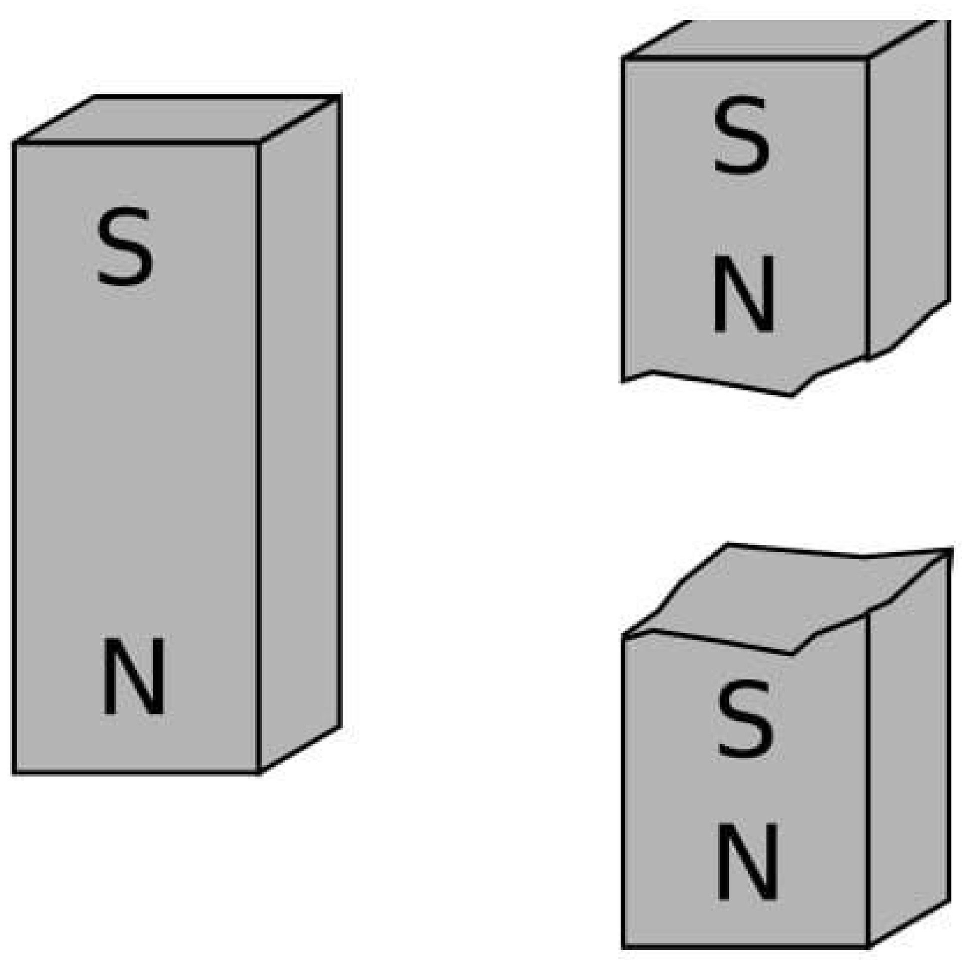
a / The first two humans to know what starlight was: James Clerk Maxwell and Katherine Maxwell, 1869.

Chapter 6
Electromagnetism

In this chapter we discuss the intimate relationship between magnetism and electricity discovered by James Clerk Maxwell. Maxwell realized that light was a wave made up of electric and magnetic fields linked to each other. He is said to have gone for a walk with his wife one night and told her that she was the only other person in the world who knew what starlight really was.

6.1 The Magnetic Field

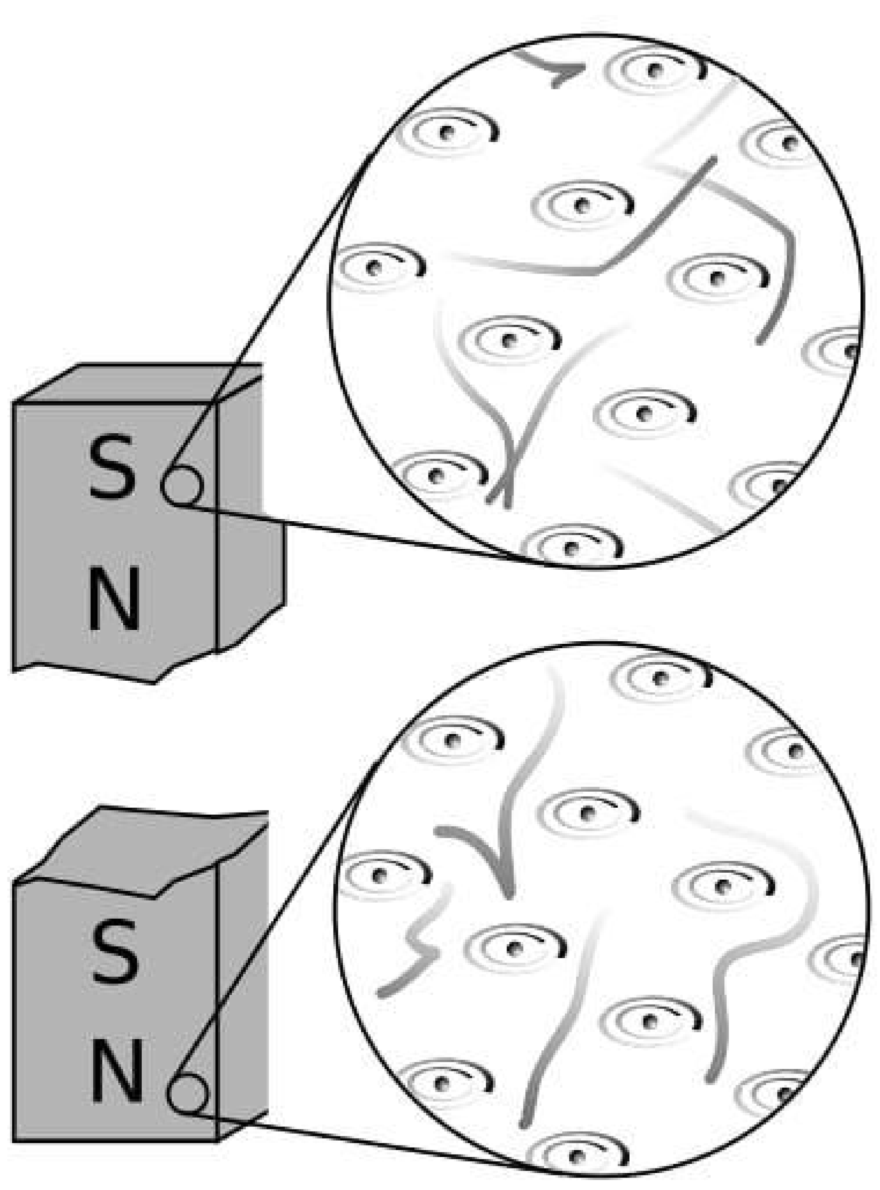
No magnetic monopoles



b / Breaking a bar magnet in half doesn't create two monopoles, it creates two smaller dipoles.

If you could play with a handful of electric dipoles and a handful of bar magnets, they would appear very similar. For instance, a pair of bar magnets wants to align themselves head-to-tail, and a pair of electric dipoles does the same thing. (It is unfortunately not that easy to make a permanent electric dipole that can be handled like this, since the charge tends to leak.)

You would eventually notice an important difference between the two types of objects, however. The electric dipoles can be broken apart to form isolated positive charges and negative charges. The two-ended device can be broken into parts that are not two-ended. But if you break a bar magnet in half, b, you will find that you have simply made two smaller two-ended objects.



c / An explanation at the atomic level.

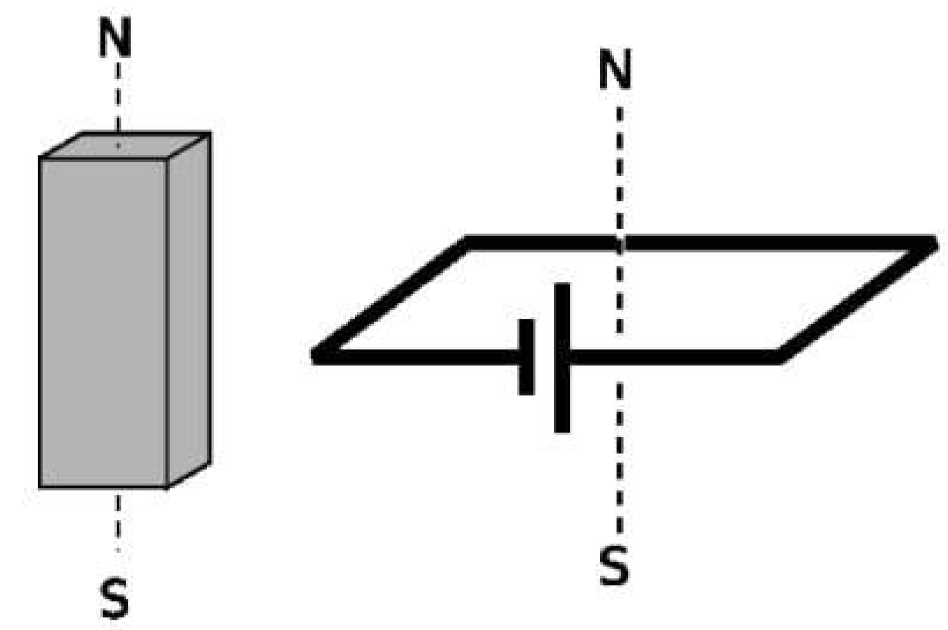
The reason for this behavior is not hard to divine from our microscopic picture of permanent iron magnets. An electric dipole has extra positive "stuff" concentrated in one end and extra negative in the other. The bar magnet, on the other hand, gets its magnetic properties not from an imbalance of magnetic "stuff" at the two ends but from the orientation of the rotation of its electrons. One end is the one from which we could look down the axis and see the electrons rotating clockwise, and the other is the one from which they would appear to go counterclockwise. There is no difference between the "stuff" in one end of the magnet and the other, c.

Nobody has ever succeeded in isolating a single magnetic pole. In technical language, we say that magnetic monopoles do not seem to exist. Electric monopoles *do* exist — that's what charges are.

Electric and magnetic forces seem similar in many ways. Both act at a distance, both can be either attractive or repulsive, and both are intimately related to the property of matter called charge. (Recall that magnetism is an interaction between moving charges.) Physicists's aesthetic senses have been offended for a long time because this seeming symmetry is broken by the existence of electric monopoles and the absence of magnetic ones. Perhaps some exotic form of matter exists, composed of particles that are magnetic monopoles. If such particles could be found in cosmic rays or moon rocks, it would be evidence that the apparent asymmetry was only an asymmetry in the composition of the universe, not in the laws of physics. For these admittedly subjective reasons, there have been several searches for magnetic monopoles. Experiments have been performed, with negative results, to look for magnetic monopoles embedded in ordinary matter. Soviet physicists in the 1960s made exciting claims that they had created and detected magnetic monopoles in particle accelerators, but there was no success in attempts to reproduce the results there or at other accelerators. The most recent search for magnetic monopoles, done by reanalyzing data from the search for the top quark at Fermilab, turned up no candidates, which shows that either monopoles don't exist in nature or they are extremely massive and thus hard to create in accelerators.

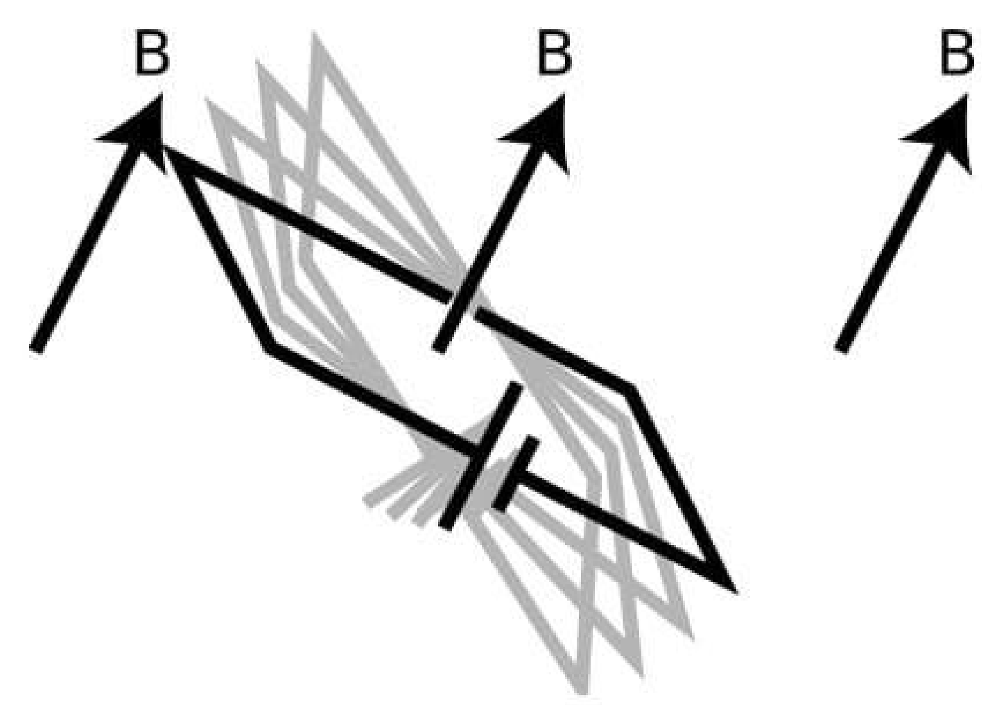
Definition of the magnetic field

Since magnetic monopoles don't seem to exist, it would not make much sense to define a magnetic field in terms of the force on a test monopole. Instead, we follow the philosophy of the alternative definition of the electric field, and define the field in terms of the torque on a magnetic test dipole. This is exactly what a magnetic compass does: the needle is a little iron magnet which acts like a magnetic dipole and shows us the direction of the earth's magnetic field.



d / A standard dipole made from a square loop of wire shorting across a battery. It acts very much like a bar magnet, but its strength is more easily quantified.

To define the strength of a magnetic field, however, we need some way of defining the strength of a test dipole, i.e., we need a definition of the magnetic dipole moment. We could use an iron permanent magnet constructed according to certain specifications, but such an object is really an extremely complex system consisting of many iron atoms, only some of which are aligned. A more fundamental standard dipole is a square current loop. This could be little resistive circuit consisting of a square of wire shorting across a battery.



e / A dipole tends to align itself to the surrounding magnetic field.

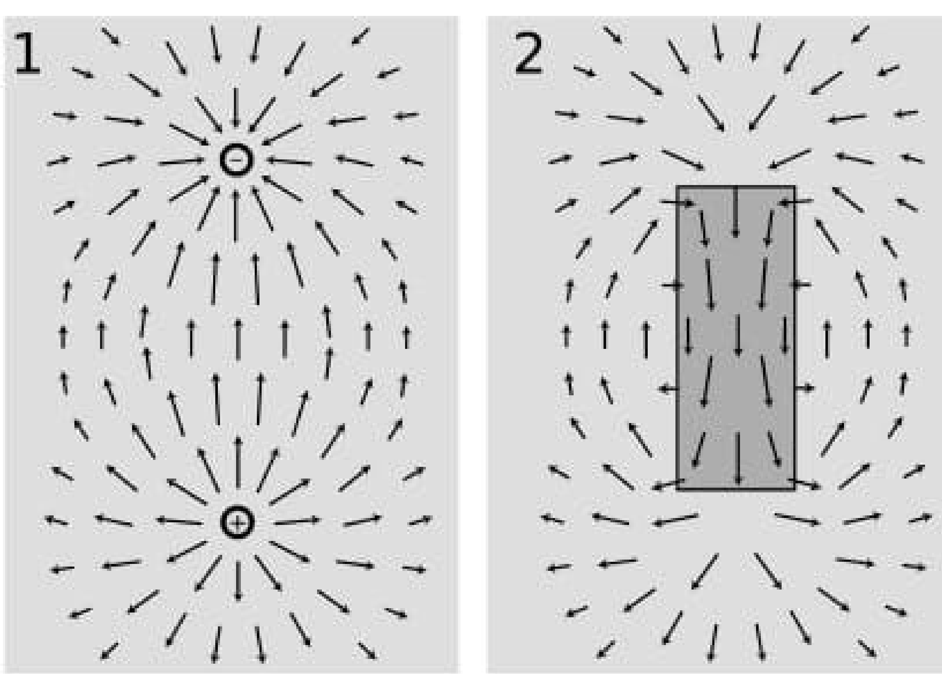
We will find that such a loop, when placed in a magnetic field, experiences a torque that tends to align plane so that its face points in a certain direction. (Since the loop is symmetric, it doesn't care if we rotate it like a wheel without changing the plane in which it lies.) It is this preferred facing direction that we will end up defining as the direction of the magnetic field.

Experiments show if the loop is out of alignment with the field, the torque on it is proportional to the amount of current, and also to the interior area of the loop. The proportionality to current makes sense, since magnetic forces are interactions between moving charges, and current is a measure of the motion of charge. The proportionality to the loop's area is also not hard to understand, because increasing the length of the sides of the square increases both the amount of charge contained in this circular "river" and the amount of leverage supplied for making torque. Two separate physical reasons for a proportionality to length result in an overall proportionality to length squared, which is the same as the area of the loop. For these reasons, we define the magnetic dipole moment of a square current loop as

$$D_m = IA \quad , \qquad \text{[definition of the magnetic dipole moment of a square current loop]}$$

We now define the magnetic field in a manner entirely analogous to the second definition of the electric field:

> **definition of the magnetic field**
> The magnetic field vector, **B**, at any location in space is defined by observing the torque exerted on a magnetic test dipole D_{mt} consisting of a square current loop. The field's magnitude is $|\mathbf{B}| = \tau / D_{mt} \sin\theta$, where θ is the angle by which the loop is misaligned. The direction of the field is perpendicular to the loop; of the two perpendiculars, we choose the one such that if we look along it, the loop's current is counterclockwise.



f / Electric fields, 1, have sources and sinks, but magnetic fields, 2, don't.

We find from this definition that the magnetic field has units of $\text{N}\cdot\text{m}/\text{A}\cdot\text{m}^2 = \text{N}/\text{A}\cdot\text{m}$. This unwieldy combination of units is abbreviated as the tesla, 1 T = 1 N/A·m. Refrain from memorizing the part about the counterclockwise direction at the end; in section 6.4 we'll see how to understand this in terms of more basic principles.

The nonexistence of magnetic monopoles means that unlike an electric field, f/1, a magnetic one, f/2, can never have sources or sinks. The magnetic field vectors lead in paths that loop back on themselves, without ever converging or diverging at a point.

6.2 Calculating Magnetic Fields and Forces

Magnetostatics

Our study of the electric field built on our previous understanding of electric forces, which was ultimately based on Coulomb's law for the electric force between two point charges. Since magnetism is ultimately an interaction between currents, i.e., between moving charges, it is reasonable to wish for a magnetic analog of Coulomb's law, an equation that would tell us the magnetic force between any two moving point charges.

Such a law, unfortunately, does not exist. Coulomb's law describes the special case of electrostatics: if a set of charges is sitting around and not moving, it tells us the interactions among them. Coulomb's law fails if the charges are in motion, since it does not incorporate any allowance for the time delay in the outward propagation of a change in the locations of the charges.

A pair of moving point charges will certainly exert magnetic forces on one another, but their magnetic fields are like the v-shaped bow waves left by boats. Each point charge experiences a magnetic field that originated from the other charge when it was at some previous position. There is no way to construct a force law that tells us the force between them based only on their current positions in space.

There is, however, a science of magnetostatics that covers a great many important cases. Magnetostatics describes magnetic forces among currents in the special case where the currents are steady and continuous, leading to magnetic fields throughout space that do not change over time.

If we cannot build a magnetostatics from a force law for point charges, then where do we start? It can be done, but the level of mathematics required (vector calculus) is inappropriate for this course. Luckily there is an alternative that is more within our reach. Physicists of generations past have used the fancy math to derive simple equations for the fields created by various static current distributions, such as a coil of wire, a circular loop, or a straight wire. Virtually all practical situations can be treated either directly using these equations or by doing vector addition, e.g., for a case like the field of two circular loops whose fields add onto one another. Figure g shows the equations for some of the more commonly encountered configurations, with illustrations of their field patterns.

Field created by a long, straight wire carrying current I:

$$B = \frac{\mu_o I}{2\pi r}$$

Here r is the distance from the center of the wire. The field vectors trace circles in planes perpendicular to the wire, going clockwise when viewed from along the direction of the current.

Field created by a single circular loop of current:
The field vectors form a dipole-like pattern, coming through the loop and back around on the outside. Each oval path traced out by the field vectors appears clockwise if viewed from along the direction the current is going when it punches through it. There is no simple equation for a field at an arbitrary point in space, but for a point lying *along the central axis* perpendicular to the loop, the field is

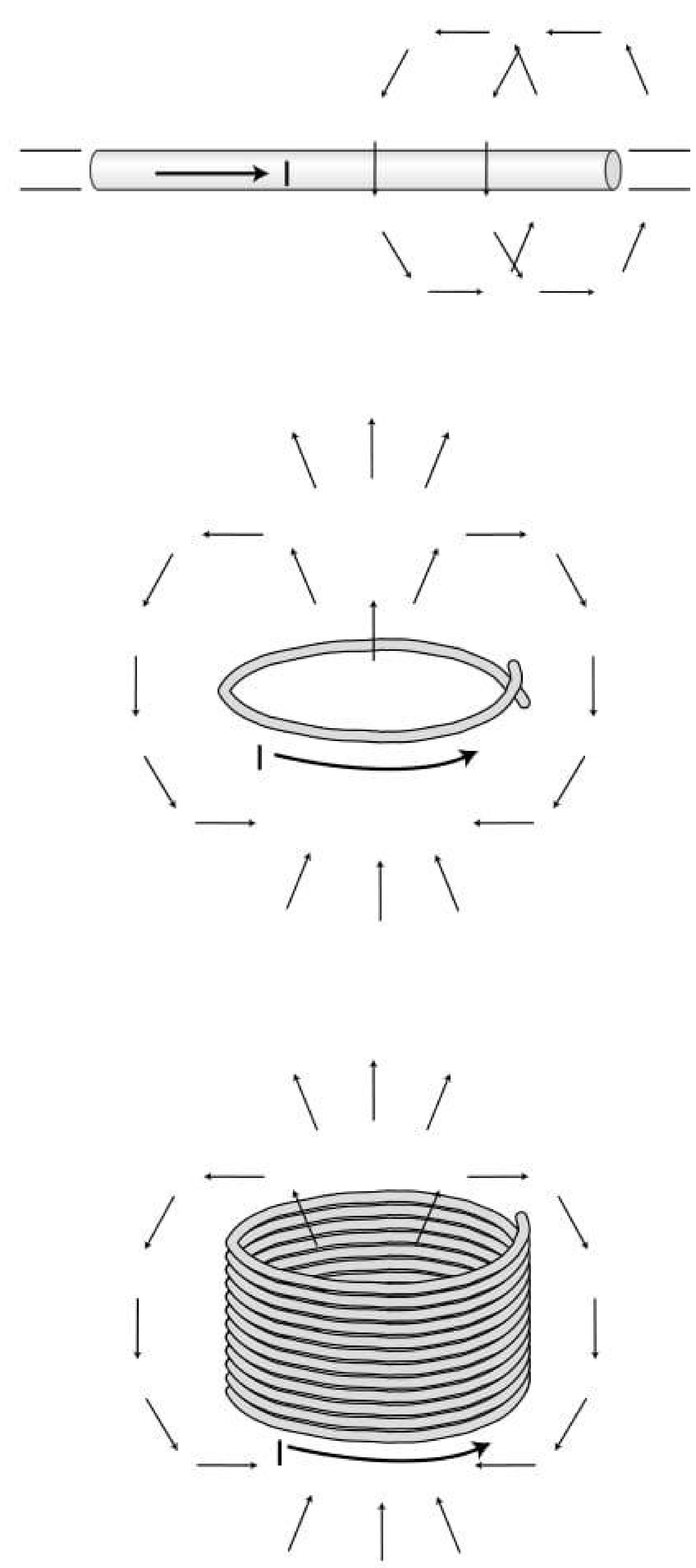
$$B = \frac{1}{2}\mu_o I b^2 \left(b^2 + z^2\right)^{-3/2} \quad ,$$

where b is the radius of the loop and z is the distance of the point from the plane of the loop.

Field created by a solenoid (cylindrical coil):
The field pattern is similar to that of a single loop, but for a long solenoid the paths of the field vectors become very straight on the inside of the coil and on the outside immediately next to the coil. For a sufficiently long solenoid, the interior field also becomes very nearly uniform, with a magnitude of

$$B = \mu_o I N/\ell \quad ,$$

where N is the number of turns of wire and ℓ is the length of the solenoid. The field near the mouths or outside the coil is not constant, and is more difficult to calculate. For a long solenoid, the exterior field is much smaller than the interior field.



g / Some magnetic fields.

Don't memorize the equations! The symbol μ_o is an abbreviation for the constant $4\pi \times 10^{-7}$ T·m/A. It is the magnetic counterpart of the Coulomb force constant k. The Coulomb constant tells us how much electric field is produced by a given amount of charge, while μ_o relates currents to magnetic fields. Unlike k, μ_o has a definite numerical value because of the design of the metric system.

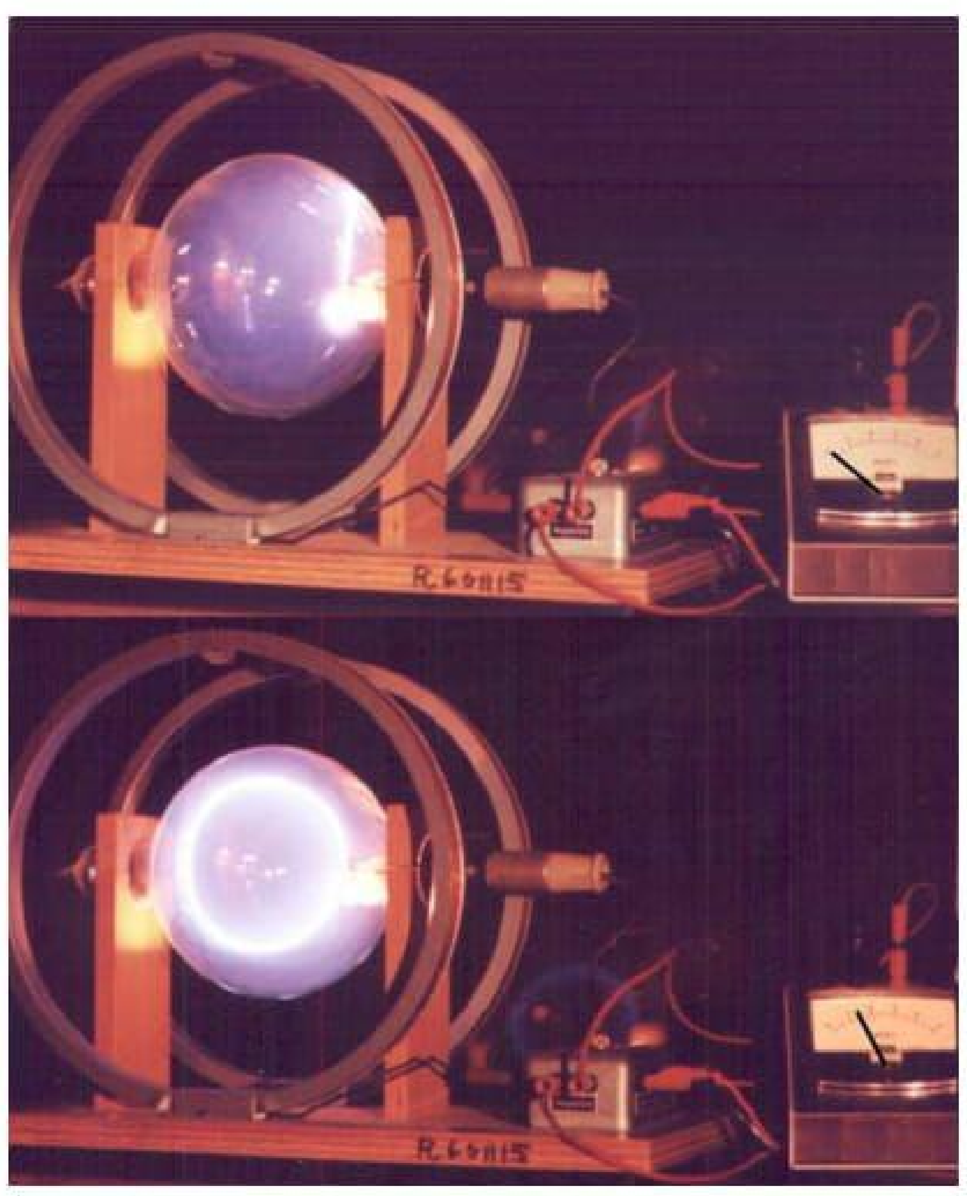
Force on a charge moving through a magnetic field

We now know how to calculate magnetic fields in some typical situations, but one might also like to be able to calculate magnetic forces, such as the force of a solenoid on a moving charged particle, or the force between two parallel current-carrying wires.

We will restrict ourselves to the case of the force on a charged particle moving through a magnetic field, which allows us to calculate the force between two objects when one is a moving charged particle and the other is one whose magnetic field we know how to find. An example is the use of solenoids inside a TV tube to guide the electron beam as it paints a picture.

Experiments show that the magnetic force on a moving charged particle has a magnitude given by

$$|\mathbf{F}| = q|\mathbf{v}||\mathbf{B}|\sin\theta \qquad ,$$



h / Magnetic forces cause a beam of electrons to move in a circle. The beam is created in a vacuum tube, in which a small amount of hydrogen gas has been left. A few of the electrons strike hydrogen molecules, creating light and letting us see the beam. A magnetic field is produced by passing a current (meter) through the circular coils of wire in front of and behind the tube. In the bottom figure, with the magnetic field turned on, the force perpendicular to the electrons' direction of motion causes them to move in a circle.

where $\mathbf{v}$ is the velocity vector of the particle, and θ is the angle between the $\mathbf{v}$ and $\mathbf{B}$ vectors. Unlike electric and gravitational forces, magnetic forces do not lie along the same line as the field vector. The force is always *perpendicular* to both $\mathbf{v}$ and $\mathbf{B}$. Given two vectors, there is only one line perpendicular to both of them, so the force vector points in one of the two possible directions along this line. For a positively charged particle, the direction of the force vector can be found as follows. First, position the $\mathbf{v}$ and $\mathbf{B}$ vectors with their tails together. The direction of $\mathbf{F}$ is such that if you sight along it, the $\mathbf{B}$ vector is clockwise from the $\mathbf{v}$ vector; for a negatively charged particle the direction of the force is reversed. Note that since the force is perpendicular to the particle's motion, the magnetic field never does work on it.

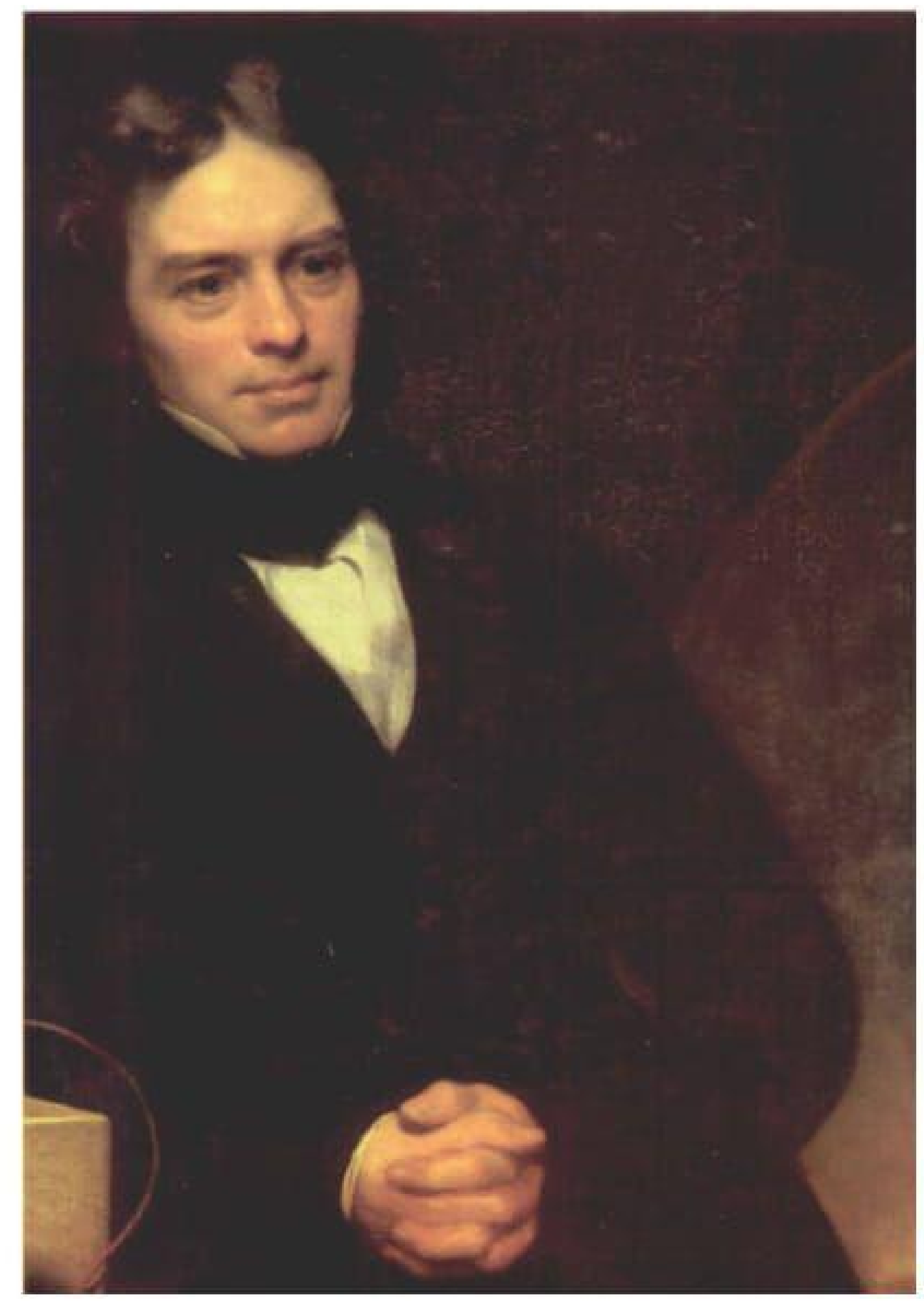
Hallucinations during an MRI scan — *example 1*

During an MRI scan of the head, the patient's nervous system is exposed to intense magnetic fields. The average velocities of the charge-carrying ions in the nerve cells is fairly low, but if the patient moves her head suddenly, the velocity can be high enough that the magnetic field makes significant forces on the ions. This can result in visual and auditory hallucinations, e.g., frying bacon sounds.

6.3 Induction

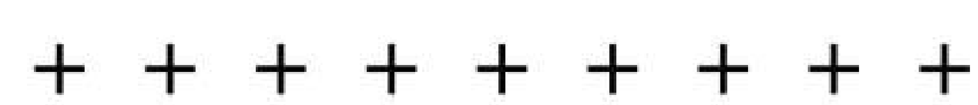
Electromagnetism and relative motion

The theory of electric and magnetic fields constructed up to this point contains a paradox. One of the most basic principles of physics, dating back to Newton and Galileo and still going strong today, states that motion is relative, not absolute. Thus the laws of physics should not function any differently in a moving frame of reference, or else we would be able to tell which frame of reference was the one in an absolute state of rest. As an example from mechanics, imagine that a child is tossing a ball up and down in the back seat of a moving car. In the child's frame of reference, the car is at rest and the landscape is moving by; in this frame, the ball goes straight up and down, and obeys Newton's laws of motion and Newton's law of gravity. In the frame of reference of an observer watching from the sidewalk, the car is moving and the sidewalk isn't. In this frame, the ball follows a parabolic arc, but it still obeys Newton's laws.



i / Michael Faraday (1791-1867), the son of a poor blacksmith, discovered induction experimentally.

When it comes to electricity and magnetism, however, we have a problem, which was first clearly articulated by Einstein: if we state that magnetism is an interaction between moving charges, we have apparently created a law of physics that violates the principle that motion is relative, since different observers in different frames would disagree about how fast the charges were moving, or even whether they were moving at all. The incorrect solution that Einstein was taught (and disbelieved) as a student around the year 1900 was that the relative nature of motion applied only to mechanics, not to electricity and magnetism. The full story of how Einstein restored the principle of relative motion to its rightful place in physics involves his theory of special relativity, which we will not take up until book 6 of this series. However, a few simple and qualitative thought experiments will suffice to show how, based on the principle that motion is relative, there must be some new and previously unsuspected relationships between electricity and magnetism. These relationships form the basis for many practical, everyday devices, such as generators and transformers, and they also lead to an explanation of light itself as an electromagnetic phenomenon.

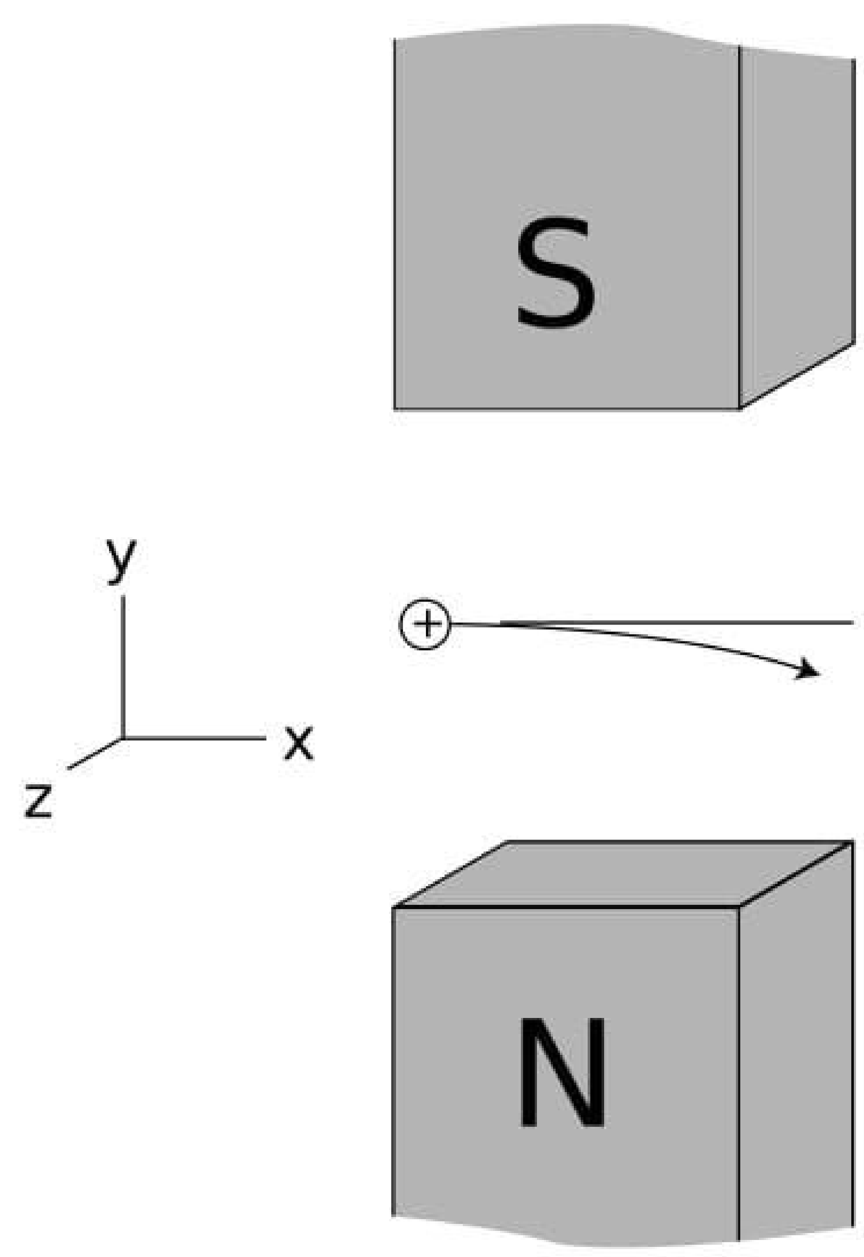


j / A line of positive charges.

Let's imagine an electrical example of relative motion in the same spirit as the story of the child in the back of the car. Suppose we have a line of positive charges, j. Observer A is in a frame of reference which is at rest with respect to these charges, and observes that they create an electric field pattern that points outward, away from the charges, in all directions, like a bottle brush. Suppose, however, that observer B is moving to the right with respect to the charges. As far as B is concerned, she's the one at rest, while the charges (and observer A) move to the left. In agreement with A, she observes an electric field, but since to her the charges are in motion, she must also observe a magnetic field in the same region of space, exactly like the magnetic field made by a current in a long, straight wire.

Who's right? They're both right. In A's frame of reference, there is only an **E**, while in B's frame there is both an **E** and a **B**. The principle of relative motion forces us to conclude that depending on our frame of reference we will observe a different combination of fields. Although we will not prove it (the proof requires special relativity, which we get to in book 6), it is true that either frame of reference provides a perfectly self-consistent description of things. For instance, if an electron passes through this region of space, both A and B will see it swerve, speed up, and slow down. A will successfully explain this as the result of an electric field, while B will ascribe the electron's behavior to a combination of electric and magnetic forces.

Thus, if we believe in the principle of relative motion, then we must accept that electric and magnetic fields are closely related phenomena, two sides of the same coin.



k / Observer A sees a positively charged particle moves through a region of upward magnetic field, which we assume to be uniform, between the poles of two magnets. The resulting force along the z axis causes the particle's path to curve toward us.

Now consider figure k. Observer A is at rest with respect to the bar magnets, and sees the particle swerving off in the z direction, as it should according to the rule given in section 6.2 (sighting along the force vector, i.e., from behind the page, the **B** vector is clockwise from the v vector). Suppose observer B, on the other hand, is moving to the right along the x axis, initially at the same speed as the particle. B sees the bar magnets moving to the left and the particle initially at rest but then accelerating along the z axis in a straight line. It is not possible for a magnetic field to start a particle moving if it is initially at rest, since magnetism is an interaction of moving charges with moving charges. B is thus led to the inescapable conclusion that there is an electric field in this region of space, which points along the z axis. In other words, what A perceives as a pure **B** field, B sees as a mixture of **E** and **B**.

In general, observers who are not at rest with respect to one another will perceive different mixtures of electric and magnetic fields.

The principle of induction

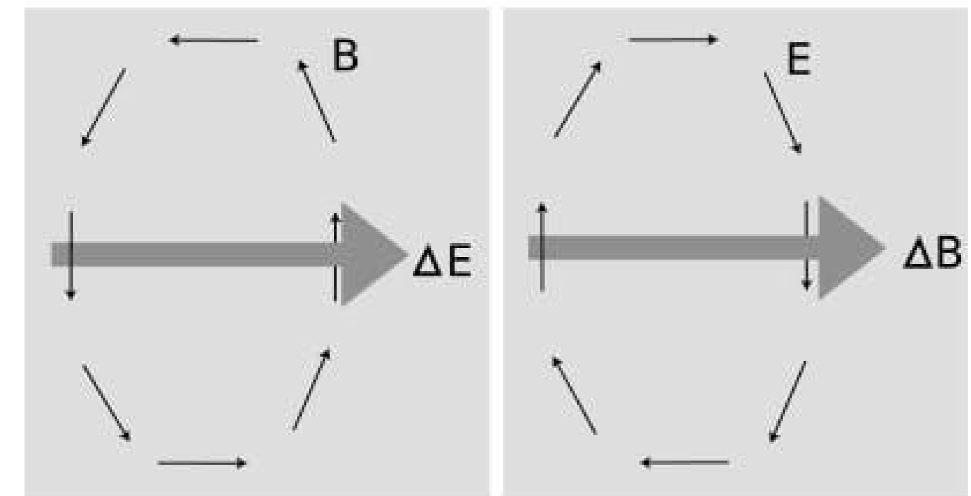
So far everything we've been doing might not seem terribly useful, since it seems that nothing surprising will happen as long as we stick to a single frame of reference, and don't worry about what people in other frames think. That isn't the whole story, however, as was discovered experimentally by Faraday in 1831 and explored mathematically by Maxwell later in the same century. Let's state Faraday's idea first, and then see how something like it must follow inevitably from the principle that motion is relative:

the principle of induction

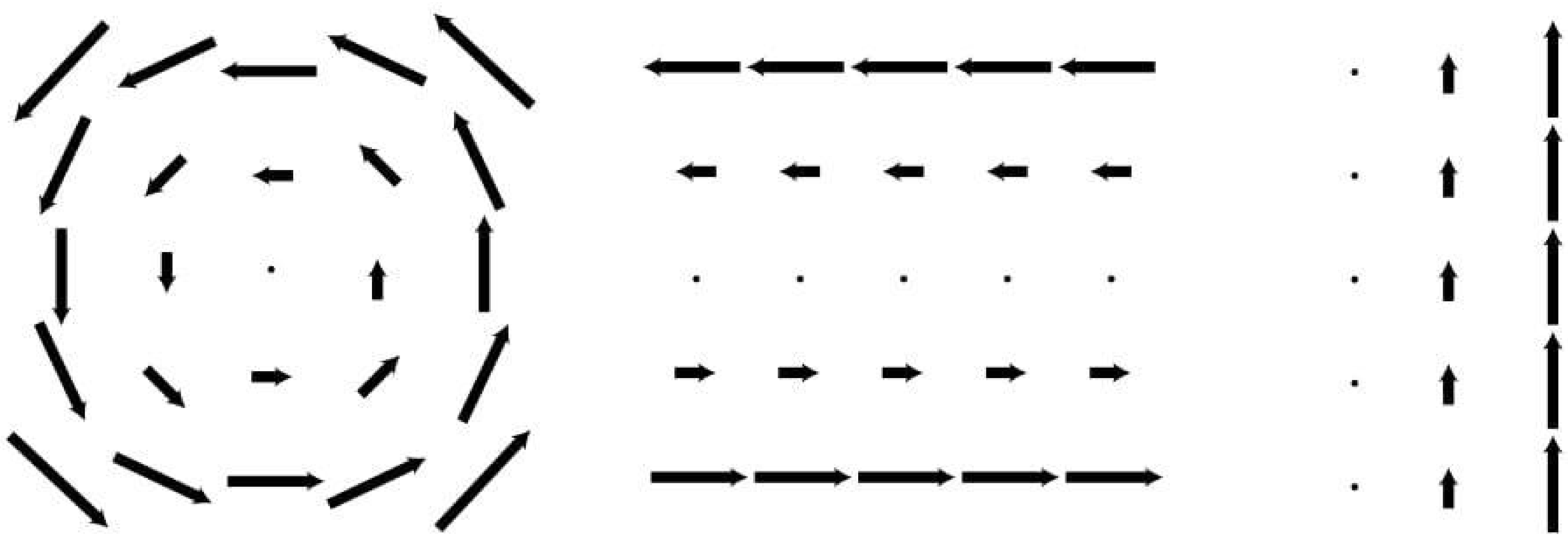
Any electric field that changes over time will produce a magnetic field in the space around it.

Any magnetic field that changes over time will produce an electric field in the space around it.

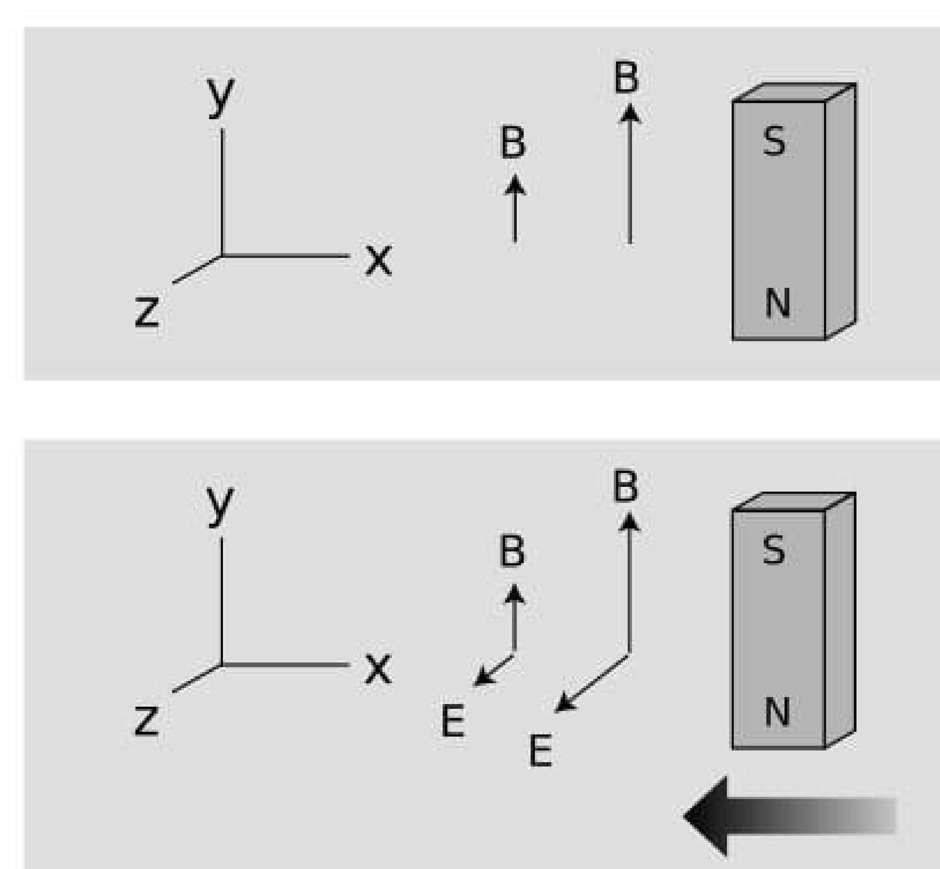
The induced field tends to have a whirlpool pattern, as shown in figure l, but the whirlpool image is not to be taken too literally; the principle of induction really just requires a field pattern such that, if one inserted a paddlewheel in it, the paddlewheel would spin. All of the field patterns shown in figure m are ones that could be created by induction; all have a counterclockwise "curl" to them.



l / The geometry of induced fields. The induced field tends to form a whirlpool pattern around the change in the vector producing it. Note how they circulate in opposite directions.



m / Three fields with counter-clockwise "curls."

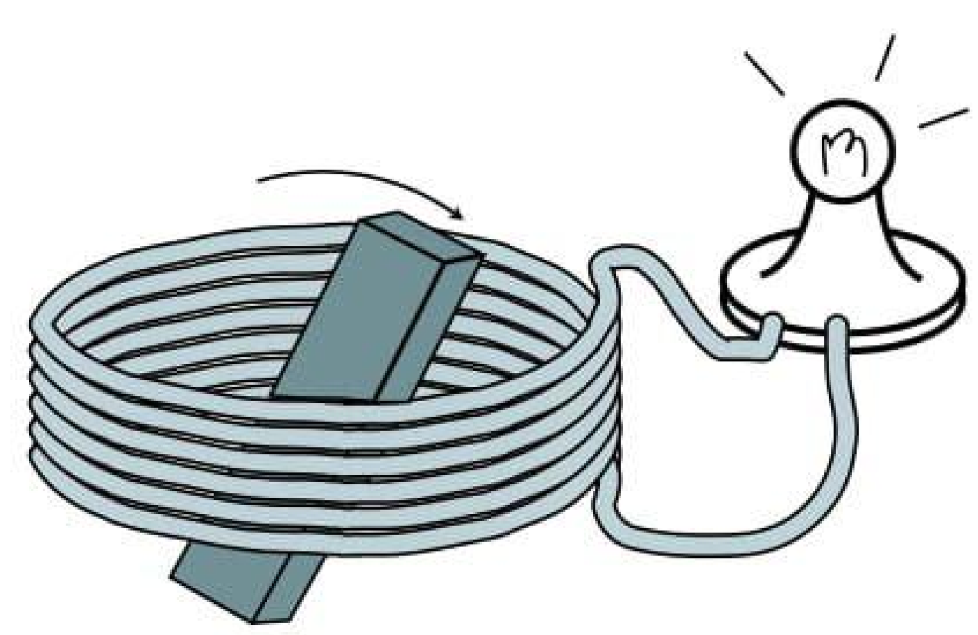


n / 1. Observer A is at rest with respect to the bar magnet, and observes magnetic fields that have different strengths at different distances from the magnet. 2. Observer B, hanging out in the region to the left of the magnet, sees the magnet moving toward her, and detects that the magnetic field in that region is getting stronger as time passes. As in 1, there is an electric field along the z axis because she's in motion with respect to the magnet. The $\Delta\mathbf{B}$ vector is upward, and the electric field has a curliness to it: a paddlewheel inserted in the electric field would spin clockwise as seen from above, since the clockwise torque made by the strong electric field on the right is greater than the counterclockwise torque made by the weaker electric field on the left.

Figure n shows an example of the fundamental reason why a changing **B** field must create an **E** field. The electric field would be inexplicable to observer B if she believed only in Coulomb's law, and thought that all electric fields are made by electric charges. If she knows about the principle of induction, however, the existence of this field is to be expected.

the generator — *example 2*

A generator, o, consists of a permanent magnet that rotates within a coil of wire. The magnet is turned by a motor or crank, (not shown). As it spins, the nearby magnetic field changes. According to the principle of induction, this changing magnetic field results in an electric field, which has a whirlpool pattern. This electric field pattern creates a current that whips around the coils of wire, and we can tap this current to light the



o / A generator

lightbulb.

self-check A

When you're driving a car, the engine recharges the battery continuously using a device called an alternator, which is really just a generator like the one shown on the previous page, except that the coil rotates while the permanent magnet is fixed in place. Why can't you use the alternator to start the engine if your car's battery is dead? ▷ Answer, p. 196

The transformer *example 3*

In section 4.3 we discussed the advantages of transmitting power over electrical lines using high voltages and low currents. However, we don't want our wall sockets to operate at 10000 volts! For this reason, the electric company uses a device called a transformer, (g), to convert to lower voltages and higher currents inside your house. The coil on the input side creates a magnetic field. Transformers work with alternating current, so the magnetic field surrounding the input coil is always changing. This induces an electric field, which drives a current around the output coil.

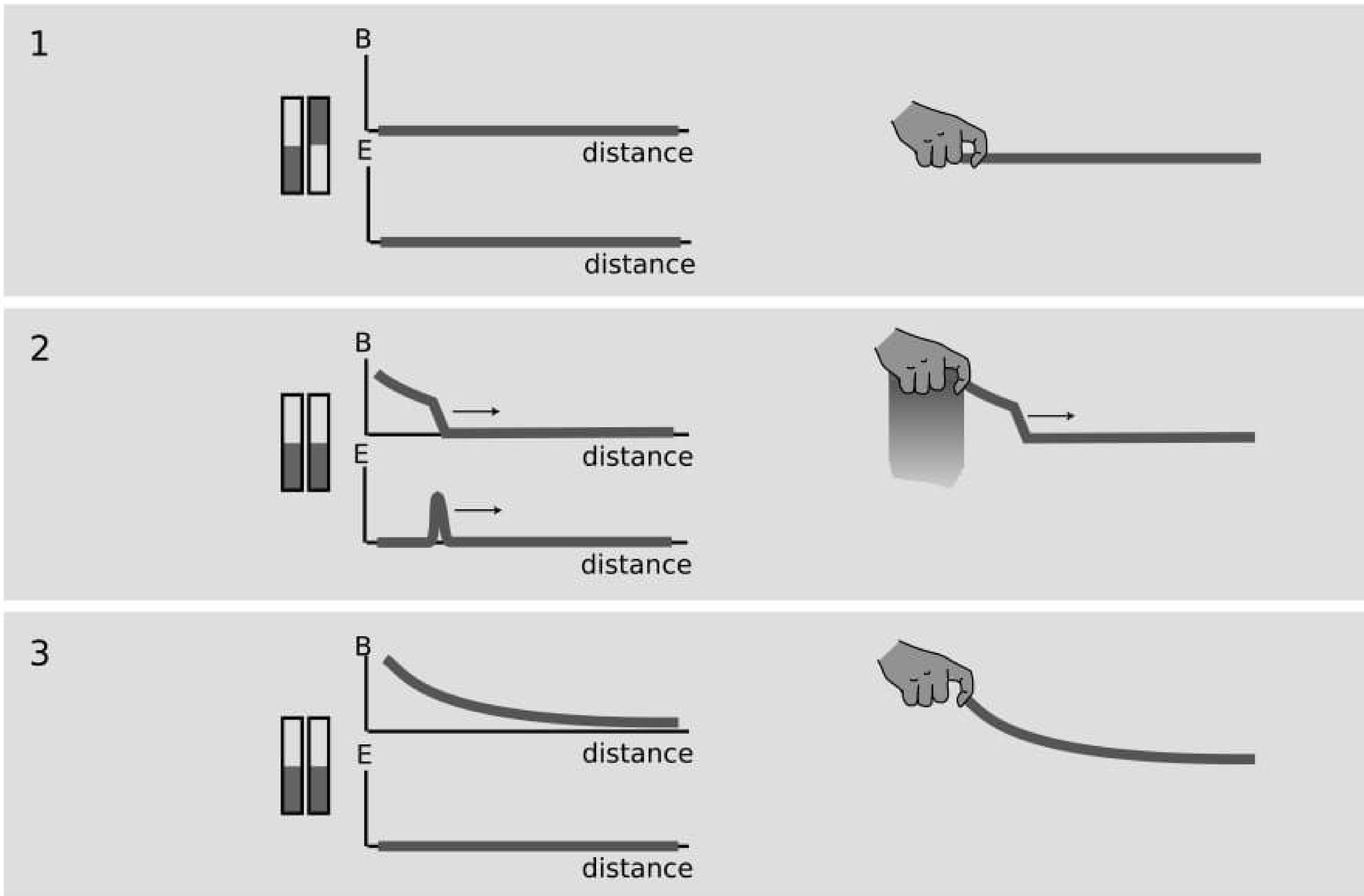
If both coils were the same, the arrangement would be symmetric, and the output would be the same as the input, but an output coil with a smaller number of coils gives the electric forces a smaller distance through which to push the electrons. Less mechanical work per unit charge means a lower voltage. Conservation of energy, however, guarantees that the amount of power on the output side must equal the amount put in originally, $I_{in}V_{in} = I_{out}V_{out}$, so this reduced voltage must be accompanied by an increased current.

A mechanical analogy *example 4*

Figure p shows an example of induction (left) with a mechanical analogy (right). The two bar magnets are initially pointing in opposite directions, 1, and their magnetic fields cancel out. If one magnet is flipped, 2, their fields reinforce, but the change in the magnetic field takes time to spread through space. Eventually, 3, the field becomes what you would expect from the theory of magnetostatics. In the mechanical analogy, the sudden motion of the hand produces a violent kink or wave pulse in the rope, the pulse travels along the rope, and it takes some time for the rope to settle down. An electric field is also induced in by the changing magnetic field, even though there is no net charge anywhere to to act as a source. (These simplified drawings are not meant to be accurate representations of the complete three-dimensional pattern of electric and magnetic fields.)

Discussion Question

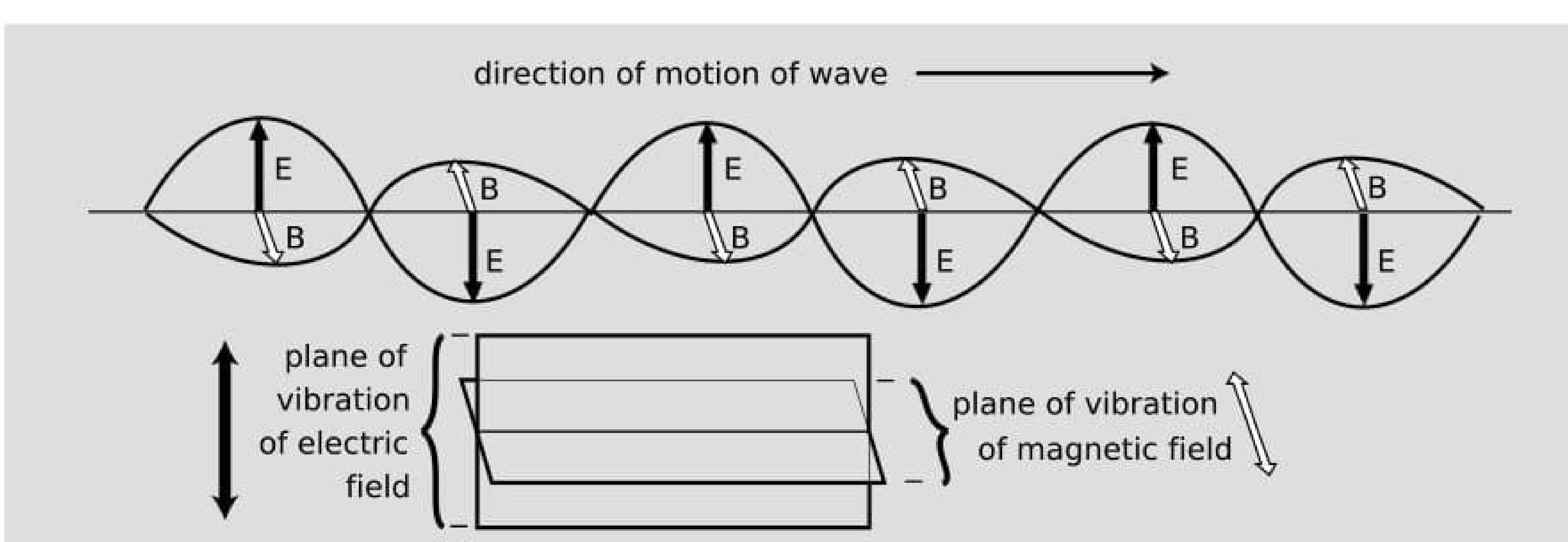
A In figures k and n, observer B is moving to the right. What would have happened if she had been moving to the left?



p / Example 4.

6.4 Electromagnetic Waves

The most important consequence of induction is the existence of electromagnetic waves. Whereas a gravitational wave would consist of nothing more than a rippling of gravitational fields, the principle of induction tells us that there can be no purely electrical or purely magnetic waves. Instead, we have waves in which there are both electric and magnetic fields, such as the sinusoidal one shown in the figure. Maxwell proved that such waves were a direct consequence of his equations, and derived their properties mathematically. The derivation would be beyond the mathematical level of this book, so we will just state the results.



q / An electromagnetic wave.

A sinusoidal electromagnetic wave has the geometry shown in figure q. The **E** and **B** fields are perpendicular to the direction of motion, and are also perpendicular to each other. If you look along the direction of motion of the wave, the **B** vector is always 90 degrees clockwise from the **E** vector. The magnitudes of the two fields are related by the equation $|\mathbf{E}| = c|\mathbf{B}|$.

How is an electromagnetic wave created? It could be emitted, for example, by an electron orbiting an atom or currents going back and forth in a transmitting antenna. In general any accelerating charge will create an electromagnetic wave, although only a current that varies sinusoidally with time will create a sinusoidal wave. Once created, the wave spreads out through space without any need for charges or currents along the way to keep it going. As the electric field oscillates back and forth, it induces the magnetic field, and the oscillating magnetic field in turn creates the electric field. The whole wave pattern propagates through empty space at a velocity $c = 3.0 \times 10^8$ m/s, which is related to the constants k and μ_o by $c = \sqrt{4\pi k/\mu_o}$.

Polarization

Two electromagnetic waves traveling in the same direction through space can differ by having their electric and magnetic fields in different directions, a property of the wave called its polarization.

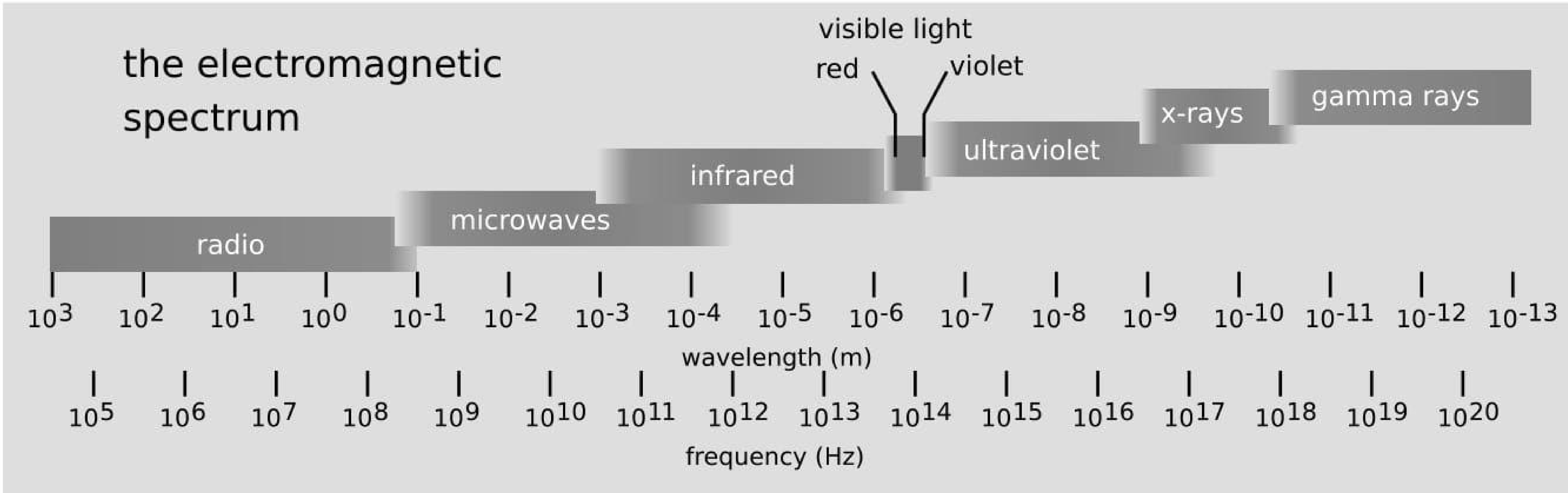
Light is an electromagnetic wave

Once Maxwell had derived the existence of electromagnetic waves, he became certain that they were the same phenomenon as light. Both are transverse waves (i.e., the vibration is perpendicular to the direction the wave is moving), and the velocity is the same.

Heinrich Hertz (for whom the unit of frequency is named) verified Maxwell's ideas experimentally. Hertz was the first to succeed in producing, detecting, and studying electromagnetic waves in detail using antennas and electric circuits. To produce the waves, he had to make electric currents oscillate very rapidly in a circuit. In fact, there was really no hope of making the current reverse directions at the frequencies of 10^{15} Hz possessed by visible light. The fastest electrical oscillations he could produce were 10^9 Hz, which would give a wavelength of about 30 cm. He succeeded in showing that, just like light, the waves he produced were polarizable, and could be reflected and refracted (i.e., bent, as by a lens), and he built devices such as parabolic mirrors that worked according to the same optical principles as those employing light. Hertz's results were convincing evidence that light and electromagnetic waves were one and the same.

The electromagnetic spectrum

Today, electromagnetic waves with frequencies in the range employed by Hertz are known as radio waves. Any remaining doubts that the "Hertzian waves," as they were then called, were the same type of wave as light waves were soon dispelled by experiments in the whole range of frequencies in between, as well as the frequencies outside that range. In analogy to the spectrum of visible light, we speak of the entire electromagnetic spectrum, of which the visible spectrum is one segment.



The terminology for the various parts of the spectrum is worth memorizing, and is most easily learned by recognizing the logical relationships between the wavelengths and the properties of the waves with which you are already familiar. Radio waves have wavelengths that are comparable to the size of a radio antenna, i.e., meters to tens of meters. Microwaves were named that because they have much shorter wavelengths than radio waves; when food heats unevenly in a microwave oven, the small distances between neighboring hot and cold spots is half of one wavelength of the standing wave the oven creates. The infrared, visible, and ultraviolet obviously have much shorter wavelengths, because otherwise the wave nature of light would have been as obvious to humans as the wave nature of ocean waves. To remember that ultraviolet, x-rays, and gamma rays all lie on the short-wavelength side of visible, recall that all three of these can cause cancer. (As we'll discuss later in the course, there is a basic physical reason why the cancer-causing disruption of DNA can only be caused by very short-wavelength electromagnetic waves. Contrary to popular belief, microwaves cannot cause cancer, which is why we have microwave ovens and not x-ray ovens!)

r / Heinrich Hertz (1857-1894).

Why the sky is blue — *example 5*

When sunlight enters the upper atmosphere, a particular air molecule finds itself being washed over by an electromagnetic wave of frequency f. The molecule's charged particles (nuclei and electrons) act like oscillators being driven by an oscillating force, and respond by vibrating at the same frequency f. Energy is sucked out of the incoming beam of sunlight and converted into the kinetic energy of the oscillating particles. However, these particles are accelerating, so they act like little radio antennas that put the energy back out as spherical waves of light that spread out in all directions. An object oscillating at a frequency f has an acceleration proportional to f^2, and an accelerating charged particle creates an electromagnetic wave whose fields are proportional to its acceleration, so the field of the reradiated spherical wave is proportional to f^2. The energy of a field is proportional to the square of the field, so the energy of the reradiated is proportional to f^4. Since blue light has about twice the frequency of red light, this process is about 2^4 = 16 times as strong for blue as for red, and that's why the sky is blue.

6.5 Calculating Energy in Fields

We have seen that the energy stored in a wave (actually the energy density) is typically proportional to the square of the wave's amplitude. Fields of force can make wave patterns, for which we might expect the same to be true. This turns out to be true not only for wave-like field patterns but for all fields:

$$\text{energy stored in the gravitational field per m}^3 = -\frac{1}{8\pi G}|\mathbf{g}|^2$$
$$\text{energy stored in the electric field per m}^3 = \frac{1}{8\pi k}|\mathbf{E}^2|$$
$$\text{energy stored in the magnetic field per m}^3 = \frac{1}{2\mu_o}|\mathbf{B}|^2$$

Although funny factors of 8π and the plus and minus signs may have initially caught your eye, they are not the main point. The important idea is that the energy density is proportional to the square of the field strength in all three cases. We first give a simple numerical example and work a little on the concepts, and then turn our attention to the factors out in front.

Getting killed by a solenoid *example 6*

Solenoids are very common electrical devices, but they can be a hazard to someone who is working on them. Imagine a solenoid that initially has a DC current passing through it. The current creates a magnetic field inside and around it, which contains energy. Now suppose that we break the circuit. Since there is no longer a complete circuit, current will quickly stop flowing, and the magnetic field will collapse very quickly. The field had energy stored in it, and even a small amount of energy can create a dangerous power surge if released over a short enough time interval. It is prudent not to fiddle with a solenoid that has current flowing through it, since breaking the circuit could be hazardous to your health.

As a typical numerical estimate, let's assume a 40 cm $\times$ 40 cm $\times$ 40 cm solenoid with an interior magnetic field of 1.0 T (quite a strong field). For the sake of this rough estimate, we ignore the exterior field, which is weak, and assume that the solenoid is cubical in shape. The energy stored in the field is

$$\begin{aligned}\text{(energy per unit volume)(volume)} &= \frac{1}{2\mu_o}|\mathbf{B}|^2 V \\ &= 3\times 10^4\ \text{J}\end{aligned}$$

That's a lot of energy!

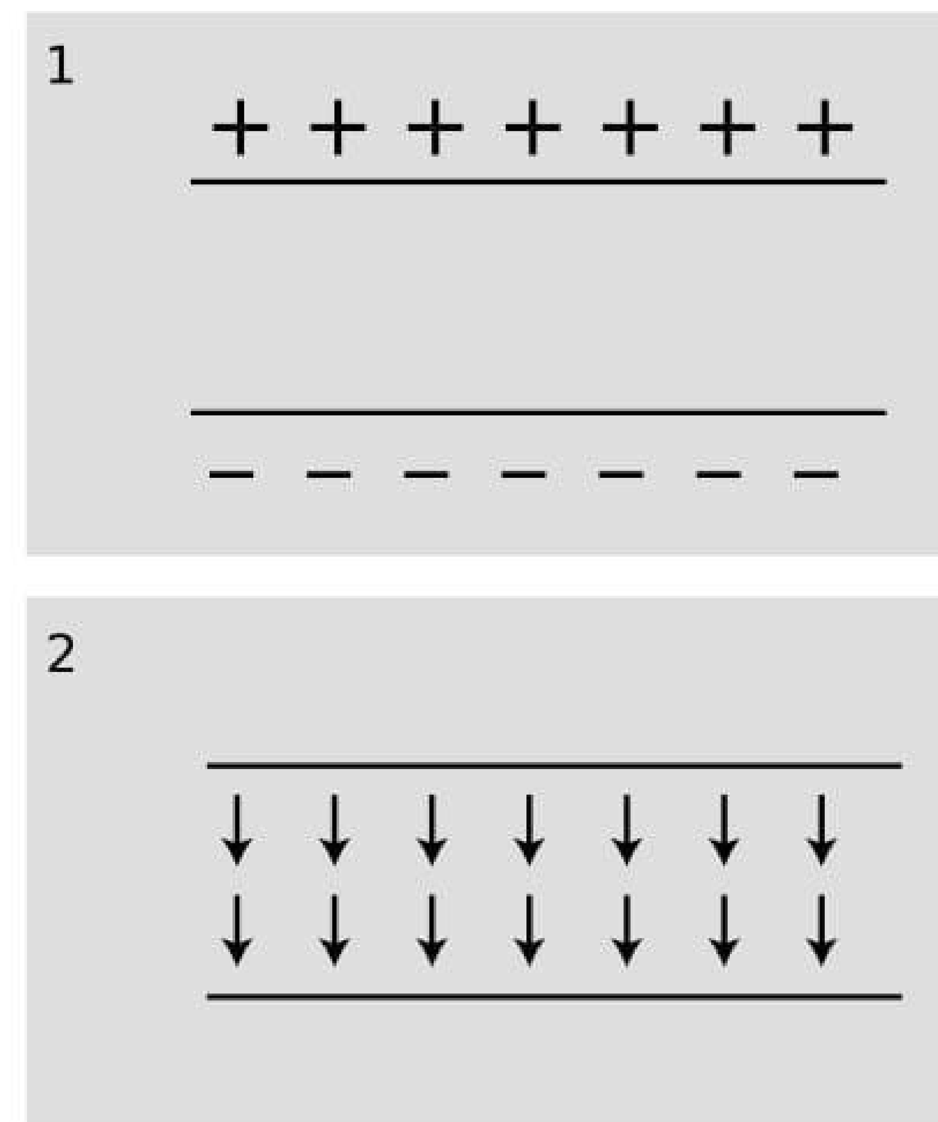
In chapter 5 when we discussed the original reason for introducing the concept of a field of force, a prime motivation was that otherwise there was no way to account for the energy transfers involved when forces were delayed by an intervening distance. We used to think of the universe's energy as consisting of

kinetic energy
+gravitational potential energy based on the distances between objects that interact gravitationally
+electric potential energy based on the distances between objects that interact electrically
+magnetic potential energy based on the distances between objects that interact magnetically ,

but in nonstatic situations we must use a different method:

kinetic energy
+gravitational potential energy stored in gravitational fields
+electric potential energy stored in electric fields
+magnetic potential stored in magnetic fields

Surprisingly, the new method still gives the same answers for the static cases.

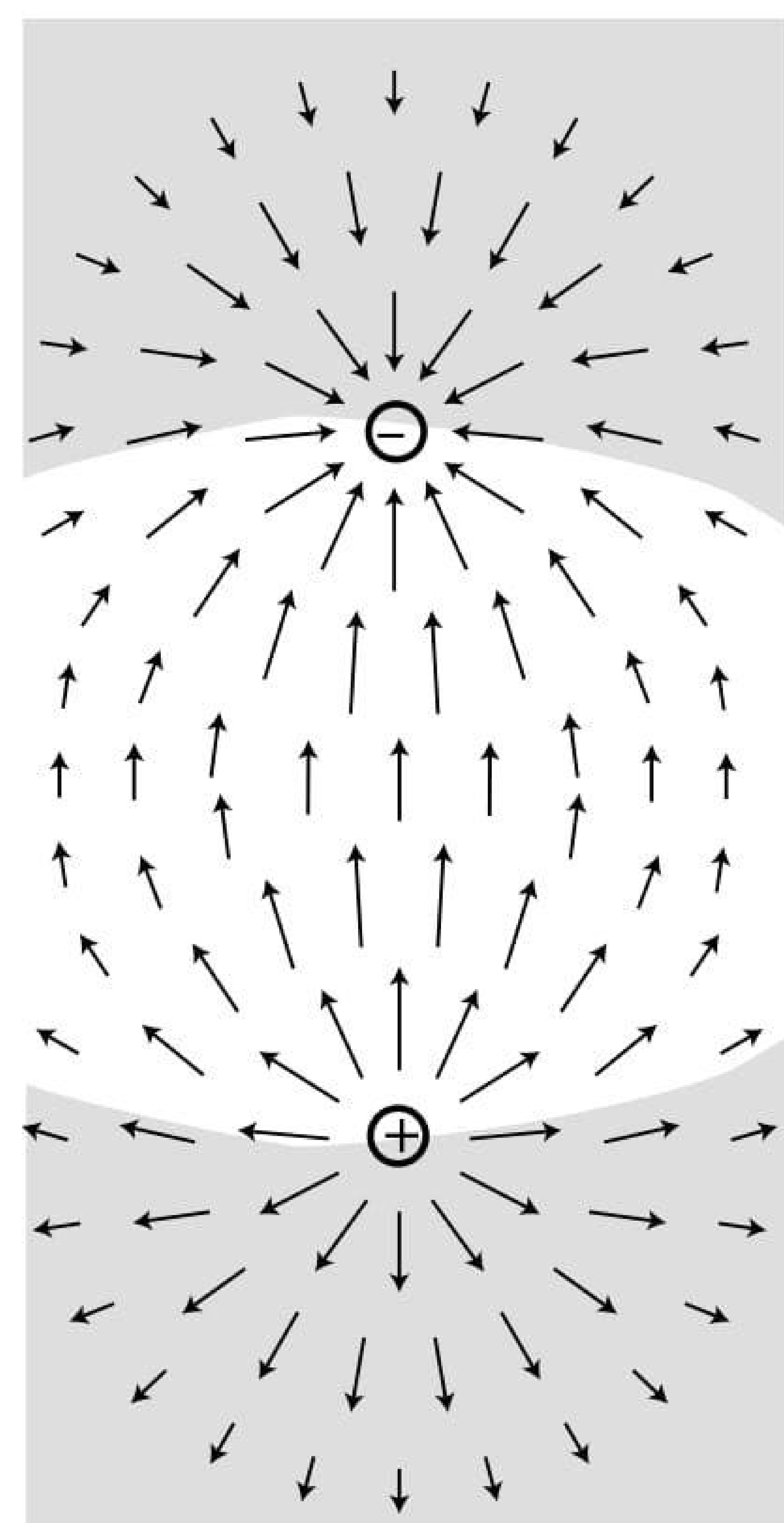


s / Example 7.

Energy stored in a capacitor *example 7*

A pair of parallel metal plates, seen from the side in figure s, can be used to store electrical energy by putting positive charge on one side and negative charge on the other. Such a device is called a capacitor. (We have encountered such an arrangement previously, but there its purpose was to deflect a beam of electrons, not to store energy.)

In the old method of describing potential energy, 1, we think in terms of the mechanical work that had to be done to separate the positive and negative charges onto the two plates, working against their electrical attraction. The new description, 2, attributes the storage of energy to the newly created electric field occupying the volume between the plates. Since this is a static case, both methods give the same, correct answer.


t / Example 8.

Potential energy of a pair of opposite charges *example 8*

Imagine taking two opposite charges, t, that were initially far apart and allowing them to come together under the influence of their electrical attraction.

According to the old method, potential energy is lost because the electric force did positive work as it brought the charges together. (This makes sense because as they come together and accelerate it is their potential energy that is being lost and converted to kinetic energy.)

By the new method, we must ask how the energy stored in the electric field has changed. In the region indicated approximately by the shading in the figure, the superposing fields of the two charges undergo partial cancellation because they are in opposing directions. The energy in the shaded region is reduced by this effect. In the unshaded region, the fields reinforce, and the energy is increased.

It would be quite a project to do an actual numerical calculation of the energy gained and lost in the two regions (this is a case where the old method of finding energy gives greater ease of computation), but it is fairly easy to convince oneself that the energy is less when the charges are closer. This is because bringing the charges together shrinks the high-energy unshaded region and enlarges the low-energy shaded region.

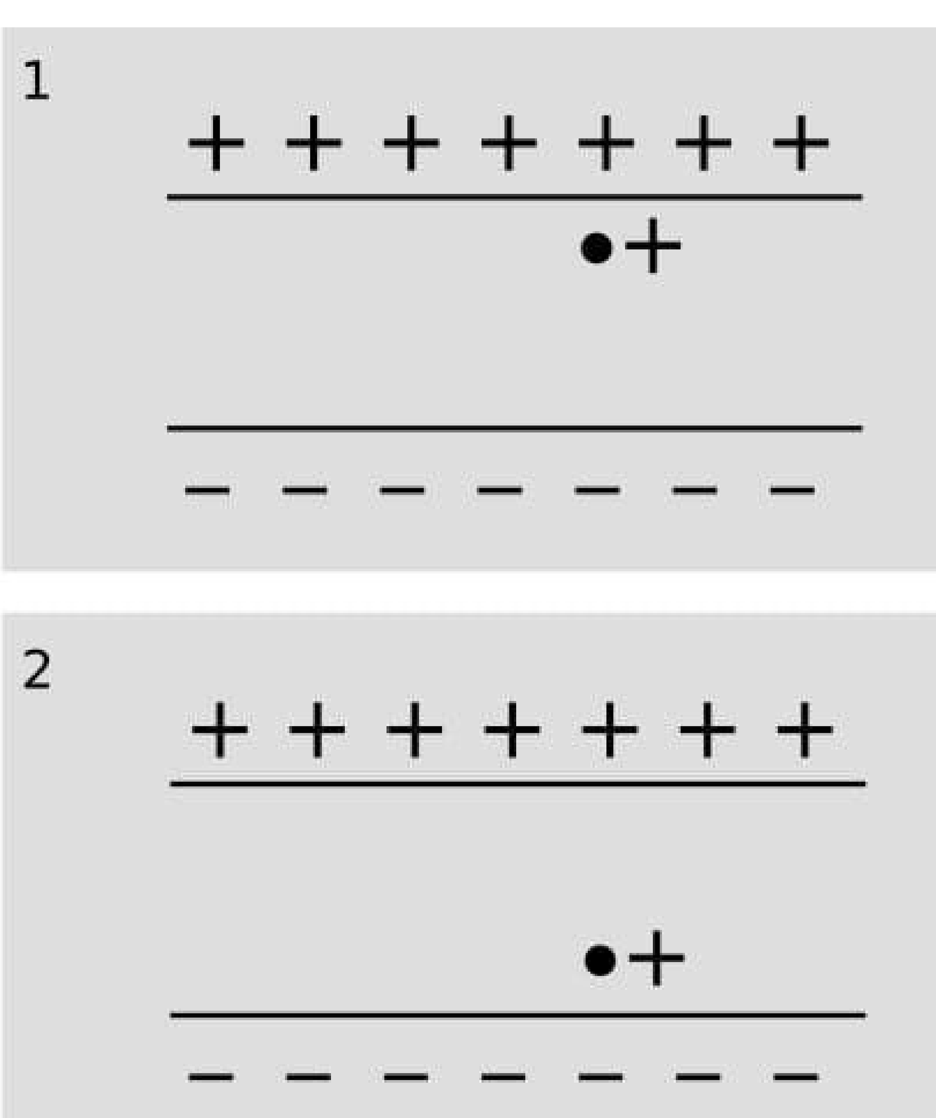
Energy in an electromagnetic wave — *example 9*

The old method would give zero energy for a region of space containing an electromagnetic wave but no charges. That would be wrong! We can only use the old method in static cases.

Now let's give at least some justification for the other features of the three expressions for energy density, $-\frac{1}{8\pi G}|\mathbf{g}|^2$, $\frac{1}{8\pi k}|\mathbf{E}^2|$, and $\frac{1}{2\mu_o}|\mathbf{B}|^2$, besides the proportionality to the square of the field strength.

First, why the different plus and minus signs? The basic idea is that the signs have to be opposite in the gravitational and electric cases because there is an attraction between two positive masses (which are the only kind that exist), but two positive charges would repel. Since we've already seen examples where the positive sign in the electric energy makes sense, the gravitational energy equation must be the one with the minus sign.

It may also seem strange that the constants G, k, and μ_o are in the denominator. They tell us how strong the three different forces are, so shouldn't they be on top? No. Consider, for instance, an alternative universe in which gravity is twice as strong as in ours. The numerical value of G is doubled. Because G is doubled, all the gravitational field strengths are doubled as well, which quadruples the quantity . In the expression $-\frac{1}{8\pi G}|\mathbf{g}|^2$, we have quadrupled something on top and doubled something on the bottom, which makes the energy twice as big. That makes perfect sense.



u / Discussion question A.

Discussion Questions

A The figure shows a positive charge in the gap between two capacitor plates. First make a large drawing of the field pattern that would be formed by the capacitor itself, without the extra charge in the middle. Next, show how the field pattern changes when you add the particle at these two positions. Compare the energy of the electric fields in the two cases. Does this agree with what you would have expected based on your knowledge of electrical forces?

B Criticize the following statement: "A solenoid makes a charge in the space surrounding it, which dissipates when you release the energy."

C In the example on the previous page, I argued that the fields surrounding a positive and negative charge contain less energy when the charges are closer together. Perhaps a simpler approach is to consider the two extreme possibilities: the case where the charges are infinitely far apart, and the one in which they are at zero distance from each other, i.e., right on top of each other. Carry out this reasoning for the case of (1) a positive charge and a negative charge of equal magnitude, (2) two positive charges of equal magnitude, (3) the gravitational energy of two equal masses.

6.6 ⋆ Symmetry and Handedness

The physicist Richard Feynman helped to get me hooked on physics with an educational film containing the following puzzle. Imagine that you establish radio contact with an alien on another planet. Neither of you even knows where the other one's planet is, and you aren't able to establish any landmarks that you both recognize. You manage to learn quite a bit of each other's languages, but you're stumped when you try to establish the definitions of left and right (or, equivalently, clockwise and counterclockwise). Is there any way to do it?

If there was any way to do it without reference to external landmarks, then it would imply that the laws of physics themselves were asymmetric, which would be strange. Why should they distinguish left from right? The gravitational field pattern surrounding a star or planet looks the same in a mirror, and the same goes for electric fields. However, the field patterns shown in section 6.2 seem to violate this principle, but do they really? Could you use these patterns to explain left and right to the alien? In fact, the answer is no. If you look back at the definition of the magnetic field in section 6.1, it also contains a reference to handedness: the counterclockwise direction of the loop's current as viewed along the magnetic field. The aliens might have reversed their definition of the magnetic field, in which case their drawings of field patterns would look like mirror images of ours.

Until the middle of the twentieth century, physicists assumed that any reasonable set of physical laws would have to have this kind of symmetry between left and right. An asymmetry would be grotesque. Whatever their aesthetic feelings, they had to change their opinions about reality when experiments showed that the weak nuclear force (section 6.5) violates right-left symmetry! It is still a mystery why right-left symmetry is observed so scrupulously in general, but is violated by one particular type of physical process.

Summary

Selected Vocabulary

magnetic field . .	a field of force, defined in terms of the torque exerted on a test dipole
magnetic dipole .	an object, such as a current loop, an atom, or a bar magnet, that experiences torques due to magnetic forces; the strength of magnetic dipoles is measured by comparison with a standard dipole consisting of a square loop of wire of a given size and carrying a given amount of current
induction	the production of an electric field by a changing magnetic field, or vice-versa

Notation

$\mathbf{B}$	the magnetic field
D_m	magnetic dipole moment

Summary

Magnetism is an interaction of moving charges with other moving charges. The magnetic field is defined in terms of the torque on a magnetic test dipole. It has no sources or sinks; magnetic field patterns never converge on or diverge from a point.

The magnetic and electric fields are intimately related. The principle of induction states that any changing electric field produces a magnetic field in the surrounding space, and vice-versa. These induced fields tend to form whirlpool patterns.

The most important consequence of the principle of induction is that there are no purely magnetic or purely electric waves. Disturbances in the electrical and magnetic fields propagate outward as combined magnetic and electric waves, with a well-defined relationship between their magnitudes and directions. These electromagnetic waves are what light is made of, but other forms of electromagnetic waves exist besides visible light, including radio waves, x-rays, and gamma rays.

Fields of force contain energy. The density of energy is proportional to the square of the magnitude of the field. In the case of static fields, we can calculate potential energy either using the previous definition in terms of mechanical work or by calculating the energy stored in the fields. If the fields are not static, the old method gives incorrect results and the new one must be used.

Problems

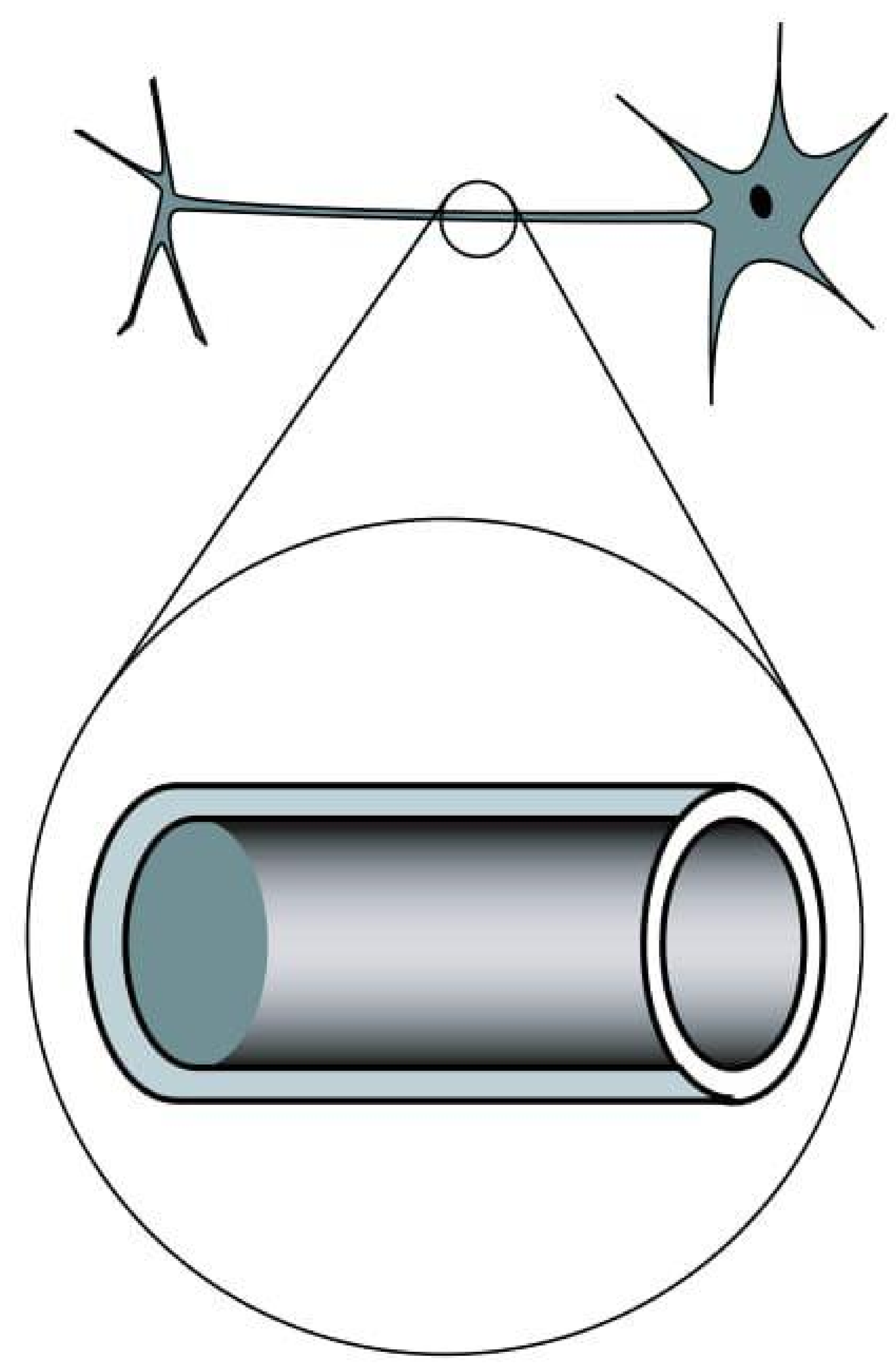
Key

- √ A computerized answer check is available online.
- ∫ A problem that requires calculus.
- ⋆ A difficult problem.

1 In an electrical storm, the cloud and the ground act like a parallel-plate capacitor, which typically charges up due to frictional electricity in collisions of ice particles in the cold upper atmosphere. Lightning occurs when the magnitude of the electric field builds up to a critical value, E_c, at which air is ionized.
(a) Treat the cloud as a flat square with sides of length L. If it is at a height h above the ground, find the amount of energy released in the lightning strike. √
(b) Based on your answer from part a, which is more dangerous, a lightning strike from a high-altitude cloud or a low-altitude one?
(c) Make an order-of-magnitude estimate of the energy released by a typical lightning bolt, assuming reasonable values for its size and altitude. E_c is about 10^6 V/m.

See problem 21 for a note on how recent research affects this estimate.

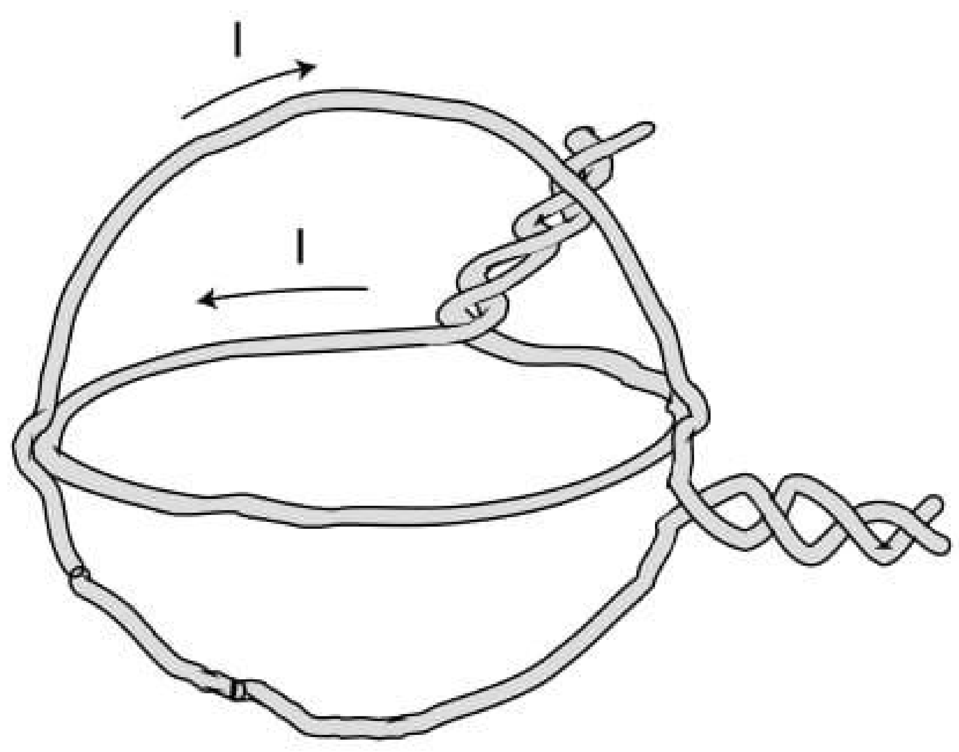
2 The neuron in the figure has been drawn fairly short, but some neurons in your spinal cord have tails (axons) up to a meter long. The inner and outer surfaces of the membrane act as the "plates" of a capacitor. (The fact that it has been rolled up into a cylinder has very little effect.) In order to function, the neuron must create a voltage difference V between the inner and outer surfaces of the membrane. Let the membrane's thickness, radius, and length be t, r, and L. (a) Calculate the energy that must be stored in the electric field for the neuron to do its job. (In real life, the membrane is made out of a substance called a dielectric, whose electrical properties increase the amount of energy that must be stored. For the sake of this analysis, ignore this fact.) [Hint: The volume of the membrane is essentially the same as if it was unrolled and flattened out.] √
(b) An organism's evolutionary fitness should be better if it needs less energy to operate its nervous system. Based on your answer to part a, what would you expect evolution to do to the dimensions t and r? What other constraints would keep these evolutionary trends from going too far?

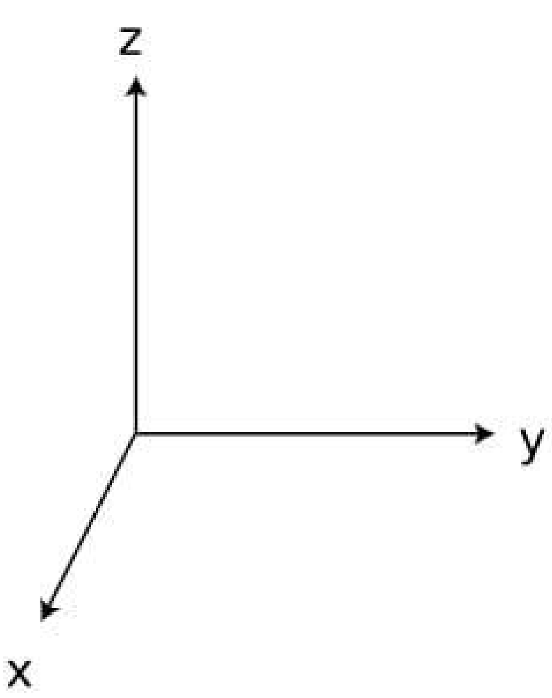


Problem 2.

3 Consider two solenoids, one of which is smaller so that it can be put inside the other. Assume they are long enough so that each one only contributes significantly to the field inside itself, and the interior fields are nearly uniform. Consider the configuration where the small one is inside the big one with their currents circulating in the same direction, and a second configuration in which the currents circulate in opposite directions. Compare the energies of these configurations with the energy when the solenoids are far apart. Based on this reasoning, which configuration is stable, and in which configuration will the little solenoid tend to get twisted around or spit out? [Hint: A stable system has low energy; energy would have to be added to change its configuration.]

4 The figure shows a nested pair of circular wire loops used to create magnetic fields. (The twisting of the leads is a practical trick for reducing the magnetic fields they contribute, so the fields are very nearly what we would expect for an ideal circular current loop.) The coordinate system below is to make it easier to discuss directions in space. One loop is in the $y-z$ plane, the other in the $x-y$ plane. Each of the loops has a radius of 1.0 cm, and carries 1.0 A in the direction indicated by the arrow.





Problem 4.

(a) Using the equation in optional section 6.2, calculate the magnetic field that would be produced by *one* such loop, at its center.

(b) Describe the direction of the magnetic field that would be produced, at its center, by the loop in the $x-y$ plane alone.

(c) Do the same for the other loop.

(d) Calculate the magnitude of the magnetic field produced by the two loops in combination, at their common center. Describe its direction. ✓ ✓

5 (a) Show that the quantity $\sqrt{4\pi k/\mu_o}$ has units of velocity.

(b) Calculate it numerically and show that it equals the speed of light.

(c) Prove that in an electromagnetic wave, half the energy is in the electric field and half in the magnetic field.

6 One model of the hydrogen atom has the electron circling around the proton at a speed of 2.2×10^6 m/s, in an orbit with a radius of 0.05 nm. (Although the electron and proton really orbit around their common center of mass, the center of mass is very close to the proton, since it is 2000 times more massive. For this problem, assume the proton is stationary.) In that previous homework problem, you calculated the electric current created.
(a) Now estimate the magnetic field created at the center of the atom by the electron. We are treating the circling electron as a current loop, even though it's only a single particle. ✓
(b) Does the proton experience a nonzero force from the electron's magnetic field? Explain.
(c) Does the electron experience a magnetic field from the proton? Explain.
(d) Does the electron experience a magnetic field created by its own current? Explain.
(e) Is there an electric force acting between the proton and electron? If so, calculate it. ✓
(f) Is there a gravitational force acting between the proton and elec-

tron? If so, calculate it.
(g) An inward force is required to keep the electron in its orbit – otherwise it would obey Newton's first law and go straight, leaving the atom. Based on your answers to the previous parts, which force or forces (electric, magnetic and gravitational) contributes significantly to this inward force?

7 [You need to have read optional section 6.2 to do this problem.] Suppose a charged particle is moving through a region of space in which there is an electric field perpendicular to its velocity vector, and also a magnetic field perpendicular to both the particle's velocity vector and the electric field. Show that there will be one particular velocity at which the particle can be moving that results in a total force of zero on it. Relate this velocity to the magnitudes of the electric and magnetic fields. (Such an arrangement, called a velocity filter, is one way of determining the speed of an unknown particle.)

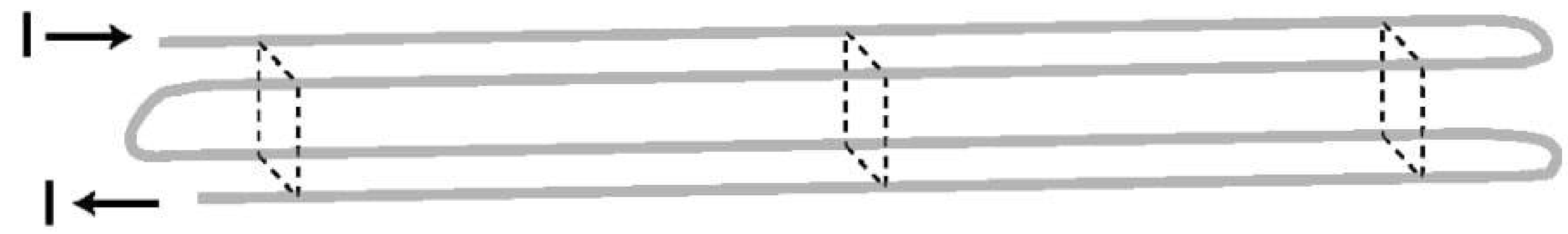
8 If you put four times more current through a solenoid, how many times more energy is stored in its magnetic field?

9 Suppose we are given a permanent magnet with a complicated, asymmetric shape. Describe how a series of measurements with a magnetic compass could be used to determine the strength and direction of its magnetic field at some point of interest. Assume that you are only able to see the direction to which the compass needle settles; you cannot measure the torque acting on it. ⋆

10 Consider two solenoids, one of which is smaller so that it can be put inside the other. Assume they are long enough to act like ideal solenoids, so that each one only contributes significantly to the field inside itself, and the interior fields are nearly uniform. Consider the configuration where the small one is partly inside and partly hanging out of the big one, with their currents circulating in the same direction. Their axes are constrained to coincide.

(a) Find the magnetic potential energy as a function of the length x of the part of the small solenoid that is inside the big one. (Your equation will include other relevant variables describing the two solenoids.)

(b) Based on your answer to part (a), find the force acting between the solenoids.



Problem 11.

11 Four long wires are arranged, as shown, so that their cross-section forms a square, with connections at the ends so that current flows through all four before exiting.

Note that the current is to the right in the two back wires, but to the left in the front wires. If the dimensions of the cross-sectional square (height and front-to-back) are b, find the magnetic field (magnitude and direction) along the long central axis.

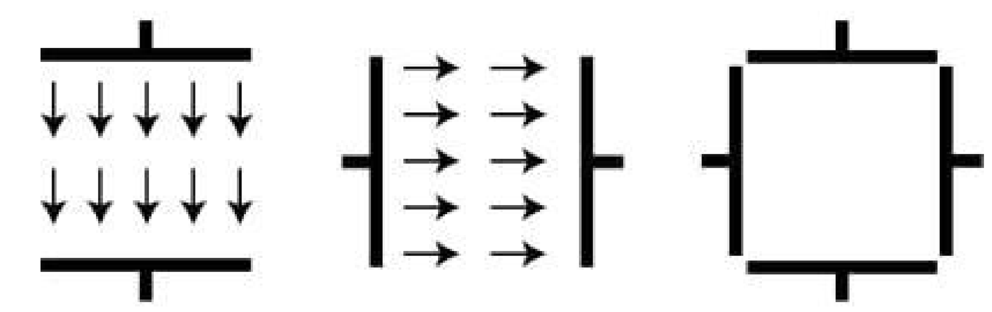
12 To do this problem, you need to understand how to do volume integrals in cylindrical and spherical coordinates. (a) Show that if you try to integrate the energy stored in the field of a long, straight wire, the resulting energy per unit length diverges both at $r \rightarrow 0$ and $r \rightarrow \infty$. Taken at face value, this would imply that a certain real-life process, the initiation of a current in a wire, would be impossible, because it would require changing from a state of zero magnetic energy to a state of infinite magnetic energy. (b) Explain why the infinities at $r \rightarrow 0$ and $r \rightarrow \infty$ don't really happen in a realistic situation. (c) Show that the electric energy of a point charge diverges at $r \rightarrow 0$, but not at $r \rightarrow \infty$.

A remark regarding part (c): Nature does seem to supply us with particles that are charged and pointlike, e.g., the electron, but one could argue that the infinite energy is not really a problem, because an electron traveling around and doing things neither gains nor loses infinite energy; only an infinite *change* in potential energy would be physically troublesome. However, there are real-life processes that create and destroy pointlike charged particles, e.g., the annihilation of an electron and antielectron with the emission of two gamma rays. Physicists have, in fact, been struggling with infinities like this since about 1950, and the issue is far from resolved. Some theorists propose that apparently pointlike particles are actually not pointlike: close up, an electron might be like a little circular loop of string. $\int$ ⋆

13 The purpose of this problem is to find the force experienced by a straight, current-carrying wire running perpendicular to a uniform magnetic field. (a) Let A be the cross-sectional area of the wire, n the number of free charged particles per unit volume, q the charge per particle, and v the average velocity of the particles. Show that the current is $I = Avnq$. (b) Show that the magnetic force per unit length is $AvnqB$. (c) Combining these results, show that the force on the wire per unit length is equal to IB. ▷ Solution, p. 199

14 Suppose two long, parallel wires are carrying current I_1 and I_2. The currents may be either in the same direction or in opposite directions. (a) Using the information from section 6.2, determine under what conditions the force is attractive, and under what conditions it is repulsive. Note that, because of the difficulties explored in problem 12, it's possible to get yourself tied up in knots if you use the energy approach of section 6.5. (b) Starting from the result of problem 13, calculate the force per unit length. ▷ Solution, p. 199

15 The figure shows cross-sectional views of two cubical capacitors, and a cross-sectional view of the same two capacitors put together so that their interiors coincide. A capacitor with the plates close together has a nearly uniform electric field between the plates, and almost zero field outside; these capacitors don't have their plates very close together compared to the dimensions of the plates, but for the purposes of this problem, assume that they still have approximately the kind of idealized field pattern shown in the figure. Each capacitor has an interior volume of 1.00 m^3, and is charged up to the point where its internal field is 1.00 V/m. (a) Calculate the energy stored in the electric field of each capacitor when they are separate. (b) Calculate the magnitude of the interior field when the two capacitors are put together in the manner shown. Ignore effects arising from the redistribution of each capacitor's charge under the influence of the other capacitor. (c) Calculate the energy of the put-together configuration. Does assembling them like this release energy, consume energy, or neither?



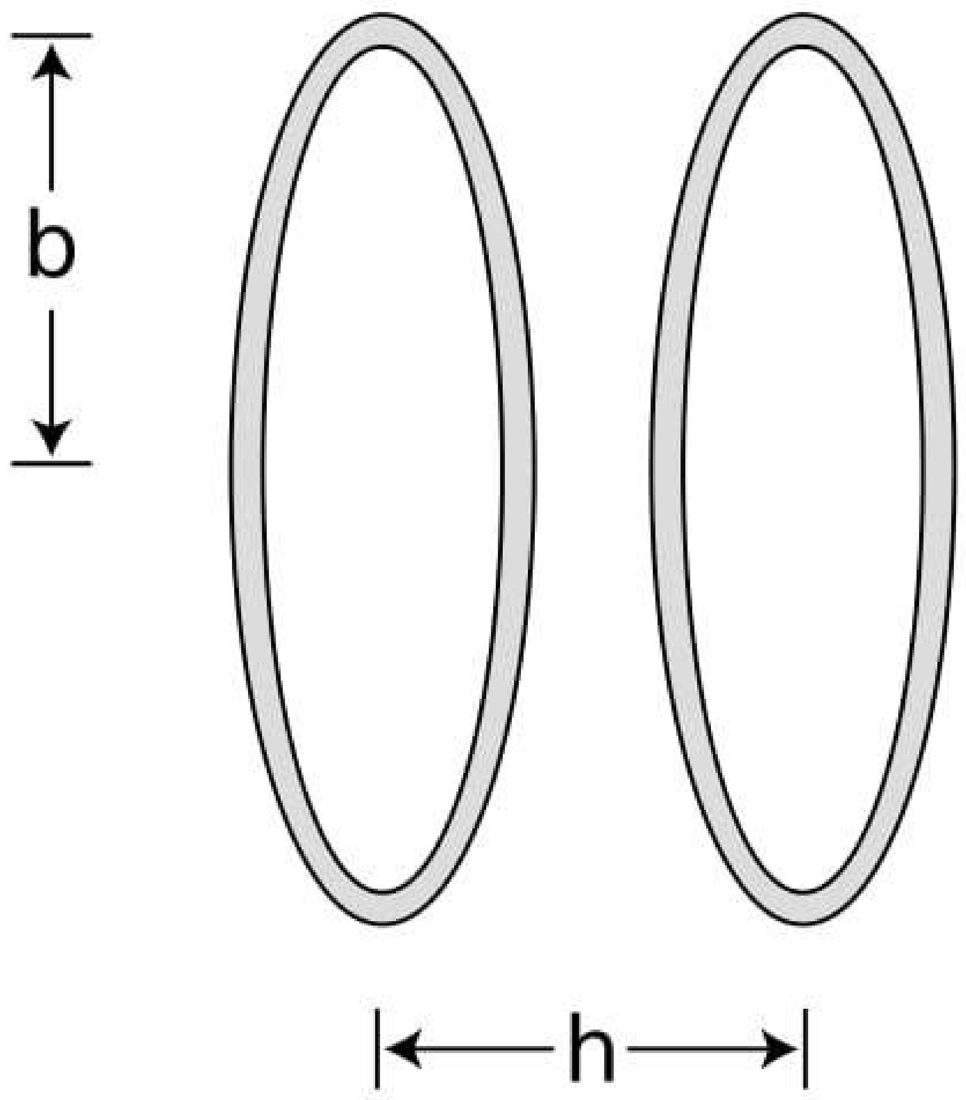
Problem 15.

16 Section 6.2 states the following rule:

For a positively charged particle, the direction of the F vector is the one such that if you sight along it, the **B** vector is clockwise from the v vector.

Make a three-dimensional model of the three vectors using pencils or rolled-up pieces of paper to represent the vectors assembled with their tails together. Now write down every possible way in which the rule could be rewritten by scrambling up the three symbols F, **B**, and v. Referring to your model, which are correct and which are incorrect?

17 Prove that any two planar current loops with the same value of IA will experience the same torque in a magnetic field, regardless of their shapes. In other words, the dipole moment of a current loop can be defined as IA, regardless of whether its shape is a square. ⋆



Problem 18.

18 A Helmholtz coil is defined as a pair of identical circular coils separated by a distance, h, equal to their radius, b. (Each coil may have more than one turn of wire.) Current circulates in the same direction in each coil, so the fields tend to reinforce each other in the interior region. This configuration has the advantage of being fairly open, so that other apparatus can be easily placed inside and subjected to the field while remaining visible from the outside. The choice of $h = b$ results in the most uniform possible field near the center. (a) Find the percentage drop in the field at the center of one coil, compared to the full strength at the center of the whole apparatus. (b) What value of h (not equal to b) would make this percentage difference equal to zero?

19 (a) In the photo of the vacuum tube apparatus in section 6.2, infer the direction of the magnetic field from the motion of the electron beam. (b) Based on your answer to a, find the direction of the currents in the coils. (c) What direction are the electrons in the coils going? (d) Are the currents in the coils repelling or attracting the currents consisting of the beam inside the tube? Compare with part a of problem 14.

20 In the photo of the vacuum tube apparatus in section 6.2, an approximately uniform magnetic field caused circular motion. Is there any other possibility besides a circle? What can happen in general? ⋆

21 In problem 1, you estimated the energy released in a bolt of lightning, based on the energy stored in the electric field immediately before the lightning occurs. The assumption was that the field would build up to a certain value, which is what is necessary to ionize air. However, real-life measurements always seemed to show electric fields strengths roughtly 10 times smaller than those required in that model. For a long time, it wasn't clear whether the field measurements were wrong, or the model was wrong. Research carried out in 2003 seems to show that the model was wrong. It is now believed that the final triggering of the bolt of lightning comes from cosmic rays that enter the atmosphere and ionize some of the air. If the field is 10 times smaller than the value assumed in problem 1, what effect does this have on the final result of problem 1?

22 In section 6.2 I gave an equation for the magnetic field in the interior of a solenoid, but that equation doesn't give the right answer near the mouths or on the outside. Although in general the computation of the field in these other regions is complicated, it is possible to find a precise, simple result for the field at the center of one of the mouths, using only symmetry and vector addition. What is it? ▷ Solution, p. 200 ⋆