Chapter 3

Circuits, Part 1

Madam, what good is a baby? *Michael Faraday, when asked by Queen Victoria what the electrical devices in his lab were good for*

A few years ago, my wife and I bought a house with Character, Character being a survival mechanism that houses have evolved in order to convince humans to agree to much larger mortgage payments than they'd originally envisioned. Anyway, one of the features that gives our house Character is that it possesses, built into the wall of the family room, a set of three pachinko machines. These are Japanese gambling devices sort of like vertical pinball machines. (The legal papers we got from the sellers hastened to tell us that they were "for amusement purposes only.") Unfortunately, only one of the three machines was working when we moved in, and it soon died on us. Having become a pachinko addict, I decided to fix it, but that was easier said than done. The inside is a veritable Rube Goldberg mechanism of levers, hooks, springs, and chutes. My hormonal pride, combined with my Ph.D. in physics, made me certain of success, and rendered my eventual utter failure all the more demoralizing.

Contemplating my defeat, I realized how few complex mechanical devices I used from day to day. Apart from our cars and my saxophone, every technological tool in our modern life-support system was electronic rather than mechanical.

3.1 Current

Unity of all types of electricity

We are surrounded by things we have been *told* are "electrical," but it's far from obvious what they have in common to justify being grouped together. What relationship is there between the way socks cling together and the way a battery lights a lightbulb? We have been told that both an electric eel and our own brains are somehow electrical in nature, but what do they have in common?

British physicist Michael Faraday (1791-1867) set out to address this problem. He investigated electricity from a variety of sources — including electric eels! — to see whether they could all produce the same effects, such as shocks and sparks, attraction and repulsion. "Heating" refers, for example, to the way a lightbulb filament gets hot enough to glow and emit light. Magnetic induction is an effect discovered by Faraday himself that connects electricity and magnetism. We will not study this effect, which is the basis for the electric generator, in detail until later in the book.

source	effect			
	shocks	sparks	attraction and repulsion	heating
rubbing	√	√	√	√
battery	√	√	√	√
animal	√	√	(√)	√
magnetically induced	√	√	√	√


a / *Gymnotus carapo*, a knifefish, uses electrical signals to sense its environment and to communicate with others of its species.

The table shows a summary of some of Faraday's results. Check marks indicate that Faraday or his close contemporaries were able to verify that a particular source of electricity was capable of producing a certain effect. (They evidently failed to demonstrate attraction and repulsion between objects charged by electric eels, although modern workers have studied these species in detail and been able to understand all their electrical characteristics on the same footing as other forms of electricity.)

Faraday's results indicate that there is nothing fundamentally different about the types of electricity supplied by the various sources. They are all able to produce a wide variety of identical effects. Wrote Faraday, "The general conclusion which must be drawn from this collection of facts is that electricity, whatever may be its source, is identical in its nature."

If the types of electricity are the same thing, what thing is that? The answer is provided by the fact that all the sources of electricity can cause objects to repel or attract each other. We use the word "charge" to describe the property of an object that allows it to participate in such electrical forces, and we have learned that charge is present in matter in the form of nuclei and electrons. Evidently all these electrical phenomena boil down to the motion of charged particles in matter.

Electric current

If the fundamental phenomenon is the motion of charged particles, then how can we define a useful numerical measurement of it? We might describe the flow of a river simply by the velocity of the water, but velocity will not be appropriate for electrical purposes because we need to take into account how much charge the moving particles have, and in any case there are no practical devices sold at Radio Shack that can tell us the velocity of charged particles. Experiments show that the intensity of various electrical effects is related to a different quantity: the number of coulombs of charge that pass by a certain point per second. By analogy with the flow of water, this quantity is called the electric *current*, I. Its units of coulombs/second are more conveniently abbreviated as amperes, 1 A=1 C/s. (In informal speech, one usually says "amps.")

b / André Marie Ampère (1775-1836).

The main subtlety involved in this definition is how to account for the two types of charge. The stream of water coming from a hose is made of atoms containing charged particles, but it produces none of the effects we associate with electric currents. For example, you do not get an electrical shock when you are sprayed by a hose. This type of experiment shows that the effect created by the motion of one type of charged particle can be canceled out by the motion of the opposite type of charge in the same direction. In water, every oxygen atom with a charge of $+8e$ is surrounded by eight electrons with charges of $-e$, and likewise for the hydrogen atoms.

We therefore refine our definition of current as follows:

> **definition of electric current**
> When charged particles are exchanged between regions of space A and B, the electric current flowing from A to B is
>
> $$I = \frac{\Delta q}{\Delta t} \quad ,$$
>
> where Δq is the change in region B's total charge occurring over a period of time Δt.

In the garden hose example, your body picks up equal amounts of positive and negative charge, resulting in no change in your total charge, so the electrical current flowing into you is zero.

Interpretation of $\Delta q/\Delta t$ *example 1*

▷ How should the expression $\Delta q/\Delta t$ be interpreted when the current isn't constant?

▷ You've seen lots of equations of this form before: $v = \Delta x/\Delta t$, $F = \Delta p/\Delta t$, etc. These are all descriptions of rates of change, and they all require that the rate of change be constant. If the rate of change isn't constant, you instead have to use the slope of the tangent line on a graph. The slope of a tangent line is equivalent to a derivative in calculus; applications of calculus are discussed in section 3.6.

Ions moving across a cell membrane *example 2*

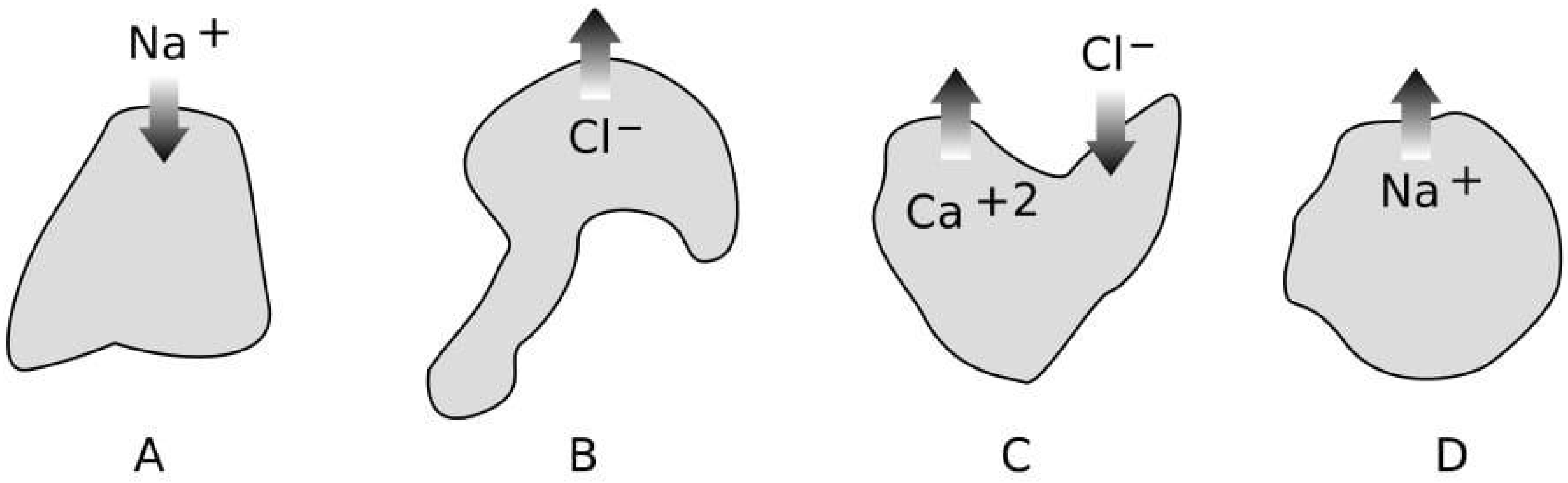
▷ Figure c shows ions, labeled with their charges, moving in or out through the membranes of three cells. If the ions all cross the membranes during the same interval of time, how would the currents into the cells compare with each other?

▷ Cell A has positive current going into it because its charge is increased, i.e., has a positive value of Δq.

Cell B has the same current as cell A, because by losing one unit of negative charge it also ends up increasing its own total charge by one unit.

Cell C's total charge is reduced by three units, so it has a large negative current going into it.

Cell D loses one unit of charge, so it has a small negative current into it.



c / Example 2

It may seem strange to say that a negatively charged particle going one way creates a current going the other way, but this is quite ordinary. As we will see, currents flow through metal wires via the motion of electrons, which are negatively charged, so the direction of motion of the electrons in a circuit is always opposite to the direction of the current. Of course it would have been convenient of Benjamin Franklin had defined the positive and negative signs of charge the opposite way, since so many electrical devices are based on metal wires.

Number of electrons flowing through a lightbulb *example 3*

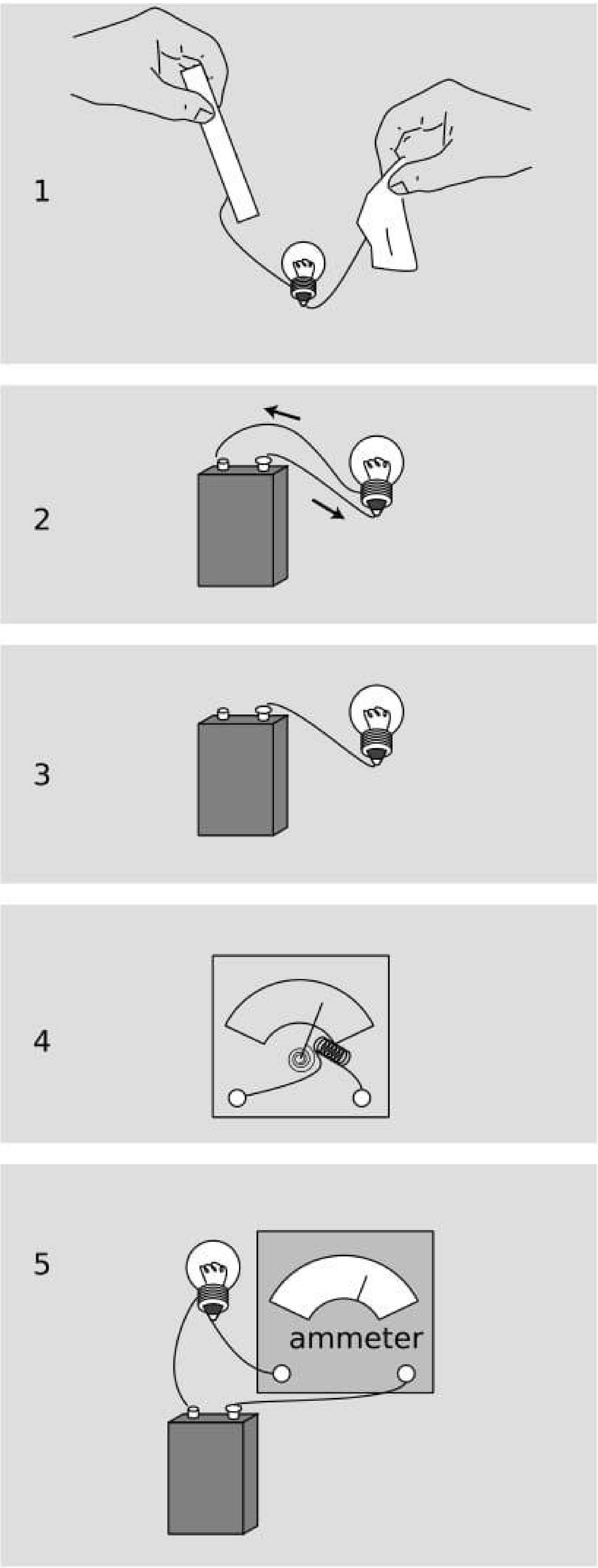
▷ If a lightbulb has 1.0 A flowing through it, how many electrons will pass through the filament in 1.0 s?

▷ We are only calculating the number of electrons that flow, so we can ignore the positive and negative signs. Solving for $\Delta q = I\Delta t$ gives a charge of 1.0 C flowing in this time interval. The number of electrons is

$$\begin{aligned}\text{number of electrons} &= \text{coulombs} \times \frac{\text{electrons}}{\text{coulomb}}\\ &= \text{coulombs} \Big/ \frac{\text{coulombs}}{\text{electron}}\\ &= 1.0\ \text{C}/e\\ &= 6.2 \times 10^{18}\end{aligned}$$

3.2 Circuits

How can we put electric currents to work? The only method of controlling electric charge we have studied so far is to charge different substances, e.g., rubber and fur, by rubbing them against each other. Figure d/1 shows an attempt to use this technique to light a lightbulb. This method is unsatisfactory. True, current will flow through the bulb, since electrons can move through metal wires, and the excess electrons on the rubber rod will therefore come through the wires and bulb due to the attraction of the positively charged fur and the repulsion of the other electrons. The problem is that after a zillionth of a second of current, the rod and fur will both have run out of charge. No more current will flow, and the lightbulb will go out.



d / 1. Static electricity runs out quickly. 2. A practical circuit. 3. An open circuit. 4. How an ammeter works. 5. Measuring the current with an ammeter.

Figure d/2 shows a setup that works. The battery pushes charge through the circuit, and recycles it over and over again. (We will have more to say later in this chapter about how batteries work.) This is called a *complete circuit.* Today, the electrical use of the word "circuit" is the only one that springs to mind for most people, but the original meaning was to travel around and make a round trip, as when a circuit court judge would ride around the boondocks, dispensing justice in each town on a certain date.

Note that an example like d/3 does not work. The wire will quickly begin acquiring a net charge, because it has no way to get rid of the charge flowing into it. The repulsion of this charge will make it more and more difficult to send any more charge in, and soon the electrical forces exerted by the battery will be canceled out completely. The whole process would be over so quickly that the filament would not even have enough time to get hot and glow. This is known as an *open circuit.* Exactly the same thing would happen if the complete circuit of figure d/2 was cut somewhere with a pair of scissors, and in fact that is essentially how an ordinary light switch works: by opening up a gap in the circuit.

The definition of electric current we have developed has the great virtue that it is easy to measure. In practical electrical work, one almost always measures current, not charge. The instrument used to measure current is called an *ammeter.* A simplified ammeter, d/4, simply consists of a coiled-wire magnet whose force twists an iron needle against the resistance of a spring. The greater the current, the greater the force. Although the construction of ammeters may differ, their use is always the same. We break into the path of the electric current and interpose the meter like a tollbooth on a road, d/5. There is still a complete circuit, and as far as the battery and bulb are concerned, the ammeter is just another segment of wire.

Does it matter where in the circuit we place the ammeter? Could we, for instance, have put it in the left side of the circuit instead of the right? Conservation of charge tells us that this can make no difference. Charge is not destroyed or "used up" by the lightbulb, so we will get the same current reading on either side of it. What is "used up" is energy stored in the battery, which is being converted into heat and light energy.

3.3 Voltage

The volt unit

Electrical circuits can be used for sending signals, storing information, or doing calculations, but their most common purpose by far is to manipulate energy, as in the battery-and-bulb example of the previous section. We know that lightbulbs are rated in units of watts, i.e., how many joules per second of energy they can convert into heat and light, but how would this relate to the flow of charge as measured in amperes? By way of analogy, suppose your friend, who didn't take physics, can't find any job better than pitching bales of hay. The number of calories he burns per hour will certainly depend on how many bales he pitches per minute, but it will also be proportional to how much mechanical work he has to do on each bale. If his job is to toss them up into a hayloft, he will got tired a lot more quickly than someone who merely tips bales off a loading dock into trucks. In metric units,

$$\frac{\text{joules}}{\text{second}} = \frac{\text{haybales}}{\text{second}} \times \frac{\text{joules}}{\text{haybale}} \qquad .$$

Similarly, the rate of energy transformation by a battery will not just depend on how many coulombs per second it pushes through a circuit but also on how much mechanical work it has to do on each coulomb of charge:

$$\frac{\text{joules}}{\text{second}} = \frac{\text{coulombs}}{\text{second}} \times \frac{\text{joules}}{\text{coulomb}}$$

or

$$\text{power} = \text{current} \times \text{work per unit charge} \qquad .$$

Units of joules per coulomb are abbreviated as *volts*, 1 V=1 J/C, named after the Italian physicist Alessandro Volta. Everyone knows that batteries are rated in units of volts, but the voltage concept is more general than that; it turns out that voltage is a property of every point in space. To gain more insight, let's think more carefully about what goes on in the battery and bulb circuit.


e / Alessandro Volta (1745-1827).

The voltage concept in general

To do work on a charged particle, the battery apparently must be exerting forces on it. How does it do this? Well, the only thing that can exert an electrical force on a charged particle is another charged particle. It's as though the haybales were pushing and pulling each other into the hayloft! This is potentially a horribly complicated situation. Even if we knew how much excess positive or negative charge there was at every point in the circuit (which realistically we don't) we would have to calculate zillions of forces using Coulomb's law, perform all the vector additions, and finally calculate how much work was being done on the charges as they moved along. To make things even more scary, there is more than one type of charged particle that moves: electrons are what move in the wires and the bulb's filament, but ions are the moving charge carriers inside the battery. Luckily, there are two ways in which we can simplify things:

> **The situation is unchanging.** Unlike the imaginary setup in which we attempted to light a bulb using a rubber rod and a piece of fur, this circuit maintains itself in a steady state (after perhaps a microsecond-long period of settling down after the circuit is first assembled). The current is steady, and as charge flows out of any area of the circuit it is replaced by the same amount of charge flowing in. The amount of excess positive or negative charge in any part of the circuit therefore stays constant. Similarly, when we watch a river flowing, the water goes by but the river doesn't disappear.
>
> **Force depends only on position.** Since the charge distribution is not changing, the total electrical force on a charged particle depends only on its own charge and on its location. If another charged particle of the same type visits the same location later on, it will feel exactly the same force.

The second observation tells us that there is nothing all that different about the experience of one charged particle as compared to another's. If we single out one particle to pay attention to, and figure out the amount of work done on it by electrical forces as it goes from point A to point B along a certain path, then this is the same amount of work that will be done on any other charged particles of the same type as it follows the same path. For the sake of visualization, let's think about the path that starts at one terminal of the battery, goes through the light bulb's filament, and ends at the other terminal. When an object experiences a force that depends only on its position (and when certain other, technical conditions are satisfied), we can define an electrical energy associated with the position of that object. The amount of work done on the particle by electrical forces as it moves from A to B equals the drop in electrical energy between A and B. This electrical energy is what is being converted into other forms of energy such as heat and light. We therefore define voltage in general as electrical energy per unit charge:

> **definition of voltage difference**
>
> The difference in voltage between two points in space is defined as
>
> $$\Delta V = \Delta U_{elec}/q \qquad ,$$

where ΔU_{elec} is the change in the electrical energy of a particle with charge q as it moves from the initial point to the final point.

The amount of power dissipated (i.e., rate at which energy is transformed by the flow of electricity) is then given by the equation

$$P = I\Delta V \quad .$$

Energy stored in a battery *example 4*

▷ The 1.2 V rechargeable battery in figure f is labeled 1800 milliamp-hours. What is the maximum amount of energy the battery can store?

▷ An ampere-hour is a unit of current multiplied by a unit of time. Current is charge per unit time, so an ampere-hour is in fact a funny unit of *charge*:

$$\begin{aligned}(1\text{ A})(1\text{ hour}) &= (1\text{ C/s})(3600\text{ s})\\ &= 3600\text{ C}\end{aligned}$$

1800 milliamp-hours is therefore $1800 \times 10^{-3} \times 3600\text{ C} = 6.5 \times 10^3\text{ C}$. That's a huge number of charged particles, but the total loss of electrical energy will just be their total charge multiplied by the voltage difference across which they move:

$$\begin{aligned}\Delta U_{elec} &= q\Delta V\\ &= (6.5 \times 10^3\text{ C})(1.2\text{ V})\\ &= 7.8\text{ kJ}\end{aligned}$$

f / Example 4.

Units of volt-amps *example 5*

▷ Doorbells are often rated in volt-amps. What does this combination of units mean?

▷ Current times voltage gives units of power, $P = I\Delta V$, so volt-amps are really just a nonstandard way of writing watts. They are telling you how much power the doorbell requires.

Power dissipated by a battery and bulb *example 6*

▷ If a 9.0-volt battery causes 1.0 A to flow through a lightbulb, how much power is dissipated?

▷ The voltage rating of a battery tells us what voltage difference ΔV it is designed to maintain between its terminals.

$$\begin{aligned}P &= I\,\Delta\text{V}\\ &= 9.0\text{ A}\cdot\text{V}\\ &= 9.0\,\frac{\text{C}}{\text{s}}\cdot\frac{\text{J}}{\text{C}}\\ &= 9.0\text{ J/s}\\ &= 9.0\text{ W}\end{aligned}$$

The only nontrivial thing in this problem was dealing with the units. One quickly gets used to translating common combinations like A · V into simpler terms.

Here are a few questions and answers about the voltage concept.

Question: OK, so what *is* voltage, really?
Answer: A device like a battery has positive and negative charges inside it that push other charges around the outside circuit. A higher-voltage battery has denser charges in it, which will do more work on each charged particle that moves through the outside circuit.

To use a gravitational analogy, we can put a paddlewheel at the bottom of either a tall waterfall or a short one, but a kg of water that falls through the greater gravitational energy difference will have more energy to give up to the paddlewheel at the bottom.

Question: Why do we define voltage as electrical energy divided by charge, instead of just defining it as electrical energy?
Answer: One answer is that it's the only definition that makes the equation $P = I\Delta V$ work. A more general answer is that we want to be able to define a voltage difference between any two points in space without having to know in advance how much charge the particles moving between them will have. If you put a nine-volt battery on your tongue, then the charged particles that move across your tongue and give you that tingly sensation are not electrons but ions, which may have charges of $+e$, $-2e$, or practically anything. The manufacturer probably expected the battery to be used mostly in circuits with metal wires, where the charged particles that flowed would be electrons with charges of $-e$. If the ones flowing across your tongue happen to have charges of $-2e$, the electrical energy difference for them will be twice as much, but dividing by their charge of $-2e$ in the definition of voltage will still give a result of 9 V.

Question: Are there two separate roles for the charged particles in the circuit, a type that sits still and exerts the forces, and another that moves under the influence of those forces?
Answer: No. Every charged particle simultaneously plays both roles. Newton's third law says that any particle that has an electrical forces acting on it must also be exerting an electrical force back on the other particle. There are no "designated movers" or "designated force-makers."

Question: Why does the definition of voltage only refer to voltage *differences*?
Answer: It's perfectly OK to define voltage as $V = U_{elec}/q$. But recall that it is only *differences* in interaction energy, U, that have direct physical meaning in physics. Similarly, voltage differences are really more useful than absolute voltages. A voltmeter measures voltage differences, not absolute voltages.

Discussion Questions

A A roller coaster is sort of like an electric circuit, but it uses gravitational forces on the cars instead of electric ones. What would a high-voltage roller coaster be like? What would a high-current roller coaster be like?

B Criticize the following statements:

> "He touched the wire, and 10000 volts went through him."
>
> "That battery has a charge of 9 volts."
>
> "You used up the charge of the battery."

C When you touch a 9-volt battery to your tongue, both positive and negative ions move through your saliva. Which ions go which way?

D I once touched a piece of physics apparatus that had been wired incorrectly, and got a several-thousand-volt voltage difference across my hand. I was not injured. For what possible reason would the shock have had insufficient power to hurt me?

3.4 Resistance

Resistance

So far we have simply presented it as an observed fact that a battery-and-bulb circuit quickly settles down to a steady flow, but why should it? Newton's second law, $a = F/m$, would seem to predict that the steady forces on the charged particles should make them whip around the circuit faster and faster. The answer is that as charged particles move through matter, there are always forces, analogous to frictional forces, that resist the motion. These forces need to be included in Newton's second law, which is really $a = F_{total}/m$, not $a = F/m$. If, by analogy, you push a crate across the floor at constant speed, i.e., with zero acceleration, the total force on it must be zero. After you get the crate going, the floor's frictional force is exactly canceling out your force. The chemical energy stored in your body is being transformed into heat in the crate and the floor, and no longer into an increase in the crate's kinetic energy. Similarly, the battery's internal chemical energy is converted into heat, not into perpetually increasing the charged particles' kinetic energy. Changing energy into heat may be a nuisance in some circuits, such as a computer chip, but it is vital in a lightbulb, which must get hot enough to glow. Whether we like it or not, this kind of heating effect is going to occur any time charged particles move through matter.

What determines the amount of heating? One flashlight bulb designed to work with a 9-volt battery might be labeled 1.0 watts, another 5.0. How does this work? Even without knowing the details of this type of friction at the atomic level, we can relate the heat dissipation to the amount of current that flows via the equation $P = I\Delta \mathrm{V}$. If the two flashlight bulbs can have two different values of P when used with a battery that maintains the same ΔV, it must be that the 5.0-watt bulb allows five times more current to flow through it.


g / Georg Simon Ohm (1787-1854).

For many substances, including the tungsten from which lightbulb filaments are made, experiments show that the amount of current that will flow through it is directly proportional to the voltage difference placed across it. For an object made of such a substance, we define its electrical *resistance* as follows:

> **definition of resistance**
> If an object inserted in a circuit displays a current flow proportional to the voltage difference across it, then we define its resistance as the constant ratio
>
> $$R = \Delta V/I$$

The units of resistance are volts/ampere, usually abbreviated as ohms, symbolized with the capital Greek letter omega, Ω.

Resistance of a lightbulb *example 7*

▷ A flashlight bulb powered by a 9-volt battery has a resistance of 10 Ω. How much current will it draw?

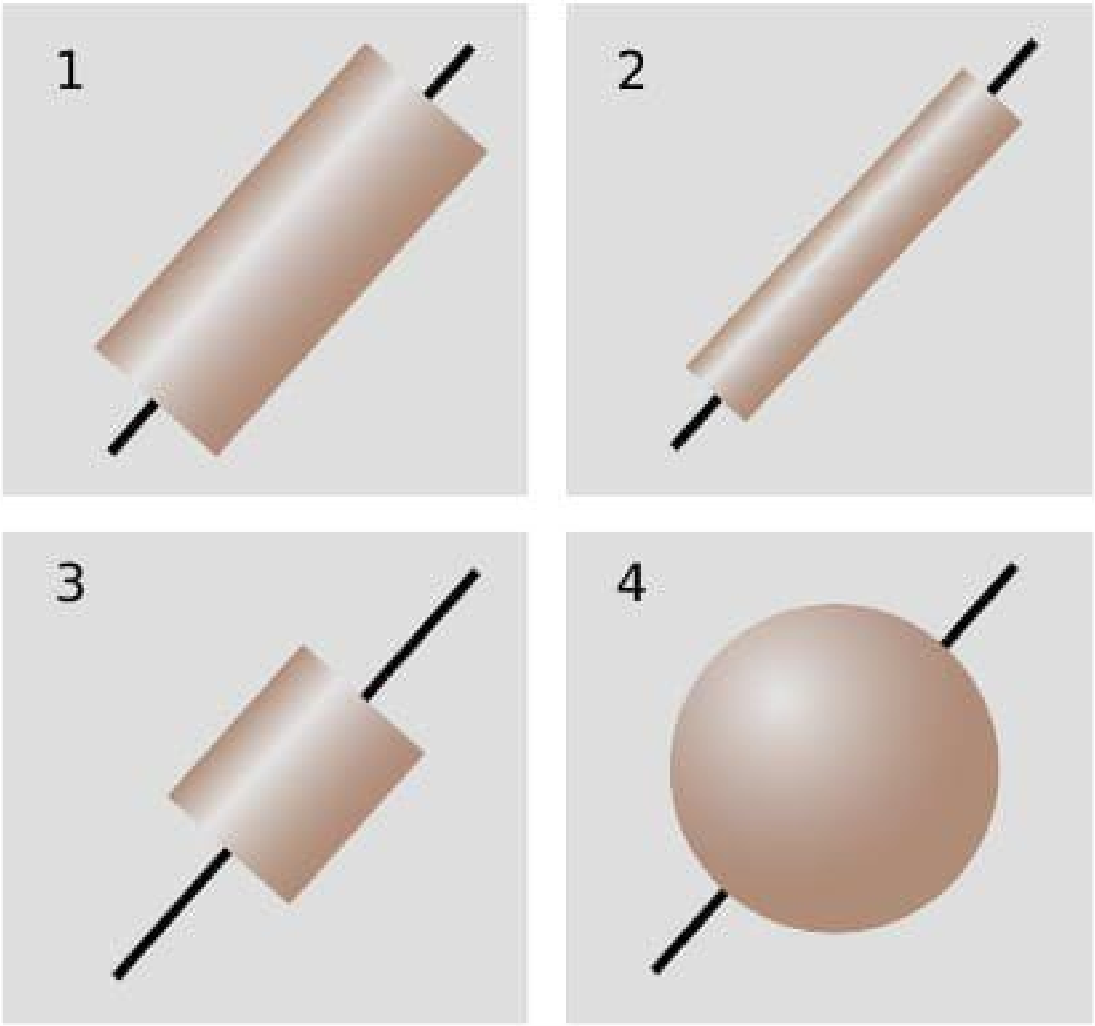
▷ Solving the definition of resistance for *I*, we find

$$\begin{aligned} I &= \Delta V/R \\ &= 0.9\ \mathrm{V}/\Omega \\ &= 0.9\ \mathrm{V}/(\mathrm{V}/\mathrm{A}) \\ &= 0.9\ \mathrm{A} \end{aligned}$$

Ohm's law states that many substances, including many solids and some liquids, display this kind of behavior, at least for voltages that are not too large. The fact that Ohm's law is called a "law" should not be taken to mean that all materials obey it, or that it has the same fundamental importance as Newton's laws, for example. Materials are called *ohmic* or *nonohmic*, depending on whether they obey Ohm's law.

If objects of the same size and shape made from two different ohmic materials have different resistances, we can say that one material is more resistive than the other, or equivalently that it is less conductive. Materials, such as metals, that are very conductive are said to be good *conductors*. Those that are extremely poor conductors, for example wood or rubber, are classified as *insulators*. There is no sharp distinction between the two classes of materials. Some, such as silicon, lie midway between the two extremes, and are called semiconductors.

On an intuitive level, we can understand the idea of resistance by making the sounds "hhhhhh" and "ffffff." To make air flow out of your mouth, you use your diaphragm to compress the air in your chest. The pressure difference between your chest and the air outside your mouth is analogous to a voltage difference. When you make the "h" sound, you form your mouth and throat in a way that allows air to flow easily. The large flow of air is like a large current. Dividing by a large current in the definition of resistance means that we get a small resistance. We say that the small resistance of your mouth and throat allows a large current to flow. When you make the "f" sound, you increase the resistance and cause a smaller current to flow.



h / Four objects made of the same substance have different resistances.

Note that although the resistance of an object depends on the substance it is made of, we cannot speak simply of the "resistance of gold" or the "resistance of wood." Figure h shows four examples of objects that have had wires attached at the ends as electrical connections. If they were made of the same substance, they would all nevertheless have different resistances because of their different sizes and shapes. A more detailed discussion will be more natural in the context of the following chapter, but it should not be too surprising that the resistance of h/2 will be greater than that of h/1

— the image of water flowing through a pipe, however incorrect, gives us the right intuition. Object h/3 will have a smaller resistance than h/1 because the charged particles have less of it to get through.

Superconductors

All materials display some variation in resistance according to temperature (a fact that is used in thermostats to make a thermometer that can be easily interfaced to an electric circuit). More spectacularly, most metals have been found to exhibit a sudden change to *zero* resistance when cooled to a certain critical temperature. They are then said to be superconductors. Theoretically, superconductors should make a great many exciting devices possible, for example coiled-wire magnets that could be used to levitate trains. In practice, the critical temperatures of all metals are very low, and the resulting need for extreme refrigeration has made their use uneconomical except for such specialized applications as particle accelerators for physics research.

But scientists have recently made the surprising discovery that certain ceramics are superconductors at less extreme temperatures. The technological barrier is now in finding practical methods for making wire out of these brittle materials. Wall Street is currently investing billions of dollars in developing superconducting devices for cellular phone relay stations based on these materials. In 2001, the city of Copenhagen replaced a short section of its electrical power trunks with superconducing cables, and they are now in operation and supplying power to customers.

There is currently no satisfactory theory of superconductivity in general, although superconductivity in metals is understood fairly well. Unfortunately I have yet to find a fundamental explanation of superconductivity in metals that works at the introductory level.

i / A superconducting segment of the ATLAS accelerator at Argonne National Laboratory near Chicago. It is used to accelerate beams of ions to a few percent of the speed of light for nuclear physics research. The shiny silver-colored surfaces are made of the element niobium, which is a superconductor at relatively high temperatures compared to other metals — relatively high meaning the temperature of liquid helium! The beam of ions passes through the holes in the two small cylinders on the ends of the curved rods. Charge is shuffled back and forth between them at a frequency of 12 million cycles per second, so that they take turns being positive and negative. The positively charged beam consists of short spurts, each timed so that when it is in one of the segments it will be pulled forward by negative charge on the cylinder in front of it and pushed forward by the positively charged one behind. The huge currents involved (see example 9 on page 99) would quickly melt any metal that was not superconducting, but in a superconductor they produce no heat at all.

Constant voltage throughout a conductor

The idea of a superconductor leads us to the question of how we should expect an object to behave if it is made of a very good conductor. Superconductors are an extreme case, but often a metal wire can be thought of as a perfect conductor, for example if the parts of the circuit other than the wire are made of much less conductive materials. What happens if R equals zero in the equation $R = \Delta V/I$? The result of dividing two numbers can only be zero if the number on top equals zero. This tells us that if we pick any two points in a perfect conductor, the voltage difference between them must be zero. In other words, the entire conductor must be at the same voltage.

Constant voltage means that no work would be done on a charge as it moved from one point in the conductor to another. If zero work was done only along a certain path between two specific points, it might mean that positive work was done along part of the path and negative work along the rest, resulting in a cancellation. But there is no way that the work could come out to be zero for all possible paths unless the electrical force on a charge was in fact zero at every point. Suppose, for example, that you build up a static charge by scuffing your feet on a carpet, and then you deposit some of that charge onto a doorknob, which is a good conductor. How can all that charge be in the doorknob without creating any electrical force at any point inside it? The only possible answer is that the charge moves around until it has spread itself into just the right configuration so that the forces exerted by all the little bits of excess surface charge on any charged particle within the doorknob exactly canceled out.

We can explain this behavior if we assume that the charge placed on the doorknob eventually settles down into a stable equilibrium. Since the doorknob is a conductor, the charge is free to move through it. If it was free to move and any part of it did experience a nonzero total force from the rest of the charge, then it would move, and we would not have an equilibrium.

It also turns out that charge placed on a conductor, once it reaches its equilibrium configuration, is entirely on the surface, not on the interior. We will not prove this fact formally, but it is intuitively reasonable. Suppose, for instance, that the net charge on the conductor is negative, i.e., it has an excess of electrons. These electrons all repel each other, and this repulsion will tend to push them onto the surface, since being on the surface allows them to be as far apart as possible.