Charge and Matter

CHAPTER 26

26–1 Electromagnetism—A Preview

The science of electricity has its roots in the observation, known to Thales of Miletus in 600 B.C., that a rubbed piece of amber will attract bits of straw. The study of magnetism goes back to the observation that naturally occurring "stones" (that is, magnetite) will attract iron. These two sciences developed quite separately until 1820, when Hans Christian Oersted (1777–1851) observed a connection between them, namely, that an *electric* current in a wire can affect a *magnetic* compass needle (Section 33–1).

The new science of electromagnetism was developed further by many workers, of whom one of the most important was Michael Faraday (1791–1867). It fell to James Clerk Maxwell (1831–1879) to put the laws of electromagnetism in the form in which we know them today. These laws, often called *Maxwell's equations*, are displayed in Table 38–3, which the student may want to examine at this time. These laws play the same role in electromagnetism that Newton's laws of motion and of gravitation do in mechanics.

Although Maxwell's synthesis of electromagnetism rests heavily on the work of his predecessors, his own contribution is central and vital. Maxwell deduced that light is electromagnetic in nature and that its speed can be found by making purely electric and magnetic measurements. Thus the science of optics was intimately connected with those of electricity and of magnetism. The scope of Maxwell's equations is remarkable, including as it does the fundamental principles of all large-scale electromagnetic and optical devices such as motors, cyclotrons, electronic computers, radio, television, microwave radar, microscopes, and telescopes.

The development of classical electromagnetism did not end with Maxwell. The English physicist Oliver Heaviside (1850–1925) and especially the Dutch physicist H. A. Lorentz (1853–1928) contributed substantially to the clarification of Maxwell's theory. Heinrich Hertz (1857–1894) * took a great step forward when, more than twenty years after Maxwell set up his theory, he produced in the laboratory electromagnetic "Maxwellian waves" of a kind that we would now call short radio waves. It remained for Marconi and others to exploit this practical application of the electromagnetic waves of Maxwell and Hertz.

Present interest in electromagnetism takes two forms. At the level of engineering applications Maxwell's equations are used constantly and universally in the solution of a wide variety of practical problems. At the level of the foundations of the theory there is a continuing effort to extend its scope in such a way that electromagnetism is revealed as a special case of a more general theory. Such a theory would also include (say) the theories of gravitation and of quantum physics. This grand synthesis has not yet been achieved.

26-2 Electric Charge

The rest of this chapter deals with electric charge and its relationship to matter. We can show that there are *two kinds* of charge by rubbing a glass rod with silk and hanging it from a long silk thread as in Fig. 26-1. If a second glass rod is rubbed with silk and held near the rubbed end of the first rod, the rods will repel each other. On the other hand, a hard-rubber rod rubbed with fur will *attract* the glass rod. Two hard-rubber rods rubbed with fur will repel each other. We explain these facts by saying that rubbing a rod gives it an *electric charge* and that the charges on the two rods exert forces on each other. Clearly the charges on the glass and on the hard rubber must be different in nature.

Benjamin Franklin (1706–1790), who, among his other achievements, was the first American physicist, named the kind of electricity that appears on the glass *positive* and the kind that appears on the hard rubber *negative;* these names have remained to this day. We can sum up these experiments by saying that *like charges repel and unlike charges attract.*

Electric effects are not limited to glass rubbed with silk or to hard rubber rubbed with fur. Any substance rubbed with any other under suitable conditions will become charged to some extent; by comparing the unknown charge with a charged glass rod or a charged hard-rubber rod, it can be labeled as either positive or negative.

The modern view of bulk matter is that, in its normal or neutral state, it contains equal amounts of positive and negative electricity. If two bodies like glass and silk are rubbed together, a small amount of charge is transferred from one to the other, upsetting the electric neutrality of each. In this case the glass would become positive, the silk negative.

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* "Heinrich Hertz," by P. and E. Morrison, *Scientific American*, December 1957.

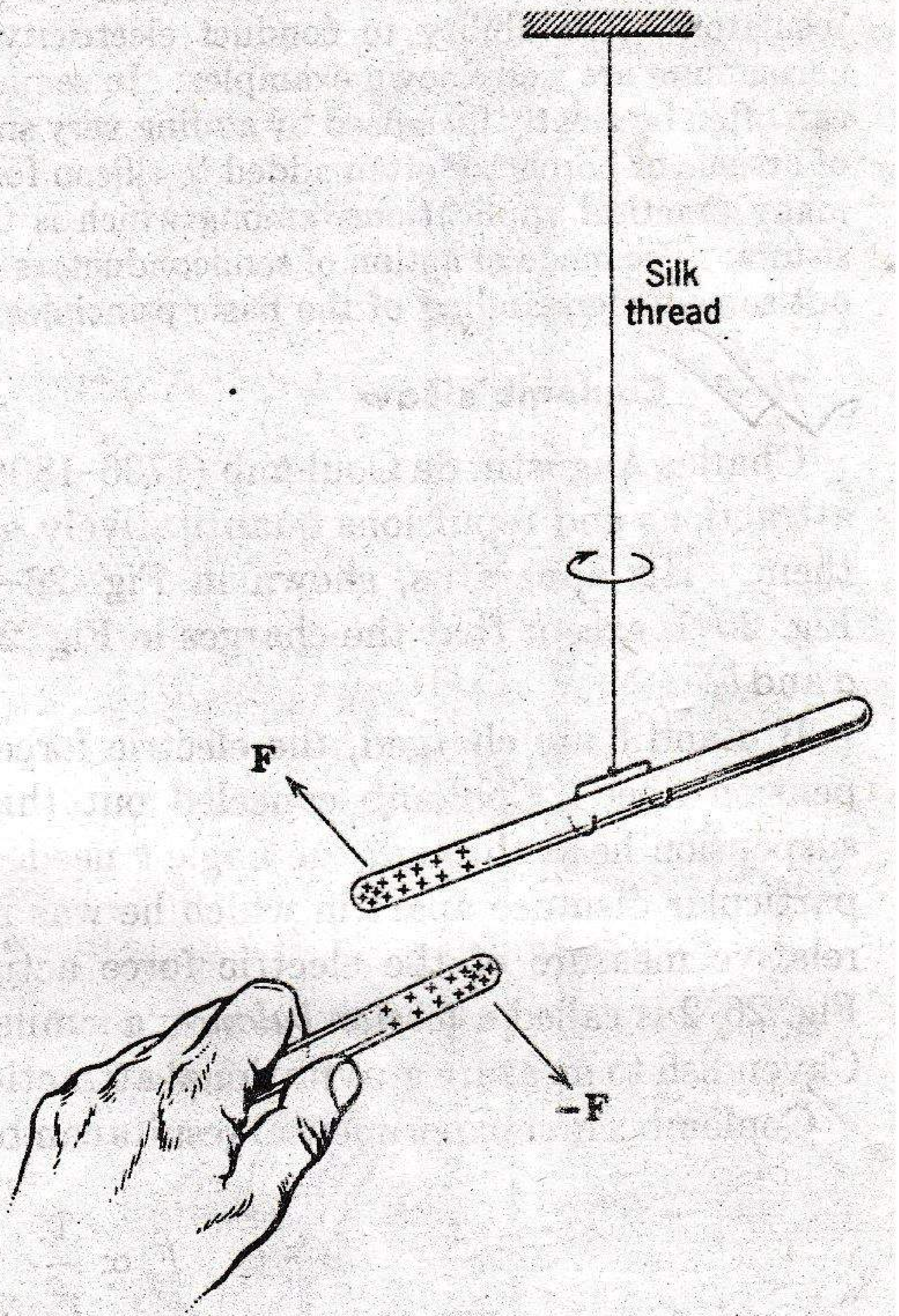

Fig. 26–1 Two positively charged glass rods repel each other.

26–3 Conductors and Insulators

A metal rod held in the hand and rubbed with fur will not seem to develop a charge. It is possible to charge such a rod, however, if it is furnished with a glass or hard-rubber handle and if the metal is not touched with the hands while rubbing it. The explanation is that metals, the human body, and the earth are *conductors* of electricity and that glass, hard rubber, plastics, etc., are *insulators* (also called *dielectrics*).

In conductors electric charges are free to move through the material, whereas in insulators they are not. Although there are no perfect insulators, the insulating ability of fused quartz is about 10^{25} times as great as that of copper, so that for many practical purposes some materials behave as if they were perfect insulators.

In metals a fairly subtle experiment called the *Hall effect* (see Section 33–5) shows that only negative charge is free to move. Positive charge is as immobile as it is in glass or in any other dielectric. The actual charge carriers in metals are the *free electrons*. When isolated atoms are combined to form a metallic solid, the outer electrons of the atom do not remain attached to individual atoms but become free to move throughout the volume of the solid. For some conductors, such as electrolytes, both positive and negative charges can move.

A class of materials called *semiconductors* is intermediate between conductors and insulators in its ability to conduct electricity. Among the elements, silicon and germanium are well-known examples. In semiconductors the electrical conductivity can often be greatly increased by adding very small amounts of other elements; traces of arsenic or boron are often added to silicon for this purpose. Semiconductors have many practical applications, among which is their use in the construction of transistors. The mode of action of semiconductors cannot be described adequately without some understanding of the basic principles of quantum physics.

26-4 Coulomb's Law

Charles Augustin de Coulomb (1736–1806) in 1785 first measured electrical attractions and repulsions quantitatively and deduced the law that governs them. His apparatus, shown in Fig. 26–2, resembles the hanging rod of Fig. 26–1, except that the charges in Fig. 26–2 are confined to small spheres *a* and *b*.

If *a* and *b* are charged, the electric force on *a* will tend to twist the suspension fiber. Coulomb canceled out this twisting effect by turning the suspension head through the angle θ needed to keep the two charges at the particular distance apart in which he was interested. The angle θ is then a relative measure of the electric force acting on charge *a*. The device of Fig. 26–2 is called a *torsion balance;* a similar arrangement was used later by Cavendish to measure gravitational attractions (Section 16–3).

Coulomb's first experimental results can be represented by

$$F \propto \frac{1}{r^2}.$$

F is the magnitude of the force that acts on each of the two charges *a* and *b*; r is their distance apart. These forces, as Newton's third law requires, act along the line joining the charges but point in opposite directions. Note that the magnitude of the force on each charge is the same, even though the charges may be different.

Coulomb also studied how the electrical force varied with the relative size of the charges on the spheres of his torsion balance. For example, if we touch a charged conducting sphere to an exactly similar but uncharged conducting sphere, the original charge must divide equally between the spheres. By such techniques Coulomb extended the inverse square relationship to

$$F \propto \frac{q_1 q_2}{r^2}, \tag{26–1}$$

where q_1 and q_2 are relative measures of the charges on spheres *a* and *b*. Equation 26–1, which is called *Coulomb's law*, holds only for charged objects whose sizes are much smaller than the distance between them. We often say that it holds only for *point charges*.

Coulomb's law resembles the inverse square law of gravitation which was already more than 100 years old at the time of Coulomb's experiments; q plays the role of m in that law. In gravity, however, the forces are always

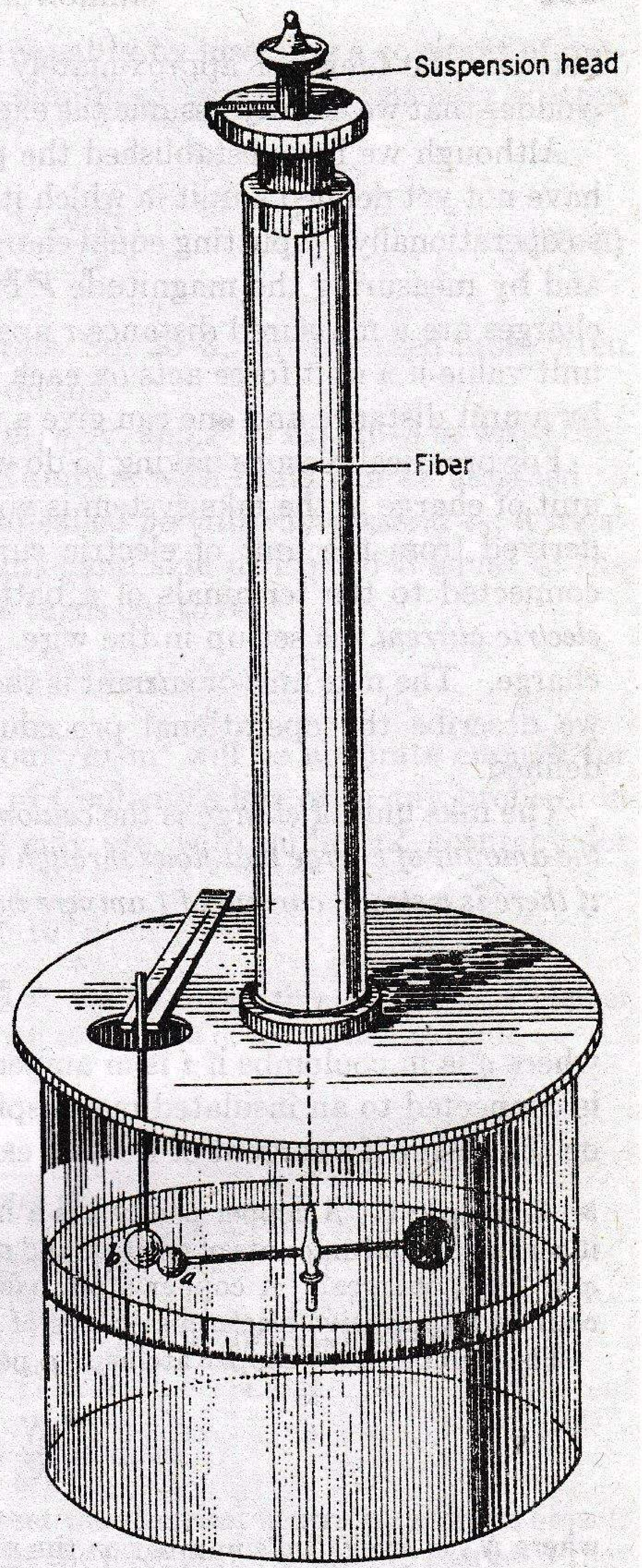

Fig. 26-2 Coulomb's torsion balance from his 1785 memoir to the French Academy of Sciences.

attractive; this corresponds to the fact that there are two kinds of electricity but (apparently) only one kind of mass.

Our belief in Coulomb's law does not rest quantitatively on Coulomb's experiments. Torsion balance measurements are difficult to make to an accuracy of better than a few per cent. Such measurements could not, for example, convince us that the exponent in Eq. 26-1 is exactly 2 and not, say, 2.01. In Section 28-5 we show that Coulomb's law can also be deduced from an indirect experiment which shows that the exponent in Eq. 26-1 lies between the limits of approximately 2.000000002 and 1.999999998. Small wonder that we usually assume the exponent to be exactly 2.

Although we have established the physical concept of electric charge, we have not yet defined a unit in which it may be measured. It is possible to do so operationally by putting equal charges q on the spheres of a torsion balance and by measuring the magnitude F of the force that acts on each when the charges are a measured distance r apart. One could then define q to have a unit value if a unit force acts on each charge when the charges are separated by a unit distance and one can give a name to the unit of charge so defined.*

For practical reasons having to do with the accuracy of measurements, the unit of charge in the mks system is not defined using a torsion balance but is derived from the unit of electric current. If the ends of a long wire are connected to the terminals of a battery, it is common knowledge that an *electric current* i is set up in the wire. We visualize this current as a flow of charge. The mks unit of current is the *ampere* (abbr. *amp*). In Section 34–4 we describe the operational procedures in terms of which the ampere is defined.

The mks unit of charge is the *coulomb* (abbr. *coul*). *A coulomb is defined as the amount of charge that flows through a given cross section of a wire in 1 second if there is a steady current of 1 ampere in the wire.* In symbols

$$q = it, \tag{26-2}$$

where q is in coulombs if i is in amperes and t is in seconds. Thus, if a wire is connected to an insulated metal sphere, a charge of 10^{-6} coul can be put on the sphere if a current of 1.0 amp exists in the wire for 10^{-6} sec.

► **Example 1** A copper penny has a mass of 3.1 gm. Being electrically neutral, it contains equal amounts of positive and negative electricity. What is the magnitude q of these charges? A copper atom has a positive nuclear charge of 4.6×10^{-18} coul and a negative electronic charge of equal magnitude.

The number N of copper atoms in a penny is found from the ratio

$$\frac{N}{N_0} = \frac{m}{M},$$

where N_0 is Avogadro's number, m the mass of the coin, and M the atomic weight of copper. This yields, solving for N,

$$N = \frac{(6.0 \times 10^{23} \text{ atoms/mole})(3.1 \text{ gm})}{64 \text{ gm/mole}} = 2.9 \times 10^{22} \text{ atoms.}$$

The charge q is

$$q = (4.6 \times 10^{-18} \text{ coul/atom})(2.9 \times 10^{22} \text{ atoms}) = 1.3 \times 10^5 \text{ coul.}$$

In a 100-watt, 110-volt light bulb the current is 0.91 amp. The student should verify that it would take 40 hr for a charge of this amount to pass through this bulb. ◄

* This scheme is the basis for the definition of the unit of charge called the *statcoulomb*. However, in this book we do not use this unit or the systems of units of which it is a part; see Appendix L, however.

Equation 26-1 can be written as an equality by inserting a constant of proportionality. Instead of writing this simply as, say, k, it is usually written in a more complex way as $1/4\pi\epsilon_0$ or

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}. \tag{26-3}$$

Certain equations that are derived from Eq. 26-3, but are used more often than it is, will be simpler in form if we do this.

In the mks system we can measure q_1, q_2, r, and F in Eq. 26-3 in ways that do not depend on Coulomb's law. Numbers with units can be assigned to them. There is no choice about the so-called *permittivity constant* ϵ_0; it must have that value which makes the right-hand side of Eq. 26-3 equal to the left-hand side. This (measured) value turns out to be *

$$\epsilon_0 = 8.85418 \times 10^{-12}\ \text{coul}^2/\text{nt-m}^2.$$

In this book the value 8.9×10^{-12} coul²/nt-m² will be accurate enough for all problems. For direct application of Coulomb's law or in any problem in which the quantity $1/4\pi\epsilon_0$ occurs we may use, with sufficient accuracy for this book,

$$1/4\pi\epsilon_0 = 9.0 \times 10^9\ \text{nt-m}^2/\text{coul}^2.$$

▶ **Example 2.** Let the total positive and the total negative charges in a copper penny be separated to a distance such that their force of attraction is 1.0 lb (= 4.5 nt). How far apart must they be?

We have (Eq. 26-3)

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}.$$

Putting $q_1q_2 = q^2$ (see Example 1) and solving for r yields

$$r = q\sqrt{\frac{1/4\pi\epsilon_0}{F}} = 1.3 \times 10^5\ \text{coul}\sqrt{\frac{9.0 \times 10^9\ \text{nt-m}^2/\text{coul}^2}{4.5\ \text{nt}}}$$

$$= 5.8 \times 10^9\ \text{meters} = 3.6 \times 10^6\ \text{miles}.$$

This suggests that it is not possible to upset the electrical neutrality of gross objects by any very large amount. What would be the force between the two charges if they were placed 1.0 meter apart? ◀

If more than two charges are present, Eq. 26-3 holds for every pair of charges. Let the charges be q_1, q_2, and q_3, etc.; we calculate the force exerted on any one (say q_1) by all the others from the vector equation

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} + \cdots, \tag{26-4}$$

where $\mathbf{F}_{12}$, for example, is the force exerted on q_1 by q_2.

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* For practical reasons this value is not actually measured by direct application of Eq. 26-3 but in an equivalent although more circuitous way that is described in Section 30-2.

▶ **Example 3.** Figure 26-3 shows three charges q_1, q_2, and q_3. What force acts on q_1? Assume that $q_1 = -1.0 \times 10^{-6}$ coul, $q_2 = +3.0 \times 10^{-6}$ coul, $q_3 = -2.0 \times 10^{-6}$ coul, $r_{12} = 15$ cm, $r_{13} = 10$ cm, and $\theta = 30°$.

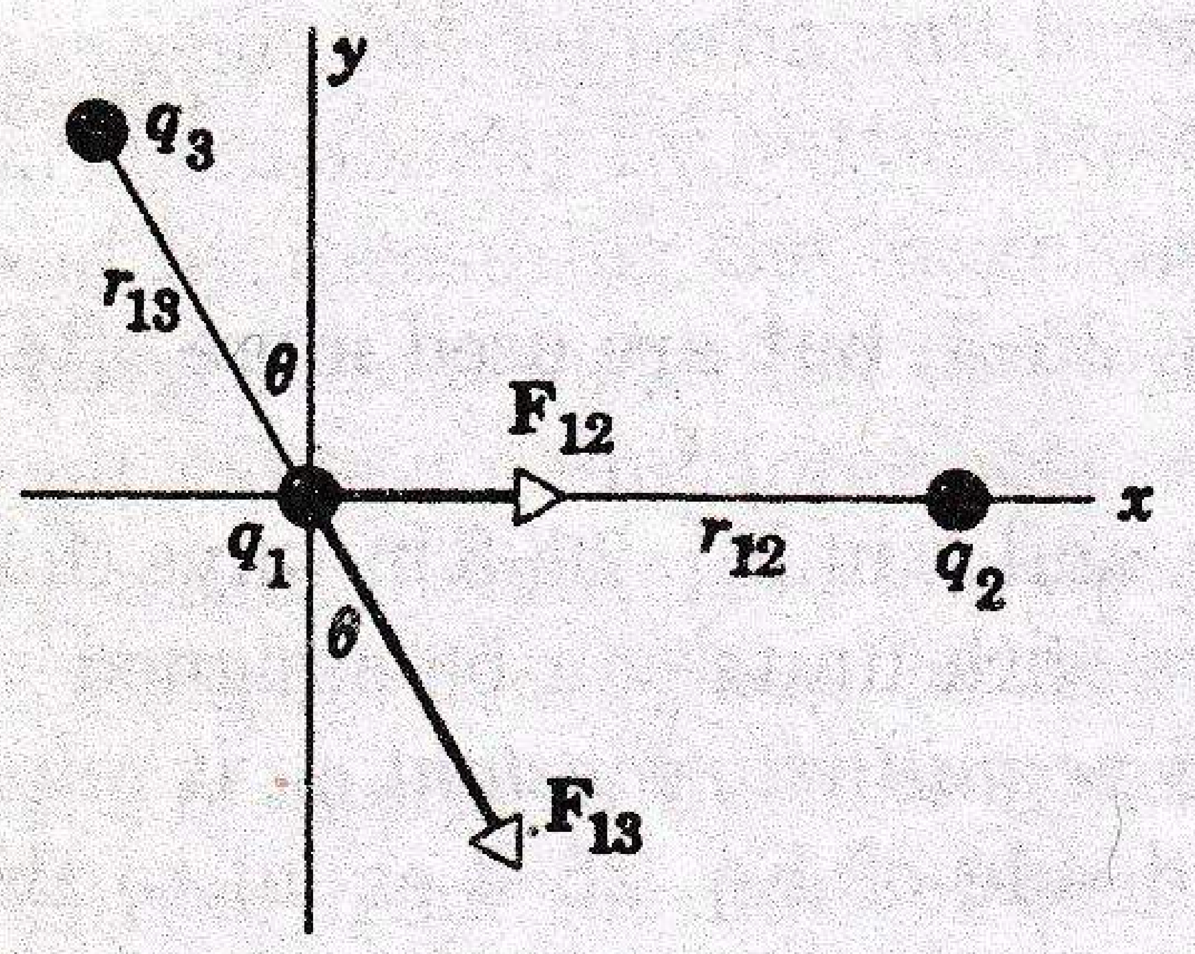

Fig. 26-3 Example 3. Showing the forces exerted on q_1 by q_2 and q_3.

From Eq. 26-3, ignoring the signs of the charges, since we are interested only in the magnitudes of the forces,

$$F_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}$$

$$= \frac{(9.0 \times 10^9 \text{ nt-m}^2/\text{coul}^2)(1.0 \times 10^{-6} \text{ coul})(3.0 \times 10^{-6} \text{ coul})}{(1.5 \times 10^{-1} \text{ meter})^2}$$

$$= 1.2 \text{ nt}$$

and

$$F_{13} = \frac{(9.0 \times 10^9 \text{ nt-m}^2/\text{coul}^2)(1.0 \times 10^{-6} \text{ coul})(2.0 \times 10^{-6} \text{ coul})}{(1.0 \times 10^{-1} \text{ meter})^2}$$

$$= 1.8 \text{ nt}.$$

The directions of $\mathbf{F}_{12}$ and $\mathbf{F}_{13}$ are as shown in the figure.

The components of the resultant force $\mathbf{F}_1$ acting on q_1 (see Eq. 26-4) are

$$F_{1x} = F_{12x} + F_{13x} = F_{12} + F_{13} \sin\theta$$

$$= 1.2 \text{ nt} + (1.8 \text{ nt})(\sin 30°) = 2.1 \text{ nt}$$

and

$$F_{1y} = F_{12y} + F_{13y} = 0 - F_{13}\cos\theta$$

$$= -(1.8 \text{ nt})(\cos 30°) = -1.6 \text{ nt}.$$

The student should find the magnitude of $\mathbf{F}_1$ and the angle it makes with the x-axis. ◀

26-5 Charge Is Quantized

In Franklin's day electric charge was thought of as a continuous fluid, an idea that was useful for many purposes. The atomic theory of matter, however, has shown that fluids themselves, such as water and air, are not continuous but are made up of atoms. Experiment shows that the "electric fluid" is not continuous either but that it is made up of integral multiples of a certain minimum electric charge. This fundamental charge, to which we give the symbol e, has the magnitude 1.60210×10^{-19} coul. Any physically existing charge q, no matter what its origin, can be written as ne where n is a positive or a negative integer.

When a physical property such as charge exists in discrete "packets" rather than in continuous amounts, the property is said to be *quantized*. Quantization is basic to modern physics. The existence of atoms and of particles like the electron and the proton indicates that *mass* is quantized also. Later the student will learn that several other properties prove to be quantized when suitably examined on the atomic scale; among them are energy and angular momentum.

The *quantum of charge* e is so small that the "graininess" of electricity does not show up in large-scale experiments, just as we do not realize that the air we breathe is made up of atoms. In an ordinary 110-volt, 100-watt light bulb, for example, 6×10^{18} elementary charges enter and leave the filament every second.

There exists today no theory that predicts the quantization of charge (or the quantization of mass, that is, the existence of fundamental particles such as protons, electrons, pions, etc.). Even assuming quantization, the classical theory of electromagnetism and Newtonian mechanics are incomplete in that they do not correctly describe the behavior of charge and matter on the atomic scale. The classical theory of electromagnetism, for example, describes correctly what happens when a bar magnet is thrust through a closed copper loop; it fails, however, if we wish to explain the magnetism of the bar in terms of the atoms that make it up. The more detailed theories of quantum physics are needed for this and similar problems.

26-6 Charge and Matter

Matter as we ordinarily experience it can be regarded as composed of three kinds of elementary particles, the proton, the neutron, and the electron. Table 26-1 shows their masses and charges. Note that the masses of the neutron and the proton are approximately equal but that the electron is lighter by a factor of about 1840.

Atoms are made up of a dense, positively charged *nucleus*, surrounded by a

Table 26-1

PROPERTIES OF THE PROTON, THE NEUTRON, AND THE ELECTRON

Particle	Symbol	Charge	Mass
Proton	p	$+e$	1.67252×10^{-27} kg
Neutron	n	0	1.67482×10^{-27} kg
Electron	e^-	$-e$	9.1091×10^{-31} kg

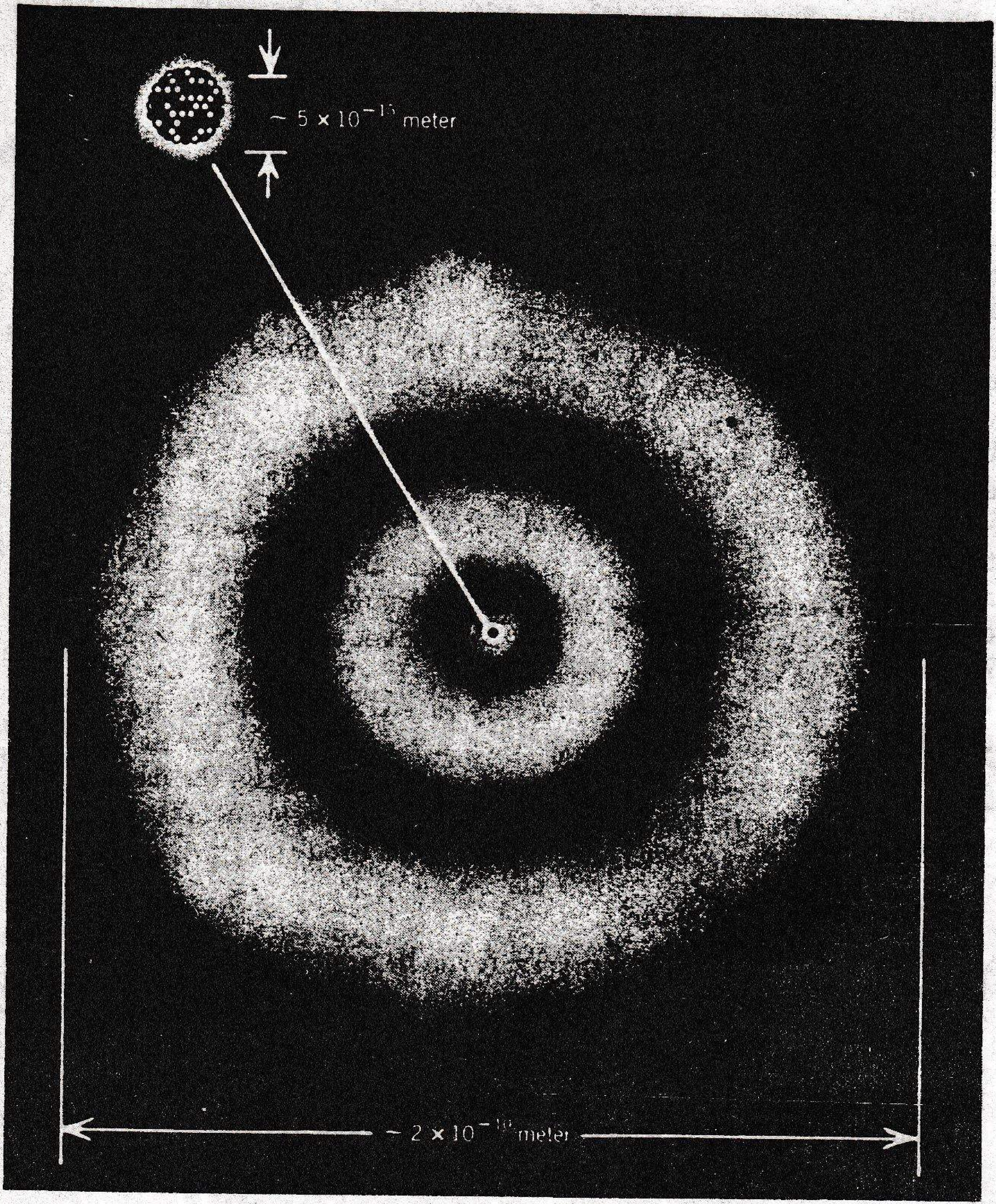

Fig. 26–4 An atom, suggesting the electron cloud and, above, an enlarged view of the nucleus.

cloud of electrons; see Fig. 26–4. The nucleus varies in radius from about 1×10^{-15} meter for hydrogen to about 7×10^{-15} meter for the heaviest atoms. The outer diameter of the electron cloud, that is, the diameter of the the atom, lies in the range 1–3 $\times 10^{-10}$ meter, about 10^5 times larger than the nuclear diameter.

▶ **Example 4.** The distance r between the electron and the proton in the hydrogen atom is about 5.3×10^{-11} meter. What are the magnitudes of (a) the electrical force and (b) the gravitational force between these two particles?

From Coulomb's law,

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

$$= \frac{(9.0 \times 10^9 \text{ nt-m}^2/\text{coul}^2)(1.6 \times 10^{-19} \text{ coul})^2}{(5.3 \times 10^{-11} \text{ meter})^2}$$

$$= 8.1 \times 10^{-8} \text{ nt.}$$

The gravitational force is given by Eq. 16-1, or

$$F_g = G\frac{m_1m_2}{r^2}$$

$$= \frac{(6.7 \times 10^{-11} \text{ nt-m}^2/\text{kg}^2)(9.1 \times 10^{-31} \text{ kg})(1.7 \times 10^{-27} \text{ kg})}{(5.3 \times 10^{-11} \text{ meter})^2}$$

$$= 3.7 \times 10^{-47} \text{ nt.}$$

Thus the electrical force is about 10^{39} times stronger than the gravitational force. ◀

The significance of Coulomb's law goes far beyond the description of the forces acting between charged balls or rods. This law, when incorporated into the structure of quantum physics, correctly describes (*a*) the electric forces that bind the electrons of an atom to its nucleus, (*b*) the forces that bind atoms together to form molecules, and (*c*) the forces that bind atoms or molecules together to form solids or liquids. Thus most of the forces of our daily experience that are not gravitational in nature are electrical. A force transmitted by a steel cable is basically an electrical force because, if we pass an imaginary plane through the cable at right angles to it, it is only the attractive electrical interatomic forces acting between atoms on opposite sides of the plane that keep the cable from parting. We ourselves are an assembly of nuclei and electrons bound together in a stable configuration by Coulomb forces.

In the atomic *nucleus* we encounter a new force which is neither gravitational nor electrical in nature. This strong attractive force, which binds together the protons and neutrons that make up the nucleus, is called simply *the nuclear force*. If this force were not present, the nucleus would fly apart at once because of the strong Coulomb repulsion force that acts between its protons. The nature of the nuclear force is only partially understood today and forms the central problem of present-day researches in nuclear physics.

▶ **Example 5.** What repulsive Coulomb force exists between two protons in a nucleus of iron? Assume a separation of 4.0×10^{-15} meter.

From Coulomb's law,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

$$= \frac{(9.0 \times 10^9 \text{ nt-m}^2/\text{coul}^2)(1.6 \times 10^{-19} \text{ coul})^2}{(4.0 \times 10^{-15} \text{ meter})^2}$$

$$= 14 \text{ nt.}$$

This enormous repulsive force must be more than compensated for by the strong attractive nuclear forces. This example, combined with Example 4, shows that nuclear binding forces are much stronger than atomic binding forces. Atomic binding forces are, in turn, much stronger than gravitational forces for the same particles separated by the same distance. ◀

The repulsive Coulomb forces acting between the protons in a nucleus make the nucleus less stable than it otherwise would be. The spontaneous emission of alpha particles from heavy nuclei and the phenomenon of nuclear fission are evidences of this instability.

The fact that heavy nuclei contain significantly more neutrons than protons is still another effect of the Coulomb forces. Consider Fig. 26–5 in which a particular atomic species is represented by a circle, the coordinates being Z, the number of protons in the nucleus (that is, the *atomic number*), and N, the number of neutrons in the nucleus (that is, the *neutron number*). Stable nuclei are represented by filled circles and radioactive nuclei, that is, nuclei that disintegrate spontaneously, emitting electrons or α-particles, by open circles. Note that all elements (iron, for example, for which $Z = 26$; see arrow) exist in a number of different forms, called *isotopes*.

Figure 26–5 shows that light nuclei, for which the Coulomb forces are relatively unimportant,* lie on or close to the line labeled "$N = Z$" and thus have about equal numbers of neutrons and protons. The heavier nuclei have a pronounced neutron excess, U^{238} having 92 protons and 238 − 92 or 146 neutrons.† In the absence of Coulomb forces we would assume, extending the $N = Z$ rule, that the most stable nucleus with 238 particles would have 119 protons and 119 neutrons. However, such a nucleus, if assembled, would fly apart at once because of Coulomb repulsion. Relative stability is found only if 27 of the protons are replaced by neutrons, thus diluting the total Coulomb repulsion. Even in U^{238} Coulomb repulsion is still very important because (*a*) this nucleus is radioactive and emits α-particles, and (*b*) it may break up into two large fragments (*fission*) if it absorbs a neutron; both processes result in separation of the nuclear charge and are Coulomb repulsion effects. Figure 26–5 shows that *all* nuclei with $Z > 83$ are unstable.

We have pointed out that matter, as we ordinarily experience it, is made up of electrons, neutrons, and protons. Nature exhibits much more variety than this, however. No fewer than 28 distinct elementary particles are now known, most of them having been discovered since 1940, either in the penetrating cosmic rays that come to us from beyond our atmosphere or in the reaction products of giant cyclotron-like devices.

Appendix E, which lists some properties of these particles, shows that, like the more familiar particles of Table 26–1, their charges are quantized, the quantum of charge again being e. An understanding of the nature of these particles and of their relationships to each other is perhaps the most significant research goal of modern physics.

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* Coulomb forces are important in relation to the strong nuclear attractive forces only for large nuclei, because Coulomb repulsion occurs between *every pair* of protons in the nucleus but the attractive nuclear force does not. In U^{238}, for example, every proton exerts a force of repulsion on each of the other 91 protons. However, each proton (and neutron) exerts a nuclear attraction on only a small number of other neutrons and protons that happen to be near it. As we proceed to larger nuclei, the amount of energy associated with the repulsive Coulomb forces increases much faster than that associated with the attractive nuclear forces.

† The superscript in this notation is the *mass number* A $(= N + Z)$. This is the total number of particles in the nucleus.

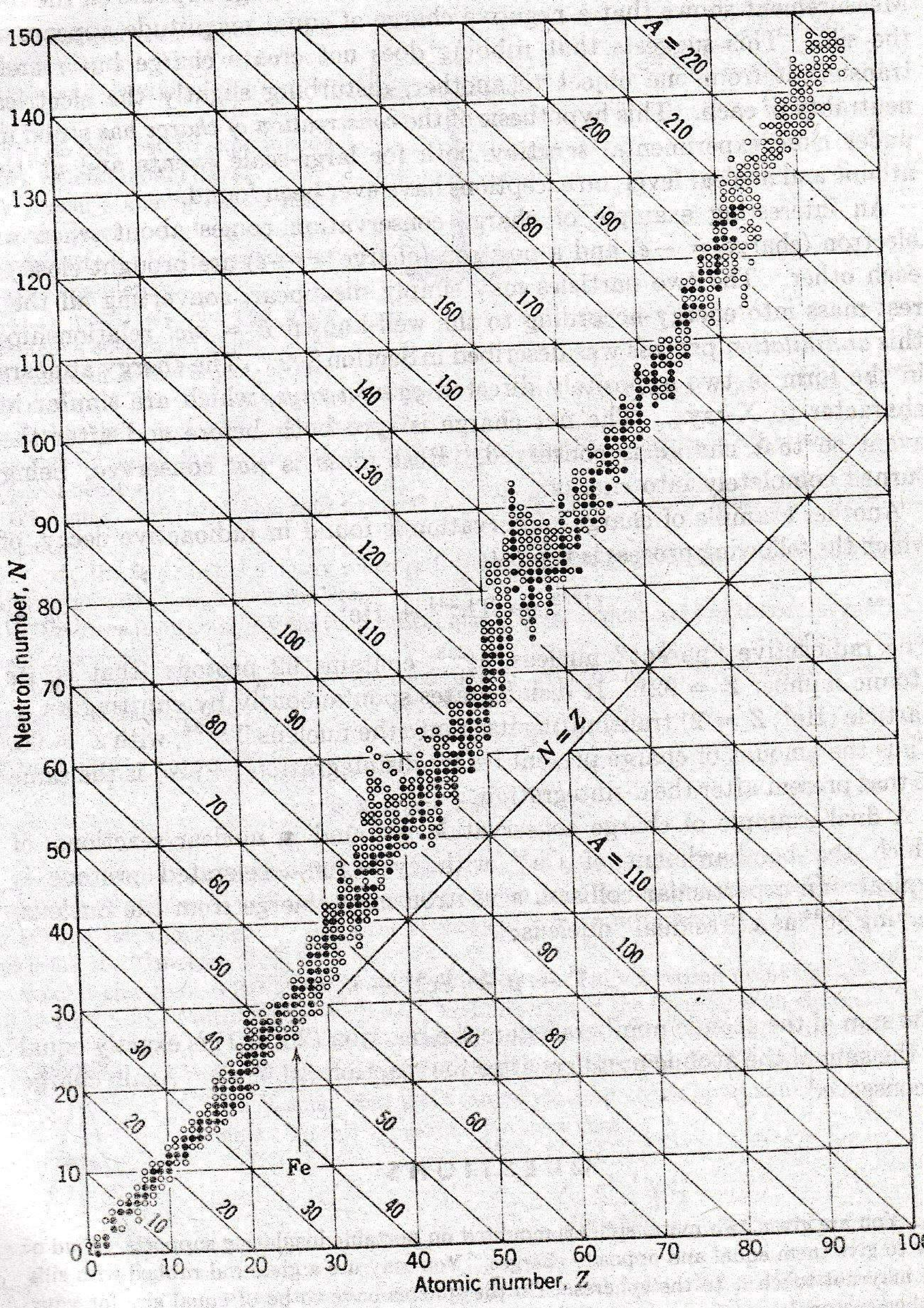

Fig. 26-5 The filled circles represent stable nuclei, the open ones radioactive nuclei. This figure does not show the more recently discovered radioactive nuclei, which appear throughout the chart, extending up to $Z = 103$.

26-7 Charge Is Conserved

When a glass rod is rubbed with silk, a positive charge appears on the rod. Measurement shows that a negative charge of equal magnitude appears on the silk. This suggests that rubbing does not create charge but merely transfers it from one object to another, disturbing slightly the electrical neutrality of each. This hypothesis of the *conservation of charge* has stood up under close experimental scrutiny both for large-scale events and at the atomic and nuclear level; no exceptions have ever been found.

An interesting example of charge conservation comes about when an electron (charge $= -e$) and a positron (charge $= +e$) are brought close to each other. The two particles may simply disappear, converting all their rest mass into energy according to the well-known $E = mc^2$ relationship; this *annihilation* process was described in Section 8–9. The energy appears in the form of two oppositely directed *gamma rays*, which are similar in character to X-rays. The net charge is zero both before and after the event so that charge is conserved. Rest mass is *not* conserved, being turned completely into energy.

Another example of charge conservation is found in radioactive decay, of which the following process is typical:

$$U^{238} \rightarrow Th^{234} + He^4. \tag{26-5}$$

The radioactive "parent" nucleus, U^{238}, contains 92 protons (that is, its atomic number $Z = 92$). It disintegrates spontaneously by emitting an α-particle (He^4; $Z = 2$) transmuting itself into the nucleus Th^{234}, with $Z = 90$. Thus the amount of charge present before disintegration ($+92e$) is the same as that present after the disintegration.

A final example of charge conservation is found in nuclear reactions, of which the bombardment of Ca^{44} with cyclotron-accelerated protons is typical. In a particular collision a neutron may emerge from the nucleus, leaving Sc^{44} as a "residual" nucleus:

$$Ca^{44} + p \rightarrow Sc^{44} + n.$$

The sum of the atomic numbers before the reaction $(20 + 1)$ is exactly equal to the sum of the atomic numbers after the reaction $(21 + 0)$. Again charge is conserved.

QUESTIONS

1. You are given two metal spheres mounted on portable insulating supports. Find a way to give them equal and opposite charges. You may use a glass rod rubbed with silk but may not touch it to the spheres. Do the spheres have to be of equal size for your method to work?

2. A charged rod attracts bits of dry cork dust which, after touching the rod, often jump violently away from it. Explain.

3. If a charged glass rod is held near one end of an insulated uncharged metal rod as in Fig. 26-6, electrons are drawn to one end, as shown. Why does the flow of electrons cease? There is an almost inexhaustible supply of them in the metal rod.

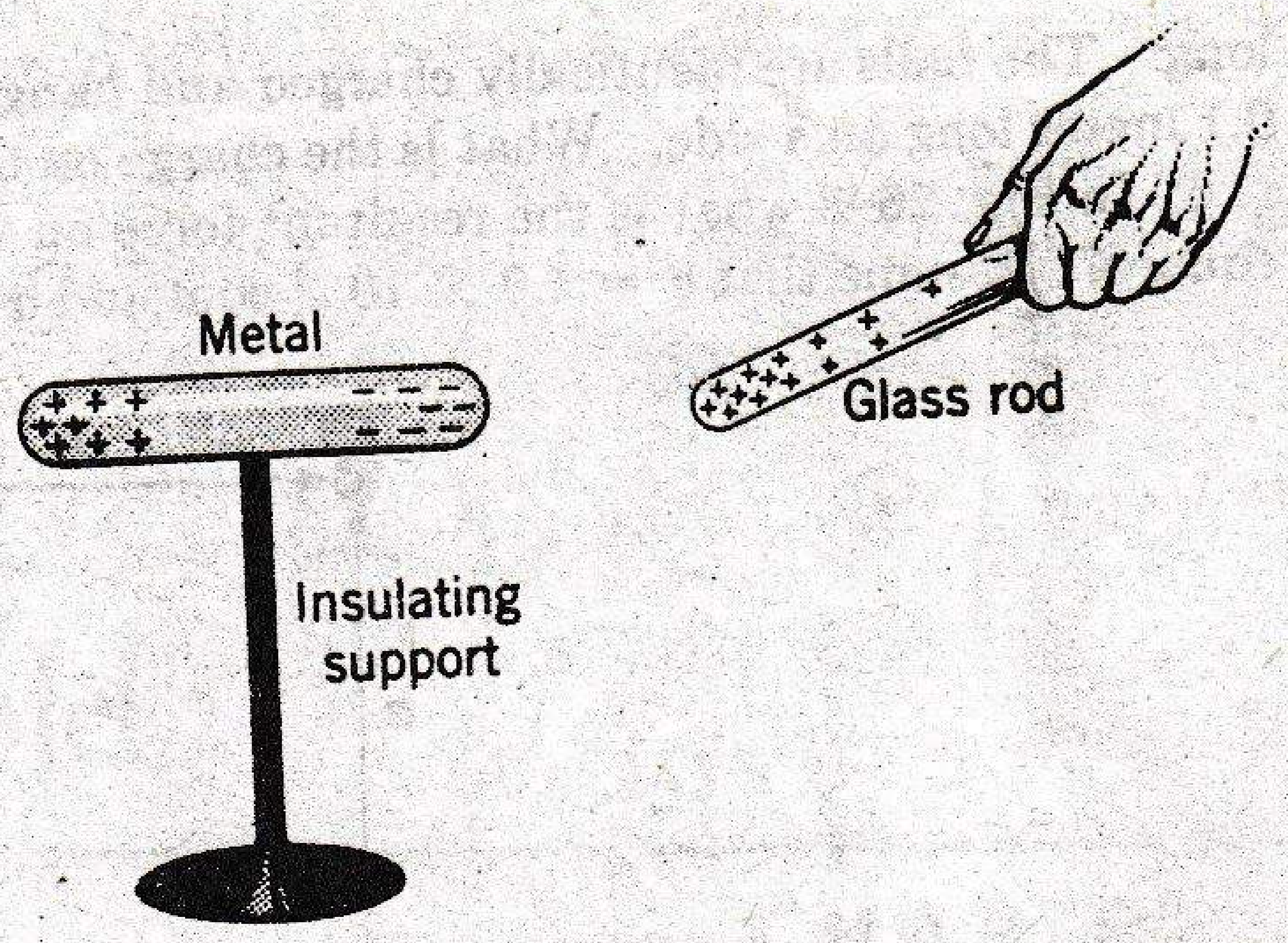

Fig. 26–6

4. In Fig. 26–6 does any net electrical force act on the metal rod? Explain.

5. An insulated rod is said to carry an electric charge. How could you verify this and determine the sign of the charge?

6. Why do electrostatic experiments not work well on humid days?

7. A person standing on an insulated stool touches a charged, insulated conductor. Is the conductor discharged completely?

8. (*a*) A positively charged glass rod attracts a suspended object. Can we conclude that the object is negatively charged? (*b*) A positively charged glass rod *repels* a suspended object. Can we conclude that the object is *positively* charged?

9. Is the Coulomb force that one charge exerts on another changed if other charges are brought nearby?

10. The quantum of charge is 1.60×10^{-19} coul. Is there a corresponding single quantum of mass?

11. Verify the fact that the decay schemes for the elementary particles in Appendix E are consistent with charge conservation.

12. What does it mean to say that a physical quantity is (*a*) quantized or (*b*) conserved? Give some examples.

13. A nucleus U^{238} splits into two identical parts. Are the nuclei so produced likely to be stable or radioactive?

PROBLEMS

1. Protons in the cosmic rays strike the earth's upper atmosphere at a rate, averaged over the earth's surface, of 0.15 protons/cm²-sec. What total current does the earth receive from beyond its atmosphere in the form of incident cosmic ray protons? The earth's radius is 6.4×10^6 meters.

2. A point charge of $+3.0 \times 10^{-6}$ coul is 12 cm distant from a second point charge of -1.5×10^{-6} coul. Calculate the magnitude and direction of the force on each charge.

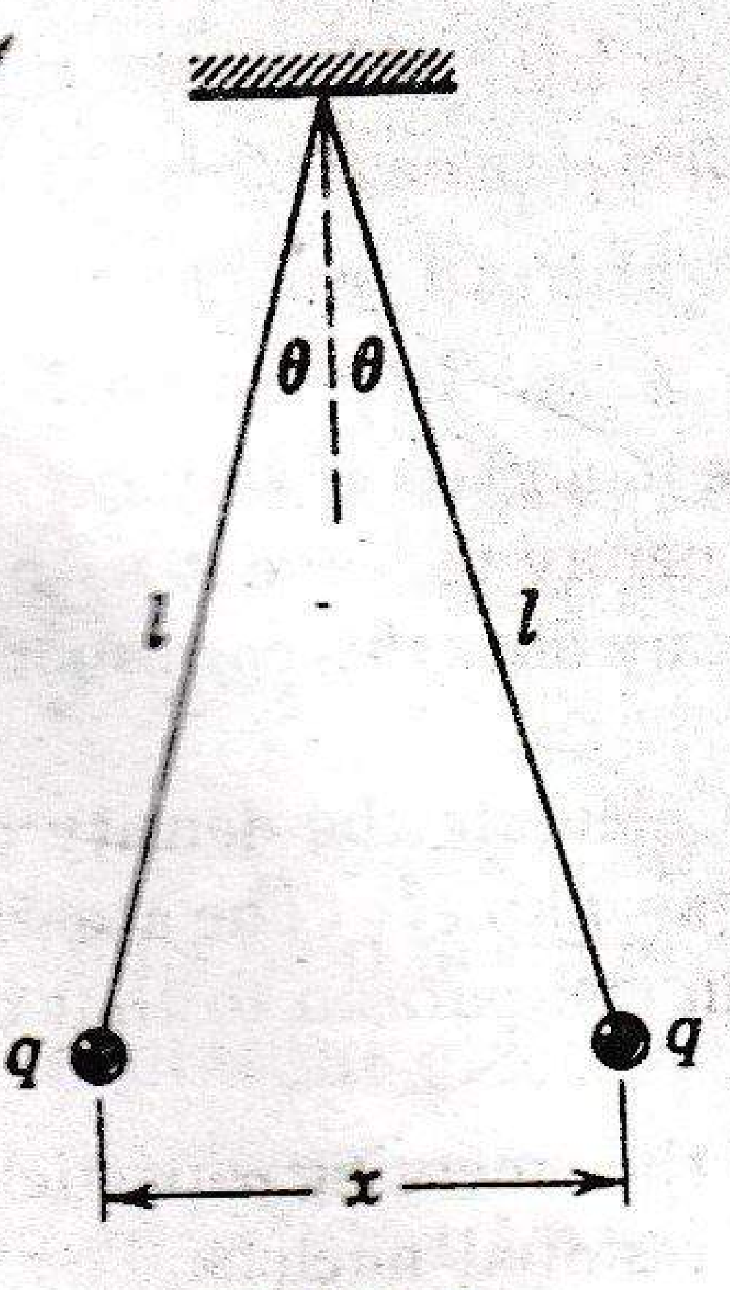

Fig. 26–7

3. Two similar balls of mass m are hung from silk threads of length l and carry similar charges q as in Fig. 26–7. Assume that θ is so small that $\tan\theta$ can be replaced by its approximate equal, $\sin\theta$. To this approximation show that

$$x = \left(\frac{q^2 l}{2\pi\epsilon_0 mg}\right)^{1/3}$$

where x is the separation between the balls. If $l = 120$ cm, $m = 10$ gm, and $x = 5.0$ cm, what is q?

4. Assume that each ball in Problem 3 is losing charge at the rate of 1.0×10^{-9} coul/sec. At what instantaneous relative speed ($= dx/dt$) do the balls approach each other initially?

5. Three small balls, each of mass 10 gm, are suspended separately from a common point by silk threads, each 1.0 meter

long. The balls are identically charged and hang at the corners of an equilateral triangle 0.1 meter long on a side. What is the charge on each ball?

6. In Fig. 26–8 what is the resultant force on the charge in the lower left corner of the square? Assume that $q = 1.0 \times 10^{-7}$ coul and $a = 5.0$ cm.

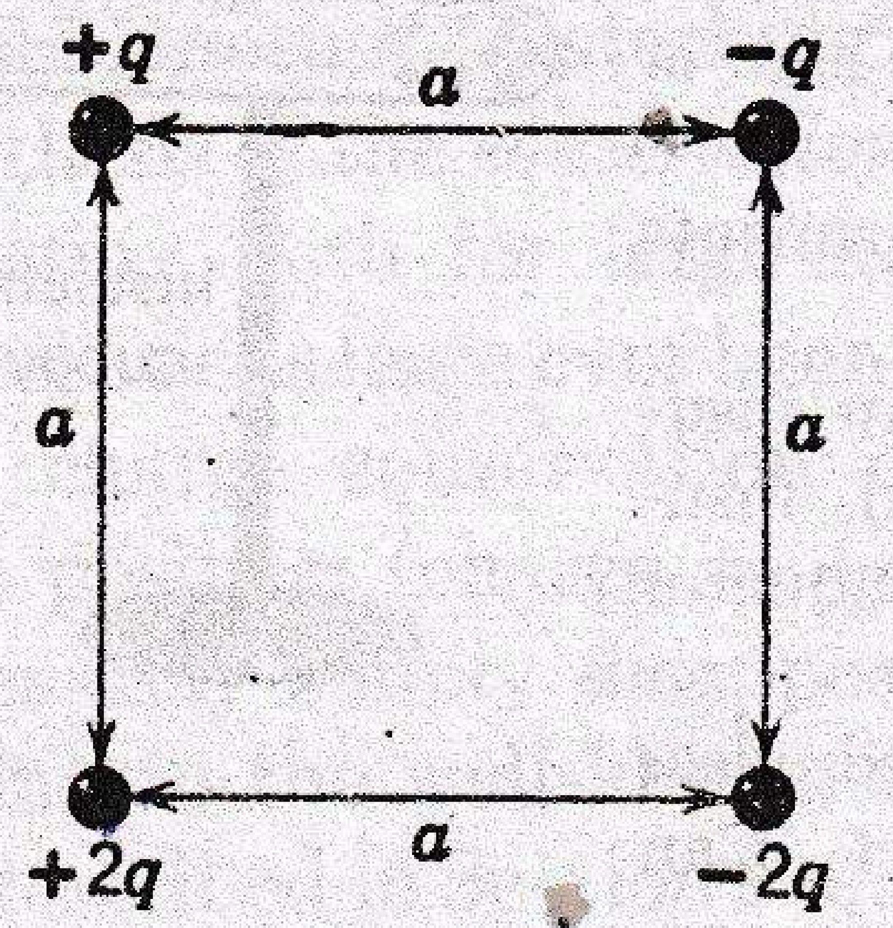

Fig. 26–8

7. A charge Q is placed at each of two opposite corners of a square. A charge q is placed at each of the other two corners. (*a*) If the resultant electrical force on Q is zero, how are Q and q related? (*b*) Could q be chosen to make the resultant force on *every* charge zero?

8. How far apart must two protons be if the electrical repulsive force acting on either one is equal to its weight?

9. (*a*) What equal positive charges would have to be placed on the earth and on the moon to neutralize their gravitational attraction? (*b*) Do you need to know the lunar distance to solve this problem? (*c*) How many pounds of hydrogen would be needed to provide the positive charge calculated in *a*?

10. A certain charge Q is to be divided into two parts, q and $Q - q$. What is the relationship of Q to q if the two parts, placed a given distance apart, are to have a maximum Coulomb repulsion?

11. Each of two small spheres is charged positively, the combined charge being 5.0×10^{-5} coul. If each sphere is repelled from the other by a force of 1.0 newton when the spheres are 2.0 meters apart, how is the total charge distributed between the spheres?

12. Two equal positive point charges are separated by a distance $2a$. A point test charge is located in a plane which is normal to the line joining these charges and midway between them. (*a*) Calculate the radius r of the circle of symmetry in this plane for which the force on the test charge has a maximum value. (*b*) What is the direction of this force, assuming a positive test charge.

13. A cube of edge a carries a point charge q at each corner. (*a*) Show that the magnitude of the resultant force on any one of the charges is

$$F = \frac{0.261q^2}{\epsilon_0 a^2}.$$

(*b*) What is the direction of **F** relative to the cube edges?

14. Estimate roughly the number of coulombs of positive charge in a glass of water.

15. (*a*) How many electrons would have to be removed from a penny to leave it with a charge of $+10^{-7}$ coul? (*b*) What fraction of the electrons in the penny does this correspond to?

16. The radius of a copper nucleus is about 1.9×10^{-13} cm. Calculate the density of the material that makes up the nucleus. Does your answer seem reasonable? (The atomic weight of copper is 64 gm/mole; ignore the mass of the electrons in comparison to that of the nucleus.)

17. In the radioactive decay of U^{238} (see Eq. 26–5) the center of the emerging α-particle is, at a certain instant, 9×10^{-15} meter from the center of the residual nucleus Th^{234} At this instant (*a*) what is the force on the α-particle and (*b*) what is its acceleration?

The Electric Field

CHAPTER 27

27–1 The Electric Field

With every point in space near the earth we can associate a *gravitational field strength* vector **g** (see Eq. 16–12). This is the gravitational acceleration that a test body, placed at that point and released, would experience. If m is the mass of the body and **F** the gravitational force acting on it, **g** is given by

$$\mathbf{g} = \mathbf{F}/m. \tag{27–1}$$

This is an example of a *vector field*. For points near the surface of the earth the field is often taken as *uniform;* that is, **g** is the same for all points.

The flow of water in a river provides another example of a vector field, called a *flow field* (see Section 18–7). Every point in the water has associated with it a vector quantity, the velocity **v** with which the water flows past the point. If **g** and **v** do not change with time, the corresponding fields are described as *stationary*. In the case of the river note that even though the water is moving the vector **v** at any point does not change with time for steady-flow conditions.

The space surrounding a charged rod is affected by the presence of the rod, and we speak of an *electric field* in this space. In the same way we speak of a *magnetic field* in the space around a bar magnet. In the classical theory of electromagnetism the electric and magnetic fields are central concepts.

Before Faraday's time, the force acting between charged particles was thought of as a direct and instantaneous interaction between the two particles. This *action-at-a-distance* view was also held for magnetic and for gravitational forces. Today we prefer to think in terms of electric fields as follows:

1. Charge q_1 in Fig. 27–1 sets up an electric field in the space around itself. This field is suggested by the shading in the figure; later we shall show how to represent electric fields more concretely.

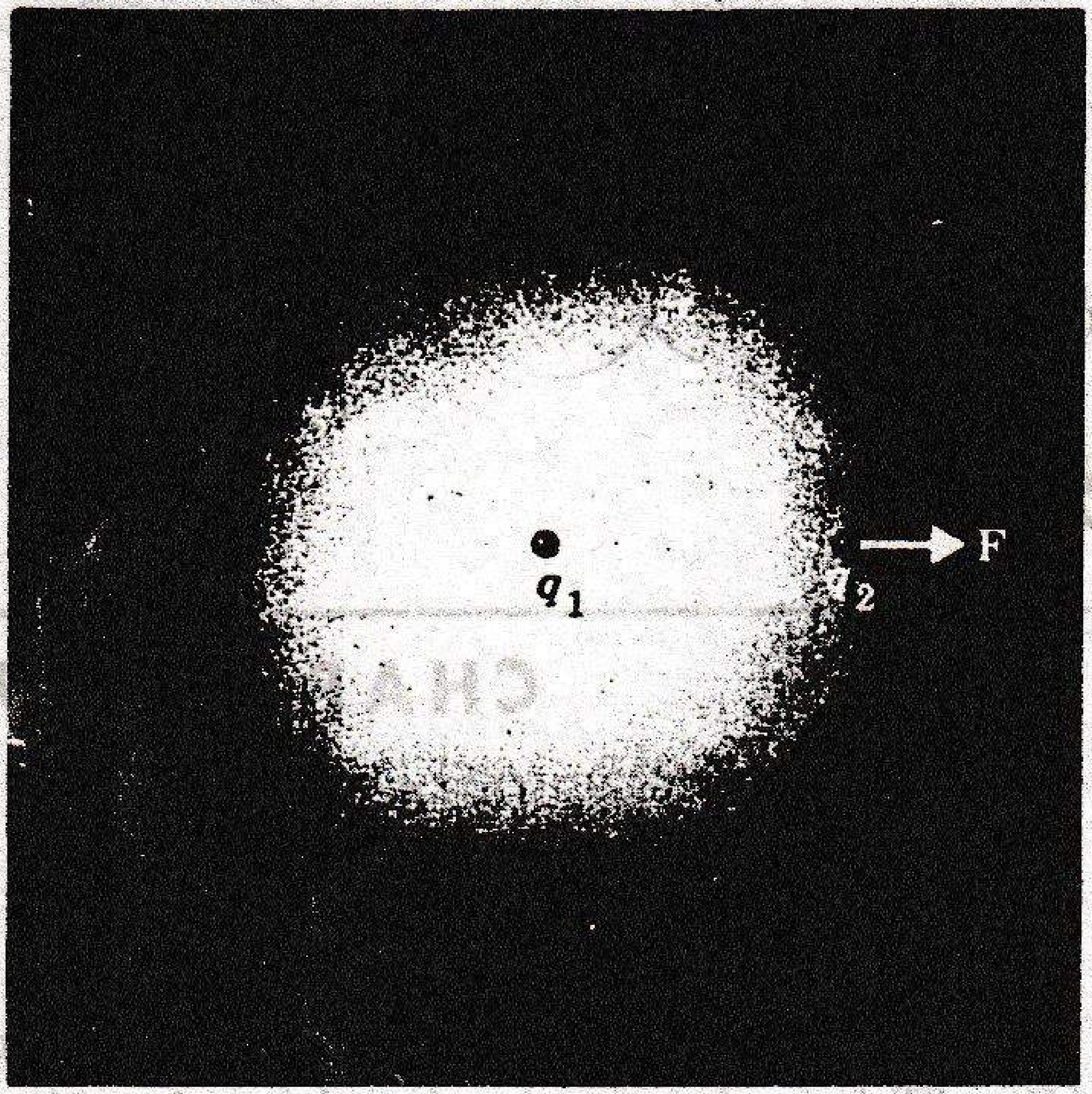

Fig. 27–1 Charge q_1 sets up a field that exerts a force **F** on charge q_2.

2. The field acts on charge q_2; this shows up in the force **F** that q_2 experiences.

The field plays an intermediary role in the forces between charges. There are two separate problems: (*a*) calculating the fields that are set up by given distributions of charge and (*b*) calculating the forces that given fields will exert on charges placed in them. We think in terms of

$$\text{charge} \rightleftharpoons \text{field}$$

and not, as in the action-at-a-distance point of view, in terms of

$$\text{charge} \rightleftharpoons \text{charge}.$$

In Fig. 27–1 we can also imagine that q_2 sets up a field and that this field acts on q_1, producing a force $-\mathbf{F}$ on it. The situation is completely symmetrical, each charge being immersed in a field associated with the other charge.

If the only problem in electromagnetism was that of the forces between stationary charges, the field and the action-at-a-distance points of view would be perfectly equivalent. Suppose, however, that q_1 in Fig. 27–1 suddenly accelerates to the right. How quickly does the charge q_2 learn that q_1 has moved and that the force which it (q_2) experiences must increase? Electromagnetic theory predicts that q_2 learns about q_1's motion by a *field disturbance* that emanates from q_1, traveling with the speed of light. The action-at-a-distance point of view requires that information about q_1's acceleration be communicated *instantaneously* to q_2; this is not in accord with experiment. Accelerating electrons in the antenna of a radio transmitter influence electrons in a distant receiving antenna only after a time l/c where l is the separation of the antennas and c is the speed of light.

27-2 The Electric Field Strength E

To define the electric field operationally, we place a small test body carrying a test charge q_0 (assumed positive for convenience) at the point in space that is to be examined, and we measure the electrical force **F** (if any) that acts on this body. The *electric field strength* **E** at the point is defined as *

$$\mathbf{E} = \mathbf{F}/q_0. \qquad (27\text{-}2)$$

Here **E** is a vector because **F** is one, q_0 being a scalar. The direction of **E** is the direction of **F**, that is, it is the direction in which a resting positive charge placed at the point would tend to move.

The definition of gravitational field strength **g** is much like that of electric field strength, except that the mass of the test body rather than its charge is the property of interest. Although the units of **g** are usually written as meters/sec², they could also be written as nt/kg (Eq. 27-1); those for **E** are nt/coul (Eq. 27-2). Thus both **g** and **E** are expressed as a force divided by a property (mass or charge) of the test body.

▶ **Example 1.** What is the magnitude of the electric field strength **E** such that an electron, placed in the field, would experience an electrical force equal to its weight?

From Eq. 27-2, replacing q_0 by e and F by mg, where m is the electron mass, we have

$$E = \frac{F}{q_0} = \frac{mg}{e}$$

$$= \frac{(9.1 \times 10^{-31}\ \text{kg})(9.8\ \text{meters/sec}^2)}{1.6 \times 10^{-19}\ \text{coul}}$$

$$= 5.6 \times 10^{-11}\ \text{nt/coul}.$$

This is a very weak electric field. Which way will **E** have to point if the electric force is to cancel the gravitational force? ◀

In applying Eq. 27-2 we must use a test charge as small as possible. A large test charge might disturb the primary charges that are responsible for the field, thus changing the very quantity that we are trying to measure. Equation 27-2 should, strictly, be replaced by

$$\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0}. \qquad (27\text{-}3)$$

This equation instructs us to use a smaller and smaller test charge q_0, evaluating the ratio $\mathbf{F}/q_0$ at every step. The electric field **E** is then the limit of this ratio as the size of the test charge approaches zero.

27-3 Lines of Force

The concept of the electric field as a vector was not appreciated by Michael Faraday, who always thought in terms of *lines of force*. The lines of force still form a convenient way of visualizing electric-field patterns. We shall use them for this purpose but we shall not employ them quantitatively.

---

* This definition of **E**, though conceptually sound and quite appropriate to our present purpose, is rarely carried out in practice because of experimental difficulties. **E** is normally found by calculation from more readily measurable quantities such as the electric potential; see Section 29-7.

The relationship between the (imaginary) lines of force and the electric field strength vector is this:

1. The tangent to a line of force at any point gives the *direction* of **E** at that point.
2. The lines of force are drawn so that the number of lines per unit cross-sectional area is proportional to the *magnitude* of **E**. Where the lines are close together E is large and where they are far apart E is small.

It is not obvious that it is possible to draw a continuous set of lines to meet these requirements. Indeed, it turns out that if Coulomb's law were not true it would *not* be possible to do so; see Problem 4.

Figure 27–2 shows the lines of force for a uniform sheet of positive charge. We assume that the sheet is infinitely large, which, for a sheet of finite dimensions, is equivalent to considering only those points whose distance from the sheet is small compared to the distance to the nearest edge of the sheet. A positive test charge, released in front of such a sheet, would move away from the sheet along a perpendicular line. Thus the electric field strength vector at any point near the sheet must be at right angles to the sheet. The lines of force are uniformly spaced, which means that **E** has the same magnitude for all points near the sheet.

Figure 27–3 shows the lines of force for a negatively charged sphere. From symmetry, the lines must lie along radii. They point inward because a free positive charge would be accelerated in this direction. The electric field E is not constant but decreases with increasing distance from the charge. This is evident in the lines of force, which are farther apart at greater distances. From symmetry, E is the same for all points that lie a given distance from the center of the charge.

► **Example 2.** In Fig. 27–3 how does E vary with the distance r from the center of the charged sphere?

Suppose that N lines originate on the sphere. Draw an imaginary concentric sphere of radius r; the number of lines per unit cross-sectional area at every point on the sphere is $N/4\pi r^2$. Since E is proportional to this, we can write that

$$E \propto 1/r^2.$$

We derive an exact relationship in Section 27–4. How should E vary with distance from an infinitely long uniform cylinder of charge? ◄

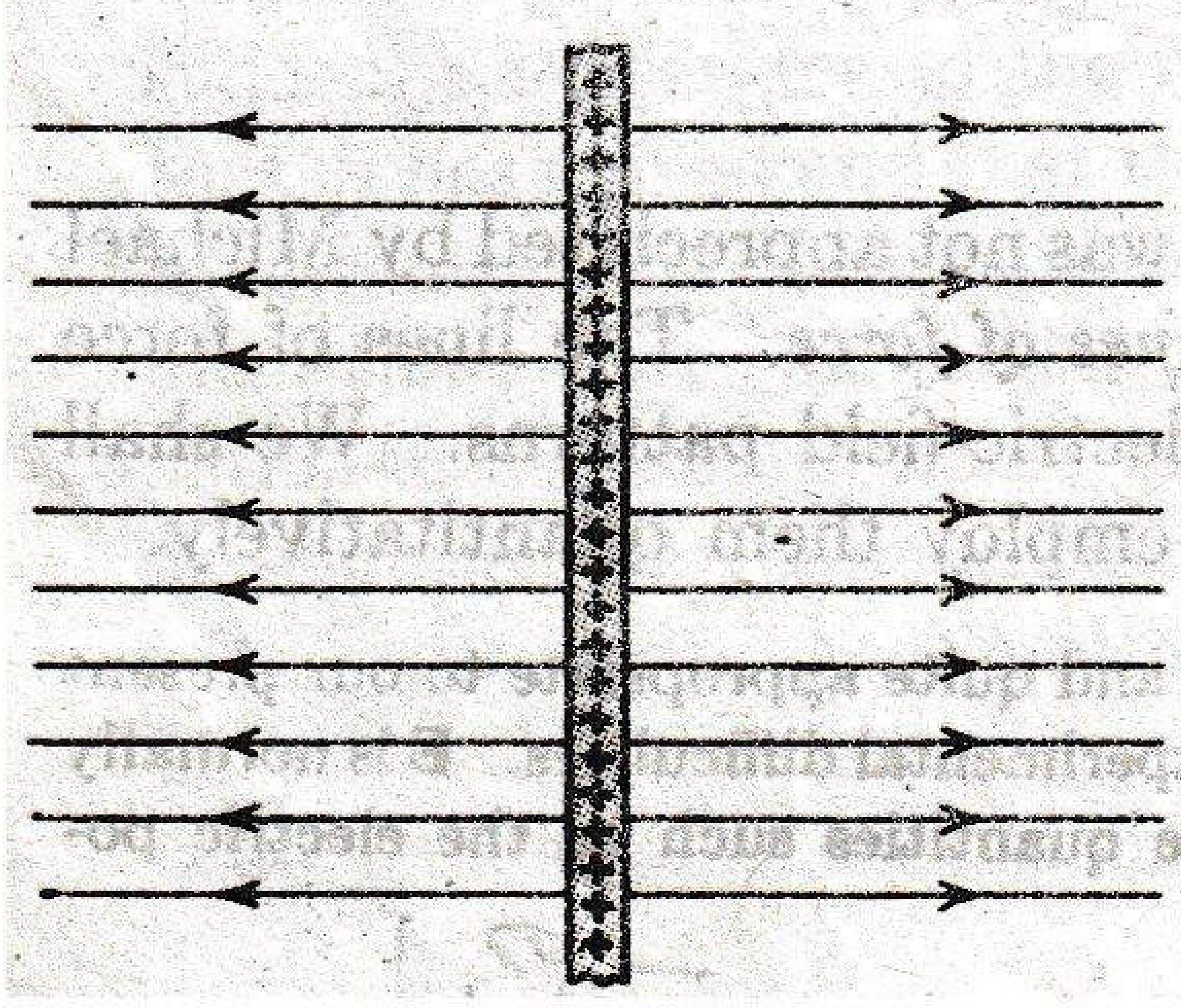

Fig. 27–2 Lines of force for a section of an infinitely large sheet of positive charge.

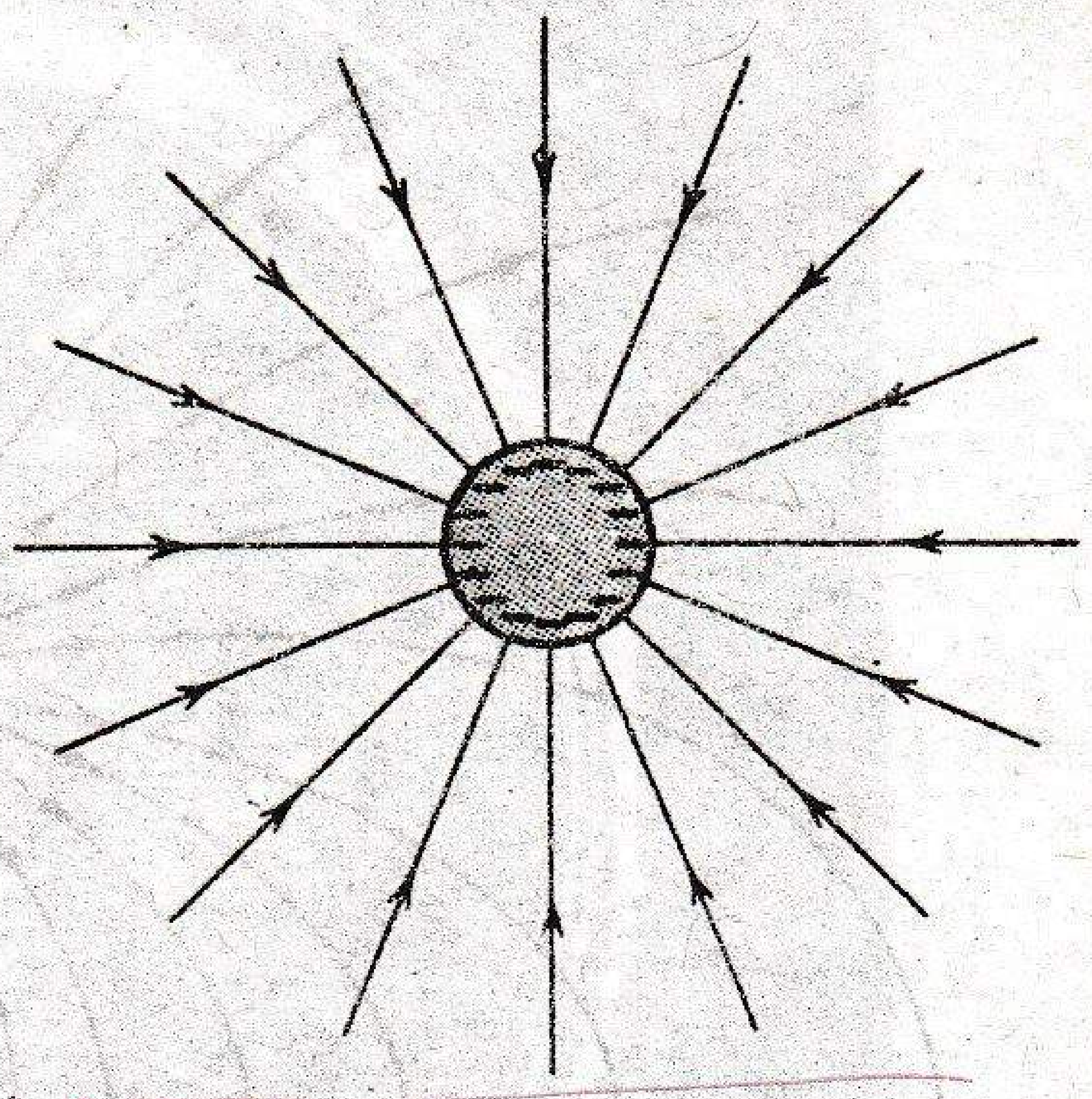

Fig. 27–3 Lines of force for a negatively charged sphere.

Figures 27–4 and 27–5 show the lines of force for two equal like charges and for two equal unlike charges, respectively. Michael Faraday, as we have said, used lines of force a great deal in his thinking. They were more real for him than they are for most scientists and engineers today. It is possible

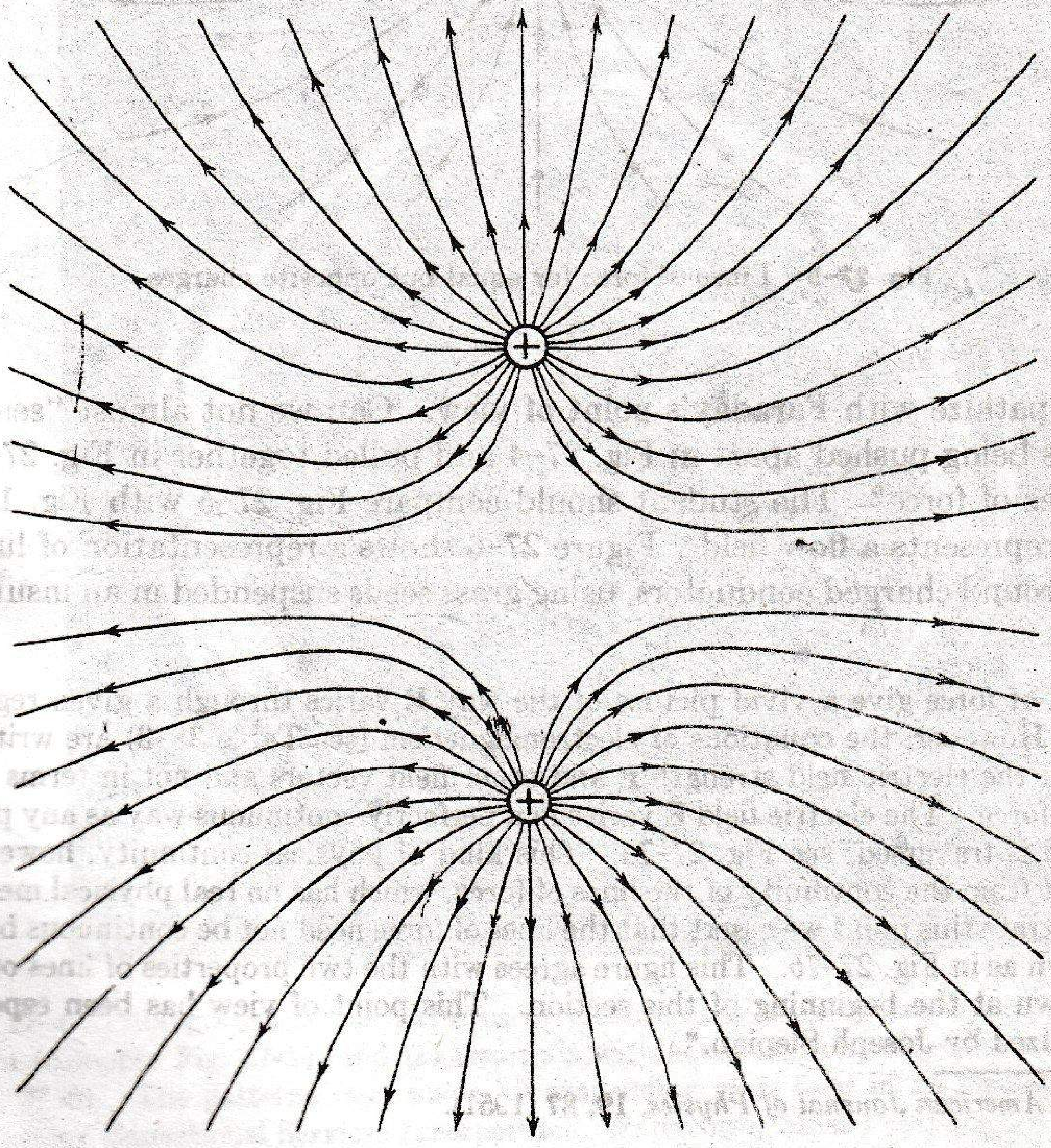

Fig. 27–4 Lines of force for two equal positive charges.

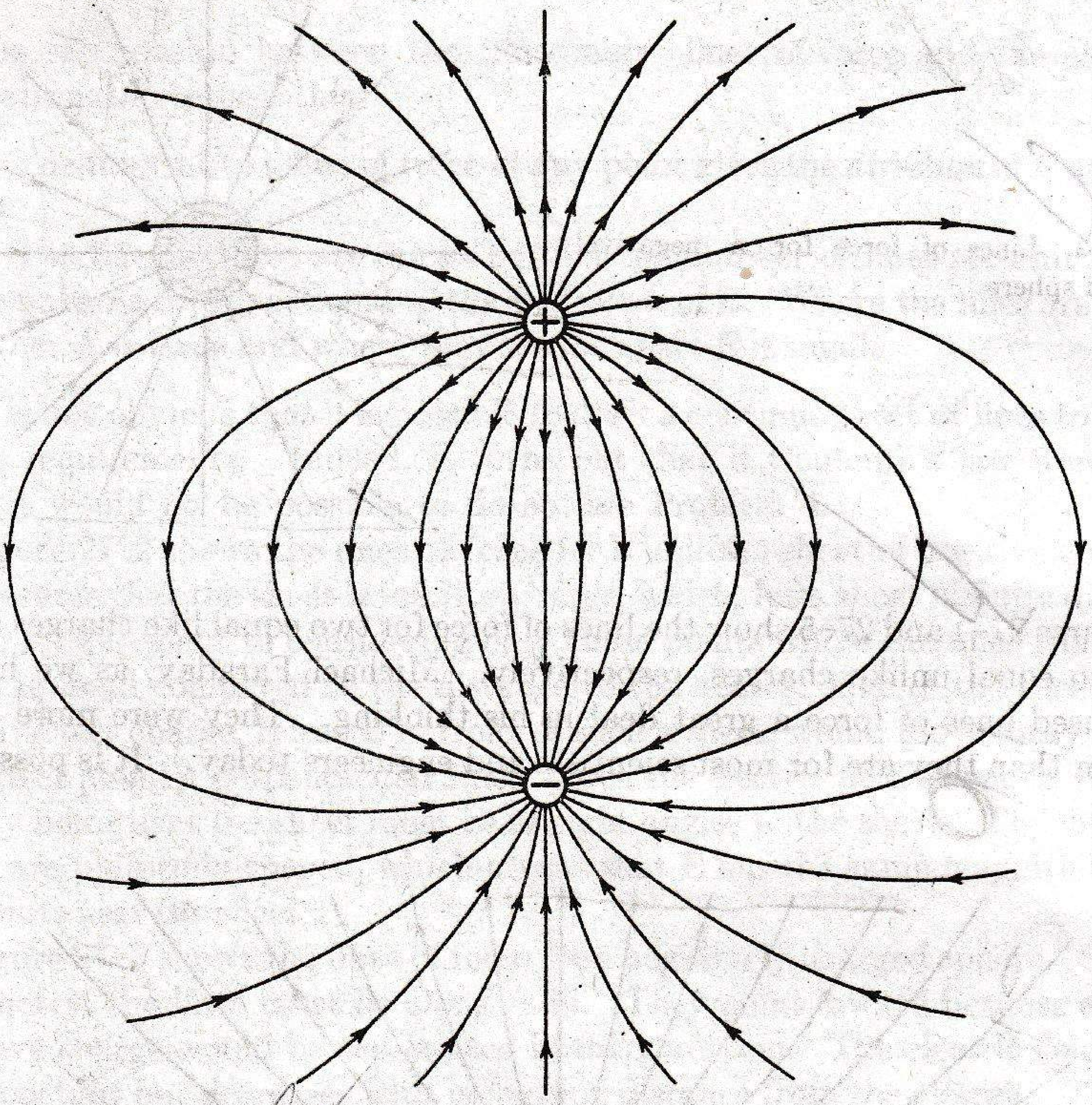

Fig. 27–5 Lines of force for equal but opposite charges.

to sympathize with Faraday's point of view. Can we not almost "see" the charges being pushed apart in Fig. 27–4 and pulled together in Fig. 27–5 by the lines of force? The student should compare Fig. 27–5 with Fig. 18–15, which represents a flow field. Figure 27–6 shows a representation of lines of force around charged conductors, using grass seeds suspended in an insulating liquid.

Lines of force give a vivid picture of the way **E** varies through a given region of space. However, the equations of electromagnetism (see Table 38–3) are written in terms of the electric field strength **E** and other field vectors and not in terms of the lines of force. The electric field **E** varies in a perfectly continuous way as any path in the field is traversed; see Fig. 27–7*a*. This kind of physical continuity, however, is different from the continuity of the lines of force, which has no real physical meaning. To illustrate this point we assert that the lines of force need not be continuous but can be drawn as in Fig. 27–7*b*. This figure agrees with the two properties of lines of force laid down at the beginning of this section. This point of view has been especially emphasized by Joseph Slepian.*

* See *American Journal of Physics*, **19**, 87 (1951).

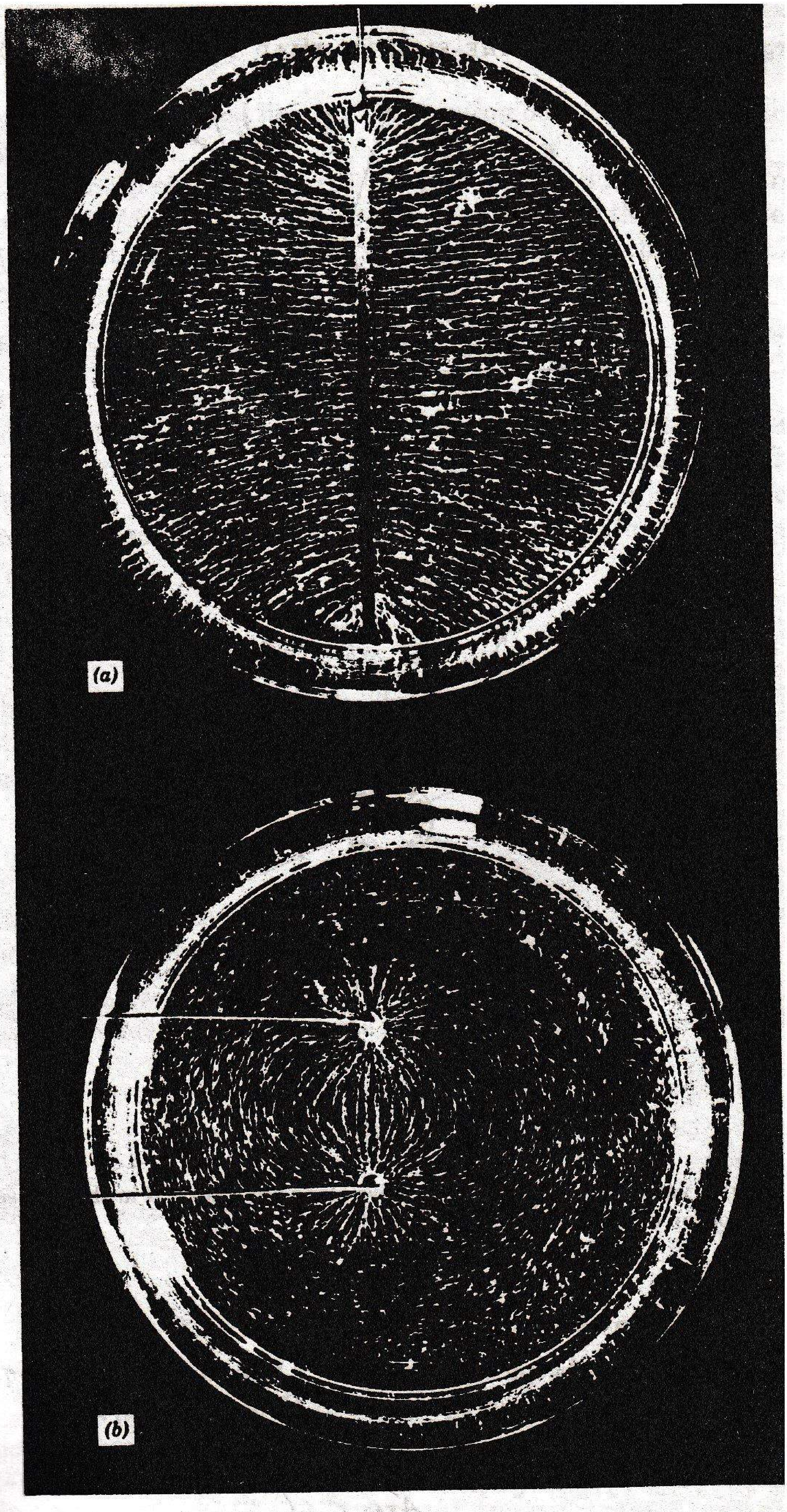

Fig. 27–6 Photographs of the patterns of electric lines of force around (*a*) a charged plate (compare Fig. 27–2), and (*b*) two rods with equal and opposite charges (compare Fig. 27–5). The patterns were made by suspending grass seed in an insulating liquid. (Courtesy Educational Services Incorporated, Watertown, Mass.)

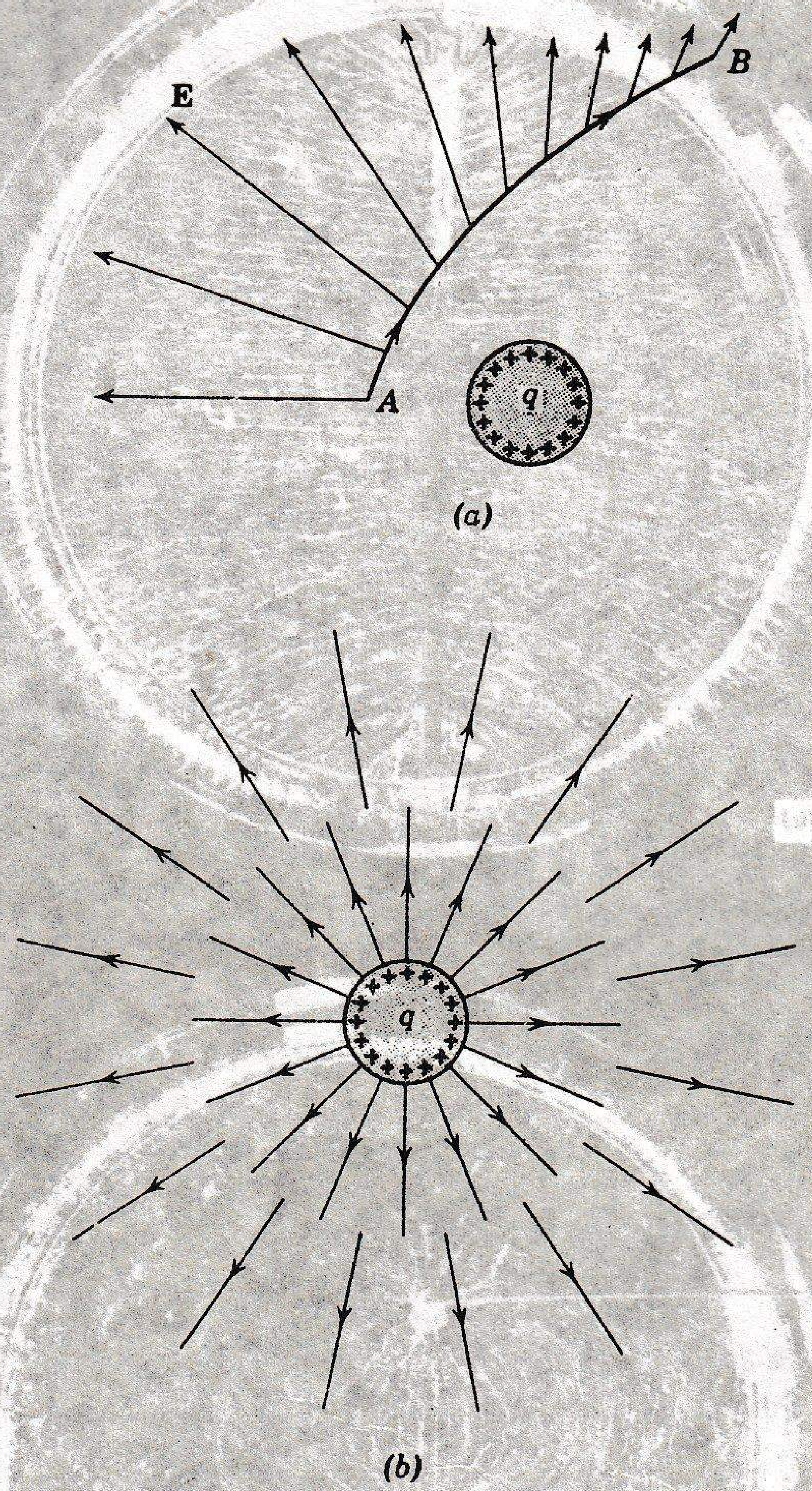

Fig. 27-7 (*a*) **E** varies continuously as we move along any path AB in the field set up by q. (*b*) Lines of force need not be continuous.

27-4 Calculation of E

Let a test charge q_0 be placed a distance r from a point charge q. The magnitude of the force acting on q_0 is given by Coulomb's law, or

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}.$$

The electric field strength at the site of the test charge is given by Eq. 27-2, or

$$E = \frac{F}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}. \tag{27-4}$$

The direction of **E** is on a radial line from q, pointing outward if q is positive and inward if q is negative.

To find **E** for a group of point charges: (*a*) Calculate $\mathbf{E}_n$ due to each charge at the given point *as if it were the only charge present.* (*b*) Add these separately calculated fields vectorially to find the resultant field **E** at the point. In equation form,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \cdots = \Sigma\mathbf{E}_n \qquad n = 1, 2, 3, \ldots. \tag{27-5}$$

The sum is a vector sum, taken over all the charges.

If the charge distribution is a continuous one, the field it sets up at any point P can be computed by dividing the charge into infinitesimal elements dq. The field $d\mathbf{E}$ due to each element at the point in question is then calculated, treating the elements as point charges. The magnitude of dE (see Eq. 27-4) is given by

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}, \tag{27-6}$$

where r is the distance from the charge element dq to the point P. The resultant field at P is then found by adding (that is, integrating) the field contributions due to all the charge elements, or,

$$\mathbf{E} = \int d\mathbf{E}. \tag{27-7}$$

The integration, like the sum in Eq. 27-5, is a vector operation; in Example 5 we will see how such an integral is handled in a simple case.

▶ **Example 3.** *An electric dipole.* Figure 27-8 shows a positive and a negative charge of equal magnitude q placed a distance $2a$ apart, a configuration called an electric dipole. The pattern of lines of force is that of Fig. 27-5, which also shows an electric dipole. What is the field **E** due to these charges at point P, a distance r along the perpendicular bisector of the line joining the charges? Assume $r \gg a$.

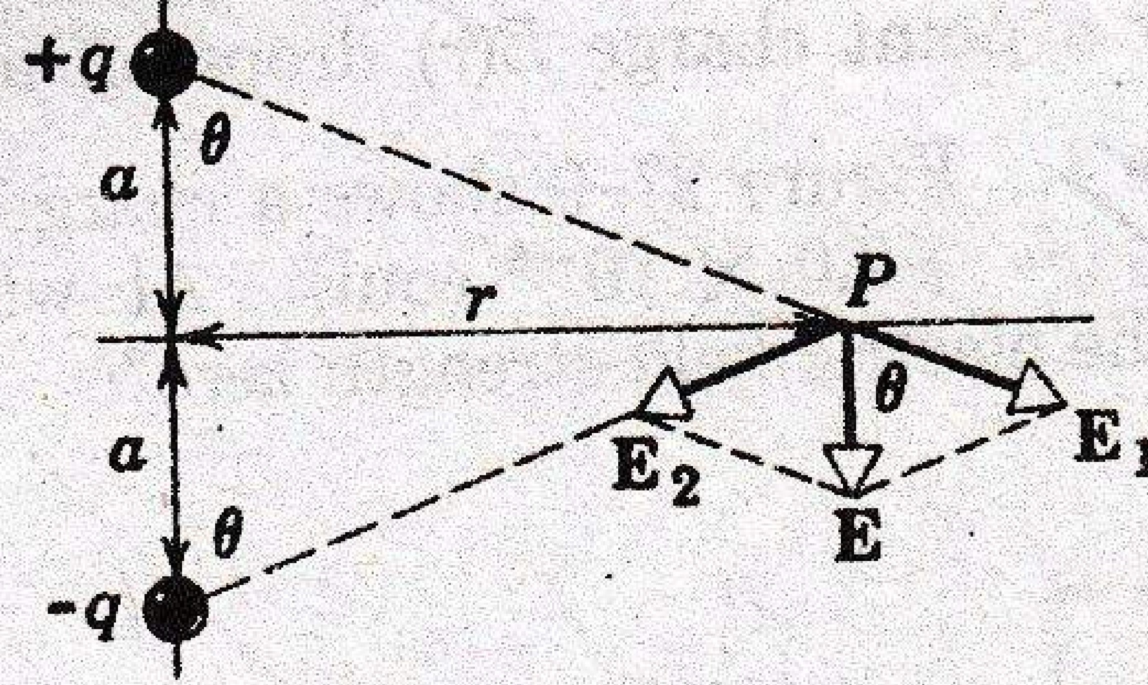

Fig. 27-8 Example 3.

Equation 27-5 gives the vector equation

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2,$$

where, from Eq. 27-4,*

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{a^2 + r^2}.$$

* Note that the r's in Eq. 27-4 and in this equation have different meanings.

The vector sum of $\mathbf{E}_1$ and $\mathbf{E}_2$ points vertically downward and has the magnitude

$$E = 2E_1 \cos\theta.$$

From the figure we see that

$$\cos\theta = \frac{a}{\sqrt{a^2 + r^2}}.$$

Substituting the expressions for E_1 and for $\cos\theta$ into that for E yields

$$E = \frac{2}{4\pi\epsilon_0} \frac{q}{(a^2 + r^2)} \frac{a}{\sqrt{a^2 + r^2}} = \frac{1}{4\pi\epsilon_0} \frac{2aq}{(a^2 + r^2)^{3/2}}.$$

If $r \gg a$, we can neglect a in the denominator; this equation then reduces to

$$E \cong \frac{1}{4\pi\epsilon_0} \frac{(2a)(q)}{r^3}. \tag{27-8a}$$

The essential properties of the charge distribution in Fig. 27–8, the magnitude of the charge q and the separation $2a$ between the charges, enter Eq. 27–8a only as a product. This means that, if we measure $\mathbf{E}$ at various distances from the electric dipole (assuming $r \gg a$), we can never deduce q and $2a$ separately but only the product $2aq$; if q were doubled and a simultaneously cut in half, the electric field *at large distances from the dipole* would not change.

The product $2aq$ is called the *electric dipole moment p*. Thus we can rewrite this equation for E, *for distant points along the perpendicular bisector*, as

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}. \tag{27-8b}$$

The result for distant points *along the dipole axis* (see Problem 10) and the general result for any distant point (see Problem 23) also contain the quantities $2a$ and q only as the product $2aq$ ($= p$). The variation of E with r in the general result for distant points is also as $1/r^3$, as in Eq. 27–8b.

The dipole of Fig. 27–8 is two equal and opposite charges placed close to each other so that their separate fields at distant points almost, but not quite, cancel. On this point of view it is easy to understand that $E(r)$ for a dipole varies as $1/r^3$ (Eq. 27–8b), whereas for a point charge $E(r)$ drops off more slowly, namely as $1/r^2$ (Eq. 27–4).

Example 4. Figure 27–9 shows a charge q_1 ($= +1.0 \times 10^{-6}$ coul) 10 cm from a charge q_2 ($= +2.0 \times 10^{-6}$ coul). At what point on the line joining the two charges is the electric field strength zero?

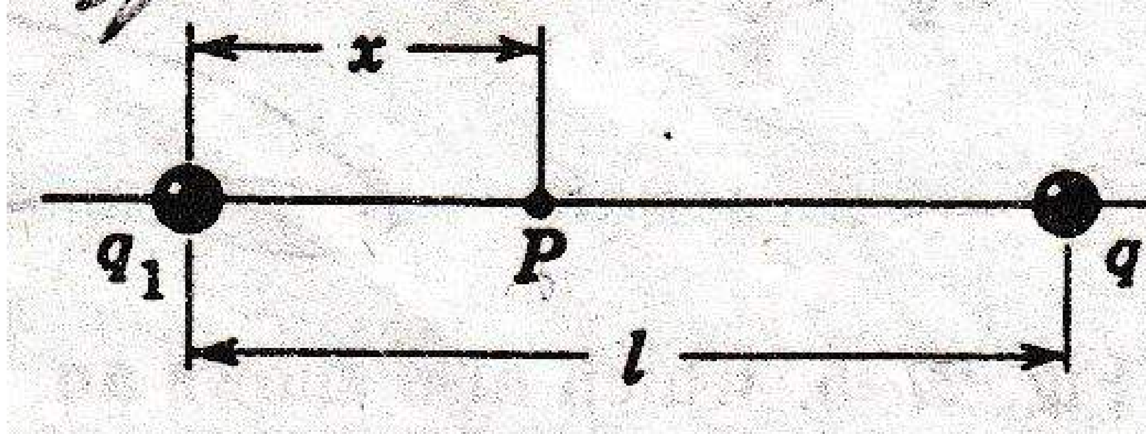

Fig. 27–9 Example 4.

The point must lie between the charges because only here do the forces exerted by q_1 and q_2 on a test charge oppose each other. If $\mathbf{E}_1$ is the electric field strength due to q_1 and $\mathbf{E}_2$ that due to q_2, we must have

$$E_1 = E_2$$

or (see Eq. 27–4)

$$\frac{1}{4\pi\epsilon_0} \frac{q_1}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{(l - x)^2}$$

where x is the distance from q_1 and l equals 10 cm. Solving for x yields

$$x = \frac{l}{1 + \sqrt{q_2/q_1}} = \frac{10 \text{ cm}}{1 + \sqrt{2}} = 4.1 \text{ cm}.$$

The student should supply the missing steps. On what basis was the second root of the resulting quadratic equation discarded?

Example 5. Figure 27–10 shows a ring of charge q and of radius a. Calculate **E** for points on the axis of the ring a distance x from its center.

Consider a differential element of the ring of length ds, located at the top of the ring in Fig. 27–10. It contains an element of charge given by

$$dq = q\,\frac{ds}{2\pi a}$$

where $2\pi a$ is the circumference of the ring. This element sets up a differential electric field $d\mathbf{E}$ at point P.

The resultant field **E** at P is found by integrating the effects of all the elements that make up the ring. From symmetry this resultant field must lie along the ring axis. Thus only the component of $d\mathbf{E}$ parallel to this axis contributes to the final result. The component perpendicular to the axis is canceled out by an equal but opposite component established by the charge element on the opposite side of the ring.

Thus the general vector integral (Eq. 27–7)

$$\mathbf{E} = \int d\mathbf{E}$$

becomes a scalar integral $$E = \int dE \cos\theta.$$

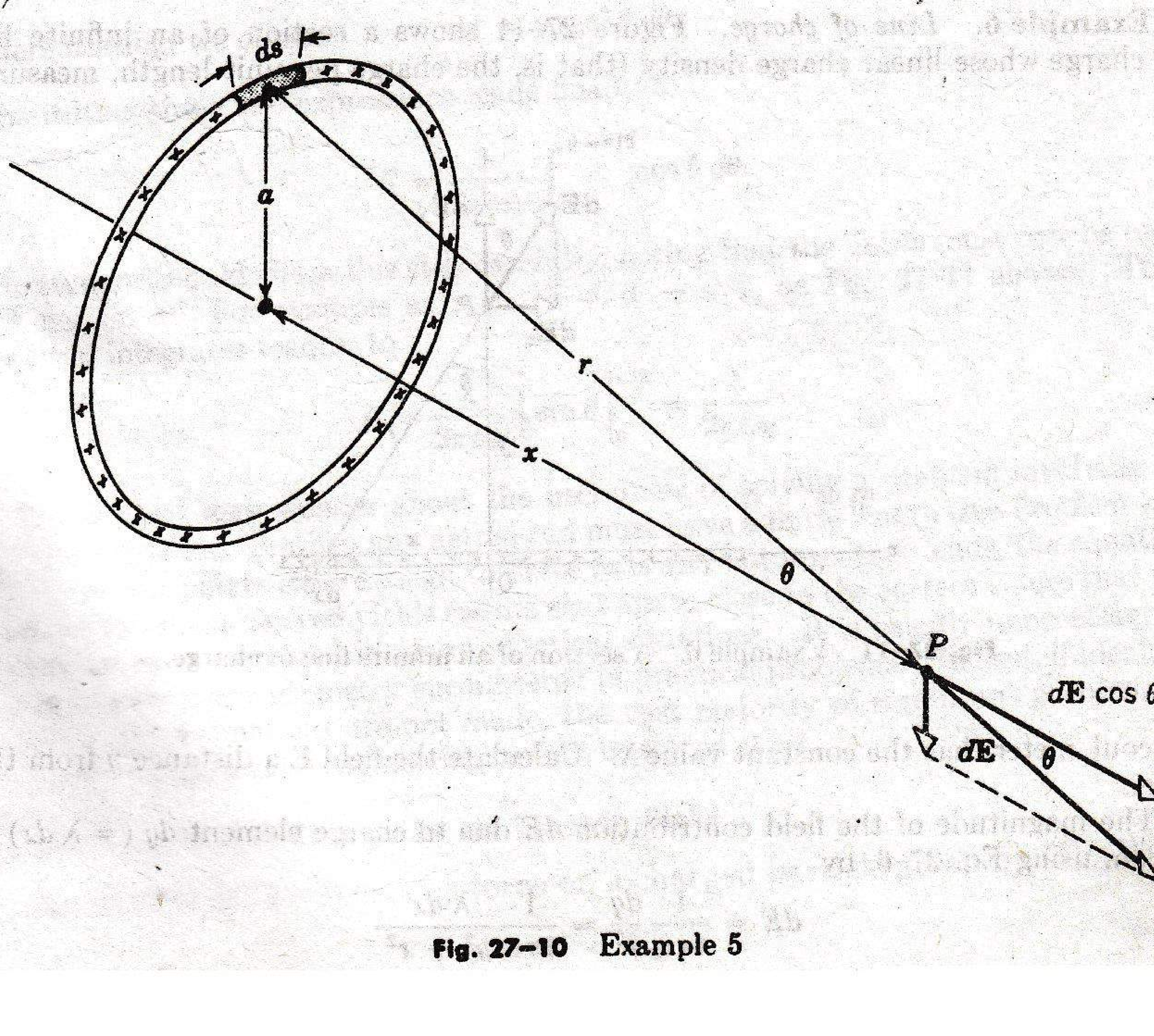

Fig. 27–10 Example 5

The quantity dE follows from Eq. 27-6, or

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\left(\frac{q\,ds}{2\pi a}\right)\frac{1}{a^2 + x^2}.$$

From Fig. 27-10 we have $\cos\theta = \dfrac{x}{\sqrt{a^2 + x^2}}.$

Noting that, for a given point P, x has the same value for all charge elements and is not a variable, we obtain

$$E = \int dE\cos\theta = \int \frac{1}{4\pi\epsilon_0}\frac{q\,ds}{(2\pi a)(a^2 + x^2)}\frac{x}{\sqrt{a^2 + x^2}}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{qx}{(2\pi a)(a^2 + x^2)^{3/2}}\int ds.$$

The integral is simply the circumference of the ring ($= 2\pi a$), so that

$$E = \frac{1}{4\pi\epsilon_0}\frac{qx}{(a^2 + x^2)^{3/2}}.$$

Does this expression for E reduce to an expected result for $x = 0$? For $x \gg a$ we can neglect a in the denominator of this equation, yielding

$$E \cong \frac{1}{4\pi\epsilon_0}\frac{q}{x^2}.$$

This is an expected result (compare Eq. 27-4) because at great enough distances the ring behaves like a point charge q.

Example 6. *Line of charge.* Figure 27-11 shows a section of an infinite line of charge whose linear charge density (that is, the charge per unit length, measured

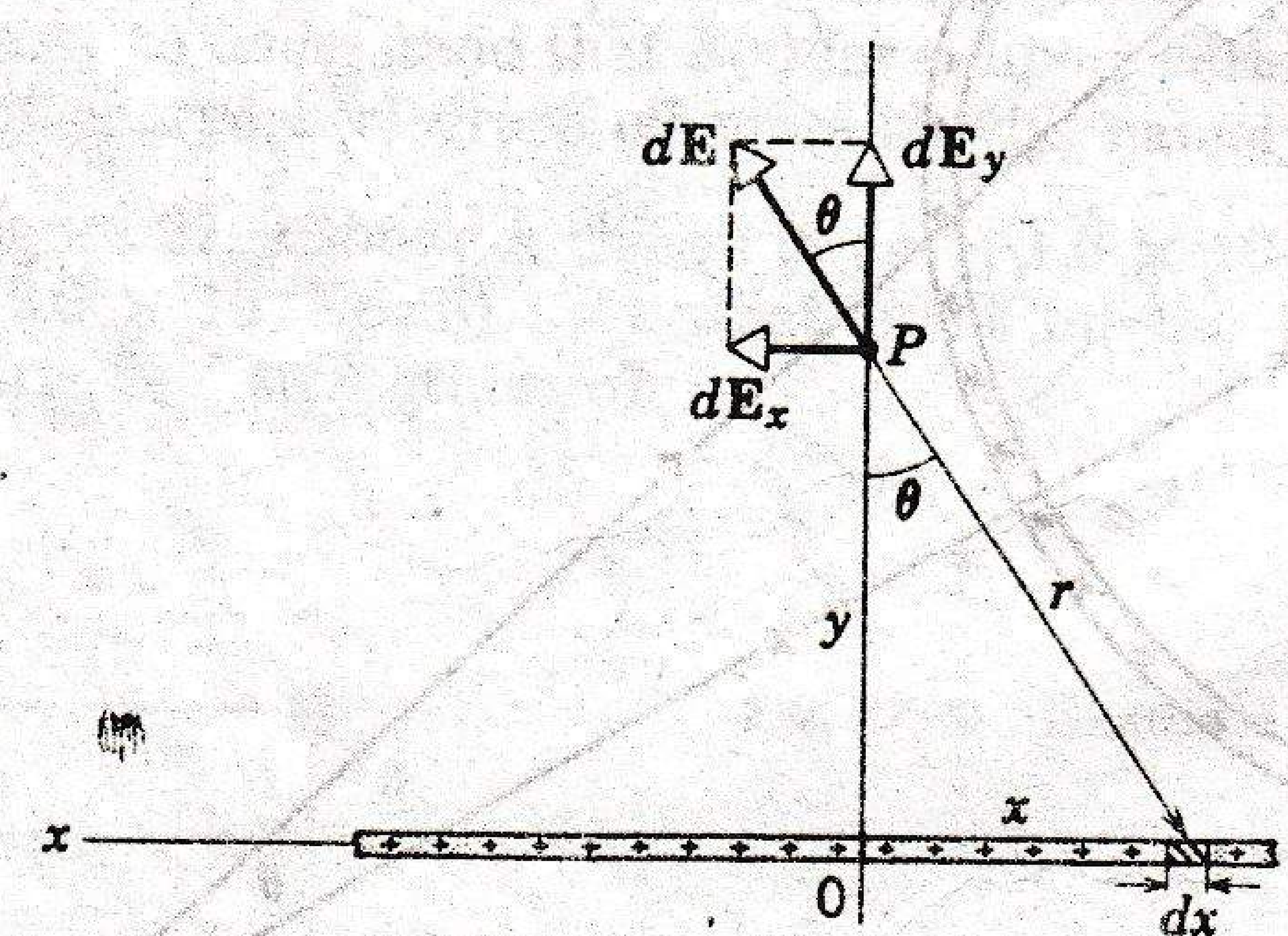

Fig. 27-11 Example 6. A section of an infinite line of charge.

in coul meter) has the constant value λ. Calculate the field $\mathbf{E}$ a distance y from the line.

The magnitude of the field contribution dE due to charge element dq ($= \lambda\,dx$) is given, using Eq. 27-6, by

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dx}{y^2 + x^2}.$$

The vector $d\mathbf{E}$, as Fig. 27-11 shows, has the components

$$dE_x = -dE \sin\theta \quad \text{and} \quad dE_y = dE \cos\theta.$$

The minus sign in front of dE_x indicates that $d\mathbf{E}_x$ points in the negative x direction. The x and y components of the resultant vector $\mathbf{E}$ at point P are given by

$$E_x = \int dE_x = -\int_{x=-\infty}^{x=+\infty} \sin\theta \, dE \quad \text{and} \quad E_y = \int dE_y = \int_{x=-\infty}^{x=+\infty} \cos\theta \, dE.$$

E_x must be zero because every charge element on the right has a corresponding element on the left such that their field contributions in the x direction cancel. Thus $\mathbf{E}$ points entirely in the y direction. Because the contributions to E_y from the right- and left-hand halves of the rod are equal, we can write

$$E = E_y = 2\int_{x=0}^{x=+\infty} \cos\theta \, dE.$$

Note that we have changed the lower limit of integration and have introduced a factor of two.

Substituting the expression for dE into this equation gives

$$E = \frac{\lambda}{2\pi\epsilon_0}\int_{x=0}^{x=\infty} \cos\theta \frac{dx}{y^2 + x^2}.$$

From Fig. 27-11 we see that the quantities θ and x are not independent. We must eliminate one of them, say x. The relation between x and θ is (see figure)

$$x = y \tan\theta.$$

Differentiating, we obtain $\quad dx = y \sec^2\theta \, d\theta.$

Substituting these two expressions leads finally to

$$E = \frac{\lambda}{2\pi\epsilon_0 y}\int_{\theta=0}^{\theta=\pi/2} \cos\theta \, d\theta.$$

The student should check this step carefully, noting that the limits must now be on θ and not on x. For example as $x \to +\infty$, $\theta \to \pi/2$, as Fig. 27-11 shows. This equation integrates readily to

$$E = \frac{\lambda}{2\pi\epsilon_0 y}\Big|\sin\theta\Big|_0^{\pi/2} = \frac{\lambda}{2\pi\epsilon_0 y}.$$

The student may wonder about the usefulness of solving a problem involving an infinite rod of charge when any actual rod must have a finite length (see Problem 15). However, for points close enough to finite rods and not near their ends, the equation that we have just derived yields results that are so close to the correct values that the difference can be ignored in many practical situations. It is usually unnecessary to solve exactly every geometry encountered in practical problems. Indeed, if idealizations or approximations are not made, the vast majority of significant problems of all kinds in physics and engineering cannot be solved at all. ◀

27-5 A Point Charge in an Electric Field

An electric field will exert a force on a charged particle given by (Eq. 27-2)

$$\mathbf{F} = \mathbf{E}q.$$

This force will produce an acceleration

$$\mathbf{a} = \mathbf{F}/m,$$

where m is the mass of the particle. We will consider two examples of the acceleration of a charged particle in a uniform electric field. Such a field can be produced by connecting the terminals of a battery to two parallel metal plates which are otherwise insulated from each other. If the spacing between the plates is small compared with the dimensions of the plates, the field between them will be fairly uniform except near the edges. Note that in calculating the motion of a particle in a field set up by external charges the field due to the particle itself (that is, its *self-field*) is ignored. For example, the earth's gravitational field can have no effect on the earth itself but only on a second object, say a stone, placed in that field.

▶ **Example 7.** A particle of mass m and charge q is placed at rest in a uniform electric field (Fig. 27-12) and released. Describe its motion.

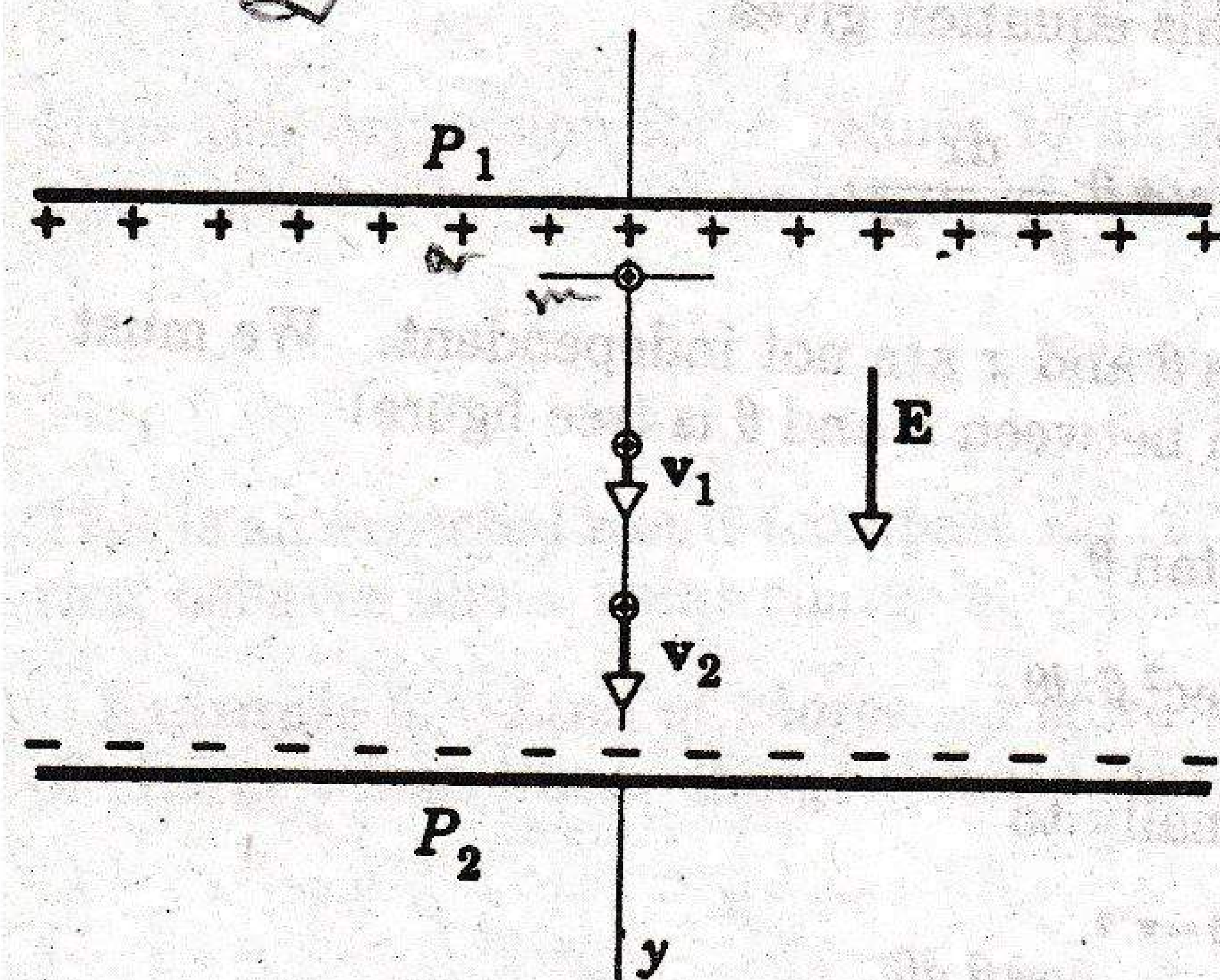

Fig. 27-12 A charge is released from rest in a uniform electric field set up between two oppositely charged metal plates P_1 and P_2.

The motion resembles that of a body falling in the earth's gravitational field. The (constant) acceleration is given by

$$a = \frac{F}{m} = \frac{qE}{m}.$$

The equations for uniformly accelerated motion (Table 3-1) then apply. With $v_0 = 0$, they are

$$v = at = \frac{qEt}{m},$$

$$y = \tfrac{1}{2}at^2 = \frac{qEt^2}{2m},$$

and

$$v^2 = 2ay = \frac{2qEy}{m}.$$

The kinetic energy attained after moving a distance y is found from

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{2qEy}{m}\right) = qEy.$$

This result also follows directly from the work-energy theorem because a constant force qE acts over a distance y.

Example 8. *Deflecting an electron beam.* Figure 27–13 shows an electron of mass m and charge e projected with speed v_0 at right angles to a uniform field **E**. Describe its motion.

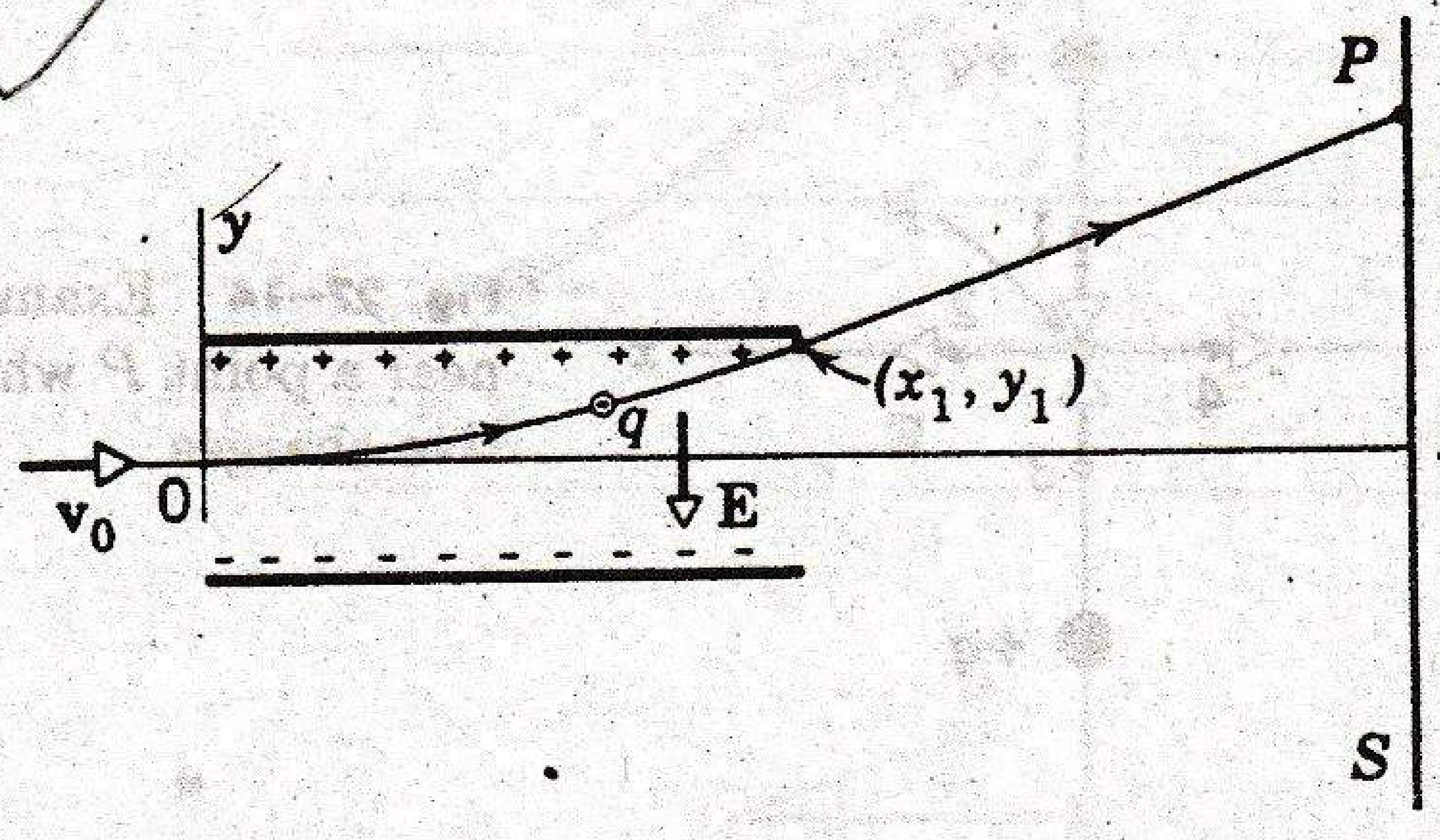

Fig. 27–13 Example 8. An electron is projected into a uniform electric field.

The motion is like that of a projectile fired horizontally in the earth's gravitational field. The considerations of Section 4–3 apply, the horizontal (x) and vertical (y) motions being given by

$$x = v_0 t$$

and

$$y = \tfrac{1}{2}at^2 = \frac{eE}{2m}t^2.$$

Eliminating t yields

$$y = \frac{eE}{2mv_0^2}x^2 \tag{27–9}$$

for the equation of the trajectory.

When the electron emerges from the plates in Fig. 27–13, it travels (neglecting gravity) in a straight line tangent to the parabola at the exit point. We can let it fall on a fluorescent screen S placed some distance beyond the plates. Together with other electrons following the same path, it will then make itself visible as a small luminous spot; this is the principle of the electrostatic *cathode-ray oscilloscope.*

Example 9. The electric field between the plates of a cathode-ray oscilloscope is 1.2×10^4 nt/coul. What deflection will an electron experience if it enters at right angles to the field with a kinetic energy of 2000 ev ($= 3.2 \times 10^{-16}$ joule), a typical value? The deflecting assembly is 1.5 cm long.

Recalling that $K_0 = \frac{1}{2}mv_0^2$, we can rewrite Eq. 27–9 as

$$y = \frac{eEx^2}{4K_0}.$$

If x_1 is the horizontal position of the far edge of the plate, y_1 will be the corresponding deflection (see Fig. 27–13), or

$$y_1 = \frac{eEx_1^2}{4K_0}$$

$$= \frac{(1.6 \times 10^{-19}\ \text{coul})(1.2 \times 10^4\ \text{nt/coul})(1.5 \times 10^{-2}\ \text{meter})^2}{(4)(3.2 \times 10^{-16}\ \text{joule})}$$

$$= 3.4 \times 10^{-4}\ \text{meter} = 0.34\ \text{mm}.$$

The deflection measured, not at the deflecting plates but at the fluorescent screen, is much larger.

Example 10. A positive point test charge q_0 is placed halfway between two equal positive charges q. What force acts on it at or near this point P?

From symmetry the force *at* the point is zero so that the particle is in equilibrium; the nature of the equilibrium remains to be found. Figure 27–14 (compare Fig. 27–4)

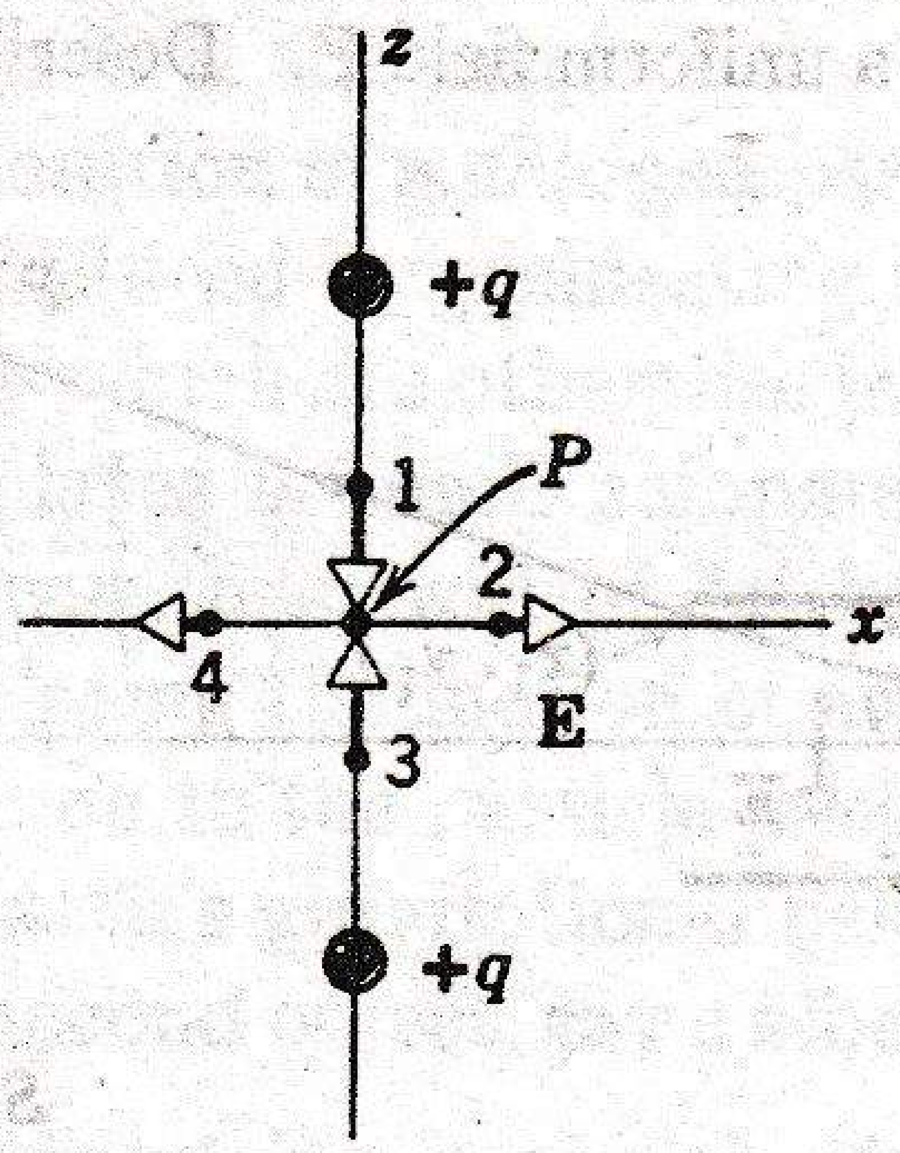

Fig. 27–14 Example 10. The electric field at four points near a point P which is centered between two equal positive charges.

shows the **E** vectors for four points near P. If the test charge is moved along the z axis, a *restoring* force is brought into play; however, the equilibrium is unstable for motion in the x-y plane. Thus we have the three-dimensional equivalent of *saddle point equilibrium;* see Fig. 14–8. What is the nature of the equilibrium for a negative test charge? ◀

27–6 A Dipole in an Electric Field

An electric dipole moment can be regarded as a vector **p** whose magnitude p, for a dipole like that described in Example 3, is the product $2aq$ of the magnitude of either charge q and the distance $2a$ between the charges. The direction of **p** for such a dipole is from the negative to the positive charge. The vector nature of the electric dipole moment permits us to cast many expressions involving electric dipoles into concise form, as we shall see.

Figure 27–15a shows an electric dipole formed by placing two charges $+q$ and $-q$ a fixed distance $2a$ apart. The arrangement is placed in a uniform *external* electric field **E**, its dipole moment **p** making an angle θ with this field. Two equal and opposite forces **F** and $-$**F** act as shown, where

$$F = qE.$$

The net force is clearly zero, but there is a net torque about an axis through O (see Eq. 12–2) given by

$$\tau = 2F(a \sin \theta) = 2aF \sin \theta.$$

Combining these two equations and recalling that $p = (2a)(q)$, we obtain

$$\tau = 2aqE \sin \theta = pE \sin \theta. \tag{27-10}$$

Thus an electric dipole placed in an external electric field **E** experiences a torque tending to align it with the field. Equation 27–10 can be written in vector form as

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}, \tag{27-11}$$

the appropriate vectors being shown in Fig. 27–15b.

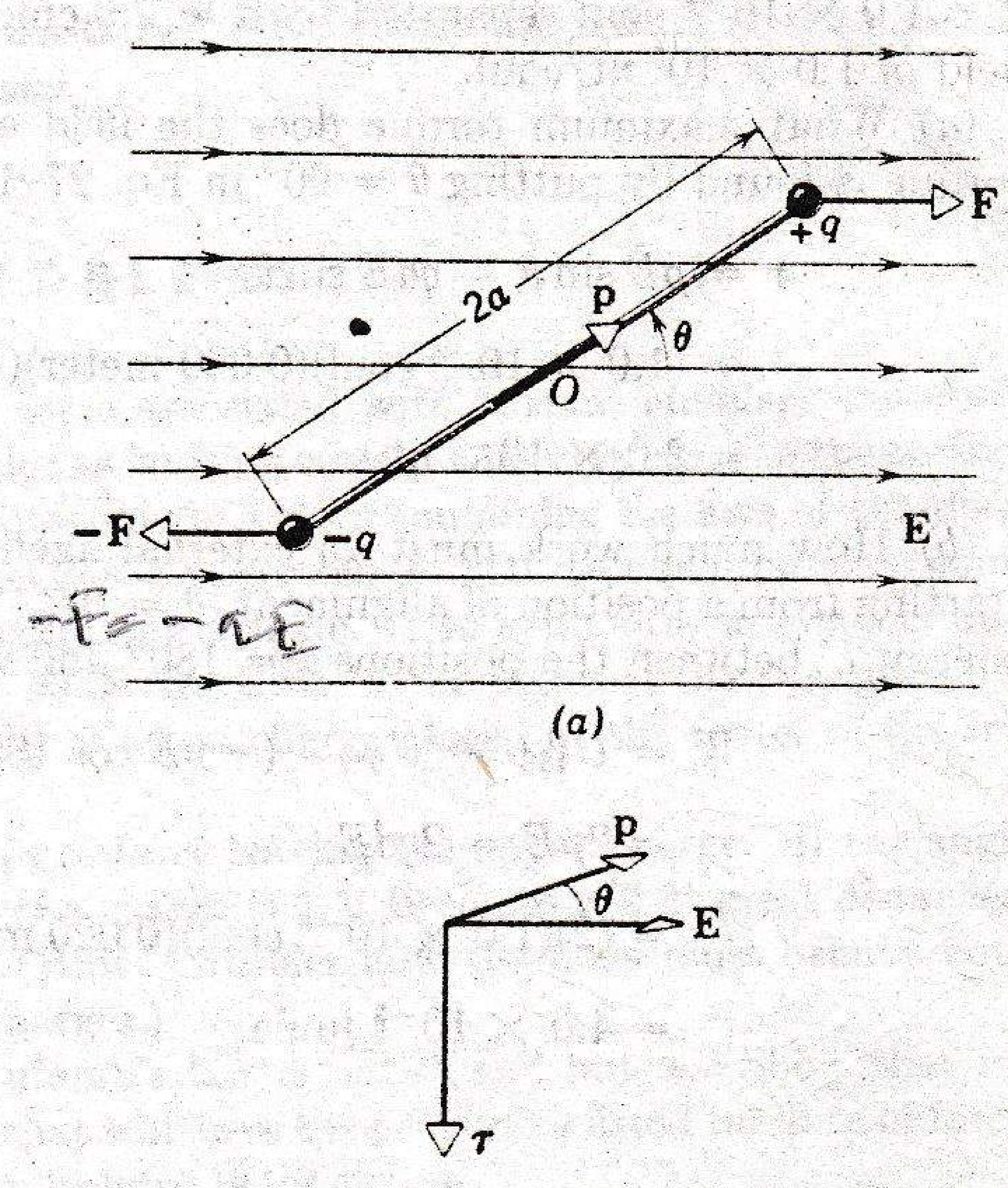

Fig. 27–15 (*a*) An electric dipole in a uniform external field. (*b*) An oblique view, illustrating $\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$.

Work (positive or negative) must be done by an external agent to change the orientation of an electric dipole in an external field. This work is stored as potential energy U in the system consisting of the dipole and the arrangement used to set up the external field. If θ in Fig. 27–15*a* has the initial value θ_0, the work required to turn the dipole axis to an angle θ is given (see Table 12–2) from

$$W = \int dW = \int_{\theta_0}^{\theta} \tau \, d\theta = U,$$

where τ is the torque exerted by the agent that does the work. Combining this equation with Eq. 27–10 yields

$$U = \int_{\theta_0}^{\theta} pE \sin\theta \, d\theta = pE \int_{\theta_0}^{\theta} \sin\theta \, d\theta$$

$$= pE \left| -\cos\theta \right|_{\theta_0}^{\theta}$$

Since we are interested only in *changes* in potential energy, we choose the reference orientation θ_0 to have any convenient value, in this case 90°. This gives

$$U = -pE\cos\theta \qquad (27\text{–}12)$$

or, in vector form,

$$U = -\mathbf{p}\cdot\mathbf{E}. \qquad (27\text{–}13)$$

Example 11. An electric dipole consists of two opposite charges of magnitude $q = 1.0 \times 10^{-6}$ coul separated by $d = 2.0$ cm. The dipole is placed in an external field of 1.0×10^5 nt/coul.

(*a*) What maximum torque does the field exert on the dipole? The maximum torque is found by putting $\theta = 90°$ in Eq. 27–10 or

$$\begin{aligned}\tau &= pE \sin\theta = qdE \sin\theta \\ &= (1.0 \times 10^{-6}\text{ coul})(0.020\text{ meter})(1.0 \times 10^5\text{ nt/coul})(\sin 90°) \\ &= 2.0 \times 10^{-3}\text{ nt-m.}\end{aligned}$$

(*b*) How much work must an external agent do to turn the dipole end for end, starting from a position of alignment ($\theta = 0$)? The work is the difference in potential energy U between the positions $\theta = 180°$ and $\theta = 0$. From Eq. 27–12,

$$\begin{aligned}W &= U_{180°} - U_{0°} = (-pE\cos 180°) - (-pE\cos 0) \\ &= 2pE = 2qdE \\ &= (2)(1.0 \times 10^{-6}\text{ coul})(0.020\text{ meter})(1.0 \times 10^5\text{ nt/coul}) \\ &= 4.0 \times 10^{-3}\text{ joule.}\end{aligned}$$

◀

QUESTIONS

1. Name as many scalar fields and vector fields as you can.

2. (*a*) In the gravitational attraction between the earth and a stone, can we say that the earth lies in the gravitational field of the stone? (*b*) How is the gravitational field due to the stone related to that due to the earth?

3. A positively charged ball hangs from a long silk thread. We wish to measure **E** at a point in the same horizontal plane as that of the hanging charge. To do so, we put a positive test charge q_0 at the point and measure $\mathbf{F}/q_0$. Will F/q_0 be less than, equal to, or greater than E at the point in question?

4. Taking into account the quantization of electric charge (the single electron providing the basic charge unit), how can we justify the procedure suggested by Eq. 27–3?

5. In Fig. 27–5 the force on the lower charge points up and is finite. The crowding of the lines of force, however, suggests that E is infinitely great at the site of this (point) charge. A charge immersed in an infinitely great field should have an infinitely great force acting on it. What is the solution to this dilemma?

6. Electric lines of force never cross. Why?

7. In Fig. 27–4 why do the lines of force around the edge of the figure appear, when extended backwards, to radiate uniformly from the center of the figure?

8. Figure 27–2 shows that **E** has the same value for all points in front of an infinite uniformly charged sheet. Is this reasonable? One might think that the field should be stronger near the sheet because the charges are so much closer.

9. Two point charges of unknown magnitude and sign are a distance d apart. The electric field strength is zero at one point between them, on the line joining them. What can you conclude about the charges?

10. Compare the way E varies with r for (*a*) a point charge (Eq. 27–4), (*b*) a dipole (Eq. 27–8*a*), and (*c*) a quadrupole (Problem 18).

11. If a point charge q of mass m is released from rest in a nonuniform field, will it follow a line of force?

12. An electric dipole is placed in a *nonuniform* electric field. Is there a net force on it?

13. An electric dipole is placed at rest in a uniform external electric field, as in Fig. 27–15*a*, and released. Discuss its motion.

14. An electric dipole has its dipole moment **p** aligned with a uniform external electric field **E**. (*a*) Is the equilibrium stable or unstable? (*b*) Discuss the nature of the equilibrium if **p** and **E** point in opposite directions.

PROBLEMS

1. Sketch qualitatively the lines of force associated with a thin, circular, uniformly charged disk of radius R. (Hint: Consider as limiting cases points very close to the surface and points very far from it.) Show the lines only in a plane containing the axis of the disk.

2. (*a*) Sketch qualitatively the lines of force associated with three equal positive point charges placed at the corners of an equilateral triangle. Use symmetry arguments and limiting cases (see hint in Problem 1). Show the lines in the plane of the triangle only. (*b*) Discuss the nature of the equilibrium of a test charge placed at the center of the triangle.

3. In Fig. 27–4 consider any two lines of force leaving the upper charge. If the angle between their tangents for points near the charge is θ, it becomes $\theta/2$ at great distances. Verify this statement and explain it. (Hint: Consider how the lines must behave both close to either charge and far from the charges.)

4. Assume that the exponent in Coulomb's law is not "two" but n. Show that for $n \neq 2$ it is impossible to construct lines that will have the properties listed for lines of force in Section 27–3. For simplicity, treat an isolated point charge.

5. What is the magnitude of a point charge chosen so that the electric field 50 cm away has the magnitude 2.0 nt/coul?

6. Two equal and opposite charges of magnitude 2.0×10^{-7} coul are 15 cm apart. (*a*) What are the magnitude and direction of **E** at a point midway between the charges? (*b*) What force (magnitude and direction) would act on an electron placed there?

7. Two point charges are a distance d apart (Fig. 27–16). Plot $E(x)$, assuming $x = 0$ at the left-hand charge. Consider both positive and negative values of x. Plot E as positive if **E** points to the right and negative if **E** points to the left. Assume $q_1 = +1.0 \times 10^{-6}$ coul, $q_2 = +3.0 \times 10^{-6}$ coul, and $d = 10$ cm.

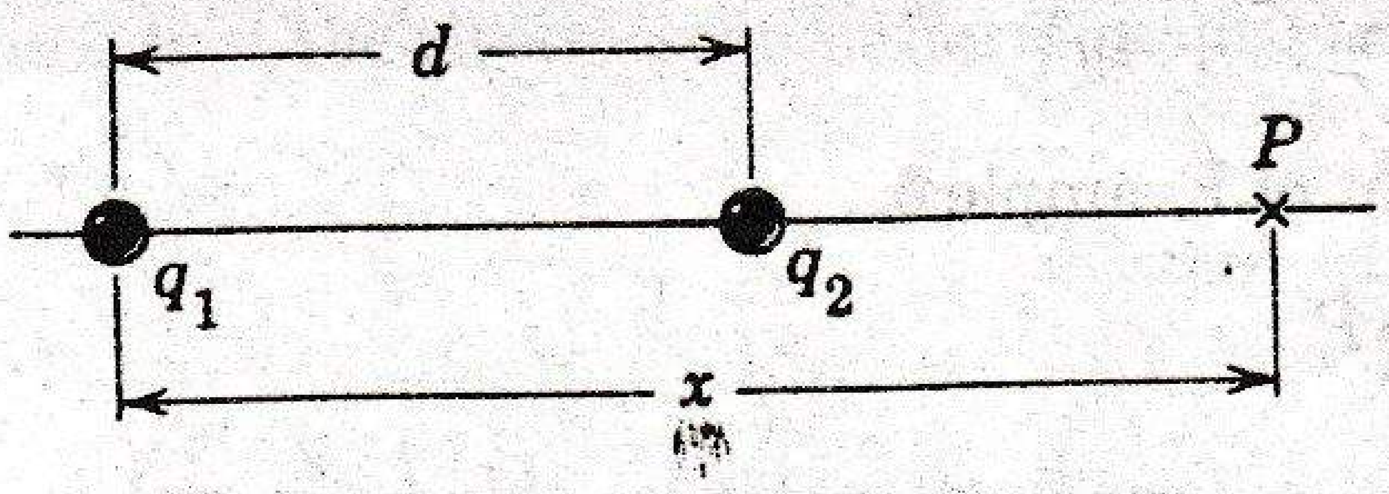

Fig. 27–16

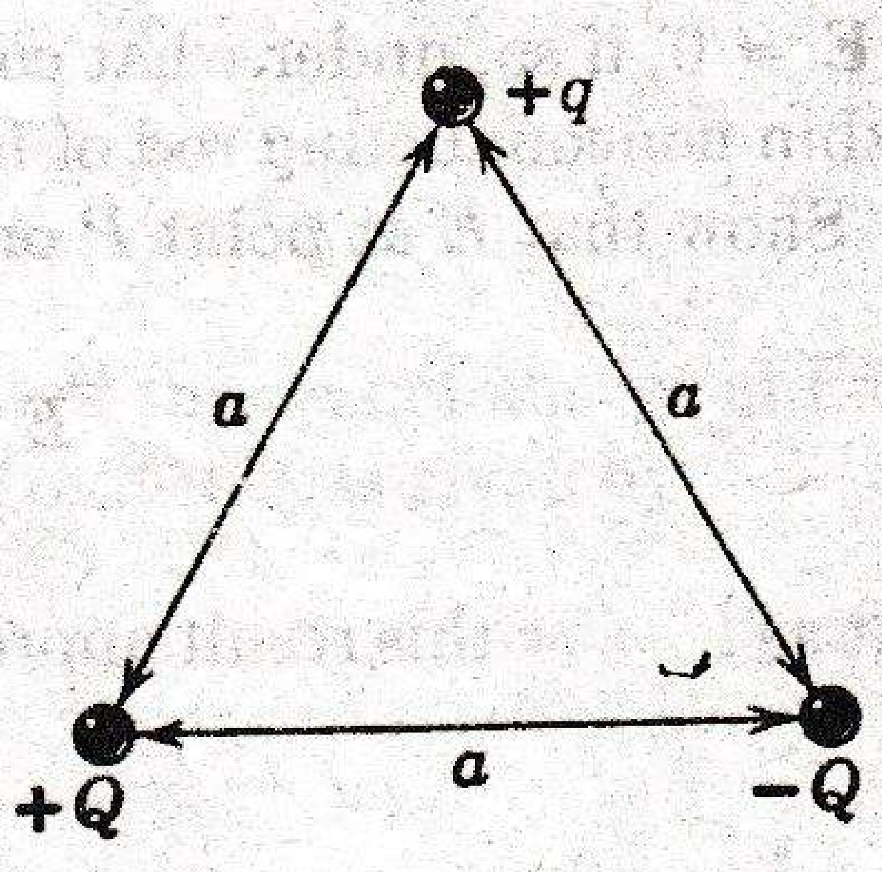

Fig. 27–17

8. Three charges are arranged in an equilateral triangle as in Fig. 27–17. What is the direction of the force on $+q$?

9. Two point charges of magnitude $+2.0 \times 10^{-7}$ coul and $+8.5 \times 10^{-8}$ coul are 12 cm apart. (*a*) What electric field does each produce at the site of the other? (*b*) What force acts on each?

10. *Axial field due to an electric dipole.* In Fig. 27–8, consider a point a distance r from the center of the dipole *along its axis.* (*a*) Show that, at large values of r, the electric field is

$$E = \frac{1}{2\pi\epsilon_0}\frac{p}{r^3},$$

which is twice the value given for the conditions of Example 3. (*b*) What is the direction of **E**?

11. In Fig. 27–8 assume that both charges are positive. (*a*) Show that E at point P in that figure, assuming $r \gg a$, is given by

$$E = \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}.$$

(*b*) What is the direction of **E**? (*c*) Is it reasonable that E should vary as r^{-2} here and as r^{-3} for the dipole of Fig. 27–8?

12. (*a*) In Fig. 27–18 locate the point (or points) at which the electric field strength is zero. (*b*) Sketch qualitatively the lines of force. Take $a = 50$ cm.

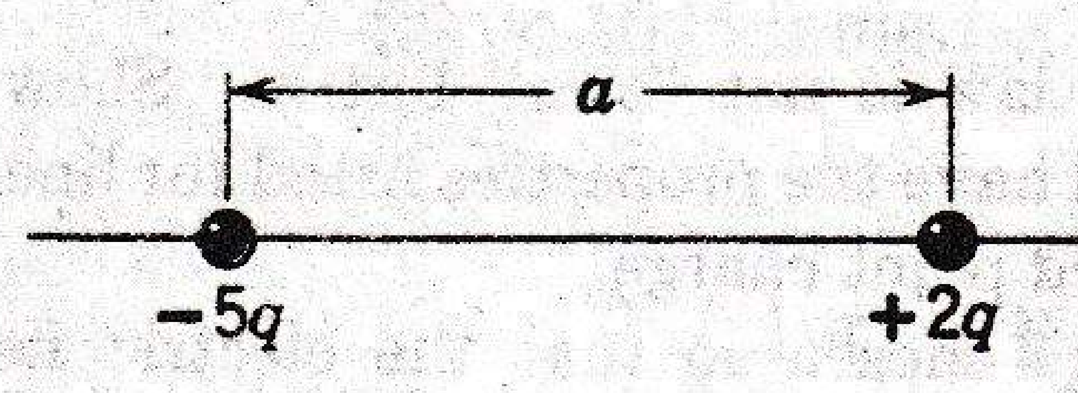

Fig. 27–18

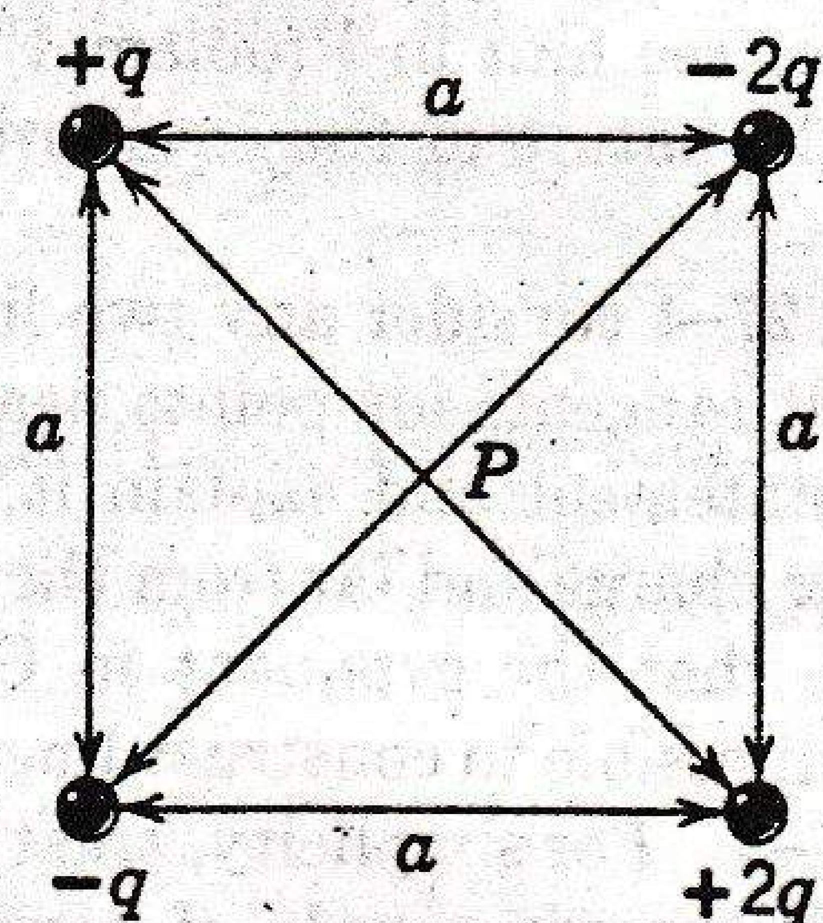

Fig. 27–19

13. What is **E** in magnitude and direction at the center of the square of Fig. 27–19? Assume that $q = 1.0 \times 10^{-8}$ coul and $a = 5.0$ cm.

14. Two point charges of unknown magnitude and sign are placed a distance d apart. (*a*) If it is possible to have $\mathbf{E} = 0$ at any point *not* between the charges but on the line joining them, what are the necessary conditions and where is the point located? (*b*) Is it possible, for any arrangement of two point charges, to find *two* points (neither at infinity) at which $\mathbf{E} = 0$; if so, under what conditions?

15. A thin nonconducting rod of finite length l carries a total charge q, spread uniformly along it. Show that E at point P on the perpendicular bisector in Fig. 27–20 is given by

$$E = \frac{q}{2\pi\epsilon_0 y}\frac{1}{\sqrt{l^2 + 4y^2}}.$$

Show that as $l \to \infty$ this result approaches that of Example 6.

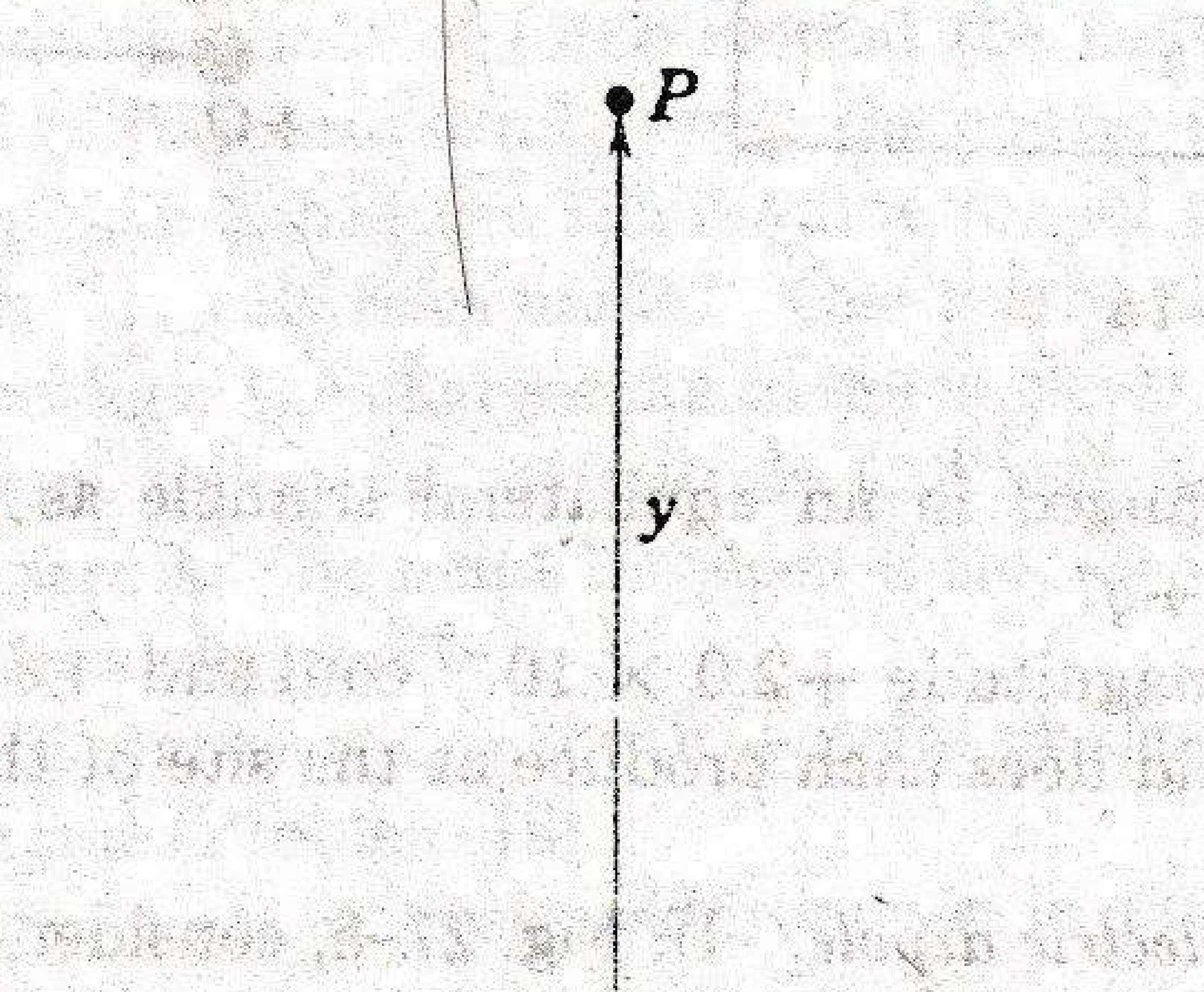

Fig. 27–20

16. A thin nonconducting rod is bent to form the arc of a circle of radius a and subtends an angle θ_0 at the center of the circle. A total charge q is spread uniformly along its length. Find the electric field strength at the center of the circle in terms of a, q, and θ_0.

17. A nonconducting hemispherical cup of inner radius a has a total charge q spread uniformly over its inner surface. Find the electric field at the center of curvature.

18. *Electric quadrupole.* Figure 27–21 shows a typical electric quadrupole. It consists of two dipoles whose effects at external points do not quite cancel. Show that the value of E on the axis of the quadrupole for points distant r from its center (assume $r \gg a$) is given by

$$E = \frac{3Q}{4\pi\epsilon_0 r^4}$$

where Q ($= 2qa^2$) is called the *quadrupole moment* of the charge distribution.

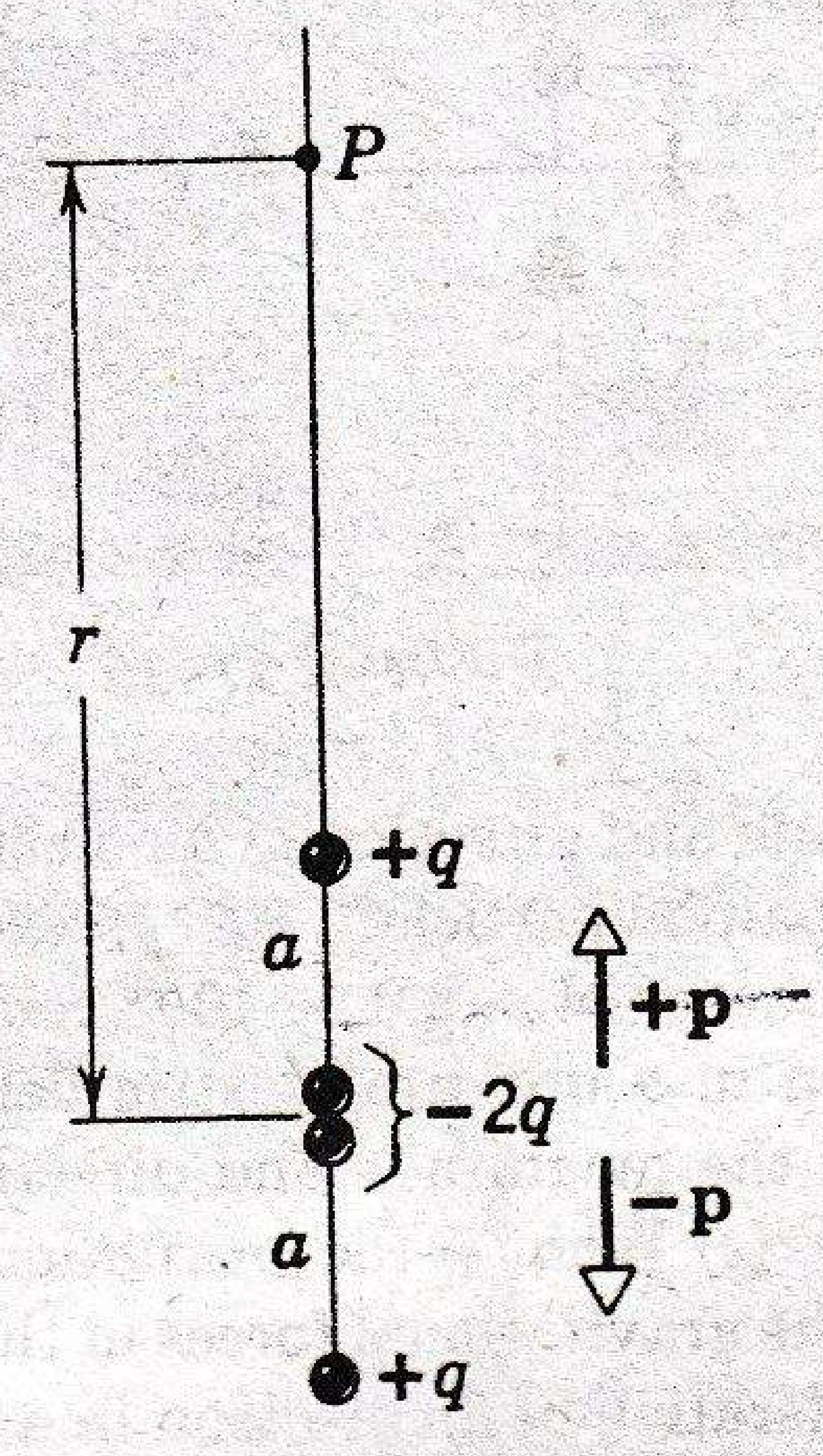

Fig. 27–21

19. An electron is constrained to move along the axis of the ring of charge in Example 5. Show that the electron can perform oscillations whose frequency is given by

$$\omega = \sqrt{\frac{eq}{4\pi\epsilon_0 ma^3}}.$$

This formula holds only for small oscillations, that is, for $x \ll a$ in Fig. 27–10. (Hint: Show that the motion is simple harmonic and use Eq. 15–11.)

20. For the ring of charge in Example 5, show that the maximum value of E occurs at $x = a/\sqrt{2}$.

21. Consider the ring of charge of Example 5. Suppose that the charge q is not distributed uniformly over the ring but that charge q_1 is distributed uniformly over half the circumference and charge q_2 is distributed uniformly over the other half. Let $q_1 + q_2 = q$. (*a*) Find the *component* of the electric field at any point on the axis directed *along* the axis and compare with the uniform case of Example 5. (*b*) Find the *component* of the electric field at any point on the axis *perpendicular* to the axis and compare with the uniform case of Example 5.

22. A thin circular disk of radius a is charged uniformly so as to have a charge per unit area of σ. Find the electric field on the axis of the disk at a distance r from the disk.

23. *Field due to an electric dipole.* Show that the components of **E** due to a dipole are given, at distant points, by

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{3pxy}{(x^2 + y^2)^{5/2}}$$

$$E_y = \frac{1}{4\pi\epsilon_0} \frac{p(2y^2 - x^2)}{(x^2 + y^2)^{5/2}},$$

where x and y are coordinates of a point in Fig. 27–22. Show that this general result includes the special results of Eq. 27–8*b* and of Problem 10.

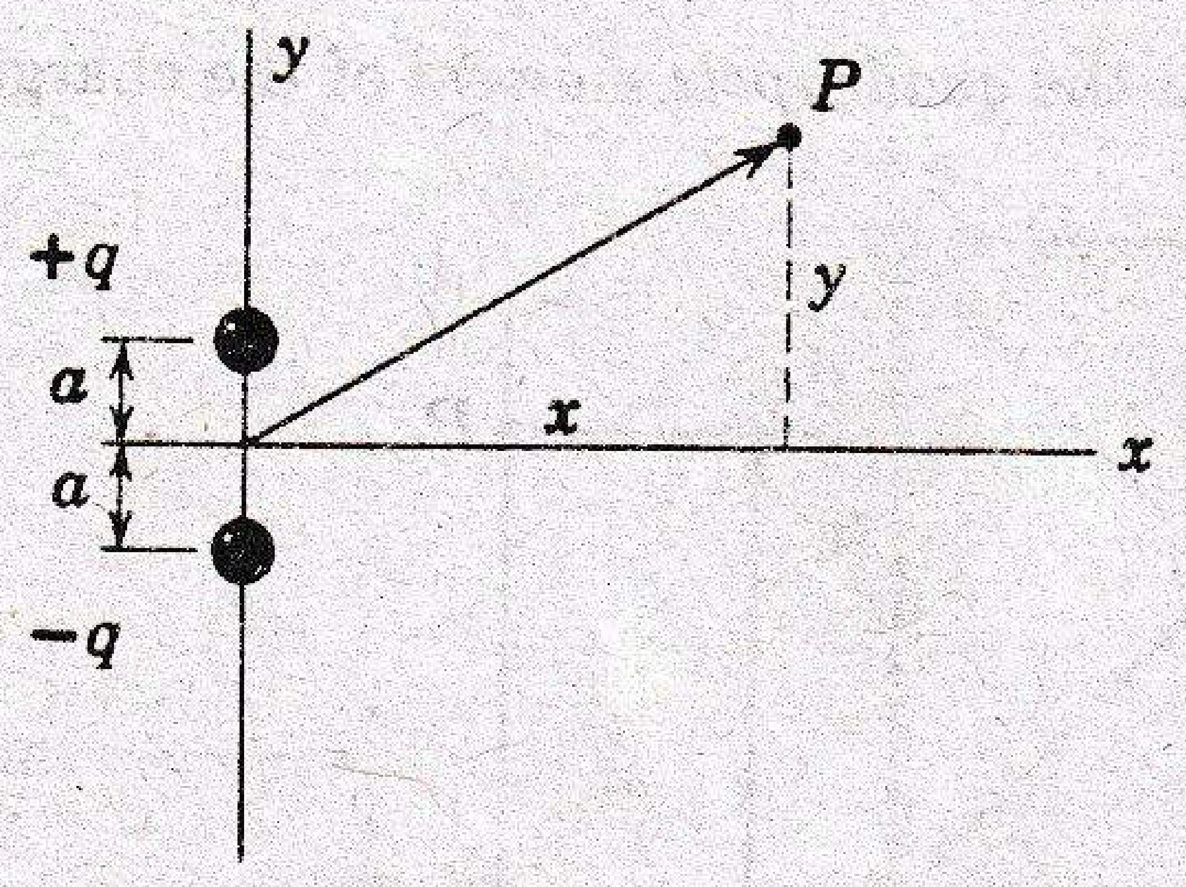

Fig. 27–22

24. What is the magnitude and direction of an electric field that will balance the weight of (a) an electron and (b) an alpha particle?

25. A particle having a charge of -2.0×10^{-9} coul is acted on by a downward electric force of 3.0×10^{-6} newton in a uniform electric field. (*a*) What is the strength of the electric field? (*b*) What is the magnitude and direction of the *electric* force exerted on a proton placed in this field? (*c*) What is the *gravitational* force on the proton? (*d*) What is the ratio of the electric to the gravitational forces in this case?

26. (*a*) What is the acceleration of an electron in a uniform electric field of 10^6 nt/coul? (*b*) How long would it take for the electron, starting from rest, to attain one-tenth the speed of light? (*c*) What considerations limit the applicability of Newtonian mechanics to such problems?

27. An electron moving with a speed of 5.0×10^8 cm/sec is shot parallel to an electric field of strength 1.0×10^3 nt/coul arranged so as to retard its motion. (*a*) How far will the electron travel in the field before coming (momentarily) to rest, and (*b*) how much time will elapse? (*c*) If the electric field ends abruptly after 0.8 cm, what fraction of its initial energy will the electron lose in traversing it?

28. An electron is projected as in Fig. 27–23 at a speed of 6.0×10^6 meters/sec and at an angle θ of 45°; $E = 2.0 \times 10^3$ nt/coul (directed upward), $d = 2.0$ cm, and $l = 10.0$ cm (*a*) Will the electron strike either of the plates? (*b*) If it strikes a plate, where does it do so?

29. *Dipole in a nonuniform field.* Derive an expression for dE/dz at a point midway between two equal positive charges, where z is the distance from one of the charges, measured along the line joining them. Would there be a force on a small dipole placed at this point, its axis being aligned with the z axis? Recall that **E** = 0 at this point.

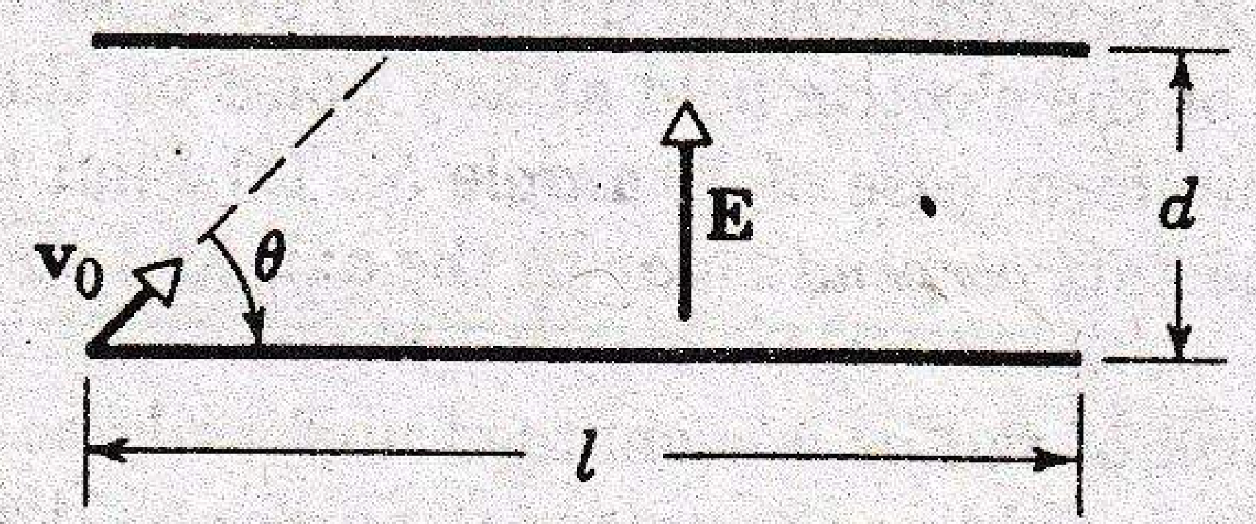

Fig. 27–23

30. *Oil drop experiment.* R. A. Millikan set up an apparatus (Fig. 27-24) in which a tiny, charged oil drop, placed in an electric field **E**, could be "balanced" by adjusting E until the electric force on the drop was equal and opposite to its weight. If the radius of the drop is 1.64×10^{-4} cm and E at balance is 1.92×10^{5} nt/coul, (*a*) what charge is on the drop? (*b*) Why did Millikan not try to balance electrons in his apparatus instead of oil drops? The density of the oil is 0.851 gm/cm^3. (Millikan first measured the electronic charge in this way. He measured the drop radius by observing the limiting speed that the drops attained when they fell in air with the electric field turned off. He charged the oil drops by irradiating them with bursts of X-rays.)

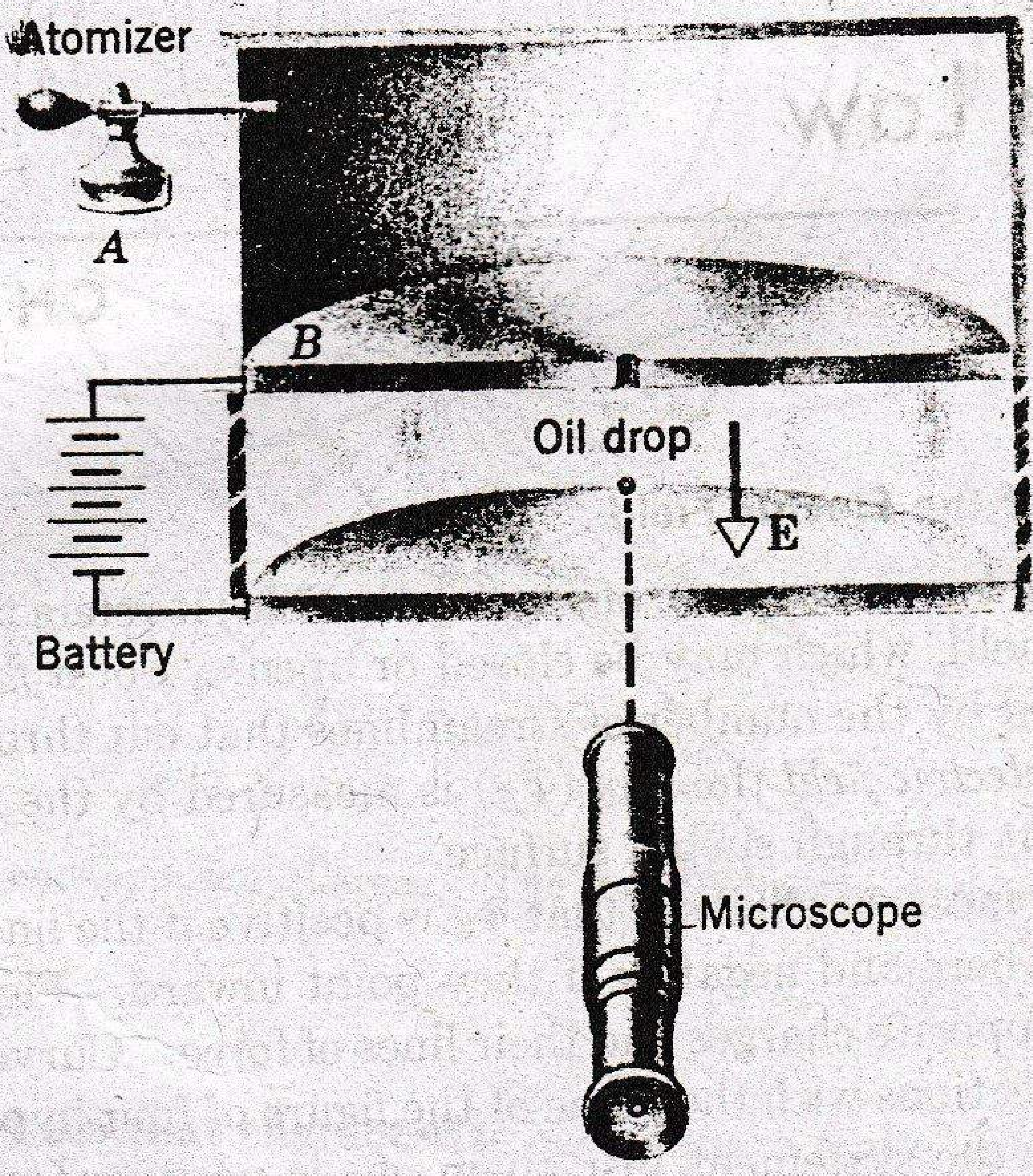

Fig. 27-24 Millikan's oil drop apparatus. Charged oil drops from atomizer A fall through the hole in plate B.

31. In a particular early run (1911), Millikan observed that the following measured charges, among others, appeared at different times on a single drop:

6.563×10^{-19} coul	13.13×10^{-19} coul	19.71×10^{-19} coul
8.204×10^{-19} coul	16.48×10^{-19} coul	22.89×10^{-19} coul
11.50×10^{-19} coul	18.08×10^{-19} coul	26.13×10^{-19} coul

What value for the elementary charge e can be deduced from these data?

32. An electric field **E** with an average magnitude of about 150 nt/coul points upward in the earth's atmosphere. We wish to "float" a sulfur sphere weighing 1.0 lb in this field by charging it. (*a*) What charge (sign and magnitude) must be used? (*b*) Why is the experiment not practical? Give a qualitative reason supported by a very rough numerical calculation to prove your point.

Gauss's Law

CHAPTER 28

28–1 Flux of the Electric Field

Flux (symbol Φ) is a property of any vector field; it refers to a hypothetical surface in the field, which may be closed or open. For a *flow field* the flux (Φ_v) is measured by the number of streamlines that cut through such a surface. For an *electric field* the flux (Φ_E) is measured by the number of lines of force that cut through such a surface.

For closed surfaces we shall see that Φ_E is positive if the lines of force point outward everywhere and negative if they point inward. Figure 28–1 shows two equal and opposite charges and their lines of force. Curves S_1, S_2, S_3, and S_4 are the intersections with the plane of the figure of four hypothetical closed surfaces. From the statement just given, Φ_E is positive for surface S_1 and negative for S_2. The flux of the electric field is important because Gauss's law, one of the four basic equations of electromagnetism (see Table 38–3), is expressed in terms of it. Although the concept of flux may seem a little abstract at first, the student will soon see its value in solving problems.

To define Φ_E precisely, consider Fig. 28–2, which shows an arbitrary closed surface immersed in an electric field. Let the surface be divided into elementary squares ΔS, each of which is small enough so that it may be considered to be plane. Such an element of area can be represented as a vector $\Delta\mathbf{S}$, whose magnitude is the area ΔS; the direction of $\Delta\mathbf{S}$ is taken as the *outward-drawn normal* to the surface.

At every square in Fig. 28–2 we can also construct an electric field vector $\mathbf{E}$. Since the squares have been taken to be arbitrarily small, $\mathbf{E}$ may be taken as constant for all points in a given square.

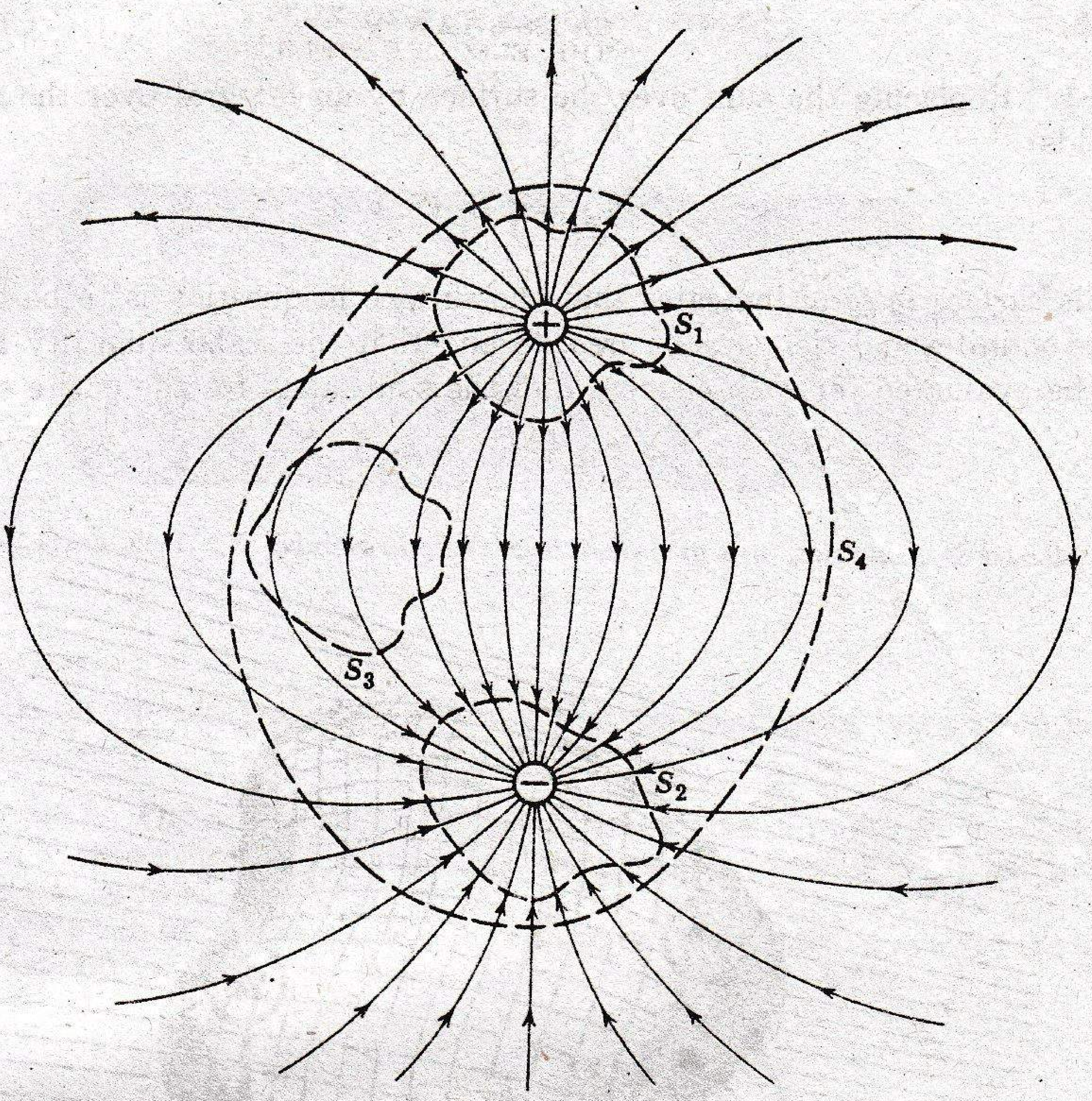

Fig. 28-1 Two equal and opposite charges. The dashed lines represent hypothetical closed surfaces.

The vectors **E** and $\Delta\mathbf{S}$ that characterize each square make an angle θ with each other. Figure 28-2*b* shows an enlarged view of the three squares on the surface of Fig. 28-2*a* marked *x*, *y*, and *z*. Note that at *x*, $\theta > 90°$; at *y*, $\theta = 90°$; and at *z*, $\theta < 90°$.

A semiquantitative definition of flux is

$$\Phi_E \cong \Sigma \mathbf{E}\cdot\Delta\mathbf{S}, \tag{28-1}$$

which instructs us to add up the scalar quantity $\mathbf{E}\cdot\Delta\mathbf{S}$ for all elements of area into which the surface has been divided. For points such as *x* in Fig. 28-2 the contribution to the flux is negative; at *y* it is zero and at *z* it is positive. Thus if **E** is everywhere outward, $\theta < 90°$, $\mathbf{E}\cdot\Delta\mathbf{S}$ will be positive, and Φ_E for the entire surface will be positive; see Fig. 28-1, surface S_1. If **E** is everywhere inward, $\theta > 90°$, $\mathbf{E}\cdot\Delta\mathbf{S}$ will be negative, and Φ_E for the surface will be negative; see Fig. 28-1, surface S_2. From Eq. 28-1 we see that the appropriate mks unit for Φ_E is the newton-meter^2/coul.

The exact definition of electric flux is found in the differential limit of Eq.

28–1. Replacing the sum over the surface by an integral over the surface yields

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{S}. \tag{28–2}$$

This *surface integral* indicates that the surface in question is to be divided into infinitesimal elements of area $d\mathbf{S}$ and that the scalar quantity $\mathbf{E} \cdot d\mathbf{S}$ is to be evaluated for each element and the sum taken for the entire surface.

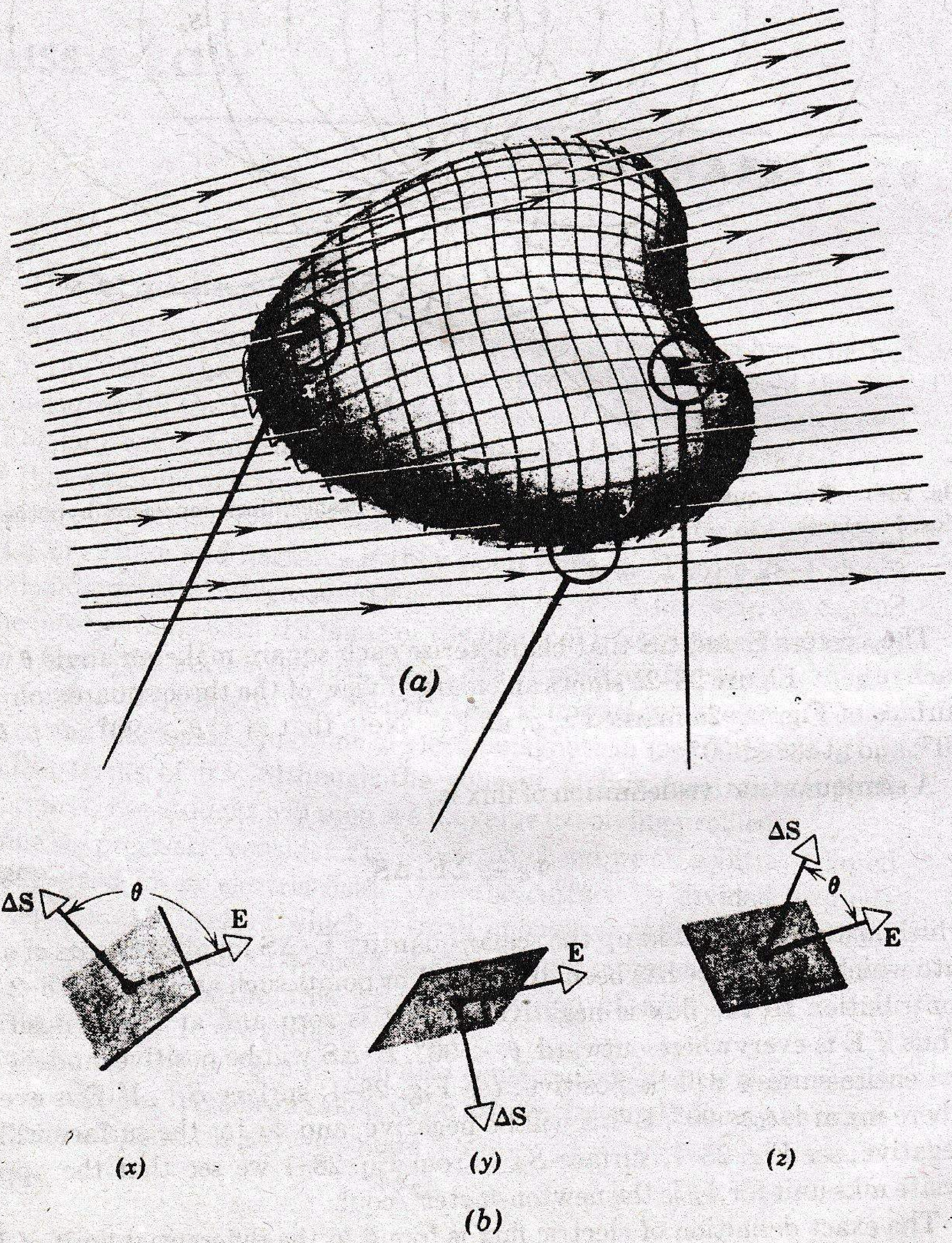

Fig. 28–2 (*a*) A hypothetical surface immersed in an electric field. (*b*) Three elements of area on this surface, shown enlarged.

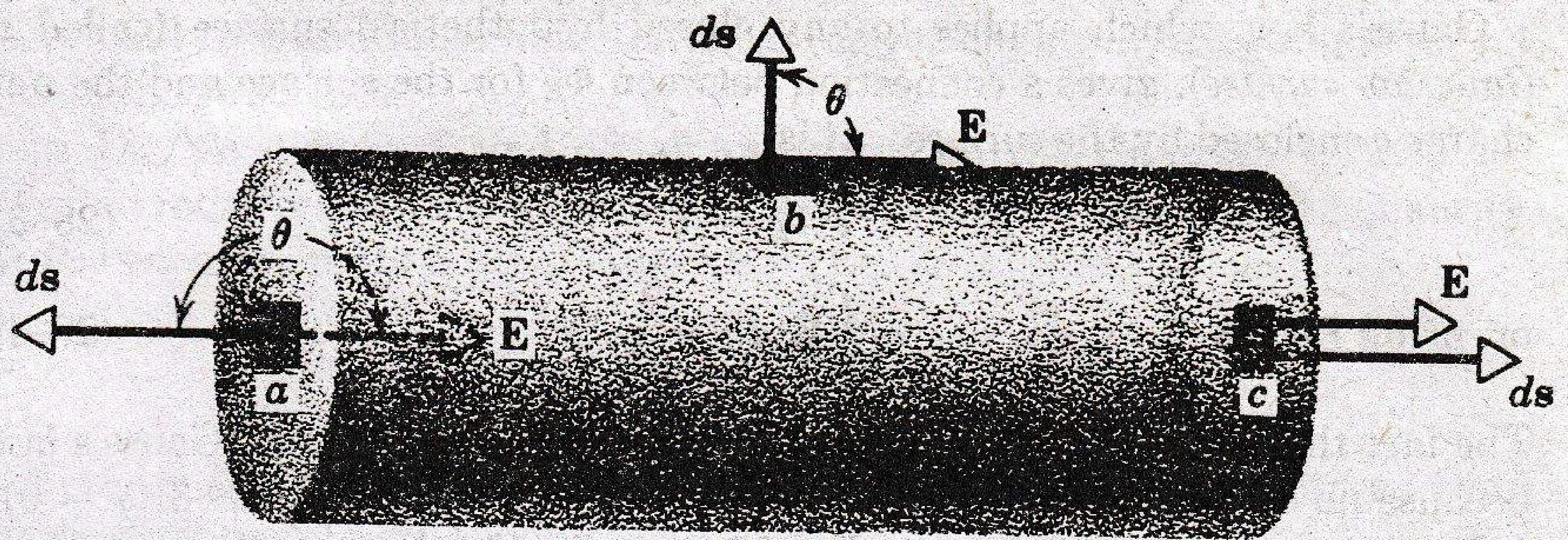

Fig. 28–3 Example 1. A cylindrical surface immersed in a uniform field **E** parallel to its axis.

The circle on the integral sign indicates that the surface of integration is a closed surface.*

▶ **Example 1.** Figure 28–3 shows a hypothetical cylinder of radius R immersed in a uniform electric field **E**, the cylinder axis being parallel to the field. What is Φ_E for this closed surface?

The flux Φ_E can be written as the sum of three terms, an integral over (a) the left cylinder cap, (b) the cylindrical surface, and (c) the right cap. Thus

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{S}$$

$$= \int_{(a)} \mathbf{E} \cdot d\mathbf{S} + \int_{(b)} \mathbf{E} \cdot d\mathbf{S} + \int_{(c)} \mathbf{E} \cdot d\mathbf{S}.$$

For the left cap, the angle θ for all points is 180°, **E** has a constant value, and the vectors $d\mathbf{S}$ are all parallel. Thus

$$\int_{(a)} \mathbf{E} \cdot d\mathbf{S} = \int E \cos 180° \, dS$$

$$= -E \int dS = -ES,$$

where $S\ (= \pi R^2)$ is the cap area. Similarly, for the right cap,

$$\int_{(c)} \mathbf{E} \cdot d\mathbf{S} = +ES,$$

the angle θ for all points being zero here. Finally, for the cylinder wall,

$$\int_{(b)} \mathbf{E} \cdot d\mathbf{S} = 0,$$

because $\theta = 90°$, hence $\mathbf{E} \cdot d\mathbf{S} = 0$ for all points on the cylindrical surface. Thus

$$\Phi_E = -ES + 0 + ES = 0.$$ ◀

* Similarly, a circle on a *line* integral sign indicates a closed *path*. It will be clear from the context and from the differential element ($d\mathbf{S}$ in this case) whether we are dealing with a surface integral or a line integral.

28–2 Gauss's Law

Gauss's law, which applies to any closed hypothetical surface (called a *Gaussian surface*), gives a connection between Φ_E for the surface and the net charge q enclosed by the surface. It is

$$\epsilon_0 \Phi_E = q \qquad (28\text{–}3)$$

or, using Eq. 28–2,

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = q. \qquad (28\text{–}4)$$

The fact that Φ_E proves to be zero in Example 1 is predicted by Gauss's law because no charge is enclosed by the Gaussian surface in Fig. 28-3 ($q = 0$).

Note that q in Eq. 28-3 (or in Eq. 28–4) is the *net* charge, taking its algebraic sign into account. If a surface encloses equal and opposite charges, the flux Φ_E is zero. Charge outside the surface makes no contribution to the value of q, nor does the exact location of the inside charges affect this value.

Gauss's law can be used to evaluate **E** if the charge distribution is so symmetric that by proper choice of a Gaussian surface we can easily evaluate the integral in Eq. 28–4. Conversely, if **E** is known for all points on a given closed surface, Gauss's law can be used to compute the charge inside. If **E** has an outward component for every point on a closed surface, Φ_E, as Eq. 28–2 shows, will be positive and, from Eq. 28–4, there must be a net positive charge within the surface (see Fig. 28–1, surface S_1). If **E** has an inward component for every point on a closed surface, there must be a net negative charge within the surface (see Fig. 28–1, surface S_2). Surface S_3 in Fig. 28–1 encloses no charge, so that Gauss's law predicts that $\Phi_E = 0$. This is consistent with the fact that lines of **E** pass directly through surface S_3, the contribution to the integral on one side canceling that on the other. What would be the value of Φ_E for surface S_4 in Fig. 28–1, which encloses both charges?

28–3 Gauss's Law and Coulomb's Law

Coulomb's law can be deduced from Gauss's law and symmetry considerations. To do so, let us apply Gauss's law to an isolated point charge q as in Fig. 28–4. Although Gauss's law holds for any surface whatever, information can most readily be extracted for a spherical surface of radius r centered

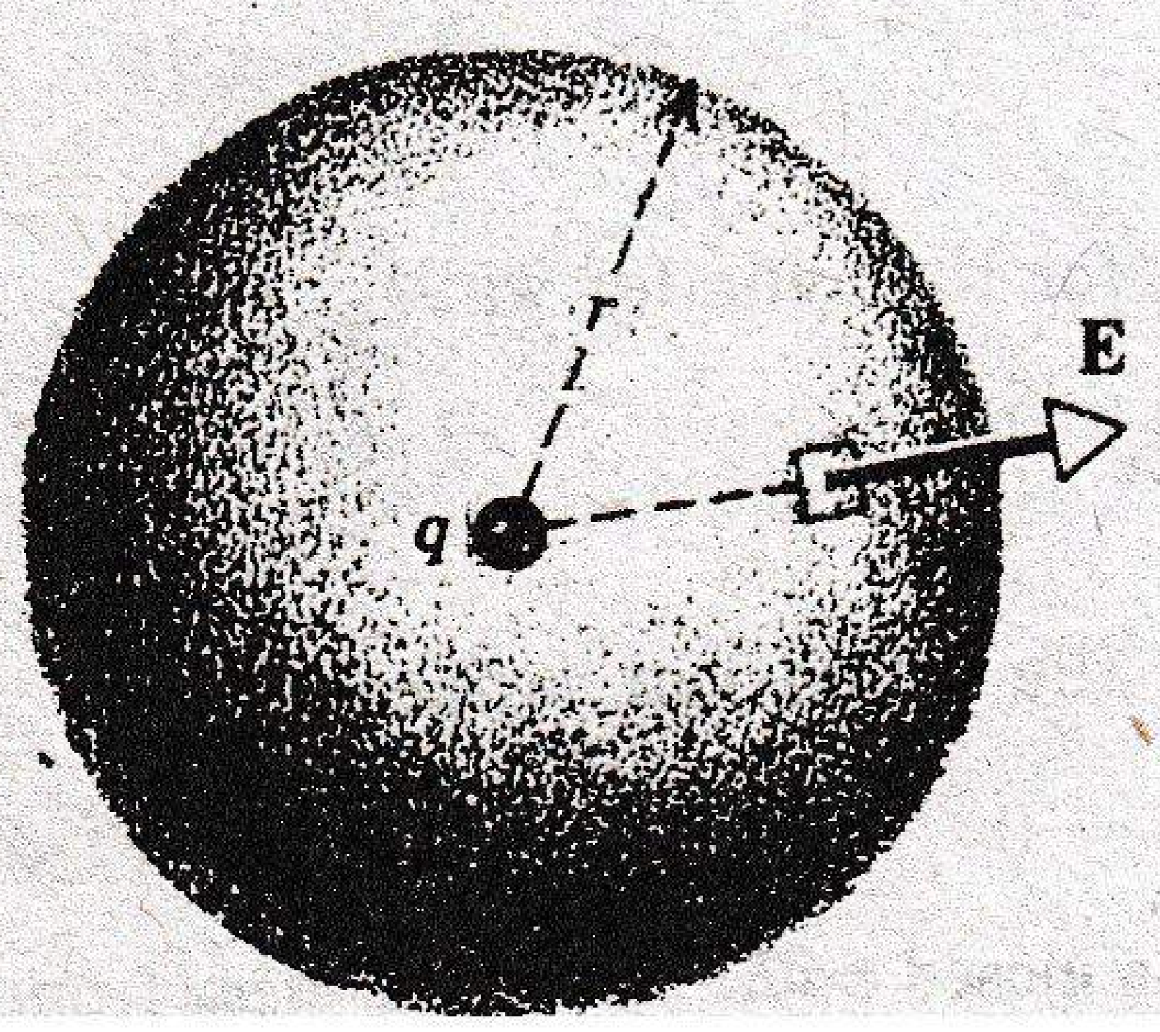

Fig. 28–4 A spherical Gaussian surface of radius r surrounding a point charge q.

on the charge. The advantage of this surface is that, from symmetry, **E** must be normal to it and must have the same (as yet unknown) magnitude for all points on the surface.

In Fig. 28-4 both **E** and $d\mathbf{S}$ at any point on the Gaussian surface are directed radially outward. The angle between them is zero and the quantity $\mathbf{E}\cdot d\mathbf{S}$ becomes simply $E\,dS$. Gauss's law (Eq. 28-4) thus reduces to

$$\epsilon_0 \oint \mathbf{E}\cdot d\mathbf{S} = \epsilon_0 \oint E\,dS = q.$$

Because E is constant for all points on the sphere, it can be factored from inside the integral sign, leaving

$$\epsilon_0 E \oint dS = q,$$

where the integral is simply the area of the sphere.* This equation gives

$$\epsilon_0 E(4\pi r^2) = q$$

or

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}. \tag{28-5}$$

Equation 28-5 gives the magnitude of the electric field strength **E** at any point a distance r from an isolated point charge q. The direction of **E** is already known from symmetry.

Let us put a second point charge q_0 at the point at which **E** is calculated. The magnitude of the force that acts on it (see Eq. 27-2) is

$$F = Eq_0.$$

Combining with Eq. 28-5 gives

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2},$$

which is precisely Coulomb's law. Thus we have deduced Coulomb's law from Gauss's law and considerations of symmetry.

Gauss's law is one of the fundamental equations of electromagnetic theory and is displayed in Table 38-3 as one of Maxwell's equations. Coulomb's law is not listed in that table because, as we have just proved, it can be deduced from Gauss's law and from simple assumptions about the symmetry of **E** due to a point charge.

It is interesting to note that writing the proportionality constant in Coulomb's law as $1/4\pi\epsilon_0$ (see Eq. 26-3) permits a particularly simple form for Gauss's law (Eq. 28-3). If we had written the Coulomb law constant simply as k, Gauss's law would have to be written as $(1/4\pi k)\Phi_E = q$. We prefer to leave the factor 4π in Coulomb's law so that it will not appear in Gauss's law or in other much used relations that will be derived later.

* The usefulness of Gauss's law depends on our ability to find a surface over which, from symmetry, both E and θ (see Fig. 28-2) have constant values. Then $E \cos\theta$ can be factored out of the integral and E can be found simply, as in this example.

28–4 An Insulated Conductor

Gauss's law can be used to make an important prediction, namely: *An excess charge, placed on an insulated conductor, resides entirely on its outer surface.* This hypothesis was shown to be true by experiment (see Section 28–5) before either Gauss's law or Coulomb's law were advanced. Indeed, the experimental proof of the hypothesis is the experimental foundation upon which both laws rest: We have already pointed out that Coulomb's torsion balance experiments, although direct and convincing, are not capable of great accuracy. In showing that the italicized hypothesis is predicted by Gauss's law, we are simply reversing the historical situation.

Figure 28–5 is a cross section of an insulated conductor of arbitrary shape carrying an excess charge q. The dashed lines show a Gaussian surface that lies a small distance below the actual surface of the conductor. Although the Gaussian surface can be as close to the actual surface as we wish, it is important to keep in mind that the Gaussian surface is *inside* the conductor.

When an excess charge is placed at random on an insulated conductor, it will set up electric fields inside the conductor. These fields act on the charge carriers of the conductor and cause them to move, that is, they set up internal currents. These currents redistribute the excess charge in such a way that the internal electric fields are automatically reduced in magnitude. Eventually the electric fields inside the conductor become zero everywhere, the currents automatically stop, and electrostatic conditions prevail. This redistribution of charge normally takes place in a time that is negligible for most purposes. What can be said about the distribution of the excess charge when such electrostatic conditions have been achieved?

If, at electrostatic equilibrium, **E** is zero everywhere inside the conductor, it must be zero for every point on the Gaussian surface. This means that the flux Φ_E for this surface must be zero. Gauss's law then predicts (see Eq. 28–3) that there must be no net charge inside the Gaussian surface. If the excess charge q is not *inside* this surface, it can only be *outside* it, that is, *it must be on the actual surface of the conductor*.

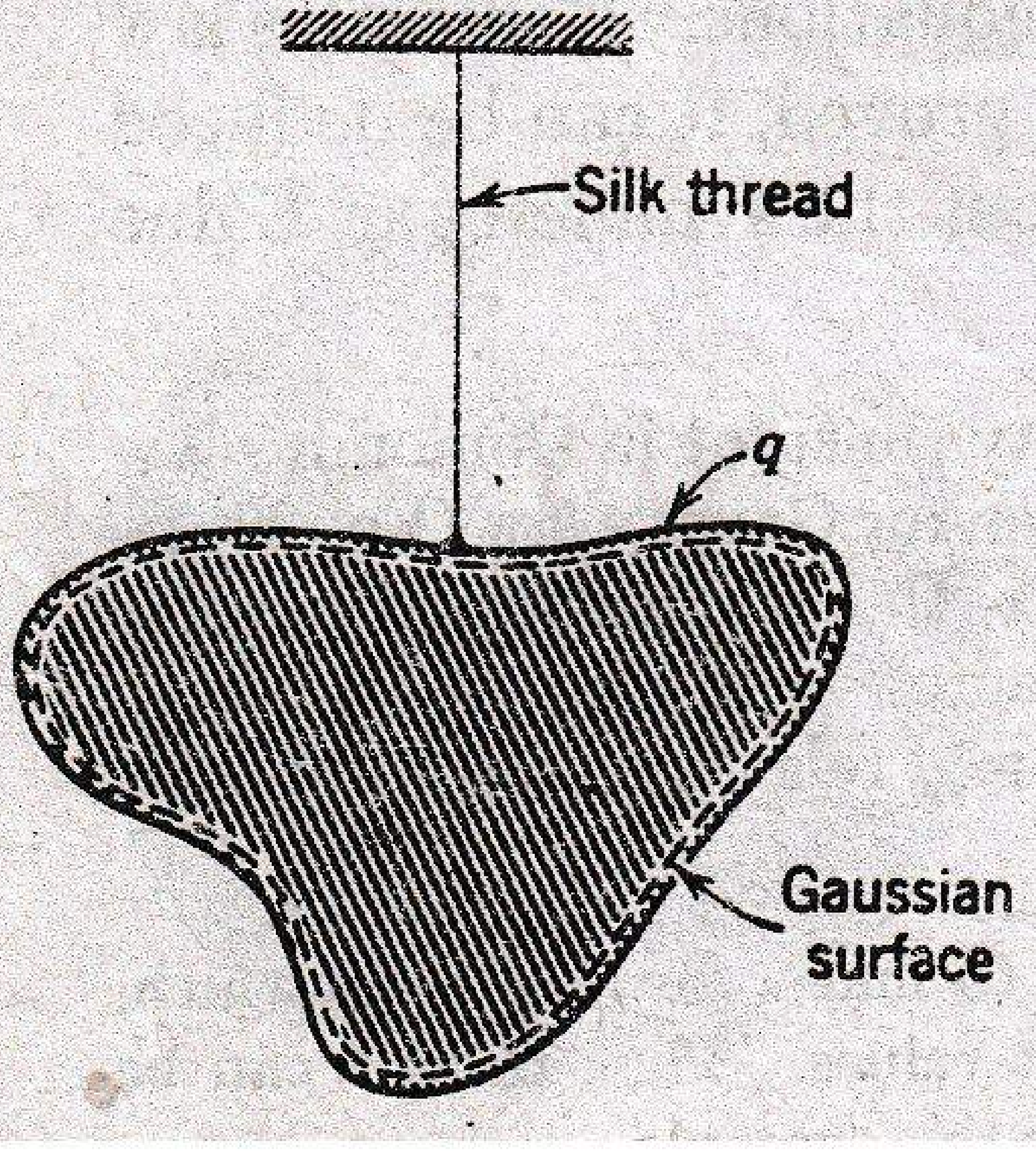

Fig. 28–5 An insulated conductor.

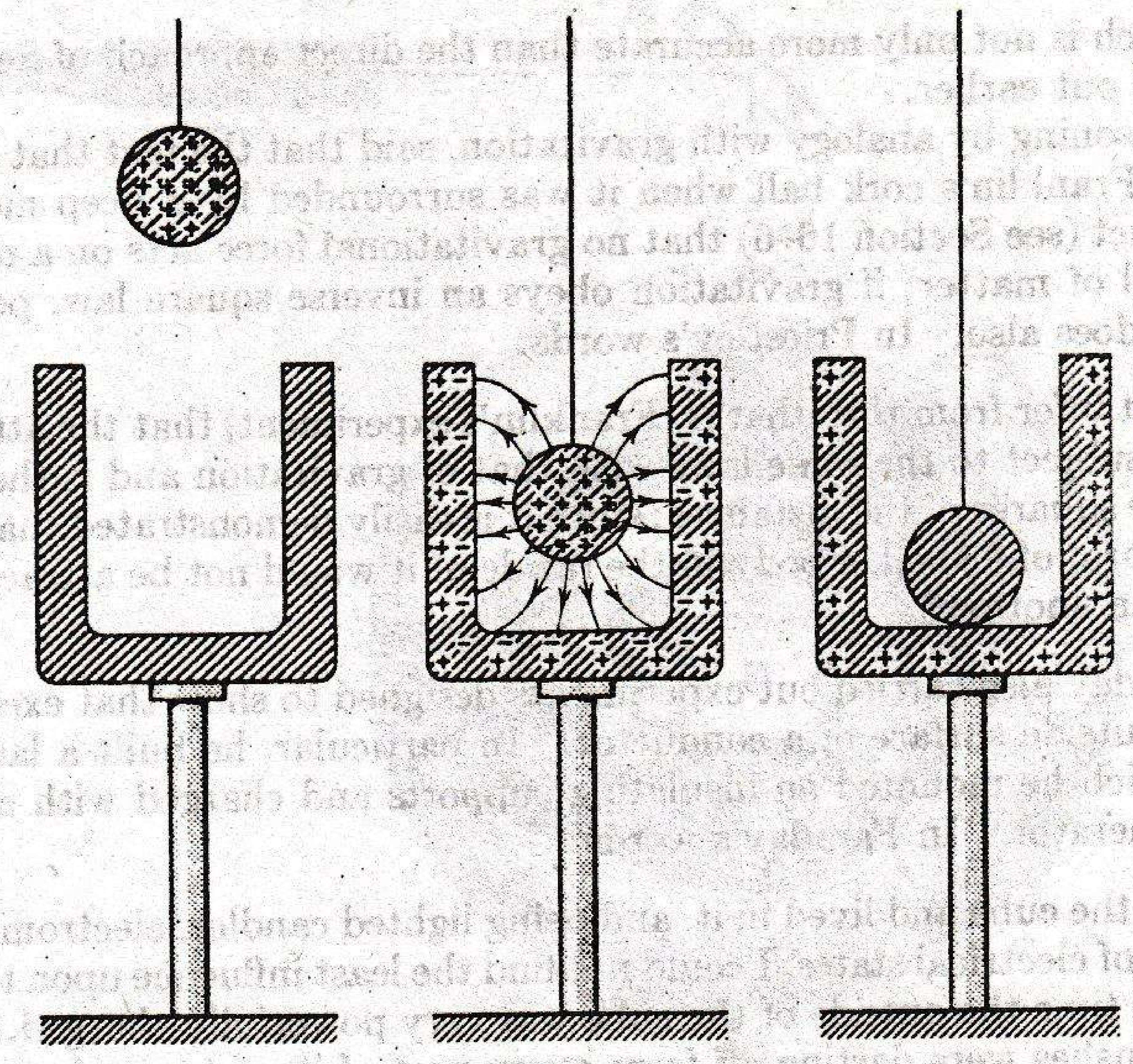

Fig. 28–6 The *entire* charge on the ball is transferred to the *outside* of the can. This statement and the discussion of the first paragraph of Section 28–5 are strictly correct only if the can is provided with a conducting lid which can be closed after the ball is inserted.

28–5 Experimental Proof of Gauss's and Coulomb's Laws

Let us turn to the experiments that prove that the hypothesis of Section 28–4. is true. For a simple test, charge a metal ball and lower it with a silk thread deep into a metal can as in Fig. 28–6. Touch the ball to the inside of the can; when the ball is removed from the can, all its charge will have vanished. When the metal ball touches the can, the ball and can together form an "insulated conductor" to which the hypothesis of Section 28–4 applies. That the charge moves entirely to the outside surface of the can can be shown by touching a small insulated metal object to the can; only on the *outside* of the can will it be possible to pick up a charge.

Benjamin Franklin seems to have been the first to notice that there can be no charge inside an insulated metal can. In 1755 he wrote to a friend:

> I electrified a silver pint cann, on an electric stand, and then lowered into it a cork-ball, of about an inch diameter, hanging by a silk string, till the cork touched the bottom of the cann. The cork was not attracted to the inside of the cann as it would have been to the outside, and though it touched the bottom, yet when drawn out, it was not found to be electrified by that touch, as it would have been by touching the outside. The fact is singular. You require the reason; I do not know it. . . .

About ten years later Franklin recommended this "singular fact" to the attention of his friend Joseph Priestley (1733–1804). In 1767 (about twenty years before Coulomb's experiments) Priestley checked Franklin's observation and, with remarkable insight, realized that the inverse square law of force followed from it. Thus the indirect approach is not only more accurate than the direct approach of Section 26–4 but was carried out earlier.

Priestley, reasoning by analogy with gravitation, said that the fact that no electric force acted on Franklin's cork ball when it was surrounded by a deep metal can is similar to the fact (see Section 16–6) that no gravitational force acts on a mass inside a spherical shell of matter; if gravitation obeys an inverse square law, perhaps the electrical force does also. In Priestley's words:

> May we not infer from this [that is, Franklin's experiment] that the attraction of electricity is subject to the same laws with that of gravitation and is therefore according to the squares of the distances; since it is easily demonstrated that were the earth in the form of a shell, a body in the inside of it would not be attracted to one side more than another?

Michael Faraday also carried out experiments designed to show that excess charge resides on the outside surface of a conductor. In particular, he built a large metal-covered box which he mounted on insulating supports and charged with a powerful electrostatic generator. In Faraday's words:

> I went into the cube and lived in it, and using lighted candles, electrometers, and all other tests of electrical states, I could not find the least influence upon them . . . though all the time the outside of the cube was very powerfully charged, and large sparks and brushes were darting off from every part of its outer surface.

Henry Cavendish (1731–1810) carried out an improved version of the experiment of Fig. 28–6. With the instruments available to him, Cavendish proved experimentally that the exponent in the force law lay, with high probability, between 2.02 and 1.98. Cavendish, however, did not publish his results so that almost nobody knew about them at the time. Maxwell repeated Cavendish's experiment with more accuracy and set these limits as 2.00005 and 1.99995. In 1936 Plimpton and Lawton repeated the experiment again; they set the probability limits as 2.000000002 and 1.999999998.

Figure 28–7 is an idealized sketch of the apparatus of Plimpton and Lawton. It consists in principle of two concentric metal shells, A and B, the former being 5 ft in diameter. The inner shell contains a sensitive electrometer E connected so that it will indicate whether any charge moves between shells A and B.

By throwing switch S to the left, a substantial charge can be placed on the sphere assembly. If any of this charge moves to shell B, it will have to pass through the electrometer and will cause a deflection, which can be observed optically using telescope T, mirror M, and windows W.

However, when the switch S is thrown alternately from left to right, thus connecting the shell assembly either to the battery or to the ground, no effect is observed on the galvanometer. This is the strongest experimental evidence to date that the hypothesis of Section 28–4 is correct. Knowing the sensitivity of their electrometer, Plimpton and Lawton calculated that the exponent in Coulomb's law lies, with high probability, between the limits already stated.

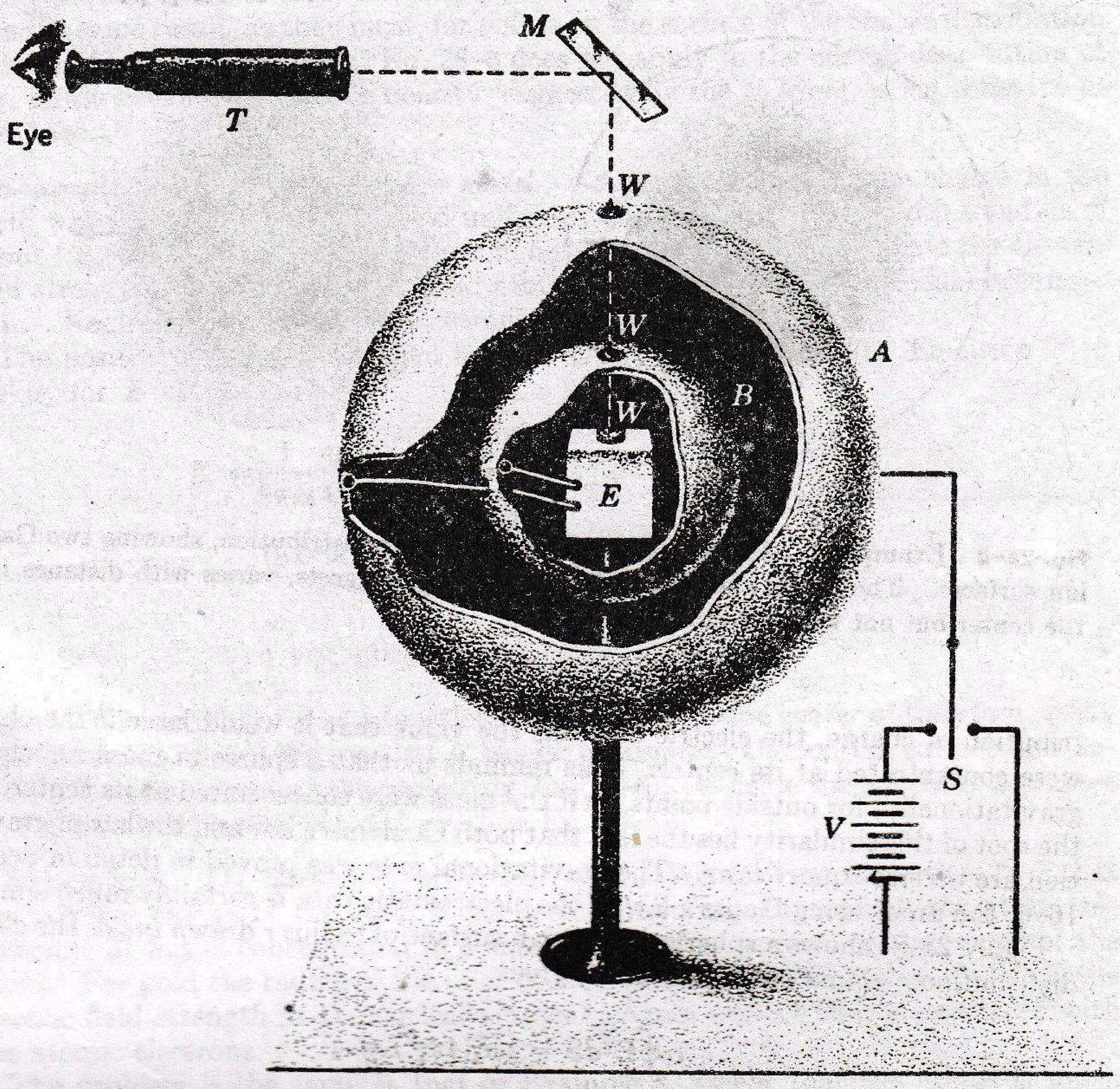

Fig. 28-7 The apparatus of Plimpton and Lawton.

28-6 Gauss's Law—Some Applications

Gauss's law can be used to calculate **E** if the symmetry of the charge distribution is high. One example of this, the calculation of **E** for a point charge, has already been discussed (Eq. 28-5). Here we present other examples.

▶ **Example 2.** *Spherically symmetric charge distribution.* Figure 28-8 shows a spherical distribution of charge of radius R. The *charge density* ρ (that is, the charge per unit volume, measured in coul/meter^3) at any point depends only on the distance of the point from the center and not on the direction, a condition called *spherical symmetry*. Find an expression for E for points (a) outside and (b) inside the charge distribution. Note that the object in Fig. 28-8 cannot be a conductor or, as we have seen, the excess charge will reside on its surface.

Applying Gauss's law to a spherical Gaussian surface of radius r in Fig. 28-8a (see Section 28-3) leads exactly to Eq. 28-5, or

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}, \tag{28-5}$$

where q is the total charge. Thus for points outside a spherically symmetric dis-

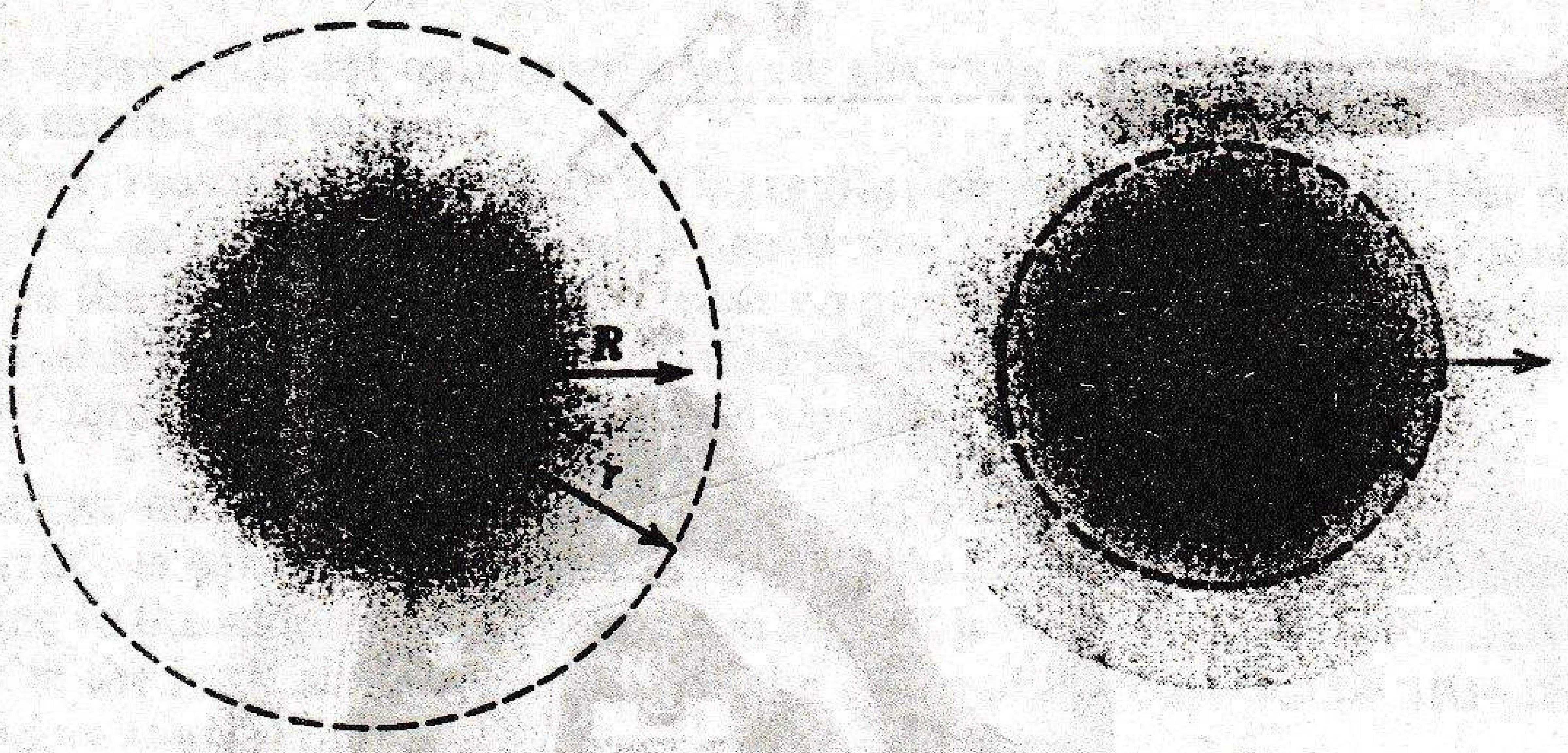

Fig. 28–8 Example 2. A spherically symmetric charge distribution, showing two Gaussian surfaces. The density of charge, as the shading suggests, varies with distance from the center but not with direction.

tribution of charge, the electric field has the value that it would have if the charge were concentrated at its center. This reminds us that a sphere of mass m behaves gravitationally, for outside points, as if the mass were concentrated at its center. At the root of this similarity lies the fact that both Coulomb's law and the law of gravitation are inverse square laws. The gravitational case was proved in detail in Section 16–6; the proof using Gauss's law in the electrostatic case is certainly much simpler.

Figure 28–8*b* shows a spherical Gaussian surface of radius r drawn *inside* the charge distribution. Gauss's law (Eq. 28–4) gives

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = \epsilon_0 E(4\pi r^2) = q'$$

or

$$E = \frac{1}{4\pi\epsilon_0} \frac{q'}{r^2},$$

in which q' is that part of q contained within the sphere of radius r. The part of q that lies outside this sphere makes no contribution to $\mathbf{E}$ at radius r. This corresponds, in the gravitational case (Section 16–6), to the fact that a spherical shell of matter exerts no gravitational force on a body inside it.

An interesting special case of a spherically symmetric charge distribution is a uniform sphere of charge. For such a sphere, which would be suggested by uniform shading in Fig. 28–8, the charge density ρ would have a constant value for all points within a sphere of radius R and would be zero for all points outside this sphere. For points inside such a uniform sphere of charge we can put

$$q' = q \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}$$

or

$$q' = q \left(\frac{r}{R}\right)^3$$

where $\frac{4}{3}\pi R^3$ is the volume of the spherical charge distribution. The expression for E then becomes

$$E = \frac{1}{4\pi\epsilon_0} \frac{qr}{R^3}. \qquad (28\text{–}6)$$

This equation becomes zero, as it should, for $r = 0$. Note that Eqs. 28-5 and 28-6 give the same result, as they must, for points on the surface of the charge distribution (that is, if $r = R$). Note that Eq. 28-6 does not apply to the charge distribution of Fig. 28-8*b* because the charge density, suggested by the shading, is *not* constant in that case.

Example 3. *The Thomson atom model.* At one time the positive charge in the atom was thought to be distributed uniformly throughout a sphere with a radius of about 1.0×10^{-10} meter, that is, throughout the entire atom. Calculate the electric field strength at the surface of a gold atom ($Z = 79$) on this (erroneous) assumption. Neglect the effect of the electrons.

The positive charge of the atom is Ze or $(79)(1.6 \times 10^{-19}$ coul). Equation 28-5 yields, for E at the surface,

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$= \frac{(9.0 \times 10^9 \text{ nt-m}^2/\text{coul}^2)(79)(1.6 \times 10^{-19} \text{ coul})}{(1.0 \times 10^{-10} \text{ meter})^2}$$

$$= 1.1 \times 10^{13} \text{ nt/coul.}$$

Figure 28-9 is a plot of E as a function of distance from the center of the atom, using Eqs. 28-5 and 28-6. We see that E has its maximum value on the surface and decreases linearly to zero at the center (see Eq. 28-6). Outside the sphere E decreases as the inverse square of the distance (see Eq. 28-5).

Example 4. *The Rutherford, or nuclear, atom.* We shall see in Section 28-7 that the positive charge of the atom is *not* spread uniformly throughout the atom (see Example 3) but is concentrated in a small region (the *nucleus*) at the center of the atom. For gold the radius of the nucleus is about 6.9×10^{-15} meter. What is the electric field strength at the nuclear surface? Again neglect effects associated with the atomic electrons.

The problem is the same as that of Example 3, except that the radius is much smaller. This will make the electric field strength at the surface larger, in proportion to the ratio of the squares of the radii. Thus

$$E = (1.1 \times 10^{13} \text{ nt/coul})\frac{(1.0 \times 10^{-10} \text{ meter})^2}{(6.9 \times 10^{-15} \text{ meter})^2}$$

$$= 2.3 \times 10^{21} \text{ nt/coul.}$$

This is an enormous electric field, much stronger than could be produced and maintained in the laboratory. It is about 10^8 times as large as the field calculated in Example 3.

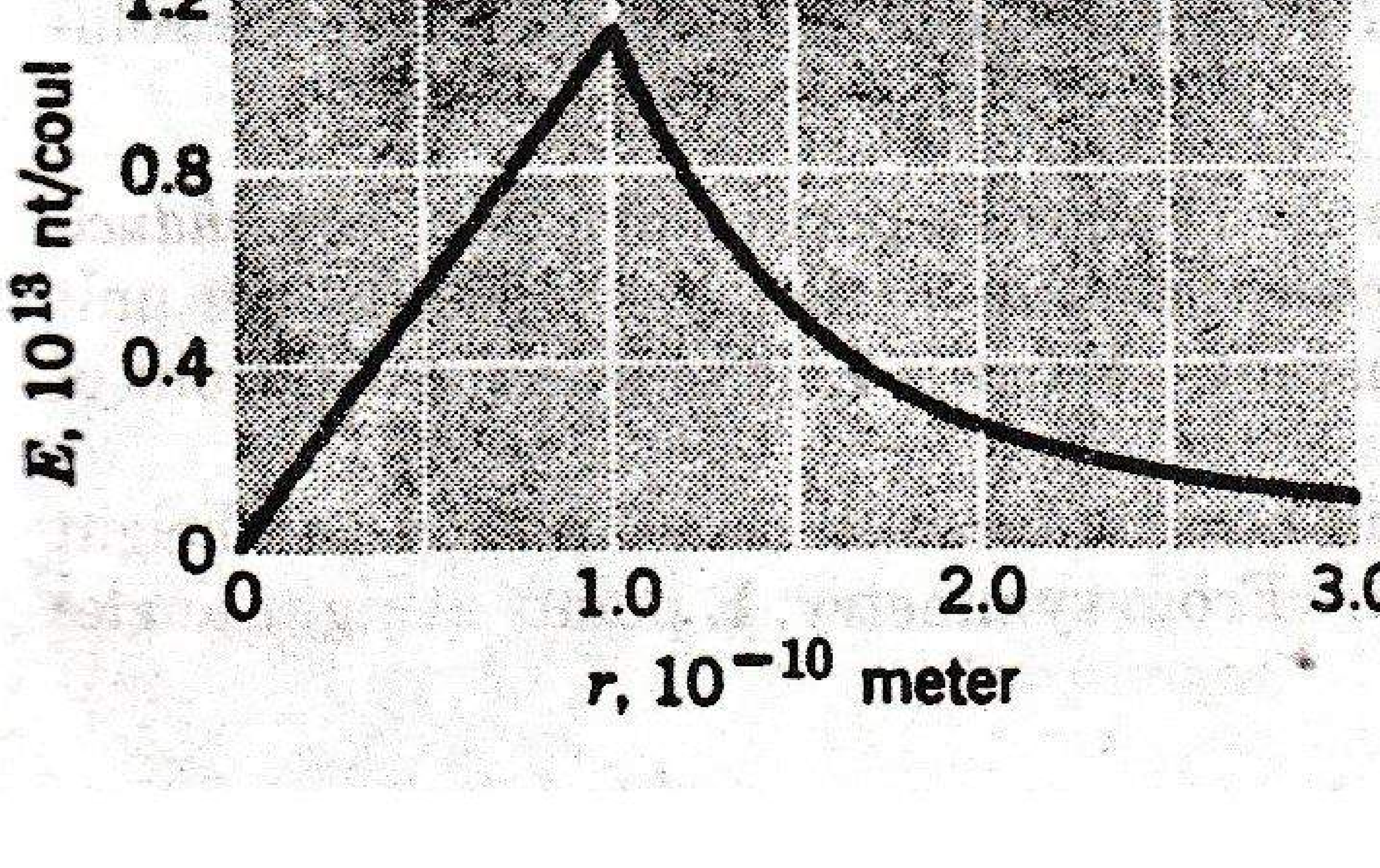

Fig. 28-9 Example 3. The electric field due to the positive charge in a gold atom, according to the (erroneous) Thomson model.

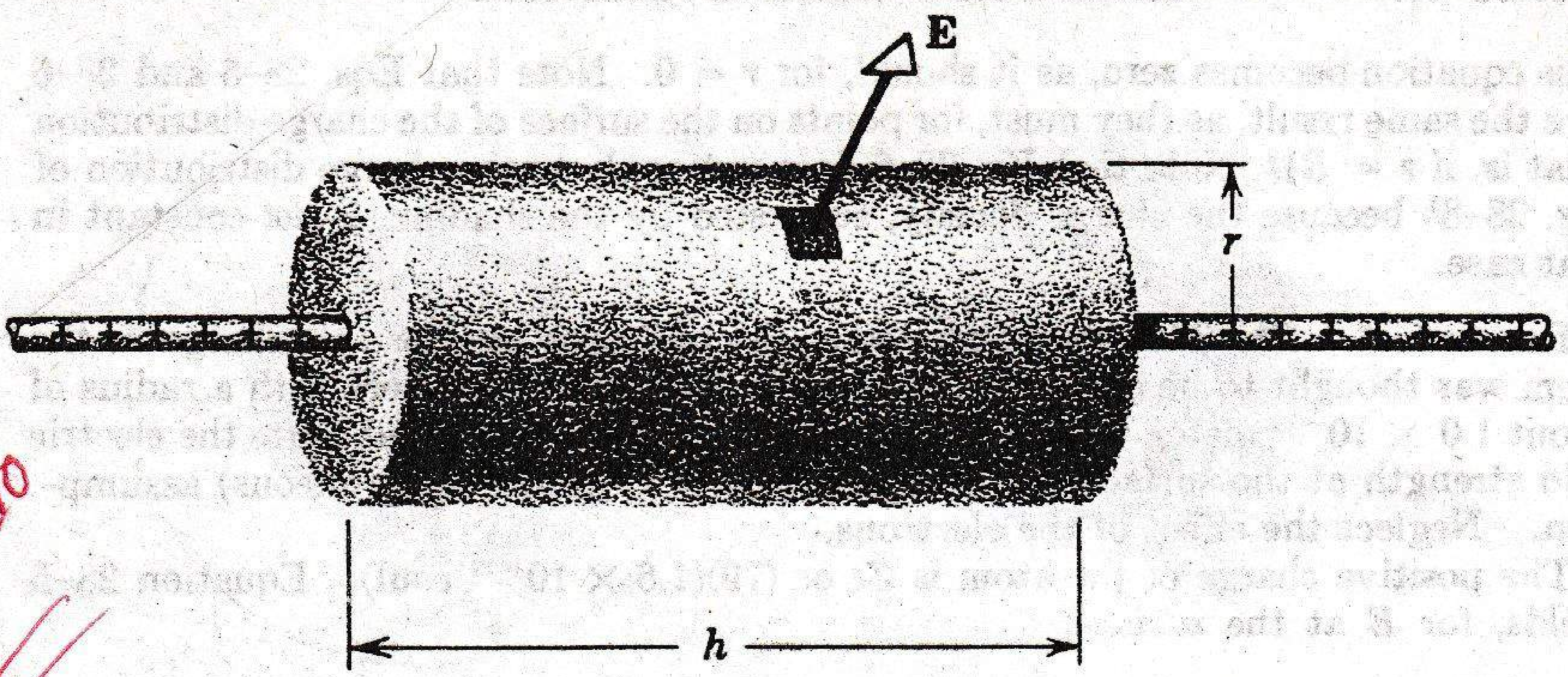

Fig. 28–10 Example 5. An infinite rod of charge, showing a cylindrical Gaussian surface.

Example 5. *Line of charge.* Figure 28–10 shows a section of an infinite rod of charge, the *linear charge density* λ (that is, the charge per unit length, measured in coul/meter) being constant for all points on the line. Find an expression for E at a distance r from the line.

From symmetry, $\mathbf{E}$ due to a uniform linear charge can only be radially directed. As a Gaussian surface we choose a circular cylinder of radius r and length h, closed at each end by plane caps normal to the axis. E is constant over the cylindrical surface and the flux of $\mathbf{E}$ through this surface is $E(2\pi rh)$ where $2\pi rh$ is the area of the surface. There is no flux through the circular caps because $\mathbf{E}$ here lies in the surface at every point.

The charge enclosed by the Gaussian surface of Fig. 28–10 is λh. Gauss's law (Eq. 28–4),

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = q,$$

then becomes

$$\epsilon_0 E(2\pi rh) = \lambda h,$$

whence

$$E = \frac{\lambda}{2\pi\epsilon_0 r}. \tag{28–7}$$

The direction of $\mathbf{E}$ is radially outward for a line of positive charge.

Note how much simpler the solution using Gauss's law is than that using integration methods, as in Example 6, Chapter 27. Note too that the solution using Gauss's law is possible only if we choose our Gaussian surface to take full advantage of the radial symmetry of the electric field set up by a long line of charge. We are free to choose any surface, such as a cube or a sphere, for a Gaussian surface. Even though Gauss's law holds for all such surfaces, they are not all useful for the problem at hand; only the cylindrical surface of Fig. 28–10 is appropriate in this case.

Gauss's law has the property that it provides a useful technique for calculation only in problems that have a certain degree of symmetry, but in these problems the solutions are strikingly simple.

Example 6. *A sheet of charge.* Figure 28–11 shows a portion of a thin, *nonconducting*, infinite sheet of charge, the *surface charge density* σ (that is, the charge per unit area, measured in coul/meter²) being constant. What is $\mathbf{E}$ at a distance r in front of the plane?

A convenient Gaussian surface is a "pill box" of cross-sectional area A and height $2r$, arranged to pierce the plane as shown. From symmetry, $\mathbf{E}$ points at right angles

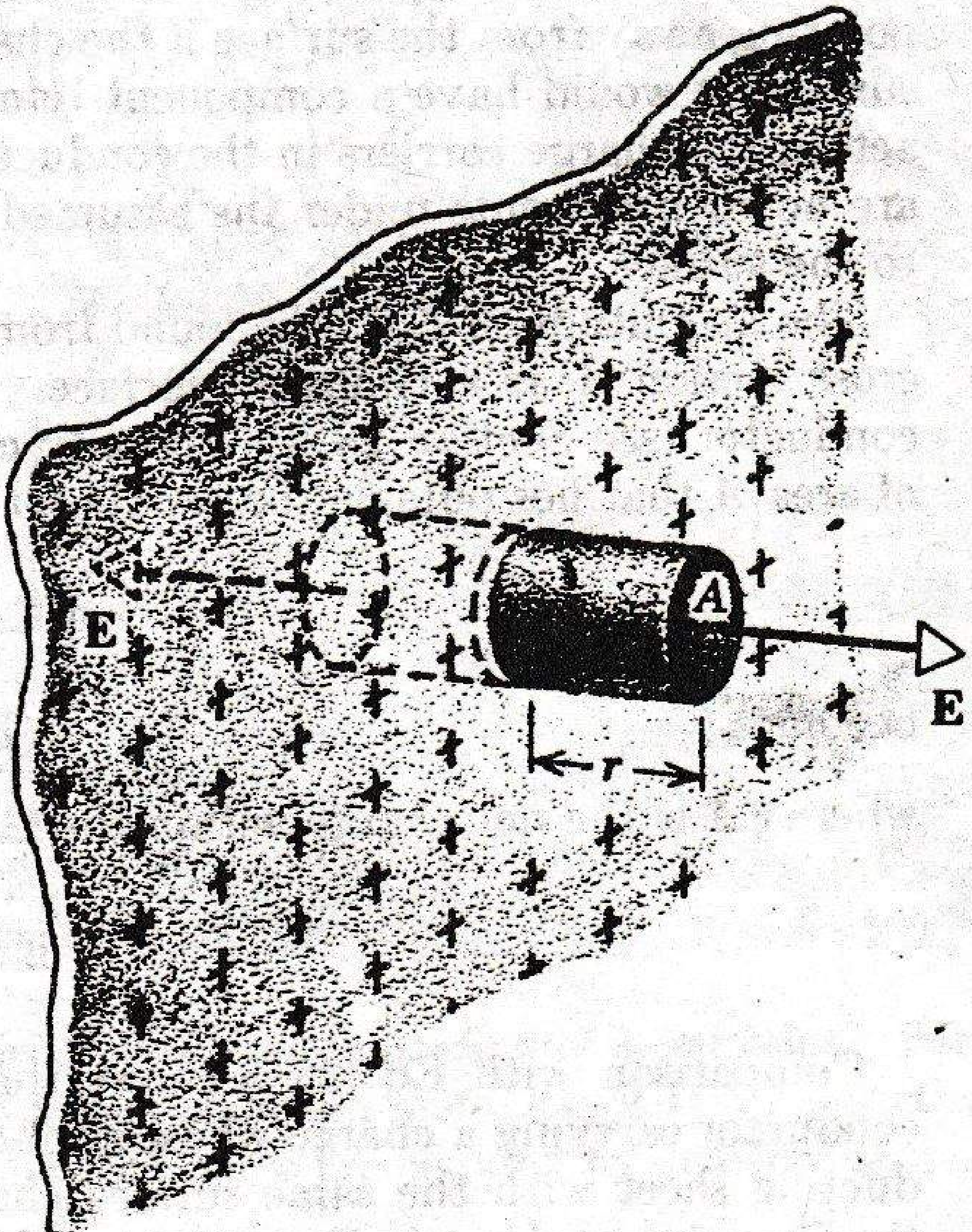

Fig. 28-11 Example 6. An infinite sheet of charge pierced by a cylindrical Gaussian surface. The cross section of the cylinder need not be circular, as shown, but can have an arbitrary shape.

to the end caps and away from the plane. Since **E** does not pierce the cylindrical surface, there is no contribution to the flux from this source. Thus Gauss's law,

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = q$$

$$\epsilon_0(EA + EA) = \sigma A$$

where σA is the enclosed charge. This gives

$$E = \frac{\sigma}{2\epsilon_0}. \tag{28-8}$$

Note that E is the same for all points on each side of the plane; compare Fig. 27-2. Although an infinite sheet of charge cannot exist physically, this derivation is still useful in that Eq. 28-8 yields substantially correct results for real (not infinite) charge sheets if we consider only points not near the edges whose distance from the sheet is small compared to the dimensions of the sheet.

Example 7. *A charged conductor.* Figure 28-12 shows a *conductor* carrying on its surface a charge whose surface charge density at any point is σ; in general σ will vary from point to point. What is **E** for points a short distance above the surface?

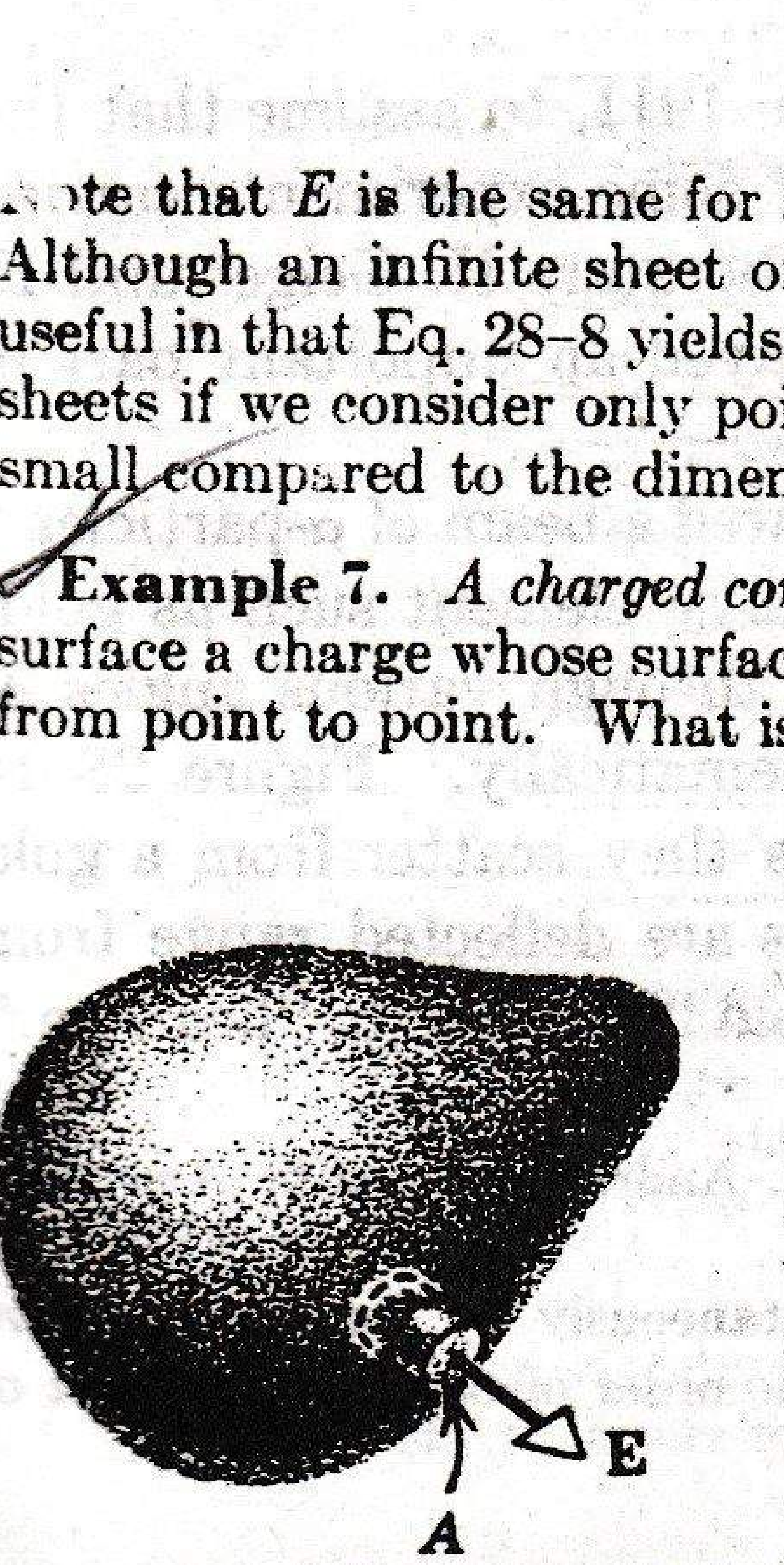

Fig. 28-12 Example 7. A charged insulated conductor, showing a Gaussian surface. The cross section of the surface need not be circular, as shown, but can have an arbitrary shape.

The direction of **E** *for points close to the surface* is at right angles to the surface, pointing away from the surface if the charge is positive. If **E** were *not* normal to the surface, it would have a component lying in the surface. Such a component would act on the charge carriers in the conductor and set up surface currents. Since there are no such currents under the assumed electrostatic conditions, **E** must be normal to the surface.

The magnitude of **E** can be found from Gauss's law using a small flat "pill box" of cross section A as a Gaussian surface. Since **E** equals zero everywhere inside the conductor (see Section 28–4), the only contribution to Φ_E is through the plane cap of area A that lies outside the conductor. Gauss's law

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = q$$

becomes

$$\epsilon_0(EA) = \sigma A$$

where σA is the net charge within the Gaussian surface. This yields

$$E = \frac{\sigma}{\epsilon_0}. \tag{28–9}$$

Comparison with Eq. 28–8 shows that the electric field is *twice as great* near a conductor carrying a charge whose surface charge density is σ as that near a nonconducting sheet with the same surface charge density. The student should compare the Gaussian surfaces in Figs. 28–11 and 28–12 carefully. In Fig. 28–11 lines of force leave the surface through *each* end cap, an electric field existing on *both* sides of the sheet. In Fig. 28–12 the lines of force leave only through the *outside* end cap, the inner end cap being inside the conductor where no electric field exists. If we assume the same surface charge density and cross-sectional area A for the two Gaussian surfaces, the enclosed charge ($= \sigma A$) will be the same. Since, from Gauss's law, the flux Φ_E must then be the same in each case, it follows that E ($= \Phi_E/A$) must be twice as large in Fig. 28–12 as in Fig. 28–11. It is helpful to note that in Fig. 28–11 half the flux emerges from one side of the surface and half from the other, whereas in Fig. 28–12 all the flux emerges from the outside surface. ◀

28–7 The Nuclear Model of the Atom

Ernest Rutherford (1871–1937) was first led, in 1911, to assume that the atomic nucleus existed when he tried to interpret some experiments carried out at the University of Manchester by his collaborators H. Geiger and E. Marsden.* The results of Examples 3 and 4 played an important part in Rutherford's analysis of these experiments.

These workers, at Rutherford's suggestion, allowed a beam of α-particles † to strike and be deflected by a thin film of a heavy element such as gold. They counted the number of particles deflected through various angles ϕ. Figure 28–13 shows the experimental setup schematically. Figure 28–14 shows the paths taken by typical α-particles as they scatter from a gold atom; the angles ϕ through which the α-particles are deflected range from 0 to 180° as the character of the collision varies from "grazing" to "head-on."

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* See "The Birth of the Nuclear Atom," E. N. da C. Andrade, *Scientific American*, November 1956. See also Example 5, Chapter 10.

† α-Particles are helium nuclei that are emitted spontaneously by some radioactive materials such as radium. They move with speeds of the order of one-thirtieth that of light when so emitted.

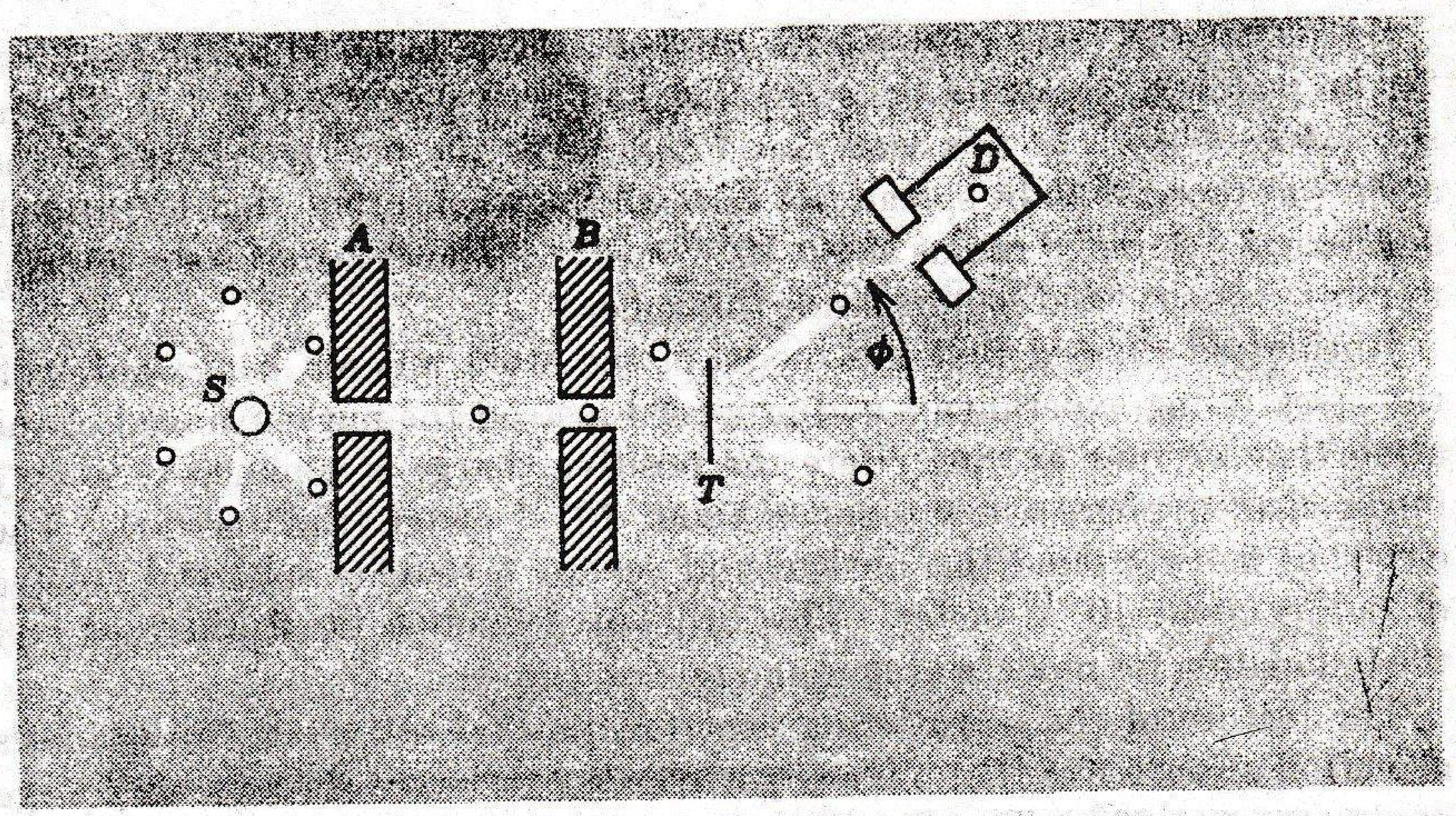

Fig. 28-13 Experimental arrangement for studying the scattering of α-particles. Particles from radioactive source S are allowed to fall on a thin metal "target" T; α-particles scattered by the target through an (adjustable) angle ϕ are counted by detector D.

The electrons in the gold atom, being so light, have almost no effect on the motion of an oncoming α-particle; the electrons are themselves strongly deflected, just as a swarm of insects would be by a stone hurled through them. Any deflection of the α-particle must be caused by the repulsive action of the positive charge of the gold atom, which is known to possess most of the mass of the atom.

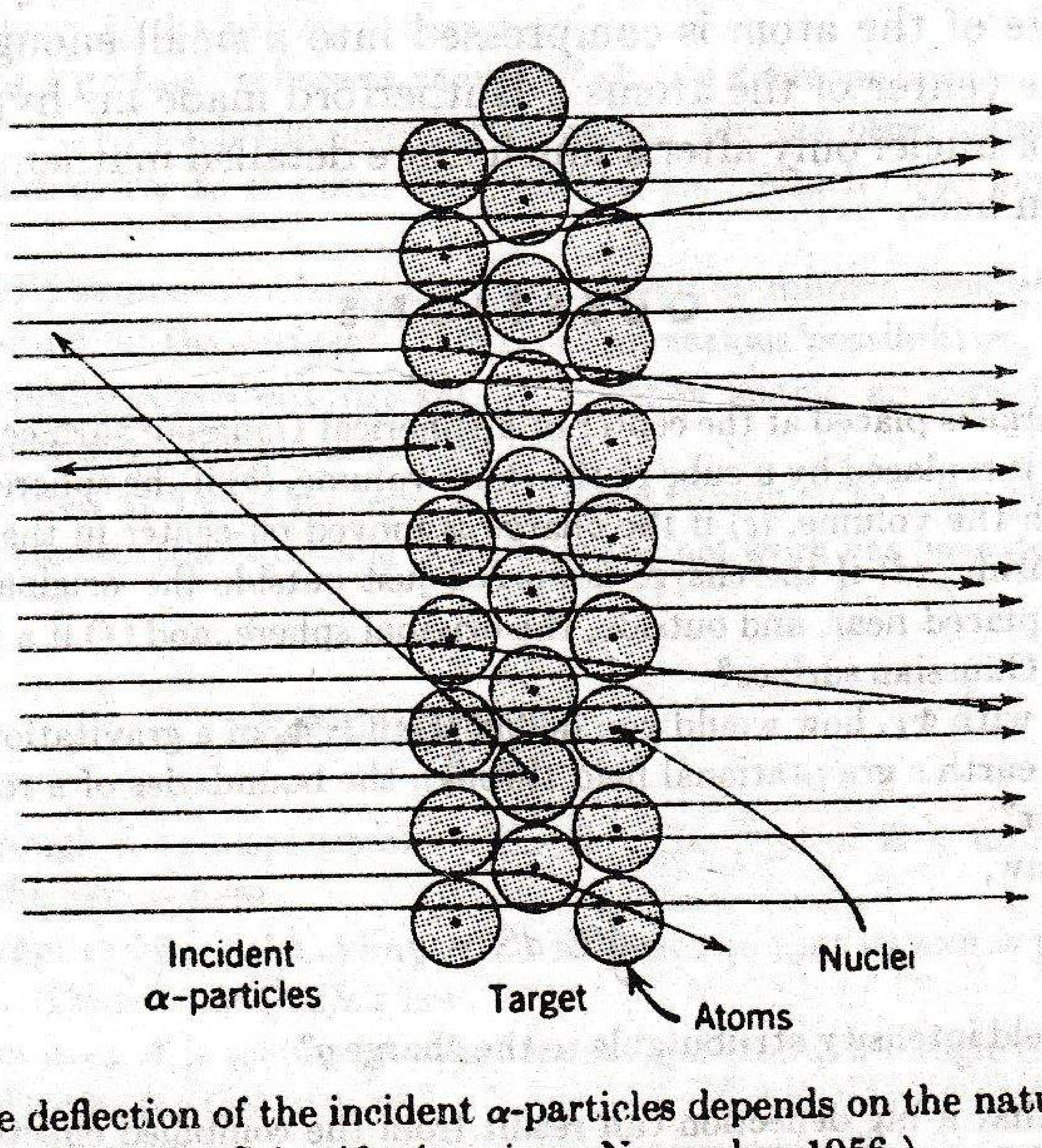

Fig. 28-14 The deflection of the incident α-particles depends on the nature of the nucle collision. (From Andrade, *Scientific American*, November 1956.)

At the time of these experiments most physicists believed in the so-called "plum pudding" model of the atom that had been suggested by J. J. Thomson (1856–1940). In this view (see Example 3) the positive charge of the atom was thought to be spread out through the whole atom, that is, through a spherical volume of radius about 10^{-10} meter. The electrons were thought to vibrate about fixed centers inside this sphere.

Rutherford showed that this model of the atom was not consistent with the α-scattering experiments and proposed instead the nuclear model of the atom that we now accept. Here the positive charge is confined to a very much smaller sphere whose radius is about 10^{-14} meter (the *nucleus*). The electrons move around this nucleus and occupy a roughly spherical volume of radius about 10^{-10} meter. This brilliant deduction by Rutherford laid the foundation for modern atomic and nuclear physics.

The feature of the α-scattering experiments that attracted Rutherford's attention at once was that a few α-particles are deflected through very large angles, up to 180°. To scientists accustomed to thinking in terms of the "plum pudding" model, this was a very surprising result. In Rutherford's words: "It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

The α-particle must pass through a region in which the electric field strength is very high indeed in order to be deflected so strongly.* Example 3 shows that, in Thomson's model, the maximum electric field strength is 1.1×10^{13} nt/coul. Compare this with the value calculated in Example 4 for a point on the surface of a gold nucleus (2.3×10^{21} nt/coul). Thus the deflecting force acting on an α-particle can be up to 10^8 times as great if the positive charge of the atom is compressed into a small enough region (the nucleus) at the center of the atoms. Rutherford made his hypothesis about the existence of nuclei only after a much more detailed mathematical analysis than that given here.

QUESTIONS

1. A point charge is placed at the center of a spherical Gaussian surface. Is Φ_E changed (*a*) if the surface is replaced by a cube of the same volume, (*b*) if the sphere is replaced by a cube of one-tenth the volume, (*c*) if the charge is moved off-center in the original sphere, still remaining inside, (*d*) if the charge is moved just outside the original sphere, (*e*) if a second charge is placed near, and outside, the original sphere, and (*f*) if a second charge is placed inside the Gaussian surface?

2. By analogy with Φ_E, how would you define the flux Φ_g of a gravitational field? What is the flux of the earth's gravitational field through the boundaries of a room, assumed to contain no matter?

3. In Gauss's law,

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = q,$$

is $\mathbf{E}$ the electric field intensity attributable to the charge q?

* The chance that a big deflection can result from the combined effects of many small deflections can be shown to be very small.

4. Show that Eq. 18–3 illustrates what might be called *Gauss's law for incompressible fluids*, or

$$\Phi_v = \oint \mathbf{v} \cdot d\mathbf{S} = 0.$$

5. A surface encloses an electric dipole. What can you say about Φ_E for this surface?

6. Suppose that a Gaussian surface encloses no net charge. Does Gauss's law require that **E** equal zero for all points on the surface? Is the converse of this statement true, that is, if **E** equals zero everywhere on the surface, does Gauss's law require that there be no net charge inside?

7. Would Gauss's law hold if the exponent in Coulomb's law were not exactly two?

8. Does Gauss's law, as applied in Section 28–4, require that all the conduction electrons in an insulated conductor reside on the surface?

9. In Section 28–4 we assumed that **E** equals zero everywhere inside a conductor. However, there are certainly very large electric fields inside the conductor, at points close to the electrons or to the nuclei. Does this invalidate the proof of Section 28–4?

10. It is sometimes said that excess charge resides entirely on the outer surface of a conductor because like charges repel and try to get as far away as possible from one another. Comment on this plausibility argument.

11. Is Gauss's law useful in calculating the field due to three equal charges located at the corners of an equilateral triangle? Explain.

12. The use of line, surface, and volume densities of charge to calculate the charge contained in an element of a charged object implies a continuous distribution of charge, whereas, in fact, charge on the microscopic scale is discontinuous. How, then, is this procedure justified?

13. Is **E** necessarily zero inside a charged rubber balloon if the balloon is (*a*) spherical or (*b*) sausage-shaped? For each shape assume the charge to be distributed uniformly over the surface.

14. A spherical rubber balloon carries a charge that is uniformly distributed over its surface. How does E vary for points (*a*) inside the balloon, (*b*) at the surface of the balloon, and (*c*) outside the balloon, as the balloon is blown up?

15. As you penetrate a uniform sphere of charge, E should decrease because less charge lies inside a sphere drawn through the observation point. On the other hand, E should increase because you are closer to the center of this charge. Which effect predominates and why?

16. Given a spherically symmetric charge distribution (not of uniform density of charge), is E necessarily a maximum at the surface? Comment on various possibilities.

17. An atom is normally *electrically neutral*. Why then should an α-particle be deflected by the atom under any circumstances?

18. If an α-particle, fired at a gold nucleus, is deflected through 135°, can you conclude (*a*) that any force has acted on the α-particle or (*b*) that any net work has been done on it?

PROBLEMS

1. Calculate Φ_E through a hemisphere of radius R. The field of **E** is uniform and is parallel to the axis of the hemisphere.

2. In Example 1 compute Φ_E for the cylinder if it is turned so that its axis is perpendicular to the electric field. Do not use Gauss's law.

3. A plane surface of area A is inclined so that its normal makes an angle θ with a uniform field of **E**. Calculate Φ_E for this surface.

4. A point charge of 1.0×10^{-6} coul is at the center of a cubical Gaussian surface 0.50 meter on edge. What is Φ_E for the surface?

5. Charge on an originally uncharged insulated conductor is separated by holding a positively charged rod nearby, as in Fig. 28–15. What can you learn from Gauss's law about the flux for the five Gaussian surfaces shown? The induced negative charge on the conductor is equal to the positive charge on the rod.

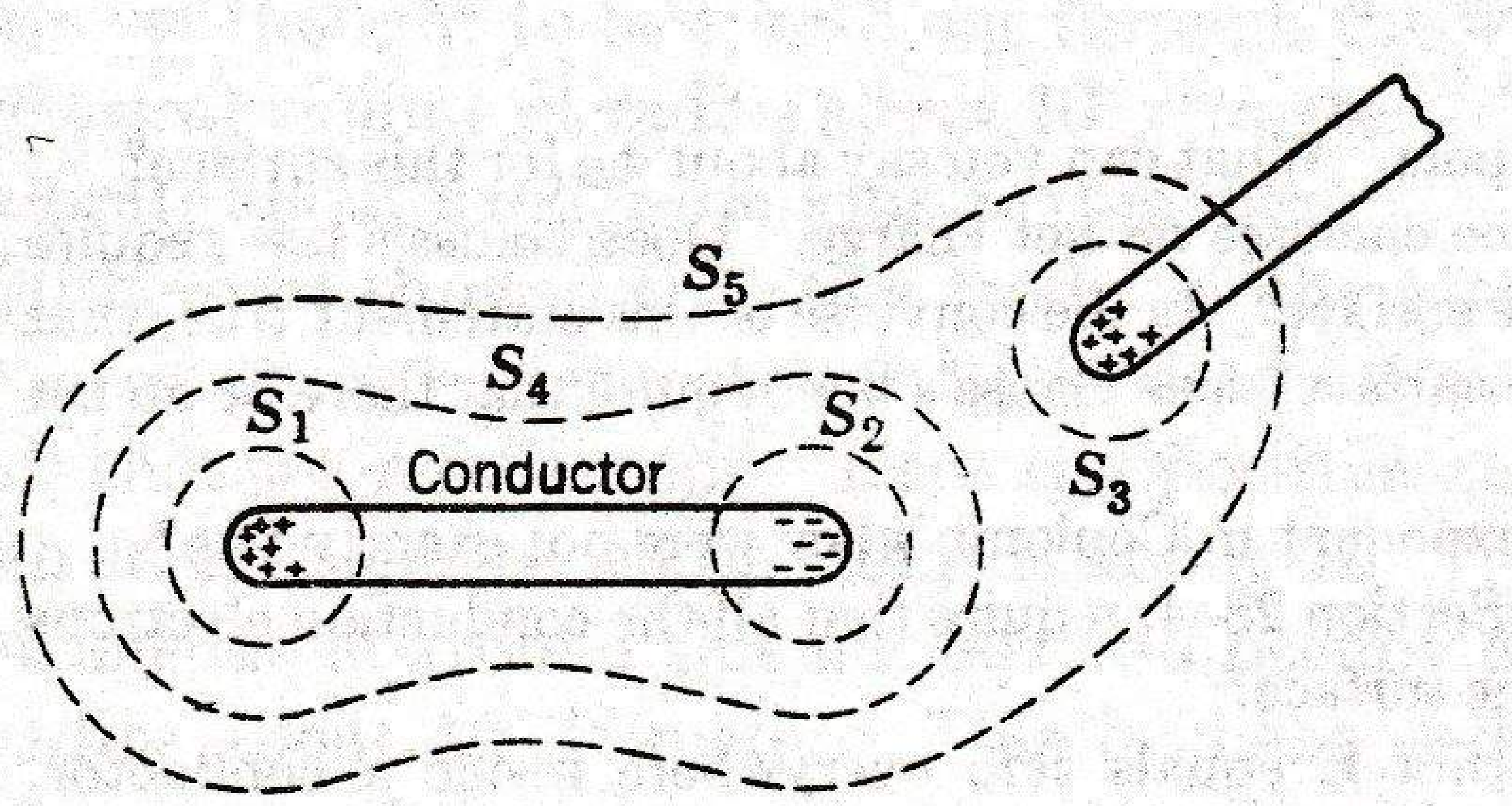

Fig. 28–15

6. "Gauss's law for gravitation" is

$$\frac{1}{4\pi G}\Phi_g = \frac{1}{4\pi G}\oint \mathbf{g}\cdot d\mathbf{S} = m,$$

where m is the enclosed mass and G is the universal gravitation constant (Section 16–3). Derive Newton's law of gravitation from this.

7. Figure 28–16 shows a point charge of 1.0×10^{-7} coul at the center of a spherical cavity of radius 3.0 cm in a piece of metal. Use Gauss's law to find the electric field at point a, halfway from the center to the surface, and at point b.

Fig. 28–16

8. An uncharged spherical thin metallic shell has a point charge q at its center. Give expressions for the electric field (*a*) inside the shell, and (*b*) outside the shell, using Gauss's law. (*c*) Has the shell any effect on the field due to q? (*d*) Has the presence of q any effect on the shell? (*e*) If a second point charge is held outside the shell, does this outside charge experience a force? (*f*) Does the inside charge experience a force? (*g*) Is there a contradiction with Newton's third law here?

9. Two large nonconducting sheets of positive charge face each other as in Fig. 28–17. What is **E** at points (*a*) to the left of the sheets, (*b*) between them, and (*c*) to the right of the sheets? Assume the same surface charge density σ for each sheet. Consider only points not near the edges whose distance from the sheets is small compared to the dimensions of the sheet. (Hint: **E** at any point is the vector sum of the separate electric field strengths set up by each sheet.)

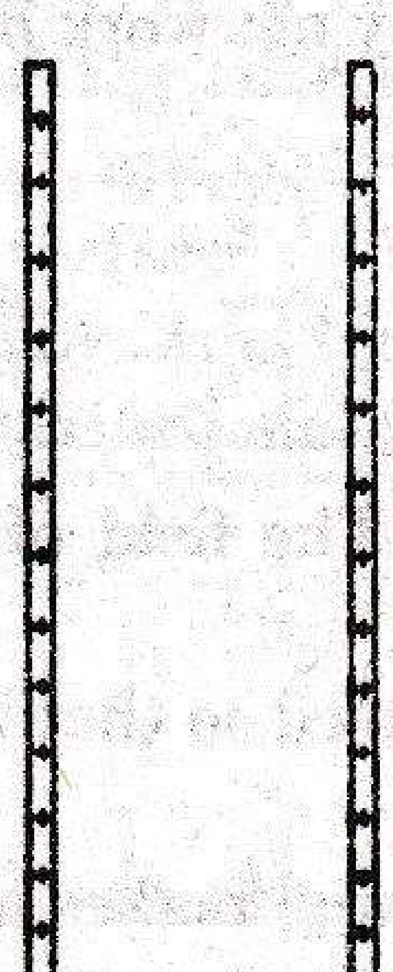

Fig. 28–17

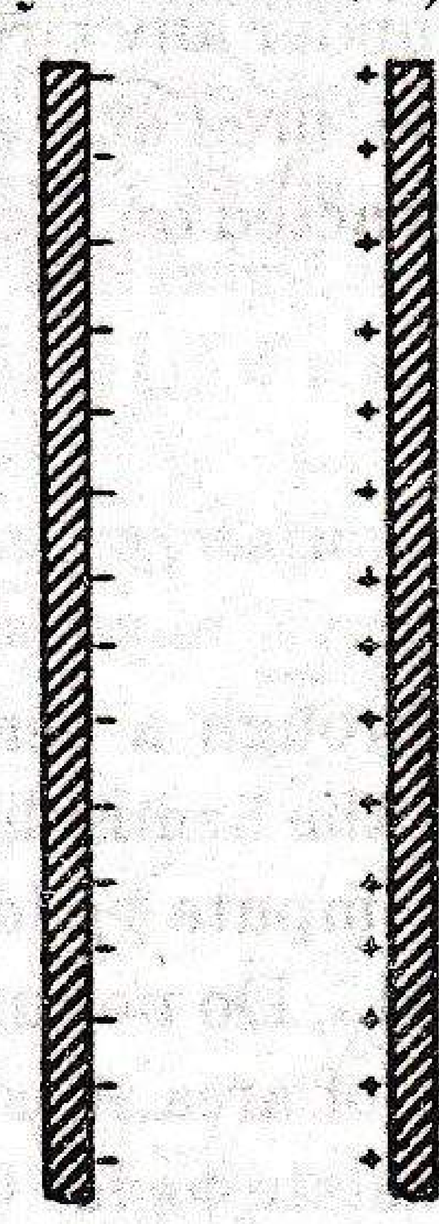

Fig. 28–18

10. Two large metal plates face each other as in Fig. 28–18 and carry charges with surface charge density $+\sigma$ and $-\sigma$, respectively, on their inner surfaces. What is **E** at points (*a*) to the left of the sheets, (*b*) between them, and (*c*) to the right of the sheets. Consider only points not near the edges whose distance from the sheets is small compared to the dimensions of the sheet.

11. Two large metal plates of area 1.0 meter^2 face each other. They are 5.0 cm apart and carry equal and opposite charges on their inner surfaces. If E between the plates is 55 nt/coul, what is the charge on the plates? Neglect edge effects. See Problem 10.

12. A thin-walled metal sphere has a radius of 25 cm and carries a charge of 2.0×10^{-7} coul. Find E for a point (*a*) inside the sphere, (*b*) just outside the sphere, and (*c*) 3.0 meters from the center of the sphere.

13. A 100-ev electron is fired directly toward a large metal plate that has a surface charge density of -2.0×10^{-6} coul/meter^2. From what distance must the electron be fired if it is to just fail to strike the plate?

14. Charge is distributed uniformly throughout an infinitely long cylinder of radius R. Show that E at a distance r from the cylinder axis ($r < R$) is given by

$$E = \frac{\rho r}{2\epsilon_0}$$

where ρ is the density of charge (coul/meter^3). What result do you expect for $r > R$?

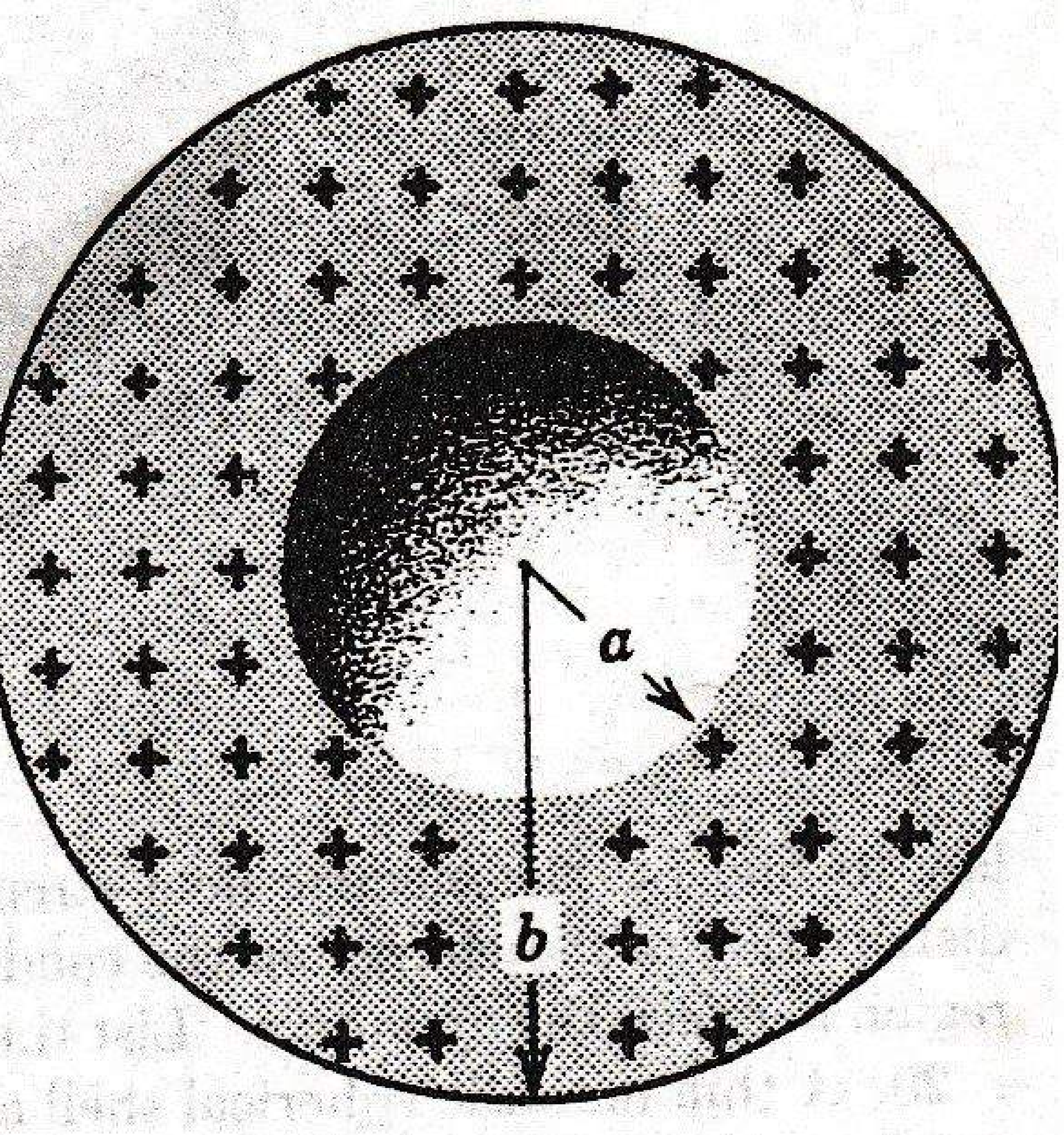

Fig. 28–19

15. Figure 28–19 shows a spherical nonconducting shell of charge of uniform density ρ (coul/meter^3). Plot E for distances r from the center of the shell ranging from zero to 30 cm. Assume that $\rho = 1.0 \times 10^{-6}$ coul/meter^3, a = 10 cm, and b = 20 cm.

16. Figure 28–20 shows a section through a long, thin-walled metal tube of radius R, carrying a charge per unit length λ on its surface. Derive expressions for E for various distances r from the tube axis, considering both $r > R$ and $r < R$. Plot your results for the range $r = 0$ to $r = 5$ cm, assuming that $\lambda = 2.0 \times 10^{-8}$ coul/meter and R = 3.0 cm.

17. Figure 28–21 shows a section through two long concentric cylinders of radii a and b. The cylinders carry equal and opposite charges per unit length λ. Using Gauss's law,

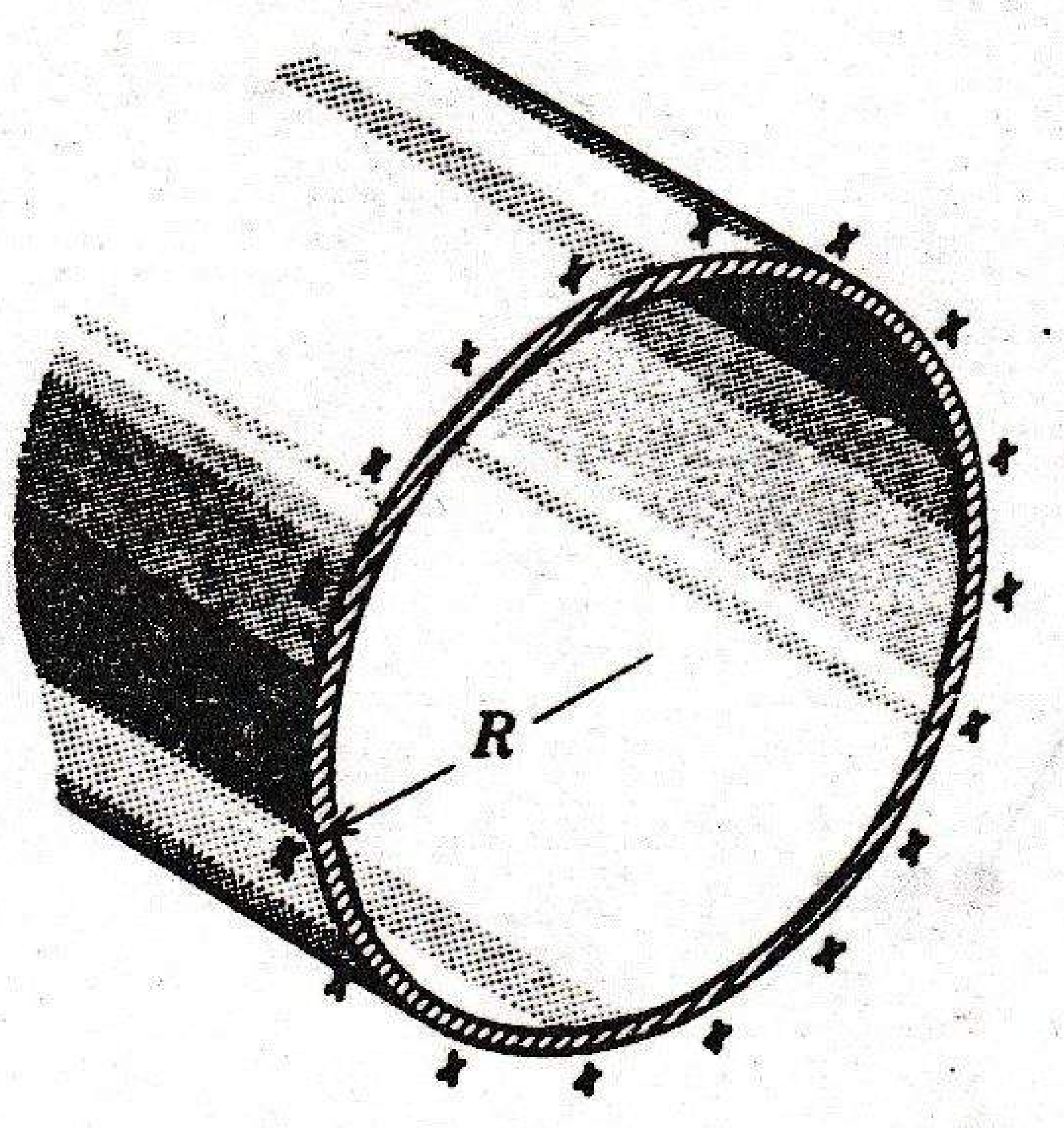

Fig. 28–20

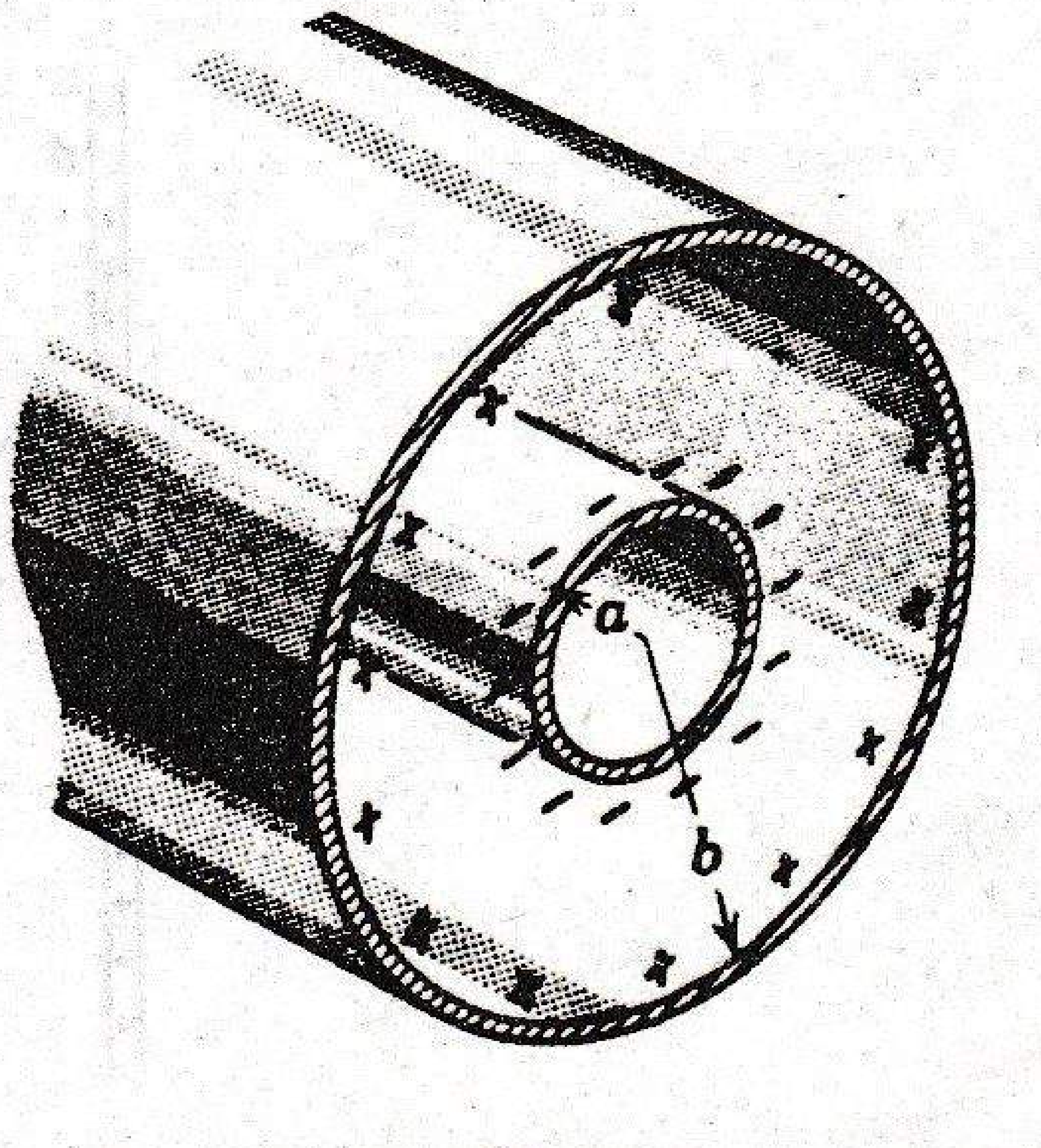

Fig. 28–21

prove (*a*) that $E = 0$ for $r > b$ and for $r < a$ and (*b*) that between the cylinders E is given by

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}.$$

18. In Problem 17 a positron revolves in a circular path of radius r, between and concentric with the cylinders. What must be its kinetic energy K? Assume $a = 2.0$ cm, $b = 3.0$ cm, and $\lambda = 3.0 \times 10^{-8}$ coul/meter.

19. A long conducting cylinder carrying a total charge $+q$ is surrounded by a conducting cylindrical shell of total charge $-2q$, as shown in cross section in Fig. 28-22. Use Gauss's

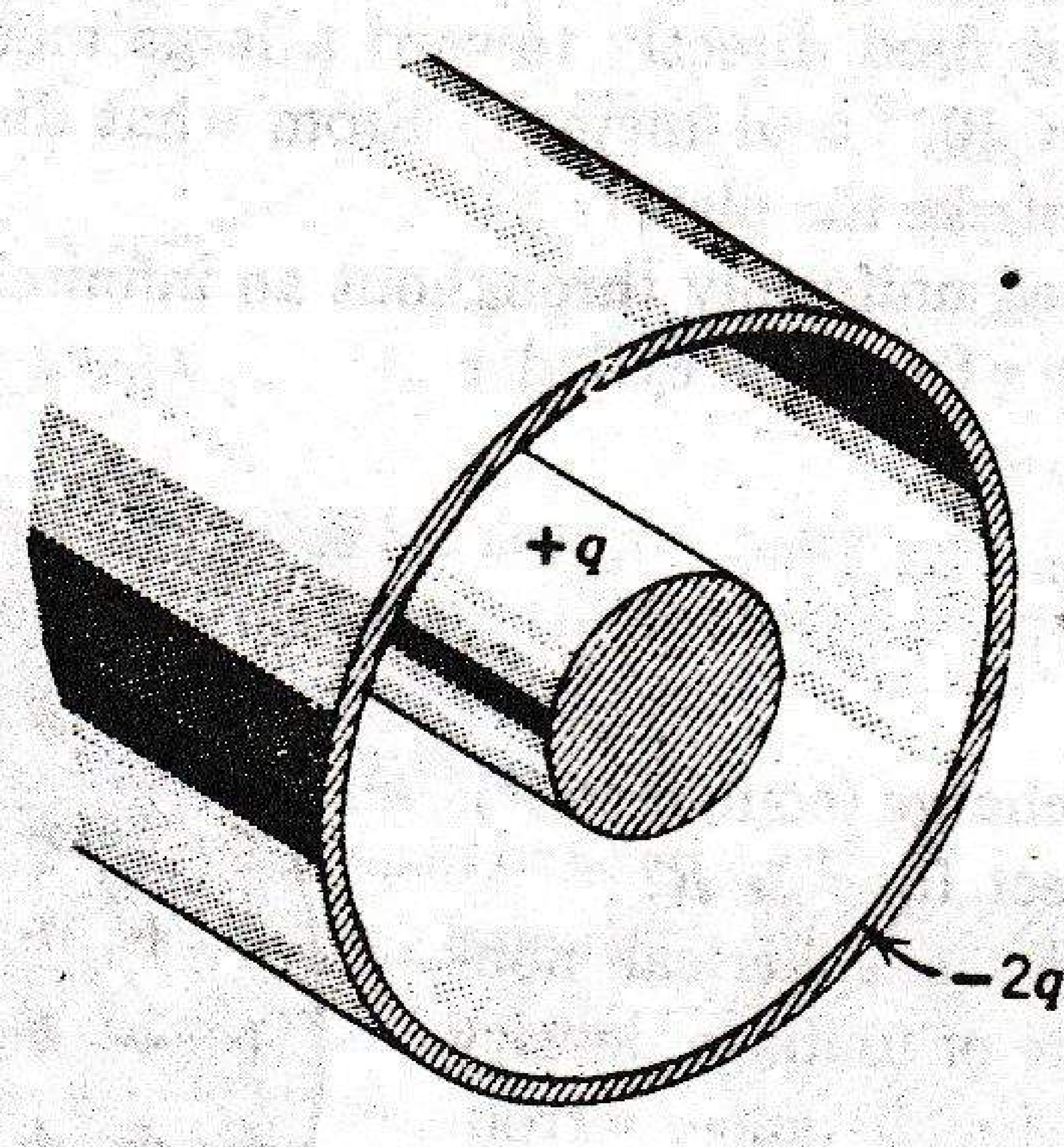

Fig. 28-22

law to find (*a*) the electric field strength at points outside the conducting shell, (*b*) the distribution of the charge on the conducting shell, and (*c*) the electric field strength in the region between the cylinders. List the assumptions made in arriving at your answers.

20. A thin metallic spherical shell of radius a carries a charge q_a. Concentric with it is another thin metallic spherical shell of radius b ($b > a$) carrying a charge q_b. Use Gauss's law to find the electric field strength at radial points r where (*a*) $r < a$; (*b*) $a < r < b$; (*c*) $r > b$. (*d*) Discuss the criterion one would use to determine how the charges are distributed on the inner and outer surfaces of each shell.

21. A small sphere whose mass m is 1.0×10^{-3} gm carries a charge q of 2.0×10^{-8} coul. It hangs from a silk thread which makes an angle of 30° with a large, charged conducting sheet as in Fig. 28-23. Calculate the surface charge density σ for the sheet.

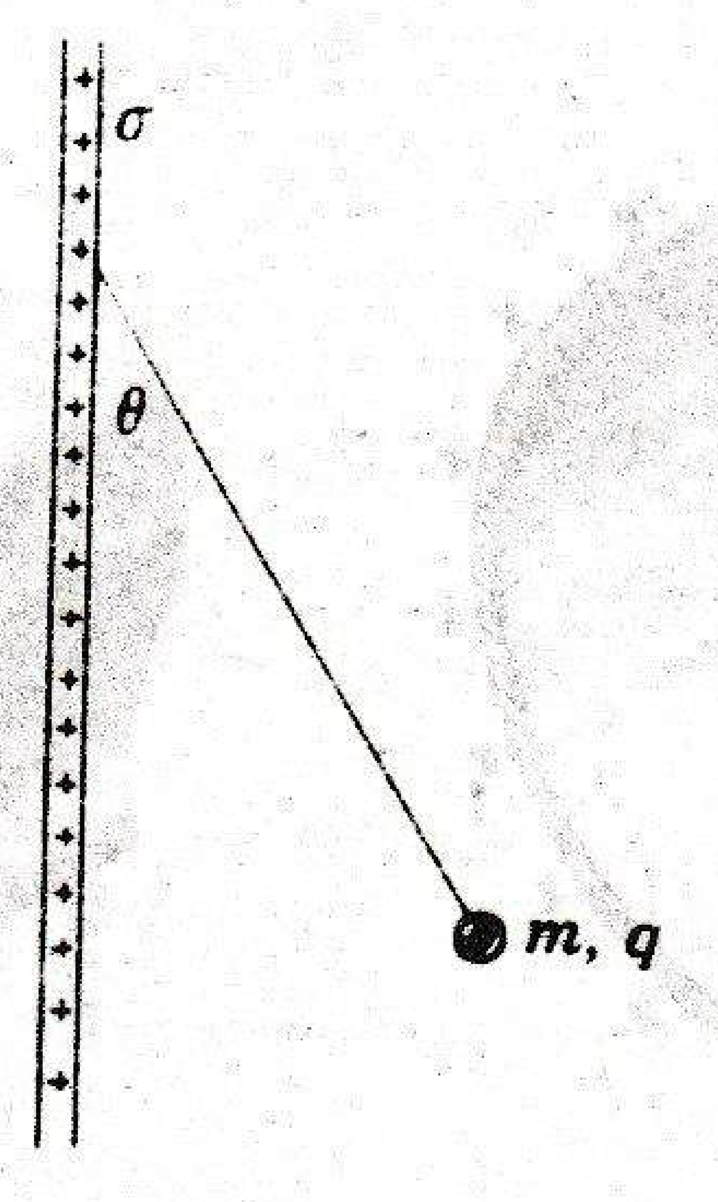

Fig. 28-23

22. Equation 28–9 ($E = \sigma/\epsilon_0$) gives the electric field at points near a charged conducting surface. Show that this equation leads to a familiar result when applied to a conducting sphere of radius r, carrying a charge q.

23. An α-particle, approaching the surface of a nucleus of gold, is a distance equal to one nuclear radius (6.9×10^{-15} meter) away from that surface. What are the forces on the α-particle and its acceleration at that point? The mass of the α-particle, which may be treated here as a point, is 6.7×10^{-27} kg.

24. A gold foil used in a Rutherford scattering experiment is 3×10^{-5} cm thick. (*a*) What fraction of its surface area is "blocked out" by gold nuclei, assuming a nuclear radius of 6.9×10^{-15} meter? Assume that no nucleus is screened by any other. (*b*) What fraction of the volume of the foil is occupied by the nuclei? (*c*) What fills all the rest of the space in the foil?

25. The electric field components in Fig. 28–24 are $E_x = bx^{1/2}$, $E_y = E_z = 0$, in which $b = 800$ nt/coul-m$^{1/2}$. Calculate (*a*) the flux Φ_E through the cube and (*b*) the charge within the cube. Assume that $a = 10$ cm.

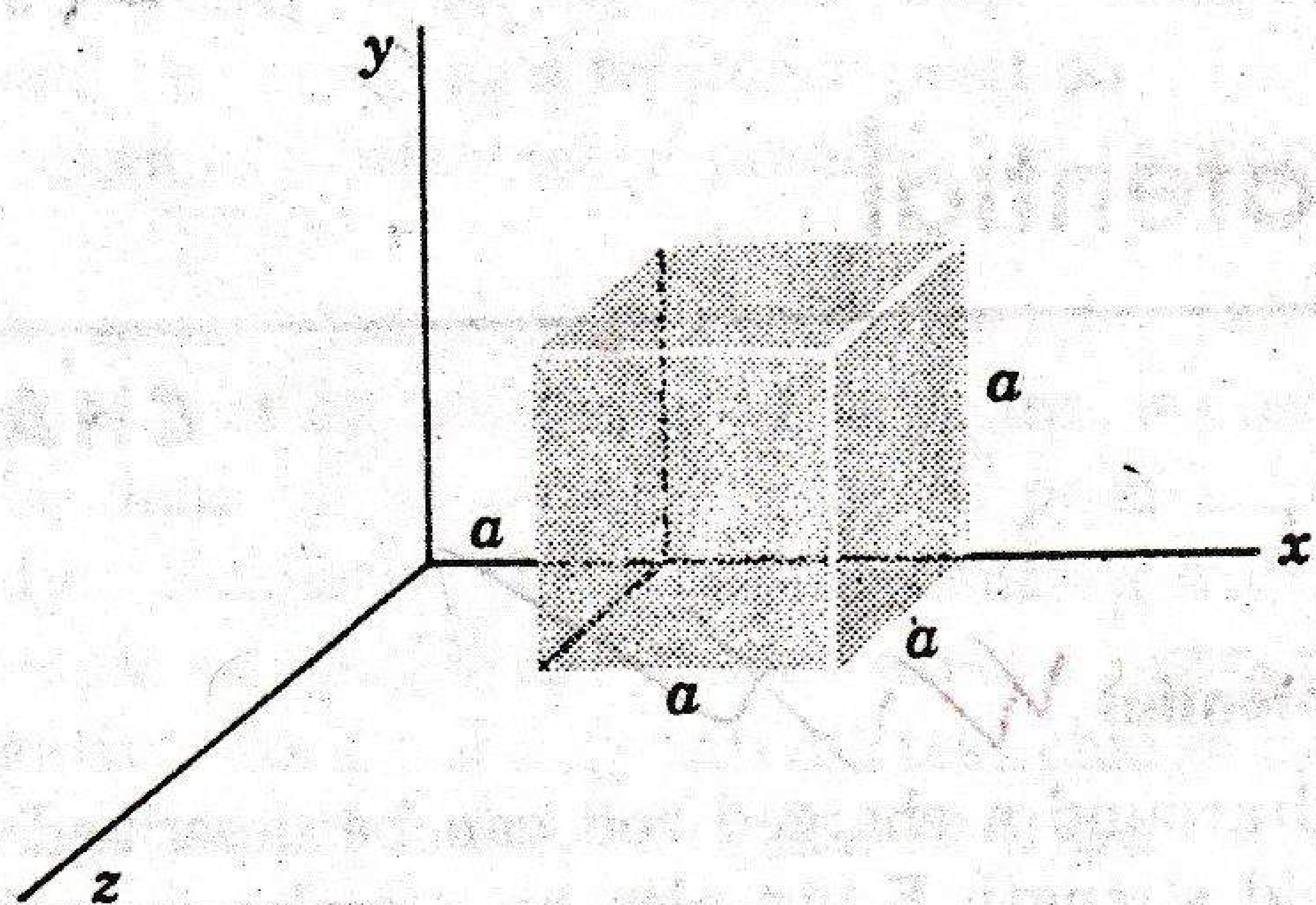

Fig. 28–24