Fourth Edition

# BUSINESS MATHEMATICS

Dr. Md. Rafiqul Islam
Mohammad Osman Gani

CBO PUBLICATIONS

Written as a Text for the students of B.B.A (Hons.) in Accounting, Management, Marketing and Finance & Banking and BBA Professional under National University and all other Public and Private Universities in Bangladesh.

# Business Mathematics

## Fourth Edition


**Dr. Md. Rafiqul Islam**
Professor
Department of Banking
University of Dhaka

**Mohammad Osman Gani**
Assistant Professor
Department of Mathematics
Jahangirnagar University



CBO
Campus Based Organization

**CBO Publications**
**38, Banglabazar (2nd Floor) Dhaka-1100**
**www.cbopublications.com.bd**

*Published by*
**CBO PUBLICATIONS**
Concord Emporium,
253-54, Elephant Road, Dhaka-1205
Phone: 9612117, 01552-488117.
www.cbopublications.com.bd



First Edition : **March, 2003**
Fourth Edition : **March, 2012**

ISBN : 984-70062-0007-9

Cover Design : Fatema Akter Shethi

Price : Tk. 330.00 (Three Hundred Thirty) Only.

**Business Mathematics by Professor Dr. Md. Rafiqul Islam & Mohammad Osman Gani** published in 2010 by CBO Publications, Concord Emporium, Suite-12-13, 253-54, Elephant Road, Dhaka-1205, Phone: 02-9612117, 01552-488117 web: www.cbopublications.com.bd

**To my Lovely Daughter**

**----------Lamia**

**Prof. Dr. Md. Rafiqul Islam**

**To my pretty Son**

**-----Dhrubo**

**Mohammad Osman Gani**

# PREFACE TO THE FOURTH EDITION

We are happy to present a book on Business Mathematics to the readers of the subject. There are several books on Business Mathematics by eminent authors already available in the market. Yet we cannot help feeling that there is scarcely any that truly meets the requirements of B.B.S Honors students of the National University for the session 2009-2010. This humble effort for writing a book on Business Mathematics is directed precisely towards this goal. Hence, the approach in this book is governed strictly by that requirement.

Each chapter starts with a simple and lucid discussion of the topic followed by properly arranged worked-out problems and ends with theoretical questions and exercises. In any mathematics examination it is of utmost importance to practice answering questions so as to develop an answering technique. The purpose of this book is to help students acquire this technique. The more they practice, the more confident they will be. In fact, constant practice is the key to success in this field.

We would like to express our gratitude to our colleagues and students for their continuous encouragement and support to write the book.

We would like to thank Mrs. Fatema Akter of CBO Publications for his valuable contributions in turning the manuscript into a bound book. Much of the attractiveness of the presentation and layout of the book is due to his untiring efforts.

Care has been taken to make the book free from errors. In spite of the sincerest efforts on our part and also on the part of the printers and publishers, if any error still remains, we beg to be excused for the same.

We shall consider ourselves amply rewarded, should the book prove helpful to those for whom it is intended. Constructive criticisms and suggestions for improvement of the book are most welcome.

**Prof. Dr. Md. Rafiqul Islam**
**Mohammad Osman Gani**
**May 2012**

# PREFACE TO THE FIRST EDITION

The primary objective of this textbook is to provide the basic concepts and applications of mathematics suited to the needs of students appearing in the examinations of BBS (Hons.), BBA and other professional examinations. This book reflects a special concern to present mathematical concepts in terms that the reader can understand and particle with a variety of examples. Thus the approach is to communicate the practical aspects of mathematics in a way that derives out the common fear of learning any mathematical subject.

Although there are several topics that could be included in this text, however, only those have been selected which are included in Business Mathematics course under the National University BBS (Hons) syllabus. The ordering of the chapter reflects the most desirable sequence of topic coverage, while at the same time allowing for flexibility in the choice of topics.

Dividing into nine chapters, each chapter explains in details, the important concepts and definition. Each concept and technique is properly supported by examples followed by numerous and varied types of solved problems to provide students and integrated view of theory and business applications of mathematics. A large number of problems with solutions have been added in each chapter to enable students to learn at their own pace. Most of the questions conform to the trend questions appearing in the university examinations.

I gratefully acknowledge the inspiration, encouragement and valuable suggestions received form the well wishers during the preparation of this book. I would like to thank Mr. Mujahid, owner of the CBO publications, for publishing the book. Specially, I would like to thank my honorable teacher Dr. Hairpada Bhattacharjee, Professor of Marketing, University of Dhaka for his valuable comments.

Finally, I would like to thank my wife and daughter for their patience, understanding, I love and assistance in making this book a reality. It is to my lovely daughter Lamia I dedicate this book.

I hope tried may best to make this book free from errors and omission. If you have any suggestions or material requiring clarification or potential errors, please write to me. Your suggestions and comments for the improvement of this text will be thankfully received and duly incorporated in the subsequent editions.

**Md. Rafiqul Islam**

**March, 2003**
**Faculty of Business Studies**
**University of Dhaka**

# NATIONAL UNIVERSITY

**Syllabus for First Year BBA (Honours) in Accounting, Management, Marketing and Finance & Banking**
**Effective From the session: 2009-2010**

## BUSINESS MATHEMATICS

1. **Number System:** Introduction, Prime, Rational and Irrational numbers, even and odd numbers and their properties. Imaginary and complex numbers; sequence, series, Divergence of a sequence, sigma notation. Sum of natural numbers.
2. **Indices and Logarithms:** Indices, laws of indices, positive and fractional indices, operation with power functions. Introduction, Laws of operations, use of logarithm tables, Antilogarithm, Exponential series, logarithmic series.
3. **Theory of Sets:** Different types of sets, union, difference intersection, Cartesian products, Venn diagram, Application in business problems.
4. **The Straight Lines, Linear Equations and Inequalities:** Different forms of Equations- Parallel and perpendicular form of Equations-Point of intersection of lines-Interpretation of Slope of different business problems and Marginal Concept analysis, Solution of linear equations, solution of simultaneous linear equations. Identities and Inequalities and their solutions. Graphical representation of solution sets. Determination of Breakeven Point having linear relationship among the variables.
5. **Quadratic Equations:** Relationship between roots+- and coefficients of a quadratic equation, Nature of roots. Solution of quadratic equations, formation of quadratic equations with given roots. Curve sketching of different linear and non linear equations.
6. **Binomial theorem:** Expansion of binomial terms with positive integral index – Determination of general term and middle terms of a Binomial expansion and exponential expansion.
7. **Permutations and Combinations:** Fundamental principles of permutations, permutations of things, all different and things, not all different, repeated and circular permutations. Combinations, relationship between permutation and combinations.
8. **Progression:** Introduction, Sum of Series in AP and GP, Use of concept of progression to find the present value and future value, Business problem solution.
9. **Mathematics of Finance:** Introduction, Annuities, Sinking Fund, Discount, Compound interest, Simple interest, Amortization.
10. **Matrices & Determinants:** Introduction, types of matrices, operation on matrices. Solution of simultaneous linear equations. Introduction, properties of determinants, operations on determinants. Expansions of determinants Sarrus diagram and Crammers rule and use in business.Leontiff input-output model analysis. Application to business problem.
11. **Differentiation, Integration & Use in Business Problems:** Differentiation of Algebraic functions, transcendental function. Definition of transcendental functions-Geometric interpretation of dy/dx and marginal concept analysis. Introduction, Different rules of integration, Indefinite and Definite integral, calculation of area of irregular curves and figures - Analysis of total concept by integration, Determination of maxima and minima of different functions using differentiation rules. Business problem solution. Conditions of maxima and minima, Test for maxima and minima. Point of inflexion.

**Books Recommended:**

Gordon D. Prichett and John C. Saber, *Mathematics with Application in Management and Economics*, 7th Edition, Irwin, USA.

**Reference Books:**


1. Sanchetti, D. C. and V. K. Kapoor: *Business Mathematics*. Published by S. Chand and Company Ltd.
2. Ann J. Hughes, *Applied Mathematics for: Business, Economics and the Social Sciences*, Latest Edition, Irwin, USA.

# NATIONAL UNIVERSITY
## BBA Professional Program
## BUSINESS MATHEMATICS

1. **Basic Concepts of Number:** Introduction, Prime, Rational and irrational numbers, even and odd numbers, imaginary and complex number.
2. **Indices, Exponents and Logarithmic Functions:** Indices, laws of Indices, positive and fractional indices with operations, logarithmic and natural logarithm.
3. **Theory of sets:** definition of set, different types of set, Union, difference, intersection, Cartesian product, Venn diagram, Application to business problems.
4. **Linear and Quadratic functions:** Solution of linear and simultaneous linear equations, Relationship between roots and co-efficient of a quadratic equation, Nature of roots, solution of quadratic equation, formation of quadratic equation with given roots, Sketching of different linear and non-linear equations.
5. **Mathematics of Finance:** Annuities, sinking fund, discount, simple and compound interest, amortization, calculation of present and future value of annuities.
6. **Co-ordinate Geometry:** Cartesian co-ordinate system, distance between Points, Straight line, slopes, intercepts, Equation of cost line, business applications.
7. **Differential calculus:** Concepts, derivatives and differentiation, higher order differentiation, exponential and logarithmic differentiation, partial differentiation, optimization, business applications.
8. **Integral Calculus:** Meaning of integration, Rules of integration, Indefinite and definite integration, area determination, application to business.

# NATIONAL UNIVERSITY

**Syllabus for First Year Honours in Accounting, Management, Marketing and Finance & Banking**
**Effective From the session: 2004-2005**

## BUSINESS MATHEMATICS

**Subject Code: 7231**

1. Basic Concepts: Concepts of number system, fractions, exponents, equations, factoring, polynomials, ordered pairs, relations, functions, types of functions.
2. Set Theory: Sets, set notation, operations with sets, lays of set operations, Venn diagrams, application of set theory.
3. Logarithms: Rules for logarithms, common logarithms, calculation of logarithm or a number, natural logarithm.
4. Equation System: Solution of equations, Simultaneous equation system, solution of simultaneous equation systems with specific applications to business problems, inequalities.
5. Geometry: Cartesian co-ordinate system, distance between two points, straight line-slopes-intercepts, equation of a line, application of linear equations.
6. Differential Calculus: Explanation of the concepts of limits and continuity, derivative and differentiation, rules of differentiation, higher order differentiation, chain order differentiation, exponential and logarithmic differentiation, partial differentiation, optimization, rate of growth and decays, business application.
7. Integral Calculus: Meaning of integration, rules of integration, indefinite integral, definite integral, resource depletion, resource accumulation, area between curves, business applications of integration in business decisions.
8. Matrix Algebra: Vectors, matrices, laws and operations transposes, inverse, adjoins, creamer's rule, determinants, solution of system of equations, application of matrix algebra in business.
9. Mathematics of Finance: Annuities, sinking fund, discount, simple and compound interest, amortization, calculation of present value and future value of annuities.

Books Recommended:

1. Gordon D.Prichett and John C. Saber, : Mathematics with application in Management and Economics, (Edition 7), Irwin, USA.
2. D.C. Sancheti & V.K. Kapoor : Business Mathematics, S. Chand & Sons, New Delhi, India.
3. Dr. Md. Rafiqul Islam : Business Mathematics, CBO Publications, 2003, Dhaka

[Any standard Bengali book covering the Syllabus will be considered as reference book]

# CONTENTS


**Chapter 1: Number System** 1-36

1.1 Introduction 1
1.2 Real number system 1
1.3 Absolute value of a number 6
1.4 Special properties of zero 6
1.5 Fractions 8
1.6 Complex and imaginary numbers 10
1.7 Sequence and series 11
1.8 Sigma notation 11
1.9 Principal of mathematical induction 12
1.10 Sum of natural numbers 19
1.11 Relations and functions 22
1.12 Business applications 29
*Brief Review* 32
*Quiz Questions* 34
*Conceptual, Analytical & Numerical Questions* 35

**Chapter 2: Indices and Logarithms** 37-82

2.1 Introduction 37
2.2 Index and base of a number 37
2.3 Laws of indices 37
2.4 Meaning of $a^m$ 38
2.5 Positive indices 38
2.6 Fractional indices 39
2.7 Operation with power functions 39
2.8 Definition of logarithm 40
2.9 Anti-logarithm 41
2.10 Laws of logarithms 41
2.11 Common logarithm 43
2.12 Natural logarithm 44
2.13 Characteristics and mantissa 44
*Brief Review* 73
*Quiz Questions* 74
*Conceptual, Analytical & Numerical Questions* 78

**Chapter-03 Theory of Sets** 83-128

3.1 Introduction 83
3.2 Definition of a Set 83
3.3 Methods of describing a set 84
3.4 Types of sets 86
3.5 Operations on sets 89
3.6 Venn diagram 92
3.7 De-Morgan's laws 93
3.8 Some results on sets operations 96
3.9 Cartesian product of two sets 101
3.10 Business applications 113
*Brief Review* 121
*Quiz Questions* 123
*Conceptual, Analytical & Numerical Questions* 125

**Chapter-04 Linear Equations and Inequalities** 129-158

4.1 Introduction 129
4.2 Equation 129
4.3 Identity 130
4.4 Linear simultaneous equations with two variables 131
4.5 Linear inequalities 135
4.6 Business application 146
*Brief Review* 154
*Quiz Questions* 155
*Conceptual, Analytical & Numerical Questions* 157

**Chapter-05 Quadratic Equations** 159-272

5.1 Introduction 159
5.2 Quadratic equations 159
5.3 Solutions of a general quadratic equation 160
5.4 Relationship between roots and coefficients of a quadratic equation 161
5.5 Nature of the roots of a quadratic equation 162
5.6 Formation of quadratic equation with given roots 162
5.7 Curve sketching of different line and non-linear equations 163
5.8 Business application 202
*Brief Review* 205
*Quiz Questions* 206
*Conceptual, Analytical & Numerical Questions* 208

**Chapter-06 The Straight Lines** 273-254

6.1 Introduction 273
6.2 Abscissa and ordinate of point 214
6.3 Rectangular coordinates 214
6.4 Distance between two points 215
6.5 Coordinates of mid-point 215
6.6 Division or section formula 217
6.7 Centroid of a triangle 217
6.8 Area of a triangle 219
6.9 Polar coordinates 222
6.10 Relation between polar and cartesian coordinates of a point 222
6.11 The straight line and standard form of its equations. 223
6.12 General equation of a straight line 226
6.13 Intersection of two straight lines 227
6.14 Condition for concurrent of the three straight lines 227
6.15 Angle between two straight lines 229
*Brief Review* 250
*Quiz Questions* 251
*Conceptual, Analytical & Numerical Questions* 253

**Chapter-07 Permutations and Combinations** 255-272

7.1 Introduction 255
7.2 Important notation 255
7.3 Permutation 256
7.4 Fundamental principles of permutation 256
7.5 Permutation of things all different 256
7.6 Permutation of things not all different 257
7.7 Circular permutations 258
7.8 Combinations 258
7.9 Some special deduction of combinations 259
7.10 Relation between permutation and combination 259
*Brief Review* 269
*Quiz Questions* 270
*Conceptual, Analytical & Numerical Questions* 271

**Chapter-08 Binomial Theorem** 273-294

8.1 Introduction 273
8.2 Binomial theorem 276
8.3 Characteristics of Binomial expansion 274
8.4 Determination of general term 275
8.5 Middle term of a Binomial expansion 275
*Brief Review* 291
*Quiz Questions* 292
*Conceptual, Analytical & Numerical Questions* 293

**Chapter-09 Progression** 295-316

9.1 Introduction 295
9.2 Sequence and series 295
9.3 Arithmetic progression (A.P.) 296
9.4 Geometric progression (G.P.) 297
9.5 Business applications 307
*Brief Review* 310
*Quiz Questions* 310
*Conceptual, Analytical & Numerical Questions* 312

**Chapter-10 Mathematics of Finance** 317-358

10.1 Introduction 317
10.2 Interest 317
10.2.1 Simple interest 318
10.2.2 Compound interest 319
10.3 Nominal and Effective Rates of interest 323
10.4 Continuous compounding 325
10.5 Equation of value of money 325
10.6 Discounting 326
10.7 Depreciation 328
10.8 Annuity 329
10.9 Derivation of different types of annuity 330
10.10 Amortization and sinking funds 339
10.11 Business Applications 342
*Brief Review* 353
*Quiz Questions* 354
*Conceptual, Analytical & Numerical Questions* 356

**Chapter-11 Determinants & Matrices** 359-430

11.1 Introduction 359
11.2 Determinants 359
11.3 Rule for expansion of determinant 360
11.4 Properties of determinants 362
11.5 Cramer's rule 363
11.6 Definition and notations of matrix 373
11.7 Types of matrices 374
11.8 Algebra of matrices 376
11.9 Multiplication of matrices 378
11.10 Transpose of a matrix 381
11.11 Adjoint of a square matrix 382
11.12 Inverse of square matrix 384
11.13 Solution of simultaneous linear equations 385
11.14 Business applications to matrices
*Brief Review* 416
*Quiz Questions* 417
*Conceptual, Analytical & Numerical Questions* 420

**Chapter-12 Differentiation and Use in Business Problems** **431-514**

12.1 Introduction 431
12.2 Concept of slope and rate of change 432
12.2.1 Slope of a straight line 432
12.2.2 Positive or negative slopes 433
12.2.3 Slope of a curve (at a point) 433
12.3 Concept of derivative 434
12.3.1 Existence of derivative 435
12.3.2 Some standard derivatives 435
12.4 Derivative of a composite function (chain rule) 440
12.5 Derivative of exponential functions 441
12.6 Derivative of logarithmic functions 442
12.7 Derivative of inverse functions 443
12.8 Derivative of an implicit function 443
12.9 Successive (or repeated) differentiation 443
12.10 Increasing and decreasing functions 469
12.11 Determination of maxima and minima of different functions using differentiation rules 470
12.12 Conditions for profit maximization 471
12.13 Marginal concept analysis 472
12.14 Business problems solution 477
*Brief Review* 505
*Quiz Questions* 506
*Conceptual, Analytical & Numerical Questions* 508

**Chapter-13 Integration and Use in Business Problems** **515-569**

13.1 Introduction 515
13.2 Definition of integration 516
13.3 Constant of integration 517
13.4 Some standard formulas of integration 517
13.5 Fundamental rules of integration 518
13.6 Method of substitution 518
13.7 Some standard integrals 519
13.8 Integration by parts 520
13.9 Definite integrals 544
13.10 Properties of definite integral 545
13.11 Business applications of integration 547
*Brief Review* 559
*Quiz Questions* 560
*Conceptual, Analytical & Numerical Questions* 562

**APPENDICES** **570-600**
**National University Question Patterns**
**National University Sample Questions (BBA Honours Program)**
**National University Questions (BBA Professional)**
**Table**