# CHAPTER-07 PERMUTATIONS AND COMBINATIONS

## LEARNING OBJECTIVES

After studying this chapter, the student will be able to understand:

- Basic concept of permutation and combination
- Different cases of permutation such as word formation
- Different cases of combination such as formation a team
- Relationship between permutation and combination
- Application of permutation and combination in our real life

## 7.1 INTRODUCTION

Permutations refer to different arrangements of things from a given lot taken one or more at a time whereas combinations refer to different sets or groups made out of a given lot, without repeating an element, taking one or more of them at a time. The difference will be clear from the following explanation of permutation and combinations produced by a set of three elements {x, y, z}.

| | Combinations | Permutations |
|---|---|---|
| (i) one at a time : | {x}, {y), {z} | {x}, {y}, (z) |
| (ii) two at a time: | {x, y}, {y, z}, {z, x} | {x, y}, {y, x}, (y, z}, {z, y} {x, z}, {z, x) |
| (iii) three at a time: | {x, y, z} | {x, y, z}, {x, z, y}, {y, z, x} {y, x z}, ( z, x, y}, {z, y, x} |

## 7.2 IMPORTANT NOTATION

The product of first $n$ positive integers is called $n$ factorial. It is written by the symbol as $n!$ and read as $n$ factorial.

$$n! = 1.2.3.4.\cdots\cdots.n; \quad n \in N$$

$$n! = n.(n-1)! \quad n \in N$$

Remember that: $0! = 1$ (special case)

For examples: $5! = 1.2.3.4.5 = 120$, $13! = 13.12.11.10.9!$, $8! = 8.7!$. $1! = 1$.

## 7.3 PERMUTATION

Each of the different arrangement that can be made out of a given number of things by taking some or all of them at a time is called a permutation. The number of different arrangement of $r$ things taken out of $n$ dissimilar things is denoted by $^nP_r$ and it is defined as follows:

$$^nP_r = n(n-1)(n-2)\cdots\cdots\{n-(r-1)\};\quad n \geq r$$

$$\text{Or,}\quad ^nP_r = \frac{n!}{(n-r)!};\quad n \geq r$$

For Example: Suppose there are three quantities a, b, c. The different arrangements of these three quantities taking 2 at a time are ab, ba, bc, cb, ca and ac. Thus $^np_r = {}^3p_2 = 6$ Again all the arrangements at there three quantities taking 3 at a time are abc, acb, bca, bac, cab and cba. Thus $^nP_r = {}^3p_3 = 6$. Hence it is clear that the number of permutations of 3 things taken r at a time is 6.

## 7.4 FUNDAMENTAL PRINCIPLES OF PERMUTATION

If one operation can be done in p different ways and when it has been done in any one of these ways, if a second operation can be done in q different ways, then the two operations together can be done in p × q ways.

## 7.5 PERMUTATION OF THINGS ALL DIFFERENT

Permutations of 'n' different things taken 'r' at a time $^np_r$ where $r \leq n$ are n (n-1) (n-2) ..... (n – r + 1) The first place can be filled up in n ways. The first two places can be filled up in n (n-1) ways. The first three places can be filled up in n (n-1) (n-2) ways.

**Remarks:**

1. The number of permutations of n different things taken all at a time is $^np_n = n(n-1)$ (n - 2)...........3.2.1 = n!
2. $^np_{n-1} = n(n-1)$ (n - 2)...........3.2.1 = $^np_n$

3. $^{n}p_r = n(n-1)\ (n-2)\ \ldots\ldots\ldots\ldots(n-r+1)$

$$=\frac{n(n-1)(n-2)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(n-r+1)(n-r)!}{(n-r)!}$$

$$=\frac{n!}{(n-r)!}$$

4. We have $^{n}p_n = n!$

Also

$$^{n}p_n = \frac{n!}{(n-n)!}$$

$$=\frac{n!}{0!}$$

$$\Rightarrow n! = \frac{n!}{0!}$$

$$\therefore 0! = \frac{n!}{n!} = 1$$

According to the definition, 0! is meaningless. But when used as a symbol its value is 1.

## 7.6 PERMUTATION OF THINGS NOT ALL DIFFERENT

The number of permutations of $n$ things of which $p$ things are of one kind, $q$ things are of a second kind, $r$ things are of a third kind and all the rest are different is given by

$$x = \frac{n!}{p! \times q! \times r!}$$

**For example:** Find the number of permutations of the word RECURRENCE.

**Solution:** The word RECURRENCE has 10 letters, of which 3 are R, 3 are E, 2 are C, the rest are different. Therefore, the number of permutations is

$$\frac{10!}{3!\ \ 3!\ \ 2!} = \frac{10.9.8.7.6.5.4.3!}{3!.3.2.1.2.1} = 50400.$$

## 7.7 CIRCULAR PERMUTATIONS

The number of distinct permutations of $n$ objects taken $n$ at a time on a circle is (n-1)!.

In considering the arrangement of keys on a chain or beads on a necklace, two permutations are considered the same if one is obtained from the other by turning the chain or necklace over.

In that case there will be $\frac{1}{2}$ (n-1)! ways of arranging the objects.

**For example:** In how many ways can 4 Indians and 4 Bangladeshis be seated at a round table so that no two Indians may be together.

**Solution:** Put one of the Bangladeshi in a fixed position and then arrange the remaining three Bangladeshi in all possible ways. Thus the number of ways in which the four Bangladeshis be seated at a round table is 3! After they have taken their seats in any one way, there are four seats for the Indians, each between two Bangladeshis. Therefore, the Indians can be seated in 4! ways correspond to one way of seating the Bangladeshi.

Total number of arrangements is $4! \times 3! = 144$

## 7.8 COMBINATIONS

Combinations refer to different set of groups made out of a given lot, without repeating an element, taking one or more of them at a time. The mathematical formula for finding out combination requires a slight modification in the formula used for permutations.
This is as follows:

For permutation $^nP_r = \dfrac{n!}{(n-r)!}$

For combination

$$^nC_r \ or, C\,(\text{n, r}) \text{ or } \binom{n}{r} = \frac{n!}{r!\,(n-r)!}; \quad n \geq r$$

**For example:** Suppose there are three quantities a, b, c. The number of combination of 3 things or quantities taken 2 a time ab, bc, ca.

Thus $^nC_r = {}^3C_2 = 3$

## 7.9 SOME SPECIAL DEDUCTION OF COMBINATIONS

I. $^nC_0 = \dfrac{n!}{0!(n-0)!} = \dfrac{n!}{0!n!} = \dfrac{n!}{1\times n!} = 1$

II. $^nC_1 = n;\ ^nC_2 = \dfrac{n(n-1)}{2!}$ and $^nC_3 = \dfrac{n(n-1)(n-2)}{3!}$

III. $^nC_n = 1$

**Proof:** $^nC_n = \dfrac{n!}{n!\,(n-n)!} = \dfrac{n!}{n!\,o!} = \dfrac{n!}{n!\times 1} = 1$

IV. $^nC_{n-1} = {}^nC_1 = n$

V. $^nC_r = {}^nC_{n-r}$

Proof; $^nC_{n-r} = \dfrac{n!}{(n-r)!\,(n-n+r)!} = \dfrac{n!}{(n-r)!\;r!} = {}^nC_r$

$\therefore {}^nC_r = {}^nC_{n-r}$

VI. $^nC_p = {}^nC_q \Leftrightarrow p = q \ or \ p+q = n$

## 7.10 RELATION BETWEEN PERMUTATION AND COMBINATION

The number of permutations by taking r things at a time from n different thing are $^nP_r = \dfrac{n!}{(n-r)!}$ ..........................(i)

The number of combinations by taking r things at a time from n different things are $^nC_r = \dfrac{n!}{r!(n-r)!}$ ..........................(ii)

From equation (ii) we have, $^nC_r = \dfrac{n!}{r!(n-r)!} = \dfrac{1}{r!}\times\dfrac{n!}{(n-r)!}$

$\Rightarrow {}^nC_r = \dfrac{1}{r!}\times {}^nP_r$ [by using equation (i)]

$\Rightarrow r!\times {}^nC_r = {}^nP_r$

$\therefore {}^nP_r = r!\times {}^nC_r$

**Some Important Deduction**

(i) when r = 1 then $^nP_r = {}^nC_r$

(ii) $^nP_r > {}^nC_r$

**Remarks:** Permutation maintain sequence but combination does not maintain sequence.

## ILLUSTRATIONS

**Illustration-01:** Find the number of permutations of the word RECURRENCE.

**Solution:** The word RECURRENCE has 10 letters, of which 3 are R, 3 are E, 2 are C, the rest are different. Therefore, the number of permutations is

$$\frac{10!}{3!\ 3!\ 2!} = \frac{10.9.8.7.6.5.4.3!}{3!.3.2.1.2.1} = 50400.$$

**Illustration-02:** Find how many three letter words can be formed out of the word LOGARITHMS.

**Solution:** There are 10 different letters, therefore $n$ is equal to 10 and since we have to find three letter words, r is 3.

Hence the required number of words are : $^{10}P_3 = \frac{10!}{(10-3)!}$

$$= \frac{10\times9\times8\times7!}{7!} = 720.$$

**Illustration-03:** Find the number of permutations of letter in the word ENGINEERING.

**Solution:** The word ENGINEERING consists of 11 letters, in which there are 3 Es, 3 Ns, 2 Gs, 2 Is and one R. The total number of permutations is

$$\frac{11!}{3!3!2!2!} = \frac{11\times10\times9\times8\times7\times6\times5\times4\times3!}{3!\times3\times2\times1\times2\times1\times2\times1} = 277200$$

**Illustration-04:** If $^nP_4 = 12\times\ ^nP_2$, then find n.

**Solution:** We have, $^nP_4 = \frac{n!}{(n-4)!}$ and $^nP_2 = \frac{n!}{(n-2)!}$

Now, $^nP_4 = 12.\ ^nP_2$

$\Rightarrow \frac{n!}{(n-4)!} = 12.\frac{n!}{(n-2)!}$ $\quad\Rightarrow 12(n-4)! = (n-2)!$ $\quad\Rightarrow 12(n-4)! = (n-2)(n-3)(n-4)!$

$\Rightarrow 12 = (n-2)(n-3)$ $\quad\Rightarrow n^2 - 2n - 3n + 6 = 12$ $\quad\Rightarrow n^2 - 5n - 6 = 0$

$\Rightarrow n^2 - 6n + n - 6 = 0$ $\quad\Rightarrow n(n-6) + 1(n-6) = 0$

$\Rightarrow (n-6)(n+1) = 0 \quad \therefore n = 6$ or $n = -1$

Since $n$ is positive integer, we reject the second value of $n$. Therefore n = 6.

**Illustration-05:** How many permutations can be made out of the letter of the following words taken all together?

i) PERMUTATION ii) COLLEGE iii) MISSISSIPPI iv) FAILURE
v) ACCOUNTANT vi) EXAMINATION
vii) ARRANGEMENTS viii) ASSINATION

**Solution:**

(i) Since the word 'PERMUTATION' consists of 11 letters, in which there are 2 T's and the rest are all different.

So the required total no of permutations / so the required total number of arrangements is

$$\frac{11!}{2!} = \frac{39916800}{2} = 19958400 \text{ (Ans:)}$$

(ii) Since the word 'COLLEGE' consists of 7 letter, in which there are 2 L's, 2E's and the rest are all different . So the required total number of permutations

is $\frac{7!}{2!\times 2!} = \frac{5040}{2\times 2} = 1260$ (Ans:)

iii) Since the word "MISSISSIPPI" Consists 11 letters in which there are 4I's, 4s's, 2p's and the rest are all different.

So the required total number of permutation is $\frac{11!}{4!\times 4!\times 2!} = 34650$ Ans.

iv) Since the word "FAILURE" Consists of 7 letters in which there are all different. So the required total member of permutation is 7! = 5040 Ans.

v) Since the word "ACCOUNTANT" Consist of 10 letters in which there are 2A's, 2C's, 2T's, 2N's and the rest are all different.

So the required total number of permutation is $\frac{10!}{2!\times 2!\times 2!\times 2!} = 2,26,800$ Ans.

(vi) Since the word 'EXAMINATION' consists of 11 letters in which there are 2A's, 2 I's, 2N's and the rest are all different.

So the required total number of arrangements

is $\frac{11!}{2!\times 2!\times 2!} = \frac{39916800}{2\times 2\times 2} = 4989600$ Ans:

vii) Since the word "ARRANGEMENTS" Consist of 12 letters in which there are 2A's, 2R's, 2Ns, 2E's, and the rest are all different.

So the required total number of permutations is $\frac{12!}{2!\times 2!\times 2!\times 2!} = 29,937,600$ Ans.

**viii)** Since the word 'ASSASSINATION' consists of 13 letters, in which there are 3 A's, 4S's, 2I's, 2N's and the rest are different. So the required total no of permutation is- $\frac{13!}{3!\times 4!\times 2!\times 2!}$. Ans:

**llustration-06:** How many different words can be made out of the letters in the word ALLAHABAD. In how many of these will the vowels occupy the even places.

**Solution:** The word 'ALLAHABAD' consists of 9 letters of which A is repeated four times. L is repeated three times and the rest all different. So the required total number of arrangements is

$\frac{9!}{4!\times 2!} = 7560$ Ans:

**<u>2nd Part:</u>**

Since the word 'ALLAHABAD' consist of 9 letters, there are 4 even places which can be filled up by the 4 vowels in $\frac{4!}{4!}$ ways. Since all the vowels are similar.

Further, the remaining 5 places can be filed up by the 5 consonants of which 2 are similar which can be filled in $\frac{5!}{2!}$ ways .

So the required total number of arrangement is $\frac{4!}{4!} \times \frac{5!}{2!} = 1 \times \frac{120}{2} = 60$ Ans:

**Illustration-07:** How many arrangements can be made with the letters of the word MATHEMATICS and in how many of then vowels occurs together?

**Solution:**

The world 'MATHEMATICS' consists of 11 letters of which 2 are M's, 2 are A's and 2 are T's and the rest are all different.

So the required total number of arrangements is

$\frac{11!}{2!\times 2!\times 2!} = \frac{39916800}{2\times 2\times 2} = 4989600$ (Ans:)

**2nd Part:**

The word 'MATHEMATICS' has 4 vowels and 7 consonants, Let us consider 4 vowels (A,A,E,1) as one letter and 7 consonants. In this case 8 letter can be arranged as $\frac{8!}{2!\times 2!}$ ways. Again 4 vowels can be rearranged among themselves as $\frac{4!}{2!}$ ways.

So the required total number of arrangements is $\frac{8!}{2!\times 2!}\times\frac{4!}{2!}=\frac{40320}{2\times 2}\times\frac{24}{2}=120960$

**Illustration-08:** In how many ways can 4 Indians and 4 Bangladeshis be seated at a round table so that no two Indians may be together.

**Solution:** Put one of the Bangladeshi in a fixed position and then arrange the remaining three Bangladeshi in all possible ways. Thus the number of ways in which the four Bangladeshis be seated at a round table is 3! After they have taken their seats in any one way, there are four seats for the Indians, each between two Bangladeshis. Therefore, the Indians can be seated in 4! ways correspond to one way of seating the Bangladeshi.

Total number of arrangements is $4! \times 3! = 144$

**Illustration-09:** In how many ways can 5 boys and 5 girls be seated around a table so that no 2 boys are adjacent.

**Solution:** Let the girls be seated first. They can sit in 4! ways. Now since the places for the boys in between girls are fixed, the option is there for the boys to occupy the remaining 5 places. There are 5! ways for the boys to fill up the 5 places in between 5 girls seated around a table already. Thus, the total number of ways in which both girls and boys can be seated such that no 2 boys are adjacent are $4!\times 5!=2880$ ways.

**Illustration-10:** How many ways 4 men and 3 women can sit in a round table if i) 3 women do not sit altogether ii) 3 women sit altogether.

**Solution:** We have, 4 men and 3 women that is total 7 persons. Then can arranged in ${}^7p_7$ or 7! ways = 5040 ways.

**(ii) :** Let us consider 3 women as one women and 4 men. In this case 5 persons can be arranged as 5! ways. Again 3 women can be rearranged among themselves as 3! ways.

So the number of arrangements in which 3 women sit together is 5!-3!

=120×6 =720 ways Ans.

**(i) :** The number of arrangements in which 3 women do not sit together is 7! -5!×3!
=5040-720 =4320 ways Ans.

**Illustration-11:** How many different words containing all the letters of the word TRIANGLE can be formed ? How many of them. i) Begin with T ii) begin with E iii)begin with T and end with E iv) have T and E in the end places v)When consonants are never together. vi) when no two vowels are together vii) when consonants and vowels always together viii) vowels occupy odd places ix) The relative positions of the vowels and consonants remain unaltered. x) vowels occupy the second, third and fourth places.

**Solution:** Since the word "TRIANGLE" Consist of 8 letters in which all the letters are different. We have 3 vowels (I, A, I) and 5 Consonants (T, R, N, G, L)

So the required no of different words is 8 way = 5040 Ans.

**(i)** Given Condition begin with T. Since each word beginning with T, So remaining 7 letter can be arranged among themselves is 7 ways = 5040 ways Ans.

**(ii)** Given condition begin with E. Since each word beginning with E. So the remaining 7 letters can be arranged among them selves as 7! ways = 5040 ways. Ans.

**(iii)** Given condition begin with T and end with E. Since each word beginning with T and end with E, so the remaining 6 letters can be arranged among themselves in 6! ways = 720 ways Ans.

**(iv)** Given condition T and E in the end places. Since each word ending T and end E in the end two places. So the remaining 6 letters can be arranged among themselves in 6! ways. And T, E arranged themselves in 2! ways. So the required total arrangement $= 6! \times 2! = 720 \times 2 = 1440$ Ans.

**(v)** Given, Condition consonants are never together. Considering 5 consonants as 1 and other 3 vowels. There 4 letters can be arranged as 4! ways. Also 5 Consonants can be rearranged as 5! ways. So the arrangement of consonants is always to together $4! \times 5!$ ways $= 2880$ ways. Again, the number of different words made by the letters of word TRIANGLE is 8! Ways. = 40320 ways. So the required number of different words, when Consonants are never together $40320 - 2880 = 37440$ Ans.

**(vi)** Given condition, when no two vowels are together.. Since the vowels are put in the places beside of Consonants, then no two vowels are together. 5 Consonants have 6 places besides themselves. 3 vowel can be put in 6 vacant places as $6p_3$ ways. Also 5 constants can be arranged among their placed as 5! Ways. So the required arrangements $= 6p_3 \times 5!$ 14400 Ans.

**(vii)** Give condition when consonant and vowels are both always together. Considering 5 Consonant as 1 and 3 vowels as 1, there two letters can be arranged as 2! Ways. Also 5 Consonants can be rearranged among themselves as 5! ways and 3 vowels as 3! ways. So the required arrangements $= 2! \times 5! \times 3! = 1440$ Ans.

(viii) Given Condition when vowels occupy only odd. Places 3 vowels can be put in 4 odd places as $4p_3$ ways. Rest of 5 consonants can be arranged as 5! ways. ∴ So the required arrangement $= 4p_3 \times 5! = 2880$ Ans.

(ix) Given Condition the relative position of the vowels and Consonants remain unaltered places 5 Consonants may be arranged as 5! Ways and 3 vowels can be arranged as 3! Ways. So the required arrangements $= 3! \times 5! = 720$ Ans.

(x) Given Condition, vowels occupy the second, third and fourth places. 5 Consonants may be arranged as 5! Ways and 3 vowels can be arranged in the second third and fourth places as 3! Ways. So the required arrangements $= 3! \times 5! = 720$ Ans.

**Illustration-12:** Six papers are set in an examination of which two are mathematical. In how many different orders can the papers be arranged so that- i) The two mathematical papers are together and ii) The two mathematical papers are not consecutive

**Solution:** The total number of arrangements that can be made of 6 papers is 6! ways = 720 ways

(i) Let us consider the two mathematical as one paper and 4 reaming paper. In this case 5 papers can be arranged as 5! ways, Again, 2 mathematical papers can be rearranged among themselves as 2! ways. So the required total number of arrangements in which two mathematical papers is together= 5! ×2!= 240 ways.

(ii) The total number of arrangements in which two mathematical papers are not consecutive =720-240=480 Ans.

**Illustration-13:** Find the value of $r$, if ${}^{16}C_r = {}^{16}C_{r+2}$

**Solution:** Since ${}^{n}C_r = {}^{n}C_{n-r}$

But we are given ${}^{16}C_r = {}^{16}C_{r+2}$

$$\Rightarrow {}^{16}C_{16-r} = {}^{16}C_{r+2}$$

$$\Rightarrow 16 - r = r + 2 \quad (\because {}^{n}C_p = {}^{n}C_q \Leftrightarrow p = q \text{ or } p + q = n)$$

$$\Rightarrow 16 - 2 = r + r \Rightarrow 14 = 2r \Rightarrow r = 7 \quad \therefore r = 7$$

**Illustration-14:** Prove that: ${}^{n+1}C_r = {}^{n}C_r + {}^{n}C_{r-1}$, which is known as **Pascal's law**.

**Solution:** We know that ${}^{n}C_r = \dfrac{n!}{r!(n-r)!}$

$$\therefore {}^{n}C_r + {}^{n}C_{r-1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

$$= \frac{n!}{r(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)\ (n-r)!}$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{(n-r+1)}\right]$$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$

$$= \frac{n!}{(r-1)!(n-r)!}\ \frac{n+1}{r\,(n-r+1)}$$

$$= \frac{(n+1)!}{r!(n-r+1)!} = {}^{n+1}C_r$$

$$\therefore {}^{n+1}C_r = {}^{n}C_r + {}^{n}C_{r-1} \quad \text{(Proved)}$$

**Illustration-15:** In a test in paper on Business Mathematics 9 questions are set. In how many different ways can an examinee choose 6 questions.

**Solution:** The number of different choices is evidently equal to the number of ways in which 6 places can be filled up by 9 different things.

∴ The required number of ways

$${}^{9}C_6 = {}^{9}C_{9-6} \quad \left[Since\ {}^{n}C_r = {}^{n}C_{n-r}\right]$$

$$= {}^{9}C_3 = \frac{9!}{3!\ (9-3)!} = \frac{9!}{3!\ 6!} = \frac{9\times8\times7\times6!}{3\times2\times1\times6!} = 84.$$

**Illustration-16:** In how many ways 4 different things can be selected from 52 different things?

**Solution:** The number of combinations by taking 4 different things at a time from 52 different things $= {}^{52}C_4 = \frac{52!}{4!(52-4)!} = \frac{52!}{4!\,48!} = \frac{52\times51\times50\times49\times48!}{4\times3\times2\times1\times48!} = 270725$

**Illustration-17:** Find the number of diagonals of an m-sided polygon.

**Solution:** The number of straight line produced by joining any two points is ${}^{m}C_2$ but m-sides of the polygon are not diagonal of it.

∴ The required number of diagonal = ${}^{m}C_2 - m$

$$= \frac{m!}{2!\ (m-2)!} - m = \frac{m(m-1)\,(m-2)!}{2\times1\times(m-2)!} - m = \frac{m(m-1)}{2} - m = \frac{m^2 - m - 2m}{2}$$

$$= \frac{m^2 - 3m}{2} = \frac{m(m-3)}{2}$$

**Illustration-18:** A cricket team of 11 players is to be formed from16 players including 4 bowlers and 2 wicket keepers. In how many different ways can a team be formed so that the teams contains i)exactly 3 bowlers and 1 wicket keeper ii) at least 3 bowlers and at least one wicket keeper.

**Solution:** Given total players =16

Bowlers =4

Wicket keepers =2

Other players = 16-(4+2)=10

(i) The questions may be answered in the following ways

| Bowlers (4) | wicket keeper (2) | Other (10) |
|---|---|---|
| 3 | 1 | 7= 11 |

The total number of ways in which exactly 3 bowlers and 1 wicket keeper is ${}^4c_3 \times {}^2c_1 \times {}^{10}c_7$

$= 4\times2\times120 = 960$ Ans:

**(ii)**

The questions may be answered in the following way

| Bowlers (4) | wicket keeper (2) | others (10) |
|---|---|---|
| 3 | 1 | 7 =11 |
| 3 | 2 | 6=11 |
| 4 | 1 | 6=11 |
| 4 | 2 | 5=11 |

So that total no of ways in which at last 3 bowlers and at least 1 wicket keeper is

$({}^4c_3 \times {}^2c_1 \times {}^{70}c_7) + ({}^4c_3 \times {}^2c_2 \times {}^{10}c_6) + ({}^4c_4 \times {}^2c_1 \times {}^{70}c_6) + ({}^4c_4 \times {}^2c_2 \times {}^{10}c_5)$

$= 960 + 4\times1\times210 + 1\times2\times210 + 1\times1\times252$

$= 960+840+420+252$

$= 2472$. Ans:

**Illustration-19:** For an examination, a candidate has a select 7 subjects from three different groups A,B and C. The three groups A,B,C contains 4,5,6 subjects respectively. In how many different ways can a candidate make his selection if he has to select at least 2 subjects from each group.

**Solution:** The question can be answered in the following way:

| Group A(4) | Group B(5) | Group c(6) | |
|---|---|---|---|
| 2 | 2 | 3 | =7 |
| 2 | 3 | 2 | =7 |
| 3 | 2 | 2 | =7 |

So the total number of ways in which at least 2 subject from each group is-

${}^4c_2 \times {}^5c_2 \times {}^6c_3 + {}^4c_2 \times {}^5c_3 \times {}^6c_2 + {}^4c_3 \times {}^5c_2 \times {}^6c_2$

$= 1200+900+600$ $=2700$Ans:

**Illustration-20:** A party of 3 ladies and 4 gentlemen is to be formed from 8 ladies and 7 gentlemen. In how many different ways can the party be formed if Mrs. Adhora and Mr. Irtiza refuse to join the same party ?

**Solution:** 3 ladies can be selected out of 8 ladies in $8_{c3}$ ways and 4 gentle men can be selected out of 7 gentle men in ${}^7C_4$ ways.

$\therefore$ The number of ways of choosing the committee $= {}^8C_3 \times {}^7C_4 = \frac{8!}{3!(8-3)!} \times \frac{7!}{4!(7-4)!}$

$= \frac{8!}{3!5!} \times \frac{7!}{4!\times 3!} = 1960$

If both Mrs Odhera and Mr. Intiza are members, there remain to be selected 2, ladies from 7 ladies and 3 gentlemen from 6 gentlemen. This can be done in ${}^7C_2 \times {}^6C_3$ ways.
= 420 ways.
$\therefore$ The required number of ways of forming the party is which Mrs. Odhera and Mr. Intiza refuse to join $= 1960 - 420 = 1540$ Ans.

**Illustration-21:** The staff of a bank consists of the Manager, the deputy manager and 10 other officers. A committee of 4 is to be selected. Find the number of ways in which this can be done so as to always include- i)the manager ii) the manager but not the deputy manager iii) Neither the manager nor the deputy manager.

**Solution:** Given, Total persons = Manager +Deputy Manager +10 others officer $= 12$ and taken at a time = 4

**(i)** For including the manager then the remaining total persons $= 12 - 1 = 11$ and remaining taken at a person $= 4 - 1 = 3$
So the required total no of ways $= {}^{11}C_3 = 165$ Ans.

**(ii)** For including the manager and excluding the deputy manager then the remaining total persons $= 12 - 1 - 1 = 10$ and the reaming taken person
$= 4 - 1 = 3$
So the required total ways $= {}^{10}C_3 = 120$ Ans.

**(iii)** For excluding both the manager and the deputy manager then the remaining total person $= 12 - 1 - 1 = 10$ and the reaming taken person = 4
$\therefore$ So the required total ways $= {}^{10}C_4 = 210$ Ans.

## BRIEF REVIEW

### Definition

**Permutation:** The arrangement made up by taking some or all elements out of a number of things is called a permutation. The number of permutations of $n$ things taking $r$ at a time is denoted by ${}^nP_r$ and it is defined as follows:

$${}^nP_r = n(n-1)(n-2)\cdots\cdots\{n-(r-1)\};\quad n \ge r$$

$$\text{Or,}\quad {}^nP_r = \frac{n!}{(n-r)!};\quad n \ge r$$

**Combination:** The group or selection made by taking some or all elements out of a number of things, without repeating an element, is called a combination. The number of combinations of $n$ things taking $r$ at a time is denoted by ${}^nC_r$ and it is defined as under:

$${}^nC_r = \frac{n(n-1)(n-2)\cdots\cdots\{n-(r-1)\}}{r\,!};\quad n \ge r$$

$$\text{Or,}\quad {}^nC_r = \frac{n!}{r\,!\,(n-r)!};\quad n \ge r$$

**Permutations of Things not all different:** The number of permutations of $n$ things of which p things are of one kind, q things are a second kind, r things are of third kind and all the rest are different, is given by $\frac{n!}{p!\times q!\times r!}$.

**Circular Permutations:** The number of distinct permutations of $n$ objects taken $n$ at a time on a circle is $(n-1)!$. In considering the arrangement of keys on a chain or beads on a necklace, two permutations are considered the same if one is obtained from the other by turning the chain or necklace over. In that case there will be $\frac{1}{2}(n-1)!$ ways of arranging the objects.

The number of circular permutations of $n$ different objects $= (n-1)\,!$

## Quiz Questions

### Multiple Choice Questions

1. Which is the correct expression for $^nP_r$ ?

   (i) $\frac{n!}{r!(n-r)!}$ (ii) $\frac{r!}{(n-r)!}$ (iii) $\frac{n!}{(n-r)!}$ (iv) $\frac{n!}{(n+r)!}$

2. What is the value of 0! ?

   (i) –1 (ii) 0 (iii) 1 (iv) Not all above.

3. Which is the correct value of $2\times{}^5P_3$ ?

   (i) 30 (ii) 60 (iii) 40 (iv) 20

4. How many words can be formed of the letters of the words MONDAY taken all at a time?

   (i) 720 (ii) 620 (iii) 360 (iv) 480

5. What is the value of $^7P_7$ ?

   (i) 72 (ii) 42 (iii) 7! (iv) 126

6. Which is the correct expression for $^nC_r$ ?

   (i) $\frac{n!}{(n-r)!}$ (ii) $\frac{r!}{n!(n-r)!}$ (iii) $\frac{r!}{(n-r)!}$ (iv) $\frac{n!}{r!(n-r)!}$

7. What is the value of $^{12}C_4+{}^{12}C_3=$ ?

   (i) $^{12}C_4$ (ii) $^{12}C_5$ (iii) $^{13}C_4$ (iv) $^{13}C_3$

8. Which is the exact relation between $^nP_r$ and $^nC_r$ ?

   (i) $^nP_r=r!\times{}^nC_r$ (ii) $^nC_r=r!\times{}^nP_r$ (iii) $^nP_r=\frac{1}{r!}\,{}^nC_r$ (iv) $^nP_r<{}^nC_r$

9. Which is the value of $^5C_3\times{}^6P_2$ ?

   (i) 300 (ii) 150 (iii) 200(iv) 250

10. In how many ways can a gentleman invite all of his 6 friends?

    (i) 1720(ii) 120 (iii) 36 (iii) $\lfloor 6$

### Which one of the following statement is true/false?

a. Permutation refer to different arrangements of things.

b. $^nP_3=n(n-1)(n-2)$

c. $^nP_n=(n-1)!$

d. $^nC_0=n!$

e. If $^nC_r={}^nC_p$ then $r=p$.

## Brief Questions

1. What is the correct expansion of $^nP_r$ ?
2. What is the correct expansion of $^nC_r$ ?
3. What is the value of $^7P_7$ ?
4. What is the value of $0!$?
5. What is the exact relationship between $^nP_r$ and $^nC_r$?
6. What do you mean by permutation?
7. What do you mean by combination ?
8. What is the value of $^5C_3 \times ^6P_2$ ?
9. If $^nP_4 = 12\,^nP_2$ find the value of $n$.
10. If $^{18}C_r = ^{18}C_{r+2}$ find the value of $r$.

## Conceptual, Analytical & Numerical Questions

1. Define Permutation and Combination. Discuss with example.
2. Difference between Permutation and Combination.
3. Discuss the relation between Permutation and Combination.
4. Prove that the Permutations of n different things taken r at a time of them is $n(n-1)(n-2)\cdots\cdots(n-r+1)$.
5. Show that $^nP_r = \frac{n!}{(n-r)!}$ where $r \geq n$.
6. Show that $^nC_r = \frac{n!}{r!(n-r)!}$ where $r \leq n$.
7. Show that $^{n+1}C_r = ^nC_r + ^nC_{r-1}$
8. Prove that $^{n-1}C_r + ^{n-1}C_{r-1} = ^nC_r$

## Numerical questions

1. Define permutation and combination with an example.
2. Derive the relationship between permutation and combination.
3. If ${}^nP_5 = 6 \times {}^nP_3$, then find the value of n ?
4. In how many ways can the letters of the word `EQUATION' be arranged so that all the letters is used together?
5. Find the number of permutations of the word (i) ACCOUNTANT.
   (ii) BUSINESS (iii) MATHEMATICS
6. How many different numbers of 3 digits can be formed from the digits 1, 2, 3, 4, 5 and 6, if no digits may be repeated?
7. Find the value of r if ${}^{18}C_r = {}^{18}C_{r+2}$
8. Find out the number of ways in which a cricket team consisting of 11 players can be selected from 14 players.
9. Prove that ${}^6C_4 + {}^6C_3 + {}^7C_3 = 70.$
10. There are 7 men who are to made general managers at 7 branches of a supermarket chain. In many ways can the 7 men be assigned to the 7 area stores?
11. Find the number triangles formed by joining the vertices of the m-sided polygon.

**ANSWERS**

3. 6
4. 40320
5. (i) 226800
   (ii) 6720
   (iii) 515600
6. 120
7. 8
8. 364
10. 5040
11. $\frac{1}{6}$m (m - 1) (m - 2)