The Differential Form of Maxwell's Equations and the Electromagnetic Wave Equation

SUPPLEMENTARY TOPIC V*

V–1 Introduction

In Chapter 39 we sought to make the existence of electromagnetic waves plausible by showing that such waves are consistent with Maxwell's equations as expressed in Table 38–3. Here we seek to start from Maxwell's equations and derive from them a differential equation whose solutions will describe electromagnetic waves. We will show directly that the speed c of such waves is given by Eq. 39–15, or $c = 1/\sqrt{\epsilon_0\mu_0}$.

We followed a similar program in Supplementary Topic III for mechanical waves on a stretched string. Starting from Newton's laws of motion we derived a differential equation (Eq. III–1) whose solutions (Eq. III–2) described such waves. We showed further that the speed v of these waves is given by Eq. III–3, or $v = \sqrt{F/\mu}$.

In Table 38–3 we wrote Maxwell's equations as

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = q, \qquad \text{(V–1)}$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0, \qquad \text{(V–2)}$$

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(i + \epsilon_0\, d\Phi_E/dt), \qquad \text{(V–3)}$$

and

$$\oint \mathbf{E} \cdot d\mathbf{l} = -d\Phi_B/dt. \qquad \text{(V–4)}$$

* Supplementary Topics I to IV appear in Part I.

These equations are said to be written in integral form. The field variables **E** and **B**, which are usually the unknown quantities, appear in the integrands. Only in a few symmetric cases (see Sections 28–6 and 34–2, for example) can we "factor them out." In more general problems we cannot do so.

The situation is somewhat analogous to computing the density ρ of a body if we know its mass m and volume τ. In general these are related by the integral equation

$$m = \int \rho \, d\tau.$$

Only if ρ is a constant over all parts of the volume can we factor it out and write $\rho = m/\tau$.

To carry out our program it is desirable to recast Maxwell's equations in the form of equalities that apply at each *point* in space rather than as integrals that apply to various *regions* of space. In other words, we wish to convert Maxwell's equations from the integral form of Eqs. V–1 to 4 into *differential form.* We will then be able to relate **E** and **B** at a point to the charge density and current density at that point.

V–2 The Operator ∇

To transform Maxwell's equations into differential form we must deepen our understanding of vector methods and, in particular, become familiar with the vector operator ∇.

In Section 29–7 we saw how to obtain the components of the (vector) electrostatic field **E** at any point from the (scalar) potential function $V(x,y,z)$ by partial differentiation. Thus,

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad \text{and} \quad E_z = -\frac{\partial V}{\partial z}$$

so that the electrostatic field

$$\mathbf{E} = \mathbf{i}E_x + \mathbf{j}E_y + \mathbf{k}E_z$$

can be written as

$$\mathbf{E} = -\left(\mathbf{i}\frac{\partial V}{\partial x} + \mathbf{j}\frac{\partial V}{\partial y} + \mathbf{k}\frac{\partial V}{\partial z}\right). \tag{V-5}$$

We can write Eq. V–5 in compact vector notation as

$$\mathbf{E} = -\nabla V,$$

where ∇ ("del") is a vector operator defined as

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}. \tag{V-6}$$

This operator is useful in dealing with scalar and vector fields (see Sections 16–8 and 18–7 for examples of such fields). Given any scalar field ψ we may form a vector field, called the gradient of ψ and written as grad ψ or $\nabla\psi$, simply by applying the operator ∇ to ψ. Given a vector field $\mathbf{U} = U_x\mathbf{i} + U_y\mathbf{j} + U_z\mathbf{k}$ we may apply the operator ∇ to it in two different ways. One way is to take the dot product of ∇ and **U**, yielding the scalar field called the divergence of **U** and written as div **U** or $\nabla \cdot \mathbf{U}$. The other way is to take the cross product of ∇ and **U**, yielding the vector field called the curl of **U** and written curl **U** or $\nabla \times \mathbf{U}$. These operations may be summarized as

$$\text{grad } \psi \equiv \nabla\psi = \mathbf{i}\frac{\partial \psi}{\partial x} + \mathbf{j}\frac{\partial \psi}{\partial y} + \mathbf{k}\frac{\partial \psi}{\partial z}$$

$$\text{div } \mathbf{U} \equiv \nabla \cdot \mathbf{U} = \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z}$$

$$\text{curl } \mathbf{U} \equiv \nabla \times \mathbf{U} = \mathbf{i}\left(\frac{\partial U_z}{\partial y} - \frac{\partial U_y}{\partial z}\right) + \mathbf{j}\left(\frac{\partial U_x}{\partial z} - \frac{\partial U_z}{\partial x}\right) + \mathbf{k}\left(\frac{\partial U_y}{\partial x} - \frac{\partial U_x}{\partial y}\right)$$

Note that grad ψ and curl **U** are vectors, whereas div **U** is a scalar. The student can gain some familiarity with these operations by the following exercises: (1) prove that curl (grad ψ) = 0 and (2) prove that div (curl **U**) = 0.

Another frequently occuring operator is ∇^2 ("del squared"). It is simply $\nabla \cdot \nabla$, or, as the student can show from Eq. V-6,

$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

When we apply ∇^2 to a scalar field ψ, we obtain

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \tag{V-10}$$

For a vector field **U**, the operation $\nabla^2\mathbf{U}$ is defined as

$$\nabla^2\mathbf{U} = \mathbf{i}\frac{\partial^2 U_x}{\partial x^2} + \mathbf{j}\frac{\partial^2 U_y}{\partial y^2} + \mathbf{k}\frac{\partial^2 U_z}{\partial z^2}. \tag{V-11}$$

As an exercise the student can show that curl (curl **U**) = $-\nabla^2\mathbf{U}$ + grad (div **U**).

V–3 Maxwell's Equations in Differential Form—I

In this section we show how to cast the first two of Maxwell's equations (Eqs. V-1, 2) into differential form. Let us apply Eq. V-1 to a differential volume element shaped like a rectangular parallelepiped and containing a point P at (and near) which an electric field exists (see Fig. V-1*a*). Point P is located at x, y, z in the reference frame of Fig. V-1*b* and the edges of the parallelepiped have lengths dx, dy, and dz.

We can write the surface area vector for the rear face of the parallelepiped as $d\mathbf{S} = -\mathbf{i}\, dy\, dz$. The minus sign enters because $d\mathbf{S}$ is defined to point in the direction of the *outward* normal, which is defined by $-\mathbf{i}$. For the front face we have $d\mathbf{S} = +\mathbf{i}\, dy\, dz$.

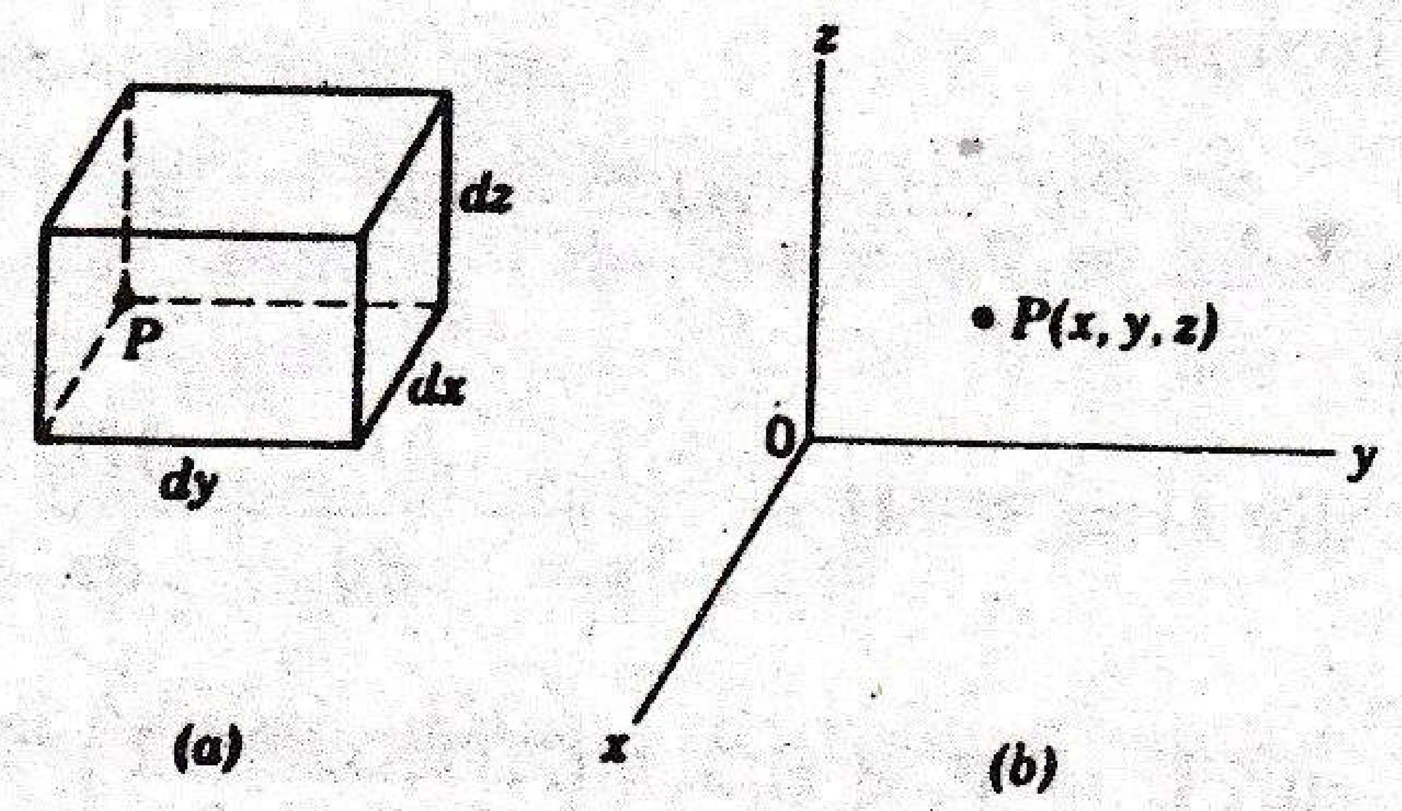


Fig. V-1

If the electric field at the rear face is **E**, that at the front face, which is a distance dx away from the rear face, is $\mathbf{E} + (\partial\mathbf{E}/\partial x)\,dx$, the latter term representing the change in **E** associated with the change dx in x.

The flux through the entire surface of the parallelepiped is $\oint \mathbf{E}\cdot d\mathbf{S}$ and the contribution to this flux due to these two faces alone is

$$(\mathbf{E})\cdot(-\mathbf{i}\,dy\,dz) + \left(\mathbf{E} + \frac{\partial\mathbf{E}}{\partial x}\,dx\right)\cdot(+\mathbf{i}\,dy\,dz)$$

$$= dx\,dy\,dz\left(\frac{\partial\mathbf{E}}{\partial x}\cdot\mathbf{i}\right) = dx\,dy\,dz\,\frac{\partial}{\partial x}(\mathbf{E}\cdot\mathbf{i})$$

$$= dx\,dy\,dz\,\frac{\partial E_x}{\partial x}.$$

With similar contributions from the other four faces the total electric flux becomes

$$\oint \mathbf{E}\cdot d\mathbf{S} = dx\,dy\,dz\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right).$$

From Eq. V-8 we may write this as

$$\oint \mathbf{E}\cdot d\mathbf{S} = dx\,dy\,dz\,\operatorname{div}\mathbf{E}. \qquad \text{(V-12)}$$

Now the right-hand side of Eq. V-1, which gives the charge enclosed by the surface, may be written in general as $q = \int\rho\,d\tau$ and, in particular, for the differential volume element at P, as

$$q = \rho\,dx\,dy\,dz, \qquad \text{(V-13)}$$

where ρ is the charge per unit volume at P. Substituting Eqs. V-12 and 13 into Eq. V-1 and canceling the common factor $dx\,dy\,dz$, we have finally

$$\epsilon_0\operatorname{div}\mathbf{E} = \rho, \qquad \text{(V-14)}$$

which is Maxwell's first equation (Eq. V-1) in differential form.

Using the same technique, we can express Maxwell's second equation (Eq. V-2) in differential form as

$$\operatorname{div}\mathbf{B} = 0. \qquad \text{(V-15)}$$

V–4 Maxwell's Equations in Differential Form—II

We now seek to transform Maxwell's third and fourth equations (Eqs. V–3, 4) into differential form. We start by applying Eq. V–3 to a differential surface element of rectangular shape at a point P in some region of a magnetic field, as shown in Fig. V–2*a*. The point P is located at x, y, z in the reference frame of Fig.

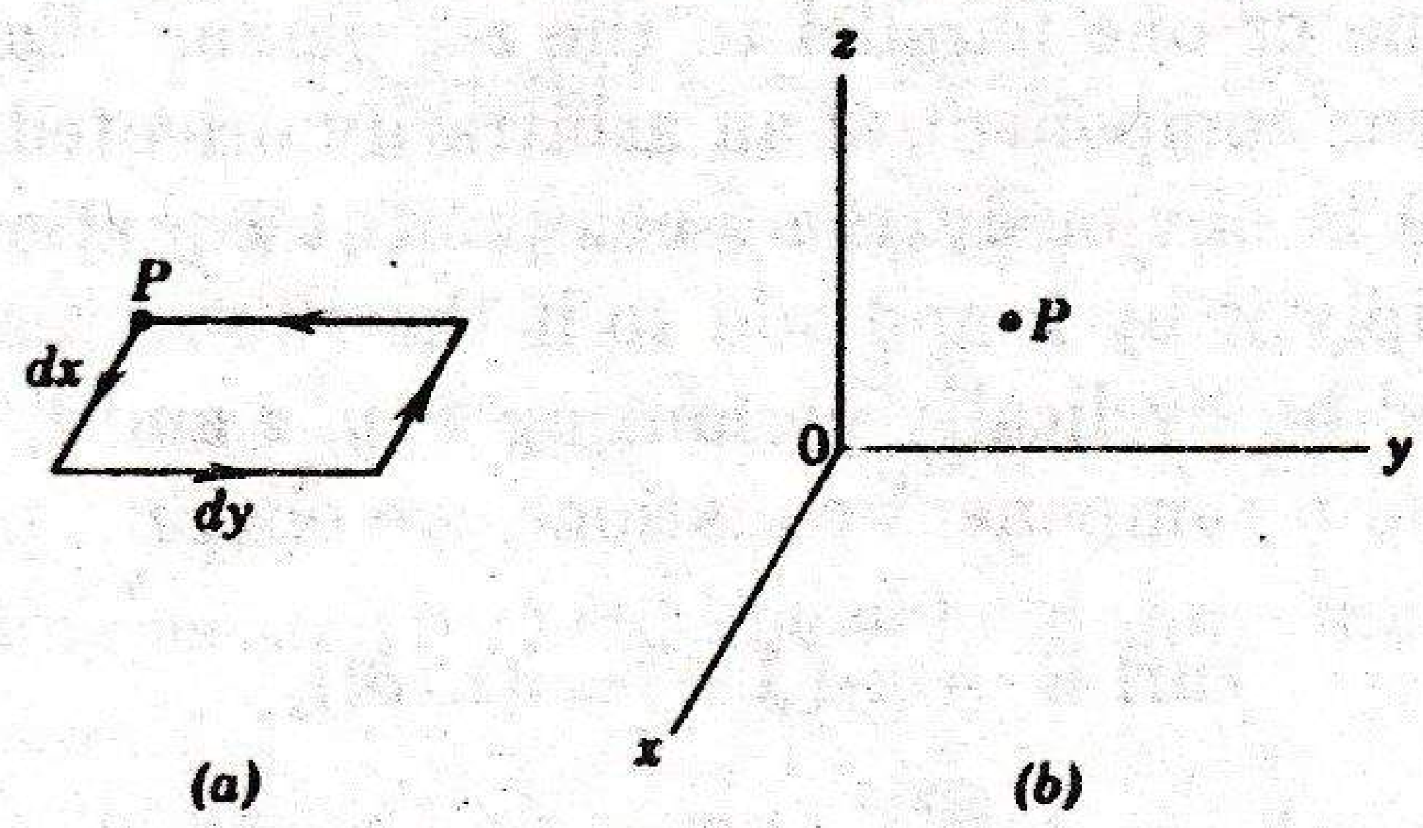


Fig. V–2

V–2*b* and the sides of the rectangle, which is parallel to the x-y plane, have lengths dx and dy. Going around the path, as shown by the arrows, we have

$$\oint \mathbf{B}\cdot d\mathbf{l} = B\cdot(-\mathbf{j}\,dy) \qquad \text{(rear side)}$$

$$+\,\mathbf{B}\cdot(+\mathbf{i}\,dx) \qquad \text{(left side)}$$

$$+\left(\mathbf{B}+\frac{\partial \mathbf{B}}{\partial x}dx\right)\cdot(+\mathbf{j}\,dy) \qquad \text{(front side)}$$

$$+\left(\mathbf{B}+\frac{\partial \mathbf{B}}{\partial y}dy\right)\cdot(-\mathbf{i}\,dx), \qquad \text{(right side)}$$

where $\mathbf{B}$ is the magnetic induction at P.

Collecting terms we obtain

$$\oint \mathbf{B}\cdot d\mathbf{l} = dx\,dy\left(\frac{\partial \mathbf{B}}{\partial x}\cdot\mathbf{j} - \frac{\partial \mathbf{B}}{\partial y}\cdot\mathbf{i}\right)$$

$$= dx\,dy\left[\frac{\partial}{\partial x}(\mathbf{B}\cdot\mathbf{j}) - \frac{\partial}{\partial y}(\mathbf{B}\cdot\mathbf{i})\right]$$

$$= dx\,dy\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right). \qquad \text{(V–16)}$$

Now in the right-hand side of Eq. V–3 i is the current enclosed by the path and $d\Phi_E/dt$ is the change in electric flux through the enclosed surface. Hence, if $\mathbf{J}$ is taken to represent the current density and $d\mathbf{S}(=\mathbf{k}\,dx\,dy)$ the surface area vector, we can write

$$i = \mathbf{J}\cdot d\mathbf{S} = \mathbf{J}\cdot(\mathbf{k}\,dx\,dy) = dx\,dy\,J_z \qquad \text{(V–17)}$$

and

$$\frac{d\Phi_E}{dt} = \frac{\partial \mathbf{E}}{\partial t}\cdot d\mathbf{S} = \frac{\partial \mathbf{E}}{\partial t}\cdot(\mathbf{k}\,dx\,dy)$$

or

$$\frac{d\Phi_E}{dt} = \frac{\partial E_z}{\partial t}dx\,dy. \qquad \text{(V–18)}$$

Substituting Eqs. V–16, 17, and 18 into Eq. V–3 and canceling the common factor $dx\,dy$, we get

$$\frac{1}{\mu_0}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) = J_z + \epsilon_0 \frac{\partial E_z}{\partial t} \qquad \text{(V–19)}$$

We could have proceeded exactly as above if we had started with a rectangle parallel to the y-z plane or one parallel to the z-x plane. Each rectangle would have given us a different component of an arbitrarily oriented differential surface at P. Equation V–19 is obviously the z-component equation corresponding to Eq. V–3. If we multiply it by $\mathbf{k}$ and add to it the two similar vector equations, which may be obtained by cyclically permuting x, y, z and $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, corresponding to the x-component and y-component equations, we obtain

$$\text{curl}\,\mathbf{B} = \mu_0(\mathbf{J} + \epsilon_0\,\partial\mathbf{E}/\partial t), \qquad \text{(V–20)}$$

which is the third Maxwell equation in differential form.

Similarly, starting with Eq. V–4, we may show that

$$\text{curl}\,\mathbf{E} = -\partial\mathbf{B}/\partial t, \qquad \text{(V–21)}$$

which is the fourth Maxwell equation in differential form.

We have derived four differential equations (see V-22 to V-25 below) from the four integral equations (V-1 to V-4). It can be shown that the integral equations can be derived from the differential equations, that is, the two sets of equations are *equivalent*

V–5 The Wave Equation

We have now obtained from their integral form the four basic equations of electromagnetism, Maxwell's equations, in differential form. Corresponding to the integral equations, Eqs. V–1, 2, 3, and 4 respectively, we have

$$\epsilon_0\,\text{div}\,\mathbf{E} = \rho, \qquad \text{(V–22)}$$

$$\text{div}\,\mathbf{B} = 0, \qquad \text{(V–23)}$$

$$\text{curl}\,\mathbf{B} = \mu_0(\mathbf{J} + \epsilon_0\,\partial\mathbf{E}/\partial t), \qquad \text{(V–24)}$$

$$\text{curl}\,\mathbf{E} = -\partial\mathbf{B}/\partial t, \qquad \text{(V–25)}$$

which are four coupled partial differential equations. They apply at each point of space in an electromagnetic field.

We will now derive the wave equation for electromagnetic waves in free space. In free space, the charge density ρ and the current density $\mathbf{J}$ are zero, so that the Maxwell equations there become

$$\text{div}\,\mathbf{E} = 0,$$

$$\text{div}\,\mathbf{B} = 0,$$

$$\text{curl}\,\mathbf{E} = -\,\partial\mathbf{B}/\partial t,$$

and

$$\text{curl}\,\mathbf{B} = \mu_0\epsilon_0\,\partial\mathbf{E}/\partial t.$$

Let us take the curl of the equation for curl $\mathbf{E}$; we obtain

$$\text{curl curl}\,\mathbf{E} = -\,\text{curl}\,\frac{\partial\mathbf{B}}{\partial t} = -\frac{\partial}{\partial t}\,\text{curl}\,\mathbf{B}.$$

But, from above, curl $\mathbf{B} = \mu_0\epsilon_0(\partial\mathbf{E}/\partial t)$, so that

$$\text{curl curl } \mathbf{E} = -\mu_0\epsilon_0 \frac{\partial^2\mathbf{E}}{\partial t^2} \tag{V-26}$$

From the exercise in V–2, we know that curl curl $\mathbf{E} = -\nabla^2 E +$ grad div $\mathbf{E}$ and from above that div $\mathbf{E} = 0$. Thus,

$$\text{curl curl } \mathbf{E} = -\nabla^2\mathbf{E}. \tag{V-27}$$

Combining Eqs. V–26 and V–27, we obtain finally

$$\nabla^2\mathbf{E} = \mu_0\epsilon_0 \frac{\partial^2\mathbf{E}}{\partial t^2}. \tag{V-28a}$$

The student, proceeding as above, should be able to show that **B** satisfies the same equation, or

$$\nabla^2\mathbf{B} = \mu_0\epsilon_0 \frac{\partial^2\mathbf{B}}{\partial t^2}. \tag{V-28b}$$

Equations V–28 are the equations of electromagnetic wave motion. Being vector equations, they are equivalent to six scalar equations, one for each component of **E** and of **B**.

There are many solutions of Eqs. V–28, corresponding to different kinds of electromagnetic waves—plane, spherical, and cylindrical waves being three examples. Let us consider a solution in which two components of **E** and two of **B** vanish, that is, in which

$$E_x = E_z = 0 \quad \text{and} \quad B_x = B_y = 0.$$

Equations V–28 are satsified for these assumptions. For the nonvanishing components, E_y and B_z, Eqs. V–28 reduce to (see Eq. V–11)

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} = \mu_0\epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \tag{V-29a}$$

and

$$\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} + \frac{\partial^2 B_z}{\partial z^2} = \mu_0\epsilon_0 \frac{\partial^2 B_z}{\partial t^2} \tag{V-29b}$$

If we make the additional assumption that E_y and B_z are functions of x and t only, the simplified wave equation that results, namely

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 B_z}{\partial t^2}, \tag{V-29c}$$

is similar to Eq. III-1 for the vibrating string.

A solution to these equations, as the student may verify by substitution, is

$$E_y = E_m \sin(kx - \omega t) \tag{V-30a}$$

and

$$B_z = B_m \sin(kx - \omega t). \tag{V-30b}$$

We interpret Eqs. V–30 as an electromagnetic wave traveling in the positive x-direction, as in Fig. 39–11, with a speed $c = \omega/k$. The student can show that substituting Eq. V–30*a* into Eq. V–29*a* (or Eq. V–30*b* into Eq. V–29*b*) yields

$$c = \omega/k = 1/\sqrt{\epsilon_0\mu_0},$$

which (see Eq. 39–15) gives the speed of electromagnetic waves in free space.

Supplementary Problems

Chapter 26

1. Two free point charges $+q$ and $+4q$ are a distance l apart. A third charge is so placed that the entire system is in equilibrium. Find the location, magnitude, and sign of the third charge.

2. If the balls of Fig. 26-7 are conducting, what happens to them after one is discharged? Find the new equilibrium separation.

3. Two identical conducting spheres, having charges of opposite sign, attract each other with a force of 0.108 nt when separated by 0.5 meter. The spheres are connected by a conducting wire, which is then removed, and thereafter repel each other with a force of 0.036 nt. What were the initial charges on the spheres?

4. A particle of charge $-q$ and mass m moves in a circular orbit about a fixed charge $+Q$. (*a*) Show that the "distance cubed $\propto$ period squared" law,

$$r^3 = \frac{Qq}{16\pi^3\epsilon_0 m} T^2$$

is satisfied. Note that the proportionality constant depends on the property (q/m) of the orbiting particle. (*b*) What is the corresponding situation when the force is gravitational rather than electrical?

5. An electron is projected with an initial speed of 3.24×10^5 meter/sec directly toward a proton which is essentially at rest. If the electron is initially a very great distance from the proton, at what distance from the proton is its speed instantaneously equal to twice its initial value? (*Hint:* Use the work-energy theorem.)

6. A "dipole" is formed from a rod of length $2a$ and two charges, $+q$ and $-q$. Two such dipoles are oriented as shown in Fig. 26-9, their centers being separated by the

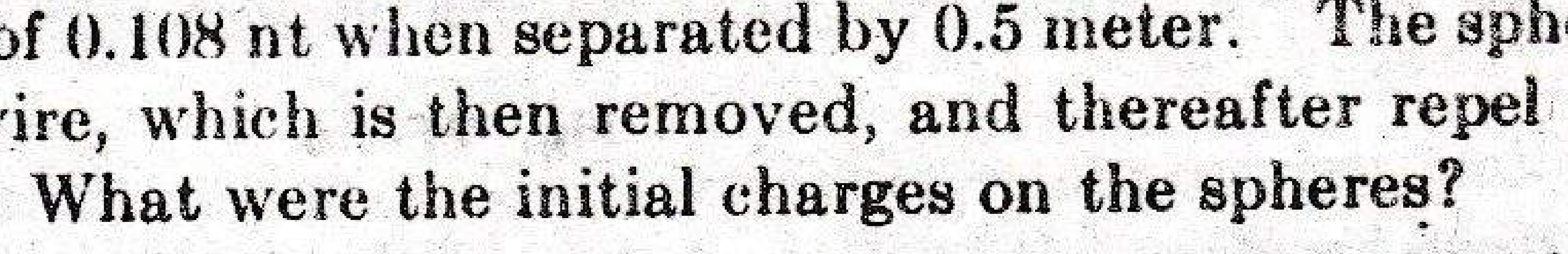


Fig. 26-9

distance R. (*a*) Calculate the force exerted on the left dipole. (*b*) For $R \gg a$, show that the magnitude of the force exerted on the left dipole is approximately given by

$$F = \frac{3p^2}{2\pi\epsilon_0 R^4}$$

where $p = 2qa$ is the "dipole moment."

Chapter 27

1. A uniform vertical field **E** is established in the space between two large parallel plates. In this field one suspends a small conducting sphere of mass m from a string of length l. Find the period of this pendulum when the sphere is given a charge $+q$ if the lower plate is charged positively; is charged negatively.

2. A charge $q = 3.0 \times 10^{-6}$ coul is 30 cm from a small dipole along its perpendicular bisector. The magnitude of the force on the charge is 5.0×10^{-6} nt. Show on a diagram (*a*) the direction of the force on the charge, (*b*) the direction of the force on the dipole, and (*c*) determine the magnitude of the force on the dipole.

3. A thin glass rod is bent into a semicircle of radius R. A charge $+Q$ is uniformly distributed along the upper half and a charge $-Q$ is uniformly distributed along the lower half, as shown in Fig. 27–25. Find the electric field **E** at P, the center of the semicircle.

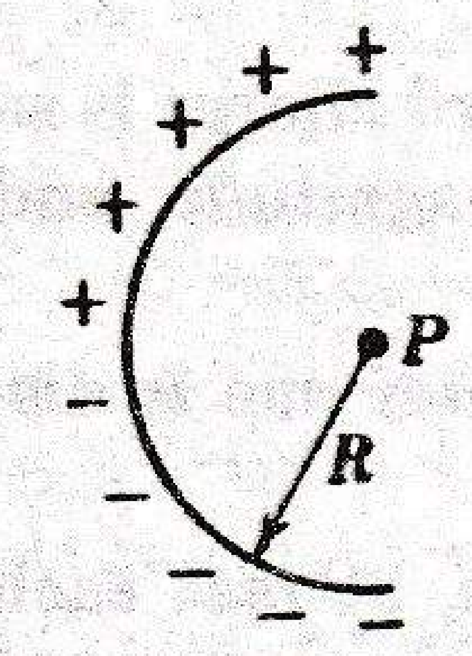


Fig. 27–25

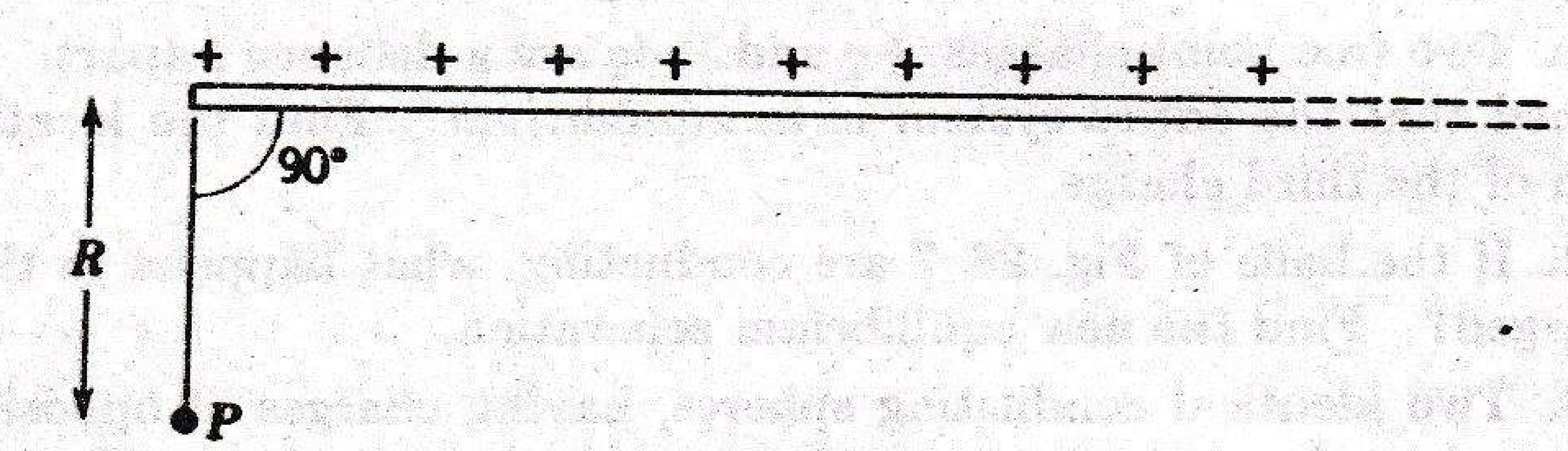


Fig. 27–26

4. A "semi-infinite" insulating rod (Fig. 27–26) carries a constant charge per unit length of λ. Show that the electric field at the point P makes an angle of 45° with the rod. This result is independent of the distance R.

5. Find the frequency of oscillation of an electric dipole, of moment p and rotational inertia I, for small amplitudes of oscillation about its equilibrium position in a uniform electric field of strength E.

6. An electric dipole of moment **p** is placed parallel to an electric field line along the y-axis in a nonuniform electric field (Fig. 27–27). The magnitude of the field **E** varies uniformly along the y-direction as shown. (*a*) Show that the magnitude of the force on the dipole is $p\,dE/dy$. (*b*) What is the direction of the force?

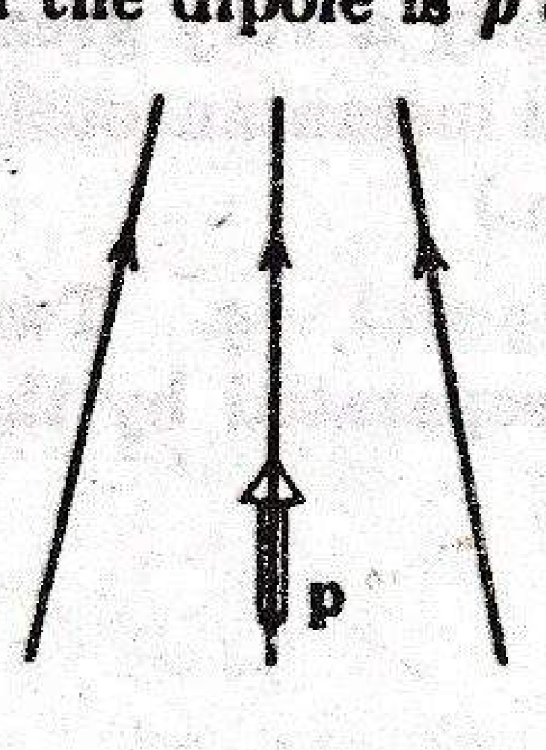


Fig. 27–27

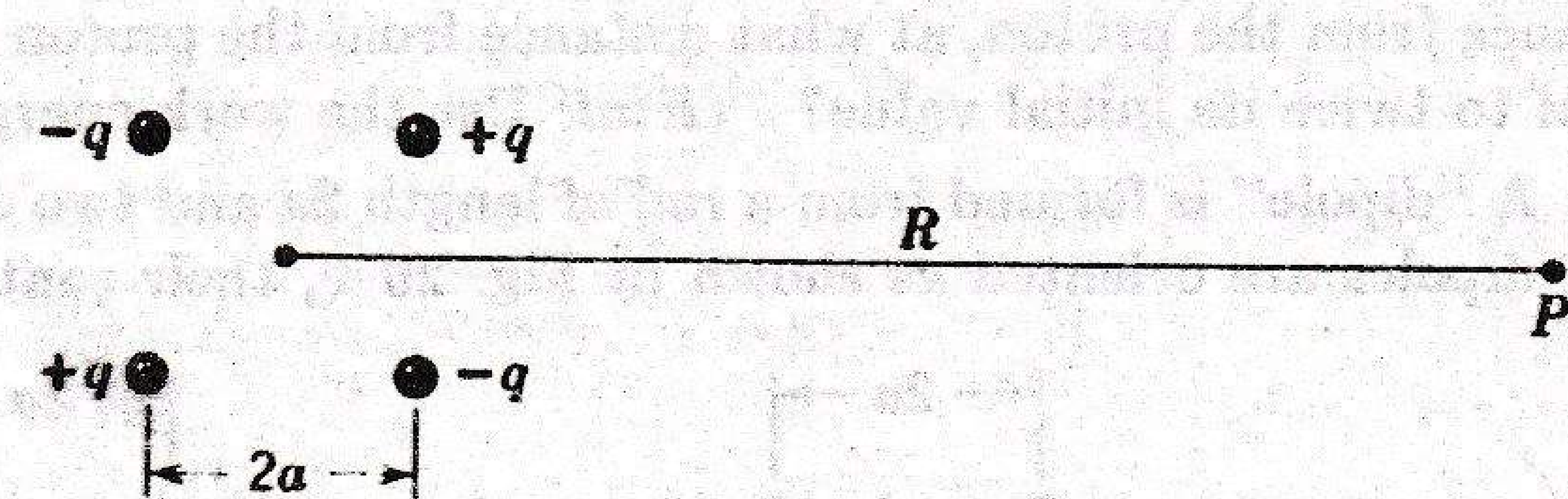


Fig. 27–28

7. One type of "electric quadrupole" is formed by four charges located at the vertices of a square of side $2a$. Point P lies a distance R from the center of the quadrupole on a line parallel to two of sides of the square as shown in Fig. 27–28. For $R \gg a$, show that the electric field at P is approximately given by

$$E = \frac{3(2qa^2)}{4\pi\epsilon_0 R^4}.$$

(*Hint:* Treat the quadrupole as two dipoles.)

Chapter 28

1. It is found experimentally that the electric field in a large region of the earth's atmosphere is directed vertically down. At an altitude of 300 meters the field is 60 volts/meter and at an altitude of 200 meters it is 100 volts/meter. Find the net amount of charge contained in a cube 100 meters on edge located between 200 and 300 meters altitude. Neglect the curvature of the earth.

2. Suppose that an electric field in some region is found to have a constant direction but to be decreasing in strength in that direction. What do you conclude about the charge in the region?

3. Two concentric conducting spherical shells have radii $R_1 = 0.145$ meter and $R_2 = 0.207$ meter. The inner sphere bears a charge -6.00×10^{-2} coul. An electron escapes from the inner sphere with negligible speed. Assuming that the region between the spheres is a vacuum, compute the speed with which the electron strikes the outer sphere.

4. (*a*) Two identical nonconducting spheres have radius r and are fixed with their centers a distance $R > 2r$ apart. If each sphere has a total charge q *uniformly distributed* on its surface, what is the magnitude of the electric force that either sphere exerts on the other? (*b*) Suppose instead that the spheres are conductors, the same total charge on each still being q. Will the electric force that either sphere exerts on the other in this case be greater than, less than, or equal to the force in case (*a*)? Explain.

5. The spherical region $a < r < b$ carries a charge per unit volume of $\rho = A/r$, where A is constant. At the center ($r = 0$) of the enclosed cavity is a point charge Q. What should the value of A be so that the electric field in the region $a < r < b$ has constant magnitude?

6. A solid insulating sphere carries a uniform charge per unit volume of ρ. Let $\mathbf{r}$ be the vector from the center of the sphere to a general point P within the sphere. (*a*) Show that the electric field at P is given by $\mathbf{E} = \rho\mathbf{r}/3\epsilon_0$. (*b*) A spherical "cavity" is removed from the above sphere, as shown in Fig. 28-25. Using superposition concepts show that the electric field at all points within the cavity is $\mathbf{E} = \rho\mathbf{a}/3\epsilon_0$ (uniform field), where $\mathbf{a}$ is the vector connecting the center of the sphere with the center of the cavity. Note that both these results are independent of the radii of the sphere and the cavity.

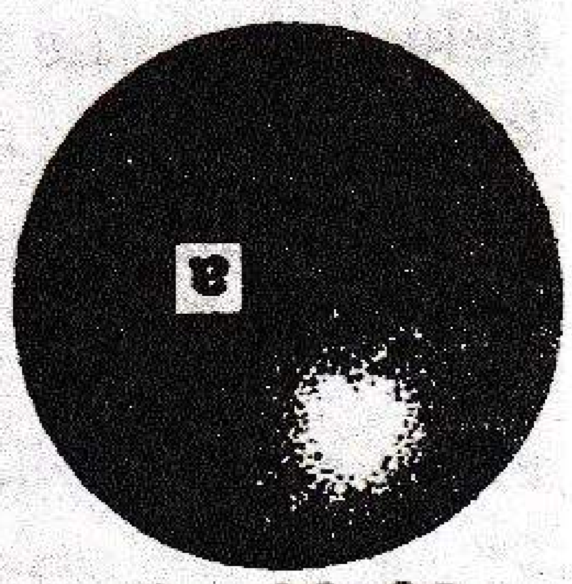


Fig. 28–25

Chapter 29

1. What is the charge density on the surface of a conducting sphere of radius 0.15 meter whose potential is 200 volts?

2. Two identical conducting spheres of radius $r = 0.15$ meter are separated by a distance $a = 10.0$ meters. What is the charge on each sphere if the potential of one is +1500 volts and if the other is −1500 volts?

3. Two conducting spheres, one of radius 6.0 cm and the other of radius 12.0 cm, each have a charge of 3×10^{-8} coul and are very far apart. If the spheres are connected by a conducting wire, find (*a*) the direction of motion and the magnitude of the charge transferred and (*b*) the final charge on and potential of each sphere.

4. In the rectangle shown in Fig. 29-29, the sides have lengths 5.0 cm and 15.0 cm, $q_1 = -5.0 \times 10^{-6}$ coul and $q_2 = +2.0 \times 10^{-6}$ coul. (*a*) What is the electric potential at corner *B*? At corner *A*? (*b*) How much work is involved in moving a third charge $q_3 = +3.0 \times 10^{-6}$ coul *from B to A* along a diagonal of the rectangle? (*c*) In this process, is external work converted into electrostatic potential energy or vice versa? Explain.

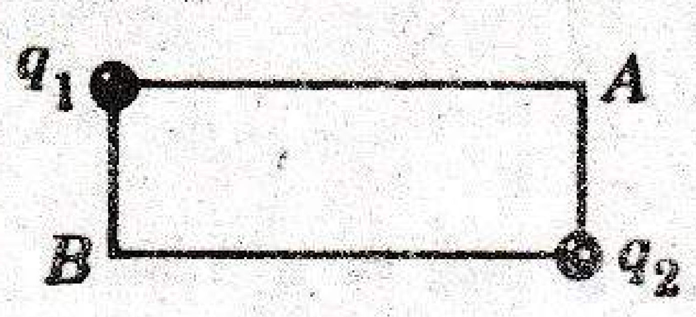


Fig. 29-29

5. Show that the potential energy of an assembly of four charges, each of charge *q*, in the configuration of a triangular pyramid, a_0 on an edge, is $6 \times (q^2/a_0)/4\pi\epsilon_0$.

6. Three charges of +0.1 coul each are placed on the corner of an equilateral triangle, 1.0 meter on a side. If energy is supplied at the rate of 1.0 kw, how many days would be required to move one of the charges onto the midpoint of the line joining the other two?

7. Show, using the fact that an electrostatic field is a conservative field, that one cannot create an electric field in which all the lines of force are straight parallel lines whose density (number per unit cross-sectional area) changes in a direction at right angles to the lines of force.

8. Two line charges are parallel to the *z*-axis; one, of charge per unit length $+\lambda$, is a distance *a* to the right of this axis, and the other, of charge per unit length $-\lambda$, is a distance *a* to the left of this axis (the lines and *z*-axis being in the same plane). Sketch some of the surfaces of equipotential.

9. A particle of (positive) charge *Q* is assumed to have a fixed position at *P*. A second particle of mass *m* and (negative) charge $-q$ moves at constant speed in a circle of radius r_1, centered at *P*. Derive an expression for the work *W* that must be done by an external agent on the second particle in order to increase the radius of the circle of motion, centered at *P*, to r_2. Express *W* in terms of quantities chosen from among *m*, r_1, r_2, *q*, *Q*, and ϵ_0 only.

10. A particle of charge *Q* is kept in a fixed position at a point *P* and a second particle of mass *m*, having the same charge *Q*, is initially held at rest a distance r_1 from *P*. The second particle is then released and is repelled from the first one. Determine its velocity at the instant it is a distance r_2 from *P*. Let $Q = 3.1 \times 10^{-6}$ coul, $m = 2.0 \times 10^{-5}$ kg, $r_1 = 9.0 \times 10^{-4}$ meter, and $r_2 = 25 \times 10^{-4}$ meter.

11. A particle of mass *m*, charge $q > 0$, and initial kinetic energy *K* is projected (from "infinity") toward a heavy nucleus of charge *Q*, assumed to have a fixed position

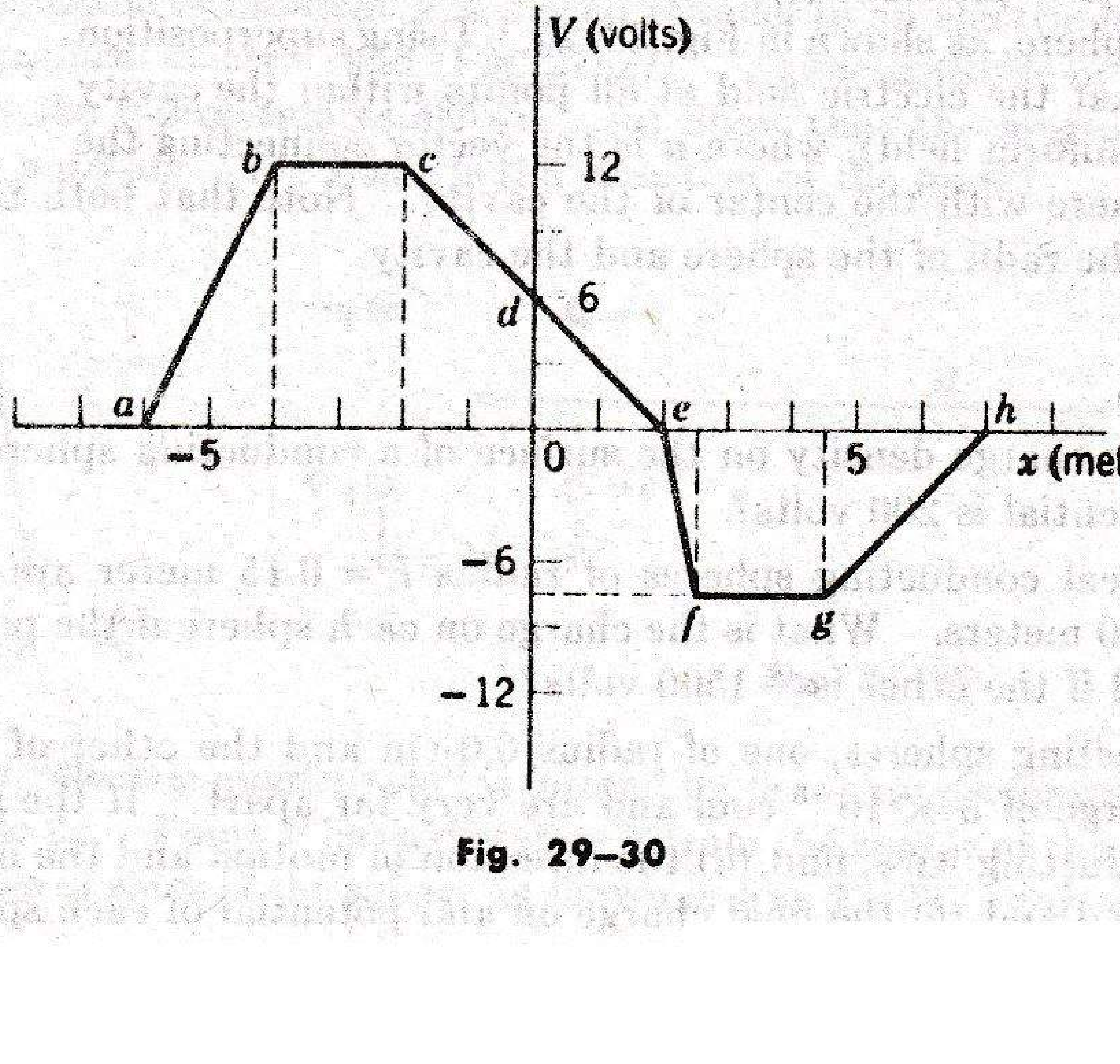


Fig. 29-30

in our reference frame. (*a*) If the "aim is perfect," how close to the center of the nucleus is the particle when it comes instantaneously to rest? (*b*) With a particular imperfect aim the particle's closest approach to the nucleus is *twice* the distance determined in part (*a*). Determine the *speed* of the particle at this closest distance of approach.

12. The electric potential varies along the x-axis as shown in the graph of Fig. 29–30. For each of the intervals shown (ignore the behavior at the end points of the intervals) determine the x-component of the electric field and plot E_x vs. x.

13. A charge per unit length λ is distributed uniformly along a straight-line segment of length L. (*a*) Determine the electrostatic potential (chosen to be zero at infinity) at a point P a distance y from one end of the charged segment and in line with it (see Fig. 29–31). (*b*) Use the result of (*a*) to compute the component of the electric field intensity at P in the y-direction (along the line). (*c*) Determine the component of the electric field intensity at P in a direction perpendicular to the straight line.

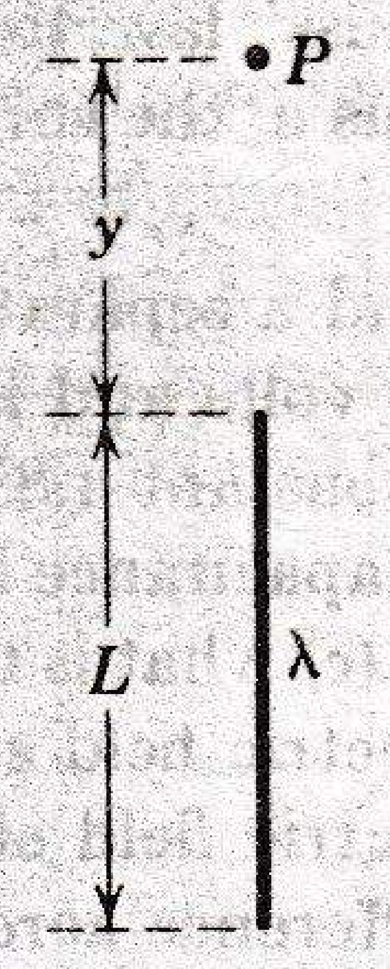


Fig. 29–31

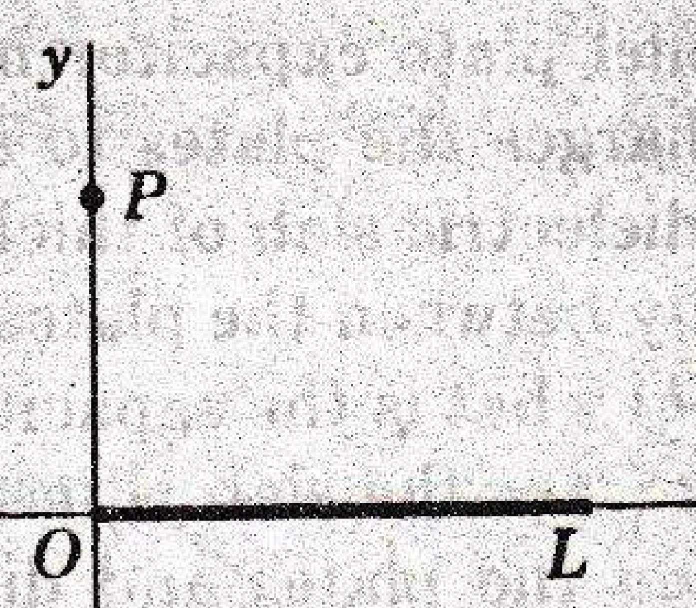


Fig. 29–32

14. On a thin rod of length L lying along the x-axis with one end at the origin ($x = 0$), as in Fig. 29–32, there is distributed a charge per unit length given by $\lambda = kx$, where k is a constant. (*a*) Taking the electrostatic potential at infinity to be zero, find V at the point P on the y-axis. (*b*) Determine the vertical component, E_y, of the electric field intensity at P from the result of part (*a*) and also by direct calculation. (*c*) Why cannot E_x, the horizontal component of the electric field at P, be found using the result of part (*a*)?

Chapter 30

1. Show that the capacitance of two oppositely charged metal spheres of the same radius, when far apart, is one-half the capacitance of one isolated sphere.

2. Two metallic spheres, radii a and b, are connected by a thin wire. Their separation is large compared with their dimensions. A charge Q is put onto this system. (*a*) How much charge resides on each sphere? (*b*) Apply the definition of capacitance to show that the capacitance of this system is $C = 4\pi\epsilon_0 (a + b)$.

3. N identical spherical drops of liquid are charged to the same potential V. One large drop is formed by combining these. Show that the potential of the large drop is $N^{2/3}V$.

4. Charges q_1, q_2, q_3 are placed on capacitors of capacitance C_1, C_2, C_3 respectively, arranged in series as shown in Fig. 30–28. Switch S is then closed. What are the final charges q_1', q_2', q_3' on the capacitors?

5. When switch S is thrown to the left in Fig. 30–29, the plates of the capacitor of capacitance C_1 acquire a potential difference V_0. C_2 and C_3 are initially uncharged.

The switch is now thrown to the right. What are the final charges q_1, q_2, q_3 on the corresponding capacitors?

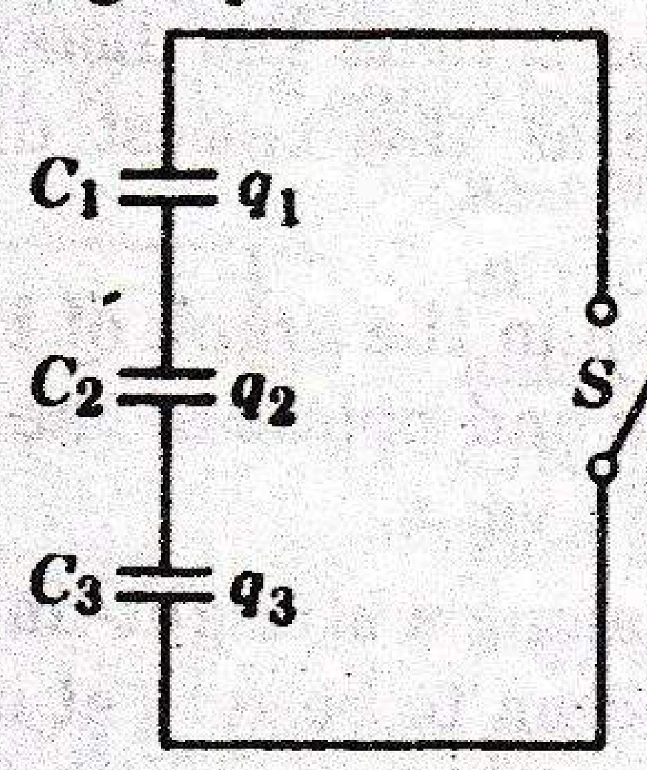


Fig. 30–28

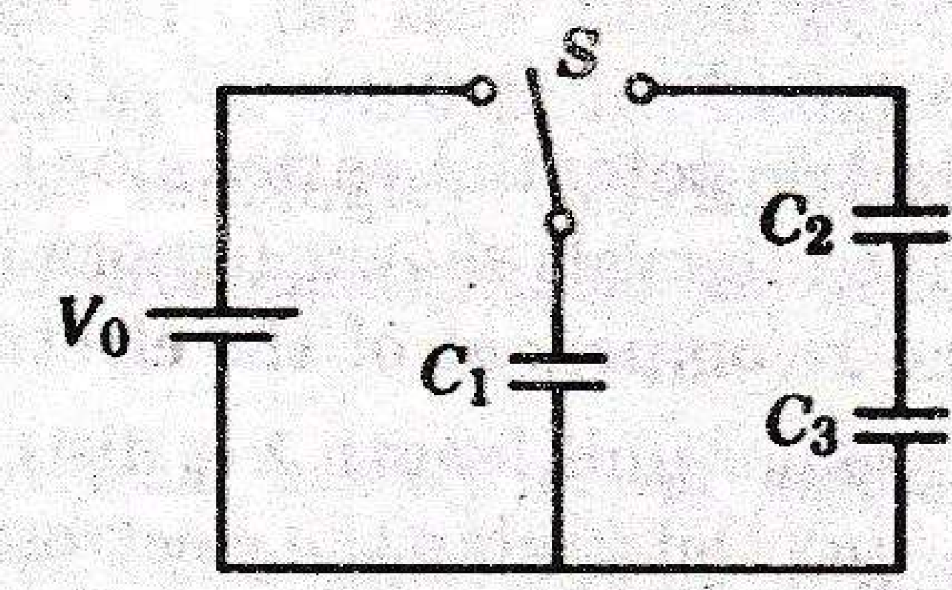


Fig. 30–29

6. A geiger tube is made of two long concentric metal cylinders with a gas of dielectric constant κ between them. Neglecting end effects, use Gauss' law to calculate the capacitance of this configuration. The center rod has a radius a, the surrounding tube a radius b, and the length $l \gg b$.

7. A parallel plate capacitor has plates of area 0.12 m^2 and a separation of 1.2 cm. A battery charges the plates to a potential difference of 120 volts and is then disconnected. A dielectric slab of thickness 0.4 cm and dielectric constant 4.8 is then placed symmetrically between the plates. In terms of ϵ_0 (*a*) find the capacitance before the slab is inserted. (*b*) what is the capacitance with the slab in place? (*c*) what is the free charge q before and after the slab is inserted? (*d*) determine the electric field strength in the space between the plates and dielectric. (*e*) what is the electric field strength in the dielectric? (*f*) with the slab in place what is the potential difference across the plates? (*g*) how much external work is involved in the process of inserting the slab?

Chapter 31

1. A steady beam of alpha particles ($q = 2e$) traveling with constant kinetic energy 20 MeV carries a current 0.25×10^{-6} ampere. (*a*) If the beam is directed perpendicular to a plane surface, how many alpha particles strike the surface in 3.0 sec? (*b*) At any instant, how many alpha particles are there in a given 20-cm length of the beam? (*c*) Through what potential difference was it necessary to accelerate each alpha particle from rest to bring it to an energy of 20 MeV?

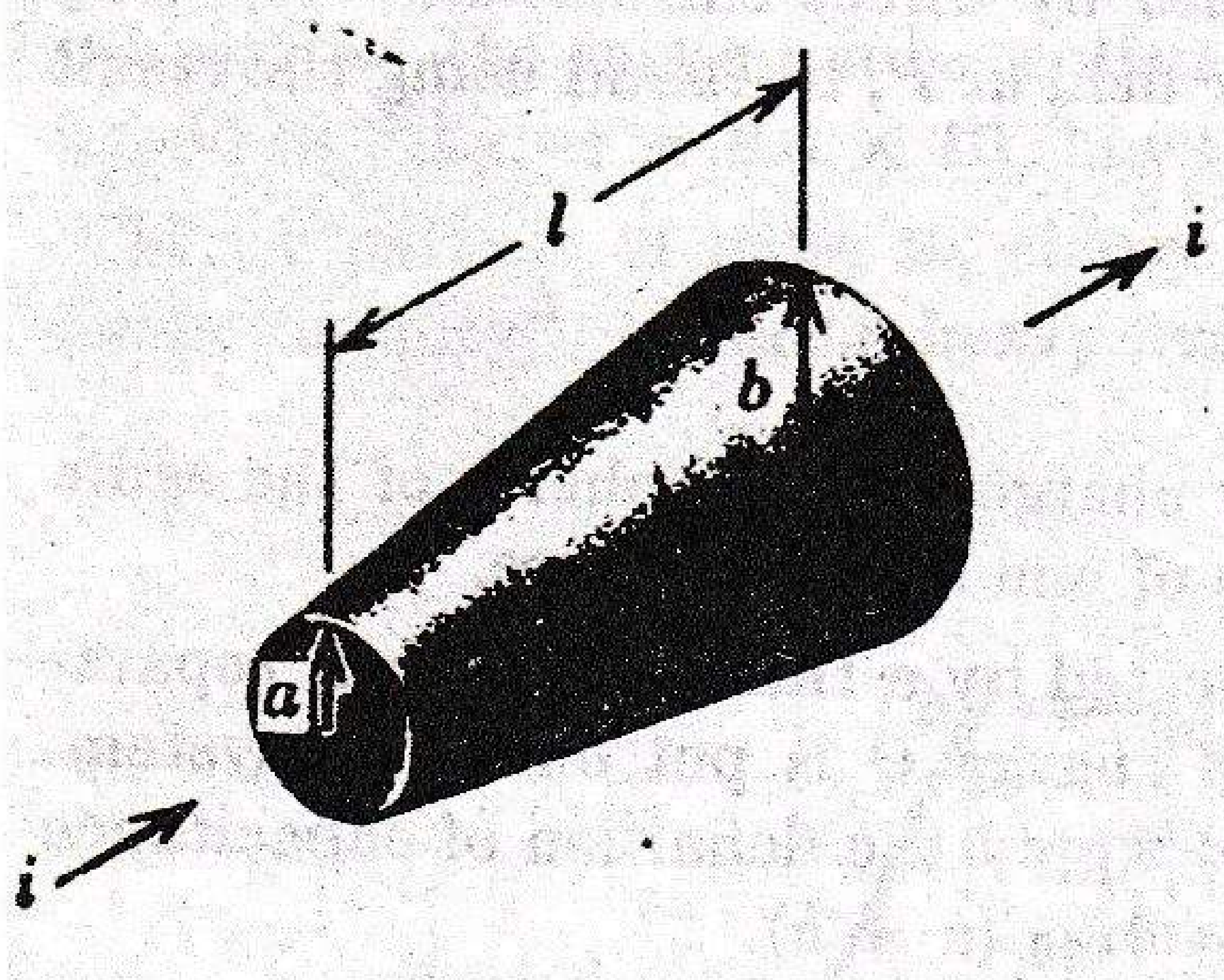


Fig. 31–10

2. List similarities and differences between the flow of charge and the flow of a fluid; between the flow of charge and the conduction of heat.

3. Explain why the momentum which conduction electrons transfer to the ions in a metal conductor does not give rise to a resultant force on the conductor.

4. A resistor is in the shape of a truncated right circular cone (Fig. 31–10). The end radii are a and b, the altitude is l. If the taper is small, we may assume that the current density is uniform across any cross section. (*a*) Calculate the resistance of this object (*b*) Show that your answer reduces to $\rho(l/A)$ for the special case of zero taper ($a = b$)

5. Conductors A and B, having equal lengths of 40 meters and a cross-sectional area of 0.10 m^2, are connected in series. A potential of 60 volts is applied across the terminal points of the connected wires. The resistances of the wires are 40 and 20 ohms respectively. Determine: (*a*) the resistivities of the two wires; (*b*) the magnitude of the elec-

tric field in each wire; (c) the current density in each wire; (d) the potential difference applied to each conductor.

6. A 1250-watt radiant heater is constructed to operate at 115 volts. (a) What will be the current in the heater? (b) What is the resistance of the heating coil? (c) How many kilocalories are created in our hour by the heater?

7. An iron wire (diameter 1 mm, length 10 cm) is placed in an evacuated chamber. Estimate the equilibrium temperature of the wire if it carries a current of 10 amp. Assume that all heat transfer is by radiation and that the surface of the wire radiates according to Eq. 47–2. Take the temperature of the chamber walls to be 27° C. List any additional assumptions that you use.

Chapter 32

1. Two batteries having the same emf $\mathcal{E}$ but different internal resistances r_1 and r_2 are connected in series to an external resistance R. Find the value of R that makes the potential difference zero between the terminals of the first battery.

2. The section of circuit AB (see Fig. 32–26) absorbs power $P = 50.0$ watts and a current $i = 1.0$ amp passes through it in the indicated direction. (a) What is the potential difference between A and B? (b) If the element C does not have internal resistance, what is its emf? (c) What is its polarity?

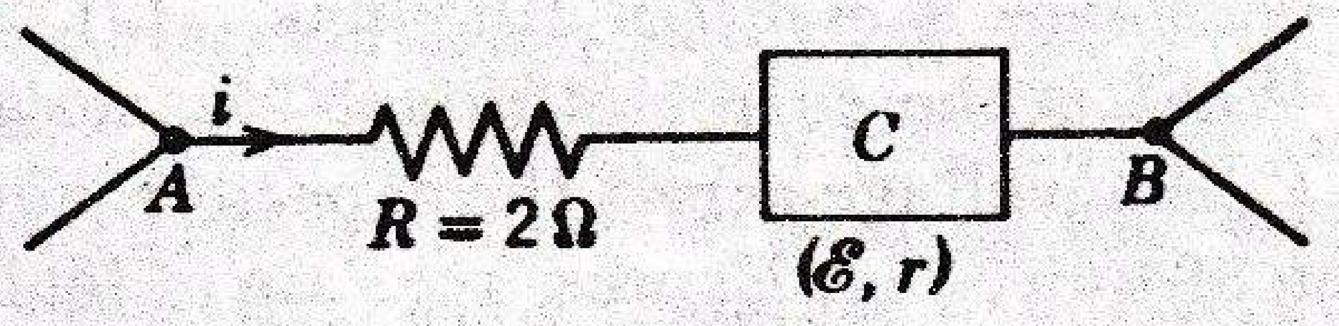


Fig. 32–26

3. Two light bulbs, one of resistance R and the other of resistance $r (< R)$, are connected (a) in parallel and (b) in series. Which bulb is brighter?

4. N identical batteries of emf $\mathcal{E}$ and internal resistance r may be connected all in series or all in parallel. Show that each arrangement will give the same current in an external resistor R if $R = r$.

5. Twelve resistors, each of resistance R ohms, form a cube (see Fig. 32–27). (a) Find R_{AB}, the resistance of an edge. (b) Find R_{BC}, the equivalent resistance of a face. (c) Find R_{AC}, the equivalent resistance of the body diagonal.

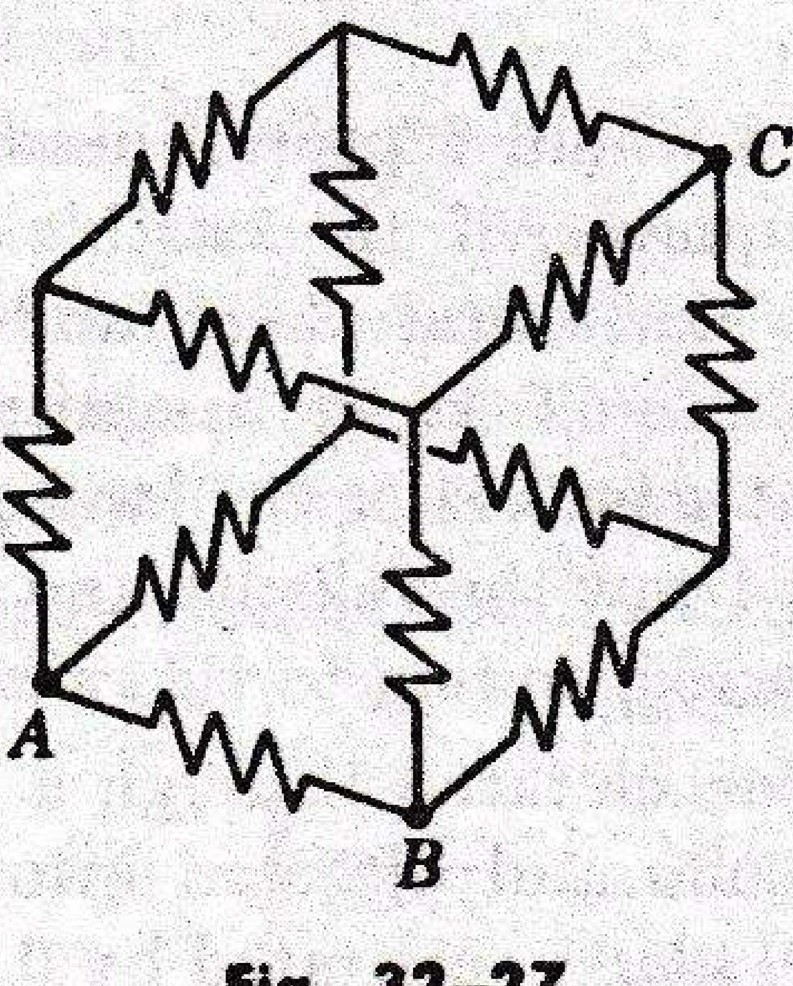


Fig. 32–27

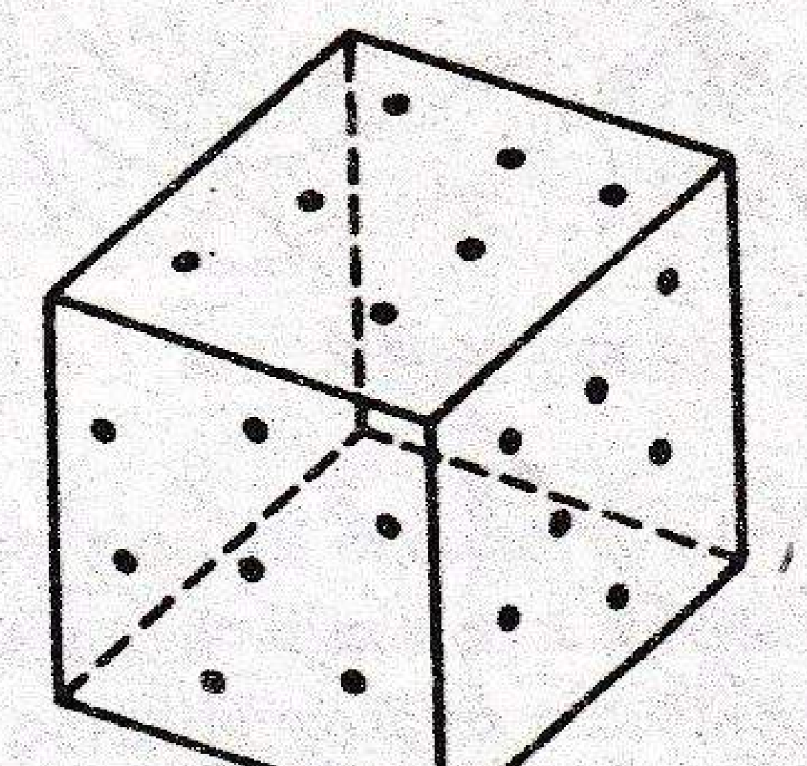


Fig. 32–28

6. What internal connections (resistors only) can account for the fact that there are 2 ohms resistance between any pair of the terminals on a box with N terminals? (See Fig. 32–28.)

7. A voltmeter and an ammeter are used to determine two unknown resistances R_1 and R_2, one determination by each of the two methods shown in Fig. 32-29. The voltmeter resistance is 307 ohms and the ammeter resistance is 3.62 ohms; in method (*a*) the ammeter reads 0.317 amp and the voltmeter reads 28.1 volts, whereas in method (*b*) the ammeter reads 0.356 amp and the voltmeter 23.7 volts. Compute R_1 and R_2.

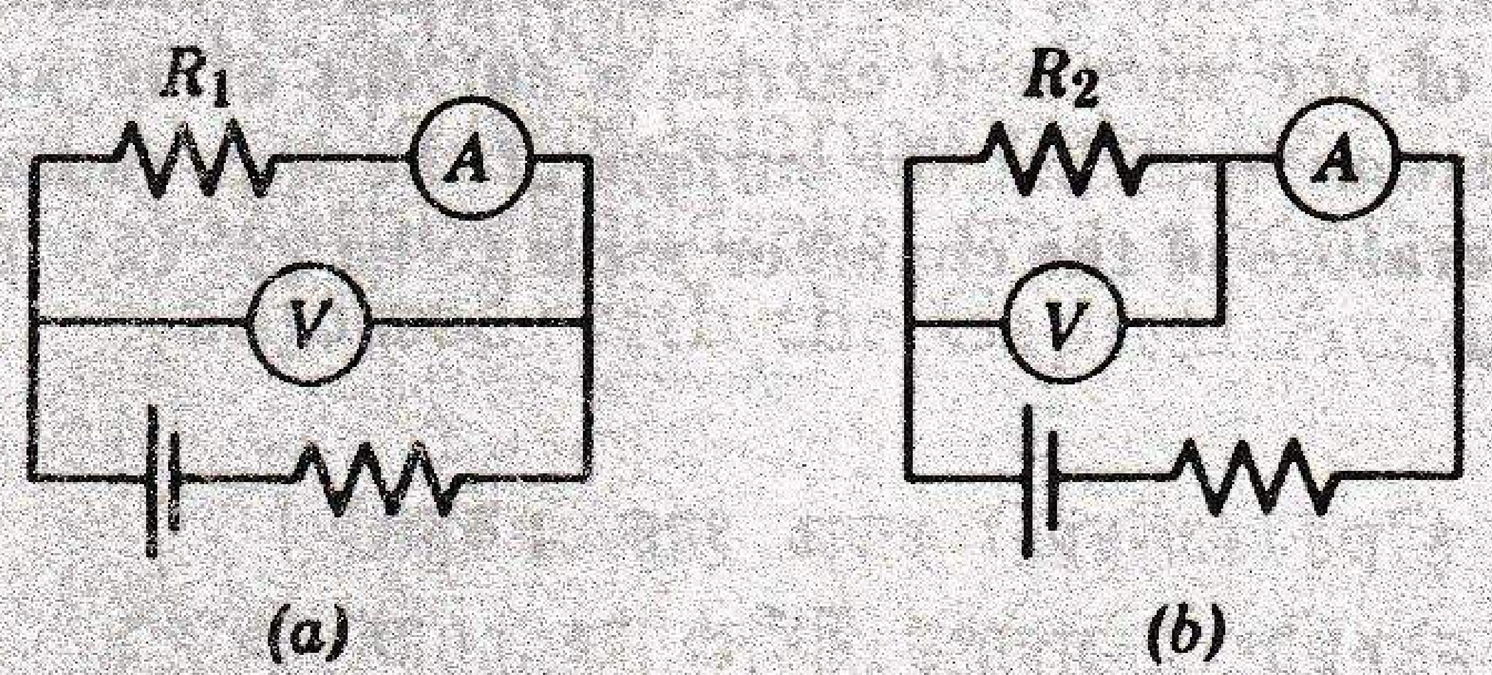


Fig. 32-29

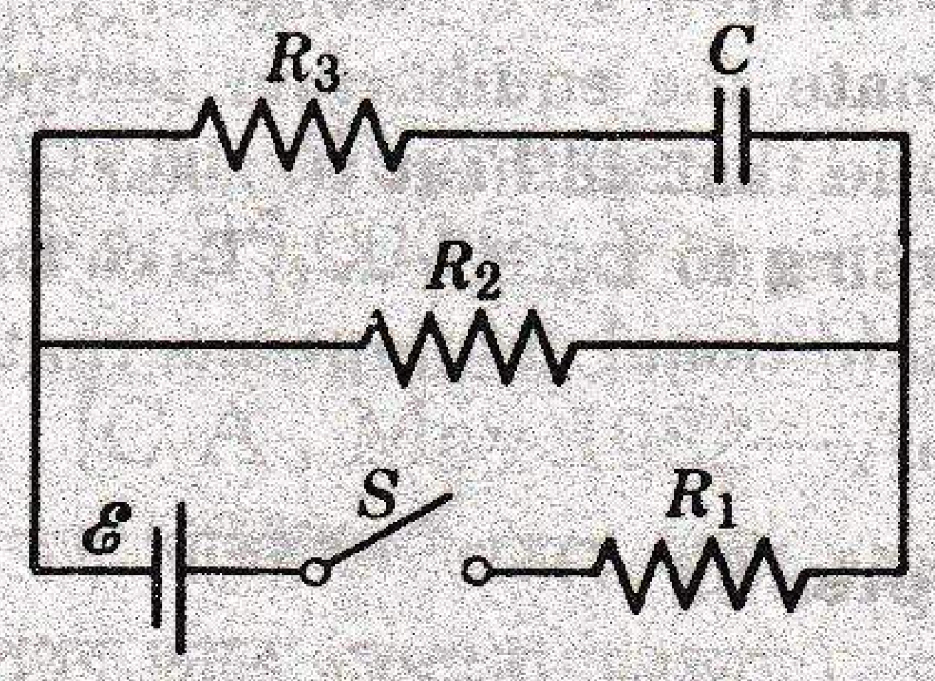


Fig. 32-30

8. An RC circuit is discharged by closing a switch at time $t = 0$. The initial potential difference across the capacitor is 100 volts. If the potential difference has decreased to 1.0 volt after 10 sec, (*a*) what will the potential difference be 20 sec after $t = 0$? (*b*) What is the time constant of the circuit?

9. A capacitor with capacitance $C = 1.0\ \mu f$ and initial stored energy $U_0 = 0.5$ joule is discharged through a resistance $R = 1.0 \times 10^6$ ohms. (*a*) What is the initial charge on the capacitor? (*b*) What is the current through the resistor when the discharge starts? (*c*) Determine V_c, the voltage across the capacitor, and V_R, the voltage across the resistor, as a function of time. (*d*) Express the rate of joule heating in the resistor as a function of time.

10. In the circuit of Fig. 32-30, $\mathcal{E} = 1200$ volts, $C = 6.50\ \mu f$, $R_1 = R_2 = R_3 = 7.30 \times 10^5$ ohms. With C completely uncharged, the switch S is suddenly closed ($t = 0$). (*a*) Determine, for $t = 0$ and $t = \infty$, the currents through each resistor. (*b*) Draw qualitatively a graph of the potential drop V_2 across R_2 from $t = 0$ to $t = \infty$. (*c*) What are the numerical values of V_2 at $t = 0$ and $t = \infty$? (*d*) Give the physical meaning of "$t = \infty$" and state a rough, but significant, numerical lower bound, in seconds, for "$t = \infty$" in this case.

Chapter 33

1. Particles 1, 2, and 3 follow the paths shown in Fig. 33-25 as they pass through the magnetic field there. What can one conclude about each particle?

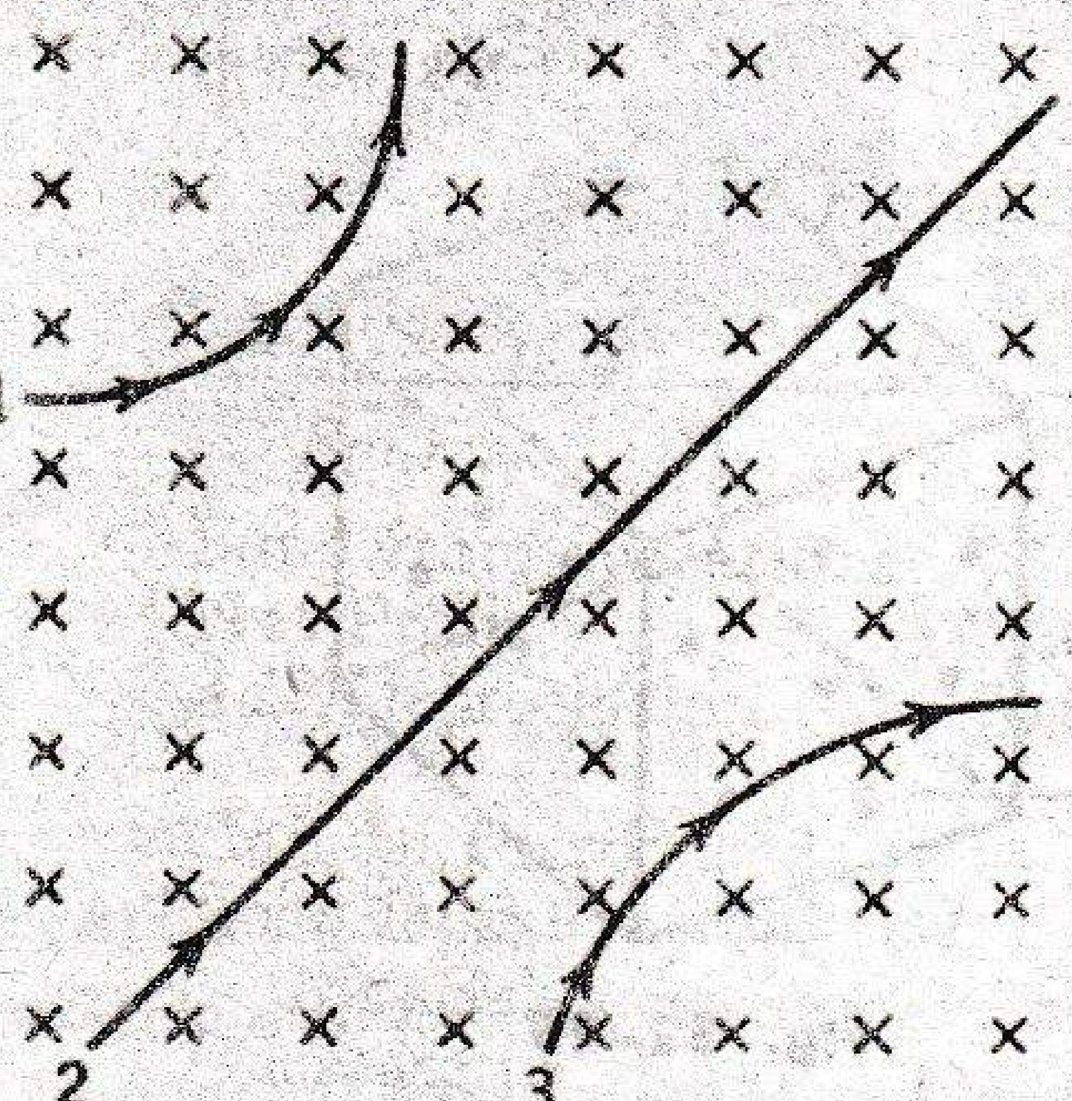


Fig. 33-25

2. (*a*) What speed would a proton need to circle the earth at the equator, if the earth's magnetic field is everywhere horizontal there and directed along longitudinal lines. Take the magnitude of the earth's magnetic field to be 0.41×10^{-4} weber/meter^2 at the equator. (*b*) Draw the velocity and magnetic induction vectors corresponding to this situation.

3. An electron is accelerated through a potential difference of 1000 volts and directed into a region between two parallel plates separated by 0.02 meter with a voltage difference of 100 volts between them. If the electron enters moving perpendicular to the electric field between the plates, what magnetic field is necessary perpendicular to both the electron path and the electric field so that the electron travels in a straight line?

4. A positive point charge Q travels in a straight line with constant speed through an evacuated region in which there is a uniform electric field **E** and a uniform magnetic field **B**. (*a*) If **E** is directed vertically up and the charge travels horizontally from north to south with speed v, determine the least value of the magnitude of **B** and the corresponding direction of **B**. (*b*) Explain why **B** is not uniquely determined when **E** and **v** alone are given. (*c*) Suppose the charge is a proton which enters the region after having been accelerated through a potential difference of 3.10×10^6 volts. If $E = 1.90 \times 10^5$ volt/meter, compute the value of B in part (*a*). (*d*) If in part (*c*) the electric field **E** is turned off, determine the radius r of the circle in which the proton now moves.

5. A certain galvanometer has a resistance of 75.3 ohms; its needle experiences a full-scale deflection when a current 1.62×10^{-3} amp passes through its coil. (*a*) Determine the value of the auxiliary resistance required to convert the galvanometer into a voltmeter that reads 1.000 volt at full-scale deflection. How is it to be connected? (*b*) Determine the value of the auxiliary resistance required to convert the galvanometer into an ammeter that reads 0.0500 amp at full-scale deflection. How is it to be connected?

6. In a Hall effect experiment a current of 3.0 amp lengthwise in a conductor 1.0 cm wide, 4.0 cm long, and 10^{-3} cm thick produced a transverse Hall voltage (across the width) of 1.0×10^{-5} volt when a magnetic field of 1.5 weber/meter^2 passed perpendicularly through the thin conductor. From these data, find (*a*) the drift velocity of the charge carriers and (*b*) the number of carriers per cubic centimeter. (*c*) Show on a diagram the polarity of the Hall voltage with a given current and magnetic field direction, assuming the charge carriers are (negative) electrons.

7. (*a*) What is the cyclotron frequency of an electron with an energy of 100 ev in the earth's magnetic field of 1.0×10^{-4} webers/meters^2? (*b*) What is the radius of curvature of the path of this electron if its velocity is perpendicular to the magnetic field?

Chapter 34

1. Eight wires cut the page perpendicularly at the points shown in Fig. 34-32. A wire labeled with the integer k ($k = 1, 2, \ldots 8$) bears the current ki_0. For those with odd k, the current flows up out of the page; for those with even k it flows down into the page. Evaluate $\oint \mathbf{B} \cdot d\mathbf{l}$ along the closed path shown in the direction indicated by the single arrowhead.

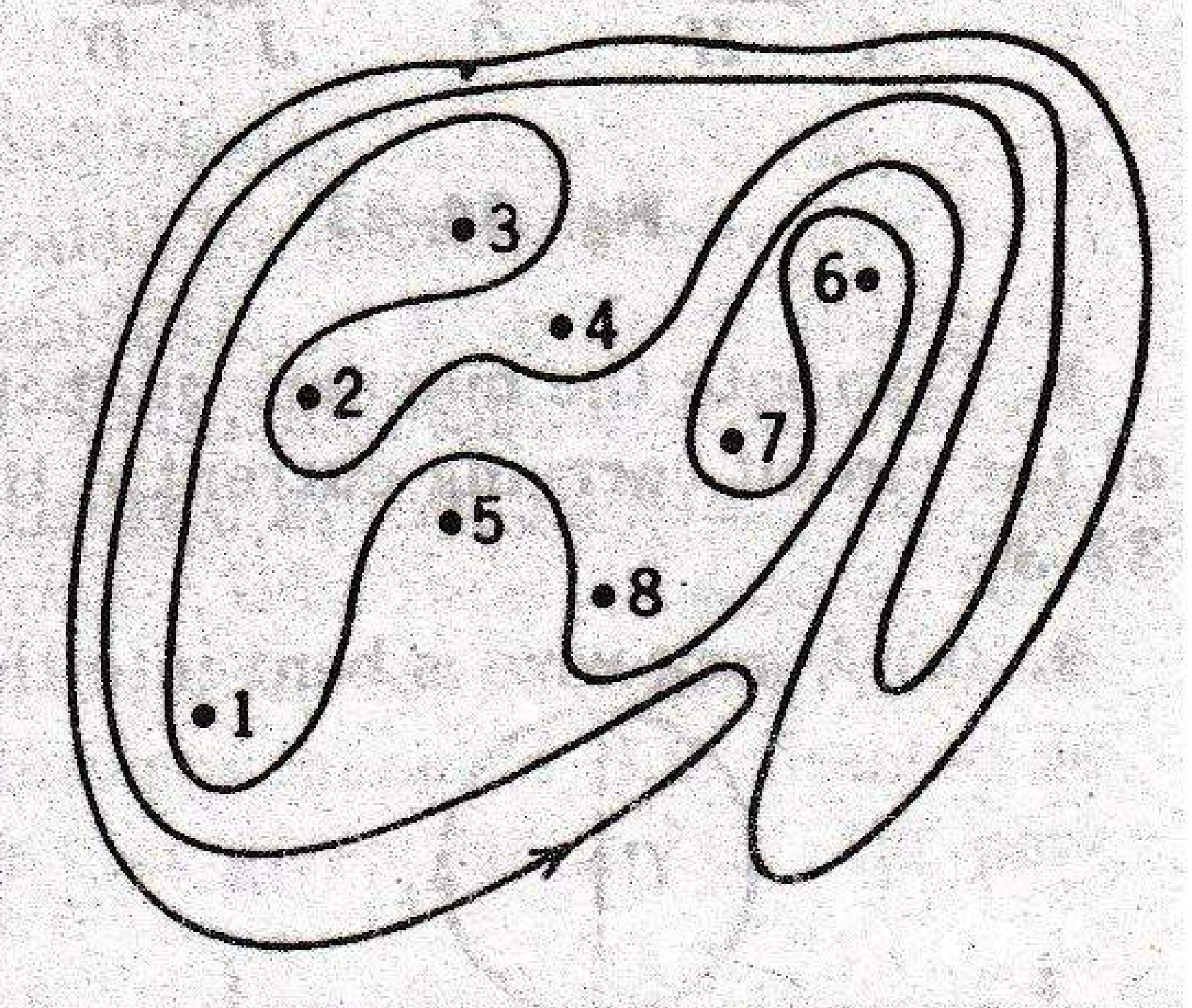


Fig. 34-32

2. Two long straight wires pass near one another at right angles. If the wires are free to move, describe what happens when currents are sent through them.

3. Suppose, in Fig. 34-25, that the currents are all in the same direction. What is the force per meter (magnitude and direction) on any one wire? In the analogous case of parallel motion of charged particles in a plasma this is known as the pinch effect.

4. A long straight conductor has a circular cross section of radius R and carries a current i. Inside the conductor there is a cylindrical hole of radius a whose axis is parallel to the conductor axis at a distance b from it. Use superposition ideas, and obtain an expression for the magnetic induction B inside the hole.

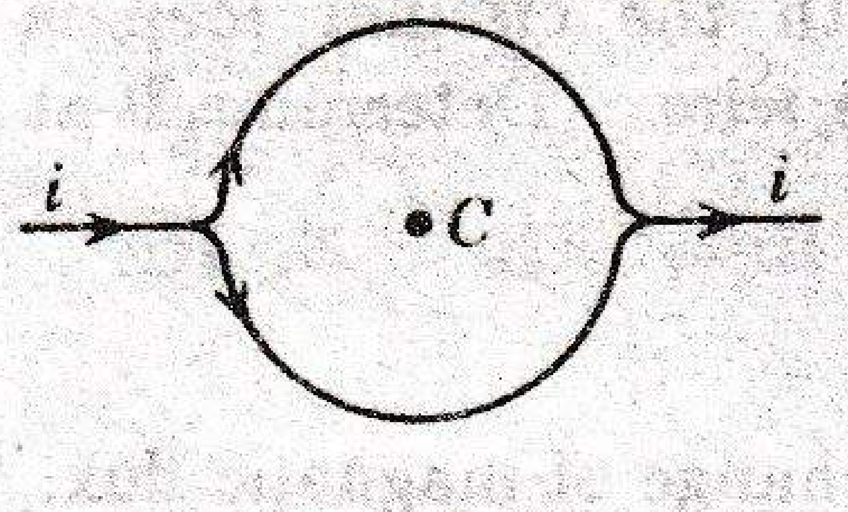
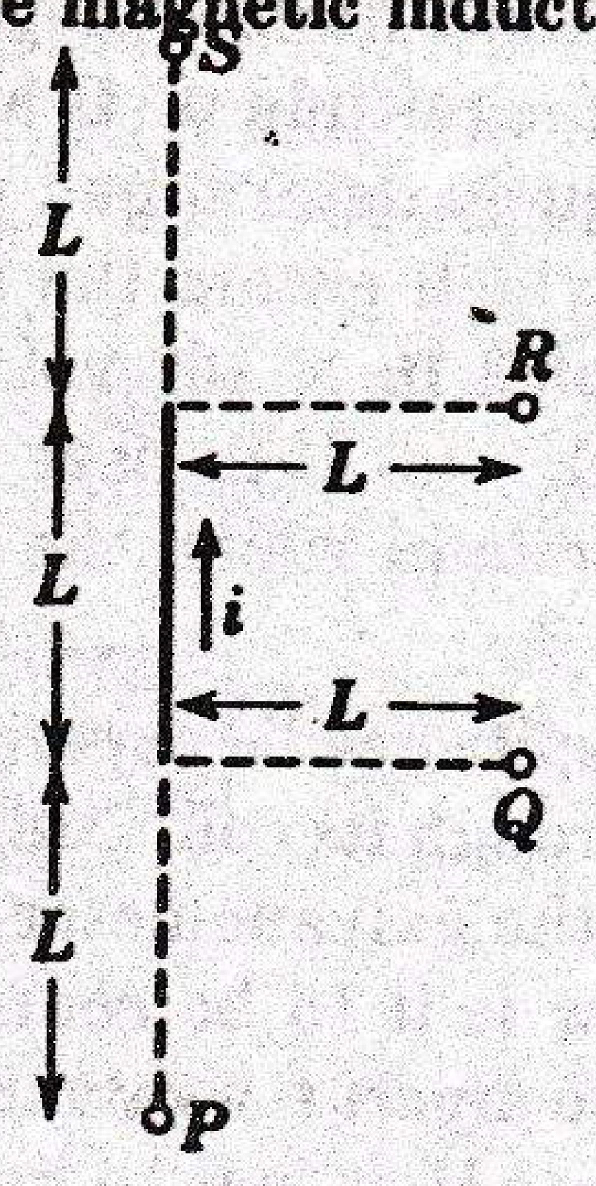


Fig. 34-33

5. A straight conductor is split into identical semicircular turns as shown in Fig. 34-33. What is the magnetic field at the center C of the circular loop so-formed?

6. (*a*) A current i flows in a straight wire of length L in the direction shown in Fig. 34-34*a*. Starting from the Biot-Savart law, determine the resulting magnetic induction ($\mathbf{B}_P$, $\mathbf{B}_Q$, $\mathbf{B}_R$, $\mathbf{B}_S$, respectively-direction and magnitude in each case) at each of the four points P, Q, R, S (all coplanar with the wire). (*b*) Using the results of part (*a*), compute the magnetic induction **B** (magnitude and direction) resulting at the point T



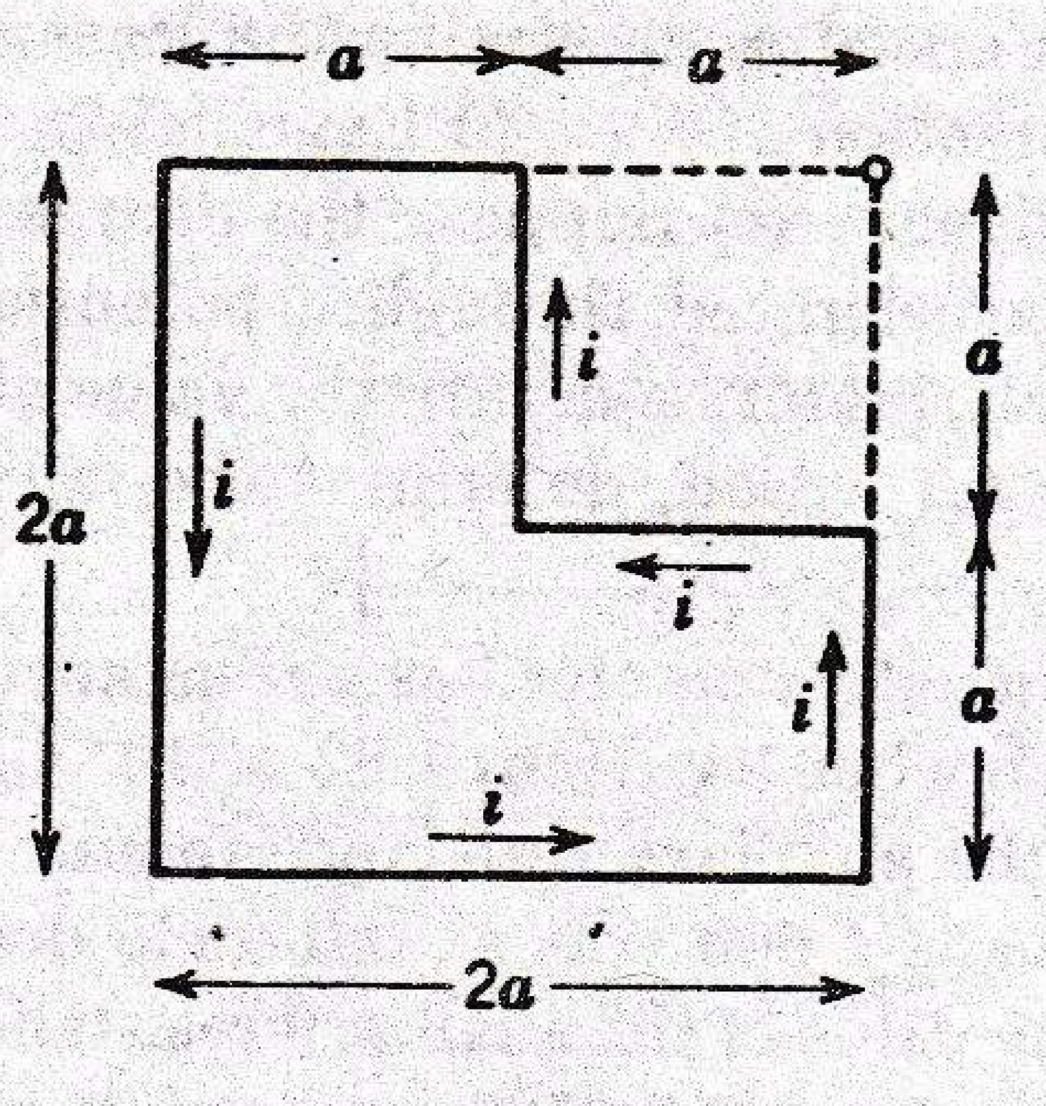


Fig. 34-34

from the current flowing as indicated in the six-sided rectilinear closed loop shown in Fig. 34-34*b*. (Everything drawn is meant to lie in the same plane and all angles are 90°.)

7. Use the Biot-Savort law to calculate the magnetic induction **B** at C, the common center of the semicircular arcs AD and HJ, of radii R_2 and R_1 respectively, forming part of the circuit $ADJHA$ carrying current i, as shown in Fig. 34-35.

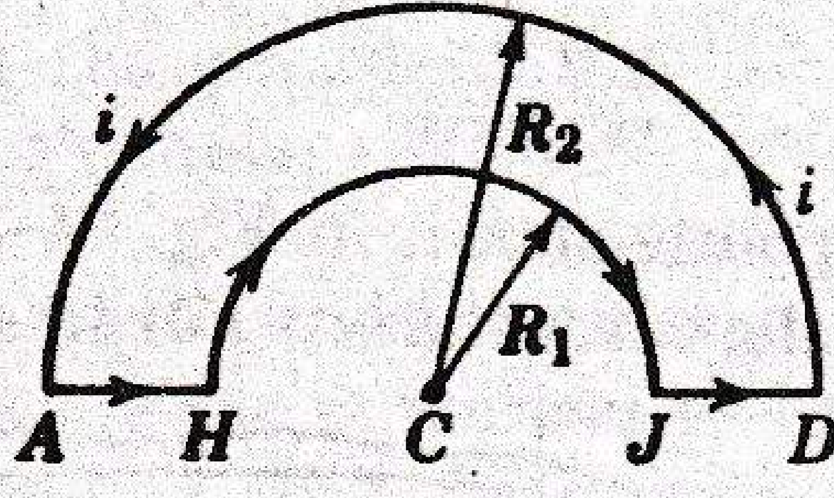


Fig. 34-35

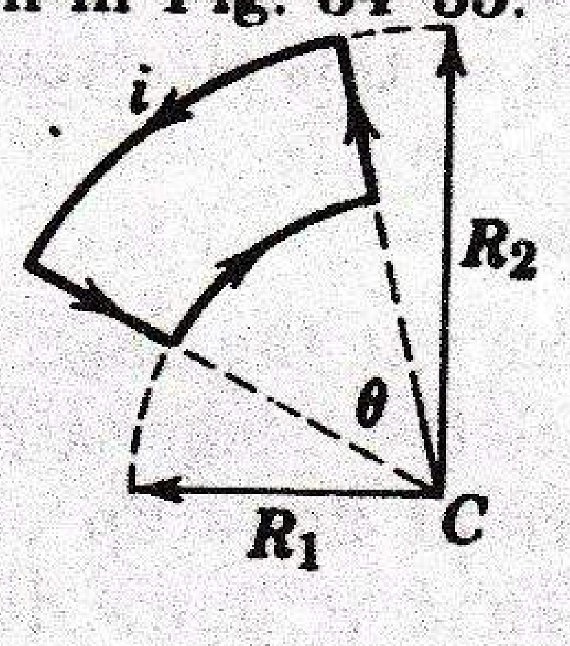


Fig. 34-36

8. Compute the magnetic induction **B** at C, the common center of the circular arcs of the circuit carrying current i, the arcs cutting a sector of angle θ, as shown in Fig. 34-36.

9. (*a*) A long wire is bent into the shape shown in Fig. 34-37, without cross-contact at P. Determine the magnitude and direction of **B** at the center C of the circular portion when the current i flows as indicated. (*b*) The circular part of the wire is rotated without distortion about its (dashed) diameter perpendicular to the straight portion of the wire. The magnetic moment associated with the circular loop is now in the direction of the current in the straight part of the wire. Determine **B** at C in this case.

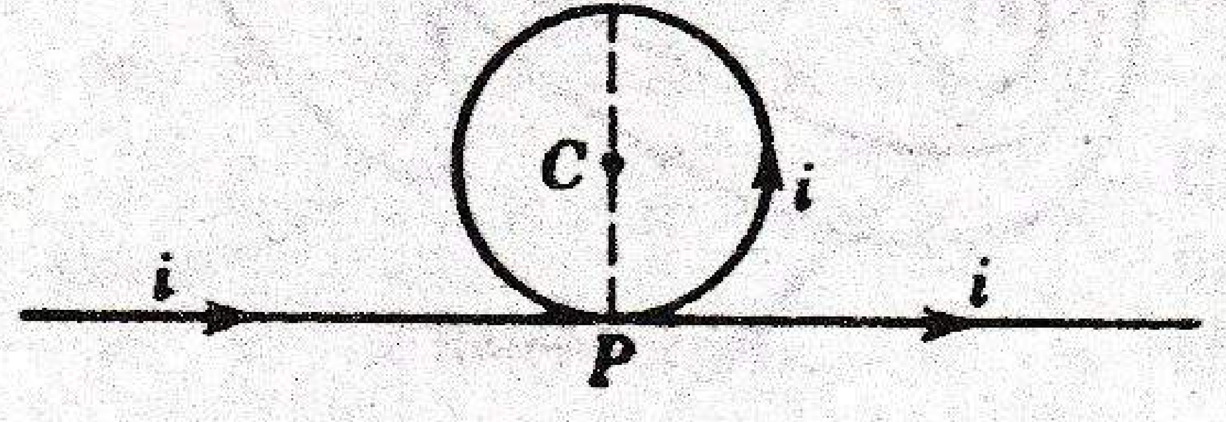


Fig. 34-37

Chapter 35

1. Show that emf has the same dimensions as time rate of change of magnetic flux.

2. In Fig. 35-39, $l = 2.0$ meters and $v = 50$ cm/sec. **B** is the earth's magnetic field, directed perpendicularly out of the page and having a magnitude 6.0×10^{-5} webers/m^2 at that place. The resistance of the circuit $ADCB$, assumed constant (explain how this may be achieved approximately), is $R = 1.2 \times 10^{-5}$ ohms. (*a*) What is the emf induced in the circuit? (*b*) What is the electric field strength in the wire AB? (*c*) What force does each electron in the wire experience due to the motion of the wire in the magnetic field? (*d*) What is the magnitude and direction of the current in the wire? (*e*) What force must an external agency exert in order to keep the wire moving with this constant velocity? (*f*) Compute the rate at which the external agency is doing work. (*g*) Compute the rate at which electrical energy is being converted into heat energy.

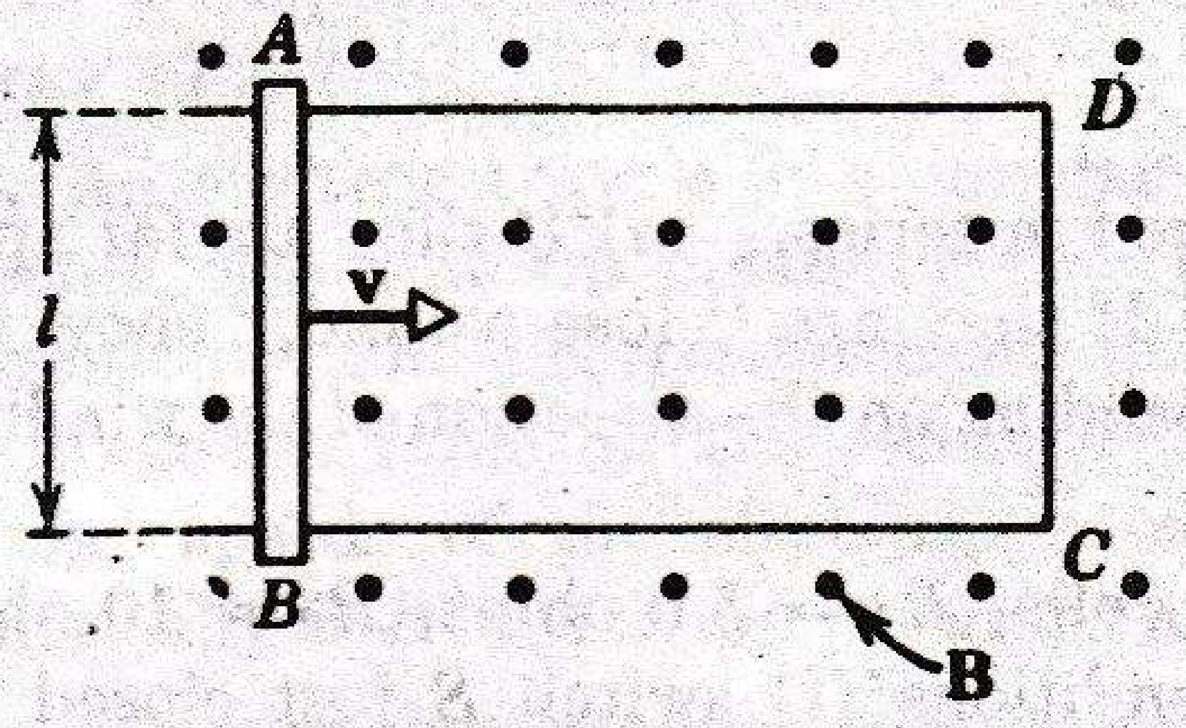


Fig. 35–39

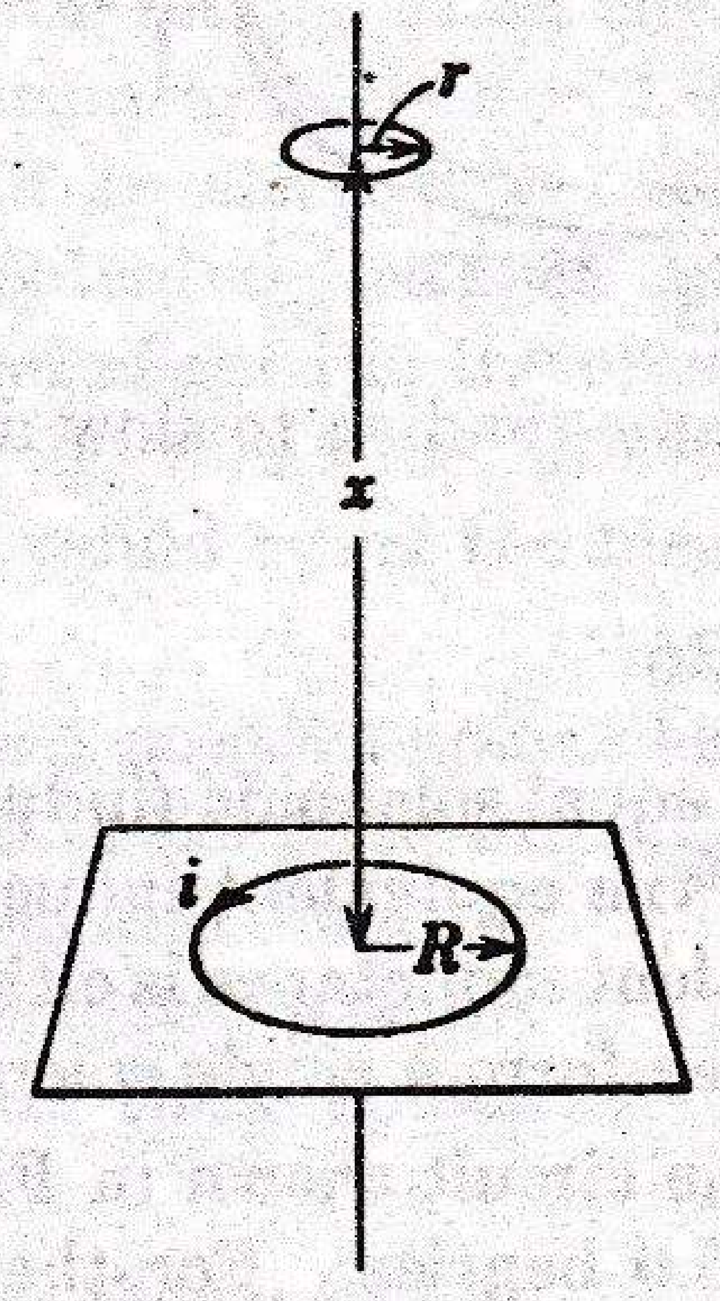


Fig. 35–40

3. Figure 35–40 shows two loops of wire having the same axis. The smaller loop is above the larger one, a distance x which is large compared to the radius R of the larger loop. Hence, with current i flowing as indicated in the larger loop, the consequent magnetic induction is nearly constant throughout the plane area πr^2 bounded by the smaller loop. Suppose now that x is not constant but is changing at the constant rate $dx/dt = v$. (*a*) Determine the magnetic flux across the area bounded by the smaller loop as a function of x. (*b*) Compute the emf generated in the smaller loop at the instant when $x = NR$. (*c*) Determine the direction of the induced current flowing in the smaller loop if $v > 0$.

4. A closed loop of wire consists of a pair of equal semicircles, radius 3.70 cm, lying in mutually perpendicular planes. The loop was formed by folding a circular loop along a diameter until the two halves became perpendicular. A uniform magnetic field **B** of magnitude 760 gauss is directed perpendicular to the fold diameter and makes equal angles (45°) with the planes of the semicircles as shown in Fig. 35–41*a*. (*a*) The magnetic field is reduced at a uniform rate to zero during a time interval 4.50×10^{-3} sec. Determine the magnitude of the induced emf and the sense of the induced current in the loop during this interval. (*b*) How would the answers change if **B** is directed as shown in Fig. 35–41*b*, perpendicular to the direction first given for it but still perpendicular to the "fold-diameter?"

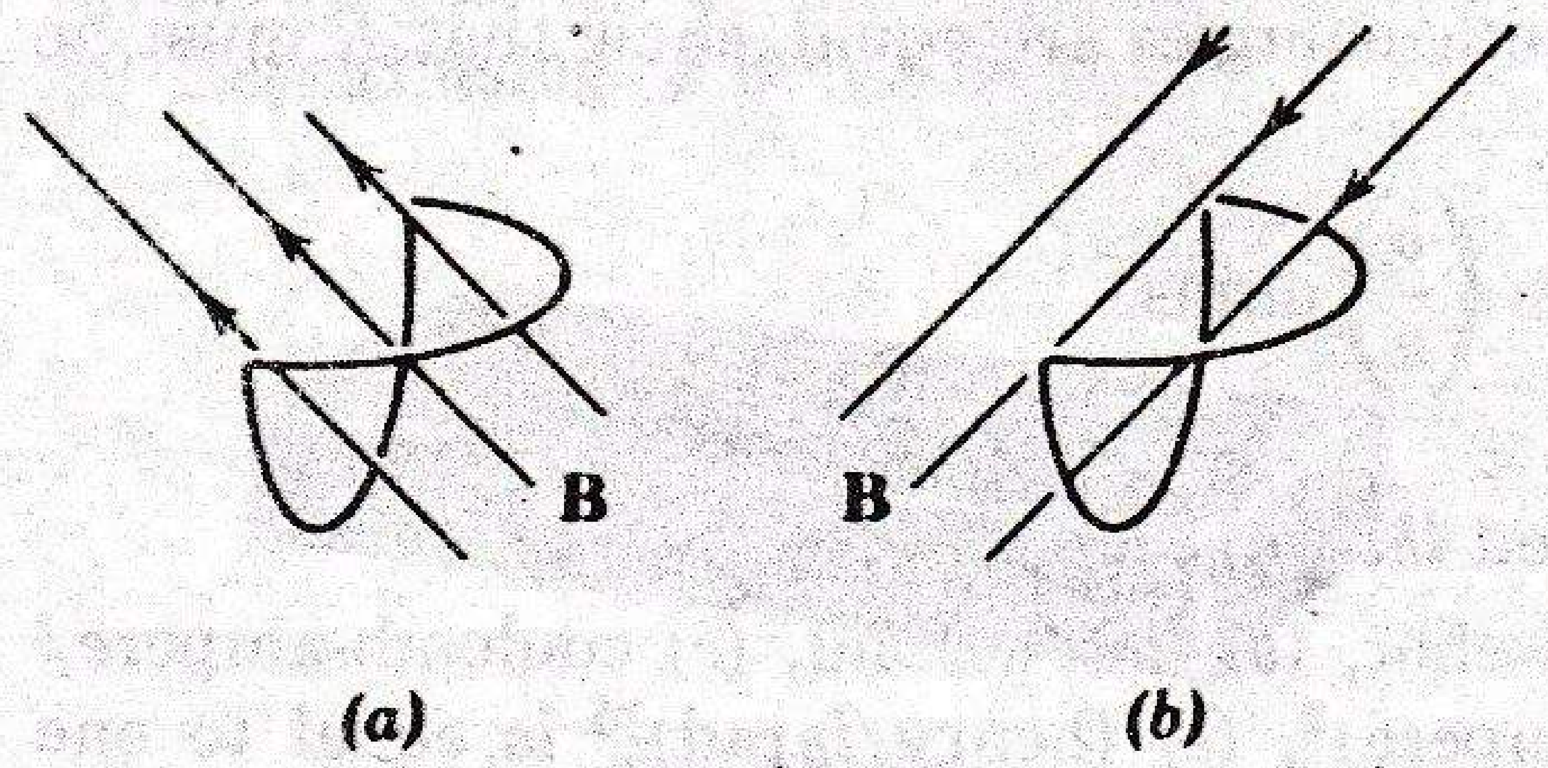


Fig. 35–41

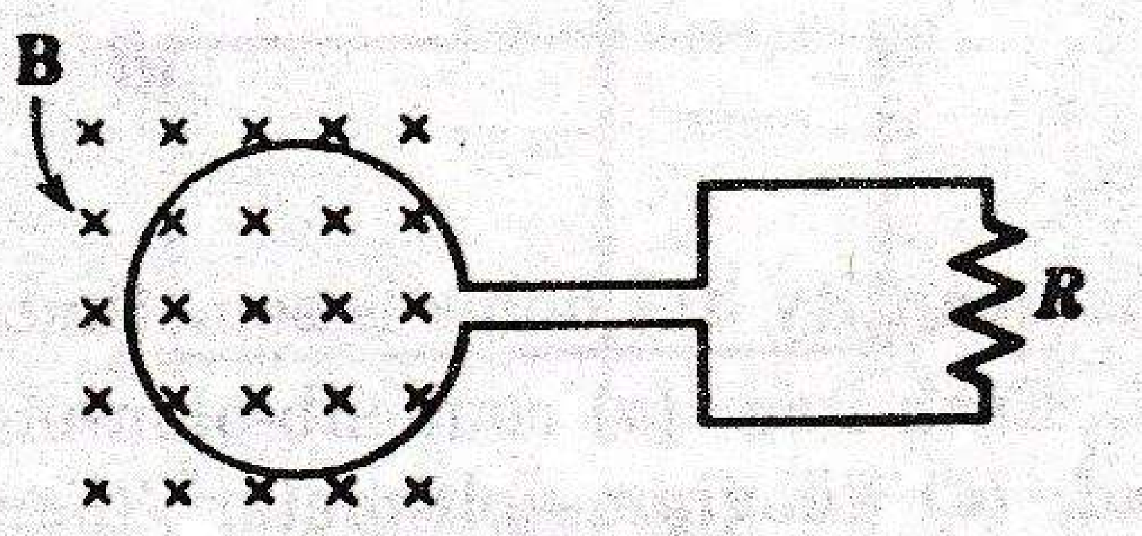


Fig. 35–42

5. A loop of wire of area A is connected to a resistance R. The loop is exposed to a time varying **B** field (see Fig. 35–42). (*a*) Derive an expression for the ***net*** charge

transferred through the resistor between $t = t_1$ and $t = t_2$. Show that your answer is proportional to the difference $\Phi_B(t_2) - \Phi_B(t_1)$, and is otherwise independent of the manner in which **B** is changing. (*b*) Suppose the change in flux, $\Phi_B(t_2) - \Phi_B(t_1)$ is zero. Does it then follow that no joule-heating occurred during this time interval?

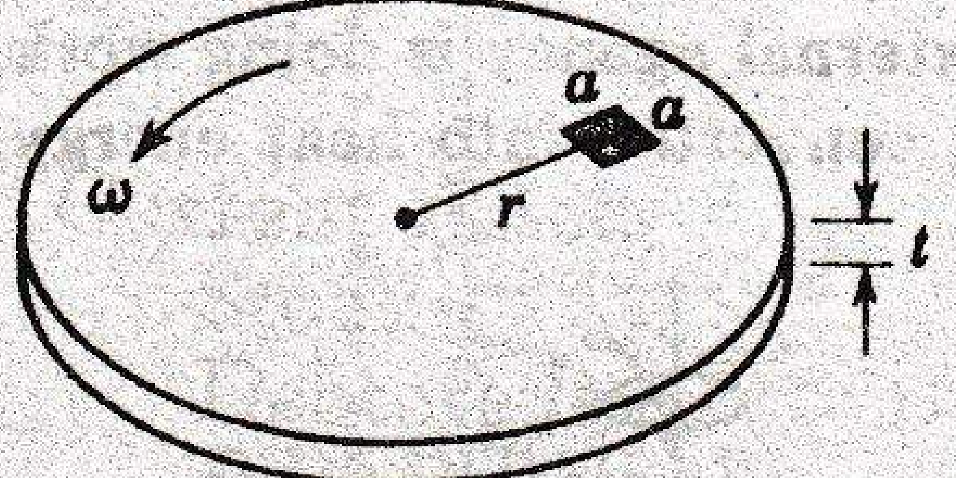


Fig. 35–43

6. An electromagnetic "eddy current" brake consists of a disk of conductivity σ and thickness t rotating about an axis through its center with a magnetic field **B** applied perpendicular to the plane of the disk over a small area a^2 (see Fig. 35–43). If the area a^2 is at a distance r from the axis, find an approximate expression for the torque tending to slow down the disk at the instant its angular velocity equals ω.

Chapter 36

1. Two equal solenoids (inductance L) are wired in series. (*a*) If they are far apart, show that the equivalent inductance is $2L$. (*b*) If they are "close wound," show that the equivalent inductance is either zero or $4L$ depending on the direction of the windings. (*Hint:* They form a single solenoid.)

2. In the circuit shown in Fig. 36–8, $\mathcal{E}$ = 10 volts, R_1 = 5.0 ohms, R_2 = 10 ohms, and L = 5.0 henries. For the two separate conditions (I) switch S just closed and (II) switch S closed for a very long time, calculate (*a*) the current i_1 through R_1, (*b*) the current L_2 through R_2, (*c*) the current i through the switch, (*d*) the voltage across R_2, (*e*) the voltage across L, and (*f*) di_2/dt.

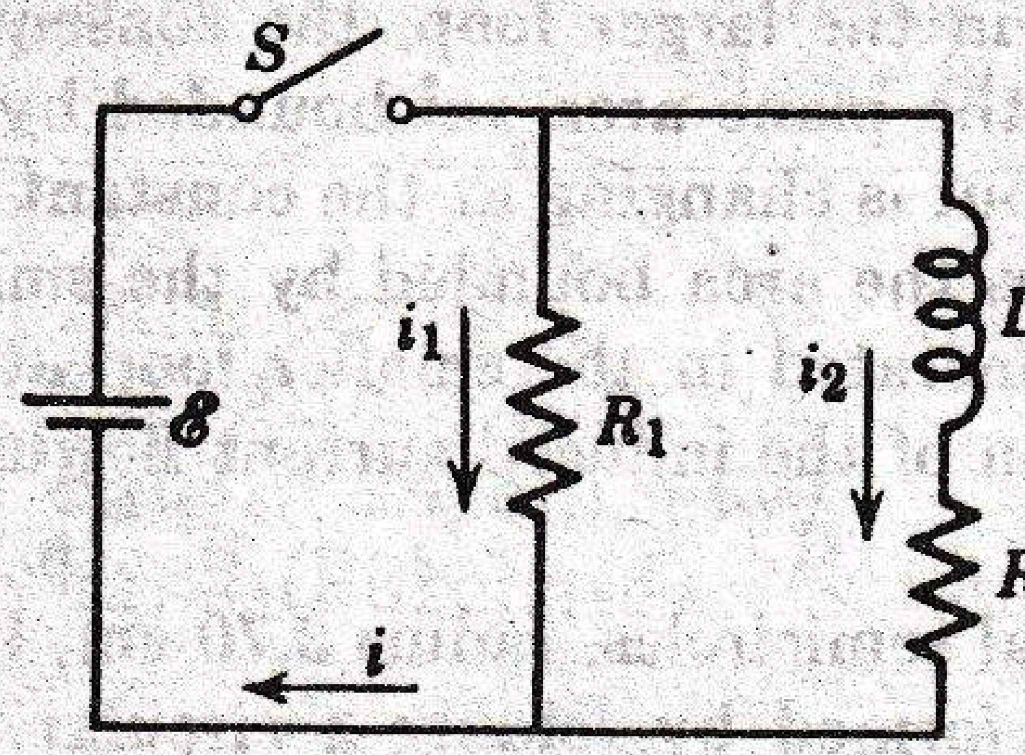


Fig. 36–8

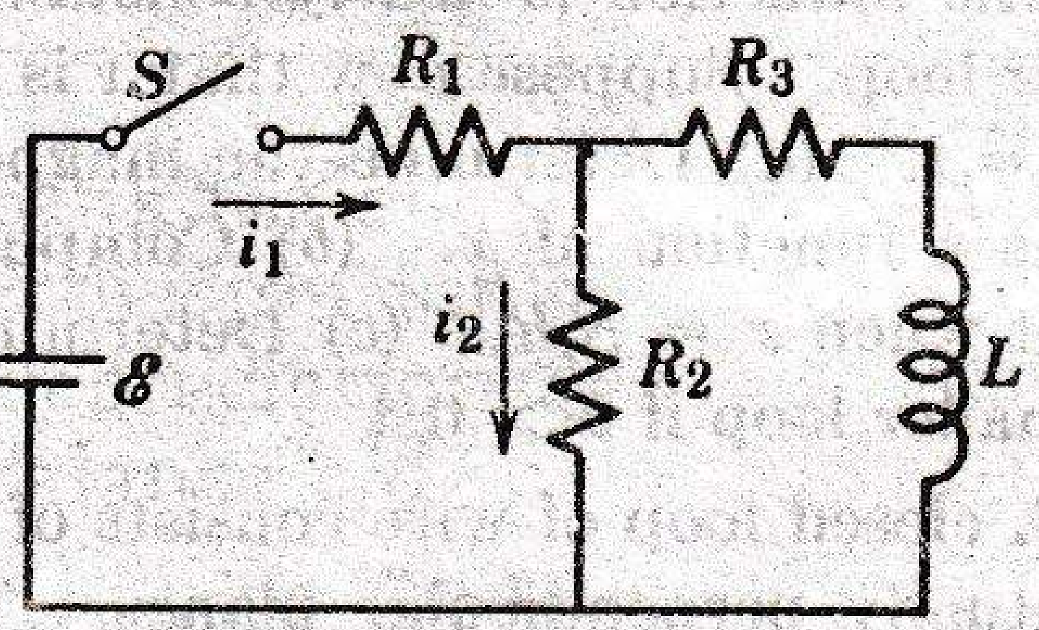


Fig. 36–9

3. In Fig. 36–9, $\mathcal{E}$ = 100 volts, $R_1 = 10\,\Omega$, $R_2 = 20\,\Omega$, $R_3 = 30\,\Omega$ and L = 2 h. Find the value of i_1 and i_2 (*a*) immediately after S is closed; (*b*) a long time later; (*c*) immediately after S is opened again; (*d*) a long time later.

4. A coaxial cable (inner radius a, outer radius b) is used as a transmission line between a battery $\mathcal{E}$ and a resistor R. The cable acts as both a capacitor and an inductor. If the stored electric and magnetic energies are equal, show that R must be

$$R = \frac{377}{2\pi} \ln\left(\frac{b}{a}\right) \text{ ohms.}$$

(The quantity 377 ohms is sometimes called *the impedance of free space.*)

5. Each item (*a*) coulomb-ohm-meter/weber, (*b*) volt-second, (*c*) coulomb-ampere/farad, (*d*) kilogram-volt-meter^2/(henry-ampere)^2, (*e*) (henry/farad)$^{1/2}$ is equal to one of the items in the following list: meter, second, kilogram, dimensionless number, newton, joule, volt, ohm, watt, coulomb, ampere, weber, henry, farad. Give the equalities.

Chapter 37

1. A total charge q is distributed uniformly on a dielectric ring of radius r. If the ring is rotated about an axis perpendicular to its plane and through its center at an angular speed ω, find the magnitude and direction of its resulting magnetic moment.

2. (*a*) What is the magnetic moment due to the orbital motion of an electron in an atom when the orbital angular momentum is one quantum unit ($= \hbar = 1.05 \times 10^{-34}$ joule-sec). (*b*) The intrinsic spin magnetic moment of an electron is 0.928×10^{-23} amp-meter^2. What is the difference in the magnetic potential energy U between the states in which the magnetic moment is aligned with and aligned in the opposite direction to an external magnetic field of 1.2 weber/meter^2? (*c*) What absolute temperature would be required so that the energy difference in (b) would equal the mean thermal energy $kT/2$?

3. An electron travels in a circular orbit about a fixed positive point charge in the presence of a uniform magnetic field **B** directed normal to the plane of its motion. The electric force has precisely N times the magnitude of the magnetic force on the electron. (*a*) Determine the two possible angular speeds of the electron's motion. (*b*) Evaluate these speeds numerically if $B = 4.27 \times 10^3$ gauss and $N = 100$.

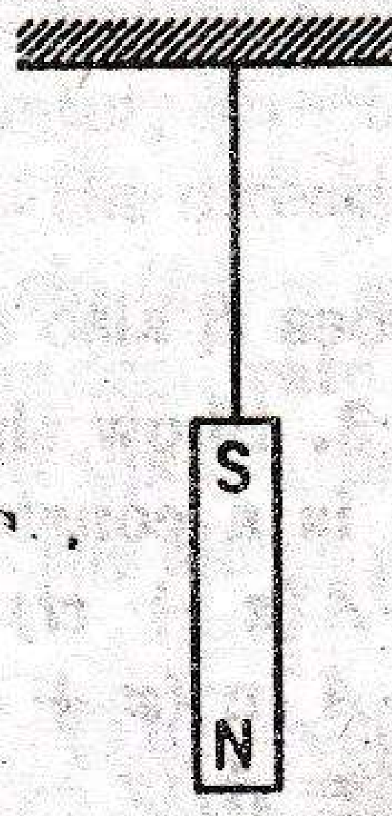


Fig. 37–23

4. A simple bar magnet is suspended by a string as shown in Fig. 37–23. If a uniform magnetic field **B** directed parallel to the ceiling is then established, show the resulting orientation of string and magnet.

5. An iron magnet containing iron of relative permeability 5000 has a flux path 1.0 meter long in the iron and an air gap 0.01 meter long each with cross-sectional areas of 0.02 meter.^{2} What current is necessary in a 500 turn coil wrapped around the iron to give a flux density in the air gap of 1.8 webers/meter^2?

Chapter 38

1. Initially the 900 μf capacitor is charged to 100 volts, and the 100 μf capacitor is uncharged in Fig. 38–13. (*a*) Describe in detail how one may charge the 100 μf capacitor to 300 volts using S_1 and S_2 appropriately. (*b*) Describe in detail the mass + spring mechanical analogy of this problem.

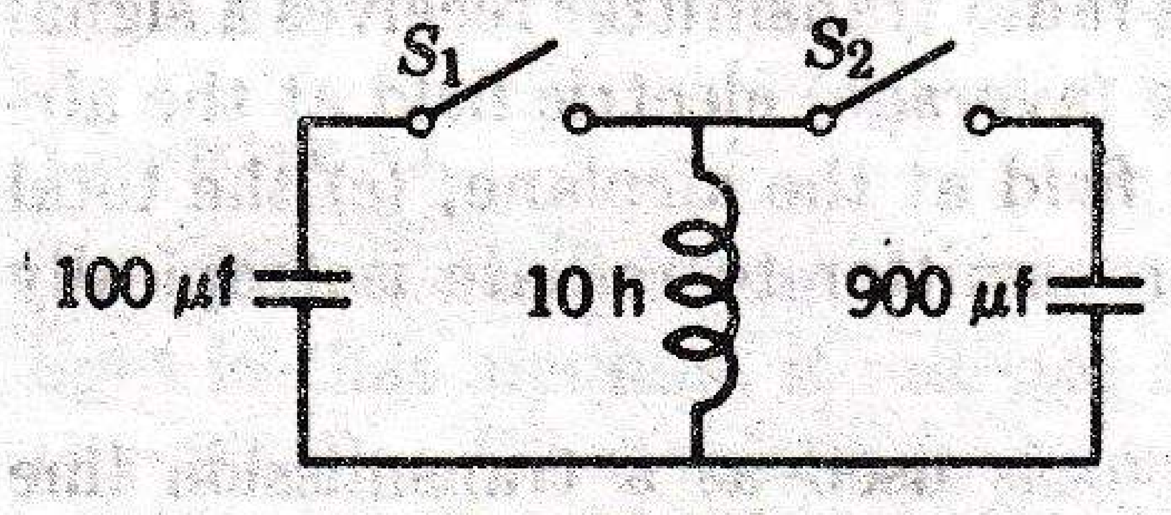


Fig. 38–13

2. *A self-consistency property of two of the Maxwell equations.* Two adjacent parallelepipeds share a common face as shown in Fig. 38–14. (*a*) We may apply $\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = q$ to each of the two closed surfaces separately. Show that, from this alone, it follows that $\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = q$ is *automatically* satisfied for the composite closed surface. (*b*) Repeat using $\oint \mathbf{B} \cdot d\mathbf{S} = 0$.

Fig. 38–14

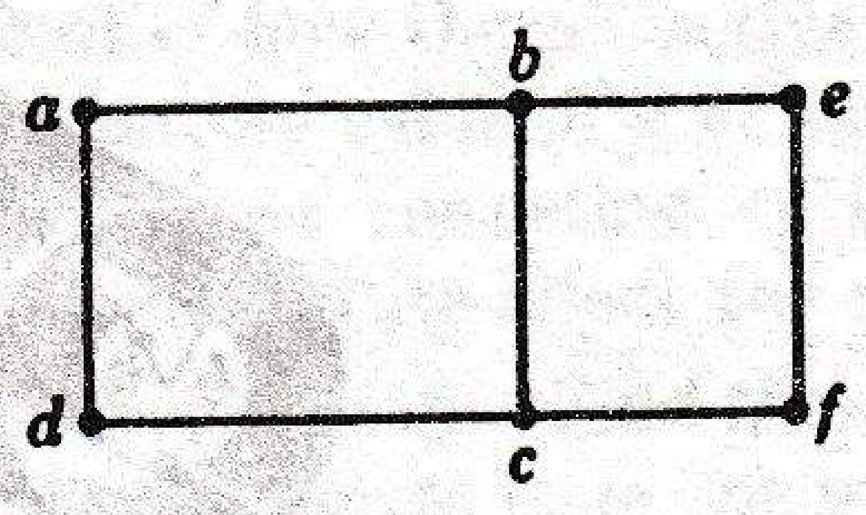


Fig. 38–15

3. *A self-consistency property of two of the Maxwell equations.* Two adjacent closed paths *abcda* and *efcbe* share the common edge *bc* as shown in Fig. 38–15. (*a*) We may apply $\oint \mathbf{E} \cdot d\mathbf{l} = -d\Phi_B/dt$ to each of these closed paths separately. Show that from this alone, $\oint \mathbf{E} \cdot d\mathbf{l} = -d\Phi_B/dt$ is *automatically* satisfied for the composite closed path *abefcda*. (*b*) Repeat using $\frac{1}{\mu_0} \oint \mathbf{B} \cdot d\mathbf{l} = i + \epsilon_0 \frac{d\Phi_E}{dt}$.

4. Using the definitions of flux, volume charge density ρ, and current density $\mathbf{j}$, write the four Maxwell equations in such a manner that all the fluxes, currents, and charges appear as volume or surface integrals.

Chapter 39

1. A coaxial cable is made of a center wire of radius a surrounded by a thin metal tube of radius b. The substance between the conductors is air. (*a*) Find the capacitance per unit length of this coaxial cable (*Hint:* Imagine equal but opposite charges to be on the wire and the tube). (*b*) Find the inductance per unit length of this coaxial cable (*Hint:* Imagine a current i flowing down the center wire and back along the tube.)

2. Starting with Eqs. 39-10 and 39-13, show that E satisfies the "wave equation"

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2}.$$

Does B also satisfy the wave equation?

3. Show that (*a*) through (*d*) below satisfy Eqs. 39-10 and 39-13. In each of these, A is a constant. (*a*) $E = Ac(x - ct)$, $B = A(x - ct)$. (*b*) $E = Ac(x + ct)^{15}$, $B = -A(x + ct)^{15}$. (*c*) $E = Ace^{(x-ct)}$, $B = Ae^{(x-ct)}$. (*d*) $E = Ac \ln (x + ct)$, $B = -A \ln (x + ct)$. (*e*) Generalize these examples to show that $E = Acf(x - ct)$, $B = Af(x - ct)$ is a solution where f is *any* function of $(x - ct)$. What is the corresponding situation for functions of $(x + ct)$?

4. Consider the possibility of "standing waves":

$$E = E_m(\sin \omega t)(\sin kx)$$
$$B = B_m(\cos \omega t)(\cos kx)$$

(*a*) Show that these satisfy Eqs. 39-10 and 39-13 if E_m is suitably related to B_m and ω suitably related to k. What are these relationships? (*b*) Find the (instantaneous) Poynting vector. (*c*) Show that the time average power flow across any area is zero. (*d*) Describe the flow of energy in this problem.

5. An airplane flying at a distance of 10 km from a radio transmitter receives a signal of power 10.0 microwatts/meter^2. Calculate (*a*) the (average) electric field at the airplane due to this signal; (*b*) the (average) magnetic field at the airplane; (*c*) the total power radiated by the transmitter, assuming the transmitter to radiate isotropically and the earth to be a perfect absorber.

6. A coaxial cable (inner radius a, outer radius b) is used as a transmission line between a battery $\mathcal{E}$ and a resistor R, as shown in Fig. 39-18. (*a*) Calculate **E**, **B** for

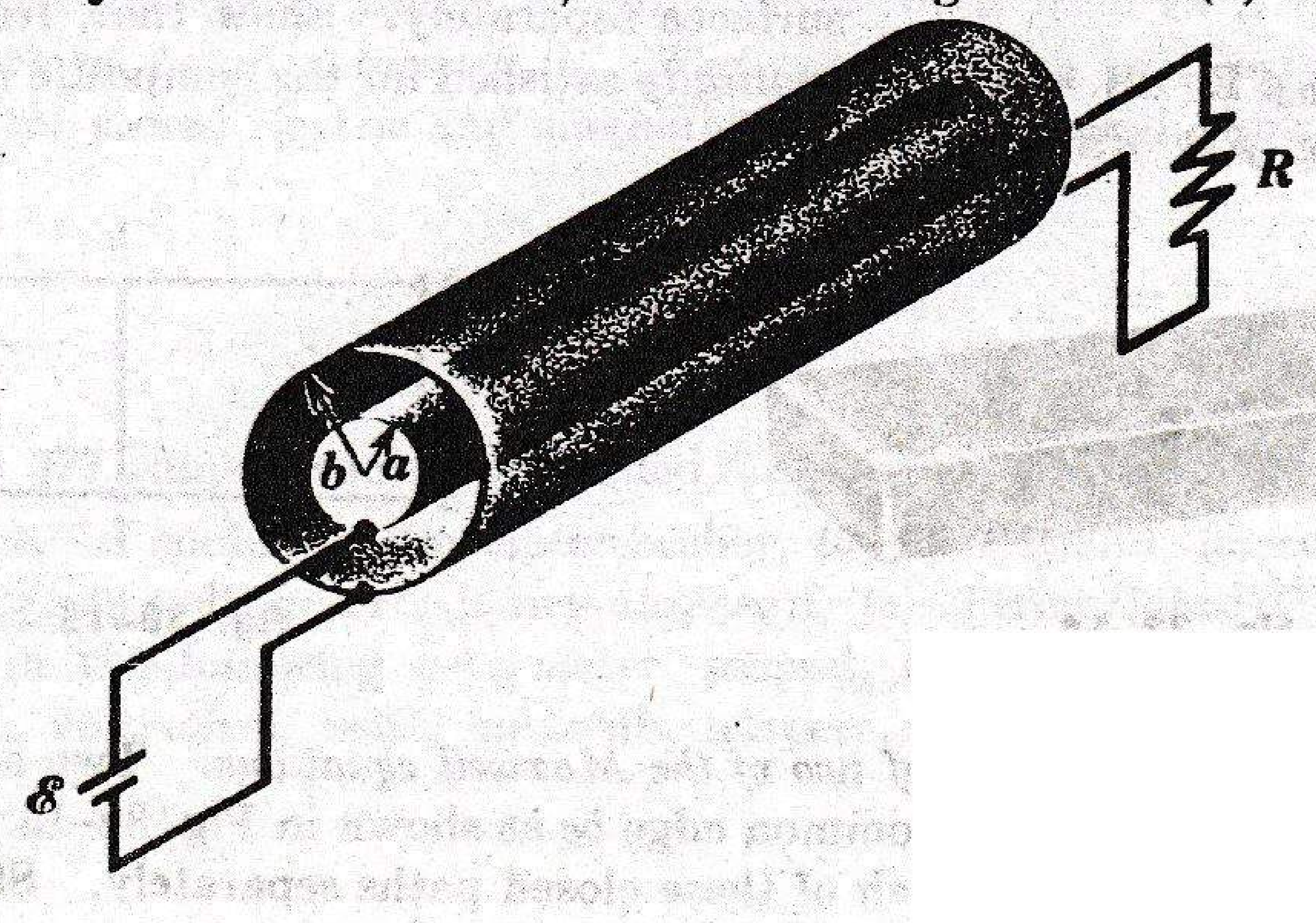


Fig. 39-18

$a < r < b$. (*b*) Calculate the Poynting vector **S** for $a < r < b$. (*c*) By suitably integrating the Poynting vector, show that the total power flowing across the annular cross section $a < r < b$ is $\mathcal{E}^2/R$. Is this reasonable? (*d*) Show that the direction of **S** is always from the battery to the resistor, no matter which way the battery is connected.

7. A long hollow cylinder (radius R, length l) carries a uniform charge per unit area of σ on its surface. An externally applied torque causes the cylinder to rotate at constant acceleration $\omega(t) = \alpha t$. (*a*) Find **B** within the cylinder (treat it as a solenoid). (*b*) Find **E** at the inner surface of the cylinder. (*c*) Find **S** at the inner surface of the cylinder. (*d*) Show that the flux of **S** entering the interior volume of the cylinder is equal to $\dfrac{d}{dt}\left(\dfrac{\pi R^2 l}{2\mu_0} B^2\right)$. (Compare with Problem 18.)

Chapter 40

1. A plane electromagnetic wave, with wavelength 3.0 meters, travels in free space in the $+x$-direction with its electric vector **E**, of amplitude 300 volts/meter, directed along the y-axis. (*a*) What is the frequency ν of the wave? (*b*) What is the direction and amplitude of the **B** field associated with the wave? (*c*) If $E = E_m \sin(kx - \omega t)$, what are the values of k and ω for this wave? (*d*) What is the time averaged rate of energy flow per unit area associated with this wave? (*e*) If the wave fell upon a perfectly absorbing sheet of area A, what momentum would be delivered to the sheet per second and what is the radiation pressure exerted on the sheet?

2. Show that $\epsilon_0 \mathbf{E} \times \mathbf{B}$ has the dimensions of momentum/volume. (The vector $\epsilon_0 \mathbf{E} \times \mathbf{B}$ may be used to compute the momentum stored in the fields in the same manner that the scalar $\frac{1}{2}\epsilon_0 \mathbf{E}^2 + \frac{1}{2\mu_0}\mathbf{B}^2$ may be used to compute the energy stored in the fields.)

3. A particle in the solar system is under the combined influence of the sun's gravitational attraction and the radiation force due to the sun's rays. Assume that the particle is a sphere of density 1.0 gm/cm^3 and that all of the incident light is absorbed. (*a*) Show that all particles with radius less than some critical radius, R_0, will be blown out of the solar system. (*b*) Calculate R_0. (*c*) Does R_0 depend on the distance from the earth to the sun? (See the appendices for the necessary constants.)

Chapter 41

1. When an electron moves through a medium at a speed exceeding the speed of light in that medium, it radiates electromagnetic energy (the Cerenkov effect, see Section 20–7). What minimum speed must an electron have in a liquid of refractive index 1.54 in order to radiate?

2. Assume that the index of refraction of the earth's atmosphere varies, with altitude only, from the value one at the edge of the atmosphere to some larger value at the surface of the earth. (*a*) Neglecting the earth's curvature, show that the apparent angle of a star from the zenith direction is independent of how the refractive index of the atmosphere varies with altitude and depends only on the value of n at the earth's surface. (*Hint:* Compare a uniform atmosphere with one consisting of layers of increasing refractive index.) (*b*) How does the earth's curvature affect the analysis?

3. A point source is 80 cm below the surface of a body of water. Find the diameter of the largest circle at the surface through which light can emerge from the water.

4. A pole extends 2.0 meters above the bottom of a swimming pool and 0.5 meters above the water. Sunlight is incident at 45°. What is the length of the shadow of the pole on the bottom of the pool?

5. A given monochromatic light ray, initially in air, strikes the 90° prism at P (see Fig. 41–24) and is refracted there and at Q to such an extent that it just grazes the right-hand prism surface after it emerges into air at Q. (*a*) Determine the index of

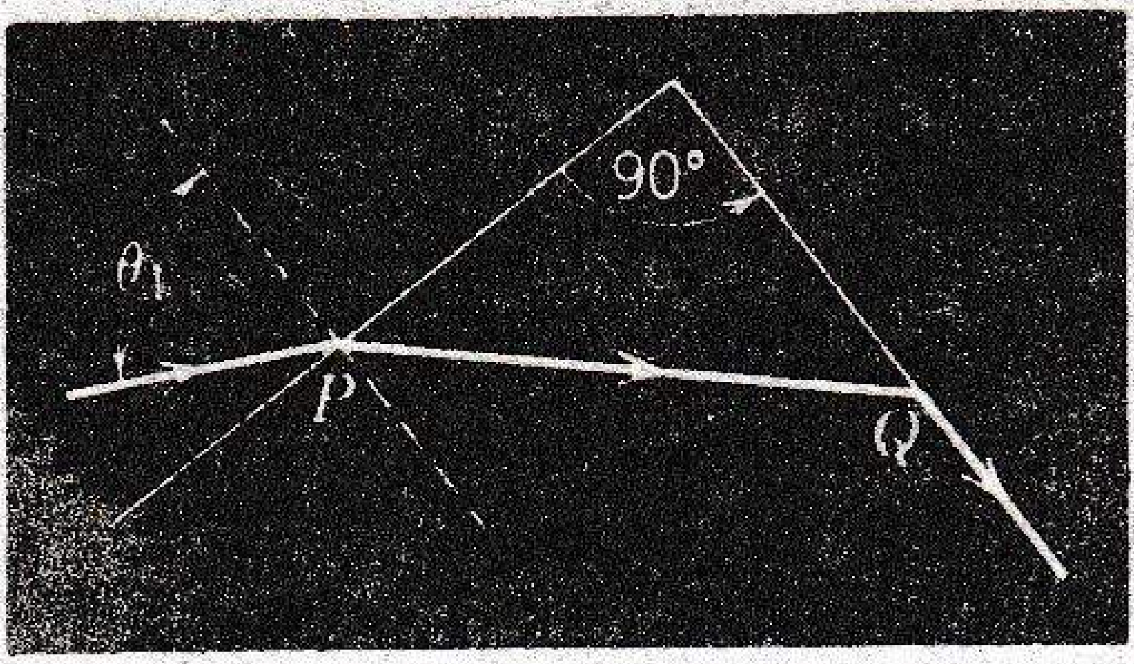


Fig. 41–24

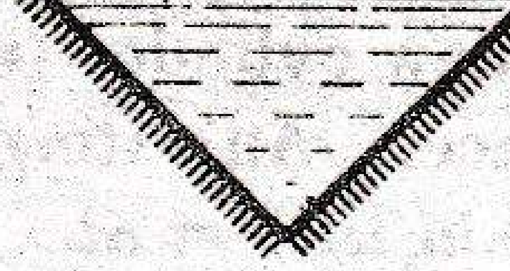


Fig. 41–25

refraction, relative to air, of the prism for this wavelength in terms of the angle of incidence θ_1 which gives rise to this situation. (b) Give a numerical upper bound for the index of refraction of the prism. (c) Show, by a ray diagram, what happens if the angle of incidence at P is slightly greater than θ_1, is slightly less than θ_1.

6. Two perpendicular mirrors form the sides of a vessel filled with water, as shown in Fig. 41–25. A light ray is incident from above, normal to the water surface. (a) Show that the emerging ray is parallel to the incident ray. Assume that there are two reflections at the mirror surfaces. (b) Repeat the analysis for the case of oblique incidence, the ray lying in the plane of the figure. (c) Using three mirrors, state and prove the three-dimensional analog to this problem.

Chapter 42

1. How many images of himself can an observer see in a room whose ceiling and two adjacent walls are mirrors? Explain.

2. A thin flat plate of glass is a distance b from a convex mirror. A point source of light S is placed a distance a in front of the plate (see Fig. 42–33) so that its image in the partially reflecting plate coincides with its image in the mirror. If $b = 7.5$ cm and the focal length of the mirror is $f = -30$ cm, find a and draw the ray diagram.

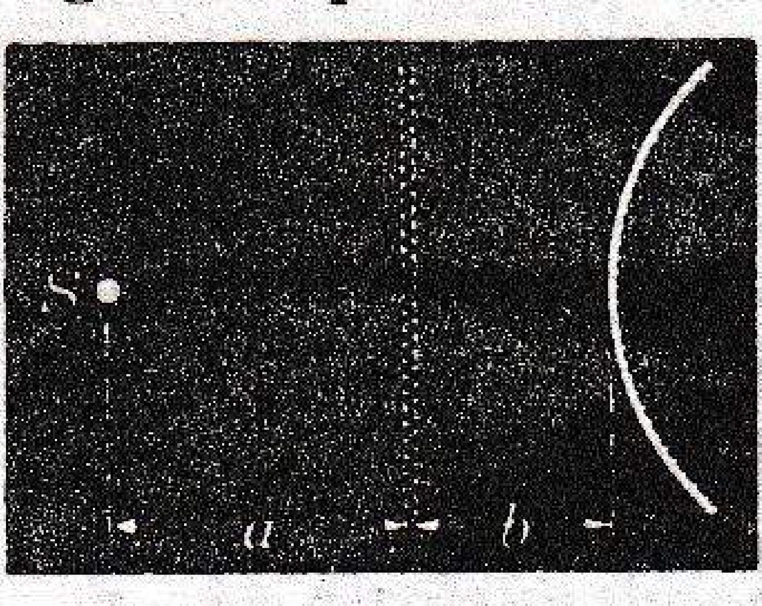


Fig. 42–33

3. (a) A simple magnifier (a converging lens) is used directly in front of the eye to examine an object. Draw a ray diagram showing the final image on the retina of the eye. (b) The smallest distance at which a normal eye can focus upon an object is 25 cm. Use this result with your ray diagram to prove that the (angular) magnification of a simple magnifier is $25/f + 1$, where f is the focal length of the magnifier lens.

4. Two thin lens, one having $f = +12.0$ cm and the other $f = -10.0$ cm, are separated by 7.0 cm. A small object is placed 43.5 cm from the center of the lens system on the principal axis first on one side and next on the other side. Find the location of the final image in each case.

5. Show that a thin converging lens of focal length f followed by a thin diverging lens of focal length $-f$ will bring parallel light to a focus beyond the second lens provided that the separation of the lenses L satisfies $0 < L < f$. (b) Does this property change if the lenses are interchanged? (c) What happens when $L = 0$?

6. A concave and a convex lens are cut out of a plane parallel block of glass as shown in Fig. 42–34. Discuss the geometry of a beam of parallel rays incident (a) on the concave lens and (b) on the convex lens as the distance between lenses is increased from contact to a large separation.

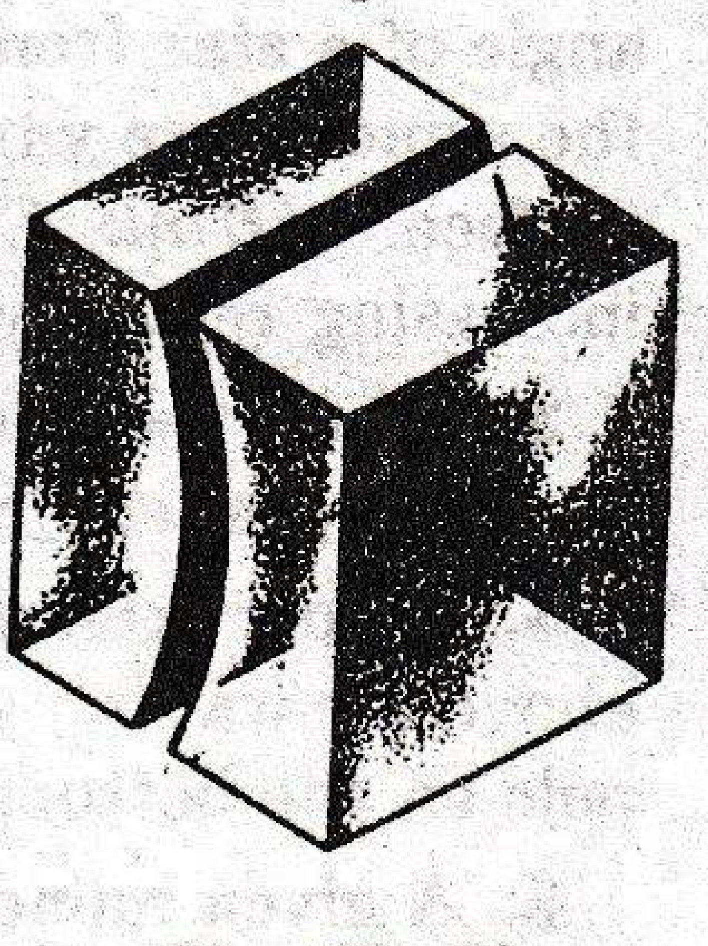


Fig. 42–34

7. An object is placed 1.0 meter in front of a convergent lens, of focal length 0.5 meter, which is 2.0 meters in front of a plane mirror. (a) Where is the final image, measured from the lens,

that would be seen by an eye looking toward the mirror through the lens? (*b*) Is the final image real or virtual? (*c*) Is the final image erect or inverted? (*d*) What is the lateral magnification?

Chapter 43

1. In a double-slit arrangement the distance between slits is 5.0 mm and the slits are 1.0 meter from the screen. Two interference patterns can be seen on the screen, one due to light of 4800 A and the other 6000 A. What is the separation on the screen between the third-order interference fringes of the two different patterns?

2. In Young's interference experiment in a large ripple tank (see Fig. 43–4) the coherent vibrating sources are placed 12.0 cm apart. The distance between maxima 2.0 meters away is 18.0 cm. If the speed of ripples is 25.0 cm/sec, find the frequency of the vibrators.

3. One slit of a double-slit arrangement is covered by a thin glass plate of refractive index 1.4, and the other by a thin glass plate of refractive index 1.7. The point on the screen where the central maximum fell before the glass plates were inserted is now occupied by what had been the fifth bright fringe before. Assume $\lambda = 4800$ A and that the plates have the same thickness t and find the value of t.

4. In Fig. 43–25, the source emits monochromatic light of wavelength λ. S is a very narrow slit in an otherwise opaque screen I. A plane mirror, whose surface includes the axis of the lens shown, is located a distance h below S. Screen II is at the focal plane of the lens. (*a*) Find the condition for maxima and minima brightness of fringes

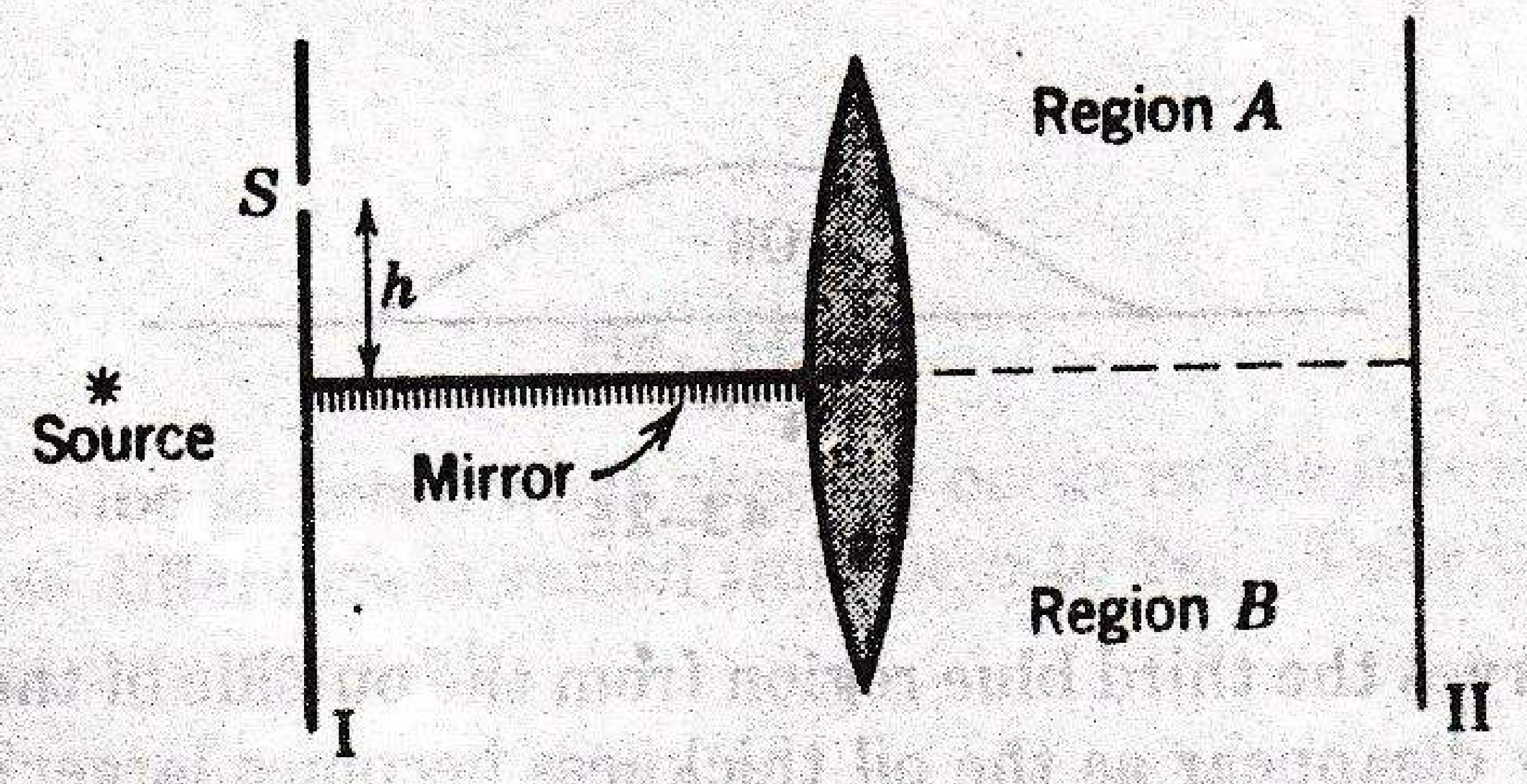


Fig. 43–25

on screen II in terms of the usual angle θ, the wavelength λ, and the distance h. (*b*) Do fringes appear only in region A (above the axis of the lens), only in region B (below the axis of the lens), or in both regions A and B? Explain. (*Hint:* Consider the image of S formed by the mirror.)

5. Let $f_1(t) = A_1 \sin(\omega t + \phi_1)$ and $f_2(t) = A_2 \sin(\omega t + \phi_2)$. Suppose we want to calculate the time-average of their product:

$$\frac{1}{T}\int_{t=0}^{T} f_1(t) f_2(t)\, dt, \qquad T = \frac{2\pi}{\omega}.$$

(Such a problem might arise if f_1 represents a current and f_2 a voltage; $f_1 f_2$ would then be the *instantaneous* power.) Show that this average is equal to one-half the dot product of the corresponding phasors.

6. Consider the problem of determining the sum

$$A_1 \sin(\omega t + \phi_1) + A_2 \sin(\omega t + \phi_2) + \cdots + A_n \sin(\omega t + \phi_n)$$

from the phasor diagram.

(a) Show that the sum may always be written in the form

$$B \sin \omega t + C \cos \omega t.$$

(b) Show that $B^2 + C^2 \leq (A_1 + A_2 + \cdots + A_n)^2$.
(c) When does the equality sign in (b) hold?

7. A thin film of acetone (refractive index 1.25) is floated on a thick glass plate (refractive index 1.50). Plane light waves of variable wavelength are incident normal to the film. When one views the reflected wave it is noted that complete destructive interference occurs at 6000 A and constructive interference at 7000 A. Calculate the thickness of the acetone film.

8. We wish to coat a flat piece of glass ($n = 1.50$) with a transparent material ($n = 1.25$) so that light of wavelength 6000 A (in vacuum) incident normally is not reflected. How can this be done?

9. Light of wavelength 6300 A is incident normally on a thin wedge-shaped film of index of refraction 1.5. There are ten bright and nine dark fringes over the length of film. By how much does the film thickness change over this length?

10. An oil drop ($n = 1.20$) floats on a water ($n = 1.33$) surface and is observed from above by reflected light (see Fig. 43–26). (a) Will the outer (thinnest) regions of the drop correspond to a bright or a dark region? (b) Approximately how thick is the oil

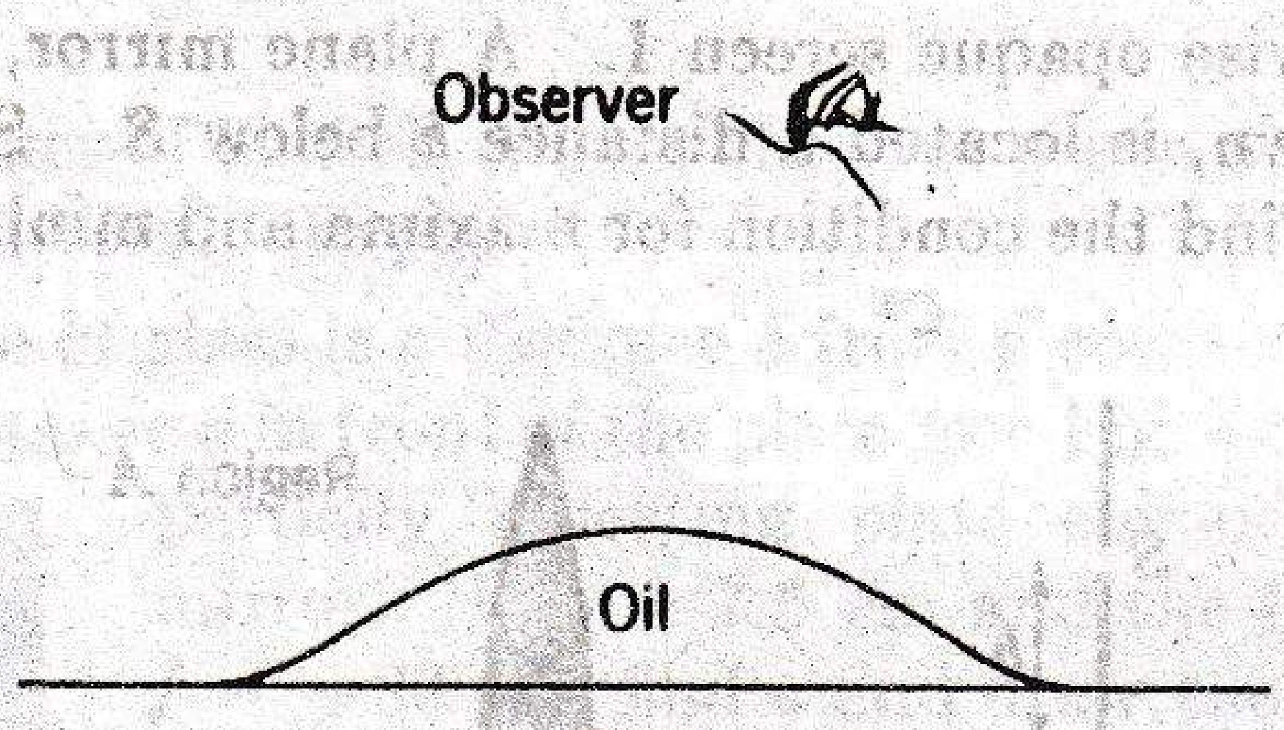


Fig. 43–26

film where one observes the third blue region from the outside of the drop? (c) Why do the colors gradually disappear as the oil thickness becomes larger?

Chapter 44

1. It can be shown that, except for $\theta = 0$, a circular obstacle produces the same diffraction pattern as a circular hole of the same diameter. Furthermore, if there are many such obstacles located randomly, then the interference effects vanish leaving only the diffraction associated with a *single* obstacle. (a) Explain why one sees a "ring" around the moon on a foggy day. (b) Calculate the size of the water droplet in the air if the bright ring around the moon appears to have a diameter 1.5 times that of the moon.

2. A monochromatic beam of parallel light is incident on a "collimating" hole of diameter $x \gg \lambda$. Point P lies in the geometrical shadow region on a distance screen, as shown in Fig. 44–15a. Two obstacles, shown in Fig. 44–15b, are placed in turn over the collimating hole. A is an opaque circle with a hole in it and B is the "photographic negative" of A. Using superposition concepts, show that the intensity at P is identical for each of the two diffracting objects A and B (*Babinet's principle*).

3. An astronaut in a satellite claims he can just barely resolve two point sources on the earth, 100 miles below him. What is their separation, assuming ideal conditions? Take $\lambda = 5500$ A, and the pupil diameter to be 5.0 mm.

4. Under ideal conditions, estimate the linear separation of two objects on the planet Mars which can just be resolved by an observer on earth (a) using the naked eye,

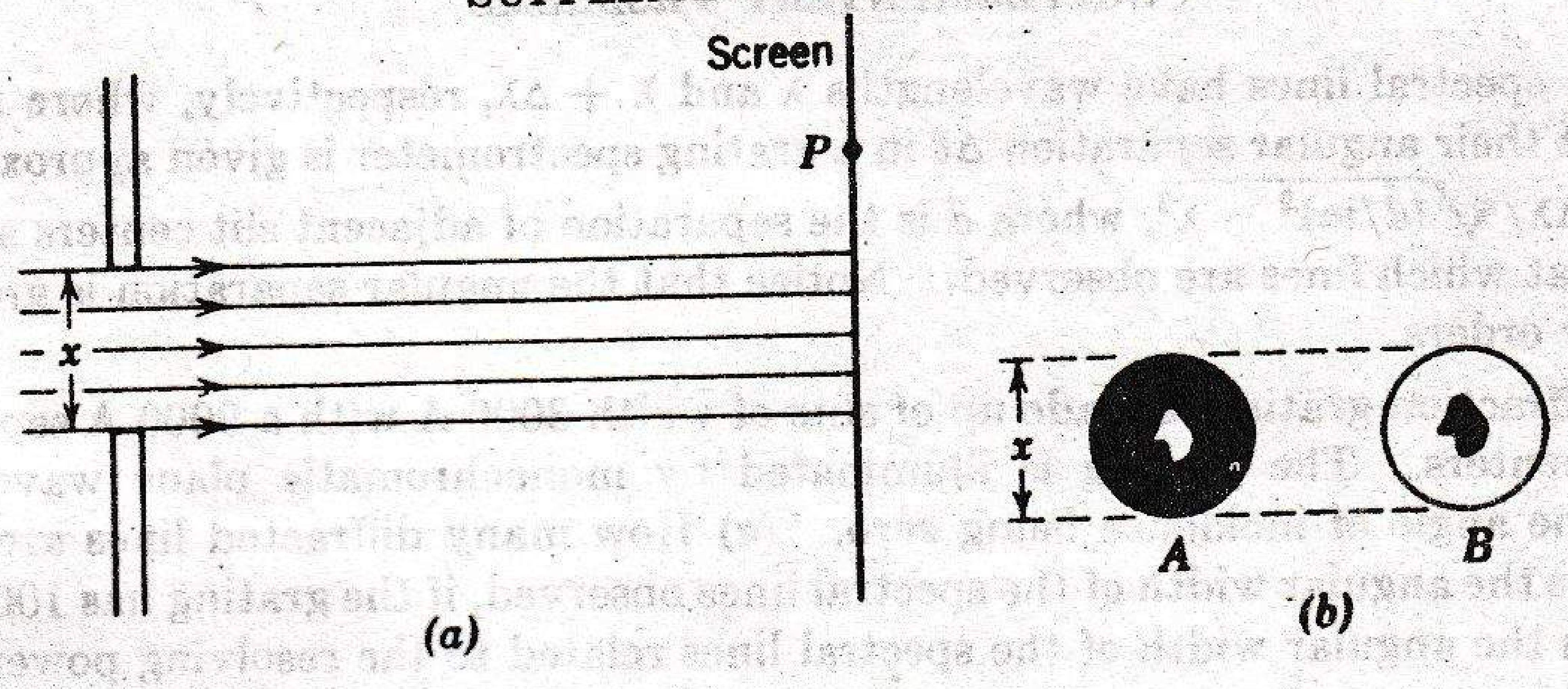


Fig. 44–15

(b) using the 200-in. Mt. Palomar telescope. Use the following data: distance to Mars = 50 million miles; diameter of pupil = 5.0 mm; wavelength of light = 5500 A.

5. A double-slit system (slit separation d, slit width a) is driven by two loudspeakers as shown in Fig. 44–16. By use of a variable delay line, the phase of one of the speakers

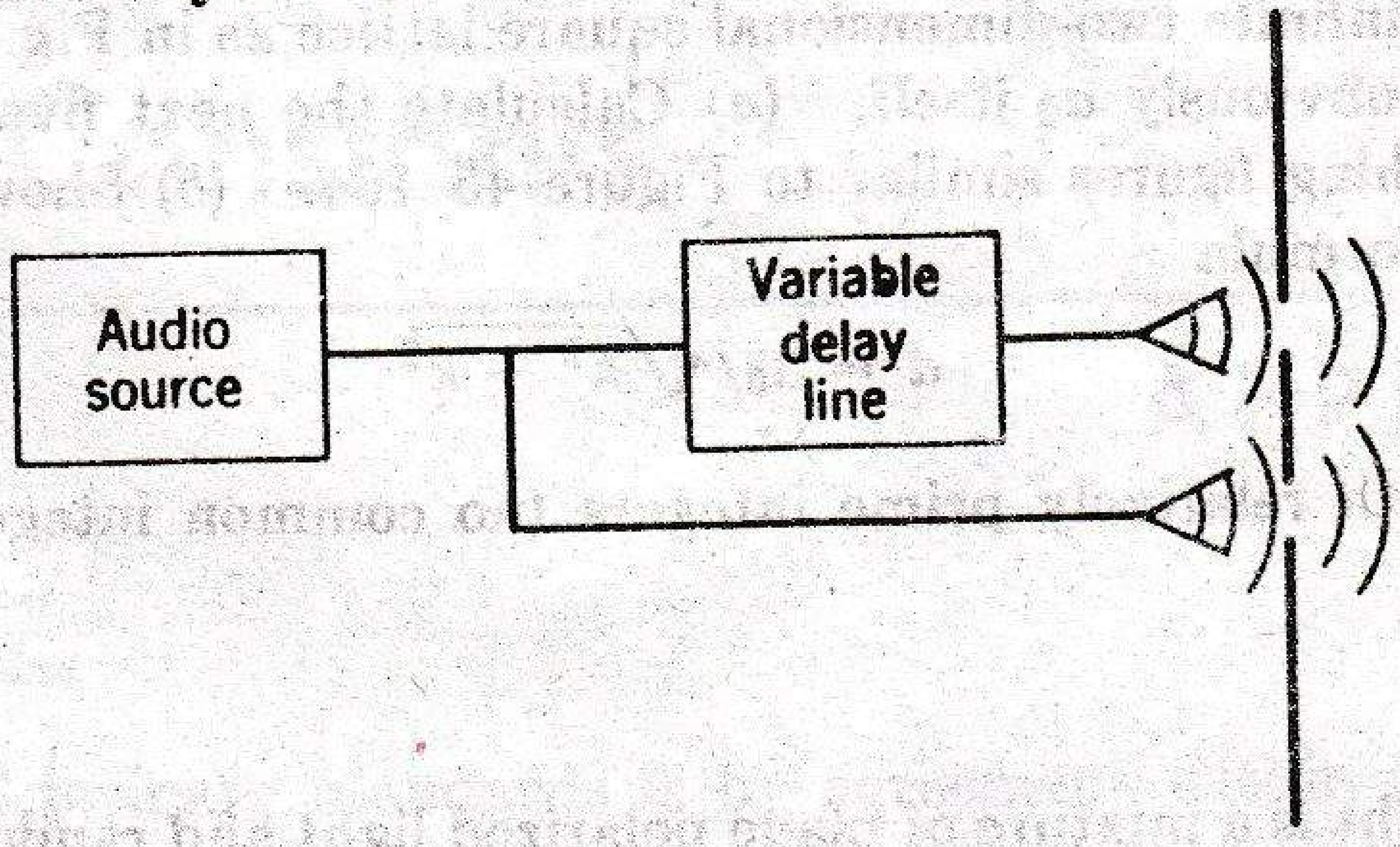


Fig. 44–16

may be varied. Describe in detail what changes occur in the intensity pattern at large distances as this phase difference is varied from zero to 2π. Take both the interference and diffraction effects into account.

Chapter 45

1. A diffraction grating has a large number N of slits, each of width d. Let I_{max} denote the intensity at some principal maximum, and let I_k denote the intensity of the kth adjacent secondary maxima. (a) If $k \ll N$, show from the phasor diagram that, approximately, $I_k/I_{max} = 1/(k + \frac{1}{2})^2\pi^2$. (Compare this with the single-slit formula.) (b) For those secondary maxima which lie roughly midway between two adjacent principal maxima, show that roughly $I_k/I_{max} = 1/N^2$. (c) Consider the central principal maximum and those adjacent secondary maxima for which $k \ll N$. Show that this part of the diffraction pattern quantitatively resembles that of one single slit of width Nd.

2. With light from a gaseous discharge tube incident normally on a grating with a distance 1.732×10^{-4} cm between adjacent slit centers, a green line appears with sharp maxima at measured transmission angles $\theta = \pm 17.6°, 37.3°, -37.1°, 65.2°$, and $-65.0°$. (a) Compute the wavelength of the green line that best fits the data.

3. A narrow beam of monochromatic light strikes a grating at normal incidence and produces sharp maxima at the following angles from the normal: 6°40′, 13°30′, 20°20′, 35°40′. No other maxima appear at any angle between 0° and 35°40′. The separation between adjacent slit centers in the grating is 5.04×10^{-4} cm. (a) Compute the wavelength of the light used. (b) Make the most complete *quantitative* statement that can be inferred from the above data concerning the width of each slit.

4. Two spectral lines have wavelengths λ and $\lambda + \Delta\lambda$, respectively, where $\Delta\lambda \ll \lambda$. Show that their angular separation $\Delta\theta$ in a grating spectrometer is given approximately by $\Delta\theta = \Delta\lambda/\sqrt{(d/m)^2 - \lambda^2}$, where d is the separation of adjacent slit centers and m is the order at which lines are observed. Notice that the angular separation is greater in the higher orders.

5. A diffraction grating is made up of slits of width 3000 A with a 9000 A separation between centers. The grating is illuminated by monochromatic plane waves, $\lambda =$ 6000 A, the angle of incidence being zero. (*a*) How many diffracted lines are there? (*b*) What is the angular width of the spectral lines observed, if the grating has 1000 slits? (*c*) How is the angular width of the spectral lines related to the resolving power of the grating?

6. A diffraction grating has a resolving power $R = \lambda/\Delta\lambda = Nm$. (*a*) Show that the corresponding frequency range, $\Delta\nu$, that can just be resolved is given by $\Delta\nu = c/Nm\lambda$. (*b*) From Fig. 45–1, show that the "times of flight" of the two extreme rays differ by an amount $\Delta t = Nd \sin\theta/c$. (*c*) Show that $(\Delta\nu)(\Delta t) = 1$, this relation being independent of the various grating parameters.

7. Consider an infinite two-dimensional square lattice as in Fig. 45–14*b*. One interplanar spacing is obviously a_0 itself. (*a*) Calculate the next five smaller interplanar spacings by sketching figures similar to Figure 45–16*a*. (*b*) Show that your answers obey the general formula

$$d = a_0/\sqrt{h^2 + k^2}$$

where h, k are both relatively prime integers (no common integer factor other than unity).

Chapter 46

1. A beam of light is a mixture of plane polarized light and randomly polarized light. When it is sent through a Polaroid sheet, it is found that the transmitted intensity can be varied by a factor of five depending on the orientation of the Polaroid. Find the relative intensities of these two components of the incident beam.

2. It is desired to rotate the plane of polarization of a beam of plane polarized light by 90°. (*a*) How might this be done using only Polaroid sheets? (*b*) How many sheets are required in order that the total intensity loss is less than 5%? Assume that each Polaroid sheet is ideal.

3. A sheet of Polaroid and a quarter-wave plate are glued together in such a way that, if the combination is placed with face *A* against a shiny coin, the face of the coin can be seen when illuminated with light of appropriate wavelength. When the combination is placed with face *A* away from the coin, the coin cannot be seen. Which component is on face *A* and what is the relative orientation of the components?

4. A beam of right circularly polarized light is reflected from a mirror. (*a*) Is the reflected beam right or left circularly polarized? (*b*) Has the direction of the associated linear momentum of the light changed? (*c*) Has the direction of the associated angular momentum of light changed? (*d*) Describe the reaction "felt" by the mirror.