amount of non-defective items in the inventory.
- C_o : Set up cost per cycle.
- C_p : Production cost per unit item irrespective of production lot size.
- C_h : Inventory holding cost is per item per unit time.
- C_R : Raw material cost per item.
- C_r : Recycle cost per item.
- TC** : Average total Inventory cost per unit time.

4 MODEL ANALYSIS

In presence of defective items of imperfect production system of single product manufacturing process, we consider that raw materials of defective items after being recycled is usable for the same product in the same company and are added to the next production cycle. If 'W' units of defective items among the lot-size 'Q' units of items, then 'Q-W' units of fresh items are sold and all defective 'W' units of items are recycled during the cycle time 'T' and added to the raw material in the next production cycle. Therefore, procurement of raw material for the next production cycle will be 'Q - W' units. Material flow is also depicted in the figure 1.

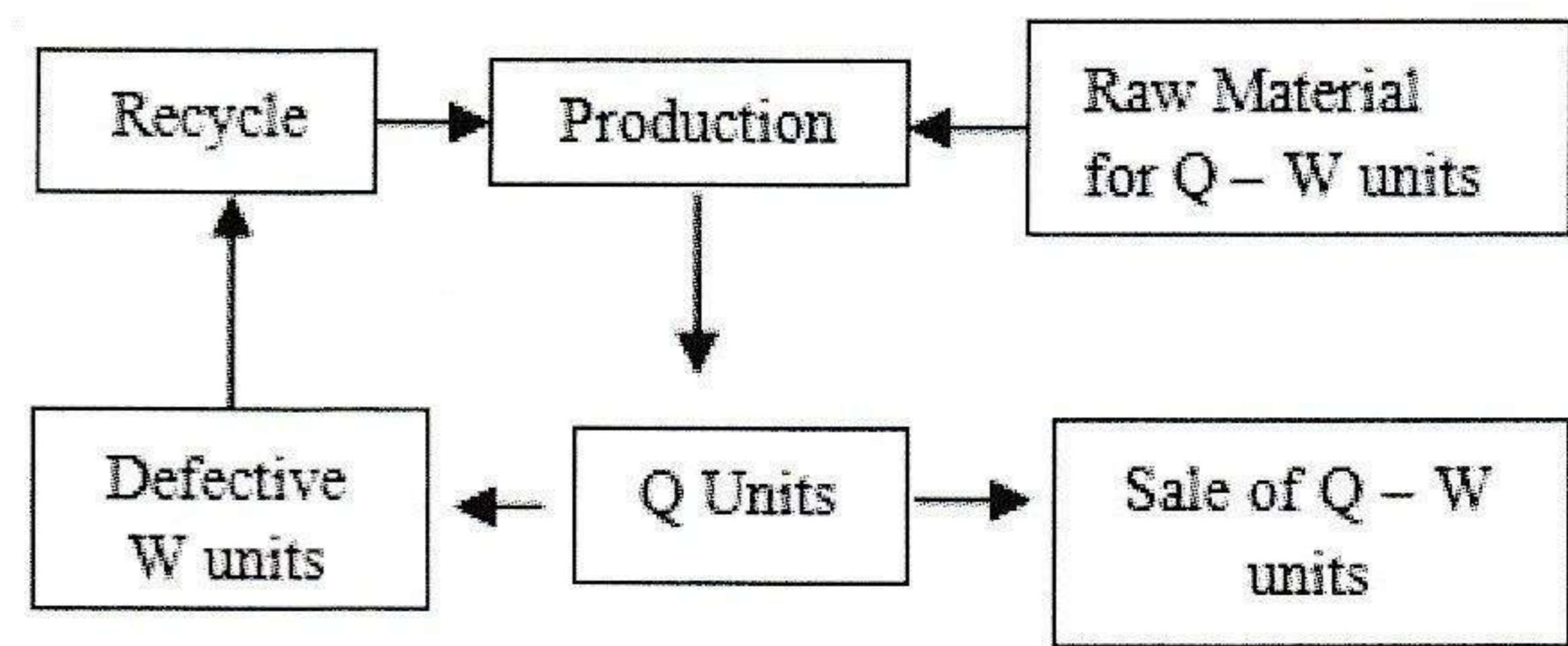


Fig. 1 Material flow of the process

The inventory is depicted in figure 2 below. Production starts at the point O from time $t = 0$ with constant production rate P, demand rate D and defective rate d per unit time and stops at the time $t = t_1$ producing a total of 'Q' units of item where 'W' units are defective items. Due to demand 'D' some Dt_1 amounts are sold during t_1 time and the rest amount is included maximum Q_d units of non-defective items and 'W' units of defective items. The inventory is decreased at a constant rate of D units per unit time through the line AB during next time t_2 . The production restarts as soon as the inventory reaches to zero at the point B where the next cycle starts.

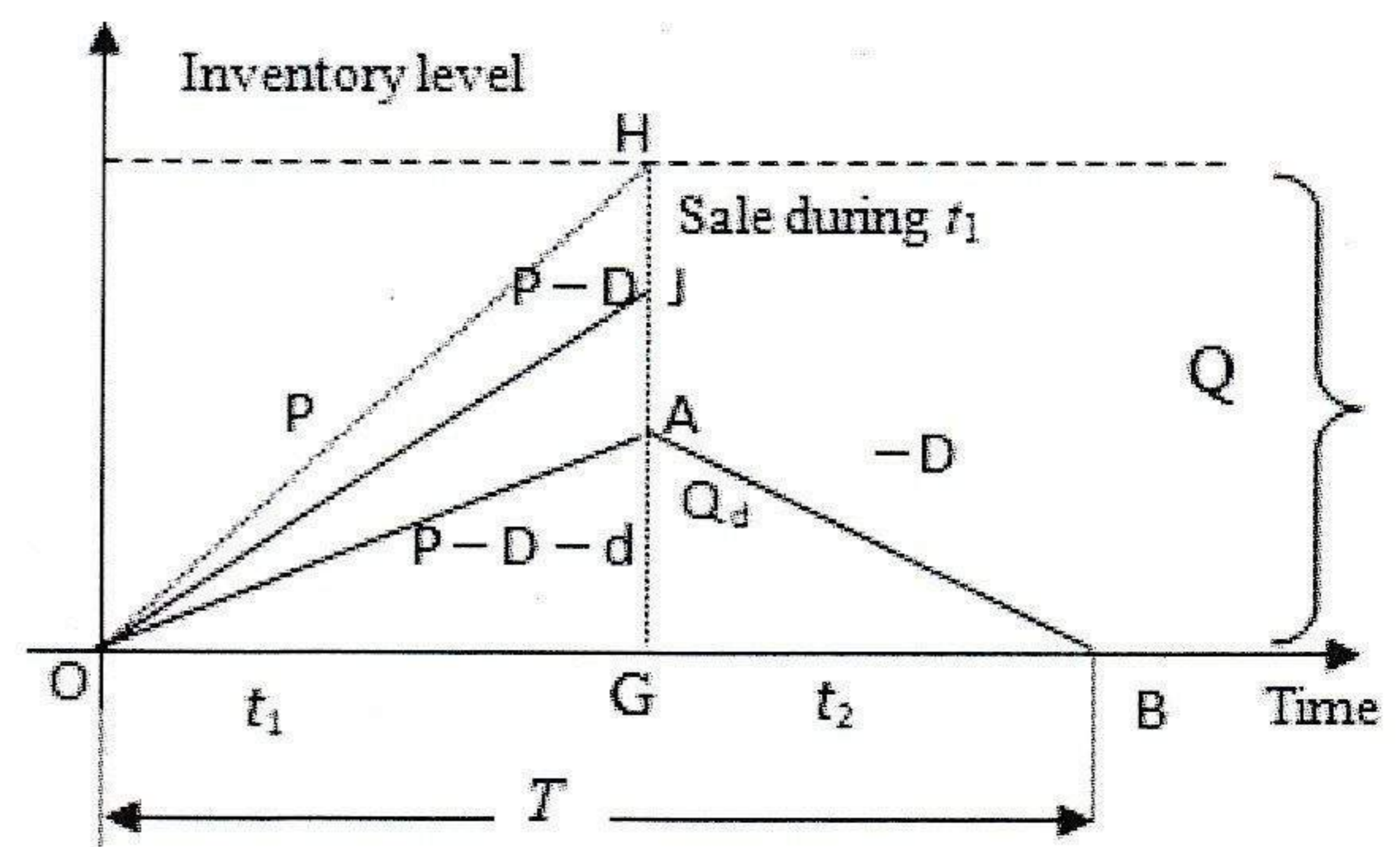


Figure 2: Time -Inventory graph

5 MATHEMATICAL FORMULATION

In figure 2, $Q = GH$, $Q_d = AG$
 & $Q_D = JG = JA + AG = W + Q_d$.
 Production rate is P and the defective rate is d during time t_1 time. Therefore,

$$t_1 = \frac{W}{d} = \frac{Q}{P} \quad (1.1)$$

$$\& \quad Q = P \times t_1 = P \frac{W}{d} \quad (1.2)$$

The inventory is accumulated at a rate $(P - D - d)$ during time t_1 . Therefore,

$$Q_d = (P - D - d) \times t_1 = (P - D - d) \frac{W}{d} \quad (1.3)$$

And inventory amount Q_d is decreased at a constant rate of D units per unit time during time t_2 .

$$\therefore t_2 = \frac{Q_d}{D} = (P - D - d) \frac{W}{dD} \quad (1.4)$$

$$\therefore T = t_1 + t_2 = \frac{W}{d} + (P - D - d) \frac{W}{dD}$$

$$\therefore T = \frac{P-d}{dD} W \quad (1.5)$$

Holding cost of defective items in the inventory is considered for t_1 time. Therefore,

Total Inventory

$$= \frac{1}{2} (Q_d + W) \times t_1 + \frac{1}{2} Q_d \times t_2 = \frac{1}{2} Q_d T + \frac{1}{2} W \times t_1$$

Average Inventory

$$\begin{aligned} &= \frac{1}{T} \left(\frac{1}{2} Q_d T + \frac{1}{2} W \times t_1 \right) \\ &= \frac{1}{2} Q_d + \frac{DW}{2(P-d)} = (P - D - d) \frac{W}{2d} + \frac{DW}{2(P-d)} \\ &= \frac{(P-D-d)(P-d)+dD}{2d(P-d)} W \end{aligned}$$

Costs of the model are:

$$\text{Set up cost} = \frac{1}{T} C_o = \frac{dD}{(P-d)W} \times C_o \quad (1.6)$$

$$\text{Raw material cost} = \frac{1}{T} (Q - W) C_R = DC_R \quad (1.7)$$

Production cost

$$= \frac{1}{T} QC_p = \frac{PD}{(P-d)Q} \times QC_p = \frac{PD}{(P-d)} C_p \quad (1.8)$$

$$\text{Holding cost} = \frac{(P-D-d)(P-d)+dD}{2d(P-d)} W \times C_h \quad (1.9)$$

$$\text{Recycle cost per} = \frac{1}{T} WC_r = \frac{dD}{(P-d)} C_r \quad (1.10)$$

TC

$$\begin{aligned} &= \frac{dD}{(P-d)W} \times C_o + DC_R + \frac{PD}{(P-d)} C_p + \frac{(P-D-d)(P-d)+dD}{2d(P-d)} W \times \\ &C_h + \frac{dD}{(P-d)} \times C_r \quad (1.11) \end{aligned}$$

The total cost is a convex function of 'W' and therefore,

$$\frac{d(TC)}{dW} = -\frac{dD}{(P-d)W^2} \times C_o + \frac{(P-D-d)(P-d)+dD}{2d(P-d)} \times C_h$$

$$\frac{d^2(TC)}{dW^2} = \frac{2dD}{(P-d)W^3} \times C_o > 0$$

$$\text{So, } \frac{d(TC)}{dW} = 0 \quad \text{gives}$$

$$W = d \sqrt{\frac{2DC_o}{\{(P-D-d)(P-d)+dD\}C_h}} ; \text{ which is minimum.}$$

Therefore, the optimal values are:

$$W^* = d \sqrt{\frac{2DC_o}{\{(P-D-d)(P-d)+dD\}C_h}} \quad (1.12)$$

$$t_1^* = \frac{W^*}{d} \quad (1.13)$$

$$t_2^* = \frac{(P-D-d)}{D} t_1^* \quad (1.14)$$

$$T^* = \frac{P-d}{D} t_1^* \quad (1.15)$$

$$Q^* = Pt_1^* \quad (1.16)$$

$$Q_D^* = (P - D)t_1^* \quad (1.17)$$

$$Q_d^* = (P - D - d)t_1^* \quad (1.18)$$

If $d = 0$, then,

$$(i) \quad W^* = 0,$$

$$(ii) \quad Q^* = \sqrt{\frac{2PDC_o}{(P-D)C_h}}$$

$$(iii) \quad T^* = \sqrt{\frac{2PC_o}{D(P-D)C_h}}$$

$$(iv) \quad Q_d^* = Q_D^* = \sqrt{\frac{2D(P-D)C_o}{PC_h}}$$

Therefore, 'model 1' is conformable with the classical EPQ model.

6 NUMERICAL ILLUSTRATIONS

Let $P = 5000$ units per year, $D = 4500$ Units per year, $d = 50$ Units per year, $C_o = 100$, $C_h = 10$, $C_p = 50$, $C_R = 50$, $C_r = 5$

Solution:

Then, from equation (1.6) to (1.18), we get,

$$W^* = 9.578 \text{ units}, \quad Q^* = 957.826 \text{ units},$$

$$t_1^* = 0.1915 \text{ year} = 2 \text{ Month } 8.94 \text{ days}$$

$$t_2^* = 0.0191 \text{ year} = 6.88 \text{ Days}$$

$$T^* = 0.2107 \text{ year} = 2 \text{ Month } 15.85 \text{ days}$$

$$Q_D^* = 95.783 \text{ Units}, \quad Q_d^* = 86.204 \text{ Units}$$

- Set up cost per year = 474.55,
- Raw material cost per year = 225000
- Production cost per year = 227272.72
- Holding cost per year = 474.54
- Recycle cost per year = 227.27

Total cost per year = 453,449

Cycle Time Verification: From the above results,

$$t_1^* + t_2^* = 0.1915 + 0.0191 = 0.2106 \text{ year} = T^*$$

Hence, it proves the model.

Using above values of parameters and changing the values of 'd' and variation of optimum values in the table 1, it is concluded that when the rate of defective items increases, the optimum production quantity, amount of defective items, production period, set up cost, holding cost and the total cost increases. Hence there is a positive relation between them. When the defective rate increases, then the cycle time as well as the non-production period decreases and maximum inventory level decreases. Hence, there is a inverse relation between them.

Table 1: Variation of defective Items and optimum values

d	100	200	300	Remarks
W	19.32	39.22	59.47	increasing
t_1	0.1932	0.1961	0.1982	increasing
t_2	0.0172	0.0131	0.0088	decreasing
T	0.2104	0.2092	0.2071	decreasing
Q	966.23	980.58	991.22	increasing
Q _D	96.62	98.06	99.12	increasing

Q _d	77.30	58.83	39.65	decreasing
Set up Cost	475.34	478.07	483	increasing
Raw Mat. Cost	225000	225000	225000	constant
Prod. Cost	229592	234375	239362	increasing
Holding Cost	475	478	483	increasing
Recycle Cost	449	937	1463	increasing
TC	456001	461269	466764	increasing

Concavity of 'TC' with respect to 'W'

P = 5000 units per year, D = 4500 Units per year, d = 100 Units per year, C_o = 100, C_h = 10, C_p = 50, C_R = 50, C_r = 5 and 10 ≤ W ≤ 30, the following figure 3 & table 2 shows the convexity of TC with respect to 'W' and the optimum value W* ≈ 20.

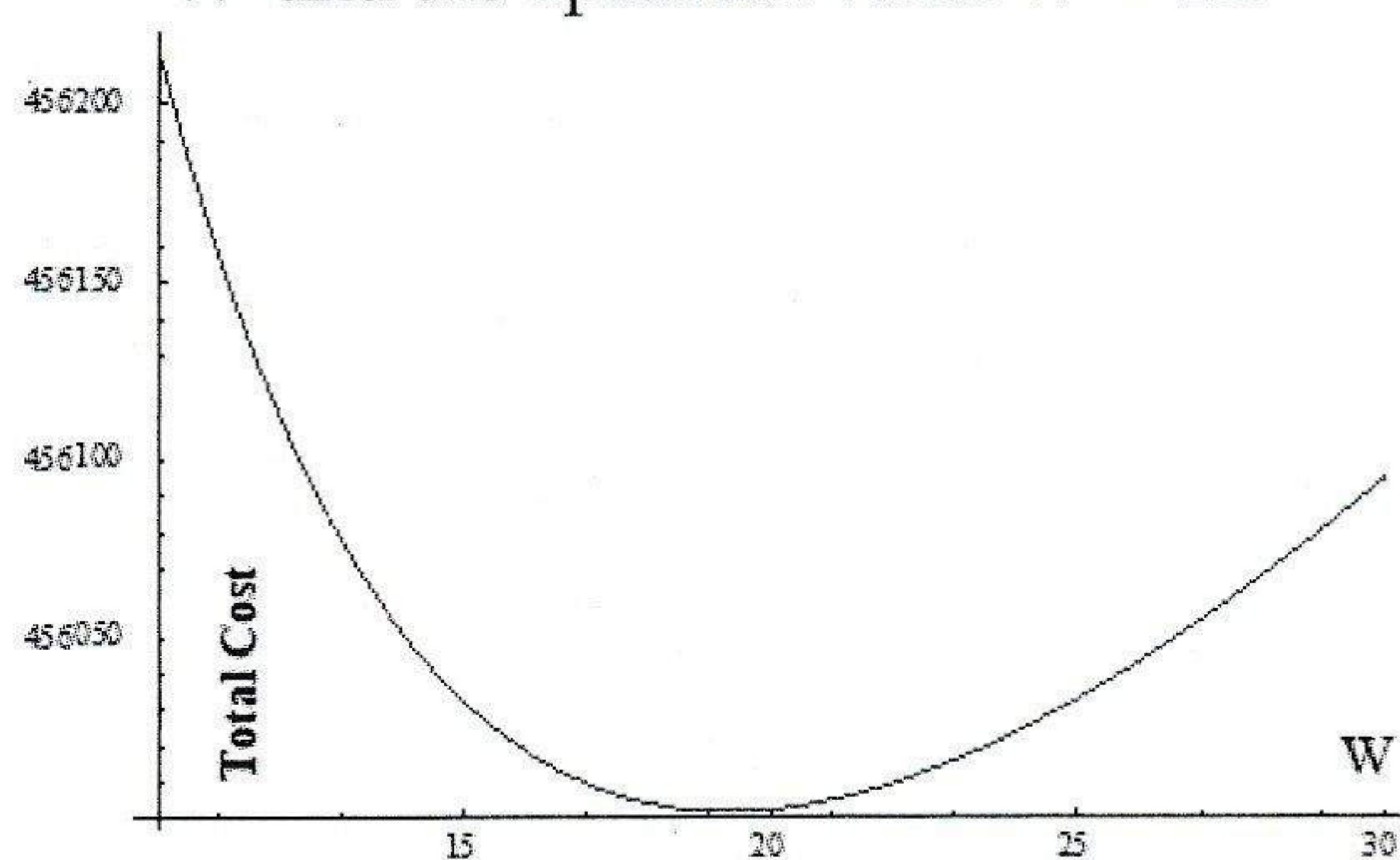


Figure 3: Convexity of TC with defective items

Table 2: Convexity of TC with respect to 'W'

W	TC
10	456215
13	456077
15	456032
18	456004
20	456001
22	456009
25	456033
27	456055

7 SENSITIVITY ANALYSIS

Sensitivity analysis of various system parameters for the model is required to observe. The total cost is a function of all model parameters and 'W'. Therefore, any change in any parameter has sensitivity over total inventory cost. But our target is to find the more sensitive parameters of total cost function. Sensitivities of related parameters over

different optimal solutions and other single cost are also measured. While sensitivity of a parameter is analyzed, change of only that parameter is considered and other parameters considered static.

Table 3: Variation of set up cost and effect on optimum values

C _o	100	200	300	comments
W	19.32	27.3239	33.4714	Increase
t ₁	0.1932	0.27323	0.33471	Increase
t ₂	0.0171	0.02429	0.02975	Increase
T	0.21042	0.29758	0.36446	Increase
Q	966.23	1366.46	1673.57	Increase
Q _D	96.62	136.646	167.357	Increase
Q _d	77.30	109.317	133.885	Increase
Set up Cost	475.34	672.075	823.121	Increase
Raw Mat. Cost	225000	225000	225000	Const.
Prod Cost	229592	229592	229592	Const.
Holding cost	475.11	672.078	823.123	Increase
Recycle Cost	459.184	459.184	459.184	Const.
TC	456001	456395	456697	Increase

Table 4: Variation of holding cost and effect on optimum values

P = 5000, D = 4500, d = 100, C_o = 100, C_p = 50, C_R = 50, C_r = 5

C _h	10	20	30	40	comments
W	19.32	13.6646	11.1571	9.66	Decrease
t ₁	0.1932	0.13664	0.11157	0.09662	Decrease
t ₂	0.0172	0.02114	0.00991	0.00858	Decrease
T	0.2104	0.14879	0.12148	0.10521	Decrease
Q	966.23	683.231	557.856	483.117	Decrease
Q _D	96.62	68.3231	55.78	48.3117	Decrease
Q _d	77.30	54.6585	44.62	38.6494	Decrease
Set up Cost	475.34	672.076	822.91	950.691	Increase
Raw Mat. Cost	225000	225000	225000	225000	Constant
Prod. Cost	229592	229592	229592	229592	Constant
Holding Cost	475.11	672.076	823.335	950.229	Increase
Recycle Cost	449.20	459.18	459.18	459.18	Increase
TC	456001	456395	456697	456952	Increase

Table 5: Variation of C_p and effect on TC

C_p	Production Cost	TC
50	229592	456001
60	275510	501920
70	321429	547838
80	367347	593757
90	413265	639675
100	459184	685593

Table 6 : Variation of C_R and effect on TC

C_R	Raw Mat. Cost	TC
50	225000	456001
60	270000	501001
70	315000	546001
80	360000	591001
90	405000	636001
100	450000	681001

From table 5, we can say, 100% increase in production cost per unit item causes 100% increase in average production cost and 50% increase in average total cost per year.

From table 6, we can say, 100% increase in raw material cost per unit item causes 100% increase in average raw material cost and 49% increase in average total cost per year.

Therefore, sensitivity for production cost and raw material cost is same.

8 COST-BENEFIT ANALYSIS

Cost-Benefit Analysis is a tool to inform the decision-making process only. In the cost-benefit analysis, we consider the comparative total cost of two inventory systems; system without recycle of defective items and system with recycle of defective items. If in the production inventory model without recycle of defective items, raw material for 'Q' items is procured for each production cycle and holding cost for defective items is not be considered. Then,

Average raw material cost

$$= \frac{1}{T}(Q)C_R = \frac{PD}{P-d}C_R$$

Average holding cost

$$= (P - D - d) \frac{W}{2d} \times C_h$$

$$TC = \frac{dD}{(P-d)W} \times C_o + \frac{PD}{P-d}C_R + \frac{PD}{(P-d)}C_p$$

$$+ (P - D - d) \frac{W}{2d} \times C_h \quad (1.19)$$

Equation (1.11) describes the average total cost of a system with recycle of defective items. Therefore, subtracting (1.11) from (1.19), we get,

Average Cost Benefit

$$= \frac{dD}{P-d}C_R - \frac{DW}{2(P-d)}C_h - \frac{dD}{(P-d)}C_r$$

$$= \frac{dD}{P-d}(C_R - C_r) - \frac{DW}{2(P-d)}C_h \quad (1.20)$$

Therefore, from the equation (1.20), we can say that cost benefit increases when cost of raw material increases and cost benefit decreases when holding cost or recycle or both increases.

The following table 7 shows the cost benefit with respect to the increase of defective rate.

Table 7: Variation of recycle rate and effect on different costs

[P = 5000, D = 4500, C_o = 100, C_p = 50, C_R = 50, C_h = 10, C_r = 5]

Recycle Rate	TC without Recycle	TC with Recycle	Cost Benefit
100	460045	456001	4054
200	469522	461269	8254
300	479405	466764	12614
350	484507	469602	14904
400	489720	472502	17218
450	495050	475468	19582
500	500500	478500	22000

Table 8: Cost benefit with respect to recycle cost

C_r	Cost benefit
05	4043
10	3584
15	3125
20	2666
25	2207
30	1748
35	1288

Remarks : Cost benefit decreases while recycle cost increases.

Table 9: Cost benefit with respect to holding cost

C_h	Cost benefit
10	4043
20	4007
30	3987
40	3955
50	3934
60	3915

Remarks: Cost benefit decreases slowly while holding cost increases.

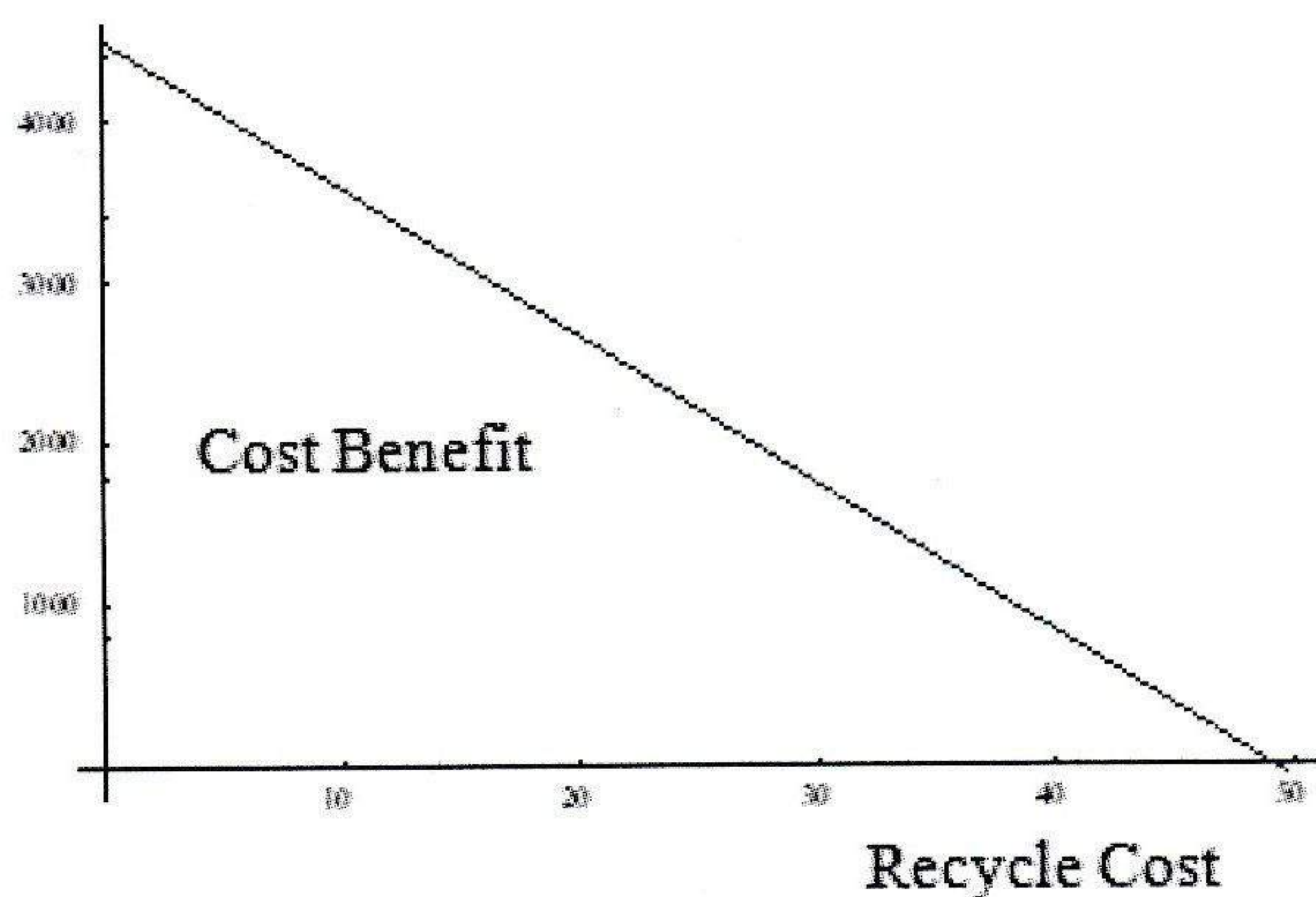


Fig. 4 Cost Benefit versus Recycle Cost

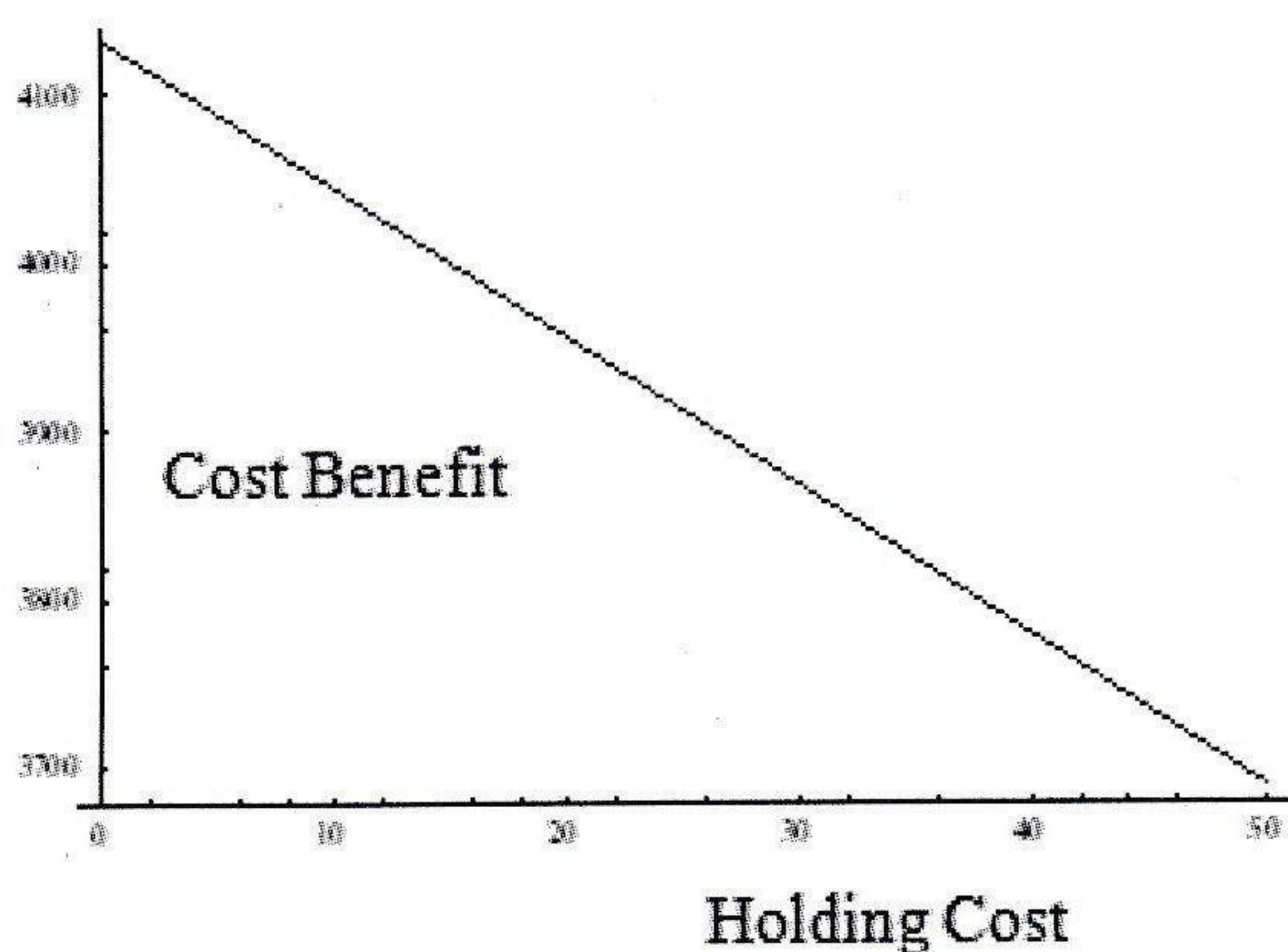


Fig. 5 Cost Benefit versus Holding Cost

Figure 4 & 5 shows that the cost benefit decreases linearly with the increase of recycle cost and holding cost. But recycle cost is more sensitive than holding cost, because cost-benefit decreases faster with the increase of recycle cost than with the increase of holding cost.

9 RESULTS AND DISCUSSION

In this Inventory Model, we have developed the mathematical model for finding the optimal value of the wastage. From the data and graph, it is very much clear that total cost function vs wastage amount form a convex curve. Further in certain condition our model when is converted to the classical lot size inventory model shows the strength of model. We have also introduced a new concept of Recyclable Inventory Model in this paper.

10 CONCLUSION

An important new trend in production management is repair, remanufacturing, recycling or reuse of

defective products. From the models, we observed that the total cost is favorable to the organization where defective items can be recycled into raw materials that can be used in the production of same product in the next production cycle when the price of fresh raw material is more than the price of recycled raw material. Therefore, these models can be considered as 'Green Inventory Model' or 'Sustainable Inventory Model' because they are cost beneficial and environment friendly.

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12 ACKNOWLEDGEMENT

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