# CHAPTER-01 NUMBER SYSTEM

## LEARNING OBJECTIVES

This chapter will enable you to learn the concepts and application of:

- Real number system
- Complex and imaginary numbers
- Definition of sequence and series
- About sigma notation
- Principal of Mathematical Induction
- Relations and functions

## 1.1 INTRODUCTION

Numbers are so fundamental that we are using them every time in the form of units of measurements of mass, space and time. In primitive society, perhaps, the number began with the counting of people, animals, various articles and possessions of man. We shall not try to define what numbers are, but taking them as known, we make attempt to classify them and state some of their properties.

## 1.2 REAL NUMBER SYSTEM

### NATURAL NUMBERS

The numbers first invented are those used for counting. We too, as young children first learnt to count and thus became acquainted with the counting numbers:

$$1, 2, 3, 4, 5, 6, 7, \ldots\ldots\ldots\ldots$$

These are the positive whole numbers, which are called **the natural numbers.** The smallest natural number is 1, but there is no largest natural number, because regardless of how large at number is chosen, there exist larger ones. Thus we say that there are infinitely many natural numbers.

If any two natural numbers are added, the result will be another natural number. For example,

$$5+5=10 \text{ and } 3+8=11.$$

Similarly, if any two natural numbers are multiplied, the product will be a natural number. For example,

$$4\times4=16 \text{ and } 6\times7=42.$$

These two properties are stated by saying that natural numbers are closed under addition and multiplication. Zero is not a natural number.

*Therefore, the set of all natural numbers is denoted by* **N** *and is defined by*

$$N = \{1, 2, 3, \cdots\cdots, +\infty\}.$$

## INTEGERS

If we subtract a natural number from another, we do not always get a natural number. For example: $3-3=0, 4-9=-5$ *i.e.* $0$ and $-5$ are not natural numbers. Then we extend the numbers to include zero and the negative integers: $0, -1, -2, -3, -4, -5, \cdots\cdots$

The above numbers taken together with the natural numbers form the integers or whole numbers: $\cdots\cdots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \cdots\cdots$

Therefore, the set of integers consists of natural numbers (or positive integers), zero and the negative integers. Thus, the integers are closed under addition, subtraction and multiplication.

*The set of all integers is denoted by* **Z** *and is defined by* $Z = \{-\infty \cdots\cdots, -2, -1, 0, 1, 2, \cdots\cdots + \infty\}$

$$= \{o, \pm 1, \pm 2, \pm 3, \pm 4, \cdots\cdots, \pm\infty\}.$$

## RATIONAL NUMBERS

If we divide an integer by another integer, we do not always get an integer. For Example, $\frac{3}{4}, \frac{2}{9}$ and $\frac{7}{3}$ are not integers. These are called **fractions.** Thus the entire collection of such fractions including the integers is called the **rational numbers.** Hence, a rational number is a number which can be put in the form $\frac{p}{q}$, where p and q are integers and q is not equal to zero.

*The set of all rational numbers is denoted by* **Q** *and is defined by* $Q = \{\frac{p}{q} : p, q \in Z \, and \, q \neq 0\}$.

Here p and q are termed as numerator and denominator of the rational number $\frac{p}{q}$. Rational numbers can also be represented by decimals. The representations of some fractions are terminating decimal numbers. For example, $\frac{1}{2} = 0.5, \frac{2}{5} = 0.4, \frac{1}{8} = 0.125$. Other fractions, however, have repeating decimal representation. For example,

$$\frac{1}{3} = 0.3333........ = 0.\dot{3}, \frac{5}{11} = 0.454545............ = .\dot{4}\dot{5}, \frac{2}{7} = 0.285714285714..... = 0.\dot{2}8571\dot{4}.$$

So, every terminating and repeating decimal number is rational number.

## FRACTIONS

If $p$ and $q$ are integers, with $q \neq 0$, then $\frac{p}{q}$ is called a **fraction** (or a rational number). Here $p$ and $q$ are termed as numerator and denominator respectively. In the fraction $\frac{5}{8}$, 5 is the numerator and 8 is the denominator. If the numerator is less than the denominator, the fraction is called a **proper fraction.** Here $\frac{5}{8}$ is a proper fraction. If the numerator is greater than the denominator, the fraction is called an **improper fraction**. For example, $\frac{9}{8}$ and $\frac{17}{5}$ are improper fractions.

## IRRATIONAL NUMBERS

During the process of extracting square roots of numbers 2,3,5,7,31 etc., we find that the results are not rational numbers, as they cannot be put in the form of $\frac{p}{q}$. *Thus a number which cannot be expressed as* $\frac{p}{q}$, *where p and q are integers and* $q \neq 0$ *is called* ***an irrational number*** *and it is denoted by* $Q'$. That is $Q \cap Q' = \varphi$. For example, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{31}$, ...are irrational numbers.

**Note:** The irrational numbers can also be expressed in non-terminating decimals. For example, $\sqrt{2} = 1.4142135.......$, $\pi = 3.14159625.........$, $\sqrt{7} = 2.6457513.......$

**Remarks:** For rational numbers, the infinite decimals have a repeating pattern of a number or a group of number. For example, $\frac{1}{3} = 0.3333.......$, $\frac{5}{11} = 0.4545........$, where as for irrational numbers, the non-terminating decimals have no such pattern. For example, $\sqrt{3} = 1.3720508.......$, $\sqrt{5} = 2.2360680......$

Therefore every non-terminating and non-repeating decimal number is irrational number.

## REAL NUMBERS

The collection of all the rational and irrational numbers is called the system of real numbers. It is denoted by $R$ and so $R = Q \cup Q'$. A diagram of the set of real numbers is shown in the following figure:

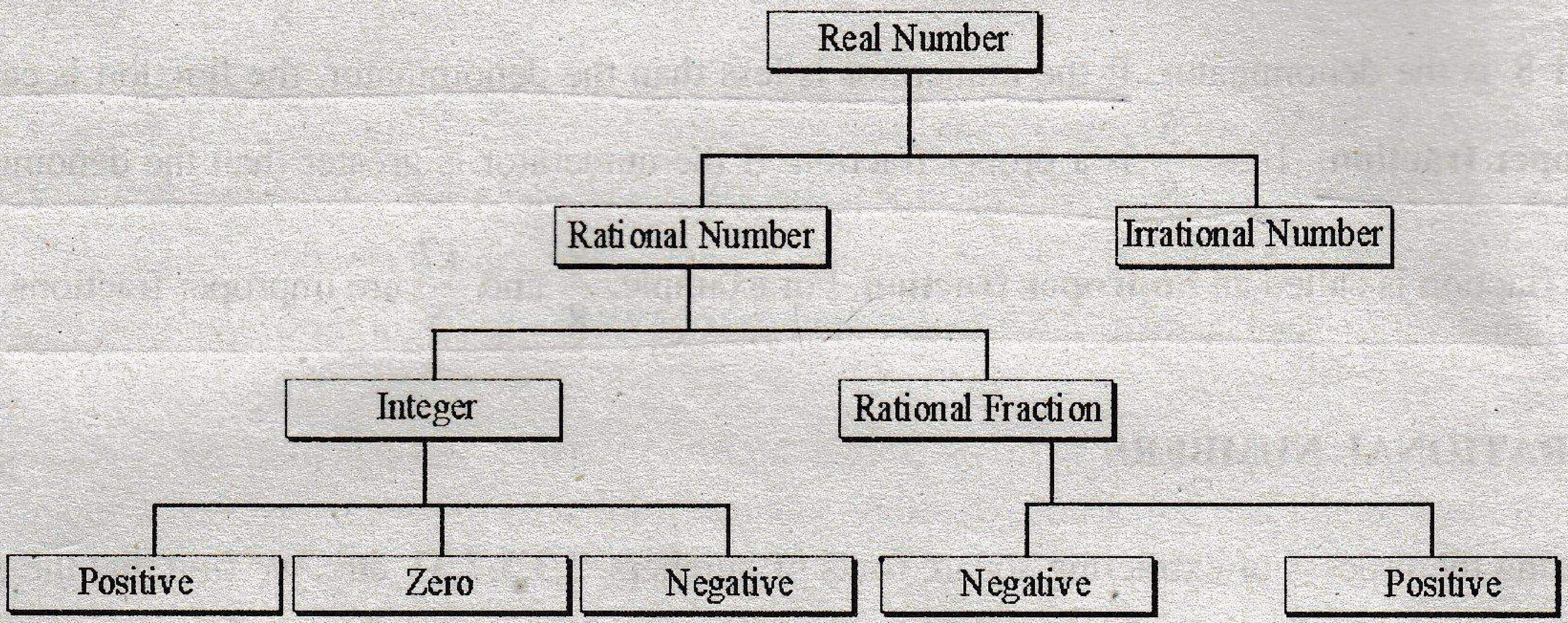


**Figure-1.1 Real Number System**

## EVEN AND ODD NUMBERS

An integer is said to be **even** if it is divisible by 2, otherwise it is said to be an **odd** integer. Thus the even integers are: .........., -6, -4, -2, 0, 2, 4, 6,...........; and the odd integers are: .........-7, -5, -3, -1, 1, 3, 5, 7,......... Since an even integer is divisible by 2, we can write every even integer in the form 2n, where odd integer can be written in the form (2n+1). The other forms of writing the odd integers are: 2n-1, 2n+3, 2n-3 or, 2n+5, 2n-5, etc. If two even integers or two odd integers are added or subtracted, the result is an even integer. For example: 6+10 = 16, 5+9 = 14, 18-24 = -6, 13-3 = 10. Similarly, if two even integers or two odd integers are multiplied the result is a respective number. For example, $4 \times 8 = 32$ (even integer)

$7 \times 13 = 91$ (odd integer)

On the other hand, if an even and an odd integer are multiplied together, the result is an even integer, for example, $9 \times 14 = 126$, (even integer)

$11 \times 8 = 88$ (even integer)

If an even and an odd integer are added or subtracted, the result is an odd integer, for example,

21+14 = 35 (odd integer)

94-27 = 67 (odd integer)

## PRIME NUMBERS

*A number which is not exactly divisible by any number except itself and unity is called a prime number or a prime.* The first few primes are:

$$2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,\ldots\ldots\ldots$$

**Remark:** It should be clearly noted that 1 is not included in the prime numbers. The fact that 1 is not a prime is a mathematical convention. The details are not within in the scope of this book.

## LIST OF SOME PRIMES

15 primes between 1 and 50
25 primes between 1 and 100
168 primes between 1 and 1000
303 primes between 1 and 2000
430 primes between 1 and 3000
550 primes between 1 and 4000
669 primes between 1 and 5000

Nevertheless, the list of primes is endless; i.e. there are infinitely many prime Numbers.

## COMPOSITE NUMBERS

*A number, which is divisible, by other numbers besides itself and unity is called a* ***composite number***. For example, 35 is a composite number because it has divisors 1, 5,7,35. Two numbers which have no common factor except unity are said to be **relatively prime** to each other. Thus 24 is prime to 77 and 35 is prime to 48. Every composite number can be represented uniquely as a product of prime factors. For example,

$$36 = 2\times2\times3\times3 = 2^2\times3^2,$$

$$45 = 3\times3\times5 = 3^2\times5,$$

$$150 = 2\times3\times5\times5 = 2\times3\times5^2.$$

For the factorization, the following technique can be used.

## 1.3 ABSOLUTE VALUE OF A NUMBER

The absolute value of a real number `$a$' is denoted by $|a|$ and is defined by the following:

(i) If `a' is positive or zero, then $|a| = a$

(ii) If `a' is negative, then $|a| = -a$

Symbolically, we can write $|a| = \begin{cases} a & when\ a \geq 0 \\ -a & when\ a < 0 \end{cases}$

Absolute value of a number can never be negative.

For example, $|5| = 5$, $|-6| = -(-6) = 6$, $|-0| = 0$.

$$12 - |-12| = 12 - 12 = 0, \quad 13 + |-4| = 13 + 4 = 17.$$

## 1.4 SPECIAL PROPERTIES OF ZERO

The behavior of zero is one of the most troublesome parts of the study of real numbers. It appears as the numerator or denominator of a fraction in three possible situations:

**Situation-1:** $\frac{0}{d}$, where $d \neq 0$. **Situation-2 :** $\frac{a}{0}$, where $a \neq 0$. **Situation-3 :** $\frac{0}{0}$.

(i) Let us suppose $\frac{0}{d} = c \Rightarrow 0 = c \times d, \quad d \neq 0$

Since $d \neq 0$, it follows that $c = 0$, hence $\frac{0}{d} = 0$

(ii) Again let us assume $\frac{a}{0} = c \Rightarrow a = 0 \times c, a \neq 0$

However, $0 \times c$ is equal to $0$ for values of $c$, and hence cannot be equal to $a$, which is not zero. Therefore, $\frac{a}{0}$ is meaningless. That is, **division by zero is not permissible.**

(iii) Finally, let us take $\frac{0}{0} = c \Rightarrow 0 = 0 \times c$

But this is satisfied for any real number c. For this reason we say that $\frac{0}{0}$ is indeterminate.

(iv) $a \pm 0 = a$ for all $a \in R$

(v) $a.0 = 0$ for all $a \in R$

(vi) If $xy = 0$, then either $x = 0$ or $y = 0$ or both.

**REMARK:** Do not confuse $\frac{0}{0}$ with $\frac{5}{5}$ which is equal to 1.

## INEQUALITY OF NUMBERS

Let a and b be two numbers. We say that `a' is greater than `b' (written as $a > b$) if $a - b$ is positive, similarly `a' is less than `b' (written as $a < b$) if $a - b$ is negative. The symbols $\geq$ and $\leq$ means "greater than or equal to" and "less than or equal to" respectively.

**Example-:** 1. 6 and $8 > 5$.

2. Since $-5 < 1$ and $1 < 7$, it follows that $-5 < 7$ or, $7 > -5$.
3. Since $4 > 2$, it follows that $4 + 5 > 2 + 5$ or, $9 > 7$.
   Also we have $4 - 8 > 2 - 8 \Rightarrow -4 > -6 \Rightarrow -6 < -4$.
4. Since $2 < 7$, it follows that $2 \times 5 < 7 \times 5 \Rightarrow 10 < 35 \Rightarrow 35 > 10$

## FORMAL LAWS OF NUMBERS

In short, we state that following are the laws of the arithmetic of numbers. If $a, b, c$ stand for arbitrary real numbers, we have.

**Addition:**

1. $a + b$ is a unique number [Closure law]
2. $(a + b) + c = a + (b + c)$ [Associative law]
3. $a + 0 = 0 + a = a$ [Identity law]
4. $a + (-a) = (-a) + a = 0$ [Inverse law]
5. $a + b = b + a$ [Commutative law]

**MULTIPLICATION:**

6. $a \times b$ is a unique number [Closure law]
7. $(a \times b) \times c = a \times (b \times c)$ [Associative law]
8. $a \times 1 = 1 \times a = a$ [Identity law]
9. $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$ [Inverse law]
10. $a \times b = b \times a$ [Commutative law]
11. $a \times (b + c) = (a \times b) + (a \times c)$ [Distributive law]

## RAPID METHODS FOR DIVISION

1. **Division by 2.** A number is divisible by 2 if its last digit is divisible by 2(i.e., last digit is 0,2,4,6 or 8).
2. **Division by 3.** Only such numbers are divisible by 3 the sum of whose digits is divisible by3.
3. **Division by 4.** A number is divisible by 4 if the last two digits are divisible by 4.
4. **Division by 5.** Numbers ending in 0 or 5 are divisible by 5
5. **Division by 8.** A Number is divisible by 8 if the last three digits are divisible by 8.

6. **Division by 9.** If the sum of the digits of a number is divisible by 9, the number is divisible by 9.
7. **Division by 10.** A Number is divisible by 10 if its last digit is zero.
8. **Division by 11.** If the difference of sums of digits occupying even and odd positions in the given number is divisible by 11, the number is divisible by 11.

For example-the number 1,03,785 is divisible by 11 since the sum of the digits in odd positions $(1+3+8=12)$ minus the sum of the digits in even positions $(0+7+5=12)$ is 0, which is divisible by 11.

## 1.5 FRACTIONS

We already know that when a number can be expressed as exact number of units of any kind, then it is called a whole number or an integer. But, when the unit is supposed to be divided into any number of equal parts, and one or more of these part are taken, then the number is called a **fraction.**

Thus, if one taka is the unit and we suppose the unit to be divided into one-hundred equal parts, each part will be one paisa (p) and $5p, 15p, 25p, 75p$, will be respectively five hundredths, fifteen hundredths, twenty-five hundredths, seventy-five hundredths of the unit, i.e. one Taka. These fractions are written as: $\frac{5}{100}, \frac{15}{100}, \frac{25}{100}, \frac{75}{100}$. Thus a fraction is expressed by two numbers one over the other, separated by a horizontal line (bar). The number one-half, one-third, one-fourth or one-fifth etc. of a thing may be expressed as $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ etc. When we speak of three-quarters of a meter, we regard a length of one meter as having been divided into 4 equal parts and that 3 of these parts are taken. If a thing is divided into 7 equal parts and out of these, 4 parts are taken, it is $\frac{4}{7}$ of a thing. The lower number, which expresses the number of equal parts into which the unit is divided, is called the **denominator.** The upper number, which expresses the number of parts taken in any fraction, is called the **numerator.** The numerator and denominator are sometimes called the terms of a fraction. The fractions expressed in this manner are known as **vulgar or common fractions.** When the numerator and denominator of a fraction are equal, its value is unity. A fraction may also be defined as the result of dividing the numerator by the denominator. For example, $\frac{4}{5}$ denotes four times the fifth part of the unit. But, if we divide 4 units by 5 we get a result, which is four times as great as the fifth part of 1 unit. Hence the fraction $\frac{4}{5}$ is the result of dividing 4 units by 5. The following mode of reading fractions is sometimes used : $\frac{5}{8}$ is read five over eight, $\frac{10}{13}$ is read ten over thirteen. Instead of using the horizontal line to separate numerator and denominator, it is sometimes convenient to write fractions with two numbers before and after a slanting line in the following form: 1/2, 2/3, 5/7, 7/16, ......etc. A fraction is said to be **proper fraction** if the numerator is smaller than the denominator. For example, the fractions $\frac{3}{4},\frac{5}{8},\frac{7}{16}$ etc. are proper fractions. An **improper fraction** is one which the numerator is equal to or is larger than the denominator, Thus $\frac{3}{3},\frac{4}{3},\frac{7}{4},\frac{16}{7},\frac{32}{16}$ etc. are improper fractions. A whole number is a number containing no fractions or a fraction whose denominator is unity as 3,15,48 etc. For changing an improper fraction to a whole number or to a mixed number, we divide the numerator of the fraction by its denominator. If nothing is left over, the result of this division is a whole number, which is equal to the improper fraction. If there is a remainder, or a number left over after the division then this remainder becomes the numerator of fraction whose denominator is the same as the denominator in the original fraction. This new fraction, together with the whole number resulting from the division forms the new mixed number. For example, $\frac{85}{5}=17$ (whole number) and $\frac{217}{9}=24\frac{1}{9}$ (mixed number)

**REMARK:** When the numerator and the denominator of a fraction have no common factor, the fraction is said to be in its lowest terms.

**COMPARISON OF FRACTIONS**

**Rule 1.** When two fractions have the same denominator, the greater is that which has the greater numerator. Example, $\frac{5}{9}$ is greater than $\frac{2}{9}\left(\frac{5}{9}>\frac{2}{9}\right)$.

**Rule 2.** When two or more fractions with different denominators are to be compared, They must first be reduced to equivalent fractions having the least common denominator and rule 1 is applied.

**REMARK:** The LCM (Least Common Multiple) of the denominators of a series of fractions is called their (LCD) least common denominator. It is usually best to keep the LCD in factors, then the multipliers required to reduce the given fractions to their LCD are found by mentally crossing out the factors of the given denominators.

## ADDITION AND SUBTRACTION OF FRACTIONS

(i) When the fractions are of the same denominators, the value of sum or difference of the fractions is the sum or difference of the numerators retaining the common denominator.

(ii) When the fractions have different denominators they must first be expressed with a common denominator, and the work will be simplified by taking the least common denominator (LCD)

(iii) In addition a series of fractions, some of which are mixed numbers, it is convenient to add the whole numbers and fractional parts separately, and finally add the sum of the fractions to the sum of the whole numbers. Any improper fractions should first be expressed as mixed number.

## MULTIPLICATION AND DIVISION OF FRACTION

(i) To multiply a fraction by an integer, multiply the numerator by the integer and keep the denominator unchanged.

(ii) To multiply one fraction by another fraction, multiply the numerators together for the acquired numerator and the denominators together for the required denominator of the product.

(iii) To divide a fraction by an integer, either divide the numerator if the numerator is exactly divisible by the integer or multiply the denominator by the integer.

(iv) To divide a fraction by a fraction, first interchange numerator and the denominator of the divisor and then multiply by this changed fraction.

(v) Continued product is the product obtained by multiplying more than two numbers together.

## 1.6 COMPLEX AND IMAGINARY NUMBERS

**Complex Numbers:** The set of all complex numbers is denoted by $C$ and is defined by $C = \{a + ib : a, b \in R \text{ and } i^2 = -1\}$, where $a$ is called real part of complex number and $b$ is called imaginary part of complex number and $i$ is the imaginary unit.

**For examples:** 2 + i3, 2 - i3, i3, 2, 4 + i6 etc.

**Note:** Every real number is a complex number having imaginary part 0 (zero), because $2 = 2 + i.0$.

**Imaginary Numbers:** Square root of any negative numbers is called imaginary number.

In other words, any number of the form: $ib$, where $b \in R$ and $i^2 = -1$ is called an imaginary number.

For Example: $2i$, $\sqrt{-2}$, $\sqrt{-5}$, $\sqrt{-3}$ *etc.*

## 1.7 SEQUENCE AND SERIES

A list of numbers such as 1, 4, 9, 16, .........., $x^2$,..............; $x \in$ **N** is called a Sequence. In other words, a sequence in a function with domain a set of successive integers.

### SERIES

The sum of all the terms of a sequence is called series.

Notes:

1, 2, 3, 4, 5, 6, 7, 8 Finite sequence

1, 2, 3, 4, 5, 6,...................... Infinite sequence

1+2+3+4+5+6+7+8 Finite series

1+2+3+4+5+6+........... Infinite series

### DISCOVERY OF SEQUENCE

By the method of trial and error we have to determine the sequence, ensuring that at least first few terms are observing the rule so as to generalize ultimately.

**For example:** 2, 4, 6, 8, ......... can be written in the form:

2.1, 2.2, 2.3, 2.4, ..........

Hence we can express n-th term as, $u_n = 2n$ or $\{2n\}$.

## 1.8 SIGMA NOTATION

We use a simple summation notation which considerably simplifies the formulae and makes handling of complicated expressions simpler. The symbol "$\sum$ " of Greek alphabet is used to denote the summation of a given series and this symbol is known as sigma notation or summation notation. We write $\sum_{r=1}^{n} u_r$ to denote the sum of $n$ terms of the sequence $u_r$. If we want to sum up to $u_r$ for values of $r$ corresponding to $r = 1, 2, 3, ......, 5$; we denote the sum by $\sum_{r=1}^{5} u_r$ Or simply by $\sum_{1}^{5} u_r$ . $\therefore \sum_{r=1}^{5} u_r = u_1 + u_2 + u_3 + u_4 + u_5$

### PROPERTIES OF SIGMA NOTATION

1. $\sum_{r=1}^{n} a u_r = a \sum_{r=1}^{n} u_r$ ; where a is constant

2. $\sum_{r=1}^{n} (u_r + v_r) = \sum_{r=1}^{n} u_r + \sum_{r=1}^{n} v_r$

3. $\sum_{r=1}^{n} (ar^3 + br^2 + cr + d) = a\sum_{r=1}^{n} r^3 + b\sum_{r=1}^{n} r^2 + c\sum_{r=1}^{n} r + nd \quad (\because \sum_{r=1}^{n} d = dn)$

## 1.9 PRINCIPLE OF MATHEMATICAL INDUCTION

Let the formula or the proposition be denoted by $P(n)$. The principal of mathematical induction states that if

1. The formula or the given proposition $P(n)$ involving $n$ is true for $n = 1, 2$ and
2. if the truth of $P(m)$ for $n = m$ implies it is true for $P(m+1)$, then $P(n)$ is true for all positive integer values of $n$.

## ILLUSTRATIONS

**Illustration-01:** Which of the following integers are odd or even?

$$158, -1715, 26170, 987003, 201, -2402$$

**Solution:** By definition of even integers, we know that the numbers, which are divisible by 2,are called even integers.

$\therefore$ 158,26170,–2402, are even integers.

The remaining integers are odd, i.e. $-1715, 987003, 201$ are odd integers.

**Illustration-02:** Factorize the number 1421 into prime factors.

**Solution:** We take the primes, one after the other, and stop at the prime, which is divisor of the given number. Using this process, we see that the primes 2,3 and 5 are not the divisors of 1421. Again we attempt to divide by 7 and then see that 1421 is dividable by 7 giving a quotient of 203. Test 203 in the same manner ignoring the numbers 2,3,5 that proved ineffective in the first trial. Only begin with 7. It turns out that 7 is a divisor of 203. The quotient 29 is the prime. This completes the factorization. Thus, we have $1421 = 7 \times 7 \times 29 = 7^2 \times 29$.

**Illustration-03:** What meaning is to be attached to each of the following?

$$\frac{0}{3}, \frac{3}{0}, \frac{3}{3}, \frac{0}{\frac{1}{3}}, \frac{0}{0}.$$

**Solution:** $\frac{0}{3} = 0$. $\frac{3}{0}$ is meaningless, $\frac{3}{3} = 1$, $\frac{0}{1/3} = 0$, $\frac{0}{0}$ is indeterminate.

**Illustration-04:** Reduce $\frac{36}{48}$ to its lowest terms.

**Solution:** $\frac{36}{48} = \frac{18}{24} = \frac{9}{12} = \frac{3}{4}$ $\therefore \frac{36}{48} = \frac{3}{4}$.

Sometimes it is difficult to find out the common factors of the terms of the given fraction by observation. In that case find out the GCD (Greatest Common Divisor) of the terms and then divide the terms by the GCD to reduce the given fraction to its lowest terms.

**Illustration-05:** Reduce $\frac{129}{215}$ to its lowest terms.

**Solution:** Dividing & multiplying the given fraction $\frac{129}{215}$ by 43, we get, $\frac{129}{215} = \frac{3}{5}$.

**Illustration-06:** Which is greater $\frac{2}{3}$ or $\frac{5}{7}$?

**Solution:** The LCM of 3 and 7 is 21

$$\therefore \frac{2}{3} = \frac{2\times7}{3\times7} = \frac{14}{21} \text{ and } \frac{5}{7} = \frac{5\times3}{7\times3} = \frac{15}{21}$$

$\frac{15}{21}$ is greater than $\frac{14}{21}$, Hence $\frac{5}{7}$ is greater.

**Illustration-07:** Arrange the fractions $\frac{7}{10}, \frac{3}{8}, \frac{4}{5}$ in ascending order of magnitude.

**Solution:** The L.C.M. of 10,8,5 is 40.

$$\therefore \frac{7}{10} = \frac{7\times4}{10\times4} = \frac{28}{40}, \frac{3}{8} = \frac{3\times5}{8\times5} = \frac{15}{40} \text{ and } \frac{4}{5} = \frac{4\times8}{5\times8} = \frac{32}{40}$$

Hence the given fractions can be arranged in ascending order of magnitude as:

$$\frac{15}{40}, \frac{28}{40}, \frac{32}{40} \text{ i.e. } \frac{3}{8}, \frac{7}{10}, \frac{4}{5}.$$

**Illustration-08:** Arrange the fractions $\frac{2}{3}, \frac{5}{6}, \frac{3}{4}$ in descending order of magnitude

**Solution:** The LCM. of 3,6,4 is 12

$$\therefore \frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12}, \quad \frac{5}{6} = \frac{5\times2}{6\times2} = \frac{10}{12}, \quad \frac{3}{4} = \frac{3\times3}{4\times3} = \frac{9}{12}.$$

Hence the given fractions can be arranged in descending order of magnitude as:

$$\frac{10}{12}, \frac{9}{12}, \frac{8}{12} \text{ i.e. } \frac{5}{6}, \frac{3}{4}, \frac{2}{3}.$$

**Illustration-09:** Find the value of $\frac{5}{6}+\frac{7}{8}+\frac{3}{4}$.

**Solution:** $\frac{5}{6}+\frac{7}{8}+\frac{3}{4}=\frac{20+21+18}{24}=\frac{59}{24}=2\frac{11}{24}$.

**Illustration-10:** Find the value of $2\frac{2}{5}+1\frac{3}{4}+2\frac{1}{2}$.

**Solution:** Method-1: $2\frac{2}{5}+1\frac{3}{4}+2\frac{1}{2}=\frac{12}{5}+\frac{7}{4}+\frac{5}{2}=\frac{48+35+50}{20}=\frac{133}{20}=6\frac{13}{20}$.

Method-2: $2\frac{2}{5}+1\frac{3}{4}+2\frac{1}{2}=2+1+2+\frac{2}{5}+\frac{3}{4}+\frac{1}{2}=5+\frac{8+15+10}{20}=5+\frac{33}{20}=5+1\frac{13}{20}=6\frac{13}{20}$.

**Illustration-11:** Find the value of (i) $\frac{12}{35}-\frac{4}{21}+\frac{7}{15}$, (ii) $3\frac{10}{11}+5\frac{7}{15}-2\frac{9}{22}-4\frac{9}{10}$.

**Solution:** (i) $\frac{12}{35}-\frac{4}{21}+\frac{7}{15}=\frac{36-20+49}{105}=\frac{65}{105}=\frac{13}{21}$.

(ii) Method-1: $3\frac{10}{11}+5\frac{7}{15}-2\frac{9}{22}-4\frac{9}{10}=\frac{43}{11}+\frac{82}{15}-\frac{53}{22}-\frac{49}{10}$

$=\frac{1290+1804-795-1617}{330}=\frac{3094-2412}{330}=\frac{682}{330}=\frac{31}{15}=2\frac{1}{15}$.

Method-2: $3\frac{10}{11}+5\frac{7}{15}-2\frac{9}{22}-4\frac{9}{10}=3+5-2-4+\frac{10}{11}+\frac{7}{15}-\frac{9}{22}-\frac{9}{10}$

$=2+\frac{300+154-135-297}{330}=2+\frac{454-432}{330}=2+\frac{22}{330}=2+\frac{1}{15}=2\frac{1}{15}$.

**Illustration-12**: Multiply $2\frac{1}{4}$ by 3.

**Solution:** $2\frac{1}{4}\times 3=\frac{9}{4}\times 3=\frac{27}{4}=6\frac{3}{4}$.

**Illustration-13:** Find the cost of 5 ink-pots if an ink-pot cost Tk. $\frac{3}{8}$.

**Solution:** Cost of one inkpot= Tk. $\frac{3}{8}$.

Cost of 5 ink-pots= $5\times Tk\frac{3}{8}=Tk\frac{15}{8}=Tk.1\frac{7}{8}$.

**Illustration-14:** Simplify $\frac{5+3i}{2+5i}$

Solution: $\frac{5+3i}{2+5i} = \frac{(5+3i)(2-5i)}{(2+5i)(2-5i)}$ [Multiplying denominator and numerator by $(2-5i)$]

$$= \frac{10-25i+6i-15i^2}{2^2-(5i)^2}$$

$$= \frac{10-19i+15}{4-25i^2} (\because i^2 = -1)$$

$$= \frac{10+15-19i}{4+25}$$

$$= \frac{25-19i}{29} = \frac{25}{29} - \frac{19i}{29} \text{ Ans.}$$

**Illustration-15:** Simplify $\frac{3+2i}{5-3i}$

Solution: $\frac{3+2i}{5-3i} = \frac{(3+2i)(5+3i)}{(5-3i)(5+3i)}$ [Multiplying denominator and numerator by $(5+3i)$]

$$= \frac{15+9i+10i+6i^2}{(5)^2-(3i)^2} = \frac{15+19i+6(-1)}{25-9(-1)} \quad (\because i^2 = -1)$$

$$= \frac{15-6+19i}{25+9} = \frac{9+19i}{34} = \frac{9}{34} + i\frac{19}{34} \quad \text{(Ans.)}$$

**Illustration-16:** Simplify: $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$.

Solution: $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$

$$= \frac{(2+5i)(3+2i)+(2-5i)(3-2i)}{(2-5i)(2+5i)}$$

$$= \frac{6+15i+4i+10i^2+6-4i-15i+10i^2}{2^2-(5i)^2}$$

$$= \frac{12+20i^2}{4-25i^2} = \frac{12-20}{4+25} (\because i^2 = -1) = -\frac{8}{29} \text{ (Ans.)}$$

**Illustration-17:** Simplify: $\frac{9-7i}{2-3i}$

Solution: $\frac{9-7i}{2-3i}$

$=\frac{(9-7i)(2+3i)}{(2-3i)(2+3i)}$ [Multiplying denominator and numerator by $(2+3i)$]

$=\frac{18+27i-14i-21i^2}{2^2-(3i)^2}=\frac{18+13i-21i^2}{4-9i^2}=\frac{18+13i+21}{4+9}\ (\because i^2=-1)$

$=\frac{39+13i}{13}=\frac{39}{13}+\frac{13i}{13}=3+i$ *Ans.*

**Illustration-18:** Prove that $\sqrt{2}$ is irrational.

**Proof:** Suppose that $\sqrt{2}$ is a rational number.

$\therefore \sqrt{2}=\frac{p}{q}$, where p and q are integers, $q\neq 0$.

Again, suppose that the rational number $\frac{p}{q}$ is in its lowest terms i.e., p and q have no common factors, without 1. Now, Squaring the above equation, we get $2=\frac{p^2}{q^2}$ or, $p^2=2q^2$. The term $2q^2$ represents an even integer, so $p^2$ is an even integer, and hence p is an even integer, say $p=2r$ where r is also an integer. Replacing p by 2r in the equation $p^2=2q^2$, we get $(2r)^2=2q^2 \Rightarrow 4r^2=2q^2 \Rightarrow 2r^2=q^2$.

The term $2r^2$ represents an even integer, so $q^2$ is an even integer, and hence q is an even integer. Thus we have seen that both p and q are even integers, i.e. they have a common factor 2 which contradicts our assumption that p and q have no common factors. Hence, it follows that $\sqrt{2}$ is not a rational number. Therefore, $\sqrt{2}$ is an irrational number.

**Illustration-19:** Prove that $\sqrt{3}$ is an irrational number.

**Solution:** Suppose $\sqrt{3}$ is a rational number

i.e. $\sqrt{3}=\frac{p}{q}$ (say)......(1)

where p and $q \neq 0$ are integers and $\frac{p}{q}$ is in lowest terms, so that p and q are not divisible by 3.

Squaring both sides of (1) we get $3 = \frac{p^2}{q^2} \Rightarrow p^2 = 3q^2$.

The integer $3q^2$ is divisible by 3; i.e. $p^2$ is divisible by 3. Therefore p itself is divisible by 3, say $p = 3r$, where r is an integer.

Replacing p by 3r in the above equation, we get

$$p^2 = 3q^2 \Rightarrow (3r)^2 = 3q^2 \Rightarrow 9r^2 = 3q^2 \Rightarrow 3r^2 = q^2.$$

This shows that $q^2$ is divisible by 3, and hence q is divisible by 3.

Thus we have seen that both p and q are divisible by 3, which contradicts our assumption.

Hence $\sqrt{3}$ is not a rational number.

Therefore, $\sqrt{3}$ is an irrational number.

**Illustration-20:** Find n-th term of 2, 6, 12, 20, 30,.........and hence express the sequence as generalized form.

**Solution:**

2, 6, 12, 20, 30,....... can be written in the form $1+1^2, 2+2^2, 3+3^2, 4+4^2, 5+5^2, \ldots\ldots\ldots$

∴ n-th term of the sequence is $n+n^2$

∴ The generalized form of the sequence is $\{n+n^2\}$ for all $n \in N$

**Illustration-21:** Write the following series without summation or sigma notation, $\Sigma$:

1. $\sum_{K=1}^{5} a_k = a_1 + a_2 + a_3 + a_4 + a_5$

2. $\sum_{i=1}^{4} a_i = a_1 + a_2 + a_3 + a_4$

3. $\sum_{i=1}^{3} a^i = a^1 + a^2 + a^3$

4. $\sum_{r=2}^{5} (-1)^r \left(\frac{x^r}{r}\right) = (-1)^2 \left(\frac{x^2}{2}\right) + (-1)^3 \left(\frac{x^3}{3}\right) + (-1)^4 \left(\frac{x^4}{4}\right) + (-1)^5 \left(\frac{x^5}{5}\right)$

$$= \frac{x^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} - \frac{x^5}{5} \text{ Ans.}$$

5. $\sum_{k=1}^{5} \frac{k-1}{k}$

$$=\frac{1-1}{1}+\frac{2-1}{2}+\frac{3-1}{3}+\frac{4-1}{4}+\frac{5-1}{5}$$

$$=\frac{0}{1}+\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5} \quad =0+\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}$$

$$=\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5} \ \textit{Ans.}$$

6. $\sum_{k=0}^{5}(-1)^k \frac{1}{2k+1}$

$$=(-1)^0\frac{1}{2.0+1}+(-1)^1\frac{1}{2.1+1}+(-1)^2\frac{1}{2.2+1}+(-1)^3\frac{1}{2.3+1}+(-1)^4\frac{1}{2.4+1}+(-1)^5\frac{1}{2.5+1}$$

$$=1\frac{1}{0+1}-\frac{1}{2+1}+1\frac{1}{4+1}-\frac{1}{6+1}+\frac{1}{8+1}-\frac{1}{10+1}$$

$$=\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11} \text{ Ans.}$$

7. $\sum_{k=1}^{4}\frac{(-2)^{k+1}}{k}$

$$=\frac{(-2)^{1+1}}{1}+\frac{(-2)^{2+1}}{2}+\frac{(-2)^{3+1}}{3}+\frac{(-2)^{4+1}}{4}$$

$$=\frac{(-2)^2}{1}+\frac{(-2)^3}{2}+\frac{(-2)^4}{3}+\frac{(-2)^5}{4}$$

$$=4-\frac{8}{2}+\frac{16}{3}-\frac{32}{4} \ =4-4+\frac{16}{3}-8 \ =\frac{16}{3}-8 \ =\frac{16-24}{3} \ =\frac{-8}{3} \text{ Ans.}$$

8. $\sum_{k=1}^{5}(-1)^{k+1}(2k-1)^2$

$$=(-1)^{1+1}(2.1-1)^2+(-1)^{2+1}(2.2-1)^2+(-1)^{3+1}(2.3-1)^2+(-1)^{4+1}(2.4-1)^2+(-1)^{5+1}(2.5-1)^2$$

$$=(-1)^2(2-1)^2+(-1)^3(4-1)^2+(-1)^4(6-1)^2+(-1)^5(8-1)^2+(-1)^6(10-1)^2$$

$$=1^2-3^2+5^2-7^2+9^2=1-9+25-49+81=107-58 \ =49 \ \textit{Ans.}$$

9. $\sum_{k=1}^{4}\frac{(-1)^{k+1}}{k}x^k$

$$=\frac{(-1)^{1+1}}{1}x^1+\frac{(-1)^{2+1}}{2}x^2+\frac{(-1)^{3+1}}{3}x^3+\frac{(-1)^{4+1}}{4}x^4+\frac{(-1)^{5+1}}{5}x^5$$

$$=\frac{(-1)^2}{1}x+\frac{(-1)^3}{2}x^2+\frac{(-1)^4}{3}x^3+\frac{(-1)^5}{4}x^4+\frac{(-1)^6}{5}x^5$$

$$=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5} \text{ Ans.}$$

10. $\sum_{k=0}^{5}\frac{(-1)^k x^{2k+1}}{2k+1}$

$$=\frac{(-1)^0 x^{2.0+1}}{2.0+1}\frac{(-1)^1 x^{2.1+1}}{2.1+1}+\frac{(-1)^2 x^{2.2+1}}{2.2+1}+\frac{(-1)^3 x^{2.3+1}}{2.3+1}+\frac{(-1)^4 x^{2.4+1}}{2.4+1}$$

$$=\frac{1\,x^{0+1}}{0+1}-\frac{x^{2+1}}{2+1}+\frac{x^{4+1}}{4+1}-\frac{x^{6+1}}{6+1}+\frac{x^{8+1}}{8+1}$$

$$=\frac{x^1}{1}-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9} \text{ Ans.}$$

## 1.10 SUM OF NATURAL NUMBERS

**Illustration-22:** By using Mathematical induction show that

$1+2+3+\dots\dots\dots\dots+n=\frac{n(n+1)}{2}$, for all $n\in N$

**Proof:**

**Step-1**: Here, $1+2+3+\dots\dots\dots+n=\frac{n(n+1)}{2}$ ………………… (1)

(1) is true for $n = 1$ because L. H. S. = 1 and R.H.S. = $\frac{1(1+1)}{2}=1$

**Step-2:** Let, (1) is true for $n=m$,

i. e. $1+2+3+\dots\dots\dots+m=\frac{m(m+1)}{2}$ ………. (2)

Now (1) is true for $n=m+1$,

if $1+2+3+\dots\dots\dots+(m+1))=\frac{(m+1)(m+2)}{2}$ ……… (3) is true.

Now adding $(m+1)$ in both sides of equations (2) we get,

$$1+2+3+\dots\dots\dots+m+(m+1)=\frac{m(m+1)}{2}+(m+1)$$

$$=\frac{m(m+1)+2(m+1)}{2}$$

$$=\frac{(m+1)(m+2)}{2}$$

$\therefore$ (3) is true,

i.e. (1) is true for $n = m+1$.

Since the statement (1) is true for $n=1$

So (1) is true for $n = 1+1 = 2, n = 2+1 = 3$........ and so on.

Hence the statement (1) is true for all values of $n \in$ N.

**Illustration-23:** By using Mathematical induction show that,

$1+3+5+\cdots\cdots+(2n-1) = n^2$, where $n$ is any natural number.

**Proof:**

**Step-1:** Here, $1+3+5+\cdots\cdots+(2n-1) = n^2 \cdots\cdots(1)$

(1) is true for $n=1$, because

L.H.S = 1 and R.H.S = $1^2 = 1$

**Step-2:** Let, (1) is true for $n = m$,

i.e. $1+3+5+.....+(2m-1) = m^2 \cdots\cdots$ (2)

Now (1) is true for $n = m+1$,

if $1+3+5+\cdots\cdots+(2m+1) = (m+1)^2 \cdots\cdots$ (3) is true

Adding $(2m+1)$ in both sides of (2) we get,

$1+3+5+\cdots\cdots+(2m-1)+(2m+1) = m^2+(2m+1) = (m+1)^2$

$\therefore$ (3) is true

i.e. (1) is true for $n = m+1$

Since the statement (1) is true for $n=1$

So (1) is true for $n = 1+1 = 2, n = 2+1 = 3 \cdots\cdots$ etc.

Hence the statement (1) is true for all values of $n \in$ N.

**Illustration-24:** By using Mathematical induction show that

$1^2+2^2+3^2+\cdots\cdots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$ for all $n \in$ N.

**Proof:**

**Step-1:** Here $1^2+2^2+3^2+\cdots\cdots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$ ........ (1)

(1) is true for $n=1$ because,

L.H.S = $1^2 = 1$ and R.H.S = $\dfrac{1(1+1)(2.1+1)}{6} = \dfrac{2\times 3}{6} = 1$

**Step-2:** Let (1) is true for $n = m$,

i.e. $1^2+2^2+3^2+\cdots\cdots+m^2 = \dfrac{m(m+1)(2m+1)}{6}$ ........ (2)

Now (1) is true for $n = m+1$, if

$$1^2 + 2^2 + 3^2 + \cdots\cdots + (m+1)^2 = \frac{(m+1)(m+2)(2m+3)}{6} \quad \text{........ (3) is true.}$$

Adding $(m+1)^2$ in both sides of (2), we get,

$$1^2 + 2^2 + 3^2 + \cdots\cdots + m^2 + (m+1)^2 = \frac{m(m+1)(2m+1)}{6} + (m+1)^2$$

$$= \frac{m(m+1)(2m+1) + 6(m+1)^2}{6}$$

$$= \frac{(m+1)\{m(2m+1) + 6(m+1)\}}{6}$$

$$= \frac{(m+1)(2m^2 + m + 6m + 6)}{6}$$

$$= \frac{(m+1)(2m^2 + 7m + 6)}{6}$$

$$= \frac{(m+1)(2m^2 + 4m + 3m + 6)}{6}$$

$$= \frac{(m+1)\{2m(m+2) + 3(m+2)\}}{6}$$

$$= \frac{(m+1)(m+2)(2m+3)}{6}$$

$\therefore$ (3) is true, i.e. (1) is true for $n = m+1$

Since the statement (1) is true for $n = 1$

So (1) is true for $n = 1+1 = 2$, $n = 2+1 = 3$ ..... etc,

Hence the statement (1) is true for all values of $n \in \mathrm{N}$.

**Illustration-25:** By using Mathematical induction show that

$$1^3 + 2^3 + 3^3 + \cdots\cdots + n^3 = \frac{n^2(n+1)^2}{4} \text{ for all } n \in \mathrm{N}.$$

**Proof:**

**Step-1:** Here, $1^3 + 2^3 + 3^3 + \cdots\cdots + n^3 = \dfrac{n^2(n+1)^2}{4}$ ..... (1)

For $n = 1$, L.H.S and R.H.S of (1) are equal,

i.e. L.H.S = $1^3 = 1$ and R.H.S $= \dfrac{1^2(1+1)^2}{4}$

$= \dfrac{4}{4} = 1 \qquad \therefore$ (1) is true for $n = 1$

**Step-2:** Let (1) is true for $n = m$,

i.e. $1^3 + 2^3 + 3^3 + \cdots\cdots + m^3 = \frac{m^2(m+1)^2}{4}$ ------ (2)

Now (1) is true for $n = m+1$,

if $1^3 + 2^3 + 3^3 + \cdots\cdots (m+1)^3 = \frac{(m+1)^2(m+2)^2}{4}$ ----- (3) is true,

Adding $(m+1)^3$ both sides of (2) we get,

$$1^3 + 2^3 + 3^3 + \cdots\cdots + m^3 + (m+1)^3 = \frac{m^2(m+1)^2}{4} + (m+1)^3$$

$$= \frac{m^2(m+1)^2 + 4(m+1)^3}{4}$$

$$= \frac{(m+1)^2\{m^2 + 4(m+1)\}}{4}$$

$$= \frac{(m+1)^2(m^2 + 4m + 4)}{4}$$

$$= \frac{(m+1)^2(m+2)^2}{4}$$

$\therefore$ (3) is true, i.e. (1) is true for $n = m+1$

Since the statement (1) is true for $n = 1$,

So (1) is true for $n = 1+1 = 2, n = 2+1 = 3$ ----- etc.

Hence the statement (1) is true for all value of $n \in$ N.

## 1.11 RELATIONS AND FUNCTIONS

A relation in its ordinary meaning signifies some ties. The word relation means the association between two numbers or objects. This type of relation is called binary relation. For example, the relation of marriage between man and woman is a binary relation. Suppose R is a relation and $(x, y)$ is an element of the relation R and $(x, y) \in R$, then such relation is expressed as: x R y which is read as, "x is related to y" and $(x, y) \notin R$ is read as, "x is not related to y".

### TYPES OF RELATION

Some special types of relations are as follows:

(a) **Reflexive Relation :** A relation R in a set A is said to be reflexive if and only if each element in A is related to itself, i.e.a R a for all $a \in A$.

For example, the relation $R=\{(1,1),(2,2),(3,3)\}$ is a reflexive relation in the set $A=\{1,2,3\}$.

(b) **Symmetric Relation:** A relation R in a set A is said to be symmetric if and only if $a\ R\ b \Rightarrow b\ R\ a$ for all $a,b\in A$ i.e $(a,b)\in R\Rightarrow(b,a)\in R$. For example, "x is the neighbor of y" or, "x is 100 miles away from y" represents symmetric relation. But "x is the father of y" and "x is 100 miles south of y" are not symmetric relation.

(c) **Transitive Relation:** A relation R in a set A is said to be transitive if and only if $a\ R\ b$ and $b\ R\ c \Rightarrow a\ R\ c$ for all $a,b,c\in A$. The relation R defined by "x is less than y" is a transitive relation because $a<b$ and $b<c\Rightarrow a<c$.

(d) **Equivalence Relation :** A relation R in a set A is said to be an equivalence relation (denoted by~) if and only if

i. R is reflexive, ii. R is symmetric and, iii. R is transitive.

For example, suppose $A=\{a,b,c\}$ is any set of numbers. The relation R in A defined by "is equal to" is reflexive since every number is equal to itself. It is symmetric because $a=b\Rightarrow b=a$. Also if $a=b$ and $b=c$, then $a=c$ so that R is transitive as well. Hence R is an equivalence relation.

**Illustration-26:** Suppose $A\times B=\{(1,2),(1,4),(2,2),(2,4)\}$ and $R=\{(1,2),(2,4)(2,1)\}$. State whether R is relation from A to B or not. Give reasons.

**Solution:** $R=\{(1,2),(2,4),(2,1)\}$ is not the subset of $A\times B$ because $(2,1)\notin A\times B$. Therefore R is not a relation from A to B.

## FUNCTIONS

A function is a technical term used to symbolize relationship between variables. A function explains the nature of correspondences between variables indicated by some formula, graph or mathematical equation.

Suppose A and B are two non-empty sets. If there exists a rule 'f' which associates with each element x of A a unique (i.e., one and only one) element y in B, then the rule 'f' is called a function of x over the set A. We write

$$f:A\rightarrow B \text{ or, } A\xrightarrow{f}B.$$

If for each x in A, two or more elements in B are associated under the rule 'f' then we shall not call it a function.

The notation of function is given by the following examples:

i. $f:x\rightarrow y=2x+3$ means $f(x)=2x+3$ for all $x\in R$;

ii. $f:x\rightarrow .x^2$ means $f(x)=x^2$;

iii. $f:x\rightarrow \sin x$ means $f(x)=\sin x$.

**Another Definition of Function**

If there exists a relation between two real variables x and y such that for every value of x in a given set X, we get a definite value of y, then y is called a **function** of x and we write $y = f(x)$. Here x is called the independent variable and y is the dependent variable. The set X in which x varies is called the domain of the function. The set of all values of 'f' is called **range of** 'f'.

**Domain and range:** The domain of a function is a set of possible values of the independent variables and the range is the corresponding set of values of the dependent variable.

**Illustration-27:** (a) Find the domain of the function defined below:

| **X** | 1 | 2 | 3 | 5 | 6 |
|---|---|---|---|---|---|
| F(x) | 0 | 1 | 2 | 4 | 5 |

What is the range of the function?

(b) Consider the function $y = 2x + 3$. What is the domain of the function?

**Solution: (a)** If A be the set of value of x, then $A = \{1,2,3,5,6\}$. The set A is the domain of the function $f(x)$.

The set B of all values of $f(x)$ is $B = \{0,2,4,5\}$, which is the range of $f(x)$.

**(b)** $y = 2x + 3, \text{i.e.}, f(x) = 2x + 3$.

Here the function $f(x)$ is defined for all real values for x.

$\therefore$ The domain of $f(x)$ is the set R of all real numbers.

## TYPES OF FUNCTIONS

**(a) Explicit and Implicit Functions:** If a function is expressed directly in terms of the independent variable alone, then it is called an explicit function. Otherwise it is an implicit function.

For example,

i. $y = 3x^2 + 2$ is an explicit function for all real x, but $3x - 5y = 2$ is an implicit function.

ii. $x^2 + y^2 = 16$ is an implicit function, but $y = \sqrt{16 - x^2}$ is an explicit function.

But from an implicit function, it is not always possible to find y as an explicit function of x.

**(b) Algebraic Function**

An expression containing fixed number of terms involving a variable, say x, formed by the operations of addition, subtraction, multiplication, division, involution (powers) and evolution (roots) is called an **algebraic function** of x.

For example, $3x^2+2, \frac{x+2}{x^2-3x+4}, \sqrt{x^2-5}, \frac{x}{\sqrt{x^2+6x+5}}$ etc. are algebraic functions.

An algebraic function may be either a polynomial or rational or an irrational function.

**i. Polynomial Function:** An expression of the form

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ......a_{n-1}x + a_n$,

where $a_0, a_1, a_2........., a_n$ are constants and n is a positive integer, is called a polynomial function or a rational integral function of x degree n and it is denoted by $P(x)$. Clearly to each real number a, we get the number $P(a)$.

Examples: $x+2$, $x^2+3x+5, x^3+2$ etc. are polynomial functions of x of degree 1,2,3 respectively.

**ii. Rational Function:** A rational function is the ratio of two polynomial functions of the same variable. If $P(x)$ and $Q(x)$ be two polynomial functions of x, then $R(x)=\frac{P(x)}{Q(x)}$ is a rational function of x. Clearly this function associates to each real number a, the value $R(a)$, except those values of 'a' for which $Q(a)=0$.

Examples: $\frac{2x+3}{x^2+3x-4}, \frac{x^2+5x+6}{x^2-9x+20}$, etc rational functions of x.

**iii. Irrational Function:** Algebraic functions which are not rational, i.e. functions like $\sqrt{2x+3}, \sqrt{x^2-5x+6}$ are called irrational function.

**(c) Transcendental Functions**

Functions, which are not algebraic, are called transcendental functions. Trigonometric functions, inverse trigonometric functions, logarithmic functions, exponential functions etc. are all transcendental functions.

Examples: $\text{Sin } x, \cos x, \tan x, \sin^{-1}x, \cos^{-1} x, \tan^{-1} x$ $\log x$, $\log(1+x), e^x, a^x$ etc. are all transcendental functions.

**Trigonometric Functions**

$\sin x$, $\cos x, \tan x$ and their reciprocals $\text{cos}ec x, \sec x, \cot x$ are called trigonometric functions. In these functions the angles x are measured in radians (or circular measures), where $180^0 = \pi$ radians$=\pi^c$.

**Inverse trigonometric functions:**

$\sin^{-1}x, \cos^{-1}x, \tan^{-1}x, \sec^{-1}x, \operatorname{cosec}^{-1}x, \cot^{-1}x$, are called inverse circular functions. Here $-1 \le x \le 1$ for $\sin^{-1}x, \cos^{-1}x$ and $x \ge 1$ or, $x \le -1$ for $\sec^{-1}x$, $\operatorname{cosec}^{-1}x$.

**Exponential Function**

The function $e^x$, where $2 < e < 3 (e = 2.71828....)$, and $a^x$, where $a > 0, a \ne 1$, are called exponential functions.

**Logarithmic Function**

$\log_e x, \log_a x$ where $a > 0, a \ne 1$ are called the logarithmic functions of x. Thus $\log_2 x (x > 0), \log_e (1 + x)(x > -1)$, $\log_e (\sin x)$, etc. are examples of logarithmic functions.

**(d) Modulus Function**

For each real number x, let $|x|$ denotes the absolute value of x. then we have

$$f(x) = |x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

The function $f(x) = |x|$ is called the modulus function of x. In this case, for every value of $x \in R$, we get a unique value of $f(x)$. Hence the domain of $f(x)$ is the set R of all real numbers.

**(e) Even Function and Odd Function**

A function $f(x)$ is said to be an even function of x if $f(-x) = f(x)$.

**Example:** i. $f(x) = x^2$ is an even function of x , since $f(-x) = (-x^2) = x^2 = f(x)$,

ii. $f(x) = \cos x$ is an even function of x, since $f(-x) = \cos(-x) = \cos x = f(x)$.

A function $f(x)$ is said to be an odd function of x if $f(-x) = -f(x)$.

**Example:** i. $f(x) = x^3$ is an odd function of x, since $f(-x) = (-x)^3 = -x^3 = -f(x)$

ii. $f(x) = \sin x$ is an odd function of x, since $f(-x) = \sin(-x) = -\sin x = -f(x)$.

**Illustration-28:** Identify the type of the function $f(x) = \dfrac{x^3 - x^2 + 4x - 7}{3x + 11}$ and determine its domain.

**Solution:** We have $f(x) = \dfrac{x^3 - x^2 + 4x - 3}{3x + 11}$ ............(1)

From (1), we see that $f(x)$ is the quotient of two polynomial function $x^3 - x^2 + 4x - 7$ and $3x + 11$. Hence by definition, $f(x)$ is a rational function.

From (1) we see that $f(x)$ is undefined when $3x+11=0$, i.e., when $x=-\frac{11}{3}$ and $f(x)$ is defined at all other points. Hence the domain of $f(x)$ is the set of all real numbers except $x=-\frac{11}{3}$ Therefore, the domain of $f(x)=R-\left\{\frac{11}{3}\right\}$.

**Illustration-29:** Find the domain of each of the following functions:

$$\text{a. } f(x)=\frac{1}{x-3},\quad \text{b. } g(x)=\sqrt{x-2}.$$

**Solution:**

**a.** Since division by any real number except zero is possible, the only value of x for which $f(x)=\frac{1}{x-3}$ cannot be evaluated is $x=3$, the value that makes the denominator of $f(x)$ equal to zero. Hence, the domain of $f(x)$ consists of all real numbers except 3

i.e. $D_f=R-\{3\}$

**b.** Since negative numbers do not have square roots, the only value of x for which $g(x)=\sqrt{x-2}$ can be evaluated are those for which $x-2$ is non-negative. That is, $x-2\geq 0 \Rightarrow x\geq 2$.

Thus, the domain of $g(x)$ consists of all real numbers that greater than or equal to 2.

i.e. $D_g=\{x: x\in R, x\geq 2\}$

**Illustration-30:** If $f(x)=2x^2+3x^4$ and $g(x)=3x^3+5x$, show that $f(x)$ is an even function and $g(x)$ is an odd function of x.

**Solution:** We have $f(x)=2x^2+3x^4$.

$$\therefore f(-x)=2(-x)^2+3(-x)^4=2x^2+3x^4=f(x).$$

Hence $f(x)$ is an even function of x.

Again $g(x)=3x^3+5x$

$$\therefore g(-x)=3(-x)^3+5(-x)=-3x^3-5x=-(3x^3+5x)=-g(x)$$

Hence $g(x)$ is an odd function of x.

**Illustration-31:** Find $f\left(-\frac{1}{2}\right)$, $f(1)$ and $f(2)$ where $f(x)=\begin{cases}\frac{1}{x-1} & ;x<1\\ 3x^2+1 & ;x\geq 1\end{cases}$

**Solution:** Since $-\frac{1}{2}<1$, we use the top formula in the description of $f(x)$ to find $f\left(-\frac{1}{2}\right)$.

$$\therefore f\left(-\frac{1}{2}\right)=\frac{1}{-\frac{1}{2}-1}=\frac{1}{-\frac{3}{2}}=-\frac{2}{3}.$$

Again, since both $x=1$ and $x=2$ satisfy $x\geq 1$, we use the bottom formula in the description of $f(x)$ to obtain both $f(1)$ and $f(2)$.

$$\therefore f(1)=3(1)^2+1=4 \text{ and } f(2)=3(2)^2+1=13.$$

**Illustration-32:** If $f(x)=e^{px+q}$ where p and q are constants, show that

$$f(a)f(b)f(c)=f(a+b+c)e^{2q}.$$

**Solution:** We have $f(x)=e^{px+q}$.

$$\therefore f(a)=e^{pa+q}, f(b)=e^{pb+q} \text{ and } f(c)=e^{pc+q}$$

$$\therefore \text{LHS}= f(a)f(b)f(c)=e^{pa+pb+pc+3q}=e^{p(a+b+c)+3q}$$

Now RHS= $f(a+b+c).e^{2q}$

$$e^{p(a+b+c)+q}.e^{2q}=e^{p(a+b+c)+3q}$$

Hence LHS=RHS

**Illustration-33:** If $f(x)=2x^2-5x+4$, for what values of $x$ is $2f(x)=f(2x)$?

**Solution:** We have $f(x)=2x^2-5x+4$

$$\therefore f(2x)=2(2x)^2-5(2x)+4=8x^2-10x+4$$

From the condition $2f(x)=f(2x)$, we have

$$\Rightarrow 2(2x^2-5x+4)=8x^2-10x+4 \Rightarrow 4x^2-10x+8=8x^2-10x+4$$

$$\Rightarrow 4x^2=4 \Rightarrow x^2=1 \Rightarrow x=\pm 1.$$

## 1.12 BUSINESS APPLICATION

**Problem-01:** A frog, who falls in a well 8 meters deep tries to jump out of it. Every time the frog jumps 70 cm upwards, and falls back 20 cm. What is the net result of one jump? How many jumps would the frog need to get out of the well?

**Solution:** The frog jumps 70 cm upwards and falls back 20 cm.

$\therefore$ Net result of one jump= 70 cm- 20 cm= 50 cm upwards

No. of jumps $= 8\text{m} \div 50\text{cm} = 800\text{cm} \div 50\text{cm} = 16.$

**Problem-02:** If the value of an investment triples each year, what percent of its value today will the investment be worth in 4 years?

**Solution:** The value increases by a factor of 3 each year. Since the time is 4 years, there will be four factors of 3. So the investment will be worth $3\times3\times3\times3 = 3^4$ as much as it is today. $3^4 = 81$, So the investment will be worth 8,100% of its value today in four years.

**Problem-03.** A company has 6,435 bars of soap. If the company sells 20% of its bars of soap, how many bars of soap did it sell?

**Solution:** $20\% = 20\times\frac{1}{100} = 0.20$. Thus, the company sold $(.20)(6{,}435) = 1{,}287$ bars of soap.

**Alternative Method:** $20\% = 20\times\frac{1}{100} = \frac{1}{5}$. Then $\frac{1}{5}\times 6{,}435 = 1{,}287.$

**Problem-04:** How much interest will Tk. 10,000 earn in 9 months at an annual rate of 6%?

**Solution:** 9 months$= 9\times\frac{1}{12}$ year $= \frac{3}{4}$ year and $6\% = 6\times\frac{1}{100} = \frac{3}{50}$

$\therefore \text{Interest} = \text{Tk.}10{,}000\times\frac{3}{4}\times\frac{3}{50} = \text{Tk.}50\times 9 = \text{Tk.}450.$

**Problem-05:** In a class of 60 students, 18 students received a grade of B. what percentage of the class received a grade of B?

**Solution:** $\frac{18}{60}$ of the class received a grade of B. $\therefore \frac{18}{60} = \frac{3}{10} = .3 = 30\%$

Therefore, 30% of the class received a grade of B.

**Problem-06:** If the population of a city was 10,000 in 1980 and the population of that city increased by 15% between 1980 and 1990, what was the population of the city in 1990?

**Solution:** The population increased by 15% between 1980 and 1990.

Therefore, the increase was (.15)(10,000)=1,500.

Hence, the population in 1990 was 10,000+1,500=11,500.

**Problem-07:** What was the rate of discount if a boat which cost \$5,000 was sold for \$4,800?

**Solution:** The rate of discount$=\dfrac{5,000-4,800}{5,000}=\dfrac{200}{5,000}=\dfrac{1}{25}=.04=4\%$

**Problem-08:** If the price for a product is Tk. P per unit, the demand for the product, that is, the number of units consumers will be willing to purchase, is given by the equation $d=200,000-800p$. Find the demand d for the product if its price per unit is Tk. 25.

**Solution:** Demand function: $d=200,000-800p$, where p is per unit price of the product

For $p=25$, demand $d=200,000-800\times 25=180,000$ units.

**Problem-09:** A salesman received a base salary of Tk. 800 a month plus a commission of 10 percent of the Taka amount of his monthly sales. Use functional notation to specify the relationship between his total monthly earnings and the Taka amount of goods that he sells. Define each variable used.

**Solution:** Suppose x denotes the monthly sales in taka.

R denotes the total earnings in taka.

Therefore, according to question: $R=800+10\%x$

or, $R=800+0.10x$

**Problem-10:** A firm has determined that the total revenue R, in Taka, from the sales of q units of a product is: $R=f(q)=12q$.

a. What will be the total revenue generated by the sale of 800 units of the product?
b. How many units must be sold in order to generate Tk. 24,000 in revenue?

**Solution:** Revenue, $R=f(q)=12q$

a. For 800 units, $R=12\times 800=$ Tk.9,600

b. For revenue Tk. 24,000, we have $24,000=12q.\Rightarrow q=\dfrac{24,000}{12}=2,000$ units.

**Problem-11:** Suppose the total cost in Taka of manufacturing q units of a certain commodity is given by the function $C(q)=q^3-30q^2+500q+200$.

a. Compute the cost of manufacturing 10 units of the commodity
b. Compute the cost of manufacturing the tenth unit of the commodity.

**Solution:** Given cost function, $C(q)=q^3-30q^2+500q+200$

a. The cost of manufacturing 10 units is the value of the total cost function when q=10. That is, cost of 10 units $=C(10)=(10)^3-30(10)^2+500(10)+200=$ Tk.3,200.

b. The cost of manufacturing the tenth unit is the difference between the cost of manufacturing 10 units and the cost of manufacturing 9 units.

$\therefore$Cost of tenth unit $= C(10) - C(9)$

Now $C(10) = \text{Tk.}3{,}200$

$C(9) = 9^3 - 30\times 9^2 + 500\times 9 + 200 = \text{Tk.}2{,}999$

$\therefore$Cost of tenth unit $= C(10) - C(9) = \text{Tk.}(3{,}200 - 2{,}999) = \text{Tk.}201$.

**Problem-12:** If a firm produces and sells x units of its product, its profit is $p(x) = -x^2 + 30x - 200$ thousands Taka. Find the firm's profit if it makes and sells 20 units.

**Solution:** Profit function for x units, $p(x) = -x^2 + 30x - 200$

For 20 units, profit $p(20) = -(20)^2 + 30\times 20 - 200 = \text{Tk.}0$

**Problem-13:** The total cost C of a factory per week is a function of its weekly output Q given by the equation $C = 500 + 12Q$. The factory has a capacity limit of 600 units of output per week. Find the domain of definition and range of the cost function.

**Solution:** Cost function: $C = 500 + 12Q$

Since the factory has a capacity limit of 600 units of output per week, therefore we have domain of definition: $0 \le Q \le 600$.

When output is zero, then cost $C = 500 + 12\times 0 = 500$

Again, the factory can produce maximum of 600 units.

For 600 units, $C = 500 + 12\times 600 = 7{,}700$

$\therefore$Range of the cost function: $500 \le Q \le 7{,}700$.

**Problem-14:** Total revenue from selling a particular product depends upon the price charged per unit. Specifically, the revenue function is $R = f(p) = 1{,}500p - 50p^2$, where R equals total revenue in Taka and P equals price in Taka.

a. What type of function is Taka?
b. What is total revenue expected to equal if the price equals Tk. 10?
c. What price(s) would result in total revenue equaling zero?

**Solution:** Revenue function $R = 1{,}500p - 50p^2$

a. Here highest power of P is 2, hence this is a quadratic function.
b. If p=10, then $R = 1{,}500\times 10 - 50(10)^2 = \text{Tk.}10{,}000$.
c. If R=0, then $0 = 1{,}500p - 50p^2 \Rightarrow p(1500 - 50p) = 0$

$$\therefore \text{ either } p = 0, \text{ or } 1500 - 50p = 0 \Rightarrow p = \frac{1500}{50} = 30$$

Therefore, price $p = \text{Tk.}0, \text{Tk.}30$ [Ans.]

## BRIEF REVIEW

### Definition

**Natural Number:** The set of all natural numbers is denoted by **N** and is defined by

$$N = \{1, 2, 3, \cdots\cdots, +\infty\}.$$

**Integers:** The set of integers consists of natural numbers (or positive integers), zero and the negative integers. The set of all integers is denoted by **Z** and is defined by

$$\mathbf{Z} = \{-\infty \cdots\cdots, -2, -1, 0, 1, 2, \cdots\cdots + \infty\}$$
$$= \{o, \pm 1, \pm 2, \pm 3, \pm 4, \cdots\cdots, \pm\infty\}.$$

**Rational Number:** A number which can be expressed as a fraction of integers (assuming that the denominator is never zero) is called a rational number. For examples: $\frac{5}{2}, \frac{-9}{2}$, 2, -2, 1.5, 1.52, 1.523, 0.3333……., 1.525252…….., $\sqrt{4}$, 1.532532532….., etc.

**Irrational Number: A** number which can not be expressed as a fraction of two integers, is called an irrational number. For examples:

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt[3]{7}$, $\pi$, $e$, 1.01001000100001…….., 2.003000300003……., etc.

**Complex Number:** The set of all complex numbers is denoted by $C$ and is defined by $C = \{a + ib : a, b \in R \text{ and } i^2 = -1\}$, where $a$ is called real part of complex number and $b$ is called imaginary part of complex number and $i$ is the imaginary unit.

**For examples:** 2 + i3, 2 - i3, i3, 2, 4 + i6 etc.

**Imaginary Number:** Square root of any negative numbers is called imaginary number. In other words, any number of the form: $ib$, where $b \in R$ and $i^2 = -1$ is called an imaginary number. For Examples: $2i$, $\sqrt{-2}$, $\sqrt{-5}$, $\sqrt{-3}$ *etc.*

**Absolute value of a Number:** The absolute value of a real number "$a$" is denoted by $|a|$ and is defined by the following way (i) If "$a$" is positive or zero than $|a| = a$ (ii) If "$a$" negative, then $|a| = -a$. For example, $|5| = 5$, $|-6| = -(-6) = 6$.

**Real Number:** The set of all rational and irrational numbers is called the set of all real numbers. It is denoted by R. Therefore $R = Q \cup Q'$.

**Prime Number:** An integer greater than 1 is prime if its factors only 1 and itself. The first few primes are: $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, \ldots\ldots\ldots$

**Even Number:** An Integer or whole number, which is divisible by 2 is called an even number and generally it can be expressed in the form of $2n$.

**Odd Number:** An integer or whole number is said to be odd number if it is not divisible by 2 and it can be expressed in the form of $(2n+1)$.

**Composite Number:** An integer greater than 1 is composite if its factors not only 1 and itself. For examples: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, ………………….

**Sequence:** A list of numbers such as $1, 8, 27, 64, \ldots\ldots, x^3, \ldots\ldots\ldots\ldots; x \in \mathbf{N}$ is called a Sequence.

**Series:** The sum of all the terms of a sequence is called series.

## Quiz Questions

### Multiple Choice Questions

1. Which is the natural number? (i) -1; (ii) 0; (iii) 1; (iv) -5
2. Which is the proper fraction? (i) $\frac{9}{8}$; (ii) $\frac{3}{8}$; (iii) $\frac{11}{2}$; (iv) $\frac{10}{7}$.
3. Which one is not rational? (i) 1.5555.....; (ii) 1.5; (iii) 1.010010001....; (iv) 1/3.
4. Which is the irrational number? (i) $\sqrt{3}$; (ii) $\sqrt{4}$; (iii) 7; (iv) 3.
5. Which one is not a real number? (i) $\sqrt{2}$, (ii) 2; (iii) 0; (iv) $\sqrt{-2}$
6. Absolute value of -8 is: (i) 8; (ii) -8; (iii) 0 (iv) None of the above.
7. Which is the only even prime? (i) 5; (ii) 4; (iii) 11; (iv) 2.
8. How many prime numbers lies between 1 & 50? (i) 15; (ii)20; (iii)18; (iv)16.
9. Which is undefined form? (i) $0^0$; (ii) $2^0$; (iii) $9^{-2}$; (iv) $(-5)^4$.
10. Which is a prime number? (i) 0; (ii) 1; (iii) 2; (iv) 4.

### Which one of the following statement is true/false?

(i) Every real number is a rational number.
(ii) Every irrational number is a real number.
(iii) A real number is either rational or irrational.
(iv) Every natural number is integer.
(v) If $x$ is rational and $y$ is irrational then $xy$ is irrational.
(vi) The product of two odd integers is an even integer.
(vii) Every real number is a complex number having imaginary part is zero
(viii) The sum of two rational numbers is a rational number.
(ix) There is no rational number whose square is 2.
(x) 2 is a complex number
(xi) Division by zero is sometimes allowed
(xii) Every complex number is a natural number.
(xiii) 1 is the smallest prime number
(xiv) 123 is a prime number

## Conceptual, Analytical & Numerical Questions

1. Define natural number, integers, rational number and irrational number.
2. Show that $\sqrt{7}$ is not a rational number.
3. Suppose you have the following categories of numbers: a. Positive integers, b. Negative integers, c. Integers d. Rational numbers, e. Irrational numbers, f. Real numbers g. None of the above categories. Under which category or categories do the following numbers belong:

   $0, \quad 3, -2, \frac{1}{2}, -\frac{3}{4}, \pi, \sqrt{3}, \sqrt{1}, \ 2.71828.$
4. Write down the principle of mathematical induction.
5. Define sequence and series with example.
6. What do you mean by real number and complex number?
7. Find the n-th term of the sequence 1,6,15, 28······ and hence generalize the sequence.
8. What is the value of each of the expressions and why? $\frac{2}{0}, \frac{0}{2}, \frac{2}{2}, \frac{0}{0}$.
9. Place an appropriate inequality sign '>'or '<' between each pair of real numbers given below: a. $\frac{1}{2}, \frac{2}{3}$ b. $-4, -5$ c. $-\frac{3}{5}, -\frac{1}{2}$ d. $-\frac{5}{9}, -\frac{6}{7}$.
10. A bicycle originally cost Tk.100 and was discounted 10%. After three months it was sold and being discounted 15%. How much was the bicycle sold for?
11. Two numbers are in the ratio 5:4 and their difference is 10. What is the larger number?
12. Define prime number and composite number with examples.
13. Give the definition of odd number and even number with examples.
14. Show that $\sqrt{3}$ is an irrational number.
15. Define Absolute value of a number. Discuss the special properties of zero.
16. Simplify: (i) $\frac{9-7i}{2-3i}$ (ii) $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$ (iii) $\frac{3+2i}{5-3i}$ (iv) $\frac{5+3i}{2+5i}$ (v) $\frac{(1+i)^3}{3+i}$
17. Write the following series without summation or sigma notation:

   (a) $\sum_{r=2}^{5}(-1)^r\left(\frac{x^r}{r}\right)$ (b) $\sum_{k=1}^{5}\frac{k-1}{k}$ (c) $\sum_{k=0}^{5}(-1)^k\frac{1}{2k+1}$

   (d) $\sum_{k=0}^{5}\frac{(-1)^k x^{2k+1}}{2k+1}$ (e) $\sum_{k=1}^{5}(-1)^{k+1}(2k-1)^2$ (f) $\sum_{k=1}^{4}\frac{(-1)^{k+1}}{k}x^k$

**18.** By using mathematical induction show that

(a) $1+2+3+----+n=\frac{n(n+1)}{2}$, where $n$ is positive integer.

(b) $1+3+5+-----+(2n-1)=n^2$. where $n$ is any natural number.

(c) $1^2+2^2+3^2+------n^2=\frac{n(n+1)(2n+1)}{6}$ for al $n\in N$.

(d) $1^3+2^3+3^3+------+n^3=\frac{n^2(n+1)^2}{4}$ for all $n\in N$.

**19.** Use the method of induction prove that $2+6+10+\cdots\cdots+(4n-2)=2n^2$

**20.** Use the method of induction show that $2+4+6+\cdots\cdots+2n=n(n+1)$

**21.** If $f(x)=ax^2+bx+c$ and $f(1)=6, f(2)=11$ and $f(3)=18$, find the value of *a, b* and *c*. Hence find $f(-1)$.

**22.** If $f(x)=3x^2-6x+4$, for what values of x is $3f(x)=f(3x)$?

**23.** If $g(x)=\begin{cases}-2x+4, & x\le 1\\ x^2+1, & x>1\end{cases}$, then find the value of $g(3), g(1), g(0), g(-3)$. Consider the function $y=f(x)=x^2+2$.

a. Find the value of y when x is zero.

b. What is $f(3)$?

c. What values of $x$ give $y$ a value of 11?

d. Are there any values of $x$ that give $y$ a value of 1?

**24.** Storage cost of an item is given by the function $S=g(C)=0.05C+2.50$, where C is the cost of the item. (i) Find the cost of storing an item that costs Tk. 12.

(ii) Find the cost of an item whose storage cost is Tk.3

**25.** The profit function for a firm is $p(q)=-10q^2+36{,}000q-45{,}000$, where q equals the numbers of units sold and P equals annual profit in Taka.

a. What type of function is this?

b. What is the expected profit if 1,500 units are sold?

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