Contents

CHAPTER

15

Levers

1. Introduction.
2. Application of Levers in Engineering Practice.
3. Design of a Lever.
4. Hand Lever.
5. Foot Lever.
6. Cranked Lever.
7. Lever for a Lever Safety Valve.
8. Bell Crank Lever.
9. Rocker Arm for Exhaust Valve.
10. Miscellaneous Levers.

15.1 Introduction

A lever is a rigid rod or bar capable of turning about a fixed point called ***fulcrum***. It is used as a machine to lift a load by the application of a small effort. The ratio of load lifted to the effort applied is called ***mechanical advantage***. Sometimes, a lever is merely used to facilitate the application of force in a desired direction. A lever may be ***straight*** or ***curved*** and the forces applied on the lever (or by the lever) may be parallel or inclined to one another. The principle on which the lever works is same as that of moments.

Consider a straight lever with parallel forces acting in the same plane as shown in Fig 15.1. The points *A* and *B* through which the load and effort is applied are known as load and effort points respectively. *F* is the fulcrum about which the lever is capable of turning. The perpendicular distance between the load point and fulcrum (l_1) is known as ***load arm*** and the perpendicular distance between the

Top

Contents

effort point and fulcrum (l_2) is called ***effort arm***. According to the principle of moments,

$$W \times l_1 = P \times l_2 \quad \text{or} \quad \frac{W}{P} = \frac{l_2}{l_1}$$

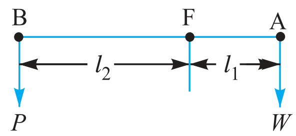


Fig. 15.1. Straight lever.

i.e. Mechanical advantage,

$$M.A. = \frac{W}{P} = \frac{l_2}{l_1}$$

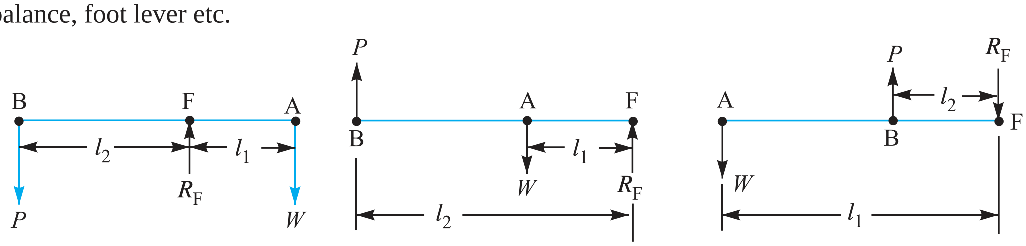
The ratio of the effort arm to the load arm *i.e.* l_2 / l_1 is called ***leverage***.

A little consideration will show that if a large load is to be lifted by a small effort, then the effort arm should be much greater than the load arm. In some cases, it may not be possible to provide a lever with large effort arm due to space limitations. Therefore in order to obtain a great leverage, ***compound levers*** may be used. The compound levers may be made of straight pieces, which may be attached to one another with pin joints. The bell cranked levers may be used instead of a number of jointed levers. In a compound lever, the leverage is the product of leverages of various levers.

15.2 Application of Levers in Engineering Practice

The load *W* and the effort *P* may be applied to the lever in three different ways as shown in Fig. 15.2. The levers shown at (*a*), (*b*) and (*c*) in Fig. 15.2 are called ***first type***, ***second type*** and ***third type*** of levers respectively.

In the ***first type*** of levers, the fulcrum is in between the load and effort. In this case, the effort arm is greater than load arm, therefore mechanical advantage obtained is more than one. Such type of levers are commonly found in bell cranked levers used in railway signalling arrangement, rocker arm in internal combustion engines, handle of a hand pump, hand wheel of a punching press, beam of a balance, foot lever etc.



(*a*) First type of lever. (*b*) Second type of lever. (*c*) Third type of lever.

Fig. 15.2. Type of levers.

In the ***second type*** of levers, the load is in between the fulcrum and effort. In this case, the effort arm is more than load arm, therefore the mechanical advantage is more than one. The application of such type of levers is found in levers of loaded safety valves.

In the ***third type*** of levers, the effort is in between the fulcrum and load. Since the effort arm, in this case, is less than the load arm, therefore the mechanical advantage is less that one. The use of such type of levers is not recommended in engineering practice. However a pair of tongs, the treadle of a sewing machine etc. are examples of this type of lever.

15.3 Design of a Lever

The design of a lever consists in determining the physical dimensions of a lever when forces acting on the lever are given. The forces acting on the lever are

1. Load (*W*), **2.** Effort (*P*), and **3.** Reaction at the fulcrum *F* (R_F).

Top

The load and effort cause moments in opposite directions about the fulcrum.

The following procedure is usually adopted in the design of a lever :

1. Generally the load W is given. Find the value of the effort (P) required to resist this load by taking moments about the fulcrum. When the load arm is equal to the effort arm, the effort required will be equal to the load provided the friction at bearings is neglected.

2. Find the reaction at the fulcrum (R_F), as discussed below :

(i) When W and P are parallel and their direction is same as shown in Fig. 15.2 (*a*), then

$$R_F = W + P$$

The direction of R_F will be opposite to that of W and P.

(ii) When W and P are parallel and acts in opposite directions as shown in Fig. 15.2 (*b*) and (*c*), then R_F will be the difference of W and P. For load positions as shown in Fig. 15.2 (*b*),

$$R_F = W - P$$

and for load positions as shown in Fig. 15.2 (*c*),

$$R_F = P - W$$

The direction of R_F will be opposite to that of W or P whichever is greater.

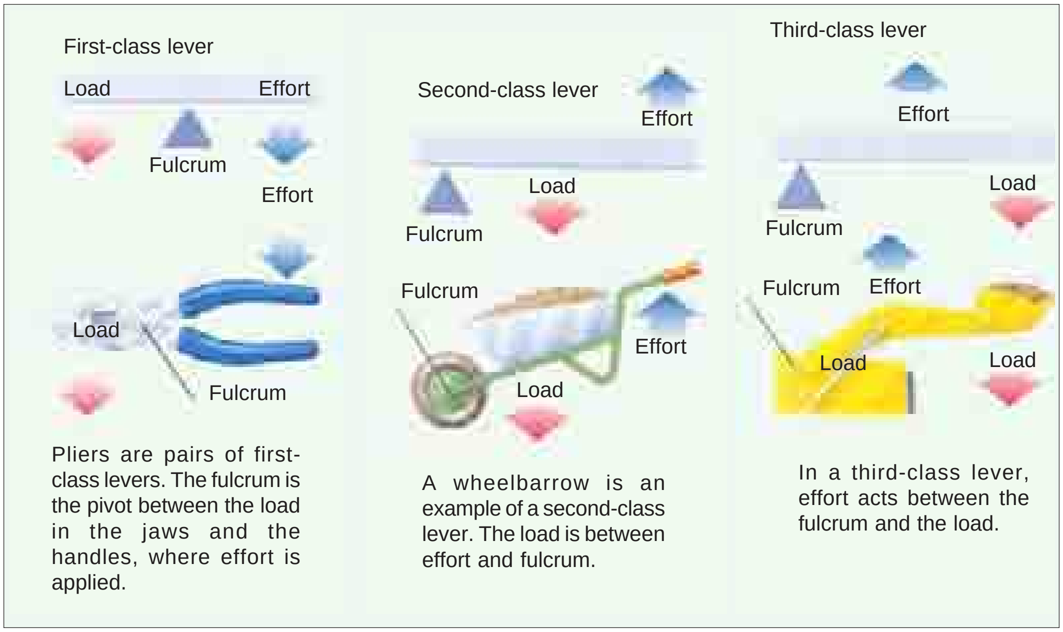
(iii) When W and P are inclined to each other as shown in Fig. 15.3 (*a*), then R_F, which is equal to the resultant of W and P, is determined by parallelogram law of forces. The line of action of R_F passes through the intersection of W and P and also through F. The direction of R_F depends upon the direction of W and P.

(iv) When W and P acts at right angles and the arms are inclined at an angle θ as shown in Fig. 15.3 (*b*), then R_F is determined by using the following relation :

$$R_F = \sqrt{W^2 + P^2 - 2W \times P \cos \theta}$$

In case the arms are at right angles as shown in Fig. 15.3 (*c*), then

$$R_F = \sqrt{W^2 + P^2}$$



There are three classes of levers.

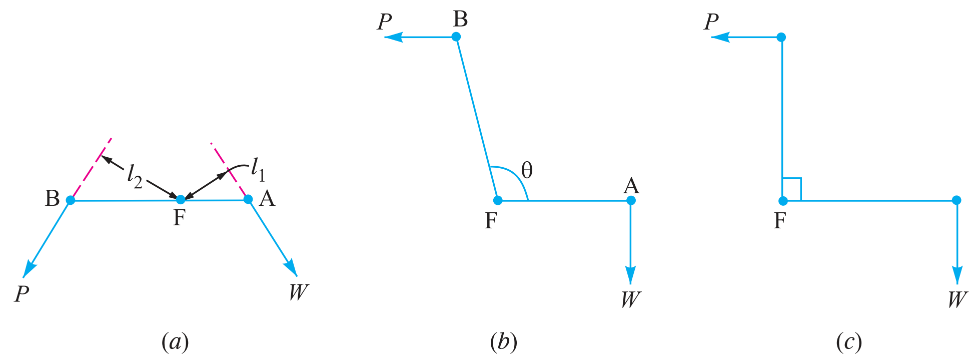


Fig. 15.3

3. Knowing the forces acting on the lever, the cross-section of the arm may be determined by considering the section of the lever at which the maximum bending moment occurs. In case of levers having two arms as shown in Fig. 15.4 (*a*) and cranked levers, the maximum bending moment occurs at the boss. The cross-section of the arm may be rectangular, elliptical or *I*-section as shown in Fig. 15.4 (*b*). We know that section modulus for rectangular section,

$$Z = \frac{1}{6} \times t \times h^2$$

where t = Breadth or thickness of the lever, and

h = Depth or height of the lever.

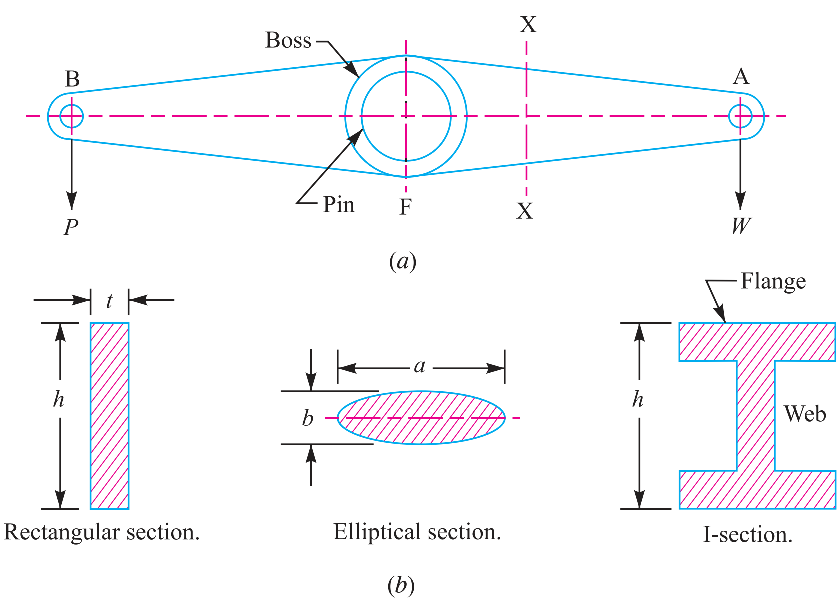


Fig. 15.4. Cross-sections of lever arm (Section at *X-X*).

The height of the lever is usually taken as 2 to 5 times the thickness of the lever.

For elliptical section, section modulus,

$$Z = \frac{\pi}{32} \times b \times a^2$$

where a = Major axis, and b = Minor axis.

The major axis is usually taken as 2 to 2.5 times the minor axis.

Contents

For *I*-section, it is assumed that the bending moment is taken by flanges only. With this assumption, the section modulus is given by

$$Z = \text{Flange area} \times \text{depth of section}$$

The section of the arm is usually tapered from the fulcrum to the ends. The dimensions of the arm at the ends depends upon the manner in which the load is applied. If the load at the end is applied by forked connections, then the dimensions of the lever at the end can be proportioned as a knuckle joint.

4. The dimensions of the fulcrum pin are obtained from bearing considerations and then checked for shear. The allowable bearing pressure depends upon the amount of relative motion between the pin and the lever. The length of pin is usually taken from 1 to 1.25 times the diameter of pin. If the forces on the lever do not differ much, the diameter of the pins at load and effort point shall be taken equal to the diameter of the fulcrum pin so that the spares are reduced. Instead of choosing a thick lever, the pins are provided with a boss in order to provide sufficient bearing length.

5. The diameter of the boss is taken twice the diameter of pin and length of the boss equal to the length of pin. The boss is usually provided with a 3 mm thick phosphor bronze bush with a dust proof lubricating arrangement in order to reduce wear and to increase the life of lever.

Example 15.1. *A handle for turning the spindle of a large valve is shown in Fig. 15.5. The length of the handle from the centre of the spindle is 450 mm. The handle is attached to the spindle by means of a round tapered pin.*

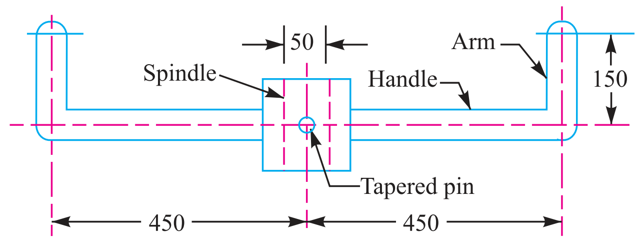


Fig. 15.5

If an effort of 400 N is applied at the end of the handle, find: 1. mean diameter of the tapered pin, and 2. diameter of the handle.

The allowable stresses for the handle and pin are 100 MPa in tension and 55 MPa in shear.

Solution. Given : $L = 450$ mm ; $P = 400$ N ; $\sigma_t = 100$ MPa $= 100$ N/mm^2 ; $\tau = 55$ MPa$= 55$ N/mm^2

1. Mean diameter of the tapered pin

Let d_1 = Mean diameter of the tapered pin, and

d = Diameter of the spindle = 50 mm ...(Given)

We know that the torque acting on the spindle,

$$T = P \times 2L = 400 \times 2 \times 450 = 360 \times 10^3 \text{ N-mm} \quad ...(i)$$

Since the pin is in double shear and resists the same torque as that on the spindle, therefore resisting torque,

$$T = 2 \times \frac{\pi}{4}(d_1)^2 \,\tau \times \frac{d}{2} = 2 \times \frac{\pi}{4}(d_1)^2\, 55 \times \frac{50}{2} \text{ N-mm}$$

$$= 2160\,(d_1)^2 \text{ N-mm} \quad ...(ii)$$

From equations ***(i)*** and ***(ii)***, we get

$$(d_1)^2 = 360 \times 10^3 / 2160 = 166.7 \text{ or } d_1 = 12.9 \text{ say } 13 \text{ mm}$$ **Ans.**

Top

2. *Diameter of the handle*

Let D = Diameter of the handle.

Since the handle is subjected to both bending moment and twisting moment, therefore the design will be based on either equivalent twisting moment or equivalent bending moment. We know that bending moment,

$$M = P \times L = 400 \times 450 = 180 \times 10^3 \text{ N-mm}$$

The twisting moment depends upon the point of application of the effort. Assuming that the effort acts at a distance 100 mm from the end of the handle, we have twisting moment,

$$T = 400 \times 100 = 40 \times 10^3 \text{ N-mm}$$

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(180 \times 10^3)^2 + (40 \times 10^3)^2} = 184.4 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$184.4 \times 10^3 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 55 \times D^3 = 10.8\, D^3$$

$\therefore$ $D^3 = 184.4 \times 10^3 / 10.8 = 17.1 \times 10^3$ or $D = 25.7$ mm

Again we know that equivalent bending moment,

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(180 \times 10^3 + 184.4 \times 10^3) = 182.2 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$182.2 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times D^3 = \frac{\pi}{32} \times 100 \times D^3 = 9.82\, D^3 \quad \ldots(\because \sigma_b = \sigma_t)$$

$\therefore$ $D^3 = 182.2 \times 10^3 / 9.82 = 18.6 \times 10^3$ or $D = 26.5$ mm

Taking larger of the two values, we have

$D = 26.5$ mm **Ans.**

Example 15.2. *A vertical lever PQR, 15 mm thick is attached by a fulcrum pin at R and to a horizontal rod at Q, as shown in Fig. 15.6.*

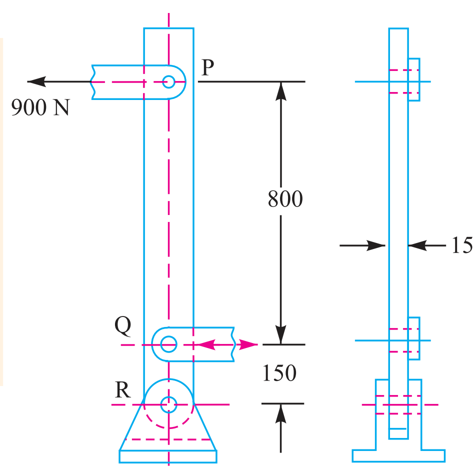
An operating force of 900 N is applied horizontally at P. Find :

1. Reactions at Q and R,

2. Tensile stress in 12 mm diameter tie rod at Q

3. Shear stress in 12 mm diameter pins at P, Q and R, and

4. Bearing stress on the lever at Q.



All dimensions in mm.

Fig. 15.6

Solution. Given : $t = 15$ mm ; $F_P = 900$ N

1. *Reactions at Q and R*

Let R_Q = Reaction at Q, and

R_R = Reaction at R,

Taking moments about R, we have

$$R_Q \times 150 = 900 \times 950 = 855\,000$$

$\therefore$ $R_Q = 855\,000 / 150 = 5700$ N **Ans.**

These levers are used to change railway tracks.

Since the forces at P and Q are parallel and opposite as shown in Fig. 15.7, therefore reaction at R,

$$R_R = R_Q - 900 = 5700 - 900 = 4800 \text{ N} \quad \textbf{Ans.}$$

2. *Tensile stress in the tie rod at Q*

Let $\quad d_t$ = Diameter of tie rod = 12 mm ...(Given)

$\therefore$ Area, $A_t = \frac{\pi}{4}(12)^2 = 113 \text{ mm}^2$

We know that tensile stress in the tie rod,

$$\sigma_t = \frac{\text{Force at } Q\ (R_Q)}{\text{Cross-sectional area } (A_t)} = \frac{5700}{113}$$
$$= 50.4 \text{ N/mm}^2 = 50.4 \text{ MPa} \quad \textbf{Ans.}$$

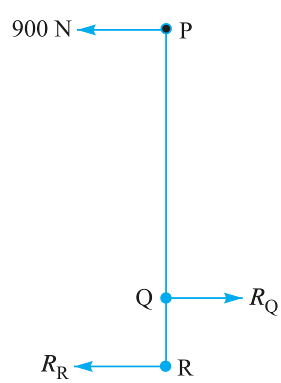


Fig. 15.7

3. *Shear stress in pins at P, Q and R*

Given : Diameter of pins at P, Q and R,

$$d_P = d_Q = d_R = 12 \text{ mm}$$

$\therefore$ Cross-sectional area of pins at P, Q and R,

$$A_P = A_Q = A_R = \frac{\pi}{4}(12)^2 = 113 \text{ mm}^2$$

Since the pin at P is in single shear and pins at Q and R are in double shear, therefore shear stress in pin at P,

$$\tau_P = \frac{F_P}{A_P} = \frac{900}{113} = 7.96 \text{ N/mm}^2 = 7.96 \text{ MPa} \quad \textbf{Ans.}$$

Shear stress in pin at Q,

$$\tau_Q = \frac{R_Q}{2A_Q} = \frac{5700}{2 \times 113} = 25.2 \text{ N/mm}^2 = 25.2 \text{ MPa} \quad \textbf{Ans.}$$

and shear stress in pin at R,

$$\tau_R = \frac{R_R}{2A_R} = \frac{4800}{2 \times 113} = 21.2 \text{ N/mm}^2 = 21.2 \text{ MPa} \quad \textbf{Ans.}$$

4. *Bearing stress on the lever at Q*

Bearing area of the lever at the pin Q,

$$A_b = \text{Thickness of lever} \times \text{Diameter of pin} = 15 \times 12 = 180 \text{ mm}^2$$

Contents

∴ Bearing stress on the lever at Q,

$$\sigma_b = \frac{R_Q}{A_b} = \frac{5700}{180} = 31.7 \text{ N/mm}^2 = 31.7 \text{ MPa} \quad \textbf{Ans.}$$

15.4 Hand Levers

A hand lever with suitable dimensions and proportions is shown in Fig. 15.8.

Let P = Force applied at the handle,

L = Effective length of the lever,

σ_t = Permissible tensile stress, and

τ = Permissible shear stress.

For wrought iron, σ_t may be taken as 70 MPa and τ as 60 MPa.

In designing hand levers, the following procedure may be followed :

1. The diameter of the shaft (d) is obtained by considering the shaft under pure torsion. We know that twisting moment on the shaft,

$$T = P \times L$$

and resisting torque,

$$T = \frac{\pi}{16} \times \tau \times d^3$$

From this relation, the diameter of the shaft (d) may be obtained.

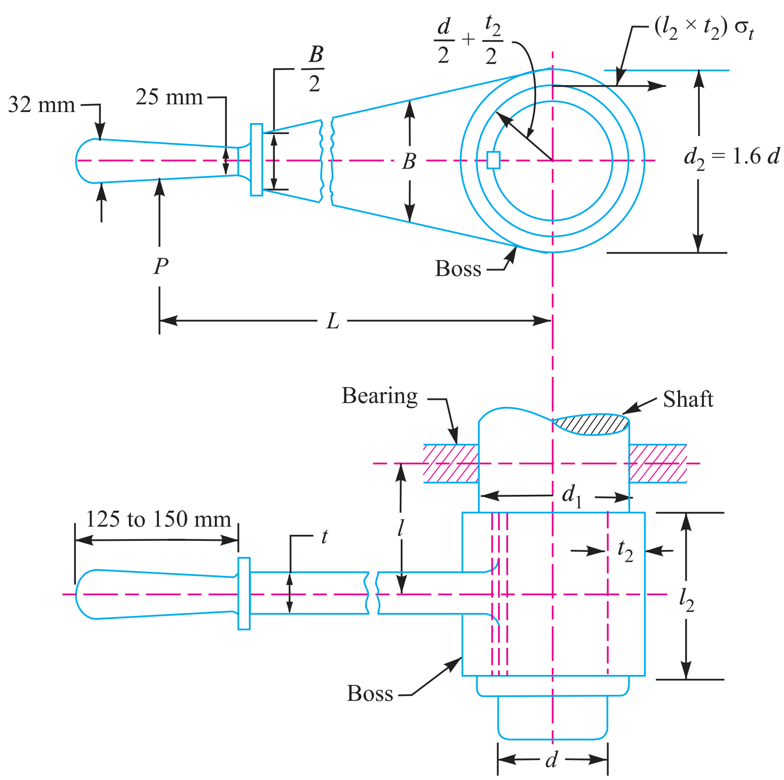


Fig. 15.8. Hand lever.

2. The diameter of the boss (d_2) is taken as 1.6 d and thickness of the boss (t_2) as 0.3 d.

3. The length of the boss (l_2) may be taken from d to 1.25 d. It may be checked for a trial thickness t_2 by taking moments about the axis. Equating the twisting moment ($P \times L$) to the moment

Top

Contents

of resistance to tearing parallel to the axis, we get

$$P \times L = l_2\, t_2\, \sigma_t \left(\frac{d + t_2}{2}\right) \quad \text{or} \quad l_2 = \frac{2\,P \times L}{t_2\, \sigma_t\, (d + t_2)}$$

4. The diameter of the shaft at the centre of the bearing (d_1) is obtained by considering the shaft in combined bending and twisting.

We know that bending moment on the shaft,

$$M = P \times l$$

and twisting moment, $T = P \times L$

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(P \times l)^2 + (P \times L)^2} = P\sqrt{l^2 + L^2}$$

We also know that equivalent twisting moment,

$$T_e = \frac{\pi}{16} \times \tau\,(d_1)^3 \quad \text{or} \quad P\sqrt{l^2 + L^2} = \frac{\pi}{16} \times \tau\,(d_1)^3$$

The length l may be taken as $2\,l_2$.

From the above expression, the value of d_1 may be determined.

5. The key for the shaft is designed as usual for transmitting a torque of $P \times L$.

6. The cross-section of the lever near the boss may be determined by considering the lever in bending. It is assumed that the lever extends to the centre of the shaft which results in a stronger section of the lever.

Let t = Thickness of lever near the boss, and

B = Width or height of lever near the boss.

We know that the bending moment on the lever,

$$M = P \times L$$

Section modulus, $Z = \dfrac{1}{6} \times t \times B^2$

We know that the bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{P \times L}{\frac{1}{6} \times t \times B^2} = \frac{6\,P \times L}{t \times B^2}$$

The width of the lever near the boss may be taken from 4 to 5 times the thickness of lever, *i.e.* $B = 4\,t$ to $5\,t$. The width of the lever is tapered but the thickness (t) is kept constant. The width of the lever near the handle is $B/2$.

Note: For hand levers, about 400 N is considered as full force which a man is capable of exerting. About 100 N is the mean force which a man can exert on the working handle of a machine, off and on for a full working day.

15.5 Foot Lever

A foot lever, as shown in Fig. 15.9, is similar to hand lever but in this case a foot plate is provided instead of handle. The foot lever may be designed in a similar way as discussed for hand lever. For foot levers, about 800 N is considered as full force which a man can exert in pushing a foot lever. The proportions of the foot plate are shown in Fig. 15.9.

Example 15.3. *A foot lever is 1 m from the centre of shaft to the point of application of 800 N load. Find :*

1. Diameter of the shaft, 2. Dimensions of the key, and 3. Dimensions of rectangular arm of the foot lever at 60 mm from the centre of shaft assuming width of the arm as 3 times thickness.

The allowable tensile stress may be taken as 73 MPa and allowable shear stress as 70 MPa.

Top

Solution. Given : L = 1 m = 1000 mm ; P = 800 N ; σ_t = 73 MPa = 73 N/mm^2 ; τ = 70 MPa = 70 N/mm^2

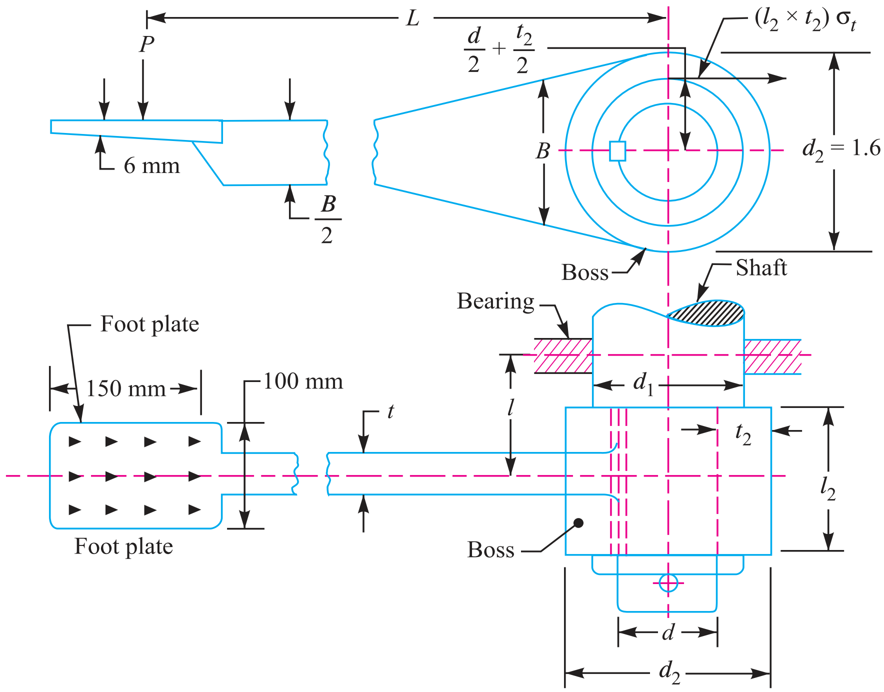


Fig. 15.9. Foot lever.

1. *Diameter of the shaft*

Let d = Diameter of the shaft.

We know that the twisting moment on the shaft,

$$T = P \times L = 800 \times 1000 = 800 \times 10^3 \text{ N-mm}$$

We also know that the twisting moment on the shaft (T),

$$800 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\, d^3$$

$$\therefore \quad d^3 = 800 \times 10^3 / 13.75 = 58.2 \times 10^3$$

or $\quad d$ = 38.8 say 40 mm **Ans.**

We know that diameter of the boss,

$$d_2 = 1.6\, d = 1.6 \times 40 = 64 \text{ mm}$$

Thickness of the boss,

$$t_2 = 0.3\, d = 0.3 \times 40 = 12 \text{ mm}$$

and length of the boss, $\quad l_2 = 1.25\, d = 1.25 \times 40 = 50$ mm

Now considering the shaft under combined bending and twisting, the diameter of the shaft at the centre of the bearing (d_1) is given by the relation

$$\frac{\pi}{16} \times \tau\, (d_1)^3 = P\sqrt{l^2 + L^2}$$

$$\frac{\pi}{16} \times 70 \times (d_1)^3 = 800\sqrt{(100)^2 + (1000)^2} \qquad \text{...(Taking } l = 2\, l_2\text{)}$$

or $\quad 13.75\, (d_1)^3 = 804 \times 10^3$

$\therefore \quad (d_1)^3 = 804 \times 10^3 / 13.75 = 58.5 \times 10^3$ or d_1 = 38.8 say 40 mm **Ans.**

2. Dimensions of the key

The standard dimensions of the key for a 40 mm diameter shaft are :

Width of key, w = 12 mm **Ans.**

and thickness of key = 8 mm **Ans.**

The length of the key (l_1) is obtained by considering the shearing of the key.

We know that twisting moment (T),

$$800 \times 10^3 = l_1 \times w \times \tau \times \frac{d}{2}$$

$$= l_1 \times 12 \times 70 \times \frac{40}{2} = 16\,800\, l_1$$

$\therefore$ $l_1 = 800 \times 10^3 / 16\,800 = 47.6$ mm

It may be taken as equal to the length of boss (l_2).

$\therefore$ $l_1 = l_2 = 50$ mm **Ans.**

Accelerator and brake levers inside an automobile.

3. Dimensions of the rectangular arm at 60 mm from the centre of shaft

Let t = Thickness of arm in mm, and

B = Width of arm in mm = 3 t ...(Given)

$\therefore$ Bending moment at 60 mm from the centre of shaft,

$$M = 800\,(1000 - 60) = 752 \times 10^3 \text{ N-mm}$$

and section modulus, $Z = \frac{1}{6} \times t \times B^2 = \frac{1}{6} \times t\,(3t)^2 = 1.5\, t^3 \text{ mm}^3$

We know that the tensile bending stress (σ_t),

$$73 = \frac{M}{Z} = \frac{752 \times 10^3}{1.5\, t^3} = \frac{501.3 \times 10^3}{t^3}$$

$\therefore$ $t^3 = 501.3 \times 10^3/73 = 6.87 \times 10^3$

or t = 19 say 20 mm **Ans.**

and $B = 3\,t = 3 \times 20 = 60$ mm **Ans.**

The width of the arm is tapered while the thickness is kept constant throughout. The width of the arm on the foot plate side,

$$B_1 = B / 2 = 30 \text{ mm } \textbf{Ans.}$$

15.6 Cranked Lever

A cranked lever, as shown in Fig. 15.10, is a hand lever commonly used for operating hoisting winches.

The lever can be operated either by a single person or by two persons. The maximum force in order to operate the lever may be taken as 400 N and the length of handle as 300 mm. In case the lever is operated by two persons, the maximum force of operation will be doubled and length of handle may be taken as 500 mm. The handle is covered in a pipe to prevent hand scoring. The end of the shaft is usually squared so that the lever may be easily fixed and removed. The length (L) is usually from 400 to 450 mm and the height of the shaft centre line from the ground is usually one metre. In order to design such levers, the following procedure may be adopted :

1. The diameter of the handle (*d*) is obtained from bending considerations. It is assumed that the effort (*P*) applied on the handle acts at $\frac{2}{3}$rd of its length (*l*).

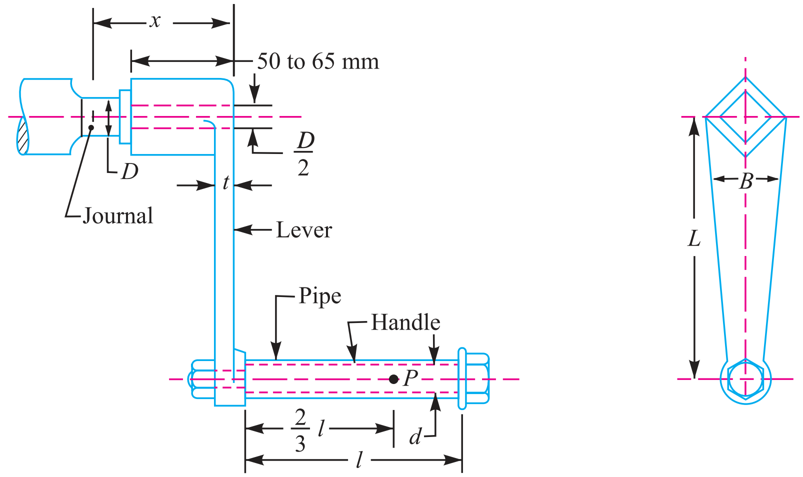


Fig. 15.10. Cranked lever.

∴ Maximum bending moment,

$$M = P \times \frac{2l}{3} = \frac{2}{3} \times P \times l$$

and section modulus,

$$Z = \frac{\pi}{32} \times d^3$$

∴ Resisting moment $= \sigma_b \times Z = \sigma_b \times \frac{\pi}{32} \times d^3$

where σ_b = Permissible bending stress for the material of the handle.

Equating resisting moment to the maximum bending moment, we have

$$\sigma_b \times \frac{\pi}{32} \times d^3 = \frac{2}{3} \times P \times l$$

From this expression, the diameter of the handle (*d*) may be evaluated. The diameter of the handle is usually proportioned as 25 mm for single person and 40 mm for two persons.

2. The cross-section of the lever arm is usually rectangular having uniform thickness throughout. The width of the lever arm is tapered from the boss to the handle. The arm is subjected to constant twisting moment, $T = \frac{2}{3} \times P \times l$ and a varying bending moment which is maximum near the boss. It is assumed that the arm of the lever extends upto the centre of shaft, which results in a slightly stronger lever.

∴ Maximum bending moment = $P \times L$

Since, at present time, there is insufficient information on the subject of combined bending and twisting of rectangular sections to enable us to find equivalent bending or twisting, with sufficient accuracy, therefore the indirect procedure is adopted.

We shall design the lever arm for 25% more bending moment.

∴ Maximum bending moment

$$M = 1.25\, P \times L$$

Let t = Thickness of the lever arm, and

B = Width of the lever arm near the boss.

∴ Section modulus for the lever arm,

$$Z = \frac{1}{6} \times t \times B^2$$

Now by using the relation, $\sigma_b = M / Z$, we can find t and B. The width of the lever arm near the boss is taken as twice the thickness *i.e.* $B = 2\,t$.

After finding the value of t and B, the induced bending stress may be checked which should not exceed the permissible value.

3. The induced shear stress in the section of the lever arm near the boss, caused by the twisting moment, $T = \frac{2}{3} \times P \times l$ may be checked by using the following relations :

$$T = \frac{2}{9} \times B \times t^2 \times \tau \qquad \text{...(For rectangular section)}$$

$$= \frac{2}{9} \times t^3 \times \tau \qquad \text{...(For square section of side } t\text{)}$$

$$= \frac{\pi}{16} \times B \times t^2 \times \tau \qquad \text{...(For elliptical section having major axis } B \text{ and minor axis } t\text{)}$$

4. Knowing the values of σ_b and τ, the maximum principal or shear stress induced may be checked by using the following relations :

Maximum principal stress,

$$\sigma_{b(max)} = \frac{1}{2}\left[\sigma_b + \sqrt{(\sigma_b)^2 + 4\tau^2}\right]$$

Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

5. Since the journal of the shaft is subjected to twisting moment and bending moment, therefore its diameter is obtained from equivalent twisting moment.

We know that twisting moment on the journal of the shaft,

$$T = P \times L$$

and bending moment on the journal of the shaft,

$$M = P\left(\frac{2l}{3} + x\right)$$

where x = Distance from the end of boss to the centre of journal.

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = P\sqrt{\left(\frac{2l}{3} + x\right)^2 + L^2}$$

We know that equivalent twisting moment,

$$T_e = \frac{\pi}{16} \times \tau \times D^3$$

From this expression, we can find the diameter (D) of the journal.

The diameter of the journal is usually taken as

D = 30 to 40 mm, for single person

= 40 to 45 mm, for two persons.

Note: The above procedure may be used in the design of overhung cranks of engines.

Example 15.4. *A cranked lever, as shown in 15.10, has the following dimensions :*

Length of the handle = 300 mm

Length of the lever arm = 400 mm

Overhang of the journal = 100 mm

If the lever is operated by a single person exerting a maximum force of 400 N at a distance of $\frac{1}{3}$ rd length of the handle from its free end, find : 1. Diameter of the handle, 2. Cross-section of the lever arm, and 3. Diameter of the journal.

The permissible bending stress for the lever material may be taken as 50 MPa and shear stress for shaft material as 40 MPa.

Solution. Given : l = 300 mm ; L = 400 mm ; x = 100 mm ; P = 400 N ; σ_b = 50 MPa = 50 N/mm^2 ; τ = 40 MPa = 40 N/mm^2

1. *Diameter of the handle*

Let d = Diameter of the handle in mm.

Since the force applied acts at a distance of 1/3 rd length of the handle from its free end,therefore maximum bending moment,

$$M = \left(1 - \frac{1}{3}\right) P \times l = \frac{2}{3} \times P \times l = \frac{2}{3} \times 400 \times 300 \text{ N-mm}$$

$$= 80 \times 10^3 \text{ N-mm} \quad ...(i)$$

Section modulus, $Z = \dfrac{\pi}{32} \times d^3 = 0.0982\ d^3$

$\therefore$ Resisting bending moment,

$$M = \sigma_b \times Z = 50 \times 0.0982\ d^3 = 4.91\ d^3 \text{ N-mm} \quad ..(ii)$$

From equations ***(i)*** and ***(ii)***, we get

$$d^3 = 80 \times 10^3 / 4.91 = 16.3 \times 10^3 \quad \text{or} \quad d = 25.4 \text{ mm } \textbf{Ans.}$$

2. *Cross-section of the lever arm*

Let t = Thickness of the lever arm in mm, and

B = Width of the lever arm near the boss, in mm.

Since the lever arm is designed for 25% more bending moment, therefore maximum bending moment,

$$M = 1.25\ P \times L = 1.25 \times 400 \times 400 = 200 \times 10^3 \text{ N-mm}$$

Section modulus, $Z = \dfrac{1}{6} \times t \times B^2 = \dfrac{1}{6} \times t\ (2t)^2 = 0.667\ t^3$ ...(Assuming $B = 2t$)

We know that bending stress (σ_b),

$$50 = \frac{M}{Z} = \frac{200 \times 10^3}{0.667\ t^3} = \frac{300 \times 10^3}{t^3}$$

$\therefore$ $t^3 = 300 \times 10^3/50 = 6 \times 10^3$ or t = 18.2 say 20 mm **Ans.**

and $B = 2\ t = 2 \times 20 = 40$ mm **Ans.**

Let us now check the lever arm for induced bending and shear stresses.

Bending moment on the lever arm near the boss (assuming that the length of the arm extends upto the centre of shaft) is given by

$$M = P \times L = 400 \times 400 = 160 \times 10^3 \text{ N-mm}$$

and section modulus, $Z = \dfrac{1}{6} \times t \times B^2 = \dfrac{1}{6} \times 20\ (40)^2 = 5333 \text{ mm}^3$

∴ Induced bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{160 \times 10^3}{5333} = 30 \text{ N/mm}^2 = 30 \text{ MPa}$$

The induced bending stress is within safe limits.

We know that the twisting moment,

$$T = \frac{2}{3} \times P \times l = \frac{2}{3} \times 400 \times 300 = 80 \times 10^3 \text{ N-mm}$$

We also know that the twisting moment (T),

$$80 \times 10^3 = \frac{2}{9} \times B \times t^2 \times \tau = \frac{2}{9} \times 40\ (20)^2\ \tau = 3556\ \tau$$

$$\therefore \quad \tau = 80 \times 10^3 / 3556 = 22.5 \text{ N/mm}^2 = 22.5 \text{ MPa}$$

The induced shear stress is also within safe limits.

Let us now check the cross-section of lever arm for maximum principal or shear stress.

We know that maximum principal stress,

$$\sigma_{b\,(max)} = \frac{1}{2}\left[\sigma_b + \sqrt{(\sigma_b)^2 + 4\tau^2}\right] = \frac{1}{2}\left[30 + \sqrt{(30)^2 + 4\ (22.5)^2}\right]$$

$$= \frac{1}{2}(30 + 54) = 42 \text{ N/mm}^2 = 42 \text{ MPa}$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} = \frac{1}{2}\sqrt{(30)^2 + 4\ (22.5)^2} = 27 \text{ N/mm}^2 = 27 \text{ MPa}$$

The maximum principal and shear stresses are also within safe limits.

3. Diameter of the journal

Let D = Diameter of the journal.

Since the journal of the shaft is subjected to twisting moment and bending moment, therefore its diameter is obtained from equivalent twisting moment.

We know that equivalent twisting moment,

$$T_e = P\sqrt{\left(\frac{2l}{3} + x\right)^2 + L^2} = 400\sqrt{\left(\frac{2 \times 300}{3} + 100\right)^2 + (400)^2}$$

$$= 200 \times 10^3 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$200 \times 10^3 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 40 \times D^3 = 7.86\ D^3$$

$$\therefore \quad D^3 = 200 \times 10^3 / 7.86 = 25.4 \times 10^3 \text{ or } D = 29.4 \text{ say } 30 \text{ mm}$$ **Ans.**

15.7 Lever for a Lever Safety Valve

A lever safety valve is shown in Fig. 15.11. It is used to maintain a constant safe pressure inside the boiler. When the pressure inside the boiler increases the safe value, the excess steam blows off through the valve automatically. The valve rests over the gunmetal seat which is secured to a casing fixed upon the boiler. One end of the lever is pivoted at the fulcrum F by a pin to the toggle, while the other end carries the weights. The valve is held on its seat against the upward steam pressure by the force P provided by the weights at B. The weights and its distance from the fulcrum are so adjusted that when the steam pressure acting upward on the valve exceeds the normal limit, it lifts the valve and the lever with its weights. The excess steam thus escapes until the pressure falls to the required limit.

The lever may be designed in the similar way as discussed earlier. The maximum steam load (W), at which the valve blows off, is given by

$$W = \frac{\pi}{4} \times D^2 \times p$$

where D = Diameter of the valve, and

p = Steam pressure.

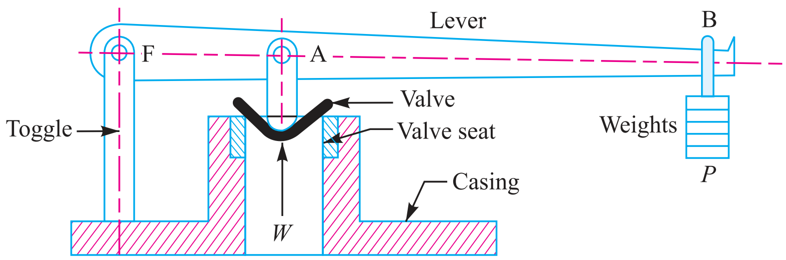


Fig. 15.11. Lever safety valve.

Example 15.5. *A lever loaded safety valve is 70 mm in diameter and is to be designed for a boiler to blow-off at pressure of 1 N/mm² gauge. Design a suitable mild steel lever of rectangular cross-section using the following permissible stresses :*

Tensile stress = 70 MPa; Shear stress = 50 MPa; Bearing pressure intensity = 25 N/mm².

The pin is also made of mild steel. The distance from the fulcrum to the weight of the lever is 880 mm and the distance between the fulcrum and pin connecting the valve spindle links to the lever is 80 mm.

Solution. Given : D = 70 mm ; p = 1 N/mm² ; σ_t = 70 MPa = 70 N/mm² ; τ = 50 MPa = 50 N/mm² ; p_b = 25 N/mm² ; FB = 880 mm ; FA = 80 mm

We know that the maximum steam load at which the valve blows off,

$$W = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} (70)^2 \times 1 = 3850 \text{ N} \quad ...(i)$$

Taking moments about the fulcrum F, we have

$$P \times 880 = 3850 \times 80 = 308 \times 10^3 \quad \text{or} \quad P = 308 \times 10^3 / 880 = 350 \text{ N}$$

Since the load (W) and the effort (P) in the form of dead weight are parallel and opposite, therefore reaction at F,

$$R_F = W - P = 3850 - 350 = 3500 \text{ N}$$

This rection will act vertically downward as shown in Fig. 15.12.

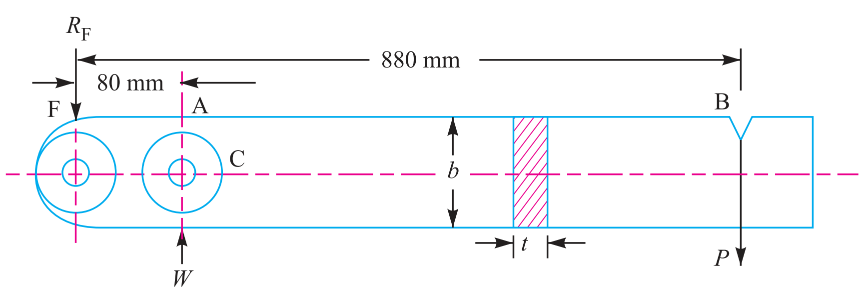


Fig. 15.12

First of all, let us find the diameter of the pin at A from bearing considerations.

Let d_p = Diameter of the pin at A, and

l_p = Length of the pin at A.

∴ Bearing area of the pin at A

$$= d_p \times l_p = 1.25\,(d_p)^2 \qquad \text{...(Assuming } l_p = 1.25\,d_p\text{)}$$

and load on the pin at A = Bearing area × Bearing pressure

$$= 1.25\,(d_p)^2\,p_b = 1.25\,(d_p)^2\,25 = 31.25\,(d_p)^2 \qquad \text{...(ii)}$$

Since the load acting on the pin at A is W = 3850 N, therefore from equations **(*i*)** and **(*ii*),** we get

$$(d_p)^2 = 3850 / 31.25 = 123.2 \quad \text{or} \quad d_p = 11.1 \text{ say } 12 \text{ mm } \textbf{Ans.}$$

and $$l_p = 1.25\,d_p = 1.25 \times 12 = 15 \text{ mm } \textbf{Ans.}$$

Let us now check the pin for shearing. Since the pin is in double shear, therefore load on the pin at A (W),

$$3850 = 2 \times \frac{\pi}{4}\,(d_p)^2\,\tau = 2 \times \frac{\pi}{4}\,(12)^2\,\tau = 226.2\,\tau$$

∴ $$\tau = 3850 / 226.2 = 17.02 \text{ N/mm}^2 = 17.02 \text{ MPa}$$

This value of shear stress is less than the permissible value of 50 MPa, therefore the design for pin at A is safe. Since the load at F does not very much differ with the load at A, therefore the same diameter of pin may be used at F, in order to facilitate the interchangeability of parts.

∴ Diameter of the fulcrum pin at F

$$= 12 \text{ mm}$$

A gun metal bush of 2 mm thickness is provided in the pin holes at A and F in order to reduce wear and to increase the life of lever.

∴ Diameter of hole at A and F

$$= 12 + 2 \times 2 = 16 \text{ mm}$$

and outside diameter of the boss

$$= 2 \times \text{Dia. of hole} = 2 \times 16 = 32 \text{ mm}$$

Power clamp of an excavator.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Contents

Now let us find out the cross-section of the lever considering the bending moment near the boss at *A*.

Let t = Thickness of the lever, and

b = Width of the lever.

Bending moment near the boss at *A i.e.* at point *C*,

$$M = P \times BC = P\,(BF - AF - AC) = 350\left(880 - 80 - \frac{16}{2}\right) \text{ N-mm}$$

$$= 277\ 200 \text{ N-mm}$$

and section modulus, $$Z = \frac{1}{6} \times t.b^2 = \frac{1}{6} \times t\,(4t)^2 = 2.67\,t^3 \quad \text{...(Assuming } b = 4\,t)$$

We know that the bending stress (σ_b)

$$70 = \frac{M}{Z} = \frac{277\ 200}{2.67\,t^3} = \frac{104 \times 10^3}{t^3} \quad \text{...}(\because \sigma_b = \sigma_t)$$

$\therefore$ $t^3 = 104 \times 10^3 / 70 = 1.5 \times 10^3$ or $t = 11.4$ say 12 mm **Ans.**

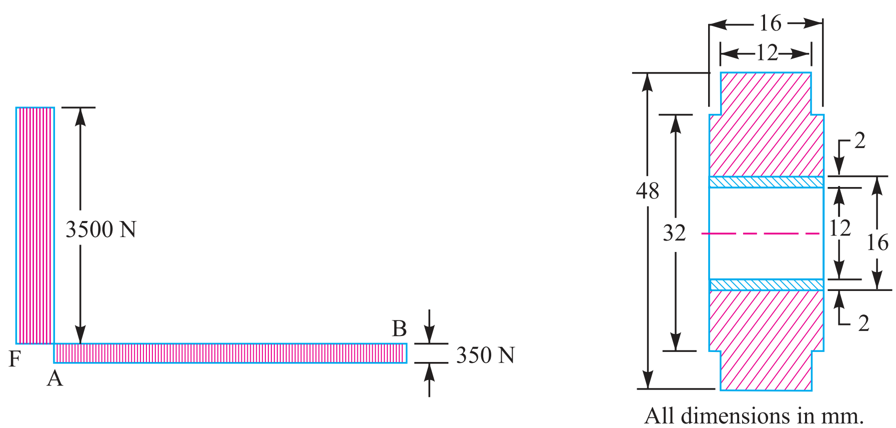
and $b = 4\,t = 4 \times 12 = 48$ mm **Ans.**

Now let us check for the maximum shear stress induced in the lever. From the shear force diagram as shown in Fig. 15.13 (*a*), we see that the maximum shear force on the lever is ($W - P$) *i.e.* 3500 N.

$\therefore$ Maximum shear stress induced,

$$\tau_{max} = \frac{\text{Maximum shear force}}{\text{Cross-sectional area of the lever}} = \frac{3500}{12 \times 48}$$

$$= 6.07 \text{ N/mm}^2 = 6.07 \text{ MPa}$$



(*a*) Shear force diagram.

(*b*) Section at A through the centre of hole.

Fig. 15.13

Since this value of maximum shear stress is much below the permissible shear stress of 50 MPa therefore the design for lever is safe.

Again checking for the bending stress induced at the section passing through the centre of hole at *A*. The section at *A* through the centre of the hole is shown in Fig. 15.13 (*b*).

$\therefore$ Maximum bending moment at the centre of hole at *A*,

$$M = 350\,(880 - 80) = 280 \times 10^3 \text{ N-mm}$$

Top

Section modulus,

$$Z = \frac{\frac{1}{12} \times 12 \left[(48)^3 - (16)^3\right] + 2 \times \frac{1}{12} \times 2 \left[(32)^3 - (16)^3\right]}{48/2}$$

$$= \frac{106\ 496 + 9557}{24} = 4836 \text{ mm}^3$$

∴ Maximum bending stress induced,

$$\sigma_t = \frac{M}{Z} = \frac{280 \times 10^3}{4836} = 58 \text{ N/mm}^2 = 58 \text{ MPa}$$

Since this maximum stress is below the permissible value of 70 MPa, therefore the design in safe.

15.8 Bell Crank Lever

In a bell crank lever, the two arms of the lever are at right angles. Such type of levers are used in railway signalling, governors of Hartnell type, the drive for the air pump of condensors etc. The bell crank lever is designed in a similar way as discussed earlier. The arms of the bell crank lever may be assumed of rectangular, elliptical or I-section. The complete design procedure for the bell crank lever is given in the following example.

Example 15.6. *Design a right angled bell crank lever. The horizontal arm is 500 mm long and a load of 4.5 kN acts vertically downward through a pin in the forked end of this arm. At the end of the 150 mm long arm which is perpendicular to the 500 mm long arm, a force P act at right angles to the axis of 150 mm arm through a pin into a forked end. The lever consists of forged steel material and a pin at the fulcrum. Take the following data for both the pins and lever material:*

Safe stress in tension = 75 MPa

Safe stress in shear = 60 MPa

Safe bearing pressure on pins = 10 N/mm²

Solution. Given : FB = 500 mm ; W = 4.5 kN = 4500 N ; FA = 150 mm ; σ_t = 75 MPa = 75 N/mm^2 ; τ = 60 MPa = 60 N/mm^2 ; p_b = 10 N/mm^2

The bell crank lever is shown in Fig. 15.14.

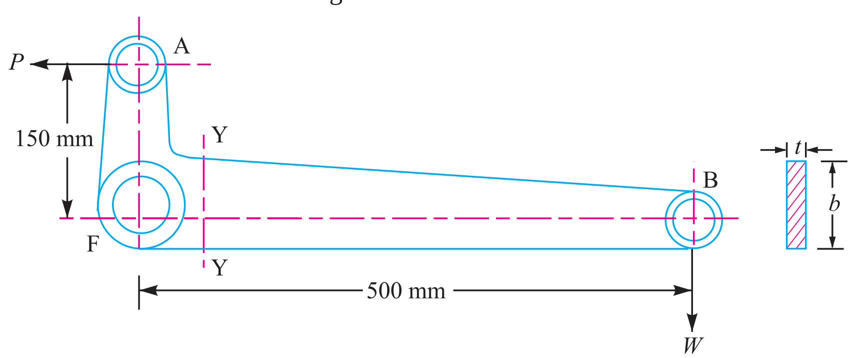


Fig. 15.14

First of all, let us find the effort (P) required to raise the load (W). Taking moments about the fulcrum F, we have

$$W \times 500 = P \times 150$$

$$\therefore \quad P = \frac{W \times 500}{150} = \frac{4500 \times 500}{150} = 15\ 000 \text{ N}$$

and reaction at the fulcrum pin at F,

$$R_F = \sqrt{W^2 + P^2} = \sqrt{(4500)^2 + (15\,000)^2} = 15\,660 \text{ N}$$

1. *Design for fulcrum pin*

Let d = Diameter of the fulcrum pin, and

l = Length of the fulcrum pin.

Considering the fulcrum pin in bearing. We know that load on the fulcrum pin (R_F),

$$15\,660 = d \times l \times p_b = d \times 1.25\,d \times 10 = 12.5\,d^2 \qquad \text{...(Assuming } l = 1.25\,d)$$

$\therefore$ $d^2 = 15\,660 / 12.5 = 1253$ or $d = 35.4$ say 36 mm **Ans.**

and $l = 1.25\,d = 1.25 \times 36 = 45$ mm **Ans.**

Let us now check for the shear stress induced in the fulcrum pin. Since the pin is in double shear, therefore load on the fulcrum pin (R_F),

$$15\,660 = 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4}(36)^2\,\tau = 2036\,\tau$$

$\therefore$ $\tau = 15\,660/2036 = 7.7 \text{ N/mm}^2 = 7.7$ MPa

Since the shear stress induced in the fulcrum pin is less than the given value of 60 MPa, therefore design for the fulcrum pin is safe.

A brass bush of 3 mm thickness is pressed into the boss of fulcrum as a bearing so that the renewal become simple when wear occurs.

$\therefore$ Diameter of hole in the lever

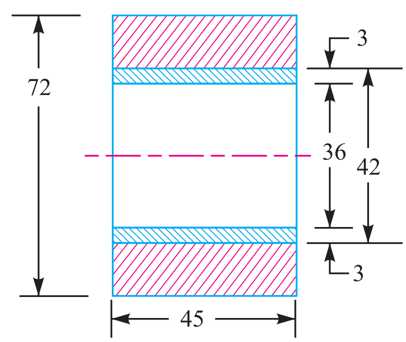
$= d + 2 \times 3$

$= 36 + 6 = 42$ mm

and diameter of boss at fulcrum

$= 2\,d = 2 \times 36 = 72$ mm

Now let us check the bending stress induced in the lever arm at the fulcrum. The section of the fulcrum is shown in Fig. 15.15.



All dimensions in mm.

Fig. 15.15

Bending moment at the fulcrum

$$M = W \times FB = 4500 \times 500 = 2250 \times 10^3 \text{ N-mm}$$

Section modulus,

$$Z = \frac{\frac{1}{12} \times 45\left[(72)^3 - (42)^3\right]}{72/2} = 311\,625 \text{ mm}^3$$

$\therefore$ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{2250 \times 10^3}{311\,625} = 7.22 \text{ N/mm}^2 = 7.22 \text{ MPa}$$

Since the bending stress induced in the lever arm at the fulcrum is less than the given value of 85 MPa, therefore it is safe.

2. *Design for pin at A*

Since the effort at A (which is 15 000 N), is not very much different from the reaction at fulcrum (which is 15 660 N), therefore the same dimensions for the pin and boss may be used as for fulcrum pin to reduce spares.

$\therefore$ Diameter of pin at A = 36 mm **Ans.**

Length of pin at A = 45 mm **Ans.**

and diameter of boss at A = 72 mm **Ans.**

Top

3. *Design for pin at B*

Let d_1 = Diameter of the pin at *B*, and

l_1 = Length of the pin at *B*.

Considering the bearing of the pin at *B*. We know that load on the pin at *B* (*W*),

$$4500 = d_1 \times l_1 \times p_b = d_1 \times 1.25\, d_1 \times 10 = 12.5\, (d_1)^2$$

... (Assuming $l_1 = 1.25\, d_1$)

$\therefore$ $(d_1)^2 = 4500 / 12.5 = 360$ or $d_1 = 18.97$ say 20 mm **Ans.**

and $l_1 = 1.25\, d_1 = 1.25 \times 20 = 25$ mm **Ans.**

Let us now check for the shear stress induced in the pin at *B*. Since the pin is in double shear, therefore load on the pin at *B* (*W*),

$$4500 = 2 \times \frac{\pi}{4} (d_1)^2\, \tau = 2 \times \frac{\pi}{4} (20)^2\, \tau = 628.4\, \tau$$

$\therefore$ $\tau = 4500 / 628.4 = 7.16 \text{ N/mm}^2 = 7.16$ MPa

Since the shear stress induced in the pin at *B* is within permissible limits, therefore the design is safe.

Since the end *B* is a forked end, therefore thickness of each eye,

$$t_1 = \frac{l_1}{2} = \frac{25}{2} = 12.5 \text{ mm}$$

In order to reduce wear, chilled phosphor bronze bushes of 3 mm thickness are provided in the eyes.

$\therefore$ Inner diameter of each eye

$$= d_1 + 2 \times 3 = 20 + 6 = 26 \text{ mm}$$

and outer diameter of eye,

$$D = 2\, d_1 = 2 \times 20 = 40 \text{ mm}$$

Let us now check the induced bending stress in the pin. The pin is neither simply supported nor rigidly fixed at its ends. Therefore the common practice is to assume the load distribution as shown in Fig. 15.16. The maximum bending moment will occur at *Y-Y.*

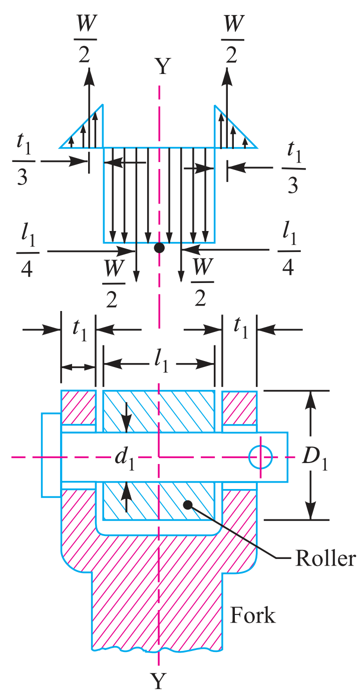


Fig. 15.16

$\therefore$ Maximum bending moment at *Y-Y*,

$$M = \frac{W}{2}\left(\frac{l_1}{2} + \frac{t_1}{3}\right) - \frac{W}{2} \times \frac{l_1}{4}$$

$$= \frac{5}{24} W \times l_1$$

...($\because t_1 = l_1/2$)

$$= \frac{5}{24} \times 4500 \times 25 = 23\,438 \text{ N-mm}$$

and section modulus,

$$Z = \frac{\pi}{32} (d_1)^3 = \frac{\pi}{32} (20)^3 = 786 \text{ mm}^3$$

$\therefore$ Bending stress induced,

$$\sigma_b = \frac{M}{Z} = \frac{23\,438}{786} = 29.8 \text{ N/mm}^2 = 29.8 \text{ MPa}$$

This induced bending stress is within safe limits.

4. *Design of lever*

It is assumed that the lever extends upto the centre of the fulcrum from the point of application of the load. This assumption is commonly made and results in a slightly stronger section. Considering the weakest section of failure at *Y-Y*.

Let t = Thickness of the lever at *Y-Y*, and

b = Width or depth of the lever at *Y-Y*.

Taking distance from the centre of the fulcrum to *Y-Y* as 50 mm, therefore maximum bending moment at *Y-Y*,

$$= 4500\,(500 - 50) = 2025 \times 10^3 \text{ N-mm}$$

and section modulus, $Z = \dfrac{1}{6} \times t \times b^2 = \dfrac{1}{6} \times t\,(3t)^2 = 1.5\,t^3$ ...(Assuming $b = 3\,t$)

We know that the bending stress (σ_b),

$$75 = \frac{M}{Z} = \frac{2025 \times 10^3}{1.5t^3} = \frac{1350 \times 10^3}{t^3}$$

$\therefore$ $t^3 = 1350 \times 10^3 / 75 = 18 \times 10^3$ or $t = 26$ mm **Ans.**

and $b = 3\,t = 3 \times 26 = 78$ mm **Ans.**

Bucket of a bulldozer.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Example 15.7. *In a Hartnell governor, the length of the ball arm is 190 mm, that of the sleeve arm is 140 mm, and the mass of each ball is 2.7 kg. The distance of the pivot of each bell crank lever from the axis of rotation is 170 mm and the speed when the ball arm is vertical, is 300 r.p.m. The speed is to increase 0.6 per cent for a lift of 12 mm of the sleeve.*

(*a*) *Find the necessary stiffness of the spring.*

(*b*) *Design the bell crank lever. The permissible tensile stress for the material of the lever may be taken as 80 MPa and the allowable bearing pressure at the pins is 8 N/mm^2.*

Solution. Given : x = 190 mm ; y = 140 mm ; m = 2.7 kg ; r_2 = 170 mm = 0.17 m ; N_2 = 300 r.p.m. ; h = 12 mm ; σ_t = 80 MPa = 80 N/mm^2 ; p_b = 8 N/mm^2

A Hartnell governor is shown in Fig. 15.17.

(a) Stiffness of the spring

Let s_1 = Stiffness of the spring.

We know that minimum angular speed of the ball arm (*i.e.* when the ball arm is vertical),

$$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 300}{60} = 31.42 \text{ rad/s}$$

Since the increase in speed is 0.6 per cent, therefore maximum angular speed of the ball arm,

$$\omega_1 = \omega_2 + \frac{0.6}{100} \times \omega_2 = 1.006\ \omega_2 = 1.006 \times 31.42 = 31.6 \text{ rad/s}$$

We know that radius of rotation at the maximum speed,

$$r_1 = r_2 + h \times \frac{x}{y} = 170 + 12 \times \frac{190}{140} = 186.3 \text{ mm} = 0.1863 \text{ m}$$

$$\ldots\left[\because h = (r_1 - r_2)\frac{y}{x}\right]$$

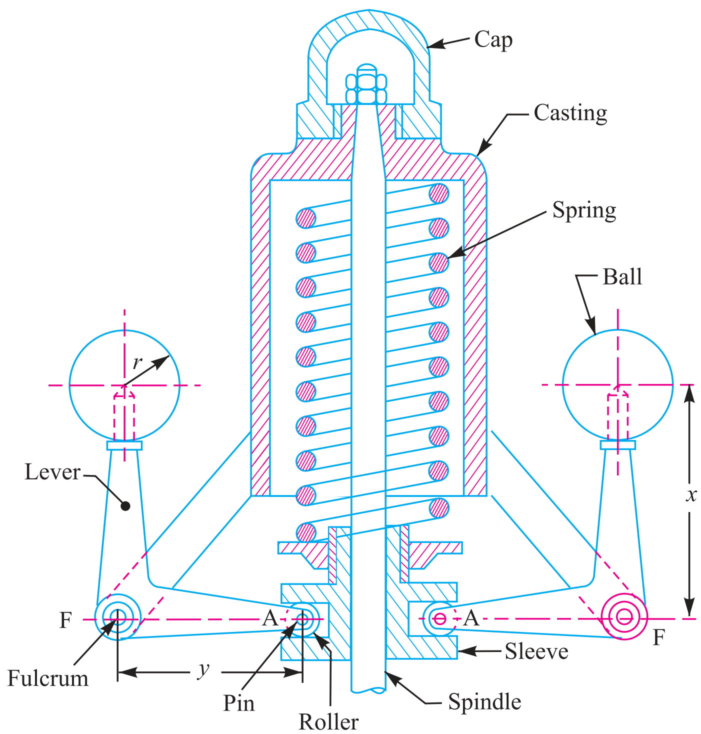


Fig. 15.17

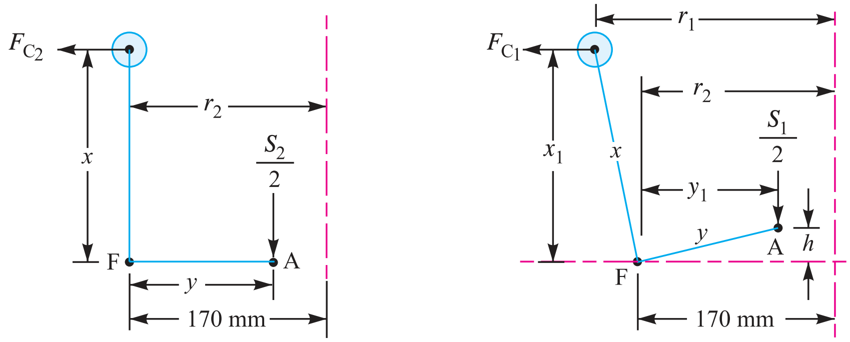
The minimum and maximum position of the ball arm and sleeve arm is shown in Fig. 15.18 (*a*) and (*b*) respectively.

Let F_{C1} = Centrifugal force at the maximum speed = $m\,(\omega_1)^2\, r_1$,

F_{C2} = Centrifugal force at the minimum speed = $m\,(\omega_2)^2\, r_2$,

S_1 = Spring force at the maximum speed (ω_1), and

S_2 = Spring force at the minimum speed (ω_2).



(*a*) Minimum position. (*b*) Maximum position.

Fig. 15.18

Taking moments about the fulcrum *F* of the bell crank lever, neglecting the obliquity effect of the arms (*i.e.* taking $x_1 = x$ and $y_1 = y$) and the moment due to mass of the balls, we have for *maximum position,

$$S_1 = 2F_{C1} \times \frac{x}{y} = 2m\,(\omega_1)^2\; r_1 \times \frac{x}{y} \qquad \ldots\left(\because\ \frac{S_1}{2} \times y = F_{C1} \times x\right)$$

$$= 2 \times 2.7\,(31.6)^2\; 0.1863 \times \frac{190}{140} = 1364 \text{ N}$$

Similarly

$$S_2 = 2F_{C2} \times \frac{x}{y} = 2m\,(\omega_2)^2\; r_2 \times \frac{x}{y}$$

$$= 2 \times 2.7\,(31.42)^2\; 0.17 \times \frac{190}{140} = 1230 \text{ N}$$

We know that

$$S_1 - S_2 = h \times s_1$$

$$\therefore \qquad s_1 = \frac{S_1 - S_2}{h} = \frac{1364 - 1230}{12} = 11.16 \text{ N/mm} \quad \textbf{Ans.}$$

(b) Design of bell crank lever

The bell crank lever is shown in Fig. 15.19. First of all, let us find the centrifugal force (or the effort *P*) required at the ball end to resist the load at *A*.

We know that the maximum load on the roller arm at *A*,

$$W = \frac{S_1}{2} = \frac{1364}{2} = 682 \text{ N}$$

Taking moments about *F*, we have

$$P \times x = W \times y$$

$$\therefore \qquad P = \frac{W \times y}{x} = \frac{682 \times 140}{190}$$

$$= 502 \text{ N}$$

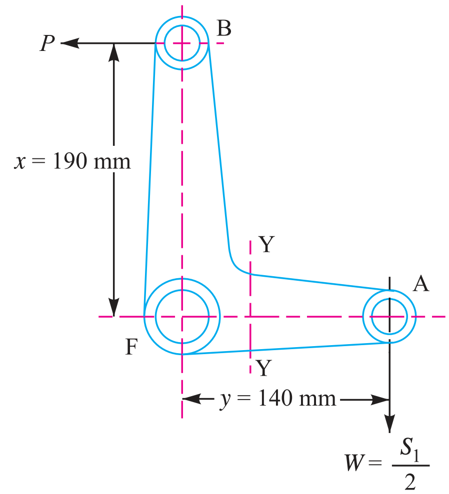


Fig. 15.19

* For further details, please refer chapter on 'Governors' of authors' popular book on **'Theory of Machines'.**

We know that reaction at the fulcrum *F*,

$$R_F = \sqrt{W^2 + P^2} = \sqrt{(682)^2 + (502)^2} = 847 \text{ N}$$

1. *Design for fulcrum pin*

Let d = Diameter of the fulcrum pin, and

l = Length of the fulcrum pin = 1.25 d ...(Assume)

The fulcrum pin is supported in the eye which is integral with the frame for the spring. Considering the fulcrum pin in bearing. We know that load on the fulcrum pin (R_F),

$$847 = d \times l \times p_b = d \times 1.25\, d \times 8 = 10\, d^2$$

$$\therefore \quad d^2 = 847 / 10 = 84.7 \text{ or } d = 9.2 \text{ say } 10 \text{ mm}$$ **Ans.**

and $l = 1.25\, d = 1.25 \times 10 = 12.5\, d = 12.5$ mm **Ans.**

Let us now check for the induced shear stress in the pin. Since the pin is in double shear, therefore load on the fulcrum pin (R_F),

$$847 = 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (10)^2\, \tau = 157.1\, \tau$$

$$\therefore \quad \tau = 847 / 157.1 = 5.4 \text{ N/mm}^2 = 5.4 \text{ MPa}$$

This induced shear stress is very much within safe limits.

A brass bush of 3 mm thick may be pressed into the boss. Therefore diameter of hole in the lever or inner diameter of boss

$= 10 + 2 \times 3 = 16$ mm

and outer diameter of boss

$= 2\, d = 2 \times 10 = 20$ mm

2. *Design for lever*

The cross-section of the lever is obtained by considering the lever in bending. It is assumed that the lever arm extends upto the centre of the fulcrum from the point of application of load. This assumption results in a slightly stronger lever. Considering the weakest section of failure at *Y-Y* (40 mm from the centre of the fulcrum).

Lapping is a surface finishing process for finishing gears, etc.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Contents

∴ Maximum bending moment at *Y-Y*,

$$= 682\ (140 - 40) = 68\ 200 \text{ N-mm}$$

Let t = Thickness of the lever, and

B = Depth or width of the lever.

∴ Section modulus,

$$Z = \frac{1}{6} \times t \times B^2 = \frac{1}{6} \times t\ (3t)^2 = 1.5\ t^3 \qquad \text{...(Assuming } B = 3\ t)$$

We know that bending stress (σ_b),

$$80 = \frac{M}{Z} = \frac{68\ 200}{1.5 t^3} = \frac{45\ 467}{t^3}$$

$$\therefore \quad t^3 = 45\ 467 / 80 = 568 \quad \text{or} \quad t = 8.28 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

and $B = 3\ t = 3 \times 10 = 30$ mm **Ans.**

3. Design for ball

Let r = Radius of the ball.

The balls are made of cast iron, whose density is 7200 kg/m^3. We know that mass of the ball (m),

$$2.7 = \text{Volume} \times \text{density} = \frac{4}{3} \pi r^3 \times 7200 = 30\ 163\ r^3$$

$$\therefore \quad r^3 = 2.7 / 30\ 163 = 0.089/10^3$$

or $r = 0.0447$ m = 44.7 say 45 mm **Ans.**

The ball is screwed to the end of the lever. The screwed length of lever will be equal to the radius of ball.

∴ Maximum bending moment on the screwed end of the lever,

$$M = P \times r = 502 \times 45 = 22\ 590 \text{ N-mm}$$

Let d_c = Core diameter of the screwed length of the lever.

∴ Section modulus,

$$Z = \frac{\pi}{32} (d_c)^3 = 0.0982\ (d_c)^3$$

We know that bending stress (σ_b),

$$80 = \frac{M}{Z} = \frac{22\ 590}{0.0982\ (d_c)^3} = \frac{230 \times 10^3}{(d_c)^3}$$

$$\therefore \quad (d_c)^3 = 230 \times 10^3 / 80 = 2876 \quad \text{or} \quad d_c = 14.2 \text{ mm}$$

We shall take nominal diameter of the screwed length of lever as 16 mm. **Ans.**

4. Design for roller end A

Let d_1 = Diameter of the pin at *A*, and

l_1 = Length of the pin at $A = 1.25\ d_1$ ...(Assume)

We know that the maximum load on the roller at *A*,

$$W = S_1 / 2 = 1364 / 2 = 682 \text{ N}$$

Considering the pin in bearing. We know that load on the pin at *A* (*W*),

$$682 = d_1.l_1.p_b = d_1 \times 1.25\ d_1 \times 8 = 10\ (d_1)^2$$

$$\therefore \quad (d_1)^2 = 682 / 10 = 68.2 \quad \text{or} \quad d_1 = 8.26 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

and $l_1 = 1.25\ d_1 = 1.25 \times 10 = 12.5$ mm **Ans.**

Top

Let us now check the pin for induced shear stress. Since the pin is in double shear, therefore load on the pin at A (W),

$$682 = 2 \times \frac{\pi}{4} (d_1)^2 \tau = 2 \times \frac{\pi}{4} (10)^2 \tau = 157.1 \tau$$

$$\therefore \quad \tau = 682 / 157.1 = 4.35 \text{ N/mm}^2 = 4.35 \text{ MPa}$$

This induced stress is very much within safe limits.

The roller pin is fixed in the forked end of the bell crank lever and the roller moves freely on the pin.

Let us now check the pin for induced bending stress. We know that maximum bending moment,

$$M = \frac{5}{24} \times W \times l_1 = \frac{5}{24} \times 682 \times 12.5 = 1776 \text{ N-mm}$$

and section modulus of the pin,

$$Z = \frac{\pi}{32} (d_1)^3 = \frac{\pi}{32} (10)^3 = 98.2 \text{ mm}^3$$

$\therefore$ Bending stress induced

$$= \frac{M}{Z} = \frac{1776}{98.2} = 18.1 \text{ N/mm}^2 = 18.1 \text{ MPa}$$

This induced bending stress is within safe limits.

We know that the thickness of each eye of the fork,

$$t_1 = \frac{l_1}{2} = \frac{12.5}{2} = 6.25 \text{ mm}$$

and outer diameter of the eye,

$$D = 2\, d_1 = 2 \times 10 = 20 \text{ mm}$$

The outer diameter of the roller is taken slightly larger (at least 3 mm more) than the outer diameter of the eye. In the present case, 23 mm outer diameter of the roller will be sufficient. The roller is not provided with bush because after sufficient service, the roller has to be replaced due to wear on the profile. A clearance of 1.5 mm is provided between the roller and fork on either side of roller.

$\therefore$ Total length of the pin,

$$l_2 = l_1 + 2\, t_1 + 2 \times 1.5 = 12.5 + 2 \times 6.25 + 3 = 28 \text{ mm}$$ **Ans.**

15.9 Rocker Arm for Exhaust Valve

A rocker arm for operating the exhaust valve is shown in Fig. 15.20. In designing a rocker arm, the following procedure may be followed :

1. The rocker arm is usually of I-section. Due to the load on the valve, it is subjected to bending moment. In order to find the bending moment, it is assumed that the arm of the lever extends from the point of application of the load to the centre of the pivot which acts as a fulcrum of the rocker arm. This assumption results in a slightly stronger lever near the boss.
2. The ratio of the length to the diameter of the fulcrum and roller pin is taken as 1.25. The permissible bearing pressure on this pin is taken from 3.5 to 6 N/mm^2.
3. The outside diameter of the boss at fulcrum is usually taken as twice the diameter of the pin at fulcrum. The boss is provided with a 3 mm thick phosphor bronze bush to take up wear.
4. One end of the rocker arm has a forked end to receive the roller. The roller is carried on a pin and is free to revolve in an eye to reduce wear. The pin or roller is not provided with a bush because after sufficient service the roller has to be discarded due to wear at the profile.
5. The outside diameter of the eye at the forked end is also taken as twice the diameter of pin. The diameter of the roller is taken slightly larger (at least 3 mm more) than the diameter of

Contents

eye at the forked end. The radial thickness of each eye of the forked end is taken as half the diameter of pin. Some clearance, about 1.5 mm, must be provided between the roller and eye at the forked end so that the roller can move freely. The pin should, therefore, be checked for bending.

6. The other end of the rocker arm (*i.e.* tappet end) is made circular to receive the tappet which is a stud with a lock nut. The outside diameter of the circular arm is taken as twice the diameter of the stud. The depth of the section is also taken equal to twice the diameter of the stud.

Example 15.8. *For operating the exhaust valve of a petrol engine, the maximum load required on the valve is 5000 N. The rocker arm oscillates around a pin whose centre line is 250 mm away from the valve axis. The two arms of the rocker are equal and make an included angle of 160°. Design the rocker arm with the fulcrum if the tensile stress is 70 MPa and the bearing pressure is 7 N/mm². Assume the cross-section of the rocker arm as rectangular.*

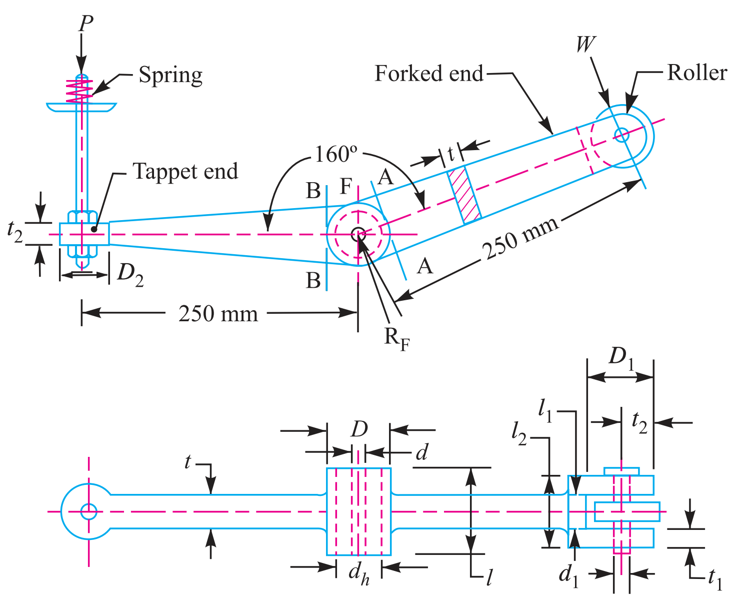


Fig. 15.20

Solution. Given : $W = 5000$ N ; $\theta = 160°$; $\sigma_t = 70$ MPa $= 70$ N/mm²; $p_b = 7$N/mm²

A rocker arm for operating the exhaust valve is shown in Fig. 15.20.

First of all, let us find out the reaction at the fulcrum pin.

Let R_F = Reaction at the fulcrum pin.

Since the two arms of the rocker are equal, therefore the load at the two ends of the arm are equal *i.e.* $W = P = 5000$ N.

We know that

$$R_F = \sqrt{W^2 + P^2 - 2W \times P \times \cos\theta}$$

$$= \sqrt{(5000)^2 + (5000)^2 - 2 \times 5000 \times 5000 \times \cos 160°}$$

$$= \sqrt{25 \times 10^6 + 25 \times 10^6 + 47 \times 10^6} = 9850 \text{ N}$$

Top

Design of fulcrum

Let d = Diameter of the fulcrum pin, and

l = Length of the fulcrum pin = 1.25 d ...(Assume)

Considering the bearing of the fulcrum pin. We know that load on the fulcrum pin (R_F),

$$9850 = d \times l \times p_b = d \times 1.25\, d \times 7 = 8.75\, d^2$$

$$\therefore \quad d^2 = 9850 / 8.75 = 1126 \text{ or } d = 33.6 \text{ say } 35 \text{ mm} \quad \textbf{Ans.}$$

and $l = 1.25\, d = 1.25 \times 35 = 43.75$ say 45 mm **Ans.**

Now let us check the average shear stress induced in the pin. Since the pin is in double shear, therefore load on the fulcrum pin (R_F),

$$9850 = 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (35)^2\, \tau = 1924.5\, \tau$$

$$\therefore \quad \tau = 9850 / 1924.5 = 5.12 \text{ N/mm}^2 = 5.12 \text{ MPa}$$

The induced shear stess is quite safe.

Now external diameter of the boss,

$$D = 2\, d = 2 \times 35 = 70 \text{ mm}$$

Assuming a phosphor bronze bush of 3 mm thick, the internal diameter of the hole in the lever,

$$d_h = d + 2 \times 3 = 35 + 6 = 41 \text{ mm}$$

Now let us check the induced bending stress for the section of the boss at the fulcrum which is shown in Fig. 15.21.

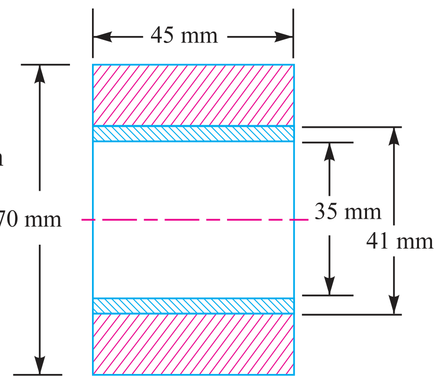


Fig. 15.21

Bending moment at this section

$$= W \times 250$$
$$= 5000 \times 250 \text{ N-mm}$$
$$= 1250 \times 10^3 \text{ N-mm}$$

Section modulus, $$Z = \frac{\frac{1}{12} \times 45 \left[(70)^3 - (41)^3 \right]}{70/2} = 29\,365 \text{ mm}^3$$

$\therefore$ Induced bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{1250 \times 10^3}{29\,365} = 42.6 \text{ N/mm}^2 = 42.6 \text{ MPa}$$

Since the induced bending stress is less than the permissible value of 70 MPa, therefore it is safe.

Design for forked end

Let d_1 = Diameter of the roller pin, and

l_1 = Length of the roller pin = 1.25 d_1 ...(Assume)

Considering bearing of the roller pin. We know that load on the roller pin (W),

$$5000 = d_1 \times l_1 \times p_b = d_1 \times 1.25\, d_1 \times 7 = 8.75\, (d_1)^2$$

$$\therefore \quad (d_1)^2 = 5000 / 8.75 = 571.4 \text{ or } d_1 = 24 \text{ mm} \quad \textbf{Ans.}$$

and $l_1 = 1.25\, d_1 = 1.25 \times 24 = 30$ mm **Ans.**

Let us now check the roller pin for induced shearing stress. Since the pin is in double shear, therefore load on the roller pin (W),

$$5000 = 2 \times \frac{\pi}{4} (d_1)^2\, \tau = 2 \times \frac{\pi}{4} (24)^2\, \tau = 905\, \tau$$

$\therefore \qquad \tau = 5000/905 = 5.5 \text{ N/mm}^2 = 5.5 \text{ MPa}$

This induced shear stress is quite safe.

The roller pin is fixed in eye and the thickness of each eye is taken as half the length of the roller pin.

$\therefore$ Thickness of each eye,

$$t_1 = \frac{l_1}{2} = \frac{30}{2} = 15 \text{ mm}$$

Let us now check the induced bending stress in the roller pin. The pin is neither simply supported in fork nor rigidly fixed at the end. Therefore the common practice is to assume the load distribution as shown in Fig. 15.22.

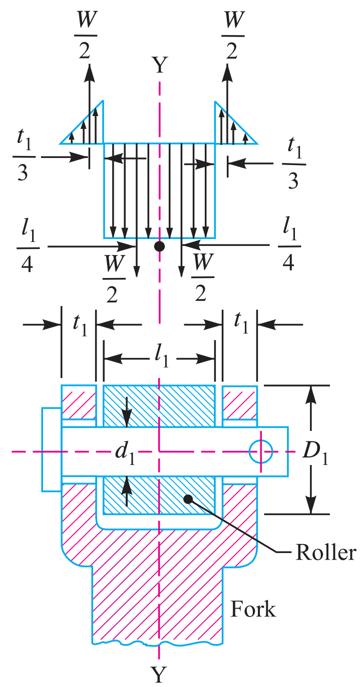


Fig. 15.22

The maximum bending moment will occur at *Y-Y*.

Neglecting the effect of clearance, we have

Maximum bending moment at *Y–Y*,

$$M = \frac{W}{2}\left(\frac{l_1}{2} + \frac{t_1}{3}\right) - \frac{W}{2} \times \frac{l_1}{4}$$

$$= \frac{W}{2}\left(\frac{l_1}{2} + \frac{l_1}{6}\right) - \frac{W}{2} \times \frac{l_1}{4} \qquad \ldots(\because t_1 = l_1 / 2)$$

$$= \frac{5}{24} W \times l_1 = \frac{5}{24} \times 5000 \times 30 \text{ N-mm}$$

$$= 31\,250 \text{ N-mm}$$

and section modulus of the pin,

$$Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(24)^3 = 1357 \text{ mm}^3$$

$\therefore$ Bending stress induced in the pin

$$= \frac{M}{Z} = \frac{31\,250}{1357} = 23 \text{ N/mm}^2 = 23 \text{ MPa}$$

The bending stress induced in the pin is within permissible limit of 70 MPa.

Since the radial thickness of eye (t_2) is taken as $d_1 / 2$, therefore overall diameter of the eye,

$$D_1 = 2d_1 = 2 \times 24 = 48 \text{ mm}$$

The outer diameter of the roller is taken slightly larger (at least 3 mm more) than the outer diameter of the eye.

In the present case, 54 mm outer diameter of the roller will be sufficient.

Providing a clearance of 1.5 mm between the roller and the fork on either side of the roller, we have

$$l_2 = l_1 + 2 \times \frac{t_1}{2} + 2 \times 1.5 = 30 + 2 \times \frac{15}{2} + 3 = 48 \text{ mm}$$

Design of lever arm

The cross-section of the lever arm is obtained by considering the bending of the sections just near the boss of fulcrum on both sides, such as section *A-A* and *B-B*.

Let t = Thickness of the lever arm which is uniform throughout.

B = Width or depth of the lever arm which varies from boss diameter of fulcrum to outside diameter of the eye (for the forked end side) and from boss diameter of fulcrum to thickness t_2 (for the tappet or stud end side).

Now bending moment on sections *A-A* and *B-B*,

$$M = 5000\left(250 - \frac{D}{2}\right) = 5000\left(250 - \frac{70}{2}\right) = 1075 \times 10^3 \text{ N-mm}$$

and section modulus at *A-A* and *B-B*,

$$Z = \frac{1}{6} \times t \times B^2 = \frac{1}{6} \times t \times D^2 = \frac{1}{6} \times t\ (70)^2 = 817\ t \text{ mm}^3$$

...(At sections *A-A* and *B-B*, $B = D$)

We know that bending stress (σ_b),

$$70 = \frac{M}{Z} = \frac{1075 \times 10^3}{817\ t} = \frac{1316}{t}$$

$\therefore$ t = 1316 / 70 = 18.8 say 20 mm **Ans.**

Design for tappet screw

The adjustable tappet screw carries a compressive load of 5000 N. Assuming the screw is made of mild steel for which the allowable compressive stress (σ_c) may be taken as 50 N/mm^2.

Let d_c = Core diameter of the screw.

We know that load on the tappet screw (W),

$$5000 = \frac{\pi}{4}(d_c)^2\sigma_c = \frac{\pi}{4}(d_c)^2\ 50 = 39.3\,(d_c)^2$$

$\therefore$ $(d_c)^2$ = 5000 / 39.3 = 127 or d_c = 11.3 mm

and outer or nominal diameter of the screw,

$d = d_c$ / 0.84 = 11.3 / 0.84

= 13.5 say 14 mm **Ans.**

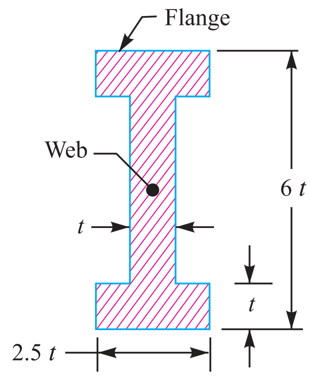


Fig. 15.23

We shall use 14 mm stud and it is provided with a lock nut. The diameter of the circular end of the lever arm (D_2) and its depth (t_2) is taken twice the diameter of stud.

$\therefore$ D_2 = 2 × 14 = 28 mm

and t_2 = 2 × 14 = 28 mm

If the lever arm is assumed to be of *I*-section with proportions as shown in Fig. 15.23 at *A-A* and *B-B*, then section modulus,

$$Z = \frac{\frac{1}{12}\left[2.5\ t\ (6t)^3 - 1.5\ t\ (4t)^3\right]}{6t/2} = \frac{37t^4}{3t} = 12.3\ t^3$$

We know that the maximum bending moment at *A-A* and *B-B*,

$$M = 5000\left(250 - \frac{70}{2}\right) = 1075 \times 10^3 \text{ N-mm}$$

$\therefore$ Bending stress (σ_b),

$$70 = \frac{M}{Z} = \frac{1075 \times 10^3}{12.3t^3} = \frac{87.4 \times 10^3}{t^3}$$

$\therefore$ t^3 = 87.4 × 10^3 / 70 = 1248 or t = 10.77 say 12 mm

Contents

We have assumed that width of the flange

$$= 2.5\,t = 2.5 \times 12 = 30 \text{ mm } \textbf{Ans.}$$

Depth of the web $= 4\,t = 4 \times 12 = 48$ mm **Ans.**

and depth of the section $= 6\,t = 6 \times 12 = 72$ mm **Ans.**

Normally thickness of the flange and web is constant throughout whereas the width and the depth is tapered.

15.10 Miscellaneous Levers

In the previous articles, we have discussed the design of various types of levers used in engineering practice. Some more types of levers designed on the same principle are discussed in the following examples.

Example 15.9. *A pressure vessel as shown in Fig. 15.24, is used as a digester in a chemical process. It is designed to withstand a pressure of 0.2 N/mm² by gauge. The diameter of the pressure vessel is 600 mm. The vessel and its cover are made of cast iron. All other parts are made of steel. The cover is held tightly against the vessel by a screw B which is turned down through the tapped hole in the beam A, so that the end of the screw presses against the cover. The beam A is of rectangular section in which $b_1 = 2\,t_1$.*

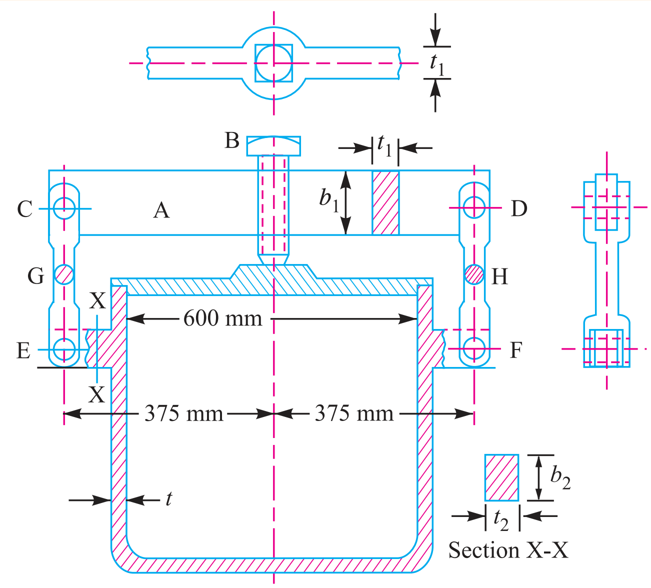


Fig. 15.24

The rectangular section is opened up at the centre to take the tapped hole as shown in the figure. The beam is attached by pins C and D to the links G and H which are secured by pins E and F to the extensions cast on the vessel.

Assume allowable stresses as under :

Material	*Tension*	*Compression*	*Shear*
Cast iron	*17.5 MPa*	—	—
Steel	*52.5 MPa*	*52.5 MPa*	*42 MPa*

Find: 1. Thickness of the vessel; 2. Diameter of the screw; 3. Cross-section of beam A; 3. Diameter of pins C and D; 5. Diameter of pins E and F; 6. Diameter of pins G and H; and 7. Cross-section of the supports of pins E and F.

Top

Contents

This massive crane is used for construction work.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Solution. Given : $p = 0.2$ N/mm^2 ; $d = 600$ mm ; $\sigma_{tc} = 17.5$ MPa = 17.5 N/mm^2 ; $\sigma_{ts} = 52.5$ MPa = 52.5 N/mm^2 ; $\sigma_{cs} = 52.5$ MPa = 52.5 N/mm^2 ; $\tau_s = 42$ MPa = 42 N/mm^2

1. ***Thickness of the vessel***

We know that thickness of the vessel,

$$t = \frac{p \times d}{2\,\sigma_{tc}} = \frac{0.2 \times 600}{2 \times 17.5} = 3.43 \text{ mm}$$

Since the thickness of cast iron casting should not be less than 6 mm, therefore we shall take thickness of the vessel, t = 6 mm. **Ans.**

2. ***Diameter of the screw***

Let d_c = Core diameter of the screw.

We know that load acting on the cover,

$$W = \text{Pressure} \times \text{Cross-sectional area of the cover}$$

$$= p \times \frac{\pi}{4}\,d^2 = 0.2 \times \frac{\pi}{4}\,(600)^2 = 56\,556 \text{ N} \qquad ...(i)$$

We also know that load acting on the cover (W),

$$56\,556 = \frac{\pi}{4}\,(d_c)^2\,\sigma_{ts} = \frac{\pi}{4}\,(d_c)^2\,52.5 = 41.24\,(d_c)^2 \qquad ...(ii)$$

From equations ***(i)*** and ***(ii)***, we have

$\therefore$ $(d_c)^2 = 56\,556 / 41.24 = 1372$ or $d_c = 37$ mm

We shall use a standard screw of size M 48 with core diameter 41.5 mm and outer diameter 48 mm. **Ans.**

3. ***Cross-section of the beam A***

Let t_1 = Thickness of the beam, and

b_1 = Width of the beam = 2 t_1 ...(Given)

Since it is a simply supported beam supported at C and D and the load W acts in the centre, therefore the reactions at C and D (R_C and R_D) will be W/2.

$$\therefore \quad R_C = R_D = \frac{W}{2} = \frac{56\,556}{2} = 28\,278 \text{ N}$$

Top

Maximum bending moment at the centre of beam,

$$M = \frac{W}{2} \times \frac{l}{2} = \frac{56\ 556}{2} \times \frac{750}{2} = 10.6 \times 10^6 \text{ N-mm}$$

and section modulus of the beam,

$$Z = \frac{1}{6} \times t_1 (b_1)^2 = \frac{1}{6} \times t_1 (2t_1)^2 = \frac{2}{3}(t_1)^3 \qquad ...(\because b_1 = 2\,t_1)$$

We know that bending stress (σ_b),

$$52.5 = \frac{M}{Z} = \frac{10.6 \times 10^6 \times 3}{2\,(t_1)^3} = \frac{15.9 \times 10^6}{(t_1)^3} \qquad ...(\text{Substituting } \sigma_b = \sigma_{ts})$$

$\therefore$ $(t_1)^3 = 15.9 \times 10^6 / 52.5 = 303 \times 10^3$ or $t_1 = 67.5$ mm **Ans.**

and $b = 2\,t = 2 \times 67.5 = 135$ mm **Ans.**

4. Diameter of pins C and D

Let d_1 = Diameter of pins *C* and *D*.

The load acting on the pins *C* and *D* are reactions at *C* and *D* due to the load acting on the beam. Since the pins at *C* and *D* are in double shear, therefore load acting on the pins (R_C or R_D).

$$28\ 278 = 2 \times \frac{\pi}{4}(d_1)^2\,\tau_s = 2 \times \frac{\pi}{4}(d_1)^2\,42 = 66\,(d_1)^2$$

$\therefore$ $(d_1)^2 = 28\ 278 / 66 = 428.5$ or $d_1 = 20.7$ say 21 mm **Ans.**

5. Diameter of pins E and F

Since the load on pins *E* and *F* is same as that of *C* and *D*, therefore diameter of pins *E* and *F* will be of same diameter *i.e.* 21 mm. **Ans.**

6. Diameter of links G and H

Let d_2 = Diameter of links *G* and *H*.

A little consideration will show that the links are in tension and the load acting on each link

$$= \frac{W}{2} = \frac{56\ 556}{2} = 28\ 278 \text{ N}$$

We also know that load acting on each link,

$$28\ 278 = \frac{\pi}{4}(d_2)^2\,\sigma_{ts} = \frac{\pi}{4}(d_2)^2\,52.5 = 41(d_2)^2$$

$\therefore$ $(d_2)^2 = 28\ 278 / 41 = 689.7$ or $d_2 = 26.3$ mm **Ans.**

7. Cross-section of the supports of pins E and F

Let t_2 = Thickness of the support, and

b_2 = Width of the support.

The supports are a part of casting with a vessel and acts as a cantilever, therefore maximum bending moment at the support,

$$M = R_C \times x = R_C\,[375 - (300 + t)]$$
$$= 28\ 278\,[375 - (300 + 6)] = 1.95 \times 10^6 \text{ N-mm}$$

and section modulus, $$Z = \frac{1}{6} \times t_2 (b_2)^2 = \frac{1}{6} \times t_2 (2t_2)^2 = \frac{2}{3}(t_2)^3 \qquad ...(\text{Assuming } b_2 = 2t_2)$$

We know that bending stress (σ_b),

$$17.5 = \frac{M}{Z} = \frac{1.95 \times 10^6 \times 3}{2\,(t_2)^3} = \frac{2.9 \times 10^6}{(t_2)^3} \qquad ...(\text{Substituting } \sigma_b = \sigma_{tc})$$

$\therefore$ $(t_2)^3 = 2.9 \times 10^6 / 17.5 = 165.7 \times 10^3$ or $t_2 = 55$ mm **Ans.**

and $b_2 = 2\,t_2 = 2 \times 55 = 110$ mm **Ans.**

Example 15.10. *A cross-lever to operate a double cylinder double acting pump is shown in Fig. 15.25. Find*

1. *Dimension of pins at L, M, N and Q,*
2. *Cross-section for the vertical arm of the lever, and*
3. *Cross-section for the horizontal arm of the lever.*

The permissible shear stress for the material of the pin is 40 MPa. The bearing pressure on the pins should not exeed 17.5 N/mm^2.

The permissible bending stress for the material of the lever should not exceed 70 MPa.

Solution. Given : W_L = 3 kN ; W_N = 5 kN ; τ = 40 MPa = 40 N/mm^2 ; p_b = 17.5 N/mm^2 ; σ_b = 70 MPa = 70 N/mm^2

First of all, let us find the effort *P* applied at *Q*. Taking moments about the fulcrum *M*, we have

$$P \times 800 = 5 \times 300 + 3 \times 300 = 2400 \quad \text{or} \quad P = 2400 / 800 = 3 \text{ kN}$$

When both sides of pump operate, then load on the fulcrum pin *M*,

$$W_M = 5 - 3 = 2 \text{ kN}$$

∴ Resultant force on the fulcrum pin *M*,

$$R_M = \sqrt{(W_M)^2 + P^2} = \sqrt{2^2 + 3^2} = 3.6 \text{ kN}$$

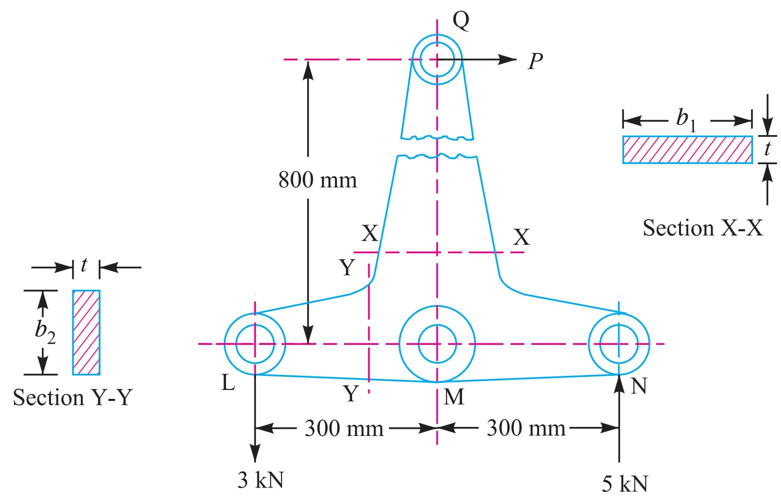


Fig. 15.25

The worst condition arises when one side of the pump does not work. At that time, the effort required increases. Taking moments about *M*, we get

$$P \times 800 = 5 \times 300 = 1500 \quad \text{or} \quad P = 1500 / 800 = 1.875 \text{ kN}$$

∴ In worst condition, the resultant force on the fulcrum pin *M*,

$$R_{M1} = \sqrt{(1.875)^2 + 5^2} = 5.34 \text{ kN} = 5340 \text{ N}$$

Therefore the fulcrum pin *M* will be designed for a maximum load of 5.34 kN.

A little consideration will show that the load on the pins *L* and *Q* is 3 kN each; therefore the pins *L* and *Q* will be of the same size. Since the load on pin *N* (5 kN) do not differ much with the maximum load on pin *M i.e.* 5.34 kN, therefore the pins at *N* and *M* may be taken of the same size.

Contents

1. *Dimension of pins at L, M, N and Q*

First of all, let us find the diameter of pins at *M* and *N*. These pins will be designed for a maximum load of 5.34 kN or 5340 N.

Let d = Diameter of pins at *M* and *N*, and

l = Length of pins at *M* and $N = 1.25\ d$ ...(Assume)

Considering the bearing of the pins. We know that load on the pins,

$$5340 = d \times l \times p_b = d \times 1.25\, d \times 17.5 = 21.87\, d^2$$

$\therefore$ $d^2 = 5340 / 21.87 = 244$ or $d = 15.6$ say 16 mm **Ans.**

and $l = 1.25\, d = 1.25 \times 16 = 20$ mm **Ans.**

Let us check the pin for induced shear stress. Since the pin is in double shear, therefore load on the pin,

$$5340 = 2 \times \frac{\pi}{4}\, d^2 \times \tau = 2 \times \frac{\pi}{4}\,(16)^2\,\tau = 402\,\tau$$

$\therefore$ $\tau = 5340 / 402 = 13.3\ \text{N/mm}^2 = 13.3$ MPa

The induced shear stress is within safe limits.

A 3 mm thick bush may be inserted so that the diameter of hole in the lever is 22 mm. The outside diameter of the boss may be taken as twice the diameter of hole.

$\therefore$ Outside diameter of boss = 2 × 22 = 44 mm

Let us now check the section of the lever for induced bending stress. The section at the fulcrum is shown in Fig. 15.26.

We know the maximum bending moment,

$$M = \frac{5}{24} \times W \times l = \frac{5}{24} \times 5340 \times 20 = 22\ 250\ \text{N}$$

and section modulus,

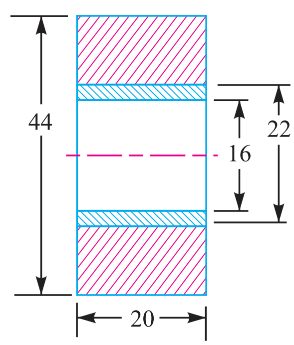
$$Z = \frac{\frac{1}{12} \times 20\left[(44)^3 - (22)^3\right]}{44/2} = 5647\ \text{mm}^3$$

$\therefore$ Bending stress induced,

$$\sigma_b = \frac{M}{Z} = \frac{22\ 250}{5647} = 3.94\ \text{N/mm}^2 = 3.94\ \text{MPa}$$

The bending stress induced is very much within safe limits.

It should be remembered that the direction of the load will be reversed, consequently the loads will be changed. Hence the pins at *L* and *N* must be identical. We shall provide the pin at *Q* of the same size as for *L*, *M* and *N* in order to avoid extra storage. Thus the diameter of pins at *L*, *M*, *N* and *Q* is 16 mm and length 20 mm. **Ans.**



All dimensions in mm.

Fig. 15.26

2. *Cross-section for the vertical arm of the lever*

Considering the cross-section of the vertical arm at *X-X*,

Let t = Thickness of the arm, and

b_1 = Width of the arm = $3\,t$ ... (Assume)

It is assumed that the length of the arm extends upto the centre of the fulcrum. This assumption results in a slightly stronger arm.

$\therefore$ Maximum bending moment,

$$M = P \times 800 = 3 \times 800 = 2400\ \text{kN-mm} = 2.4 \times 10^6\ \text{N-mm}$$

Top

and section modulus,

$$Z = \frac{1}{6} \times t\,(b_1)^2 = \frac{1}{6} \times t\,(3t)^2 = 1.5\,t^3 \text{ mm}^3$$

We know that bending stress (σ_b),

$$70 = \frac{M}{Z} = \frac{2.4 \times 10^6}{1.5\,t^3} = \frac{1.6 \times 10^6}{t^3}$$

$\therefore \quad t^3 = 1.6 \times 10^6 / 70 = 23 \times 10^3$ or $t = 28.4$ say 30 mm **Ans.**

and $\quad b_1 = 3\,t = 3 \times 30 = 90$ mm **Ans.**

Lasers supported by computer controls can cut the metal very accurately

Note : This picture is given as additional information and is not a direct example of the current chapter.

3. Cross-section of horizontal arm of the lever

Considering the cross-section of the arm at *Y-Y*.

Let $\quad t$ = Thickness of the arm. The thickness of the horizontal arm will be same as that of vertical arm.

b_2 = Width of the arm.

Again, assuming that the length of arm extends upto the centre of the fulcrum, therefore maximum bending moment,

$$M = W_N \times 300 = 5 \times 300$$
$$= 1500 \text{ kN-mm} = 1.5 \times 10^6 \text{ N-mm}$$

and section modulus,

$$Z = \frac{1}{6} \times t\,(b_2)^2 = \frac{1}{6} \times 30\,(b_2)^2 = 5\,(b_2)^2$$

We know that bending stress (σ_b),

$$70 = \frac{M}{Z} = \frac{1.5 \times 10^6}{5\,(b_2)^2} = \frac{0.3 \times 10^6}{(b_2)^2}$$

$\therefore \quad (b_2)^2 = 0.3 \times 10^6 / 70 = 43 \times 10^2$

or $\quad b_2 = 65.5$ say 66 mm **Ans.**

Example 15.11. *A bench shearing machine as shown in Fig. 15.27, is used to shear mild steel bars of 5 mm × 3 mm. The ultimate shearing strength of the mild steel is 400 MPa. The permissible tensile stress for pins, links and lever is 80 MPa.*

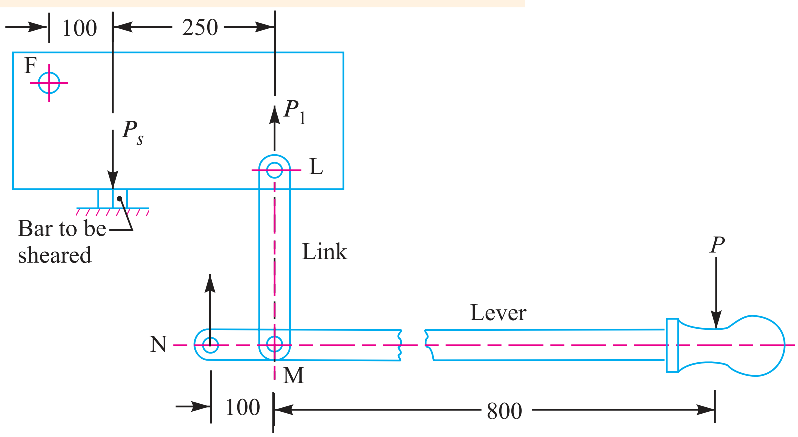


Fig. 15.27

The allowable bearing pressure on pins may be taken as 20 N/mm². Design the pins at L, M and N; the link and the lever.

Solution. Given : $A_s = 5 \times 3 = 15$ mm² ; $\tau_u = 400$ MPa = 400 N/mm² ; $\sigma_t = 80$ MPa = 80 N/mm² ; $p_b = 20$ N/mm²

We know that maximum shearing force required,

$$P_s = \text{Area sheared} \times \text{Ultimate shearing strength}$$

$$= A_s \times \tau_u = 15 \times 400 = 6000 \text{ N}$$

Let P_1 = Force in the link LM.

Taking moments about F, we have

$$P_1 \times 350 = P_s \times 100 = 6000 \times 100 = 600 \times 10^3$$

$$\therefore \quad P_1 = 600 \times 10^3/350 = 1715 \text{ N}$$

Again taking moments about N to find the force P required to operate the handle, we have

$$P \times 900 = P_1 \times 100 = 1715 \times 100 = 171\,500$$

$$\therefore \quad P = 171\,500/900 = 191 \text{ N}$$

and force in the pin at $N = P_1 + P = 1715 + 191 = 1906$ N

Design of pins at L, M and N

We see that the force in the pins at L and M is equal to 1715 N and force in the pin at N is 1906 N. Since the forces in pins at L, M and N do not differ very much, therefore the same size of pins may be used. These pins will be designed for a maximum load of 1906 N.

Let d = Diameter of the pins at L, M and N, and

l = Length of the pins = 1.25 d ...(Assume)

Considering the pins in bearing. We know that load on the pins,

$$1906 = d \times l \times p_b = d \times 1.25\,d \times 20 = 25\,d^2$$

$$\therefore \quad d^2 = 1906 / 25 = 76.2 \text{ or } d = 8.73 \text{ say } 10 \text{ mm} \quad \textbf{Ans.}$$

and $l = 1.25\,d = 1.25 \times 10 = 12.5$ mm **Ans.**

Let us check the pins for induced shear stress. Since the pins are in double shear, therefore load on the pins,

$$1906 = 2 \times \frac{\pi}{4}\,d^2 \times \tau = 2 \times \frac{\pi}{4}\,(10)^2\,\tau = 157.1\,\tau$$

$$\therefore \quad \tau = 1906 / 157.1 = 12.1 \text{ N/mm}^2 = 12.1 \text{ MPa}$$

The induced shear stress is within safe limits.

A 3 mm thick bush in inserted in the hole. Therefore, diameter of the hole in the link and lever

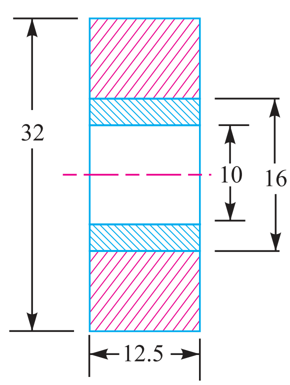
$$= d + 2 \times 3 = 10 + 6 = 16 \text{ mm}$$

The diameter of the boss may be taken as twice the diameter of hole.

∴ Diameter of the boss

$$= 2 \times 16 = 32 \text{ mm}$$

Let us now check the induced bending stress for the cross-section of the lever at N. The cross-section at N is shown in Fig. 15.28.



All dimensions in mm.

Fig. 15.28

We know that maximum bending moment,

$$M = \frac{5}{24} \times W \times l$$

Contents

$$= \frac{5}{24} \times 1906 \times 12.5 = 4964 \text{ N-mm}$$

and section modulus,

$$Z = \frac{\frac{1}{12} \times 12.5 \left[(32)^3 - (16)^3 \right]}{32/2} = 1867 \text{ mm}^3$$

$\therefore$ Induced bending stress

$$= \frac{M}{Z} = \frac{4964}{1867} = 2.66 \text{ N/mm}^2 = 2.66 \text{ MPa}$$

The induced bending stress is very much within safe limits.

Design for link

The link is of circular cross-section with ends forked.

Let d_1 = Diameter of the link.

The link is designed for a maximum load of 1906 N. Since the link is under tension, therefore load on the link,

$$1906 = \frac{\pi}{4} (d_1)^2 \sigma_t = \frac{\pi}{4} (d_1)^2 \, 80 = 62.84 \, (d_1)^2$$

$$\therefore \quad (d_1)^2 = 1906 / 62.84 = 30.3 \quad \text{or} \quad d_1 = 5.5 \text{ mm}$$

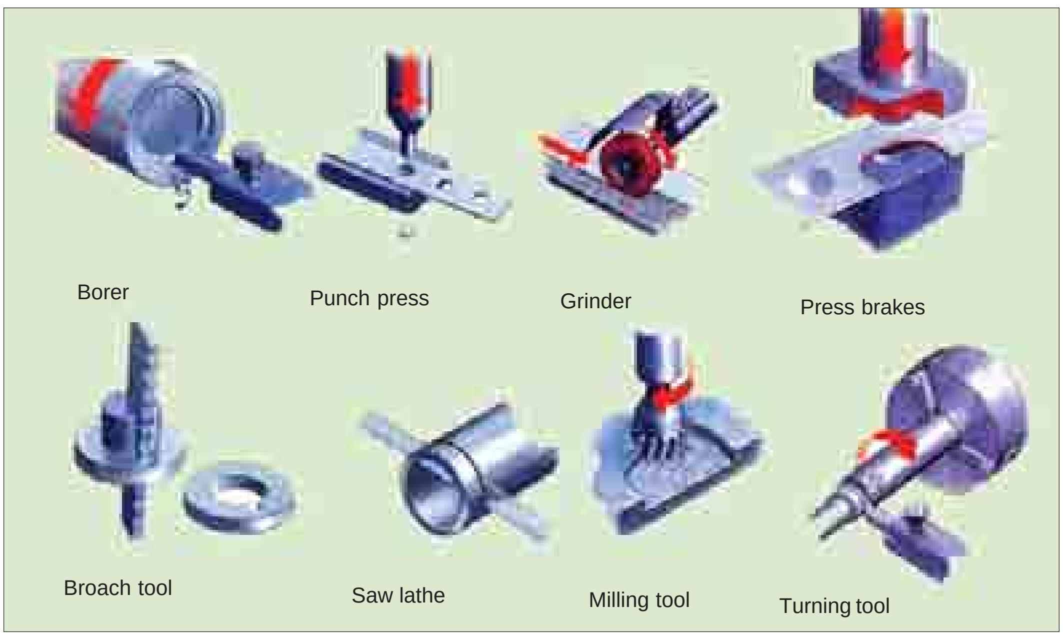
We shall provide the diameter of links (d_1) as 10 mm because forks are to be made at each end. **Ans.**

Design for lever

Assuming the lever to be rectangular.

Let t = Thickness of the lever. The thickness of the lever will be same as that of length of pin *i.e.* 12.5 mm.

B = Width of the lever.



Common machine tools.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Top

We know that maximum bending moment on the lever,

$$M = 1906 \times 100 = 190\,600 \text{ N-mm}$$

Section modulus, $$Z = \frac{1}{6} \times t\ B^2 = \frac{1}{6} \times 12.5\ B^2 = 2.1\ B^2$$

We know that bending stress (σ_b),

$$80 = \frac{M}{Z} = \frac{190\,600}{2.1\ B^2} = \frac{90\,762}{B^2}$$

$\therefore$ $B^2 = 90\,762 / 80 = 1134$ or $B = 33.7$ say 34 mm **Ans.**

The handle at the end of the lever is made 125 mm long with maximum diameter 32 mm and minimum diameter 25 mm. **Ans.**

EXERCISES

1. The spindle of a large valve is turned by a handle as shown in Fig. 15.5. The length of the handle from the centre of the spindle is 500 mm. The handle is attached to the spindle by means of a round tapered pin. If the spindle diameter is 60 mm and an effort of 300 N is applied at the end of the handle, find the dimensions for the tapered pin and the handle. The grip length of the handle may be taken as 200 mm. The allowable stresses for the handle and key are 100 MPa and 55 MPa in shear.

   [Ans. d_1 = 12 mm ; D = 25 mm]

2. A vertical lever PQR of length 1 m is attached by a fulcrum pin at R and to a horizontal rod at Q. An operating force of 700 N is applied horizontally at P. The distance of the horizontal rod Q from the fulcrum pin R is 140 mm. If the permissible stresses are 52.5 MPa in tension and compression and 32 MPa in shear; find the diameter of the pins, tie rod at Q and thickness of the lever. The bearing pressure on the pins may be taken as 22 N/mm^2.

   [Ans. 5.4 mm ; 14 mm ; 13 mm ; 11 mm ; 16.5 mm]

3. A hand lever for a brake is 0.8 m long from the centre of gravity of the spindle to the point of application of the pull of 300 N. The effective overhang from the nearest bearing is 100 mm. If the permissible stress in tension, shear and crushing is not to exceed 66 MPa, design the spindle, key and lever. Assume the arm of the lever to be rectangular having width twice of its thickness.

   [Ans. d = 45 mm ; l_1 = 45 mm ; t = 13 mm]

4. Design a foot brake lever from the following data :

   Length of lever from the centre of gravity of the spindle
   to the point of application of load = 1 metre
   Maximum load on the foot plate = 800 N
   Overhang from the nearest bearing = 100 mm
   Permissible tensile and shear stress = 70 MPa **[Ans. d = 40 mm ; l_1 = 40 mm ; t = 20 mm]**

5. Design a cranked lever for the following dimensions :

   Length of the handle = 320 mm
   Length of the lever arm = 450 mm
   Overhang of the journal = 120 mm

   The lever is operated by a single person exerting a maximum force of 400 N at a distance of 1/3rd length of the handle from its free end. The permissible stresses may be taken as 50 MPa for lever material and 40 MPa for shaft material.

   [Ans. d = 42 mm ; t = 20 mm ; B = 40 mm ; D = 32 mm]

6. A lever safety valve is 75 mm in diameter. It is required to blow off at 1.3 N/mm^2. Design the mild steel lever of rectangular cross-section if the permissible stresses are 70 MPa in tension, 52.5 MPa in shear and 24.5 MPa in bearing. The pin is made of the same material as that of the lever. The distance from the fulcrum to the dead weight of the lever is 800 mm and the distance between the fulcrum pin and the valve spindle link pin is 80 mm. **[Ans. t = 7.25 mm ; b = 21.75 mm]**

7. The line sketch of a lever of loaded safety valve is shown in Fig. 15.29. The maximum force at which the valve blows is 4000 N. The weight at the end of the lever is 300 N and the distance between the fulcrum point and the line of action of valve force is 'a'. The following permissible values may be used :

   For lever : $\sigma_t = \sigma_c$ = 40 MPa and τ = 25 MPa

   For pins : $\sigma_t = \sigma_c$ = 60 MPa and τ = 35 MPa

   Find the distance 'a'. Design the lever and make a neat sketch of the lever. Take lever cross-section as ($t \times 3\,t$). The permissible bearing stress is 20 MPa. **[Ans. 73.5 mm]**

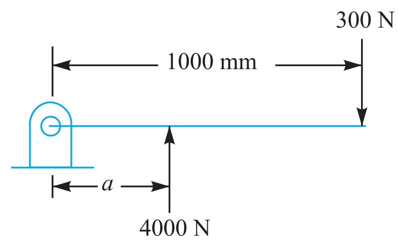


Fig. 15.29

8. Design a right angled bell crank lever having one arm 500 mm and the other 150 mm long. The load of 5 kN is to be raised acting on a pin at the end of 500 mm arm and the effort is applied at the end of 150 mm arm. The lever consists of a steel forgings, turning on a point at the fulcrum. The permissible stresses for the pin and lever are 84 MPa in tension and compression and 70 MPa in shear. The bearing pressure on the pin is not to exceed 10 N/mm^2. **[Ans. t = 27 mm ; b = 81 mm]**

9. Design a bell crank lever to apply a load of 5 kN (vertical) at the end A of an horizontal arm of length 400 mm. The end of the vertical arm C and the fulcrum B are to be fixed with the help of pins inside forked shaped supports. The end A is itself forked. Determine the cross-section of the arms and the dimensions of the pins. The lever is to have mechanical advantage of 4 with a shorter vertical arm BC. The ultimate stresses in shear and tension for the lever and pins are 400 MPa and 500 MPa respectively. The allowable bearing pressure for the pins is 12 N/mm^2. Make a sketch of the lever to scale and give all the dimensions. **[Ans. t = 22 mm ; b = 66 mm]**

   [**Hint.** Assume a factor of safety as 4 and the cross-section of the lever as rectangular with depth (b) as three times the thickness (t).]

10. A Hartnell type governor as shown in Fig. 15.17 has the ball arm length 120 mm and sleeve arm length 90 mm. The maximum and minimum distances of the balls from the axis of governor are 150 mm and 75 mm. The mass of each ball is 2.2 kg. The speed of the governor fluctuates between 310 r.p.m. and 290 r.p.m. Design the cast iron ball and mild steel lever. The permissible tensile stress for the lever material may be taken as 100 MPa. The bearing pressure for the roller and pin should not exceed 7 N/mm^2. **[Ans. r = 42 mm ; t = 12 mm ; b = 36 mm]**

11. The pivots of the bell crank levers of a spring loaded governor of Hartnell type are fixed at 100 mm radius from the spindle axis. The length of the ball arm of each lever is 150 mm, the length of the sleeve arm is 75 mm and the two arms are at right angles. The mass of each ball is 2 kg. The equilibrium speed in the lowest position of the governor is 300 r.p.m. when the radius of rotation of the ball path is 82 mm. The speed is to be limited to 6% more than the lowest equilibrium speed. The lift of the sleeve, for the operating speed range, is 15 mm. Design and draw a bell crank lever for the governor. **[Ans. t = 7 mm ; b = 21 mm]**

12. The maximum load at the roller end of a rocker arm is 2000 N. The distance between the centre of boss and the load line is 200 mm. Suggest suitable I-section of the rocker arm, if the permissible normal stress is limited to 70 MPa. **[Ans. t = 10 mm]**

    [**Hint.** The dimensions of I-section may be taken as follows :

    Top and bottom flanges = 2.5 $t \times t$ and Web = 4 $t \times t$]

QUESTIONS

1. What is a lever ? Explain the principle on which it works.
2. What do you understand by leverage ?
3. Why are levers usually tapered ?
4. (*a*) Why are bushes of softer material inserted in the eyes of levers ?
   (*b*) Why is a boss generally needed at the fulcrum of the levers.
5. State the application of hand and foot levers. Discuss the procedure for designing a hand or foot lever.
6. A lever is to be designed for a hoisting winch. Write the procedure for designing a lever for such operation.
7. Explain the design procedure of a lever for a lever safety valve.
8. Discuss the design procedure of a rocker arm for operating the exhaust valve.

OBJECTIVE TYPE QUESTIONS

1. In levers, the leverage is the ratio of
   (*a*) load lifted to the effort applied (*b*) mechanical advantage to the velocity ratio
   (*c*) load arm to the effort arm (*d*) effort arm to the load arm
2. In the levers of first type, the mechanical advantage is................ one.
   (*a*) less than (*b*) equal to
   (*c*) more than
3. The bell crank levers used in railway signalling arrangement are of
   (*a*) first type of levers (*b*) second type of levers
   (*c*) third type of levers
4. The rocker arm in internal combustion engines are of .... type of levers.
   (*a*) first (*b*) second
   (*c*) third
5. The cross-section of the arm of a bell crank lever is
   (*a*) rectangular (*b*) elliptical
   (*c*) I-section (*d*) any one of these
6. All the types of levers are subjected to
   (*a*) twisting moment (*b*) bending moment
   (*c*) direct axial load (*d*) combined twisting and bending moment
7. The method of manufacturing usually adopted for levers is
   (*a*) casting (*b*) fabrication
   (*c*) forging (*d*) machining
8. An *I*-section is more suitable for a
   (*a*) rocker arm (*b*) cranked lever
   (*c*) foot lever (*d*) lever of lever safety valve
9. The design of the pin of a rocker arm of an I.C. Engine is based on
   (*a*) tensile, creep and bearing failure (*b*) creep, bearing and shearing failure
   (*c*) bearing, shearing and bending failure (*d*) none of these
10. In designing a rocker arm for operating the exhaust valve, the ratio of the length to the diameter of the fulcrum and roller pin is taken as
    (*a*) 1.25 (*b*) 1.5 (*c*) 1.75 (*d*) 2

ANSWERS

1. (*d*)	**2.** (*c*)	**3.** (*c*)	**4.** (*a*)	**5.** (*d*)
6. (*b*)	**7.** (*c*)	**8.** (*a*)	**9.** (*c*)	**10.** (*a*)

Contents

CHAPTER

16

Columns and Struts

1. Introduction.
2. Failure of a Column or Strut.
3. Types of End Conditions of Columns.
4. Euler's Column Theory.
5. Assumptions in Euler's Column Theory.
6. Euler's Formula.
7. Slenderness Ratio.
8. Limitations of Euler's Formula.
9. Equivalent Length of a Column.
10. Rankine's Formula for Columns.
11. Johnson's Formula for Columns.
12. Long Columns Subjected to Eccentric Loading.
13. Design of Piston Rod.
14. Design of Push Rods.
15. Design of Connecting Rod.
16. Forces Acting on a Connecting Rod.

16.1 Introduction

A machine part subjected to an axial compressive force is called a ***strut.*** A strut may be horizontal, inclined or even vertical. But a vertical strut is known as a ***column, pillar*** or ***stanchion.*** The machine members that must be investigated for column action are piston rods, valve push rods, connecting rods, screw jack, side links of toggle jack etc. In this chapter, we shall discuss the design of piston rods, valve push rods and connecting rods.

Note: The design of screw jack and toggle jack is discussed in the next chapter on 'Power screws'.

16.2 Failure of a Column or Strut

It has been observed that when a column or a strut is subjected to a compressive load and the load is gradually increased, a stage will reach when the column will be subjected to ultimate load. Beyond this, the column will fail by crushing and the load will be known as ***crushing load.***

Top

Depending on the end conditions, different columns have different crippling loads

It has also been experienced, that sometimes, a compression member does not fail entirely by crushing, but also by bending *i.e.* buckling. This happens in the case of long columns. It has also been observed, that all the *short columns fail due to their crushing. But, if a **long column is subjected to a compressive load, it is subjected to a compressive stress. If the load is gradually increased, the column will reach a stage, when it will start buckling. The load, at which the column tends to have lateral displacement or tends to buckle is called ***buckling load, critical load,*** or ***crippling load*** and the column is said to have developed an elastic instability. The buckling takes place about the axis having minimum radius of gyration or least moment of inertia. It may be noted that for a long column, the value of buckling load will be less than the crushing load. Moreover, the value of buckling load is low for long columns, and relatively high for short columns.

16.3 Types of End Conditions of Columns

In actual practice, there are a number of end conditions for columns. But we shall study the Euler's column theory on the following four types of end conditions which are important from the subject point of view:

1. Both the ends hinged or pin jointed as shown in Fig. 16.1 (*a*),
2. Both the ends fixed as shown in Fig. 16.1 (*b*),
3. One end is fixed and the other hinged as shown in Fig. 16.1 (*c*), and
4. One end is fixed and the other free as shown in Fig. 16.1 (*d*).

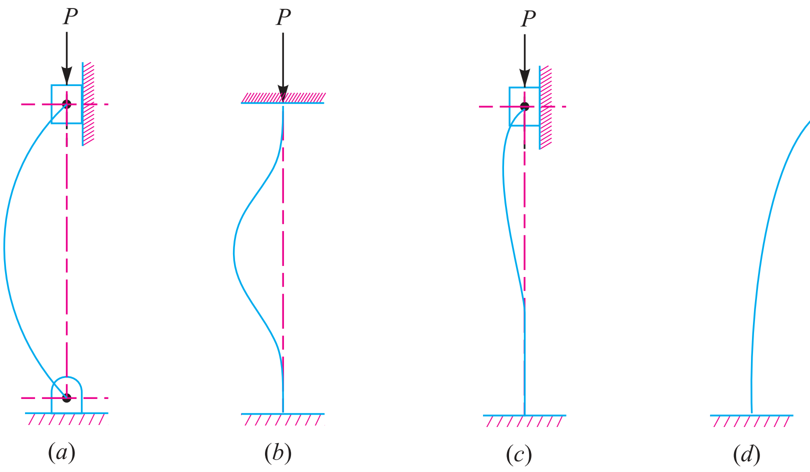


Fig. 16.1. Types of end conditions of columns.

16.4 Euler's Column Theory

The first rational attempt, to study the stability of long columns, was made by Mr. Euler. He

* The columns which have lengths less than 8 times their diameter, are called ***short columns*** (see also Art 16.8).

** The columns which have lengths more than 30 times their diameter are called ***long columns.***

Contents

derived an equation, for the buckling load of long columns based on the bending stress. While deriving this equation, the effect of direct stress is neglected. This may be justified with the statement, that the direct stress induced in a long column is negligible as compared to the bending stress. It may be noted that Euler's formula cannot be used in the case of short columns, because the direct stress is considerable, and hence cannot be neglected.

16.5 Assumptions in Euler's Column Theory

The following simplifying assumptions are made in Euler's column theory :

1. Initially the column is perfectly straight, and the load applied is truly axial.
2. The cross-section of the column is uniform throughout its length.
3. The column material is perfectly elastic, homogeneous and isotropic, and thus obeys Hooke's law.
4. The length of column is very large as compared to its cross-sectional dimensions.
5. The shortening of column, due to direct compression (being very small) is neglected.
6. The failure of column occurs due to buckling alone.
7. The weight of the column itself is neglected.

16.6 Euler's Formula

According to Euler's theory, the crippling or buckling load (W_{cr}) under various end conditions is represented by a general equation,

$$W_{cr} = \frac{C\,\pi^2\,E\,I}{l^2} = \frac{C\,\pi^2\,E\,A\,k^2}{l^2} \qquad \ldots (\because I = A.k^2)$$

$$= \frac{C\,\pi^2\,E\,A}{(l/k)^2}$$

where E = Modulus of elasticity or Young's modulus for the material of the column,

A = Area of cross-section,

k = Least radius of gyration of the cross-section,

l = Length of the column, and

C = Constant, representing the end conditions of the column or end fixity coefficient.

The following table shows the values of end fixity coefficient (C) for various end conditions.

Table 16.1. Values of end fixity coefficient (C).

S. No.	End conditions	End fixity coefficient (C)
1.	Both ends hinged	1
2.	Both ends fixed	4
3.	One end fixed and other hinged	2
4.	One end fixed and other end free	0.25

Notes : 1. The vertical column will have two moment of inertias (*viz.* I_{xx} and I_{yy}). Since the column will tend to buckle in the direction of least moment of inertia, therefore the least value of the two moment of inertias is to be used in the relation.

2. In the above formula for crippling load, we have not taken into account the direct stresses induced in the material due to the load which increases gradually from zero to the crippling value. As a matter of fact, the combined stresses (due to the direct load and slight bending), reaches its allowable value at a load lower than that required for buckling and therefore this will be the limiting value of the safe load.

Top

Contents

16.7 Slenderness Ratio

In Euler's formula, the ratio l / k is known as ***slenderness ratio***. It may be defined as the ratio of the effective length of the column to the least radius of gyration of the section.

It may be noted that the formula for crippling load, in the previous article is based on the assumption that the slenderness ratio l / k is so large, that the failure of the column occurs only due to bending, the effect of direct stress (*i.e.* W / A) being negligible.

This equipment is used to determine the crippling load for axially loaded long struts.

16.8 Limitations of Euler's Formula

We have discussed in Art. 16.6 that the general equation for the crippling load is

$$W_{cr} = \frac{C\,\pi^2\,E\,A}{(l/k)^2}$$

∴ Crippling stress,

$$\sigma_{cr} = \frac{W_{cr}}{A} = \frac{C\,\pi^2\,E}{(l/k)^2}$$

A little consideration will show that the crippling stress will be high, when the slenderness ratio is small. We know that the crippling stress for a column cannot be more than the crushing stress of the column material. It is thus obvious that the Euler's fromula will give the value of crippling stress of the column (equal to the crushing stress of the column material) corresponding to the slenderness ratio. Now consider a mild steel column. We know that the crushing stress for mild steel is 330 N/mm^2 and Young's modulus for mild steel is 0.21×10^6 N/mm^2.

Now equating the crippling stress to the crushing stress, we have

$$\frac{C\,\pi^2\,E}{(l/k)^2} = 330$$

$$\frac{1 \times 9.87 \times 0.21 \times 10^6}{(l/k)^2} = 330 \qquad \text{... (Taking } C = 1)$$

Top

Contents

or $$(l / k)^2 = 6281$$

∴ $$l / k = 79.25 \text{ say } 80$$

Hence if the slenderness ratio is less than 80, Euler's formula for a mild steel column is not valid.

Sometimes, the columns whose slenderness ratio is more than 80, are known as ***long columns,*** and those whose slenderness ratio is less than 80 are known as ***short columns.*** It is thus obvious that the Euler's formula holds good only for long columns.

16.9 Equivalent Length of a Column

Sometimes, the crippling load according to Euler's formula may be written as

$$W_{cr} = \frac{\pi^2 E I}{L^2}$$

where L is the equivalent length or effective length of the column. The equivalent length of a given column with given end conditions is the length of an equivalent column of the same material and cross-section with hinged ends to that of the given column. The relation between the equivalent length and actual length for the given end conditions is shown in the following table.

Table 16.2. Relation between equivalent length (*L*) and actual length (*l*).

S.No.	*End Conditions*	*Relation between equivalent length (L) and actual length (l)*
1.	Both ends hinged	$L = l$
2.	Both ends fixed	$L = \frac{l}{2}$
3.	One end fixed and other end hinged	$L = \frac{l}{\sqrt{2}}$
4.	One end fixed and other end free	$L = 2l$

Example 16.1. *A T-section 150 mm × 120 mm × 20 mm is used as a strut of 4 m long hinged at both ends. Calculate the crippling load, if Young's modulus for the material of the section is 200 kN/mm².*

Solution. Given : l = 4 m = 4000 mm ; E = 200 kN/mm^2 = 200 × 10^3 N/mm^2

First of all, let us find the centre of gravity (G) of the T-section as shown in Fig. 16.2.

Let $\bar{y}$ be the distance between the centre of gravity (G) and top of the flange,

We know that the area of flange,

$$a_1 = 150 \times 20 = 3000 \text{ mm}^2$$

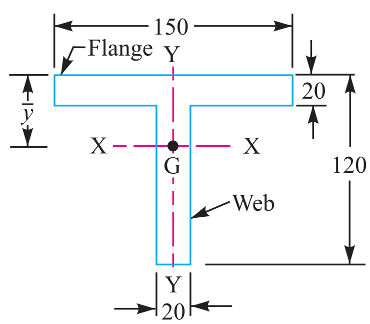
Its distance of centre of gravity from top of the flange,

$$y_1 = 20 / 2 = 10 \text{ mm}$$

Area of web, $$a_2 = (120 - 20)\, 20 = 2000 \text{ mm}^2$$

Its distance of centre of gravity from top of the flange,

$$y_2 = 20 + 100 / 2 = 70 \text{ mm}$$



All dimensions in mm.

Fig. 16.2

∴ $$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{3000 \times 10 + 2000 \times 70}{3000 + 2000} = 34 \text{ mm}$$

Top

We know that the moment of inertia of the section about X-X,

$$I_{XX} = \left[\frac{150\ (20)^3}{12} + 3000\ (34 - 10)^2 + \frac{20\ (100)^3}{12} + 2000\ (70 - 34)^2\right]$$

$$= 6.1 \times 10^6 \text{ mm}^4$$

and $$I_{YY} = \frac{20\ (150)^3}{12} + \frac{100\ (20)^3}{12} = 5.7 \times 10^6 \text{ mm}^4$$

Since I_{YY} is less than I_{XX}, therefore the column will tend to buckle in Y-Y direction. Thus we shall take the value of I as $I_{YY} = 5.7 \times 10^6$ mm^4.

Moreover as the column is hinged at its both ends, therefore equivalent length,

$$L = l = 4000 \text{ mm}$$

We know that the crippling load,

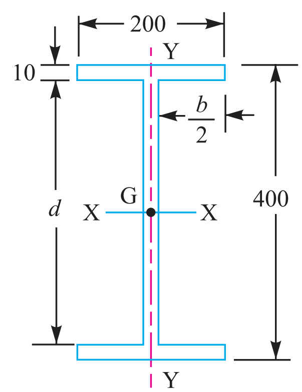
$$W_{cr} = \frac{\pi^2 E\ I}{L^2} = \frac{9.87 \times 200 \times 10^3 \times 5.7 \times 10^6}{(4000)^2} = 703 \times 10^3 \text{ N} = 703 \text{ kN} \quad \textbf{Ans.}$$

Example 16.2. *An I-section 400 mm × 200 mm × 10 mm and 6 m long is used as a strut with both ends fixed. Find Euler's crippling load. Take Young's modulus for the material of the section as 200 kN/mm^2.*

Solution. Given : $D = 400$ mm ; $B = 200$ mm ; $t = 10$ mm ; $l = 6$ m $= 6000$ mm ; $E = 200$ kN/mm^2 $= 200 \times 10^3$ N/mm^2

The I-section is shown in Fig. 16.3.

Crippling load.



All dimensions in mm.

Fig. 16.3

We know that the moment of inertia of the I-section about X-X,

$$I_{XX} = \frac{B.D^3}{12} - \frac{b.d^3}{12}$$

$$= \frac{200\ (400)^3}{12} - \frac{(200 - 10)\ (400 - 20)^3}{12}$$

$$= 200 \times 10^6 \text{ mm}^4$$

and moment of inertia of the I-section about Y-Y,

$$I_{YY} = 2\left(\frac{t.B^3}{12}\right) + \frac{d.t^3}{12}$$

$$= 2\left[\frac{10\ (200)^3}{12}\right] + \frac{(400 - 20)\ 10^3}{12}$$

$$= 13.36 \times 10^6 \text{ mm}^4$$

Since I_{YY} is less than I_{XX}, therefore the section will tend to buckle about Y-Y axis. Thus we shall take I as I_{YY} = 13.36 × 10^4 mm^4.

Since the column is fixed at its both ends, therefore equivalent length,

$$L = l / 2 = 6000 / 2 = 3000 \text{ mm}$$

We know that the crippling load,

$$W_{cr} = \frac{\pi^2 E I}{L^2} = \frac{9.87 \times 200 \times 10^3 \times 13.36 \times 10^6}{(3000)^2} = 2.93 \times 10^6 \text{ N}$$

$$= 2930 \text{ kN}$$ **Ans.**

16.10 Rankine's Formula for Columns

We have already discussed that Euler's formula gives correct results only for very long columns. Though this formula is applicable for columns, ranging from very long to short ones, yet it does not give reliable results. Prof. Rankine, after a number of experiments, gave the following empirical formula for columns.

$$\frac{1}{W_{cr}} = \frac{1}{W_C} + \frac{1}{W_E} \quad \text{...}(i)$$

where W_{cr} = Crippling load by Rankine's formula,

W_C = Ultimate crushing load for the column = $\sigma_c \times A$,

W_E = Crippling load, obtained by Euler's formula = $\dfrac{\pi^2 E I}{L^2}$

A little consideration will show, that the value of W_C will remain constant irrespective of the fact whether the column is a long one or short one. Moreover, in the case of short columns, the value of W_E will be very high, therefore the value of 1 / W_E will be quite negligible as compared to 1 / W_C. It is thus obvious, that the Rankine's formula will give the value of its crippling load (*i.e.* W_{cr}) approximately equal to the ultimate crushing load (*i.e.* W_C). In case of long columns, the value of W_E will be very small, therefore the value of 1 / W_E will be quite considerable as compared to 1 / W_C. It is thus obvious, that the Rankine's formula will give the value of its crippling load (*i.e.* W_{cr}) approximately equal to the crippling load by Euler's formula (*i.e.* W_E). Thus, we see that Rankine's formula gives a fairly correct result for all cases of columns, ranging from short to long columns.

From equation (*i*), we know that

$$\frac{1}{W_{cr}} = \frac{1}{W_C} + \frac{1}{W_E} = \frac{W_E + W_C}{W_C \times W_E}$$

$$\therefore \quad W_{cr} = \frac{W_C \times W_E}{W_C + W_E} = \frac{W_C}{1 + \dfrac{W_C}{W_E}}$$

Now substituting the value of W_C and W_E in the above equation, we have

$$W_{cr} = \frac{\sigma_c \times A}{1 + \dfrac{\sigma_c \times A \times L^2}{\pi^2 E I}} = \frac{\sigma_c \times A}{1 + \dfrac{\sigma_c}{\pi^2 E} \times \dfrac{A.L^2}{A.k^2}} \quad \text{... } (\because I = A.k^2)$$

$$= \frac{\sigma_c \times A}{1 + a\left(\dfrac{L}{k}\right)^2} = \frac{\text{Crushing load}}{1 + a\left(\dfrac{L}{k}\right)^2}$$

where σ_c = Crushing stress or yield stress in compression,

A = Cross-sectional area of the column,

a = Rankine's constant = $\dfrac{\sigma_c}{\pi^2 E}$,

Contents

L = Equivalent length of the column, and

k = Least radius of gyration.

The following table gives the values of crushing stress and Rankine's constant for various materials.

Table 16.3. Values of crushing stress (σ_c) and Rankine's constant (a) for various materials.

S.No.	*Material*	σ_c *in MPa*	$a = \dfrac{\sigma_c}{\pi^2 E}$
1.	Wrought iron	250	$\dfrac{1}{9000}$
2.	Cast iron	550	$\dfrac{1}{1600}$
3.	Mild steel	320	$\dfrac{1}{7500}$
4.	Timber	50	$\dfrac{1}{750}$

16.11 Johnson's Formulae for Columns

Prof. J.B. Johnson proposed the following two formula for short columns.

1. *Straight line formula.* According to straight line formula proposed by Johnson, the critical or crippling load is

$$W_{cr} = A\left[\sigma_y - \frac{2\,\sigma_y}{3\pi}\left(\frac{L}{k}\right)\sqrt{\frac{\sigma_y}{3C \times E}}\right] = A\left[\sigma_y - C_1\left(\frac{L}{k}\right)\right]$$

where A = Cross-sectional area of column,

σ_y = Yield point stress,

$$C_1 = \frac{2\,\sigma_y}{3\pi}\sqrt{\frac{\sigma_y}{3C.E}}$$

= A constant, whose value depends upon the type of material as well as the type of ends, and

$\dfrac{L}{k}$ = Slenderness ratio.

If the safe stress (W_{cr} / A) is plotted against slenderness ratio (L / k), it works out to be a straight line, so it is known as straight line formula.

2. *Parabolic formula.* Prof. Johnson after proposing the straight line formula found that the results obtained by this formula are very approximate. He then proposed another formula, according to which the critical or crippling load,

$$W_{cr} = A \times \sigma_y\left[1 - \frac{\sigma_y}{4\,C\,\pi^2\,E}\left(\frac{L}{k}\right)^2\right] \text{ with usual notations.}$$

If a curve of safe stress (W_{cr} / A) is plotted against (L / k), it works out to be a parabolic, so it is known as parabolic formula.

Fig. 16.4 shows the relationship of safe stress (W_{cr} / A) and the slenderness ratio (L / k) as given by Johnson's formula and Euler's formula for a column made of mild steel with both ends hinged (*i.e.* C = 1), having a yield strength, σ_y = 210 MPa. We see from the figure that point A (the point of tangency between the Johnson's straight line formula and Euler's formula) describes the use of two formulae. In other words, Johnson's straight line formula may be used when L / k < 180 and the Euler's formula is used when L / k > 180.

Top

Similarly, the point *B* (the point of tangency between the Johnson's parabolic formula and Euler's formula) describes the use of two formulae. In other words, Johnson's parabolic formula is used when $L / k < 140$ and the Euler's formula is used when $L / k > 140$.

Note : For short columns made of ductile materials, the Johnson's parabolic formula is used.

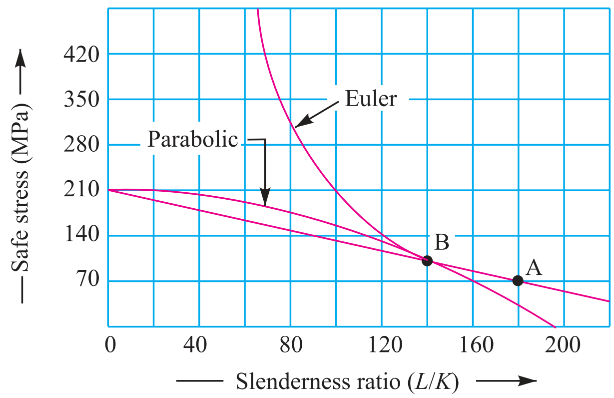


Fig. 16.4. Relation between slendeness ratio and safe stress.

16.12 Long Columns Subjected to Eccentric Loading

In the previous articles, we have discussed the effect of loading on long columns. We have always referred the cases when the load acts axially on the column (*i.e.* the line of action of the load coincides with the axis of the column). But in actual practice it is not always possible to have an axial load on the column, and eccentric loading takes place. Here we shall discuss the effect of eccentric loading on the Rankine's and Euler's formula for long columns.

Consider a long column hinged at both ends and subjected to an eccentric load as shown in Fig. 16.5.

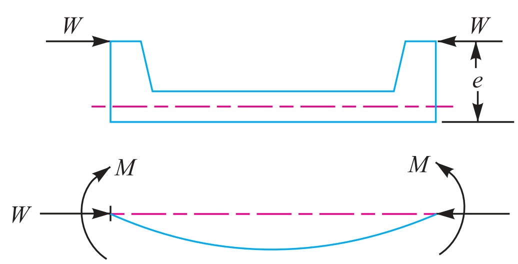


Fig. 16.5. Long column subjected to eccentric loading.

Let
W = Load on the column,
A = Area of cross-section,
e = Eccentricity of the load,
Z = Section modulus,
y_c = Distance of the extreme fibre (on compression side) from the axis of the column,
k = Least radius of gyration,
I = Moment of inertia = $A.k^2$,
E = Young's modulus, and
l = Length of the column.

We have already discussed that when a column is subjected to an eccentric load, the maximum intensity of compressive stress is given by the relation

$$\sigma_{max} = \frac{W}{A} + \frac{M}{Z}$$

The maximum bending moment for a column hinged at both ends and with eccentric loading is given by

$$M = W.e.\sec \frac{l}{2}\sqrt{\frac{W}{E.I}} = W.e.\sec \frac{l}{2\,k}\sqrt{\frac{W}{E.A}} \qquad \ldots (\because\ I = A.k^2)$$

$$\therefore \qquad \sigma_{max} = \frac{W}{A} + \frac{W.e.\sec \frac{l}{2\,k}\sqrt{\frac{W}{E.A}}}{Z}$$

$$= \frac{W}{A} + \frac{W.e.y_c.\sec \frac{l}{2\,k}\sqrt{\frac{W}{E.A}}}{A.k^2} \qquad \ldots (\because\ Z = I/y_c = A.k^2/y_c)$$

$$= \frac{W}{A}\left[1 + \frac{e.y_c}{k^2}\sec \frac{l}{2\,k}\sqrt{\frac{W}{E.A}}\right]$$

$$= {}^{*}\frac{W}{A}\left[1 + \frac{e.y_c}{k^2}\sec \frac{L}{2\,k}\sqrt{\frac{W}{E.A}}\right]$$

... (Substituting $l = L$, equivalent length for both ends hinged).

16.13 Design of Piston Rod

Since a piston rod moves forward and backward in the engine cylinder, therefore it is subjected to alternate tensile and compressive forces. It is usually made of mild steel. One end of the piston rod is secured to the piston by means of tapered rod provided with nut. The other end of the piston rod is joined to crosshead by means of a cotter.

Piston rod is made of mild steel.

* The expression $\sigma_{max} = \frac{W}{A}\left[1 + \frac{e.y_c}{k^2}\sec \frac{L}{2k}\sqrt{\frac{W}{E.A}}\right]$ may also be written as follows:

$$\sigma_{max} = \frac{W}{A} + \frac{W}{A} \times \frac{e.y_c}{\frac{I}{A}}\sec \frac{L}{2k}\sqrt{\frac{W}{E \times \frac{I}{k^2}}} \qquad \ldots\left(\text{Substituting } k^2 = \frac{I}{A} \text{ and } A = \frac{I}{k^2}\right)$$

$$= \frac{W}{A} + \frac{W.e}{Z}\sec \frac{L}{2}\sqrt{\frac{W}{E.I}}$$

Contents

Let p = Pressure acting on the piston,
D = Diameter of the piston,
d = Diameter of the piston rod,
W = Load acting on the piston rod,
W_{cr} = Buckling or crippling load = W × Factor of safety,
σ_t = Allowable tensile stress for the material of rod,
σ_c = Compressive yield stress,
A = Cross-sectional area of the rod,
l = Length of the rod, and
k = Least radius of gyration of the rod section.

The diameter of the piston rod is obtained as discussed below:

1. When the length of the piston rod is small *i.e.* when slenderness ratio (l / k) is less than 40, then the diameter of piston rod may be obtained by equating the load acting on the piston rod to its tensile strength, *i.e.*

$$W = \frac{\pi}{4} \times d^2 \times \sigma_t$$

or
$$\frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} \times d^2 \times \sigma_t$$

$$\therefore \quad d = D\sqrt{\frac{p}{\sigma_t}}$$

2. When the length of the piston rod is large, then the diameter of the piston rod is obtained by using Euler's formula or Rankine's formula. Since the piston rod is securely fastened to the piston and cross head, therefore it may be considered as fixed ends. The Euler's formula is

$$W_{cr} = \frac{\pi^2 E\ I}{L^2}$$

and Rankine's formula is,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k}\right)^2}$$

Example 16.3. *Calculate the diameter of a piston rod for a cylinder of 1.5 m diameter in which the greatest difference of steam pressure on the two sides of the piston may be assumed to be 0.2 N/mm². The rod is made of mild steel and is secured to the piston by a tapered rod and nut and to the crosshead by a cotter. Assume modulus of elasticity as 200 kN/mm² and factor of safety as 8. The length of rod may be assumed as 3 metres.*

Solution. Given : D = 1.5 m = 1500 mm ; p = 0.2 N/mm² ; E = 200 kN/mm² = 200 × 10^3 N/mm² ; l = 3 m = 3000 mm

We know that the load acting on the piston,

$$W = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} (1500)^2 \times 0.2 = 353\ 475 \text{ N}$$

∴ Buckling load on the piston rod,

$$W_{cr} = W \times \text{Factor of safety} = 353\ 475 \times 8 = 2.83 \times 10^6 \text{ N}$$

Since the piston rod is considered to have both ends fixed, therefore from Table 16.2, the equivalent length of the piston rod,

$$L = \frac{l}{2} = \frac{3000}{2} = 1500 \text{ mm}$$

Top

Contents

Let d = Diameter of piston rod in mm, and

I = Moment of inertia of the cross-section of the rod $= \dfrac{\pi}{64} \times d^4$

According to Euler's formula, buckling load (W_{cr}),

$$2.83 \times 10^6 = \frac{\pi^2 E I}{L^2} = \frac{9.87 \times 200 \times 10^3 \times \pi d^4}{(1500)^2 \times 64} = 0.043 \, d^4$$

$\therefore$ $d^4 = 2.83 \times 10^6 / 0.043 = 65.8 \times 10^6$ or $d = 90$ mm

According to Rankine's formula, buckling load,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\dfrac{L}{k}\right)^2} \quad ...(i)$$

We know that for mild steel, the crushing stress,

$$\sigma_c = 320 \text{ MPa} = 320 \text{ N/mm}^2, \text{ and } a = \frac{1}{7500}$$

and least radius of gyration for the piston rod section,

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{\pi d^4}{64} \times \frac{4}{\pi d^2}} = \frac{d}{4}$$

Substituting these values in the above equation *(i)*, we have

$$2.83 \times 10^6 = \frac{320 \times \dfrac{\pi d^2}{4}}{1 + \dfrac{1}{7500}\left(\dfrac{1500 \times 4}{d}\right)^2} = \frac{251.4 \, d^2}{1 + \dfrac{4800}{d^2}} = \frac{251.4 \, d^4}{d^2 + 4800}$$

$$251.4 \, d^4 - 2.83 \times 10^6 \, d^2 - 2.83 \times 10^6 \times 4800 = 0$$

or $$d^4 - 11\,257 \, d^2 - 54 \times 10^6 = 0$$

$\therefore$ $$d^2 = \frac{11\,250 \pm \sqrt{(11\,257)^2 + 4 \times 1 \times 54 \times 10^6}}{2} = \frac{11\,257 \pm 18\,512}{2}$$

$= 14\,885$ ... (Taking +ve sign)

or $d = 122$ mm

Taking larger of the two values, we have

$d = 122$ mm **Ans.**

16.14 Design of Push Rods

The push rods are used in overhead valve and side valve engines. Since these are designed as long columns, therefore Euler's formula should be used. The push rods may be treated as pin end columns because they use spherical seated bearings.

Let W = Load acting on the push rod,

D = Diameter of the push rod,

d = Diameter of the hole through the push rod,

I = Moment of inertia of the push rod,

$= \dfrac{\pi}{64} \times D^4$, for solid rod

These rods are used in overhead valve and side valve engines.

Top

$$= \frac{\pi}{64}(D^4 - d^4), \text{ for tubular section}$$

l = Length of the push rod, and

E = Young's modulus for the material of push rod.

If m is the factor of safety for the long columns, then the critical or crippling load on the rod is given by

$$W_{cr} = m \times W$$

Now using Euler's formula, $W_{cr} = \dfrac{\pi^2 EI}{L^2}$, the diameter of the push rod (D) can be obtained.

Notes: 1. Generally the diameter of the hole through the push rod is 0.8 times the diameter of push rod, *i.e.*

$$d = 0.8\, D$$

2. Since the push rods are treated as pin end columns, therefore the equivalent length of the rod (L) is equal to the actual length of the rod (l).

Example 16.4. *The maximum load on a petrol engine push rod 300 mm long is 1400 N. It is hollow having the outer diameter 1.25 times the inner diameter. Spherical seated bearings are used for the push rod. The modulus of elasticity for the material of the push rod is 210 kN/mm². Find a suitable size for the push rod, taking a factor of safety of 2.5.*

Solution. Given : l = 300 mm ; W = 1400 N ; D = 1.25 d ; E = 210 kN/mm² = 210 × 10³ N/mm² ; m = 2.5

Let d = Inner diameter of push rod in mm, and

D = Outer diameter of the push rod in mm = 1.25 d ...(Given)

∴ Moment of inertia of the push rod section,

$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}[(1.25\, d)^4 - d^4] = 0.07\, d^4 \text{ mm}^4$$

We know that the crippling load on the push rod,

$$W_{cr} = m \times W = 2.5 \times 1400 = 3500 \text{ N}$$

Now according to Euler's formula, crippling load (W_{cr}),

$$3500 = \frac{\pi^2 E\, I}{L^2} = \frac{9.87 \times 210 \times 10^3 \times 0.07\, d^4}{(300)^2} = 1.6\, d^4 \quad ...(\because L = l)$$

∴ $d^4 = 3500 / 1.6 = 2188$ or d = 6.84 mm **Ans.**

and $D = 1.25\, d = 1.25 \times 6.84 = 8.55$ mm **Ans.**

16.15 Design of Connecting Rod

A connecting rod is a machine member which is subjected to alternating direct compressive and tensile forces. Since the compressive forces are much higher than the tensile forces, therefore the cross-section of the connecting rod is designed as a strut and the Rankine's formula is used.

A connecting rod subjected to an axial load W may buckle with X-axis as neutral axis (*i.e.* in the plane of motion of the connecting rod) or Y-axis as neutral axis (*i.e.* in the plane perpendicular to the plane of motion). The connecting rod is considered like both ends hinged for buckling about X-axis and both ends fixed for buckling about Y-axis. A connecting rod should be equally strong in buckling about either axes.

Let A = Cross-sectional area of the connecting rod,

l = Length of the connecting rod,

σ_c = Compressive yield stress,

W_{cr} = Crippling or buckling load,

Contents

I_{xx} and I_{yy} = Moment of inertia of the section about X-axis and Y-axis respectively, and

k_{xx} and k_{yy} = Radius of gyration of the section about X-axis and Y-axis respectively.

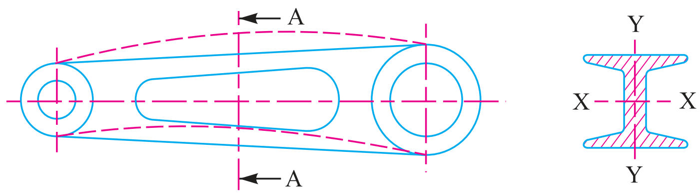


Fig. 16.6. Buckling of connecting rod.

According to Rankine's formula,

$$W_{cr} \text{ about } X\text{-axis} = \frac{\sigma_c \times A}{1 + a\left(\dfrac{L}{k_{xx}}\right)^2} = \frac{\sigma_c \times A}{1 + a\left(\dfrac{l}{k_{xx}}\right)^2} \qquad \ldots (\because \text{ For both ends hinged, } L = l)$$

and

$$W_{cr} \text{ about } Y\text{-axis} = \frac{\sigma_c \times A}{1 + a\left(\dfrac{L}{k_{yy}}\right)^2} = \frac{\sigma_c \times A}{1 + a\left(\dfrac{l}{2k_{yy}}\right)^2} \qquad \ldots \left(\because \text{ For both ends fixed, } L = \frac{l}{2}\right)$$

In order to have a connecting rod equally strong in buckling about both the axes, the buckling loads must be equal, *i.e.*

$$\frac{\sigma_c \times A}{1 + a\left(\dfrac{l}{k_{xx}}\right)^2} = \frac{\sigma_c \times A}{1 + a\left(\dfrac{l}{2\,k_{yy}}\right)^2} \quad \text{or} \quad \left(\frac{l}{k_{xx}}\right)^2 = \left(\frac{l}{2\,k_{yy}}\right)^2$$

$$\therefore \quad k^2_{xx} = 4\,k^2_{yy} \quad \text{or} \quad I_{xx} = 4\,I_{yy} \qquad \ldots (\because I = A \times k^2)$$

This shows that the connecting rod is four times strong in buckling about Y-axis than about X-axis. If $I_{xx} > 4\,I_{yy}$, then buckling will occur about Y-axis and if $I_{xx} < 4\,I_{yy}$, buckling will occur about X-axis. In actual practice, I_{xx} is kept slightly less than $4\,I_{yy}$. It is usually taken between 3 and 3.5 and the connecting rod is designed for buckling about X-axis. The design will alwyas be satisfactory for buckling about Y-axis.

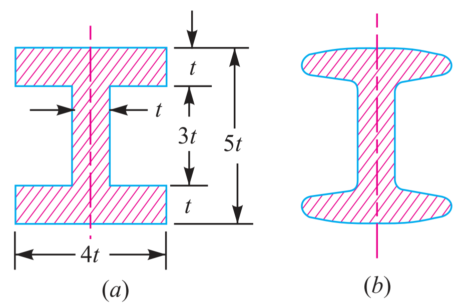


Fig. 16.7. I-section of connecting rod.

The most suitable section for the connecting rod is I-section with the proportions as shown in Fig. 16.7 (*a*).

Area of the section

$$= 2\,(4\,t \times t) + 3\,t \times t = 11\,t^2$$

$\therefore$ Moment of inertia about X-axis,

$$I_{xx} = \frac{1}{12}\left[4\,t\,(5\,t)^3 - 3\,t\,(3\,t)^3\right] = \frac{419}{12}\,t^4$$

and moment of inertia about Y-axis,

$$I_{yy} = \left[2 \times \frac{1}{12}\,t \times (4\,t)^3 + \frac{1}{12}\,(3\,t)\,t^3\right] = \frac{131}{12}\,t^4$$

Top

$$\therefore \qquad \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

Since the value of $\frac{I_{xx}}{I_{yy}}$ lies between 3 and 3.5, therefore *I*-section chosen is quite satisfactory.

Notes : 1. The *I*-section of the connecting rod is used due to its lightness and to keep the inertia forces as low as possible. It can also withstand high gas pressure.

2. Sometimes a connecting rod may have rectangular section. For slow speed engines, circular sections may be used.

3. Since connecting rod is manufactured by forging, therefore the sharp corners of *I*-section are rounded off as shown in Fig. 16.7 (*b*) for easy removal of the section from the dies.

Example 16.5. *A connecting rod of length l may be considered as a strut with the ends free to turn on the crank pin and the gudgeon pin. In the directions of the axes of these pins, however, it may be considered as having fixed ends. Assuming that Euler's formula is applicable, determine the ratio of the sides of the rectangular cross-section so that the connecting rod is equally strong in both planes of buckling.*

Solution. The rectangular cross-section of the connecting rod is shown in Fig. 16.8.

Let b = Width of rectangular cross-section, and

h = Depth of rectangular cross-section.

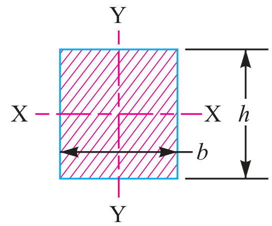


Fig. 16.8

$\therefore$ Moment of inertia about *X-X*,

$$I_{xx} = \frac{b.h^3}{12}$$

and moment of inertia about *Y-Y*,

$$I_{yy} = \frac{h.b^3}{12}$$

According to Euler's formula, buckling load,

$$W_{cr} = \frac{\pi^2 E I}{L^2}$$

$\therefore$ Buckling load about *X-X*,

$$W_{cr}\,(X\text{-axis}) = \frac{\pi^2 E I_{xx}}{l^2} \qquad \ldots (\because L = l, \text{ for both ends free to turn})$$

and buckling load about *Y-Y*,

$$W_{cr}\,(Y\text{-axis}) = \frac{\pi^2 E I_{yy}}{(l/2)^2} = \frac{4\pi^2 E I_{yy}}{l^2} \qquad \ldots (\because L = l/2, \text{ for both ends fixed})$$

In order to have the connecting rod equally strong in both the planes of buckling,

$$W_{cr}\,(X\text{-axis}) = W_{cr}\,(Y\text{-axis})$$

$$\frac{\pi^2 E I_{xx}}{l^2} = \frac{4\pi^2 E I_{yy}}{l^2} \qquad \text{or} \qquad I_{xx} = 4 I_{yy}$$

$$\therefore \qquad \frac{b\,h^3}{12} = \frac{4\,h\,b^3}{12} \qquad \text{or} \qquad h^2 = 4\,b^2$$

and $\qquad h^2 / b^2 = 4 \qquad$ or $\qquad h / b = 2$ **Ans.**

16.16 Forces Acting on a Connecting Rod

A connecting rod is subjected to the following forces :

1. Force due to gas or steam pressure and inertia of reciprocating parts, and

2. Inertia bending forces.

We shall now derive the expressions for the forces acting on a horizontal engine, as discussed below:

Contents

1. *Force due to gas or steam pressure and inertia of reciprocating parts*

Consider a connecting rod *PC* as shown in Fig. 16.9.

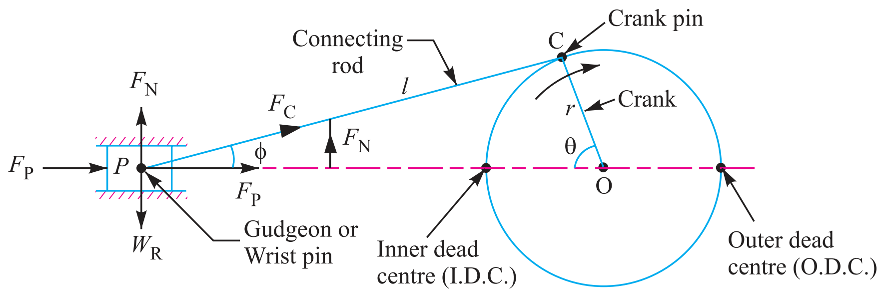


Fig. 16.9. Forces on a connecting rod.

Let
p = Pressure of gas or steam,
A = Area of piston,
m_R = Mass of reciprocating parts,
= Mass of piston, gudgeon pin etc. + $\frac{1}{3}$ rd mass of connecting rod,
ω = Angular speed of crank,
ϕ = Angle of inclination of the connecting rod with the line of stroke,
θ = Angle of inclination of the crank from inner dead centre,
r = Radius of crank,
l = Length of connecting rod, and
n = Ratio of length of connecting rod to radius of crank = l / r.

We know that the force on the piston due to pressure of gas or steam,

$$F_L = \text{Pressure} \times \text{Area} = p \times A$$

and inertia force of reciprocating parts,

$$F_I = \text{Mass} \times {}^*\text{Acceleration} = m_R \times \omega^2 \times r \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

It may be noted that in a horizontal engine, reciprocating parts are accelerated from rest during the first half of the stroke (*i.e.* when the piston moves from inner dead centre to outer dead centre). It is then retarted during the latter half of the stroke (*i.e.* when the piston moves from outer dead centre to inner dead centre). The inertia force due to the acceleration of reciprocating parts, opposes the force on the piston. On the other hand, the inertia force due to retardation of the reciprocating parts, helps the force on the piston.

∴ Net force acting on the piston pin (or gudgeon or wrist pin),

$$F_P = \text{Force due to pressure of gas or steam} \pm \text{Inertia force}$$
$$= F_L \pm F_I$$

The –ve sign is used when the piston is accelerated and +ve sign is used when the piston is retarted.

* Acceleration of reciprocating parts = $\omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

Top

Contents

The force F_P gives rise to a force F_C in the connecting rod and a thrust F_N on the sides of the cylinder walls (or normal reaction on crosshead guides). From Fig. 16.9, we see that force in the connecting rod at any instant.

$$F_C = \frac{F_P}{\cos \phi} = \frac{{}^{*}\ F_P}{\sqrt{1 - \frac{\sin^2 \theta}{n^2}}}$$

The force in the connecting rod will be maximum when the crank and the connecting rod are perpendicular to each other (*i.e.* when θ = 90°). But at this position, the gas pressure would be decreased considerably. Thus, for all practical purposes, the force in the connecting rod (F_C) is taken equal to the maximum force on the piston due to pressure of gas or steam (F_L), neglecting piston inertia effects.

Connecting rod.

2. *Inertia bending forces*

Consider a connecting rod *PC* and a crank *OC* rotating with uniform angular velocity ω rad /s. In order to find the acceleration of various points on the connecting rod, draw the Klien's acceleration diagram *CQNO* as shown in Fig. 16.10 (*a*). *CO* represents the acceleration of *C* towards *O* and *NO* represents the acceleration of *P* towards *O*. The acceleration of other points such as *D*, *E*, *F* and *G* etc. on the connecting rod *PC* may be found by drawing horizontal lines from these points to intersect *CN* at *d*, *e*, *f* and *g* respectively. Now *dO*, *eO*, *fO* and *gO* represents the acceleration of *D*, *E*, *F* and *G* all towards *O*. The inertia force acting on each point will be as follows :

Inertia force at $C = m \times \omega^2 \times CO$

Inertia force at $D = m \times \omega^2 \times dO$

Inertia force at $E = m \times \omega^2 \times eO$, and so on.

The inertia forces will be opposite to the direction of acceleration or centrifugal forces. The inertia forces can be resolved into two components, one parallel to the connecting rod and the other perpendicular to the rod. The parallel (or longitudinal) components adds up algebraically to the force acting on the connecting rod (F_C) and produces thrust on the pins. The perpendicular (or transverse) components produces bending action (also called whipping action) and the stress induced in the connecting rod is called ***whipping stress*.**

* For derivation, please refer to author's popular book on **'Theory of Machines'.**

Top

Contents

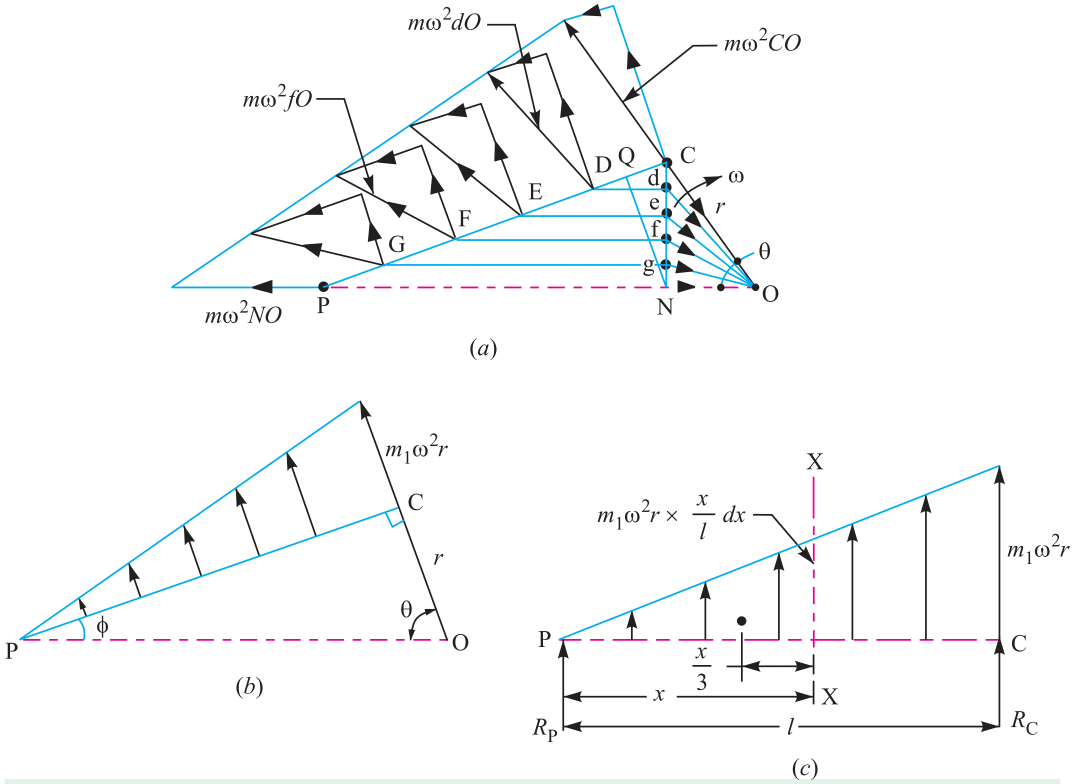


Fig. 16.10. Inertia bending forces.

A little consideration will show that the perpendicular components will be maximum, when the crank and connecting rod are at right angles to each other.

The variation of the inertia force on the connecting rod is linear and is like a simply supported beam of variable loading as shown in Fig. 16.10 (*b*) and (*c*). Assuming that the connecting rod is of uniform cross-section and has mass m_1 kg per unit length, therefore

Inertia force per unit length at the crank pin

$$= m_1 \times \omega^2 r$$

and inertia force per unit length at the gudgeon pin

$$= 0$$

Inertia forces due to small element of length dx at a distance x from the gudgeon pin P,

$$dF_I = m_1 \times \omega^2 r \times \frac{x}{l} \times dx$$

∴ Resultant inertia force,

$$F_I = \int_0^l m_1 \times \omega^2 r \times \frac{x}{l} \times dx = \frac{m_1 \times \omega^2 r}{l}\left[\frac{x^2}{2}\right]_0^l$$

$$= \frac{m_1 \times l}{2} \times \omega^2\, r = \frac{m}{2} \times \omega^2\, r \qquad \text{... (Substituting } m_1.l = m\text{)}$$

This resultant inertia force acts at a distance of $2l / 3$ from the gudgeon pin P.

Since it has been assumed that $\frac{1}{3}$ rd mass of the connecting rod is concentrated at gudgeon pin P (*i.e.* small end of connecting rod) and $\frac{2}{3}$ rd at the crank pin (*i.e.* big end of connecting rod),

Top

therefore the reactions at these two ends will be in the same proportion, *i.e.*

$$R_P = \frac{1}{3} F_I, \text{ and } R_C = \frac{2}{3} F_I$$

Now the bending moment acting on the rod at section *X-X* at a distance *x* from *P*,

$$M_X = R_P \times x - {}^*m_1 \times \omega^2 r \times \frac{x}{l} \times \frac{1}{2} x \times \frac{x}{3}$$

$$= \frac{1}{3} F_I \times x - \frac{m_1 . l}{2} \times \omega^2 r \times \frac{x^3}{3\,l^2} \qquad \ldots\left(\because R_P = \frac{1}{3} F_I\right)$$

... (Multiplying and dividing the latter expression by *l*)

$$= \frac{F_I \times x}{3} - F_I \times \frac{x^3}{3l^2} = \frac{F_I}{3}\left(x - \frac{x^3}{l^2}\right) \qquad \ldots(i)$$

For maximum bending moment, differentiate M_X with respect to *x* and equate to zero, *i.e.*

$$\frac{d_{MX}}{dx} = 0 \quad \text{or} \quad \frac{F_I}{3}\left[1 - \frac{3x^2}{l^2}\right] = 0$$

$$\therefore \quad 1 - \frac{3x^2}{l^2} = 0 \quad \text{or} \quad 3x^2 = l^2 \quad \text{or} \quad x = \frac{l}{\sqrt{3}}$$

Substituting this value of *x* in the above equation (*i*), we have maximum bending moment,

$$M_{max} = \frac{F_I}{3}\left[\frac{l}{\sqrt{3}} - \frac{\left(\frac{l}{\sqrt{3}}\right)^3}{l^2}\right] = \frac{F_I}{3}\left[\frac{l}{\sqrt{3}} - \frac{l}{3\sqrt{3}}\right]$$

$$= \frac{F_I}{3} \times \frac{2l}{3\sqrt{3}} = \frac{2\,F_I \times l}{9\sqrt{3}}$$

$$= 2 \times \frac{m}{2} \times \omega^2 r \times \frac{l}{9\sqrt{3}} = m \times \omega^2 r \times \frac{l}{9\sqrt{3}} \qquad \ldots\left(\because F_I = \frac{m}{2} \times \omega^2 r\right)$$

and the maximum bending stress, due to inertia of the connecting rod,

$$\sigma_{max} = \frac{M_{max}}{Z}$$

where Z = Section modulus.

From above we see that the maximum bending moment varies as the square of speed, therefore, the bending stress due to high speed will be dangerous. It may be noted that the maximum axial force and the maximum bending stress do not occur simultaneously. In an I.C. engine, the maximum gas load occurs close to top dead centre whereas the maximum bending stress occurs when the crank angle θ = 65° to 70° from top dead centre. The pressure of gas falls suddenly as the piston moves from dead centre. In steam engines, even though the pressure is maintained till cut off occurs, the speed is low and therefore the bending stress due to inertia is small. Thus the general practice is to design a connecting rod by assuming the force in the connecting rod (F_C) equal to the maximum force on the piston due to pressure of gas or steam (F_L), neglecting piston inertia effects and then checked for bending stress due to inertia force (*i.e.* whipping stress).

* B.M. due to variable loading from $\left(0 \text{ to } m_1\ \omega^2 r \times \frac{x}{l}\right)$ is equal to the area of triangle multiplied by distance of C.G. from *X-X* $\left(i.e.\ \frac{x}{3}\right)$.

Example 16.6. *Determine the dimensions of an I-section connecting rod for a petrol engine from the following data :*

Diameter of the piston = 110 mm

Mass of the reciprocating parts = 2 kg

Length of the connecting rod from centre to centre = 325 mm

Stroke length = 150 mm

R.P.M. = 1500 with possible overspeed of 2500

Compression ratio = 4 : 1

Maximum explosion pressure = 2.5 N/mm²

Connecting rod of a petrol engine.

Solution. Given : $D = 110$ mm $= 0.11$ m ; $m_R = 2$ kg ; $l = 325$ mm $= 0.325$ m ; Stroke length $= 150$ mm $= 0.15$ m ; $N_{min} = 1500$ r.p.m. ; $N_{max} = 2500$ r.p.m. ; *Compression ratio = 4 : 1 ; $p = 2.5$ N/mm²

We know that the radius of crank,

$$r = \frac{\text{Stroke length}}{2} = \frac{150}{2} = 75 \text{ mm} = 0.075 \text{ m}$$

and ratio of the length of connecting rod to the radius of crank,

$$n = \frac{l}{r} = \frac{325}{75} = 4.3$$

We know that the maximum force on the piston due to pressure,

$$F_L = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} (110)^2\, 2.5 = 23\ 760 \text{ N}$$

and maximum angular speed,

$$\omega_{max} = \frac{2\pi \times N_{max}}{60} = \frac{2\pi \times 2500}{60} = 261.8 \text{ rad/s}$$

We know that maximum inertia force of reciprocating parts,

$$F_I = m_R\ (\omega_{max})^2\, r \left(\cos\theta + \frac{\cos 2\theta}{n}\right) \qquad \text{... }(i)$$

The inertia force of reciprocating parts is maximum, when the crank is at inner dead centre, *i.e.* when $\theta = 0°$.

$$\therefore \quad F_I = m_R\ (\omega_{max})^2 r \left(1 + \frac{1}{n}\right) \qquad \text{... [From equation } (i)]$$

$$= 2(261.8)^2\ 0.075 \left(1 + \frac{1}{4.3}\right) = 12\ 672 \text{ N}$$

Since the connecting rod is designed by taking the force in the connecting rod (F_C) equal to the maximum force on the piston due to gas pressure (F_L), therefore

Force in the connecting rod,

$$F_C = F_L = 23\ 760 \text{ N}$$

* Superfluous data.

Consider the *I*-section of the connecting rod with the proportions as shown in Fig. 16.11. We have discussed in Art. 16.15 that for such a section

$$\frac{I_{xx}}{I_{yy}} = 3.2$$

or $$\frac{k^2_{xx}}{k^2_{yy}} = 3.2, \text{ which is satisfactory.}$$

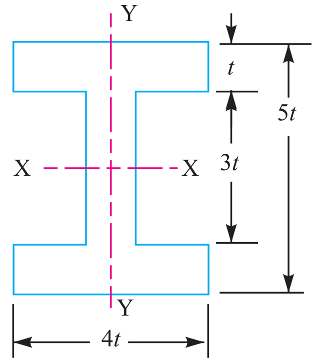


Fig. 16.11

We have also discussed that the connecting rod is designed for buckling about *X*-axis (*i.e.* in a plane of motion of the connecting rod), assuming both ends hinged. Taking a factor of safety as 6, the buckling load,

$$W_{cr} = F_C \times 6 = 23\,760 \times 6 = 142\,560 \text{ N}$$

and area of cross-section,

$$A = 2\,(4t \times t) + t \times 3t = 11\,t^2 \text{ mm}^2$$

Moment of inertia about *X*-axis,

$$I_{xx} = \left[\frac{4\,t\,(5\,t)^3}{12} - \frac{3\,t\,(3\,t)^3}{12}\right] = \frac{419\,t^4}{12} \text{ mm}^4$$

∴ Radius of gyration,

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{419\,t^4}{12} \times \frac{1}{11\,t^2}} = 1.78\,t$$

We know that equivalent length of the rod for both ends hinged,

$$L = l = 325 \text{ mm}$$

Taking for mild steel, σ_c = 320 MPa = 320 N/mm² and a = 1 / 7500, we have from Rankine's formula,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k_{xx}}\right)^2}$$

$$142\,560 = \frac{320 \times 11\,t^2}{1 + \frac{1}{7500}\left(\frac{325}{1.78\,t}\right)^2}$$

$$40.5 = \frac{t^2}{1 + \frac{4.44}{t^2}} = \frac{t^4}{t^2 + 4.44}$$

or $$t^4 - 40.5\,t^2 - 179.8 = 0$$

∴ $$t^2 = \frac{40.5 \pm \sqrt{(40.5)^2 + 4 \times 179.8}}{2} = \frac{40.5 \pm 48.6}{2} = 44.55$$

... (Taking +ve sign)

or $$t = 6.67 \text{ say } 6.8 \text{ mm}$$

Therefore, dimensions of cross-section of the connecting rod are

Height = 5 t = 5 × 6.8 = 34 mm **Ans.**

Width = 4 t = 4 × 6.8 = 27.2 mm **Ans.**

Thickness of flange and web

$$= t = 6.8 \text{ mm} = 0.0068 \text{ m} \quad \textbf{Ans.}$$

Now let us find the bending stress due to inertia force on the connecting rod.

We know that the mass of the connecting rod per metre length,

$$m_1 = \text{Volume} \times \text{density} = \text{Area} \times \text{length} \times \text{density}$$

$$= A \times l \times \rho = 11\, t^2 \times l \times \rho \qquad ..(\because A = 11\, t^2)$$

$$= 11(0.0068)^2\, 1 \times 7800 = 3.97 \text{ kg} \qquad \text{... (Taking } \rho = 7800 \text{ kg / m}^3)$$

∴ Maximum bending moment,

$$M_{max} = m\, \omega^2\, r \times \frac{l}{9\sqrt{3}} = m_1\, \omega^2\, r \times \frac{l^2}{9\sqrt{3}} \qquad \text{... } (\because m = m_1.l\,)$$

$$= 3.97\,(261.8)^2\,(0.075) \times \frac{(0.325)^2}{9\sqrt{3}} = 138.3 \text{ N-m}$$

and section modulus,

$$Z_{xx} = \frac{I_{xx}}{5\,t/2} = \frac{419\,t^4}{12} \times \frac{2}{5\,t} = \frac{419}{30}\,t^3$$

$$= \frac{419}{30}\,(0.0068)^3 = 4.4 \times 10^{-6} \text{ m}^3$$

∴ Maximum bending or whipping stress due to inertia bending forces,

$$\sigma_{b(max)} = \frac{M_{max}}{Z_{xx}} = \frac{138.3}{4.4 \times 10^{-6}} = 31.4 \times 10^6 \text{ N/m}^2$$

$$= 31.4 \text{ MPa, which is safe}$$

Note : The maximum compressive stress in the connecting rod will be,

$$\sigma_{c(max)} = \text{Direct compressive stress} + \text{Maximum bending stress}$$

$$= \frac{320}{6} + 31.4 = 84.7 \text{ MPa}$$

EXERCISES

1. Compare the ratio of strength of a solid steel column to that of a hollow column of internal diameter equal to 3/4th of its external diameter. Both the columns have the same cross-sectional areas, lengths and end conditions. **[Ans. 25/7]**

2. Find the Euler's crippling load for a hollow cylindrical steel column of 38 mm external diameter and 35 mm thick. The length of the column is 2.3 m and hinged at its both ends. Take $E = 200$ GN/m^2. Also determine the crippling load by Rankine's formula, using

$$\sigma_c = 320 \text{ MPa ; and } a = \frac{1}{7500}$$

[Ans. 17.25 kN ; 17.4 kN]

3. Determine the diameter of the pistion rod of the hydraulic cylinder of 100 mm bore when the maximum hydraulic pressure in the cylinder is limited to 14 N/mm^2. The length of the piston rod is 1.2 m. The factor of safety may be taken as 5 and the end fixity coefficient as 2. **[Ans. 45 mm]**

4. Find the diameter of a piston rod for an engine of 200 mm diameter. The length of the piston rod is 0.9 m and the stroke is 0.5 m. The pressure of steam is 1 N/mm^2. Assume factor of safety as 5. **[Ans. 31 mm]**

Contents

5. Determine the diameter of the push rod made of mild steel of an I.C. engine if the maximum force exerted by the push rod is 1500 N. The length of the push rod is 0.5 m. Take the factor of safety as 2.5 and the end fixity coefficient as 2. **[Ans. 10 mm]**

6. The eccentric rod to drive the D-slide valve mechanism of a steam engine carries a maximum compressive load of 10 kN. The length of the rod is 1.5 m. Assuming the eccentric rod hinged at both the ends, find

   (*a*) diameter of the rod, and

   (*b*) dimensions of the cross-section of the rod if it is of rectangular section. The depth of the section is twice its thickness.

   Take factor of safety = 40 and E = 210 kN/mm^2. **[Ans. 60 mm ; 30 × 60 mm]**

7. Determine the dimensions of an *I*-section connecting rod for an internal combustion engine having the following specifications :

Diameter of the piston	=	120 mm
Mass of reciprocating parts piston	=	350 kg/m^2 of area
Length of connecting rod	=	350 mm
Engine revolutions per minute	=	1800
Maximum explosion pressure	=	3 N/mm^2
Stroke length	=	180 mm

   The flange width and the depth of the *I*-section rod are in the ratio of 4 t : 6 t where t is the thickness of the flange and web. Assume yield stress in compression for the material as 330 MPa and a factor of safety as 6. **[Ans. t = 7.5 mm]**

Screwjacks

8. The connecting rod of a four stroke cycle Diesel engine is of circular section and of length 550 mm. The diameter and stroke of the cylinder are 150 mm and 240 mm respectively. The maximum combustion pressure is 4.7 N/mm^2. Determine the diameter of the rod to be used, for a factor of safety of 3 with a material having a yield point of 330 MPa.

   Find also the maximum bending stress in the connecting rod due to whipping action if the engine runs at 1000 r.p.m. The specific weight of the material is 7800 kg/m^3.

   [Ans. 33.2 mm ; 48 MPa]

QUESTIONS

1. What do you understand by a column or strut ? Explain the various end conditions of a column or strut.
2. State the assumptions used in Euler’s column theory.
3. Define ‘slenderness ratio’. How it is used to define long and short columns ?
4. What is equivalent length of a column ? Write the relations between equivalent length and actual length of a column for various end conditions.
5. Explain Johnson’s formula for columns. Describe the use of Johnson’s formula and Euler’s formula.
6. Write the formula for obtaining a maximum stress in a long column subjected to eccentric loading.

Top

7. How the piston rod is designed ?
8. Explain the design procedure of valve push rods.
9. Why an *I*-Section is usually preferred to a round section in case of connecting rods?

OBJECTIVE TYPE QUESTIONS

1. A machine part is designed as a strut, when it is subjected to
   (*a*) an axial tensile force (*b*) an axial compressive force
   (*c*) a tangential force (*d*) any one of these
2. Slenderness ratio is the ratio of
   (*a*) maximum size of a column to minimum size of column
   (*b*) width of column to depth of column
   (*c*) effective length of column to least radius of gyration of the column
   (*d*) effective length of column to width of column
3. A connecting rod is designed as a
   (*a*) long column (*b*) short column
   (*c*) strut (*d*) any one of these
4. Which of the following formula is used in designing a connecting rod ?
   (*a*) Euler's formula (*b*) Rankine's formula
   (*c*) Johnson's straight line formula (*d*) Johnson's parabolic formula
5. A connecting rod subjected to an axial load may buckle with
   (*a*) *X*-axis as neutral axis (*b*) *Y*-axis as neutral axis
   (*c*) *X*-axis or *Y*-axis as neutral axis (*d*) *Z*-axis
6. In designing a connecting rod, it is considered like .... for buckling about *X*-axis.
   (*a*) both ends hinged (*b*) both ends fixed
   (*c*) one end fixed and the other end hinged (*d*) one end fixed and the other end free
7. A connecting rod should be
   (*a*) strong in buckling about *X*-axis
   (*b*) strong in buckling about *Y*-axis
   (*c*) equally strong in buckling about *X*-axis and *Y*-axis
   (*d*) any one of the above
8. The buckling will occur about *Y*-axis,if
   (*a*) $I_{xx} = I_{yy}$ (*b*) $I_{xx} = 4I_{yy}$
   (*c*) $I_{xx} > 4I_{yy}$ (*d*) $I_{xx} < 4I_{yy}$
9. The connecting rod will be equally strong in buckling about X-axis and Y-axis, if
   (*a*) $I_{xx} = I_{yy}$ (*b*) $I_{xx} = 2I_{yy}$
   (*c*) $I_{xx} = 3I_{yy}$ (*d*) $I_{xx} = 4I_{yy}$
10. The most suitable section for the connecting rod is
   (*a*) *L*-section (*b*) *T*-section
   (*c*) *I*-section (*d*) *C*-section

ANSWERS

1. (*b*)	**2.** (*c*)	**3.** (*c*)	**4.** (*b*)	**5.** (*c*)
6. (*a*)	**7.** (*c*)	**8.** (*c*)	**9.** (*d*)	**10.** (*c*)