CHAPTER-09	PROGRESSION

LEARNING OBJECTIVES

After studying this chapter, the student will be able to understand:

- Definition of sequence and series
- Arithmetic progression and its formulae
- Geometric progression and its formulae
- Business application of progression

9.1 INTRODUCTION

In this chapter we shall study a few special types of finite and infinite sequences which are useful in interest calculations and some applications in finance and other areas. We shall discuss two types of series with sequence increasing or decreasing by an absolute quantity or a certain ratio designed as arithmetic and geometric progressions respectively.

9.2 SEQUENCE AND SERIES

A sequence is a special types of function whose domain is natural numbers $(N = 1,2,3,....)$ and range is a subset of real numbers $(R = (-\infty, \infty))$.

For example: 2, 4, 6, 8, ...

Where the function $f: N \rightarrow R$ is defined as $f(n) = 2n, \text{for } n \in N$

$\therefore f(1) = 2.1 = 2,\ f(2) = 2.2 = 4,\ f(3) = 2.3 = 6$ etc.

Definition (Series): A series in succession of numbers which are formed in order, according to some defined rule, called the law of the series.

Examples of series are

i) 1, 3, 5, 7, ... *ii)* 10, 5, 0, −5, −10, ...

iii) 2, 4, 8, 16, ... *iv)* $1, \frac{1}{3}, \frac{1}{3^2}, \frac{1}{3^3}, ...$

9.3 ARITHMETIC PROGRESSION (A.P.): Numbers are said to be in Arithmetic progression when they increase or decrease by a common difference.

For examples: (*i*) 2, 5, 8, 11,... (*ii*) 0, −2, −4, −6, ...

(iii) Suppose we invest Tk. 100 at a simple interest of 10% per annum the amount at the end of each year gives rise to the arithmetic progression.

110, 120, 130, 140, ...

GENERAL TERM (OR *n*-th TERM) OF AN ARITHMETIC PROGRESSION

Let the series be $a,\ a+d,\ a+2d,\ a+3d,$..................... to n terms

Here,

Common difference = d

The first term $=a=a+(1-1)d$

2^{nd} term $=a+d=a+(2-1)d$

3^{rd} term $=a+2d=a+(3-1)d$

4^{th} term $=a+3d=a+(4-1)d$

$\vdots$ ....

Hence the n^{th} term = $a+(n-1)d$

SUM OF THE FIRST *n* TERMS OF AN A.P.

Let the arithmetic progression be $a,\ a+d,\ a+2d,$ ....

$\therefore$ The nth term is = $a+(n-1)d$

Let S_n denote the sum of first n terms of the progression.

$$\therefore S_n = a+(a+d)+(a+2d)+...+(n-1)th\ \ term + nth\ \ term$$

$$= a+(a+d)+(a+2d)+...+\{a+(n-2)d\}+\{a+(n-1)d\} \quad ....(1)$$

Writing the sum of the series in the reverse order, we get

$$S_n = \{a+(n-1)d\}+\{a+(n-2)d\}+......+(a+d)+a \quad ....(2)$$

Adding (1) and (2) we get

$$2S_n = \{2a+(n-1)d\}+\{2a+(n-2)d+d\}+.......+\{2a+(n-2)d+d\}$$

$$+\{2a+(n-1)d\}$$

$$=\{2a+(n-1)d\}+\{2a+(n-1)d\}+......+\{2a+(n-1)d\}$$

$$+\{2a+(n-1)d\}$$

$$=n\{2a+(n-1)d\}$$

$$\therefore S_n = \frac{n}{2}\{2a+(n-1)d\}$$

SUM OF SOME SPECIAL SERIES

(i) $1+2+3+...+n=\frac{n(n+1)}{2}$

(ii) $1^2+2^3+3^2+...+n^2=\frac{1}{6}n(n+1)(2n+1)$

(iii) $1^3+2^3+3^3+...+n^3=\left\{\frac{n(n+1)}{2}\right\}^2$

(iv) $1+3+5+...+(2n-1)=n^2$

9.4 GEOMETRIC PROGRESSION (G.P.):

A geometric progression is a sequence whose terms increase or decrease by a constant ratio, called the common ratio.

Examples of geometric progression:

(i) $3, 6, 12, 24, ...$ (ii) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, ...$ (iii) $5, -10, 20, -40, ...$

GENERAL TERM (or n-th term) OF A GEOMETRIC PROGRESSION:

Let a geometric progression be

$$a,\ ar,\ ar^2,\ ar^3, ...$$

Whose first term is a and the common ratio is r.

Now,

First term $= a = ar^{1-1}$; 2nd term $= ar = ar^{2-1}$

3rd term $= ar^2 = ar^{3-1}$; 4th term $= ar^3 = ar^{4-1}$

....

Hence the n-th term $= ar^{n-1}$

SUM OF A SERIES OF THE FIRST n-TERMS AT A G.P

Let a geometric progression be $a, ar, ar^2, ..., ar^{n-1}$

Where, first term $= a$, number of term $= n$ and common ratio $= r$

If S_n denotes the sum to n terms then

$$S_n = a + ar + ar^2 + ... + ar^{n-2} + ar^{n-1} \quad ....(1)$$

Multiplying both sides by r we get

$$rS_n = ar + ar^2 + ar^3 + ... + ar^{n-1} + ar^n \text{.................................}(2)$$

By subtracting (2) from (1) we get

$S_n - rS_n = a - ar^n$ or, $(1-r)S_n = a(1-r^n)$

or, $S_n = a.\frac{1-r^n}{1-r}; r \neq 1$......................(3)

Changing the sign of the numerator and denominator, we can also write

$$S_n = a.\frac{r^n - 1}{r-1}; r \neq 1 \text{..........................}(4)$$

(3) may be use when $r < 1$ and (4) may be used when $r > 1$

Note:

1) When $r = 1$ then $S_n = na$

2) Sum of an infinite series in geometric progression

let $S = a + ar + ar^2 +$......................... to infinity

$\therefore S_\infty = \frac{a}{1-r}$ *if* $r < 1$

$= \frac{a}{r-1}$, if $r > 1$.

ILLUSTRATIONS

Illustration-01: Find the general term of the following arithmetic progressions:
1, 4, 7, 10, ...

Solution: Here, the first term, $a = 1$,

Common difference, d = 2nd term -1st term $= 4 - 1 = 3$

$\therefore n^{th}$ term of the progression $= a + (n-1)d$

$= 1 + (n-1)3$

$= 3n - 2$

$\therefore 20^{th}$ term $= 3 \times 20 - 2 = 58$

25^{th} term $= 3 \times 25 - 2 = 73$

Illustration-02: Which term of series 12, 9, 6, ... is equal to $(i)\ -30$ $(ii) -40$

Solution: The series is an arithmetic progression.

Where, first term. $a = 12$ and common difference, $d = 9 - 12 = -3$

(i) If n-th term $= -30$

We have, $a + (n-1)d = -30$

or, $12 + (n-1)(-3) = -30$

or, $12 - 3n + 3 = -30$ or, $3n = 45$ $\therefore n = 15$ $\therefore -30$ is the 15^{th} term.

(ii) If n^{th} term $= -40$

Then $a + (n-1)d = -40$

$\Rightarrow 12 + (n-1)(-3) = -40$

$\Rightarrow 3n = 55$

$\therefore n = \dfrac{55}{3}$

Which is impossible since n is not an integer.

$\therefore -40$ does not exist in the series.

Illustration-03: The 7^{th} and 13^{th} terms of an A.P. are 34 and 64 respectively. Find the series.

Solution:

Let the first term of the series = a

and common difference of the A.P = d

Then the 7^{th} term $= a + (7-1)d$

$\Rightarrow a + 6d = 34$..............................(1)

and 13^{th} term $= a + (13-1)d$

$\Rightarrow a + 12d = 64$................................(2)

Subtracting (1) from (2) we get: $6d = 30 \Rightarrow d = 5$

Putting the value of d in (1) we get

a = 4

$\therefore$ The series a, a + d, a + 2d, a + 3d,............... becomes

4, 9, 14, 19, 24, ...

Illustration-04: If a, b, c are in A. P., then show that $(b+c), (c+a), (a+b)$ are also in A.P

Solution:

Since a, b, c are in A.P. So from the definition of A.P. we get

$b-a=c-b \dots \dots (1)$

Now $(b+c)$, $(b+a)$, $(a+b)$ will be in A.P.

if $(c+a)-(b+c)=(a+b)-(c+a)$

if $a-b=b-c$

if $b-a=c-b$

Which if true by (1)

Hence, $(b+c), (c+a), (a+b)$ are in A.P.

Illustration-05: Find the sum of the following:

(a) $8+13+18+23+\dots$ to 25 terms $\quad$ (b) $-3+3+9+\dots+117$

(c) $10+9\frac{1}{2}+9+\dots$ to 10 terms $\quad$ (d) $21+15+9+3+\dots+(-93)$

Solution:

(a) The given series is in arithmetic progression:

First term, $a=8$

Common difference, $d=13-8=5$

The numbers of terms, $n=25.$

$\therefore$ The sum of the n terms of the series is

$$S_n=\frac{n}{2}\{2a+(n-1)d\}$$

$$\therefore S_{25}=\frac{25}{2}\{2\times 8+(25-1)\times 5\}$$

$$=\frac{25}{2}(16+120) \qquad =1700$$

(b) The given series is in arithmetic progression.

The first term, $a=-3$

Common difference, $d=3-(-3)=6$

n^{th} term is -117

We know,

nth term $a+(n-1)d$

or, $\quad 117=-3+(n-1)6$

or, $117 = -3 + 6n - 6$

$6n - 9$

or, $6n = 117 + 9 = 126$

$\therefore n = 21$

$\therefore$ Sum of the series of n term is $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\therefore S_{21} = \frac{21}{2}\{2\times(-3) + (21-1)\times 6\}$

$= \frac{21}{2}(-6 + 120) = 1197$

(c) Here, first term, $a = 10$

Common difference, $d = 9\frac{1}{2} - 10 = \frac{19}{2} - 10 = -\frac{1}{2}$

number of terms, $n = 10$

$\therefore$ Sum of the $n = 10$ term in arithmetic progression is $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\therefore S_{10} = \frac{10}{2}\left\{2\times 10 + (10-1)(-\frac{1}{2})\right\}$

$= 5\left\{20 - \frac{9}{2}\right\} = \frac{155}{2} = 77\frac{1}{2}$

(d) The series is in A.P.

The first term is, $a = 21$

Common difference, $d = 15 - 21 = -6$

The n^{th} term $= -93$

We know,

n^{th} term $= a + (n-1)d$

$\Rightarrow -93 = 21 + (n-1)(-6)$

or, $-93 = 21 - 6n + 6$

or, $6n = 120$

$\therefore n = 20$

Hence the sum of the series be

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$$\therefore S_{20} = \frac{20}{2}\{2\times 21+(20-1)(-6)\}$$

$$= \frac{20}{2}(42+19\times(-6)) = -720$$

Illustration-06: The first term of a series in A.P. is 17, the last term $-12\frac{3}{8}$ and the sum $35\frac{7}{16}$; find the number of term and common difference.

Solution: Let the number of terms = n

Common difference = d and the given first term $a = 17$

∴ We know that

$$\text{nth term} = a+(n-1)d$$

or, $$-12\frac{3}{8} = 17+(n-1)d$$

or, $$(n-1)d = -\frac{99}{8} - 17$$

$$= \frac{-235}{8} \text{ ....}(i)$$

If S_n be the sum of n terms then

$$S_n = \frac{n}{2}\{2a+(n-1)d\}$$

$$\Rightarrow 25\frac{7}{16} = \frac{n}{2}\left\{2\times 17 - \frac{235}{8}\right\} \quad [u\sin(i)$$

$$\Rightarrow \frac{407}{16} = \frac{n}{2}\left\{\frac{37}{8}\right\} \qquad \therefore n = \frac{407\times 2\times 8}{37\times 16} = 11$$

Putting $n = 11$ in (i) we have

$$10d = -\frac{235}{8} \qquad \Rightarrow d = -\frac{235}{80} = -\frac{47}{16}$$

∴ The number of terms is 11 and common difference is $-\frac{47}{16}$

Illustration-07: How many terms of the series $-8, -6, -4, ...etc.$ amount to 52?

Solution: Here first term, $a = -8$, common difference, $d = -6 + 8 = 2$

Let the required number of terms = n

The we have,

$$52 = \frac{n}{2}\{2\times(-8) + (n-1).2\} = \frac{n}{2}(-16 + 2n - 2) = \frac{n}{2}(2n - 18) \quad = n^2 - 9n$$

or, $n^2 - 9n + 52 = 0$ or, $(n-13)(n+4) = 0 \therefore n = 13 \ or, \ n = -4$

Since n must be positive, then the required number of term is 13.

Illustration-08: Fine the general term of the following geometric progression.

4,12,36,...........

Hence find its 15^{th} term

Solution: The first term, $a = 4$ common ratio, $r = \frac{12}{4} = 3$

$\therefore$ The n^{th} term $= a r^{n-1} \quad = 4.3^{n-1}$

$\therefore$ 15^{th} term $= 4\times 3^{15-1} = 4\times 3^{14}$

Illustration-09: If the 4^{th} and 7^{th} terms at a G. P. be $\frac{1}{18}$ and $-\frac{1}{486}$ respectively. Find the series.

Solution: Let the first term of the series, = a and common ratio = r.

$\therefore$ 4^{th} term $= ar^{4-1}$

$$\Rightarrow ar^3 = \frac{1}{18} ....(i)$$

and 7^{th} term $= ar^{7-1}$

$$\Rightarrow ar^6 = -\frac{1}{486} ....(ii)$$

Dividing (ii) from (i) we get

$$r^3 = -\frac{1}{486}\times 18 = -\frac{1}{27}$$

$$\therefore r = -\frac{1}{3} \ldots\ldots\ldots\ldots$$

Putting the value of r in (i) we get

$$a\left(-\frac{1}{3}\right)^3 = \frac{1}{18} \Rightarrow a = -\frac{27}{18} \qquad \therefore a = -\frac{3}{2}$$

$\therefore$ The series is $-\frac{3}{2}, \frac{1}{2}, -\frac{1}{6}, \frac{1}{18}, \ldots\ldots\ldots\ldots$

Illustration-10: Which term of the series $4\frac{1}{2}, 9, 18, \ldots\ldots\ldots\ldots\ldots\ldots$ is 1152?

Solution: Here, 1st term, $a = 4\frac{1}{2} = \frac{9}{2}$; Common ratio, $r = \frac{9}{4\frac{1}{2}} = 2$

If n^{th} term = 1152 then $ar^{n-1} = 1152 \Rightarrow \frac{9}{2}.2^{n-1} = 1152 \Rightarrow 2^{n-1} = \frac{2}{9} \times 1152 \qquad = 256 \quad = 2^8$

$$\therefore n - 1 = 8 \qquad \Rightarrow n = 9$$

$\therefore$ 9^{th} term of the series is 1152.

Illustration-11: Find the sum of the series : 1024, 512, 256 .... to 15 terms

Solution: Here, the first term, $a = 1024$

common ratio, $r = \frac{512}{1024} = \frac{1}{2} < 1 \qquad \therefore$ the sum of n term is $S_n = a.\frac{1-r^n}{1-r}$

$$\therefore S_{15} = 1024 \times \frac{1-\left(\frac{1}{2}\right)^{15}}{1-\frac{1}{2}} \qquad = 1024 \times \frac{1-\frac{1}{32768}}{\frac{1}{2}} \qquad = 2048 \times \frac{32767}{32768} \qquad = \frac{32767}{16}$$

Illustration-12: Find the sum of the series

$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots\ldots\ldots\ldots\ldots\ldots$ to 12 terms

Solution: Here, the first term, $a = 1$, common ratio, $r = -\frac{1}{2} < 1$

$\therefore$ the sum of n term is $S_n = a.\frac{1-r^n}{1-r}$

$$\therefore S_{12} = 1.\frac{1-\left(-\frac{1}{2}\right)^{12}}{1-\left(-\frac{1}{2}\right)} = \frac{1-\frac{1}{4096}}{\frac{3}{2}} = \frac{2}{3}\times\frac{4095}{4096} = \frac{1365}{2048}$$

Illustration-13: Find sum of the following series: $8, 4\sqrt{2}, 4, \ldots\ldots\ldots\ldots\ldots\ldots$ to ∞

Solution: Here the first term, a = 8, common ratio,

Here the first term, a = 8, common ratio, $r = \frac{4\sqrt{2}}{8} = \frac{1}{\sqrt{2}} < 1$

$\therefore$ Sum of the series to the infinity is

$$S = \frac{a}{1-r} = \frac{8}{1-\frac{1}{\sqrt{2}}} = \frac{8}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{8\sqrt{2}}{\sqrt{2}-1} = \frac{8\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = 8(2+\sqrt{2})$$

Illustration-14: Find the sum to the n terms of the series 7, 77, 777, ...

Solution: Let S_n be the sum to the n terms of the series,

$\therefore S_n = 7 + 77 + 777 + \ldots\ldots\ldots\ldots\ldots\ldots$ to n terms

$= 7(1 + 11 + 111 + \ldots\ldots\ldots$ to n terms)

$= \frac{7}{9}(9 + 99 + 999 + \ldots\ldots\ldots$ to n terms)

$= \frac{7}{9}\{(10-1) + (100-1) + (1000-1) + \ldots\ldots\ldots\ldots\ldots$ to n terms$\}$

$$=\frac{7}{9}\{(10+100+1000+\ldots\ldots\ldots\ldots \text{ to n terms}) \quad -(1+1+1+\ldots\ldots\ldots\ldots \text{ to n terms})$$

$$=\frac{7}{9}\{(10+10^2+10^3+\ldots\ldots\ldots+10^n)-n\} \qquad =\frac{7}{9}\left\{10.\frac{10^n-1}{10-1}-n\right\}$$

$$=\frac{7}{9}\left\{\frac{10}{9}(10^n-1)-n\right\} =\frac{70}{81}(10^n-1)-\frac{7n}{9}$$

Illustration-15: Find the sum upto n terms of the series: 0.3, 0.33, 0.333, ...

Solution: Let $S=0.3+0.33+0.333+\ldots\ldots\ldots$ to n terms

$$=\frac{3}{9}(.9+.99+.999+\ldots\ldots to\ n\,terms)=\frac{3}{9}\{(1-.1)+(1-.01)+(1-.001)+\ldots\ldots to\ n\ terms\}$$

$$=\frac{3}{9}\{(1+1+1+\ldots\ldots.to\ n\ terms)-(.1+.01+.001+\ldots\ldots to\ n\ terms)\}$$

$$=\frac{3}{9}\left\{n-\left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+\ldots\ldots\ldots\text{to n terms}\right)\right\} =\frac{3}{9}\left\{n-\frac{1}{10}\cdot\frac{1-\left(\frac{1}{10}\right)^n}{1-\frac{1}{10}}\right\}$$

$$=\frac{1}{3}\left(n-\frac{1}{10}\,\frac{1-\left(\frac{1}{10}\right)^n}{\frac{9}{10}}\right) \qquad =\frac{1}{3}\left\{n-\frac{1}{9}\left(1-\left(\frac{1}{10}\right)^n\right)\right\}$$

Illustration-16: Find the sum up to n term of the series 11, 102, 1003, 10004, ...

Solution: Let $S=11+102+1003+10004+\ldots\ldots\ldots\ldots\ldots$ to n terms

$=(10+1)+(100+2)+(1000+3)+(10000+4)+\ldots\ldots\ldots$ to n terms

$=(10+100+1000+10000+\ldots\ldots$ to n terms $+(1+2+3+4+\ldots\ldots\ldots\ldots$ to n terms

$$=10.\frac{10^n-1}{10-1}+\frac{1}{2}n(n+1) \quad =\frac{10}{9}(10^n-1)+\frac{1}{2}n(n+1)$$

9.5 BUSINESS APPLICATION

Problem-01: A piece of equipment cost a certain factory Tk 600000. If it depreciates in value, 15% in the first year, $13\frac{1}{2}$% in the next year, 12% in the third year and so on, what will be its value at the end of 10 years, all percentages applying to the original cost?

Solution: Suppose the cost of an equipment is Tk. 100. Now the percentages to depreciation at the end of 1st, 2nd, 3rd years are $15, 13\frac{1}{2}, 12$.

Which are in A.P.

Where first term, $a = 15$ and common difference $d = -\frac{3}{2}$

Hence the percentage of depreciation in the 10th year $= a + (10-1)d = 15 + 9\times\left(-\frac{3}{2}\right) = \frac{3}{2}$

Also total value depreciated in 10 years

$$= 15 + 13\frac{1}{2} + 12 + \dots\dots\dots\dots + \frac{3}{2} = \frac{10}{2}\left\{2\times 15 + (10-1)\left(-\frac{3}{2}\right)\right\} = \frac{10}{2}\left(30 - \frac{27}{2}\right) = \frac{165}{2}$$

Hence the value of equipment at the end of 10 years

$$= 100 - \frac{165}{2} = \frac{35}{2}$$

The total cost being Tk. 600000, its value at the end of years

$$= Tk\frac{600000}{100}\times\frac{35}{2} = Tk\,105000$$

Problem-02: A firm pays Tk. 4000 to its manager. The manager is given an increment of Tk. 500 every year.

(i) Find the total salary paid to the manager in 10 years.

(ii) Salary at the end of 10 years.

Solution:

Initial salary, $a = Tk.\ 4000$

Increment, $d = Tk\ 500$

Total number of years, $n = 10$

(i) Total sum paid in 10 years;

$$= \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{10}{2}\{2 \times 4000 + (10-1) \times 500\} = Tk.62500$$

(ii) Salary at the end of 10 years $= a + (10-1)d = 4000 + 9 \times 500 = Tk.8500$

Problem-03: The production of gold from the gold mine in the 1st year was worth Tk 35,000, next year it was $\frac{7}{5}$ of the first production. In the 3rd year $\frac{7}{5}$ of the second years production and so on. What will be the production in the 10th year and what will be the total production up to the 8th year.

Solution: Production of the 1st, 2nd and 3rd year are $35000, 35000 \times \frac{7}{5}, 35000 \times \left(\frac{7}{5}\right)^2$ respectively.

In successive years the production form a G.P. with first term, a = 35000 and common ratio, $r = \frac{7}{5} > 1$.

$\therefore$ Production in the 10th year $= ar^{10-1} = 35000 \times \left(\frac{7}{5}\right)^9$

Again the total production upto the end of the 8th year $= a.\frac{r^8 - 1}{r - 1}$

$$= 35000.\frac{\left(\frac{7}{5}\right)^8 - 1}{7/5 - 1} = 87500\left\{\left(\frac{7}{5}\right)^8 - 1\right\}$$

Problem-04: Author receives a royalty of Tk 2000 in the first year. Next year it falls to Tk 1500 and further to Tk. 1125 in the following year. Assuming that his royalty in the following years continue to falls in this geometrical progression; calculate the maximum possible royalty that he is likely to receive.

Solution: Royalty in the 1st, 2nd and 3rd year are 2000, 1500, 1125 respectively.
Then the royalty in successive years form a G.P. whose first term is, $a = 2000$ and common ratio,

$$r = \frac{1500}{2000} = \frac{3}{4} < 1$$

$\therefore$ Maximum possible royally $= 2000 + 1500 + 1125 + \ldots\ldots$ to infinity $= \dfrac{2000}{1-\dfrac{3}{4}} = 8000$

Problem-05: It the value of Fiat car depreciated by 25 percent annually, what will be its estimated value at the end of 8 years if its present value is Tk. 2048.

Solution: Since given the present value is Tk. 2048, then the value after one year

$$= 2048 \times (100 - 25)\% = 2048 \times \frac{75}{100} = 1536$$

$\therefore$ The values at the end of second, third, fourth year form a G.P with first term $a = 1536$

Common ratio $\dfrac{100-25}{100} = \dfrac{3}{4}$

$\therefore$ Value at the end of eight years $= 1536 \times \left(\dfrac{3}{4}\right)^{8-1} = 205.03$

Problem-06: A person has two parents, four grand parents, eight great grand parents etc. Find the number of ancestors which a person has in the 12th generation back and total number of all ancestors in these preceding 12 generations, assuming that there are no inter marriages.

Solution: Number of parents of 1st, 2nd, 3rd generations are 2,4, 8 respectively.

$\therefore$ Number of parent of 12th generation, $= ar^{12-1} = 2.2^{11} = 4096$

Total number of ancestors in these preceding 12 generation is $= 2 + 4 + 8 + \ldots\ldots$ to 12 terms

$$= 2.\frac{2^{12}-1}{2-1} = 8190.$$

BRIEF REVIEW

Definition

Sequence: A sequence is a special types of function whose domain is natural numbers $(N = 1,2,3,....)$ and range is a subset of real numbers $(R = (-\infty, \infty))$. For ex. : 2, 4, 6, 8, ...

Series: A series in succession of numbers which are formed in order, according to some defined rule, called the law of the series.

Arithmetic progression: Numbers are said to be in arithmetic progression when they increase or decrease by a common difference. For example: 2, 5, 8, 11,...

Geometric progression: A geometric progression is a sequence whose terms increase or decrease by a constant ratio, called the common ratio. For example: 3, 6, 12, 24,.........

Quiz Questions

Multiple Choice Questions

1. Which one is in arithmetic progression?
   (a) 1, 2, 4, 8, ... (b) 3, 5, 9, 17, ...
   (c) 2, 4, 6, ... (d) $3, 3^2, 3^3$, ...
2. Sum of 1+2+3+.............+10 is
   (a) 55 (b) 5050 (c) 550 (d) 50
3. nth term of a, a + d, a + 2d, ... is
   (a) a + nd (b) a + (n-1)d (c) a-(n-1)d (d) $\frac{n}{2}\{2a+(n-1)d\}$
4. If a, b, c is in arithmetic progression then which is not correct
   (a) $b-a=c-b$ (b) $\frac{a+c}{2}=b$
   (c) $b-c=a-b$ (d) $b+a=c+b$

5. $1+3+5+.........+(2n-1)=?$

(a) $\frac{n(n+1)}{2}$ (b) $n(n-1)$ (c) n^2 (d) n^2-1

6. What is the sum of $a+ar^2+ar^3+..........+ar^{n-1}$ When $r=1$

(a) $\frac{a}{1-r}$ (b) na (c) $a\frac{1-r^n}{1-r}$ (d) $a.\frac{r^{n-1}}{r-1}$

7. What is the n^{th} term of 2, 4, 8, ...?

(a) 2^n (b) 2^{n-1} (c) n^2 (d) $2n$

8. If $r<1$, then $a+ar+ar^2+.............$ to infinity = ?

(a) $a.\frac{r^n-1}{r-1}$ (b) $\frac{a}{1-r}$ (c) $a(r-1)$ (d) ar^{n-1}

9. $1+2+4+.............+32=?$

(a) 2^6-1 (b) 6 (c) 64 (d) 2^5-1

10. $2+1+\frac{1}{2}+...............+$ to infinity =?

(a) $3\frac{1}{2}$ (b) infinity (c) 4 (d) 6

Which one of the following statement is true/false?

a. The n^{th} term of an $A.P.$, $u_n=a+(n-2).d$.

b. Progression is special types of function whose domain is natural number.

c. If a,b,c are in $A.P$ Then $(b+c)(c+a)(a+b)$ are not is $A.P$

d. The n^{th} term of a $G.P$ is: ar^{n-1}.

e. The 14^{th} term of the $G.P$: 12, 36, 108, ……… is 4×3^{14}.

Brief Questions

1. Write down the general term of A.P.
2. Write down the general term of G.P.
3. Which term of series 12,9,6........... is equal to -30?
4. $1+2+4+\cdots\cdots\cdots+32=?$
5. What is the n^{th} term of $2,4,8\cdots\cdots\cdots$?
6. When $r=0$ then what is the sum of $a+ar^2+ar^3+\cdots\cdots+ar^{n-1}=?$
7. If $r<1$ then $a+ar+ar^2+\cdots\cdots\cdots$ to infinity =?
8. $1+3+5+\cdots\cdots\cdots+(2n-1)=?$
9. $1^2+2^2+3^2+\cdots\cdots\cdots+n^2=?$
10. $1+2+3+\cdots\cdots\cdots+10=?$

Conceptual, Analytical & Numerical Questions

1. Define progression (or series) with example
2. What is arithmetic progression? Find the general term and sum to first n terms of the following arithmetic progression.
   $a, a+d, a+2d, a+3d, \ldots\ldots\ldots\ldots\ldots\ldots\ldots$
3. Define geometric progression. Consider a geometric progression:
   $a, ar, ar^2, ar^3, \ldots\ldots\ldots\ldots\ldots$
   (i) Find the general term of the progression.
   (ii) Find the sum of the series up to n terms.
4. Find the general term, 10^{th} term, 15^{th} term of the following A.P.: 3, 8, 13, 18, ...
5. Find the 19^{th} term of the series: 10, 8, 6, 4, ...
6. What term of the series: 5, 7, 9, 11, ... is 25?
7. Find the general term and 25^{th} term of the arithmetic progression:
   $1, \frac{5}{6}, \frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\ldots\ldots$
8. The 5^{th} term of an A.P. is 10, while the 15^{th} term is 40. Find the series.

9. Find the sum of the series:

   (*i*) 72, 70, 68, ..., 40 (*ii*) 2, 3.5, 5, 6.5, ... *to* 25 *term*

   (*iii*) 7, 14, 21, ... *to* 20 *terms* (*iv*) −4, −1, 2, 5,.............. *to* 21 *terms*

10. Find the 20th term of the arithmetic 15, 13, 11, ... calculate the number of terms required to make the sum equal to zero.

11. Find which term of the series 0.004, 0.02, 0.1, ... is 12.5

12. Find the 12th term of $2, -2\sqrt{3}, 6$..........

13. Find the nth term of the series $9, -6, 4$,..........

ANSWERS

(4) 5n − 2; 48, 73 (5) −26 (6) 25 (7) $\frac{7-n}{6}, -3$ (8) −2,1,4,7,10,.............

(9) (i)952 (ii) 500 (iii)1470 (iv) 546 (10) −25, 16 (11) 6th term

(12) $-486\sqrt{3}$ (13) $\frac{(-2)^{n-1}}{3^{n-3}}$

Numerical questions

1. How many terms of the series 20, 20, 16, ... must be taken so that the sum may be 72.
2. The first term of A.P. is 2, the nth term is 32 and the sum of first n terms is 119. Find the series.
3. Find the three numbers in A.P, whose sum is 9 and the product is -165.
4. Find the four numbers in A.P, whose sum is 120 and the sum of whose squares in 120.
5. Find the three numbers in A.P, Where the sum of the numbers in 24 and the sum of their cubes is 1968.
6. Find the five numbers in A.P such that their sum is zero and the sum of squares is 135.
7. The third term of a G.P. is 2/3 and the 6th term is 2/81 find the 8th term.
8. The fourth term of a G.P. is 9 and its tenth term 6561

   a) find the series b) find the sum of first six terms

9. Find the sum of the G.P. to infinity:

$\sqrt{3}, \frac{1}{\sqrt{3}}, \frac{1}{3\sqrt{3}}, \ldots\ldots\ldots\ldots$

10. Sum to n terms the series: $(a)\ 5+55+555+\ldots\ldots\ldots\ldots\ldots\ldots\ldots to\ n\ term$

$(b)\ 0.8+0.88+0.888+\ldots\ldots\ldots\ldots to\ n\ term$

11. Find the infinite G.P. whose term is 1/4 and the sum is 1/3

12. The cost of boring a tube-well 100 meter is as follows: The cost of first meter is tk 50, and for each subsequent meter the cost is 2 taka, more than the preceding meter. Find the cost of 50^{th} meter and the total cost.

13. A man borrows Tk. 2000 and agrees to repay a total interest of tk 340 in 12 monthly installments, each installment being less than the preceding by tk 10. Find separately the amounts of the first and the last installments.

14. A machine was purchased for tk 4000 on account of constant use, the value of the machine depreciates at the rate of tk 100 per year. What will be its value at the end of the 9^{th} year?

15. A man saves Tk 100 in the first year. In the succeeding year he save tk 20 more than the last. How much has he collected at the end of 20 years.

16. A man repay a Loan of Tk 3250 by paying tk 20 in the first month and then increases the payment by tk 15 every month. How long will it take to deas his loan?

17. The yearly output of coal from a certain mine was 150000 tons in a particular year. If the production of coal goes on decreasing by 30 percent per annum, find

(a) The output in 10 years and in the 10^{th} year (b) The total production of coal in the mine.

18. A factory plant depreciates at the rate of 4% every year. Find the cost of the plant ten years, hence, if the cost at the time of purchase was Tk. 2000.

19. A machine depreciates its value each year by 8% of its value at the beginning of the year. After how many years will its value be less than half its original value?

20. The early output of a coal mine is found to be decreasing by 20% of its previous year output. If in a certain year its output was Tk. 20000. What could be reckoned as its total future output?

21. Mr. Irtiza saved Tk. 16,500 in ten years. In each year of ten the first he saved Tk. 100 more than he did in the preceding year. How much did he save in the first year?

22. Mr. Towhid buys national savings certificates of values exceeding of the last years purchase by Tk. 100. After 10 years he finds that the total value of the certificates purchased by him is Tk. 5000. Find the value of the certificates purchased by him
    (i) in the first year
    (ii) in the 8^{th} year.

23. Mr. Ornob agrees to pay a debt of Tk. 2500 in a number of installments, each installment (beginning with the second) exceeding the previous one by Tk. 2. If the first installment by of Tk. 1, Find how many installments will be necessary to wipe out the loan completely?

24. By arranges to pay off a debt of Tk. 9600 in 48 annual installments which form an arithmetic series. When 40 of these installments are paid, B becomes instrument and his creditor finds that Tk. 2400 still remains unpaid. Find the value of each of the first three installments of B, Ignore interest.

25. A money lender lends Tk. 1000 and charges an overall interest of Tk. 140. He recovers the loan and interest by 12 monthly installments each less by Tk. 10 than the preceding. Find the amount of each installment.

26. An enterprise produced 600 units in the 3^{rd} year of existence and 70 units in its 7^{th} year.
    (i) What was the initial production in the 1^{st} year?
    (ii) What was the production in the fifth year?
    (iii) What was the total production in the first five year.

27. At 10% per annum compound internet, a sum of money accumulates to Tk. 8750 in 4 years. Find the sum invested initially.

28. Calculate the population in 1985 if the population in 1975 is 55 crore and is growing at a compound rate of 2% annually.

ANSWERS

(1) 4 or 9

(2) 2, 7, 12, 17, 22, ...

(7) Tk. 148, Tk. 14900

(8) Tk. 250, Tk. 140

(9) Tk. 3100

(10) Tk. 5800

(11) 20 months

(12) $\frac{2}{729}$

(13) (a) $\frac{1}{3}$, 1, 3, 9, ... (b) $\frac{364}{3}$

(14) $\frac{3\sqrt{3}}{2}$

(15) (a) $\frac{50}{81}(10^n - 1) - \frac{5}{9}n$

(b) $\frac{8}{9}n - \frac{8}{81}\left(1 - \frac{1}{10^n}\right)$

16. $\frac{1}{4}, \frac{1}{16}, \frac{1}{64}$, ...

17. (a)

$5\left(\frac{10^{10} - 7^{10}}{10^5}\right)$, $15 \times 7^9 \times 10^{-5}$ *tons*

(b) 500000 tons

18. 13306 Tk.

19. 9 years.

21. Tk. 1200