CHAPTER

17

Power Screws

17.1 Introduction

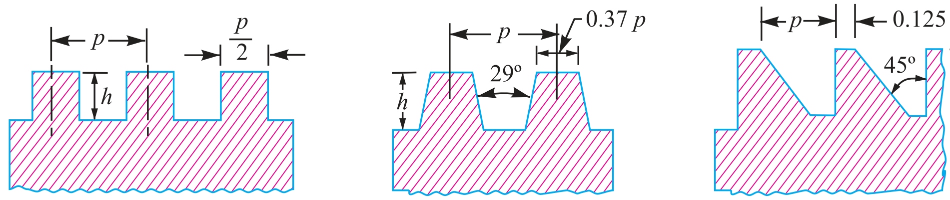
The power screws (also known as ***translation screws***) are used to convert rotary motion into translatory motion. For example, in the case of the lead screw of lathe, the rotary motion is available but the tool has to be advanced in the direction of the cut against the cutting resistance of the material. In case of screw jack, a small force applied in the horizontal plane is used to raise or lower a large load. Power screws are also used in vices, testing machines, presses, etc.

In most of the power screws, the nut has axial motion against the resisting axial force while the screw rotates in its bearings. In some screws, the screw rotates and moves axially against the resisting force while the nut is stationary and in others the nut rotates while the screw moves axially with no rotation.

17.2 Types of Screw Threads used for Power Screws

Following are the three types of screw threads mostly used for power screws :

1. *Square thread.* A square thread, as shown in Fig. 17.1 (*a*), is adapted for the transmission of power in either direction. This thread results in maximum efficiency and minimum radial or bursting



$h = 0.5\,p$
(*a*) Square thread.

$h = 0.5\,p + 0.25$ mm
(*b*) Acme thread.

$h = 0.75\,p$
(*c*) Buttress thread.

Fig. 17.1. Types of power screws.

pressure on the nut. It is difficult to cut with taps and dies. It is usually cut on a lathe with a single point tool and it can not be easily compensated for wear. The square threads are employed in screw jacks, presses and clamping devices. The standard dimensions for square threads according to IS : 4694 – 1968 (Reaffirmed 1996), are shown in Table 17.1 to 17.3.

Screw jacks

2. *Acme or trapezoidal thread.* An acme or trapezoidal thread, as shown in Fig. 17.1 (*b*), is a modification of square thread. The slight slope given to its sides lowers the efficiency slightly than square thread and it also introduce some bursting pressure on the nut, but increases its area in shear. It is used where a split nut is required and where provision is made to take up wear as in the lead screw of a lathe. Wear may be taken up by means of an adjustable split nut. An acme thread may be cut by means of dies and hence it is more easily manufactured than square thread. The standard dimensions for acme or trapezoidal threads are shown in Table 17.4 (Page 630).

3. *Buttress thread.* A buttress thread, as shown in Fig. 17.1 (*c*), is used when large forces act along the screw axis in one direction only. This thread combines the higher efficiency of square thread and the ease of cutting and the adaptability to a split nut of acme thread. It is stronger than other threads because of greater thickness at the base of the thread. The buttress thread has limited use for power transmission. It is employed as the thread for light jack screws and vices.

Table 17.1. Basic dimensions for square threads in mm (Fine series) according to IS : 4694 – 1968 (Reaffirmed 1996)

Nominal diameter (d_1)	*Major diameter*		*Minor diameter* (d_c)	*Pitch* (p)	*Depth of thread*		*Area of core* (A_c) mm^2
	Bolt (d)	*Nut* (D)			*Bolt* (h)	*Nut* (H)	
10	10	10.5	8	2	1	1.25	50.3
12	12	12.5	10				78.5

d_1	d	D	d_c	p	h	H	A_c
14	14	14.5	12				113
16	16	16.5	14	2	1	1.25	154
18	18	18.5	16				201
20	20	20.5	18				254
22	22	22.5	19				284
24	24	24.5	21				346
26	26	26.5	23				415
28	28	28.5	25				491
30	30	30.5	27				573
32	32	32.5	29				661
(34)	34	34.5	31				755
36	36	36.5	33	3	1.5	1.75	855
(38)	38	38.5	35				962
40	40	40.5	37				1075
42	42	42.5	39				1195
44	44	44.5	41				1320
(46)	46	46.5	43				1452
48	48	48.5	45				1590
50	50	50.5	47				1735
52	52	52.5	49				1886
55	55	55.5	52				2124
(58)	58	58.5	55				2376
60	60	60.5	57				2552
(62)	62	62.5	59				2734
65	65	65.5	61				2922
(68)	68	68.5	64				3217
70	70	70.5	66				3421
(72)	72	72.5	68				3632
75	75	75.5	71				3959
(78)	78	78.5	74				4301
80	80	80.5	76				4536
(82)	82	82.5	78				4778
(85)	85	85.5	81	4	2	2.25	5153
(88)	88	88.5	84				5542
90	90	90.5	86				5809
(92)	92	92.5	88				6082
95	95	95.5	91				6504
(98)	98	98.5	94				6960

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d_1	d	D	d_c	p	h	H	A_c
100	100	100.5	96				7238
(105)	105	105.5	101	4	2	2.25	8012
110	110	110.5	106				8825
(115)	115	115.5	109				9331
120	120	120.5	114				10207
(125)	125	125.5	119				11 122
130	130	130.5	124				12 076
(135)	135	135.5	129				13 070
140	140	140.5	134				14 103
(145)	145	145.5	139	6	3	3.25	15 175
150	150	150.5	144				16 286
(155)	155	155.5	149				17437
160	160	160.5	154				18 627
(165)	165	165.5	159				19 856
170	170	170.5	164				21124
(175)	175	175.5	169				22 432

Note : Diameter within brackets are of second preference.

Table 17.2. Basic dimensions for square threads in mm (Normal series)according to IS : 4694 – 1968 (Reaffirmed 1996)

Nominal diameter (d_1)	*Major diameter*		*Minor diameter*	*Pitch*	*Depth of thread*		*Area of core* (A_c) mm^2
	Bolt (d)	*Nut* (D)	(d_c)	(p)	*Bolt* (h)	*Nut* (H)	
22	22	22.5	17				227
24	24	24.5	19				284
26	26	26.5	21	5	2.5	2.75	346
28	28	28.5	23				415
30	30	30.5	24				452
32	32	32.5	26	6	3	3.25	531
(34)	34	34.5	28				616
36	36	36.5	30				707
(38)	38	38.5	31				755
40	40	40.5	33	7	3.5	3.75	855
(42)	42	42.5	35				962
44	44	44.5	37				1075

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d_1	d	D	d_c	p	h	H	A_c
(46)	46	46.5	38				1134
48	48	48.5	40	8	4	4.25	1257
50	50	50.5	42				1385
52	52	52.5	44				1521
55	55	55.5	46				1662
(58)	58	58.5	49	9	4.5	5.25	1886
(60)	60	60.5	51				2043
(62)	62	62.5	53				2206
65	65	65.5	55				2376
(68)	68	68.5	58	10	5	5.25	2642
70	70	70.5	60				2827
(72)	72	72.5	62				3019
75	75	75.5	65				3318
(78)	78	78.5	68				3632
80	80	80.5	70				3848
(82)	82	82.5	72				4072
85	85	85.5	73				41.85
(88)	88	88.5	76				4536
90	90	85.5	78	12	6	6.25	4778
(92)	92	92.5	80				5027
95	95	95.5	83				5411
(98)	98	98.5	86				5809
100	100	100.5	88				6082
(105)	105	105.5	93				6793
110	110	110.5	98				7543
(115)	115	116	101				8012
120	120	121	106				882
(125)	125	126	111	14	7	7.5	9677
130	130	131	116				10 568
(135)	135	136	121				11 499
140	140	141	126				12 469
(145)	145	146	131				13 478
150	150	151	134				14 103
(155)	155	156	139	16	8	8.5	15 175
160	160	161	144				16 286

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d_1	d	D	d_c	p	h	H	A_c
(165)	165	166	149				17 437
170	170	171	154	16	8	8.5	18 627
(175)	175	176	159				19 856

Note : Diameter within brackets are of second preference.

Table 17.3. Basic dimensions for square threads in mm (Coarse series) according toIS : 4694 – 1968 (Reaffirmed 1996)

Nominal diameter	*Major diameter*		*Minor diameter*	*Pitch*	*Depth of thread*		*Area of core*
(d_1)	*Bolt* (d)	*Nut* (D)	(d_c)	(p)	*Bolt* (h)	*Nut* (H)	(A_c) mm^2
22	22	22.5	14				164
24	24	24.5	16	8	4	4.25	204
26	26	26.5	18				254
28	28	28.5	20				314
30	30	30.5	20				314
32	32	32.5	22				380
(34)	34	34.5	24	10	5	5.25	452
36	36	36.5	26				531
(38)	38	38.5	28				616
40	40	40.5	28				616
(42)	42	42.5	30				707
44	44	44.5	32				804
(46)	46	46.5	34	12	6	6.25	908
48	48	48.5	36				1018
50	50	50.5	38				1134
52	52	52.5	40				1257
55	55	56	41				1320
(58)	58	59	44	14	7	7.25	1521
60	60	61	46				1662
(62)	62	63	48				1810
65	65	66	49				1886
(68)	68	69	52	16	8	8.5	2124
70	70	71	54				2290
(72)	72	73	56				2463
75	75	76	59				2734
(78)	78	79	62				3019
80	80	81	64				3217
(82)	82	83	66				3421

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d_1	d	D	d_c	p	h	H	A_c
85	85	86	67				3526
(88)	88	89	70				3848
90	90	91	72				4072
(92)	92	93	74	18	9	9.5	4301
95	95	96	77				4657
(96)	96	99	80				5027
100	100	101	80				5027
(105)	105	106	85	20	10	10.5	5675
110	110	111	90				6362
(115)	115	116	93				6793
120	120	121	98				7543
(125)	125	126	103	22	11	11.5	8332
130	130	131	108				9161
(135)	135	136	111				9667
140	140	141	116	24	12	12.5	10 568
(145)	145	146	121				11 499
150	150	151	126				12 469
(155)	155	156	131				13 478
160	160	161	132				13 635
(165)	165	166	137				14 741
170	170	171	142	28	14	14.5	15 837
(175)	175	176	147				16 972

Note : Diameters within brackets are of second preference.

Table 17.4. Basic dimensions for trapezoidal/Acme threads.

Nominal or major diameter (d) mm.	*Minor or core diameter (d_c) mm*	*Pitch (p) mm*	*Area of core (A_c) mm^2*
10	6.5	3	33
12	8.5		57
14	9.5		71
16	11.5	4	105
18	13.5		143
20	15.5		189
22	16.5		214
24	18.5	5	269
26	20.5		330
28	22.5		389
30	23.5		434
32	25.5	6	511
34	27.5		594
36	29.5		683

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d	d_c	p	A_c
38	30.5		731
40	32.5	7	830
42	34.5		935
44	36.5		1046
46	37.5		1104
48	39.5	8	1225
50	41.5		1353
52	43.5		1486
55	45.5		1626
58	48.5	9	1847
60	50.5		2003
62	52.5		2165
65	54.5		2333
68	57.5		2597
70	59.5	10	2781
72	61.5		2971
75	64.5		3267
78	67.5		3578
80	69.5		3794
82	71.5		4015
85	72.5		4128
88	75.5		4477
90	77.5		4717
92	79.5		4964
95	82.5	12	5346
98	85.5		5741
100	87.5		6013
105	92.5		6720
110	97.5		7466
115	100		7854
120	105		8659
125	110		9503
130	115	14	10 387
135	120		11 310
140	125		12 272
145	130		13 273
150	133		13 893
155	138		14 957
160	143		16 061
165	148	16	17 203
170	153		18 385
175	158		19 607

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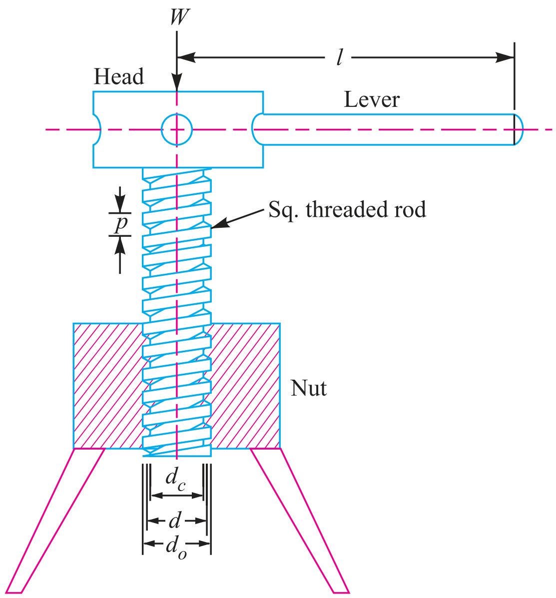
17.3 Multiple Threads

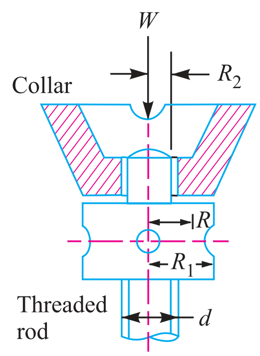
The power screws with multiple threads such as double, triple etc. are employed when it is desired to secure a large lead with fine threads or high efficiency. Such type of threads are usually found in high speed actuators.

17.4 Torque Required to Raise Load by Square Threaded Screws

The torque required to raise a load by means of square threaded screw may be determined by considering a screw jack as shown in Fig. 17.2 (*a*). The load to be raised or lowered is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of lever for lifting or lowering the load.



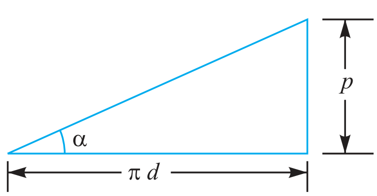
(*a*) Screw jack.



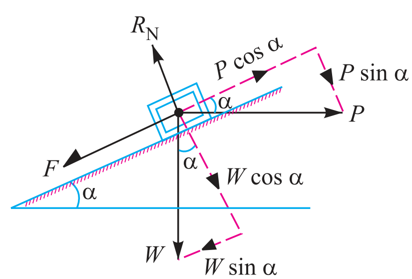
(*b*) Thrust collar.

Fig. 17.2

A little consideration will show that if one complete turn of a screw thread be imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig. 17.3 (*a*).



(*a*) Development of a screw.



(*b*) Forces acting on the screw.

Fig. 17.3

Let p = Pitch of the screw,

d = Mean diameter of the screw,

α = Helix angle,

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P = Effort applied at the circumference of the screw to lift the load,

W = Load to be lifted, and

μ = Coefficient of friction, between the screw and nut

= $\tan \phi$, where ϕ is the friction angle.

From the geometry of the Fig. 17.3 (*a*), we find that

$$\tan \alpha = p / \pi d$$

Since the principle, on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the circumference of a screw jack may be considered to be horizontal as shown in Fig. 17.3 (*b*).

Since the load is being lifted, therefore the force of friction ($F = \mu.R_N$) will act downwards. All the forces acting on the body are shown in Fig. 17.3 (*b*).

Screw jack

Resolving the forces along the plane,

$$P \cos \alpha = W \sin \alpha + F = W \sin \alpha + \mu.R_N \quad \text{...}(i)$$

and resolving the forces perpendicular to the plane,

$$R_N = P \sin \alpha + W \cos \alpha \quad \text{...}(ii)$$

Substituting this value of R_N in equation (*i*), we have

$$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$$

$$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$$

or $$P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

or $$P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$$

$$\therefore \quad P = W \times \frac{(\sin \alpha + \mu \cos \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

or $$P = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by $\cos \phi$, we have

$$P = W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \alpha \sin \phi}$$

$$= W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)} = W \tan (\alpha + \phi)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2}$$

When the axial load is taken up by a thrust collar as shown in Fig. 17.2 (*b*), so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \frac{2}{3} \times \mu_1 \times W \left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2} \right]$$

... (Assuming uniform pressure conditions)

$$= \mu_1 \times W \left(\frac{R_1 + R_2}{2} \right) = \mu_1 W R$$

....(Assuming uniform wear conditions)

where R_1 and R_2 = Outside and inside radii of collar,

R = Mean radius of collar = $\dfrac{R_1 + R_2}{2}$, and

μ_1 = Coefficient of friction for the collar.

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∴ Total torque required to overcome friction (*i.e.* to rotate the screw),

$$T = T_1 + T_2$$

If an effort P_1 is applied at the end of a lever of arm length l, then the total torque required to overcome friction must be equal to the torque applied at the end of lever, *i.e.*

$$T = P \times \frac{d}{2} = P_1 \times l$$

Notes: 1. When the *nominal diameter (d_o) and the **core diameter (d_c) of the screw is given, then

Mean diameter of screw, $$d = \frac{d_o + d_c}{2} = d_o - \frac{p}{2} = d_c + \frac{p}{2}$$

2. Since the mechanical advantage is the ratio of the load lifted (W) to the effort applied (P_1) at the end of the lever, therefore mechanical advantage,

$$\text{M.A.} = \frac{W}{P_1} = \frac{W \times 2\,l}{P \times d} \qquad \ldots\left(\because P \times \frac{d}{2} = P_1 \times l \text{ or } P_1 = \frac{P \times d}{2l}\right)$$

$$= \frac{W \times 2\,l}{W \tan(\alpha + \phi)\, d} = \frac{2\,l}{d \tan(\alpha + \phi)}$$

17.5 Torque Required to Lower Load by Square Threaded Screws

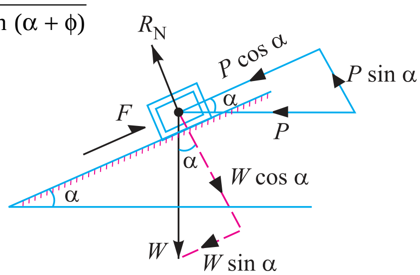

Fig. 17.4

A little consideration will show that when the load is being lowered, the force of friction ($F = \mu.R_N$) will act upwards. All the forces acting on the body are shown in Fig. 17.4.

Resolving the forces along the plane,

$$P \cos \alpha = F - W \sin \alpha$$

$$= \mu\, R_N - W \sin \alpha \qquad \ldots(i)$$

and resolving the forces perpendicular to the plane,

$$R_N = W \cos \alpha - P \sin \alpha \qquad \ldots(ii)$$

Substituting this value of R_N in equation (*i*), we have,

$$P \cos \alpha = \mu\,(W \cos \alpha - P \sin \alpha) - W \sin \alpha$$

$$= \mu\, W \cos \alpha - \mu\, P \sin \alpha - W \sin \alpha$$

or $$P \cos \alpha + \mu\, P \sin \alpha = \mu W \cos \alpha - W \sin \alpha$$

$$P\,(\cos \alpha + \mu \sin \alpha) = W\,(\mu \cos \alpha - \sin \alpha)$$

or $$P = W \times \frac{(\mu \cos \alpha - \sin \alpha)}{(\cos \alpha + \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we have

$$P = W \times \frac{(\tan \phi \cos \alpha - \sin \alpha)}{(\cos \alpha + \tan \phi \sin \alpha)}$$

Multiplying the numerator and denominator by $\cos \phi$, we have

$$P = W \times \frac{(\sin \phi \cos \alpha - \cos \phi \sin \alpha)}{(\cos \phi \cos \alpha + \sin \phi \sin \alpha)}$$

$$= W \times \frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)} = W \tan(\phi - \alpha)$$

* The nominal diameter of a screw thread is also known as ***outside diameter*** or ***major diameter***.

** The core diameter of a screw thread is also known as ***inner diameter*** or ***root diameter*** or ***minor diameter***.

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∴ Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W \tan(\phi - \alpha)\frac{d}{2}$$

Note : When $\alpha > \phi$, then $P = W \tan(\alpha - \phi)$.

17.6 Efficiency of Square Threaded Screws

The efficiency of square threaded screws may be defined as the ratio between the ideal effort (*i.e.* the effort required to move the load, neglecting friction) to the actual effort (*i.e.* the effort required to move the load taking friction into account).

We have seen in Art. 17.4 that the effort applied at the circumference of the screw to lift the load is

$$P = W \tan(\alpha + \phi) \qquad \text{...}(i)$$

where W = Load to be lifted,

α = Helix angle,

ϕ = Angle of friction, and

μ = Coefficient of friction between the screw and nut = $\tan \phi$.

If there would have been no friction between the screw and the nut, then ϕ will be equal to zero. The value of effort P_0 necessary to raise the load, will then be given by the equation,

$$P_0 = W \tan \alpha \qquad \text{[Substituting } \phi = 0 \text{ in equation } (i)]$$

$$\therefore \text{ Efficiency,} \quad \eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W \tan \alpha}{W \tan(\alpha + \phi)} = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

This shows that the efficiency of a screw jack, is independent of the load raised.

In the above expression for efficiency, only the screw friction is considered. However, if the screw friction and collar friction is taken into account, then

$$\eta = \frac{\text{Torque required to move the load, neglecting friction}}{\text{Torque required to move the load, including screw and collar friction}}$$

$$= \frac{T_0}{T} = \frac{P_0 \times d/2}{P \times d/2 + \mu_1 . W . R}$$

Note: The efficiency may also be defined as the ratio of mechanical advantage to the velocity ratio.

We know that mechanical advantage,

$$\text{M.A.} = \frac{W}{P_1} = \frac{W \times 2l}{P \times d} = \frac{W \times 2l}{W \tan(\alpha + \phi)\, d} = \frac{2l}{d \tan(\alpha + \phi)} \qquad \text{...(Refer Art .17.4)}$$

and velocity ratio,

$$V.R. = \frac{\text{Distance moved by the effort } (P_1) \text{ in one revolution}}{\text{Distance moved by the load } (W) \text{ in one revolution}}$$

$$= \frac{2\pi l}{p} = \frac{2\pi l}{\tan \alpha \times \pi d} = \frac{2l}{d \tan \alpha} \qquad \text{... } (\because \tan \alpha = p/\pi d)$$

$$\therefore \text{ Efficiency,} \quad \eta = \frac{M.A.}{V.R.} = \frac{2l}{d \tan(\alpha + \phi)} \times \frac{d \tan \alpha}{2l} = \frac{\tan \alpha}{\tan(\alpha + \phi)}$$

17.7 Maximum Efficiency of a Square Threaded Screw

We have seen in Art. 17.6 that the efficiency of a square threaded screw,

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)} = \frac{\sin \alpha / \cos \alpha}{\sin(\alpha + \phi) / \cos(\alpha + \phi)} = \frac{\sin \alpha \times \cos(\alpha + \phi)}{\cos \alpha \times \sin(\alpha + \phi)} \qquad \text{...}(i)$$

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Multiplying the numerator and denominator by 2, we have,

$$\eta = \frac{2 \sin \alpha \times \cos (\alpha + \phi)}{2 \cos \alpha \times \sin (\alpha + \phi)} = \frac{\sin (2\alpha + \phi) - \sin \phi}{\sin (2\alpha + \phi) + \sin \phi} \quad \text{...}(ii)$$

$$\text{...}\left[\begin{array}{l} \because 2 \sin A \cos B = \sin (A + B) + \sin (A - B) \\ 2 \cos A \sin B = \sin (A + B) - \sin (A - B) \end{array}\right]$$

The efficiency given by equation ***(ii)*** will be maximum when $\sin (2\alpha + \phi)$ is maximum, *i.e.* when

$\sin (2\alpha + \phi) = 1$ or when $2\alpha + \phi = 90°$

$\therefore \quad 2\alpha = 90° - \phi$ or $\alpha = 45° - \phi / 2$

Substituting the value of 2α in equation ***(ii)***, we have maximum efficiency,

$$\eta_{max} = \frac{\sin (90° - \phi + \phi) - \sin \phi}{\sin (90° - \phi + \phi) + \sin \phi} = \frac{\sin 90° - \sin \phi}{\sin 90° + \sin \phi} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

Example 17.1. *A vertical screw with single start square threads of 50 mm mean diameter and 12.5 mm pitch is raised against a load of 10 kN by means of a hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up by a thrust collar which supports the wheel boss and has a mean diameter of 60 mm. The coefficient of friction is 0.15 for the screw and 0.18 for the collar. If the tangential force applied by each hand to the wheel is 100 N, find suitable diameter of the hand wheel.*

Solution. Given : d = 50 mm ; p = 12.5 mm ; W = 10 kN = 10×10^3 N ; D = 60 mm or R = 30 mm ; $\mu = \tan \phi = 0.15$; $\mu_1 = 0.18$; $P_1 = 100$ N

We know that $\tan \alpha = \dfrac{p}{\pi d} = \dfrac{12.5}{\pi \times 50} = 0.08$

and the tangential force required at the circumference of the screw,

$$P = W \tan (\alpha + \phi) = W \left(\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi}\right)$$

$$= 10 \times 10^3 \left[\frac{0.08 + 0.15}{1 - 0.08 \times 0.15}\right] = 2328 \text{ N}$$

We also know that the total torque required to turn the hand wheel,

$$T = P \times \frac{d}{2} + \mu_1 W R = 2328 \times \frac{50}{2} + 0.18 \times 10 \times 10^3 \times 30 \text{ N-mm}$$

$$= 58\,200 + 54\,000 = 112\,200 \text{ N-mm} \quad \text{...}(i)$$

Let D_1 = Diameter of the hand wheel in mm.

We know that the torque applied to the handwheel,

$$T = 2 P_1 \times \frac{D_1}{2} = 2 \times 100 \times \frac{D_1}{2} = 100 D_1 \text{ N-mm} \quad \text{...}(ii)$$

Equating equations ***(i)*** and ***(ii)***,

D_1 = 112 200 / 100 = 1122 mm = 1.122 m **Ans.**

Example 17.2. *An electric motor driven power screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 300 mm / min. The screw has a single square thread of 6 mm pitch on a major diameter of 40 mm. The coefficient of friction at screw threads is 0.1. Estimate power of the motor.*

Solution. Given : W = 75 kN = 75×10^3 N ; v = 300 mm/min ; p = 6 mm ; d_o = 40 mm ; $\mu = \tan \phi = 0.1$

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We know that mean diameter of the screw,

$$d = d_o - p/2 = 40 - 6/2 = 37 \text{ mm}$$

and $$\tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 37} = 0.0516$$

We know that tangential force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right]$$

$$= 75 \times 10^3 \left[\frac{0.0516 + 0.1}{1 - 0.0516 \times 0.1}\right] = 11.43 \times 10^3 \text{ N}$$

and torque required to operate the screw,

$$T = P \times \frac{d}{2} = 11.43 \times 10^3 \times \frac{37}{2} = 211.45 \times 10^3 \text{ N-mm} = 211.45 \text{ N-m}$$

Since the screw moves in a nut at a speed of 300 mm / min and the pitch of the screw is 6 mm, therefore speed of the screw in revolutions per minute (r.p.m.),

$$N = \frac{\text{Speed in mm/min.}}{\text{Pitch in mm}} = \frac{300}{6} = 50 \text{ r.p.m.}$$

and angular speed, $$\omega = 2\pi N/60 = 2\pi \times 50/60 = 5.24 \text{ rad/s}$$

$\therefore$ Power of the motor $= T.\omega = 211.45 \times 5.24 = 1108$ W $= 1.108$ kW **Ans.**

Example. 17.3. *The cutter of a broaching machine is pulled by square threaded screw of 55 mm external diameter and 10 mm pitch. The operating nut takes the axial load of 400 N on a flat surface of 60 mm and 90 mm internal and external diameters respectively. If the coefficient of friction is 0.15 for all contact surfaces on the nut, determine the power required to rotate the operating nut when the cutting speed is 6 m/min. Also find the efficiency of the screw.*

Solution. Given : d_o = 55 mm ; p = 10 mm = 0.01 m ; W = 400 N ; D_2 = 60 mm or R_2 = 30 mm ; D_1 = 90 mm or R_1 = 45 mm ; $\mu = \tan\phi = \mu_1 = 0.15$; Cutting speed = 6 m / min

Power required to operate the nut

We know that the mean diameter of the screw,

$$d = d_o - p/2 = 55 - 10/2 = 50 \text{ mm}$$

$\therefore$ $$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$$

and force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right]$$

$$= 400\left[\frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15}\right] = 86.4 \text{ N}$$

We know that mean radius of the flat surface,

$$R = \frac{R_1 + R_2}{2} = \frac{45 + 30}{2} = 37.5 \text{ mm}$$

$\therefore$ Total torque required,

$$T = P \times \frac{d}{2} + \mu_1 W R = 86.4 \times \frac{50}{2} + 0.15 \times 400 \times 37.5 \text{ N-mm}$$

$$= 4410 \text{ N-mm} = 4.41 \text{ N-m}$$

We know that speed of the screw,

$$N = \frac{\text{Cutting speed}}{\text{Pitch}} = \frac{6}{0.01} = 600 \text{ r.p.m}$$

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and angular speed, $\omega = 2\,\pi\,N / 60 = 2\pi \times 600 / 60 = 62.84$ rad / s

∴ Power required to operate the nut

$= T.\omega = 4.41 \times 62.84 = 277$ W $= 0.277$ kW **Ans.**

Efficiency of the screw

We know that the efficiency of the screw,

$$\eta = \frac{T_0}{T} = \frac{W \tan \alpha \times d/2}{T} = \frac{400 \times 0.0637 \times 50/2}{4410}$$

$= 0.144$ or 14.4% **Ans.**

Example 17.4. *A vertical two start square threaded screw of a 100 mm mean diameter and 20 mm pitch supports a vertical load of 18 kN. The axial thrust on the screw is taken by a collar bearing of 250 mm outside diameter and 100 mm inside diameter. Find the force required at the end of a lever which is 400 mm long in order to lift and lower the load. The coefficient of friction for the vertical screw and nut is 0.15 and that for collar bearing is 0.20.*

Solution. Given : $d = 100$ mm ; $p = 20$ mm ; $W = 18$ kN $= 18 \times 10^3$N ; $D_1 = 250$ mm or $R_1 = 125$ mm ; $D_2 = 100$ mm or $R_2 = 50$ mm ; $l = 400$ mm ; $\mu = \tan \phi = 0.15$; $\mu_1 = 0.20$

Force required at the end of lever

Let P = Force required at the end of lever.

Since the screw is a two start square threaded screw, therefore lead of the screw

$= 2\,p = 2 \times 20 = 40$ mm

We know that $\tan \alpha = \dfrac{\text{Lead}}{\pi d} = \dfrac{40}{\pi \times 100} = 0.127$

1. ***For raising the load***

We know that tangential force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W \left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \tan\phi}\right]$$

$$= 18 \times 10^3 \left[\frac{0.127 + 0.15}{1 - 0.127 \times 0.15}\right] = 5083 \text{ N}$$

and mean radius of the collar,

$$R = \frac{R_1 + R_2}{2} = \frac{125 + 50}{2} = 87.5 \text{ mm}$$

∴ Total torque required at the end of lever,

$$T = P \times \frac{d}{2} + \mu_1 WR$$

$$= 5083 \times \frac{100}{2} + 0.20 \times 18 \times 10^3 \times 87.5 = 569\,150 \text{ N-mm}$$

We know that torque required at the end of lever (T),

$569\,150 = P_1 \times l = P_1 \times 400$ or $P_1 = 569\,150/400 = 1423$ N **Ans.**

2. ***For lowering the load***

We know that tangential force required at the circumference of the screw,

$$P = W \tan(\phi - \alpha) = W \left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi \tan\alpha}\right]$$

$$= 18 \times 10^3 \left[\frac{0.15 - 0.127}{1 + 0.15 \times 0.127}\right] = 406.3 \text{ N}$$

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and the total torque required the end of lever,

$$T = P \times \frac{d}{2} + \mu_1 W R$$

$$= 406.3 \times \frac{100}{2} + 0.20 \times 18 \times 10^3 \times 87.5 = 335\ 315 \text{ N-mm}$$

We know that torque required at the end of lever (T),

$$335\ 315 = P_1 \times l = P_1 \times 400 \quad \text{or} \quad P_1 = 335\ 315 / 400 = 838.3 \text{ N} \quad \textbf{Ans.}$$

Example 17.5. The *mean diameter of the square threaded screw having pitch of 10 mm is 50 mm. A load of 20 kN is lifted through a distance of 170 mm. Find the work done in lifting the load and the efficiency of the screw, when*

1. *The load rotates with the screw, and*
2. *The load rests on the loose head which does not rotate with the screw.*

The external and internal diameter of the bearing surface of the loose head are 60 mm and 10 mm respectively. The coefficient of friction for the screw and the bearing surface may be taken as 0.08.

Solution. Given : p = 10 mm ; d = 50 mm ; W = 20 kN = 20 × 10^3 N ; D_1 = 60 mm or R_2 = 30 mm ; D_2 = 10 mm or R_2 = 5 mm ; $\mu = \tan \phi = \mu_1 = 0.08$

We know that $$\tan \alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$$

∴ Force required at the circumference of the screw to lift the load,

$$P = W \tan (\alpha + \phi) = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right]$$

$$= 20 \times 10^3 \left[\frac{0.0637 + 0.08}{1 - 0.0673 \times 0.08} \right] = 2890 \text{ N}$$

and torque required to overcome friction at the screw,

$$T = P \times d / 2 = 2890 \times 50 / 2 = 72\ 250 \text{ N-mm} = 72.25 \text{ N-m}$$

Since the load is lifted through a vertical distance of 170 mm and the distance moved by the screw in one rotation is 10 mm (equal to pitch), therefore number of rotations made by the screw,

$$N = 170 / 10 = 17$$

1. *When the load rotates with the screw*

We know that workdone in lifting the load

$$= T \times 2 \pi N = 72.25 \times 2\pi \times 17 = 7718 \text{ N-m} \quad \textbf{Ans.}$$

and efficiency of the screw,

$$\eta = \frac{\tan \alpha}{\tan (\alpha + \phi)} = \frac{\tan \alpha (1 - \tan \alpha \tan \phi)}{\tan \alpha + \tan \phi}$$

$$= \frac{0.0637 (1 - 0.0637 \times 0.08)}{0.0637 + 0.08} = 0.441 \text{ or } 44.1\% \quad \textbf{Ans.}$$

2. *When the load does not rotate with the screw*

We know that mean radius of the bearing surface,

$$R = \frac{R_1 + R_2}{2} = \frac{30 + 5}{2} = 17.5 \text{ mm}$$

and torque required to overcome friction at the screw and the collar,

$$T = P \times \frac{d}{2} + \mu_1 W R$$

$$= 2890 \times \frac{50}{2} + 0.08 \times 20 \times 10^3 \times 17.5 = 100\ 250 \text{ N-mm}$$

$$= 100.25 \text{ N-m}$$

∴ Workdone by the torque in lifting the load

$$= T \times 2\pi\, N = 100.25 \times 2\pi \times 17 = 10\ 710 \text{ N-m} \textbf{ Ans.}$$

We know that torque required to lift the load, neglecting friction,

$$T_0 = P_0 \times d/2 = W \tan\alpha \times d/2 \qquad \text{... } (P_o = W \tan\alpha)$$

$$= 20 \times 10^3 \times 0.0637 \times 50 / 2 = 31\ 850 \text{ N-mm} = 31.85 \text{ N-m}$$

∴ Efficiency of the screw,

$$\eta = \frac{T_0}{T} = \frac{31.85}{100.25} = 0.318 \text{ or } 31.8\% \quad \textbf{Ans.}$$

17.8 Efficiency Vs Helix Angle

We have seen in Art. 17.6 that the efficiency of a square threaded screw depends upon the helix angle α and the friction angle ϕ. The variation of efficiency of a square threaded screw for raising the load with the helix angle α is shown in Fig. 17.5. We see that the efficiency of a square threaded screw increases rapidly upto helix angle of 20°, after which the increase in efficiency is slow. The efficiency is maximum for helix angle between 40 to 45°.

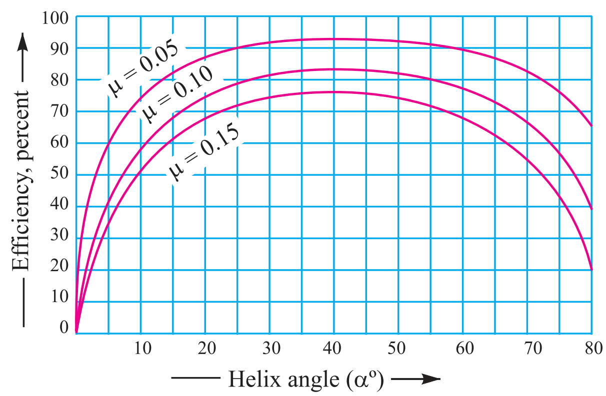

Fig. 17.5. Graph between efficiency and helix angle.

When the helix angle further increases say 70°, the efficiency drops. This is due to the fact that the normal thread force becomes large and thus the force of friction and the work of friction becomes large as compared with the useful work. This results in low efficiency.

17.9 Over Hauling and Self Locking Screws

We have seen in Art. 17.5 that the effort required at the circumference of the screw to lower the load is

$$P = W \tan(\phi - \alpha)$$

and the torque required to lower the load,

$$T = P \times \frac{d}{2} = W \tan(\phi - \alpha)\frac{d}{2}$$

In the above expression, if $\phi < \alpha$, then torque required to lower the load will be ***negative***. In other words, the load will start moving downward without the application of any torque. Such a condition is known as ***over hauling of screws.*** If however, $\phi > \alpha$, the torque required to lower the load will be ***positive,*** indicating that an effort is applied to lower the load. Such a screw is known as

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Mechanical power screw driver

self locking screw. In other words, a screw will be self locking if the friction angle is greater than helix angle or coefficient of friction is greater than tangent of helix angle *i.e.* μ or $\tan\phi > \tan\alpha$.

17.10 Efficiency of Self Locking Screws

We know that the efficiency of screw,

$$\eta = \frac{\tan\phi}{\tan(\alpha + \phi)}$$

and for self locking screws, $\phi \geq \alpha$ or $\alpha \leq \phi$.

$\therefore$ Efficiency for self locking screws,

$$\eta \leq \frac{\tan\phi}{\tan(\phi+\phi)} \leq \frac{\tan\phi}{\tan 2\phi} \leq \frac{\tan\phi\,(1-\tan^2\phi)}{2\tan\phi} \leq \frac{1}{2} - \frac{\tan^2\phi}{2}$$

$$\ldots\left(\because \tan 2\phi = \frac{2\tan\phi}{1-\tan^2\phi}\right)$$

From this expression we see that efficiency of self locking screws is less than $\frac{1}{2}$ or 50%. If the efficiency is more than 50%, then the screw is said to be overhauling.

Note: It can be proved as follows:

Let W = Load to be lifted, and

h = Distance through which the load is lifted.

$\therefore$ Output = $W.h$

and Input = $\frac{\text{Output}}{\eta} = \frac{W.h}{\eta}$

$\therefore$ Work lost in overcoming friction

$$= \text{Input} - \text{Output} = \frac{W.h}{\eta} - W.h = W.h\left(\frac{1}{\eta} - 1\right)$$

For self locking,

$$W.h\left(\frac{1}{\eta} - 1\right) \leq W.h$$

$$\therefore \quad \frac{1}{\eta} - 1 \leq 1 \quad \text{or} \quad \eta \leq \frac{1}{2} \quad \text{or} \quad 50\%$$

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17.11 Coefficient of Friction

The coefficient of friction depends upon various factors like *material of screw and nut, workmanship in cutting screw, quality of lubrication, unit bearing pressure and the rubbing speeds. The value of coefficient of friction does not vary much with different combination of material, load or rubbing speed, except under starting conditions. The coefficient of friction, with good lubrication and average workmanship, may be assumed between 0.10 and 0.15. The various values for coefficient of friction for steel screw and cast iron or bronze nut, under different conditions are shown in the following table.

Table 17.5. Coefficient of friction under different conditions.

S.No.	*Condition*	*Average coefficient of friction*	
		Starting	*Running*
1.	High grade materials and workmanship and best running conditions.	0.14	0.10
2.	Average quality of materials and workmanship and average running conditions.	0.18	0.13
3.	Poor workmanship or very slow and in frequent motion with indifferent lubrication or newly machined surface.	0.21	0.15

If the thrust collars are used, the values of coefficient of friction may be taken as shown in the following table.

Table 17.6. Coefficient of friction when thrust collars are used.

S.No.	*Materials*	*Average coefficient of friction*	
		Starting	*Running*
1.	Soft steel on cast iron	0.17	0.12
2.	Hardened steel on cast iron	0.15	0.09
3.	Soft steel on bronze	0.10	0.08
4.	Hardened steel on bronze	0.08	0.06

17.12 Acme or Trapezoidal Threads

We know that the normal reaction in case of a square threaded screw is

$$R_N = W \cos \alpha,$$

where α is the helix angle.

But in case of Acme or trapezoidal thread, the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load (W).

Consider an Acme or trapezoidal thread as shown in Fig. 17.6.

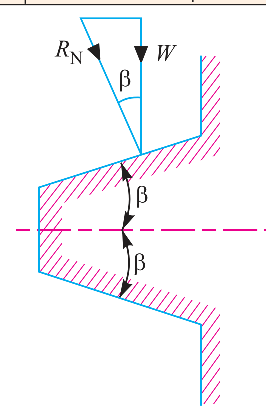

Fig. 17.6. Acme or trapeonidal threads.

Let **2β = Angle of the Acme thread, and

β = Semi-angle of the thread.

* The material of screw is usually steel and the nut is made of cast iron, gun metal, phosphor bronze in order to keep the wear to a mininum.

** For Acme threads, $2\beta = 29°$, and for trapezoidal threads, $2\beta = 30°$.

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$$\therefore \qquad R_N = \frac{W}{\cos\beta}$$

and frictional force, $F = \mu . R_N = \mu \times \frac{W}{\cos\beta} = \mu_1 . W$

where $\mu / \cos\beta = \mu_1$, known as ***virtual coefficient of friction.***

Notes : 1. When coefficient of friction, $\mu_1 = \frac{\mu}{\cos\beta}$ is considered, then the Acme thread is equivalent to a square thread.

2. All equations of square threaded screw also hold good for Acme threads. In case of Acme threads, μ_1 (*i.e.* $\tan\phi_1$) may be substituted in place of μ (*i.e.* $\tan\phi$). Thus for Acme threads,

$$P = W \tan(\alpha + \phi_1)$$

where ϕ_1 = Virtual friction angle, and $\tan\phi_1 = \mu_1$.

Example 17.6. *The lead screw of a lathe has Acme threads of 50 mm outside diameter and 8 mm pitch. The screw must exert an axial pressure of 2500 N in order to drive the tool carriage. The thrust is carried on a collar 110 mm outside diameter and 55 mm inside diameter and the lead screw rotates at 30 r.p.m. Determine* (a) *the power required to drive the screw; and* (b) *the efficiency of the lead screw. Assume a coefficient of friction of 0.15 for the screw and 0.12 for the collar.*

Solution. Given : d_o = 50 mm ; p = 8 mm ; W = 2500 N ; D_1 = 110 mm or R_1 = 55 mm ; D_2 = 55 mm or R_2 = 27.5 mm ; N = 30 r.p.m. ; $\mu = \tan\phi = 0.15$; $\mu_2 = 0.12$

(a) Power required to drive the screw

We know that mean diameter of the screw,

$$d = d_o - p / 2 = 50 - 8 / 2 = 46 \text{ mm}$$

$$\therefore \qquad \tan\alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 46} = 0.055$$

Since the angle for Acme threads is $2\beta = 29°$ or $\beta = 14.5°$, therefore virtual coefficient of friction,

$$\mu_1 = \tan\phi_1 = \frac{\mu}{\cos\beta} = \frac{0.15}{\cos 14.5°} = \frac{0.15}{0.9681} = 0.155$$

We know that the force required to overcome friction at the screw,

$$P = W \tan(\alpha + \phi_1) = W\left[\frac{\tan\alpha + \tan\phi_1}{1 - \tan\alpha \tan\phi_1}\right]$$

$$= 2500\left[\frac{0.055 + 0.155}{1 - 0.055 \times 0.155}\right] = 530 \text{ N}$$

and torque required to overcome friction at the screw.

$$T_1 = P \times d / 2 = 530 \times 46 / 2 = 12\,190 \text{ N-mm}$$

We know that mean radius of collar,

$$R = \frac{R_1 + R_2}{2} = \frac{55 + 27.5}{2} = 41.25 \text{ mm}$$

Assuming uniform wear, the torque required to overcome friction at collars,

$$T_2 = \mu_2 W R = 0.12 \times 2500 \times 41.25 = 12\,375 \text{ N-mm}$$

$\therefore$ Total torque required to overcome friction,

$$T = T_1 + T_2 = 12\,190 + 12\,375 = 24\,565 \text{ N-mm} = 24.565 \text{ N-m}$$

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We know that power required to drive the screw

$$= T.\omega = \frac{T \times 2\pi N}{60} = \frac{24.565 \times 2\pi \times 30}{60} = 77\text{ W} = 0.077\text{ kW} \quad \textbf{Ans.}$$

... ($\because$ $\omega = 2\pi N / 60$)

(b) Efficiency of the lead screw

We know that the torque required to drive the screw with no friction,

$$T_o = W \tan\alpha \times \frac{d}{2} = 2500 \times 0.055 \times \frac{46}{2} = 3163\text{ N-mm} = 3.163\text{ N-m}$$

$\therefore$ Efficiency of the lead screw,

$$\eta = \frac{T_o}{T} = \frac{3.163}{24.565} = 0.13 \text{ or } 13\% \quad \textbf{Ans.}$$

17.13 Stresses in Power Screws

A power screw must have adequate strength to withstand axial load and the applied torque. Following types of stresses are induced in the screw.

1. *Direct tensile or compressive stress due to an axial load.* The direct stress due to the axial load may be determined by dividing the axial load (W) by the minimum cross-sectional area of the screw (A_c) *i.e.* area corresponding to minor or core diameter (d_c).

$\therefore$ Direct stress (tensile or compressive)

$$= \frac{W}{A_c}$$

This is only applicable when the axial load is compressive and the unsupported length of the screw between the load and the nut is short. But when the screw is axially loaded in compression and the unsupported length of the screw between the load and the nut is too great, then the design must be based on column theory assuming suitable end conditions. In such cases, the cross-sectional area corresponding to core diameter may be obtained by using Rankine-Gordon formula or J.B. Johnson's formula. According to this,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4 C \pi^2 E}\left(\frac{L}{k}\right)^2\right]$$

$$\therefore \qquad \sigma_c = \frac{W}{A_c}\left[\frac{1}{1 - \frac{\sigma_y}{4 C \pi^2 E}\left(\frac{L}{k}\right)^2}\right]$$

where
W_{cr} = Critical load,
σ_y = Yield stress,
L = Length of screw,
k = Least radius of gyration,
C = End-fixity coefficient,
E = Modulus of elasticity, and
σ_c = Stress induced due to load W.

Note : In actual practice, the core diameter is first obtained by considering the screw under simple compression and then checked for critical load or buckling load for stability of the screw.

2. *Torsional shear stress.* Since the screw is subjected to a twisting moment, therefore torsional shear stress is induced. This is obtained by considering the minimum cross-section of the screw. We know that torque transmitted by the screw,

$$T = \frac{\pi}{16} \times \tau (d_c)^3$$

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or shear stress induced,

$$\tau = \frac{16\,T}{\pi (d_c)^3}$$

When the screw is subjected to both direct stress and torsional shear stress, then the design must be based on maximum shear stress theory, according to which maximum shear stress on the minor diameter section,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_t \text{ or } \sigma_c)^2 + 4\,\tau^2}$$

It may be noted that when the unsupported length of the screw is short, then failure will take place when the maximum shear stress is equal to the shear yield strength of the material. In this case, shear yield strength,

$$\tau_y = \tau_{max} \times \text{Factor of safety}$$

Friction between the threads of screw and nut plays important role in determining the efficiency and locking properties of a screw

3. *Shear stress due to axial load.* The threads of the screw at the core or root diameter and the threads of the nut at the major diameter may shear due to the axial load. Assuming that the load is uniformly distributed over the threads in contact, we have

Shear stress for screw,

$$\tau_{(screw)} = \frac{W}{\pi\, n\, .d_c\, .\, t}$$

and shear stress for nut,

$$\tau_{(nut)} = \frac{W}{\pi\, n\, .d_o\, .\, t}$$

where W = Axial load on the screw,

n = Number of threads in engagement,

d_c = Core or root diameter of the screw,

d_o = Outside or major diameter of nut or screw, and

t = Thickness or width of thread.

4. *Bearing pressure.* In order to reduce wear of the screw and nut, the bearing pressure on the thread surfaces must be within limits. In the design of power screws, the bearing pressure depends upon the materials of the screw and nut, relative velocity between the nut and screw and the nature of lubrication. Assuming that the load is uniformly distributed over the threads in contact, the bearing pressure on the threads is given by

$$p_b = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{{}^{*}W}{\pi\, d\,.t\,.n}$$

where d = Mean diameter of screw,

t = Thickness or width of screw = p / 2, and

n = Number of threads in contact with the nut

$$= \frac{\text{Height of the nut}}{\text{Pitch of threads}} = \frac{h}{p}$$

Therefore, from the above expression, the height of nut or the length of thread engagement of the screw and nut may be obtained.

The following table shows some limiting values of bearing pressures.

* We know that $\dfrac{(d_o)^2 - (d_c)^2}{4} = \dfrac{d_o + d_c}{2} \times \dfrac{d_o - d_c}{2} = d \times \dfrac{p}{2} = d.t$

Table 17.7. Limiting values of bearing pressures.

Application of screw	*Material*		*Safe bearing pressure in N/mm²*	*Rubbing speed at thread pitch diameter*
	Screw	*Nut*		
1. Hand press	Steel	Bronze	17.5 - 24.5	Low speed, well lubricated
2. Screw jack	Steel	Cast iron	12.6 – 17.5	Low speed < 2.4 m / min
	Steel	Bronze	11.2 – 17.5	Low speed < 3 m / min
3. Hoisting screw	Steel	Cast iron	4.2 – 7.0	Medium speed 6 – 12 m / min
	Steel	Bronze	5.6 – 9.8	Medium speed 6 – 12 m / min
4. Lead screw	Steel	Bronze	1.05 – 1.7	High speed > 15 m / min

Example 17.7. *A power screw having double start square threads of 25 mm nominal diameter and 5 mm pitch is acted upon by an axial load of 10 kN. The outer and inner diameters of screw collar are 50 mm and 20 mm respectively. The coefficient of thread friction and collar friction may be assumed as 0.2 and 0.15 respectively. The screw rotates at 12 r.p.m. Assuming uniform wear condition at the collar and allowable thread bearing pressure of 5.8 N/mm², find: 1. the torque required to rotate the screw; 2. the stress in the screw; and 3. the number of threads of nut in engagement with screw.*

Solution. Given : d_o = 25 mm ; p = 5 mm ; W = 10 kN = 10 × 10³ N ; D_1 = 50 mm or R_1 = 25 mm ; D_2 = 20 mm or R_2 = 10 mm ; $\mu = \tan\phi = 0.2$; $\mu_1 = 0.15$; N = 12 r.p.m. ; p_b = 5.8 N/mm²

1. *Torque required to rotate the screw*

We know that mean diameter of the screw,

$$d = d_o - p/2 = 25 - 5/2 = 22.5 \text{ mm}$$

Since the screw is a double start square threaded screw, therefore lead of the screw,

$$= 2\,p = 2 \times 5 = 10 \text{ mm}$$

$$\therefore \qquad \tan\alpha = \frac{\text{Lead}}{\pi d} = \frac{10}{\pi \times 22.5} = 0.1414$$

We know that tangential force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right]$$

$$= 10 \times 10^3 \left[\frac{0.1414 + 0.2}{1 - 0.1414 \times 0.2}\right] = 3513 \text{ N}$$

and mean radius of the screw collar,

$$R = \frac{R_1 + R_2}{2} = \frac{25 + 10}{2} = 17.5$$

∴ Total torque required to rotate the screw,

$$T = P \times \frac{d}{2} + \mu_1 \, W \, R = 3513 \times \frac{22.5}{2} + 0.15 \times 10 \times 10^3 \times 17.5 \text{ N-mm}$$

$$= 65\,771 \text{ N-mm} = 65.771 \text{ N-m}$$ **Ans.**

2. *Stress in the screw*

We know that the inner diameter or core diameter of the screw,

$$d_c = d_o - p = 25 - 5 = 20 \text{ mm}$$

∴ Corresponding cross-sectional area of the screw,

$$A_c = \frac{\pi}{4}(d_c)^2 = \frac{\pi}{4}(20)^2 = 314.2 \text{ mm}^2$$

We know that direct stress,

$$\sigma_c = \frac{W}{A_c} = \frac{10 \times 10^3}{314.2} = 31.83 \text{ N/mm}^2$$

and shear stress,
$$\tau = \frac{16\,T}{\pi (d_c)^3} = \frac{16 \times 65771}{\pi (20)^3} = 41.86 \text{ N/mm}^2$$

We know that maximum shear stress in the screw,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2} = \frac{1}{2}\sqrt{(31.83)^2 + 4(41.86)^2}$$

$$= 44.8 \text{ N/mm}^2 = 44.8 \text{ MPa}$$ **Ans.**

3. *Number of threads of nut in engagement with screw*

Let n = Number of threads of nut in engagement with screw, and

t = Thickness of threads = $p / 2 = 5 / 2 = 2.5$ mm

We know that bearing pressure on the threads (p_b),

$$5.8 = \frac{W}{\pi d \times t \times n} = \frac{10 \times 10^3}{\pi \times 22.5 \times 2.5 \times n} = \frac{56.6}{n}$$

∴ $n = 56.6 / 5.8 = 9.76$ say 10 **Ans.**

Example 17.8. *The screw of a shaft straightener exerts a load of 30 kN as shown in Fig. 17.7. The screw is square threaded of outside diameter 75 mm and 6 mm pitch. Determine:*

1. Force required at the rim of a 300 mm diameter hand wheel, assuming the coefficient of friction for the threads as 0.12;

2. Maximum compressive stress in the screw, bearing pressure on the threads and maximum shear stress in threads; and

3. Efficiency of the straightner.

Solution. Given : $W = 30$ kN $= 30 \times 10^3$ N ; $d_o = 75$ mm ; $p = 6$ mm ; $D = 300$ mm ; $\mu = \tan \phi = 0.12$

1. *Force required at the rim of handwheel*

Let P_1 = Force required at the rim of handwheel.

We know that the inner diameter or core diameter of the screw,

$$d_c = d_o - p = 75 - 6 = 69 \text{ mm}$$

Mean diameter of the screw,

$$*d = \frac{d_o + d_c}{2} = \frac{75 + 69}{2} = 72 \text{ mm}$$

and $$\tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 72} = 0.0265$$

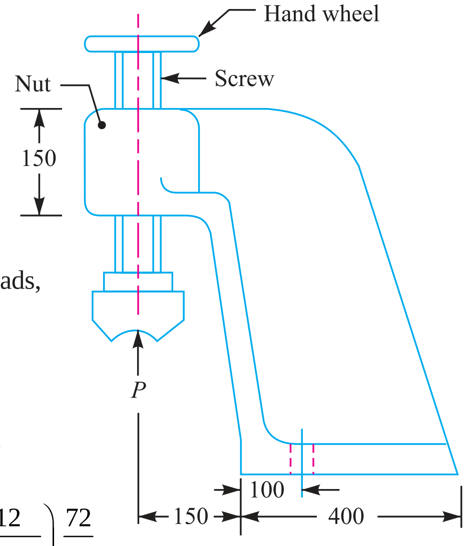

Fig. 17.7

∴ Torque required to overcome friction at the threads,

$$T = P \times \frac{d}{2}$$

$$= W \tan(\alpha + \phi) \frac{d}{2}$$

$$= W \left(\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \tan\phi} \right) \frac{d}{2}$$

$$= 30 \times 10^3 \left(\frac{0.0265 + 0.12}{1 - 0.0265 \times 0.12} \right) \frac{72}{2}$$

$$= 158\ 728 \text{ N-mm}$$

We know that the torque required at the rim of handwheel (T),

$$158\ 728 = P_1 \times \frac{D}{2} = P_1 \times \frac{300}{2} = 150\ P_1$$

∴ $P_1 = 158\ 728 / 150 = 1058$ N **Ans.**

2. *Maximum compressive stress in the screw*

We know that maximum compressive stress in the screw,

$$\sigma_c = \frac{W}{A_c} = \frac{W}{\frac{\pi}{4}(d_c)^2} = \frac{30 \times 10^3}{\frac{\pi}{4}(69)^2} = 8.02 \text{ N/mm}^2 = 8.02 \text{ MPa} \quad \textbf{Ans.}$$

Bearing pressure on the threads

We know that number of threads in contact with the nut,

$$n = \frac{\text{Height of nut}}{\text{Pitch of threads}} = \frac{150}{6} = 25 \text{ threads}$$

and thickness of threads, $t = p/2 = 6/2 = 3$ mm

We know that bearing pressure on the threads,

$$p_b = \frac{W}{\pi\ d\ .\ t . n} = \frac{30 \times 10^3}{\pi \times 72 \times 3 \times 25} = 1.77 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Maximum shear stress in the threads

We know that shear stress in the threads,

$$\tau = \frac{16\ T}{\pi (d_c)^3} = \frac{16 \times 158\ 728}{\pi\ (69)^3} = 2.46 \text{ N/mm}^2 \quad \textbf{Ans.}$$

* The mean diameter of the screw (d) is also given by

$$d = d_o - p/2 = 75 - 6/2 = 72 \text{ mm}$$

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∴ Maximum shear stress in the threads,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{(8.02)^2 + 4\,(2.46)^2}$$

$$= 4.7 \text{ N/mm}^2 = 4.7 \text{ MPa} \quad \textbf{Ans.}$$

3. *Efficiency of the straightener*

We know that the torque required with no friction,

$$T_0 = W \tan\alpha \times \frac{d}{2} = 30 \times 10^3 \times 0.0265 \times \frac{72}{2} = 28\,620 \text{ N-mm}$$

∴ Efficiency of the straightener,

$$\eta = \frac{T_0}{T} = \frac{28\,620}{158\,728} = 0.18 \text{ or } 18\% \quad \textbf{Ans.}$$

Example 17.9. *A sluice gate weighing 18 kN is raised and lowered by means of square threaded screws, as shown in Fig. 17.8. The frictional resistance induced by water pressure against the gate when it is in its lowest position is 4000 N.*

The outside diameter of the screw is 60 mm and pitch is 10 mm. The outside and inside diameter of washer is 150 mm and 50 mm respectively. The coefficient of friction between the screw and nut is 0.1 and for the washer and seat is 0.12. Find :

1. The maximum force to be exerted at the ends of the lever raising and lowering the gate, 2. Efficiency of the arrangement, and 3. Number of threads and height of nut, for an allowable bearing pressure of 7 N/mm².

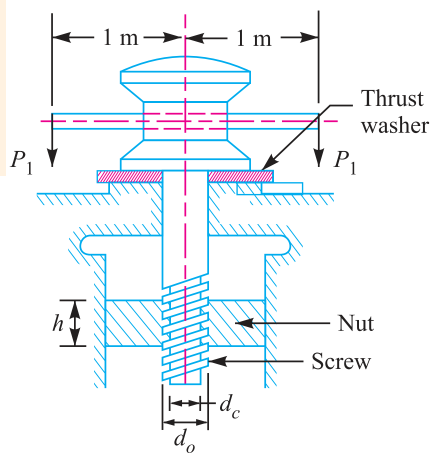

Fig. 17.8

Solution. Given : W_1 = 18 kN = 18 000 N ; F = 4000 N ; d_o = 60 mm ; p = 10 mm ; D_1 = 150 mm or R_1 = 75 mm ; D_2 = 50 mm or R_2 = 25 mm ; $\mu = \tan\phi$ = 0.1 ; μ_1 = 0.12 ; p_b = 7 N/mm²

1. *Maximum force to be exerted at the ends of lever*

Let P_1 = Maximum force exerted at each end of the lever 1 m (1000 mm) long.

We know that inner diameter or core diameter of the screw,

$$d_c = d_o - p = 60 - 10 = 50 \text{ mm}$$

Mean diameter of the screw,

$$d = \frac{d_o + d_c}{2} = \frac{60 + 50}{2} = 55 \text{ mm}$$

and
$$\tan\alpha = \frac{p}{\pi\, d} = \frac{10}{\pi \times 55} = 0.058$$

(a) For raising the gate

Since the frictional resistance acts in the opposite direction to the motion of screw, therefore for raising the gate, the frictional resistance (F) will act downwards.

∴ Total load acting on the screw,

$$W = W_1 + F = 18\,000 + 4000 = 22\,000 \text{ N}$$

and torque required to overcome friction at the screw,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi)\frac{d}{2} = W\left(\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right)\frac{d}{2}$$

Top

$$= 22\ 000 \left(\frac{0.058 + 0.1}{1 - 0.058 \times 0.1} \right) \frac{55}{2} = 96\ 148 \text{ N-mm}$$

Mean radius of washer,

$$R = \frac{R_1 + R_2}{2} = \frac{75 + 25}{2} = 50 \text{ mm}$$

∴ Torque required to overcome friction at the washer,

$$T_2 = \mu_1 W R = 0.12 \times 22\ 000 \times 50 = 132\ 000 \text{ N-mm}$$

and total torque required to overcome friction,

$$T = T_1 + T_2 = 96\ 148 + 132\ 000 = 228\ 148 \text{ N-mm}$$

We know that the torque required at the end of lever (T),

$$228\ 148 = 2P_1 \times \text{Length of lever} = 2P_1 \times 1000 = 2000\ P_1$$

∴ $P_1 = 228\ 148 / 2000 = 141.1$ N **Ans.**

(b) For lowering the gate

Since the gate is being lowered, therefore the frictional resistance (F) will act upwards,

∴ Total load acting on the screw,

$$W = W_1 - F = 18\ 000 - 4000 = 14\ 000 \text{ N}$$

We know that torque required to overcome friction at the screw,

$$T_1 = P \times \frac{d}{2} = W \tan (\phi - \alpha) \frac{d}{2} = W \left(\frac{\tan \phi - \tan \alpha}{1 + \tan \phi \tan \alpha} \right) \frac{d}{2}$$

$$= 14\ 000 \left(\frac{0.1 - 0.058}{1 + 0.1 \times 0.058} \right) \frac{55}{2} = 16\ 077 \text{ N-mm}$$

and torque required to overcome friction at the washer,

$$T_2 = \mu_1 W R = 0.12 \times 14\ 000 \times 50 = 84\ 000 \text{ N-mm}$$

∴ Total torque required to overcome friction,

$$T = T_1 + T_2 = 16\ 077 + 84\ 000 = 100\ 077 \text{ N-mm}$$

We know that the torque required at the end of lever (T),

$100\ 077 = 2P_1 \times 1000 = 2000\ P_1$ or $P_1 = 100\ 077/2000 = 50.04$ N **Ans.**

2. Efficiency of the arrangement

We know that the torque required for raising the load, with no friction,

$$T_0 = W \tan \alpha \times \frac{d}{2} = 22\ 000 \times 0.058 \times \frac{55}{2} = 35\ 090 \text{ N-mm}$$

∴ Efficiency of the arrangement,

$$\eta = \frac{T_0}{T} = \frac{35\ 090}{228\ 148} = 0.154 \text{ or } 15.4\%$$ **Ans.**

3. Number of threads and height of nut

Let n = Number of threads in contact with the nut,

h = Height of nut = $n \times p$, and

t = Thickness of thread = $p / 2 = 10 / 2 = 5$ mm.

We know that the bearing pressure (p_b),

$$7 = \frac{W}{\pi . d . t . n} = \frac{22000}{\pi \times 55 \times 5 \times n} = \frac{25.46}{n}$$

∴ $n = 25.46 / 7 = 3.64$ say 4 threads **Ans.**

and $h = n \times p = 4 \times 10 = 40$ mm **Ans.**

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Example 17.10. *The screw, as shown in Fig. 17.9 is operated by a torque applied to the lower end. The nut is loaded and prevented from turning by guides. Assume friction in the ball bearing to be negligible. The screw is a triple start trapezoidal thread. The outside diameter of the screw is 48 mm and pitch is 8 mm. The coefficient of friction of the threads is 0.15. Find :*

1. Load which can be raised by a torque of 40 N-m ;

2. Whether the screw is overhauling ; and

3. Average bearing pressure between the screw and nut thread surface.

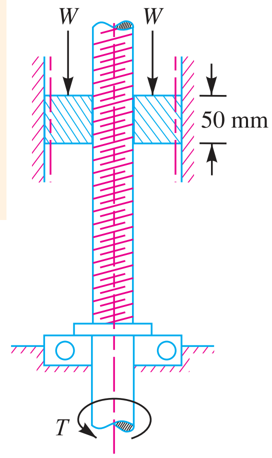


Fig. 17.9

Solution. Given : d_o = 48 mm ; p = 8 mm ; $\mu = \tan \phi = 0.15$; T = 40 N-m = 40 000 N-mm

1. *Load which can be raised*

Let W = Load which can be raised.

We know that mean diameter of the screw,

$$d = d_o - p/2 = 48 - 8/2 = 44 \text{ mm}$$

Since the screw is a triple start, therefore lead of the screw

$$= 3\,p = 3 \times 8 = 24 \text{ mm}$$

$$\therefore \quad \tan \alpha = \frac{\text{Lead}}{\pi d} = \frac{24}{\pi \times 44} = 0.174$$

and virtual coefficient of friction,

$$\mu_1 = \tan \phi_1 = \frac{\mu}{\cos \beta} = \frac{0.15}{\cos 15°} = \frac{0.15}{0.9659} = 0.155$$

... (∵ For trapezoidal threads, 2 β = 30°)

We know that the torque required to raise the load,

$$T = P \times \frac{d}{2} = W \tan(\alpha + \phi_1)\frac{d}{2} = W\left[\frac{\tan\alpha + \tan\phi_1}{1 - \tan\alpha \tan\phi_1}\right]\frac{d}{2}$$

$$40\,000 = W\left(\frac{0.174 + 0.155}{1 - 0.174 \times 0.155}\right)\frac{44}{2} = 7.436\,W$$

$$\therefore \quad W = 40\,000 / 7.436 = 5380 \text{ N}$$ **Ans.**

2. *Whether the screw is overhauling*

We know that torque required to lower the load,

$$T = W \tan(\phi_1 - \alpha)\frac{d}{2}$$

We have discussed in Art. 17.9 that if ϕ_1 is less than α, then the torque required to lower the load will be ***negative,*** *i.e.* the load will start moving downward without the application of any torque. Such a condition is known as overhauling of screws.

In the present case, tan ϕ_1 = 0.155 and tan α = 0.174. Since ϕ_1 is less than α, therefore the screw is overhauling. **Ans.**

3. *Average bearing pressure between the screw and nut thread surfaces*

We know that height of the nut,

$$h = n \times p = 50 \text{ mm}$$ ...(Given)

∴ Number of threads in contact,

$$n = h/p = 50/8 = 6.25$$

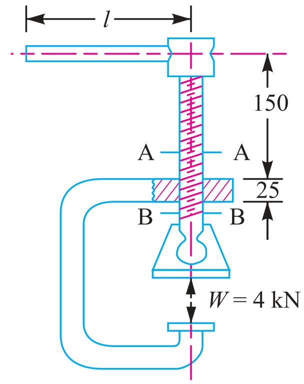
and thickness of thread, $t = p/2 = 8/2 = 4$ mm

Top

We know that the average bearing pressure,

$$p_b = \frac{W}{\pi . d . t . n} = \frac{5380}{\pi \times 44 \times 4 \times 6.25} = 1.56 \text{ N/mm}^2 \textbf{ Ans.}$$

Example 17.11. *A C-clamp, as shown in Fig. 17.10, has trapezoidal threads of 12 mm outside diameter and 2 mm pitch. The coefficient of friction for screw threads is 0.12 and for the collar is 0.25. The mean radius of the collar is 6 mm. If the force exerted by the operator at the end of the handle is 80 N, find: 1. The length of handle; 2. The maximum shear stress in the body of the screw and where does this exist; and 3. The bearing pressure on the threads.*



All dimensions in mm.

Fig. 17.10

Solution. Given : d_o = 12 mm ; p = 2 mm ; $\mu = \tan \phi = 0.12$; μ_2 = 0.25 ; R = 6 mm ; P_1 = 80 N ; W = 4 kN = 4000 N

1. Length of handle

Let l = Length of handle.

We know that the mean diameter of the screw,

$$d = d_o - p / 2 = 12 - 2 / 2 = 11 \text{ mm}$$

$$\therefore \quad \tan \alpha = \frac{p}{\pi d} = \frac{2}{\pi \times 11} = 0.058$$

Since the angle for trapezoidal threads is $2\beta = 30°$ or $\beta = 15°$, therefore virtual coefficient of friction,

$$\mu_1 = \tan \phi_1 = \frac{\mu}{\cos \beta} = \frac{0.12}{\cos 15°} = \frac{0.12}{0.9659} = 0.124$$

We know that the torque required to overcome friction at the screw,

$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi_1) \frac{d}{2} = W \left(\frac{\tan \alpha + \tan \phi_1}{1 - \tan \alpha \tan \phi_1} \right) \frac{d}{2}$$

$$= 4000 \left(\frac{0.058 + 0.124}{1 - 0.058 \times 0.124} \right) \frac{11}{2} = 4033 \text{ N-mm}$$

Assuming uniform wear, the torque required to overcome friction at the collar,

$$T_2 = \mu_2 \, W R = 0.25 \times 4000 \times 6 = 6000 \text{ N-mm}$$

∴ Total torque required at the end of handle,

$$T = T_1 + T_2 = 4033 + 6000 = 10\,033 \text{ N-mm}$$

We know that the torque required at the end of handle (T),

$$10\,033 = P_1 \times l = 80 \times l \quad \text{or} \quad l = 10\,033 / 80 = 125.4 \text{ mm} \textbf{ Ans.}$$

2. Maximum shear stress in the body of the screw

Consider two sections *A-A* and *B-B*. The section *A-A* just above the nut, is subjected to torque and bending. The section *B-B* just below the nut is subjected to collar friction torque and direct compressive load. Thus, both the sections must be checked for maximum shear stress.

Considering section A-A

We know that the core diameter of the screw,

$$d_c = d_o - p = 12 - 2 = 10 \text{ mm}$$

and torque transmitted at *A-A*,

$$T = \frac{\pi}{16} \times \tau \, (d_c)^3$$

Top

$\therefore$ Shear stress, $\tau = \dfrac{16\,T}{\pi\,(d_c)^3} = \dfrac{16 \times 10\,033}{\pi \times 10^3} = 51.1 \text{ N/mm}^2$

Bending moment at *A-A*,

$$M = P_1 \times 150 = 80 \times 150 = 12\,000 \text{ N-mm}$$

$$= \frac{\pi}{32} \times \sigma_b\,(d_c)^3$$

$\therefore$ Bending stress, $\sigma_b = \dfrac{32\,M}{\pi\,(d_c)^3} = \dfrac{32 \times 12\,000}{\pi\,(10)^3} = 122.2 \text{ N/mm}^2$

We know that the maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} = \frac{1}{2}\sqrt{(122.2)^2 + 4(51.1)^2} = 79.65 \text{ N/mm}^2$$

$$= 79.65 \text{ MPa}$$

Considering section B-B

Since the section *B-B* is subjected to collar friction torque (T_2), therefore the shear stress,

$$\tau = \frac{16\,T_2}{\pi\,(d_c)^3} = \frac{16 \times 6000}{\pi \times 10^3} = 30.6 \text{ N/mm}^2$$

and direct compressive stress,

$$\sigma_c = \frac{W}{A_c} = \frac{4W}{\pi(d_c)^2} = \frac{4 \times 4000}{\pi \times 10^2} = 51 \text{ N/mm}^2$$

$\therefore$ Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{(51)^2 + 4(30.6)^2} = 39.83 \text{ N/mm}^2 = 39.83 \text{ MPa}$$

From above, we see that the maximum shear stress is 79.65 MPa and occurs at section *A-A*. **Ans.**

3. Bearing pressure on the threads

We know that height of the nut,

$$h = n \times p = 25 \text{ mm} \quad \text{...(Given)}$$

$\therefore$ Number of threads in contact,

$$n = h / p = 25 / 2 = 12.5$$

and thickness of threads, $t = p / 2 = 2 / 2 = 1$ mm

We know that bearing pressure on the threads,

$$p_b = \frac{W}{\pi\,d.t.n} = \frac{4000}{\pi \times 11 \times 1 \times 12.5} = 9.26 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Example 17.12. *A power transmission screw of a screw press is required to transmit maximum load of 100 kN and rotates at 60 r.p.m. Trapezoidal threads are as under :*

Nominal dia, mm	*40*	*50*	*60*	*70*
Core dia, mm	*32.5*	*41.5*	*50.5*	*59.5*
Mean dia, mm	*36.5*	*46*	*55.5*	*65*
Core area, mm²	*830*	*1353*	*2003*	*2781*
Pitch, mm	*7*	*8*	*9*	*10*

The screw thread friction coefficient is 0.12. Torque required for collar friction and journal bearing is about 10% of the torque to drive the load considering screw friction. Determine screw dimensions and its efficiency. Also determine motor power required to drive the screw. Maximum permissible compressive stress in screw is 100 MPa.

Solution. Given : W = 100 kN = 100 × 10^3 N ; N = 60 r.p.m. ; μ = 0.12 ; σ_c = 100 MPa = 100 N/mm^2

Dimensions of the screw

Let A_c = Core area of threads.

We know that the direct compressive stress (σ_c),

$$100 = \frac{W}{A_c} = \frac{100 \times 10^3}{A_c}$$

or $$A_c = 100 \times 10^3 / 100 = 1000 \text{ mm}^2$$

Since the core area is 1000 mm^2, therefore we shall use the following dimensions for the screw (for core area 1353 mm^2).

Nominal diameter, d_o = 50 mm ;

Core diameter, d_c = 41.5 mm ;

Mean diameter, d = 46 mm ;

Pitch, p = 8 mm. **Ans.**

This screw press was made in 1735 and installed in the Segovia Mint to strike a new series of coper coins which began in 1772. This press is presently on display in the Alcazar castle of Segovia.

Efficiency of the screw

We know that $\tan \alpha = \frac{p}{\pi \, d} = \frac{8}{\pi \times 46} = 0.055$

and virtual coefficient of friction,

$$\mu_1 = \tan \phi_1 = \frac{\mu}{\cos \beta} = \frac{0.12}{\cos 15°}$$

$$= \frac{0.12}{0.9659} = 0.124 \qquad \text{... (} \because \text{ For trapezoidal threads, } 2\beta = 30°\text{)}$$

∴ Force required at the circumference of the screw,

$$P = W \tan (\alpha + \phi_1) = W \left[\frac{\tan \alpha + \tan \phi_1}{1 - \tan \alpha \tan \phi_1} \right]$$

$$= 100 \times 10^3 \left[\frac{0.055 + 0.124}{1 - 0.055 \times 0.124} \right] = 18\ 023 \text{ N}$$

and the torque required to drive the load,

$$T_1 = P \times d / 2 = 18\ 023 \times 46 / 2 = 414\ 530 \text{ N-mm}$$

We know that the torque required for collar friction,

$$T_2 = 10\% \ T_1 = 0.1 \times 414\ 530 = 41\ 453 \text{ N-mm}$$

∴ Total torque required,

$$T = T_1 + T_2 = 414\ 530 + 41\ 453 = 455\ 983 \text{ N-mm} = 455.983 \text{ N-m}$$

We know that the torque required with no friction,

$$T_0 = W \tan \alpha \times \frac{d}{2} = 100 \times 10^3 \times 0.055 \times \frac{46}{2} = 126\ 500 \text{ N-mm}$$

∴ Efficiency of the screw,

$$\eta = \frac{T_0}{T} = \frac{126\ 500}{455\ 983} = 0.278 \text{ or } 27.8\% \text{ **Ans.**}$$

Power required to drive the screw

We know that the power required to drive the screw,

$$= T \times \omega = \frac{T \times 2\,\pi N}{60} = \frac{455.683 \times 2\pi \times 60}{60} = 2865 \text{ W}$$

$= 2.865$ kW **Ans.**

Example 17.13. *A vertical two start square threaded screw of 100 mm mean diameter and 20 mm pitch supports a vertical load of 18 kN. The nut of the screw is fitted in the hub of a gear wheel having 80 teeth which meshes with a pinion of 20 teeth. The mechanical efficiency of the pinion and gear wheel drive is 90 percent. The axial thrust on the screw is taken by a collar bearing 250 mm outside diameter and 100 mm inside diameter. Assuming uniform pressure conditions, find, minimum diameter of pinion shaft and height of nut, when coefficient of friction for the vertical screw and nut is 0.15 and that for the collar bearing is 0.20. The permissible shear stress in the shaft material is 56 MPa and allowable bearing pressure is 1.4 N/mm^2.*

Solution. Given : $d = 100$ mm ; $p = 20$ mm ; $W = 18$ kN $= 18 \times 10^3$N ; No. of teeth on gear wheel $= 80$; No. of teeth on pinion $= 20$; $\eta_m = 90\% = 0.9$; $D_1 = 250$ mm or $R_1 = 125$ mm ; $D_2 = 100$ mm or $R_2 = 50$ mm ; $\mu = \tan\phi = 0.15$; $\mu_1 = 0.20$; $\tau = 56$ MPa $= 56$ N/mm^2 ; $p_b = 1.4$ N/mm^2

Minimum diameter of pinion shaft

Let D = Minimum diameter of pinion shaft.

Since the screw is a two start square threaded screw, therefore lead of the screw

$= 2\,p = 2 \times 20 = 40$ mm

$$\therefore \qquad \tan\alpha = \frac{\text{Lead}}{\pi d} = \frac{40}{\pi \times 100} = 0.127$$

and torque required to overcome friction at the screw and nut,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi)\frac{d}{2} = W\left(\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right)\frac{d}{2}$$

$$= 18 \times 10^3\left(\frac{0.127 + 0.15}{1 - 0.127 \times 0.15}\right)\frac{100}{2} = 254\,160 \text{ N-mm}$$

$= 254.16$ N-m

We know that, for uniform pressure conditions, torque required to overcome friction at the collar bearing,

$$T_2 = \frac{2}{3} \times \mu_1 W\left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2}\right]$$

$$= \frac{2}{3} \times 0.20 \times 18 \times 10^3\left[\frac{(125)^3 - (50)^3}{(125)^2 - (50)^2}\right] \text{ N-mm}$$

$= 334\,290$ N-mm $= 334.29$ N-m

Since the nut of the screw is fixed in the hub of a gear wheel, therefore the total torque required at the gear wheel,

$$T_w = T_1 + T_2 = 254.16 + 334.29 = 588.45 \text{ N-m}$$

Also the gear wheel having 80 teeth meshes with pinion having 20 teeth and the torque is proportional to the number of teeth, therefore torque required at the pinion shaft,

$$= \frac{T_w \times 20}{80} = 588.45 \times \frac{20}{80} = 147.11 \text{ N-m}$$

Since the mechanical efficiency of the pinion and gear wheel is 90%, therefore net torque required at the pinion shaft,

$$T_p = \frac{147.11 \times 100}{90} = 163.46 \text{ N-m} = 163\,460 \text{ N-mm}$$

We know that the torque required at the pinion shaft (T_p),

$$163\ 460 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 56 \times D^3 = 11\ D^3$$

$\therefore$ $D^3 = 163\ 460 / 11 = 14\ 860$ or $D = 24.6$ say 25 mm **Ans.**

Height of nut

Let h = Height of nut,

n = Number of threads in contact, and

t = Thickness or width of thread = $p/2 = 20/2 = 10$ mm

We know that the bearing pressure (p_b),

$$1.4 = \frac{W}{\pi\, d.t.n} = \frac{18 \times 10^3}{\pi \times 100 \times 10 \times n} = \frac{5.73}{n}$$

$\therefore$ $n = 5.73 / 1.4 = 4.09$ say 5 threads

and height of nut, $h = n \times p = 5 \times 20 = 100$ mm **Ans.**

Example 17.14. *A screw press is to exert a force of 40 kN. The unsupported length of the screw is 400 mm. Nominal diameter of screw is 50 mm. The screw has square threads with pitch equal to 10 mm. The material of the screw and nut are medium carbon steel and cast iron respectively. For the steel used take ultimate crushing stress as 320 MPa, yield stress in tension or compression as 200 MPa and that in shear as 120 MPa. Allowable shear stress for cast iron is 20 MPa and allowable bearing pressure between screw and nut is 12 N/mm². Young's modulus for steel = 210 kN/mm². Determine the factor of safety of screw against failure. Find the dimensions of the nut. What is the efficiency of the arrangement? Take coefficient of friction between steel and cast iron as 0.13.*

Solution. Given : $W = 40$ kN $= 40 \times 10^3$ N ; $L = 400$ mm $= 0.4$ m ; $d_o = 50$ mm ; $p = 10$ mm ; $\sigma_{cu} = 320$ MPa $= 320$ N/mm² ; $\sigma_y = 200$ MPa $= 200$ N/mm² ; $\tau_y = 120$ MPa $= 120$ N/mm² ; $\tau_c = 20$ MPa $= 20$ N/mm² ; $p_b = 12$ N/mm² ; $E = 210$ kN/mm² $= 210 \times 10^3$ N/mm² ; $\mu = \tan\phi = 0.13$

We know that the inner diameter or core diameter of the screw,

$$d_c = d_o - p = 50 - 10 = 40 \text{ mm}$$

and core area of the screw,

$$A_c = \frac{\pi}{4}(d_c)^2 = \frac{\pi}{4}(40)^2 = 1257 \text{ mm}^2$$

$\therefore$ Direct compressive stress on the screw due to axial load,

$$\sigma_c = \frac{W}{A_c} = \frac{40 \times 10^3}{1257} = 31.8 \text{ N/mm}^2$$

We know that the mean diameter of the screw,

$$d = \frac{d_o + d_c}{2} = \frac{50 + 40}{2} = 45 \text{ mm}$$

and $$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 45} = 0.07$$

$\therefore$ Torque required to move the screw,

$$T = P \times \frac{d}{2} = W \tan(\alpha + \phi)\frac{d}{2} = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right]\frac{d}{2}$$

$$= 40 \times 10^3\left[\frac{0.07 + 0.13}{1 - 0.07 \times 0.13}\right]\frac{45}{2} = 181.6 \times 10^3 \text{ N-mm}$$

We know that torque transmitted by the screw (T),

$$181.6 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_c)^3 = \frac{\pi}{16} \times \tau\,(40)^3 = 12\ 568\ \tau$$

$\therefore$ $\tau = 181.6 \times 10^3 / 12\ 568 = 14.45$ N/mm²

According to maximum shear stress theory, we have

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2} = \frac{1}{2}\sqrt{(31.8)^2 + 4\,(14.45)^2} = 21.5 \text{ N/mm}^2$$

Factor of safety

We know that factor of safety

$$= \frac{\tau_y}{\tau_{max}} = \frac{120}{21.5} = 5.58$$

Now considering the screw as a column, assuming one end fixed and other end free. According to J.B. Johnson's formula, critical load,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4C\,\pi^2\,E}\left(\frac{L}{k}\right)^2\right]$$

For one end fixed and other end free, $C = 0.25$.

$$\therefore \quad W_{cr} = 1257 \times 200 \left[1 - \frac{200}{4 \times 0.25 \times \pi^2 \times 210 \times 10^3}\left(\frac{400}{10}\right)^2\right] \text{N}$$

... ($\because$ $k = d_c / 4 = 40 / 4 = 10$ mm)

$$= 212\ 700 \text{ N}$$

$$\therefore \quad \text{Factor of safety} = \frac{W_{cr}}{W} = \frac{212\,700}{40 \times 10^3} = 5.3$$

We shall take larger value of the factor of safety.

$\therefore$ Factor of safety = 5.58 say 6 **Ans.**

Dimensions of the nut

Let n = Number of threads in contact with nut, and

h = Height of nut = $p \times n$

Assume that the load is uniformly distributed over the threads in contact.

We know that the bearing pressure (p_b),

$$12 = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{40 \times 10^3}{\frac{\pi}{4}\left[(50)^2 - (40)^2\right] n} = \frac{56.6}{n}$$

$\therefore$ n = 56.6 / 12 = 4.7 say 5 threads **Ans.**

and $h = p \times n = 10 \times 5 = 50$ mm **Ans.**

Now let us check for the shear stress induced in the nut which is of cast iron. We know that

$$\tau_{nut} = \frac{W}{\pi\, n.d_o\, t} = \frac{40 \times 10^3}{\pi \times 5 \times 50 \times 5} = 10.2 \text{ N/mm}^2 = 10.2 \text{ MPa}$$

...($\because$ $t = p / 2 = 10 / 2 = 5$ mm)

This value is less than the given value of τ_c = 20 MPa, hence the nut is safe.

Efficiency of the arrangement

We know that torque required to move the screw with no friction,

$$T_0 = W \tan\alpha \times \frac{d}{2} = 40 \times 10^3 \times 0.07 \times \frac{45}{2} = 63 \times 10^3 \text{ N-mm}$$

$\therefore$ Efficiency of the arrangement

$$\eta = \frac{T_0}{T} = \frac{63 \times 10^3}{181.6 \times 10^3} = 0.347 \text{ or } 34.7\%$$ **Ans.**

17.14 Design of Screw Jack

A bottle screw jack for lifting loads is shown in Fig. 17.11. The various parts of the screw jack are as follows:

1. Screwed spindle having square threaded screws,
2. Nut and collar for nut,
3. Head at the top of the screwed spindle for handle,
4. Cup at the top of head for the load, and
5. Body of the screw jack.

In order to design a screw jack for a load *W*, the following procedure may be adopted:

1. First of all, find the core diameter (d_c) by considering that the screw is under pure compression, *i.e.*

$$W = \sigma_c \times A_c = \sigma_c \times \frac{\pi}{4} (d_c)^2$$

The standard proportions of the square threaded screw are fixed from Table 17.1.

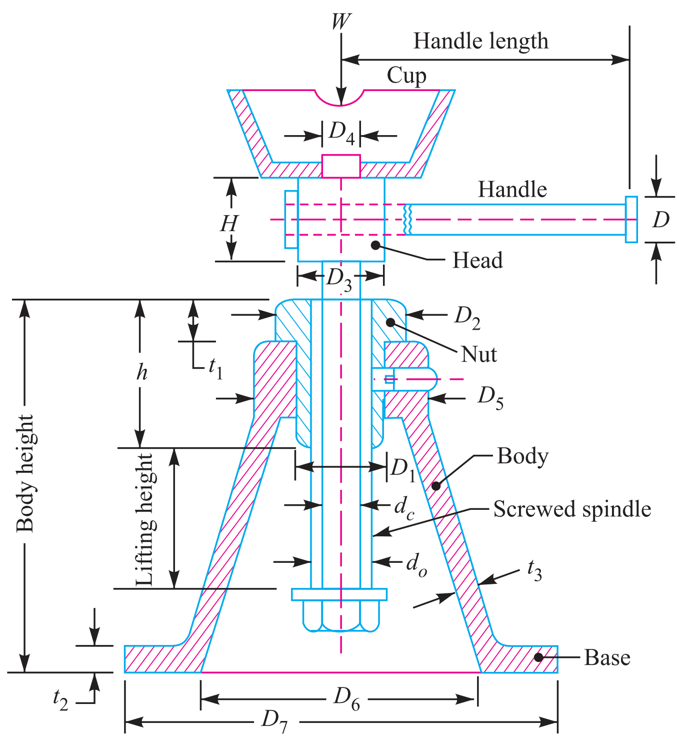


Fig. 17.11. Screw jack.

2. Find the torque (T_1) required to rotate the screw and find the shear stress (τ) due to this torque.

We know that the torque required to lift the load,

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi) \frac{d}{2}$$

where P = Effort required at the circumference of the screw, and

d = Mean diameter of the screw.

∴ Shear stress due to torque T_1,

$$\tau = \frac{16\, T_1}{\pi (d_c)^3}$$

Also find direct compressive stress (σ_c) due to axial load, *i.e.*

$$\sigma_c = \frac{W}{\frac{\pi}{4}(d_c)^2}$$

3. Find the principal stresses as follows:

   Maximum principal stress (tensile or compressive),

$$\sigma_{c(max)} = \frac{1}{2}\left[\sigma_c + \sqrt{(\sigma_c)^2 + 4\tau^2}\right]$$

   and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2}$$

   These stresses should be less than the permissible stresses.

4. Find the height of nut (h), considering the bearing pressure on the nut. We know that the bearing pressure on the nut,

$$p_b = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n}$$

   where n = Number of threads in contact with screwed spindle.

   ∴ Height of nut, $h = n \times p$

   where p = Pitch of threads.

5. Check the stressess in the screw and nut as follows :

$$\tau_{(screw)} = \frac{W}{\pi\, n.d_c.t}$$

$$\tau_{(nut)} = \frac{W}{\pi\, n.d_o.t}$$

   where t = Thickness of screw $= p / 2$

6. Find inner diameter (D_1), outer diameter (D_2) and thickness (t_1) of the nut collar.

   The inner diameter (D_1) is found by considering the tearing strength of the nut. We know that

$$W = \frac{\pi}{4}\left[(D_1)^2 - (d_o)^2\right]\sigma_t$$

   The outer diameter (D_2) is found by considering the crushing strength of the nut collar. We know that

$$W = \frac{\pi}{4}\left[(D_2)^2 - (D_1)^2\right]\sigma_c$$

   The thickness (t_1) of the nut collar is found by considering the shearing strength of the nut collar. We know that

$$W = \pi\, D_1.t_1.\tau$$

7. Fix the dimensions for the diameter of head (D_3) on the top of the screw and for the cup. Take $D_3 = 1.75\, d_o$. The seat for the cup is made equal to the diameter of head and it is chamfered at the top. The cup is fitted with a pin of diameter $D_4 = D_3 / 4$ approximately. This pin remains a loose fit in the cup.

8. Find the torque required (T_2) to overcome friction at the top of screw. We know that

$$T_2 = \frac{2}{3} \times \mu_1 W \left[\frac{(R_3)^3 - (R_4)^3}{(R_3)^2 - (R_4)^2}\right] \quad \text{... (Assuming uniform pressure conditions)}$$

$$= \mu_1 W \left[\frac{R_3 + R_4}{2}\right] = \mu_1 \, W \, R \quad \text{... (Assuming uniform wear conditions)}$$

where R_3 = Radius of head, and

R_4 = Radius of pin.

9. Now the total torque to which the handle will be subjected is given by

$$T = T_1 + T_2$$

Assuming that a person can apply a force of 300 – 400 N intermittently, the length of handle required

$$= T / 300$$

The length of handle may be fixed by giving some allowance for gripping.

10. The diameter of handle (D) may be obtained by considering bending effects. We know that bending moment,

$$M = \frac{\pi}{32} \times \sigma_b \times D^3 \quad \text{... } (\because \sigma_b = \sigma_t \text{ or } \sigma_c)$$

11. The height of head (H) is usually taken as twice the diameter of handle, *i.e.* $H = 2D$.

12. Now check the screw for buckling load.

Effective length or unsupported length of the screw,

$$L = \text{Lift of screw} + \frac{1}{2}\text{ Height of nut}$$

We know that buckling or critical load,

$$W_{cr} = A_c.\sigma_y \left[1 - \frac{\sigma_y}{4\, C\, \pi^2\, E}\left(\frac{L}{k}\right)^2\right]$$

where σ_y = Yield stress,

C = End fixity coefficient. The screw is considered to be a strut with lower end fixed and load end free. For one end fixed and the other end free, $C = 0.25$

k = Radius of gyration = $0.25\, d_c$

The buckling load as obtained by the above expression must be higher than the load at which the screw is designed.

13. Fix the dimensions for the body of the screw jack.

14. Find efficiency of the screw jack.

Screw jack

Example 17.15. *A screw jack is to lift a load of 80 kN through a height of 400 mm. The elastic strength of screw material in tension and compression is 200 MPa and in shear 120 MPa. The material for nut is phosphor-bronze for which the elastic limit may be taken as 100 MPa in tension, 90 MPa in compression and 80 MPa in shear. The bearing pressure between the nut and the screw is not to exceed 18 N/mm^2. Design and draw the screw jack. The design should include the design of 1. screw, 2. nut, 3. handle and cup,and 4. body.*

Solution. Given : W = 80 kN = 80 × 10^3N ; H_1 = 400 mm = 0.4 m ; $\sigma_{et} = \sigma_{ec}$ = 200 MPa = 200 N/mm^2 ; τ_e = 120 MPa = 120 N/mm^2 ; $\sigma_{et(nut)}$ = 100 MPa = 100 N/mm^2 ; $\sigma_{ec\ (nut)}$ = 90 MPa = 90 N/mm^2 ; $\tau_{e(nut)}$ = 80 MPa = 80N/mm^2 ; p_b = 18 N/mm^2

The various parts of a screw jack are designed as discussed below:

1. Design of screw for spindle

Let d_c = Core diameter of the screw.

Since the screw is under compression, therefore load (W),

$$80 \times 10^3 = \frac{\pi}{4}(d_c)^2 \times \frac{\sigma_{ec}}{F.S.} = \frac{\pi}{4}(d_c)^2 \frac{200}{2} = 78.55\,(d_c)^2$$

... (Taking factor of safety, $F.S.$ = 2)

$\therefore$ $(d_c)^2 = 80 \times 10^3 / 78.55 = 1018.5$ or $d_c = 32$ mm

For square threads of normal series, the following dimensions of the screw are selected from Table 17.2.

*Core diameter, d_c = 38 mm **Ans.**

Nominal or outside diameter of spindle,

d_o = 46 mm **Ans.**

Pitch of threads, p = 8 mm **Ans.**

Now let us check for principal stresses:

We know that the mean diameter of screw,

$$d = \frac{d_o + d_c}{2} = \frac{46 + 38}{2} = 42 \text{ mm}$$

and
$$\tan\alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 42} = 0.0606$$

Assuming coefficient of friction between screw and nut,

$\mu = \tan\phi = 0.14$

$\therefore$ Torque required to rotate the screw in the nut,

$$T_1 = P \times \frac{d}{2} = W\tan(\alpha + \phi)\frac{d}{2} = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right]\frac{d}{2}$$

$$= 80 \times 10^3 \left[\frac{0.0606 + 0.14}{1 - 0.0606 \times 0.14}\right]\frac{42}{2} = 340 \times 10^3 \text{ N-mm}$$

Now compressive stress due to axial load,

$$\sigma_c = \frac{W}{A_c} = \frac{W}{\frac{\pi}{4}(d_c)^2} = \frac{80 \times 10^3}{\frac{\pi}{4}(38)^2} = 70.53 \text{ N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16\,T_1}{\pi (d_c)^3} = \frac{16 \times 340 \times 10^3}{\pi\,(38)^3} = 31.55 \text{ N/mm}^2$$

$\therefore$ Maximum principal stress (tensile or compressive),

$$\sigma_{c(max)} = \frac{1}{2}\left[\sigma_c + \sqrt{(\sigma_c)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[70.53 + \sqrt{(70.53)^2 + 4\,(31.55)^2}\right]$$

$$= \frac{1}{2}[70.53 + 94.63] = 82.58 \text{ N/mm}^2$$

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* From Table 17.2, we see that next higher value of 32 mm for the core diameter is 33 mm. By taking d_c = 33 mm, gives higher principal stresses than the permissible values. So core diameter is chosen as 38 mm.

The given value of σ_c is equal to $\dfrac{\sigma_{ec}}{F.S.}$, *i.e.* $\dfrac{200}{2} = 100\ \text{N/mm}^2$

We know that maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(70.53)^2 + 4\,(31.55)^2}\right]$$

$$= \frac{1}{2} \times 94.63 = 47.315\ \text{N/mm}^2$$

The given value of τ is equal to $\dfrac{\tau_e}{F.S.}$, *i.e.* $\dfrac{120}{2} = 60\ \text{N/mm}^2$.

Since these maximum stresses are within limits, therefore design of screw for spindle is safe.

2. *Design for nut*

Let n = Number of threads in contact with the screwed spindle,

h = Height of nut = $n \times p$, and

t = Thickness of screw = $p / 2 = 8 / 2 = 4$ mm

Assume that the load is distributed uniformly over the cross-sectional area of nut.

We know that the bearing pressure (p_b),

$$18 = \frac{W}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{80 \times 10^3}{\frac{\pi}{4}\left[(46)^2 - (38)^2\right] n} = \frac{151.6}{n}$$

$\therefore$ $n = 151.6 / 18 = 8.4$ say 10 threads **Ans.**

and height of nut, $h = n \times p = 10 \times 8 = 80$ mm **Ans.**

Now, let us check the stresses induced in the screw and nut.

We know that shear stress in the screw,

$$\tau_{(screw)} = \frac{W}{\pi\, n.d_c.t} = \frac{80 \times 10^3}{\pi \times 10 \times 38 \times 4} = 16.15\ \text{N/mm}^2$$

... ($\because$ $t = p / 2 = 4$ mm)

and shear stress in the nut,

$$\tau_{(nut)} = \frac{W}{\pi\, n.d_o.t} = \frac{80 \times 10^3}{\pi \times 10 \times 46 \times 4} = 13.84\ \text{N/mm}^2$$

Since these stresses are within permissible limit, therefore design for nut is safe.

Let D_1 = Outer diameter of nut,

D_2 = Outside diameter for nut collar, and

t_1 = Thickness of nut collar.

First of all considering the tearing strength of nut, we have

$$W = \frac{\pi}{4}\left[(D_1)^2 - (d_o)^2\right]\sigma_t$$

$$80 \times 10^3 = \frac{\pi}{4}\left[(D_1)^2 - (46)^2\right]\frac{100}{2} = 39.3\left[(D_1)^2 - 2116\right] \quad \ldots\left[\because \sigma_t = \frac{\sigma_{et(nut)}}{F.S.}\right]$$

or $(D_1)^2 - 2116 = 80 \times 10^3 / 39.3 = 2036$

$\therefore$ $(D_1)^2 = 2036 + 2116 = 4152$ or $D_1 = 65$ mm **Ans.**

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Now considering the crushing of the collar of the nut, we have

$$W = \frac{\pi}{4}\left[(D_2)^2 - (D_1)^2\right]\sigma_c$$

$$80 \times 10^3 = \frac{\pi}{4}\left[(D_2)^2 - (65)^2\right]\frac{90}{2} = 35.3\left[(D_2)^2 - 4225\right] \qquad \ldots\left[\sigma_c = \frac{\sigma_{ec(nut)}}{F.S.}\right]$$

or $\quad (D_2)^2 - 4225 = 80 \times 10^3 / 35.3 = 2266$

$\therefore \quad (D_2)^2 = 2266 + 4225 = 6491 \quad$ or $\quad D_2 = 80.6$ say 82 mm **Ans.**

Considering the shearing of the collar of the nut, we have

$$W = \pi\, D_1 \times t_1 \times \tau$$

$$80 \times 10^3 = \pi \times 65 \times t_1 \times \frac{80}{2} = 8170\, t_1 \qquad \ldots\left[\tau = \frac{\tau_{e(nut)}}{F.S.}\right]$$

$\therefore \quad t_1 = 80 \times 10^3/8170 = 9.8$ say 10 mm **Ans.**

3. *Design for handle and cup*

The diameter of the head (D_3) on the top of the screwed rod is usually taken as 1.75 times the outside diameter of the screw (d_o).

$\therefore \quad D_3 = 1.75\, d_o = 1.75 \times 46 = 80.5$ say 82 mm **Ans.**

The head is provided with two holes at the right angles to receive the handle for rotating the screw. The seat for the cup is made equal to the diameter of head, *i.e.* 82 mm and it is given chamfer at the top. The cup prevents the load from rotating. The cup is fitted to the head with a pin of diameter D_4 = 20 mm. The pin remains loose fit in the cup. Other dimensions for the cup may be taken as follows :

Height of cup = 50 mm **Ans.**

Thickness of cup = 10 mm **Ans.**

Diameter at the top of cup = 160 mm **Ans.**

Now let us find out the torque required (T_2) to overcome friction at the top of the screw.

Assuming uniform pressure conditions, we have

$$T_2 = \frac{2}{3} \times \mu_1\, W\left[\frac{(R_3)^3 - (R_4)^3}{(R_3)^2 - (R_4)^2}\right]$$

$$= \frac{2}{3} \times 0.14 \times 80 \times 10^3\left[\frac{\left(\frac{82}{2}\right)^3 - \left(\frac{20}{2}\right)^3}{\left(\frac{82}{2}\right)^2 - \left(\frac{20}{2}\right)^2}\right] \qquad \ldots \text{(Assuming } \mu_1 = \mu)$$

$$= 7.47 \times 10^3\left[\frac{(41)^3 - (10)^3}{(41)^2 - (10)^2}\right] = 321 \times 10^3 \text{ N-mm}$$

$\therefore$ Total torque to which the handle is subjected,

$$T = T_1 + T_2 = 340 \times 10^3 + 321 \times 10^3 = 661 \times 10^3 \text{ N-mm}$$

Assuming that a force of 300 N is applied by a person intermittently, therefore length of handle required

$$= 661 \times 10^3 / 300 = 2203 \text{ mm}$$

Allowing some length for gripping, we shall take the length of handle as 2250 mm.

Top

A little consideration will show that an excessive force applied at the end of lever will cause bending. Considering bending effect, the maximum bending moment on the handle,

$$M = \text{Force applied} \times \text{Length of lever}$$
$$= 300 \times 2250 = 675 \times 10^3 \text{ N-mm}$$

Let D = Diameter of the handle.

Assuming that the material of the handle is same as that of screw, therefore taking bending stress $\sigma_b = \sigma_t = \sigma_{et} / 2 = 100$ N/mm^2.

We know that the bending moment (M),

$$675 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times D^3 = \frac{\pi}{32} \times 100 \times D^3 = 9.82\ D^3$$

$\therefore$ $D^3 = 675 \times 10^3 / 9.82 = 68.74 \times 10^3$ or $D = 40.96$ say 42 mm **Ans.**

The height of head (H) is taken as $2D$.

$\therefore$ $H = 2\,D = 2 \times 42 = 84$ mm **Ans.**

Now let us check the screw for buckling load.

We know that the effective length for the buckling of screw,

$$L = \text{Lift of screw} + \frac{1}{2}\text{ Height of nut} = H_1 + h / 2$$
$$= 400 + 80 / 2 = 440 \text{ mm}$$

When the screw reaches the maximum lift, it can be regarded as a strut whose lower end is fixed and the load end is free. We know that critical load,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4C\ \pi^2\ E}\left(\frac{L}{k}\right)^2\right]$$

For one end fixed and other end free, $C = 0.25$.

Also $k = 0.25\ d_c = 0.25 \times 38 = 9.5$ mm

$$\therefore \quad W_{cr} = \frac{\pi}{4}(38)^2\ 200\left[1 - \frac{200}{4 \times 0.25 \times \pi^2 \times 210 \times 10^3}\left(\frac{440}{9.5}\right)^2\right]$$

... (Taking $\sigma_y = \sigma_{et}$)

$$= 226\ 852\ (1 - 0.207) = 179\ 894 \text{ N}$$

Since the critical load is more than the load at which the screw is designed (*i.e.* 80×10^3 N), therefore there is no chance of the screw to buckle.

4. Design of body

The various dimensions of the body may be fixed as follows:

Diameter of the body at the top,

$$D_5 = 1.5\ D_2 = 1.5 \times 82 = 123 \text{ mm}$$ **Ans.**

Thickness of the body,

$$t_3 = 0.25\ d_o = 0.25 \times 46 = 11.5 \text{ say } 12 \text{ mm}$$ **Ans.**

Inside diameter at the bottom,

$$D_6 = 2.25\ D_2 = 2.25 \times 82 = 185 \text{ mm}$$ **Ans.**

Outer diameter at the bottom,

$$D_7 = 1.75\ D_6 = 1.75 \times 185 = 320 \text{ mm}$$ **Ans.**

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Thickness of base, $t_2 = 2\,t_1 = 2 \times 10 = 20$ mm **Ans.**

Height of the body = Max. lift + Height of nut + 100 mm extra

= 400 + 80 + 100 = 580 mm **Ans.**

The body is made tapered in order to achieve stability of jack.

Let us now find out the efficiency of the screw jack. We know that the torque required to rotate the screw with no friction,

$$T_0 = W \tan \alpha \times \frac{d}{2} = 80 \times 10^3 \times 0.0606 \times \frac{42}{2} = 101\,808 \text{ N-mm}$$

∴ Efficiency of the screw jack,

$$\eta = \frac{T_0}{T} = \frac{101808}{661 \times 10^3} = 0.154 \text{ or } 15.4\% \quad \textbf{Ans.}$$

Example 17.16. *A toggle jack as shown in Fig. 17.12, is to be designed for lifting a load of 4 kN. When the jack is in the top position, the distance between the centre lines of nuts is 50 mm and in the bottom position this distance is 210 mm. The eight links of the jack are symmetrical and 110 mm long. The link pins in the base are set 30 mm apart. The links, screw and pins are made from mild steel for which the permissible stresses are 100 MPa in tension and 50 MPa in shear. The bearing pressure on the pins is limited to 20 N/mm².*

Assume the pitch of the square threads as 6 mm and the coefficient of friction between threads as 0.20.

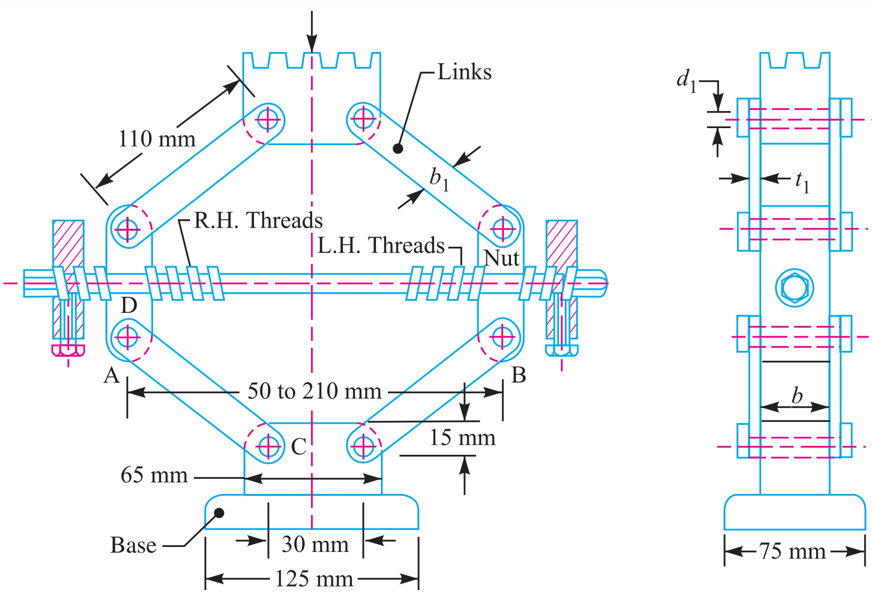

Fig. 17.12

Solution. Given : $W = 4$ kN $= 4000$ N ; $l = 110$ mm ; $\sigma_t = 100$ MPa $= 100$ N/mm² ; $\tau = 50$ MPa $= 50$ N/mm² ; $p_b = 20$ N/mm² ; $p = 6$ mm ; $\mu = \tan \phi = 0.20$

The toggle jack may be designed as discussed below :

1. *Design of square threaded screw*

A little consideration will show that the maximum load on the square threaded screw occurs when the jack is in the bottom position. The position of the link *CD* in the bottom position is shown in Fig. 17.13 (*a*).

Let θ be the angle of inclination of the link *CD* with the horizontal.

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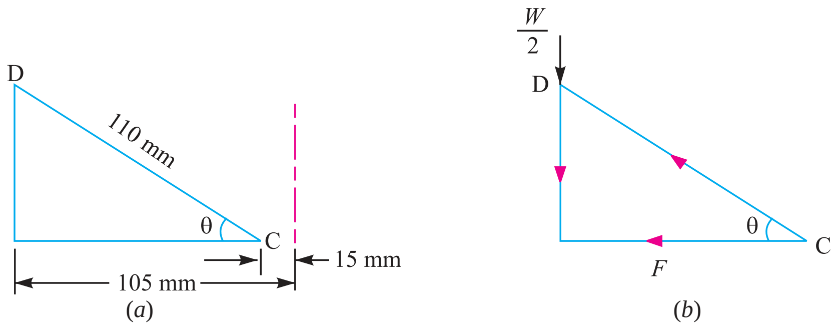

Fig. 17.13

From the geometry of the figure, we find that

$$\cos\theta = \frac{105 - 15}{110} = 0.8112 \text{ or } \theta = 35.1°$$

Each nut carries half the total load on the jack and due to this, the link *CD* is subjected to tension while the square threaded screw is under pull as shown in Fig. 17.13 (*b*). The magnitude of the pull on the square threaded screw is given by

$$F = \frac{W}{2\tan\theta} = \frac{W}{2\tan 35.1°}$$

$$= \frac{4000}{2 \times 0.7028} = 2846 \text{ N}$$

The rotational speed of the lead screw relative to the spindle speed can be adjusted manually by adding and removing gears to and from the gear

Since a similar pull acts on the other nut, therefore total tensile pull on the square threaded rod,

$$W_1 = 2F = 2 \times 2846 = 5692 \text{ N}$$

Let d_c = Core diameter of the screw,

We know that load on the screw (W_1),

$$5692 = \frac{\pi}{4}(d_c)^2\,\sigma_t = \frac{\pi}{4}(d_c)^2\,100$$

$$= 78.55\,(d_c)^2$$

$$\therefore \quad (d_c)^2 = 5692 / 78.55 = 72.5 \quad \text{or} \quad d_c = 8.5 \text{ say } 10 \text{ mm}$$

Since the screw is also subjected to torsional shear stress, therefore to account for this, let us adopt

$$d_c = 14 \text{ mm}$$ **Ans.**

∴ Nominal or outer diameter of the screw,

$$d_o = d_c + p = 14 + 6 = 20 \text{ mm}$$ **Ans.**

and mean diameter of the screw,

$$d = d_o - p/2 = 20 - 6/2 = 17 \text{ mm}$$

Let us now check for principal stresses. We know that

$$\tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 17} = 0.1123 \qquad \text{... (where } \alpha \text{ is the helix angle)}$$

We know that effort required to rotate the screw,

$$P = W_1 \tan(\alpha + \phi) = W_1\left(\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}\right)$$

$$= 5692\left(\frac{0.1123 + 0.20}{1 - 0.1123 \times 0.20}\right) = 1822 \text{ N}$$

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∴ Torque required to rotate the screw,

$$T = P \times \frac{d}{2} = 1822 \times \frac{17}{2} = 15\ 487 \text{ N-mm}$$

and shear stress in the screw due to torque,

$$\tau = \frac{16\,T}{\pi\,(d_c)^3} = \frac{16 \times 15\ 487}{\pi\,(14)^3} = 28.7 \text{ N/mm}^2$$

We know that direct tensile stress in the screw,

$$\sigma_t = \frac{W_1}{\frac{\pi}{4}(d_c)^2} = \frac{W_1}{0.7855\,(d_c)^2} = \frac{5692}{0.7855\,(14)^2} = 37 \text{ N/mm}^2$$

∴ Maximum principal (tensile) stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\,\tau^2} = \frac{37}{2} + \frac{1}{2}\sqrt{(37)^2 + 4\,(28.7)^2}$$

$$= 18.5 + 34.1 = 52.6 \text{ N/mm}^2$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2} = \frac{1}{2}\sqrt{(37)^2 + 4(28.7)^2} = 34.1 \text{ N/mm}^2$$

Since the maximum stresses are within safe limits, therefore the design of square threaded screw is satisfactory.

2. *Design of nut*

Let n = Number of threads in contact with the screw (*i.e.* square threaded rod).

Assuming that the load W_1 is distributed uniformly over the cross-sectional area of the nut, therefore bearing pressure between the threads (p_b),

$$20 = \frac{W_1}{\frac{\pi}{4}\left[(d_o)^2 - (d_c)^2\right] n} = \frac{5692}{\frac{\pi}{4}\left[(20)^2 - (14)^2\right] n} = \frac{35.5}{n}$$

∴ $n = 35.5 / 20 = 1.776$

In order to have good stability and also to prevent rocking of the screw in the nut, we shall provide $n = 4$ threads in the nut. The thickness of the nut,

$t = n \times p = 4 \times 6 = 24$ mm **Ans.**

The width of the nut (b) is taken as 1.5 d_o.

∴ $b = 1.5\,d_o = 1.5 \times 20 = 30$ mm **Ans.**

To control the movement of the nuts beyond 210 mm (the maximum distance between the centre lines of nuts), rings of 8 mm thickness are fitted on the screw with the help of set screws.

∴ Length of screwed portion of the screw

= 210 + t + 2 × Thickness of rings

= 210 + 24 + 2 × 8 = 250 mm **Ans.**

The central length (about 25 mm) of screwed rod is kept equal to core diameter of the screw *i.e.* 14 mm. Since the toggle jack is operated by means of spanners on both sides of the square threaded rod, therefore the ends of the rod may be reduced to 10 mm square and 15 mm long.

∴ Toal length of the screw

= 250 + 2 × 15 = 280 mm **Ans.**

Assuming that a force of 150 N is applied by each person at each end of the rod, therefore length of the spanner required

$$= \frac{T}{2 \times 150} = \frac{15\,487}{300} = 51.62 \text{ mm}$$

We shall take the length of the spanner as 200 mm in order to facilitate the operation and even a single person can operate it.

3. *Design of pins in the nuts*

Let d_1 = Diameter of pins in the nuts.

Since the pins are in double shear, therefore load on the pins (F),

$$2846 = 2 \times \frac{\pi}{4}(d_1)^2 \; \tau = 2 \times \frac{\pi}{4}(d_1)^2 \; 50 = 78.55\,(d_1)^2$$

$\therefore$ $(d_1)^2 = 2846 / 78.55 = 36.23$ or $d_1 = 6.02$ say 8 mm **Ans.**

The diameter of pin head is taken as 1.5 d_1 (*i.e.* 12 mm) and thickness 4 mm. The pins in the nuts are kept in position by separate rings 4 mm thick and 1.5 mm split pins passing through the rings and pins.

4. *Design of links*

Due to the load, the links may buckle in two planes at right angles to each other. For buckling in the vertical plane (*i.e.* in the plane of the links), the links are considered as hinged at both ends and for buckling in a plane perpendicular to the vertical plane, it is considered as fixed at both ends. We know that load on the link

$$= F / 2 = 2846 / 2 = 1423 \text{ N}$$

Assuming a factor of safety = 5, the links must be designed for a buckling load of

$$W_{cr} = 1423 \times 5 = 7115 \text{ N}$$

Let t_1 = Thickness of the link, and

b_1 = Width of the link.

Assuming that the width of the link is three times the thickness of the link, *i.e.* $b_1 = 3\,t_1$, therefore cross-sectional area of the link,

$$A = t_1 \times 3t_1 = 3(t_1)^2$$

and moment of inertia of the cross-section of the link,

$$I = \frac{1}{12} \times t_1\,(b_1)^3 = \frac{1}{12} \times t_1\,(3t_1)^3 = 2.25\,(t_1)^4$$

We know that the radius of gyration,

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.25\,(t_1)^4}{3(t_1)^2}} = 0.866\,t_1$$

Since for buckling of the link in the vertical plane, the ends are considered as hinged, therefore equivalent length of the link,

$$L = l = 110 \text{ mm}$$

and Rankine's constant, $a = \dfrac{1}{7500}$

According to Rankine's formula, buckling load (W_{cr}),

$$7115 = \frac{\sigma_c \times A}{1 + a\left(\dfrac{L}{k}\right)^2} = \frac{100 \times 3(t_1)^2}{1 + \dfrac{1}{7500}\left(\dfrac{110}{0.866\,t_1}\right)^2} = \frac{300\,(t_1)^2}{1 + \dfrac{2.15}{(t_1)^2}}$$

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or $$\frac{7115}{300} = \frac{(t_1)^4}{(t_1)^2 + 2.15}$$

$$(t_1)^4 - 23.7\,(t_1)^2 - 51 = 0$$

$$\therefore \quad (t_1)^2 = \frac{23.7 \pm \sqrt{(23.7)^2 + 4 \times 51}}{2} = \frac{23.7 + 27.7}{2} = 25.7$$

or $t_1 = 5.07$ say 6 mm ... (Taking + ve sign)

and $b_1 = 3\,t_1 = 3 \times 6 = 18$ mm

Now let us consider the buckling of the link in a plane perpendicular to the vertical plane.

Moment of inertia of the cross-section of the link,

$$I = \frac{1}{12} \times b_1\,(t_1)^3 = \frac{1}{12} \times 3t_1\,(t_1)^3 = 0.25\,(t_1)^4$$

and cross-sectional area of the link,

$$A = t_1.b_1 = t_1 \times 3\,t_1 = 3\,(t_1)^2$$

$\therefore$ Radius of gyration,

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.25\,(t_l)^4}{3\,(t_1)^2}} = 0.29\,t_1$$

Since for buckling of the link in a plane perpendicular to the vertical plane, the ends are considered as fixed, therefore

Equivalent length of the link,

$$L = l / 2 = 110 / 2 = 55 \text{ mm}$$

Again according to Rankine's formula, buckling load,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a\left(\frac{L}{k}\right)^2} = \frac{100 \times 3(t_1)^2}{1 + \frac{1}{7500}\left(\frac{55}{0.29\,t_1}\right)^2} = \frac{300\,(t_1)^2}{1 + \frac{4.8}{(t_1)^2}}$$

Substituting the value of $t_1 = 6$ mm, we have

$$W_{cr} = \frac{300 \times 6^2}{1 + \frac{4.8}{6^2}} = 9532 \text{ N}$$

Since this buckling load is more than the calculated value (*i.e.* 7115 N), therefore the link is safe for buckling in a plane perpendicular to the vertical plane.

$\therefore$ We may take $t_1 = 6$ mm ; and $b_1 = 18$ mm **Ans.**

17.15 Differential and Compound Screws

There are certain cases in which a very slow movement of the screw is required whereas in other cases, a very fast movement of the screw is needed. The slow movement of the screw may be obtained by using a small pitch of the threads, but it results in weak threads. The fast movement of the screw may be obtained by using multiple-start threads, but this method requires expensive machning and the loss of self-locking property. In order to overcome these difficulties, differential or compound screws, as discussed below, are used.

1. *Differential screw.* When a slow movement or fine adjustment is desired in precision equipments, then a differential screw is used. It consists of two threads of the same hand (*i.e.* right handed or left handed) but of different pitches, wound on the same cylinder or different cylinders as shown in Fig. 17.14. It may be noted that when the threads are wound on the same cylinder, then two

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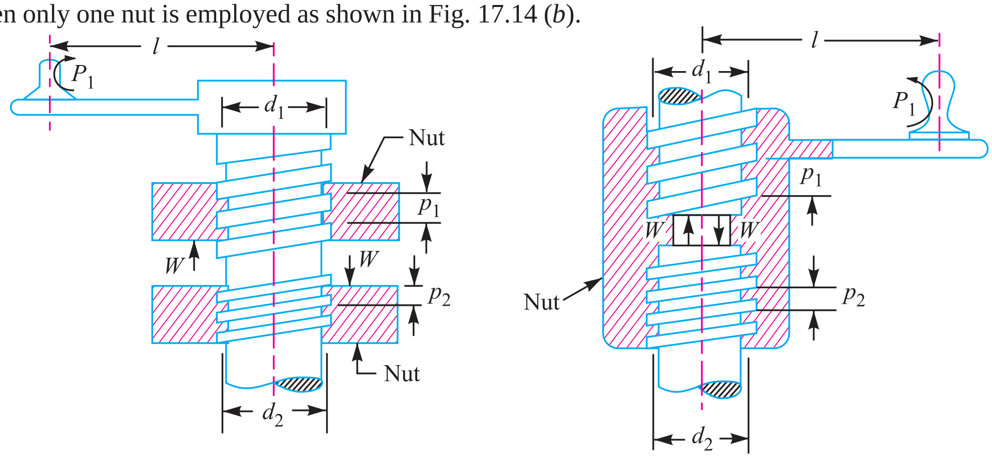
nuts are employed as shown in Fig. 17.14 (*a*) and when the threads are wound on different cylinders, then only one nut is employed as shown in Fig. 17.14 (*b*).



(*a*) Threads wound on the same cylinder. (*b*) Threads wound on the different cylinders.

Fig. 17.14

In this case, each revolution of the screw causes the nuts to move towards or away from each other by a distance equal to the difference of the pitches.

Let p_1 = Pitch of the upper screw,

d_1 = Mean diameter of the upper screw,

α_1 = Helix angle of the upper screw, and

μ_1 = Coefficient of friction between the upper screw and the upper nut

= $\tan \phi_1$, where ϕ_1 is the friction angle.

p_2, d_2, α_2 and μ_2 = Corresponding values for the lower screw.

We know that torque required to overcome friction at the upper screw,

$$T_1 = W \tan(\alpha_1 + \phi_1)\frac{d_1}{2} = W\left[\frac{\tan\alpha_1 + \tan\phi_1}{1 - \tan\alpha_1 \tan\phi_1}\right]\frac{d_1}{2} \quad \text{...}(i)$$

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W \tan(\alpha_2 + \phi_2)\frac{d_2}{2} = W\left[\frac{\tan\alpha_2 + \tan\phi_2}{1 - \tan\alpha_2 \tan\phi_2}\right]\frac{d_2}{2} \quad \text{...}(ii)$$

∴ Total torque required to overcome friction at the thread surfaces,

$$T = P_1 \times l = T_1 - T_2$$

When there is no friction between the thread surfaces, then $\mu_1 = \tan\phi_1 = 0$ and $\mu_2 = \tan\phi_2 = 0$. Substituting these values in the above expressions, we have

$$\therefore \quad T_1' = W \tan\alpha_1 \times \frac{d_1}{2}$$

and
$$T_2' = W \tan\alpha_2 \times \frac{d_2}{2}$$

∴ Total torque required when there is no friction,

$$T_0 = T_1' - T_2'$$

$$= W \tan\alpha_1 \times \frac{d_1}{2} - W \tan\alpha_2 \times \frac{d_2}{2}$$

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$$= W\left[\frac{p_1}{\pi d_1}\times\frac{d_1}{2}-\frac{p_2}{\pi d_2}\times\frac{d_2}{2}\right]=\frac{W}{2\pi}(p_1-p_2)$$

$$\left[\because \tan\alpha_1=\frac{p_1}{\pi d_1};\text{ and }\tan\alpha_2=\frac{p_2}{\pi d_2}\right]$$

We know that efficiency of the differential screw,

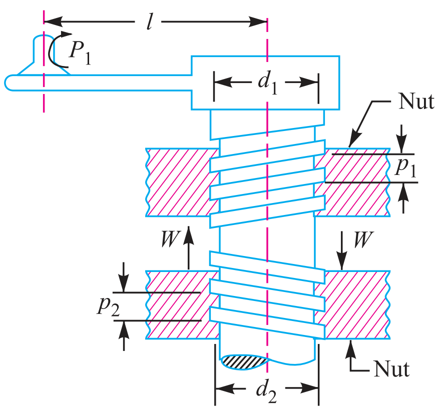
$$\eta = \frac{T_0}{T}$$

2. *Compound screw.* When a fast movement is desired, then a compound screw is employed. It consists of two threads of opposite hands (*i.e.* one right handed and the other left handed) wound on the same cylinder or different cylinders, as shown in Fig. 17.15 (*a*) and (*b*) respectively.

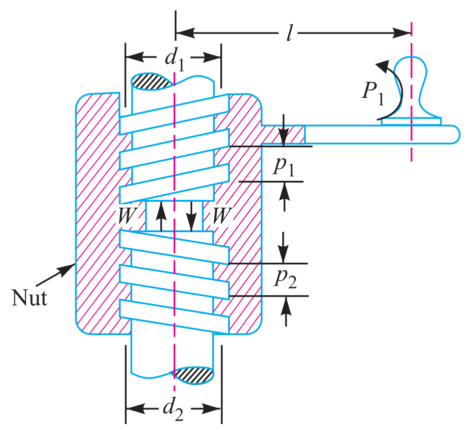
In this case, each revolution of the screw causes the nuts to move towards one another equal to the sum of the pitches of the threads. Usually the pitch of both the threads are made equal.

We know that torque required to overcome friction at the upper screw,

$$T_1 = W\tan(\alpha_1+\phi_1)\frac{d_1}{2}=W\left[\frac{\tan\alpha_1+\tan\phi_1}{1-\tan\alpha_1\tan\phi_1}\right]\frac{d_1}{2} \quad ...(i)$$



(*a*) Threads wound on the same cylinder.



(*b*) Threads wound on the different cylinders.

Fig. 17.15

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W\tan(\alpha_2+\phi_2)\frac{d_2}{2}=W\left[\frac{\tan\alpha_2+\tan\phi_2}{1-\tan\alpha_2\tan\phi_2}\right]\frac{d_2}{2} \quad ...(ii)$$

∴ Total torque required to overcome friction at the thread surfaces,

$$T = P_1\times l = T_1+T_2$$

When there is no friction between the thread surfaces, then $\mu_1 = \tan\phi_1 = 0$ and $\mu_2 = \tan\phi_2 = 0$. Substituting these values in the above expressions, we have

$$T_1' = W\tan\alpha_1\times\frac{d_1}{2}$$

$$T_2' = W\tan\alpha_2\times\frac{d_2}{2}$$

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∴ Total torque required when there is no friction,

$$T_0 = T_1' + T_2'$$

$$= W \tan \alpha_1 \times \frac{d_2}{2} + W \tan \alpha_2 \times \frac{d_2}{2}$$

$$= W \left[\frac{p_1}{\pi\, d_1} \times \frac{d_1}{2} + \frac{p_2}{\pi\, d_2} \times \frac{d_2}{2} \right] = \frac{W}{2\,\pi} (p_1 + p_2)$$

We know that efficiency of the compound screw,

$$\eta = \frac{T_0}{T}$$

Example 17.17. *A differential screw jack is to be made as shown in Fig. 17.16. Neither screw rotates. The outside screw diameter is 50 mm. The screw threads are of square form single start and the coefficient of thread friction is 0.15.*

Determine : 1. Efficiency of the screw jack; 2. Load that can be lifted if the shear stress in the body of the screw is limited to 28 MPa.

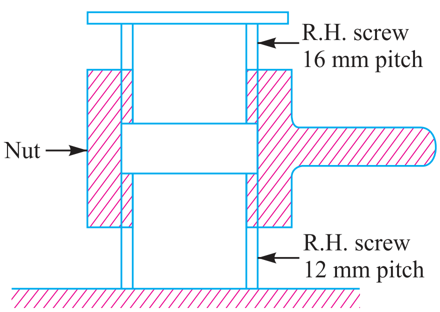

Fig. 17.16. Differential screw.

Solution. Given : $d_o = 50$ mm ; $\mu = \tan \phi = 0.15$; $p_1 = 16$ mm ; $p_2 = 12$ mm ; $\tau_{max} = 28$ MPa = 28 N/mm^2

1. Efficiency of the screw jack

We know that the mean diameter of the upper screw,

$$d_1 = d_o - p_1 / 2 = 50 - 16 / 2 = 42 \text{ mm}$$

and mean diameter of the lower screw,

$$d_2 = d_o - p_2 / 2 = 50 - 12 / 2 = 44 \text{ mm}$$

$$\therefore \quad \tan \alpha_1 = \frac{p_1}{\pi d_1} = \frac{16}{\pi \times 42} = 0.1212$$

and

$$\tan \alpha_2 = \frac{p_2}{\pi d_2} = \frac{12}{\pi \times 44} = 0.0868$$

Let W = Load that can be lifted in N.

We know that torque required to overcome friction at the upper screw,

$$T_1 = W \tan (\alpha_1 + \phi) \frac{d_1}{2} = W \left[\frac{\tan \alpha_1 + \tan \phi}{1 - \tan \alpha_1 \tan \phi} \right] \frac{d_1}{2}$$

$$= W \left[\frac{0.1212 + 0.15}{1 - 0.1212 \times 0.15} \right] \frac{42}{2} = 5.8\, W \text{ N-mm}$$

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W \tan (\alpha_2 - \phi) \frac{d_2}{2} = W \left[\frac{\tan \alpha_2 - \tan \phi}{1 + \tan \alpha_2 \tan \phi} \right] \frac{d_2}{2}$$

$$= W \left[\frac{0.0868 - 0.15}{1 + 0.0868 \times 0.15} \right] \frac{44}{2} = -1.37\, W \text{ N-mm}$$

∴ Total torque required to overcome friction,

$$T = T_1 - T_2 = 5.8\, W - (-1.37\, W) = 7.17\, W \text{ N-mm}$$

We know that the torque required when there is no friction,

$$T_0 = \frac{W}{2\pi} (p_1 - p_2) = \frac{W}{2\pi} (16 - 12) = 0.636\, W \text{ N-mm}$$

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∴ Efficiency of the screw jack,

$$\eta = \frac{T_0}{T} = \frac{0.636\,W}{7.17\,W} = 0.0887 \text{ or } 8.87\% \quad \textbf{Ans.}$$

2. *Load that can be lifted*

Since the upper screw is subjected to a larger torque, therefore the load to be lifted (W) will be calculated on the basis of larger torque (T_1).

We know that core diameter of the upper screw,

$$d_{c1} = d_o - p_1 = 50 - 16 = 34 \text{ mm}$$

Since the screw is subjected to direct compressive stress due to load W and shear stress due to torque T_1, therefore

Direct compressive stress,

$$\sigma_c = \frac{W}{A_{c1}} = \frac{W}{\frac{\pi}{4}(d_{c1})^2} = \frac{W}{\frac{\pi}{4}(34)^2} = \frac{W}{908} \text{ N/mm}^2$$

and shear stress,

$$\tau = \frac{16\,T_1}{\pi\,(d_{c1})^3} = \frac{16 \times 5.8\,W}{\pi(34)^3} = \frac{W}{1331} \text{ N/mm}^2$$

We know that maximum shear stress (τ_{max}),

$$28 = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{W}{908}\right)^2 + 4\left(\frac{W}{1331}\right)^2}$$

$$= \frac{1}{2}\sqrt{1.213 \times 10^{-6}\,W^2 + 2.258 \times 10^{-6}\,W^2} = \frac{1}{2}\,1.863 \times 10^{-3}\,W$$

$$\therefore \quad W = \frac{28 \times 2}{1.863 \times 10^{-3}} = 30\,060 \text{ N} = 30.06 \text{ kN} \quad \textbf{Ans.}$$

EXERCISES

1. In a hand vice, the screw has double start square threads of 24 mm outside diameter. If the lever is 200 mm long and the maximum force that can be applied at the end of lever is 250 N, find the force with which the job is held in the jaws of the vice. Assume a coefficient of friction of 0.12. **[Ans. 17 420 N]**
2. A square threaded bolt of mean diameter 24 mm and pitch 5 mm is tightened by screwing a nut whose mean diameter of bearing surface is 50 mm. If the coefficient of friction for the nut and bolt is 0.1 and for the nut and bearing surfaces 0.16, find the force required at the end of a spanner 0.5 m long when the load on the bolt is 10 kN. **[Ans. 120 N]**
3. The spindle of a screw jack has a single start square thread with an outside diameter of 45 mm and a pitch of 10 mm. The spindle moves in a fixed nut. The load is carried on a swivel head but is not free to rotate. The bearing surface of the swivel head has a mean diameter of 60 mm. The coefficient of friction between the nut and screw is 0.12 and that between the swivel head and the spindle is 0.10. Calculate the load which can be raised by efforts of 100 N each applied at the end of two levers each of effective length of 350 mm. Also determine the efficiency of the lifting arrangement. **[Ans. 9945 N ; 22.7%]**

Lead screw supported by collar bearing.

4. The cross bar of a planner weighing 12 kN is raised and lowered by means of two square threaded screws of 38 mm outside diameter and 7 mm pitch. The screw is made of steel and a bronze nut of 38 mm thick. A steel collar has 75 mm outside diameter and 38 mm inside diameter. The coefficient of friction at the threads is assumed as 0.11 and at the collar 0.13. Find the force required at a radius of 100 mm to raise and lower the load. **[Ans. 402.5 N ; 267 N]**
5. The lead screw of a lathe has square threads of 24 mm outside diameter and 5 mm pitch. In order to drive the tool carriage, the screw exerts an axial pressure of 2.5 kN. Find the efficiency of the screw and the power required to drive the screw, if it is to rotate at 30 r.p.m. Neglect bearing friction. Assume coefficient of friction of screw threads as 0.12. **[Ans. 37.76% ; 16.55 W]**
6. The lead screw of a lathe has Acme threads of 60 mm outside diameter and 8 mm pitch. It supplies drive to a tool carriage which needs an axial force of 2000 N. A collar bearing with inner and outer radius as 30 mm and 60 mm respectively is provided. The coefficient of friction for the screw threads is 0.12 and for the collar it is 0.10. Find the torque required to drive the screw and the efficiency of the screw. **[Ans. 18.5 N-m ; 13.6%]**
7. A cross bar of a planer weighing 9 kN is raised and lowered by means of two square threaded screws of 40 mm outside diameter and 6 mm pitch. The screw is made of steel and nut of phosphor bronze having 42 mm height. A steel collar bearing with 30 mm mean radius takes the axial thrust. The coefficient of friction at the threads and at the collar may be assumed as 0.14 and 0.10 respectively. Find the force required at a radius of 120 mm of a handwheel to raise and lower the load. Find also the shear stress in the nut material and the bearing pressure on the threads. **[Ans. 495 N, 346 N ; 1.7 MPa ; 1.84 N/mm^2]**
8. A machine slide weighing 3000 N is elevated by a double start acme threaded screw at the rate of 840 mm/min. If the coefficient of friction be 0.12, calculate the power to drive the slide. The end of the screw is carried on a thrust collar of 32 mm inside diameter and 58 mm outside diameter. The pitch of the screw thread is 6 mm and outside diameter of the screw is 40 mm. If the screw is of steel, is it strong enough to sustain the load? Draw a neat sketch of the system. **[Ans. 0.165 kW]**
9. A sluice valve, used in water pipe lines, consists of a gate raised by the spindle, which is rotated by the hand wheel. The spindle has single start square threads. The nominal diameter of the spindle is 36 mm and the pitch is 6 mm. The friction collar has inner and outer diameters of 32 mm and 50 mm respectively. The coefficient of friction at the threads and the collar are 0.12 and 0.18 respectively. The weight of the gate is 7.5 kN and the frictional resistance to open the valve due to water pressure is 2.75 kN. Using uniform wear theory, determine : 1. torque required to raise the gate; and 2. overall efficiency. **[Ans. 136.85 N-m ; 7.1%]**
10. A vertical square threads screw of a 70 mm mean diameter and 10 mm pitch supports a vertical load of 50 kN. It passes through the boss of a spur gear wheel of 70 teeth which acts as a nut. In order to raise the load, the spur gear wheel is turned by means of a pinion having 20 teeth. The mechanical efficiency of pinion and gear wheel drive is 90%. The axial thrust on the screw is taken up by a collar bearing having a mean radius of 100 mm. The coefficient of friction for the screw and nut is 0.15 and that for collar bearing is 0.12. Find:
    (*a*) Torque to be applied to the pinion shaft,
    (*b*) Maximum principal and shear stresses in the screw ; and
    (*c*) Height of nut, if the bearing pressure is limited to 12 N/mm^2.
    [Ans. 299.6 N-m ; 26.6 N/mm^2, 19 N/mm^2 ; 40 mm]
11. A single start square threaded screw is to be designed for a C-clamp. The axial load on the screw may be assumed to be 10 kN. A thrust pad is attached at the end of the screw whose mean diameter may be taken as 30 mm. The coefficient of friction for the screw threads and for the thrust pads is 0.12 and 0.08 respectively. The allowable tensile strength of the screw is 60 MPa and the allowable bearing pressure is 12 N/mm^2. Design the screw and nut. The square threads are as under :

Nominal diameter, mm	16	18	20	22
Core diameter, mm	13	15	17	19
Pitch, mm	3	3	3	3

[Ans. d_c = 17 mm ; n = 10, h = 30 mm]

12. Design a screw jack for lifting a load of 50 kN through a height of 0.4 m. The screw is made of steel and nut of bronze. Sketch the front sectional view. The following allowable stresses may be assumed

For steel : Compressive stress = 80 MPa ; Shear stress = 45 MPa

For bronze : Tensile stress = 40 MPa ; Bearing stress = 15 MPa

Shear stress = 25 MPa.

The coefficient of friction between the steel and bronze pair is 0.12. The dimensions of the swivel base may be assumed proportionately. The screw should have square threads. Design the screw, nut and handle. The handle is made of steel having bending stress 150 MPa (allowable).

13. A screw jack carries a load of 22 kN. Assuming the coefficient of friction between screw and nut as 0.15, design the screw and nut. Neglect collar friction and column action. The permissible compressive and shear stresses in the screw should not exceed 42 MPa and 28 MPa respectively. The shear stress in the nut should not exceed 21 MPa. The bearing pressure on the nut is 14 N/mm^2. Also determine the effort required at the handle of 200 mm length in order to raise and lower the load. What will be the efficiency of screw? **[Ans. d_c = 30 mm ; h = 36 mm ; 381 N ; 166 N ; 27.6%]**

14. Design and draw a screw jack for lifting a safe load of 150 kN through a maximum lift of 350 mm.

The elastic strength of the material of the screw may be taken as 240 MPa in compression and 160 MPa in shear. The nut is to be made of phosphor bronze for which the elastic strengths in tension, compression and shear are respectively 130, 115 and 100 MPa. Bearing pressure between the threads of the screw and the nut may be taken as 18 N/mm^2. Safe crushing stress for the material of the body is 100 MPa. Coefficient of friction for the screw as well as collar may be taken as 0.15.

15. Design a toggle jack to lift a load of 5 kN. The jack is to be so designed that the distance between the centre lines of nuts varies from 50 to 220 mm. The eight links are symmetrical and 120 mm long. The link pins in the base are set 30 mm apart. The links, screw and pins are made from mild steel for which the stresses are 90 MPa in tension and 50 MPa in shear. The bearing pressure on the pin is 20 N/mm^2. Assume the coefficient of friction between screw and nut as 0.15 and pitch of the square threaded screw as 6 mm.

[Ans. d_c = 10 mm : d_o = 22 mm ; d = 19 mm ; n = 4; t = 24 mm ; b = 33 mm ; d_1 = 10 mm ; t_1 = 7 mm ; b_1 = 21 mm]

QUESTIONS

1. Discuss the various types of power threads. Give atleast two practical applications for each type. Discuss their relative advantages and disadvantages.

2. Why are square threads preferable to V-threads for power transmission?

3. How does the helix angle influence on the efficiency of square threaded screw?

4. What do you understand by overhauling of screw?

5. What is self locking property of threads and where it is necessary?

6. Show that the efficiency of self locking screws is less than 50 percent.

7. In the design of power screws, on what factors does the thread bearing pressure depend? Explain.

8. Why is a separate nut preferable to an integral nut with the body of a screw jack?

9. Differentiate between differential screw and compound screw.

Screw jack building-block system

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OBJECTIVE TYPE QUESTIONS

1. Which of the following screw thread is adopted for power transmission in either direction?
   (*a*) Acme threads (*b*) Square threads
   (*c*) Buttress threads (*d*) Multiple threads
2. Multiple threads are used to secure
   (*a*) low efficiency (*b*) high efficiency
   (*c*) high load lifting capacity (*d*) high mechanical advantage
3. Screws used for power transmission should have
   (*a*) low efficiency (*b*) high efficiency
   (*c*) very fine threads (*d*) strong teeth
4. If α denotes the lead angle and ϕ, the angle of friction, then the efficiency of the screw is written as
   (*a*) $\dfrac{\tan(\alpha - \phi)}{\tan \alpha}$ (*b*) $\dfrac{\tan \alpha}{\tan(\alpha - \phi)}$
   (*c*) $\dfrac{\tan(\alpha + \phi)}{\tan \alpha}$ (*d*) $\dfrac{\tan \alpha}{\tan(\alpha + \phi)}$
5. A screw jack has square threads and the lead angle of the thread is α. The screw jack will be self-locking when the coefficient of friction (μ) is
   (*a*) $\mu > \tan \alpha$ (*b*) $\mu = \sin \alpha$
   (*c*) $\mu = \cot \alpha$ (*d*) $\mu = \text{cosec}\ \alpha$
6. To ensure self locking in a screw jack, it is essential that the helix angle is
   (*a*) larger than friction angle (*b*) smaller than friction angle
   (*c*) equal to friction angle (*d*) such as to give maximum efficiency in lifting
7. A screw is said to be self locking screw, if its efficiency is
   (*a*) less than 50% (*b*) more than 50%
   (*c*) equal to 50% (*d*) none of these
8. A screw is said to be over hauling screw, if its efficiency is
   (*a*) less than 50% (*b*) more than 50%
   (*c*) equal to 50% (*d*) none of these
9. While designing a screw in a screw jack against buckling failure, the end conditions for the screw are taken as
   (*a*) both ends fixed (*b*) both ends hinged
   (*c*) one end fixed and other end hinged (*d*) one end fixed and other end free.
10. The load cup of a screw jack is made separate from the head of the spindle to
   (*a*) enhance the load carrying capacity of the jack
   (*b*) reduce the effort needed for lifting the working load
   (*c*) reduce the value of frictional torque required to be countered for lifting the load
   (*d*) prevent the rotation of load being lifted

ANSWERS

1. (*b*)	**2.** (*b*)	**3.** (*b*)	**4.** (*d*)	**5.** (*a*)
6. (*b*)	**7.** (*a*)	**8.** (*b*)	**9.** (*d*)	**10.** (*d*)

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CHAPTER

18

Flat Belt Drives

18.1 Introduction

The belts or *ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds. The amount of power transmitted depends upon the following factors :

1. The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The arc of contact between the belt and the smaller pulley.
4. The conditions under which the belt is used.

It may be noted that

(*a*) The shafts should be properly in line to insure uniform tension across the belt section.

(*b*) The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.

* Rope drives are discussed in Chapter 20.

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(*c*) The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thus increasing the friction load on the bearings.

(*d*) A long belt tends to swing from side to side, causing the belt to run out of the pulleys, which in turn develops crooked spots in the belt.

(*e*) The tight side of the belt should be at the bottom, so that whatever sag is present on the loose side will increase the arc of contact at the pulleys.

(*f*) In order to obtain good results with flat belts, the maximum distance between the shafts should not exceed 10 metres and the minimum should not be less than 3.5 times the diameter of the larger pulley.

18.2 Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,
2. Speed reduction ratio,
3. Power to be transmitted,
4. Centre distance between the shafts,
5. Positive drive requirements,
6. Shafts layout,
7. Space available, and
8. Service conditions.

18.3 Types of Belt Drives

The belt drives are usually classified into the following three groups:

1. ***Light drives.*** These are used to transmit small powers at belt speeds upto about 10 m/s as in agricultural machines and small machine tools.

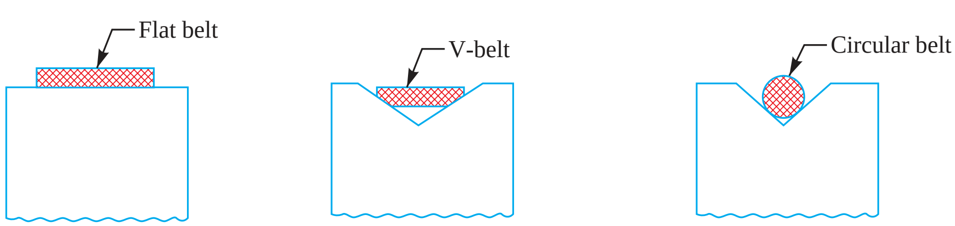
2. ***Medium drives.*** These are used to transmit medium powers at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

3. ***Heavy drives.*** These are used to transmit large powers at belt speeds above 22 m/s as in compressors and generators.

18.4 Types of Belts

Though there are many types of belts used these days, yet the following are important from the subject point of view:

1. ***Flat belt.*** The flat belt as shown in Fig. 18.1 (*a*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 metres apart.



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

Fig. 18.1. Types of belts

2. ***V- belt.*** The V-belt as shown in Fig. 18.1 (*b*), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

3. ***Circular belt or rope.*** The circular belt or rope as shown in Fig. 18.1 (*c*) is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 metres apart.

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If a huge amount of power is to be transmitted, then a single belt may not be sufficient. In such a case, wide pulleys (for V-belts or circular belts) with a number of grooves are used. Then a belt in each groove is provided to transmit the required amount of power from one pulley to another.

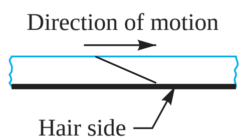
Note : The V-belt and rope drives are discussed in Chapter 20.

18.5 Material used for Belts

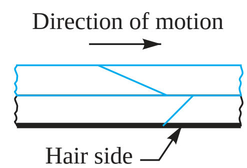
The material used for belts and ropes must be strong, flexible, and durable. It must have a high coefficient of friction. The belts, according to the material used, are classified as follows:

1. ***Leather belts.*** The most important material for flat belt is leather. The best leather belts are made from 1.2 metres to 1.5 metres long strips cut from either side of the back bone of the top grade steer hides. The hair side of the leather is smoother and harder than the flesh side, but the flesh side is stronger. The fibres on the hair side are perpendicular to the surface, while those on the flesh side are interwoven and parallel to the surface. Therefore for these reasons the hair side of a belt should be in contact with the pulley surface as shown in Fig. 18.2. This gives a more intimate contact between belt and pulley and places the greatest tensile strength of the belt section on the outside, where the tension is maximum as the belt passes over the pulley.

The leather may be either oak-tanned or mineral salt-tanned *e.g.* chrome-tanned. In order to increase the thickness of belt, the strips are cemented together. The belts are specified according to the number of layers *e.g.* single, double or triple ply and according to the thickness of hides used *e.g.* light, medium or heavy.



(*a*) Single layer belt.



(*b*) Double layer belt.

Fig. 18.2. Leather belts.

The leather belts must be periodically cleaned and dressed or treated with a compound or dressing containing neats foot or other suitable oils so that the belt will remain soft and flexible.

2. ***Cotton or fabric belts.*** Most of the fabric belts are made by folding convass or cotton duck to three or more layers (depending upon the thickness desired) and stitching together. These belts are woven also into a strip of the desired width and thickness. They are impregnated with some filler like linseed oil in order to make the belt water-proof and to prevent injury to the fibres. The cotton belts are cheaper and suitable in warm climates, in damp atmospheres and in exposed positions. Since the cotton belts require little attention, therefore these belts are mostly used in farm machinery, belt conveyor etc.

3. ***Rubber belt.*** The rubber belts are made of layers of fabric impregnated with rubber composition and have a thin layer of rubber on the faces. These belts are very flexible but are quickly destroyed if allowed to come into contact with heat, oil or grease. One of the principle advantage of these belts is that they may be easily made endless. These belts are found suitable for saw mills, paper mills where they are exposed to moisture.

4. ***Balata belts.*** These belts are similar to rubber belts except that balata gum is used in place of rubber. These belts are acid proof and water proof and it is not effected by animal oils or alkalies. The balata belts should not be at temperatures above 40°C because at this temperature the balata begins to soften and becomes sticky. The strength of balata belts is 25 per cent higher than rubber belts.

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18.6 Working Stresses in Belts

The ultimate strength of leather belt varies from 21 to 35 MPa and a factor of safety may be taken as 8 to 10. However, the wear life of a belt is more important than actual strength. It has been shown by experience that under average conditions an allowable stress of 2.8 MPa or less will give a reasonable belt life. An allowable stress of 1.75 MPa may be expected to give a belt life of about 15 years.

18.7 Density of Belt Materials

The density of various belt materials are given in the following table.

Table 18.1. Density of belt materials.

Material of belt	*Mass density in kg / m*^{3}
Leather	1000
Convass	1220
Rubber	1140
Balata	1110
Single woven belt	1170
Double woven belt	1250

18.8 Belt Speed

A little consideration will show that when the speed of belt increases, the centrifugal force also increases which tries to pull the belt away from the pulley. This will result in the decrease of power transmitted by the belt. It has been found that for the efficient transmission of power, the belt speed 20 m/s to 22.5 m/s may be used.

18.9 Coefficient of Friction Between Belt and Pulley

The coefficient of friction between the belt and the pulley depends upon the following factors:

1. The material of belt;
2. The material of pulley;
3. The slip of belt; and
4. The speed of belt.

According to C.G. Barth, the coefficient of friction (μ) for oak tanned leather belts on cast iron pulley, at the point of slipping, is given by the following relation, *i.e.*

Belts used to drive wheels

$$\mu = 0.54 - \frac{42.6}{152.6 + v}$$

where v = Speed of the belt in metres per minute.

The following table shows the values of coefficient of friction for various materials of belt and pulley.

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Table 18.2. Coefficient of friction between belt and pulley.

Belt material	*Pulley material*						
	Cast iron, steel			*Wood*	*Compressed paper*	*Leather face*	*Rubber face*
	Dry	*Wet*	*Greasy*				
1. Leather oak tanned	0.25	0.2	0.15	0.3	0.33	0.38	0.40
2. Leather chrome tanned	0.35	0.32	0.22	0.4	0.45	0.48	0.50
3. Convass-stitched	0.20	0.15	0.12	0.23	0.25	0.27	0.30
4. Cotton woven	0.22	0.15	0.12	0.25	0.28	0.27	0.30
5. Rubber	0.30	0.18	—	0.32	0.35	0.40	0.42
6. Balata	0.32	0.20	—	0.35	0.38	0.40	0.42

18.10 Standard Belt Thicknesses and Widths

The standard flat belt thicknesses are 5, 6.5, 8, 10 and 12 mm. The preferred values of thicknesses are as follows:

(*a*) 5 mm for nominal belt widths of 35 to 63 mm,

(*b*) 6.5 mm for nominal belt widths of 50 to 140 mm,

(*c*) 8 mm for nominal belt widths of 90 to 224 mm,

(*d*) 10 mm for nominal belt widths of 125 to 400 mm, and

(*e*) 12 mm for nominal belt widths of 250 to 600 mm.

The standard values of nominal belt widths are in R10 series, starting from 25 mm upto 63 mm and in R 20 series starting from 71 mm up to 600 mm. Thus, the standard widths will be 25, 32, 40, 50, 63, 71, 80, 90, 100, 112, 125, 140, 160, 180, 200, 224, 250, 280, 315, 355, 400, 450, 500, 560 and 600 mm.

18.11 Belt Joints

When the endless belts are not available, then the belts are cut from big rolls and the ends are joined together by fasteners. The various types of joints are

1. Cemented joint, **2.** Laced joint, and **3.** Hinged joint.

The **cemented joint,** as shown in Fig. 18.3 (*a*), made by the manufacturer to form an endless belt, is preferred than other joints. The **laced joint** is formed by punching holes in line across the belt, leaving a margin between the edge and the holes. A raw hide strip is used for lacing the two ends together to form a joint. This type of joint is known as ***straight-stitch raw hide laced joint,*** as shown in Fig. 18.3 (*b*).

Metal laced joint as shown in Fig. 18.3 (*c*), is made like a staple connection. The points are driven through the flesh side of the belt and clinched on the inside.

Sometimes, **metal hinges** may be fastened to the belt ends and connected by a steel or fibre pin as shown in Fig. 18.3 (*d*).

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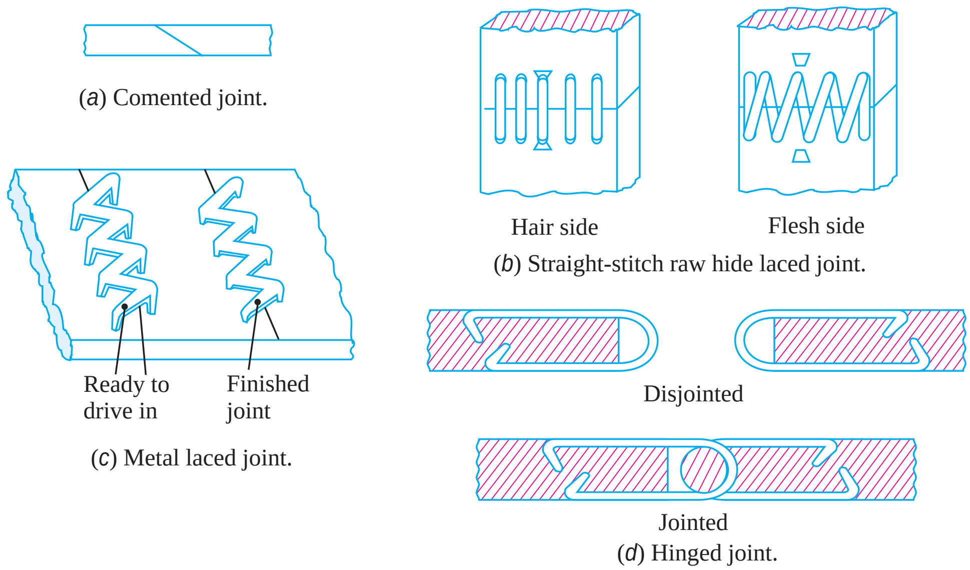

Fig. 18.3. Belt joints.

The following table shows the efficiencies of these joints.

Table 18.3. Efficiencies of belt joints.

Type of joint	*Efficiency* (%)	*Type of joint*	*Efficiency* (%)
1. Cemented, endless, cemented at factory	90 to 100	4. Wire laced by hand	70 to 80
2. Cemented in shop	80 to 90	5. Raw-hide laced	60 to 70
3. Wire laced by machine	75 to 85	6. Metal belt hooks	35 to 40

18.12 Types of Flat Belt Drives

The power from one pulley to another may be transmitted by any of the following types of belt drives.

Cross or twist belt drive

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1. *Open belt drive.* The open belt drive, as shown in Fig. 18.4, is used with shafts arranged parallel and rotating in the same direction. In this case, the driver *A* pulls the belt from one side (*i.e.* lower side *RQ*) and delivers it to the other side (*i.e.* upper side *LM*). Thus the tension in the lower side belt will be more than that in the upper side belt. The lower side belt (because of more tension) is known as ***tight side*** whereas the upper side belt (because of less tension) is known as ***slack side,*** as shown in Fig. 18.4.

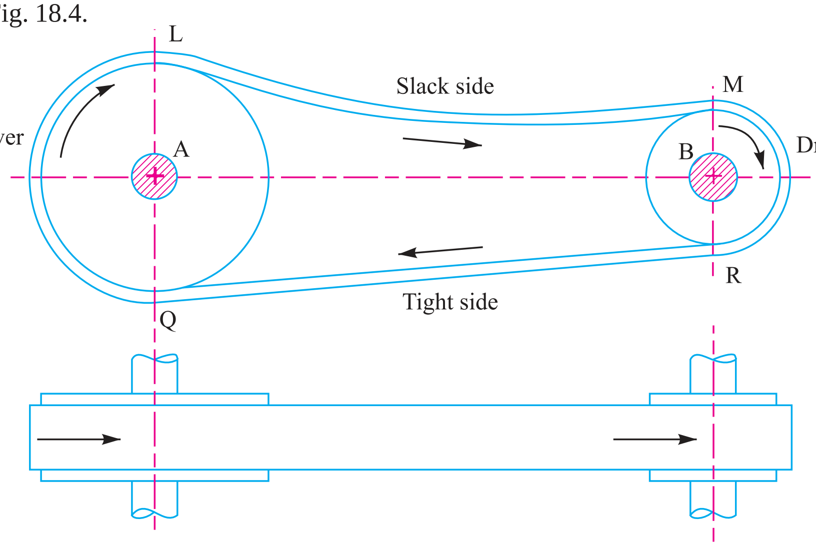

Fig. 18.4. Open belt drive.

2. *Crossed or twist belt drive.* The crossed or twist belt drive, as shown in Fig. 18.5, is used with shafts arranged parallel and rotating in the opposite directions. In this case, the driver pulls the belt from one side (*i.e. RQ*) and delivers it to the other side (*i.e. LM*). Thus, the tension in the belt *RQ* will be more than that in the belt *LM*. The belt *RQ* (because of more tension) is known as ***tight side,*** whereas the belt *LM* (because of less tension) is known as ***slack side,*** as shown in Fig. 18.5.

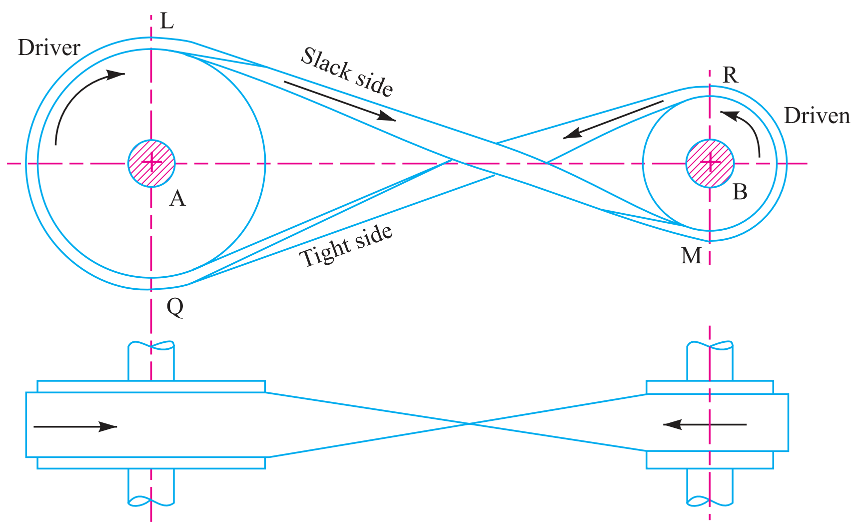

Fig. 18.5. Crossed or twist belt drive.

A little consideration will show that at a point where the belt crosses, it rubs against each other and there will be excessive wear and tear. In order to avoid this, the shafts should be placed at a

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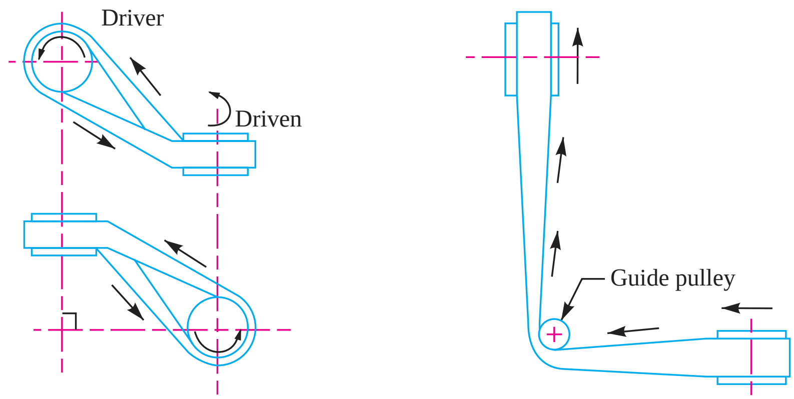
maximum distance of 20 *b*, where *b* is the width of belt and the speed of the belt should be less than 15 m/s.

3. *Quarter turn belt drive.* The quarter turn belt drive (also known as ***right angle belt drive***) as shown in Fig. 18.6 (*a*), is used with shafts arranged at right angles and rotating in one definite direction. In order to prevent the belt from leaving the pulley, the width of the face of the pulley should be greater or equal to 1.4 *b*, where *b* is width of belt.

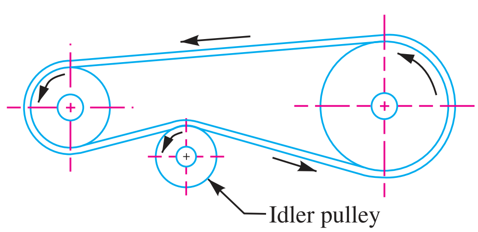
In case the pulleys cannot be arranged as shown in Fig. 18.6 (*a*) or when the reversible motion is desired, then a ***quarter turn belt drive with a guide pulley,*** as shown in Fig. 18.6 (*b*), may be used.



(*a*) Quarter turn belt drive. (*b*) Quarter turn belt drive with guide pulley.

Fig. 18.6

4. *Belt drive with idler pulleys.* A belt drive with an idler pulley (also known as ***jockey pulley drive***) as shown in Fig. 18.7, is used with shafts arranged parallel and when an open belt drive can not be used due to small angle of contact on the smaller pulley. This type of drive is provided to obtain high velocity ratio and when the required belt tension can not be obtained by other means.



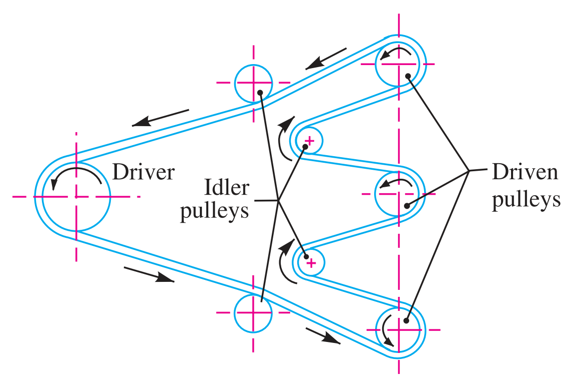


Fig. 18.7. Belt drive with single idler pulley. **Fig. 18.8.** Belt drive with many idler pulleys.

When it is desired to transmit motion from one shaft to several shafts, all arranged in parallel, a belt drive with many idler pulleys, as shown in Fig. 18.8, may be employed.

5. *Compound belt drive.* A compound belt drive as shown in Fig. 18.9, is used when power is transmitted from one shaft to another through a number of pulleys.

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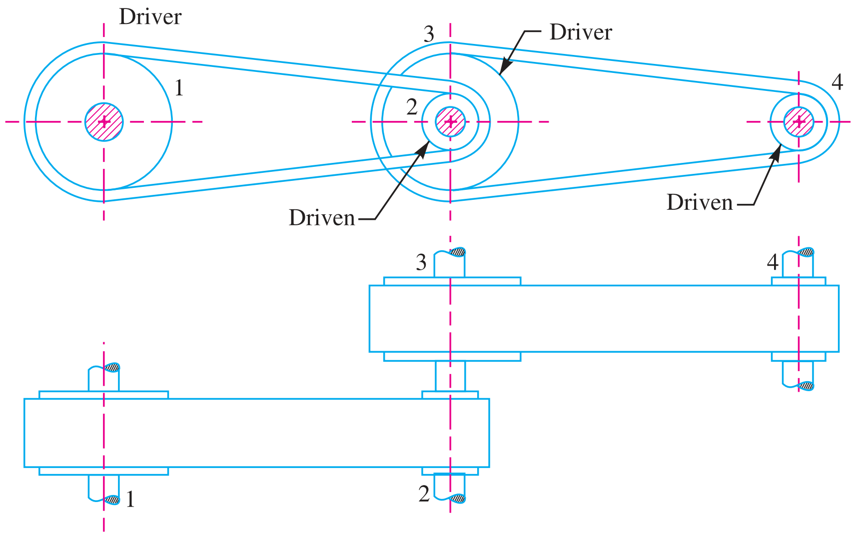

Fig. 18.9. Compound belt drive.

6. *Stepped or cone pulley drive.* A stepped or cone pulley drive, as shown in Fig. 18.10, is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed. This is accomplished by shifting the belt from one part of the steps to the other.

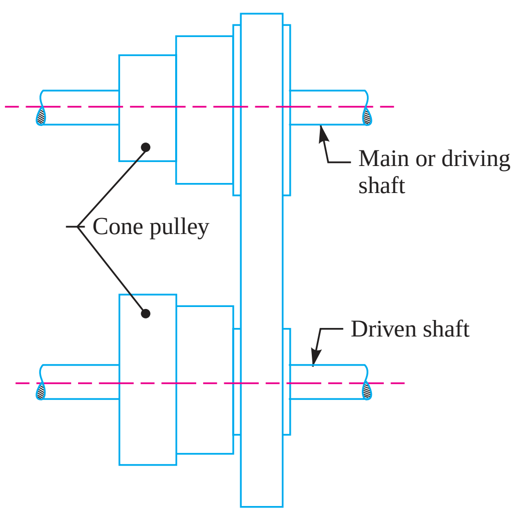

Fig. 18.10. Stepped or cone pulley drive.

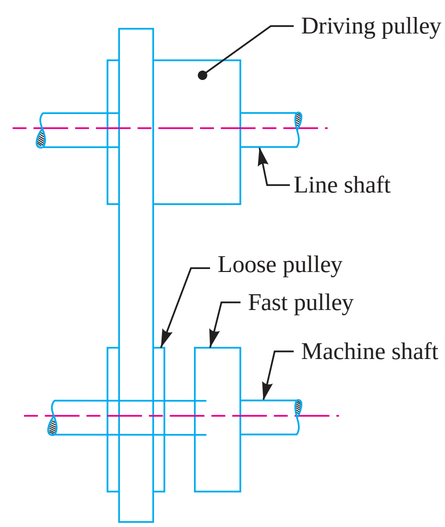

Fig. 18.11. Fast and loose pulley drive.

7. *Fast and loose pulley drive.* A fast and loose pulley drive, as shown in Fig. 18.11, is used when the driven or machine shaft is to be started or stopped whenever desired without interferring with the driving shaft. A pulley which is keyed to the machine shaft is called fast pulley and runs at the same speed as that of machine shaft. A loose pulley runs freely over the machine shaft and is incapable of transmitting any power. When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar having belt forks.

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18.13 Velocity Ratio of a Belt Drive

It is the ratio between the velocities of the driver and the follower or driven. It may be expressed, mathematically, as discussed below:

Let d_1 = Diameter of the driver,

d_2 = Diameter of the follower,

N_1 = Speed of the driver in r.p.m.,

N_2 = Speed of the follower in r.p.m.,

∴ Length of the belt that passes over the driver, in one minute

$$= \pi\, d_1\, N_1$$

Similarly, length of the belt that passes over the follower, in one minute

$$= \pi\, d_2\, N_2$$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that passes over the follower in one minute, therefore

$$\because \quad \pi\, d_1\, N_1 = \pi\, d_2\, N_2$$

and velocity ratio, $$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

When thickness of the belt (t) is considered, then velocity ratio,

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Notes : 1. The velocity ratio of a belt drive may also be obtained as discussed below:

We know that the peripheral velocity of the belt on the driving pulley,

$$v_1 = \frac{\pi\, d_1\, N_1}{60} \text{ m/s}$$

and peripheral velocity of the belt on the driven pulley,

$$v_2 = \frac{\pi\, d_2\, N_2}{60} \text{ m/s}$$

When there is no slip, then $v_1 = v_2$.

$$\therefore \quad \frac{\pi\, d_1\, N_1}{60} = \frac{\pi\, d_2\, N_2}{60} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

2. In case of a compound belt drive as shown in Fig. 18.7, the velocity ratio is given by

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{or} \quad \frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

18.14 Slip of the Belt

In the previous articles we have discussed the motion of belts and pulleys assuming a firm frictional grip between the belts and the pulleys. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This is called ***slip of the belt*** and is generally expressed as a percentage.

The result of the belt slipping is to reduce the velocity ratio of the system. As the slipping of the belt is a common phenomenon, thus the belt should never be used where a definite velocity ratio is of importance (as in the case of hour, minute and second arms in a watch).

Let s_1 % = Slip between the driver and the belt, and

s_2 % = Slip between the belt and follower,

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$\therefore$ Velocity of the belt passing over the driver per second,

$$v = \frac{\pi\, d_1\, N_1}{60} - \frac{\pi\, d_1\, N_1}{60} \times \frac{s_1}{100}$$

$$= \frac{\pi\, d_1\, N_1}{60}\left(1 - \frac{s_1}{100}\right) \qquad ...(i)$$

and velocity of the belt passing over the follower per second

$$\frac{\pi\, d_2\, N_2}{60} = v - v\left(\frac{s_2}{100}\right) = v\left(1 - \frac{s_2}{100}\right)$$

Substituting the value of v from equation (*i*), we have

Belt slip indicator is used to indicate that the belt is slipping.

$$\frac{\pi\, d_2\, N_2}{60} = \frac{\pi\, d_1\, N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right) \qquad ...\left(\text{Neglecting } \frac{s_1 \times s_2}{100 \times 100}\right)$$

$$= \frac{d_1}{d_2}\left[1 - \left(\frac{s_1 + s_2}{100}\right)\right] = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

...(where $s = s_1 + s_2$ *i.e.* total percentage of slip)

If thickness of the belt (t) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

18.15 Creep of Belt

When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to the slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as ***creep.*** The total effect of creep is to reduce slightly the speed of the driven pulley or follower. Considering creep, the velocity ratio is given by

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

where σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively, and

E = Young's modulus for the material of the belt.

Note: Since the effect of creep is very small, therefore it is generally neglected.

Example 18.1. *An engine running at 150 r.p.m. drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft is 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Fine the speed of dynamo shaft, when 1. there is no slip, and 2. there is a slip of 2% at each drive.*

Solution. Given : N_1 = 150 r.p.m. ; d_1 = 750 mm ; d_2 = 450 mm ; d_3 = 900 mm ; d_4 = 150 mm ; $s_1 = s_2$ = 2%

The arrangement of belt drive is shown in Fig. 18.12.

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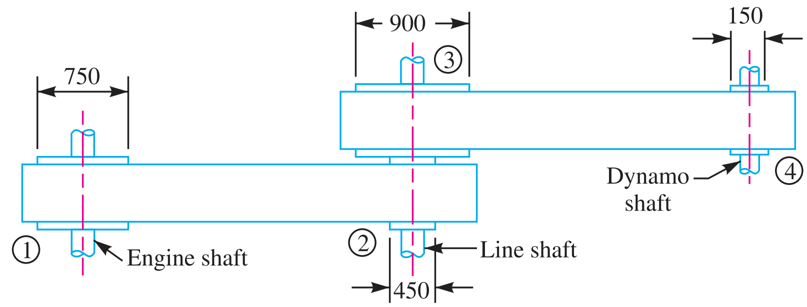
Let N_4 = Speed of the dynamo shaft.

1. *When there is no slip*

We know that

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{or} \quad \frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} = 10$$

∴ N_4 = 150 × 10 = 1500 r.p.m. **Ans.**



All dimensions in mm.

Fig. 18.12

2. *When there is a slip of 2% at each drive*

We know that

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

or
$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right) = 9.6$$

∴ N_4 = 150 × 9.6 = 1440 r.p.m. **Ans.**

18.16 Length of an Open Belt Drive

We have discussed in Art. 18.12, that in an open belt drive, both the pulleys rotate in the same direction as shown in Fig. 18.13.

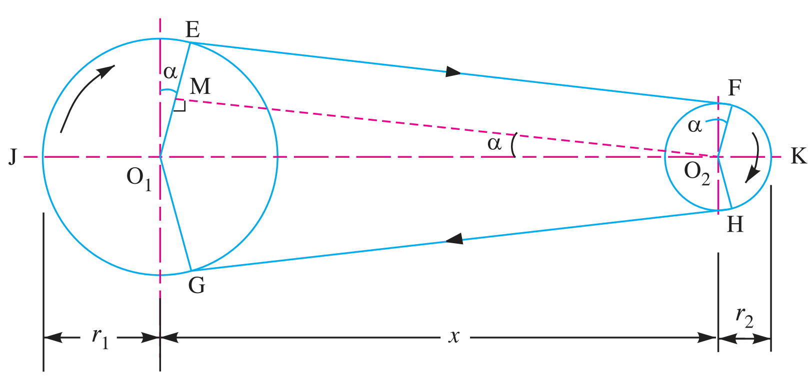


Fig. 18.13. Open belt drive.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* O_1O_2), and

L = Total length of the belt.

Let the belt leaves the larger pulley at *E* and *G* and the smaller pulley at *F* and *H* as shown in Fig. 18.13. Through O_2 draw O_2M parallel to *FE*.

From the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the angle $MO_2O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \qquad \text{...}(i)$$

From the geometry of the figure, we also find that

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x}$$

Since the angle α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x} \qquad \text{...}(ii)$$

$$\therefore \qquad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \qquad \text{...}(iii)$$

Similarly,

$$\text{arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right) \qquad \text{...}(iv)$$

and

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem, we have

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \ldots\right] = x - \frac{(r_1 - r_2)^2}{2x} \qquad \text{...}(v)$$

Substituting the values of arc *JE* from equation ***(iii)***, arc *FK* from equation ***(iv)*** and *EF* from equation ***(v)*** in equation ***(i)***, we get

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \frac{\pi}{2} - r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha\,(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi\,(r_1 + r_2) + 2\alpha\,(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

Substituting the value of $\alpha = \dfrac{(r_1 - r_2)}{x}$ from equation ***(ii)***, we get

$$L = \pi\,(r_1 + r_2) + 2 \times \frac{(r_1 - r_2)}{x}\,(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi\,(r_1 + r_2) + \frac{2\,(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

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$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$ ... (in terms of pulley radii)

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$ ... (in terms of pulley diameters)

18.17 Length of a Cross Belt Drive

We have discussed in Art. 18.12 that in a cross belt drive, both the pulleys rotate in the opposite directions as shown in Fig. 18.14.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* O_1O_2), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. 18.14.

Through O_2 draw O_2M parallel to FE.

From the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the angle $MO_2O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + FE + \text{Arc } FK) \qquad ...(i)$$

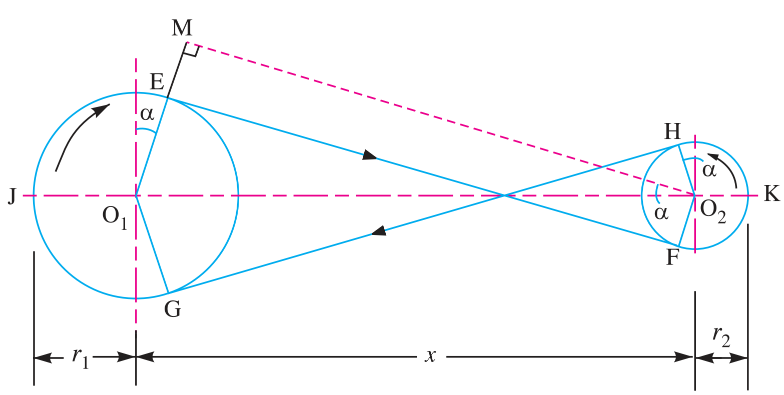

Fig. 18.14. Crossed belt drive.

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2} = \frac{r_1 + r_2}{x}$$

Since the angle α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 + r_2}{x} \qquad ...(ii)$$

$$\therefore \qquad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \qquad ...(iii)$$

Similarly, $$\text{arc } FK = r_2\left(\frac{\pi}{2} + \alpha\right) \qquad ...(iv)$$

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and
$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$
$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem, we have

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + ...\right] = x - \frac{(r_1 + r_2)^2}{2x} \quad ...(v)$$

In the above conveyor belt is used to transport material as well as to drive the rollers

Substituting the values of arc JE from equation ***(iii)***, arc FK from equation ***(iv)*** and EF from equation ***(v)*** in equation ***(i)***, we get,

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$
$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \times \frac{\pi}{2} + r_2.\alpha\right]$$
$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$
$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

Substituting the value of $\alpha = \dfrac{(r_1 + r_2)}{x}$ from equation ***(ii)***, we get

$$L = \pi(r_1 + r_2) + 2 \times \frac{(r_1 + r_2)}{x}(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$
$$= \pi(r_1 + r_2) + \frac{2(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

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$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \quad \text{... (in terms of pulley radii)}$$

$$= \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \quad \text{... (in terms of pulley diameters)}$$

It may be noted that the above expression is a function of $(r_1 + r_2)$. It is thus obvious, that if sum of the radii of the two pulleys be constant, length of the belt required will also remain constant, provided the distance between centres of the pulleys remain unchanged.

18.18 Power Transmitted by a Belt

Fig. 18.15 shows the driving pulley (or driver) *A* and the driven pulley (or follower) *B*. As already discussed, the driving pulley pulls the belt from one side and delivers it to the other side. It is thus obvious that the tension on the former side (*i.e.* tight side) will be greater than the latter side (*i.e.* slack side) as shown in Fig. 18.15.

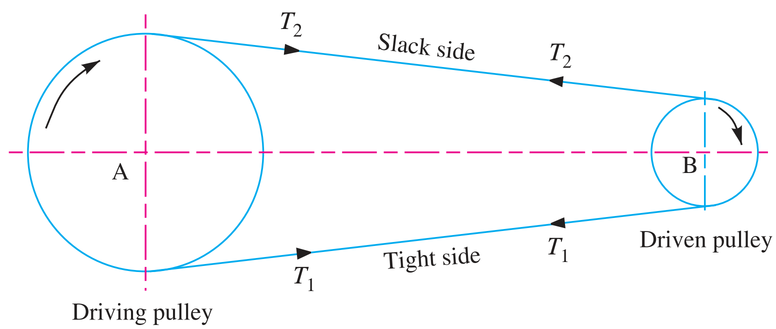

Fig. 18.15. Power transmitted by a belt.

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively in newtons,

r_1 and r_2 = Radii of the driving and driven pulleys respectively in metres,

and v = Velocity of the belt in m/s.

The effective turning (driving) force at the circumference of the driven pulley or follower is the difference between the two tensions (*i.e.* $T_1 - T_2$).

This massive shaft-like pulley drives the conveyor belt.

$\therefore$ Work done per second = $(T_1 - T_2)\, v$ N-m/s

and power transmitted = $(T_1 - T_2)\, v$ W ... ($\because$ 1 N-m/s = 1W)

A little consideration will show that torque exerted on the driving pulley is $(T_1 - T_2)\, r_1$. Similarly, the torque exerted on the driven pulley is $(T_1 - T_2)\, r_2$.

18.19 Ratio of Driving Tensions for Flat Belt Drive

Consider a driven pulley rotating in the clockwise direction as shown in Fig. 18.16.

Let T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtended by the arc *AB*, along which the belt touches the pulley, at the centre).

Now consider a small portion of the belt *PQ*, subtending an angle $\delta\theta$ at the centre of the pulley as shown in Fig. 18.16. The belt *PQ* is in equilibrium under the following forces:

1. Tension *T* in the belt at *P*,
2. Tension $(T + \delta T)$ in the belt at *Q*,
3. Normal reaction R_N, and
4. Frictional force $F = \mu \times R_N$, where μ is the coefficient of friction between the belt and pulley.

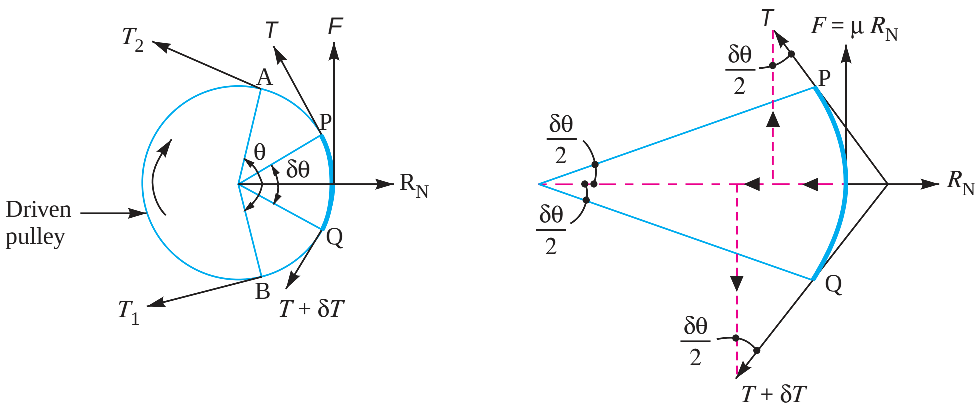

Fig. 18.16. Ratio of driving tensions for flat belt.

Resolving all the forces horizontally, we have

$$R_N = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \qquad \text{...}(i)$$

Since the angle $\delta\theta$ is very small, therefore putting $\sin \delta\theta/2 = \delta\theta/2$ in equation (*i*), we have

$$R_N = (T + \delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} = \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2}$$

$$= T.\delta\theta \qquad \text{...}\left(\text{Neglecting } \frac{\delta T.\delta\theta}{2}\right) \qquad \text{...}(ii)$$

Now resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \qquad \text{...}(iii)$$

Since the angle $\delta\theta$ is very small, therefore putting $\cos \delta\theta/2 = 1$ in equation (*iii*), we have

$$\mu \times R_N = T + \delta T - T = \delta T \quad \text{or} \quad R_N = \frac{\delta T}{\mu} \qquad \text{...}(iv)$$

Equating the values of R_N from equations ***(ii)*** and ***(iv)***, we get

$$T.\delta\theta = \frac{\delta T}{\mu} \quad \text{or} \quad \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating the above equation between the limits T_2 and T_1 and from 0 to θ, we have

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta$$

$$\therefore \qquad \log_e \left(\frac{T_1}{T_2}\right) = \mu.\theta \quad \text{or} \quad \frac{T_1}{T_2} = e^{\mu.\theta} \qquad ...(v)$$

The equation ***(v)*** can be expressed in terms of corresponding logarithm to the base 10, *i.e.*

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta$$

The above expression gives the relation between the tight side and slack side tensions, in terms of coefficient of friction and the angle of contact.

Notes : 1. While determining the angle of contact, it must be remembered that it is the angle of contact at the smaller pulley, if both the pulleys are of the same material. We know that

$$\sin\alpha = \frac{r_1 - r_2}{x} \qquad \text{... (for open belt drive)}$$

$$= \frac{r_1 + r_2}{x} \qquad \text{... (for cross-belt drive)}$$

∴ Angle of contact or lap,

$$\theta = (180° - 2\alpha)\frac{\pi}{180} \text{ rad} \qquad \text{...(for open belt drive)}$$

$$= (180° + 2\alpha)\frac{\pi}{180} \text{ rad} \qquad \text{... (for cross-belt drive)}$$

2. When the pulleys are made of different material (*i.e.* when the coefficient of friction of the pulleys or the angle of contact are different), then the design will refer to the pulley for which μ.θ is small.

Example 18.2. *Two pulleys, one 450 mm diameter and the other 200 mm diameter, on parallel shafts 1.95 m apart are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley.*

What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25?

Solution. Given : d_1 = 450 mm = 0.45 m or r_1 = 0.225 m ; d_2 = 200 mm = 0.2 m or r_2 = 0.1 m ; x = 1.95 m ; N_1 = 200 r.p.m. ; T_1 = 1 kN = 1000 N ; μ = 0.25

The arrangement of crossed belt drive is shown in Fig. 18.17.

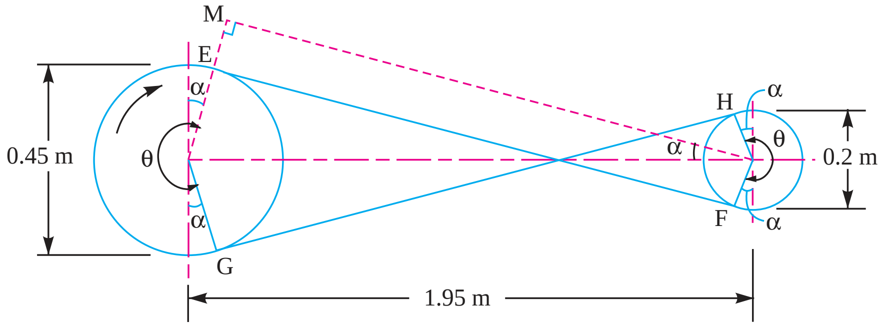

Fig. 18.17

Length of the belt

We know that length of the belt,

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95}$$

$$= 1.02 + 3.9 + 0.054 = 4.974 \text{ m}$$ **Ans.**

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667$$

$\therefore$ $\alpha = 9.6°$

and $\theta = 180° + 2\alpha = 180 + 2 \times 9.6 = 199.2°$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad}$$ **Ans.**

Power transmitted

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 3.477 = 0.8693$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8693}{2.3} = 0.378 \quad \text{or} \quad \frac{T_1}{T_2} = 2.387$$ ... (Taking antilog of 0.378)

$$\therefore \quad T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419 \text{ N}$$

We know that the velocity of belt,

$$v = \frac{\pi\, d_1 N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.713 \text{ m/s}$$

$\therefore$ Power transmitted,

$$P = (T_1 - T_2)\, v = (1000 - 419)\, 4.713 = 2738 \text{ W} = 2.738 \text{ kW}$$ **Ans.**

18.20 Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both the tight as well as the slack sides. The tension caused by centrifugal force is called ***centrifugal tension.*** At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

Consider a small portion *PQ* of the belt subtending an angle $d\theta$ at the centre of the pulley, as shown in Fig. 18.18.

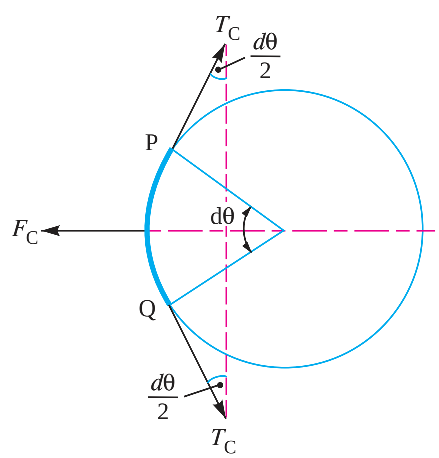

Fig. 18.18. Centrifugal tension.

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Let m = Mass of belt per unit length in kg,

v = Linear velocity of belt in m/s,

r = Radius of pulley over which the belt runs in metres, and

T_C = Centrifugal tension acting tangentially at P and Q in newtons.

We know that length of the belt PQ

$= r.d\theta$

and mass of the belt PQ $= m.r.d\theta$

$\therefore$ Centrifugal force acting on the belt PQ,

$$F_C = m.r.d\theta \times \frac{v^2}{r} = m.d\theta.v^2$$

Belt drive on a lathe

The centrifugal tension T_C acting tangentially at P and Q keeps the belt in equilibrium. Now resolving the forces (*i.e.* centrifugal force and centrifugal tension) horizontally, we have

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C = m.d\theta.v^2 \qquad ...(i)$$

Since the angle $d\theta$ is very small, therefore putting $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$ in equation (*i*), we have

$$2T_C\left(\frac{d\theta}{2}\right) = m.d\theta.v^2$$

$$\therefore \quad T_C = m.v^2$$

Notes : 1. When centrifugal tension is taken into account, then total tension in the tight side,

$$T_{t1} = T_1 + T_C$$

and total tension in the slack side,

$$T_{t2} = T_2 + T_C$$

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2. Power transmitted,

$$P = (T_{t1} - T_{t2})\ v \quad \text{...(in watts)}$$

$$= [(T_1 + T_C) - (T_2 + T_C)]v = (T_1 - T_2)\ v \quad \text{... (same as before)}$$

Thus we see that the centrifugal tension has no effect on the power transmitted.

3. The ratio of driving tensions may also be written as

$$2.3 \log \left(\frac{T_{t1} - T_C}{T_{t2} - T_C}\right) = \mu.\theta$$

where T_{t1} = Maximum or total tension in the belt.

18.21 Maximum Tension in the Belt

A little consideration will show that the maximum tension in the belt (T) is equal to the total tension in the tight side of the belt (T_{t1}).

Let σ = Maximum safe stress,

b = Width of the belt, and

t = Thickness of the belt.

We know that the maximum tension in the belt,

$$T = \text{Maximum stress} \times \text{Cross-sectional area of belt} = \sigma.b.t$$

When centrifugal tension is neglected, then

$$T \text{ (or } T_{t1}) = T_1, \textit{ i.e.} \text{ Tension in the tight side of the belt.}$$

When centrifugal tension is considered, then

$$T \text{ (or } T_{t1}) = T_1 + T_C$$

18.22 Condition for the Transmission of Maximum Power

We know that the power transmitted by a belt,

$$P = (T_1 - T_2)\ v \quad \text{...}(i)$$

where T_1 = Tension in the tight side in newtons,

T_2 = Tension in the slack side in newtons, and

v = Velocity of the belt in m/s.

From Art. 18.19, ratio of driving tensions is

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{or} \quad T_2 = \frac{T_1}{e^{\mu\theta}} \quad \text{...}(ii)$$

Substituting the value of T_2 in equation (*i*), we have

$$P = \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right) v = T_1 \left(1 - \frac{1}{e^{\mu\theta}}\right) v = T_1.v.C \quad \text{...}(iii)$$

where $C = \left(1 - \dfrac{1}{e^{\mu\theta}}\right)$

We know that

$$T_1 = T - T_C$$

where T = Maximum tension to which the belt can be subjected in newtons, and

T_C = Centrifugal tension in newtons.

Substituting the value of T_1 in equation (*iii*), we have

$$P = (T - T_C)\ v \times C$$

$$= (T - mv^2)\ v \times C = (T.v - m.v^3)\ C \quad \text{... (Substituting } T_C = m.v^2)$$

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For maximum power, differentiate the above expression with respect to v and equate to zero, *i.e.*

$$\frac{dP}{dv} = 0 \quad \text{or} \quad \frac{d}{dv}(T.v - m.v^3)\, C = 0$$

or $$T - 3\, m.v^2 = 0 \qquad \ldots(iv)$$

$$\therefore \quad T - 3\, T_C = 0 \quad \text{or} \quad T = 3T_C \qquad \ldots (\because m.v^2 = T_C)$$

It shows that when the power transmitted is maximum, 1/3rd of the maximum tension is absorbed as centrifugal tension.

Notes : 1. We know that $T_1 = T - T_C$ and for maximum power, $T_C = \frac{T}{3}$.

$$\therefore \quad T_1 = T - \frac{T}{3} = \frac{2T}{3}$$

2. From equation **(*iv*)**, we find that the velocity of the belt for maximum power,

$$v = \sqrt{\frac{T}{3m}}$$

Example 18.3. *A leather belt 9 mm × 250 mm is used to drive a cast iron pulley 900 mm in diameter at 336 r.p.m. If the active arc on the smaller pulley is 120° and the stress in tight side is 2 MPa, find the power capacity of the belt. The density of leather may be taken as 980 kg/m³, and the coefficient of friction of leather on cast iron is 0.35.*

Solution. Given: t = 9 mm = 0.009 m ; b = 250 mm = 0.25 m; d = 900 mm = 0.9 m ; N = 336 r.p.m ; $\theta = 120° = 120 \times \frac{\pi}{180} = 2.1$ rad ; σ = 2 MPa = 2 N/mm² ; ρ = 980 kg/m³ ; μ = 0.35

We know that the velocity of the belt,

$$v = \frac{\pi\, d.N}{60} = \frac{\pi \times 0.9 \times 336}{60} = 15.8 \text{ m/s}$$

and cross-sectional area of the belt,

$$a = b.t = 9 \times 250 = 2250 \text{ mm}^2$$

∴ Maximum or total tension in the tight side of the belt,

$$T = T_{t1} = \sigma.a = 2 \times 2250 = 4500 \text{ N}$$

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.t.l.\rho = 0.25 \times 0.009 \times 1 \times 980 \text{ kg/m}$$
$$= 2.2 \text{ kg/m}$$

∴ Centrifugal tension,

$$^*T_C = m.v^2 = 2.2\,(15.8)^2 = 550 \text{ N}$$

and tension in the tight side of the belt,

$$T_1 = T - T_C = 4500 - 550 = 3950 \text{ N}$$

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.35 \times 2.1 = 0.735$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.735}{2.3} = 0.3196 \quad \text{or} \quad \frac{T_1}{T_2} = 2.085 \qquad \ldots \text{(Taking antilog of 0.3196)}$$

* $T_C = m.v^2 = \frac{\text{kg}}{\text{m}} \times \frac{\text{m}^2}{\text{s}^2} = \text{kg-m/s}^2$ or N ...($\because$ 1 N = 1 kg-m/s²)

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and $$T_2 = \frac{T_1}{2.085} = \frac{3950}{2.085} = 1895 \text{ N}$$

We know that the power capacity of the belt,

$$P = (T_1 - T_2)\, v = (3950 - 1895)\, 15.8 = 32\,470 \text{ W} = 32.47 \text{ kW} \quad \textbf{Ans.}$$

Notes : The power capacity of the belt, when centrifugal tension is taken into account, may also be obtained as discussed below :

1. We know that the maximum tension in the tight side of the belt,

$$T_{t1} = T = 4500 \text{ N}$$

Centrifugal tension, $T_C = 550$ N

and tension in the slack side of the belt,

$$T_2 = 1895 \text{ N}$$

∴ Total tension in the slack side of the belt,

$$T_{t2} = T_2 + T_C = 1895 + 550 = 2445 \text{ N}$$

We know that the power capacity of the belt,

$$P = (T_{t1} - T_{t2})\, v = (4500 - 2445)\, 15.8 = 32\,470 \text{ W} = 32.47 \text{ kW} \quad \textbf{Ans.}$$

2. The value of total tension in the slack side of the belt (T_{t2}) may also be obtained by using the relation as discussed in Art. 18.20, *i.e.*

$$2.3 \log \left(\frac{T_{t1} - T_C}{T_{t2} - T_C} \right) = \mu.\theta$$

Example 18.4. *A flat belt is required to transmit 30 kW from a pulley of 1.5 m effective diameter running at 300 r.p.m. The angle of contact is spread over $\frac{11}{24}$ of the circumference. The coefficient of friction between the belt and pulley surface is 0.3. Determine, taking centrifugal tension into account, width of the belt required. It is given that the belt thickness is 9.5 mm, density of its material is 1100 kg / m^3 and the related permissible working stress is 2.5 MPa.*

Solution. Given : $P = 30$ kW $= 30 \times 10^3$ W ; $d = 1.5$ m ; $N = 300$ r.p.m. ; $\theta = \frac{11}{24} \times 360 = 165°$ $= 165 \times \pi / 180 = 2.88$ rad ; $\mu = 0.3$; $t = 9.5$ mm $= 0.0095$ m ; $\rho = 1100$ kg/m^3 ; $\sigma = 2.5$ MPa $= 2.5 \times 10^6$ N/m^2

Let T_1 = Tension in the tight side of the belt in newtons, and

T_2 = Tension in the slack side of the belt in newtons.

We know that the velocity of the belt,

$$v = \frac{\pi\, d\, N}{60} = \frac{\pi \times 1.5 \times 300}{60} = 23.57 \text{ m/s}$$

and power transmitted (P),

$$30 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 23.57$$

$$\therefore \quad T_1 - T_2 = 30 \times 10^3 / 23.57 = 1273 \text{ N} \qquad ...(i)$$

We know that

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu.\theta = 0.3 \times 2.88 = 0.864$$

$$\therefore \quad \log \left(\frac{T_1}{T_2} \right) = \frac{0.864}{2.3} = 0.3756 \quad \text{or} \quad \frac{T_1}{T_2} = 2.375 \qquad ...(ii)$$

... (Taking antilog of 0.3756)

From equations ***(i)*** and ***(ii)***, we find that

$$T_1 = 2199 \text{ N} ; \quad \text{and} \quad T_2 = 926 \text{ N}$$

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Let b = Width of the belt required in metres.

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$
$$= b \times 0.0095 \times 1 \times 1100 = 10.45\, b \text{ kg/m}$$

and centrifugal tension, $T_C = m.v^2 = 10.45\, b\, (23.57)^2 = 5805\, b$ N

We know that maximum tension in the belt,

$$T = T_1 + T_C = \text{Stress} \times \text{Area} = \sigma.b.t$$

or $2199 + 5805\, b = 2.5 \times 10^6 \times b \times 0.0095 = 23\,750\, b$

$\therefore$ $23\,750\, b - 5805\, b = 2199$ or $b = 0.122$ m or 122 mm

The standard width of the belt is 125 mm. **Ans.**

Example 18.5. *An electric motor drives an exhaust fan. Following data are provided :*

	Motor pulley	*Fan pulley*
Diameter	*400 mm*	*1600 mm*
Angle of warp	*2.5 radians*	*3.78 radians*
Coefficient of friction	*0.3*	*0.25*
Speed	*700 r.p.m.*	—
Power transmitted	*22.5 kW*	—

Calculate the width of 5 mm thick flat belt. Take permissible stress for the belt material as 2.3 MPa.

Solution. Given : $d_1 = 400$ mm or $r_1 = 200$ mm ; $d_2 = 1600$ mm or $r_2 = 800$ mm ; $\theta_1 = 2.5$ rad ; $\theta_2 = 3.78$ rad ; $\mu_1 = 0.3$; $\mu_2 = 0.25$; $N_1 = 700$ r.p.m. ; $P = 22.5$ kW $= 22.5 \times 10^3$ W ; $t = 5$ mm $= 0.005$ m ; $\sigma = 2.3$ MPa $= 2.3 \times 10^6$ N/m^2

Fig. 18.19 shows a system of flat belt drive. Suffix 1 refers to motor pulley and suffix 2 refers to fan pulley.

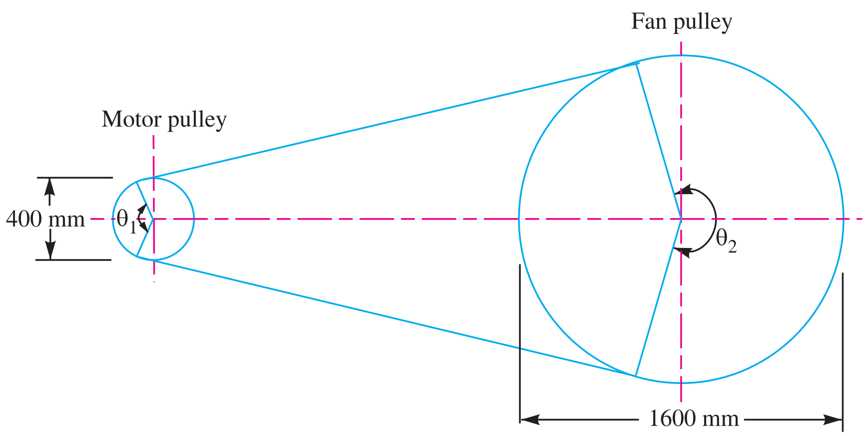

Fig. 18.19

We have discussed in Art. 18.19 (Note 2) that when the pulleys are made of different material [*i.e.* when the pulleys have different coefficient of friction (μ) or different angle of contact (θ), then the design will refer to a pulley for which μ.θ is small.

$\therefore$ For motor pulley, $\mu_1.\theta_1 = 0.3 \times 2.5 = 0.75$

and for fan pulley, $\mu_2.\theta_2 = 0.25 \times 3.78 = 0.945$

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Since $\mu_1.\theta_1$ for the motor pulley is small, therefore the design is based on the motor pulley.

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the velocity of the belt,

$$v = \frac{\pi\, d_1.N_1}{60} = \frac{\pi \times 0.4 \times 700}{60} = 14.7 \text{ m/s} \quad \text{... } (d_1 \text{ is taken in metres})$$

and the power transmitted (P),

$$22.5 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 14.7$$

$$\therefore \quad T_1 - T_2 = 22.5 \times 10^3 / 14.7 = 1530 \text{ N} \quad \text{...}(i)$$

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu_1.\theta_1 = 0.3 \times 2.5 = 0.75$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{0.75}{2.3} = 0.3261 \quad \text{or} \quad \frac{T_1}{T_2} = 2.12 \quad \text{...}(ii)$$

... (Taking antilog of 0.3261)

From equations (*i*) and (*ii*), we find that

T_1 = 2896 N ; and T_2 = 1366 N

Let b = Width of the belt in metres.

Since the velocity of the belt is more than 10 m/s, therefore centrifugal tension must be taken into consideration. Assuming a leather belt for which the density may be taken as 1000 kg / m^3.

∴ Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$
$$= b \times 0.005 \times 1 \times 1000 = 5\, b \text{ kg/m}$$

and centrifugal tension, $T_C = m.v^2 = 5\, b\, (14.7)^2 = 1080\, b$ N

We know that the maximum (or total) tension in the belt,

$$T = T_1 + T_C = \text{Stress} \times \text{Area} = \sigma.b.t$$

or $$2896 + 1080\, b = 2.3 \times 10^6\, b \times 0.005 = 11\,500\, b$$

$$\therefore \quad 11\,500\, b - 1080\, b = 2896 \quad \text{or} \quad b = 0.278 \text{ say } 0.28 \text{ m or } 280 \text{ mm}$$ **Ans.**

Example 18.6. *Design a rubber belt to drive a dynamo generating 20 kW at 2250 r.p.m. and fitted with a pulley 200 mm diameter. Assume dynamo efficiency to be 85%.*

Allowable stress for belt = *2.1 MPa*

Density of rubber = *1000 kg / m^3*

Angle of contact for dynamo pulley = *165°*

Coefficient of friction between belt and pulley = *0.3*

Solution. Given : P = 20 kW = 20 × 10^3 W ; N = 2250 r.p.m. ; d = 200 mm = 0.2 m ; η_d = 85% = 0.85 ; σ = 2.1 MPa = 2.1 × 10^6 N/m^2 ; ρ = 1000 kg/m^3 ; θ = 165° = 165 × π/180 = 2.88 rad ; μ = 0.3

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that velocity of the belt,

$$v = \frac{\pi\, d.N}{60} = \frac{\pi \times 0.2 \times 2250}{60} = 23.6 \text{ m/s}$$

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and power transmitted (P),

$$20 \times 10^3 = (T_1 - T_2)\, v.\eta_d$$
$$= (T_1 - T_2)\, 23.6 \times 0.85$$
$$= 20.1\,(T_1 - T_2)$$

$\therefore \quad T_1 - T_2 = 20 \times 10^3 / 20.1 = 995$ N ...*(i)*

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.88 = 0.864$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{0.864}{2.3} = 0.3756$$

or
$$\frac{T_1}{T_2} = 2.375 \qquad \text{... }(ii)$$

... (Taking antilog of 0.3756)

Rubber belt

From equations *(i)* and *(ii)*, we find that

T_1 = 1719 N ; and T_2 = 724 N

Let b = Width of the belt in metres, and

t = Thickness of the belt in metres.

Assuming thickness of the belt, t = 10 mm = 0.01 m, we have

Cross-sectional area of the belt

$$= b \times t = b \times 0.01 = 0.01\, b \text{ m}^2$$

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = 0.01\, b \times 1 \times 1000 = 10\, b \text{ kg / m}$$

$\therefore$ Centrifugal tension,

$$T_C = m.v^2 = 10\, b\,(23.6)^2 = 5570\, b \text{ N}$$

We know that maximum tension in the belt,

$$T = \sigma.b.t = 2.1 \times 10^6 \times b \times 0.01 = 21\,000\, b \text{ N}$$

and tension in the tight side of belt (T_1),

$$1719 = T - T_C = 21\,000\, b - 5570\, b = 15\,430\, b$$

$$\therefore \quad b = 1719 / 15\,430 = 0.1114 \text{ m} = 111.4 \text{ mm}$$

The standard width of the belt (b) is 112 mm. **Ans.**

Example 18.7. *Design a belt drive to transmit 110 kW for a system consisting of two pulleys of diameters 0.9 m and 1.2 m, centre distance of 3.6 m, a belt speed 20 m / s, coefficient of friction 0.3, a slip of 1.2% at each pulley and 5% friction loss at each shaft, 20% over load.*

Solution. Given : P = 110 kW = 110 × 10^3 W ; d_1 = 0.9 m or r_1 = 0.45 m ; d_2 = 1.2 m or r_2 = 0.6 m ; x = 3.6 m ; v = 20 m/s ; μ = 0.3 ; $s_1 = s_2$ = 1.2%

Fig 18.20 shows a system of flat belt drive consisting of two pulleys.

Let N_1 = Speed of the smaller or driving pulley in r.p.m., and

and N_2 = Speed of the larger or driven pulley in r.p.m.

We know that speed of the belt (v),

$$20 = \frac{\pi\, d_1.N_1}{60}\left(1 - \frac{s_1}{100}\right) = \frac{\pi \times 0.9\; N_1}{60}\left(1 - \frac{1.2}{100}\right) = 0.0466\; N_1$$

$$\therefore \quad N_1 = 20 / 0.0466 = 430 \text{ r.p.m.}$$

and peripheral velocity of the driven pulley,

$$\frac{\pi\, d_2 . N_2}{60} = \text{Belt speed in m/s}\left(1 - \frac{s_2}{100}\right) = v\left(1 - \frac{s_2}{100}\right)$$

or

$$\frac{\pi \times 1.2 \times N_2}{60} = 20\left(1 - \frac{1.2}{100}\right) = 19.76$$

$$\therefore \quad N_2 = \frac{19.76 \times 60}{\pi \times 1.2} = 315 \text{ r.p.m.}$$

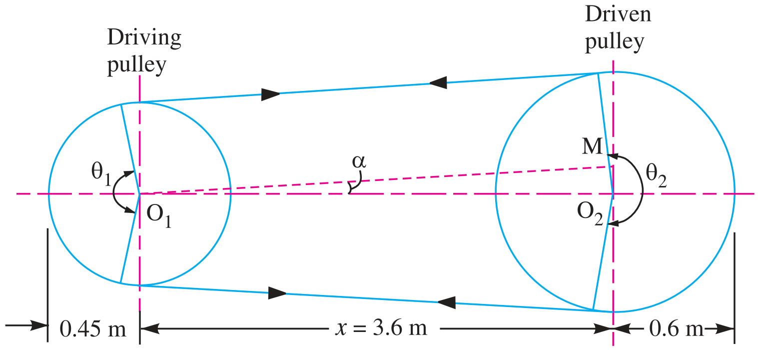

Fig. 18.20

We know that the torque acting on the driven shaft

$$= \frac{\text{Power transmitted} \times 60}{2\,\pi\, N_2} = \frac{110 \times 10^3 \times 60}{2\,\pi \times 315} = 3334 \text{ N-m}$$

Since there is a 5% friction loss at each shaft, therefore torque acting on the belt

$$= 1.05 \times 3334 = 3500 \text{ N-m}$$

Since the belt is to be designed for 20% overload, therefore design torque

$$= 1.2 \times 3500 = 4200 \text{ N-m}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the torque exerted on the driven pulley

$$= (T_1 - T_2)\, r_2 = (T_1 - T_2)\, 0.6 = 0.6\,(T_1 - T_2) \text{ N-m}$$

Equating this to the design torque, we have

$$0.6\,(T_1 - T_2) = 4200 \quad \text{or} \quad T_1 - T_2 = 4200 / 0.6 = 7000 \text{ N} \qquad ...(i)$$

Now let us find out the angle of contact (θ_1) of the belt on the smaller or driving pulley.

From the geometry of the Fig. 18.20, we find that

$$\sin \alpha = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{0.6 - 0.45}{3.6} = 0.0417 \quad \text{or} \quad \alpha = 2.4°$$

$$\therefore \quad \theta_1 = 180° - 2\alpha = 180 - 2 \times 2.4 = 175.2° = 175.2 \times \frac{\pi}{180} = 3.06 \text{ rad}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta_1 = 0.3 \times 3.06 = 0.918$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.918}{2.3} = 0.3991 \quad \text{or} \quad \frac{T_1}{T_2} = 2.51 \text{ ... (Taking antilog of 0.3991)} \qquad ...(ii)$$

From equations **(*i*)** and **(*ii*)**, we find that

$$T_1 = 11\ 636 \text{ N} \; ; \text{ and } T_2 = 4636 \text{ N}$$

Let σ = Safe stress for the belt = 2.5 MPa = 2.5×10^6 N/m^2 ...(Assume)

t = Thickness of the belt = 15 mm = 0.015 m, and ...(Assume)

b = Width of the belt in metres.

Since the belt speed is more than 10 m/s, therefore centrifugal tension must be taken into consideration. Assuming a leather belt for which the density may be taken as 1000 kg / m^3.

∴ Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$

$$= b \times 0.015 \times 1 \times 1000 = 15\,b \text{ kg/m}$$

and centrifugal tension,

$$T_C = m.v^2 = 15\,b\,(20)^2 = 6000\,b \text{ N}$$

We know that maximum tension in the belt,

$$T = T_1 + T_C = \sigma.b.t$$

or
$$11\ 636 + 6000\,b = 2.5 \times 10^6 \times b \times 0.015 = 37\ 500\,b$$

$$\therefore \quad 37\ 500\,b - 6000\,b = 11\ 636 \quad \text{or} \quad b = 0.37 \text{ m or } 370 \text{ mm}$$

The standard width of the belt (b) is 400 mm. **Ans.**

We know that length of the belt,

$$L = \pi\,(r_2 + r_1) + 2\,x + \frac{(r_2 - r_1)^2}{x}$$

$$= \pi\,(0.6 + 0.45) + 2 \times 3.6 + \frac{(0.6 - 0.45)^2}{3.6}$$

$$= 3.3 + 7.2 + 0.006 = 10.506 \text{ m}$$ **Ans.**

Example 18.8. *A belt 100 mm wide and 10 mm thick is transmitting power at 1000 metres/min. The net driving tension is 1.8 times the tension on the slack side. If the safe permissible stress on the belt section in 1.6 MPa, calculate the maximum power, that can be transmitted at this speed. Assume density of the leather as 1000 kg/m^3.*

Calculate the absolute maximum power that can be transmitted by this belt and the speed at which this can be transmitted.

Solution. Given : b = 100 mm = 0.1 m ; t = 10 mm = 0.01 m ; v = 1000 m/min = 16.67 m/s ; $T_1 - T_2 = 1.8\ T_2$; σ = 1.6 MPa = 1.6 N/mm^2 ; ρ = 1000 kg/m^3

Power transmitted

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the maximum tension in the belt,

$$T = \sigma.b.t = 1.6 \times 100 \times 10 = 1600 \text{ N}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$

$$= 0.1 \times 0.01 \times 1 \times 1000 = 1 \text{ kg/m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 1\,(16.67)^2 = 278 \text{ N}$$

We know that

$$T_1 = T - T_C = 1600 - 278 = 1322 \text{ N}$$

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and $$T_1 - T_2 = 1.8\, T_2 \qquad \text{...(Given)}$$

$$\therefore \qquad T_2 = \frac{T_1}{2.8} = \frac{1322}{2.8} = 472 \text{ N}$$

We know that the power transmitted.

$$P = (T_1 - T_2)\, v = (1322 - 472)\, 16.67 = 14\,170 \text{ W} = 14.17 \text{ kW} \quad \textbf{Ans.}$$

Speed at which absolute maximum power can be transmitted

We know that the speed of the belt for maximum power,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{1600}{3 \times 1}} = 23.1 \text{ m/s} \quad \textbf{Ans.}$$

Absolute maximum power

We know that for absolute maximum power, the centrifugal tension,

$$T_C = T/3 = 1600/3 = 533 \text{ N}$$

∴ Tension in the tight side,

$$T_1 = T - T_C = 1600 - 533 = 1067 \text{ N}$$

and tension in the slack side,

$$T_2 = \frac{T_1}{2.8} = \frac{1067}{2.8} = 381 \text{ N}$$

∴ Absolute maximum power transmitted,

$$P = (T_1 - T_2)\, v = (1067 - 381)\, 23.1 = 15\,850 \text{ W} = 15.85 \text{ kW} \quad \textbf{Ans.}$$

18.23 Initial Tension in the Belt

When a belt is wound round the two pulleys (*i.e.* driver and follower), its two ends are joined together, so that the belt may continuously move over the pulleys, since the motion of the belt (from the driver) and the follower (from the belt) is governed by a firm grip due to friction between the belt and the pulleys. In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tension, called ***initial tension***.

When the driver starts rotating, it pulls the belt from one side (increasing tension in the belt on this side) and delivers to the other side (decreasing tension in the belt on that side). The increased tension in one side of the belt is called tension in tight side and the decreased tension in the other side of the belt is called tension in the slack side.

Let T_0 = Initial tension in the belt,

T_1 = Tension in the tight side of the belt,

T_2 = Tension in the slack side of the belt, and

α = Coefficient of increase of the belt length per unit force.

A little consideration will show that the increase of tension in the tight side

$$= T_1 - T_0$$

and increase in the length of the belt on the tight side

$$= \alpha\,(T_1 - T_0) \qquad \text{...}(i)$$

Similarly, decrease in tension in the slack side

$$= T_0 - T_2$$

and decrease in the length of the belt on the slack side

$$= \alpha\,(T_0 - T_2) \qquad \text{...}(ii)$$

Assuming that the belt material is perfectly elastic such that the length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to decrease in the length on the slack side. Thus, equating equations **(*i*)** and **(*ii*)**, we have

$$\alpha\,(T_1 - T_0) = \alpha\,(T_0 - T_2)$$

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or $$T_1 - T_0 = T_0 - T_2$$

$\therefore$ $$T_0 = \frac{T_1 + T_2}{2} \quad \text{... (Neglecting centrifugal tension)}$$

$$= \frac{T_1 + T_2 + 2T_C}{2} \quad \text{... (Considering centrifugal tension)}$$

Note: In actual practice, the belt material is not perfectly elastic. Therefore, the sum of the tensions T_1 and T_2, when the belt is transmitting power, is always greater than twice the initial tension. According to C.G. Barth, the relation between T_0, T_1 and T_2 is given by

$$\sqrt{T_1} + \sqrt{T_2} = 2\sqrt{T_0}$$

Example 18.9. *Two parallel shafts whose centre lines are 4.8 m apart, are connected by an open belt drive. The diameter of the larger pulley is 1.5 m and that of smaller pulley 1 m. The initial tension in the belt when stationary is 3 kN. The mass of the belt is 1.5 kg / m length. The coefficient of friction between the belt and the pulley is 0.3. Taking centrifugal tension into account, calculate the power transmitted, when the smaller pulley rotates at 400 r.p.m.*

Solution. Given : x = 4.8 m ; d_1 = 1.5 m ; d_2 = 1 m ; T_0 = 3 kN = 3000 N ; m = 1.5 kg/m ; μ = 0.3 ; N_2 = 400 r.p.m.

We know that the velocity of the belt,

$$v = \frac{\pi\, d_2.N_2}{60} = \frac{\pi \times 1 \times 400}{60} = 21 \text{ m/s}$$

$\therefore$ Centrifugal tension,

$$T_C = m.v^2 = 1.5\,(21)^2 = 661.5 \text{ N}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the initial tension (T_0),

$$3000 = \frac{T_1 + T_2 + 2T_C}{2} = \frac{T_1 + T_2 + 2 \times 661.5}{2}$$

$\therefore$ $$T_1 + T_2 = 3000 \times 2 - 2 \times 661.5 = 4677 \text{ N} \quad \text{...}(i)$$

For an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2\,x} = \frac{1.5 - 1}{2 \times 4.8} = 0.0521 \text{ or } \alpha = 3°$$

$\therefore$ Angle of lap on the smaller pulley,

$$\theta = 180° - 2\alpha = 180 - 2 \times 3 = 174°$$

$$= 174 \times \frac{\pi}{180} = 3.04 \text{ rad}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.04 = 0.912$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.912}{2.3} = 0.3965 \text{ or } \frac{T_1}{T_2} = 2.5 \quad \text{... (Taking antilog of 0.3965) ...}(ii)$$

From equations ***(i)*** and ***(ii)***, we have

$$T_1 = 3341 \text{ N ; and } T_2 = 1336 \text{ N}$$

We know that the power transmitted,

$$P = (T_1 - T_2)\,v = (3341 - 1336)\,21 = 42\,100 \text{ W} = 42.1 \text{ kW}$$ **Ans.**

Example 18.10. *In a horizontal belt drive for a centrifugal blower, the blower is belt driven at 600 r.p.m. by a 15 kW, 1750 r.p.m. electric motor. The centre distance is twice the diameter of the larger pulley. The density of the belt material = 1500 kg/m^3; maximum allowable stress = 4 MPa; μ_1 = 0.5 (motor pulley); μ_2 = 0.4 (blower pulley); peripheral velocity of the belt = 20 m/s. Determine the following:*

1. Pulley diameters; 2. belt length; 3. cross-sectional area of the belt; 4. minimum initial tension for operation without slip; and 5. resultant force in the plane of the blower when operating with an initial tension 50 per cent greater than the minimum value.

Solution. Given : N_2 = 600 r.p.m. ; P = 15 kW = 15 × 10^3 W ; N_1 = 1750 r.p.m . ; ρ = 1500 kg/m^3 ; σ = 4 MPa = 4 × 10^6 N/m^2 ; μ_1 = 0.5 ; μ_2 = 0.4 ; v = 20 m/s

Fig. 18.21 shows a horizontal belt drive. Suffix 1 refers to a motor pulley and suffix 2 refers to a blower pulley.

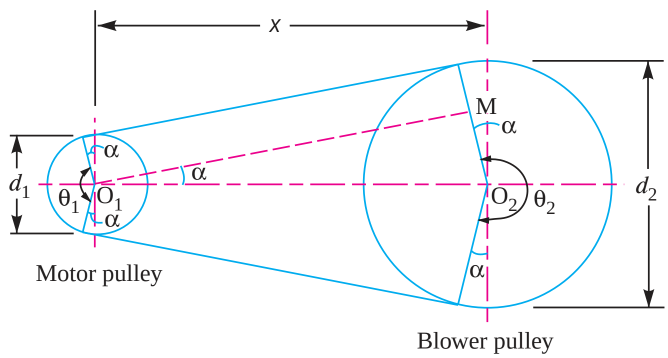

Fig. 18.21

1. *Pulley diameters*

Let d_1 = Diameter of the motor pulley, and

d_2 = Diameter of the blower pulley.

We know that peripheral velocity of the belt (v),

$$20 = \frac{\pi\, d_1 N_1}{60} = \frac{\pi d_1 \times 1750}{60} = 91.64\, d_1$$

$\therefore$ d_1 = 20 / 91.64 = 0.218 m = 218 mm **Ans.**

We also know that $\dfrac{N_2}{N_1} = \dfrac{d_1}{d_2}$

$$\therefore \quad d_2 = \frac{d_1 \times N_1}{N_2} = \frac{218 \times 1750}{600} = 636 \text{ mm} \quad \textbf{Ans.}$$

2. *Belt length*

Since the centre distance (x) between the two pulleys is twice the diameter of the larger pulley (*i.e.* 2 d_2), therefore centre distance,

$$x = 2\, d_2 = 2 \times 636 = 1272 \text{ mm}$$

We know that length of belt,

$$L = \frac{\pi}{2}(d_1 + d_2) + 2\,x + \frac{(d_1 - d_2)^2}{4x}$$

$$= \frac{\pi}{2}(218 + 636) + 2 \times 1272 + \frac{(218 - 636)^2}{4 \times 1272}$$

$$= 1342 + 2544 + 34 = 3920 \text{ mm} = 3.92 \text{ m} \quad \textbf{Ans.}$$

3. *Cross-sectional area of the belt*

Let a = Cross-sectional area of the belt.

First of all, let us find the angle of contact for both the pulleys. From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2\,x} = \frac{636 - 218}{2 \times 1272} = 0.1643$$

$\therefore$ $\alpha = 9.46°$

We know that angle of contact on the motor pulley,

$$\theta_1 = 180° - 2\alpha = 180 - 2 \times 9.46 = 161.08°$$
$$= 161.08 \times \pi / 180 = 2.8 \text{ rad}$$

and angle of contact on the blower pulley,

$$\theta_2 = 180° + 2\alpha = 180 + 2 \times 9.46 = 198.92°$$
$$= 198.92 \times \pi / 180 = 3.47 \text{ rad}$$

Since both the pulleys have different coefficient of friction (μ), therefore the design will refer to a pulley for which $\mu.\theta$ is small.

$\therefore$ For motor pulley,

$$\mu_1.\theta_1 = 0.5 \times 2.8 = 1.4$$

and for blower pulley, $\mu_2.\theta_2 = 0.4 \times 3.47 = 1.388$

Since $\mu_2.\theta_2$ for the blower pulley is less then $\mu_1.\theta_1$, therefore the design is based on the blower pulley.

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that power transmitted (P),

$$15 \times 10^3 = (T_1 - T_2)\,v = (T_1 - T_2)\,20$$

$\therefore$ $T_1 - T_2 = 15 \times 10^3 / 20 = 750 \text{ N}$ ...(*i*)

We also know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu_2.\theta_2 = 0.4 \times 3.47 = 1.388$$

$\therefore$ $\log\left(\frac{T_1}{T_2}\right) = \frac{1.388}{2.3} = 0.6035$ or $\frac{T_1}{T_2} = 4$ ...(*ii*)

... (Taking antilog of 0.6035)

From equations (*i*) and (*ii*),

$$T_1 = 1000 \text{ N} \; ; \text{ and } \; T_2 = 250 \text{ N}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = a \times l \times \rho$$
$$= a \times 1 \times 1500 = 1500\,a \text{ kg / m}$$

$\therefore$ Centrifugal tension,

$$T_C = m.v^2 = 1500\,a\,(20)^2 = 0.6 \times 10^6\,a \text{ N}$$

We know that maximum or total tension in the belt,

$T = T_1 + T_C = 1000 + 0.6 \times 10^6\,a$ N ...(*iii*)

We also know that maximum tension in the belt,

T = Stress × area = $\sigma \times a = 4 \times 10^6\,a$ N ...(*iv*)

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From equations ***(iii)*** and ***(iv)*,**

$$1000 + 0.6 \times 10^6 a = 4 \times 10^6\, a \quad \text{or} \quad 3.4 \times 10^6 a = 1000$$

$$\therefore \quad a = 1000 / 3.4 \times 10^6 = 294 \times 10^{-6}\ \text{m}^2 = 294\ \text{mm}^2 \quad \textbf{Ans.}$$

4. Minimum initial tension for operation without slip

We know that centrifugal tension,

$$T_C = 0.6 \times 10^6\, a = 0.6 \times 10^6 \times 294 \times 10^{-6} = 176.4\ \text{N}$$

∴ Minimum initial tension for operation without slip,

$$T_0 = \frac{T_1 + T_2 + 2T_C}{2} = \frac{1000 + 250 + 2 \times 176.4}{2} = 801.4\ \text{N} \quad \textbf{Ans.}$$

5. Resultant force in the plane of the blower when operating with an initial tension 50 per cent greater than the minimum value

We have calculated above that the minimum initial tension,

$$T_0 = 801.4\ \text{N}$$

∴ Increased initial tension,

$$T_0' = 801.4 + 801.4 \times \frac{50}{100} = 1202\ \text{N}$$

Let T_1' and T_2' be the corresponding tensions in the tight side and slack side of the belt respectively.

We know that increased initial tension (T_0'),

$$1202 = \frac{T_1' + T_2' + 2T_C}{2} = \frac{T_1' + T_2' + 2 \times 176.4}{2}$$

$$\therefore \quad T_1' + T_2' = 1202 \times 2 - 2 \times 176.4 = 2051.2\ \text{N} \qquad \text{...}(v)$$

Since the ratio of tensions will be constant, *i.e.* $\dfrac{T_1'}{T_2'} = \dfrac{T_1}{T_2} = 4$, therefore from equation ***(v)***, we have

$$4T_2' + T_2' = 2051.2 \quad \text{or} \quad T_2' = 2051.2 / 5 = 410.24\ \text{N}$$

and

$$T_1' = 4\,T_2' = 4 \times 410.24 = 1640.96\ \text{N}$$

∴ Resultant force in the plane of the blower

$$= T_1' - T_2' = 1640.96 - 410.24 = 1230.72\ \text{N} \quad \textbf{Ans.}$$

Example 18.11. *An open belt connects two flat pulleys. The pulley diameters are 300 mm and 450 mm and the corresponding angles of lap are 160° and 210°. The smaller pulley runs at 200 r.p.m. The coefficient of friction between the belt and pulley is 0.25. It is found that the belt is on the point of slipping when 3 kW is transmitted. To increase the power transmitted two alternatives are suggested, namely (i) increasing the initial tension by 10%, and (ii) increasing the coefficient of friction by 10% by the application of a suitable dressing to the belt.*

Which of these two methods would be more effective? Find the percentage increase in power possible in each case.

Solution. Given : $d_1 = 300$ mm = 0.3 m ; $d_2 = 450$ mm = 0.45 m ; $\theta_1 = 160° = 160 \times \dfrac{\pi}{180} = 2.8$ rad ; $\theta_2 = 210° = 210 \times \dfrac{\pi}{180} = 3.66$ rad ; $N_1 = 200$ r.p.m.; $\mu = 0.25$; $P = 3$ kW = 3000 W

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We have discussed in Art 18.19 (Note 2) that when the pulleys are made of different material [*i.e.* when the pulleys have different coefficient of friction (μ) or different angle of contact (θ)], then the design will be refer to a pulley for which μ.θ is small.

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∴ For smaller pulley, $\mu.\theta_1 = 0.25 \times 2.8 = 0.7$

and for larger pulley, $\mu.\theta_2 = 0.25 \times 3.66 = 0.915$

Since $\mu.\theta_1$ for the smaller pulley is less than $\mu.\theta_2$, therefore the design is based on the smaller pulley.

We know that velocity of the belt,

$$v = \frac{\pi\, d_1.N_1}{60} = \frac{\pi \times 0.3 \times 200}{60} = 3.142 \text{ m/s}$$

and power transmitted (P),

$$3000 = (T_1 - T_2)\, v = (T_1 - T_2)\, 3.142$$

$$\therefore \quad T_1 - T_2 = 3000 / 3.142 = 955 \text{ N} \quad \text{...}(i)$$

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta_1 = 0.25 \times 2.8 = 0.7$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{0.7}{2.3} = 0.3043 \quad \text{or} \quad \frac{T_1}{T_2} = 2.015 \quad \text{...}(ii)$$

... (Taking antilog of 0.3043)

From equations (*i*) and (*ii*), we find that

$$T_1 = 1896 \text{ N, and } T_2 = 941 \text{ N}$$

(i) Power transmitted when initial tension is increased by 10%

We know that the initial tension,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1896 + 941}{2} = 1418.5 \text{ N}$$

∴ Increased initial tension,

$$T_0' = 1418.5 + 1418.5 \times \frac{10}{100} = 1560.35 \text{ N}$$

Let T_1 and T_2 be the corresponding tensions in the tight side and slack side of the belt respectively.

$$\therefore \quad T_0' = \frac{T_1 + T_2}{2}$$

or

$$T_1 + T_2 = 2\, T_0' = 2 \times 1560.35 = 3120.7 \text{ N} \quad \text{...}(iii)$$

Since the ratio of the tensions is constant, *i.e.* $T_1 / T_2 = 2.015$ or $T_1 = 2.015\, T_2$, therefore from equation (*iii*),

$$2.015\, T_2 + T_2 = 3120.7 \quad \text{or} \quad T_2 = 3120.7 / 3.015 = 1035 \text{ N}$$

and

$$T_1 = 2.015\, T_2 = 2.015 \times 1035 = 2085.7 \text{ N}$$

∴ Power transmitted,

$$P = (T_1 - T_2)\, v = (2085.7 - 1035)\, 3.142 = 3300 \text{ W} = 3.3 \text{ kW}$$

(ii) Power transmitted when the coefficient of friction is increased by 10%

We know that the coefficient of friction,

$$\mu = 0.25$$

∴ Increased coefficient of friction,

$$\mu' = 0.25 + 0.25 \times \frac{10}{100} = 0.275$$

Let T_1 and T_2 be the corresponding tensions in the tight side and slack side of the belt respectively. We know that

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$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu'.\theta_1 = 0.275 \times 2.8 = 0.77$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{0.77}{2.3} = 0.3348 \quad \text{or} \quad \frac{T_1}{T_2} = 2.16 \quad \ldots(iv)$$

... (Taking antilog of 0.3348)

Here the initial tension is constant, *i.e*.

$$T_0 = \frac{T_1 + T_2}{2}$$

$$\therefore \quad T_1 + T_2 = 2\,T_0 = 2 \times 1418.5 = 2837 \text{ N} \quad \ldots(v)$$

From equations (*iv*) and (*v*), we find that

$$T_1 = 1939 \text{ N, and } T_2 = 898 \text{ N}$$

∴ Power transmitted,

$$P = (T_1 - T_2)\,v = (1939 - 898)\,3.142 = 3271 \text{ W} = 3.217 \text{ kW}$$

Since the power transmitted by increasing the initial tension is more, therefore in order to increase the power transmitted, we shall adopt the method of increasing the initial tension. **Ans.**

Percentage increase in power

Percentage increase in power when the initial tension is increased

$$= \frac{3.3 - 3}{3} \times 100 = 10\% \quad \textbf{Ans.}$$

Percentage increase in power when coefficient of friction is increased,

$$= \frac{3.271 - 3}{3} \times 100 = 9.03\% \quad \textbf{Ans.}$$

EXERCISES

1. An engine shaft running at 120 r.p.m. is required to drive a machine shaft by means of a belt. The pulley on the engine shaft is of 2 m diameter and that of the machine shaft is 1 m diameter. If the belt thickness is 5 mm; determine the speed of the machine shaft, when

1. there is no slip; and 2. there is a slip of 3%. **[Ans. 239.4 r.p.m. ; 232.3 r.p.m.]**

2. A pulley is driven by a flat belt running at a speed of 600 m/min. The coefficient of friction between the pulley and the belt is 0.3 and the angle of lap is 160°. If the maximum tension in the belt is 700 N; find the power transmitted by a belt. **[Ans. 3.974 kW]**

3. Find the width of the belt necessary to transmit 10 kW to a pulley 300 mm diameter, if the pulley makes 1600 r.p.m. and the coefficient of friction between the belt and the pulley is 0.22.

Assume the angle of contact as 210° and the maximum tension in the belt is not to exceed 8N/mm width. **[Ans. 90 mm]**

4. An open belt 100 mm wide connects two pulleys mounted on parallel shafts with their centres 2.4 m apart. The diameter of the larger pulley is 450 mm and that of the smaller pulley 300 mm. The coefficient of friction between the belt and the pulley is 0.3 and the maximum stress in the belt is limited to 14 N/mm width. If the larger pulley rotates at 120 r.p.m., find the maximum power that can be transmitted. **[Ans. 2.387 kW]**

5. A rough rule for leather belt is that effective tension in it, shall not exceed 15 N/mm of width for a belt of 10 mm thickness. This rule is applied to determine width of belt required to transmit 37 kW, under the following conditions :

Angle of lap = 165°; Coefficient of friction = 0.3; Velocity of belt = 1500 m/min; Density of leather = 950 kg/m^3.

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Find the width of belt required.

Assuming limiting friction between belt and pulley rim, find the stress in the belt.

[Ans. 140 mm ; 1.48 MPa]

6. A leather belt, 125 mm wide and 6 mm thick, transmits power from a pulley 750 mm diameter which runs at 500 r.p.m. The angle of lap is 150° and $\mu = 0.3$. If the mass of 1 m^3 of leather is 1 Mg and the stress in the belt is not to exceed 2.75 MN/m^2, find the maximum power that can be transmitted. **[Ans. 18.97 kW]**

7. An exhaust fan fitted with 900 mm diameter pulley is driven by a flat belt from a 30 kW, 950 r.p.m. squirrel cage motor. The pulley on the motor shaft is 250 mm in diameter and the centre distance between the fan and motor is 2.25 m. The belt is 100 mm wide with a coefficient of friction of 0.25. If the allowable stress in the belt material is not to exceed 2 MPa, determine the necessary thickness of the belt and its total length. Take centrifugal force effect into consideration for density of belt being 950 kg/m^3. **[Ans. 26 mm ; 6.35 m]**

8. A cross belt arrangement has centre distance between pulleys as 1.5 m. The diameter of bigger and smaller pulleys are '*D*' and '*d*' respectively. The smaller pulley rotates at 1000 r.p.m. and the bigger pulley at 500 r.p.m. The flat belt is 6 mm thick and transmits 7.5 kW power at belt speed of 13 m/s approximately. The coefficient of belt friction is 0.3 and the density of belt material is 950 kg/m^3. If the permissible tensile stress for the belt material is 1.75 MPa, calculate: 1. Diameters of pulleys; 2. Length and width of belt.

[Ans. 500 mm, 250 mm ; 4.272 m, 90 mm]

9. A blower is driven by an electric motor through a belt drive. The motor runs at 450 r.p.m. For this power transmission, a flat belt of 8 mm thickness and 250 mm width is used. The diameter of the motor pulley is 350 mm and that of the blower pulley is 1350 mm. The centre distance between these pulleys is 1850 mm and an open belt configuration is adopted. The pulleys are made of cast iron. The coefficient of friction between the belt and pulley is 0.35 and the permissible stress for the belt material can be taken as 2.5 N/mm^2. The mass of the belt is 2 kg/metre length. Find the maximum power transmitted without belt slipping in any one of the pulley. **[Ans. 38 kW]**

Blower driven by electric motor

10. A 18 kW, 900 r.p.m. motor drives a centrifugal pump at 290 r.p.m. by means of a leather belt. The pulleys are of cast iron and are 1.2 metre centre distance. The pulleys of diameter less than 150 mm should not be used. The coefficient of friction between the leather belt and the cast iron pulley is 0.35, and the mass of the belt is 9 kg/m width/m length. The maximum permissible tension per mm width of the belt is 10 N. The drive is to be designed for 20% overload.

Determine the pulley diameters, the required width and length of the belt. Also find the initial tension with which the belt is to be mounted on the pulleys. **[Ans. 460 mm ; 270 mm ; 3.4 m ; 2970 N]**

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11. A flat belt, 8 mm thick and 100 mm wide transmits power between two pulleys, running at 1600 m/min. The mass of the belt is 0.9 kg/m length. The angle of lap in the smaller pulley is 165° and the coefficient of friction between the belt and pulleys is 0.3. If the maximum permissible stress in the belt is 2 MN/m^2, find (*i*) Maximum power transmitted, and (*ii*) Initial tension in the belt.

[Ans. 14.821 kW; 1.322 kN]

12. Design a flat belt drive to transmit 110 kW at a belt speed of 25 m/s between two pulleys of diameters 250 mm and 400 mm having a pulley centre distance of 1 metre. The allowable belt stress is 8.5 MPa and the belts are available having a thickness to width ratio of 0.1 and a material density of 1100 kg/m^3. Given that the coefficient of friction between the belt and pulleys is 0.3, determine the minimum required belt width.

What would be the necessary installation force between the pulley bearings and what will be the force between the pulley bearings when the full power is transmitted?

13. A 8 mm thick leather open belt connects two flat pulleys. The smaller pulley is 300 mm diameter and runs at 200 r.p.m. The angle of lap of this pulley is 160° and the coefficient of friction between the belt and the pulley is 0.25. The belt is on the point of slipping when 3 kW is transmitted. The safe working stress in the belt material is 1.6 N/mm^2. Determine the required width of the belt for 20% overload capacity. The initial tension may be taken equal to the mean of the driving tensions. It is proposed to increase the power transmitting capacity of the drive by adopting one of the following alternatives :

1. by increasing initial tension by 10%, and
2. by increasing the coefficient of friction to 0.3 by applying a dressing to the belt.

Examine the two alternatives and recommend the one which will be more effective. How much power would the drive transmit adopting either of the two alternatives?

QUESTIONS

1. Discuss the different types of belts and their material used for power transmission.

2. Discuss the various important parameters necessary for the selection of a particular drive for power transmission.

3. What are the factors upon which the coefficient of friction between the belt and the pulley depends?

4. How are ends of belts joined? For horizontal belts which side (tight or slack) of the belt should run on the top and why?

5. Explain, with the help of neat sketches, the types of various flat belt drives.

6. List and discuss briefly the factors that control the power transmission capacity of a belt.

7. Prove that the ratio of the driving tensions on the two sides of a pulley is

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

where T_1 = Tension in the tight side of the belt,

T_2 = Tension in the slack side of the belt,

μ = Coefficient of friction between the belt and the pulley, and

θ = Angle of contact in radians.

8. In a belt drive, how will you decide the pulley governing design?

9. It is stated that the speed at which a belt should be run to transmit maximum power is that at which the maximum allowable tension is three times the centrifugal tension in the belt at that speed. Prove the statement.

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OBJECTIVE TYPE QUESTIONS

1. The material suitable for the belts used in agricultural equipments is
   (a) cotton (b) rubber
   (c) leather (d) balata gum
2. The power transmitted by means of a belt depends upon
   (a) velocity of the belt
   (b) tension under which the belt is placed on the pulleys
   (c) arc of contact between the belt and the smaller pulley
   (d) all of the above
3. When the speed of belt increases,
   (a) the coefficient of friction between the belt and pulley increases
   (b) the coefficient of friction between the belt and pulley decreases
   (c) the power transmitted will decrease
   (d) the power transmitted will increase
4. In a crossed belt drive, the shafts are arranged parallel and rotate in the .... directions.
   (a) same (b) opposite
5. The tension in the slack side of the belt is .... the tension in the tight side of the belt.
   (a) equal to (b) less than (c) greater than
6. In a flat belt drive, the belt can be subjected to a maximum tension (T) and centrifugal tension (T_C). The condition for transmission of maximum power is given by
   (a) $T = T_C$ (b) $T = 2\,T_C$
   (c) $T = 3\,T_C$ (d) $T = \sqrt{3}\,T_C$
7. When a belt drive is transmitting maximum power,
   (a) effective tension is equal to the centrifugal tension
   (b) effective tension is half of the centrifugal tension
   (c) driving tension in slack side is equal to the centrifugal tension
   (d) driving tension in tight side is twice the centrifugal tension
8. All stresses produced in a belt are
   (a) compressive stresses (b) tensile stresses
   (c) both tensile and compressive stresses (d) shear stresses
9. For maximum power, the velocity of the belt will be
   (a) $\sqrt{\dfrac{T}{m}}$ (b) $\sqrt{\dfrac{T}{2m}}$ (c) $\sqrt{\dfrac{T}{3m}}$
10. The centrifugal tension in the belt
   (a) increases the power transmitted (b) decreases the power transmitted
   (c) has no effect on the power transmitted (d) is equal to maximum tension on the belt

ANSWERS

1. (b)	2. (d)	3. (d)	4. (b)	5. (b)
6. (c)	7. (d)	8. (b)	9. (c)	10. (c)