CHAPTER-03 THEORY OF SETS

LEARNING OBJECTIVES

This chapter will enable you to learn the concepts and application of:

- The concept of sets
- Types of sets and operation of sets
- Venn diagram
- Application of sets in business problem

3.1 INTRODUCTION

George Cantor, a great mathematician developed the theory of set. This is one of the most fundamental concepts of mathematics and is now used in almost all branches of science, technology, commerce and economics. The theory has established co-ordination among different branches of mathematics namely geometry, algebra, calculus etc. The solutions of complicated problems of science and commerce become easier by the application of set theory. Not only that, the theory helps to base the subject probability on a solid logical foundation. In this chapter we shall elaborate and discuss the importance of set theory in the application of mathematics to real life problems. We often come across phases such as: a bunch of keys, a pack of cards, a class of students, a team of players etc. The words bunch, pack, class, and team all denote collections. Also the dictionary meaning of set is a group or a collection. In mathematics, the word set is used in the context of a well-defined collection of objects.

3.2 DEFINITION OF A SET

Definition: A well-defined and well-distinguished collection of objects is called **a set**.

By a well-defined collection, we mean that there exists a rule with the help of which we should be able to say whether any given object or entity belongs to the collection under consideration.

The following are some examples of a set:

i. The vowels in English alphabets.
ii. Rivers in Bangladesh.
iii. The planets: Sun, Moon, and Earth.
iv. The positive even integers from 1 to 50.

The basic characteristics of a set are that it should be well-defined, its objects or elements should be well distinguished for easy recognition by description.

ELEMENTS OF A SET

The objects that make up a set are called the elements or members of a set. The elements in the set must be distinct and distinguishable. By 'distinct' we mean that no element is repeated and by 'distinguishable' we mean that given any object, it is either in the set or not in the set. The set is generally denoted by capital letters A, B, C, X, Y, Z, etc and its elements by small letters a, b, c, x, y, z, etc.

The symbol $\in$ (epsilon, a Greek alphabet) is used to indicate that a particular element or object 'belongs to a set' or 'a member of a set'. For example, if x is an element of a set A, then symbolically we write $x \in A$, which is read as "x is an element of the set A" or "x belongs to the set A". The symbol $\notin$ is used to indicate that a particular element or object 'does not belong to a set' or 'is not a member of a set'. Thus, if x is not an element of a set A, then symbolically we write $x \notin A$, which is read as "x is not an element of the set A". Thus given a set A and an object x, one and only one of the following statements is true: (i) $x \in A$, (ii) $x \notin A$.

DIFFERENCE BETWEEN A COLLECTION AND A SET

Let us make a clear Difference between a 'collection' and a 'set'.

1. Every collection is not a set. For a collection to be a set, it must be well-defined. For a collection to be well-defined, it should be known whether a given object does or does not belong to that collection. For example, consider the following collection:
   A = the collection of any five positive integers.
   B = the collection of first five positive integers.
   The collection A is not well-defined because, the positive integer 4 (say) may or may not belong to A. On the other hand, collection B is well-defined because, the members of the collection are 1,2,3,4,5. Therefore, the collection B is a set.
2. Collection of qualitative attributes in certain objects such as honest persons, rich persons, beautiful women, good players, etc do not form set.

3.3 METHODS OF DESCRIBING A SET

The expression of the set has to be compact and clear, otherwise the basic quality of the set being well defined and distinctive is lost. Following are the most common methods of describing a set:

a. Tabular, or Roster, or Enumeration method and
b. Selector, or Property builder, or Rule method.

a. Tabular method: Under this method, a set can be described by actual listing all the objects belonging to it within braces i.e.{ }. For example, the elements of the set V of all the vowels in the English alphabets can be represented as:

$$V = \{a, e, i, o, u\}.$$

Sometimes, it is not possible to list all the elements, but after knowing a few elements we can see as to what the other elements are. For example, the set N of natural numbers may be written as:

$N = \{1,2,3,...\}$

Here the dots indicate that the set contains all the natural numbers. Similarly, the set of squares of positive odd integers is written as:

$$S = \{1,9,25,49,...\}$$

Remark: No element in the set should be repeated. For example, we never write $\{3,5,5\}$, but rather write $\{3,5\}$. Also became a set is a collection of objects in a well-defined Manner, the order in which the elements of a set are listed does not make any difference. The three sets: $\{2,4,3\},\{4,2,3\}$ and $\{3,2,4\}$ are different listings of the same set. The elements of a set distinguish the set but not the order in which the elements are written.

b. Selector method: In this method, the elements of a set are represented by mentioning their qualitative or quantitative or both characteristics. We may state some characteristics, which an object must possess in order to be an element in the set. To understand this method, let us consider the sets:

i. $A = \{2,4,6,8,...\}$ ii. $B = \{a,e,i,o,u\}$

If we try to search out some property among the elements of the sets A and B, we find that all elements of set A are even positive integers; all elements of set B are vowels of English alphabets. Thus we can use the letter 'x' to represent an arbitrary element of the set together with the property of x. Therefore sets A and B may be represented as:

$A=\{x|x$ is an even positive integer$\}$.

$B=\{x|x$ is a vowel in English alphabet$\}$.

The symbol '$|$' after x stands for 'such that'. Sometimes we use ':' to denote 'such that'. For example:

$A=\{x : x$ is an even positive integer$\}$.

It will be clear from the above discussion that the tabular method is particularly useful when the elements are few in number while the set builder method is more suitable when the elements are numerous.

SOME STANDARD SETS

i. $N = \{1,2,3,4,...\}$, the set of all natural numbers.

ii. $I = \{...,-3,-2,-1,0,1,2,3,...\}$, the set of all integers.

iii. $I_+ = \{1,2,3,...\}$, the set of all positive integers.

iv. $Q = \{x : x = p/q$, where p and q are integers and $q \neq 0\}$, the set of all rational numbers.

v. Q_+= the set of all positive-rational numbers.

vi. Q' = the set of all irrational numbers.

vii. R = the set of all real numbers.
viii. P = the set of all prime numbers.
ix. C = $\{x : x = a + ib; a, b \in R, i = \sqrt{-1}\}$, the set of all complex numbers.

For example-1: Represent the following sets in set notation:
i. Set of all alphabets in English language,
ii. The set of all odd integers less than 25,
iii. The set of all odd integers,
iv. The set of positive integers satisfying the equation $x^2 + 5x + 7 = 0$.

Solution:
i. $A = \{x : x$ is a alphabet in English language$\}$,
ii. $I = \{x : x$ is an odd integer $< 25\}$,
iii. $I = \{x : x = 1,3,5,7...\}$,
iv. $I = \{x : x$ is the $+ve$ integers satisfying the equation $x^2 + 5x + 7 = 0\}$.

For example-2: Rewrite the following sets in a set-builder form:
i. $A = \{a, e, i, o, u\}$, ii. $B = \{1,2,3,4,...\}$ iii. C is a set of integers between -15 and $+15$.

Solution:
i. $A = \{x : x$ is a vowel in English alphabet$\}$,
ii. $B = \{x : x$ is a $+ve$ integer$\}$,
iii. $C = \{x : -15 < x < 15,$ and x is an integer$\}$.

3.4 TYPES OF SETS

1. **Finite set:** A set which contains finite numbers of different elements is called finite set. The following are the examples of finite sets:
$A = \{1,2,3,4,5\}$
$B = \{a, e, i, o, u\}$
$C = \{x : x$ is a river of Bangladesh$\}$
2. **Infinite set:** A set which contains infinite numbers of elements is known as infinite set. The following are the examples of infinite sets:
$N = \{1,2,3,4,5.....\}$
$A = \{x : x$ is the star in the sky$\}$.
$B = \{x / x$ is a positive integers divisible by $5\}$.
3. **Singleton set:** A set having only one element is called singleton or unit set. For example:
$A = \{5\}$
$B = \{x : 4 < x < 6, x$ is an integer$\}$

$C = \{x / x^2 = 9$ and x is a negative integer$\}$.

$D = \{x : x$ is a Bangladeshi noble laureate$\}$.

4. **Empty set:** A set having no element is called an empty set or null set or void set. this set is denoted by a Greek letter φ (read as phi). For example:

$A = \{x : x$ is a man aged 300 years $\} = \varphi$ (empty set.)

$B =$ set of all integers whose square is 5

$= \{x : x^2 = 5$ and x is an integer$\} = \varphi$.

5. **Equal sets:** Two sets A and B are said to be equal if every element of A is an element of B and also every element of B is an element of A. The equality to A and B is denoted by $A = B$. Symbolically, we write

$A = B$ if and only if $x \in A \Leftrightarrow x \in B$

Example:

(i) Let $A = \{4,3,1,2\}$ and $B = \{1,2,3,4\}$. then $A = B$, since each of the elements of A belongs to B and each of the elements of B belongs to A

(ii) Let $A = \{2,3\}, B = \{3,2\}$ and $C = \{x : x^2 - 5x + 6 = 0\}$. Then $A = B = C$, because each element which belongs to any one of the sets also belongs to the other two sets.

(iii) Let $A = \{x / x$ is a letter in the word 'march'$\}$

$B = \{x / x$ is a letter in the word 'charm'$\}$

$C = \{a, c, h, m, r\}$.

Here each set contains the same elements namely a, c, h, m, r irrespective of their order.

Hence the sets are equal i.e. $A = B = C$.

6. **Equivalent sets:** Two sets having same number of distinct elements are called equivalent sets. That is, if the total number of elements in one set are equal to the total number of elements in another set, then the two sets are equivalent. We write $A \cong B$ to mean that the sets A and B are equivalent.

Example:

(i) Let $A = \{a, b, c, d,\}$ and $B = \{1,2,3,4\}$. Clearly A is not equal to B, but the set A is equivalent to the set B, because total number of elements of the two sets are equal but the elements are not exactly same.

(ii) Let $A = \{x : x$ is a letter in the word 'BOAT'$\}$

$B = \{x : x$ is a letter in the word 'CART'$\}$

Here the sets A and B contain the same number of elements. Therefore $A \cong B$.

7. **Subset:** If A and B are two non-empty sets such that every element of A is also an element of B, then A is said to be a subset of B (or A is contained in B). In other words, A is a subset of B if $x \in A \Rightarrow x \in B$. Symbolically, this relation is written as:

$A \subseteq B$ if $x \in A \Rightarrow x \in B$, which is read as 'A is a subset of B' or 'A is contained in B'.

If $A \subseteq B$, then B is called the superset of A and we write: $B \supseteq A$ which is read as 'B is a superset of A' or 'B contains A'.

If A is not a subset of B, we write: $A \not\subset B$ if $x \in A \Rightarrow x \notin B$, which is read as 'A is not a subset of B.

Examples:

a. Suppose $A = \{1,3,5\}$, and $B = \{5,3,4,2,1\}$. Here each element of A are also the elements of B. Therefore, $A \subseteq B$.

b. Let $A = \{3,5,6\}$ and $B = \{5,3,6\}$. Since each element of A belongs to B and vice-versa, therefore $A = B$. Thus it follows that every set is a subset of itself.

c. The set $A = \{1,3,5\}$ is not the subset of the set $B = \{1,3,6,7\}$, since $5 \in A$ but $5 \notin B$.

d. Null set ϕ is a subset of every set.

8. Proper subset: The set A is a proper subset of the set B if and only if every element of the set A is also an element of the set B and there is at least one element in the set B that is not in the set A. Symbolically, this is written as '$A \subset B$' and is read as 'A is a proper subset of superset B'.

For example: Let $A = \{1,2,3,5,8,9\}$ and $B = \{1,2,3,4,5,7,8,9\}$. Here $A \subset B$.

Remarks: (i) If A is a subset of B, then B is called the superset of A.

(ii) A set is always a subset of itself.

(iii) The null set is unique and is a subset of every set.

Example: $A = \{1,2,3,8,9\}$, $B = \{2,4,6,8\}$, $C = \{1,3,5,7,9\}$, $D = \{3,4,5\}$, $E = \{3,5\}$.

What is the S if (i) $S \subset D$ and $S \not\subset A$, (ii) $S \subset B$ and $S \not\subset C$.

Solution: (i) When $S = \{4,5\}$, then S is a proper subset of D, while S is not a proper subset of A.

$\therefore S = \{4,5\}$ [Ans.]

(ii) When $S = \{2,4,6\}$, then S is a proper subset of B, while S is not a proper subset of C.

$\therefore S = \{2,4,6\}$ [Ans.]

Example: Write down all the subsets of the sets: $B = \{6,8,11\}$.

Solution: Given $B = \{6,8,11\}$.

Required all possible subsets of set B are

$\phi, \{6\}, \{8\}, \{11\}, \{6,8\}, \{6,11\}, \{8,11\}, \{6,8,11\}$ [Ans.]

9. Family of sets: If all the elements of a set are sets themselves, then such a set is called 'family of set' or 'set of sets'.

Example: If $A = \{a,b\}$, then the set $\{\phi, \{a\}, \{b\}, \{a,b\}\}$ is the family of sets whose elements are subsets of the set A.

Note: $\in$ connects an element and a set, while $\subset$ and $\subseteq$ connect two sets, i.e. if $A=\{a,b,c\}$ then $a\in A$, $\{a\}\subset A$ are correct statements while $a\subset A$ and $\{a\}\in A$ are incorrect statements.

10. Power set: The family of all subsets of a set is called power set of that set. If S is any set, then the family of all subsets of S is called the power set of S and is denoted by P(S).

Example: If $A=\{a,b\}$, then its subsets are: $\phi,\{a\},\{b\},\{a,b\}$.

Therefore power set, $P(S)=\{\phi,\{a\},\{b\},\{a,b\}\}$.

If A is a finite set of n elements, then the total number of subsets of A are 2^n. In other words, the power set of A has 2^n elements.

Example: Let $A=\{a,b,c,d\}$, where a,b,c,d represent the members of a decision-making body, say a committee. List the elements of power set $P(A)$.

Solution: Given $A=\{a,b,c,d\}$

$\therefore P(A)=\{\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},$
$\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\},\{a,b,c,d\}\}$ [*Ans.*]

11. Universal set: In applications of set theory, all the sets are considered as subsets of a fixed set. This fixed set is called the universal set. This set is usually denoted by U. Obviously, the set U is the superset of every set.

Examples: (i) All people in the world constitute the universal set in any study of human population.

(ii) A set of integers may be considered as a universal set for a set of odd or even integers.

(iii) A deck of cards may be universal set for a set of spade.

(iv) If $A=\{a\},B=\{b\}$, then $U=\{a,b\}$.

3.5 OPERATIONS ON SETS

Two or more sets can be associated, obeying some rules to give rise to new set. The method of assigning such a new set is called operation on sets. These operations on sets are defined to develop an algebra of sets. Different set operations are defined below:

1. **Union of sets:** The union of two sets A and B is the set of all elements which belong either in A or in B or in both A and B. The union of the sets A and B is denoted by $A\cup B$. It is sually read as "A union B" or "A cup B". Union is also known as "join" or "logical sum" of A and B. Symbolically, we write $A\cup B=\{x: x\in A \text{ or } x\in B\}$. In other words, $x\in A\cup B \Rightarrow x\in A$ or $x\in B$.

   Example: If $A=\{1,2,3,4\}$ and $B=\{2,6,8\}$, then $A\cup B=\{1,2,3,4,6,8\}$.

Properties of union of sets:

(a) The individual sets composing a union are members of the union.
If A and B are any sets, then $A \subseteq A \cup B$ and $B \subseteq A \cup B$.

(b) Union of sets has an identity property in an empty set.
If A is any set, then $A \cup \varphi = A$, where φ is the empty set.

(c) Union of sets is commutative.
If A and B are any two sets, then $A \cup B = B \cup A$.

(d) Union of a set with itself is the set itself.
If A is any set, then $A \cup A = A$.

(e) The union of sets is associative.
If A, B, and C are any three sets, then $A \cup (B \cup C) = (A \cup B) \cup C$.

2. **Intersection of sets:** The intersection of two given sets A and B is the set of all elements which are common to both the sets A and B. In other words, the intersection of two sets A and B is the set consisting of all the elements which belong to both A and B. Intersection of the sets A and B is denoted by $A \cap B$. It is usually read as "A intersection B" or "A cap B". Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$. In other words, $x \in A \cap B \Rightarrow x \in A$ and $x \in B$.

Example: (i) If $A = \{1,2,3,4,5,6,7\}$ and $B = \{3,5,7,9,11\}$, then $A \cap B = \{3,5,7\}$.

(ii) Let $A = \{x : 0 \le x \le 3\}$ and $B = \{x : 1 \le x \le 5\}$. Thus $A \cap B = \{x : 1 \le x \le 3\}$.

Properties of intersection of sets:

(a) Intersection of sets is commutative.
If A and B are any two sets, then $A \cap B = B \cap A$.

(b) The intersection of sets is associative.
If A, B, and C are any three sets, then $A \cap (B \cap C) = (A \cap B) \cap C$.

(c) Intersection of a set with itself is the set itself.
If A is any set, then $A \cap A = A$.

(d) Intersection of any set with an empty set is the null set.
If A is any set, then $A \cap \phi = \phi$, where ϕ is the null set.

3. **Disjoint sets:** Two sets A and B are said to be disjoint or mutually exclusive if they have no common elements. Clearly, if the sets A and B are disjoint then their intersection $(A \cap B)$ is a null set. Therefore, the sets A and B are disjoint when $A \cap B = \phi$, where ϕ is the null set.

Examples: (i) If $A = \{1,2,3,4\}$ and $B = \{5,6,7\}$, then $A \cap B = \phi$.

(ii) If A = set of all positive integers and B = set of all negative integers, then $A \cap B = \phi$, i.e. there is no integer which is positive and negative both.

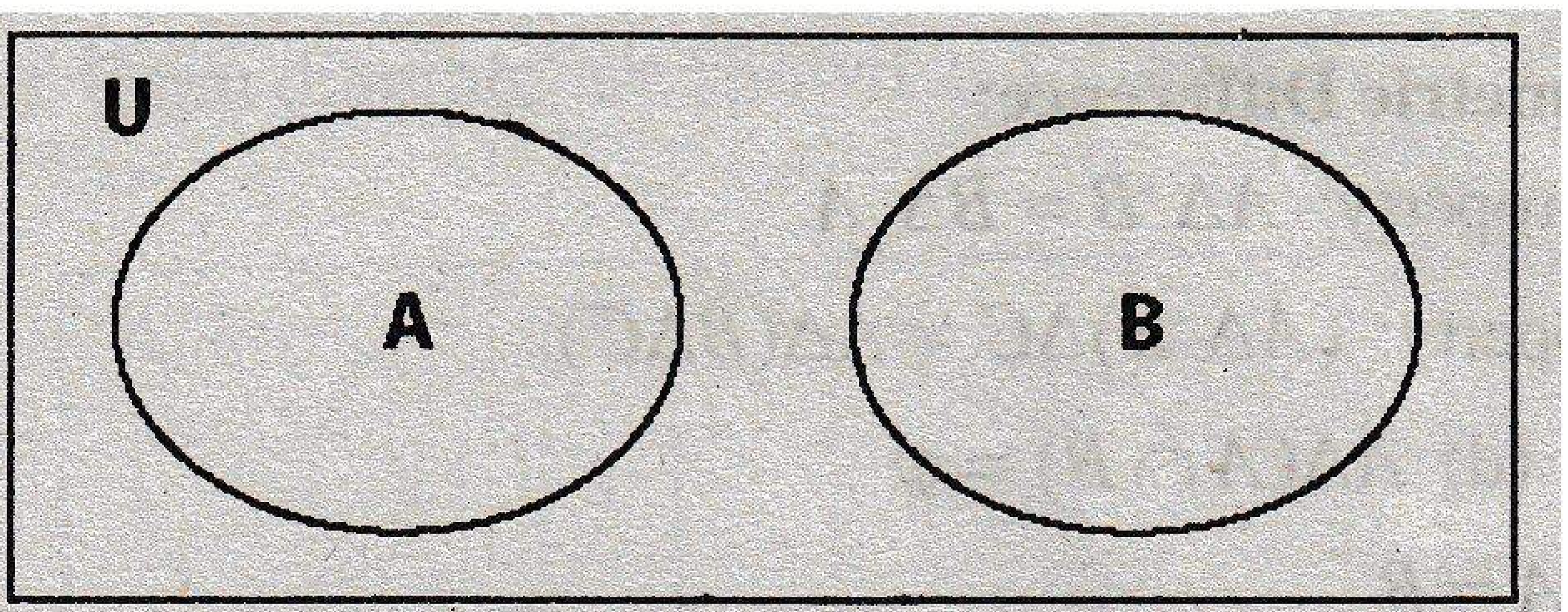


Figure: Disjoint sets A and B;

4. **Difference of two sets:** If A and B are any two sets, then the difference of A and B is the set of elements which belongs to A but does not belong to B. The difference of A and B is denoted by $A-B$ or $A\sim B$. It is read as: " A difference B" or simply "A minus B". Symbolically, we write $A-B=\{x: x\in A \text{ and } x\notin B\}$.

   Example: If $A=\{0,2,4,9\}$ and $B=\{0,3,6,8,9\}$, then $A-B=\{2,4\}$ and $B-A=\{3,6,8\}$.

   Properties of Difference of two sets:

   (a) $A-A=\phi$ (b) $A-\phi=A$

   (c) $A-B\subseteq A$

   (d) $A-B, A\cap B$ and $B-A$ are mutually disjoint.

   (e) $(A-B)\cap B=\phi$ (f) $(A-B)\cup A=A$.

5. **Complement of a set:** Complement of a set A is defined with respect to its universal set U. It is the set of all elements of the universal set U which do not belong to A, that is, the difference of the universal set U and the set A. It is also called negation of A. We denote the complement of A by A^c or, $\overline{A}$ or, A' or, $\sim A$. Symbolically, if A^c is the complement of A, then $A^c=U-A=\{x: x\in U \text{ and } x\notin A\}$.

   Example: Let $U=\{1,2,3,4,5,6,7,8,9\}$ and $A=\{1,3,5,7,\}$.

   Then complement of $A=A^c=U-A=\{2,4,6,8,9\}$.

   Properties of Complement:

   (a) The intersection of a set A and its complement A^c is a null set, i.e. $A\cap A^C=\phi$.

   (b) The union of a set A and its complement A^C is the universal set, i.e. $A\cup A^C=U$.

   (c) The complement of the universal set is the empty set and the complement of the empty set is the universal set, i.e. $U^c=\phi$ and the $\phi'=U$.

   (d) The complement of the complement of a set is the set itself, i.e. $(A^c)^c=A$.

6. **Symmetric Difference of sets:** The set having only the uncommon elements of two sets is called the symmetric difference of the sets. Let A and B be two sets. Then the difference set of A and B is called symmetric difference if it contains all those elements which either belong to A or to B, but not in both. This symmetric difference of A and B is denoted by $A\Delta B=(A-B)\cup(B-A)$. **Example**, if $A=\{1,2,3\}$ and $B=\{3,4,5\}$, then $A\Delta B=(A-B)\cup(B-A)=\{1,2,\}\cup\{4,5\}=\{1,2,4,5\}$.

Properties of Symmetric Difference:

(a) Commutative property: $A\Delta B = B\Delta A$.

(b) Associative property: $(A\Delta B)\Delta C = A\Delta(B\Delta C)$.

(c) $A\Delta A = \varphi$ (d) $A\Delta(A \cap B) = \varphi$

(e) $A\Delta B = \varphi \Leftrightarrow A = B$

7. **Partition of a set:** When a universal set U is sub-divided into subsets, which are disjoint but make into a union, then it is called partition of a set. We can say $U = A_1 \cup A_2 \cup A_3 \cup ..... \cup A_n$ is such that if $a \in U$, then it belongs to one and only one of the subsets. Therefore, $A_i \cap A_j = \varphi$.

Example, Let $U = \{1,2,3,4,5,6,7,8,9\}$. Now

(i) $[\{1,2,3\},\{2,4\},\{5,6,7\}]$ is not a partition, because 2 is in both the first and the second subsets.

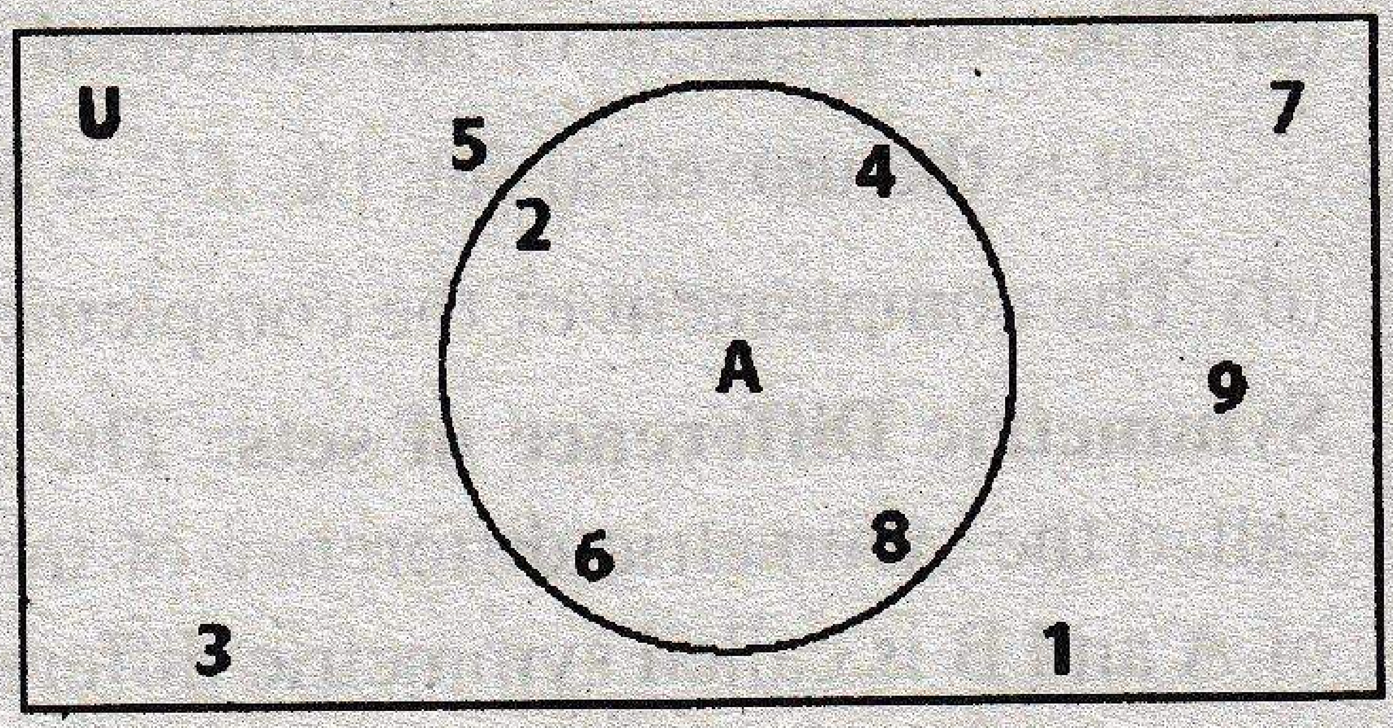
(ii) $[\{1,5\},\{3,7\},\{2,4,6\}]$ is a partition where no element is common and the union of the three subsets makes the set.

3.6 VENN DIAGRAM

The relationship between sets can be better understood with the help of pictorial representation. These diagrams are called Venn-Euler diagrams or simply Venn diagrams. These diagrams consist of rectangles and circles. In Venn-diagrams, a universal set U is represented by a large rectangle and each subset of U is represented by the circle within the rectangle. Venn diagrams are used to illustrate the set relations such as the subset and the operations with sets. But these cannot be used to prove any statement(s) regarding sets.

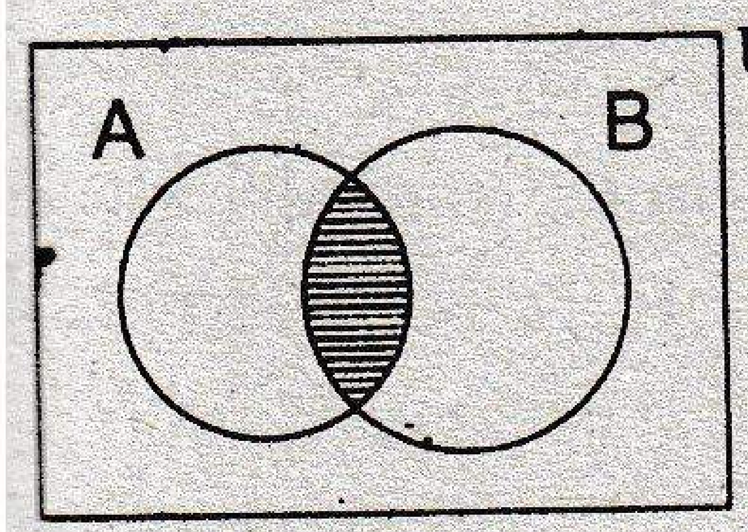
The universal set U is generally represented by points inside a rectangle, its subsets (A, B, etc) are represented by points inside the circles drawn within the rectangle. The circles representing the subsets A and B of U overlap each other if they have some common element, these circles are non-intersecting when A and B have no common elements.

Example: Let $U = \{1,2,3,4,5,6,7,8,9\}$ be the universal set and $A = \{2,4,6,8\}$ be one of its subsets. Venn diagram of U and A has been shown in the besides figure:

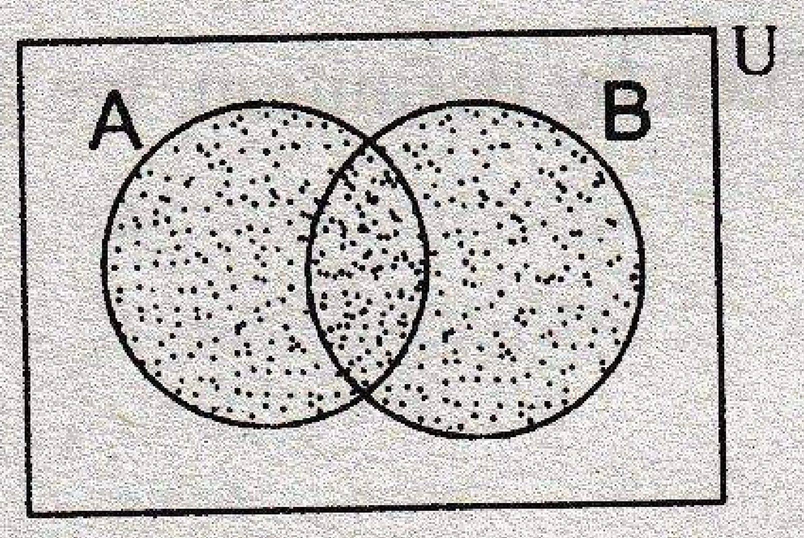


Venn Diagram

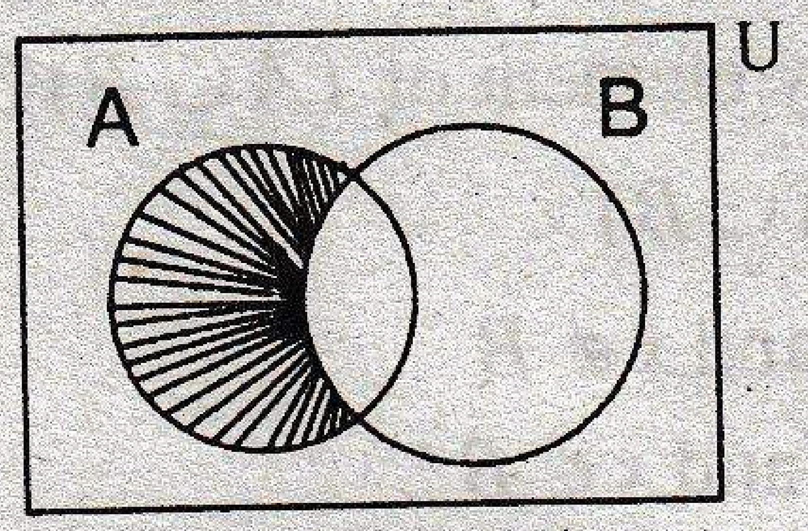
Some other examples of Venn diagram are shown as follows:-



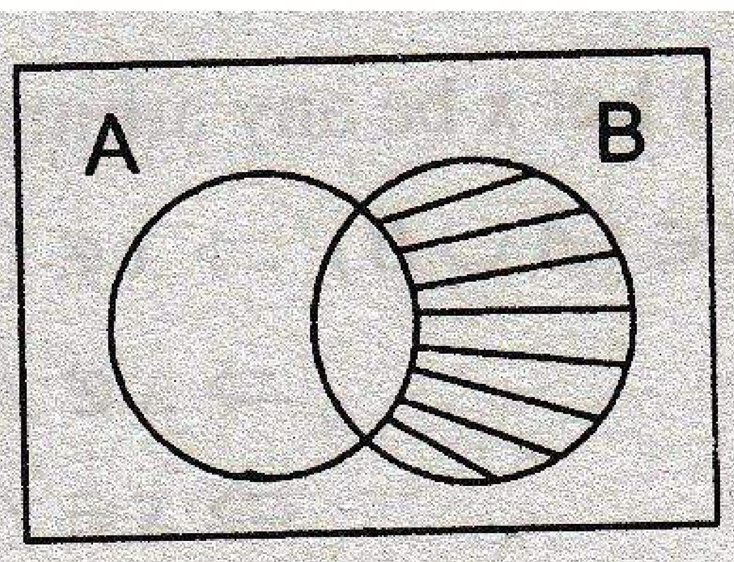
$A \cap B$



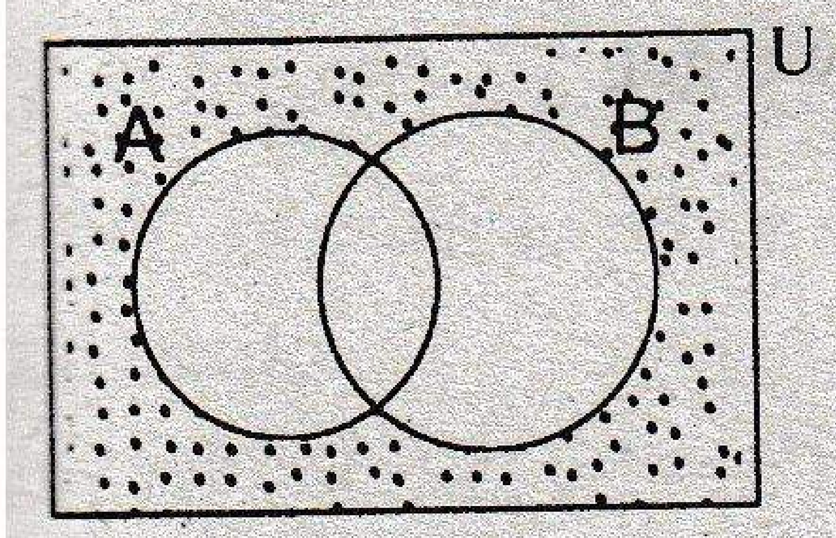
$A \cup B$



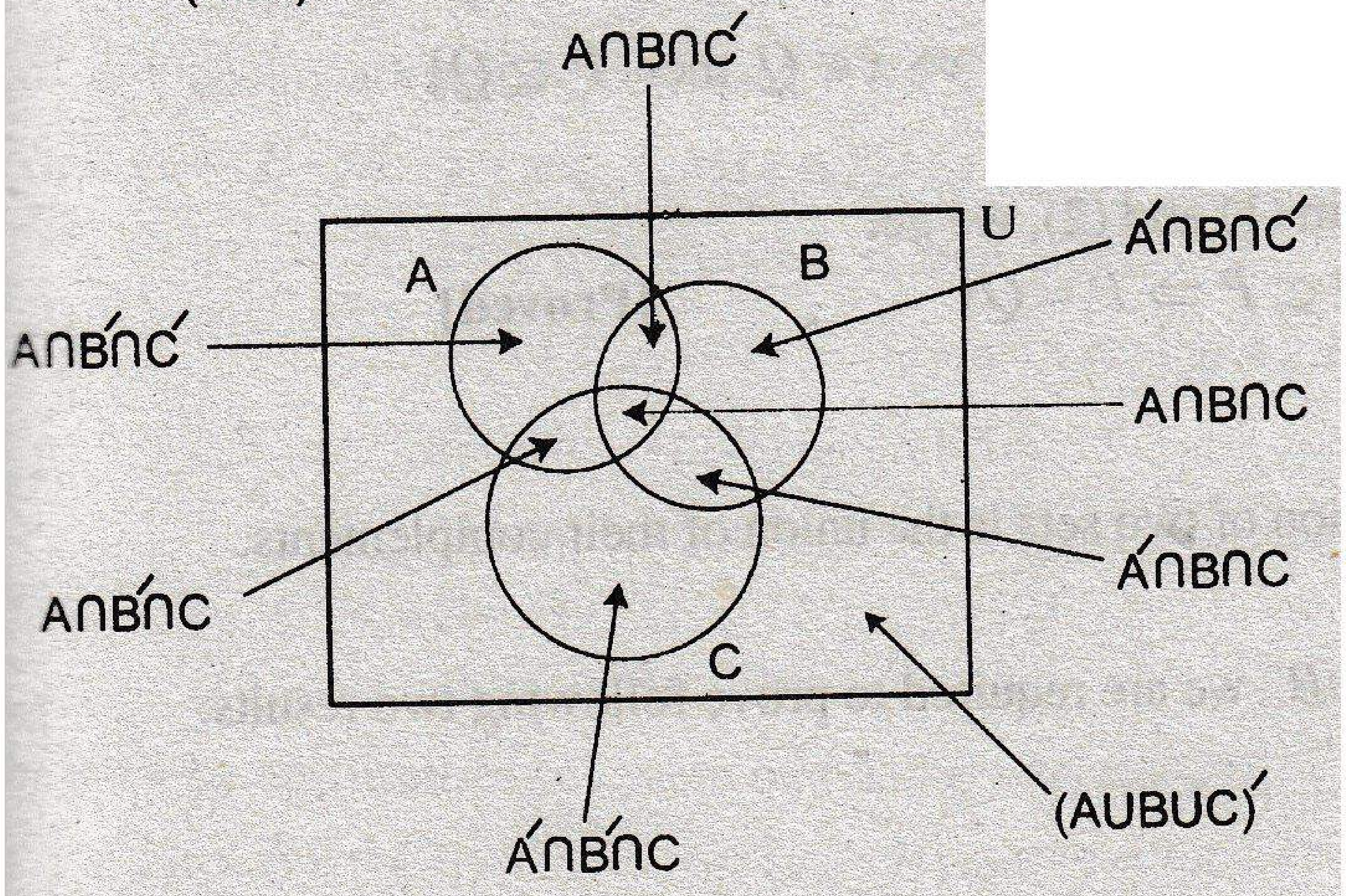
$A - B$ or $A \cap B'$



$A' \cap B$



$(A \cup B)'$



3.7 DE-MORGAN'S LAWS

(a) De-Morgan's Laws on complement of sets

Statement: For any two sets A and B,

(i) Complement of the union of two sets is the intersection of their complements,

i.e. $(A \cup B)' = A' \cap B'$

(ii) Complement of the intersection of two sets is the union of their complements,

i.e. $(A \cap B)' = A' \cup B'$

(i) De-Morgan's first law

Statement: Complement of the union of two sets is the intersection of their complements,

i.e. $(A \cup B)' = A' \cap B'$

Proof: In order to prove $(A \cup B)' = A' \cap B'$, we are required to prove following two results:

$(A \cup B)' \subseteq A' \cap B'$ ....(a)

$A' \cap B' \subseteq (A \cup B)'$ ....(b)

(a) **Let** x be any arbitrary element of $(A \cup B)'$. Then by definition, we get

$x \in (A \cup B)' \Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in (A' \cap B')$

$\therefore (A \cup B)' \subseteq A' \cap B'$ ....(1) $[\because$ if $x \in P \Rightarrow x \in Q$, then $P \subseteq Q]$

(b) Again, let y be any arbitrary element of $(A' \cap B')$. Then by definition, we get

$y \in (A' \cap B') \Rightarrow y \in A'$ and $y \in B'$

$\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin (A \cup B)$

$\Rightarrow y \in (A \cup B)'$

$\therefore A' \cap B' \subseteq (A \cup B)'$ ....(2) $[\because$ if $x \in P \Rightarrow x \in Q$, then $P \subseteq Q]$

By the definition of equality of sets, from (1) and (2), we get

$(A \cup B)' = A' \cap B'$ $[\because P \subseteq Q$ and $Q \subseteq P \Rightarrow P = Q]$ **[Proved]**

(ii) De-Morgan's second law

Statement: Complement of the intersection of two sets is the union of their complements, i.e. $(A \cap B)' = A' \cup B'$

Proof: In order to prove $(A \cap B)' = A' \cup B'$, we are required to prove following two results:

$(A \cap B)' \subseteq A' \cup B'$ ....(a)

$A' \cup B' \subseteq (A \cap B)'$ ....(b)

(a) Let x be any arbitrary element of $(A \cap B)'$. Then by definition, we get

$x \in (A \cap B)' \Rightarrow x \notin (A \cap B)$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in (A' \cup B')$

$\therefore (A \cap B)' \subseteq A' \cup B'$ ....(1) $[\because$ if $x \in P \Rightarrow x \in Q$, then $P \subseteq Q]$

(b) Again, let y be any arbitrary element of $A' \cup B'$. Then by definition, we get

$y \in (A' \cup B') \Rightarrow y \in A'$ or $y \in B'$

$\Rightarrow y \notin A$ or $y \notin B$

$\Rightarrow y \notin (A \cap B)$

$\Rightarrow y \in (A \cap B)'$

$\therefore A' \cup B' \subseteq (A \cap B)'$..........(2) $[\because$ if $x \in P \Rightarrow x \in Q$, then $P \subseteq Q]$

By the definition of equality of sets, from (1) and (2), we get

$(A \cap B)' = A' \cup B'$ $[\because P \subseteq Q$ and $Q \subseteq P \Rightarrow P = Q]$ [Proved]

(b) De-Morgan's laws on difference of sets.

Statement: Let A, B, and C be any three sets, then

(i) $A - (B \cup C) = (A - B) \cap (A - C)$.

(ii) $A - (B \cap C) = (A - B) \cup (A - C)$.

(i) De-Morgan's first law:

Statement: Difference of a union is the intersection of differences, i.e.

$A - (B \cup C) = (A - B) \cap (A - C)$

Proof: In order to prove $A - (B \cup C) = (A - B) \cap (A - C)$, we are required to prove following two results:

$A - (B \cup C) \subseteq (A - B) \cap (A - C)$...............$(a)$

$(A - B) \cap (A - C) \subseteq A - (B \cup C)$...............$(b)$

(a) Let x be any arbitrary element of $A - (B \cup C)$.

$\therefore x \in A - (B \cup C) \Rightarrow x \in A$ and $x \notin (B \cup C)$

$\Rightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Rightarrow (x \in A$ and $x \notin B)$ and $(x \in A$ and $x \notin C)$

$\Rightarrow x \in (A - B)$ and $x \in (A - C)$

$\Rightarrow x \in (A - B) \cap (A - C)$

$\therefore A - (B \cup C) \subseteq (A - B) \cap (A - C)$...........(1)

(b) Let y be any arbitrary element of $(A - B) \cap (A - C)$.

$\therefore y \in (A - B) \cap (A - C) \Rightarrow y \in (A - B)$ and $y \in (A - C)$

$\Rightarrow (y \in A$ and $y \notin B)$ and $(y \in A$ and $y \notin C)$

$\Rightarrow y \in A$ and $(y \notin B$ and $y \notin C)$

$\Rightarrow y \in A$ and $y \notin (B \cup C)$

$\Rightarrow y \in A - (B \cup C)$

$\therefore (A - B) \cap (A - C) \subseteq A - (B \cup C)$................(2)

Hence from (1) and (2), by the definition of equality of sets, we get

$A - (B \cup C) = (A - B) \cap (A - C)$ [Proved]

(ii) De-Morgan's second law:

Statement: Difference of an intersection is the union of the differences, i.e.

$A - (B \cap C) = (A - B) \cup (A - C)$

Proof: In order to prove $A-(B\cap C)=(A-B)\cup(A-C)$, we are required to prove following two results

$A-(B\cap C)\subseteq(A-B)\cup(A-C)$.................(a)

$(A-B)\cup(A-C)\subseteq A-(B\cap C)$.................(b)

(a) Let x be any arbitrary element of $A-(B\cap C)$.

$\therefore x\in A-(B\cap C)\Rightarrow x\in A$ and $x\notin(B\cap C)$

$\Rightarrow x\in A$ and $(x\notin B$ or $x\notin C)$

$\Rightarrow (x\in A$ and $x\notin B)$ or $(x\in A$ and $x\notin C)$

$\Rightarrow x\in(A-B)$ or $x\in(A-C)$

$\Rightarrow x\in(A-B)\cup(A-C)$

$\therefore A-(B\cap C)\subseteq(A-B)\cup(A-C)$...............(1)

(b) Let y be any arbitrary element of $(A-B)\cap(A-C)$.

$\therefore y\in(A-B)\cup(A-C)\Rightarrow y\in(A-B)$ or $y\in(A-C)$.

$\Rightarrow (y\in A$ and $y\notin B)$ or $(y\in A$ and $y\notin C)$

$\Rightarrow y\in A$ and $(y\notin B$ or $y\notin C)$

$\Rightarrow y\in A$ and $y\notin(B\cap C)$

$\Rightarrow y\in A-(B\cap C)$

$\therefore (A-B)\cup(A-C)\subseteq A-(B\cap C)$.............(2)

Hence from (1) and (2), by the definition of equality of sets, we get

$A-(B\cap C)=(A-B)\cup(A-C)$ [Proved]

3.8 SOME RESULTS ON SET OPERATIONS

Result-1: Let A, B, and C be any three sets, prove that $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Proof: In order to prove $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$, we are required to prove following two results:

$A\cup(B\cap C)\subseteq(A\cup B)\cap(A\cup C)$..................(1)

$(A\cup B)\cap(A\cup C)\subseteq A\cup(B\cap C)$..................(2)

(1) Let x be any arbitrary element of $A\cup(B\cap C)$.

$\therefore x\in A\cup(B\cap C)\Rightarrow x\in A$ or $x\in(B\cap C)$

$\Rightarrow x\in A$ or $(x\in B$ and $x\in C)$

$\Rightarrow (x\in A$ or $x\in B)$ and $(x\in A$ or $x\in C)$

$\Rightarrow x\in(A\cup B)$ and $x\in(A\cup C)$

$\Rightarrow x\in(A\cup B)\cap(A\cup C)$

$\therefore A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$..........$(a)$

(2) Let y be any arbitrary element of $(A \cup B) \cap (A \cup C)$.

$\therefore y \in (A \cup B) \cap (A \cup C) \Rightarrow y \in (A \cup B)$ and $y \in (A \cup C)$

$\Rightarrow (y \in A$ or $y \in B)$ and $(y \in A$ or $y \in C)$

$\Rightarrow y \in A$ or $(y \in B$ and $y \in C)$

$\Rightarrow y \in A$ or $y \in (B \cap C)$

$\Rightarrow y \in A \cup (B \cap C)$

$\therefore (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$..................$(b)$

Hence from (a) and (b), by the definition of equality of sets, we get

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ [Proved]

Result-2: Prove that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

Proof: In order to prove $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$, we are required to prove following two results:

$(A - B) \cup (B - A) \subseteq (A \cup B) - (A \cap B)$.................(1)

$(A \cup B) - (A \cap B) \subseteq (A - B) \cup (B - A)$.................(2)

(1) Let x be any arbitrary element of $(A - B) \cup (B - A)$.

$\therefore x \in (A - B) \cup (B - A) \Rightarrow x \in (A - B)$ or $x \in (B - A)$

$\Rightarrow (x \in A$ and $x \notin B)$ or $(x \in B$ and $x \notin A)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \notin A$ or $x \notin B)$

$\Rightarrow x \in (A \cup B)$ and $x \notin (A \cap B)$

$\Rightarrow x \in (A \cup B) - (A \cap B)$

$\therefore (A - B) \cup (B - A) \subseteq (A \cup B) - (A \cap B)$..............$(a)$

(2) Let y be any arbitrary element of $(A \cup B) - (A \cap B)$.

$\therefore y \in (A \cup B) - (A \cap B) \Rightarrow y \in (A \cup B)$ and $y \notin (A \cap B)$

$\Rightarrow (y \in A$ or $y \in B)$ and $(y \notin A$ or $y \notin B)$

$\Rightarrow (y \in A$ and $y \notin B)$ or $(y \in B$ and $y \notin A)$

$\Rightarrow y \in (A - B)$ or $y \in (B - A)$

$\Rightarrow y \in (A - B) \cup (B - A)$

$\therefore (A \cup B) - (A \cap B) \subseteq (A - B) \cup (B - A)$..............$(b)$

Hence from (a) and (b), by the definition of equality of sets, we get

$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$ [Proved]

Result-3: If A and B are two sets, then
$(i) A-B \subseteq A, (ii) A \cap (B-A) = \varphi$
$(iii) A \cup B = (A-B) \cup B, (iv) A-B = A \cap B' = B' - A'$.

Proof: (i) Let x be any arbitrary element of $(A-B)$.

$\therefore x \in (A-B) \Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A$ or $x \notin B$ $[\because A \cap B = \varphi]$

$\Rightarrow x \in A$ [$\because x$ is an element of set A, not set B.]

$\therefore A-B \subseteq A$ [Proved]

(ii) Let x be any arbitrary element of $A \cap (B-A)$.

$\therefore x \in A \cap (B-A) \Rightarrow x \in A$ and $x \in (B-A)$

$\Rightarrow x \in A$ and $(x \in B$ and $x \notin A)$ $\Rightarrow (x \in A$ and $x \notin A)$ and $x \in B$ $\Rightarrow \phi$ and $x \in B$ $\Rightarrow \phi$

$\therefore A \cap (B-A) \subseteq \phi$.................(1)

Also, since ϕ is a subset of every set,

$\therefore \phi \subseteq A \cap (B-A)$..............(2)

Hence from (1) and (2), by the definition of equality of sets, we get

$A \cap (B-A) = \phi$ [Proved]

(iii) In order to prove $A \cup B = (A-B) \cup B$, we are required to prove following two results:

$A \cup B \subseteq (A-B) \cup B$..................(1)

$(A-B) \cup B \subseteq A \cup B$..................(2)

(1) Let x be any arbitrary element of $A \cup B$.

$\therefore x \in (A \cup B) \Rightarrow x \in A$ or $x \in B$

$\Rightarrow (x \in A$ or $x \in B)$ or ϕ $\Rightarrow (x \in A$ or $x \in B)$ or $(x \in B$ and $x \notin B)$

$\Rightarrow (x \in A$ and $x \notin B)$ or $x \in B$ $\Rightarrow x \in (A-B)$ or $x \in B$

$\Rightarrow x \in (A-B) \cup B$

$\therefore A \cup B \subseteq (A-B) \cup B$..............$(a)$

(2) Let y be any arbitrary element of $(A-B) \cup B$.

$\therefore y \in (A-B) \cup B \Rightarrow y \in (A-B)$ or $y \in B$

$\Rightarrow (y \in A$ and $y \notin B)$ or $y \in B$ $\Rightarrow (y \in A$ or $y \in B)$ or $(y \in B$ and $y \notin B)$

$\Rightarrow (y \in A$ or $y \in B)$ or ϕ $\Rightarrow y \in A$ or $y \in B$

$\Rightarrow y \in (A \cup B)$ $\therefore (A-B) \cup B \subseteq A \cup B$.............$(b)$

Hence from (a) and (b), by the definition of equality of sets, we get

$\therefore A-B = A \cap B'$ **[Proved]**

(iv) In order to prove $A-B = A \cap B' = B' - A'$, we are required to prove following two results:

$A-B = A \cap B'$....................(1)

$A \cap B' = B' - A'$....................(2)

(1) Let $x \in (A - B) \Rightarrow (x \in A \text{ and } x \notin B)$

$\Rightarrow (x \in A \text{ and } x \in B')$ $\qquad \Rightarrow x \in (A \cap B')$

$\therefore (A - B) \subseteq (A \cap B')$.............(a)

Also, let $y \in (A \cap B') \Rightarrow (y \in A \text{ and } y \in B')$

$\Rightarrow (y \in A \text{ and } y \notin B)$ $\qquad \Rightarrow y \in (A - B)$

$\therefore (A \cap B') \subseteq (A - B)$.............(b)

Hence from (a) and (b), by the definition of equality of sets, we get

$A - B = A \cup B'$........................(A)

(2) Let $x \in A \cap B' \Rightarrow (x \in A \text{ and } x \in B')$

$\Rightarrow (x \in B' \text{ and } x \notin A')$ $\qquad \Rightarrow x \in (B' - A')$

$\therefore (A \cap B') \subseteq (B' - A')$......................(c)

Also, let $y \in (B' - A') \Rightarrow (y \in B' \text{ and } y \notin A')$

$\Rightarrow (y \in A \text{ and } y \in B')$ $\qquad \Rightarrow y \in (A \cap B')$

$\therefore (B' - A') \subseteq (A \cap B')$.....................(d)

Hence from (c) and (d), by the definition of equality of sets, we get

$A \cap B' = B' - A'$...........................(B)

Thus, from (A) and (B), we get

$A - B = A \cap B' = B' - A'$ **[Proved]**

Result-4: Prove that $(i)(A \cup B) \cap (A \cup B') = A, (ii)(A \cap B) \cup (A \cap B') = A.$

Proof: **(i)** In order to prove $(A \cup B) \cap (A \cup B') = A,$ we are required to prove following two results:

$(A \cup B) \cap (A \cup B') \subseteq A$...............(1)

$A \subseteq (A \cup B) \cap (A \cup B')$...............(2)

(1) Let x be any arbitrary element of $(A \cup B) \cap (A \cup B')$.

$\therefore x \in (A \cup B) \cap (A \cup B') \Rightarrow x \in (A \cup B) \text{ and } x \in (A \cup B')$

$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ and } x \notin B)$ $\qquad \Rightarrow x \in A \text{ or } (x \in B \text{ and } x \notin B)$

$\Rightarrow x \in A \text{ or } \phi$ $\qquad \Rightarrow x \in A$

$\therefore (A \cup B) \cap (A \cup B') \subseteq A$..................(a)

(2) Let y be any arbitrary element of A.

$\therefore y \in A \Rightarrow y \in A \text{ or } \phi$

$\Rightarrow y \in A \text{ or } (y \in B \text{ and } y \notin B)$ $\qquad \Rightarrow (y \in A \text{ or } y \in B) \text{ and } (y \in A \text{ and } y \notin B)$

$\Rightarrow y \in (A \cup B) \text{ and } y \in (A \cup B')$ $\qquad \Rightarrow y \in (A \cup B) \cap (A \cup B')$

$\therefore A \subseteq (A \cup B) \cap (A \cup B')$................(b)

Hence from (a) and (b), by the definition of equality of sets, we get

$(A\cup B)\cap(A\cup B')=A$ [Proved]

(ii) In order to prove $(A\cap B)\cup(A\cap B')=A$, we are required to prove following two results:

$(A\cap B)\cup(A\cap B')\subseteq A$...................(1)

$A\subseteq(A\cap B)\cup(A\cap B')$..................(2)

(1) Let x be any arbitrary element of $(A\cap B)\cup(A\cap B')$.

$\therefore x\in(A\cap B)\cup(A\cap B')\Rightarrow x\in(A\cap B)$ or $x\in(A\cap B')$

$\Rightarrow(x\in A$ and $x\in B)$ or $(x\in A$ and $x\notin B)$ $\Rightarrow x\in A$ or $(x\in B$ and $x\notin B)$

$\Rightarrow x\in A$ or ϕ $\Rightarrow x\in A$

$\therefore(A\cap B)\cup(A\cap B')\subseteq A$....................$(a)$

(2) Let y be any arbitrary element of A.

$\therefore y\in A\Rightarrow y\in A$ or ϕ

$\Rightarrow y\in A$ or $(y\in B$ and $y\notin B)$ $\Rightarrow(y\in A$ and $y\in B)$ or $(y\in A$ and $y\notin B)$

$\Rightarrow y\in(A\cap B)$ or $y\in(A\cap B')$ $\Rightarrow y\in(A\cap B)\cup(A\cap B')$

$\therefore A\subseteq(A\cap B)\cup(A\cap B')$..................$(b)$

Hence from (a) and (b), by the definition of equality of sets, we get

$(A\cap B)\cup(A\cap B')=A$ **[Proved]**

Result-5: Prove that $(B-A')=B\cap A$.

Proof: In order to prove $(B-A')=B\cap A$, we are required to prove following two results:

$(B-A')\subseteq B\cap A$................(1)

$B\cap A\subseteq(B-A')$...............(2)

(1) Let x be any arbitrary element of $(B-A')$

$\therefore x\in(B-A')\Rightarrow x\in B$ and $x\notin A'$

$\Rightarrow x\in B$ and $x\in A$ $\Rightarrow x\in(B\cap A)$

$\therefore(B-A')\subseteq(B\cap A)$............$(a)$

(2) Let y be any arbitrary element of $(B\cap A)$.

$\therefore y\in(B\cap A)\Rightarrow y\in B$ and $y\in A$

$\Rightarrow y\in B$ and $y\notin A'$ $\Rightarrow y\in(B-A')$

$\therefore(B\cap A)\subseteq(B-A')$................$(b)$

Hence from (a) and (b), by the definition of equality of sets, we get

$(B-A')=B\cap A$ **[Proved]**

Result-6: (a) Prove that null set is a subset of every set.
(b) Prove that every set is a subset of its own.
(c) Prove that null set is unique.
(d) Prove that every set has a subset.
(e) Prove that every set has not a proper subset.

Proof:

(a) If possible, let $\phi \not\subseteq A$. Since ϕ is not a subset of set A, ϕ has at least one element which does not belong to set A. But from definition we know, ϕ has no element. This is a contradiction. So $\phi \not\subseteq A$ is not true. Hence null set ϕ is a subset of every set. **[Proved]**

(b) If possible, let $A \not\subseteq A$ Since set A is not a subset of set A, set A has at least one element, which does not belong to set A. But it is impossible. So $A \not\subseteq A$ is not true. Hence every set is a subset of its own. **[Proved]**

(c) Let ϕ_1 and ϕ_2 are two null sets. Since null set is a subset of every set, $\phi_1 \subseteq \phi_1$, and $\phi_2 \subseteq \phi_1$.
$\therefore \phi_1 = \phi_2$. So null set is unique. **[Proved]**

(d) Let the set A has no subset. But we know, every set is a subset of its own. $\therefore$ let $A \subseteq A$. Since A has at least one subset as itself, any set has at least a subset as itself. So every set has a subset. **[Proved]**

(e) Since null set has no element, its proper subset is impossible. Since null set is a set and it has no proper subset, every set does not contain a proper subset. So every set has not a proper subset. **[Proved]**

Result-7 : Show that $A \cap B = \phi$ if and only if $A - B = A$.

Solution: Here $A - B = A$

$\Rightarrow -B = A - A \Rightarrow -B = \{\} \Rightarrow B = \{\}$

$\therefore L.H.S. = A \cap B = A \cap \{\} = \{\} = \phi = R.H.S.$ [Proved]

3.9 CARTESIAN PRODUCT OF TWO SETS

Ordered Pair: An ordered pair consists of two elements, say a and b in which one of them say a is designated as the first element and the other as the second element. An ordered pair is usually denoted by (a, b).

The object a is called the first member or first coordinate of the ordered pair (a, b) and the object b is called the second member or second coordinate of the ordered pair (a, b). There can be ordered pair which have the same first and second elements such as (1, 1), (a, a), etc. Two ordered pairs (a, b) and (c, d) are said to be equal if and only if a = c and b = d. In other words:

$(a,b) = (c,d) \Leftrightarrow a = c \text{ and } b = d.$

Thus, the ordered pairs (2,3) and (2,3) are equal while the ordered pairs (2,3) and (3,2) are different. The reader must note the difference between the set {2,3} and the ordered pair (2,3). We have $\{2,3\}=\{3,2\}$, but $(2,3)\neq(3,2)$.

In coordinate geometry, the points in the Cartesian plane can be represented by an ordered pair (x, y) of real numbers, where the first member x is called the abscissa and the second member y called the ordinate.

Cartesian Product of Two sets: If A and B be any two sets, then the set of all distinct ordered pairs whose first member belongs to set A and second member belongs to set B is called the Cartesian product of A and B in that order. It is denoted by $A\times B$, to be read as '*A* cross *B*.'

Symbolically, $A\times B=\{(a,b): a\in A \text{ and } b\in B\}$

The Cartesian product $A\times B$ is also called the product set of A and B.

Remarks:

1. In the product set $B\times A$, the first member is of B and second is of A.
2. If A and B are finite sets, then $(A\times B)=n(A)\times n(B)$.
3. If set A has m elements and the set B has n elements, then the product set $A\times B$ has mn elements.
   For example: If $A=\{a,b\}$, and $B=\{1,2\}$, then $A\times B=\{(a,1),(a,2),(b,1),(b,2)\}$

ILLUSTRATIONS

Illustration-01: Write down the following statements in set-theoretic notations:

1. 3 is an element of a set A,
2. 4 does not belong to a set B,
3. C is a subset of D,
4. P and Q are disjoint sets.

Solution: Set-theoretic notations of the given statements are:

1. $3\in A$ 2. $4\notin B$ 3. $C\subset D$ 4. $P\cap Q=\phi$, where ϕ is the null set.

Illustration-02: Let $A=\{x: x \text{ is a letter in English alphabet}\}$ be the universal set.

$V=\{x: x \text{ is a vowel}\}$, $C=\{x: x \text{ is a consonant}\}$,

$N=\{x: x \text{ is a letter in your full name}\}$.

Describe the four sets by listing the elements of each set.

Solution: $A=\{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z\}$,

$V=\{a,e,i,o,u\}$,

$C=\{b,c,d,f,g,h,j,k,l,m,n,p,q,r,s,t,v,w,x,y,z\}$,

$N=\{rafique\}=\{a,e,f,i,q,r,u\}$,

Illustration-03: Represent the following sets in Tabular (or Roster) form

1. Set of all factors of 30,
2. $X = \{a : a$ is a perfect square and $2 < a \leq 49\}$,
3. $Y = \{x : x$ is an even natural number greater than $20\}$.

Solution:

1. Here, each of the numbers $1,2,3,5,6,10,15,30$ is a factor of 30. Let A be the set of factors of 30. Then the tabular form of A is: $A = \{1,2,3,5,6,10,15,30\}$.
2. By definition $'a'$ is a perfect square and $2 < a \leq 49$. Clearly, the values of $'a'$ satisfying the given rules are: 4,9,16,25,36, and 49. Therefore, the Roster form of set X is, $X = \{4,9,16,25,36,49\}$.
3. By given condition, even natural numbers greater than 20 are: $22,24,26,28,30,...$ which are the elements of the set Y. Therefore, the tabular form of set Y is: $Y = \{22,24,26,28,30,......\}$

Illustration-04: Write the following sets in set-builder form:

I. set of letters in the word 'Statistics',

II. $A = \{3,6,9,12,15,.....\}$,

III. set of integers either equal or greater than 3 but less than 25.

Solution: (i) Let S be the set of letters in the word 'Statistics'.

Set-builder form of the set S is given by:

$S = \{x : x$ is a letter in the word 'Statistics;$\}$

(ii) Set-builder form of the set A is given below:

$A = \{x : x \in N$ and x is a multiple of $3\}$, where N is the set of natural numbers.

(iii) Denoting the given set by P, the set-builder form of P is given below:

$P = \{x : x$ is an integer and $3 < x < 25\}$.

Illustration-05: State whether each of the following sets is finite or infinite:

(i) $A = \{x : x$ is an odd integer greater than $100\}$, (ii) $B = \{x : x$ is real and $-1 \leq x < 1\}$,

(iii) $C = \{x : x$ is an odd negative integer greater than $(-150)\}$.

Solution: (i) By definition of set A, we have $A = \{101,103,105,107,.......\}$.

Clearly, the set A does not contain a specific number of elements. Therefore, the set A is infinite.

(ii) By definition of set B, it is clear that any real value of x lying in the interval $-1 < x < 1$ is an element of set B. Hence, it is evident that the number of elements contained in set B cannot be stated by a specific number. Therefore, the set B is infinite.

(iii) By definition of set C, we have $C = \{-1,-3,-5,-7,....,-147,-149\}$.

Clearly, the set C contains a specific number of elements. Therefore, the set C is finite.

Illustration-06: If $A = \{1,2,3\}$, then there exist eight subsets of A. Determine all these subsets of A.

Solution: We know that null set ϕ is a subset of every set. $\therefore \phi \subseteq A$.

Number of subsets of A containing one element are: $\{1\},\{2\},\{3\}$.

Number of subsets of A containing two elements are: $\{1,2\},\{2,3\},\{1,3\}$.

Finally, every set is a subset of itself; hence $A \subseteq A$.

Therefore, eight subsets of A are: ϕ, $\{1\},\{2\},\{3\},\{1,2\},\{2,3\},\{1,3\},\{1,2,3\}$.

Illustration-07: Suppose $V = \{x : x+2=0\}, R = \{x : x^2+2x=0\}, S = \{x : x^2+x-2=0\}$.

Find the value of x for which the sets V, R, and S are equal.

Solution: $x+2=0 \Rightarrow x=-2$

$$x^2+2x=0 \Rightarrow x(x+2)=0 \Rightarrow x=0,-2.$$

$$x^2+x-2=0 \Rightarrow x^2+2x-x-2 \Rightarrow x(x+2)-1(x+2)=0 \Rightarrow x=1,-2.$$

Therefore, the sets V, R, and S will be equal if $x=-2$ [Ans.]

Illustration-08: $A = \{1,3,a,\{1\},\{1,a\}\}$. State whether the following statements are true or false: (i) $1 \in A$, (ii) $\{1\} \in A$, (iii) $\{1\} \subset A$, (iv) $\phi \in A$, (v) $\phi \subset A$, (vi) $\{1,a\} \subset A$, (vii) $\{3,a\} \in A$, (viii) $\{1,a\} \in A$.

Solution: (i) $1 \in A$ is true, because 1 is an element of set A.

(ii) $\{1\} \in A$ is true, because $\{1\}$ is also an element of set A.

(iii) $\{1\} \subset A$ is true, because $\{1\}$ is contained in set A.

(iv) $\phi \in A$ is false, because ϕ is not an element of set A.

(v) $\phi \subset A$ is true, because ϕ is contained in set A.

(vi) $\{1,a\} \subset A$ is true, because $\{1,a\}$ is contained in set A.

(vii) $\{3,a\} \in A$ is false, because $\{3,a\}$ is not contained in set A.

(viii) $\{1,a\} \in A$ is true, because $\{1,a\}$ is an element of set A.

Illustration-09: Write, in words, the following set notation:

$A \subset B; x \notin A; A \supset B; \{0\}; A \not\subset B; A = \phi$.

Solution: $A \subset B : A$ is a proper subset of B.

$x \notin A : x$ is not an element of the set A.

$A \supset B : A$ contains B or, B is a subset of the set A.

$\{0\}$: Singleton set with only one element zero.

$A \not\subset B : A$ is not contained in B or, A is not a subset of B.

$A = \varphi$: A is a null set.

Illustration-10: Given $A=\{2,3,4\}$, and $B=\{4,5\}$, which of the following statements are correct and why? (i) $5\in A$, (ii) $\{5\}\subset A$, (iii) $4\in A$.

Solution: Given $A=\{2,3,4\}, B=\{4,5\}$

(i) $5\in A$ is not correct, because 5 is not an element of the set A.

(ii) $\{5\}\subset A$ is not correct, because the set $\{5\}$ is not a subset of the set A.

(iii) $4\in A$ is correct, because 4 is an element of the set A.

Illustration-11: If $A=\{2,3,4\}$, and $U=\{0,1,2,3,4\}$, which of the following statements are correct or incorrect. Give reasons. (i) $\{0\}\in A'$, (ii) $\phi\in A'$, (iii) $\{0\}\subset A'$, (iv) $0\in A'$, (v) $0\subset A'$.

Solution: Given $A=\{2,3,4\}, U=\{0,1,2,3,4\}$

$\therefore A'=U-A=\{0,1,2,3,4\}-\{2,3,4\}=\{0,1\}$

(i) $\{0\}\in A'$ is not correct, because $\{0\}$ is not an element of the set A'

(ii) $\phi\in A'$ is not correct, because ϕ is not an element of the set A'.

(iii) $\{0\}\subset A'$ is correct, because $\{0\}$ is a subset of the set A'.

(iv) $0\in A'$ is correct, because 0 is an element of the set A'

(v) $0\subset A'$ is not correct, because 0 is not a subset of the set A'.

Illustration-12: State whether each of these statements is correct or incorrect:
(i) $\{a,b,c\}=\{c,b,a\}$, (ii) $\{a,c,a,d,c,d\}\subseteq\{a,c,d\}$, (iii) $\{b\}\in\{\{b\}\}$, (iv) $\{b\}\subset\{\{b\}\}$, and (v) $\phi\subset\{\{b\}\}$.

Solution: (i) Given $\{a,b,c\}=\{c,b,a\}$

This statement is correct, because all the elements of both the sets are exactly same.

(ii) Given $\{a,c,a,d,c,d\}\subseteq\{a,c,d\}$

This statement is correct, because all the elements of both the sets are same. Two sets having same elements can be subsets of each other.

(iii) Given $\{b\}\in\{\{b\}\}$

This statement is correct, because $\{b\}$ is an element of set $\{\{b\}\}$.

(iv) Given $\{b\}\subset\{\{b\}\}$

This statement is not correct, because both sets have different elements.

(v) Given $\phi\subset\{\{b\}\}$

This statement is correct, because null set is a subset of any set.

Illustration-13: Let $A=\{0\}, B=\{0,1\}, C=\phi,\ D=\{\phi\}$,
$E=\{x: x \text{ is a human being 200 years old}\}$, $F=\{x: x\in A, \text{and } x\in B\}$.
State which of the followings are true, and which are false:
(i) $A\subset B$, (ii) $B=F$, (iii) $C\subset D$, (iv) $C=E$, (v) $A=F$, (vi) $F=1$, (vii) $E=C=D$

Solution:

(i) $A\subset B$ is true, because set B has only one more element than set A.

(ii) Given $A=\{0\}, B=\{0,1\}$. Since $F=\{x: x\in A, \text{and } x\in B\}, F=\{0\}$. So $B=F$ is false, because the numbers of elements of sets B and F are not equal.

(iii) Given $C\subset D \Rightarrow \phi\subset\{\phi\}\Rightarrow\{\ \}\subset\{\phi\}$. So $C\subset D$ is true, because set D has only one more element than set C.

(iv) Given $C=E\Rightarrow \phi=\{\ \}\Rightarrow\{\ \}=\{\ \}$. So $C=E$ is true, because both sets have no element.

(v) Given $A=F\Rightarrow\{0\}=\{0\}$. So $A=F$ is true, because both sets have same element.

(vi) Given $F=1\Rightarrow\{0\}=1$. So $F=1$ is true, because F is a set while 1 is an element.

(vii) Given $E=C=D\Rightarrow\{\ \}=\phi=\{\phi\}\Rightarrow\{\ \}=\{\ \}=\{\phi\}$.

So $E=C=D$ is false, because E and C are null sets while D is a singleton set.

Illustration-14: What is the relationship between the following sets:

$A=\{x: x$ is a letter in the word flower$\}$,

$B=\{x: x$ is a letter in the word flow$\}$,

$C=\{x: x$ is a letter in the word wolf$\}$,

$D=\{x: x$ is a letter in the word follow$\}$,

Solution: We can write the given sets as follows:

$A=\{f,l,o,w,e,r\}$, $B=\{f,l,o,w\}$,

$C=\{w,o,l,f\}$, $D=\{f,o,l,l,o,w\}$.

Here all the elements of the above sets except A are same. So B, C, D, are equal sets.

$\therefore B=C=D$

Also all these sets are subsets of the set A.

Illustration-15: Let $A=\{a,b,c\}, B=\{a,b\}, C=\{a,b,d\}, D=\{c,d\}$ and $E=\{d\}$.
State whether each of these statements is correct and give reasons:
$(i) B\subset A$, $(ii) D\neq C$, $(iii) C\supset E$, $(iv) D\supset E$, $(v) D\subset B$, $(vi) D=A$, $(vii) B\not\subset C$,
$(viii) E\subset A$, $(ix) E\not\subset B$, $(x) a\in A$, $(xi) a\subset A$, $(xii) \{a\}\in A$, $(xiii) \{a\}\subset A$.

Solution: Given $A=\{a,b,c\}, B=\{a,b\}, C=\{a,b,d\}, D=\{c,d\}, E=\{d\}$.

(i) $B\subset A$ is correct, because set A has only one more element than set B.

(ii) $D \neq C$ is correct, because the elements of set C are not same to the elements of set D except the element 'd'.
(iii) $C \supset E$ is correct, because set C contains the element of set E.
(iv) $D \supset E$ is correct, because set D contains the element of set E.
(v) $D \subset B$ is not correct, because both sets have different elements.
(vi) $D = A$ is not correct, because the elements of set D are not same to the elements of set A except the element 'c'.
(vii) $B \not\subset C$ is not correct, because the elements of set B are contained in set C.
(viii) $E \subset A$ is not correct, because both sets have different elements.
(ix) $E \not\subset B$ is correct, because the elements of set E are not contained in set B.
(x) $a \in A$ is correct, because 'a' is an element of set A.
(xi) $a \subset A$ is not correct, because 'a' is a proper subset of set A.
(xii) $\{a\} \in A$ is not correct, because $\{a\}$ is not an element of set A.
(xiii) $\{a\} \subset A$ is correct, because $\{a\}$ is a proper subset of set A.

Illustration-16: If $A = \{0,1\}$, state which of the following statements is true, and which are false: (i) $\{1\} \subset A$, (ii) $\{1\} \in A$, (iii) $\phi \in A$, (iv) $0 \in A$, (v) $1 \subset A$, (vi) $\{0\} \in A$, (vii) $\phi \subset A$.

Solution: Given $A = \{0,1\}$
(i) $\{1\} \subset A$ is true, because set A has only one more element than set $\{1\}$.
(ii) $\{1\} \in A$ is false, because $\{1\}$ is not an element of set A.
(iii) $\phi \in A$ is true, because ϕ is an element of every set.
(iv) $0 \in A$ is true, because 0 is an element of set A.
(v) $1 \subset A$ is false, because 1 is not a proper subset of set A.
(vi) $\{0\} \in A$ is false, because $\{0\}$ is not an element of set A.
(vii) Given $\phi \subset A \Rightarrow \{\ \} \subset A$.
So $\phi \subset A$ is false, because ϕ is not a proper subset of set A.

Illustration-17: Let $V = \{0,1,2,3,4,5,6,7,8,9,\}$, $X = \{0,2,4,6\}$, $Y = \{3,5,7\}$, and $Z = \{3,7\}$. Find: (i) $Y \cup Z$, (ii) $(V \cup Y) \cap X$, (iii) $(X \cup Z) \cup V$, (iv) $(X \cup Y) \cap Z$, (v) $(\phi \cup V) \cap \phi$.

Solution: (i) $Y \cup Z = \{3,5,7\} \cup \{3,7\} = \{3,5,7\}$ [Ans.]
(ii) $(V \cup Y) \cap X$
$= [\{0,1,2,3,45,6,7,8,9\} \cup \{3,5,7\}] \cap \{0,2,4,6\} \quad = \{0,1,2,3,4,5,6,7,8,9\} \cap \{0,2,4,6\}$
$= \{0,2,4,6\}$ [Ans.]
(iii) $(X \cup Z) \cup V$
$= [\{0,2,4,6\} \cup \{3,7\}] \cup \{0,1,2,3,45,6,7,8,9\} \quad = \{0,2,3,4,6,7,8\} \cup \{0,1,2,3,4,56,7,8,9\}$

$= \{0,1,2,3,4,5,6,7,8,9\}$ [Ans.]

(iv) $(X \cup Y) \cap Z = [\{0,2,4,6\} \cup \{3,5,7\}] \cap \{3,7\}$

$= \{0,2,3,4,5,6,7,8\} \cap \{3,7\} = \{3,7\}$ [Ans.]

(v) $(\phi \cup V) \cap \phi = [\{\} \cup \{0,1,2,3,4,5,6,7,8,9\}] \cap \{\} = \{0,1,2,3,4,5,6,7,8,9\} \cap \{\} = \{\} = \phi$ [Ans.]

Illustration-18: If $A = \{a,b,c,d,e,f\}, B = \{a,e,i,o,u\}$, and $C = \{m,n,o,p,q,r,s,t,u,\}$ (i) $A \cup B$, (ii) $A \cup C$, (iii) $B \cup C$, (iv) $A - B$, (v) $A \cap B$, (vi) $B \cap C$, (vii) $A \cup (B - C)$, (viii) $A \cup B \cup C$, and (ix) $A \cap B \cap C$.

Solution: (i) $A \cup B = \{a,b,c,d,e,f\} \cup \{a,e,i,o,u,\} = \{a,b,c,d,e,f,i,o,u\}$ [Ans.]

(ii) $A \cup C = \{a,b,c,d,e,f\} \cup \{m,n,o,p,q,r,s,t,u\}$

$= \{a,b,c,d,e,f,m,n,o,p,q,r,s,t,u\}$ [Ans.]

(iii) $B \cup C = \{a,e,i,o,u\} \cup \{m,n,o,p,q,r,s,t,u\} = \{a,e,i,m,n,o,p,q,r,s,t,u,\}$ [Ans.]

(iv) $A - B = \{a,b,c,d,e,f\} - \{a,e,i,o,u\} = \{b,c,d,f\}$ [Ans.]

(v) $A \cap B = \{a,b,c,d,e,f\} \cap \{a,e,i,o,u\} = \{a,e\}$ [Ans.]

(vi) $B \cap C = \{a,e,i,o,u\} \cap \{m,n,o,p,q,r,s,t,u\} = \{o,u\}$ [Ans.]

(vii) Here $B - C = \{a,e,i,o,u\} - \{m,n,o,p,q,r,s,t,u\} = \{a,e,i\}$

$\therefore A \cup (B - C) = \{a,b,c,d,e,f\} \cup \{a,e,i\} = \{a,b,c,d,e,f,i\}$ [Ans.]

(viii) $A \cup B \cup C = \{a,b,c,d,e,f\} \cup \{a,e,i,o,u\} \cup \{m,n,o,p,q,r,s,t,u\}$

$= \{a,b,c,d,e,f,i,m,n,o,p,q,r,s,t,u\}$ [Ans.]

(ix) $A \cap B \cap C = \{a,b,c,d,e,f\} \cap \{a,e,i,o,u\} \cap \{m,n,o,p,q,r,s,t,u\} = \{\} = \phi$ [Ans.]

Illustration-19: Let the sets A and B be given by $A = \{1,2,3,4\}, B = \{2,4,6,8,10\}$ and the universal set $S = \{1,2,3,4,5,6,7,8,9,10\}$. Find $(A \cup B)'$ and $(A \cap B)'$.

Solution: By definition of union and intersection of sets, we get

$A \cup B = \{1,2,3,4,6,8,10\}$ and $A \cap B = \{2,4\}$.

Now, by definition of complement of a set, we get

$(A \cup B)' = \{5,7,9\}$ and $(A \cap B)' = \{1,3,5,6,7,8,9,10\}$.

Illustration-20: Let the universal set $U = \{3,4,5,6,7,8,9,10,11,12,13\}$, $A = \{3,4,5,6\}, B = \{3,7,9,5\}$, and $C = \{6,8,10,12,7\}$.

Write down the following sets:

(i) A', (ii) B', (iii) C', (iv) $(A')'$, (v) $(B')'$, (vi) $(A \cup B)'$, (vii) $(A \cap B)'$, and (viii) $A' \cup C'$.

Solution: Given $U = \{3,4,5,6,7,8,9,10,11,12,13\}$,

$A = \{3,4,5,6\}, B = \{3,7,9,5\}, C = \{6,8,10,12,7\}$.

(i) $A' = U - A = \{3,4,5,6,7,8,9,10,11,12,13\} - \{3,4,5,6\} = \{7,8,9,10,11,12,13\}$ [Ans.]

(ii) $B' = U - B = \{3,4,5,6,7,8,9,10,11,12,13\} - \{3,7,9,5\} = \{4,6,8,10,11,12,13\}$ [Ans.]

(iii) $C' = U - C = \{3,4,5,6,7,8,9,10,11,12,13\} - \{6,8,10,12,7\} = \{3,4,5,9,11,13\}$ [Ans.]

(iv) $(A')' = U - A' = U - (U - A) = U - U + A = A = \{3,4,5,6\}$ [Ans.]

(v) $(B')' = U - B' = U - (U - B) = U - U + B = B = \{3,7,9,5\}$ [Ans.]

(vi) Here $(A \cup B) = \{3,4,5,6\} \cup \{3,7,9,5\} = \{3,4,5,6.,7,9\}$

$\therefore (A \cup B)' = U - (A \cup B) = \{3,4,5,6,7,8,9,10,11,12,13\} - \{3,4,5,6,7,9\} = \{8,10,11,12,13\}$ [Ans.]

(vii) Here $(A \cap B) = \{3,4,5,6\} \cap \{3,7,9,5\} = \{3,5\}$

$\therefore (A \cap B)' = U - (A \cap B) = \{3,4,5,6,7,8,9,10,11,12,13\} - \{3,5\} = \{4,6,7,8,9,10,11,12,13\}$ [Ans.]

(viii) $A' \cup C' = \{7,8,9,10,11,12,13\} \cup \{3,4,5,9,11,13\} = \{3,4,5,7,8,9,10,11,12,13\}$ [Ans.]

Illustration-21: (a) If P has three elements, Q four and R two, how many elements does the Cartesian product set $P \times Q \times R$ will have?

(b) Identify the elements of B, if $Q = \{1,2,3\}$, and

$B \times Q = \{(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\}$

Solution: (a) Given $n(P) = 3, n(Q) = 4, n(R) = 2$. $\therefore P \times Q \times R = 3 \times 4 \times 2 = 24$ **[Ans.]**

(b) Given $B \times Q = \{(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\}$

$\Rightarrow B = \{(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\} \div Q$

$\Rightarrow B = \{(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\} \div \{1,2,3\} \Rightarrow B = \{4,5,6\}$ **[Ans.]**

Illustration-22: If $A = \{1,2,3\}$, and $B = \{2,3\}$, prove that $A \times B \neq B \times A$.

Solution: Given $A = \{1,2,3\}$, and $B = \{2,3\}$

LHS $= A \times B = \{1,2,3\} \times \{2,3\} = \{(1,2),(1,3),(2,2),(2,3),(3,2),(3,3)\}$

RHS $= B \times A = \{2,3\} \times \{1,2,3\} = \{(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

According to Cartesian product rule, both sides do not contain same ordered pairs.

$\therefore A \times B \neq B \times A$

Illustration-23: If $A = \{1,4\}, B, = \{2,3\}, C = \{3,5\}$ prove that $A \times B \neq B \times A$.

Also find $(A \times B) \cap (A \times C)$.

Solution: (i) Given $A = \{1,4\}, B = \{2,3\}$, and $C = \{3,5\}$

LHS $= A \times B = \{1,4\} \times \{2,3\} = \{(1,2),(1,3),(4,2),(4,3)\}$

RHS $= B\times A = \{2,3\}\times\{1,4\} = \{(2,1),(2,4),(3,1),(3,4)\}$

According to Cartesian product rule, both sides do not contain same ordered pairs.

$\therefore A\times B \neq B\times A$

(ii) $(A\times B)\cap(A\times C) = [\{1,4\}\times\{2,3\}]\cap[\{1,4\}\times\{3,5\}]$

$= \{(1,2),(1,3),(4,2),(4,3)\}\cap\{(1,3),(1,5),(4,3),(4,5)\}$

$= \{(1,3),(4,3)\}$ [Ans.]

Illustration-24: Given $A=\{2,3\}, B=\{4,5\}, C=\{5,6\}$. Find:

$A\times(B\cup C), A\times(B\cap C), (A\times B)\cup(B\times C)$.

Solution: (i) Given $A=\{2,3\}, B=\{4,5\}, C=\{5,6\}$

Here $(B\cup C)=\{4,5\}\cup\{5,6\}=\{4,5,6\}$

$\therefore A\times(B\cup C)=\{2,3\}\times\{4,5,6\}=\{(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$ [Ans.]

(ii) Here $(B\cap C)=\{4,5\}\cap\{5,6\}=\{5\}$

$\therefore A\times(B\cap C)=\{2,3\}\times\{5\}=\{(2,5),(3,5),\}$ [Ans.]

(iii) Here $(A\times B)=\{2,3\}\times\{4,5\}=\{(2,4),(2,5),(3,4),(3,5)\}$

$(B\times C)=\{4,5\}\times\{5,6\}=\{(4,5),(4,6),(5,5),(5,6)\}$

$\therefore (A\times B)\cup(B\times C)=\{(2,4),(2,5),(3,4),(3,5)\}\cup\{(4,5),(4,6),(5,5),(5,6)\}$

$=\{(2,4),(2,5),(3,4),(3,5),(4,5),(4,6),(5,5),(5,6)\}$ [Ans.]

Illustration-25: List the sets A, B, and C given that

$A\cup B=\{p,q,r,s\}; A\cup C=\{q,r,s,t\}; A\cap B=\{q,r\}; A\cap C=\{q,s\}$.

Solution: Since $A\cap B=\{q,r\}$, and $A\cap C=\{q,s\}$,

$q,r,s,\in A; q,r\in B$ and $q,s\in C$....................(1)

Since $A\cup C=\{q,r,s,t\}; q,s\in C$ and $t\notin A$,

$q,s,t\in C$..(2)

Since $A\cup B=\{p,q,r,s\}; q,r\in B$ and $p\notin A$,

$p,q,r\in B$..(3)

From (1),(2), and (3), we get

$A=\{q,r,s\}, B=\{p,q,r\}, C=\{q,s,t\}$ [Ans.]

Illustration-26: If $A=\{a,b\}, B=\{p,q\}, C=\{q,r\}$. Find **(a)** $A\times(B\cup C)$ **(b)** $(A\times B)\cup(A\times C)$ **(c)** $A\times(B\cap C)$ **(d)** $(A\times B)\cap(A\times C)$

Solution: Given $A=\{a,b\}, B=\{p,q\}, C=\{q,r\}$

(a) $A\times(B\cup C) = \{a,b\}\times[\{p,q\}\cup\{q,r\}] = \{a,b\}\times\{p,q,r\}$

$=\{(a,p),(a,q),(a,r),(b,p),(b,q),(b,r)\}$ [Ans.]

(b) $(A\times B)\cup(A\times C)=[\{a,b\}\times\{p,q\}]\cup[\{a,b\}\times\{q,r\}]$

$=\{(a,p),(a,q),(b,p),(b,q)\}\cup\{(a,q),(a,r),(b,q),(b,r)\}$

$=\{(a,p),(a,q),(b,p),(b,q),(a,r),(b,r)\}$ [Ans.]

(c) $A\times(B\cap C)=\{a,b\}\times[\{p,q\}\cap\{q,r\}]=\{a,b\}\times\{q\}=\{(a,q),(b,q)\}$ [Ans.]

(d) $(A\times B)\cap(A\times C)=[\{a,b\}\times\{p,q\}]\cap[\{a,b\}\times\{q,r\}]$

$=\{(a,p),(a,q),(b,p),(b,q)\}\cap\{(a,q),(a,r),(b,q),(b,r)\}=\{(a,q),(b,q)\}$ [Ans.]

Illustration-27: Let $A=\{x: x$ is a letter in English alphabet$\}$ be the universal set. $V=\{x: x$ is a vowel$\}$, $C=\{x: x$ is a consonant$\}$, $N=\{x: x$ is a letter in your full name$\}$. List the elements of the following sets:

1. $N\cup V$, 2. $N\cap C$, 3. $V\cup C$, 4. $N\cap V'$, 5. $C\cap N'$, 6. C'.

Solution: Given $A=\{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z\}$,

$V=\{a,e,i,o,u\}$, $C=\{b,c,d,f,g,h,j,k,l,m,n,p,q,r,s,t,v,w,x,y,z\}$,

$N=\{rafique\}=\{a,e,f,i,,q,r,u\}$.

1. $N\cup V=\{a,e,f,i,q,r,u,\}\cup\{a,e,i,o,u\}=\{a,e,f,i,o,q,r,u\}$ [Ans.]

2. $N\cap C=\{a,e,f,i,q,r,u\}\cap\{b,c,d,f,g,h,j,k,l,m,n,p,q,r,s,t,v,w,x,y,z\}$

   $=\{f,q,r\}$ [Ans.]

3. $V\cup C=\{a,e,i,o,u\}\cup\{b,c,d,f,g,h,j,k,l,m,n,p,q,r,s,t,v,w,x,y,z\}$

   $=\{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z\}$ [Ans.]

4. Here $V'=A-V=C$

   $\therefore N\cap V'=N\cap C$

   $=\{a,e,f,i,q,r,u\}\cap\{b,c,d,f,g,h,j,k,l,m,n,p,q,r,s,t,v,w,x,y,z\}$

   $=\{f,q,r\}$ [Ans.]

5. Here $N'=A-N$

   $=\{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z\}-\{a,e,f,i,q,r,u\}$

   $=\{b,c,d,g,h,j,k,l,m,n,o,p,s,t,v,w,x,y,z\}$

   $\therefore C\cap N'$

   $=\{b,c,d,f,g,h,j,k,l,m,n,p,q,r,s,t,v,w,x,y,z\}\cap\{b,c,d,g,h,j,k,l,m,n,o,p,s,t,v,$

   $w,x,y,z\}=\{b,c,d,g,h,j,k,l,m,n,p,s,t,v,w,x,y,z\}$ [*Ans.*]

6. $C'=A-C=V=\{a,e,i,o,u,\}$ [Ans.]

Illustration-28: If the universal set is $X = \{x : x \in N, 1 \le x \le 12\}$, and $A = \{1,9,10\}$, $B = \{3,4,6,11,12\}$, and $C = \{2,5,6\}$ are the subsets of X, find the sets $A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$.

Solution: Given $A = \{1,9,10\}, B = \{3,4,6,11,12\}, C = \{2,5,6\}$.

(i) Here $(B \cap C) = \{3,4,6,11,12\} \cap \{2,5,6\} = \{6\}$

$\therefore A \cup (B \cap C) = \{1,9,10\} \cup \{6\} = \{1,6,9,10\}$ [Ans.]

(ii)Here $(A \cup B) = \{1,9,10\} \cup \{3,4,6,11,12\} = \{1,3,4,6,9,10,11,12\}$ and

$(A \cup C) = \{1,9,10\} \cup \{2,5,6\} = \{1,2,5,6,9,10\}$

$\therefore (A \cup B) \cap (A \cup C) = \{1,3,4,6,9,10,11,12\} \cap \{1,2,5,6,9,10\} = \{1,6,9,10\}$ [Ans.]

Illustration-29: Find $A \cup B$, $A \cap B$, $A - B$ and $B - A$

(i) If $A = \{x : x^2 = 9,\ 2x = 4\}$, $B = \{x : x + 8 = 8\}$

(ii) If $A = \{x : x^2 - 7x + 12 = 0\}$, $B = \{x : x^2 - 10x + 21 = 0\}$

Solution:(ii) Here solving $x^2 - 7x + 12 = 0$ we get, $x = 3,4$

$\therefore A = \{3,4\}$

Again If $x^2 - 10x + 21 = 0$ we get, $x = 3,7$

$\therefore B = \{3,7\}$

$\therefore A \cup B = \{3,4,7\},\ A \cap B = \{3\},\ A - B = \{4\},\ B - A = \{7\}$.

CARDINAL NUMBER OF A SET

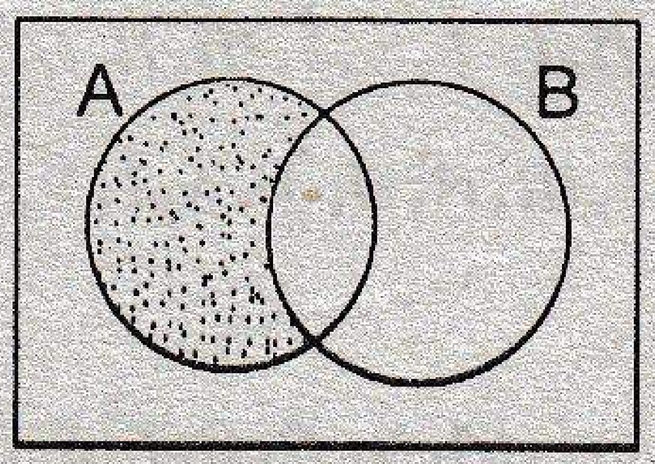
The number of elements in a finite set A is called the cardinal number of the set A. It is denoted by n (A). For example, if $A = \{1,2,3,4\}$, then $n(A) = 4$ as A contains only four elements.

Some important Results on Cardinal Number of a Set

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

2. $n(A') = n(U) - n(A)$

3. $n(A \cup B)' = n(U) - n(A \cup B)$



A∩B′ or A−B

4. $n(A \cap B') = n(A') - n(A \cap B)$

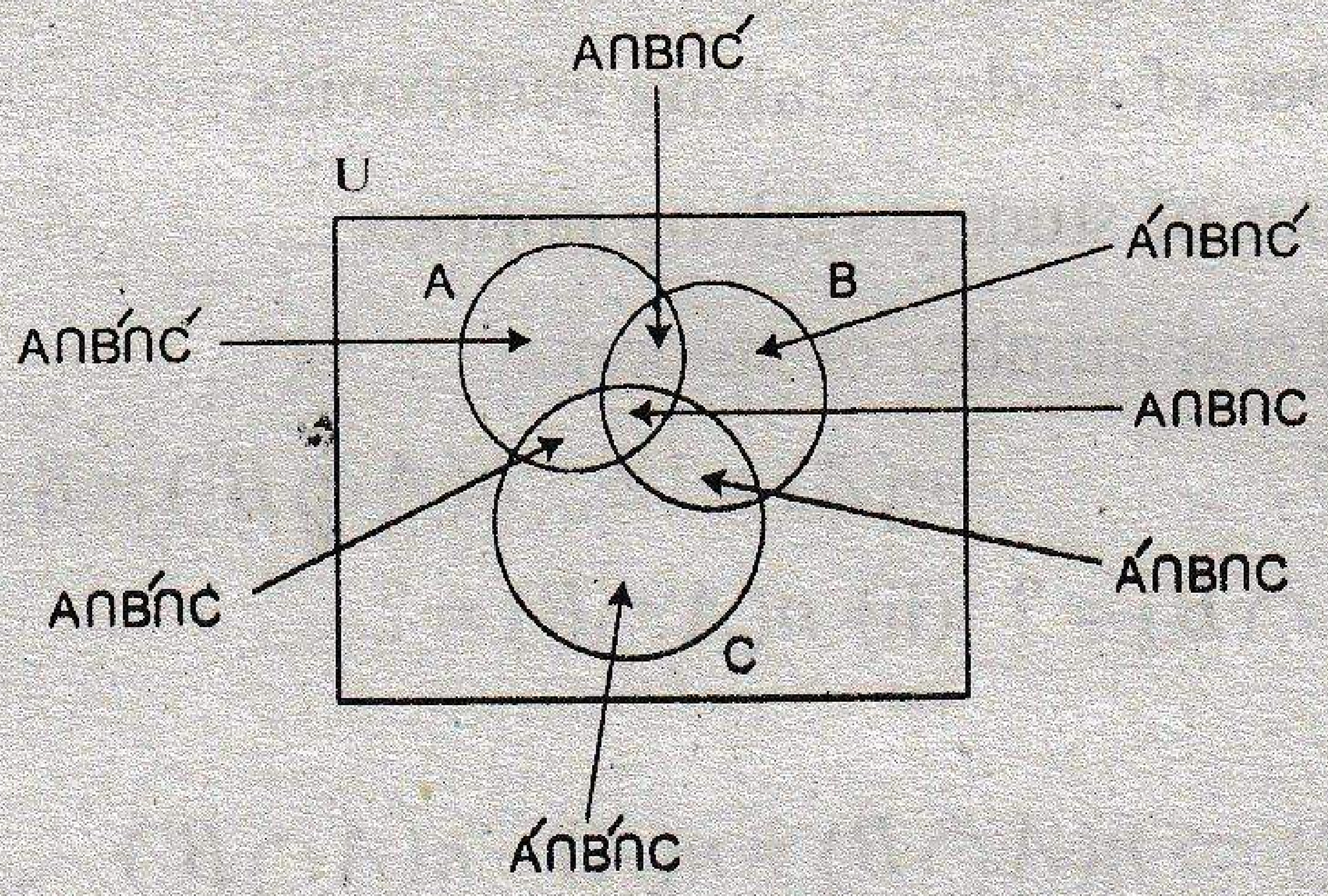
5. $n(A' \cap B) = n(B) - n(A \cap B)$

6. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

8. $n(A \cup B)' = n(U) - n(A) - n(B) + n(A \cap B)$

6. $n(A \cup B \cup C)' = n(U) - n(A) - n(B) - n(C) + n(A \cap B) + n(B \cap C) + n(C \cap A) - n(A \cap B \cap C)$

7. $n(A \cup B \cup C)' = n(U) - n(A \cup B \cup C)$
8. $n(A \cap B' \cap C') = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$
9. $n(A' \cap B \cap C') = n(B) - n(B \cap A) - n(B \cap C) + n(A \cap B \cap C)$
10. $n(A' \cap B' \cap C) = n(C) - n(C \cap A) - n(C \cap B) + n(A \cap B \cap C)$
11. $n(A \cap B \cap C') = n(A \cap B) - n(A \cap B \cap C)$
12. $n(A \cap B' \cap C) = n(A \cap C) - n(A \cap B \cap C)$
13. $n(A' \cap B \cap C) = n(B \cap C) - n(A \cap B \cap C)$
14. $n(A) = n(A \cap B' \cap C') + n(A \cap B \cap C') + n(A \cap B' \cap C) + n(A \cap B \cap C)$
15. $n(B) = n(A' \cap B \cap C') + n(A \cap B \cap C') + n(A' \cap B \cap C) + n(A \cap B \cap C)$
16. $n(C) = n(A' \cap B' \cap C) + n(A' \cap B \cap C) + n(A \cap B' \cap C) + n(A \cap B \cap C)$



3.10 APPLICATION OF SETS IN BUSINESS PROBLEM

Problem-01: A has 32 elements, and B has 42 elements and $A \cup B$ has 62 elements, indicate the number of elements in $A \cap B$.

Solution: Given $n(A) = 32$, $n(B) = 42$, $n(A \cup B) = 62$, Requirement: $n(A \cap B) = ?$

We know, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\Rightarrow 62 = 32 + 42 - n(A \cap B) \Rightarrow n(A \cap B) = 32 + 42 - 62$$

$$\therefore n(A \cap B) = 12 \text{ (Ans.)}$$

Problem-02: A town has a total population of 50,000. Out of it 28,000 read Patriot and 23,000 read Daily Times while 4,000 read both the papers. Indicate how many read neither Patriot nor Daily Times?

Solution: Let U=Total Population

P= Set of all population who read Patriot

T= Set of all population who read Daily Times

So we have, $n(U)=50{,}000$, $n(P)=28{,}000$, $n(T)=23{,}000$

$n(P\cap T)=4{,}000$ Requirement: $n(P\cup T)'=?$

We know, $n(P\cup T)'=n(U)-n(P\cup T)=n(U)-n(P)-n(T)+n(P\cap T)$

$=50000-28000-23000+4000,\ =3000$ (Ans.)

Problem-03: In a group of 52 persons, 16 drink tea but not coffee and 33 drink tea. Find:

(i) how many drink tea and coffee both.

(ii) how many drink coffee but not tea.

Solution: (Try yourself). (Ans.(i) 17 (ii) 19)

Problem-04: In a class of 25 students, 12 students have taken economics; 8 have taken economics but not politics. Find the number of students who have taken economics, and those who have taken politics, but not economics.

Solution: Let $n(A)=$ Number of students taking economics

$n(B)=$ Number of students taking politics

$n(A\cup B)=$ Total number of students

Given $n(A\cup B)=25$, $n(A)=12$, $n(A\cap B')=8$

Requirement: (i) $n(A\cap B)=?$ (ii) $n(A'\cap B)=?$

(i) We know,

$n(A\cap B')=n(A)-n(A\cap B)\Rightarrow n(A\cap B)=n(A)-n(A\cap B')$

$\Rightarrow n(A\cap B)=12-8\quad \Rightarrow n(A\cap B)=4$ [Ans.]

(ii) We know,

$n(A\cup B)=n(A)+n(B)-n(A\cap B)\quad \Rightarrow 25=12+n(B)-4\Rightarrow n(B)=17$

We also know, $n(A'\cap B)=n(B)-n(A\cap B)=17-4=13$ [Ans.]

Problem-05: A company studies the product preferences of 20,000 consumers. It was found that each of the products A, B, C was liked by 7020, 6230, and 5980 respectively, and all the products were liked by 1500; products A and B were liked by 2580, products A and C were liked by 1200, and products B and C were liked by 1950. Prove that the study results are not correct.

Solution: Given $n(A)=7020$ $n(A\cap B)=2580$

$n(B)=6230$ $n(B\cap C)=1950$

$n(C)=5980$ $n(A\cap C)=1200$

$n(A\cap B\cap C)=1500$

Requirement: To verify $n(A\cup B\cup C)=20000$.

We know,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$
$$= 7020 + 6230 + 5980 - 2580 - 1950 - 1200 + 1500$$
$$= 15000$$

But $15000 \neq 20000$, So the study results are not correct. **[Proved]**

Problem-06: Out of 880 boys in a school, 224 like cricket, 240 like hockey and 336 like basket ball; Of the boys total 64 like both basket ball and hockey; 80 like cricket and basket ball and 40 like cricket and hockey; 24 boys like all the three games.

(i) How many boys do not play any game?

(ii) How many like only one game?

Solution: Given

$n(U) = 880$, $n(C) = 224$, $n(H) = 240$, $n(B) = 336$, $n(H \cap B) = 64$, $n(C \cap B) = 80$,

$n(C \cap H) = 40$, $n(C \cap H \cap B) = 24$, Requirements: (i) $n(C \cup H \cup B)' = ?$ and

$n(C \cap H' \cap B') + n(C' \cap H \cap B') + n(C' \cap H' \cap B) = ?$

We know that,

$$n(C \cup H \cup B) = n(C) + n(H) + n(B) - n(C \cap H) - n(H \cap B) - n(C \cap B) + n(C \cap H \cap B)$$
$$= 224 + 240 + 336 - 40 - 64 - 80 + 24$$
$$= 640$$

$$\therefore n(C \cup H \cup B)' = n(U) - n(C \cup H \cup B)$$
$$= 880 - 640$$
$$= 240$$

i.e. 240 boy's do not play any game (Ans.)

(ii) We know that,

$$n(C \cap H' \cap B') = n(C) - n(C \cap H) - n(C \cap B) + n(C \cap H \cap B)$$
$$= 224 - 40 - 80 + 24$$
$$= 248 - 120$$
$$= 128$$

Similarly have to calculate: $n(C' \cap H \cap B') = 160$ and $n(C' \cap H' \cap B) = 216$

$\therefore 128 + 160 + 216 = 504$

i.e. 504 boys like only one game. (Ans.)

Problem-07: A TV survey gives the following data for TV viewing:- 60% see program A, 50% program B, 50% program C, 30% Program A and B, 20% Program B and C, 30% program A and C and 10% do not view any program. Draw a Venn-diagram and find the following:-

a) What percent view A, B and C?
b) What percent view exactly two programs?
c) What percent view only A?

Solution: (a) Let x% view program A, B and C.

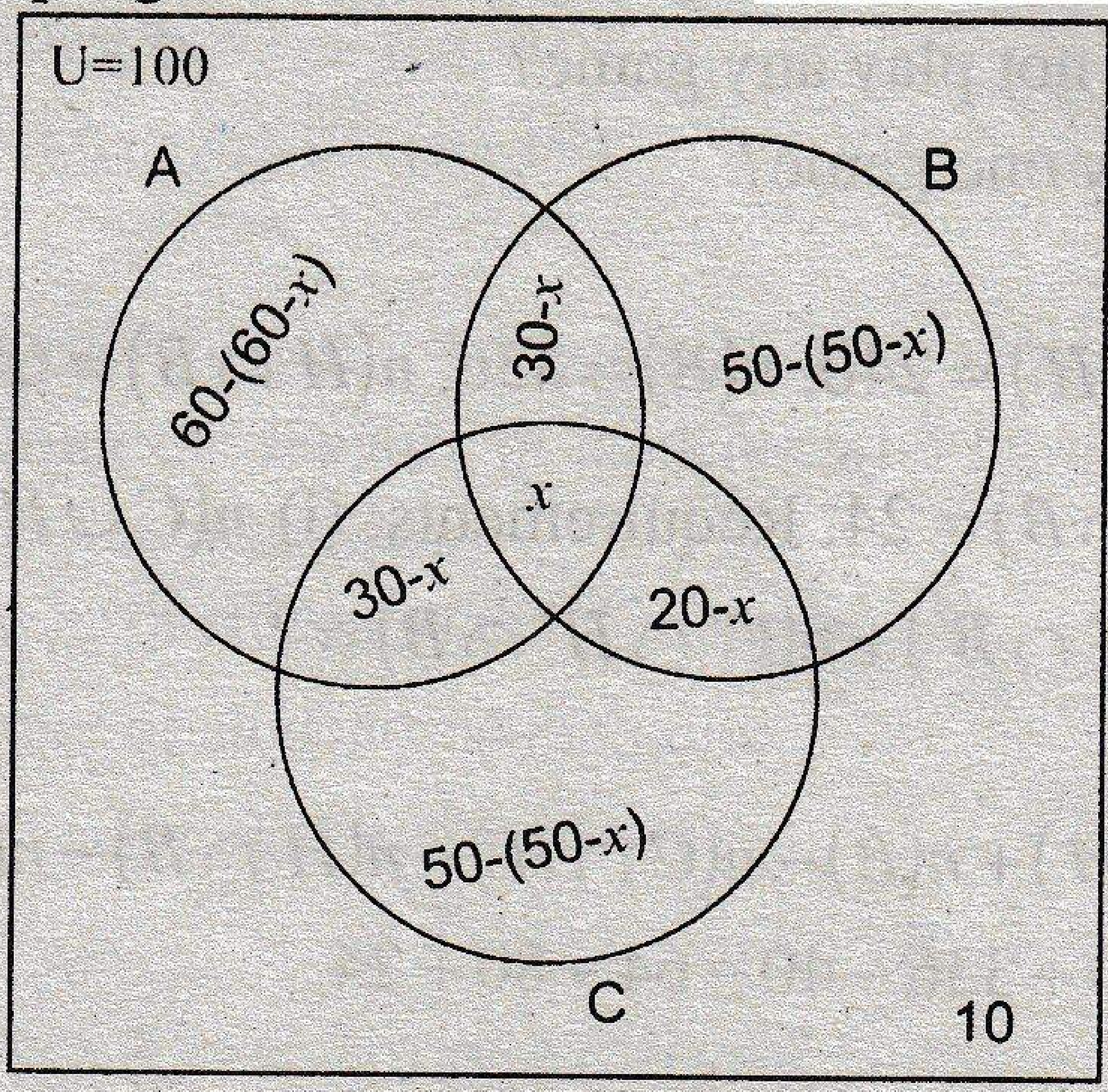


Fig: Venn-diagram

We know that,

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$\Rightarrow 100 - 10 = 60 + 50 + 50 - 30 - 20 - 30 + x$

$\Rightarrow 90 = 160 - 80 + x$

$\Rightarrow x = 90 + 80 - 160 = 10$ i.e. 10% view programs A, B and C Ans.

b) $n(A \cap B \cap C') = 30 - x = 30 - 10 = 20$ (from Venn diagram)

$n(A' \cap B \cap C) = 20 - x = 20 - 10 = 10$

and $n(A \cap B' \cap C) = 30 - x = 30 - 10 = 20$

$\therefore n(A \cap B \cap C') + n(A' \cap B \cap C) + n(A \cap B' \cap C)$

$= 20 + 10 + 20 = 50$

Hence 50% view exactly two programs. Ans.

c) Only $A = 60 - [(30 - x) + x + (30 - x)]$ (from diagram)

$= 60 - (60 - x)$

$= 60 - (60 - 10)$

$= 60 - 50 = 10$ i.e. 10% view only A. (Ans.)

Problem-08: In a survey conducted of 2000 clerks in an office it was found that 48% preferred coffee (C), 54% liked tea (T) and 64% used to smoke (S).Of the total 28% used C and T, 32% used T and S and 30% preferred C and S. Only 6% did none of these. Find (i) the number having all the three, (ii) T and S but not C, (iii) only C.

Solution: Given $n(U) = 2000$

$n(C) = 48\% \times 2000 = 960$, $n(T) = 54\% \times 2000 = 1080$

$n(S) = 64\% \times 2000 = 1280$, $n(C \cap T) = 28\% \times 2000 = 560$

$n(T \cap S) = 32\% \times 2000 = 640$, $n(C \cap S = 30\% \times 2000 = 600$

$n(C \cup T \cup S)' = 6\% \times 2000 = 120$

Requirement: (i) $n(C \cap T \cap S) = ?$ (ii) $n(C' \cap T \cap S) = ?$ (iii) $n(C \cap T' \cap S') = ?$

(i) We know,

$n(C \cup T \cup S)' = n(U) - n(C) - (T) - (S) + n(C \cap T) + n(T \cap S) + n(C \cap S) - n(C \cap T \cap S)$

$\Rightarrow 120 = 2000 - 960 - 1080 - 1280 + 560 + 640 + 600 - n(C \cap T \cap S)$

$\Rightarrow n(C \cap T \cap S) = 2000 - 960 - 1080 - 1280 + 560 + 640 + 600 - 120$

$\Rightarrow n(C \cap T \cap S) = 360$ [Ans.]

(ii) $n(C' \cap T \cap S) = n(T \cap S) - n(C \cap T \cap S) = 640 - 360 = 280$ [Ans.]

(iii) $n(C \cap T \cap S') = n(C) - n(C \cap T) - n(C \cap S) + n(C \cap T \cap S)$

$= 960 - 560 - 600 + 360 = 160$ [Ans.]

Problem-09: Complaints about work canteen fell into three categories. Complaints are about (i) Mess (M), (ii) Food (F), (iii) Service (S). Total complaints 173 were received as follows:

$n(M) = 110, n(S) = 67, n(M \cap F \cap S)' = 20, n(M \cap S \cap F') = 11, n(F \cap S \cap M') = 16, n(F) = 55.$

Determine the complaints about (i) all the three, (ii) about two or more than two.

Solution: Given $n(M \cup F \cup S) = 173$

$n(M) = 110$, $n(F) = 55$, $n(S) = 67$, $n(M \cap F \cap S') = 20$, $n(M \cap S \cap F') = 11$

$n(F \cap S \cap M') = 16$

Requirement: (i) $n(M \cap F \cap S) = ?$

(ii) $n(M \cap F \cap S') + n(M \cap S \cap F') + n(F \cap S \cap M') + n(M \cap F \cap S) = ?$

(i) We know,

$n(M \cup F \cup S) = n(M) + n(F) + n(S) - n(M \cap F \cap S') - n(M \cap S \cap F') - n(F \cap S \cap M')$

$- 2n(M \cap F \cap S)$

$\Rightarrow 173 = 110 + 55 + 67 - 20 - 11 - 16 - 2n(M \cap F \cap S)$

$\Rightarrow 2n(M \cap F \cap S) = 110 + 55 + 67 - 20 - 11 - 16 - 173$

$\Rightarrow 2n(M \cap F \cap S) = 12$, $\Rightarrow n(M \cap F \cap S) = 6$ [Ans.]

(ii) $n(M \cap F \cap S') + n(M \cap S \cap F') + n(F \cap S \cap M') + n(M \cap F \cap S)$

$= 20 + 11 + 16 + 6 = 53$ [Ans.]

Problem-10: Out of the total 150 students who appeared for MBA Examination from a center, 45 failed in Accounting, 50 failed in Business Mathematics, and 30 failed in Costing. Those who failed both in Accounting and Business Mathematics were 30, those failed both in Business Mathematics and Costing were 32, and those who failed both in Accounting and Costing were 35. The students who in all the subjects were 25. Find out the number who failed at least in any one of the subjects.

Solution: Let U=Total Students, A=Accounting, M=Business Mathematics, C=Costing

Given $n(U) = 150$ $n(A \cap M) = 30$

$n(A) = 45$ $n(M \cap C) = 32$

$n(M) = 50$ $n(A \cap C) = 35$

$n(C) = 30$ $n(A \cap M \cap C) = 25$

Requirement: (i) $n(A \cup M \cup C) = ?$

We know,

$n(A \cup M \cup C) = n(A) + n(M) + n(C) - n(A \cap M) - n(M \cap C) - n(A \cap C) + n(A \cap M \cap C)$

$= 45 + 50 + 30 - 30 - 32 - 35 + 25 = 53$ [Ans.]

Problem-11: A survey of 400 recently qualified Chartered Accountants revealed that 112 joined industry, 120 started practice and 160 joined the firms of practicing chartered accountants as paid assistant. There were 32 who joined service and also did practice; 40 had both practices and assistantship and 20 had both job in industry and assistantship. There were 12 who did all the three. Indicate how many could not get any of these and how many did only one of these.

Solution: Let U=Total Chartered Accountants

A=Industry, B=Practice, C=Assistant

Given $n(U) = 400$ $n(A \cap B) = 32$

$n(A) = 112$ $n(B \cap C) = 40$

$n(B) = 120$ $n(A \cap C) = 20$

$n(C) = 160$ $n(A \cap B \cap C) = 12$

Requirements: (i) $n(A \cup B \cup C)' = ?$

(ii) $n(A \cap B' \cap C') + n(A' \cap B \cap C') + n(A' \cap B' \cap C) = ?$

(i) We know,

$n(A\cup B\cup C)' = n(U) - n(A) - n(B) - n(C) + n(A\cap B) + n(B\cap C) + n(A\cap C) - n(A\cap B\cap C)$

$= 400 - 112 - 120 - 160 + 32 + 40 + 20 - 12$

$= 88$ [Ans.]

(ii) $n(A\cap B'\cap C') + n(A'\cap B\cap C') + n(A'\cap B'\cap C)$

$= [n(A) - n(A\cap B) - n(A\cap C) + n(A\cap B\cap C)]$

$+ [(B) - n(A\cap B) - n(B\cap C) + n(A\cap B\cap C)]$

$+ [(C) - n(A\cap C) - n(B\cap C) + n(A\cap B\cap C)]$

$= [112 - 32 - 20 + 12] + [120 - 32 - 40 + 12] + [160 - 20 - 40 + 12]$

$= 70 + 60 + 112 = 244$ [Ans.]

Problem-12: (i) In a survey of 100 students it was found that 50 used the college library, 40 had their own and 30 borrowed books, 20 used both college library and their own, 15 borrowed books and used their own books, whereas 10 used borrowed books and college library, Assuming that all students use either college library books or their own or borrowed books, find the number of students using all the three sources.

(ii) If the number of students using no book at all is 10, and the number of students using all the three is 20, show that the information is inconsistent.

Solution: Let A=College books, B=Own books, C=Borrowed books

Given $n(A\cup M\cup C) = 100$ $n(A\cap M) = 20$

$n(A) = 50$ $n(M\cap C) = 15$

$n(B) = 40$ $n(A\cap C) = 10$

$n(C) = 30$

Requirement: (i) $n(A\cap B\cap C) = ?$

We know,

$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

$\Rightarrow 100 = 50 + 40 + 30 - 20 - 15 - 10 + n(A\cap B\cap C) \Rightarrow n(A\cap B\cap C) = 25$ [Ans.]

(ii) In this case, we are given

$n(A\cup B\cup C)' = 10$, $n(A\cap B\cap C) = 20$

Requirement: To show $n(U) \neq 100$.

We know,

$n(A\cup B\cup C)' = n(U) - n(A) - n(B) - n(C) + n(A\cap B) + n(B\cap C) + n(A\cap C) - n(A\cap B\cap C)$

$\Rightarrow 10 = n(U) - 50 - 40 - 30 + 20 + 15 + 10 - 20$

$\Rightarrow n(U) = 105$, Here $105 \neq 100$. Hence the information is inconsistent. [Proved]

Problem-13: A class of 60 students appeared for an examination of Business Law, Statistics and Accounting. 25 students failed in Business Law, 24 failed in Statistics, 32 failed in Accounting, 9 failed in Business Law alone, 6 failed in Statistics alone, 5 failed in Accounting and Statistics only and 3 failed in Business Law and Statistics only. Find:

(i) How many failed in all three subjects? (ii) How many passed in all the three subjects?

Solution: Let M=Business Law

S=Statistics

A=Accounting

$n(U)=60$ $\quad$ $n(M\cap S'\cap A')=9$

$n(M)=25$ $\quad$ $n(M'\cap S\cap A')=6$

$n(S)=24$ $\quad$ $n(M'\cap S\cap A)=5$

$n(A)=32$ $\quad$ $n(M\cap S\cap A')=3$

Requirement: (i) $n(M\cap S\cap A)=?$ (ii) $n(M\cup S\cup A)'=?$

(i) From the above figure, we get

$n(M\cap S\cap A)=n(S)-3-6-5=24-3-6-5=10$

(ii) $n(M\cup S\cup A)'=n(U)-n(A)-9-3-6 \quad =60-32-9-3-6=10$ [Ans.]

Problem-14: A market research team interviews 100 people asking each whether he smokes any or all of items; A – cigarettes, B – cigars, C – pipe tobacco. The team returns the following data:

Category	Number	Category	Number
ABC	3	A	42
AB	7	B	17
BC	13	C	27
AC	18	Total	100

Are the returns consistent?

Solution: Given $n(A\cup B\cup C)=100$ $\quad$ $n(A\cap C)=18$

$n(A\cap B\cap C)=3$ $\quad$ $n(A)=42$

$n(A\cap B)=7$ $\quad$ $n(B)=17$

$n(B\cap C)=13$ $\quad$ $n(C)=27$

Requirements: To verify $n(A\cup M\cup C)=100$.

We know, $n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(A\cap C)+n(A\cap B\cap C)$

$=42+17+27-7-13-18+3 \quad =51$

But $51\neq100$, Hence the returns (results) are inconsistent. [Ans.]

BRIEF REVIEW

Definition

Set: Every well–defined and well-distinguished collection, class or list of objects is called a set. The objects in a set are called its elements or members. Sets are usually be denoted by capital letters A, B, C, X, Y, Z etc whereas the elements of a set are denoted by small letters a, b, c, x, y, z etc. If x is an element of a set A we say x belongs to A and we write $x \in A$. On the other hand if x does not belong to A, we write $x \notin A$.

For Example: A = {a , e, i, o ,u} is a set of vowels.

Finite Set: A set having finite number of different elements is called finite set.

Infinite Set: A set having infinite number of different elements is called infinite set.

Null Set: A set which has no elements is called null set or empty set . It is denoted by φ (phi).

Singleton Set: A set which contains only one element is called singleton set.

Equal Sets: Two sets A and B are said to be equal if every element of A is also an element of B, and every element of B is also an element of A and we write $A = B$.

Equivalent sets: Two sets having same number of different elements are called equivalent sets. If sets A and B are equivalent then we write $A \cong B$.

Subset: If every element of a set A is also an element of a set B, then A is called a subset of B and is written as $A \subseteq B$.

Proper Subset: If $A \subseteq B$ then it is still possible that A = B. When $A \subseteq B$ but $A \neq B$, we say that A is a proper subset of B and we write $A \subset B$.

Family of sets: If all the elements of a set are sets themselves, then it is called a set of sets or family of sets.

Power set: The family of all the subset of a given set A , is called power set of A.. Power set of A is denoted by P(A).

Universal Set: In any discussion of set theory, all the sets are considered as subsets of a fixed set. This fixed set is called the universal set. It is denoted by U.

Union of two Sets: The union of two sets A and B, denoted by $A \cup B$, is the set of all those elements which belongs to A or B or both. Mathematically,

$$A \cup B = \{x : x \in A \ or \ x \in B\}$$

Intersection of two Sets: The intersection of two sets A and B, denoted by $A \cap B$, is the set of all those elements which belongs to both A and B. Mathematically,

$$A \cap B = \{x : x \in A \ and \ x \in B\}$$

Disjoint Set: Two sets A and B are said to be disjoint or mutually exclusive if they have no common elements, i.e. if $A \cap B = \varphi$.

Difference of two Sets: The difference of two sets A and B, denoted by $A - B$, is the set of all those elements which belongs to A but does not belong to B.

Symbolically, $A - B = \{x : x \in A \ and \ x \notin B\}$ or, $B - A = \{x : x \in B \ and \ x \notin A\}$

Complements of Set: The complement of a set A, denoted by A' or A^C, is the set of all those elements which belongs to the universal set U but does not belong to A.

Mathematically, $A' = U - A = \{x : x \in U \ and \ x \notin A\}$

Symmetric difference of Set: The symmetric difference of two sets A and B, denoted by $A \Delta B$ or $A \oplus B$, is the set of all uncommon elements of $A \ and \ B$. Mathematically

$$A \Delta B = (A - B) \cup (B - A)$$

Ordered pair: An ordered pair consists of two elements, say a and b, in which one of them, say a, is designated as the first element and the other as the second element. An ordered pair is usually denoted by (a, b).

Cartesian product of two Sets: The Cartesian product of two sets A and B, denoted by A × B, is the set of all distinct ordered pairs whose first member belongs to A and second member belongs to B. Symbolically, $A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$.

Venn diagram: Venn diagram is the pictorial representation of sets named after English logician John Venn. In this case, the universal set is shown by a rectangle and one or more sets are shown through circles or closed curves within the rectangle.

Quiz Questions

Multiple Choice Questions (Indices)

1. Who developed the theory of sets?
   (a) De-Morgan's (b) George Cantor
   (c) Euclid (d) Lagrange
2. Which types of set A= $\{0\}$?
   (a) Infinite set (b) Empty set (c) Equivalent set (d) Singleton set
3. For the sets $A=\{2,4,6\}$ *and* $B=\{2,4,7,8\}$, which is true of the following?
   (a) $A \not\subset B$ (b) $A \subset B$ (c) $A \subseteq B$ (d) $A = B$
4. If A is a finite set of n element, then how many number of elements of power set A?
   (a) n (b) $2n$ (c) 2^n (d) n^2
5. If $A=\{1,3,5\}$ and $B=\{2,3,5\}$, then what is the value of $A \cup B$?
   (a) $\{1,3,5\}$ (b) $\{1,2,3\}$ (c) $\{3\}$ (d) $\{1,2,3,5\}$.
6. If $U=\{1,2,3,4,5\}$ and $A=\{2,3,5\}$, then what is the Value of A'
   (a) $\{2,3\}$ (b) $\{1,2,3,4,5\}$ (c) ϕ (d) $\{1,4\}$
7. If $p=\{a,b,c\}$ *and* $q=\{3,d,c\}$, then what is the value of $p \cap q$?
   (a) ϕ (b) $\{d,c\}$ (c) $\{3\}$ (d) $\{c\}$
8. What is true of the following of $(A^C)^C$?
   (a) A (b) A^{2C} (c) A^C (d) A'
9. Which is the correct meaning of $B \cap B^C$?
   (a) B (b) ϕ (c) U (d) B^C
10. Which is the meaning of $A^C \cup A$?
    (a) A (b) A^C (c) U (d) ϕ
11. If A is a set then which is correct of $n(A')$?
    (a) $n(A)-n(U)$ (b) $n(U)-n(A)$
    (c) $n(U)-A$ (d) $U-n(A)$
12. If $A=\{0\}$, $B=\{0,1\}, C=\phi, D=\{\phi\}$, then which is false of the following?
    (a) $A \subset B$ (b) $C \subset D$ (c) $A \not\subset B$ (d) $A \cap B=\{0\}$
13. If $A=\{0,1\}$, then which is true of the following?
    (a) $0 \in A-\{0\}$ (b) $\{1\} \in A$ (c) $1 \subset A$ (d) $\phi \subseteq A$
14. If $A=\{a,b\}$, $B=\{p,q\}$. $C=\{q,r\}$, then what is value of $A \times (B \cap C)$?
    (a) $\{(a,p),(b,p)\}$ (b) $\{(a,p),(b,q)\}$ (c) $\{(a,r),(b,r)\}$ (d) $\{(a,q),(b,q)\}$

Which one of the following statement is true/false?

Problem Set-I

(i) $\{a,b,c\}=\{c,b,a\}$, (ii) $\{a,c,a,d,c,d\}\subseteq\{a,c,d\}$, (iii) $\{b\}\in\{\{b\}\}$, (iv) $\{b\}\subset\{\{b\}\}$, and (v) $\phi\subset\{\{b\}\}$.

Problem Set-II

Let $A=\{0\}, B=\{0,1\}, C=\phi, D=\{\phi\}$,

$E=\{x: x \text{ is a human being 200 years old}\}$, $F=\{x: x\in A \text{ and } x\in B\}$.

State which of the followings are true, and which are false:

(i) $A\subset B$, (ii) $B=F$, (iii) $C\subset D$, (iv) $C=E$, (v) $A=F$, (vi) $F=1$, (vii) $E=C=D$ Let $A=\{a,b,c\}, B=\{a,b\}, C=\{a,b,d\}, D=\{c,d\}$ and $E=\{d\}$.

Problem Set-III

If $A=\{0,1\}$, state which of the following statements is true, and which are false: (i) $\{1\}\subset A$, (ii) $\{1\}\in A$, (iii) $\phi\in A$, (iv) $0\in A$, (v) $1\subset A$, (vi) $\{0\}\in A$, (vii) $\phi\subset A$.

Brief Questions

1. Who developed the theory of sets?
2. Write down the name of the Set $\{\phi\}$.
3. If $A=\{a,b\}, B=\{p,q\}, C=\{q,r\}$. How many elements are in $A\times B\times C$?
4. What is the value of $\phi\cap\{a,b\}$?
5. If A is a finite set of n element, then how many members are in the power set of A?
6. What is the value of $(A')'$?
7. Write down the methods of describing a set.
8. Who invented Venn diagram?
9. Which of the following is not a set: $\{\phi\}, \phi, \{0\}, 0$
10. $A=\{a,b,c\cdots\cdots\cdots\}$. Is it finite or infinite set?
11. If $A=\{1,3\}$ and $B=\{1,3,5,7\}$ find the value of $A-B$.
12. Rewrite the following sets in a set builder from.
    (i) $A=\{a,e,i,o,u\}$ (ii) $B=\{1,2,3\cdots\cdots\cdots\}$
13. Given $A=\{x,y,z\}$. State with reasons which of the following statements are correct:
    (i) $\{x\}\in A$, (ii) $x\in A$, (iii) $\{x\}\subset A$, (iv) $y\subseteq A$, (v) $\phi\in A$, (vi) $\phi\subseteq A$, (vii) $\{x,y,z\}\subseteq A$ (viii) $\{z\}\in P(A)$.
14. State with reasons whether the following statements are true or false:
    (i) If $A\cup B=B\cup C$ then $A=C$, (ii) If $A\subset B$ then $A\cup B=B$.

Conceptual, Analytical & Numerical Questions

1. What is Set?
2. Discuss the method of describing a Set.
3. Define with examples:
   (i) Finite and Infinite sets, (ii) Null set, (iii) Universal set, (iv) Singleton set, (v) Equal sets, (vi) Subset and proper subset, (vii) Union of two sets, (viii) Intersection of two sets, (ix) Disjoint sets, (x) Complement of a set, (xi) Difference of two sets, (xii) Power set.
4. Write short notes on:
   (i) set and subset, (ii) equality of two sets, (iii) universal set and null set, (iv) finite and infinite sets, (v) union, intersection and difference of two sets, (vi) universal set and subset, (vii) the three set operations (union, intersection and complementation), (viii) ordered pair and Cartesian product.
5. Define union and intersection of two Set.
6. Distinguish between collection and Set.
7. Distinguish between finite and infinite Set.
8. Distinguish between subset and proper Subset.
9. Distinguish between equal and equivalent Set.
10. Distinguish between Universal Set and Null Set.
11. Distinguish between Union and Intersection of two Sets.
12. Write the differences between $\{\phi\}$ and Null Set.
13. Distinguish between $\phi, \{\phi\}, \{0\}, 0$.
14. Define Venn diagram. Discuss the following sets with Venn diagram, union of two sets, intersection of two sets, difference of two sets, complements of a set, disjoint sets.
15. State and prove De Morgan's Laws on two sets.
16. If A is a finite set and contains n elements, prove that the power set of A has 2^n element.
17. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. Also, verify this relation for the sets
    $A = \{1,2,3,5\}, B = \{2,3,4.6\}$, and $C = \{1,2,4,5,7\}$.
18. Let $A = \{1,3,5,7,9\}, B = \{2,4,6,8,10\}, C = \{3,4,7,8,11,12\}$. Show that:
    (i) $A \cup B) \cup C = A \cup (B \cup C)$, (ii) $(A \cap B) \cap C = A \cap (B \cap C)$,
    (iii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, (iv) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

19. If $U = \{1,2,3,........,8,9\}$ be the universal set; $A = \{1,2,3,4\}$, and $B = \{2,4,6,8\}$.

Write down the following sets:

(i) $A \cup B$, (ii) $A \cap B$, (iii) A', (iv) $(A \cup B)'$, (v) $(A \cap B)'$.

20. (a) If $A = \{5,6,7,8,9\}, B = \{2,4,6,8,10,12\}, C = \{3,6,9,12\}$,

verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(b) Given $U = \{a,b,c,d,e,f,x,y,z,w\}, A = \{a,b,c,d,e\}, B = \{b,d,x,y,z\}$.

If $A - B$ is defined as $A \cap B'$, verify that $(A \cap B)' = A' \cup B'$.

21. If the universal set $U = \{x : x$ is a positive integer $< 25\}, A = \{2,6,8,14,22\}, B = \{4,8,10,14\}, C = \{6,10,12,14,18,20\}$; establish the following relation:

(i) $(A \cap B)' = A' \cup B'$, (ii) $(B' \cap C) \cup (A' \cap C) = C \cap (A' \cup B')$.

22. If $A = \{1,2,3\}, B = \{2,3,4,5\}$, and $C = \{2,4,6,8\}$, verify that

(i) $(A \cup B) = (A - B) \cup B$, (ii) $A - (A - B) = A \cap B$,

(iii) $A \cap (B - C) = (A \cap B) - (A \cap B)$.

23. Let A, B and C be three subsets of the universal set U, prove each of the following:

(i) $A \cap (A \cup B) = A$, (ii) $(A \cap B) \cup (A \cap B') = A$, (iii) $(A \cup B) = (A' \cap B')'$,

(iv) $(A \cup B) \cap (A \cup B') = A$, (v) $[A' \cup (B \cup C)]' = A \cap B' \cap C'$,

(vi) $A \cup B = (A \cap B) \cup (A \cap B') \cup (A' \cap B)$, (vii) $[A' \cup (A \cap B')]' = A \cap B$,

(viii) $B \cup (A \cup B) = (A \cup B)$, (ix) $(A' \cup B') \cup (A' \cup B)' = A$.

24. If $A = \{1,2.3\}$, and $B = \{2,3\}$, prove that $A \times B \neq B \times A$.

25. If $A = \{1,4\}, B = \{2,3\}, C = \{3,5\}$ prove that $A \times B \neq B \times A$. Also find $(A \times B) \cap (A \times C)$.

26. If $A = \{1,4\}; B = \{4,5\}; C = \{5,7\}$ verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

27. If $A = \{1,2,3\}; B = \{2,3,4\}; S = \{1,3,4\}; T = \{2,4,5\}$, verify that

$(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$.

28. Let $A = \{a,b,c,d,e,f\}, B = \{x : x$ is a vowel$\}$, $C = \{x, y, z\}$.

Give the following Cartesian product sets and count the number of elements in each.

(i) $A \times B$, (ii) $B \times A$, (iii) $C \times B$, (iv) $(A \times B) \times C$, (v) $A \times (B \times C)$.

29. Let $P=\{1,2,x\}, Q=\{a,x,y\}, R=\{x,y,z\}$. Find:
(i) $P\times Q$, (ii) $P\times R$, (iii) $Q\times R$, (iv) $(P\times Q)\cap(P\times R)$,
(v) $(R\times Q)\cap(R\times P)$, (vi) $(P\times Q)\cup(R\times P)$.

30. If $A=\{1,2,3,4\}, B=\{2,4,6\}, C=\{1,2,5\}$, and $U=\{1,2,3,4,5,6,7,8\}$. Compute:
(i) $A\cap(B-C)$, (ii) $A\cup(B\cap C)$, (iii) $A'\cup(B-C)$, (iv) $A'\cap(B-C)$,
(v) $A-(B'-C')$, (vi) $A'\cap(B'\cap C')$, (vii) $A'\cup(B'-C')$, (viii) $(A'-B')\cap(B'-C')$,
(ix) $(A'\cup B')\cap C'$, (x) $A'-(B\cap C)'$, (xi) $A\times B$, (xii) $A\times(B\cup C)$, (xiii) $(A\cup B)-(B\cup C)$,
(xiv) $(A\cap B)\times(B\cap C)$, (xv) $(A-B)\times(B-C)$, (xvi) $(A'-B')\times(B-C)'$,
(xvii) $A\times(B\cup C)'$, (xviii) $A\times(B\times C)$, (xix) $A\Delta B$, (xx) $(A\Delta B)\Delta C$.

31. In an examination 45% of the candidates have passed in English, 40% have passed in Bengali, while 30% have passed in both the subjects. Find the total number of candidates if 90 of them have failed in both the subjects.

32. In a group of people, each of whom speaks at least one of the languages English, Hindi and Bengali, 31 speak English, 36 speak Hindi and 27 speak Bengali. Ten speak both English and Hindi, 9 both English and Bengali, 11 both Hindi and Bengali. Using a Venn diagram or otherwise, prove that the group contains at least 64 people and not more than 73 people.

33. In a survey concerning the smoking habits of consumers, it was found that 55% smoke cigarette A, 50% smoke B, 42% smoke C, 28% smoke A and B, 20% smoke B and C, 12% smoke C and A, 10% smoke all the three brands. What percentage (i) do not smoke? (ii) smoke exactly two brands of cigarettes?

34. A factory inspector examined the defects in hardness, finish and dimension of an item. After examining 100 items he gave the following report:

35. All three defects 5, defect in hardness and finish 10, defect in dimensions and finish 8, defect in dimension and hardness 20. Defect in finish 30, in hardness 23 and in dimension 50. The inspector was fined, why?

36. A market research group conducted a survey of 1000 consumers and reported that 720 consumers liked product A and 450 consumers liked product B. What is the least number that must have liked both products?

37. In a survey of college students, it was found that 40% use their own books, 50% use library books, 30% use borrowed books, 20% use both their own books and library books, 15% use their own books and borrowed books, 10% use library books and borrowed books, and 4% use their own books, library books and borrowed books.

Calculate the percentage of students who do not use a book at all.

38. In a city three daily newspapers A, B, C are published; 65% of the citizens read A, 54% read B, 45% read C, 38% read A and B, 32% read B and C, 28% read A and C, 12% do not read any one of these three papers. If the total number of people in the city be 10,000, find the number of citizens who read all the three newspapers.

39. A company studies the product preferences of 300 consumers. It was found that 226 liked product A, 51 liked product B, 54 liked product C; 21 liked products A and B, 54 liked products A and C, 39 liked products B and C and 9 liked all the three products. Prove that, the study results are not correct. [Assume that each consumer likes at least one of the three products].

40. In a group of 200 people, each of whom is at least accountant or management consultant or sales manager. It was found that 80 are accountants, 110 are management consultants and 130 are sales managers, 25 are accountants as well as sales managers, 70 are management consultants as well as sales managers, 10 are accountants as well as management consultants as well as sales managers. Find the number of those people who are accountants as well as management consultants but not sales managers.

41. In a pollution study of 1,500 rivers, the following data were reported: 520 were polluted by sulphur compounds, 335 were polluted by phosphates, 425 were polluted by crude oil, 100 were polluted by both crude oil and sulphur compounds, 180 were polluted by sulphur compounds and phosphates, 150 were polluted by both phosphates and crude oil and 28 were polluted by sulphur compounds, phosphates, and crude oil. How many of the rivers (i) were polluted by at least one of the three impurities? (ii) were polluted by exactly one of the three impurities? (iii) were not polluted?

42. Of a group of 20 persons, 10 are interested in music, 7 are interested in photography, and 4 like swimming. Furthermore, 4 are interested in both music and photography, 3 are interested in both music and swimming, 2 are interested in both photography and swimming and one is interested in music, photography and swimming. How many are interested in photography but not in music and swimming?

ANSWERS

19. (i) $A \cup B = \{1,2,3,4,6,8\}$, (ii) $A \cap B = \{2,4\}$, (iii) $A' = \{5,6,7,8,9\}$
(iv) $(A \cup B)' = \{5,7,9\}$, (v) $(A \cap B)' = \{1,3,5,6,7,8,9\}$

25. $\{(1,3),(4,3)\}$, 31. 200 33. 3%, 30%. 34. incorrect data.

35. 170, 37. 21%, 38. 2200, 40. 25, 41. (i) 878, (ii) 504, (iii) 622. 4 2. 2.