Contents

CHAPTER

19

Flat Belt Pulleys

19.1 Introduction

The pulleys are used to transmit power from one shaft to another by means of flat belts, V-belts or ropes. Since the velocity ratio is the inverse ratio of the diameters of driving and driven pulleys, therefore the pulley diameters should be carefully selected in order to have a desired velocity ratio. The pulleys must be in perfect alignment in order to allow the belt to travel in a line normal to the pulley faces.

The pulleys may be made of cast iron, cast steel or pressed steel, wood and paper. The cast materials should have good friction and wear characteristics. The pulleys made of pressed steel are lighter than cast pulleys, but in many cases they have lower friction and may produce excessive wear.

Top

Contents

19.2 Types of Pulleys for Flat Belts

Following are the various types of pulleys for flat belts :

1. Cast iron pulleys, **2.** Steel pulleys, **3.** Wooden pulleys, **4.** Paper pulleys, and **5.** Fast and loose pulleys.

We shall now discuss, the above mentioned pulleys in the following pages.

19.3 Cast Iron Pulleys

The pulleys are generally made of *cast iron, because of their low cost. The rim is held in place by web from the central boss or by arms or spokes. The arms may be straight or curved as shown in Fig. 19.1 (*a*) and (*b*) and the cross-section is usually elliptical.

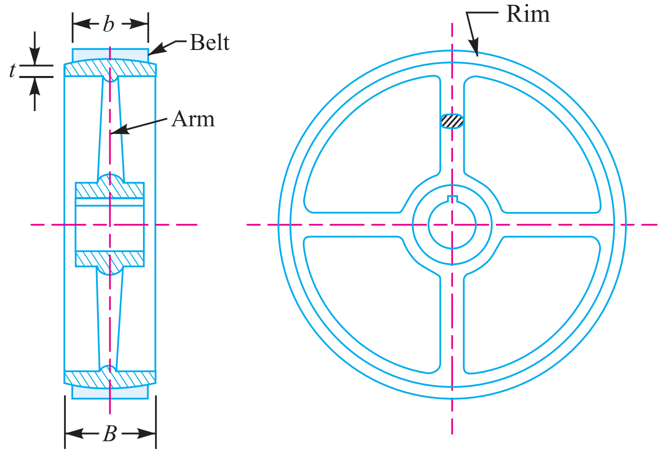


Fig. 19.1. Solid cast iron pulleys.

When a cast pulley contracts in the mould, the arms are in a state of stress and very liable to break. The curved arms tend to yield rather than to break. The arms are near the hub.

The cast iron pulleys are generally made with rounded rims. This slight convexity is known as ***crowning.*** The crowning tends to keep the belt in centre on a pulley rim while in motion. The crowning may be 9 mm for 300 mm width of pulley face.

The cast iron pulleys may be solid as shown in Fig. 19.1 or split type as shown in Fig. 19.2. When it is necessary to mount a pulley on a shaft which already carrying pulleys etc. or have its ends swelled, it is easier to use a split-pulley. There is a clearance between the faces and the two halves are readily tightened upon the shafts by the bolts as shown in Fig. 19.2. A sunk key is used for heavy drives.

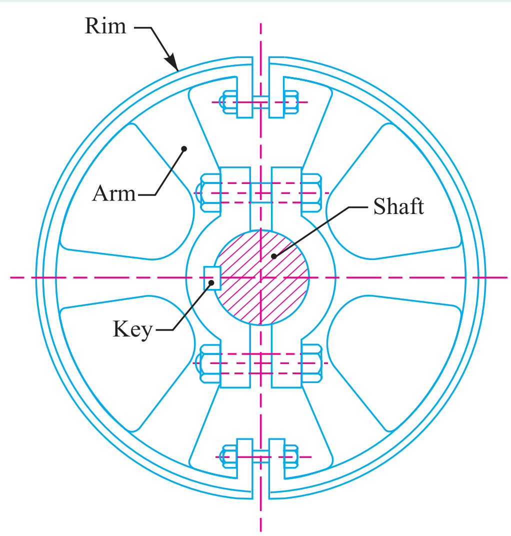


Fig. 19.2. Split cast iron pulley.

* For further details, please refer IS : 1691 – 1980 (Reaffirmed 1990).

Top

19.4 Steel Pulleys

Flat belt drive in an aircraft engine.

Steel pulleys are made from pressed steel sheets and have great strength and durability. These pulleys are lighter in weight (about 40 to 60% less) than cast iron pulleys of the same capacity and are designed to run at high speeds. They present a coefficient of friction with leather belting which is atleast equal to that obtained by cast iron pulleys.

Steel pulleys are generally made in two halves which are bolted together. The clamping action of the hub holds the pulley to its shaft, thus no key is required except for most severe service. Steel pulleys are generally equipped with interchangeable bushings to permit their use with shafts of different sizes. The following table shows the number of spokes and their sizes according to Indian Standards, IS : 1691 – 1980 (Reaffirmed 1990).

Table 19.1. Standard number of spokes and their sizes according to IS : 1691 – 1980 (Reaffirmed 1990).

Diameter of pulley (*mm*)	*No. of spokes*	*Diameter of spokes* (*mm*)
280 – 500	6	19
560 – 710	8	19
800 – 1000	10	22
1120	12	22
1250	14	22
1400	16	22
1600	18	22
1800	18	22

Other proportions for the steel pulleys are :

$$\text{Length of hub} = \frac{\text{Width of face}}{2}$$

The length of hub should not be less than 100 mm for 19 mm diameter spokes and 138 mm for 22 mm diameter of spokes.

Thickness of rim = 5 mm for all sizes.

A single row of spokes is used for pulleys having width upto 300 mm and double row of spokes for widths above 300 mm.

19.5 Wooden Pulleys

Wooden pulleys are lighter and possesses higher coefficient of friction than cast iron or steel pulleys. These pulleys have 2/3rd of the weight of cast iron pulleys of similar size. They are generally made from selected maple which is laid in segments and glued together under heavy pressure. They are kept from absorbing moisture by protective coatings of shellac or varnish so that warping may not occur. These pulleys are made both solid or split with cast iron hubs with keyways or have adjustable bushings which prevents relative rotation between them and the shaft by the frictional resistance set up. These pulleys are used for motor drives in which the contact arc between the pulley face and belt is restricted.

Wooden pulleys.

19.6 Paper Pulleys

Paper pulleys are made from compressed paper fibre and are formed with a metal in the centre. These pulleys are usually used for belt transmission from electric motors, when the centre to centre shaft distance is small.

19.7 Fast and Loose Pulleys

A fast and loose pulley, as shown in Fig. 19.3, used on shafts enables machine to be started or stopped at will. A fast pulley is keyed to the machine shaft while the loose pulley runs freely. The belt runs over the fast pulley to transmit power by the machine and it is shifted to the loose pulley when the machine is not required to transmit power. By this way, stopping of one machine does not interfere with the other machines which run by the same line shaft.

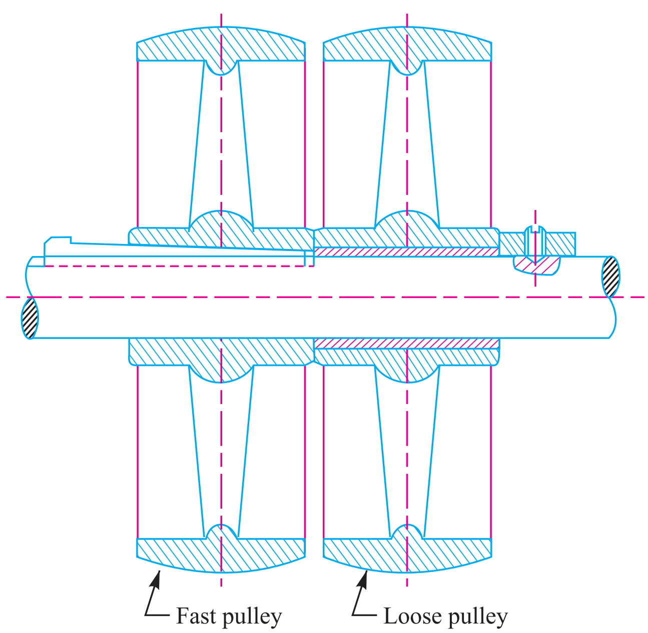


Fig. 19.3. Fast and loose pulley.

The loose pulley is provided with a cast iron or gun-metal bush with a collar at one end to prevent axial movement.

The rim of the fast pulley is made larger than the loose pulley so that the belt may run slackly on the loose pulley. The loose pulley usually have longer hub in order to reduce wear and friction and it requires proper lubrication.

19.8 Design of Cast Iron Pulleys

The following procedure may be adopted for the design of cast iron pulleys.

1. *Dimensions of pulley*

(i) The diameter of the pulley (D) may be obtained either from velocity ratio consideration or centrifugal stress consideration. We know that the centrifugal stress induced in the rim of the pulley,

$$\sigma_t = \rho . v^2$$

where ρ = Density of the rim material

= 7200 kg/m^3 for cast iron

v = Velocity of the rim = $\pi DN / 60$, D being the diameter of pulley and N is speed of the pulley.

The following are the diameter of pulleys in mm for flat and V-belts.

20, 22, 25, 28, 32, 36, 40, 45, 50, 56, 63, 71, 80, 90, 100, 112, 125, 140, 160, 180, 200, 224, 250, 280, 315, 355, 400, 450, 500, 560, 630, 710, 800, 900, 1000, 1120, 1250, 1400, 1600, 1800, 2000, 2240, 2500, 2800, 3150, 3550, 4000, 5000, 5400.

The first six sizes (20 to 36 mm) are used for V-belts only.

(ii) If the width of the belt is known, then width of the pulley or face of the pulley (B) is taken 25% greater than the width of belt.

$$\therefore \quad B = 1.25\, b \text{ ; where } b = \text{Width of belt.}$$

According to Indian Standards, IS : 2122 (Part I) – 1973 (Reaffirmed 1990), the width of pulley is fixed as given in the following table :

Table 19.2. Standard width of pulley.

Belt width in mm	*Width of pulley to be greater than belt width by (mm)*
upto 125	13
125-250	25
250-375	38
475-500	50

The following are the width of flat cast iron and mild steel pulleys in mm :

16, 20, 25, 32, 40, 50, 63, 71, 80, 90, 100, 112, 125, 140, 160, 180, 200, 224, 250, 315, 355, 400, 450, 560, 630.

(iii) The thickness of the pulley rim (t) varies from $\frac{D}{300} + 2$ mm to $\frac{D}{200} + 3$ mm for single belt and $\frac{D}{200} + 6$ mm for double belt. The diameter of the pulley (D) is in mm.

2. *Dimensions of arms*

(i) The number of arms may be taken as 4 for pulley diameter from 200 mm to 600 mm and 6 for diameter from 600 mm to 1500 mm.

Note : The pulleys less than 200 mm diameter are made with solid disc instead of arms. The thickness of the solid web is taken equal to the thickness of rim measured at the centre of the pulley face.

(ii) The cross-section of the arms is usually elliptical with major axis (a_1) equal to twice the minor axis (b_1). The cross-section of the arm is obtained by considering the arm as cantilever *i.e.* fixed at the hub end and carrying a concentrated load at the rim end. The length of the cantilever is taken equal to the radius of the pulley. It is further assumed that at any given time, the power is transmitted from the hub to the rim or ***vice versa,*** through only half the total number of arms.

Let T = Torque transmitted,

R = Radius of pulley, and

n = Number of arms,

∴ Tangential load per arm,

$$W_T = \frac{T}{R \times n / 2} = \frac{2\,T}{R \cdot n}$$

Maximum bending moment on the arm at the hub end,

$$M = \frac{2\,T}{R \times n} \times R = \frac{2\,T}{n}$$

and section modulus,

$$Z = \frac{\pi}{32} \times b_1\,(a_1)^2$$

Now using the relation,

σ_b or σ_t = M/Z, the cross-section of the arms is obtained.

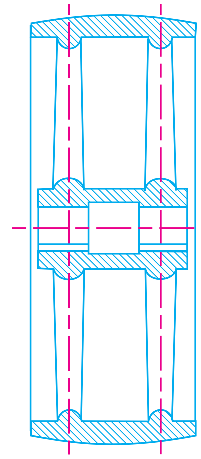


Fig. 19.4. Cast iron pulley with two rows of arms.

(iii) The arms are tapered from hub to rim. The taper is usually 1/48 to 1/32.

(iv) When the width of the pulley exceeds the diameter of the pulley, then two rows of arms are provided, as shown in Fig. 19.4. This is done to avoid heavy arms in one row.

3. *Dimensions of hub*

(i) The diameter of the hub (d_1) in terms of shaft diameter (d) may be fixed by the following relation :

$$d_1 = 1.5\,d + 25 \text{ mm}$$

The diameter of the hub should not be greater than 2 d.

(ii) The length of the hub,

$$L = \frac{\pi}{2} \times d$$

The minimum length of the hub is $\frac{2}{3}$ B but it should not be more than width of the pulley (B).

Example 19.1. *A cast iron pulley transmits 20 kW at 300 r.p.m. The diameter of pulley is 550 mm and has four straight arms of elliptical cross-section in which the major axis is twice the minor axis. Find the dimensions of the arm if the allowable bending stress is 15 MPa. Mention the plane in which the major axis of the arm should lie.*

Solution. Given : P = 20 kW = 20 × 10^3 W ; N = 300 r.p.m. ; *d = 550 mm ; n = 4 ; σ_b = 15 MPa = 15 N/mm^2

Let b_1 = Minor axis, and

a_1 = Major axis = $2b_1$ ...(Given)

We know that the torque transmitted by the pulley,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{20 \times 10^3 \times 60}{2\,\pi \times 300} = 636 \text{ N-m}$$

* Superfluous data.

∴ Maximum bending moment per arm at the hub end,

$$M = \frac{2T}{n} = \frac{2 \times 636}{4}$$

$$= 318 \text{ N-m} = 318 \times 10^3 \text{ N-mm}$$

and section modulus,

$$Z = \frac{\pi}{32} \times b_1 (a_1)^2 = \frac{\pi}{32} \times b_1 (2b_1)^2$$

$$= \frac{\pi (b_1)^3}{8}$$

Cast iron pulley.

We know that the bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{318 \times 10^3 \times 8}{\pi (b_1)^3} = \frac{810 \times 10^3}{(b_1)^3}$$

∴ $(b_1)^3 = 810 \times 10^3 / 15 = 54 \times 10^3$ or $b_1 = 37.8$ mm **Ans.**

and $a_1 = 2\, b_1 = 2 \times 37.8 = 75.6$ mm **Ans.**

The major axis will be in the plane of rotation which is also the plane of bending.

Example 19.2. *An overhung pulley transmits 35 kW at 240 r.p.m. The belt drive is vertical and the angle of wrap may be taken as 180°. The distance of the pulley centre line from the nearest bearing is 350 mm. µ = 0.25. Determine :*

1. *Diameter of the pulley ;*
2. *Width of the belt assuming thickness of 10 mm ;*
3. *Diameter of the shaft ;*
4. *Dimensions of the key for securing the pulley on to the shaft ; and*
5. *Size of the arms six in number.*

The section of the arm may be taken as elliptical, the major axis being twice the minor axis.

The following stresses may be taken for design purposes :

Shaft }	*Tension and compression*	—	*80 MPa*
Key }	*Shear*	—	*50 MPa*
Belt : Tension		—	*2.5 MPa*
Pulley rim : Tension		—	*4.5 MPa*
Pulley arms : Tension		—	*15 MPa*

Steel pulley.

Solution. Given : $P = 35$ kW $= 35 \times 10^3$ W ; $N = 240$ r.p.m. ; $\theta = 180° = \pi$ rad ; $L = 350$ mm $= 0.35$ m ; $\mu = 0.25$; $t = 10$ mm ; $n = 6$; $\sigma_{ts} = \sigma_{tk} = 80$ MPa $= 80$ N/mm^2 ; $\tau_s = \tau_k = 50$ MPa $= 50$ N/mm^2 ; $\sigma = 2.5$ MPa $= 2.5$ N/mm^2 ; $\sigma_t = 4.5$ MPa $= 4.5$ N/mm^2 ; $\sigma_b = 15$ MPa $= 15$ N/mm^2

1. Diameter of the pulley

Let D = Diameter of the pulley,

σ_t = Centrifugal stress or tensile stress in the pulley rim

$= 4.5$ MPa $= 4.5 \times 10^6$ N/m^2 ...(Given)

ρ = Density of the pulley material (*i.e.* cast iron) which may be taken as 7200 kg/m^3.

We know that centrifugal stress (σ_t),

$$4.5 \times 10^6 = \rho.v^2 = 7200 \times v^2$$

$$\therefore \quad v^2 = 4.5 \times 10^6 / 7200 = 625 \quad \text{or} \quad v = 25 \text{ m/s}$$

and velocity of the pulley (v),

$$25 = \frac{\pi\, D\,.\,N}{60} = \frac{\pi\, D \times 240}{60} = 12.568\, D$$

$$\therefore \quad D = 25 / 12.568 = 2 \text{ m } \textbf{Ans.}$$

2. Width of the belt

Let b = Width of the belt in mm,

T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the power transmitted (P),

$$35 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 25$$

$$\therefore \quad T_1 - T_2 = 35 \times 10^3 / 25 = 1400 \text{ N} \qquad \text{...}(i)$$

We also know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times \pi = 0.7855$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{0.7855}{2.3} = 0.3415 \quad \text{or} \quad \frac{T_1}{T_2} = 2.195 \qquad \text{...}(ii)$$

... (Taking antilog of 0.3415)

From equations (*i*) and (*ii*), we find that

$$T_1 = 2572 \text{ N ; and } T_2 = 1172 \text{ N}$$

Since the velocity of the belt (or pulley) is more than 10 m/s, therefore centrifugal tension must be taken into consideration. Assuming a leather belt for which the density may be taken as 1000 kg/m^3.

We know that cross-sectional area of the belt,

$$= b \times t = b \times 10 = 10\, b \text{ mm}^2 = \frac{10\, b}{10^6} \text{ m}^2$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density}$$

$$= \frac{10\, b}{10^6} \times 1 \times 1000 = 0.01\, b \text{ kg / m}$$

We know that centrifugal tension,

$$T_C = m.v^2 = 0.01\, b\, (25)^2 = 6.25\, b \text{ N}$$

and maximum tension in the belt,

$$T = \sigma.b.t = 2.5 \times b \times 10 = 25\, b \text{ N}$$

We know that tension in the tight side of the belt (T_1),

$$2572 = T - T_C = 25\, b - 6.25\, b = 18.75\, b$$

$$\therefore \quad b = 2572 / 18.75 = 137 \text{ mm}$$

The standard width of the belt (b) is 140 mm. **Ans.**

3. Diameter of the shaft

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\, \pi\, N} = \frac{35 \times 10^3 \times 60}{2\, \pi \times 240} = 1393 \text{ N-m} = 1393 \times 10^3 \text{ N-mn}$$

Top

Contents

and bending moment on the shaft due to the tensions of the belt,

$$M = (T_1 + T_2 + 2T_C)\,L = (2572 + 1172 + 2 \times 6.25 \times 140) \times 0.35 \text{ N-m}$$

$$= 1923 \text{ N-m} \qquad \ldots (\because T_C = 6.25\,b)$$

We know that equivalent twisting moment,

$$T_e = \sqrt{T^2 + M^2} = \sqrt{(1393)^2 + (1923)^2} = 2375 \text{ N-m}$$

$$= 2375 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting momnt (T_e),

$$2375 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 50 \times d^3 = 9.82\,d^3$$

$$\therefore \quad d^3 = 2375 \times 10^3 / 9.82 = 242 \times 10^3 \quad \text{or} \quad d = 62.3 \text{ say } 65 \text{ mm}$$ **Ans.**

4. Dimensions of the key

The standard dimensions of the key for 65 mm diameter shaft are :

Width of key, w = 20 mm **Ans.**

Thickness of key = 12 mm **Ans.**

Let l = Length of the key.

Considering shearing of the key. We know that the torque transmitted (T),

$$1393 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = l \times 20 \times 50 \times \frac{65}{2} = 32\,500\,l$$

$$\therefore \quad l = 1393 \times 10^3 / 32\,500 = 42.8 \text{ mm}$$

The length of key should be atleast equal to hub length. The length of hub is taken as $\frac{\pi}{2} \times d$.

$$\therefore \quad \text{Length of key} = \frac{\pi}{2} \times 65 = 102 \text{ mm}$$ **Ans.**

5. Size of arms

Let b_1 = Minor axis, and

a = Major axis = $2b_1$ ... (Given)

We know that the maximum bending moment per arm at the hub end,

$$M = \frac{2\,T}{n} = \frac{2 \times 1393}{6} = 464.33 \text{ N-m} = 464\,330 \text{ N-mm}$$

and section modulus,

$$Z = \frac{\pi}{32} \times b_1\,(a_1)^2 = \frac{\pi}{32} \times b_1\,(2b_1)^2 = 0.393\,(b_1)^3$$

We know that bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{464\,330}{0.393 \times (b_1)^3} = \frac{1.18 \times 10^6}{(b_1)^3}$$

$$\therefore \quad (b_1)^3 = 1.18 \times 10^6 / 15 = 78.7 \times 10^3 \quad \text{or} \quad b_1 = 42.8 \text{ say } 45 \text{ mm}$$ **Ans.**

and $a_1 = 2b_1 = 2 \times 45 = 90$ mm **Ans.**

Example 19.3. *A pulley of 0.9 m diameter revolving at 200 r.p.m. is to transmit 7.5 kW. Find the width of a leather belt if the maximum tension is not to exceed 145 N in 10 mm width. The tension in the tight side is twice that in the slack side. Determine the diameter of the shaft and the dimensions of the various parts of the pulley, assuming it to have six arms. Maximum shear stress is not to exceed 63 MPa.*

Solution. Given : D = 0.9 m ; N = 200 r.p.m. ; P = 7.5 kW = 7500 W ; T = 145 N in 10 mm width ; $T_1 = 2T_2$; n = 6 ; τ = 63 MPa = 63 N/mm^2

Top

Contents

We know that velocity of the pulley or belt,

$$v = \frac{\pi\, D.N}{60} = \frac{\pi \times 0.9 \times 200}{60} = 9.426 \text{ m/s}$$

Let T_1 = Tension in the tight of the belt, and

T_2 = Tension in the slack side of the belt.

We know that the power transmitted (P),

$$7500 = (T_1 - T_2)\, v = (T_1 - T_2)\, 9.426$$

$$T_1 - T_2 = 7500 / 9.426 = 796 \text{ N}$$

or $$2T_2 - T_2 = 796 \text{ N} \qquad \ldots (\because T_1 = 2T_2)$$

$\therefore$ $$T_2 = 796 \text{ N};$$

and $$T_1 = 2T_2 = 2 \times 796 = 1592 \text{ N}$$

Note : Since the velocity of belt is less than 10 m/s, therefore the centrifugal tension need not to be considered.

Width of belt

Let b = Width of belt.

Since the maximum tension is 145 N in 10 mm width or 14.5 N/mm width, therefore width of belt,

$$b = T_1 / 14.5 = 1592 / 14.5 = 109.8 \text{ mm}$$

The standard width of the belt (b) is 112 mm. **Ans.**

Diameter of the shaft

Let d = Diameter of the shaft,

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi\, N} = \frac{7500 \times 60}{2\,\pi \times 200} = 358 \text{ N-m} = 358\,000 \text{ N-mm}$$

We also know the torque transmitted by the shaft (T),

$$358\,000 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 63 \times d^3 = 12.4\, d^3$$

$\therefore$ $$d^3 = 358\,000 / 12.4 = 28\,871 \quad \text{or} \quad d = 30.67 \text{ say } 35 \text{ mm}$$ **Ans.**

Dimensions of the various parts of the pulley

1. Width and thickness of pulley

Since the width of the belt is 112 mm, therefore width of the pulley,

$$B = 112 + 13 = 125 \text{ mm}$$ **Ans.**

and thickness of the pulley rim for single belt,

$$t = \frac{D}{300} + 2 \text{ mm} = \frac{900}{300} + 2 = 5 \text{ mm}$$ **Ans.**

Top

2. Dimensions of arm

Assuming the cross-section of the arms as elliptical with major axis equal to twice the minor axis.

Let b_1 = Minor axis, and

a_1 = Major axis = $2b_1$

We know that maximum bending moment on the arm at the hub end,

$$M = \frac{2T}{n} = \frac{2 \times 358\,000}{6} = 119\,333 \text{ N-mm}$$

and section modulus,

$$Z = \frac{\pi}{32} \times b_1 (a_1)^2 = \frac{\pi}{32} \times b_1 (2b_1)^2 = 0.393 (b_1)^3$$

Assume the arms of cast iron for which the tensile stress may be taken as 15 N/mm². We know that the tensile stress (σ_t),

$$15 = \frac{M}{Z} = \frac{119\,333}{0.393 \times (b_1)^3} = \frac{303\,646}{(b_1)^3}$$

$\therefore$ $(b_1)^3 = 303\,646 / 15 = 20\,243$ or $b_1 = 27.3$ say 30 mm **Ans.**

and $a_1 = 2\,b_1 = 2 \times 30 = 60$ mm **Ans.**

Dimensions of the hub

Diameter of the hub $= 2\,d = 2 \times 35 = 70$ mm **Ans.**

$$\text{Length of the hub} = \frac{\pi}{2} \times d = \frac{\pi}{2} \times 35 = 55 \text{ mm}$$

Since the length of the hub should not be less than $\frac{2}{3}$ B, therefore the length of hub

$$= \frac{2}{3} \times B = \frac{2}{3} \times 125 = 83.3 \text{ say } 85 \text{ mm } \textbf{Ans.}$$

EXERCISES

1. Design the elliptical cross-section of a belt pulley arm near the hub for the following specifications:

   The mean pulley diameter is 300 mm and the number of pulley arms are 4. The elliptical section has major axis twice the minor axis length. The tight and slack sides tension in the belt are 600 N and 200 N respectively. Assume half number of arms transmit torque at any time and the load factor of 1.75 to account for dynamic effects on the pulley while transmitting torque. The permissible tensile stress for cast iron pulley material is 15 MPa. The pulley hub diameter is 60 mm.

   [Ans. a_1 = 40 mm, b_1 = 20 mm]

2. Design a cast iron driven pulley to transmit 20 kW at 300 r.p.m. The diameter of the pulley is 500 mm and the angle of lap is 180°. The pulley has four arms of elliptical cross-section with major axis twice the minor axis. The coefficient of friction between the belt and the pulley surface is 0.3. The allowable tension per metre width of the belt is 2.5 N. The following allowable stresses may be taken :

   Shear stress for the shaft material = 50 MPa, and

   Bending stress for the pulley arms = 15 MPa.

3. An overhung cast iron pulley transmits 7.5 kW at 400 r.p.m. The belt drive is vertical and the angle of wrap may be taken as 180º. Find :

   (*a*) Diameter of the pulley. The density of cast iron is 7200 kg/m³.

   (b) Width of the belt, if the coefficient of friction between the belt and the pulley is 0.25.

   (*c*) Diameter of the shaft, if the distance of the pulley centre line from the nearest bearing is 300 mm.

Contents

(*d*) Dimensions of the key for securing the pulley on to the shaft.

(*e*) Size of the arms six in number.

The section of the arms may be taken as elliptical, the major axis being twice the minor axis. The following stresses may be taken for design purposes :

Shaft and key :	Tension – 80 MPa
	Shear – 50 MPa
Belt :	Tension – 2.5 MPa
Pulley rim :	Tension – 4.5 MPa
Pulley arms :	Tension – 15 MPa

QUESTIONS

1. Discuss the different types of pulleys used in belt drives.
2. Why the face of a pulley is crowned?
3. When a split pulley is used and how it is tightened on a shaft?
4. Explain the 'fast and loose pulley' with the help of a neat sketch.
5. Discuss the procedure used in designing a cast iron pulley.

OBJECTIVE TYPE QUESTIONS

1. The crowning on a 300 mm width of pulley face should be
   (*a*) 9 mm (*b*) 12 mm
   (*c*) 15 mm (*d*) 18 mm
2. The steel pulleys are .... in weight than cast iron pulleys of the same capacity.
   (*a*) heavier (*b*) lighter
3. For a steel pulley of 500 mm, the recommended number of spokes are
   (*a*) 2 (*b*) 4
   (*c*) 6 (*d*) 8
4. The thickness of rim for all sizes of steel pulleys should be
   (*a*) 5 mm (*b*) 10 mm
   (*c*) 15 mm (*d*) 20 mm
5. The width of the pulley should be
   (*a*) equal to the width of belt (*b*) less than the width of belt
   (*c*) greater than the width of belt

ANSWERS

1. (*a*) **2.** (*b*) **3.** (*c*) **4.** (*a*) **5.** (*c*)

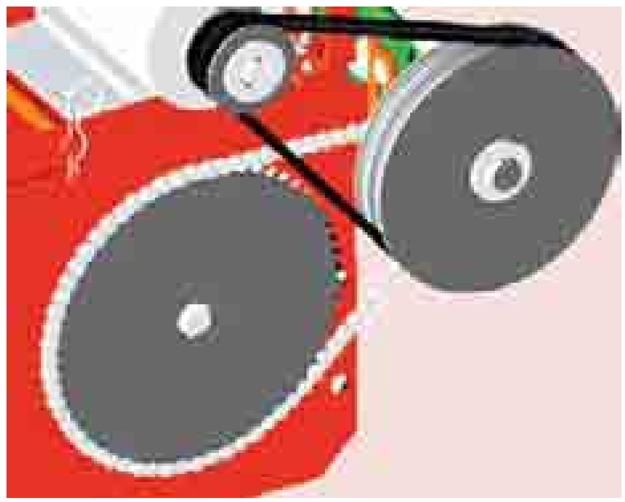
Top

Contents

CHAPTER

20

V-Belt and Rope Drives



20.1 Introduction

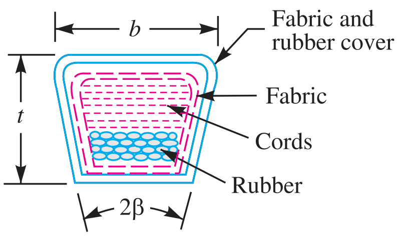
We have already discussed that a *V*-belt is mostly used in factories and workshops where a great amount of power is to be transmitted from one pulley to another when the two pulleys are very near to each other.

The *V*-belts are made of fabric and cords moulded in rubber and covered with fabric and rubber as shown in Fig. 20.1 (*a*). These belts are moulded to a trapezoidal shape and are made endless. These are particularly suitable for short drives. The included angle for the *V*-belt is usually from 30° to 40°. The power is transmitted by the *wedging

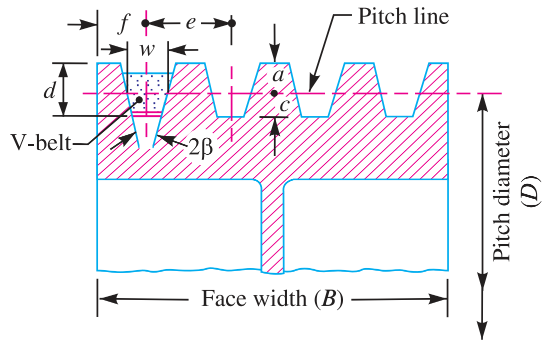
* The wedging action of the *V*-belt in the groove of the pulley results in higher forces of friction. A little consideration will show that the wedging action and the transmitted torque will be more if the groove angle of the pulley is small. But a small groove angle will require more force to pull the belt out of the groove which will result in loss of power and excessive belt wear due to friction and heat. Hence the selected groove angle is a compromise between the two. Usually the groove angles of 32° to 38° are used.

Top

action between the belt and the *V*-groove in the pulley or sheave. A clearance must be provided at the bottom of the groove as shown in Fig. 20.1 (*b*), in order to prevent touching of the bottom as it becomes narrower from wear. The *V*-belt drive may be inclined at any angle with tight side either at top or bottom. In order to increase the power output, several *V*-belts may be operated side by side. It may be noted that in multiple *V*-belt drive, all the belts should stretch at the same rate so that the load is equally divided between them. When one of the set of belts break, the entire set should be replaced at the same time. If only one belt is replaced, the new unworn and unstretched belt will be more tightly stretched and will move with different velocity.



(*a*) Cross-section of a V-belt.



(*b*) Cross-section of a V-grooved pulley.

Fig. 20.1. V-Belt and V-grooved pulley.

20.2 Types of V-belts and Pulleys

According to Indian Standards (IS: 2494 – 1974), the *V*-belts are made in five types *i.e. A*, *B*, *C*, *D* and *E*. The dimensions for standard *V*-belts are shown in Table 20.1. The pulleys for *V*-belts may be made of cast iron or pressed steel in order to reduce weight. The dimensions for the standard *V*-grooved pulley according to IS: 2494 – 1974, are shown in Table 20.2.

Table 20.1. Dimensions of standard V-belts according to IS: 2494 – 1974.

Type of belt	*Power ranges in kW*	*Minimum pitch diameter of pulley (D) mm*	*Top width (b) mm*	*Thickness (t) mm*	*Weight per metre length in newton*
A	0.7 – 3.5	75	13	8	1.06
B	2 – 15	125	17	11	1.89
C	7.5 – 75	200	22	14	3.43
D	20 – 150	355	32	19	5.96
E	30 – 350	500	38	23	–

Table 20.2. Dimensions of standard V-grooved pulleys according to IS : 2494–1974. (All dimensions in mm)

Type of belt	*w*	*d*	*a*	*c*	*f*	*e*	*No. of sheave grooves (n)*	*Groove angle (2β) in degrees*
A	11	12	3.3	8.7	10	15	6	32, 34, 38
B	14	15	4.2	10.8	12.5	19	9	32, 34, 38
C	19	20	5.7	14.3	17	25.5	14	34, 36, 38
D	27	28	8.1	19.9	24	37	14	34, 36, 38
E	32	33	9.6	23.4	29	44.5	20	–

Note : Face width $(B) = (n - 1)\, e + 2\, f$

20.3 Standard Pitch Lengths of V-belts

According to IS: 2494-1974, the V-belts are designated by its type and nominal inside length. For example, a *V*-belt of type *A* and inside length 914 mm is designated as *A* 914–IS: 2494. The standard inside lengths of *V*-belts in mm are as follows :

610, 660, 711, 787, 813, 889, 914, 965, 991, 1016, 1067, 1092, 1168, 1219, 1295, 1372, 1397, 1422, 1473, 1524, 1600, 1626, 1651, 1727, 1778, 1905, 1981, 2032, 2057, 2159, 2286, 2438, 2464, 2540, 2667, 2845, 3048, 3150, 3251, 3404, 3658, 4013, 4115, 4394, 4572, 4953, 5334, 6045, 6807, 7569, 8331, 9093, 9885, 10 617, 12 141, 13 665, 15 189, 16 713

Material handler.

According to IS: 2494-1974, the pitch length is defined as the circumferential length of the belt at the pitch width (*i.e.* the width at the neutral axis) of the belt. The value of the pitch width remains constant for each type of belt irrespective of the groove angle.

The pitch lengths are obtained by adding to inside length: 36 mm for type *A*, 43 mm for type *B*, 56 mm for type *C*, 79 mm for type *D* and 92 mm for type *E*. The following table shows the standard pitch lengths for the various types of belt.

Table 20.3. Standard pitch lengths of V-belts according to IS: 2494-1974.

Type of belt	*Standard pitch lengths of V-belts in mm*
A	645, 696, 747, 823, 848, 925, 950, 1001, 1026, 1051, 1102 1128, 1204, 1255, 1331, 1433, 1458, 1509, 1560, 1636, 1661, 1687, 1763, 1814, 1941, 2017, 2068, 2093, 2195, 2322, 2474, 2703, 2880, 3084, 3287, 3693.
B	932, 1008, 1059, 1110, 1212, 1262, 1339, 1415, 1440, 1466, 1567, 1694, 1770, 1821, 1948, 2024, 2101, 2202, 2329, 2507, 2583, 2710, 2888, 3091, 3294, 3701, 4056, 4158, 4437, 4615, 4996, 5377.
C	1275, 1351, 1453, 1580, 1681, 1783, 1834, 1961, 2088, 2113, 2215, 2342, 2494, 2723, 2901, 3104, 3205, 3307, 3459, 3713, 4069, 4171, 4450, 4628, 5009, 5390, 6101, 6863, 7625, 8387, 9149.
D	3127, 3330, 3736, 4092, 4194, 4473, 4651, 5032, 5413, 6124, 6886, 7648, 8410, 9172, 9934, 10 696, 12 220, 13 744, 15 268, 16 792.
E	5426, 6137, 6899, 7661, 8423, 9185, 9947, 10 709, 12 233, 13 757, 15 283, 16 805.

Note: The *V*-belts are also manufactured in non-standard pitch lengths (*i.e.* in oversize and undersize). The standard pitch length belt is designated by grade number 50. The oversize belts are designated by a grade

Contents

number more than 50, while the undersize belts are designated by a grade number less than 50. It may be noted that one unit of a grade number represents 2.5 mm in length from nominal pitch length. For example, a *V*-belt marked *A* – 914 – 50 denotes a standard belt of inside length 914 mm and a pitch length 950 mm. A belt marked *A* – 914 – 52 denotes an oversize belt by an amount of (52 – 50) = 2 units of grade number. Since one unit of grade number represents 2.5 mm, therefore the pitch length of this belt will be 950 + 2 × 2.5 = 955 mm. Similarly, a belt marked *A* – 914 – 48 denotes an undersize belt, whose pitch length will be 950 – 2 × 2.5 = 945 mm.

20.4 Advantages and Disadvantages of V-belt Drive over Flat Belt Drive

Following are the advantages and disadvantages of the *V*-belt drive over flat belt drive :

Advantages

1. The *V*-belt drive gives compactness due to the small distance between centres of pulleys.
2. The drive is positive, because the slip between the belt and the pulley groove is negligible.
3. Since the *V*-belts are made endless and there is no joint trouble, therefore the drive is smooth.
4. It provides longer life, 3 to 5 years.
5. It can be easily installed and removed.
6. The operation of the belt and pulley is quiet.
7. The belts have the ability to cushion the shock when machines are started.
8. The high velocity ratio (maximum 10) may be obtained.
9. The wedging action of the belt in the groove gives high value of limiting *ratio of tensions. Therefore the power transmitted by *V*-belts is more than flat belts for the same coefficient of friction, arc of contact and allowable tension in the belts.
10. The *V*-belt may be operated in either direction, with tight side of the belt at the top or bottom. The centre line may be horizontal, vertical or inclined.

Disadvantages

1. The *V*-belt drive can not be used with large centre distances, because of larger weight per unit length.
2. The *V*-belts are not so durable as flat belts.
3. The construction of pulleys for V-belts is more complicated than pulleys of flat belts.
4. Since the *V*-belts are subjected to certain amount of creep, therefore these are not suitable for constant speed applications such as synchronous machines and timing devices.
5. The belt life is greatly influenced with temperature changes, improper belt tension and mismatching of belt lengths.
6. The centrifugal tension prevents the use of *V*-belts at speeds below 5 m / s and above 50 m / s.

20.5 Ratio of Driving Tensions for V-belt

A *V*-belt with a grooved pulley is shown in Fig. 20.2.

Let R_1 = Normal reactions between belts and sides of the groove.

R = Total reaction in the plane of the groove.

μ = Coefficient of friction between the belt and sides of the groove.

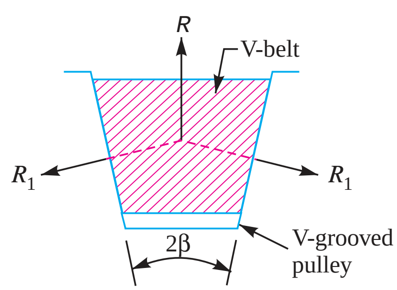


Fig. 20.2. V-belt with pulley.

Resolving the reactions vertically to the groove, we have

$$R = R_1 \sin \beta + R_1 \sin \beta = 2R_1 \sin \beta$$

* The ratio of tensions in *V*-belt drive is cosec β times the flat belt drive.

Top

or $$R_1 = \frac{R}{2 \sin\beta}$$

We know that the frictional force

$$= 2\,\mu.R_1 = 2\,\mu \times \frac{R}{2\sin\beta} = \frac{\mu.R}{\sin\beta} = \mu.R.\text{cosec}\,\beta$$

Consider a small portion of the belt, as in Art. 18.19, subtending an angle $\delta\theta$ at the centre, the tension on one side will be T and on the other side $(T + \delta T)$. Now proceeding in the same way as in Art. 18.19, we get the frictional resistance equal to $\mu\,R$. cosec β against $\mu.R$. Thus the relation between T_1 and T_2 for the V-belt drive will be

$$2.3 \log (T_1 / T_2) = \mu.\theta \text{ cosec } \beta$$

20.6 V-flat Drives

In many cases, particularly, when a flat belt is replaced by V-belt, it is economical to use flat-faced pulley, instead of large grooved pulley, as shown in Fig. 20.3. The cost of cutting the grooves is thereby eliminated. Such a drive is known as ***V-flat drive.***

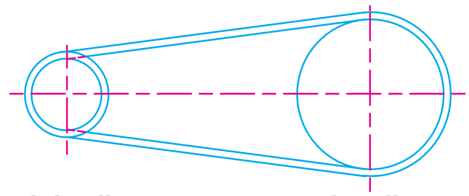


Fig. 20.3. V-flat drive.

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Example 20.1. *A compressor, requiring 90 kW, is to run at about 250 r.p.m. The drive is by V-belts from an electric motor running at 750 r.p.m. The diameter of the pulley on the compressor shaft must not be greater than 1 metre while the centre distance between the pulleys is limited to 1.75 metre. The belt speed should not exceed 1600 m / min.*

Determine the number of V-belts required to transmit the power if each belt has a cross-sectional area of 375 mm^2, density 1000 kg / m^3 and an allowable tensile stress of 2.5 MPa. The groove angle of the pulleys is 35°. The coefficient of friction between the belt and the pulley is 0.25. Calculate also the length required of each belt.

Solution. Given : P = 90 kW = 90 × 10^3 W ; N_2 = 250 r.p.m. ; N_1 = 750 r.p.m. ; d_2 = 1 m ; x = 1.75 m ; v = 1600 m/min = 26.67 m/s ; a = 375 mm^2 = 375 × 10^{-6} m^2 ; ρ = 1000 kg / m^3 ; σ = 2.5 MPa = 2.5 N/mm^2 ; 2β = 35° or β = 17.5° ; μ = 0.25

First of all, let us find the diameter of pulley on the motor shaft (d_1). We know that

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} \quad \text{or} \quad d_1 = \frac{d_2 N_2}{N_1} = \frac{1 \times 250}{750} = 0.33 \text{ m}$$

Contents

For an open belt drive, as shown in Fig. 20.4,

$$\sin\alpha = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{1 - 0.33}{2 \times 1.75} = 0.1914$$

$$\therefore \quad \alpha = 11.04°$$

and angle of lap on the smaller pulley (*i.e.* pulley on the motor shaft),

$$\theta = 180° - 2\alpha = 180 - 2 \times 11.04 = 157.92°$$

$$= 157.92 \times \frac{\pi}{180} = 2.76 \text{ rad}$$

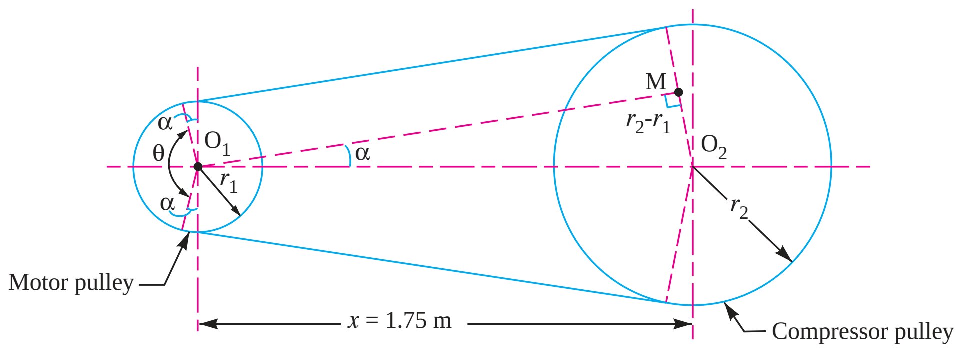


Fig. 20.4

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = 375 \times 10^{-6} \times 1 \times 1000 = 0.375 \text{ kg / m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 0.375\,(26.67)^2 = 267 \text{ N}$$

and maximum tension in the belt,

$$T = \sigma \times a = 2.5 \times 375 = 937.5 \text{ N}$$

∴ Tension in the tight side of the belt,

$$T_1 = T - T_C = 937.5 - 267 = 670.5 \text{ N}$$

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec}\beta = 0.25 \times 2.76 \times \operatorname{cosec} 17.5°$$

$$= 0.69 \times 3.3255 = 2.295$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{2.295}{2.3} = 0.9976 \quad \text{or} \quad \frac{T_1}{T_2} = 9.95 \qquad \text{...(Taking antilog of 0.9976)}$$

and $T_2 = T_1 / 9.95 = 670.5 / 9.95 = 67.4$ N

Number of V-belts

We know that the power transmitted per belt,

$$= (T_1 - T_2)\, v = (670.5 - 67.4)\, 26.67 = 16\,085 \text{ W} = 16.085 \text{ kW}$$

∴ Number of *V*-belts

$$= \frac{\text{Total power transmitted}}{\text{Power transmitted per belt}} = \frac{90}{16.085} = 5.6 \text{ say } 6$$ **Ans.**

Top

Length of each belt

We know that radius of pulley on motor shaft,

$$r_1 = d_1 / 2 = 0.33 / 2 = 0.165 \text{ m}$$

and radius of pulley on compressor shaft,

$$r_2 = d_2 / 2 = 1 / 2 = 0.5 \text{ m}$$

We know that length of each belt,

$$L = \pi (r_2 + r_1) + 2x + \frac{(r_2 - r_1)^2}{x}$$

$$= \pi (0.5 + 0.165) + 2 \times 1.75 + \frac{(0.5 - 0.165)^2}{1.75}$$

$$= 2.09 + 3.5 + 0.064 = 5.654 \text{ m}$$ **Ans.**

Example 20.2. *A belt drive consists of two V-belts in parallel, on grooved pulleys of the same size. The angle of the groove is 30°. The cross-sectional area of each belt is 750 mm² and μ = 0.12. The density of the belt material is 1.2 Mg / m³ and the maximum safe stress in the material is 7 MPa. Calculate the power that can be transmitted between pulleys of 300 mm diameter rotating at 1500 r.p.m. Find also the shaft speed in r.p.m. at which the power transmitted would be a maximum.*

Solution. Given : $n = 2$; $2\beta = 30°$ or $\beta = 15°$; $a = 750 \text{ mm}^2 = 750 \times 10^{-6} \text{ m}^2$; $\mu = 0.12$; $\rho = 1.2 \text{ Mg/m}^3 = 1200 \text{ kg/m}^3$; $\sigma = 7 \text{ MPa} = 7 \times 10^6 \text{ N/m}^2$; $d = 300 \text{ mm} = 0.3 \text{ m}$; $N = 1500$ r.p.m.

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = 750 \times 10^{-6} \times 1 \times 1200 = 0.9 \text{ kg/m}$$

and speed of the belt, $$v = \frac{\pi d N}{60} = \frac{\pi \times 0.3 \times 1500}{60} = 23.56 \text{ m/s}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 0.9 (23.56)^2 = 500 \text{ N}$$

and maximum tension, $$T = \sigma \times a = 7 \times 10^6 \times 750 \times 10^{-6} = 5250 \text{ N}$$

We know that tension in the tight side of the belt,

$$T_1 = T - T_C = 5250 - 500 = 4750 \text{ N}$$

Let T_2 = Tension in the slack side of the belt.

Since the pulleys are of the same size, therefore angle of lap (θ) = 180° = π rad.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec} \beta = 0.12 \times \pi \times \operatorname{cosec} 15° = 0.377 \times 3.8637 = 1.457$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{1.457}{2.3} = 0.6335 \quad \text{or} \quad \frac{T_1}{T_2} = 4.3 \qquad \text{...(Taking antilog of 0.6335)}$$

and $$T_2 = T_1 / 4.3 = 4750 / 4.3 = 1105 \text{ N}$$

Power transmitted

We know that power transmitted,

$$P = (T_1 - T_2)\, v \times n = (4750 - 1105)\, 23.56 \times 2 = 171\,750 \text{ W}$$

$$= 171.75 \text{ kW}$$ **Ans.**

Shaft speed

Let N_1 = Shaft speed in r.p.m., and

v_1 = Belt speed in m/s.

We know that for maximum power, centrifugal tension,

$$T_C = T/3 \quad \text{or} \quad m(v_1)^2 = T/3$$

or $$0.9\,(v_1)^2 = 5250/3 = 1750$$

$\therefore$ $$(v_1)^2 = 1750/0.9 = 1944.4 \quad \text{or} \quad v_1 = 44.1 \text{ m/s}$$

We know that belt speed (v_1),

$$44.1 = \frac{\pi\, d\, N_1}{60} = \frac{\pi \times 0.3 \times N_1}{60} = 0.0157\, N_1$$

$\therefore$ $$N_1 = 44.1/0.0157 = 2809 \text{ r.p.m.}$$ **Ans.**

Example 20.3. *Two shafts whose centres are 1 metre apart are connected by a V-belt drive. The driving pulley is supplied with 95 kW power and has an effective diameter of 300 mm. It runs at 1000 r.p.m. while the driven pulley runs at 375 r.p.m. The angle of groove on the pulleys is 40°. Permissible tension in 400 mm² cross-sectional area belt is 2.1 MPa. The material of the belt has density of 1100 kg / m³. The driven pulley is overhung, the distance of the centre from the nearest bearing being 200 mm. The coefficient of friction between belt and pulley rim is 0.28. Estimate: 1. The number of belts required ; and 2. Diameter of driven pulley shaft, if permissible shear stress is 42 MPa.*

Solution. Given : $x = 1$ m ; $P = 95$ kW $= 95 \times 10^3$ W ; $d_1 = 300$ mm $= 0.3$ m ; $N_1 = 1000$ r.p.m. ; $N_2 = 375$ r.p.m ; $2\,\beta = 40°$ or $\beta = 20°$; $a = 400$ mm² $= 400 \times 10^{-6}$ m² ; $\sigma = 2.1$ MPa $= 2.1$ N/mm² ; $\rho = 1100$ kg/m³; $\mu = 0.28$; $\tau = 42$ MPa $= 42$ N/mm²

First of all, let us find the diameter of the driven pulley (d_2). We know that

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} \quad \text{or} \quad d_2 = \frac{N_1 \times d_1}{N_2} = \frac{1000 \times 300}{375} = 800 \text{ mm} = 0.8 \text{ m}$$

For an open belt drive,

$$\sin\alpha = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{0.8 - 0.3}{2 \times 1} = 0.25$$

$\therefore$ $$\alpha = 14.5°$$

and angle of lap on the smaller or driving pulley,

$$\theta = 180° - 2\,\alpha = 180° - 2 \times 14.5 = 151°$$

$$= 151 \times \frac{\pi}{180} = 2.64 \text{ rad}$$

We know that the mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = 400 \times 10^{-6} \times 1 \times 1100 = 0.44 \text{ kg / m}$$

and velocity of the belt,

$$v = \frac{\pi\, d_1 . N_1}{60} = \frac{\pi \times 0.3 \times 1000}{60} = 15.71 \text{ m/s}$$

$\therefore$ Centrifugal tension,

$$T_C = m.v^2 = 0.44\,(15.71)^2 = 108.6 \text{ N}$$

and maximum tension in the belt,

$$T = \sigma \times a = 2.1 \times 400 = 840 \text{ N}$$

$\therefore$ Tension in the tight side of the belt,

$$T_1 = T - T_C = 840 - 108.6 = 731.4 \text{ N}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec}\beta = 0.28 \times 2.64 \operatorname{cosec} 20° = 0.74 \times 2.9238 = 2.164$$

Contents

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{2.164}{2.3} = 0.9407 \quad \text{or} \quad \frac{T_1}{T_2} = 8.72 \qquad \text{...(Taking antilog of 0.9407)}$$

and
$$T_2 = \frac{T_1}{8.72} = \frac{731.4}{8.72} = 83.9 \text{ N}$$

1. *Number of belts required*

We know that the power transmitted per belt

$$= (T_1 - T_2)\, v = (731.4 - 83.9)\, 15.71 = 10\,172 \text{ W} = 10.172 \text{ kW}$$

$\therefore$ Number of belts required

$$= \frac{\text{Total power transmitted}}{\text{Power transmitted per belt}} = \frac{95}{10.172} = 9.34 \text{ say } 10 \quad \textbf{Ans.}$$

2. *Diameter of driven pulley shaft*

Let D = Diameter of driven pulley shaft.

We know that torque transmitted by the driven pulley shaft,

$$T = \frac{P \times 60}{2\pi N_2} = \frac{95 \times 10^3 \times 60}{2\pi \times 375} = 2420 \text{ N-m} = 2420 \times 10^3 \text{ N-mm}$$

Since the driven pulley is overhung and the distance of the centre from the nearest bearing is 200 mm, therefore bending moment on the shaft due to the pull on the belt,

$$M = (T_1 + T_2 + 2T_C)\, 200 \times 10 \qquad \text{...}(\because \text{ No. of belts} = 10)$$
$$= (731.4 + 83.9 + 2 \times 108.6)\, 200 \times 10 = 2065 \times 10^3 \text{ N-mm}$$

$\therefore$ Equivalent twisting moment,

$$T_e = \sqrt{T^2 + M^2} = \sqrt{(2420 \times 10^3)^2 + (2065 \times 10^3)^2} \text{ N-mm}$$
$$= 3181 \times 10^3 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$3181 \times 10^3 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 42\, D^3 = 8.25\, D^3$$

$$\therefore \quad D^3 = 3181 \times 10^3 / 8.25 = 386 \times 10^3$$

or
$$D = 72.8 \text{ say } 75 \text{ mm} \quad \textbf{Ans.}$$

Example 20.4. *Power of 60 kW at 750 r.p.m. is to be transmitted from an electric motor to compressor shaft at 300 r.p.m. by V-belts. The approximate larger pulley diameter is 1500 mm. The approximate centre distance is 1650 mm, and overload factor is to be taken as 1.5. Give a complete design of the belt drive. A belt with cross-sectional area of 350 mm^2 and density 1000 kg / m^3 and having an allowable tensile strength 2 MPa is available for use. The coefficient of friction between the belt and the pulley may be taken as 0.28. The driven pulley is overhung to the extent of 300 mm from the nearest bearing and is mounted on a shaft having a permissible shear stress of 40 MPa with the help of a key. The shaft, the pulley and the key are also to be designed.*

Solution. Given : P = 60 kW ; N_1 = 750 r.p.m. ; N_2 = 300 r.p.m. ; d_2 = 1500 mm ; x = 1650 mm ; Overload factor = 1.5 ; a = 350 mm^2 = 350×10^{-6} m^2 ; ρ = 1000 kg/m^3 ; σ = 2 MPa = 2 N/mm^2 ; μ = 0.28 ; τ = 40 MPa = 40 N/mm^2

1. *Design of the belt drive*

First of all, let us find the diameter (d_1) of the motor pulley. We know that

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} \quad \text{or} \quad d_1 = \frac{d_2 \times N_2}{N_1} = \frac{1500 \times 300}{750} = 600 \text{ mm} = 0.6 \text{ m}$$

and
$$\sin\alpha = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{1500 - 600}{2 \times 1650} = 0.2727 \quad \text{or} \quad \alpha = 15.83°$$

Top

We know that the angle of contact,

$$\theta = 180° - 2\alpha = 180 - 2 \times 15.83 = 148.34°$$

$$= 148.34 \times \pi / 180 = 2.6 \text{ rad}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

Assume the groove angle of the pulley, $2\beta = 35°$ or $\beta = 17.5°$. We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec} \beta = 0.28 \times 2.6 \times \operatorname{cosec} 17.5° = 2.42$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = 2.42 / 2.3 = 1.0526 \quad \text{or} \quad \frac{T_1}{T_2} = 11.28 \quad \text{...}(i)$$

...(Taking antilog of 1.0526)

We know that the velocity of the belt,

$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.6 \times 750}{60} = 23.66 \text{ m/s}$$

and mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = 350 \times 10^{-6} \times 1 \times 1000 = 0.35 \text{ kg / m}$$

∴ Centrifugal tension in the belt,

$$T_C = m.v^2 = 0.35\,(23.66)^2 = 196 \text{ N}$$

and maximum tension in the belt,

$$T = \text{Stress} \times \text{area} = \sigma \times a = 2 \times 350 = 700 \text{ N}$$

∴ Tension in the tight side of the belt,

$$T_1 = T - T_C = 700 - 196 = 504 \text{ N}$$

and

$$T_2 = \frac{T_1}{11.28} = \frac{504}{11.28} = 44.7 \text{ N} \quad \text{...[From equation } (i)]$$

We know that the power transmitted per belt

$$= (T_1 - T_2)\, v = (504 - 44.7)\, 23.66 = 10\,867 \text{ W} = 10.867 \text{ kW}$$

Since the over load factor is 1.5, therefore the belt is to be designed for 1.5 × 60 = 90 kW.

∴ Number of belts required

$$= \frac{\text{Designed power}}{\text{Power transmitted per belt}} = \frac{90}{10.867} = 8.3 \text{ say } 9 \quad \textbf{Ans.}$$

Since the *V*-belt is to be designed for 90 kW, therefore from Table 20.1, we find that a '*D*' type of belt should be used.

We know that the pitch length of the belt,

$$L = \pi\,(r_2 + r_1) + 2x + \frac{(r_2 - r_1)^2}{x} = \frac{\pi}{2}\,(d_2 + d_1) + 2x + \frac{(d_2 - d_1)^2}{4x}$$

$$= \frac{\pi}{2}\,(1500 + 600) + 2 \times 1650 + \frac{(1500 - 600)^2}{4 \times 1650}$$

$$= 3300 + 3300 + 123 = 6723 \text{ mm}$$

Subtracting 79 mm for '*D*' type belt, we find that inside length of the belt

$$= 6723 - 79 = 6644 \text{ mm}$$

According to IS: 2494 – 1974, the nearest standard inside length of *V*-belt is 6807 mm.

∴ Pitch length of the belt,

$$L_1 = 6807 + 79 = 6886 \text{ mm } \textbf{Ans.}$$

Now let us find out the new centre distance (x_1) between the two pulleys. We know that

$$L_1 = \frac{\pi}{2}(d_2 + d_1) + 2x_1 + \frac{(d_2 - d_1)^2}{4x_1}$$

$$6886 = \frac{\pi}{2}(1500 + 600) + 2x_1 + \frac{(1500 - 600)^2}{4x_1}$$

$$= 3300 + 2x_1 + \frac{810\ 000}{4x_1} \quad \text{....(}d_1 \text{ and } d_2 \text{ are taken in mm)}$$

$$6886 \times 4x_1 = 3300 \times 4x_1 + 2x_1 \times 4x_1 + 810\ 000$$

or $$3443\, x_1 = 1650\, x_1 + x_1^2 + 101\ 250$$

or $$x_1^2 - 1793\, x_1 + 101\ 250 = 0$$

$$\therefore \quad x_1 = \frac{1793 \pm \sqrt{(1793)^2 - 4 \times 101\ 250}}{2}$$

$$= \frac{1793 \pm 1677}{2} = 1735 \text{ mm } \textbf{Ans.} \quad \text{...(Taking + ve sign)}$$

2. *Design of shaft*

Let D = Diameter of the shaft.

We know that the torque transmitted by the driven or compressor pulley shaft,

$$T = \frac{\text{Designed power} \times 60}{2\pi N_2} = \frac{90 \times 10^3 \times 60}{2\pi \times 300} = 2865 \text{ N-m}$$

$$= 2865 \times 10^3 \text{ N-mm}$$

Since the overhang of the pulley is 300 mm, therefore bending moment on the shaft due to the belt tensions,

$$M = (T_1 + T_2 + 2T_C)\, 300 \times 9 \quad \text{...}(\because \text{No. of belts} = 9)$$

$$= (504 + 44.7 + 2 \times 196)\, 300 \times 9 = 2540 \times 10^3 \text{ N-mm}$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{T^2 + M^2} = \sqrt{(2865 \times 10^3)^2 + (2540 \times 10^3)^2}$$

$$= 3830 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3830 \times 10^3 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 40 \times D^3 = 7.855\, D^3$$

$$\therefore \quad D^3 = 3830 \times 10^3 / 7.855 = 487.6 \times 10^3 \quad \text{or} \quad D = 78.7 \text{ say } 80 \text{ mm } \textbf{Ans.}$$

3. *Design of the pulley*

The dimensions for the standard *V*-grooved pulley (Refer Fig. 20.1) are shown in Table 20.2, from which we find that for '*D*' type belt

w = 27 mm, d = 28 mm, a = 8.1 mm, c = 19.9 mm, f = 24 mm, and e = 37 mm.

We know that face width of the pulley,

$$B = (n - 1)\, e + 2f = (9 - 1)\, 37 + 2 \times 24 = 344 \text{ mm } \textbf{Ans.}$$

4. *Design for key*

The standard dimensions of key for a shaft of 80 mm diameter are

Width of key = 25 mm **Ans.**

and thickness of key = 14 mm **Ans.**

Contents

Example 20.5. *A V-belt is driven on a flat pulley and a V-pulley. The drive transmits 20 kW from a 250 mm diameter V-pulley operating at 1800 r.p.m. to a 900 mm diameter flat pulley. The centre distance is 1 m, the angle of groove 40° and μ = 0.2. If density of belting is 1110 kg / m^3 and allowable stress is 2.1 MPa for belt material, what will be the number of belts required if C-size V-belts having 230 mm^2 cross-sectional area are used.*

Solution. Given : $P = 20$ kW ; $d_1 = 250$ mm $= 0.25$ m ; $N_1 = 1800$ r.p.m. ; $d_2 = 900$ mm $= 0.9$ m ; $x = 1$ m $= 1000$ mm ; $2\beta = 40°$ or $\beta = 20°$; $\mu = 0.2$; $\rho = 1110$ kg/m^3 ; $\sigma = 2.1$ MPa $= 2.1$ N/mm^2 ; $a = 230$ mm^2 $= 230 \times 10^{-6}$ m^2

Fig. 20.5 shows a *V*-flat drive. First of all, let us find the angle of contact for both the pulleys. From the geometry of the Fig. 20.5, we find that

$$\sin \alpha = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{d_2 - d_1}{2x} = \frac{900 - 250}{2 \times 1000} = 0.325$$

$$\therefore \quad \alpha = 18.96°$$

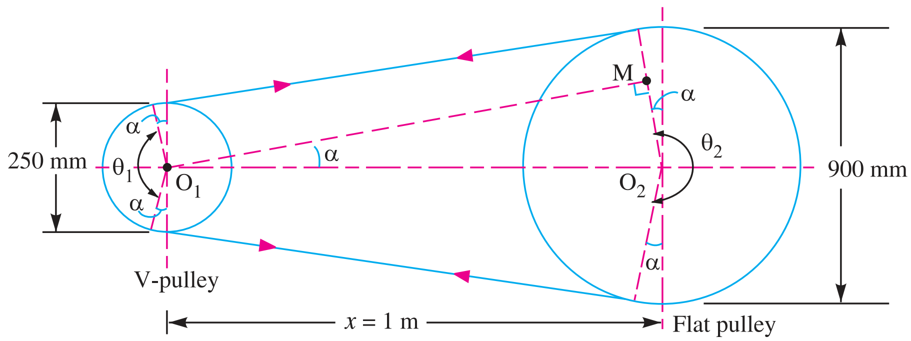


Fig. 20.5

We know that angle of contact on the smaller or *V*-pulley,

$$\theta_1 = 180° - 2\,\alpha = 180° - 2 \times 18.96 = 142.08°$$
$$= 142.08 \times \pi/180 = 2.48 \text{ rad}$$

and angle of contact on the larger or flat pulley,

$$\theta_2 = 180° + 2\alpha = 180° + 2 \times 18.96 = 217.92°$$
$$= 217.92 \times \pi \;/\; 180 = 3.8 \text{ rad}$$

We have already discussed that when the pulleys have different angle of contact (θ), then the design will refer to a pulley for which μ.θ is small.

We know that for a smaller or *V*-pulley,

$$\mu.\theta = \mu.\theta_1 \operatorname{cosec} \beta = 0.2 \times 2.48 \times \operatorname{cosec} 20° = 1.45$$

and for larger or flat pulley,

$$\mu.\theta = \mu.\theta_2 = 0.2 \times 3.8 = 0.76$$

Since (μ.θ) for the larger or flat pulley is small, therefore the design is based on the larger or flat pulley.

We know that peripheral velocity of the belt,

$$v = \frac{\pi\, d_1\, N_1}{60} = \frac{\pi \times 0.25 \times 1800}{60} = 23.56 \text{ m/s}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = a \times l \times \rho$$
$$= 230 \times 10^{-6} \times 1 \times 1100 = 0.253 \text{ kg / m}$$

Top

∴ Centrifugal tension,

$$T_C = m.v^2 = 0.253\,(23.56)^2 = 140.4 \text{ N}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

We know that maximum tension in the belt,

$$T = \text{Stress} \times \text{area} = \sigma \times a = 2.1 \times 230 = 483 \text{ N}$$

We also know that maximum or total tension in the belt,

$$T = T_1 + T_C$$

$$\therefore \quad T_1 = T - T_C = 483 - 140.4 = 342.6 \text{ N}$$

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta_2 = 0.2 \times 3.8 = 0.76$$

$$\log \left(\frac{T_1}{T_2}\right) = 0.76 / 2.3 = 0.3304 \quad \text{or} \quad \frac{T_1}{T_2} = 2.14 \quad \text{...(Taking antilog of 0.3304)}$$

and

$$T_2 = T_1 / 2.14 = 342.6 / 2.14 = 160 \text{ N}$$

∴ Power transmitted per belt

$$= (T_1 - T_2)\, v = (342.6 - 160)\, 23.56 = 4302 \text{ W} = 4.302 \text{ kW}$$

We know that number of belts required

$$= \frac{\text{Total power transmitted}}{\text{Power transmitted per belt}} = \frac{20}{4.302} = 4.65 \text{ say } 5 \quad \textbf{Ans.}$$

20.7 Rope Drives

The rope drives are widely used where a large amount of power is to be transmitted, from one pulley to another, over a considerable distance. It may be noted that the use of flat belts is limited for the transmission of moderate power from one pulley to another when the two pulleys are not more than 8 metres apart. If large amounts of power are to be transmitted, by the flat belt, then it would result in excessive belt cross-section.

The ropes drives use the following two types of ropes :

1. Fibre ropes, and **2.** *Wire ropes.

The fibre ropes operate successfully when the pulleys are about 60 metres apart, while the wire ropes are used when the pulleys are upto 150 metres apart.

20.8 Fibre Ropes

The ropes for transmitting power are usually made from fibrous materials such as hemp, manila and cotton. Since the hemp and manila fibres are rough, therefore the ropes made from these fibres are not very flexible and possesses poor mechanical properties. The hemp ropes have less strength as compared to manila ropes. When the hemp and manila ropes are bent over the sheave, there is some sliding of the fibres, causing the rope to wear and chafe internally. In order to minimise this defect, the rope fibres are lubricated with a tar, tallow or graphite. The lubrication also makes the rope moisture proof. The hemp ropes are suitable only for hand operated hoisting machinery and as tie ropes for lifting tackle, hooks etc.

The cotton ropes are very soft and smooth. The lubrication of cotton ropes is not necessary. But if it is done, it reduces the external wear between the rope and the grooves of its sheaves. It may be noted that the manila ropes are more durable and stronger than cotton ropes. The cotton ropes are costlier than manila ropes.

* Wire ropes are discussed in Art. 20.12.

Contents

Notes : 1. The diameter of manila and cotton ropes usually ranges from 38 mm to 50 mm. The size of the rope is usually designated by its circumference or ***'girth'***.

2. The ultimate tensile breaking load of the fibre ropes varies greatly. For manila ropes, the average value of the ultimate tensile breaking load may be taken as 500 d^2 kN and for cotton ropes, it may be taken as 350 d^2 kN, where d is the diameter of rope in mm.

20.9 Advantages of Fibre Rope Drives

The fibre rope drives have the following advantages :

1. They give smooth, steady and quiet service.

2. They are little affected by out door conditions.

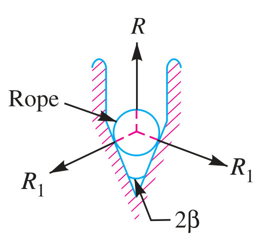
3. The shafts may be out of strict alignment.

4. The power may be taken off in any direction and in fractional parts of the whole amount.

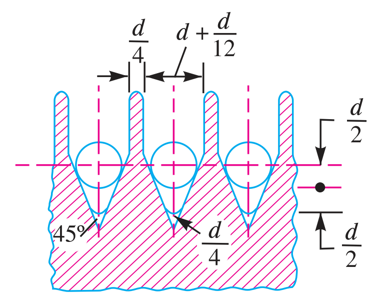
5. They give high mechanical efficiency.

20.10 Sheave for Fibre Ropes

The fibre ropes are usually circular in cross-section as shown in Fig. 20.6 (*a*). The sheave for the fibre ropes, is shown in Fig. 20.6 (*b*). The groove angle of the pulley for rope drives is usually 45°.



(*a*) Cross-section of a rope.



(*b*) Sheave (grooved pulley) for ropes.

Fig. 20.6. Rope and sheave.

The grooves in the pulleys are made narrow at the bottom and the rope is pinched between the edges of the *V*-groove to increase the holding power of the rope on the pulley. The grooves should be finished smooth to avoid chafing of the rope. The diameter of the sheaves should be large to reduce the wear on the rope due to internal friction and bending stresses. The proper size of sheave wheels is 40 d and the minimum size is 36 d, where d is the diameter of rope in cm.

Note : The number of grooves should not be more than 24.

Rope drives

20.11 Ratio of Driving Tensions for Fibre Rope

A fibre rope with a grooved pulley is shown in Fig. 20.6 (*a*). The fibre ropes are designed in the similar way as *V*-belts. We have discussed in Art. 20.5, that the ratio of driving tensions is

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec} \beta$$

where μ, θ and β have usual meanings.

Top

Example 20.6. *A pulley used to transmit power by means of ropes has a diameter of 3.6 metres and has 15 grooves of 45° angle. The angle of contact is 170° and the coefficient of friction between the ropes and the groove sides is 0.28. The maximum possible tension in the ropes is 960 N and the mass of the rope is 1.5 kg per metre length. Determine the speed of the pulley in r.p.m. and the power transmitted if the condition of maximum power prevail.*

Solution. Given : d = 3.6 m ; n = 15 ; 2 β = 45° or β = 22.5° ; θ = 170° = 170 × π / 180 = 2.967 rad ; μ = 0.28 ; T = 960 N ; m = 1.5 kg / m

Speed of the pulley

Let N = Speed of the pulley in r.p.m.

We know that for maximum power, speed of the pulley,

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{960}{3 \times 1.5}} = 14.6 \text{ m/s}$$

We also know that speed of the pulley (v),

$$14.6 = \frac{\pi d . N}{60} = \frac{\pi \times 3.6 \times N}{60} = 0.19\ N$$

$\therefore$ N = 14.6 / 0.19 = 76.8 r.p.m. **Ans.**

Power transmitted

We know that for maximum power, centrifugal tension,

$$T_C = T / 3 = 960 / 3 = 320 \text{ N}$$

$\therefore$ Tension in the tight side of the rope,

$$T_1 = T - T_C = 960 - 320 = 640 \text{ N}$$

Let T_2 = Tension in the slack side of the rope.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec} \beta = 0.28 \times 2.967 \times \operatorname{cosec} 22.5° = 2.17$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{2.17}{2.3} = 0.9435 \quad \text{or} \quad \frac{T_1}{T_2} = 8.78 \qquad \text{...(Taking antilog of 0.9435)}$$

and $T_2 = T_1 / 8.78 = 640 / 8.78 = 73$ N

$\therefore$ Power transmitted,

$$P = (T_1 - T_2)\, v \times n = (640 - 73)\, 14.6 \times 15 = 124\,173 \text{ W}$$

$$= 124.173 \text{ kW}$$ **Ans.**

Example 20.7. *A rope pulley with 10 ropes and a peripheral speed of 1500 m / min transmits 115 kW. The angle of lap for each rope is 180° and the angle of groove is 45°. The coefficient of friction between the rope and pulley is 0.2. Assuming the rope to be just on the point of slipping, find the tension in the tight and slack sides of the rope. The mass of each rope is 0.6 kg per metre length.*

Solution. Given : n = 10 ; v = 1500 m/min = 25 m/s ; P = 115 kW = 115 × 10^3 W ; θ = 180° = π rad ; 2 β = 45° or β = 22.5° ; μ = 0.2 ; m = 0.6 kg / m

Let T_1 = Tension in the tight side of the rope, and

T_2 = Tension in the slack side of the rope.

We know that total power transmitted (P),

$$115 \times 10^3 = (T_1 - T_2)\, v \times n = (T_1 - T_2)\, 25 \times 10 = 250\, (T_1 - T_2)$$

$$\therefore \quad T_1 - T_2 = 115 \times 10^3 / 250 = 460 \qquad \text{...}(i)$$

We also know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec} \beta = 0.2 \times \pi \times \operatorname{cosec} 22.5° = 1.642$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{1.642}{2.3} = 0.714 \quad \text{or} \quad \frac{T_1}{T_2} = 5.18 \quad \text{...(Taking antilog of 0.714) ...}(ii)$$

From equations ***(i)*** and ***(ii)***, we find that

$$T_1 = 570 \text{ N, and } T_2 = 110 \text{ N}$$

We know that centrifugal tension,

$$T_C = m\, v^2 = 0.6\,(25)^2 = 375 \text{ N}$$

∴ Total tension in the tight side of the rope,

$$T_{t1} = T_1 + T_C = 570 + 375 = 945 \text{ N}$$ **Ans.**

and total tension in the slack side of the rope,

$$T_{t2} = T_2 + T_C = 110 + 375 = 485 \text{ N}$$ **Ans.**

Example 20.8. *A rope drive transmits 600 kW from a pulley of effective diameter 4 m, which runs at a speed of 90 r.p.m. The angle of lap is 160°; the angle of groove 45°; the coefficient of friction 0.28; the mass of rope 1.5 kg / m and the allowable tension in each rope 2400 N. Find the number of ropes required.*

Solution. Given : P = 600 kW ; d = 4 m ; N = 90 r.p.m. ; θ = 160° = 160 × π/180 = 2.8 rad ; 2β = 45° or β = 22.5° ; μ = 0.28 ; m = 1.5 kg / m ; T = 2400 N

We know that velocity of the pulley or rope,

$$v = \frac{\pi\, d\, N}{60} = \frac{\pi \times 4 \times 90}{60} = 18.85 \text{ m/s}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 1.5\,(18.85)^2 = 533 \text{ N}$$

and tension in the tight side of the rope,

$$T_1 = T - T_C = 2400 - 533 = 1867 \text{ N}$$

Let T_2 = Tension in the slack side of the rope.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec} \beta = 0.28 \times 2.8 \times \operatorname{cosec} 22.5° = 0.784 \times 2.6131 = 2.0487$$

$$\log \left(\frac{T_1}{T_2}\right) = \frac{2.0487}{2.3} = 0.8907 \quad \text{or} \quad \frac{T_1}{T_2} = 7.78 \quad \text{...(Taking antilog of 0.8907)}$$

$$\therefore \quad T_2 = T_1 / 7.78 = 1867 / 7.78 = 240 \text{ N}$$

We know that power transmitted per rope

$$= (T_1 - T_2)\, v = (1867 - 240)\, 18.85 = 30\,670 \text{ W} = 30.67 \text{ kW}$$

∴ Number of ropes required

$$= \frac{\text{Total power transmitted}}{\text{Power transmitted per rope}} = \frac{600}{30.67} = 19.56 \text{ say } 20$$ **Ans.**

Example 20.9. *A rope drive is to transmit 250 kW from a pulley of 1.2 m diameter, running at a speed of 300 r.p.m. The angle of lap may be taken as π radians. The groove half angle is 22.5°. The ropes to be used are 50 mm in diameter. The mass of the rope is 1.3 kg per metre length and each rope has a maximum pull of 2.2 kN, the coefficient of friction between rope and pulley is 0.3. Determine the number of ropes required. If the overhang of the pulley is 0.5 m, suggest suitable size for the pulley shaft if it is made of steel with a shear stress of 40 MPa.*

Solution. Given : $P = 250$ kW $= 250 \times 10^3$ W ; $d = 1.2$ m ; $N = 300$ r.p.m ; $\theta = \pi$ rad ; $\beta = 22.5°$; $d_r = 50$ mm ; $m = 1.3$ kg / m ; $T = 2.2$ kN $= 2200$ N ; $\mu = 0.3$; $\tau = 40$ MPa $= 40$ N/mm^2

We know that the velocity of belt,

$$v = \frac{\pi d.N}{60} = \frac{\pi \times 1.2 \times 300}{60} = 18.85 \text{ m/s}$$

and centrifugal tension, $T_C = m.v^2 = 1.3\,(18.85)^2 = 462$ N

∴ Tension in the tight side of the rope,

$$T_1 = T - T_C = 2200 - 462 = 1738 \text{ N}$$

Let T_2 = Tension in the slack side of the rope.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta.\text{cosec}\,\beta = 0.3 \times \pi \times \text{cosec}\,22.5° = 0.9426 \times 2.6131 = 2.463$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{2.463}{2.3} = 1.071 \quad \text{or} \quad \frac{T_1}{T_2} = 11.8 \qquad \text{...(Taking antilog of 1.071)}$$

and
$$T_2 = \frac{T_1}{11.8} = \frac{1738}{11.8} = 147.3 \text{ N}$$

Number of ropes required

We know that power transmitted per rope

$$= (T_1 - T_2)\,v = (1738 - 147.3) \times 18.85 = 29\,985 \text{ W} = 29.985 \text{ kW}$$

∴ Number of ropes required

$$= \frac{\text{Total power transmitted}}{\text{Power transmitted per rope}} = \frac{250}{29.985} = 8.34 \text{ say } 9$$ **Ans.**

Diameter for the pulley shaft

Let D = Diameter for the pulley shaft.

We know that the torque transmitted by the pulley shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{250 \times 10^3 \times 60}{2\pi \times 300} = 7957 \text{ N-m}$$

Since the overhang of the pulley is 0.5 m, therefore bending moment on the shaft due to the rope pull,

$$M = (T_1 + T_2 + 2T_C)\,0.5 \times 9 \qquad \text{...}(\because \text{ No. of ropes} = 9)$$

$$= (1738 + 147.3 + 2 \times 462)\,0.5 \times 9 = 12\,642 \text{ N-m}$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{T^2 + M^2} = \sqrt{(7957)^2 + (12\,642)^2} = 14\,938 \text{ N-m}$$

$$= 14.938 \times 10^6 \text{ N-mm}$$

We know that the equivalent twisting moment (T_e),

$$14.938 \times 10^6 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 40 \times D^3 = 7.855\,D^3$$

$$\therefore \quad D^3 = 14.938 \times 10^6 / 7.855 = 1.9 \times 10^6 \quad \text{or} \quad D = 123.89 \text{ say } 125 \text{ mm}$$ **Ans.**

Contents

20.12 Wire Ropes

When a large amount of power is to be transmitted over long distances from one pulley to another (*i.e.* when the pulleys are upto 150 metres apart), then wire ropes are used. The wire ropes are widely used in elevators, mine hoists, cranes, conveyors, hauling devices and suspension bridges. The wire ropes run on grooved pulleys but they rest on the bottom of the *grooves and are not wedged between the sides of the grooves.

The wire ropes are made from cold drawn wires in order to have increase in strength and durability. It may be noted that the strength of the wire rope increases as its size decreases. The various materials used for wire ropes in order of increasing strength are wrought iron, cast steel, extra strong cast steel, plough steel and alloy steel. For certain purposes, the wire ropes may also be made of copper, bronze, aluminium alloys and stainless steels.

20.13 Advantages of Wire Ropes

The wire ropes have the following advantages as compared to fibre ropes.

1. These are lighter in weight,
2. These offer silent operation,
3. These can withstand shock loads,
4. These are more reliable,
5. These are more durable,
6. They do not fail suddenly,
7. The efficiency is high, and
8. The cost is low.

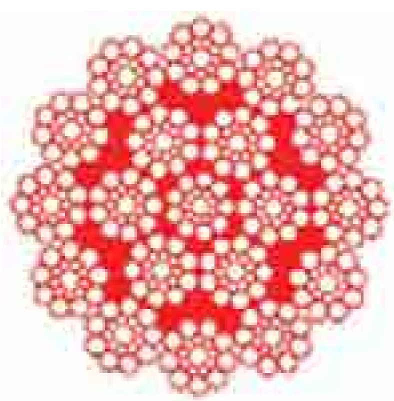
20.14 Construction of Wire Ropes

The wire ropes are made from various grades of steel wire having a tensile strength ranging from 1200 to 2400 MPa as shown in the following table :

Table 20.4. Grade and tensile strength of wires.

Grade of wire	120	140	160	180	200
Tensile strength range (MPa)	1200 – 1500	1400 – 1700	1600 – 1900	1800 – 2100	2000 – 2400

The wires are first given special heat treatment and then cold drawn in order to have high strength and durability of the rope. The steel wire ropes are manufactured by special machines. First of all, a number of wires such as 7, 19 or 37 are twisted into a strand and then a number of strands, usually 6 or 8 are twisted about a core or centre to form the rope as shown in Fig. 20.7. The core may be made of hemp, jute, asbsestos or a wire of softer steel. The core must be continuously saturated with lubricant for the long life of the core as well as the entire rope. The asbestos or soft wire core is used when ropes are subjected to radiant heat such as cranes operating near furnaces. However, a wire core reduces the flexibility of the rope and thus such ropes are used only where they are subjected to high compression as in the case of several layers wound over a rope drum.



Wire strands

* The fibre ropes do not rest at the bottom of the groove.

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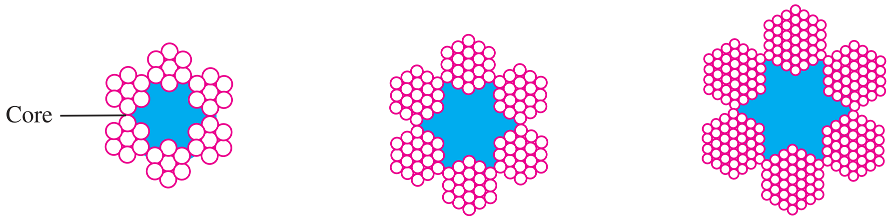


(*a*) 6 × 7 rope. (*b*) 6 × 19 rope. (*c*) 6 × 37 rope.

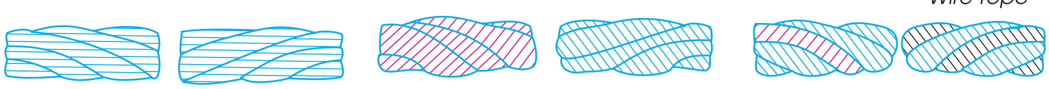
Fig. 20.7. Cross-sections of wire rope.

20.15 Classification of Wire Ropes

Wire rope

According to the direction of twist of the individual wires and that of strands, relative to each other, the wire ropes may be classified as follows :

1. ***Cross or regular lay ropes.*** In these types of ropes, the direction of twist of wires in the strands is opposite to the direction of twist of the stands, as shown in Fig. 20.8 (*a*). Such type of ropes are most popular.
2. ***Parallel or lang lay ropes.*** In these type of ropes, the direction of twist of the wires in the strands is same as that of strands in the rope, as shown in Fig. 20.8 (*b*). These ropes have better bearing surface but is harder to splice and twists more easily when loaded. These ropes are more flexible and resists wear more effectively. Since such ropes have the tendency to spin, therefore these are used in lifts and hoists with guide ways and also as haulage ropes.



(*i*) right handed (*ii*) left handed
(*a*) Cross or regular lay ropes.

(*i*) right handed (*ii*) left handed
(*b*) Parallel or lang lay ropes.

(*c*) Composite or reverse laid ropes.

Fig. 20.8. Wire ropes classified according to the direction of twist of the individual wires.

3. ***Composite or reverse laid ropes.*** In these types of ropes, the wires in the two adjacent strands are twisted in the opposite direction, as shown in Fig. 20.8 (*c*).

Note: The direction of the lay of the ropes may be right handed or left handed, depending upon whether the strands form right hand or left hand helixes, but the right hand lay ropes are most commonly used.

20.16 Designation of Wire Ropes

The wire ropes are designated by the number of strands and the number of wires in each strand. For example, a wire rope having six strands and seven wires in each strand is designated by 6 × 7 rope. Following table shows the standard designation of ropes and their applications :

Table 20.5. Standard designation of ropes and their applications.

Standard designation	*Application*
6 × 7 rope	It is a standard coarse laid rope used as haulage rope in mines, tramways, power transmission.
6 × 19 rope	It is a standard hoisting rope used for hoisting purposes in mines, quarries, cranes, dredges, elevators, tramways, well drilling.
6 × 37 rope	It is an extra flexible hoisting rope used in steel mill laddles, cranes, high speed elevators.
8 × 19 rope	It is also an extra flexible hoisting rope.

Contents

20.17 Properties of Wire Ropes

The following tables show the properties of the various types of wire ropes. In these properties, the diameter of the wire rope (d) is in mm.

Table 20.6. Steel wire ropes for haulage purposes in mines.

Type of rope	*Nominal diameter (mm)*	*Average weight (N/m)*	*Tensile strength (N)* *Tensile strength of wire*	
			1600 MPa	*1800 MPa*
6 × 7	8, 9, 10, 11, 12, 13, 14, 16 18, 19, 20, 21, 22, 24, 25 26, 27, 28, 29, 31, 35	0.0347 d^2	530 d^2	600 d^2
6 × 19	13, 14, 16, 18, 19, 20, 21 22, 24, 25, 26, 28, 29, 32 35, 36, 38	0.0363 d^2	530 d^2	595 d^2

Table 20.7. Steel wire suspension ropes for lifts, elevators and hoists.

Type of rope	*Nominal diameter (mm)*	*Average weight (N/m)*	*Tensile strength (N)* *Tensile strength of wire*	
			1100–1250 MPa	*1250–1400 MPa*
6 × 19	6, 8, 10, 12, 14, 16 18, 20, 22, 25	0.0383 d^2	385 d^2	435 d^2
8 × 19	8, 10, 12, 14,16 18, 20, 22, 25	0.034 d^2	355 d^2	445 d^2

Table 20.8. Steel wire ropes used in oil wells and oil well drilling.

Type of rope	*Nominal diameter (mm)*	*Approximate weight (N/m)*	*Ultimate tensile strength (N)* *Tensile strength of wire*		
			1600 – 1800 MPa	*1800 – 2000 MPa*	*2000 – 2250 MPa*
6 × 7	10, 11, 13, 14, 16, 19, 22, 25	0.037 d^2	550 d^2	610 d^2	–
6 × 19	13, 14, 16, 19 22, 25, 29, 32, 35, 38,	0.037 d^2	510 d^2	570 d^2	630 d^2
6 × 37	13, 14, 16, 19, 22, 25, 26, 32, 35, 38	0.037 d^2	490 d^2	540 d^2	600 d^2
8 × 19	13, 14, 16, 19, 22, 25, 29	0.0338 d^2	–	530 d^2	–

Top

Table 20.9. Steel wire ropes for general engineering purposes such as cranes, excavators etc.

Type of rope	*Nominal diameter (mm)*	*Average weight (N/m)*	*Average tensile strength (N) — Tensile strength of wire*	
			1600–1750 MPa	*1750–1900 MPa*
6 × 19	8, 9, 10, 11, 12, 13, 14, 16, 18, 20, 22, 24, 26, 28, 32, 36, 38, 40	0.0375 d^2	540 d^2	590 d^2
6 × 37	8, 9, 10, 11, 12, 13, 14 16, 18, 20, 22, 24, 26, 28, 32, 36, 40, 44, 48, 52, 56	0.038 d^2	510 d^2	550 d^2

20.18 Diameter of Wire and Area of Wire Rope

The following table shows the diameter of wire (d_w) and area of wire rope (A) for different types of wire ropes :

Table 20.10. Diameter of wire and area of wire rope.

Type of wire rope	6 × 8	6 × 19	6 × 37	8 × 19
Wire diameter (d_w)	0.106 d	0.063 d	0.045 d	0.050 d
Area of wire rope (A)	0.38 d^2	0.38 d^2	0.38 d^2	0.35 d^2

20.19 Factor of Safety for Wire Ropes

The factor of safety for wire ropes based on the ultimate strength are given in the following table.

Table 20.11. Factor of safety for wire ropes.

Application of wire rope	*Factor of safety*	*Application of wire rope*	*Factor of safety*
Track cables	4.2	Derricks	6
Guys	3.5	Haulage ropes	6
Mine hoists : Depths		Small electric and air hoists	7
upto 150 m	8	Over head and gantry cranes	6
300 – 600 m	7	Jib and pillar cranes	6
600 – 900 m	6	Hot ladle cranes	8
over 900 m	5	Slings	8
Miscellaneous hoists	5		

20.20 Wire Rope Sheaves and Drums

The sheave diameter should be fairly large in order to reduce the bending stresses in the ropes when they bend around the sheaves or pulleys. The following table shows the sheave diameters for various types of wire ropes :

Contents

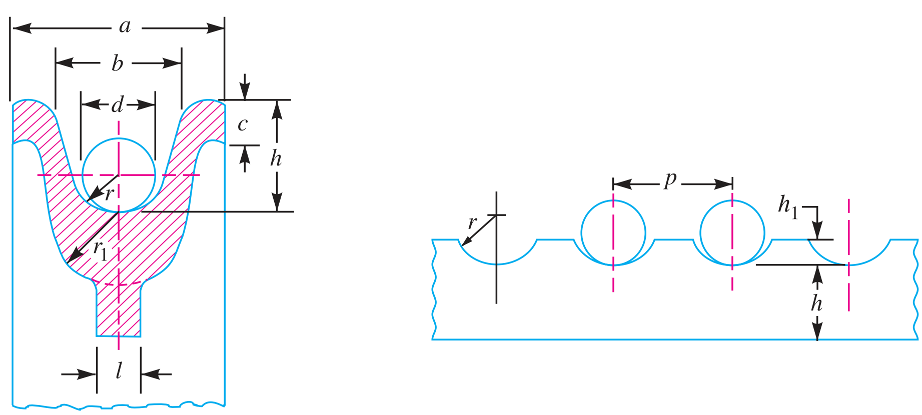
Table 20.12. Sheave diameters (*D*) for wire ropes.

Type of wire rope	*Recommended sheave diameter (D)*		*Uses*
	Minimum sheave diameter	*Preferred sheave diameter*	
6 × 7	42 *d*	72 *d*	Mines, haulage tramways.
6 × 19	30 *d*	45 *d*	Hoisting rope.
	60 *d*	100 *d*	Cargo cranes, mine hoists
	20 *d*	30 *d*	Derricks, dredges, elevators, tramways, well drilling.
6 × 37	18 *d*	27 *d*	Cranes, high speed elevators and small shears.
8 × 19	21 *d*	31 *d*	Extra flexible hoisting rope.

However, if the space allows, then the large diameters should be employed which give better and more economical service.

Sheave or pulleys for winding ropes

The sheave groove has a great influence on the life and service of the rope. If the groove is bigger than rope, there will not be sufficient support for the rope which may, therefore, flatten from its normal circular shape and increase fatigue effects. On the other hand, if the groove is too small, then the rope will be wedged into the groove and thus the normal rotation is prevented. The standard rim of a rope sheave is shown in Fig. 20.9 (*a*) and a standard grooved drum for wire ropes is shown in Fig. 20.9 (*b*).



$r = 0.53\ d;\ r_1 = 1.1\ d; a = 2.7\ d; b = 2.1\ d;$
$c = 0.4\ d; h = 1.6\ d; l = 0.75\ d$

(*a*) Wire rope sheave rim.

$p = 1.15\ d\ ;\ h_1 = 0.25\ d; r = 0.53\ d; h = 1.1\ d$

(*b*) Grooved rope drum.

Fig. 20.9

Top

Contents

For light and medium service, the sheaves are made of cast iron, but for heavy crane service they are often made of steel castings. The sheaves are usually mounted on fixed axles on antifriction bearings or bronze bushings.

The small drums in hand hoists are made plain. A hoist operated by a motor or an engine has a drum with helical grooves, as shown in Fig. 20.9 (*b*). The pitch (*p*) of the grooves must be made slightly larger than the rope diameter to avoid friction and wear between the coils.

20.21 Wire Rope Fasteners

The various types of rope fasteners are shown in Fig. 20.10. The splices in wire ropes should be avoided because it reduces the strength of the rope by 25 to 30 percent of the normal ultimate strength.

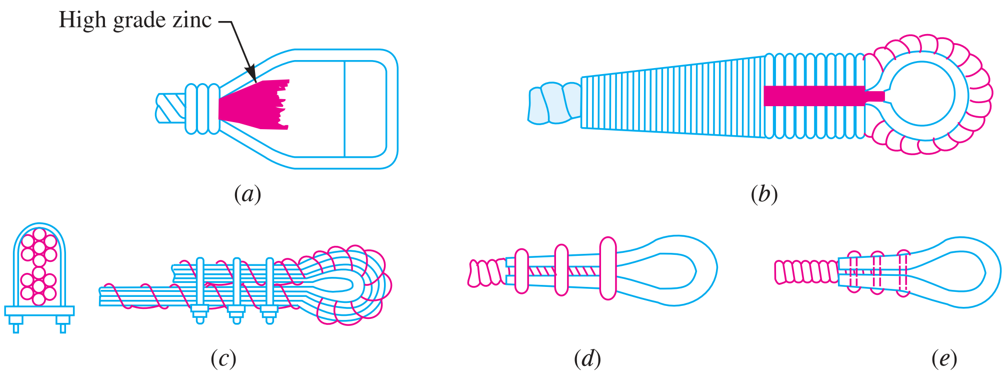


Fig. 20.10. Types of wire rope fastners.

The efficiencies of various types of fasteners are given in the following table.

Table 20.13. Efficiencies of rope fasteners.

Type of fastening	*Efficiency (%)*
(*a*) Wire rope socket with zinc, Fig. 20.10 (*a*)	100
(*b*) Thimble with four or five wire tucks, Fig. 20.10 (*b*)	90
(*c*) Special offset thimble with clips, Fig. 20.10 (*c*)	90
(*d*) Regular thimble with clips, Fig. 20.10 (*d*)	85
(*e*) Three bolt wire clamps, Fig. 20.10 (*e*)	75

20.22 Stresses in Wire Ropes

A wire rope is subjected to the following types of stresses :

1. Direct stress due to axial load lifted and weight of the rope

Let W = Load lifted,

w = Weight of the rope, and

A = Net cross-sectional area of the rope.

∴ Direct stress, $$\sigma_d = \frac{W + w}{A}$$

2. Bending stress when the rope winds round the sheave or drum. When a wire rope is wound over the sheave, then the bending stresses are induced in the wire which is tensile at the top and compressive at the lower side of the wire. The bending stress induced depends upon many factors such as construction of rope, size of wire, type of centre and the amount of restraint in the grooves. The approximate value of the bending stress in the wire as proposed by Reuleaux, is

$$\sigma_b = \frac{E_r \times d_w}{D}$$

Top

Contents

A heavy duty crane. Cranes use rope drives in addition to gear drives

and equivalent bending load on the rope,

$$W_b = \sigma_b \times A = \frac{E_r \times d_w \times A}{D}$$

where E_r = Modulus of elasticity of the wire rope,

d_w = Diameter of the wire,

D = Diameter of the sheave or drum, and

A = Net cross-sectional area of the rope.

It may be noted that E_r is not the modulus of elasticity for the wire material, but it is of the entire rope. The value of E_r may be taken as 77 kN/mm^2 for wrought iron ropes and 84 kN/mm^2 for steel ropes. It has been found experimentally that $E_r = 3/8\ E$, where E is the modulus of elasticity of the wire material.

If σ_b is the bending stress in each wire, then the load on the whole rope due to bending may be obtained from the following relation, *i.e.*

$$W_b = \frac{\pi}{4} (d_w)^2\, n \times \sigma_b$$

where n is the total number of wires in the rope section.

3. ***Stresses during starting and stopping.*** During starting and stopping, the rope and the supported load are to be accelerated. This induces additional load in the rope which is given by

$$W_a = \frac{W + w}{g} \times a \qquad \text{...(}W \text{ and } w \text{ are in newton)}$$

and the corresponding stress,

$$\sigma_a = \frac{W + w}{g} \times \frac{a}{A}$$

Top

where a = Acceleration of the rope and load, and

g = Acceleration due to gravity.

If the time (t) necessary to attain a speed (v) is known, then the value of 'a' is given by

$$a = v / 60\,t$$

The general case of starting is when the rope has a slack (h) which must be overcome before the rope is taut and starts to exert a pull on the load. This induces an impact load on the rope.

The impact load on starting may be obtained by the impact equation, *i.e.*

$$W_{st} = (W + w)\left[1 + \sqrt{1 + \frac{2a \times h \times E_r}{\sigma_d \times l \times g}}\right]$$

and velocity of the rope (v_r) at the instant when the rope is taut,

$$v_r = \sqrt{2a \times h}$$

where a = Acceleration of the rope and load,

h = Slackness in the rope, and

l = Length of the rope.

When there is no slackness in the rope, then $h = 0$ and $v_r = 0$, therefore

Impact load during starting,

$$W_{st} = 2\,(W + w)$$

and the corresponding stress,

$$\sigma_{st} = \frac{2\,(W + w)}{A}$$

4. *Stress due to change in speed.* The additional stress due to change in speed may be obtained in the similar way as discussed above in which the acceleration is given by

$$a = (v_2 - v_1) / t$$

where $(v_2 - v_1)$ is the change in speed in m/s and t is the time in seconds.

It may be noted that when the hoist drum is suddenly stopped while lowering the load, it produces a stress that is several times more than the direct or static stress because of the kinetic energy of the moving masses is suddenly made zero. This kinetic energy is absorbed by the rope and the resulting stress may be determined by equating the kinetic energy to the resilience of the rope. If during stopping, the load moves down a certain distance, the corresponding change of potential energy must be added to the kinetic energy. It is also necessary to add the work of stretching the rope during stopping, which may be obtained from the impact stress.

5. *Effective stress.* The sum of the direct stress (σ_d) and the bending stress (σ_b) is called the effective stress in the rope during normal working. Mathematically,

Effective stress in the rope during normal working

$$= \sigma_d + \sigma_b$$

Effective stress in the rope during starting

$$= \sigma_{st} + \sigma_b$$

and effective stress in the rope during acceleration of the load

$$= \sigma_d + \sigma_b + \sigma_a$$

Contents

While designing a wire rope, the sum of these stresses should be less than the ultimate strength divided by the factor of safety.

Ropes on a pile driver

20.23 Procedure for Designing a Wire Rope

The following procedure may be followed while designing a wire rope.

1. First of all, select a suitable type of rope from Tables 20.6, 20.7, 20.8 and 20.9 for the given application.

2. Find the design load by assuming a factor of safety 2 to 2.5 times the factor of safety given in Table 20.11.

3. Find the diameter of wire rope (d) by equating the tensile strength of the rope selected to the design load.

4. Find the diameter of the wire (d_w) and area of the rope (A) from Table 20.10.

5. Find the various stresses (or loads) in the rope as discussed in Art. 20.22.

6. Find the effective stresses (or loads) during normal working, during starting and during acceleration of the load.

7. Now find the actual factor of safety and compare with the factor of safety given in Table 20.11. If the actual factor of safety is within permissible limits, then the design is safe.

Wheel that winds the metal rope.

Top

Example 20.10. *Select a wire rope for a vertical mine hoist to lift a load of 55 kN from a depth 300 metres. A rope speed of 500 metres / min is to be attained in 10 seconds.*

Solution. Given : W = 55 kN = 55 000 N ; Depth = 300 m ; v = 500 m/min ; t = 10 s

The following procedure may be adopted in selecting a wire rope for a vertical mine hoist.

1. From Table 20.6, we find that the wire ropes for haulage purposes in mines are of two types, *i.e.* 6 × 7 and 6 × 19. Let us take a rope of type 6 × 19.

2. From Table 20.11, we find that the factor of safety for mine hoists from 300 to 600 m depth is 7. Since the design load is calculated by taking a factor of safety 2 to 2.5 times the factor of safety given in Table 20.11, therefore let us take the factor of safety as 15.

∴ Design load for the wire rope

$$= 15 \times 55 = 825 \text{ kN} = 825\ 000 \text{ N}$$

3. From Table 20.6, we find that the tensile strength of 6 × 19 rope made of wire with tensile strength of 1800 MPa is 595 d^2 (in newton), where d is the diameter of rope in mm. Equating this tensile strength to the design load, we get

$$595\ d^2 = 825\ 000$$

$$\therefore \quad d^2 = 825\ 000 / 595 = 1386.5 \quad \text{or} \quad d = 37.2 \text{ say } 38 \text{ mm}$$

4. From Table 20.10, we find that for a 6 × 19 rope,

Diameter of wire, $d_w = 0.063\ d = 0.063 \times 38 = 2.4$ mm

and area of rope, $A = 0.38\ d^2 = 0.38\ (38)^2 = 550 \text{ mm}^2$

5. Now let us find out the various loads in the rope as discussed below :

(*a*) From Table 20.6, we find that weight of the rope,

$$w = 0.0363\ d^2 = 0.0363\ (38)^2 = 52.4 \text{ N/m}$$

$$= 52.4 \times 300 = 15\ 720 \text{ N} \qquad \text{...}(\because \text{Depth} = 300 \text{ m})$$

(*b*) From Table 20.12, we find that diameter of the sheave (D) may be taken as 60 to 100 times the diameter of rope (d). Let us take

$$D = 100\ d = 100 \times 38 = 3800 \text{ mm}$$

∴ Bending stress,

$$\sigma_b = \frac{E_r \times d_w}{D} = \frac{84 \times 10^3 \times 2.4}{3800} = 53 \text{ N/mm}^2$$

...(Taking $E_r = 84 \times 10^3$ N/mm^2)

and the equivalent bending load on the rope,

$$W_b = \sigma_b \times A = 53 \times 550 = 29\ 150 \text{ N}$$

(*c*) We know that the acceleration of the rope and load,

$$a = v / 60t = 500 / 60 \times 10 = 0.83 \text{ m / s}^2$$

∴ Additional load due to acceleration,

$$W_a = \frac{W + w}{g} \times a = \frac{55\ 000 + 15\ 720}{9.81} \times 0.83 = 5983 \text{ N}$$

(*d*) We know that the impact load during starting (when there is no slackness in the rope),

$$W_{st} = 2\ (W + w) = 2(55\ 000 + 15\ 720) = 141\ 440 \text{ N}$$

6. We know that the effective load on the rope during normal working (*i.e.* during uniform lifting or lowering of the load)

$$= W + w + W_b = 55\ 000 + 15\ 720 + 29150 = 99\ 870 \text{ N}$$

Contents

∴ Actual factor of safety during normal working

$$= \frac{825\ 000}{99\ 870} = 8.26$$

Effective load on the rope during starting

$$= W_{st} + W_b = 141\ 440 + 29\ 150 = 170\ 590 \text{ N}$$

∴ Actual factor of safety during starting

$$= \frac{825\ 000}{170\ 590} = 4.836$$

Effective load on the rope during acceleration of the load (*i.e.* during first 10 seconds after starting)

$$= W + w + W_b + W_a$$
$$= 55\ 000 + 15\ 7\ 20 + 29\ 150 + 5983 = 105\ 853 \text{ N}$$

∴ Actual factor of safety during acceleration of the load

$$= \frac{825\ 000}{105\ 853} = 7.8$$

Since the actual factor of safety as calculated above are safe, therefore a wire rope of diameter 38 mm and 6 × 19 type is satisfactory. **Ans.**

A vertical hoist with metal ropes

Example 20.11. *An extra flexible 8 × 19 plough steel wire rope of 38 mm diameter is used with a 2m diameter hoist drum to lift 50 kN of load. Find the factor of safety (ratio of the breaking load to the maximum working load) under the following conditions of operation :*

The wire rope is required to lift from a depth of 900 metres. The maximum speed is 3 m / s and the acceleration is 1.5 m / s^2, when starting under no slack condition. The diameter of the wire may be taken as 0.05 d, where d is the diameter of wire rope. The breaking strength of plough steel is 1880 N/mm^2 and modulus of elasticity of the entire rope is 84 × 10^3 N/mm^2. The weight of the rope is 53 N/m length.

Solution. Given : d = 38 mm ; D = 2 m = 2000 mm ; W = 50 kN = 50 000 N ; Depth = 900 m ; v = 3 m/s ; a = 1.5 m/s^2 ; d_w = 0.05 d ; Breaking strength = 1880 N/mm^2 ; E_r = 84 × 10^3 N/mm^2 ; w = 53 N/m = 53 × 900 = 47 700 N

Top

Since the wire rope is 8 × 19, therefore total number of wires in the rope,

$$n = 8 \times 19 = 152$$

We know that diameter of each wire,

$$d_w = 0.05\, d = 0.05 \times 38 = 1.9 \text{ mm}$$

∴ Cross-sectional area of the wire rope,

$$A = \frac{\pi}{4}(d_w)^2\, n = \frac{\pi}{4}(1.9)^2\, 152 = 431 \text{ mm}^2$$

and minimum breaking strength of the rope

$$= \text{Breaking strength} \times \text{Area} = 1880 \times 431 = 810\ 280 \text{ N}$$

We know that bending stress,

$$\sigma_b = \frac{E_r \times d_w}{D} = \frac{84 \times 10^3 \times 1.9}{2000} = 79.8 \text{ N/mm}^2$$

and equivalent bending load on the rope,

$$W_b = \sigma_b \times A = 79.8 \times 431 = 34\ 390 \text{ N}$$

Additional load due to acceleration of the load lifted and rope,

$$W_a = \frac{W + w}{g} \times a = \frac{50\ 000 + 47\ 700}{9.81} \times 1.5 = 14\ 940 \text{ N}$$

Impact load during starting (when there is no slackness in the rope),

$$W_{st} = 2\,(W + w) = 2(50\ 000 + 47\ 700) = 195\ 400 \text{ N}$$

We know that the effective load on the rope during normal working

$$= W + w + W_b = 50\ 000 + 47\ 700 + 34\ 390 = 132\ 090 \text{ N}$$

∴ Factor of safety during normal working

$$= 810\ 280 / 132\ 090 = 6.13$$ **Ans.**

Effective load on the rope during starting

$$= W_{st} + W_b = 195\ 400 + 34\ 390 = 229\ 790 \text{ N}$$

∴ Factor of safety during starting

$$= 810\ 280 / 229\ 790 = 3.53$$ **Ans.**

Effective load on the rope during acceleration of the load (*i.e.* during the first 2 second after starting)

$$= W + w + W_b + W_a = 50\ 000 + 47\ 700 + 34\ 390 + 14\ 940$$

$$= 147\ 030 \text{ N}$$

∴ Factor of safety during acceleration of the load

$$= 810\ 280 / 147\ 030 = 5.51$$ **Ans.**

Example 20.12. *A workshop crane is lifting a load of 25 kN through a wire rope and a hook. The weight of the hook etc. is 15 kN. The rope drum diameter may be taken as 30 times the diameter of the rope. The load is to be lifted with an acceleration of 1 m/s². Calculate the diameter of the wire rope. Take a factor of safety of 6 and Young's modulus for the wire rope 80 kN/mm². The ultimate stress may be taken as 1800 MPa. The cross-sectional area of the wire rope may be taken as 0.38 times the square of the wire rope diameter.*

Solution. Given : W = 25 kN = 25 000 N ; w = 15 kN = 15 000 N ; D = 30 d ; a = 1 m/s^2 ; E_r = 80 kN/mm^2 = 80 × 10^3 N/mm^2 ; σ_u = 1800 MPa = 1800 N/mm^2 ; $A = 0.38\, d^2$

Contents

Let d = Diameter of wire rope in mm.

We know that direct load on the wire rope,

$$W_d = W + w = 25\,000 + 15\,000 = 40\,000 \text{ N}$$

Let us assume that a 6 × 19 wire rope is used. Therefore from Table 20.10, we find that the diameter of wire,

$$d_w = 0.063\, d$$

We know that bending load on the rope,

$$W_b = \frac{E_r \times d_w}{D} \times A = \frac{80 \times 10^3 \times 0.063\, d}{30\, d} \times 0.38\, d^2 = 63.84\, d^2 \text{ N}$$

and load on the rope due to acceleration,

$$W_a = \frac{W + w}{g} \times a = \frac{25\,000 + 15\,000}{9.81} \times 1 = 4080 \text{ N}$$

∴ Total load acting on the rope

$$= W_d + W_b + W_a = 40\,000 + 63.84\, d^2 + 4080$$

$$= 44\,080 + 63.84\, d^2 \qquad \ldots(i)$$

We know that total load on the rope

$$= \text{Area of wire rope} \times \text{Allowable stress}$$

$$= A \times \frac{\sigma_u}{F.S.} = 0.38\, d^2 \times \frac{1800}{6} = 114\, d^2 \qquad \ldots(ii)$$

From equations (*i*) and (*ii*), we have

$$44\,080 + 63.84\, d^2 = 114\, d^2$$

$$d^2 = \frac{44\,080}{114 - 63.84} = 879 \quad \text{or} \quad d = 29.6 \text{ mm}$$

From Table 20.9, we find that standard nominal diameter of 6 × 19 wire rope is 32 mm. **Ans.**

EXERCISES

1. A *V*-belt drive consists of three *V*-belts in parallel on grooved pulleys of the same size. The angle of groove is 30° and the coefficient of friction 0.12. The cross-sectional area of each belt is 800 mm^2 and the permissible safe stress in the material is 3 MPa. Calculate the power that can be transmitted between two pulleys 400 mm in diameter rotating at 960 r.p.m. **[Ans. 101.7 kW]**

2. Power is transmitted between two shafts by a *V*-belt whose mass is 0.9 kg/m length. The maximum permissible tension in the belt is limited to 2.2 kN. The angle of lap is 170° and the groove angle 45°. If the coefficient of friction between the belt and pulleys is 0.17; find 1. velocity of the belt for maximum power; and 2. power transmitted at this velocity.

[Ans. 28.54 m/s ; 30.66 kW]

3. A *V*-belt drive system transmits 100 kW at 475 r.p.m. The belt has a mass of 0.6 kg/m. The maximum permissible tension in the belt is 900 N. The groove angle is 38° and the angle of contact is 160°. Find minimum number of belts and pulley diameter. The coefficient of friction between belt and pulley is 0.2. **[Ans. 9 ; 0.9 m]**

Top

Contents

4. A-*V* belt is to transmit 20 kW from a 250 mm pitch diameter sheave to a 900 mm diameter pulley. The centre distance between the two shafts is 1000 mm. The groove angle is 40° and the coefficient of friction for the belt and sheave is 0.2 and the coefficient of friction between the belt and flat pulley is 0.2. The cross-section of the belt is 40 mm wide at the top, 20 mm wide at the bottom and 25 mm deep. The density of the belt is 1000 kg / m^3 and the allowable tension per belt is 1000 N. Find the number of belts required. **[Ans. 3]**

5. Determine the number of *V*-belts required to transmit 30 kW power under the following conditions :

	Smaller pulley	*Larger pulley*
Speed	1120 r.p.m.	280 r.p.m.
Pitch diameter	225 mm	900 mm
Pulley groove angle	34°	34°

Maximum working load per belt	=	560 N
Coefficient of friction	=	0.15
Centre distance between pulleys	=	875 mm
Mass of belt	=	0.3 kg/m

[Ans. 7]

6. Determine the percentage increase in power capacity made possible in changing over from a flat belt drive to a *V*-belt drive. The diameter of the flat pulley is same as the pitch diameter of the grooved pulley. The pulley rotates at the same speed as the grooved pulley. The coefficient of friction for the grooved and flat belt is same and is 0.3. The *V*-belt pulley groove angle is 60°. The belts are of the same material and have same cross-sectional area. In each case, the angle of wrap is 150°. **[Ans. 217.52 %]**

7. A rope drive is required to transmit 750 kW from a pulley of 1 m diameter running at 450 r.p.m. The safe pull in each rope is 2250 N and the mass of the rope is 1 kg / m length. The angle of lap and the groove angle is 150° and 45° respectively. Find the number of ropes required for the drive if the coefficient of friction between the rope and the pulley is 0.3. **[Ans. 22]**

8. Following data is given for a rope pulley transmitting 24 kW :

   Diameter of pulley = 400 mm ; Speed = 110 r.p.m ; Angle of groove = 45° ; Angle of lap = 160° ; Coefficient of friction = 0.28 ; Number of ropes = 10 ; Mass in kg / m length of ropes = 53 C^2.

   The working tension is limited to 122 C^2; where C = girth (*i.e.* circumference) of rope in metres. Find the initial tension and diameter of each rope. **[Ans. 675.55 N ; 31.57 mm]**

9. Select a suitable wire rope to lift a load of 10 kN of debris from a well 60 m deep. The rope should have a factor of safety equal to 6. The weight of the bucket is 5 kN. The load is lifted up with a maximum speed of 150 metres/min which is attained in 1 second.

   Find also the stress induced in the rope due to starting with an initial slack of 250 mm. The average tensile strength of the rope may be taken as 590 d^2 newtons (where d is the rope diameter in mm) for 6 × 19 wire rope. The weight of the rope is 18.5 N/m.

   Take diameter of the wire (d_w) = 0.063 d, and area of the rope (A) = 0.38 d^2. **[Ans. 20 mm, 412 MPa]**

10. Suggest the suitable size of 6 × 19 hoisting steel wire rope for an inclined mine shaft of 1000 m length and inclination of the rails 60° with the horizontal. The weight of the loaded skip is 100 kN. The maximum acceleration is limited to 1.5 m/s^2. The diameter of the drum on which the rope is being wound may be taken as 80 times the diameter of the rope. The car friction is 20 N / kN of weight normal to the incline and friction of the rope on the guide roller is 50 N / kN of weight normal to the incline. Assume a factor of safety of 5. The following properties of 6 × 19 flexible hoisting rope are given :

    The diameter of the rope (d) is in mm. The weight of the rope per metre = 0.0334 d^2 N; breaking load = 500 d^2 N; wire diameter = 0.063 d mm; area of wires in rope = 0.38 d^2 mm^2; equivalent elastic modulus = 82 kN/mm^2. **[Ans. 105 mm]**

Top

QUESTIONS

1. Sketch the cross-section of a *V*-belt and label its important parts.
2. What are the advantages and disadvantages of *V*-belt drive over flat belt drive?
3. Derive the relation for the ratio of driving tensions of a *V*-belt.
4. Describe the fibre ropes. What are its advantages? Draw a neat proportionate sketch of a sheave for fibre ropes.
5. Under what circumstances a fibre rope and a wire rope is used ? What are the advantages of a wire rope over fibre rope ?
6. Discuss the uses and construction of wire ropes. How are wire-rope ends fastened ?
7. Give the application of the following wire ropes :

   (*a*) 6 × 7 rope (*b*) 6 × 19 rope, and (*c*) 6 × 37 rope.

OBJECTIVE TYPE QUESTIONS

1. The included angle for the *V*-belt is usually
   (*a*) 20° – 30°
   (*b*) 30° – 40°
   (*c*) 40° – 60°
   (*d*) 60° – 80°
2. The *V*-belts are particularly suitable for .... drives.
   (*a*) short
   (*b*) long
3. The groove angle of the pulley for *V*-belt drive is usually
   (*a*) 20° – 25°
   (*b*) 25° – 32°
   (*c*) 32° – 38°
   (*d*) 38° – 45°
4. A *V*-belt designated by *A*-914-50 denotes
   (*a*) a standard belt
   (*b*) an oversize belt
   (*c*) an undersize belt
   (*d*) none of these
5. The wire ropes make contact at
   (*a*) bottom of groove of the pulley
   (*b*) sides of groove of the pulley
   (*c*) sides and bottom of groove of the pulley
   (*d*) any where in the groove of the pulley

This heavy duty crane moves within shopfloor on fixed rails.

ANSWERS

1. (*b*)	**2.** (*a*)	**3.** (*c*)	**4.** (*a*)	**5.** (*a*)