CHAPTER-10 MATHEMATICS OF FINANCE

LEARNING OBJECTIVES

After studying this chapter, the student will be able to understand:

- Concept of simple interest, compound interest, Nominal and effective rate of interest
- Meaning of interest compounded continuously
- Discounting and depreciation
- The meaning and different terminologies of annuity
- Derivation of formulas for different types of annuities
- The concept of amortization and sinking fund
- Business problem solution

10.1 INTRODUCTION

In this chapter we shall focus on the use of financial information as a part of decision Making process, and shall introduce a number of techniques applied specifically to the evaluation of such information. This leads into an examination of the principles involved in assessing the value of money over a period of time and seeing how this information can be used to evaluate alternative financial decision. However, a word of caution is necessary before westart, using such information. The financial decision area is a veritable minefield in the real world hedged as it is with tax implication. Nevertheless the principles of such financial decision making are established through the concepts of interest, present value and annuities, amortization and sinking funds.

10.2 INTEREST

Interest is the money that is paid for the use of money. The total amount of money borrowed initially is called the principal amount. It might be an amount borrowed by an individual from a bank in the form of a loan, or by a bank from an individual in the form of a savings account. The rate of interest is the amount charged for the use of the principal for a given period of time (usually on a yearly basis). Rates of interest are generally expressed as a percentage.

10.2.1 SIMPLE INTEREST

Simple interest is the interest computed on the principal for the entire period it is borrowed. If a principal of P rupees is borrowed at a simple interest rate of r% per year for a period of t years, then the simple interest is determined by:

S.I.=Principal×Rate×Time=Prt

Thus, the amount A due to be paid at the end of period of t years is:

$$\text{A} = \text{Principal} + \text{Interest} = P + \Pr t = P(1+rt) \text{ or } P = \frac{A}{1+rt}$$

If we move backward, then this formula is used to calculate the value of the money.

Remark: The simple interest is charged on yearly basis. But if the time period is given in months, weeks or days, the conversion formula is as given below:

$$k \text{ months} = \frac{k}{12} \text{ years; } n \text{ weeks} = \frac{n}{52} \text{ years (1 years = 52 weeks).}$$

Illustration-01: A man deposited Tk. 5000 in a bank that pays 5% per annum every six month. The man will withdraw Tk. 500 from his principal plus any interest accrued at each six-month period. How much total interest can he expect to receive?

Solution: In this case bank is borrowing Tk. 5000 at 5% interest and will pay off its debt in 10 equal installments of Tk. 500 each every six months. The interest to be paid for first six months period in

$$I = 5000(1/2)(0.05) = Tk.125$$

$$2^{nd} \text{ installment, } I = 4500(1/2)(0.05) = Tk.112.50$$

$$3^{rd} \text{ installment, } I = 4000(1/2)(0.05) = Tk.100.00$$

....

$$10^{th} \text{ installment, } I = 500(1/2)(0/05) = Tk.12.50$$

Total interest paid by the bank (received by the man) is

$$= 125.00 + 112.50 + 100.00 + \ldots\ldots + 25.00 + 12.50$$

$$= 12.50 + 12.50(2) + 12.50(3) + \ldots\ldots + (12.50)(10)$$

$$= 12.50(1 + 2 + 3 + \ldots\ldots + 10)$$

$$= 12.50 \times \frac{10(10+1)}{2} = \text{Tk.}687.50 \quad [\text{Sum of first n natural numbers} = n(n+1)/2]$$

Illustration-02: A person desires to buy a house. If the person borrowed Tk. 4 lakhs at 12% interest for 36 months, find the simple interest the person paid the first month and the portion of the house purchased with the first payment of Tk. 50,000.

Solution: $P = Tk.4,00,000, r = 12\%$ and $t = 1/12$. Using the formula, $I = Prt$, we get

$$I = 4,00,000 \times \frac{12}{100} \times \frac{1}{12} = Tk.4,000$$

Since first payment is Tk. 50,000, the person has purchased Tk. (50,000-4,000)=Tk.46,000 towards his house with his first payment. Tk. 46,000 is applied to the reduction of his debt and is called the reduction of his principal. Thus, in the next month he owes only Tk (4,00,000-46,000)=Tk. 3,54,000. Interest is then charged for the loan on this slightly smaller amount.

10.2.2 COMPOUND INTEREST

If the interest on a particular principal sum is added to it after each prefixed period, the whole amount earns interest for the next period, then the interest calculated in this manner is called compound interest. The period after which interest becomes due is called interest period or (conversion period). The interest due period may be yearly, half yearly, quarterly etc.
Let an initial amount of money P be invested at an interest rate of r percent per year for the period of n years. The amount of interest at the end of the first year would become p×r. therefore, the total amount at the end of the first year is given by

$$A_1 = P + \Pr = P(1+r)$$

Similarly, the total amount at the end of the second year is given by

$$A_2 = A_1 + A_1 r = A_1(1+r) = P(1+r)(1+r) = P(1+r)^2.$$

The amount A_n acquired on a principal P after n payment periods at r% interest is

$$A_n = P(1+r)^n, \text{.......................(i)}$$

In the compound interest formula, the rate of interest, r is given by

$$r = \frac{\text{Interest rate per year}}{\text{Number of compounding periods per year}}$$ of money at the end of n years.

I f the normal rate I is quoted together with the frequency of conversion period, t per year or at intervals of 1/t years, then the interest rate per period r is determined as:

$$r = \frac{i}{t} = \frac{\textit{Interes rate per year}}{\textit{Number of compounding periods per year}}$$

Thus, the compound interest formula for the amount A_n accrued on a principal P at annual interest rate I compounded t times year is:

$$A_n = P\left(1+\frac{i}{t}\right)^n \quad \text{..............................(ii)}$$

The equation (ii) indicates that the value of the investment at the end of a period is the value at the beginning of the period times the factor $1+(i/t)$

The table 1 shows the growth pattern of an initial investment P at different periods:

Table-1: growth pattern of Money

Period	Value at the beginning	Value at the end
1	P	$P\left(1+\frac{i}{t}\right)$
2	$P\left(1+\frac{i}{t}\right)$	$P\left(1+\frac{i}{t}\right)^2$
3	$P\left(1+\frac{i}{t}\right)^2$	$P\left(1+\frac{i}{t}\right)^3$
:	:	:
:	:	:
n	$P\left(1+\frac{i}{t}\right)^{n-1}$	$P\left(1+\frac{i}{t}\right)^n$

Table-2 shows the compound value of investment of Tk. 1 for different values of r and n.

Table-2: Compound Value of Tk. 1

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	1.010	1.020	1.030	1.040	1.050	1.060	1.070	1.080	1.090	1.100
2	1.020	1.040	1.061	1.082	1.102	1.124	1.145	1.166	1.188	1.210
3	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295	1.331
4	1.041	1.082	1.126	1.170	1.216	1.262	1.311	1.360	1.412	1.464
5	1.051	1.104	1.159	1.217	1.276	1.338	1.403	1.469	1.539	1.611
6	1.062	1.126	1.194	1.265	1.340	1.419	1.501	1.587	1.677	1.772
7	1.072	1.149	1.230	1.316	1.407	1.504	1.606	1.714	1.828	1.949
8	1.083	1.172	1.267	1.369	1.477	1.594	1.718	1.851	1.993	2.144
9	1.094	1.195	1.305	1.423	1.551	1.689	1.838	1.999	2.172	2.358
10	1.105	1.219	1.344	1.480	1.629	1.791	1.967	2.159	2.367	2.594
11	1.116	1.243	1.384	1.539	1.710	1.898	2.105	2.232	2.580	2.853
12	1.127	1.268	1.426	1.601	1.796	2.012	2.252	2.518	2.813	3.138
13	1.138	1.294	1.469	1.665	1.886	2.133	2.410	2.720	3.066	3.452
14	1.149	1.319	1.513	1.732	1.980	2.261	2.579	2.937	3.342	3.797
15	1.161	1.346	1.558	1.801	2.079	2.397	2.759	3.172	3.642	4.177

Special Cases: (i) $A_n = P\left(1+\frac{r}{2}\right)^{2n}$, if interest is compounded half yearly.

(ii) $A_n = P\left(1+\frac{r}{4}\right)^{4n}$, if interest is compounded quarterly.

In general, if the interest be compounded with a frequency t per year or at intervals of 1/t years, then $A_n = P\left(1+\frac{r}{t}\right)^{tn}$

Illustration-03: Find the number of year in which a sum of Tk. 1234 amount to Tk. 5678 at 5% per annum compound interest payable quarterly.

Solution: Given, $P=1234,\ A_n=5678,\ t=4,\ r/t=(5/4)\%=0.0125, n=?$

Now applying the formula, $A_n = P\left(1+\frac{r}{t}\right)^{tn}$, we get

$$5678=1234[1+0.0125]^{4n} \text{ or, } (1.0125)^{4n}=\frac{5678}{1234}$$

Taking logarithm on both sides, we have

$$4n\log(1.0125)=\log 5678-\log 1234$$

$$\text{or, } 4n\times 0.0086=3.7542-3.0913=0.6629 \text{ or, } 4n=\frac{0.6629}{0.0086}=77.08 \text{ or, } n=30.69 \text{ years.}$$

Illustration-04: If Tk. 500 were invested for 8 years at interest rate of 6% compounded quarterly, then what will be the compounded interest?

Solution: Given $P=500, n=8, t=4, r/t=(6/4)\%=0.015, A_n=?$

Now applying the formula, $A_n = P\left(1+\frac{r}{t}\right)^{tn}$, we get

$$A_n=500(1+0.015)^{32}=500(1.015)^{32}$$

Taking logarithm on both sides, we have

$$\log A_n=\log 500+32\log 1.015=2.6990+32\times 0.0065=2.907$$

Then $A_n=\text{antilog}(2.907)=807.24$

Hence, compound interest $= A_n - p = 807.24-500 = Tk.307.24$

Illustration-05: Find the compound interest on Tk. 10,000 for 1 year 6 months if the interest is payable half yearly at the rate of 8% per annum.

Solution: Given, $P = 10{,}000, n = 3/2, t = 2, r/t = (8/2)\% = 0.04, A_n = ?$

Now applying the formula, $A_n = P\left(1+\frac{r}{t}\right)^{tn}$, we get

$$\therefore \quad A_n = 10{,}000(1+0.04)^{2\times(3/2)} = 10{,}000(1.04)^3$$

The amount A is the principal amount for next one-month period. Then

$$B = A\left(1+\frac{8}{12\times100}\right) = A(1+0.006) = 10{,}000(1.04)^3(1.006)$$

Taking logarithm on both sides, we have

$$\log B = \log 10{,}000 + 3\log(1.04) + \log(1.006)$$
$$= \log 10{,}000 + 3\log(1.04) + \log(1.006)$$
$$= 4.000 + 0.0513 + 0.0029 = 4.0542$$

Then $\qquad B = anti\log(4.0542) = 11329$

Hence, compound interest $= 11329 - 10{,}000 = Tk. 1329$.

Illustration-06: (a) Find the present value of Tk. 6950 due in 3 years at the interest rate of 5% per annum.

(b) Find the time in which a sum of money will be double of itself at the interest rate of 5% per annum.

Solution: (a) Given, $P = ?,\ A = 6950,\ i = \frac{r}{100} = \frac{5}{100} = 0.05,\ n = 3$

Now applying the formula, $A_n = P(1+i)^n$, we get

$$6950 = P(1+0.05)^3$$

$$\text{or, } P = \frac{6950}{(1+0.05)^3} = \frac{6950}{(1.05)^3}$$

or, $\log P = \log 6950 - 3\log 1.05 = 3.8420 - 3\times0.0212 = 3.7784$

Then $\mathrm{P} = \text{anti}\log(3.7784) = 6004$

Hence, the present value of the sum is Tk. 6004

(b) Let $P = 100$. Then $A = 200,\ i = \frac{r}{100} = \frac{5}{100} = 0.05$

Applying the formula: $A = P(1+i)^n$ or, $(1+i)^n = \frac{A}{P}$, we get

$$(1+0.05)^n = \frac{200}{100} \text{ or, } (1.05)^n = 2$$

Taking logarithm on both sides, we have

$$n\log(1.05) = \log 2 \text{ or, } n = \frac{\log 2}{\log(1.05)} = \frac{0.3010}{0.0212} = 14.19 \text{ years.}$$

Illustration-07: Mr. Habib borrowed Tk. 25,000 from a money-lender but he could not repay any amount in a period of 5 years. Accordingly the money lender demands now Tk. 35,880 from him. At what rate percent per annum compound interest did the latter lend his money.

Solution: Given, $A = 35,880,\ P = 25,000,\ n = 5,\ i = r/100 = ?$

Applying the formula, $A = P(1+i)^n$, we get

$$35,880 = 25,000(1+i)^5$$

Taking logarithm on both sides, we get

$$\log 35,880 = \log 25,000 + 5\log(1+i)$$

or, $$\log(1+i) = \frac{\log 35,880 - \log 25,000}{5} = \frac{4.5549 - 4.3979}{5} = 0.0314$$

Then $(1+i) = anti\log(0.0314) = 1.075\ \ or,\ i = 1.075 - 1 = 0.075$

Hence, the required rate of interest is: $100 \times i = 100 \times 0.075 = 7.5\%$.

10.3 NONIMAL AND EFFECTIVE RATES OF INTEREST

The compound interest charged is based on annual rate of interest and the frequency of compounding when interest is compounded more than once a period (generally a year), the annual rate is called the nominal rate. The rate actually earned is called the effective rate of interest.

For example, suppose Tk. 1000 is invested for 5 years at 8% interest compounded annually. Then the compounded amount and compound interest are

$$A = 1000(1+0.08)^5 = 1000(1.46933) = \text{Tk.}1469.33$$

$$I = 1469.33 - 1000 = \text{Tk.}469.33$$

But if the same amount is invested for same period at 8% interest compounded quarterly, the compounded amount and interest are:

$$A = 1000(1+0.02)^{20} = 1000(1.485951) = Tk.1,485.95$$

$$I = 1485.95 - 1000 = Tk.485.95$$

The effective rate of interest is calculated by making the effective time period equal to the compounding period and then actually compound over a period of a year. The formula is:

$$\text{Effective rate of interest, } r_{eff} = \left(1+\frac{i}{t}\right)^t - 1 = (1+r)^t - 1$$

Where i = nominal rate in percentage

t = number of conversion (compounding periods per years)

r = i/t = interest rate per period.

Remarks: 1. The effective rate of interest depends on the nominal rate (i) and the conversion periods (t) rather than principal P.

2. The effective rate of interest is useful in comparing alternative investment opportunities.

Illustration-08: Calculate effective rate of interest for Tk. 1000 invested for 5 years at 8% interest compounded quarterly.

Solution: Given, $i = 0.08$ and $t = 4$ (conversion periods per year).

Thus, $$r_{eff} = \left(1+\frac{0.08}{4}\right)^4 - 1 = (1+0.02)^4 - 1 = 1.082432 - 1$$

$$= 0.082432 = 8.2432\%$$

Illustration-09: A person needs to borrow Tk. 3000 for two years. Which of the following loans should he take: (a) 4.10% simple interest or (b) 4% per annum compounded semi-annually.

Solution: Given, $i = 0.041$, $t = 4$ (conversion periods per years) and $r = i/t = 0.041/4 = 0.0102$.

Thus $$r_{eff} = (1+r)^4 - 1 = (1+0.0102)^4 - 1$$

$$= 1.04142 - 1 = 0.04142$$

This is an effective rate of interest of 4.142%. Hence, the simple interest of 4.1% per year is better alternative.

10.4 CONTINUOUS COMPOUNDING

For a fixed principal, time period and annual rate of interest, if the compound interest increases continuously with the increase in the frequency of compounding, then such growth in investment is called continuous compounding.

Let I be the nominal rate of interest and r_{eff} be the effective rate of interest for n compounding periods.

Then $r_{eff} = \left(1+\frac{i}{n}\right)^n - 1$ $or, 1+r_{eff} = \left(1+\frac{i}{n}\right)^n$

When $n \to \infty$, the above formula is expressed as:

$$1+r_{eff} = \lim_{n\to\infty}\left(1+\frac{i}{n}\right)^n = e^i \text{ or, } r_{eff} = e^i - 1$$

The compounded amount or future value of an original principal P using continuous compounding is $A = Pe^{in}$

Illustration-10: Suppose Tk. 1000 is invested for 5 years at 8% interest compounded continuously.

(a) What is the effective rate if interest?

(b) Determine the value of the original principal after 5 years.

Solution: Given, P = Tk.1000, i = 0.08 and n = 5 years.

(a) $r_{eff} = e^i - 1 = e^{0.08} - 1 = 1.0833 - 1 = 0.0833 = 8.33\%$.

(b) $A = Pe^{in} = 1000e^{0.08(5)} = 1000(1.49182) = Tk.1491.82$

10.5 EQUATION OF VALUE OF MONEY

The equation of value is obtained by equating the sum of the values on a certain comparison or specific date (due date) of one set of obligations to the sum of the values on the same date of another set of obligations. That is, equation of value is:

Value of loans at specific date=Value of payment at specific

Illustration-11: In return of a promise to pay Tk. 500 at the end of 10 years, a person agrees to pay Tk. 100 now, Tk. 200 at the end of 6 years and a final payment at the end of 12 years. If the rate of interest is 2% per annum effective, what should be the final payment be?

Solution: Let x be the final payment also the focal date be 12 years hence. The calculations for old and new obligations are shown in the table below.

Focal date: 12 years hence

Old obligation	Value of each at comparison date	New obligations	Value of each at comparison date
Tk. 500 to be returned at the end of 10 years.	$500(1+0.02)^2$	Tk. 100 now Tk. 200 at the end of 6 years Tk. x due in 12 years	$100(1.02)^{12}$ $200(1.02)^6$ x

The equation of value is:

$$100(1.02)^{12} + 200(1.02)^6 + x = 500(1.02)^2$$

$$\Rightarrow 100(1.268242) + 200(1.126162) + x = 500(1.0404)$$

$$\Rightarrow 126.8242 + 225.232 + x = 520.20$$

$$\Rightarrow x = 520.20 - 126.824 - 225.232 = Tk.168.143$$

10.6 DISCOUNTING

(a) Simple discount: It is often called bank discount where the rate of the discount d in percentage for a period of one year. Let

D = simple discount on a sum;
s = sum on which discount is taken
t = time in years.

The simple discount (D) and the present value (P) of a sum (s) is:

$$D = sdt$$

$$P = s - D = s - sdt = s(1 - dt).$$

Illustration-12: Find a simple discount and present value of Tk.2000 loan for six months at 8%.

Solution: Given, $s = Tk.2000$; $t = 6 \text{months} = 1/2 \text{ years and } d = 8\%$.

Thus $D = sdt = 2000 \times 0.08 \times (1/2) = Tk.80$

And $P = s(1 - dt) = 2000\{1 - 0.08(1/2)\} = Tk.1920.$

(b) Compound Discount: The present value or capital value of an amount, A discounted (or payable) for n periods at an annual interest rate i is determined by making certain changes in the compound interest formula. In other words, the percent value of Tk. A due in n periods is that principal which is invested now at an interest rate i per period will amount to A in n periods.

From compound interest formula, we have

$$A = P(1+i)^n \text{ or, } P = \frac{A}{(1+i)^n} = A\,(1+i)^{-n}$$

where P = present or capital value,

A = amount payable in n period time,

i = interest rate (as a proportion),

n = number of time periods.

Table 3 contains values for $(1+i)^{-n}$ used in determining present value if an amount Tk. 1 for various values of time period (n) and interest rate (i)

Table-3: present value of Tk.1

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	15%
1	.990	.980	.971	.962	.952	.943	.935	.926	.917	.909	.893	.877	.870
2	.980	.961	.943	.925	.907	.890	.873	.857	.843	.826	.797	.769	.756
3	.971	.942	.915	.889	.864	.840	.816	.794	.772	.751	.712	.675	.658
4	.961	.924	.889	.855	.823	.792	.763	.835	.708	.683	.636	.592	.572
5	.951	.906	.863	.822	.784	.747	.713	.681	.650	.621	.567	.519	.479
6	.942	.888	.838	.790	.746	.705	.666	.630	.596	.564	.507	.456	.432
7	.933	.871	.813	.760	.711	.665	.623	.583	.547	.513	.452	.400	.376
8	.923	.853	.789	.731	.677	.627	.582	.540	.502	.467	.404	.351	.327
9	.914	.837	.766	.703	.645	.592	.544	.500	.460	.424	.361	.308	.284
10	.905	.820	.744	.676	.614	.558	.508	.463	.422	.386	.322	.270	.247

(c) **Continuous discounting:** In discounting a single sum to determine the present value of an amount, A due at the end of n years at the annual rate i with continuous discounting, the following formula is used:

$$P = A\,e^{-in}.$$

Illustration-13: Determine the present value of Tk. 5000 due in 5 years invested at 8% compounded annually, What is the compound discount of this investment?

Solution: Given, A = Tk 5000; i = 8% and n = 5 years. Thus

$$P = A\,(1+i)^{-n} = 5000(1+0.08)^{-5} = 5000(1.08)^{-5}$$

$$= 5000(0.68058) = \text{Tk.}3402.90$$

Thus, compounded discount $= A - P = 5000 - 3402.90 = Tk.1597.10.$

Illustration-14: What is the present value of Tk. 2000 due after 5 years from now if the interest is compounded continuously at the interest rate of Tk. 8%

Solution: Given, $A = Tk.2000; i = 0.08\ and\ n = 5\ years.$ Then

$$P = A\,e^{-in} = 2000\,e^{-0.08(5)} = 2000\,e^{-0.40}$$
$$= 2000(0.67032) = \text{Tk.}1340.64.$$

10.7 DEPRECIATION

In the case of depreciation, the principal value goes on decreasing every year by a certain constant amount. In this way after a certain period the diminished value becomes the principal value. In the case of depreciation 'r' is replaced by –r, then the formula as derived in previous section reduced to

$$A_n = P(1-i)^n$$

Where P = Original value of the asset,

i = rate of depreciation

A_n = scrap value at the end of the time period.

This formula is also known as **Reducing Balance Depreciated Value Formula.**

Illustration-15: A Machine is depreciated in such a way that the value of the machine at the end of any years is 90% of the value at the beginning of the year. The actual cost of the machine was Tk. 10,000, but it was sold only for Tk. 300 due to some defects. Calculate the number of year during which the machine was in use.

Solution: Given, $P = 10{,}000,\ A_n = 300, i = \frac{r}{100} = \frac{10}{100}, n = ?$

Using the formula, $A_n = P(1-i)^n$, we get

$$300 = 10{,}000\left(1-\frac{10}{100}\right)^n = 10{,}000\left(\frac{9}{10}\right)^n$$

$$\text{or, } \left(\frac{9}{10}\right)^n = \frac{300}{10{,}000} = \frac{3}{100}$$

Taking logarithm on both sides, we get

$$n\log\left(\frac{9}{10}\right) = \log\left(\frac{3}{100}\right)$$

or, $n[\log 9 - \log 10] = \log 3 - \log 100$

or, $n[0.9542 - 1.0000] = 0.4771 - 2.0000$

or, $n(-0.458) = -1.5229,\ or\ n = 1.5229/0.458 = 33\ years$ (approx).

Illustration-16: The life of a machine is estimated 15 years. The original cost of the machine is Tk. 12,000 and the depreciation on the reducing installment system being charged at 10% per annum. Find out the scrap value of the machine after the end of its life.

Solution: Given, $P = 12{,}000, i = \frac{r}{100} = \frac{10}{100} = \frac{1}{10}, n = 15, A_n = ?$

Using formula, $A_n = (1-i)^n$ we get

$$A_n = 12{,}000\left(\frac{9}{10}\right)^{15}$$

or, $\log A = \log 12{,}000 + 15[\log 9 - \log 10] = 4.0792 + 15[0.9542 - 1]$

$= 4.0792 - 0.6870 = 3.3922$

Then $A = \text{antilog}\ (3.3922) = 2467$

Hence, the scrap value of the machine is Tk. 2470.

10.8 ANNUITY

Many transactions in every day life involve making a series of equal payments over a period of time, such as mortgage, rent, etc. In general, *a sequence of fixed annual payments (or receipts) made at uniform (or equal) time intervals is called an annuity*. The time between payments is called the period, and the time from the beginning of the first period to the end of the last period is called the term of the annuity. Annuity may be classified into two categories:

Annuity Certain: In case the first and last dates of an annuity are fixed, the annuity is called an annuity certain, for example, installment payments. That is, payment period is fixed for a certain number of years. Annuity certain may be further divided into two categories:

(a) **Annuity due:** The annuity in which the payment is made at the beginning of each period, i.e.; all payments are to be made at the beginning of successive intervals, for example, rent or leases, is called an annuity due.

(b) **Annuity ordinary (or immediate):** The annuity in which the payment is made at the end of each period, i.e. all the payments are to be made at the end of successive interval, for example, mortgages or loans, is called simple (or immediate) annuity.

Contingent Annuity: In case the term of payment depends on some uncertain event, the annuity is called contingent annuity, for example insurance premium which is terminated with the death of the insured person.

Deferred annuity: If the payments are deferred (or delayed) for a certain number of years, then it is called deferred annuity. When it is deferred for n years, it is said to commence after n years, and the first installment is made at the end of (n+1) years.

Remarks: 1. If we consider an ordinary annuity, the accumulated amount, denoted by

$$S_{n,i} = 1 + (1+i) + (1+i)^2 + ... + (1+i)^{n-1}$$

$$= \frac{(1+i)^n - 1}{i} \quad \text{[Sum of finite G.P.; with } a = 1, r = (1+i)\text{]}$$

2. If instead of Tk. 1, we use payment of Tk. P at each payment period for an annuity at i percent interest per payment period, the accumulated amount or sum of annuity after n payment periods is:

$$A = Tk.P\, S_{n,i}\,.$$

10.9 DERIVATION OF DIFFERENT TYPES OF ANNUITY

a. PRESENT VALUE OF AN ANNUITY

The present value of an annuity is the current value of the total amount of annuity at the end of the given period. In other words, present value of a given sum if money due at the end of a certain period of time is the sum of principal amount plus interest accumulated at the given rate for the same period.

1. Present Value of Immediate Annuity (or Ordinary Annuity):-

Let a denote the annual payment of an ordinary annuity, n is the number of years and i percent is the interest on one taka per year and P be the present value of the annuity. In the case of immediate annuity, payments are made periodically at the end of specified period. Since the first installment is paid at the end of first year, its present value is $\frac{a}{1+i}$, the present value of second installment is $\frac{a}{(1+i)^2}$ and so on. If the present value of last installment is $\frac{a}{(1+i)^n}$ then we have

$$P = \frac{a}{(1+i)} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \cdots + \frac{a}{(1+i)^n}$$

$$= a[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots + \frac{1}{(1+i)^n}]$$

$$= a.\frac{1}{1+i}\left[\frac{1-\frac{1}{(1+i)^n}}{1-\frac{1}{1+i}}\right] (\because \text{here first term } = \frac{1}{1+r}, \text{common ratio } = \frac{1}{1+r} < 1)$$

$(\because$ For G.P. Series $S_n = a.\frac{1-r^n}{1-r}$; $r < 1$, where a is the first term and r is the common ratio)

$$= a.\frac{1}{1+i}.\frac{1+i}{1+i-1}\left[1-\frac{1}{(1+i)^n}\right]$$

$$\therefore P \;\; or \;\; PV = \frac{a}{i}\left[1-\frac{1}{(1-i)^n}\right]$$

For convenience, values of $a_{n,r}$ for various values of n and r are given in table-4.

Table-4: Present value of an annuity of Re-1: $a_{n,r} = \{1-(1+r)^n\}/r$

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	.99010	.98039	.97007	.96154	.96238	.94340	.92558	.92593	.91743	.90909
1	.9901	.9804	.9709	.9615	.9524	.9434	.9346	.99259	.9174	.9091
2	1.704	1.9416	1.9135	1.8831	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355
3	2.9470	2.8839	2.8286	2.7751	2.7233	2.6730	2.6243	2.5771	2.5313	2.4868
4	3.9020	3.8077	3.7171	3.6299	3.5459	3.4657	3.3872	3.3121	3.2397	3.1699
5	4.8535	4.7134	4.5897	4.4518	4.3295	4.2123	4.1002	3.9927	3.8866	3.7908
6	5.7955	5.6014	5.4172	5.2421	5.0757	5.9173	4.7665	4.6229	4.4859	4.3553
7	6.7282	2.4720	6.2302	6.0020	5.7863	5.5824	5.3893	5.2064	5.0329	4.8684
8	7.6517	7.3254	7.0196	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349
9	8.5661	8.1522	7.7861	7.4352	7.1078	6.8017	6.5152	6.2469	5.9852	5.7590
10	9.4714	8.9825	8.7302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4176	6.1446
11	10.3677	9.7868	9.2526	8.7604	8.3064	7.7768	7.4987	7.1389	6.8052	6.4951
12	11.2552	10.5753	9.9539	9.3850	8.8632	8.3838	7.9427	7.5361	7.1607	6.8137
13	12.1338	11.3483	10.6349	9.9856	9.3935	8.8527	8.3576	7.9038	7.4869	7.1074
14	13.0038	12.1062	11.2960	10.5631	9.8985	9.2950	8.7454	8.2442	7.7861	7.3667
15	13.8651	12.8492	11.9379	11.1183	10.3796	9.7122	9.1079	8.5595	8.0607	7.6061

2. Present Value of Annuity Due:-

Since the first installment is paid at the beginning of the first period (year), its present value will be the same as a where a is the annual payment of annuity due. The second installment is paid at the beginning of the second year, hence its present value is given as $\frac{a}{1+i}$ and so on, the last installment is paid in the beginning of n th year period, hence its present value is given as $\frac{a}{(1+i)^{n-1}}$.

Thus if P denotes the present value of annuity due then

$$P = a + \frac{a}{1+i} + \frac{a}{(1+i)^2} + \cdots + \frac{a}{(1+i)^{n-1}}$$

$$= a\left[1 + \frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^{n-1}}\right]$$

$$= a\frac{\left\{1 - \left[\frac{1}{(1+i)}\right]^n\right\}}{1 - \frac{1}{(1+i)}} \text{(G.P. with common ratio} > 1)$$

$$= a\frac{\left\{1 - \frac{1}{(1+i)^n}\right\}}{\frac{(1+i-1)}{(1+i)}} \qquad = a(1+i)\frac{\left[1 - \frac{1}{(1+i)^n}\right]}{i}$$

$$PV = (a + \frac{a}{i})\left[1 - \frac{1}{(1+i)^n}\right]$$

b. FUTURE VALUE OF AN ANNUITY

The future value of an annuity is the sum of all payments made and interest earned on them at the end of the term of annuities.

3. Future Value or Amount of Immediate Annuity (or Ordinary Annuity):-

Let a be the ordinary and i percent be the rate of interest per period. In ordinary annuity, the first installment is paid after the end of first period. Therefore it earns interest for $(n-1)$ periods, second installment earns interest for $(n-2)$ periods and so on. The last installment earns for $(n-n)$ period, i.e,. earns no interest. The amount of first annuity for $(n-1)$ period at i percent rate per period $= a(1+i)^{n-1}$, second annuity $= a(1+i)^{n-2}$, third annuity $= a(1+i)^{n-3}$ and so on.

Thus the total annuity A for n period at i per cent rate of interest is:

$$A = (1+i)^{n-1} + a(1+i)^{n-2} + \cdots + a(1+i) + a$$

$$= a + a(1+i) + \cdots + a(1+i)^{n-2} + a(1+i)^{n-1}$$

$$= a\left[1 + (1+i) + \cdots + (1+i)^{n-2} + (1+i)^{n-1}\right]$$

$$= a.\frac{(1+i)^n - 1}{(1+i)-1} \quad \text{(Finite G.P with common ration } 1+i>1 \text{ and } 1^{st} \text{ term} = 1)$$

$$\therefore A \text{ or } FV = \frac{a}{i}\left[(1+i)^n - 1\right]$$

4. Future Value or amount of Annuity Due:-

As defined earlier, annuity due is an annuity in which the payments are made at the beginning of each period. The first installment will earn interest for n periods at the rate of i percent per period. Similarly second installment will earn interest for $(n-1)$ periods, and so on, the last installment will earn interest for one period.

Hence the amount of annuity due

$$A = a(1+i)^n + a(1+i)^{n-1} + a(1+i)^{n-2} + \cdots + a(1+i)^1$$

$$= a(1+i)\left[(1+i)^{n-1} + (1+i)^{n-2} + \cdots + 1\right]$$

$$= (1+i)a\left[1 + (1+i) + (1+i)^2 + \cdots + (1+i)^{n-1}\right]$$

$$= (1+i)a\left[\frac{(1+i)^n - 1}{1+i-1}\right]$$

$$A = (1+i)a\left[\frac{(1+i)^n - 1}{i}\right]$$

$$A = (a + \frac{a}{i})\ [(1+i)^n - 1]$$

5. Perpetual Annuity:-

Perpetual annuity is an annuity whose payment continues forever. As such the amount of perpetuity is undefined as the amount increases without any limit as time passes on. We know that the present value P of immediate annuity is given by

$$P = \frac{a}{i}\left[1 - \frac{1}{(1+i)^n}\right]$$

Now as per the definition of perpetual annuity as $n \to \infty$, we know that $\frac{1}{(1+i)^n} \to 0$ since $1+i>1$, Hence, $P = \frac{a}{i}[1-0]$ $\therefore P$ or $PV = \frac{a}{i}$.

6. Deferred Annuity:-

Amount of deferred annuity for n periods, differed m periods, is the value of the annuity at the end of its term and is given as

$$A = \frac{a}{i}\left[\frac{(1+i)^n - 1}{(1+i)^m}\right]$$

The present value of deferred annuity of n periods, deferred m periods, at the rate of i per year is given as

$$P = \frac{a}{i}\left[\frac{(1+i)^n - 1}{(1+i)^{m+n}}\right]$$

The derivation of the above formulae is consider as an exercise for the students.

Note: In all the above formulae the period is of one year. Now if the payment is made more than once in a year then i is replaced by $\frac{i}{k}$ and n is replace by nk where k is the number of payments in a year.

Illustration-17: Equipment is purchased on an installment basis, such that Tk. 5,000 is to be paid on the signing of the contract and four yearly installments of Tk. 3,000 each payable at the end of the first, second and fourth year. If interest is charged at 5% per annum, what would be the cash down price?

[Given: $\log 105 = 2.0212; \log 82.26 = 1.9152$]

Solution: To find the present value of four installment, we use the formula

$$P = \frac{A}{r}\left[1 - \frac{1}{(1+r)^n}\right],$$

Given, $A = 3000$, $r = 0.05$ *and* $n = 4$. Then

$$P = \frac{3000}{0.05}\left[1 - \frac{1}{(1+0.05)^4}\right] = \frac{3000}{0.05}\left[1-(1.05)^{-4}\right] \quad \ldots\ldots(i)$$

Let $\quad x = (1.05)^{-4}$

$$\log x = -4 \ \log 1.05 = -4 \times 0.0212 = -0.0848 = \bar{1} + 1 - 0.0848 = \bar{1}.9152$$

$\therefore \quad x = anti\log(\bar{1}.9152) = 0.8226$

Putting the value of $(1.05)^{-4}$ in (i), we get

$$P = \frac{3000}{0.05}[1 - 0.8226] = \frac{3000}{0.05} \times .1774 = 10{,}644$$

Hence the cash down price would be Tk. (5000+10,644) = Tk.15,644.

Illustration-18: A machine costs Tk. 98,000 and its effective life is estimated to 12 years. If the scrap value is Tk. 3,000 only, what should be retained out of profits at the end of each year to accumulate at compound interest at 5% per annum so that a new machine can be purchased at the same price after 12 years. [Given $\log 1.05 = 0.0212$, *and* $\log 1.797 = 0.2544$]

Solution: FV=Tk. 98,000 - Tk. 3,000=Tk. 95,000

To find the amount of money available after 12 years, we use the formula

$$FV = \frac{A}{r}[(1+r)]^n - 1\,]$$

Given, r = 0.05; n = 12, M (cost of the machine) = Tk.98,000, A = ?

$$\text{Therefore } 98{,}000 = \frac{A}{0.05}\left[(1+0.05)^{12} - 1\right] \quad \ldots(i)$$

Let $x = (1.05)^{12}$: Then

$\log x = 12 \log 1.05 = 12 \times 0.0212 = 0.2544$

$\therefore \quad x = anti \log (0.2544) = 1.797$

Putting the value of $(1.05)^{12}$ in (i), we have

$$98{,}000 = \frac{A}{0.05}[1.797 - 1] \quad or, \; A = \frac{98{,}000 \times 0.05}{0.797} = Tk.5{,}968$$

Illustration-19: If money is worth 6% compounded once in two months, find the present value and the amount of an annuity whose annual rent is Tk. 1,800 which is payable once in two months for 5 years. [Given: $\log 101 = 2.0043$, and $\log 13.458 = 1{,}129$]

Solution: To find the present value of an annuity, we use the formula

$$P = \frac{A}{r}\left[1 - \frac{1}{(1+r)^n}\right] = \frac{A}{r}\left[\frac{(1+r)^n - 1}{(1+r)^n}\right]$$

Here, $n = 5 \times 6 = 30$, $r = 0.06/6 = 0.01$ and

A (amount due after every two months) $= 1{,}800/6 = 300$.

$$\therefore \quad P = \frac{300}{0.01}\left[\frac{(1+0.01)^{30} - 1}{(1+0.01)^{30}}\right] = 300 \times 100\left[\frac{(1.01)^{30} - 1}{(1.01)^{30}}\right] \quad \ldots.(i)$$

Let $x = (1.01)^{30.}$ Then

$\log x = 30 \log 1.01 = 30 \times 2.0043 = 0.129$

$\therefore \quad x = anti \log (10.129) = 1.3458$

$$\text{Thus,} \quad P = 30{,}000\left[\frac{1.3458 - 1}{1.3458}\right] = 30{,}000\left[\frac{0.3458}{1.3458}\right] = 30{,}000 \times 0.2569 = Tk.7.707$$

Now the amount of such annuity is calculated by using the formula

$$M = \frac{A}{r}\left[(1+r)^n - 1\right] = \frac{300}{0.01}\left[(1.01)^{30} - 1\right] = 300 \times 100[1.3458 - 1] = 30{,}000 \times .3458 = Tk.10{,}378.$$

Illustration-20: A man borrows Tk. 6,000 at 6% and promises to pay both principal amount and the interest in 20 annual installments at the end of each year. What is the annual payment necessary?

Solution: Given, $P = 6{,}000$, $r = 0.06\%$ $and\, n = 20$

Applying the formula: $P = \frac{A}{r}\left[1 - \frac{1}{(1+r)^n}\right]$, we get

$$6{,}000 = \frac{A}{0.06}\left[1 - \frac{1}{(1+0.06)^{20}}\right] = \frac{A}{0.06}\left[1-(1.06)^{-20}\right] \quad \ldots(i)$$

Let $x = (1.06)^{-20}$

Or, $\log x = -20\log 1.06 = -20 \times 0.0253 = -0.5060 = -1+1-0.5060 = \bar{1}.4940$

Then $x = anti\log(\bar{1}.4940) = 0.3119$

Putting the value of (1.06) in (i), we have

$$6{,}000 = \frac{A}{0.06}(1-0.3119) = \frac{A}{0.06}(0.6881)$$

or, $A = \frac{6{,}000 \times 0.06}{0.6881} = Tk.523.179$

Illustration-21: A man retires at the age of 60 years and his employer gives him pension of Tk. 1,200 for the rest of his life. Reckoning his expectation of life to be 13 years and that interest is at 4% per annum, what single sum is equivalent to this person? [Given: $\log 104 = 2.0170$ *and* $\log 6012 = 3.7790$]

Solution: The present value P of an annuity of Tk. A paid periodically at the end of each of n Periods is given by

$$P = \frac{A}{r}\left[1 - \frac{1}{(1+r)^n}\right]$$

Here, $A = 1{,}200,\ r = 4\% = 0.04,\ and\ n = 13$. Thus,

$$P = \frac{1200}{0.04}\left[1-(1+0.04)^{-13}\right]$$

Let $x = (1.04)^{-13}$. Then

$$\log x = -13\log(1.04) = -13 \times (0.017) = -0.221$$

or, $x = anti\log(-0.221) = 0.6012$

Substituting this value of x in (i), we get

$$P = \frac{1200}{0.04} \times [1-0.6012] = 300 \times 100(0.3968) = Rs.11{,}964.$$

Hence, the single sum equivalent to his Persian is Tk. 11,964.

Illustration-22: A company intends to create a depreciation fund to replace at the end of the 20th years assets closing Tk. 5,00,000. Calculate the amount to be retained out of profits every year if the interest rate is 5%.

Solution: Given, $M = 5,00,000, n = 20, r = 5\% = 0.05, A = ?$

Applying the formula, $M = \frac{A}{r}\{(1+r)^n - 1\}$, we get

$$5,00,000 = \frac{A}{0.05}\{(1+0.05)^{20} - 1\} \qquad \text{.... (i)}$$

Let $\quad x = (1.05)^{20}$ *or* $\log x = 20\log 1.05 = 20 \times 0.0212 = 0.4240$

$\therefore \quad x = anti\log(0.4240) = 2.655$

Substituting this value of x in (i), we have

$$5,00,000 = \frac{A}{0.05}\{2.655 - 1\} = \frac{A}{0.05}(1.655)$$

or $\quad A = \frac{5,00,000 \times 0.05}{1.655} = Tk.15,105$ (approx).

Illustration-23: A machine costs a company Tk. 52,000 and its effective life is estimated to be 25 years. A sinking fund is created for replacing the machine by a new model at the end of its lifetime, when its scrap will realize a sum of Tk. 2,500 only. The price of the new model is estimated to be 25% higher than the price of the present one. Find what amount should be set aside every year out of the profits for the sinking fund, if it accumulates at 3.5% per annum.

Solution: The amount of the annuity, which will continue for 25 years, is Tk. (52,00-2,500)=Tk. 49,500.

Applying the formula, $M = \frac{A}{r}\{(1+r)^n - 1\}$, we get

$$49,500 = \frac{A}{0.035}\{(1+0.35)^{25} - 1\} \qquad \text{....(i)}$$

Let $\quad x = (1.035)^{25}$ *or* $\log x = 25\log(1.035) = 25 \times 0.0149 = 0.3725$

$\therefore \quad x = anti\log(0.3725) = 2.358$

Substituting this value of x in (i), we get

$$49,500 = \frac{A}{0.035}\{2.358 - 1\} = \frac{A}{0.035}(1.358)$$

$\therefore \quad A = \frac{49,500 \times 0.035}{1.358} = Tk.1276$ (approx)

10.10 AMORTIZATION AND SINKING FUNDS

One of the most important applications of annuities is the repayment of interest bearing debts. These debts can be paid by making periodic deposits into a sinking fund, which is used at a future date to pay the principal of the debt, or by making periodic payments that cover the outstanding interest and part of the principal. This second method is known as amortization.

Definition: *A loan with a fixed rate of interest is said to be amortized if both principal and interest are paid by a sequence of equal payments made over equal periods of time.*

Purchasing a car of other items by making a series of periodic payments is an example of a loan that is amortized, but probably the most familiar example of amortization is monthly payments made for 15 or 20 years on a loan used to buy a house. An understanding of how loans are amortized and the costs involved can save money by enabling you to make an intelligent selection of a lender and a repayment plan.

Finding the payment: When a loan of Tk. R is amortized at a particular rate r of interest per payment period over n payment periods, the question is what is the payment P? To find the amount of payment P which, after n payment periods at r percent interest per payment period, gives us a present value of an annuity equal to the amount of the loan. this present value is given by

Finding the payment: When a loan of Tk. R is amortized at a particular rate r of interest per payment period over n payment periods, the question is what is the payment P? To find the amount of payment P which, after n payment periods at r percent interest per payment period, gives us a present value of an annuity equal to the amount of the loan. this present value is given by

$$R = P\left[\frac{1(1+r)^{-n}}{r}\right] = P.a_{n,r} \text{ or } P = R\left(\frac{1}{a_{n,r}}\right)$$

For amortization problems involving long-term mortgages with monthly payment, it is convenient to use the table for mortgage Problems in which appropriate values of $a_{n,r}$ and $\frac{1}{a_{n,r}}$ are listed.

Table-5: Value of $a_{n,r}$ and $1/a_{n,r}$

Period	$a_{n,r}$			$1/a_{n,r}$		
n	2/3%	3/4%	5/6%	2/3%	3/4%	5/6%
60	49.318433	48.173374	47.065369	0.020276	0.020758	0.021247
120	82.421481	78.941693	75.671163	0.012133	0.012668	0.013215
180	104.640592	98.593409	93.057439	0.009557	0.010143	0.010746
240	119.554292	111.144954	103.624619	0.008364	0.008997	0.009650
300	129.564523	119.61622	110.047230	0.007718	0.008392	0.009087

Illustration-24: What monthly payment is necessary to pay off a loan of Tk. 800 at 18% per annum in two years? In three years?

Solution: For the two-year loan, $R = Tk.800,\ n = 24,\ r = 0.015$. The monthly payment P is

$$P = Tk.800\left(\frac{1}{a_{24,0.015}}\right) = Tk800(10.049924) = Tk.39.94$$

For the three-year loan $R = Tk.800,\ n = 36,\ r = 0.015$. The monthly payment P is:

$$P = Tk.800\left(\frac{1}{a_{36,0.015}}\right) = Tk.800(0.036152) = Tk.28.92$$

For the two-years loan, the total amount paid out is Tk. (39.94) (24)=Tk. 958.56; for the three-years loan, the total amount paid out is Tk.(28.92) (36)=Tk. 1041.12. It should be clear that the longer the term of a debt, the more it costs the borrower to pay off the loan.

Illustration-25: A person has just purchased a Tk. 70,000 house and have made a down payment of Tk. 15,000. They can amortize the balance (Tk. 55,000) at interest payment? After 20 years, what equity do they have in their house?

Solution: The monthly payment P needed to pay off the loan of Tk. 55,000 at 9% for 5 years is

$$P = Tk.55{,}000\left(\frac{1}{a_{300,0.0075}}\right) = Tk.55{,}000(0.008392) = Tk.461.56$$

The total paid out for the loan is Tk. (461.56) (300) = Tk. 138,468.00. Thus, the interest on this amount is Tk. (138,468-55,000) = Tk. 83,468.00.

After 20 years, (240 months), the present value of the loan is

$Tk.461.56)\times a_{60,0.0075} = Tk.(461.56)(48.173374) = Tk.22{,}234.90.$

Thus, the equity after 20 years is $Tk.55{,}000 - Tk.2{,}234.90 = Tk.32{,}765.10$.

Sinking Fund

Quite often, a person with a debt decides to accumulate sufficient funds to pay off his or her debt by agreeing to set aside enough money each month (or quarter or year) so that, when the debt becomes payable, the money set aside each month plus the interest earned equals the debt. This type of fund created by such a plan is called a sinking fund. Sinking funds are used to redeem bond issues, payoff debts, replace outdated equipment, or provide money for purchasing new equipment. In general, sinking funds pay only the principal (not the interest) of a debt. In case the principal and the interest are paid off by partial payments, the process is called amortization as discussed earlier. We shall limit our discussion of sinking funds to those in which equal payments are made at equal time intervals. Usually, the debtor agrees to pay interest on his debt as a separate item so that the amount necessary in a sinking fund need only equal the amount he originally borrows. If 'a' is the periodic deposits or payments, at the rate or i per year then after n years the sinking fund A is $A = \frac{a}{i}\left[(1+i)^n - 1\right]$.

Illustration-26: A man borrows Tk. 3000 and agrees to pay interest quarterly at an annual rate 8%. At the same time, he set up a sinking fund in order to repay the loan at the end of 5 years. If the sinking fund earns interest au the rate of 6% compounded semi-annually, find the size of each semi-annual sinking fund deposit.

Solution: The quarterly interest payments due on the debt are Tk. $3{,}000(0.02) = Tk.60$

The size of the sinking deposit is calculated by using the formula $A = Ps_{n.r}$ in which A represent the amount to be saved. The payment P is: $3{,}000\frac{1}{S_{n.r}}$, where $n = 10$ and $r = 0.03$. Thus

$$P = 3000\frac{1}{S_{10,0.03}} = 3{,}000(0.087231) = Tk.261.69$$

That is, a semi-annual sinking fund payment of Tk. 261.69 is needed.

10.11 BUSINESS APPLICATIONS

Problem-01: A man borrows Tk. 750 from a money-lender and the bill is renewed after every half year at an increase of 21%. What time will elapse before it reaches Tk. 7500?

[Use $\log_{10} 121 = 2.0828$]

Solution: Given $P = 750, F = 7500, i = 0.21, \log_4 64 = 3$

Requirement: $n = ?$

$\therefore F = P(1+i)^{n\times 2}$

$\Rightarrow 7500 = 750(1+0.21)^{2n}$

$\Rightarrow 7500 = 750(1+0.21)^{2n}$

$\Rightarrow \dfrac{7500}{750} = (1.21)^{2n}$

$\Rightarrow 10 = \left(\dfrac{121}{100}\right)^{2n} \Rightarrow \log 10 = 2n \log\left(\dfrac{121}{100}\right)$

$\Rightarrow 1 = 2n(\log 121 - \log 100)$

$\Rightarrow 1 = 2n(2.0828 - 2)$ [Using 'log' tables]

$\Rightarrow 1 = 2n(0.0828) \Rightarrow 2n(0.0828) = 1.$

$\Rightarrow 2n = \dfrac{1}{0.0828} \Rightarrow 2n = 12.077$

$\Rightarrow n = \dfrac{12.077}{2} n \approx 6\,years$ [Ans.]

Problem-02: A man left Tk. 18000 with the direction that it should be divided in such a way that his 3 sons aged 9, 12 and 15 years should each receive the amount when they reached the age of 25. If the rate of interest is $3^1/_2\,\%$ p.a., what should each son receive when he is 25 years old?

Solution: Given $i = 3^1/_2\,\% = 0.035$

Let the son aged 9 years would receive F_1

" " " 12 " " " F_2

" " " 15 " " " F_3

From the condition, $P_1 + P_2 + P_3 = 18000$ and $F_1 = F_2 = F_3$.

P_1 is invested for (25-9) = 16 years

$\therefore F_1 = P_1(1+i)^n$

$\Rightarrow F_1 = P_1(1+0.035)^{16} \Rightarrow P_1 = \dfrac{F_1}{(1.035)^{16}}$ ....(1)

P_2 is invested for (25-12)=13 years

$\therefore F_2 = P_2(1+i)^n$

$\Rightarrow F_2 = P_2(1+0.035)^{13} \Rightarrow P_2 = \dfrac{F_2}{(1.035)^{13}}$ ....(2)

P_3 is invested for (25-15)=10 years

$\therefore F_3 = P_3(1+i)^n$

$\Rightarrow F_3 = P_3(1+0.035)^{10} \Rightarrow P_3 = \dfrac{F_3}{(1.035)^{10}}$ ....(3)

Adding (1),(2) and (3), we get

$$P_1 + P_2 + P_3 = \frac{F_1}{(1.035)^{16}} + \frac{F_2}{(1.035)^{13}} + \frac{F_3}{(1.035)^{10}}$$

$$\Rightarrow 18000 = \frac{F_1}{(1.035)^{16}} + \frac{F_2}{(1.035)^{13}} + \frac{F_3}{(1.035)^{10}}$$

$$\Rightarrow 18000 = \frac{F_1}{(1.035)^{16}} + \frac{F_1}{(1.035)^{13}} + \frac{F_1}{(1.035)^{10}}$$

$[\therefore F_1 = F_2 = F_3]$

$$\Rightarrow 18000 = F_1\left[\frac{1}{(1.035)^{16}} + \frac{1}{(1.035)^{13}} + \frac{1}{(1.035)^{10}}\right]$$

$$\Rightarrow 18000 = F_1\left[\frac{1}{1.7314} + \frac{1}{1.5621} + \frac{1}{1.4106}\right]$$

$$\Rightarrow 18000 = F_1[.5776 + .6402 + .7089]$$

$$\Rightarrow 18000 = F_1[1.9267]$$

$\Rightarrow F_1 \dfrac{18000}{1.9267} \Rightarrow F_1 = 9342$ [Ans.]

Let $x = (1.035)^{16}$

$\Rightarrow \log x = 16 \log 1.035$

$\Rightarrow \log x = 16 \times 0.0149$

$\Rightarrow x = anti \log 0.2384$

$\Rightarrow x = 1.7314$

Let $y = (1.035)^{13}$

$\Rightarrow \log y = 13 \log 1.035$

$\Rightarrow y = anti \log 0.1937$

$\Rightarrow y = 1.5621$

Let $z = (1.035)^{10}$

$\Rightarrow \log z = 10 \log 1.035$

$\Rightarrow x = anti \log 0.1494$

$\Rightarrow x = 1.4106$

Problem-03: A owes B Tk. 1600 but it is due for payment till the end of 3 years from this date. How much should A pay B if he is willing to accept now in order to clear off the debt: (a) taking money to be worth 5% per annum simple interest (b) taking it to be worth 5% per annum compound interest, payable yearly?

Solution: $F = 1600, n = 3, i = 5\% = 0.05$

Requirement: P = ?

(a) For simple interest,

$$P = \frac{F}{1+ni} = \frac{1600}{1+3\times0.05} = \frac{1600}{1.15} = Tk.1391.30 \quad \text{[Ans.]}$$

(b) For compound interest,

$$P = \frac{F}{(1+i)^n}$$

$$= \frac{1600}{(1+0.05)^3}$$

$$= \frac{1600}{(1.05)^3}$$

$$= \frac{1600}{1.1577}$$

$$= Tk.1382 \quad \text{[Ans.]}$$

Let $x = (1.05)^3$

$\Rightarrow \log x = 3\log 1.05$

$\Rightarrow \log x = 3\times0.0212$

$\Rightarrow \log x = 0.0636$

$\Rightarrow x = anti\log 0.0636$

$\Rightarrow x = 1.1577$

Problem-04: A machine in a factory is valued at Tk. 49074 and it is decided to reduce the estimated value at the end of each year by 15 percent of the value at the beginning of that year. When will the value be (a) Tk.20000,(b)1/10th of the original value?

Solution: **(a)** Given A=20000, P=49074, d=15%=0.15

Requirement: n=?

$\therefore A = P(1-d)^n$

$\Rightarrow 20000 = 49074(1-0.15)^n$

$\Rightarrow \frac{20000}{49074} = (0.85)^n \Rightarrow 0.4075 = (0.85)^n$

$\Rightarrow \log 0.4075 = \log(0.85)^n \Rightarrow \log 0.4075 = n\log 0.85$

$\Rightarrow n = \frac{\log 0.4075}{\log 0.85} \Rightarrow n = \frac{\bar{1}.6101}{\bar{1}.9294}$ [Using 'log' tables]

$$\Rightarrow n=\frac{-1+.6101}{-1+.9294}\Rightarrow n=\frac{-0.3899}{-0.0706}$$

$\Rightarrow n=5.52$ years [Ans.]

(b) Given $A=\frac{49074}{10}=4907.4, p=49074, d=15\%=0.15$

Requirement: n=?

$\therefore A=P(1-d)^n$

$\Rightarrow 4907.4=49074(1-0.15)^n$

$$\Rightarrow \frac{4907.4}{49074}=(0.85)^n\Rightarrow 0.10=(0.85)^n$$

$\Rightarrow \log 0.10=\log(0.85)^n\Rightarrow \log 0.10=n\log 0.85$

$$\Rightarrow n=\frac{\log 0.10}{\log 0.85}\Rightarrow n=\frac{1.0000}{1.9294}$$ [Using 'log' tables]

$$\Rightarrow n=\frac{-1+.0000}{-1+.9294}\Rightarrow n=\frac{-1}{-0.0706}\Rightarrow n=14.16$$ years [Ans.]

Problem-05: A machine depreciates at the rate of 10% of its value at the beginning of a year. The machine was purchased for Tk. 5810 and scrap value realized when sold was Tk. 2250. Find the number of years that the machine was used.

Solution: Given $A=2250, P=5810, d=10\%=0.10$

Requirement: n=?

$\therefore A=P(1-d)^n$

$\Rightarrow 2250=5810(1-0.10)^n$

$$\Rightarrow \frac{2250}{5810}=(0.90)^n\Rightarrow 0.3873=(0.90)^n$$

$\Rightarrow \log 0.3873=\log(0.90)^n\Rightarrow \log 0.3873=n\log 0.90$

$$\Rightarrow n=\frac{\log 0.3873}{\log 0.90}\Rightarrow n=\frac{\bar{1}.5880}{\bar{1}.9542}$$ [Using 'log' tables]

$$\Rightarrow n=\frac{-1+.5880}{-1+.9542}\Rightarrow n=\frac{-0.4120}{-0.0458}$$

$\Rightarrow n=8.9946\Rightarrow n\approx 9$ years [Ans.]

Problem-06: A man borrows Tk. 20,000 at 4% C.I. and agrees to pay both the principal and the interest in 10 equal installments at the end of each year. Find the amount of each installment.

Solution: Given P=20,000, n = 10, i = 4%=0.04

Requirement: A=?

We have, $P = A\left[\frac{1-(1+i)^{-n}}{i}\right]$

$$\therefore 20000 = A\left[\frac{1-(1+0.04)^{-10}}{0.04}\right]$$

$$= A\left[\frac{1-(1.04)^{-10}}{0.04}\right]$$

$$= A\left[\frac{1-0.6761}{0.04}\right]$$

$$= A\left[\frac{0.3239}{0.04}\right] \quad \therefore A = \frac{20,000\times 0.04}{0.3239} = Tk.2470$$

Let $x = (1.04)^{-10}$

$\Rightarrow \log x = -10\log 1.04$

$\Rightarrow \log x = -10\times 0.0170$

$\Rightarrow \log x = -0.1700$

$\Rightarrow \log x = \bar{1}.8300$

$\Rightarrow x = anti\log \bar{1}.8300$

$\Rightarrow x = 0.6761$ [Ans.]

Problem-07: A man borrows Tk. 1500 promising to repay the sum borrowed and the proper interest by 10 equal yearly installments, the first two falling due in 1 year's time. Reckoning C.I. at 5% p.a., find the value of the annual installment [Given $(1.05)^{10} = 1.629$.]

Solution: Given $P = 1500, n = 10, i = 5\% = 0.05, (1.05)^{10} = 1.629$

Requirement: Annual Installment (A) = ?

$$\therefore P = A\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$\Rightarrow 1500 = A\left[\frac{1-(1+0.05)^{-10}}{0.05}\right]$$

$$\Rightarrow 1500 = A\left[\frac{1-(1.05)^{-10}}{0.05}\right]$$

$$\Rightarrow 1500 = A\left[\frac{1-0.6138}{0.05}\right]$$

$$\Rightarrow 1500 = A[7.724]$$

$$\Rightarrow A = \frac{1500}{7.724} = Tk.194\ 20$$ [Ans.]

Let $x = (1.05)^{-10}$

$\Rightarrow \log x = -10\log 1.05$

$\Rightarrow \log x = -10\times 0.0212$

$\Rightarrow \log x = -0.212$

$\Rightarrow \log x = \bar{1}.788$

$\Rightarrow x = anti\log \bar{1}.788$

$\Rightarrow x = 0.6138$

Problem-08: A company buys a machine for Tk. 100000. Its estimated life is 12 years and scrap value is Tk.5000. What amount is to be retained every year from the profit and allowed to accumulate at 5% C.I. for buying a new machine at the same price after 12 years.

Solution: Given F = Cost-Scrap Value = 100000-5000 = 95000, n =12, i =0.05

Requirement: A =?

$$\therefore F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\Rightarrow 95000 = A\left[\frac{(1+0.05)^{12} - 1}{0.05}\right]$$

$$\Rightarrow 95000 = A\left[\frac{(1.05)^{12} - 1}{0.05}\right]$$

$$\Rightarrow 95000 = A\left[\frac{1.7964 - 1}{0.05}\right]$$

$$\Rightarrow 95000 = A[15.928]$$

$$\Rightarrow A = \frac{95000}{15.928} = Tk.5964.34 \text{ [Ans.]}$$

$Let x = (1.05)^{12}$

$\Rightarrow \log x = 12 \log 1.05$

$\Rightarrow \log x = 12 \times 0.0212$

$\Rightarrow \log x = 0.2544$

$\Rightarrow x = anti \log 0.2544$

$\Rightarrow x = 1.7964$

Problem-09: A man borrows Tk. 1000 on the understanding that it is to be paid back in four equal installments at intervals of six months, the first payment to be made six months after the money was borrowed. Calculate the amount of each installment, reckoning compound interest at 2½% per half year.

Solution: Given P = 1000, n = 4, i = 2½% = 0.025

Requirement: Annual Installment (A) = ?

$$\therefore P = A\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$\Rightarrow 1000 = A\left[\frac{1-(1+0.025)^{-4}}{0.025}\right]$$

$$\Rightarrow 1000 = A\left[\frac{1-(1.025)^{-4}}{0.025}\right]$$

$$\Rightarrow 1000 = A\left[\frac{1-0.9061}{0.025}\right]$$

Let $x = (1.025)^{-4}$

$\Rightarrow \log x = -4 \log 1.025$

$\Rightarrow \log x = -4 \times 0.0107$

$\Rightarrow \log x = -0.0428$

$\Rightarrow \log x = \bar{1}.9572$

$\Rightarrow x = anti \log \bar{1}.9572$

$\Rightarrow x = 0.9061$

$$\Rightarrow 1000 = A[3.756]$$

$$\Rightarrow A = \frac{1000}{3.756} = Tk.266.24 \text{ [Ans.]}$$

Problem-10: A loan of Tk. 40,000 is to be repaid in equal annual installment consisting of principal and interest due in course of 30 years. Find the amount of each installment reckoning interest at 4% p.a.

Solution: Given $p = 40{,}000, n = 30, i = 4\% = 0.04,$

Requirement: Annual Installment (A) = ?

$$\therefore P = A\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$\Rightarrow 40000 = A\left[\frac{1-(1+0.04)^{-30}}{0.04}\right]$$

$$\Rightarrow 40000 = A\left[\frac{1-(1.04)^{-30}}{0.04}\right]$$

$$\Rightarrow 40000 = A\left[\frac{1-0.3090}{0.04}\right]$$

$$\Rightarrow 40000 = A[17275] = A[17.275]$$

$$\Rightarrow A = \frac{40000}{17.275} = \text{Tk.}2315.48$$

[Ans]

Let $x = (1.04)^{-30}$

$$\Rightarrow \log x = -30\log 1.04$$

$$\Rightarrow \log x = -30 \times 0.0170$$

$$\Rightarrow \log x = -0.5100$$

$$\Rightarrow \log x = \bar{1}.4900$$

$$\Rightarrow x = anti\log \bar{1}.4900$$

$$\Rightarrow x = 0.3090$$

Problem-11: (a) The annual subscription for the membership of a club is Tk. 25 and a person may become a life member by paying Tk. 1000 in a lump sum. Find the rate of interest charged. (b) A man wishes to create an endowment fund to provide an annual prize of Tk.500 out of its income. If the fund is invested in $2^1/_2$ % p.a., find the amount of this fund.

Solution: (a) Given E = 1000, n = 30, Annual Installment (A) = 25

Requirement: Rate of interest (i) = ?

$$\therefore E = \frac{A}{i}$$

$$\Rightarrow 1000 = \frac{25}{i}$$

$$\Rightarrow i = \frac{25}{1000} = 0.025 \Rightarrow i = 2.5\% \text{ [Ans.]}$$

(b) Given $A = 500, i = 2^1/_2\% = 0.025$

Requirement: Endowment Fund (E) = ?

$$\therefore E = \frac{A}{i}$$

$$= \frac{500}{0.025} = \text{Tk. } 20000 \text{ [Ans.]}$$

Problem-12: A limited company intends to create a depreciation fund to replace at the end of the 25th year assets costing Tk. 1,00,000. Calculate the amount to be retained out of profits every year if the interest rate is 3%.

Solution: Given $F = 1{,}00{,}000, n = 25, i = 3\% = 0.03$

Requirement: $A = ?$

$$\therefore F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\Rightarrow 1{,}00{,}000 = A\left[\frac{(1+0.03)^{25} - 1}{0.03}\right]$$

$$\Rightarrow 1{,}00{,}000 = A\left[\frac{(1.03)^{25} - 1}{0.03}\right]$$

$$\Rightarrow 1{,}00{,}000 = A\left[\frac{2.089 - 1}{0.03}\right]$$

$$\Rightarrow 1{,}0{,}000 = A\left[\frac{1.089}{0.03}\right]$$

$$\Rightarrow A = \frac{1{,}00{,}000 \times 0.03}{1.089} = Tk.2755 \text{ [Ans.]}$$

Let $x = (1.03)^{25}$

$$\Rightarrow \log x = 25 \log 1.03$$

$$\Rightarrow \log x = 25 \times 0.0128$$

$$\Rightarrow \log x = 0.3200$$

$$\Rightarrow x = anti\log 0.3200$$

$$\Rightarrow x = 2.089$$

Problem-13: A machine costs the company Tk.97000 and its effective life is estimated to be 12 years. If the scrap realizes Tk. 2000 only, what amount should be retained out of profits at the end of each year to accumulate at compound interest at 5% per annum?

Solution: Given $F = \text{Cost} - \text{Scrap Value} = 97000 - 2000 = 95000, n = 12, i = 0.05$

Requirement: $A = ?$

$$\therefore F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\Rightarrow 95000 = A\left[\frac{(1+0.05)^{12} - 1}{0.05}\right]$$

$$\Rightarrow 95000 = A\left[\frac{(1.05)^{12} - 1}{0.05}\right]$$

$$\Rightarrow 95000 = A\left[\frac{1.7964 - 1}{0.05}\right]$$

$$\Rightarrow A = \frac{95000}{15.928} = Tk.5964.34 \text{ [Ans.]}$$

Let $x = (1.05)^{12}$

$\Rightarrow \log x = 12 \log 1.05$

$\Rightarrow \log x = 12 \times 0.0212$

$\Rightarrow \log x = 0.2544$

$\Rightarrow x = anti \log 0.2544$

$\Rightarrow x = 1.7964$

Problem-14: A loan of Tk.1000 is to be paid in 5 annual payments interest being at 6 percent per annum composed interest and first payment being made after a year. Analyze the payment into those on account of interest and on account of amortization of the principal.

Solution: Given $P = 1000, n = 5, i = 6\% = 0.06,$

Requirement: $A = ?$

$$\therefore PV = A\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$\Rightarrow 1000 = A\left[\frac{1-(1+0.06)^{-5}}{0.06}\right]$$

$$\Rightarrow 1000 = A\left[\frac{1-(1.06)^{-5}}{0.06}\right]$$

$$\Rightarrow 1000 = A\left[\frac{1-0.7473}{0.06}\right]$$

Let $x = (1.06)^{-5}$

$\Rightarrow \log x = -5 \log 1.06$

$\Rightarrow \log x = -5 \times 0.0253$

$\Rightarrow \log x = -0.1265$

$\Rightarrow \log x = \bar{1}.8735$

$\Rightarrow \log x = anti \log \bar{1}.8735$

$\Rightarrow x = 0.7473$

$$\Rightarrow 1000 = A[4.2115]$$

$$\Rightarrow A = \frac{1000}{4.2115} = \text{Tk.}237.45 \text{ [Ans.]}$$

Problem-15: A machine costs a company Tk. 52000 and its effective life is estimated to be 25 years. A sinking fund is created for replacing the machine by a new model at the end of its life time, when its scrap realize a sum of Tk. 2500 only. The price of the new model is estimated to be 25 percent higher than the price of the present one. Find what amount should be set aside every year, out of the profits for the sinking fund, if it accumulates at $3^1/_2$ Percent per annum compound.

Solution: Given Cost (old machine) = 52000

$$\text{Cost (new machine)} = 52000 + (52000 \times 25\%) = 65000$$

$$F = \text{Cost} - \text{Scrap Value} = 65000 - 2500 = 62500, n = 25, i = 3^1/_2\% = 0.035$$

$$\therefore F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\Rightarrow 62500 = A\left[\frac{(1+0.035)^{25} - 1}{0.035}\right]$$

$$\Rightarrow 62500 = A\left[\frac{(1.035)^{25} - 1}{0.035}\right]$$

$$\Rightarrow 62500 = A\left[\frac{2.3578 - 1}{0.035}\right]$$

$$\Rightarrow 62500 = A[38.7932]$$

$$\Rightarrow A = \frac{62500}{38.7932} = \text{Tk.}1611 \text{ [Ans.]}$$

Let $x = (1.035)^{25}$

$$\Rightarrow \log x = 25 \log 1.035$$

$$\Rightarrow \log x = 25 \times 0.035$$

$$\Rightarrow \log x = 0.3725$$

$$\Rightarrow x = anti\log 0.3725$$

$$\Rightarrow x = 2.3578$$

Problem-16: A man aged 40 wishes his dependents to have Tk. 40,000 at his death. A banker agrees to pay this amount to his dependents on condition that the man makes equal annual payments of Tk. x to the bank commencing now and going on until his death. What should be the value of x, assuming that the bank pays interest at 3% p.a. compound? From the table on the expectation of life it is found that the expectation of life of a man of 40 is 30 years.

Solution: Given $F = 40{,}000, n = 30, i = 0.03$

Requirement: $A = ?$

$$\therefore F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\Rightarrow 40000 = A\left[\frac{(1+0.03)^{30} - 1}{0.03}\right]$$

$$\Rightarrow 40000 = A\left[\frac{(1.03)^{30} - 1}{0.03}\right]$$

$$\Rightarrow 40000 = A\left[\frac{2.421 - 1}{0.03}\right]$$

$$\Rightarrow 40000 = A[47.3667]$$

$$\Rightarrow A = \frac{40000}{47.3667} = \text{Tk.}844.48\text{[Ans.]}$$

Let $x = (1.03)^{30}$

$\Rightarrow \log x = 30 \log 1.03$

$\Rightarrow \log x = 30 \times 0.0128$

$\Rightarrow \log x = 0.3840$

$\Rightarrow x = anti \log 0.3840$

$\Rightarrow x = 2.421$

Problem-17: The cost of a machine is Tk. 1,00,000 and its effective life is 12 years. If the scrap realizes only Tk. 5000, what amount should be retained out of profits at the end of each year to accumulate at C.I. at 5% p.a.? [Use $\log_{10} 1.05 = 0.0212, \log_{10} 1.797 = 0.2544$]

Solution: Given F = Cost – Scrap Value = $100000 - 5000 = 95000, n = 12, i = 0.05$

Requirement: $A = ?$

$$\therefore F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\Rightarrow 95000 = A\left[\frac{(1+0.05)^{12} - 1}{0.05}\right]$$

$$\Rightarrow 95000 = A\left[\frac{(1.05)^{12} - 1}{0.05}\right]$$

$$\Rightarrow 95000 = A\left[\frac{1.7964 - 1}{0.05}\right]$$

$$\Rightarrow 95000 = A[15.9277]$$

$$\Rightarrow A = \frac{95000}{15.9277} = \text{Tk. } 5964.45\text{[Ans.]}$$

Let $x = (1.05)^{12}$

$\Rightarrow \log x = 12 \log 1.05$

$\Rightarrow \log x = 12 \times 0.0212$

$\Rightarrow \log x = 0.2544$

$\Rightarrow x = anti \log 0.2544$

$\Rightarrow x = 1.7964$

BRIEF REVIEW

Definition

Simple Interest: When interest is calculated only on the original principal, then it is called simple interest (S.I.).

Compound Interest: When interest is calculated on both principal and successive interests then it is called compound interest (C.I.).

Nominal Interest: The annual compound interest rate is called nominal rate of interest.

Effective Interest: When interest is compounded more than once in a year, then the actual percentage of interest rate per year is called effective rate of interest.

Annuity: A sequence of equal payments made at equal time intervals is called an annuity.

Annuity Certain: An annuity payable for a fixed number of years is called annuity certain.

Annuity Due: An annuity, in which all payments are made at the beginning of each period, is called annuity due. Examples: saving schemes, life insurance payments, etc.

Immediate Annuity: An annuity, in which all payments are made at the end of each period, is called immediate annuity or ordinary annuity. Examples: car loan, repayment of housing loan etc.

Annuity Contingent: In case the term of payment depends on some uncertain event, the annuity is called annuity contingent.

Deferred Annuity: If the payments are deferred or delayed for a certain number of years, then it is called deferred annuity. For ex.: pension plan etc. Many financial organizations give loan amount immediately and regular installments may start after specified time period.

Perpetual annuity: An annuity whose payments are continue forever is called perpetual annuity or perpetuity. In this case, PV = $\frac{a}{i}$; where a = payment of each installment, i= rate of interest.

Present value of an annuity: The present value of an annuity is the sum of the present values of all the payments of annuity at the beginning of the annuity.

Future value of an annuity: The future value of an annuity is the sum of all payments made and interest earned on them at the end of the term of annuities.

Sinking Fund: A type of savings fund, in which deposits are made regularly, with compound interest earned, to be used later for a specific purpose, such as purchasing equipment or buildings, is called sinking fund.

Amortization: A loan with fixed rate of interest is said to be amortized if both principal and interest are paid by a sequence of equal payments with equal time periods. Purchasing a car by making a series of periodic payments is an example of a loan that is amortized.

Quiz Questions

Multiple Choice Questions

1. Which one is incorrect?
   a. $S.I. = Pni$
   b. PV = FV $(1+i)^n$
   c. FV = PV $(1+i)^n$
   d. C.I. = FV-PV.
2. What will be the simple interest, if Tk. 10,000 invested for 4 years at 5% per annum?
   a. Tk. 2000
   b. Tk. 200
   c. Tk. 20000
   d. Tk. 3000
3. What will be the compound interest if Tk. 10,000 invested for 4 years at 5% per annum?
   a. Tk. 2255
   b. Tk. 2155
   c. Tk. 2355
   d. Tk. 2455
4. In what time will a sum of money double itself at 5% per annum C.I.?
   a. 14.2 years
   b. 15 years
   c. 16 years
   d. 17 years
5. How you classify interests?
   a. 2 types
   b. 3 types
   c. 4 types
   d. 5 types

6. Which one is true in case of depreciation?
a. $A = P(1+1)^n$
b. $A = P(1-1)^n$
c. $P = A(1+1)^n$
d. All of the above

7. Which one is the classification of annuity?
a. Annuity certain
b. Deferred annuity
c. Contingent annuity
d. All of the above

8. Which one is not true for annuity?
a. $PV = \frac{a}{i}\left\{1 - \frac{1}{(1+i)^n}\right\}$
b. $FV = \frac{a}{i}\left\{(1+i)^n - 1\right\}$
c. $PV = \frac{a}{i}\left\{1 - \frac{1}{(1+i)^{-n}}\right\}$
d. All are true

9. Which formula is true in case of sinking fund?
a. $PV = \frac{a}{i}\left\{1 - \frac{1}{(1+i)^n}\right\}$
b. $FV = \frac{a}{i}\left\{(1+i)^n - 1\right\}$
c. $FV = PV(1+i)^n$
d. None of the above

10. Which formula we use in case of amortization?
a. $PV = \frac{a}{i}\left\{1 - \frac{1}{(1+i)^n}\right\}$
b. $FV = \frac{a}{i}\left\{(1+i)^n - 1\right\}$
c. $FV = PV(1+i)^n$
d. None of the above

Which one of the following statement is true/false?

a. Rates of interest are generally expressed as a percentage.
b. The total amount of money borrowed initially is called compound interest.
c. The simple interest is charged only yearly basis.
d. The present value of an annuity is the sum of the future value of its installments.
e. Annuity certain may be divided into two categories.
f. If the conversion period is one year, then the nominal rate and effective rate are equal.
g. In case of perpetual annuity or perpetuity, beginning date is known but the terminal date is unknown.

Brief Questions

1. Write down the mathematical formula of simple interest.
2. If interest is compounded daily basis then write down the laws of amount of principal with interest?
3. Write down the formula of scrap value in case of depreciation.
4. In what time will a sum of money double it self at 5% p.a.C.I?
5. What will be simple interest if Tk. 10,000 invested for 4 years at 5% p.a?
6. Write down the example of annuity contingent.
7. Write down the formula of effective rate of interest.
8. What is principal amount?
9. Which annuity is the first payment falls due at the end of first interval?
10. What is conversion period?
11. What do you mean by amortization?
12. What do you mean by sinking fund?

Conceptual, Analytical & Numerical Questions

1. Define simple interest and compound interest with example.
2. What do you mean by Compound interest?
3. What is annuity? Discuss the different types of annuity.
4. Distinguish between nominal and effective interest rate.
5. Distinguish between annuity due and immediate annuity.
6. Distinguish between simple and compound interests.
7. Distinguish between annuity Certain and annuity contingent.
8. Distinguish between present value of annuity and future value of annuity.

Numerical Questions

1. Find the compound interest of Tk. 10,000 for 4 years at 5% per annum. What will be the simple interest in the above case?
2. Find the compound interest on Tk. 1000 for 4 years at 5% per annum. What will be the simple interest in the above case?
3. Find the difference between simple and compound interest on Tk. 5000 invested for 4 years at 5% per annum, interest payable yearly.
4. What is the present value of Tk. 10,000 due in 2 years at 8% p.a., C.I. according as the interest is paid (a) yearly, (b) half yearly.
5. What is the present value of Tk. 1000 due in 2 years at 5% p.a. compound interest, according as the interest is paid (a) yearly and (b) half yearly.
6. Find the compound interest on Tk. 6950 for 3 years, if interest is payable half yearly, the rate for the first two years being 6%, and for the third year 9% p.a.
7. What the compound interest on Tk. 25800 for 5 years if the rate of interest be 2% in the 1^{st} year, $2^1/_2$% in the second year, 3% in the 3^{rd} year and thereafter at 4% p.a.
8. Mr. Manjur borrowed Tk. 20,000 from a money-lender but he could not repay any amount in a period of 4 years. Accordingly the money-lender demands now Tk. 26,500 from him. At what rate percent per annum compound interest did the latter lend his money?
9. (a) In what time will a sum of money double itself at 5% p.a., compound interest? (b) In what time will a sum of money treble itself at 5% p.a., compound interest payable half yearly?
10. Find the number of years and the fraction of a year in which a sum of money will treble itself at compound interest at 8 percent per annum.
11. In what time will a sum of Tk. 1234 amount to Tk. 5678 at 8% p.a. compound interest payable quarterly?
12. Find the present value of an annuity of Tk. 1000 p.a. for 14 years following compound interest at 5% p.a.
13. Calculate the amount and present value of an annuity of Tk. 3000 for 15 years if the rate of interest be $4^1/_4$% p.a.
14. A man borrows Tk. 6,000 at 6% and promises to pay off the loan in 20 annual payments beginning at the end of the first year. What is the annual payment necessary?
15. What sum should be paid for an annuity of Tk. 2400 for 20 years at $4^1/_2$% compound interest p.a.? [Given log 1.045 = 0.0191 and log 4.150 = 0.6180].

16. A machine, the life of which is estimated to be 10 years, costs Tk. 10,000. Calculate its scrap value at the end of its life, depreciation on the reducing installment system being charged at 10% per annum.

17. A machine is depreciated in such a way that the value of the machine at the end of any year is 90% of the value at the beginning of the year. The cost of the machine was Tk. 12,000 and it was sold eventually as waste metal for Tk. 200. Find the number of years during which the machine was in use.

18. A wagon is purchased on installment basis, such that Tk. 5,000 is to be paid on the signing of the contract and four yearly installments of Tk. 3,000 each payable at the end of the first, second, third and fourth year. If interest is charged at 5% per annum, what would be the cash down price?

19. A sinking fund is created for the redemption of debentures of Tk. 1,00,000 at the end of 25 years. How much money should be provided out of profits each year for the sinking fund, if the investment can earn interest at the rate of 4% per annum?

20. On 48^{th} birthday Mr. Mizan decides to make a gift of Tk. 5000 to a hospital. He decides to save this amount by making equal annual payments up to and including his 60^{th} birthday to a fund, which gives $3^1/_2$ percent compound interest, the first payment being made at once. Calculate the amount of each annual payment.

21. The population in a town increases every year by 2 % of the population of the town at the beginning of the year. Then in how many years will the total increase of population be 40%.

22. What is the value of an annuity at the end of 20 years if Tk. 2000 is deposited each year into an account earning 8.5% compounded annually? How much of this value is internet?

23. The difference between compound interest and simple interest on a certain sum for 3 years at 5% per annum is Tk. 30.50. Find the sum?

24. A company wants to accumulate Tk. 100000 to purchase replacement machinery 8 years from now to accomplish this, equal semiannual payments are made to a fund that earns 7% compounded semiannually. Find the amount of each payment.

ANSWERS

1. $C.I. = Tk.2155, S.I. = 2{,}000.$	2. $C.I. = Tk.215.51, S.I. = Tk.200$	3. Tk. 78
4. Tk. 8573, $Tk.8548$	5. Tk. 906.95, $Tk.906.13$	6. Tk.1592
7. Tk. 4250	8. 7.3	9. (a) 14.2 yrs, (b) 22.30 yrs
10. 14.28 yrs.	11. 19.27 yrs.	12. Tk. 9899
13. Tk. 61306, Tk. 32810	14. Tk. 523.10	15. Tk. 31200
16. Tk. 3487.	17. 39 yrs.	18. Tk. 15,638
19. Tk. 2401	20. Tk. 434.54	21. 17 years
22. Tk. 96754 , Tk. 56754	23. Tk. 3812.50	24. Tk. 4769