CHAPTER-04	LINEAR EQUATIONS AND INEQUALITIES

LEARNING OBJECTIVES

This chapter will enable you to learn the concepts and application of:

- Equation and Identity
- Linear equation and its solution system
- Solving system of simultaneous equations
- Linear inequalities and its solution technique
- Determination of Break-even point having linear relationship between the variables

4.1 INTRODUCTION

An equation is simply the mathematical way to describe a relationship between two variables. The variables may be physical quantities, perhaps temperature and position for instance, in which case the equation tells us how one quantity depends on the other, so how the temperature varies with position. The simplest kind of relationship that two such variables can have is a linear relationship. This means that to find one quantity from the other you multiply the first by some number, and then add another number to the result. Put mathematically, if we call the quantities x and y, then they are related by the equation $y = mx + c$, where we can choose any values for m and c. This is a linear equation. Fortunately, in real physical problems, quantities often are related linearly, so this equation is very commonly used.

4.2 EQUATION

If any two algebraic expressions are connected by the sign of equality (=), then this statement is called an **equation**. In other words, an expression of the form $ax = b$ where 'x' is an unknown variable, 'a' and 'b' are two constant numbers is called an equation. However, the equality is true only for certain value or values of the variables x, y, z. For example, $5x = 10$ is an equation. It is valid for $x = 2$, since $5\times 2 = 10$ or $10 = 10$ which is always true. Again, the equation $5x + 6 = 4x + 9$ is true only for the value $x = 3$, but not for $x = 2$. Since when $x = 3$, the equation is $5\times 3 + 6 = 4\times 3 + 9$ or $21 = 21$ and when $x = 2$, the equation is $5\times 2 + 6 = 4\times 2 + 9$ or, $16 = 17$ which is not true. Thus, this equality is true for the value of $x = 3$.

4.3 IDENTITY

When equality holds true of an equation whatever be the values of the variables, then it is called an identity. For example: $(x+y)^2 = x^2 + 2xy + y^2$ is an **identity**. We can verify that this identity holds true whatever be the values of the variables x and y. Suppose (i) $x = 3$ and $y = 2$, (ii) $x = -3$ and $y = 4$.

For (i) $x = 3$ and $y = 2$, we have $(3+2)^2 = 3^2 + 2\times3\times2 + 2^2 \Rightarrow 5^2 = 9+12+4 \Rightarrow 25 = 25$

Again, for (ii) $x = -3$ and $y = 4$, we get $(-3+4)^2 = 3^2 + 2\times-3\times4 + 4^2$

$$\Rightarrow 1^2 = 9 - 24 + 16 \Rightarrow 1 = 1$$

Hence, in this identity equality holds whatever be the values of the variables x and y.

OTHER EXAMPLES OF IDENTITY:

(a) $(x-y)^2 = x^2 - 2xy + y^2$ (b) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

(c) $(x+y)(x-y) = x^2 - y^2$ etc.

Remarks: All algebraic laws are identity.

LINEAR EQUATION

An equation of the form $ax \pm b = 0$ $(a \neq 0)$, where a and b are constants and x is a variable, is called a linear equation or first degree equation in x.

In other words, An equation having highest power of the variable is 1 (one), is called a linear equation.

For examples: (i) $3x + 7 = 0$; (ii) $x + 3 = 0$; (iii) $x + (3/4) = 0$; (iv) $x - \sqrt{3} = 5$; (v) $x = 0$ are all examples of linear equations in x. Linear equations are simplest of all of equations.

DEGREE OF AN EQUATION

The highest power of unknown variable with a non-zero co-efficient in an equation is the **degree** of that equation. In general, the equation of the form:

$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 = 0, (a_n \neq 0)$ is called the nth degree equation.

An equation with the highest power as 1 $(e.g.\ x + 4 = 9)$ is of the first degree. It is also called a linear equation and its graph represents a straight line.

An equation having its highest power as 2 is called **the quadratic equation.**

For example, $x^2+6x+5=0$ is a quadratic equation in one variable. Again, the equations $x^2+y^2=16$ and $x^2+xy+y^2=5$ are quadratic equations in two variables. Further, higher order equations are **cubic** with highest power of the variable 3 and **biquadratic** with the highest power of the variable 4.

For example, $2x^3+5x^2+4x+8=0$ is a cubic equation in one variable, and $x^4-7x^2=5$ is a biquadratic equation in one variable.

Remark: While determining the degree of an equation it should be noted carefully that *x* is in the numerator only and power of *x* is a positive integer, but not a fraction number.

SOLUTION OF AN EQUATION

The particular value or values of the variable *x*, which satisfies the given equation, is called the **solution** of the given equation. It is also known as the **root** of the equation. The set of values of the variable in any equation is called the **solution set** of the equation.

For example, consider the equation $x-5=2$, which is linear equation in x. putting $x=7$ in this equation, both sides will become equal. It means that $x=7$ satisfies the equation. So $x=7$ is the root of the given equation and $\{x : x=7\}$ is the solution set of the equation.

It is clear that equation $x-5=0$ has only one root as no other value of x except 5 satisfies the equation. Hence, a linear equation will have only one root.

Remark: In a linear equation with one variable, there is only one root or one solution to the equality.

4.4 LINEAR SIMULTANEOUS EQUATIONS WITH TWO VARIABLES

The sets of equations containing two or more unknowns or, variables are known as **simultaneous equation**, if they are satisfied simultaneously by the same values of the unknowns. General expression for such a system is of the form

$$\left.\begin{cases} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{cases}\right\} \text{............}(i)$$

By a solution of (i) we mean an ordered pair (x, y) which satisfies both equations. The system (i) have got a unique solution. But individually each equation has an infinite number of solutions (x, y) corresponding to infinite number of points which lie on the graphs of the given lines. Hence, with the help of simultaneous equations, we can find unique values of variables or unknowns involved in the system. The number of equations should be equal to the number of unknowns. The system of equations (i) will have:

(a) **An unique (exactly one) solution** if $a_1b_2 - a_2b_1 \neq 0$ or $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

In this case the equations are said to be **consistent and independent.**

(b) **No solution** if $a_1b_2 - b_1a_2 = 0$ and at least one of $(a_1c_2 - a_2c_1)$ and $(b_2c_1 - b_1c_2)$ is not zero. In this case, the two equations are said to be inconsistent.

(c) **Infinite number of solutions** if $a_1b_2 - a_2b_1 = 0, a_1c_2 - a_2c_1 = 0,$ and $b_2c_1 - b_1c_2 = 0.$

SOLUTION METHODS OF LINEAR SIMULTANEOUS EQUATIONS

There are various methods of solving a set of linear simultaneous equations. In this section we shall discuss only three methods which are generally used to solve such equations. These are

1. Method of substitution
2. Method of elimination
3. Method of cross multiplication

1. **Method of Substitution :** Consider two linear simultaneous equations involving only two variables in standard form as

$$a_1x + b_1y + c_1 = 0 \quad .....(i)$$

$$a_2x + b_2y + c_2 = 0 \quad .....(ii)$$

To find the solutions of (i) and (ii), we have to find the values of x and y which will satisfy simultaneously both the equations.

From (i) we get $a_1x = -(b_1y + c_1)$ or $x = -\frac{b_1y + c_1}{a_1}$ .....(iii)

Substituting this value of x in (ii), we get

$$a_2\left\{\frac{-(b_1y + c_1)}{a_1}\right\} + b_2y + c_2 = 0 \quad \text{or } a_2(-b_1y - c_1) + a_1b_2y + a_1c_2 = 0$$

or $\quad -b_1a_2y - c_1a_2 + a_1b_2y + a_1c_2 = 0$ or $(a_1b_2 - a_2b_1)y - c_1a_2 + a_1c_2 = 0$

or $\quad (a_1b_2 - a_2b_1)y = c_1a_2 - a_1c_2$, $\quad$ or $y = \frac{c_1a_2 - a_1c_2}{a_1b_2 - a_2b_1}$

Now putting value of y in (iii), we get

$$x = -\frac{1}{a_1}\left\{b_1\left(\frac{c_1a_2 - a_1c_2}{a_1b_2 - a_2b_1}\right) + c_1\right\} = -\frac{1}{a_1}\left\{\frac{b_1(c_1a_2 - a_1c_2) + c_1(a_1b_2 - a_2b_1)}{a_1b_2 - a_2b_1}\right\}$$

$$= -\frac{1}{a_1}\left\{\frac{c_1a_1b_2 - a_1b_1c_2}{a_1b_2 - a_2b_1}\right\} = -\frac{1}{a_1}\left\{\frac{a_1(c_1b_2 - b_1c_2)}{a_1b_2 - a_2b_1}\right\} = \frac{b_1c_2 - c_1b_2}{a_1b_2 - a_2b_1}$$

For example: Solve the equations: $2x+3y=1$ and $3x-y=4$.

Solution: From second equation, we get $x=\frac{4+y}{3}$. Substituting this value of x in the first equation, we get

$$2\left(\frac{4+y}{3}\right)+3y=1 \text{ or } 8+2y+9y=3$$

or, $11y=-5, i.e.\ y=-5/11$

Putting this value of y in $x=\frac{4+y}{3}$, we get $x=\frac{4-(5/11)}{3}=\frac{39}{33}$

Hence, $x=39/33, y=-\frac{5}{11}$ is the required solution.

2. **Method of Elimination:** In this method, the two linear simultaneous equations are transformed into an equivalent system such that the co-efficient of any of the variables in both the transformed equations become equal. Thereafter by addition or subtraction of these equations, that variable is eliminated. The equation so obtained is a linear equation in one variable, and which can be solved for this variable very easily. Then, by substituting the value of this variable in either of the original equations, value of the other variable can be determined. Let us consider two linear equations:

$$a_1x+b_1y+c_1=0 \qquad \text{......(i)}$$

$$a_2x+b_2y+c_2=0 \qquad \text{......(ii)}$$

Suppose we want to eliminate x from the first system. For that we shall equate the co-efficient of x in (i) and (ii) as follows:

Multiply (i) by a_2, (ii) by a_1 and in this way a_1a_2 will be the coefficient of x in the transformed system. The equivalent system becomes

$$a_1a_2x+a_2b_1y+a_2c_1=0 \qquad \text{......(iii)}$$

$$a_1a_2x+a_1b_2y+a_1c_2=0 \qquad \text{......(iv)}$$

Subtracting (iv) from (iii), we get

$$(a_2c_1-a_1b_2)y+(a_2c_1-a_1c_2)=0$$

or $$y=-\frac{a_2c_1-a_1c_2}{a_2b_1-a_1b_2}=-\frac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}$$

Substituting this value of y in (i), we get, $x=-\frac{b_2c_1-b_1c_2}{a_1b_2-a_2b_1}$.

For example: Solve the equations: $x+3y-4=0$ and $3x-y-3=0$

Solution: To eliminate y, multiply second equation by 3 and add it to the first equation, we get,

$10x-13=0$ or $x=13/10$.

Substituting this value of x in the first equation, we get

$$(13/10)+3y-4=0 \text{ or } 13+30y-40=0$$

or $30y-27=0, i.e. y=27/30=9/10.$

3. **Method of Cross Multiplication:** Let us consider the two linear equations

$$a_1x+b_1y+c_1=0 \quad .....(i)$$

$$a_2x+b_2y+c_2=0 \quad .....(ii)$$

Multiplying (i) by a_2 and (ii) by a_1, we have

$$a_1a_2x+a_2b_1y+a_2c_1=0 \quad .....(iii)$$

$$a_1a_2x+a_1b_2y+a_1c_2=0 \quad .....(iv)$$

Subtracting (iv) from (iii), we get

$$(a_2b_1-a_1b_2)y+a_2c_1-a_1c_2=0$$

or $$\frac{y}{a_2c_1-a_1c_2}=\frac{1}{a_1b_2-a_2b_1} \quad ......(v)$$

Similarly, multiplying (i) by b_2 and (ii) by b_1, we get

$$\frac{x}{b_1c_2-b_2c_2}=\frac{1}{a_1b_2-a_2b_1} \quad ......(vi)$$

Therefore from (v) and (vi), we get

$$\frac{x}{b_1c_2-b_2c_2}=\frac{y}{a_2c_1-a_1c_2}=\frac{1}{a_1b_2-a_2b_1}$$

This is called the rule of cross multiplication.

For example: Solve the equation: $3x-10y+5=0$ and $2x+7y+20=0$

Solution: By the method of cross multiplication, we have

$$\frac{x}{-10\times20-7\times5}=\frac{y}{5\times2-20\times3}=\frac{1}{3\times7-2\times(-10)}$$

or $$\frac{x}{-200-35}=\frac{y}{10-60}=\frac{1}{21+20} \text{ or } \frac{x}{-235}=\frac{y}{-50}=\frac{1}{41}$$

or $x=-\frac{235}{41}$ and $y=-\frac{50}{41}$

4.5 LINEAR INEQUALITIES

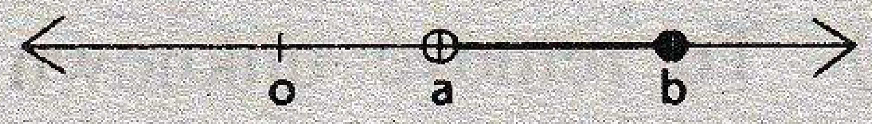
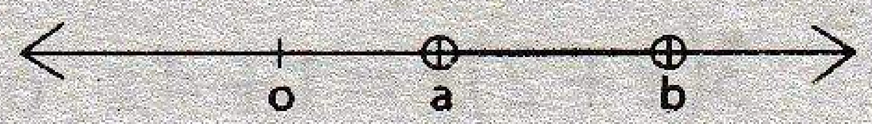
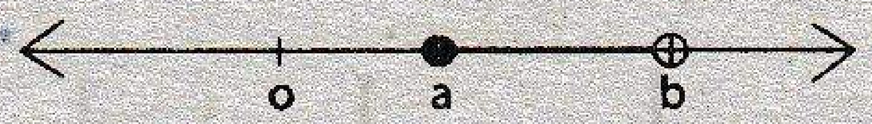
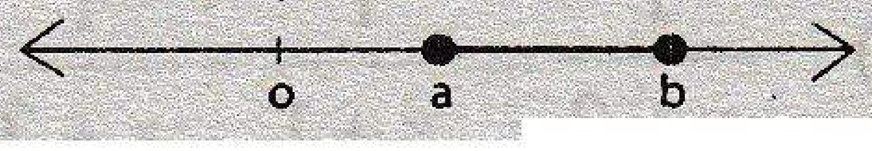
Inequalities: Between two real numbers a and b, if a and b are unequal, a may be greater than b, or a may be smaller than b. These relationships are called inequalities.

Symbolically: $a<b$ means 'a' less than 'b'

$a>b$ means 'a' greater than 'b'

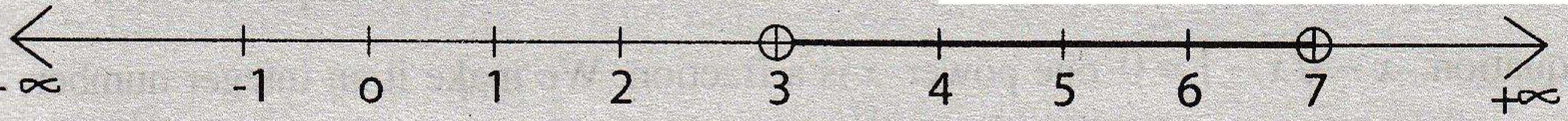
$a \le b$ means 'a' less than or equal 'b'

$a \ge b$ means 'a' greater than or equal 'b'

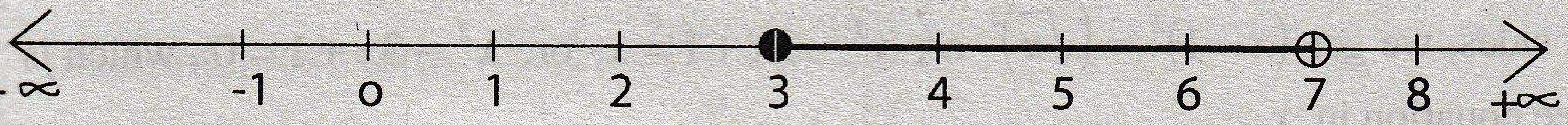
Interval notation	**Inequality notation**	**Graph**
(a,b]	$a < x \le b$	
(a,b)	$a < x < b$	
[a,b)	$a \le x < b$	
[a,b]	$a \le x \le b$	

Graphing the following interval notations:

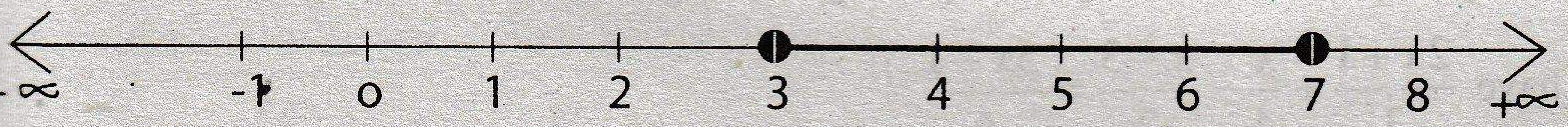
(3,7) or $3<x<7$



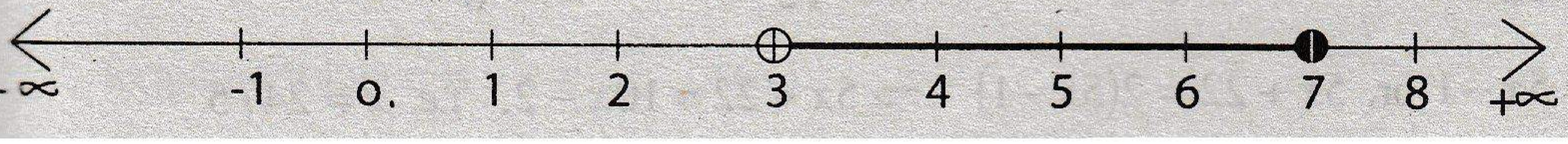
[3,7), or $3 \le x < 7$



[3,7], or $3 \le x \le 7$



(3,7], or $3 < x \le 7$



ILLUSTRATIONS

Illustration-01: Determine the **degree** of the following equations:

(i) $x^2+4=0$, (ii) $2x^2+3x=0$, (iii) $3(x+2)=\frac{1}{x}+4$, (iv) $\frac{1}{x+2}=\frac{1}{x+1}-\frac{1}{x+3}$,

(v) $x-\sqrt{x}-1=0$

Solution:

(i) This is an equation of second degree as the highest power of unknown x is 2.

(ii) This is an equation of second degree as the highest power of x is 2.

(iii) To remove unknown x from denominator, multiplying on both sides by x, we get $3(x+2)x=1+4x$ or, $3x^2+2x-1=0$ which is a second degree equation in x.

(iv) $\frac{1}{x+2}=\frac{1}{x+1}-\frac{1}{x+3}$ or, $\frac{1}{x+2}=\frac{(x+3)-(x+1)}{(x+1)(x+3)}=\frac{2}{(x+1)(x+3)}$

Multiplying both sides by $(x+1)(x+2)(x+3)$, we get

$(x+1)(x+3)=2(x+2)$

$or, x^2+3x+x+3=2x+4$ $or, x^2+2x-1=0$ which is a second degree equation in x.

(v) In the equation, $x-\sqrt{x}-1=0$ one power x is a fraction. We make it an integer number. The given equation can be written as:

$x-1=\sqrt{x}$

Squaring both sides, we get $(x-1)^2=(\sqrt{x})^2$ or, $x^2-2x+1=x$, or, $x^2-3x+1=0$, which is a second degree equation in x.

Illustration-02: Solve the equation: $\frac{2x+1}{3}+\frac{x}{4}-\frac{x-3}{2}=\frac{5x-1}{6}$

Solution: We have $\frac{2x+1}{3}+\frac{x}{4}-\frac{x-3}{2}=\frac{5x-1}{6}$

$\Rightarrow \frac{4(2x+1)+3x-6(x-3)}{12}=\frac{5x-1}{6}$ $\Rightarrow \frac{8x+4+3x-6x+18}{12}=\frac{5x-1}{6}$

$\Rightarrow \frac{5x+22}{2}=5x-1$ or, $5x+22=2(5x-1)$ $\Rightarrow 5x+22=10x-2$, *i.e.* $x=24/5$

Illustration-03: Solve the equation: $\frac{a-x}{a}+\frac{2a-x}{2a}=\frac{3a-x}{3a}$.

Solution: Given $\frac{a-x}{a}+\frac{2a-x}{2a}=\frac{3a-x}{3a}$

$\Rightarrow \frac{2(a-x)+2a-x}{2a}=\frac{3a-x}{3a} \Rightarrow 3(4a-3x)=2(3a-x)$

$\Rightarrow 12a-9x=6a-2x \Rightarrow -7x=-6a \Rightarrow x=\frac{-6a}{-7} \Rightarrow x=\frac{6a}{7}$

$\therefore$ Required solution: $x=\frac{6a}{7}$. [Ans.]

Illustration-04: Solve: $\frac{3}{x-2}+\frac{5}{x-6}=\frac{8}{x+3}$

Solution: Given $\frac{3}{x-2}+\frac{5}{x-6}=\frac{8}{x+3}$

$\Rightarrow \frac{3(x-6)+5(x-2)}{(x-2)(x-6)}=\frac{8}{x+3} \quad \Rightarrow \frac{3x-18+5x-10}{x^2-6x-2x+12}=\frac{8}{x+3}$

$\Rightarrow (x+3)(8x-28)=8(x^2-8x+12) \quad \Rightarrow 8x^2-28x+24x-84=8x^2-64x+96$

$\Rightarrow 60x=180 \Rightarrow x=\frac{180}{60} \Rightarrow x=3$ $\quad \therefore$ Required solution: $x=3$. [Ans.]

Illustration-05: Solve: $\frac{a-x}{a}+\frac{2a-x}{2a}=\frac{3a-x}{3a}$

Solution: Given $\frac{a-x}{a}+\frac{2a-x}{2a}=\frac{3a-x}{3a}$

$\Rightarrow \frac{2(a-x)+2a-x}{2a}=\frac{3a-x}{3a} \quad \Rightarrow 3(4a-3x)=2(3a-x)$

$\Rightarrow 12a-9x=6a-2x \quad \Rightarrow -7x=-6a \Rightarrow x=\frac{-6a}{-7} \Rightarrow x=\frac{6a}{7}$

$\therefore$ Required solution is : $x=\frac{6a}{7}$. [Ans.]

Illustration-06: Solve: $\frac{x-p}{q}+\frac{x-q}{p}=\frac{q}{x-p}+\frac{p}{x-q}$

Solution: Given $\frac{x-p}{q}+\frac{x-q}{p}=\frac{q}{x-p}+\frac{p}{x-q}$

$$\Rightarrow \frac{p(x-p)+q(x-q)}{pq}=\frac{q(x-q)+p(x-p)}{(x-p)(x-q)} \Rightarrow \frac{px-p^2+qx-q^2}{pq}=\frac{qx-q^2+px-p^2}{x^2-qx-px+pq}$$

$$\Rightarrow \frac{px-p^2+qx-q^2}{pq}-\frac{px-p^2+qx-q^2}{x^2-qx-px+pq}=0$$

$$\Rightarrow (px-p^2+qx-q^2)\left(\frac{1}{pq}-\frac{1}{x^2-qx-px+pq}\right)=0$$

$$\Rightarrow px-p^2+qx-q^2=0 \text{ or, } \frac{1}{pq}-\frac{1}{x^2-qx-px+pq}=0$$

$$\Rightarrow px+qx=p^2+q^2 \text{ or, } \frac{1}{pq}=\frac{1}{x^2-qx-px+pq}$$

$$\Rightarrow x(p+q)=p^2+q^2 \text{ or, } x^2-qx-px+pq=pq \quad \Rightarrow x=\frac{p^2+q^2}{p+q} \text{ or, } x^2-qx-px=0$$

$$\Rightarrow x=\frac{p^2+q^2}{p+q} \text{ or, } x(x-q-p)=0 \quad \Rightarrow x=\frac{p^2+q^2}{p+q} \text{ or, } x=0 \text{ or, } x-q-p=0$$

$$\Rightarrow x=\frac{p^2+q^2}{p+q} \text{ or } x=0 \text{ or, } x=p+q$$

$\therefore$ Required solutions are : $x=\frac{p^2+q^2}{p+q}, 0,\ p+q$. [Ans.]

Illustration-07: Solve by the method of substitution : $2x+3y=1$ and $3x-y=4$.

Solution: From second equation, we get $x=\frac{4+y}{3}$. Substituting this value of x in the first equation, we get

$$2\left(\frac{4+y}{3}\right)+3y=1 \text{ or } 8+2y+9y=3$$

or, $\quad 11y=-5, i.e.\ y=-5/11$

Putting this value of y in $x=\frac{4+y}{3}$, we get $x=\frac{4-(5/11)}{3}=\frac{39}{33}$

Hence, $x=39/33, y=-\frac{5}{11}$ is the required solution.

Illustration-08: Solve by the method of elimination: $x+3y-4=0$ and $3x-y-3=0$

Solution: To eliminate y, multiply second equation by 3 and add it to the first equation, we get, $10x-13=0$ or $x=13/10$.

Substituting this value of x in the first equation, we get

$$(13/10)+3y-4=0 \text{ or } 13+30y-40=0$$

or $30y-27=0, i.e. y=27/30=9/10.$

Therefore the required solution is: $x=13/10$ and $y=9/10$.

Illustration-09: Solve by the method of cross multiplication:

$$3x-10y+5=0 \text{ and } 2x+7y+20=0$$

Solution: By the method of cross multiplication, we have

$$\frac{x}{-10\times 20-7\times 5}=\frac{y}{5\times 2-20\times 3}=\frac{1}{3\times 7-2\times(-10)}$$

or $\dfrac{x}{-200-35}=\dfrac{y}{10-60}=\dfrac{1}{21+20}$ **or** $\dfrac{x}{-235}=\dfrac{y}{-50}=\dfrac{1}{41}$

or $x=-\dfrac{235}{41}$ and $y=-\dfrac{50}{41}$

Illustration-10: Solve the following simultaneous equations: $\dfrac{x}{6}+\dfrac{y}{15}=4,\ \dfrac{x}{3}-\dfrac{y}{12}=\dfrac{19}{4}$

Solution: Given $\dfrac{x}{6}+\dfrac{y}{15}=4, \dfrac{x}{3}-\dfrac{y}{12}=\dfrac{19}{4}$

Now $\dfrac{x}{6}+\dfrac{y}{15}=4 \Rightarrow \dfrac{15x+6y}{90}=4 \Rightarrow 15x+6y=360 \quad \Rightarrow 15x=360-6y$

$\Rightarrow x=\dfrac{360-6y}{15}$ ....(1)

Also $\dfrac{x}{3}-\dfrac{y}{12}=\dfrac{19}{4} \Rightarrow \dfrac{4x-y}{12}=\dfrac{19}{4} \Rightarrow 4x-y=57$

$\Rightarrow 4\left(\dfrac{360-6y}{15}\right)-y=57$ [From (1), we get.]

$\Rightarrow \dfrac{1440-24y-15y}{15}=57 \Rightarrow 1440-39y=855 \Rightarrow y=\dfrac{-585}{-39} \Rightarrow y=15$

Putting the value of y in (1), we get

$x=\dfrac{360-6(15)}{15}=\dfrac{360-90}{15}=\dfrac{270}{15}=18$. Required solutions are : $\left.\begin{matrix} x=18 \\ y=15 \end{matrix}\right\}$ [Ans]

Illustration-11: Solve the following simultaneous equations: $\frac{2}{x}+\frac{3}{y}=5, \frac{1}{x}-\frac{1}{2y}=\frac{1}{2}$,

Solution: Given $\frac{2}{x}+\frac{3}{y}=5,\ \frac{1}{x}-\frac{1}{2y}=\frac{1}{2}$

Now $\frac{2}{x}+\frac{3}{y}=5 \Rightarrow \frac{2}{x}=5-\frac{3}{y} \Rightarrow \frac{2}{x}=\frac{5y-3}{y}$

$\Rightarrow 2y=(5y-3)x \Rightarrow (5y-3)x=2y \Rightarrow x=\frac{2y}{5y-3}$ ....(1)

Also $\frac{1}{x}-\frac{1}{2y}=\frac{1}{2} \Rightarrow \frac{1}{\frac{2y}{5y-3}}-\frac{1}{2y}=\frac{1}{2}$ [From (1), we get]

$\Rightarrow \frac{5y-3-1}{2y}=\frac{1}{2} \Rightarrow \frac{5y-4}{y}=\frac{1}{1} \Rightarrow 5y-4=y \Rightarrow 4y=4 \Rightarrow y=1$

Putting the value of y in (1), we get

$$x=\frac{2y}{5y-3}=\frac{2(1)}{5(1)-3}=\frac{2}{5-3}=\frac{2}{2}=1$$

Required solutions: $\left.\begin{array}{l} x=1 \\ y=1 \end{array}\right\}$ [Ans.]

LINEAR SIMULTANEOUS EQUATIONS WITH THREE VARIABLES

There is no general method for solving such type of equations. But the methods used in this section for solving examples should be carefully noted.

Illustration-12: Solve the following simultaneous equations in three variables:

$2x+3y-4z=1$ ...(i)

$3x-y-2z=4$ ...(ii)

$4x-7y-6z=-7$ ...(iii)

Solution: Multiplying (ii) by 3, we have $9x-3y-6z=12$. Adding it in (i), we get

$11x-10z=13$ ...(iv)

Again, multiplying (ii), by 7, we have $21x-7y-14z=28$. Subtracting it from (iii), we get

$-17x+8z=-35$ ...(v)

Eliminating z from equations (iv) and (v), we get

Substituting this value of x in $11x-10z=13$, we get $x=3$

$$11\times3-10z=13 \text{ or } -10z=-20, \text{ i.e. } z=2$$

For getting value of y, substituting these value of x and z in (i), we get

$$2\times3+3y-4\times2=1 \text{ or } 3y=3, \text{ i.e. } y=1$$

Hence, the required solution is: $x=3, y=1, z=2$.

Illustration-13: Solve the following simultaneous equations in three variables:-

$$x+2y+2z=0 \qquad \ldots(i)$$

$$3x-4y+z=0 \qquad \ldots(ii)$$

$$x^2+3y^2+z^2=11 \qquad \ldots(iii)$$

Solution: From first two equations by cross multiplication, we have

$$\frac{x}{2\times1-2\times(-4)}=\frac{y}{2\times3-1\times1}=\frac{z}{1\times(-4)-3\times2}$$

$$\text{or, } \frac{x}{10}=\frac{y}{5}=\frac{z}{-10} \text{ or } \frac{x}{2}=\frac{y}{1}=\frac{z}{-2}=k \text{ (say)}$$

i.e. $x=2k, y=k, z=-2k$

substituting these value of x, y and z in (iii), we have

$$(2k)^2+3(k)^2+(-2k)^2=11 \text{ or } 4k^2+3k^2+4k^2=11$$

$$\text{or, } 11k^2=11, \text{ i.e. } k=\pm1$$

Taking $k=1$, we get $x=2, y=1, z=-2$. Again if $k=-1$, then $x=-2, y=-1, z=2$.

Illustration-14:

Find solution, Solution set and graphs the following inequalities:

(i) $2(2x+3)-10<6(x-2)$ (ii) $\frac{4x-3}{3}+8<6+\frac{3x}{2}$

(iii) $-3\le4-7x<18$ (iv) $-3<7-2x\le7$

Notes : 1) Let, $a\ge b$

$\Rightarrow -a\le -b$

For example : if $9>7$

then, $-9<-7$

2) Let, $a>b$

$\frac{1}{a}<\frac{1}{b}$. For example : if $\frac{4}{8}>\frac{4}{12}$

then, $\frac{8}{4}<\frac{12}{4}$

Problem-(i)

$$2(2x+3)-10<6(x-2)$$

$$\Rightarrow 4x+6-10<6x-12$$

$$\Rightarrow 4x-6x<-12+4$$

$$\Rightarrow -2x<-8 \Rightarrow x>\frac{8}{2} \Rightarrow x>4$$

So, the required solution is : x>4

The required solution set, $S=\{x\in R: x>4\}$

Graph :

Problem-(ii):

$$\frac{4x-3}{3}+8<6+\frac{3x}{2} \Rightarrow \frac{4x-3+24}{3}<\frac{12+3x}{2} \Rightarrow \frac{4x+21}{3}<\frac{3x+12}{2}$$

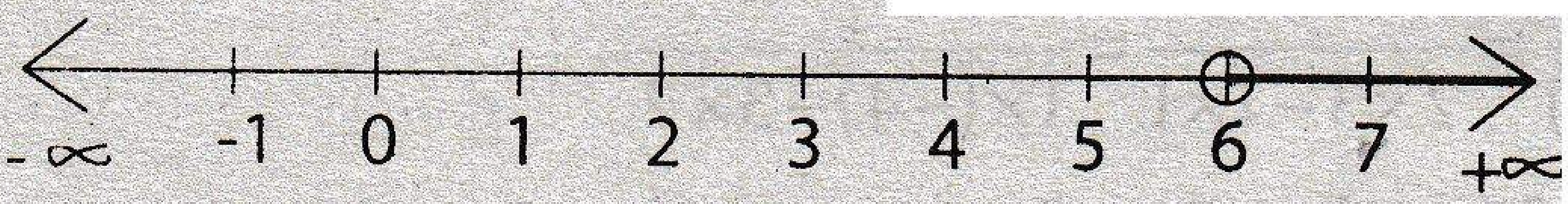
$$\Rightarrow 2(4x+21)<3(3x+12) \Rightarrow 8x+42<9x+36 \Rightarrow 8x-9x<36-42$$

$$\Rightarrow -x<-6 \Rightarrow x>6$$

So the required solution is : $x>6$

The required solution set, $S=\{x\in R: x>6\}$

Graph :



Problem-(iii): $-3\le 4-7x<18$

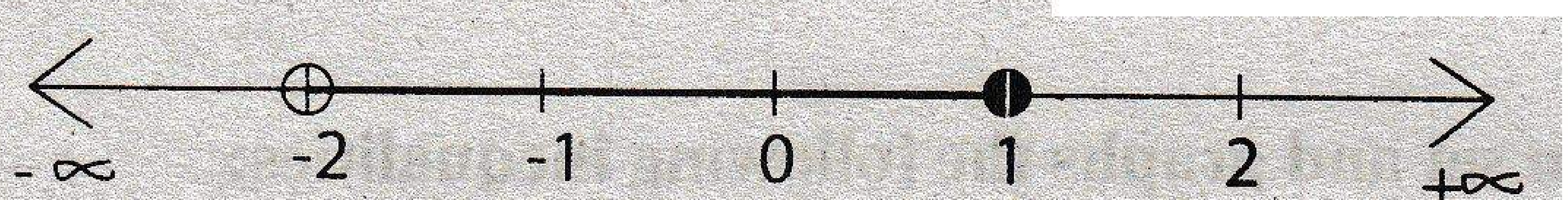
$$\Rightarrow -3-4\le -4+4-7x<-4+18 \quad \Rightarrow -7\le -7x<14$$

$$\Rightarrow \frac{-7}{7}\le -x<\frac{14}{7} \Rightarrow -1\le -x<2 \Rightarrow 1\ge x>-2$$

So the required solution is : $1\ge x>-2$

The required solution set, $S=\{x\in R: 1\ge x>-2\}$

Graph :



Problem-(iv): $-3<7-2x\le 7$

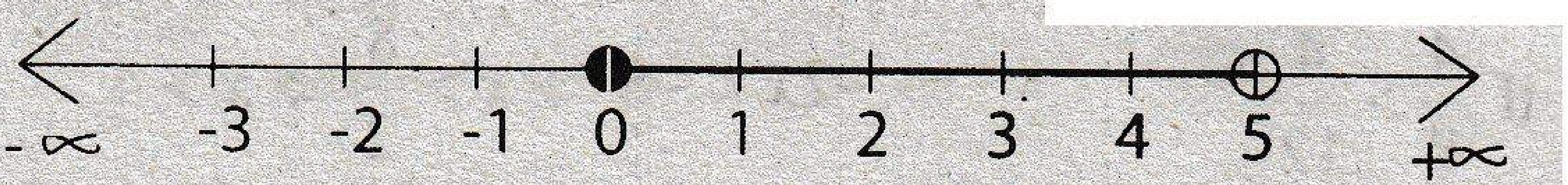
$$\Rightarrow -3-7<7-2x-7\le 7-7 \Rightarrow -10<-2x\le 0 \Rightarrow -\frac{10}{2}<-x\le \frac{0}{2}$$

$$\Rightarrow -5<-x\le 0 \Rightarrow 5>x\ge 0$$

So the required solution is : $5\ge x\ge 0$

The required solution set, $S=\{x\in R: 5>x\ge 0\}$

Graph :



ABSOLUTE VALUE

The absolute value of a real number x is denoted by $|x|$ and is defined by

$$|x| = x \text{ if } x \geq 0$$
$$= -x \text{ if } x < 0$$

For example : $|8|=8$, $|0|=0$,

$|-8|=-(-8)=8$ Note: The absolute value of a number is never negative.

Solving absolute value problem:

Illustration-15: Solve: $|3x+5|=4$.............(i)

Case I : When $(3x+5)\geq 0$

Then equation (i) $\Rightarrow 3x+5=4$

$\Rightarrow 3x=4-5 \Rightarrow 3x=-1 \Rightarrow x=-\frac{1}{3}$

Case II : when $(3x+5)<0$

Then equation (i) $\Rightarrow -(3x+5)=4$

$\Rightarrow -3x-5=4 \Rightarrow -3x=4+5 \Rightarrow -3x=9 \Rightarrow x=-3$

So, the required solutions are : $x=-3,-\frac{1}{3}$

Illustration-16: Solve: $|3x+4|=8$ .... (i)

Case I) when $(3x+4)\geq 0$

Then equation (i) $\Rightarrow 3x+4=8$

$\Rightarrow 3x=8-4 \Rightarrow 3x=4 \Rightarrow x=\frac{4}{3}$

Case II) When $(3x+4)<0$

Then equation (i) $\Rightarrow -(3x+4)=8$

$\Rightarrow -3x-4=8 \Rightarrow -3x=8+4 \Rightarrow -3x=12 \Rightarrow x=-4$

So, the required solutions are: $x=\frac{4}{3},-4.$

Illustration-17: Solve: $|2x-1| < 3$ ...... (i)

Case I : When $(2x-1) \geq o$

Then equation (i) $\Rightarrow$ $2x-1<3$

$\Rightarrow 2x<3+1 \Rightarrow 2x<4 \Rightarrow x<2$ .... (A)

Case II : When $(2x-1)<0$

The equation (i) $\Rightarrow -(2x-1)<3$

$\Rightarrow -2x+1<3 \Rightarrow -2x<3-1 \Rightarrow -2x<2 \Rightarrow -x<1 \Rightarrow x>-1$ ....(B)

From equation (A) and (B) we get $-1<x<2$

So the required solution is: $-1<x<2$.

The solution set, $S=\{x \in R: -1<x<2\}$

Graph :

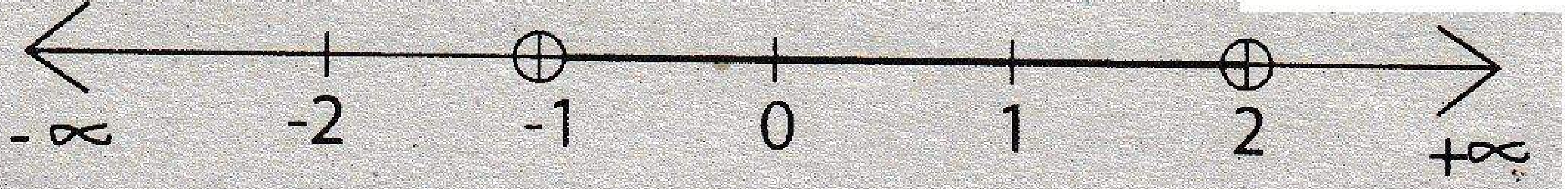


Illustration-18: Solve: $\dfrac{2x-3}{4}+6 \geq 2+\dfrac{4x}{3}$

$$\Rightarrow \frac{2x-3+24}{4} \geq \frac{6+4x}{3} \Rightarrow \frac{2x+21}{4} \geq \frac{4x+6}{3} \Rightarrow 3(2x+21) \geq 4(4x+6)$$

$$\Rightarrow 6x+63 \geq 16x+24 \Rightarrow 6x-16x \geq 24-63 \Rightarrow -10x \geq -39 \Rightarrow x \leq \frac{39}{10} \Rightarrow x \leq 3.9$$

So, the required solution is: $x \leq 3.9$

The required solution set, $S=\{x \in R: x \leq 3.9\}$

Graph :

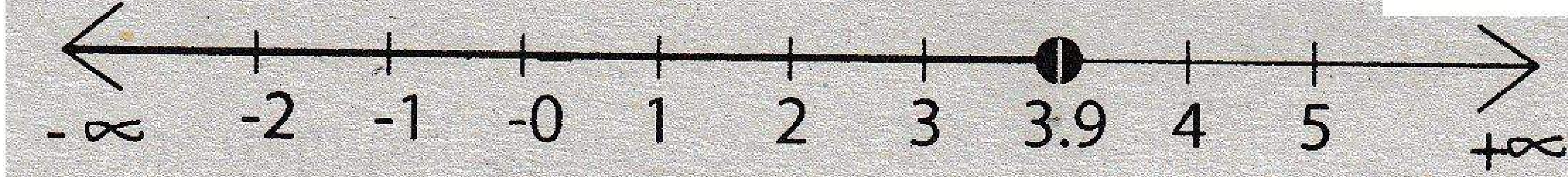


Illustration-19: Solve: $2 \leq 3m-7<14$

$$\Rightarrow 2+7 \leq 3m-7+7<14+7 \Rightarrow 9 \leq 3m<21 \Rightarrow \frac{9}{3} \leq \frac{3m}{3}<\frac{21}{3} \Rightarrow 3 \leq m<7$$

So, the required solution is: $3 \leq m<7$

The required solution set, $S=\{m \in R: 3 \leq m<7\}$

Graph:

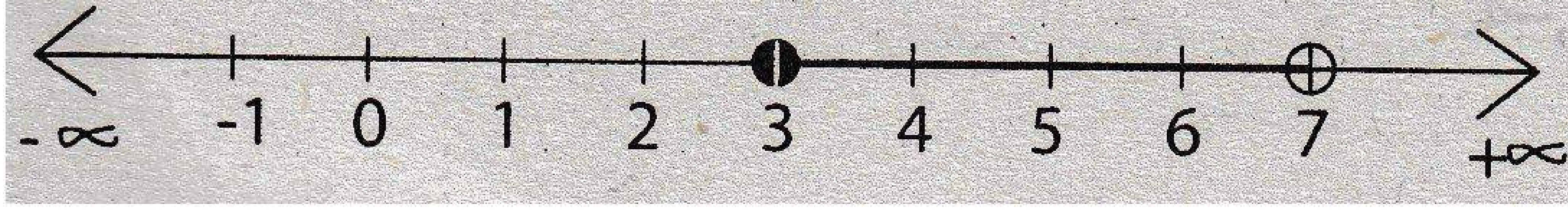


Illustration-20: Solve: $-4 < 5x + 6 \le 21$

$\Rightarrow -4 - 6 < 5x + 6 - 6 \le 21 - 6 \Rightarrow -10 < 5x \le 15 \Rightarrow \dfrac{-10}{5} < \dfrac{5x}{5} \le \dfrac{15}{5} \Rightarrow -2 < x \le 3$

So, the required solution is : $-2 < x \le 3$

The required solution set , $S = \{x \in R : -2 < x \le 3\}$

Graph:

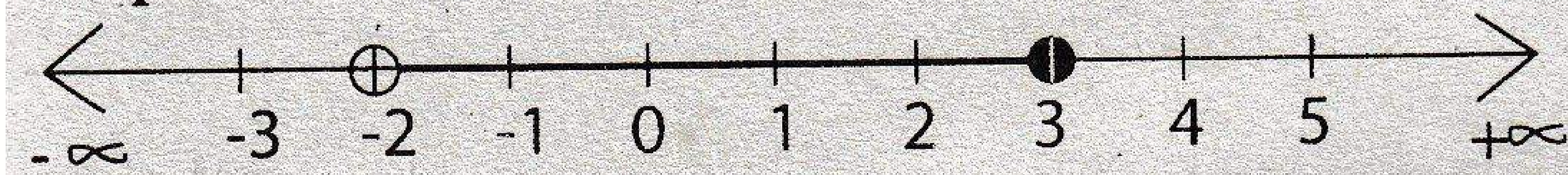


Illustration-21: Solve: $|x + 4| = 3x - 8$ .... (i)

Case I : When $(x + 4) \ge 0$

Then equation (i) $\Rightarrow x + 4 = 3x - 8 \Rightarrow x - 3x = -8 - 4 \Rightarrow -2x = -12 \Rightarrow x = 6$

Case II : When $(x + 4) < 0$

Then equation (i) $\Rightarrow -(x + 4) = 3x - 8$

$\Rightarrow -x - 4 = 3x - 8 \Rightarrow -x - 3x = -8 + 4 \Rightarrow -4x = -4 \Rightarrow x = 1$

So, the required solutions are : $x = 1,6$

Illustration-22: Solve: $|5x - 3| \le 12$ ....(i)

Case I : $(5x - 3) \ge 0$

Then equation (i) $\Rightarrow 5x - 3 \le 12$

$\Rightarrow 5x \le 12 + 3 \Rightarrow 5x \le 15 \Rightarrow x \le 3$ ....(A)

Case II : When $(5x - 3) < 0$

Then equation (i) $\Rightarrow -(5x - 3) \le 12$

$\Rightarrow -5x + 3 \le 12 \Rightarrow -5x \le 12 - 3 \Rightarrow -5x \le 9 \Rightarrow x \ge \dfrac{-9}{5}$

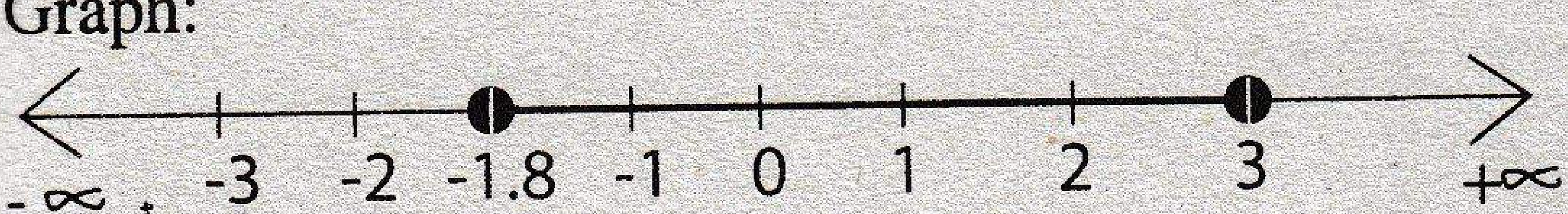
$\Rightarrow x \ge -1.8$ .... (B)

From equation (A) and (B) we get: $-1.8 \le x \le 3$

So, the required solution is: $-1.8 \le x \le 3$

The required solution set, $S = \{x \in R : -1.8 \le x \le 3\}$

Graph:



−21

Illustration-23: Solve: $|2x-3| \le 5$ ....(i)

Case I : when $(2x-3) \ge 0$

Then the equation (i) $\Rightarrow\ 2x-3 \le 5$

$\Rightarrow\ 2x \le 5+3 \Rightarrow\ 2x \le 8 \Rightarrow\ x \le 4$ .... (A)

Case II : When $(2x-3) < 0$

Then equation (i) $\Rightarrow -(2x-3) \le 5$

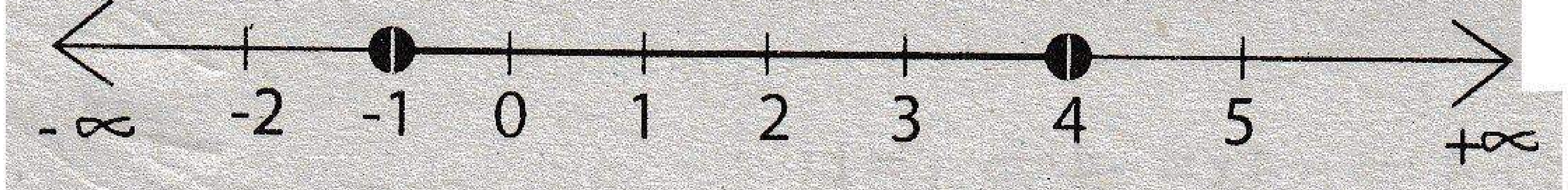
$\Rightarrow -2x+3 \le 5 \Rightarrow -2x \le 5-3 \Rightarrow\ -2x \le 2 \Rightarrow -x \le 1 \Rightarrow\ x \ge -1$ .... (B)

From equation (A) and (B) we get $-1 \le x \le 4$

So, the required solution is: $-1 \le x \le 4$

The required solution set, $S = \{x \in R : -1 \le x \le 4\}$

Graph:



4.6 BUSINESS APPLICATIONS

Problem -01: A horse and a cow were sold for Tk. 3040 making a profit of 25% on the horse and 10% on the cow. By selling them for Tk. 3070, the profit realized would have been 10% on the horse and 25% on the cow. Find the cost price of each.

Solution: Let the cost price of a horse = x

Let the cost price of a cow = y

According to the 1st condition,

selling price of horse + selling price of horse = total selling price

$\Rightarrow$(cost of horse + profit on horse)+(cost of cow + profit on cow) = total selling price

$\Rightarrow (x + 25\% \times x) + (y + 10\% \times y) = 3040$

$\Rightarrow (x + 0.25x) + (y + 0.10y) = 3040$

$\Rightarrow 1.25x + 1.10y = 3040$

$\Rightarrow 1.25x = 3040 - 1.10y$

$\Rightarrow x = \dfrac{3040 - 1.10y}{1.25}$ ....(1)

According to the 2nd condition,

selling price of horse + selling price of horse = total selling price

$\Rightarrow$ (cost of horse + profit on horse)+(cost of cow + profit on cow) = total selling price

$$\Rightarrow (x+10\%\times x)+(y+25\%\times y)=3070$$

$$\Rightarrow (x+0.10x)+(y+0.25y)=3070$$

$$\Rightarrow 1.10x+1.25y=3070$$

$$\Rightarrow 1.10\left(\frac{3040-1.10y}{1.25}\right)+1.25y=3070 \text{ [From (1), we get]}$$

$$\Rightarrow \frac{3344-1.21y}{1.25}+1.25y=3070 \qquad \Rightarrow \frac{3344-1.21y+1.5625y}{1.25}=3070$$

$$\Rightarrow 3344+0.3525y=3837.50 \qquad \Rightarrow 0.3525y=493.50 \Rightarrow y=\frac{493.50}{0.3525} \Rightarrow y=1400$$

Putting the value of y in (1), we get

$$x=\frac{3040-1.10y}{1.25} \qquad \Rightarrow x=\frac{3040-1.10(1400)}{1.25} \Rightarrow x=\frac{3040-1540}{1.25}$$

$$\Rightarrow x=\frac{1500}{1.25} \Rightarrow x=1200$$

So the required cost price of a horse (x) = Tk. 1200

and the required cost price of a cow (y) = Tk. 1400 **[Ans.]**

Problem -02: For a certain commodity the demand (D) in kg. for a price (p) in taka is given by $D=100(10-p)$. The supply equation giving the supply (S) in kg. for a price (p) in Taka is $S=75(p-3)$. The market is such that the demand equals supply. Find the market price and the quantity that will be bought and sold.

Solution: Given that, $D=S$ i.e. $100(10-p)=75(p-3)$

$$\Rightarrow 1000+225=75p+100p \Rightarrow 175p=1225, i.e.\ p=7$$

Substituting $p=7$ in the demand equation; $D=100(10-p)$, we get

$$D=100(10-p)=100\times 3=300$$

Hence, the required market price is Tk. 7 and the quantity, which will be bought and sold, is 300 kg.

Problem-03: Monthly income of two persons is in the ratio of 4:5 and their monthly expenditure is in the ratio of 7:9. If each saves Tk. 50 per month, find the monthly income of both the persons.

Solution: Let $4x$ and $5x$ be the monthly income of two persons respectively.

Then, their monthly expenditure will be $(4x - 50)$ and $(5x - 50)$ respectively.

Now, as per the given condition of their expenditure, we get

$\frac{4x-50}{5x-50} = \frac{7}{9}$ or, $9(4x-50) = 7(5x-50)$, i.e. $x = 100$

Hence, the monthly incomes of two persons are Tk. 400 and Tk. 500

Problem-04: Let speed of a boat in still water be 10 km per hour. If it can travel 24 km down stream and 14 km in the upstream, indicate the speed of the flow of stream.

Solution: Let x km / hour be the speed of flow stream. Then the speed of boat in down stream and upstream will be $(10+x)$ km/ hour and $(10-x)$ km /hour, respectively.

Now the time taken by the boat to cover 24 km in down stream and 14 km upstream will be $24/(10 \times x)$ hours and $14/(10-x)$ hours, respectively.

But it is given that both of these timings are equal.

Therefore, we have $\frac{24}{10+x} = \frac{14}{10-x}$

or, $24(10-x) = 14(10+x)$ or, $240 - 24x = 140 + 14x$

or, $240 - 140 = 14x + 24x$ *i.e.* $x = 100/38 = 2.631$

Hence, the speed of the flow of the stream is 2.631 km, per hour.

Problem-05: Total salary of A and B is equal. If A gets 65% allowance of his basic salary and B gets 80% of this basic salary, what is the basic salary of A. if the basic salary of B is Tk. 1100?

Solution: Let the basic salary of A be Tk. x.

Then total salary of A is, $x + 65\%$ of $x = x + 0.65x = 1.65x$

Given the basic salary of B as: Tk. 1100.

Therefore total salary of B is : $1100 + 80\%$ of $1100 = 1100 + 880 = 1980$

But given that, total salary of A= total salary of B,

i.e. $1.65x = 1980$ or $x = 1980 / 1.65 =$ Tk.1200.

Hence, the basic salary of A is Tk. 1200.

Problem-06: If the demand and supply laws respectively are given by the equations as:

$$4q+9p=48\text{, and } p=\frac{q}{9}+2.$$

Find the equilibrium price and quantity.

Solution: Given $4q+9p=48$

$$\Rightarrow 9p=48-4q \qquad \Rightarrow p=\frac{48-4q}{9}$$

$$\therefore p_d=\frac{48-4q}{9}$$

Also $p=\frac{q}{9}+2 \quad \Rightarrow p=\frac{q+18}{9} \qquad \therefore p_s=\frac{q+18}{9}$

Under perfect competition, equilibrium will be obtained as follows:

$$D=S$$

$$\Rightarrow p_d=p_s$$

$$\Rightarrow \frac{48-4q}{9}=\frac{q+18}{9}$$

$$\Rightarrow 5q=30$$

$\Rightarrow q=6$, which is the equilibrium quantity.

Putting the value of q in the demand or supply function, we get the equilibrium price as follows:

Equilibrium price $=\frac{q}{9}+2$

$$=\frac{6}{9}+2=\frac{24}{9}=\frac{8}{3}$$

So the required equilibrium price (p) =Tk. $\frac{8}{3}$ and

the required equilibrium quantity (q) = 6 units [Ans]

Problem-07: Demand for goods of and industry is given by the equation pq=100, where p is price and q is quantity and supply is given by the equation 20+3p=q. Find the equilibrium price and quantity.

Solution:

Given $pq=100$

$$\Rightarrow p=\frac{100}{q}$$

$$\therefore p_d=\frac{100}{q}$$

Also 20+3p=q

$$\Rightarrow 3p = q - 20$$

$$\Rightarrow p = \frac{q-20}{3}$$

$$\therefore p_s = \frac{q-20}{3}$$

Under perfect competition, equilibrium will be obtained as follows:

$$D = S$$

$$\Rightarrow p_d = p_s$$

$$\Rightarrow \frac{100}{q} = \frac{q-20}{3}$$

$$\Rightarrow 300 = q^2 - 20q$$

$$\Rightarrow -q^2 + 20q + 300 = 0$$

$$\Rightarrow q^2 - 20q - 300 = 0$$

$$\Rightarrow q^2 - 30q + 10q - 300 = 0$$

$$\Rightarrow q(q-30) + 10(q-30) = 0$$

$$\Rightarrow (q-30)(q+10) = 0$$

$$\Rightarrow (q-30) = 0 \ or \ (q+10) = 0$$

$$\Rightarrow q = 30 \ or \ q = -10$$

But quantity cannot be negative.

$\therefore q = 30$, which is the equilibrium quantity.

Putting the value of q in the demand or supply function, we get the equilibrium price as follows :

Equilibrium price $= \frac{100}{q} = \frac{100}{30} = \frac{10}{3}$

So the required equilibrium price (p) = Tk. $\frac{10}{3}$ and the required equilibrium quantity (q) = 30 units. [Ans.]

Problem-08: The sum of payment of two lecturers is Tk. 1600 per month. If the payment of one lecturer be decreased by 9% and the payment of the second be increased by 17%, their payments become equal. Find the payment of each lecturer.

Solution: let the payment of 1st lecturer = x

Let payment of 2nd lecturer = y

According to the 1st condition.

Payment of 1st lecturer + payment of 2nd lecturer = 1600

$$x + y = 1600$$

$$\Rightarrow y = 1600 - x \text{..................}(1)$$

According to the 2nd condition,

$$x - 9\% \times x = y + 17\% \times y$$

$$\Rightarrow x - 0.09x = y + 0.17y$$

$$\Rightarrow 0.91x = 1.17y$$

$$\Rightarrow 0.91x = 1.17(1600 - x) \quad \text{[From (1), we get]}$$

$$\Rightarrow 0.91x = 1872 - 1.17x$$

$$\Rightarrow 2.08x = 1872$$

$$\Rightarrow x = \frac{1872}{2.08}$$

$$\Rightarrow x = 900$$

Putting the value of x in (1), we get

$$y = 1600 - x$$

$$\Rightarrow y = 1600 - 900 \qquad \Rightarrow y = 700$$

So the required payment of 1st lecturer (x) = Tk 900

and the required payment of 2nd lecturer (y) = Tk. 700

Problem-09: A commodity is produced by using 3 units of labor and 2 units of capital. The total cost comes to 62. If the commodity is produced by using 4 units of labor and 1 units of capital, the cost comes to 56. what is the cost per unit of labor and capital?

Solution: Let the cost of 1 unit labor = L

Let the cost of 1 unit capital = C

According to the 2nd condition,

$$4L + 1C = 56$$

$$\Rightarrow C = 56 - 4L \text{..................}(1)$$

According to the 1st condition,

$$3L + 2C = 62$$

$$\Rightarrow 3L + 2(56 - 4L) = 62 \quad \text{[From (1), we get]}$$

$$\Rightarrow 3L + 112 - 8L = 62$$

$$\Rightarrow -5L = -50 \quad \Rightarrow L = 10$$

Putting the value of L in (1), we get

$$C = 56 - 4L \quad \Rightarrow C = 56 - 4(10) \Rightarrow C = 56 - 40 \Rightarrow C = 16$$

So the required cost of 1 unit labor (L) = Tk. 10

And the required cost of 1 unit capital (C) = Tk. 16 [Ans]

Problem-10: A man's income from interest and wages is Tk. 500. He Doubles his investment and also gets an increase of 50% in wages and his income increases to Tk. 800. What was his original income separately in terms of interest (I) and wages (W).

Solution: According to the 1st condition,

$$I + W = 500$$

$$\Rightarrow I = 500 - W \text{.........(1)}$$

According to the 2nd condition,

$$2I + (W + 50\%W) = 800$$

$$\Rightarrow 2(500 - W) + (W + 0.50W) = 800 \quad \text{[From (1), we get]}$$

$$\Rightarrow 1000 - 2W + 1.50W = 800$$

$$\Rightarrow -0.50W = -200 \quad \Rightarrow W = \frac{-200}{-0.50} \Rightarrow W = 400$$

Putting the value of W in (1), we get

$$I = 500 - W$$

$$\Rightarrow I = 500 - 400 \quad \Rightarrow I = 100$$

so the required income from interest (I) = Tk. 100

and the required income from wages (W) = Tk. 400 [Ans]

Problem-11: If there are two commodities X and Y with prices p_1 and p_2 demand D_1, D_2 and supplies S_1, S_2 and we have the demand and supply schedules as:

$$D_1 = 10 - p_1 + p_2, \; S_1 = 6 + p_1 + 2p_2$$

$$D_2 = 12 + 2p_1 - p_2, \; S_2 = 19 + 3p_1 + 5p_2$$

(i) Find the equilibrium prices,

(ii) Determine the equilibrium quantities exchanges in the market.

Solution: Given $D_1 = 10 - p_1 + p_2$, $S_1 = 6 + p_1 + 2p_2$

$D_2 = 12 + 2p_1 - p_2$, $S_2 = 19 + 3p_1 + 5p_2$

Under perfect competition, equilibrium will be obtained follows:

$D_1 = S_1$ and $D_2 = S_2$

$\Rightarrow 10 - p_1 + p_2 = 6 + p_1 + 2p_2$ and $12 + 2p_1 - p_2 = 19 + 3p_1 + 5p_2$

$\Rightarrow 2p_1 + p_2 - 4 = 0$..................(1) and $p_1 + 6p_2 + 7 = 0$.................(2)

Solving (1), and (2), we get $p_1 = \frac{31}{11}$ and $p_2 = -\frac{18}{11}$

Putting the values of p_1 and p_2 in the demand functions D_1 and D_1, we get

$$D_1 = 10 - p_1 + p_2 = 10 - \frac{31}{11} + \left(-\frac{18}{11}\right) = \frac{61}{11}$$

$$D_2 = 12 + 2p_1 - p_2 = 12 + 2\left(\frac{31}{11}\right) - \left(-\frac{18}{11}\right) = \frac{212}{11}$$

but price cannot be negative. $\therefore\ p_2 = \frac{18}{11}$

So the required equilibrium prices are: $p_1 = Tk\frac{31}{11}$ $p_2 = Tk.\frac{18}{11}$

And the required equilibrium quantities are: $q_1 = \frac{61}{11}$ units, $q_2 = \frac{212}{11}$ units [Ans.]

BRIEF REVIEW

Definition

Equation: If two algebraic expressions are connected by the sign of equality (=) then this statement is called an equation. For example: $3x-7=2x+1$ is an equation.

Identity: When equality holds true of an equation whatever be the values of the variables, then that equation is called an identity.

Linear equation: An equation of the form $ax \pm b = 0$ $(a \neq 0)$, where a and b are constants and x is a variable, is called a linear equation or first degree equation.

In other words, an equation having highest power of the variable is 1 (one), is called a linear equation. For example: $3x-7=0$ is a linear equation.

Linear simultaneous equation: The sets of equations containing two or more unknowns or, variables are known as **simultaneous equation**, if they are satisfied simultaneously by the same values of the unknowns.

Degree of an equation: The highest power of the unknown variable of an equation is called the degree of that equation. Ex.: $2x^2+3x-7=0$ is an equation with degree 2.

Solution of an equation: The solution of an equation is the value or values of the unknown variable, which satisfies the given equation. It is also known as the root of this equation. For Example: $3x-6=0$ is an equation with root 2.

Inequalities: Between two real numbers a and b, if a and b are unequal, a may be greater than b, or a may be smaller than b. These relationships are called inequalities.

Quiz Questions

Multiple Choice Questions

1. What is the degree of the equation $x-\sqrt{x}-1=0$?
   (i) First degree in x ; (ii) Two degree in x; (iii) Three degree in x;
   (iv) Four degree in x
2. How many root in a linear equation?
   (i) zero root; (ii) one root; (iii) two root; (iv) three root
3. If the numbers of equations and numbers of variables are equal then we get.
   (i) Unique solution; (ii) Infinite numbers of solutions; (iii) Many solutions
   (iv) No solution
4. What is the root of the equation $\frac{x}{3}+\frac{3}{x}=\frac{10}{3}$?
   (i) 9; (ii) 9, 1; (iii) 1; (iv) 1,8
5. What is the root of the equations $x+3y=4, 3x-y=3$?
   (i) (13,10) ; (ii) (9,10) (iii) $\left(\frac{13}{10},\frac{9}{10}\right)$; (iv) $\left(\frac{9}{10},\frac{13}{10}\right)$
6. Which is an identity?
   (a) $3x=15$ (b) $3x+7=10$ (c) $x^2-y^2=(x+y)(x-y)$ (d) $x^2=3x$
7. If in an equation x remains unchanged by changing x *to* $1/x$, then such of equation are known as?
   (a) Reciprocal equation (b) Quadratic equation
   (c) both a and b (d) None of these.
8. In which case a system of linear equations is said to be inconsistent?
   (a) No Solution (b) Infinite number of solution
   (c) Unique solution (d) More than one solution
9. If $\frac{x}{y}>\frac{2}{3}$, then which one of the following is true?
   a) $\frac{y}{x}<\frac{3}{2}$ b) $\frac{y}{x}>\frac{3}{2}$ c) 2y>3x d) None of these .
10. Which one is not an inequality symbol?
    a) > b) < c) = d) ≥

Which one of the following statement is true/false?

1. $x^2 + x + 1 = 0$ be a linear equation.
2. $\begin{Bmatrix} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{Bmatrix}$ has an unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.
3. Any linear equation has two roots.
4. The graph of a linear equation is curve.
5. The graph of a linear equation is a straight line.
6. $(x+y)^2 = x^2 + 2xy + y^2$ be an identity.

Brief Questions

1. What is linear equation?
2. Define degree of an equation with example.
3. What is solution of an equation? Discuss it with example.
4. Define inequities.
5. Distinguish between equation and identity.
6. Explain cross multiplication method for with example.
7. Explain elimination method with example.
8. Explain substitution method with example.
9. Determine the degree of the following equations:

   (i) $2(x^2 + y + 2) = 2x^2 - y + 3$

   (ii) $\frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x} = 0$

Conceptual, Analytical & Numerical Questions

1. Solve the following equations:

a) $6\frac{1}{3}-\frac{x-7}{3}=\frac{4x-2}{5}$

b) $4x-\frac{x-1}{3}=x+\frac{2(x-1)}{5}+3$

c) $\frac{a-x}{a}+\frac{2a-x}{2a}=\frac{3a-x}{3a}$

d) $\frac{x-bc}{b+c}+\frac{x-ca}{c+a}+\frac{x-ab}{a+b}=a+b+c$

e) $\frac{2x+1}{3}+\frac{x}{4}-\frac{x-3}{2}=\frac{5x-1}{6}$

2. Solve the following system of linear equations:

a) $5x+2y=8,\ 9x-5y=23$

b) $x-2y=1, 2x+y=-3$

c) $42x+33y=117,\ 48x+27y=123$

d) $\frac{x-1}{2}+\frac{2y+1}{3}=0, \frac{x+4}{3}-\frac{y-1}{2}=1$

e) $3x-2y=10, 5x+6y=12$

f) $2x+3y=1, 3x-y=4$

g) $3^x=9^y,\ 5^{x+y+1}=25^{xy}$

h) $\frac{2}{x}+y=3,\ \frac{1}{2x}-\frac{2y}{3}=\frac{1}{6}$

i) $8y-2x=3xy,\ \frac{10}{x}+\frac{1}{y}=2$

3. Solve the following system of linear equations / simultaneous equations:

$9x+3y-4z=35,\ x+y-z=4,\ 2x-5y-4z=-48$

4. 15% of a sale price of an article is equal to 25% of its cost and 10% of the sale price exceeds 15% of the cost by Tk. 10. Find the cost price and selling price.

5. A estate valued at Tk. 12,000 is divided among three persons A, B, C. If A surrenders 15% of his shares, then shares of B and C are respectively increased by 10% and 25% and if A surrenders 20%, then share of B and C are respectively increased by 15% and 30%. Find the shares of each.

ANSWERS

2. (d) $x=-1, \ y=1$ (i) $x=\frac{11}{13}, \ y=\frac{7}{11}$

2. (k) $x=4, \ y=-2$ 2. (j) $x=\frac{7}{4}, 2 \ ; \ y=\frac{7}{2}, \ 3$ 3. $x=5, y=6, z=7$

4. Cost price = Tk. 600, selling price = Tk. 1000

5. A = Tk. 6,000 ; B= Tk. 4,000; C= Tk. 2,000

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