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Chain Drives

21.1 Introduction

We have seen in previous chapters on belt and rope drives that slipping may occur. In order to avoid slipping, steel chains are used. The chains are made up of number of rigid links which are hinged together by pin joints in order to provide the necessary flexibility for wraping round the driving and driven wheels. These wheels have projecting teeth of special profile and fit into the corresponding recesses in the links of the chain as shown in Fig. 21.1. The toothed wheels are known as ****sprocket wheels or simply sprockets.*** The sprockets and the chain are thus constrained to move together without slipping and ensures perfect velocity ratio.

* These wheels resemble to spur gears.

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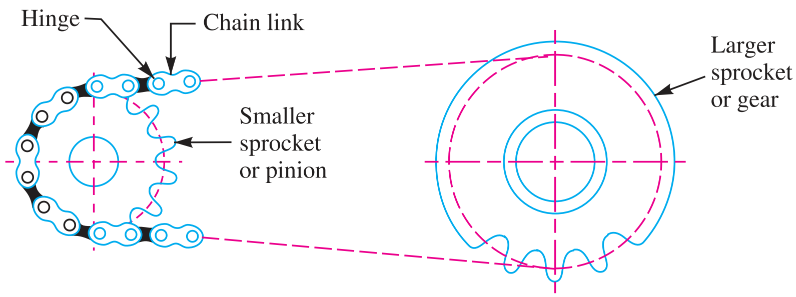


Fig. 21.1. Sprockets and chain.

The chains are mostly used to transmit motion and power from one shaft to another, when the centre distance between their shafts is short such as in bicycles, motor cycles, agricultural machinery, conveyors, rolling mills, road rollers etc. The chains may also be used for long centre distance of upto 8 metres. The chains are used for velocities up to 25 m / s and for power upto 110 kW. In some cases, higher power transmission is also possible.

21.2 Advantages and Disadvantages of Chain Drive over Belt or Rope Drive

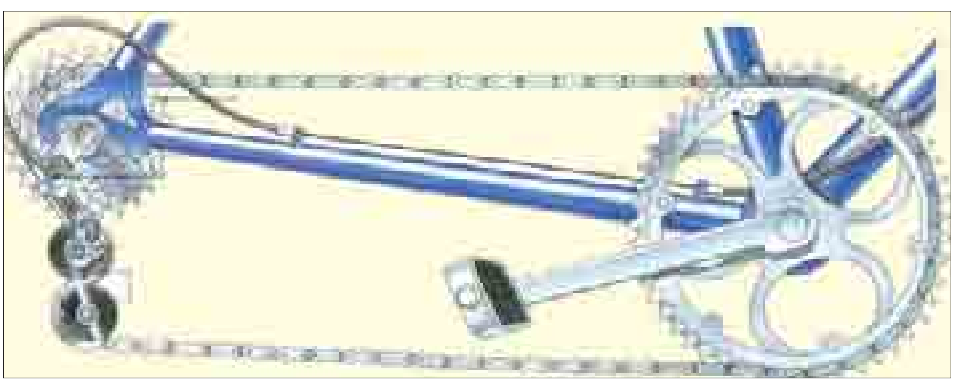
Following are the advantages and disadvantages of chain drive over belt or rope drive:

Advantages

1. As no slip takes place during chain drive, hence perfect velocity ratio is obtained.
2. Since the chains are made of metal, therefore they occupy less space in width than a belt or rope drive.
3. It may be used for both long as well as short distances.
4. It gives a high transmission efficiency (upto 98 percent).
5. It gives less load on the shafts.
6. It has the ability to transmit motion to several shafts by one chain only.
7. It transmits more power than belts.
8. It permits high speed ratio of 8 to 10 in one step.
9. It can be operated under adverse temperature and atmospheric conditions.

Disadvantages

1. The production cost of chains is relatively high.
2. The chain drive needs accurate mounting and careful maintenance, particularly lubrication and slack adjustment.
3. The chain drive has velocity fluctuations especially when unduly stretched.



Sports bicycle gear and chain drive mechanism

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21.3 Terms Used in Chain Drive

The following terms are frequently used in chain drive.

1. ***Pitch of chain.*** It is the distance between the hinge centre of a link and the corresponding hinge centre of the adjacent link, as shown in Fig. 21.2. It is usually denoted by *p*.

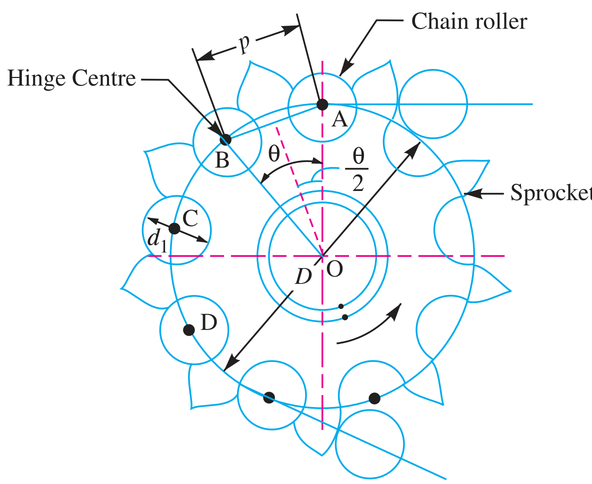

Fig. 21.2. Terms used in chain drive.

2. ***Pitch circle diameter of chain sprocket.*** It is the diameter of the circle on which the hinge centres of the chain lie, when the chain is wrapped round a sprocket as shown in Fig. 21.2. The points *A*, *B*, *C*, and *D* are the hinge centres of the chain and the circle drawn through these centres is called pitch circle and its diameter (*D*) is known as pitch circle diameter.

21.4 Relation Between Pitch and Pitch Circle Diameter

A chain wrapped round the sprocket is shown in Fig. 21.2. Since the links of the chain are rigid, therefore pitch of the chain does not lie on the arc of the pitch circle. The pitch length becomes a chord. Consider one pitch length *AB* of the chain subtending an angle θ at the centre of sprocket (or pitch circle),

Let D = Diameter of the pitch circle, and

T = Number of teeth on the sprocket.

From Fig. 21.2, we find that pitch of the chain,

$$p = AB = 2\,AO \sin\left(\frac{\theta}{2}\right) = 2 \times \left(\frac{D}{2}\right) \sin\left(\frac{\theta}{2}\right) = D \sin\left(\frac{\theta}{2}\right)$$

We know that $$\theta = \frac{360^\circ}{T}$$

$$\therefore \quad p = D \sin\left(\frac{360^\circ}{2\,T}\right) = D \sin\left(\frac{180^\circ}{T}\right)$$

or $$D = p \operatorname{cosec}\left(\frac{180^\circ}{T}\right)$$

The sprocket outside diameter (D_o), for satisfactory operation is given by

$$D_o = D + 0.8\, d_1$$

where d_1 = Diameter of the chain roller.

Note: The angle θ/2 through which the link swings as it enters contact is called **angle of articulation.**

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21.5 Velocity Ratio of Chain Drives

The velocity ratio of a chain drive is given by

$$V.R. = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

where N_1 = Speed of rotation of smaller sprocket in r.p.m.,

N_2 = Speed of rotation of larger sprocket in r.p.m.,

T_1 = Number of teeth on the smaller sprocket, and

T_2 = Number of teeth on the larger sprocket.

The average velocity of the chain is given by

$$v = \frac{\pi D N}{60} = \frac{T\ p\ N}{60}$$

where D = Pitch circle diameter of the sprocket in metres, and

p = Pitch of the chain in metres.

21.6 Length of Chain and Centre Distance

An open chain drive system connecting the two sprockets is shown in Fig. 21.3.

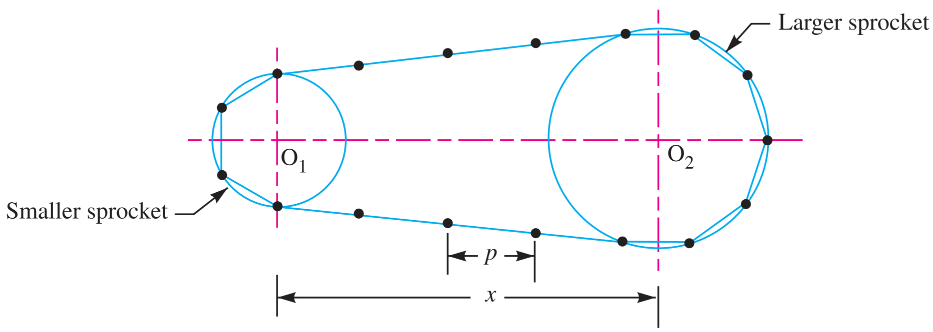


Fig. 21.3. Length of chain.

Let T_1 = Number of teeth on the smaller sprocket,

T_2 = Number of teeth on the larger sprocket,

p = Pitch of the chain, and

x = Centre distance.

The length of the chain (L) must be equal to the product of the number of chain links (K) and the pitch of the chain (p). Mathematically,

$$L = K.p$$

The number of chain links may be obtained from the following expression, *i.e.*

$$K = \frac{T_1 + T_2}{2} + \frac{2\ x}{p} + \left[\frac{T_2 - T_1}{2\ \pi}\right]^2 \frac{p}{x}$$

The value of K as obtained from the above expression must be approximated to the nearest even number.

The centre distance is given by

$$x = \frac{p}{4}\left[K - \frac{T_1 + T_2}{2} + \sqrt{\left(K - \frac{T_1 + T_2}{2}\right)^2 - 8\left(\frac{T_2 - T_1}{2\ \pi}\right)^2}\right]$$

In order to accommodate initial sag in the chain, the value of the centre distance obtained from the above equation should be decreased by 2 to 5 mm.

Notes: 1. The minimum centre distance for the velocity transmission ratio of 3, may be taken as

$$x_{min} = \frac{d_1 + d_2}{2} + 30 \text{ to } 50 \text{ mm}$$

where d_1 and d_2 are the diameters of the pitch circles of the smaller and larger sprockets.

2. For best results, the minimum centre distance should be 30 to 50 times the pitch.

3. The minimum centre distance is selected depending upon the velocity ratio so that the arc of contact of the chain on the smaller sprocket is not less than 120°. It may be noted that larger angle of arc of contact ensures a more uniform distribution of load on the sprocket teeth and better conditions of engagement.

21.7 Classification of Chains

The chains, on the basis of their use, are classified into the following three groups:

1. Hoisting and hauling (or crane) chains,

2. Conveyor (or tractive) chains, and

3. Power transmitting (or driving) chains.

These chains are discussed, in detail, in the following pages.

21.8 Hoisting and Hauling Chains

These chains are used for hoisting and hauling purposes and operate at a maximum velocity of 0.25 m / s. The hoisting and hauling chains are of the following two types:

1. *Chain with oval links*. The links of this type of chain are of oval shape, as shown in Fig. 21.4 (*a*). The joint of each link is welded. The sprockets which are used for this type of chain have receptacles to receive the links. Such type of chains are used only at low speeds such as in chain hoists and in anchors for marine works.

(*a*) Chain with oval links. (*b*) Chain with square links.

Fig. 21.4. Hoisting and hauling chains.

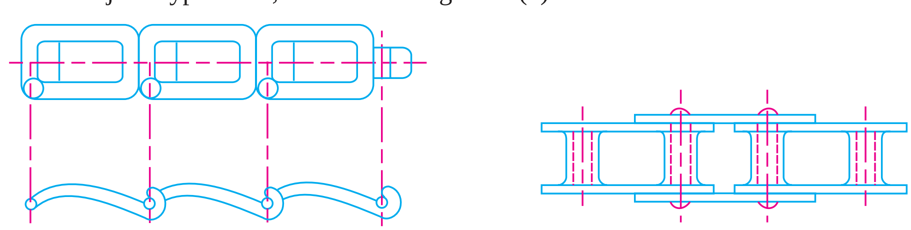
2. *Chain with square links*. The links of this type of chain are of square shape, as shown in Fig. 21.4 (*b*). Such type of chains are used in hoists, cranes, dredges. The manufacturing cost of this type of chain is less than that of chain with oval links, but in these chains, the kinking occurs easily on overloading.

21.9 Conveyor Chains

These chains are used for elevating and conveying the materials continuously at a speed upto 2 m / s. The conveyor chains are of the following two types:

1. Detachable or hook joint type chain, as shown in Fig. 21.5 (*a*), and

2. Closed joint type chain, as shown in Fig. 21.5 (*b*).



(*a*) Detachable or hook joint type chain. (*b*) Closed joint type chain.

Fig. 21.5. Conveyor chains.

The conveyor chains are usually made of malleable cast iron. These chains do not have smooth running qualities. The conveyor chains run at slow speeds of about 0.8 to 3 m / s.

21.10 Power Transmitting Chains

These chains are used for transmission of power, when the distance between the centres of shafts is short. These chains have provision for efficient lubrication. The power transmitting chains are of the following three types.

1. *Block or bush chain.* A block or bush chain is shown in Fig. 21.6. This type of chain was used in the early stages of development in the power transmission.

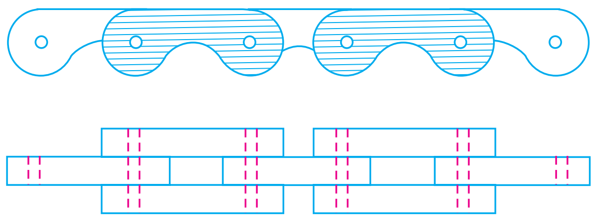


Fig. 21.6. Block or bush chain.

It produces noise when approaching or leaving the teeth of the sprocket because of rubbing between the teeth and the links. Such type of chains are used to some extent as conveyor chain at small speed.

2. *Bush roller chain.* A bush roller chain as shown in Fig. 21.7, consists of outer plates or pin link plates, inner plates or roller link plates, pins, bushes and rollers. A pin passes through the bush which is secured in the holes of the roller between the two sides of the chain. The rollers are free to rotate on the bush which protect the sprocket wheel teeth against wear. The pins, bushes and rollers are made of alloy steel.

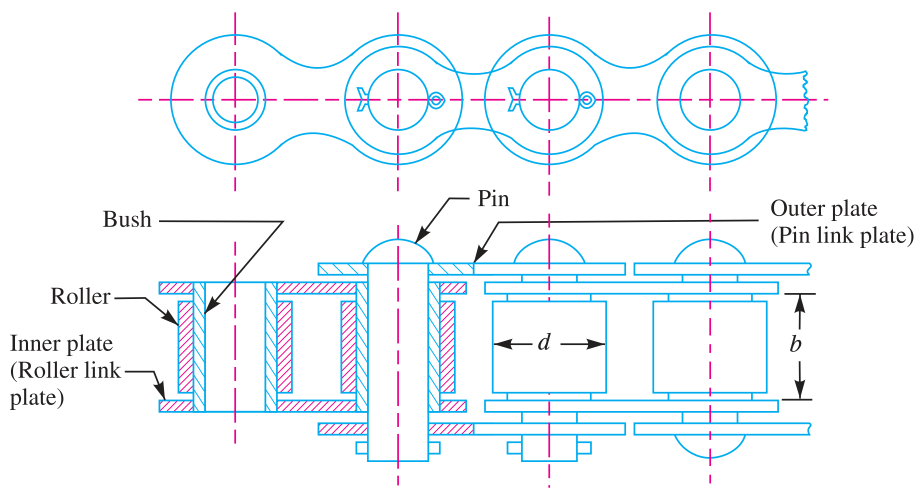


Fig. 21.7. Bush roller chain.

A bush roller chain is extremely strong and simple in construction. It gives good service under severe conditions. There is a little noise with this chain which is due to impact of the rollers on the sprocket wheel teeth. This chain may be used where there is a little lubrication. When one of these chains elongates slightly due to wear and stretching of the parts, then the extended chain is of greater pitch than the pitch of the sprocket wheel teeth. The rollers then fit unequally into the cavities of the wheel. The result is that the total load falls on one teeth or on a few teeth. The stretching of the parts increase wear of the surfaces of the roller and of the sprocket wheel teeth.

Rear wheel chain drive of a motorcycle

The roller chains are standardised and manufactured on the basis of pitch. These chains are available in single-row or multi-row roller chains such as simple, duplex or triplex strands, as shown in Fig. 21.8.

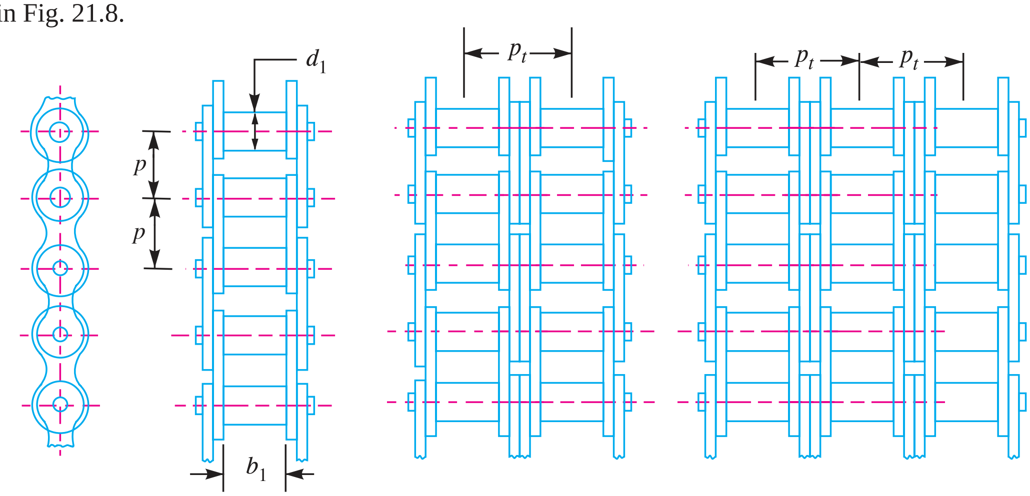


Fig. 21.8. Types of roller chain.

3. *Silent chain.* A silent chain (also known as ***inverted tooth chain***) is shown in Fig. 21.9.

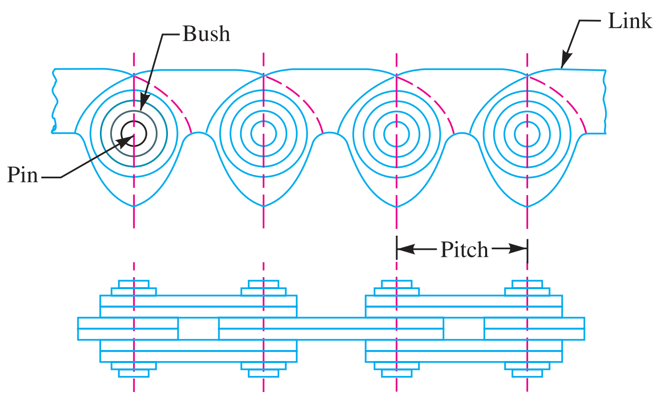


Fig. 21.9. Silent chain.

It is designed to eliminate the evil effects caused by stretching and to produce noiseless running. When the chain stretches and the pitch of the chain increases, the links ride on the teeth of the sprocket wheel at a slightly increased radius. This automatically corrects the small change in the pitch. There is no relative sliding between the teeth of the inverted tooth chain and the sprocket wheel teeth. When properly lubricated, this chain gives durable service and runs very smoothly and quietly.

The various types of joints used in a silent chain are shown in Fig. 21.10.

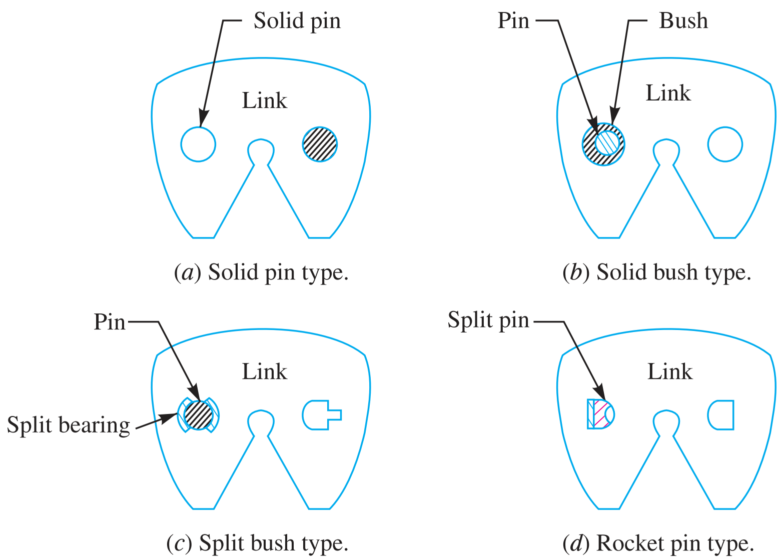


Fig. 21.10. Silent chain joints.

21.11 Characteristics of Roller Chains

According to Indian Standards (IS: 2403 —1991), the various characteristics such as pitch, roller diameter, width between inner plates, transverse pitch and breaking load for the roller chains are given in the following table.

Table 21.1. Characteristics of roller chains according to IS: 2403 — 1991.

ISO Chain number	*Pitch (p) mm*	*Roller diameter (d_1) mm Maximum*	*Width between inner plates (b_1) mm Maximum*	*Transverse pitch (p_1) mm*	*Breaking load (kN) Minimum*		
					Simple	*Duplex*	*Triplex*
05 B	8.00	5.00	3.00	5.64	4.4	7.8	11.1
06 B	9.525	6.35	5.72	10.24	8.9	16.9	24.9
08 B	12.70	8.51	7.75	13.92	17.8	31.1	44.5
10 B	15.875	10.16	9.65	16.59	22.2	44.5	66.7
12 B	19.05	12.07	11.68	19.46	28.9	57.8	86.7
16 B	25.4	15.88	17.02	31.88	42.3	84.5	126.8
20 B	31.75	19.05	19.56	36.45	64.5	129	193.5
24 B	38.10	25.40	25.40	48.36	97.9	195.7	293.6
28 B	44.45	27.94	30.99	59.56	129	258	387
32 B	50.80	29.21	30.99	68.55	169	338	507.10
40 B	63.50	39.37	38.10	72.29	262.4	524.9	787.3
48 B	76.20	48.26	45.72	91.21	400.3	800.7	1201

21.12 Factor of Safety for Chain Drives

The factor of safety for chain drives is defined as the ratio of the breaking strength (W_B) of the chain to the total load on the driving side of the chain (W). Mathematically,

$$\text{Factor of safety} = \frac{W_B}{W}$$

The breaking strength of the chain may be obtained by the following empirical relations, *i.e.*

$$W_B = 106\,p^2 \text{ (in newtons) for roller chains}$$

$$= 106\,p \text{ (in newtons) per mm width of chain for silent chains.}$$

where p is the pitch in mm.

The total load (or total tension) on the driving side of the chain is the sum of the tangential driving force (F_T), centrifugal tension in the chain (F_C) and the tension in the chain due to sagging (F_S).

We know that the tangential driving force acting on the chain,

$$F_T = \frac{\text{Power transmitted (in watts)}}{\text{Speed of chain in m/s}} = \frac{P}{v} \text{ (in newtons)}$$

Centrifugal tension in the chain,

$$F_C = m.v^2 \text{ (in newtons)}$$

and tension in the chain due to sagging,

$$F_S = k.mg.x \text{ (in newtons)}$$

where m = Mass of the chain in kg per metre length,

x = Centre distance in metres, and

k = Constant which takes into account the arrangement of chain drive

= 2 to 6, when the centre line of the chain is inclined to the horizontal at an angle less than 40º

= 1 to 1.5, when the centre line of the chain is inclined to the horizontal at an angle greater than 40º.

The following table shows the factor of safety for the bush roller and silent chains depending upon the speed of the sprocket pinion in r.p.m. and pitch of the chains.

Table 21.2. Factor of safety (n) for bush roller and silent chains.

Type of chain	*Pitch of chain (mm)*	*Speed of the sprocket pinion in r.p.m.*								
		50	200	400	600	800	1000	1200	1600	2000
Bush roller chain	12 – 15	7	7.8	8.55	9.35	10.2	11	11.7	13.2	14.8
	20 – 25	7	8.2	9.35	10.3	11.7	12.9	14	16.3	–
	30 – 35	7	8.55	10.2	13.2	14.8	16.3	19.5	–	–
Silent chain	12.7 – 15.87	20	22.2	24.4	28.7	29.0	31.0	33.4	37.8	42.0
	19.05 – 25.4	20	23.4	26.7	30.0	33.4	36.8	40.0	46.5	53.5

21.13 Permissible Speed of Smaller Sprocket

The following table shows the permissible speed of the smaller sprocket or pinion (in r.p.m.) for the bush roller and silent chain corresponding to different pitches.

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Common bicycle is the best example of a chain drive

Table 21.3. Permissible speed of smaller sprocket or pinion in r.p.m.

Type of Chain	*Number of teeth on sprocket pinion*	*Pitch of chain (p) in mm*				
		12	15	20	25	30
Bush roller chain	15	2300	1900	1350	1150	1000
	19	2400	2000	1450	1200	1050
	23	2500	2100	1500	1250	1100
	27	2550	2150	1550	1300	1100
	30	2600	2200	1550	1300	1100
Silent chain	17 – 35	3300	2650	2200	1650	1300

Note: The chain velocity for the roller chains may be as high as 20 m / s, if the chains are properly lubricated and enclosed, whereas the silent chain may be operated upto 40 m / s.

21.14 Power Transmitted by Chains

The power transmitted by the chain on the basis of breaking load is given by

$$P = \frac{W_B \times v}{n \times K_S} \quad \text{(in watts)}$$

where W_b = Breaking load in newtons,

v = Velocity of chain in m/s

n = Factor of safety, and

K_S = Service factor = $K_1.K_2.K_3$

The power transmitted by the chain on the basis of bearing stress is given by

$$P = \frac{\sigma_b \times A \times v}{K_S}$$

where σ_b = Allowable bearing stress in MPa or N/mm^2,

A = Projected bearing area in mm^2,

v = Velocity of chain in m/s, and

K_S = Service factor.

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The power rating for simple roller chains depending upon the speed of the smaller sprocket is shown in the following table.

Table 21.4. Power rating (in kW) of simple roller chain.

Speed of smaller sprocket or pinion (r.p.m.)	*Power (kW)*				
	06 B	08 B	10 B	12 B	16 B
100	0.25	0.64	1.18	2.01	4.83
200	0.47	1.18	2.19	3.75	8.94
300	0.61	1.70	3.15	5.43	13.06
500	1.09	2.72	5.01	8.53	20.57
700	1.48	3.66	6.71	11.63	27.73
1000	2.03	5.09	8.97	15.65	34.89
1400	2.73	6.81	11.67	18.15	38.47
1800	3.44	8.10	13.03	19.85	–
2000	3.80	8.67	13.49	20.57	–

The service factor (K_S) is the product of various factors, such as load factor (K_1), lubrication factor (K_2) and rating factor (K_3). The values of these factors are taken as follows:

1. Load factor (K_1) = 1, for constant load
= 1.25, for variable load with mild shock
= 1.5, for heavy shock loads

2. Lubrication factor (K_2) = 0.8, for continuous lubrication
= 1, for drop lubrication
= 1.5, for periodic lubrication

3. Rating factor (K_3) = 1, for 8 hours per day
= 1.25, for 16 hours per day
= 1.5, for continuous service

21.15 Number of Teeth on the Smaller or Driving Sprocket or Pinion

Consider an arrangement of a chain drive in which the smaller or driving sprocket has only four teeth, as shown in Fig. 21.11 (*a*). Let the sprocket rotates anticlockwise at a constant speed of *N* r.p.m. The chain link *AB* is at a distance of *d* / 2 from the centre of the sprocket and its linear speed is given by

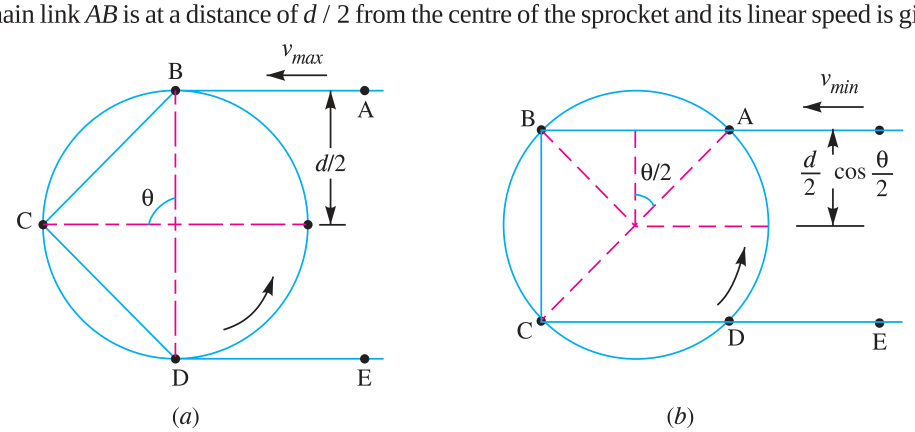

Fig. 21.11. Number of teeth on the smaller sprocket.

$$v_{max} = \frac{\pi \, d \, N}{60} \text{ m / s}$$

where d = Pitch circle diameter of the smaller or driving sprocket in metres.

When the sprocket rotates through an angle θ/2, the link *AB* occupies the position as shown in Fig. 21.11 (*b*). From the figure, we see that the link is now at a distance of $\left(\frac{d}{2} \times \cos \frac{\theta}{2}\right)$ from the centre of the sprocket and its linear velocity is given by

$$v_{min} = \frac{\pi \, d \, N \cos \theta / 2}{60} \text{ m/s}$$

From above, we see that the linear velocity of the sprocket is not uniform but varies from maximum to minimum during every cycle of tooth engagement. This results in fluctuations in chain transmission and may be minimised by reducing the angle θ or by increasing the number of teeth on the sprocket. It has been observed that for a sprocket having 11 teeth, the variation of speed is 4 percent and for the sprockets having 17 teeth and 24 teeth, the variation of speed is 1.6 percent and 1 percent respectively.

In order to have smooth operation, the minimum number of teeth on the smaller sprocket or pinion may be taken as 17 for moderate speeds and 21 for high speeds. The following table shows the number of teeth on a smaller sprocket for different velocity ratios.

Table 21.5. Number of teeth on the smaller sprocket.

Type of chain	*Number of teeth at velocity ratio*					
	1	2	3	4	5	6
Roller	31	27	25	23	21	17
Silent	40	35	31	27	23	19

Note: The number of teeth on the smaller sprocket plays an important role in deciding the performance of a chain drive. A small number of teeth tends to make the drive noisy. A large number of teeth makes chain pitch smaller which is favourable for keeping the drive silent and reducing shock, centrifugal force and friction force.

21.16 Maximum Speed for Chains

The maximum allowable speed for the roller and silent chains, depending upon the number of teeth on the smaller sprocket or pinion and the chain pitch is shown in the following table.

Table 21.6. Maximum allowable speed for chains in r.p.m.

Type of chain	*Number of teeth on the smaller sprocket (T_1)*	*Chain pitch (p) in mm*				
		12	15	20	25	30
Roller chain	15	2300	1900	1350	1150	1100
	19	2400	2000	1450	1200	1050
	23	2500	2100	1500	1250	1100
	27	2550	2150	1550	1300	1100
	30	2600	2200	1550	1300	1100
Silent chain	17–35	3300	2650	2200	1650	1300

Note: The r.p.m. of the sprocket reduces as the chain pitch increases for a given number of teeth.

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21.17 Principal Dimensions of Tooth Profile

The standard profiles for the teeth of a sprocket are shown in Fig. 21.12. According to Indian Standards (IS: 2403 – 1991), the principal dimensions of the tooth profile are as follows:

1. Tooth flank radius (r_e)

$$= 0.008\ d_1\ (T^2 + 180) \qquad \text{...(Maximum)}$$

$$= 0.12\ d_1\ (T + 2) \qquad \text{...(Minimum)}$$

where d_1 = Roller diameter, and

T = Number of teeth.

2. Roller seating radius (r_i)

$$= 0.505\ d_1 + 0.069\ \sqrt[3]{d_1} \qquad \text{...(Maximum)}$$

$$= 0.505\ d_1 \qquad \text{...(Minimum)}$$

3. Roller seating angle (α)

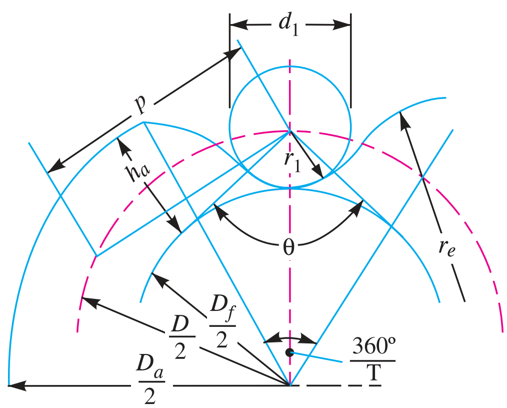
$$= 140° - \frac{90°}{T} \qquad \text{...(Maximum)}$$

$$= 120° - \frac{90°}{T} \qquad \text{...(Minimum)}$$

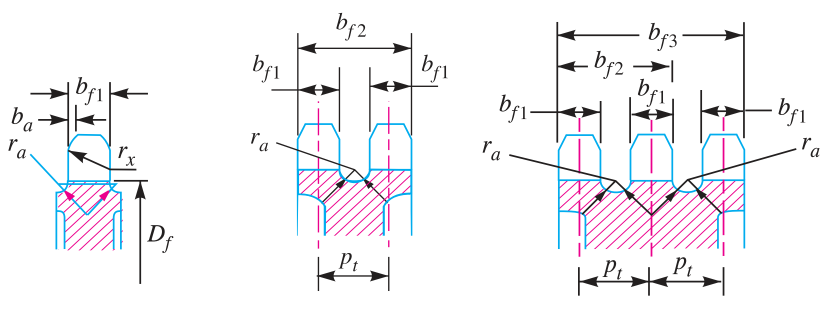
4. Tooth height above the pitch polygon (h_a)

$$= 0.625\ p - 0.5\ d_1 + \frac{0.8\ p}{T} \qquad \text{...(Maximum)}$$

$$= 0.5\ (\ p - d_1) \qquad \text{...(Minimum)}$$



(*a*) Tooth profile of sprocket.



(*b*) Rim profile of sprocket.

Fig. 21.12

Top

5. Pitch circle diameter (D)

$$= \frac{p}{\sin\left(\frac{180}{T}\right)} = p \operatorname{cosec}\left(\frac{180}{T}\right)$$

6. Top diameter (D_a)

$$= D + 1.25\,p - d_1 \qquad \text{...(Maximum)}$$

$$= D + p\left(1 - \frac{1.6}{T}\right) - d_1 \qquad \text{...(Minimum)}$$

7. Root diameter (D_f)

$$= D - 2\,r_i$$

8. Tooth width (b_{f1})

$$= 0.93\,b_1 \text{ when } p \le 12.7 \text{ mm}$$

$$= 0.95\,b_1 \text{ when } p > 12.7 \text{ mm}$$

9. Tooth side radius (r_x) = p

10. Tooth side relief (b_a)

$$= 0.1\,p \text{ to } 0.15\,p$$

11. Widths over teeth (b_{f2} and b_{f3})

$$= (\text{Number of strands} - 1)\,p_t + b_{f1}$$

Chain drive of an automobile

21.18 Design Procedure of Chain Drive

The chain drive is designed as discussed below:

1. First of all, determine the velocity ratio of the chain drive.

2. Select the minimum number of teeth on the smaller sprocket or pinion from Table 21.5.

3. Find the number of teeth on the larger sprocket.

4. Determine the design power by using the service factor, such that

Design power = Rated power × Service factor

5. Choose the type of chain, number of strands for the design power and r.p.m. of the smaller sprocket from Table 21.4.

6. Note down the parameters of the chain, such as pitch, roller diameter, minimum width of roller etc. from Table 21.1.

7. Find pitch circle diameters and pitch line velocity of the smaller sprocket.

8. Determine the load (W) on the chain by using the following relation, *i.e.*

$$W = \frac{\text{Rated power}}{\text{Pitch line velocity}}$$

9. Calculate the factor of safety by dividing the breaking load (W_B) to the load on the chain (W). This value of factor of safety should be greater than the value given in Table 21.2.

10. Fix the centre distance between the sprockets.

11. Determine the length of the chain.

12. The other dimensions may be fixed as given in Art. 21.17.

Example 21.1. *Design a chain drive to actuate a compressor from 15 kW electric motor running at 1000 r.p.m., the compressor speed being 350 r.p.m. The minimum centre distance is 500 mm. The compressor operates 16 hours per day. The chain tension may be adjusted by shifting the motor on slides.*

Solution. Given : Rated power = 15 kW ; N_1 = 1000 r.p.m ; N_2 = 350 r.p.m.

We know that the velocity ratio of chain drive,

$$V.R. = \frac{N_1}{N_2} = \frac{1000}{350} = 2.86 \text{ say } 3$$

From Table 21.5, we find that for the roller chain, the number of teeth on the smaller sprocket or pinion (T_1) for a velocity ratio of 3 are 25.

∴ Number of teeth on the larger sprocket or gear,

$$T_2 = T_1 \times \frac{N_1}{N_2} = 25 \times \frac{1000}{350} = 71.5 \text{ say } 72 \textbf{ Ans.}$$

We know that the design power

= Rated power × Service factor (K_S)

The service factor (K_S) is the product of various factors K_1, K_2 and K_3. The values of these factors are taken as follows:

Load factor (K_1) for variable load with heavy shock

= 1.5

Lubrication factor (K_2) for drop lubrication

= 1

Rating factor (K_3) for 16 hours per day

= 1.25

∴ Service factor, $K_S = K_1.K_2.K_3 = 1.5 \times 1 \times 1.25 = 1.875$

and design power = 15 × 1.875 = 28.125 kW

From Table 21.4, we find that corresponding to a pinion speed of 1000 r.p.m. the power transmitted for chain No. 12 is 15.65 kW per strand. Therefore, a chain No. 12 with two strands can be used to transmit the required power. From Table 21.1, we find that

Pitch, p = 19.05 mm

Chain drive

Roller diameter, $d = 12.07$ mm

Minimum width of roller,

$$w = 11.68 \text{ mm}$$

Breaking load, $W_B = 59 \text{ kN} = 59 \times 10^3$ N

We know that pitch circle diameter of the smaller sprocket or pinion,

$$d_1 = p \operatorname{cosec}\left(\frac{180}{T_1}\right) = 19.05 \operatorname{cosec}\left(\frac{180}{25}\right) \text{ mm}$$

$$= 19.05 \times 7.98 = 152 \text{ mm} = 0.152 \text{ m } \textbf{Ans.}$$

and pitch circle diameter of the larger sprocket or gear

$$d_2 = p \operatorname{cosec}\left(\frac{180}{T_2}\right) = 19.05 \operatorname{cosec}\left(\frac{180}{72}\right) \text{ mm}$$

$$= 19.05 \times 22.9 = 436 \text{ mm} = 0.436 \text{ m } \textbf{Ans.}$$

Pitch line velocity of the smaller sprocket,

$$v_1 = \frac{\pi\, d_1\, N_1}{60} = \frac{\pi \times 0.152 \times 1000}{60} = 7.96 \text{ m/s}$$

∴ Load on the chain,

$$W = \frac{\text{Rated power}}{\text{Pitch line velocity}} = \frac{15}{7.96} = 1.844 \text{ kN} = 1844 \text{ N}$$

and factor of safety $= \frac{W_B}{W} = \frac{59 \times 10^3}{1844} = 32$

This value is more than the value given in Table 21.2, which is equal to 11.

The minimum centre distance between the smaller and larger sprockets should be 30 to 50 times the pitch. Let us take it as 30 times the pitch.

∴ Centre distance between the sprockets,

$$= 30\, p = 30 \times 19.05 = 572 \text{ mm}$$

In order to accomodate initial sag in the chain, the value of centre distance is reduced by 2 to 5 mm.

∴ Correct centre distance

$$x = 572 - 4 = 568 \text{ mm}$$

We know that the number of chain links

$$K = \frac{T_1 + T_2}{2} + \frac{2\,x}{p} + \left[\frac{T_2 - T_1}{2\,\pi}\right]^2 \frac{p}{x}$$

$$= \frac{25 + 72}{2} + \frac{2 \times 568}{19.05} + \left[\frac{72 - 25}{2\,\pi}\right]^2 \frac{19.05}{568}$$

$$= 48.5 + 59.6 + 1.9 = 110$$

∴ Length of the chain,

$$L = K.p = 110 \times 19.05 = 2096 \text{ mm} = 2.096 \text{ m } \textbf{Ans.}$$

EXERCISES

1. Design a roller chain to transmit power from a 20 kW motor to a reciprocating pump. The pump is to operate continuously 24 hours per day. The speed of the motor is 600 r.p.m. and that of the pump is 200 r.p.m. Find: 1. number of teeth on each sprocket; 2. pitch and width of the chain.
2. Design a chain drive to run a blower at 600 r.p.m. The power to the blower is available from a 8 kW motor at 1500 r.p.m. The centre distance is to be kept at 800 mm.

3. A chain drive using bush roller chain transmits 5.6 kW of power. The driving shaft on an electric motor runs at 1440 r.p.m. and velocity ratio is 5. The centre distance of the drive is restricted to 550 ± 2% mm and allowable pressure on the pivot joint is not to exceed 10 N/mm^2. The drive is required to operate continuously with periodic lubrication and driven machine is such that load can be regarded as fairly constant with jerk and impact. Design the chain drive by calculating leading dimensions, number of teeth on the sprocket and specify the breaking strength of the chain. Assume a factor of safety of 13.

QUESTIONS

1. State the advantages and disadvantages of the chain drive over belt and rope drive.
2. Explain, with the help of a neat sketch, the construction of a roller chain.
3. What do you understand by simplex, duplex and triplex chains?
4. Write in brief on
   (*a*)Hoisting and hauling chains,
   (*b*)Conveyor chais, and
   (*c*)Silent chains.
5. Write the design procedure for a chain drive.

OBJECTIVE TYPE QUESTIONS

1. Which one of the following is a positive drive?
   (*a*) Crossed flat belt drive (*b*) Rope drive
   (*c*) *V*-belt drive (*d*) Chain drive
2. The chain drive transmits .... power as compared to belt drive.
   (*a*) more (*b*) less
3. The relation between the pitch of the chain (p) and pitch circle diameter of the sprocket (D) is given by
   (*a*) $p = D \sin\left(\frac{90°}{T}\right)$ (*b*) $p = D \sin\left(\frac{120°}{T}\right)$
   (*c*) $p = D \sin\left(\frac{180°}{T}\right)$ (*d*) $p = D \sin\left(\frac{360°}{T}\right)$
   where T = Number of teeth on the spoocket.
4. In order to have smooth operation, the minimum number of teeth on the smaller sprocket, for moderate speeds, should be
   (*a*) 15 (*b*) 17
   (*c*) 21 (*d*) 25
5. The speed of the sprocket reduces as the chain pitch .... for a given number of teeth.
   (*a*) increases (*b*) decreases

ANSWERS

1. (*d*) **2.** (*a*) **3.** (*c*) **4.** (*b*) **5.** (*a*)

C H A P T E R

22

Flywheel

22.1 Introduction

A flywheel used in machines serves as a reservior which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than supply.

In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke. For example, in I.C. engines, the energy is developed only during power stroke which is much more than the engine load, and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus

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rotating the crankshaft at a uniform speed. A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.

In machines where the operation is intermittent like punching machines, shearing machines, riveting machines, crushers etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

Note: The function of a governor in engine is entirely different from that of a flywheel. It regulates the mean speed of an engine when there are variations in the load, *e.g.* when the load on the engine increases, it becomes necessary to increase the supply of working fluid. On the other hand, when the load decreases, less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load condition and keeps the mean speed within certain limits.

Flywheel stores energy when the supply is in excess, and releases energy when the supply is in deficit.

As discussed above, the flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. In other words, a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation. It does not control the speed variations caused by the varying load.

22.2 Coefficient of Fluctuation of Speed

The difference between the maximum and minimum speeds during a cycle is called the ***maximum fluctuation of speed.*** The ratio of the maximum fluctuation of speed to the mean speed is called ***coefficient of fluctuation of speed.***

Let N_1 = Maximum speed in r.p.m. during the cycle,

N_2 = Minimum speed in r.p.m. during the cycle, and

N = Mean speed in r.p.m. = $\dfrac{N_1 + N_2}{2}$

∴ Coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{...(In terms of linear speeds)}$$

The coefficient of fluctuation of speed is a limiting factor in the design of flywheel. It varies depending upon the nature of service to which the flywheel is employed. Table 22.1 shows the permissible values for coefficient of fluctuation of speed for some machines.

Note: The reciprocal of coefficient of fluctuation of speed is known as **coefficient of steadiness** and it is denoted by m.

$$\therefore \quad m = \frac{1}{C_S} = \frac{N}{N_1 - N_2} = \frac{\omega}{\omega_1 - \omega_2} = \frac{v}{v_1 - v_2}$$

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Table 22.1. Permissible values for coefficient of fluctuation of speed (C_S).

S.No.	*Type of machine or class of service*	*Coefficient of fluctuation of speed* (C_S)
1.	Crushing machines	0.200
2.	Electrical machines	0.003
3.	Electrical machines (direct drive)	0.002
4.	Engines with belt transmission	0.030
5.	Gear wheel transmission	0.020
6.	Hammering machines	0.200
7.	Pumping machines	0.03 to 0.05
8.	Machine tools	0.030
9.	Paper making, textile and weaving machines	0.025
10.	Punching, shearing and power presses	0.10 to 0.15
11.	Spinning machinery	0.10 to 0.020
12.	Rolling mills and mining machines	0.025

22.3 Fluctuation of Energy

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider a turning moment diagram for a single cylinder double acting steam engine as shown in Fig. 22.1. The vertical ordinate represents the turning moment and the horizontal ordinate (abscissa) represents the crank angle.

A little consideration will show that the turning moment is zero when the crank angle is zero. It rises to a maximum value when crank angle reaches 90° and it is again zero when crank angle is 180°. This is shown by the curve *abc* in Fig. 22.1 and it represents the turning moment diagram for outstroke. The curve *cde* is the turning moment diagram for instroke and is somewhat similar to the curve *abc*.

Since the work done is the product of the turning moment and the angle turned, therefore the area of the turning moment diagram represents the work done per revolution. In actual practice, the engine is assumed to work against the mean resisting torque, as shown by a horizontal line *AF*. The height of the ordinate *aA* represents the mean height of the turning moment diagram. Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore the area of the rectangle *aA Fe* is proportional to the work done against the mean resisting torque.

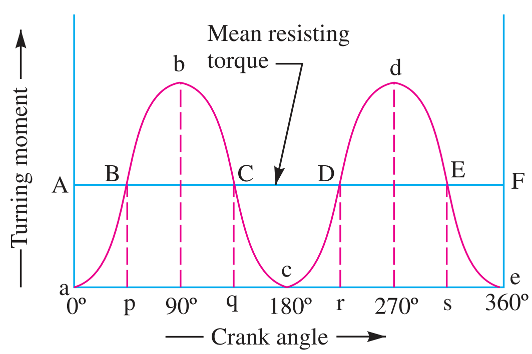

Fig. 22.1. Turning moment diagram for a single cylinder double acting steam engine.

We see in Fig. 22.1, that the mean resisting torque line *AF* cuts the turning moment diagram at points *B*, *C*, *D* and *E*. When the crank moves from '*a*' to '*p*' the work done by the engine is equal to the area *aBp*, whereas the energy required is represented by the area *aABp*. In other words, the engine has done less work (equal to the area *aAB*) than the requirement. This amount of energy is taken from the flywheel and hence the speed of the flywheel decreases. Now the crank moves from *p* to *q*, the work done by the engine is equal to the area *pBbCq*, whereas the requirement of energy is represented by the area *pBCq*. Therefore the engine has done more work than the requirement. This excess work (equal to the area *BbC*) is stored in the flywheel and hence the speed of the flywheel increases while the crank moves from *p* to *q*.

Similarly when the crank moves from *q* to *r*, more work is taken from the engine than is developed. This loss of work is represented by the area *CcD*. To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from *q* to *r*. As the crank moves from *r* to *s*, excess energy is again developed given by the area *DdE* and the speed again increases. As the piston moves from *s* to *e*, again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called ***fluctuation of energy.*** The areas *BbC*, *CcD*, *DdE* etc. represent fluctuations of energy.

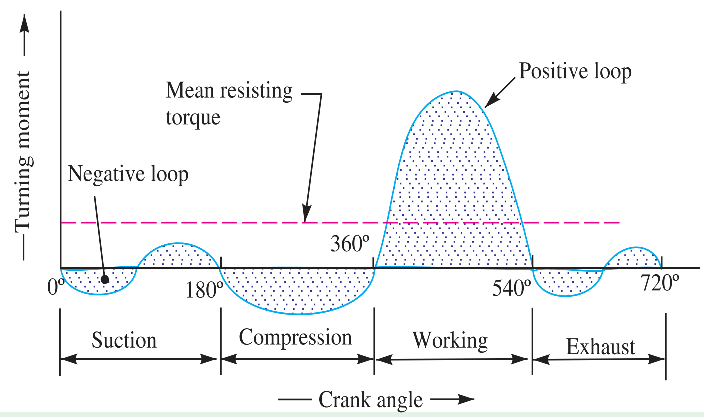

Fig. 22.2. Tunring moment diagram for a four stroke internal combustion engine.

A little consideration will show that the engine has a maximum speed either at *q* or at *s*. This is due to the fact that the flywheel absorbs energy while the crank moves from *p* to *q* and from *r* to *s*. On the other hand, the engine has a minimum speed either at *p* or at *r*. The reason is that the flywheel gives out some of its energy when the crank moves from *a* to *p* and from *q* to *r*. The difference between the maximum and the minimum energies is known as ***maximum fluctuation of energy.***

Flywheel shown as a separate part

A turning moment diagram for a four stroke internal combustion engine is shown in Fig. 22.2. We know that in a four stroke internal combustion engine, there is one working stroke after the crank has turned through 720° (or 4π radians). Since the pressure inside the engine cylinder is less than the atmospheric pressure during suction stroke, therefore a negative loop is formed as shown in Fig. 22.2. During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained. In the working stroke, the fuel burns and the gases expand, therefore a large positive loop is formed. During exhaust stroke, the work is done on the gases, therefore a negative loop is obtained.

A turning moment diagram for a compound steam engine having three cylinders and the resultant turning moment diagram is shown in Fig. 22.3. The resultant turning moment diagram is the sum of the turning moment diagrams for the three cylinders. It may be noted that the first cylinder is the high pressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low pressure cylinder. The cranks, in case of three cylinders are usually placed at 120º to each other.

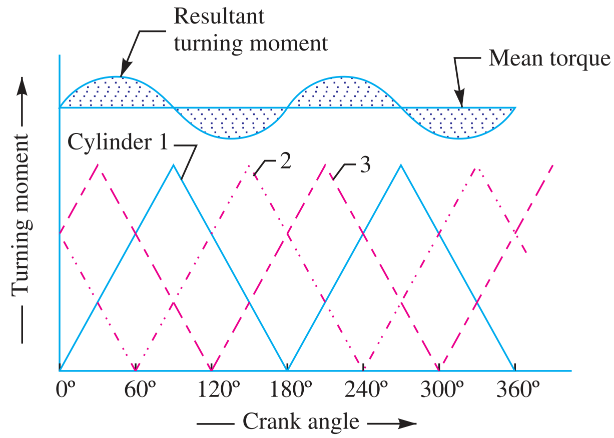

Fig. 22.3. Turning moment diagram for a compound steam engine.

22.4 Maximum Fluctuation of Energy

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. 22.4. The horizontal line *AG* represents the mean torque line. Let a_1, a_3, a_5 be the areas above the mean torque line and a_2, a_4 and a_6 be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

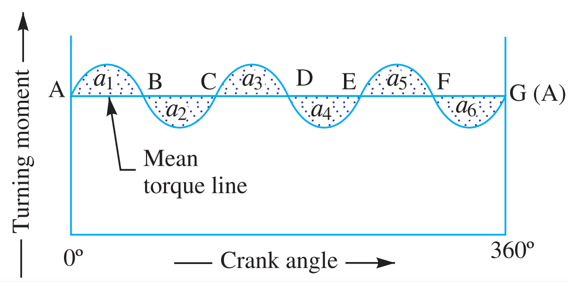

Fig. 22.4. Turning moment diagram for a multi-cylinder engine.

Let the energy in the flywheel at $A = E$, then from Fig. 22.4, we have

Energy at $B = E + a_1$

Energy at $C = E + a_1 - a_2$

Energy at $D = E + a_1 - a_2 + a_3$

Energy at $E = E + a_1 - a_2 + a_3 - a_4$

Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$ = Energy at A

Let us now suppose that the maximum of these energies is at *B* and minimum at *E*.

∴ Maximum energy in the flywheel

$$= E + a_1$$

and minimum energy in the flywheel

$$= E + a_1 - a_2 + a_3 - a_4$$

∴ Maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$$

22.5 Coefficient of Fluctuation of Energy

It is defined as the ratio of the maximum fluctuation of energy to the work done per cycle. It is usually denoted by C_E. Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

The workdone per cycle may be obtained by using the following relations:

1. Workdone / cycle $= T_{mean} \times \theta$

where T_{mean} = Mean torque, and

θ = Angle turned in radians per revolution

= 2π, in case of steam engines and two stroke internal combustion engines.

= 4π, in case of four stroke internal combustion engines.

The mean torque (T_{mean}) in N-m may be obtained by using the following relation *i.e.*

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where P = Power transmitted in watts,

N = Speed in r.p.m., and

ω = Angular speed in rad/s = $2\pi N / 60$

2. The workdone per cycle may also be obtained by using the following relation:

$$\text{Workdone / cycle} = \frac{P \times 60}{n}$$

where n = Number of working strokes per minute.

= N, in case of steam engines and two stroke internal combustion engines.

= $N / 2$, in case of four stroke internal combustion engines.

The following table shows the values of coefficient of fluctuation of energy for steam engines and internal combustion engines.

Table 22.2. Coefficient of fluctuation of energy (C_E) for steam and internal combustion engines.

S.No.	*Type of engine*	*Coefficient of fluctuation of energy* (C_E)
1.	Single cylinder, double acting steam engine	0.21
2.	Cross-compound steam engine	0.096
3.	Single cylinder, single acting, four stroke gas engine	1.93
4.	Four cylinder, single acting, four stroke gas engine	0.066
5.	Six cylinder, single acting, four stroke gas engine	0.031

22.6 Energy Stored in a Flywheel

A flywheel is shown in Fig. 22.5. We have already discussed that when a flywheel absorbs energy its speed increases and when it gives up energy its speed decreases.

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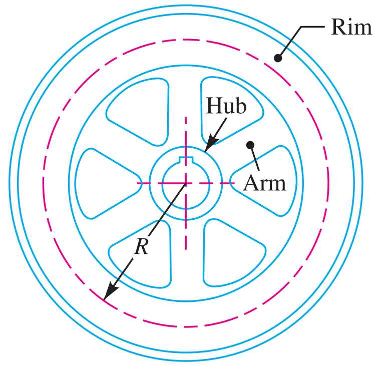

Fig. 22.5. Flywheel.

Let m = Mass of the flywheel in kg,

k = Radius of gyration of the flywheel in metres,

I = Mass moment of inertia of the flywheel about the axis of rotation in kg-m^2 $= m.k^2$,

N_1 and N_2 = Maximum and minimum speeds during the cycle in r.p.m.,

ω_1 and ω_2 = Maximum and minimum angular speeds during the cycle in rad / s,

N = Mean speed during the cycle in r.p.m. $= \dfrac{N_1 + N_2}{2}$,

ω = Mean angular speed during the cycle in rad / s $= \dfrac{\omega_1 + \omega_2}{2}$

C_S = Coefficient of fluctuation of speed $= \dfrac{N_1 - N_2}{N}$ or $\dfrac{\omega_1 - \omega_2}{\omega}$

We know that mean kinetic energy of the flywheel,

$$E = \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 \text{ (in N-m or joules)}$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = \text{Maximum K.E.} - \text{Minimum K.E.} = \frac{1}{2} \times I(\omega_1)^2 - \frac{1}{2} \times I(\omega_2)^2$$

$$= \frac{1}{2} \times I \left[(\omega_1)^2 - (\omega_2)^2\right] = \frac{1}{2} \times I\ (\omega_1 + \omega_2)\ (\omega_1 - \omega_2)$$

$$= I.\omega\ (\omega_1 - \omega_2) \qquad \ldots\left(\because \omega = \frac{\omega_1 + \omega_2}{2}\right) \ldots(i)$$

$$= I.\omega^2 \left(\frac{\omega_1 - \omega_2}{\omega}\right) \qquad \text{...[Multiplying and dividing by } \omega]$$

$$= I.\omega^2.C_S = m.k^2.\omega^2.C_S \qquad \ldots(\because I = m.k^2) \ldots(ii)$$

$$= 2\,E.C_S \qquad \ldots\left(\because E = \frac{1}{2} \times I.\omega^2\right) \ldots(iii)$$

The radius of gyration (k) may be taken equal to the mean radius of the rim (R), because the thickness of rim is very small as compared to the diameter of rim. Therefore substituting $k = R$ in equation (*ii*), we have

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S \qquad \ldots(\because v = \omega.R)$$

From this expression, the mass of the flywheel rim may be determined.

Notes: 1. In the above expression, only the mass moment of inertia of the rim is considered and the mass moment of inertia of the hub and arms is neglected. This is due to the fact that the major portion of weight of the flywheel is in the rim and a small portion is in the hub and arms. Also the hub and arms are nearer to the axis of rotation, therefore the moment of inertia of the hub and arms is very small.

2. The density of cast iron may be taken as 7260 kg / m^3 and for cast steel, it may taken as 7800 kg / m^3.

3. The mass of the flywheel rim is given by

$$m = \text{Volume} \times \text{Density} = 2\,\pi\,R \times A \times \rho$$

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From this expression, we may find the value of the cross-sectional area of the rim. Assuming the cross-section of the rim to be rectangular, then

$$A = b \times t$$

where b = Width of the rim, and

t = Thickness of the rim.

Knowing the ratio of b / t which is usually taken as 2, we may find the width and thickness of rim.

4. When the flywheel is to be used as a pulley, then the width of rim should be taken 20 to 40 mm greater than the width of belt.

Example 22.1. *The turning moment diagram for a petrol engine is drawn to the following scales:*

Turning moment, 1 mm = 5 N-m; Crank angle, 1 mm = 1º.

The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line, taken in order are 295, 685, 40, 340, 960, 270 mm^2.

Determine the mass of 300 mm diameter flywheel rim when the coefficient of fluctuation of speed is 0.3% and the engine runs at 1800 r.p.m. Also determine the cross-section of the rim when the width of the rim is twice of thickness. Assume density of rim material as 7250 kg / m^3.

Solution. Given : D = 300 mm or R = 150 mm = 0.15 m ; C_S = 0.3% = 0.003 ; N = 1800 r.p.m. or $\omega = 2\pi \times 1800 / 60 = 188.5$ rad/s ; ρ = 7250 kg / m^3

Mass of the flywheel

Let m = Mass of the flywheel in kg.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram is shown in Fig. 22.6.

Since the scale of turning moment is 1 mm = 5 N-m, and scale of the crank angle is 1 mm = 1° = π / 180 rad, therefore 1 mm^2 on the turning moment diagram.

$$= 5 \times \pi / 180 = 0.087 \text{ N-m}$$

Let the total energy at $A = E$. Therefore from Fig. 22.6, we find that

Energy at $B = E + 295$

Energy at $C = E + 295 - 685 = E - 390$

Energy at $D = E - 390 + 40 = E - 350$

Energy at $E = E - 350 - 340 = E - 690$

Energy at $F = E - 690 + 960 = E + 270$

Energy at $G = E + 270 - 270 = E$ = Energy at A

From above we see that the energy is maximum at B and minimum at E.

∴ Maximum energy $= E + 295$

and minimum energy $= E - 690$

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We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$
$$= (E + 295) - (E - 690) = 985 \text{ mm}^2$$
$$= 985 \times 0.087 = 86 \text{ N-m}$$

We also know that maximum fluctuation of energy (ΔE),

$$86 = m.R^2.\omega^2.C_S = m\,(0.15)^2\,(188.5)^2\,(0.003) = 2.4\,m$$

$\therefore$ $m = 86 / 2.4 = 35.8$ kg **Ans.**

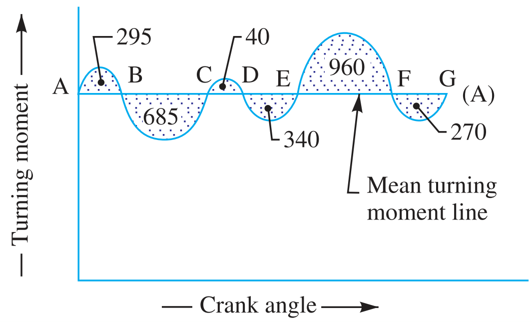

Fig. 22.6

Cross-section of the flywheel rim

Let t = Thickness of rim in metres, and

b = Width of rim in metres = $2\,t$ ...(Given)

$\therefore$ Cross-sectional area of rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim (m),

$$35.8 = A \times 2\pi R \times \rho = 2t^2 \times 2\pi \times 0.15 \times 7250 = 13\,668\,t^2$$

$\therefore$ $t^2 = 35.8 / 13\,668 = 0.0026$ or $t = 0.051$ m = 51 mm **Ans.**

and $b = 2\,t = 2 \times 51 = 102$ mm **Ans.**

Example 22.2. *The intercepted areas between the output torque curve and the mean resistance line of a turning moment diagram for a multicylinder engine, taken in order from one end are as follows:*

– 35, + 410, – 285, + 325, – 335, + 260, – 365, + 285, – 260 mm².

The diagram has been drawn to a scale of 1 mm = 70 N-m and 1 mm = 4.5°. The engine speed is 900 r.p.m. and the fluctuation in speed is not to exceed 2% of the mean speed.

Find the mass and cross-section of the flywheel rim having 650 mm mean diameter. The density of the material of the flywheel may be taken as 7200 kg / m³. The rim is rectangular with the width 2 times the thickness. Neglect effect of arms, etc.

Solution. Given : $N = 900$ r.p.m. or $\omega = 2\pi \times 900 / 60 = 94.26$ rad/s ; $\omega_1 - \omega_2 = 2\%\ \omega$ or

$$\frac{\omega_1 - \omega_2}{\omega} = C_S = 2\% = 0.02\ ;\ D = 650 \text{ mm or } R = 325 \text{ mm} = 0.325 \text{ m}\ ;\ \rho = 7200 \text{ kg / m}^3$$

Mass of the flywheel rim

Let m = Mass of the flywheel rim in kg.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram for a multi-cylinder engine is shown in Fig. 22.7.

Since the scale of turning moment is 1 mm = 70 N-m and scale of the crank angle is 1 mm = 4.5° = π / 40 rad, therefore 1 mm² on the turning moment diagram.

$$= 70 \times \pi / 40 = 5.5 \text{ N-m}$$

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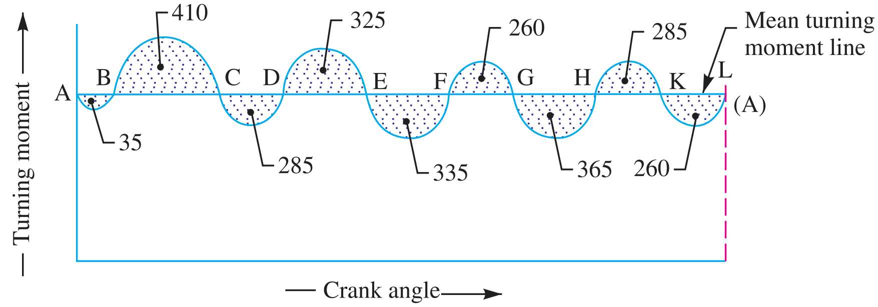

Fig. 22.7

Let the total energy at $A = E$. Therefore from Fig. 22.7, we find that

Energy at $B = E - 35$

Energy at $C = E - 35 + 410 = E + 375$

Energy at $D = E + 375 - 285 = E + 90$

Energy at $E = E + 90 + 325 = E + 415$

Energy at $F = E + 415 - 335 = E + 80$

Energy at $G = E + 80 + 260 = E + 340$

Energy at $H = E + 340 - 365 = E - 25$

Energy at $K = E - 25 + 285 = E + 260$

Energy at $L = E + 260 - 260 = E$ = Energy at A

From above, we see that the energy is maximum at E and minimum at B.

$\therefore$ Maximum energy $= E + 415$

and minimum energy $= E - 35$

We know that maximum fluctuation of energy,

$= (E + 415) - (E - 35) = 450 \text{ mm}^2$

$= 450 \times 5.5 = 2475$ N-m

We also know that maximum fluctuation of energy (ΔE),

$$2475 = \text{m}.R^2.\omega^2.C_S = m\,(0.325)^2\,(94.26)^2\,0.02 = 18.77\,m$$

$\therefore$ $m = 2475 / 18.77 = 132$ kg **Ans.**

Cross-section of the flywheel rim

Let t = Thickness of the rim in metres, and

b = Width of the rim in metres $= 2\,t$ ...(Given)

$\therefore$ Area of cross-section of the rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim (m),

$$132 = A \times 2\,\pi\,R \times \rho = 2\,t^2 \times 2\,\pi \times 0.325 \times 7200 = 29\,409\,t^2$$

$\therefore$ $t^2 = 132 / 29\,409 = 0.0044$ or $t = 0.067$ m = 67 mm **Ans.**

and $b = 2t = 2 \times 67 = 134$ mm **Ans.**

Example 22.3. *A single cylinder double acting steam engine develops 150 kW at a mean speed of 80 r.p.m. The coefficient of fluctuation of energy is 0.1 and the fluctuation of speed is ± 2% of mean speed. If the mean diameter of the flywheel rim is 2 metres and the hub and spokes provide 5 percent of the rotational inertia of the wheel, find the mass of the flywheel and cross-sectional area of the rim. Assume the density of the flywheel material (which is cast iron) as 7200 kg / m*^{3}.

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Solution. Given : P = 150 kW = 150 × 10^3 W ; N = 80 r.p.m. ; C_E = 0.1; $\omega_1 - \omega_2$ = ± 2% ω ; D = 2 m or R = 1 m ; ρ = 7200 kg/m^3

Mass of the flywheel rim

Let m = Mass of the flywheel rim in kg.

We know that the mean angular speed,

$$\omega = \frac{2\,\pi\,N}{60} = \frac{2\,\pi \times 80}{60} = 8.4 \text{ rad / s}$$

Since the fluctuation of speed is ± 2% of mean speed (ω), therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 4\,\%\,\omega = 0.04\,\omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.04$$

We know that the work done by the flywheel per cycle

$$= \frac{P \times 60}{N} = \frac{150 \times 10^3 \times 60}{80} = 112\ 500 \text{ N-m}$$

We also know that coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Workdone / cycle}}$$

∴ Maximum fluctuation of energy,

$$\Delta E = C_E \times \text{Workdone / cycle}$$

$$= 0.1 \times 112\ 500 = 11\ 250 \text{ N-m}$$

Since 5% of the rotational inertia is provided by hub and spokes, therefore the maximum fluctuation of energy of the flywheel rim will be 95% of the flywheel.

∴ Maximum fluctuation of energy of the rim,

$$(\Delta E)_{rim} = 0.95 \times 11\ 250 = 10\ 687.5 \text{ N-m}$$

We know that maximum fluctuation of energy of the rim $(\Delta E)_{rim}$,

$$10\ 687.5 = m.R^2.\omega^2.C_S = m \times 1^2\,(8.4)^2\,0.04 = 2.82\,m$$

∴ m = 10 687.5 / 2.82 = 3790 kg **Ans.**

Cross-sectional area of the rim

Let A = Cross-sectional area of the rim.

We know that the mass of the flywheel rim (m),

$$3790 = A \times 2\pi R \times \rho = A \times 2\pi \times 1 \times 7200 = 45\ 245\,A$$

∴ A = 3790 / 45 245 = 0.084 m^2 **Ans.**

Example 22.4. *A single cylinder, single acting, four stroke oil engine develops 20 kW at 300 r.p.m. The workdone by the gases during the expansion stroke is 2.3 times the workdone on the gases during the compression and the workdone during the suction and exhaust strokes is negligible. The speed is to be maintained within ± 1%. Determine the mass moment of inertia of the flywheel.*

Solution. Given : P = 20 kW = 20 × 10^3 W ; N = 300 r.p.m. or ω = 2π × 300 / 60 = 31.42 rad / s ; $\omega_1 - \omega_2$ = ± 1% ω

First of all, let us find the maximum fluctuation of energy (ΔE). The turning moment diagram for a four stroke engine is shown in Fig. 22.8. It is assumed to be triangular during compression and expansion strokes, neglecting the suction and exhaust strokes.

We know that mean torque transmitted by the engine,

$$T_{mean} = \frac{P \times 60}{2\,\pi\,N} = \frac{20 \times 10^3 \times 60}{2\,\pi \times 300} = 636.5 \text{ N-m}$$

and *workdone per cycle $= T_{mean} \times \theta = 636.5 \times 4\,\pi = 8000$ N-m ...(*i*)

Let W_C = Workdone during compression stroke, and

W_E = Workdone during expansion stroke.

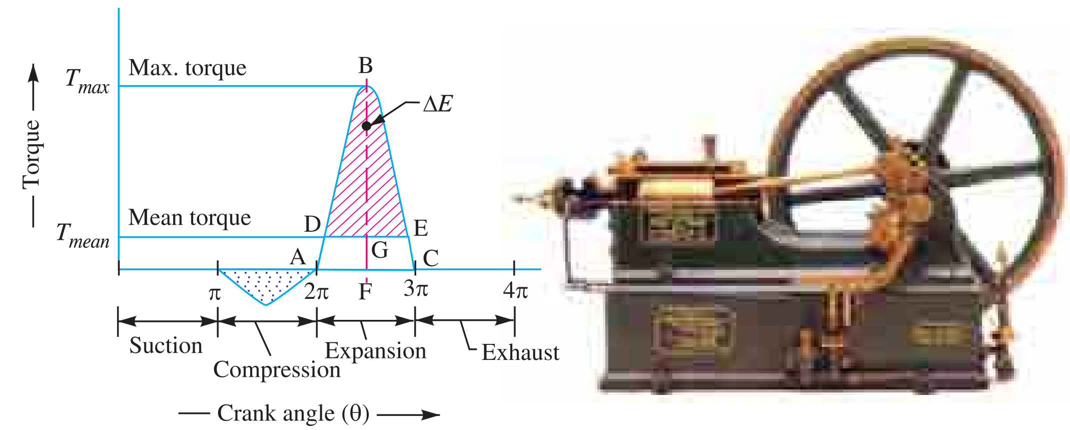

Fig. 22.8

Since the workdone during suction and exhaust strokes is negligible, therefore net work done per cycle

$$= W_E - W_C = W_E - W_E / 2.3 = 0.565\, W_E \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$W_E = 8000 / 0.565 = 14\,160 \text{ N-m}$$

The workdone during the expansion stroke is shown by triangle *ABC* in Fig. 22.8, in which base $AC = \pi$ radians and height $BF = T_{max.}$

∴ Workdone during expansion stroke (W_E),

$$14\,160 = \frac{1}{2} \times \pi \times T_{max} = 1.571\, T_{max}$$

or $T_{max} = 14\,160 / 1.571 = 9013$ N-m

We know that height above the mean torque line,

$$BG = BF - FG = T_{max} - T_{mean}$$

$$= 9013 - 636.5 = 8376.5 \text{ N-m}$$

Since the area *BDE* shown shaded in Fig. 22.8 above the mean torque line represents the maximum fluctuation of energy (Δ *E*), therefore from geometrical relation,

$$\frac{\text{Area of } \Delta\ BDE}{\text{Area of } \Delta\ ABC} = \frac{(BG)^2}{(BF)^2}\text{, we have}$$

---

* The workdone per cycle may also be calculated as follows :

We know that for a four stroke engine, number of working strokes per cycle

$$n = N / 2 = 300 / 2 = 150$$

∴ Workdone per cycle $= P \times 60 / n = 20 \times 10^3 \times 60 / 150 = 8000$ N-m

Maximum fluctuation of energy (*i.e.* area of Δ *BDE*),

$$*\Delta E = \text{Area of } \Delta\, ABC \left(\frac{BG}{BF}\right)^2 = W_E \left(\frac{BG}{BF}\right)^2$$

$$= 14\ 160 \left(\frac{8376.5}{9013}\right)^2 = 12\ 230 \text{ N-m}$$

Since the speed is to be maintained within ± 1% of the mean speed, therefore total fluctuation of speed

$$\omega_1 - \omega_2 = 2\ \%\ \omega = 0.02\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.02$$

Let I = Mass moment of inertia of the flywheel in kg-m^2.

We know that maximum fluctuation of energy (ΔE),

$$12\ 230 = I.\omega^2.C_S$$

$$= I\,(31.42)^2\ 0.02 = 19.74\ I$$

$\therefore$ $I = 12\ 230 / 19.74 = 619.5$ kg-m^2 **Ans.**

22.7 Stresses in a Flywheel Rim

A flywheel, as shown in Fig. 22.9, consists of a rim at which the major portion of the mass or weight of flywheel is concentrated, a boss or hub for fixing the flywheel on to the shaft and a number of arms for supporting the rim on the hub.

The following types of stresses are induced in the rim of a flywheel:

1. Tensile stress due to centrifugal force,

2. Tensile bending stress caused by the restraint of the arms, and

3. The shrinkage stresses due to unequal rate of cooling of casting. These stresses may be very high but there is no easy method of determining. This stress is taken care of by a factor of safety.

We shall now discuss the first two types of stresses as follows:

1. Tensile stress due to the centrifugal force

The tensile stress in the rim due to the centrifugal force, assuming that the rim is unstrained by the arms, is determined in a similar way as a thin cylinder subjected to internal pressure.

Let b = Width of rim,

t = Thickness of rim,

---

* The maximum fluctuation of energy (ΔE) may also be obtained as discussed below :
From similar triangles *BDE* and *BAC*,

$$\frac{DE}{AC} = \frac{BG}{BF} \quad \text{or} \quad DE = \frac{BG}{BF} \times AC = \frac{8376.5}{9013} \times \pi = 2.92 \text{ rad}$$

$\therefore$ Maximum fluctuation of energy (*i.e.* area of Δ *BDE*),

$$\Delta E = \frac{1}{2} \times DE \times BG = \frac{1}{2} \times 2.92 \times 8376.5 = 12\ 230 \text{ N-m}$$

A = Cross-sectional area of rim = $b \times t$,
D = Mean diameter of flywheel
R = Mean radius of flywheel,
ρ = Density of flywheel material,
ω = Angular speed of flywheel,
v = Linear velocity of flywheel, and
σ_t = Tensile or hoop stress.

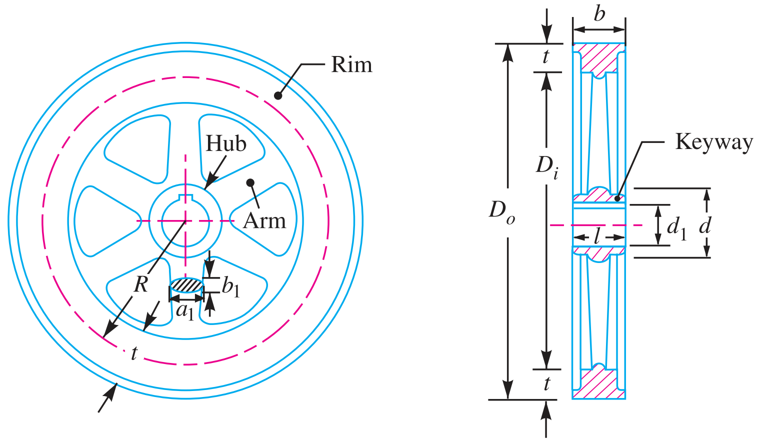

Fig. 22.9. Flywheel.

Consider a small element of the rim as shown shaded in Fig. 22.10. Let it subtends an angle $\delta\theta$ at the centre of the flywheel.

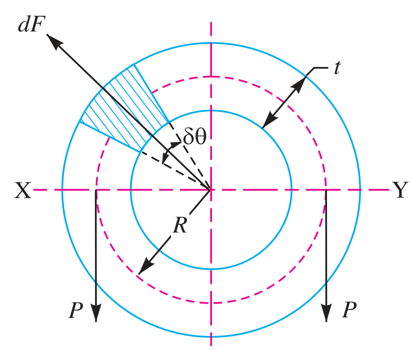

Fig. 22.10. Cross-section of a flywheel rim.

Volume of the small element

$$= A.R.\delta\theta$$

$\therefore$ Mass of the small element,

$$dm = \text{Volume} \times \text{Density}$$
$$= A.R.\delta\theta.\rho = \rho.A.R.\delta\theta$$

and centrifugal force on the element,

$$dF = dm.\omega^2.R = \rho.A.R.\delta\theta.\omega^2.R$$
$$= \rho.A.R^2.\omega^2.\delta\theta$$

Vertical component of dF

$$= dF.\sin\theta$$
$$= \rho.A.R^2.\omega^2.\delta\theta \sin\theta$$

$\therefore$ Total vertical bursting force across the rim diameter X-Y,

$$= \rho.A\,R^2.\omega^2 \int_0^{\pi} \sin\theta\, d\theta$$

$$= \rho.A.R^2.\omega^2 \left[-\cos\theta\right]_0^{\pi} = 2\,\rho.A.R^2.\omega^2 \qquad \text{...}(i)$$

This vertical force is resisted by a force of $2P$, such that

$$2P = 2\sigma_t \times A \qquad \text{...}(ii)$$

From equations (i) and (ii), we have

$$2\rho A.R^2.\omega^2 = 2\,\sigma_t \times A$$

$$\therefore \qquad \sigma_t = \rho.R^2.\omega^2 = \rho.v^2 \qquad \text{...}(\because v = \omega.R) \qquad \text{...}(iii)$$

when ρ is in kg / m^3 and v is in m / s, then σ_t will be in N / m^2 or Pa.

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Note : From the above expression, the mean diameter (D) of the flywheel may be obtained by using the relation,

$$v = \pi D.N / 60$$

2. *Tensile bending stress caused by restraint of the arms*

The tensile bending stress in the rim due to the restraint of the arms is based on the assumption that each portion of the rim between a pair of arms behaves like a beam fixed at both ends and uniformly loaded, as shown in Fig. 22.11, such that length between fixed ends,

$$l = \frac{\pi D}{n} = \frac{2 \pi R}{n}, \text{ where } n = \text{Number of arms.}$$

The uniformly distributed load (w) per metre length will be equal to the centrifugal force between a pair of arms.

$$\therefore \quad w = b.t.\rho.\omega^2.R \text{ N/m}$$

We know that maximum bending moment,

$$M = \frac{w \,.\, l^2}{12} = \frac{b \,.\, t \,\rho \,.\, \omega^2 \,.\, R}{12}\left(\frac{2 \pi R}{n}\right)^2$$

and section modulus, $\quad Z = \frac{1}{6} b \times t^2$

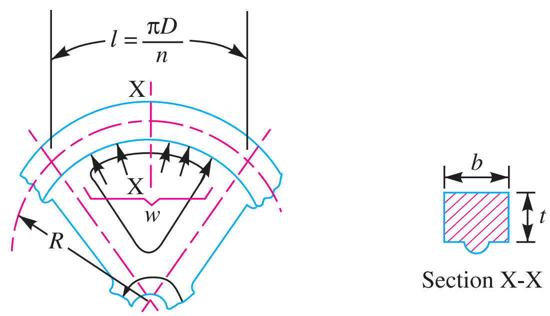

Fig. 22.11

$\therefore$ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{b \,.\, t \,\rho \,.\, \omega^2 \,.\, R}{12}\left(\frac{2 \pi R}{n}\right)^2 \times \frac{6}{b \times t^2}$$

$$= \frac{19.74 \,\rho \,.\, \omega^2 \,.\, R^3}{n^2 \,.\, t} = \frac{19.74 \,\rho \,.\, v^2 \,.\, R}{n^2 \,.\, t} \qquad ...(iv)$$

...(Substituting $\omega = v/R$)

Now total stress in the rim,

$$\sigma = \sigma_t + \sigma_b$$

If the arms of a flywheel do not stretch at all and are placed very close together, then centrifugal force will not set up stress in the rim. In other words, σ_t will be zero. On the other hand, if the arms are stretched enough to allow free expansion of the rim due to centrifugal action, there will be no restraint due to the arms, *i.e.* σ_b will be zero.

It has been shown by G. Lanza that the arms of a flywheel stretch about $\frac{3}{4}$ th of the amount necessary for free expansion. Therefore the total stress in the rim,

$$= \frac{3}{4}\sigma_t + \frac{1}{4}\sigma_b = \frac{3}{4}\rho.v^2 + \frac{1}{4} \times \frac{19.74 \,\rho \,.\, v^2 \,.\, R}{n^2 \,.\, t} \qquad ...(v)$$

$$= \rho.v^2\left(0.75 + \frac{4.935\, R}{n^2 \,.\, t}\right)$$

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Example 22.5. *A multi-cylinder engine is to run at a constant load at a speed of 600 r.p.m. On drawing the crank effort diagram to a scale of 1 m = 250 N-m and 1 mm = 3°, the areas in sq mm above and below the mean torque line are as follows:*

+ 160, – 172, + 168, – 191, + 197, – 162 sq mm

The speed is to be kept within ± 1% of the mean speed of the engine. Calculate the necessary moment of inertia of the flywheel.

Determine suitable dimensions for cast iron flywheel with a rim whose breadth is twice its radial thickness. The density of cast iron is 7250 kg / m^3, and its working stress in tension is 6 MPa. Assume that the rim contributes 92% of the flywheel effect.

Solution. Given : = N = 600 r.p.m. or $\omega = 2\pi \times 600 / 60 = 62.84$ rad / s ; $\rho = 7250$ kg / m^3 ; $\sigma_t = 6$ MPa $= 6 \times 10^6$ N/m^2

Moment of inertia of the flywheel

Let I = Moment of inertia of the flywheel.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram is shown in Fig. 22.12.

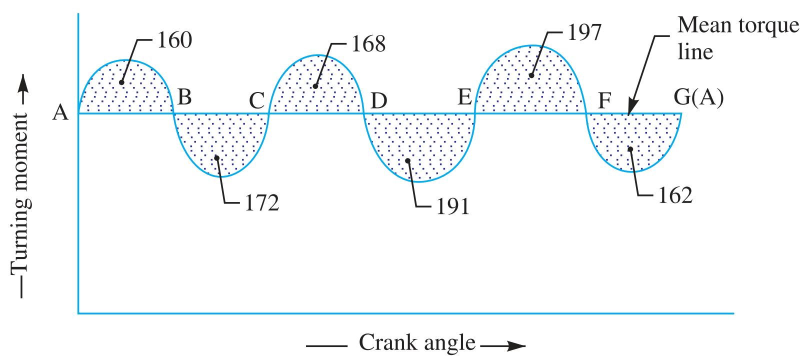


Fig. 22.12

Since the scale for the turning moment is 1 mm = 250 N-m and the scale for the crank angle is 1 mm = 3° = $\frac{\pi}{60}$ rad, therefore

1 mm^2 on the turning moment diagram

$$= 250 \times \frac{\pi}{60} = 13.1 \text{ N-m}$$

Let the total energy at $A = E$. Therefore from Fig. 22.12, we find that

Energy at B = $E + 160$

Energy at C = $E + 160 - 172 = E - 12$

Energy at D = $E - 12 + 168 = E + 156$

Energy at E = $E + 156 - 191 = E - 35$

Energy at F = $E - 35 + 197 = E + 162$

Energy at $G = E + 162 - 162 = E$ = Energy at A

From above, we find that the energy is maximum at F and minimum at E.

∴ Maximum energy $= E + 162$

and minimum energy $= E - 35$

We know that the maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + 162) - (E - 35) = 197 \text{ mm}^2 = 197 \times 13.1 = 2581 \text{ N-m}$$

Since the fluctuation of speed is ± 1% of the mean speed (ω), therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 2\%\ \omega = 0.02\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.02$$

We know that the maximum fluctuation of energy (ΔE),

$$2581 = I.\omega^2.C_S = I\,(62.84)^2\, 0.02 = 79\, I$$

$$\therefore \quad I = 2581 / 79 = 32.7 \text{ kg-m}^2$$ **Ans.**

Dimensions of a flywheel rim

Let t = Thickness of the flywheel rim in metres, and

b = Breadth of the flywheel rim in metres $= 2\,t$ ...(Given)

First of all let us find the peripheral velocity (v) and mean diameter (D) of the flywheel.

We know that tensile stress (σ_t),

$$6 \times 10^6 = \rho.v^2 = 7250 \times v^2$$

$$\therefore \quad v^2 = 6 \times 10^6 / 7250 = 827.6 \quad \text{or} \quad v = 28.76 \text{ m/s}$$

We also know that peripheral velocity (v),

$$28.76 = \frac{\pi\, D\, .\, N}{60} = \frac{\pi\, D \times 600}{60} = 31.42\, D$$

$$\therefore \quad D = 28.76 / 31.42 = 0.915 \text{ m} = 915 \text{ mm}$$ **Ans.**

Now let us find the mass of the flywheel rim. Since the rim contributes 92% of the flywheel effect, therefore the energy of the flywheel rim (E_{rim}) will be 0.92 times the total energy of the flywheel (E). We know that maximum fluctuation of energy (ΔE),

$$2581 = E \times 2\, C_S = E \times 2 \times 0.02 = 0.04\, E$$

$$\therefore \quad E = 2581 / 0.04 = 64\,525 \text{ N-m}$$

and energy of the flywheel rim,

$$E_{rim} = 0.92\, E = 0.92 \times 64\,525 = 59\,363 \text{ N-m}$$

Let m = Mass of the flywheel rim.

We know that energy of the flywheel rim (E_{rim}),

$$59\,363 = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times m\,(28.76)^2 = 413.6\, m$$

$$\therefore \quad m = 59\,363 / 413.6 = 143.5 \text{ kg}$$

We also know that mass of the flywheel rim (m),

$$143.5 = b \times t \times \pi\, D \times \rho = 2\,t \times t \times \pi \times 0.915 \times 7250 = 41\,686\, t^2$$

$\therefore \quad t^2 = 143.5 / 41\,686 = 0.003\,44$

or $\quad t = 0.0587$ say 0.06 m = 60 mm **Ans.**

and $\quad b = 2\,t = 2 \times 60 = 120$ mm **Ans.**

Notes: The mass of the flywheel rim may also be obtained by using the following relations. Since the rim contributes 92% of the flywheel effect, therefore using

1. $I_{rim} = 0.92\, I_{flywheel}$ or $m.k^2 = 0.92 \times 32.7 = 30$ kg–m²

Since radius of gyration, $k = R = D / 2 = 0.915 / 2 = 0.4575$ m, therefore

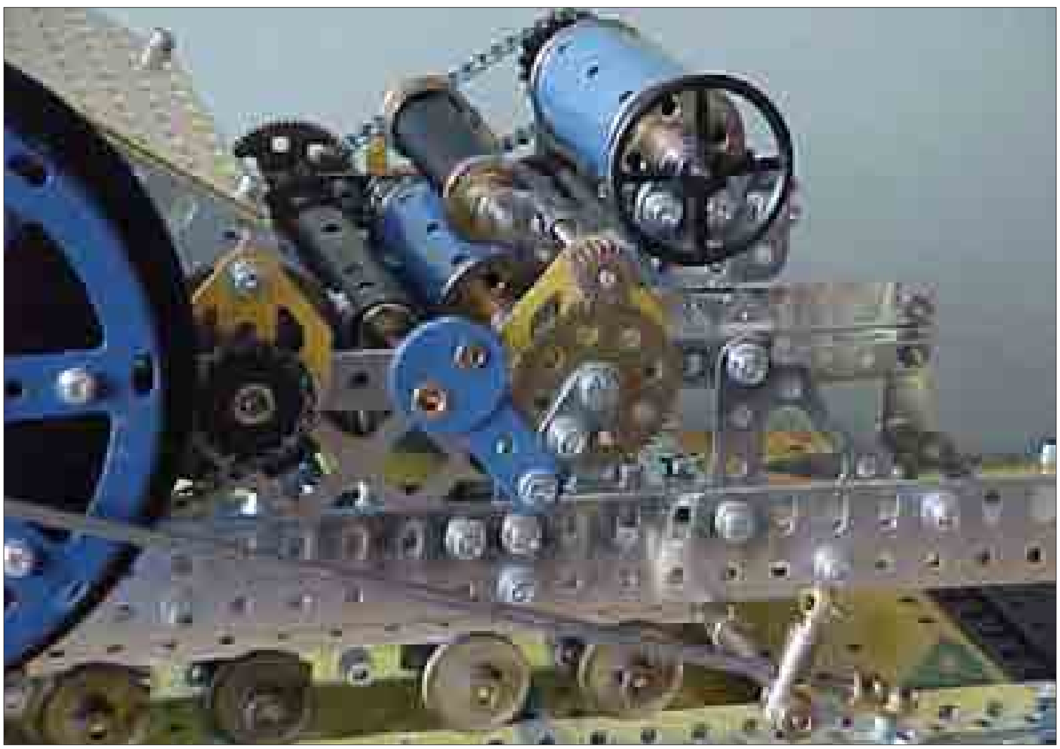
$$m = \frac{30}{k^2} = \frac{30}{(0.4575)^2} = \frac{30}{0.209} = 143.5 \text{ kg}$$

2. $(\Delta E)_{rim} = 0.92\, (\Delta E)_{flywheel}$

$m.v^2.C_S = 0.92\, (\Delta E)_{flywheel}$

$m\,(28.76)^2\, 0.02 = 0.92 \times 2581$

$16.55\, m = 2374.5$ or $m = 2374.5 / 16.55 = 143.5$ kg



Flywheel of a printing press

Example 22.6. *The areas of the turning moment diagram for one revolution of a multi-cylinder engine with reference to the mean turning moment, below and above the line, are*

– 32, + 408, – 267, + 333, – 310, + 226, – 374, + 260 and – 244 mm².

The scale for abscissa and ordinate are: 1 mm = 2.4° and 1 mm = 650 N-m respectively. The mean speed is 300 r.p.m. with a percentage speed fluctuation of ± 1.5%. If the hoop stress in the material of the rim is not to exceed 5.6 MPa, determine the suitable diameter and cross-section for the flywheel, assuming that the width is equal to 4 times the thickness. The density of the material may be taken as 7200 kg / m³. Neglect the effect of the boss and arms.

Solution. Given : N = 300 r.p.m. or $\omega = 2\,\pi \times 300/60 = 31.42$ rad/s ; σ_t = 5.6 MPa = 5.6×10^6 N/m^2 ; ρ = 7200 kg/m^3

Diameter of the flywheel

Let D = Diameter of the flywheel in metres.

We know that peripheral velocity of the flywheel,

$$v = \frac{\pi\, D . N}{60} = \frac{\pi\, D \times 300}{60} = 15.71\, D \text{ m/s}$$

We also know that hoop stress (σ_t),

$$5.6 \times 10^6 = \rho \times v^2 = 7200\,(15.71\,D)^2 = 1.8 \times 10^6\, D^2$$

$$\therefore \quad D^2 = 5.6 \times 10^6 / 1.8 \times 10^6 = 3.11 \quad \text{or} \quad D = 1.764 \text{ m}$$ **Ans.**

Cross-section of the flywheel

Let t = Thickness of the flywheel rim in metres, and

b = Width of the flywheel rim in metres = $4\,t$ ...(Given)

∴ Cross-sectional area of the rim,

$$A = b \times t = 4\,t \times t = 4\,t^2 \text{ m}^2$$

Now let us find the maximum fluctuation of energy. The turning moment diagram for one revolution of a multi-cylinder engine is shown in Fig. 22.13.

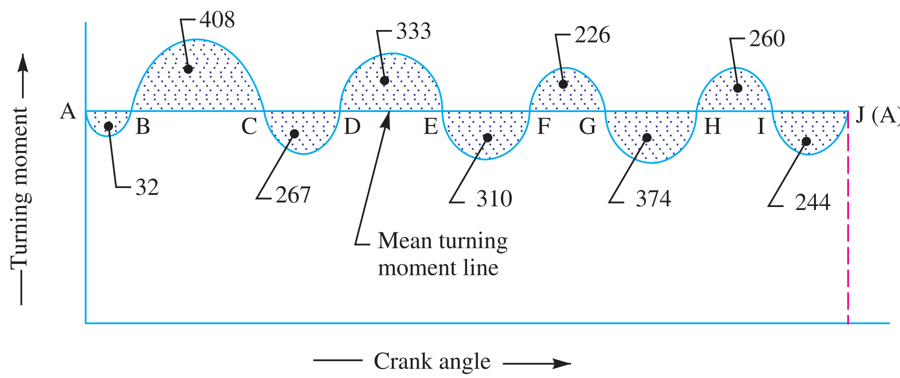


Fig. 22.13

Since the scale of crank angle is 1 mm = 2.4° = $2.4 \times \dfrac{\pi}{180}$ = 0.042 rad, and the scale of the turning moment is 1 mm = 650 N-m, therefore

1 mm^2 on the turning moment diagram

$$= 650 \times 0.042 = 27.3 \text{ N-m}$$

Let the total energy at $A = E$. Therefore from Fig. 22.13, we find that

Energy at B = $E - 32$

Energy at C = $E - 32 + 408 = E + 376$

Energy at D = $E + 376 - 267 = E + 109$

Energy at E = $E + 109 + 333 = E + 442$

Energy at F = $E + 442 - 310 = E + 132$

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Energy at $G = E + 132 + 226 = E + 358$

Energy at $H = E + 358 - 374 = E - 16$

Energy at $I = E - 16 + 260 = E + 244$

Energy at $J = E + 244 - 244 = E$ = Energy at A

From above, we see that the energy is maximum at E and minimum at B.

∴ Maximum energy $= E + 442$

and minimum energy $= E - 32$

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$
$$= (E + 442) - (E - 32) = 474 \text{ mm}^2$$
$$= 474 \times 27.3 = 12\,940 \text{ N-m}$$

Since the fluctuation of speed is ± 1.5% of the mean speed, therefore total fluctuation of speed,

$$\omega_1 - \omega_2 = 3\% \text{ of mean speed} = 0.03\,\omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.03$$

Let m = Mass of the flywheel rim.

We know that maximum fluctuation of energy (ΔE),

$$12\,940 = m.R^2.\omega^2.C_S = m\left(\frac{1.764}{2}\right)^2 (31.42)^2\, 0.03 = 23\,m$$

∴ $m = 12\,940 / 23 = 563$ kg **Ans.**

We also know that mass of the flywheel rim (m),

$$563 = A \times \pi\, D \times \rho = 4\,t^2 \times \pi \times 1.764 \times 7200 = 159\,624\,t^2$$

∴ $t^2 = 563 / 159\,624 = 0.00353$

or $t = 0.0594$ m $= 59.4$ say 60 mm **Ans.**

and $b = 4\,t = 4 \times 60 = 240$ mm **Ans.**

Example 22.7. *An otto cycle engine develops 50 kW at 150 r.p.m. with 75 explosions per minute. The change of speed from the commencement to the end of power stroke must not exceed 0.5% of mean on either side. Design a suitable rim section having width four times the depth so that the hoop stress does not exceed 4 MPa. Assume that the flywheel stores 16/15 times the energy stored by the rim and that the workdone during power stroke is 1.40 times the workdone during the cycle. Density of rim material is 7200 kg / m³.*

Solution. Given : $P = 50$ kW $= 50 \times 10^3$ W ; $N = 150$ r.p.m. ; $n = 75$; $\sigma_t = 4$ MPa $= 4 \times 10^6$ N/m² ; $\rho = 7200$ kg/m³

First of all, let us find the mean torque (T_{mean}) transmitted by the engine or flywheel. We know that the power transmitted (P),

$$50 \times 10^3 = \frac{2\,\pi\,N \times T_{mean}}{60} = 15.71\,T_{mean}$$

∴ $T_{mean} = 50 \times 10^3 / 15.71 = 3182.7$ N-m

Since the explosions per minute are equal to N/2, therefore the engine is a four stroke cycle engine. The turning moment diagram of a four stroke engine is shown in Fig. 22.14.

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We know that *workdone per cycle

$$= T_{mean} \times \theta = 3182.7 \times 4\,\pi = 40\,000 \text{ N-m}$$

∴ Workdone during power or working stroke

$$= 1.4 \times 40\,000 = 56\,000 \text{ N-m} \quad ...(i)$$

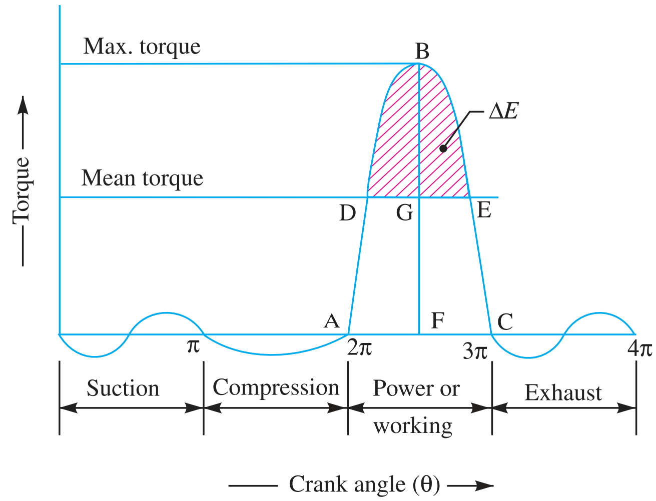

Fig. 22.14

The workdone during power or working stroke is shown by a triangle *ABC* in Fig. 22.14 in which base $AC = \pi$ radians and height $BF = T_{max.}$

∴ Workdone during working stroke

$$= \frac{1}{2} \times \pi \times T_{max} = 1.571\, T_{max} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$T_{max} = 56\,000 / 1.571 = 35\,646 \text{ N-m}$$

Height above the mean torque line,

$$BG = BF - FG = T_{max} - T_{mean} = 35\,646 - 3182.7 = 32\,463.3 \text{ N-m}$$

Since the area *BDE* (shown shaded in Fig. 22.14) above the mean torque line represents the maximum fluctuation of energy (Δ*E*), therefore from geometrical relation

$$\frac{\text{Area of } \Delta\, BDE}{\text{Area of } \Delta\, ABC} = \frac{(BG)^2}{(BF)^2}, \text{ we have}$$

Maximum fluctuation of energy (*i.e.* area of triangle *BDE*),

$$\Delta E = \text{Area of triangle } ABC \times \left(\frac{BG}{BF}\right)^2 = 56\,000 \times \left(\frac{32\,463.3}{35\,646}\right)^2$$

$$= 56\,000 \times 0.83 = 46\,480 \text{ N-m}$$

Mean diameter of the flywheel

Let D = Mean diameter of the flywheel in metres, and

v = Peripheral velocity of the flywheel in m/s.

---

* The workdone per cycle for a four stroke engine is also given by

$$\text{Workdone / cycle} = \frac{P \times 60}{\text{Number of explosion / min}} = \frac{P \times 60}{n} = \frac{50\,000 \times 60}{75} = 40\,000 \text{ N-m}$$

We know that hoop stress (σ_t),

$4 \times 10^6 = \rho . v^2 = 7200 \times v^2$

$\therefore \quad v^2 = 4 \times 10^6 / 7200 = 556$

or $\quad v = 23.58$ m/s

We also know that peripheral velocity (v),

$$23.58 = \frac{\pi \, D \, N}{60} = \frac{\pi \, D \times 150}{60} = 7.855 \, D$$

$\therefore \quad D = 23.58 / 7.855 = 3$ m **Ans.**

Flywheel of a motorcycle

Cross-sectional dimensions of the rim

Let t = Thickness of the rim in metres, and

b = Width of the rim in metres = $4\,t$ ...(Given)

$\therefore$ Cross-sectional area of the rim,

$$A = b \times t = 4\,t \times t = 4\,t^2$$

First of all, let us find the mass of the flywheel rim.

Let m = Mass of the flywheel rim, and

E = Total energy of the flywheel.

Since the fluctuation of speed is 0.5% of the mean speed on either side, therefore total fluctuation of speed,

$$N_1 - N_2 = 1\% \text{ of mean speed} = 0.01\,N$$

and coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = 0.01$$

We know that the maximum fluctuation of energy (ΔE),

$$46\,480 = E \times 2\,C_S = E \times 2 \times 0.01 = 0.02\,E$$

$$\therefore \quad E = 46\,480 / 0.02 = 2324 \times 10^3 \text{ N-m}$$

Since the energy stored by the flywheel is $\frac{16}{15}$ times the energy stored by the rim, therefore the energy of the rim,

$$E_{rim} = \frac{15}{16}\,E = \frac{15}{16} \times 2324 \times 10^3 = 2178.8 \times 10^3 \text{ N-m}$$

We know that energy of the rim (E_{rim}),

$$2178.8 \times 10^3 = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times m\,(23.58)^2 = 278\,m$$

$$\therefore \quad m = 2178.8 \times 10^3 / 278 = 7837 \text{ kg}$$

We also know that mass of the flywheel rim (m),

$$7837 = A \times \pi\,D \times \rho = 4\,t^2 \times \pi \times 3 \times 7200 = 271\,469\,t^2$$

$\therefore \quad t^2 = 7837 / 271\,469 = 0.0288$ or $t = 0.17$ m = 170 mm **Ans.**

and $\quad b = 4\,t = 4 \times 170 = 680$ mm **Ans.**

Example 22.8. *A shaft fitted with a flywheel rotates at 250 r.p.m. and drives a machine. The torque of machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 750 N-m to 3000 N-m uniformly during 1 / 2 revolution and remains constant for the following revolution. It then falls uniformly to 750 N-m during the next 1 / 2 revolution and remains constant for one revolution, the cycle being repeated thereafter. Determine the power required to drive the machine.*

If the total fluctuation of speed is not to exceed 3% of the mean speed, determine a suitable diameter and cross-section of the flywheel rim. The width of the rim is to be 4 times the thickness and the safe centrifugal stress is 6 MPa. The material density may be assumed as 7200 kg / m³.

Solution. Given : N = 250 r.p.m. or $\omega = 2\pi \times 250 / 60 = 26.2$ rad/s ; $\omega_1 - \omega_2 = 3\%\ \omega$ or

$\frac{\omega_1 - \omega_2}{\omega} = C_S = 3\% = 0.03$; $\sigma_t = 6$ MPa $= 6 \times 10^6$ N/m^2 ; $\rho = 7200$ kg / m^3

Power required to drive the machine

The turning moment diagram for the complete cycle is shown in Fig. 22.15.

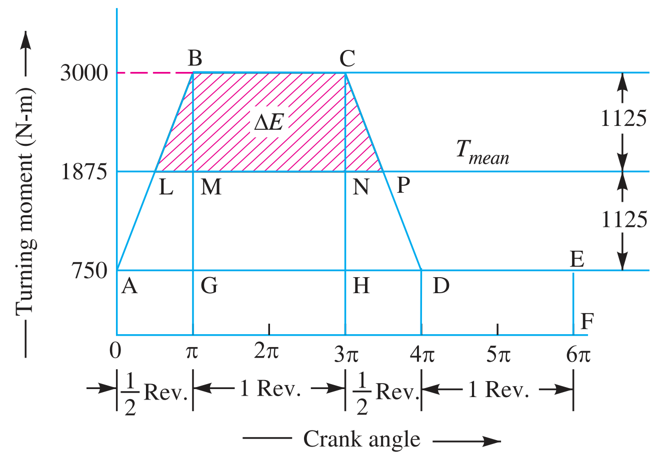

Fig. 22.15

We know that the torque required for one complete cycle

$= $ Area of figure $OABCDEF$

$= $ Area $OAEF$ + Area ABG + Area $BCHG$ + Area CDH

$$= OF \times OA + \frac{1}{2} \times AG \times BG + GH \times CH + \frac{1}{2} \times HD \times CH$$

$$= 6\pi \times 750 + \frac{1}{2} \times \pi\,(3000 - 750) + 2\pi\,(3000 - 750) + \frac{1}{2} \times \pi\,(3000 - 750)$$

$$= 4500\,\pi + 1125\,\pi + 4500\,\pi + 1125\,\pi = 11\,250\,\pi \text{ N-m} \quad ...(i)$$

If T_{mean} is the mean torque in N-m, then torque required for one complete cycle

$$= T_{mean} \times 6\,\pi \text{ N-m} \quad ...(ii)$$

From equations ***(i)*** and ***(ii)***,

$$T_{mean} = 11250\,\pi / 6\,\pi = 1875 \text{ N-m}$$

We know that power required to drive the machine,

$$P = T_{mean} \times \omega = 1875 \times 26.2 = 49\,125 \text{ W} = 49.125 \text{ kW}$$ **Ans.**

Diameter of the flywheel

Let D = Diameter of the flywheel in metres, and

v = Peripheral velocity of the flywheel in m/s.

We know that the centrifugal stress (σ_t),

$$6 \times 10^6 = \rho \times v^2 = 7200 \times v^2$$

$$\therefore \quad v^2 = 6 \times 10^6 / 7200 = 833.3 \quad \text{or} \quad v = 28.87 \text{ m/s}$$

We also know that peripheral velocity of the flywheel (v),

$$28.87 = \frac{\pi\, D\, N}{60} = \frac{\pi\, D \times 250}{60} = 13.1\, D$$

$\therefore$ $D = 28.87 / 13.1 = 2.2$ m **Ans.**

Cross-section of the flywheel rim

Let t = Thickness of the flywheel rim in metres, and

b = Width of the flywheel rim in metres = $4\,t$ ...(Given)

$\therefore$ Cross-sectional area of the flywheel rim,

$$A = b \times t = 4\,t \times t = 4\,t^2 \text{ m}^2$$

First of all, let us find the maximum fluctuation of energy (ΔE) and mass of the flywheel rim (m). In order to find ΔE, we shall calculate the values of LM and NP.

From similar triangles ABG and BLM,

$$\frac{LM}{AG} = \frac{BM}{BG} \quad \text{or} \quad \frac{LM}{\pi} = \frac{3000 - 1857}{3000 - 750} = 0.5 \quad \text{or} \quad LM = 0.5\,\pi$$

Now from similar triangles CHD and CNP,

$$\frac{NP}{HD} = \frac{CN}{CH} \quad \text{or} \quad \frac{NP}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad \text{or} \quad NP = 0.5\,\pi$$

From Fig. 22.15, we find that

$$BM = CN = 3000 - 1875 = 1125 \text{ N-m}$$

Since the area above the mean torque line represents the maximum fluctuation of energy, therefore maximum fluctuation of energy,

$$\Delta E = \text{Area } LBCP = \text{Area } LBM + \text{Area } MBCN + \text{Area } PNC$$

$$= \frac{1}{2} \times LM \times BM + MN \times \text{BM} + \frac{1}{2} \times NP \times CN$$

$$= \frac{1}{2} \times 0.5\,\pi \times 1125 + 2\,\pi \times 1125 + \frac{1}{2} \times 0.5\,\pi \times 1125$$

$$= 8837 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$8837 = m.R^2.\omega^2.C_S = m\left(\frac{2.2}{2}\right)^2 (26.2)^2\, 0.03 = 24.9\, m$$

$\therefore$ $m = 8837 / 24.9 = 355$ kg

We also know that mass of the flywheel rim (m),

$$355 = A \times \pi\, D \times \rho = 4\,t^2 \times \pi \times 2.2 \times 7200 = 199\,077\,t^2$$

$\therefore$ $t^2 = 355 / 199\,077 = 0.00178$ or $t = 0.042$ m = 42 say 45 mm **Ans.**

and $b = 4\,t = 4 \times 45 = 180$ mm **Ans.**

Example 22.9. *A punching machine makes 25 working strokes per minute and is capable of punching 25 mm diameter holes in 18 mm thick steel plates having an ultimate shear strength of 300 MPa.*

The punching operation takes place during 1/10 th of a revolution of the crank shaft.

Estimate the power needed for the driving motor, assuming a mechanical efficiency of 95 per cent. Determine suitable dimensions for the rim cross-section of the flywheel, which is to revolve at 9 times the speed of the crank shaft. The permissible coefficient of fluctuation of speed is 0.1.

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The flywheel is to be made of cast iron having a working stress (tensile) of 6 MPa and density of 7250 kg / m^3. The diameter of the flywheel must not exceed 1.4 m owing to space restrictions. The hub and the spokes may be assumed to provide 5% of the rotational inertia of the wheel.

Check for the centrifugal stress induced in the rim.

Solution. Given : n = 25 ; d_1 = 25 mm ; t_1 = 18 mm ; τ_u = 300 MPa = 300 N/mm^2 ; η_m = 95% = 0.95 ; C_S = 0.1 ; σ_t = 6 MPa = 6 N/mm^2 ; ρ = 7250 kg/m^3 ; D = 1.4 m or R = 0.7m

Punching Machine

Power needed for the driving motor

We know that the area of plate sheared,

$$A_S = \pi \, d_1 \times t_1 = \pi \times 25 \times 18 = 1414 \text{ mm}^2$$

∴ Maximum shearing force required for punching,

$$F_S = A_S \times \tau_u = 1414 \times 300 = 424\,200 \text{ N}$$

and energy required per stroke

$$= {}^*\text{Average shear force} \times \text{Thickness of plate}$$

$$= \frac{1}{2} F_S \times t_1 = \frac{1}{2} \times 424\,200 \times 18 = 3817.8 \times 10^3 \text{ N-mm}$$

∴ Energy required per min

$$= \text{Energy / stroke} \times \text{No. of working strokes / min}$$

$$= 3817.8 \times 10^3 \times 25 = 95.45 \times 10^6 \text{ N-mm} = 95\,450 \text{ N-m}$$

We know that the power needed for the driving motor

$$= \frac{\text{Energy required per min}}{60 \times \eta_m} = \frac{95\,450}{60 \times 0.95} = 1675 \text{ W}$$

$$= 1.675 \text{ kW}$$ **Ans.**

* As the hole is punched, it is assumed that the shearing force decreases uniformly from maximum value to zero.

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Dimensions for the rim cross-section

Considering the cross-section of the rim as rectangular and assuming the width of rim equal to twice the thickness of rim.

Let t = Thickness of rim in metres, and

b = Width of rim in metres = $2\,t$.

∴ Cross-sectional area of rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

Since the punching operation takes place (*i.e.* energy is consumed) during 1/10 th of a revolution of the crank shaft, therefore during 9/10 th of the revolution of a crank shaft, the energy is stored in the flywheel.

∴ Maximum fluctuation of energy,

$$\Delta E = \frac{9}{10} \times \text{Energy/stroke} = \frac{9}{10} \times 3817.8 \times 10^3$$

$$= 3436 \times 10^3 \text{ N-mm} = 3436 \text{ N-m}$$

Let m = Mass of the flywheel.

Since the hub and the spokes provide 5% of the rotational inertia of the wheel, therefore the maximum fluctuation of energy provided by the flywheel rim will be 95%.

∴ Maximum fluctuation of energy provided by the rim,

$$(\Delta E)_{rim} = 0.95 \times \Delta E = 0.95 \times 3436 = 3264 \text{ N-m}$$

Since the flywheel is to revolve at 9 times the speed of the crankshaft and there are 25 working strokes per minute, therefore mean speed of the flywheel,

$$N = 9 \times 25 = 225 \text{ r.p.m.}$$

and mean angular speed, $\omega = 2\,\pi \times 225 / 60 = 23.56$ rad/s

We know that maximum fluctuation of energy (ΔE),

$$3264 = m.R^2.\omega^2.C_S = m\,(0.7)^2\,(23.56)^2\,0.1 = 27.2\,m$$

∴ $m = 3264 / 27.2 = 120$ kg

We also know that mass of the flywheel (m),

$$120 = A \times \pi\,D \times \rho = 2\,t^2 \times \pi \times 1.4 \times 7250 = 63\,782\,t^2$$

∴ $t^2 = 120 / 63\,782 = 0.001\,88$ or $t = 0.044$ m = 44 mm **Ans.**

and $b = 2\,t = 2 \times 44 = 88$ mm **Ans.**

Check for centrifugal stress

We know that peripheral velocity of the rim,

$$v = \frac{\pi\,D\,.\,N}{60} = \frac{\pi \times 1.4 \times 225}{60} = 16.5 \text{ m/s}$$

∴ Centrifugal stress induced in the rim,

$$\sigma_t = \rho.v^2 = 7250\,(16.5)^2 = 1.97 \times 10^6 \text{ N/m}^2 = 1.97 \text{ MPa}$$

Since the centrifugal stress induced in the rim is less than the permissible value (*i.e.* 6 MPa), therefore it is safe **Ans.**

22.8 Stresses in Flywheel Arms

The following stresses are induced in the arms of a flywheel.

1. Tensile stress due to centrifugal force acting on the rim.

2. Bending stress due to the torque transmitted from the rim to the shaft or from the shaft to the rim.

3. Shrinkage stresses due to unequal rate of cooling of casting. These stresses are difficult to determine.

We shall now discuss the first two types of stresses as follows:

1. *Tensile stress due to the centrifugal force*

Due to the centrifugal force acting on the rim, the arms will be subjected to direct tensile stress whose magnitude is same as discussed in the previous article.

∴ Tensile stress in the arms,

$$\sigma_{t1} = \frac{3}{4}\ \sigma_t = \frac{3}{4}\ \rho \times v^2$$

2. *Bending stress due to the torque transmitted*

Due to the torque transmitted from the rim to the shaft or from the shaft to the rim, the arms will be subjected to bending, because they are required to carry the full torque load. In order to find out the maximum bending moment on the arms, it may be assumed as a centilever beam fixed at the hub and carrying a concentrated load at the free end of the rim as shown in Fig. 22.16.

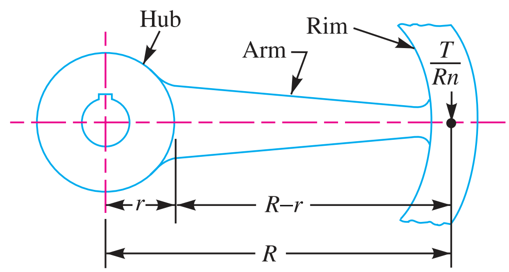

Fig. 22.16

Let T = Maximum torque transmitted by the shaft,

R = Mean radius of the rim,

r = Radius of the hub,

n = Number of arms, and

Z = Section modulus for the cross-section of arms.

We know that the load at the mean radius of the rim,

$$F = \frac{T}{R}$$

∴ Load on each arm $= \dfrac{T}{R\,.\,n}$

and maximum bending moment which lies on the arm at the hub,

$$M = \frac{T}{R\,.\,n}\ (R - r)$$

∴ Bending stress in arms,

$$\sigma_{b1} = \frac{M}{Z} = \frac{T}{R\,.\,n\,.\,Z}\ (R - r)$$

∴ Total tensile stress in the arms at the hub end,

$$\sigma = \sigma_{t1} + \sigma_{b1}$$

Notes: 1. The total stress on the arms should not exceed the allowable permissible stress.

2. If the flywheel is used as a belt pulley, then the arms are also subjected to bending due to net belt tension $(T_1 - T_2)$, where T_1 and T_2 are the tensions in the tight side and slack side of the belt respectively. Therefore the bending stress due to the belt tensions,

$$\sigma_{b2} = \frac{(T_1 - T_2)\ (R - r)}{\frac{n}{2} \times Z}$$

... (∵ Only half the number of arms are considered to be effective in transmitting the belt tensions)

∴ Total bending stress in the arms at the hub end,

$$\sigma_b = \sigma_{b1} + \sigma_{b2}$$

and the total tensile stress in the arms at the hub end,

$$\sigma = \sigma_{t1} + \sigma_{b1} + \sigma_{b2}$$

22.9 Design of Flywheel Arms

The cross-section of the arms is usually elliptical with major axis as twice the minor axis, as shown in Fig. 22.17, and it is designed for the maximum bending stress.

Let a_1 = Major axis, and

b_1 = Minor axis.

∴ Section modulus,

$$Z = \frac{\pi}{32} \times b_1 (a_1)^2 \quad \text{...(i)}$$

We know that maximum bending moment,

$$M = \frac{T}{R \,.\, n} \; (R - r)$$

∴ Maximum bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{T}{R.n.Z} \; (R - r) \quad \text{...(ii)}$$

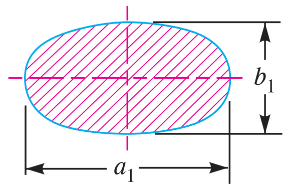

Fig. 22.17. Elliptical cross section of arms.

Assuming $a_1 = 2\, b_1$, the dimensions of the arms may be obtained from equations ***(i)*** and ***(ii)***.

Notes: 1. The arms of the flywheel have a taper from the hub to the rim. The taper is about 20 mm per metre length of the arm for the major axis and 10 mm per metre length for the minor axis.

2. The number of arms are usually 6. Sometimes the arms may be 8, 10 or 12 for very large size flywheels.

3. The arms may be curved or straight. But straight arms are easy to cast and are lighter.

4. Since arms are subjected to reversal of stresses, therefore a minimum factor of safety 8 should be used. In some cases like punching machines amd machines subjected to severe shock, a factor of safety 15 may be used.

5. The smaller flywheels (less than 600 mm diameter) are not provided with arms. They are made web type with holes in the web to facilitate handling.

22.10 Design of Shaft, Hub and Key

The diameter of shaft for flywheel is obtained from the maximum torque transmitted. We know that the maximum torque transmitted,

$$T_{max} = \frac{\pi}{16} \times \tau \, (d_1)^3$$

where d_1 = Diameter of the shaft, and

τ = Allowable shear stress for the material of the shaft.

The hub is designed as a hollow shaft, for the maximum torque transmitted. We know that the maximum torque transmitted,

$$T_{max} = \frac{\pi}{16} \times \tau \left(\frac{d^4 - d_1^4}{d} \right)$$

where d = Outer diameter of hub, and

d_1 = Inner diameter of hub or diameter of shaft.

The diameter of hub is usually taken as twice the diameter of shaft and length from 2 to 2.5 times the shaft diameter. It is generally taken equal to width of the rim.

A standard sunk key is used for the shaft and hub. The length of key is obtained by considering the failure of key in shearing. We know that torque transmitted by shaft,

$$T_{max} = L \times w \times \tau \times \frac{d_1}{2}$$

where L = Length of the key,

τ = Shear stress for the key material, and

d_1 = Diameter of shaft.

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Example 22.10. *Design and draw a cast iron flywheel used for a four stroke I.C engine developing 180 kW at 240 r.p.m. The hoop or centrifugal stress developed in the flywheel is 5.2 MPa, the total fluctuation of speed is to be limited to 3% of the mean speed. The work done during the power stroke is 1/3 more than the average work done during the whole cycle. The maximum torque on the shaft is twice the mean torque. The density of cast iron is 7220 kg/m*^{3}.

Solution. Given: $P = 180$ kW $= 180 \times 10^3$ W; $N = 240$ r.p.m. ; $\sigma_t = 5.2$ MPa $= 5.2 \times 10^6$ N/m^2 ; $N_1 - N_2 = 3\%\ N$; $\rho = 7220$ kg/m^3

First of all, let us find the maximum fluctuation of energy (ΔE). The turning moment diagram of a four stroke engine is shown in Fig. 22.18.

We know that mean torque transmitted by the flywheel,

$$T_{mean} = \frac{P \times 60}{2\ \pi\ N} = \frac{180 \times 10^3 \times 60}{2\ \pi \times 240} = 7161 \text{ N-m}$$

and *workdone per cycle $= T_{mean} \times \theta = 7161 \times 4\ \pi = 90\ 000$ N-m

Since the workdone during the power stroke is 1/3 more than the average workdone during the whole cycle, therefore,

Workdone during the power (or working) stroke

$$= 90\ 000 + \frac{1}{3} \times 90\ 000 = 120\ 000 \text{ N-m} \qquad ...(i)$$

The workdone during the power stroke is shown by a triangle *ABC* in Fig. 22.18 in which the base $AC = \pi$ radians and height $BF = T_{max.}$

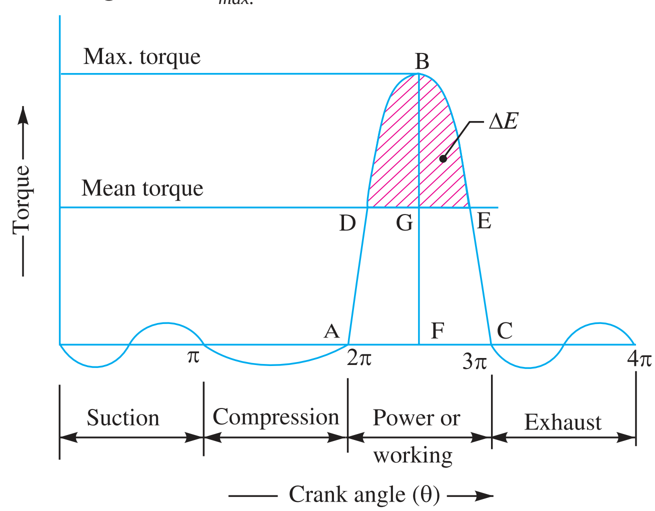

Fig. 22.18

* The workdone per cycle may also be obtained as discussed below :

Workdone per cycle $= \dfrac{P \times 60}{n}$, where n = Number of working strokes per minute

For a four stroke engine, $n = N / 2 = 240 / 2 = 120$

$\therefore$ Workdone per cycle $= \dfrac{180 \times 10^3 \times 60}{120} = 90\ 000$ N-m

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∴ Workdone during power stroke

$$= \frac{1}{2} \times \pi \times T_{max} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we have

$$\frac{1}{2} \times \pi \times T_{max} = 120\ 000$$

$$\therefore \qquad T_{max} = \frac{120\ 000 \times 2}{\pi} = 76\ 384 \text{ N-m}$$

Height above the mean torque line,

$$BG = BF - FG = T_{max} - T_{mean} = 76\ 384 - 7161 = 69\ 223 \text{ N-m}$$

Since the area *BDE* shown shaded in Fig. 22.18 above the mean torque line represents the maximum fluctuation of energy (ΔE), therefore from geometrical relation,

$$\frac{\text{Area of } \Delta\ BDE}{\text{Area of } \Delta\ ABC} = \frac{(BG)^2}{(BF)^2}, \text{ we have}$$

*Maximum fluctuation of energy (*i.e.* area of $\Delta\ BDE$),

$$\Delta E = \text{Area of } \Delta ABC \times \left(\frac{BG}{BF}\right)^2 = 120\ 000 \left(\frac{69\ 223}{76\ 384}\right)^2 = 98\ 555 \text{ N-m}$$

1. *Diameter of the flywheel rim*

Let D = Diameter of the flywheel rim in metres, and

v = Peripheral velocity of the flywheel rim in m/s.

We know that the hoop stress developed in the flywheel rim (σ_t),

$$5.2 \times 10^6 = \rho.v^2 = 7220 \times v^2$$

$$\therefore \qquad v^2 = 5.2 \times 10^6 / 7220 = 720 \quad \text{or} \quad v = 26.8 \text{ m/s}$$

We also know that peripheral velocity (v),

$$26.8 = \frac{\pi\ D\ .N}{60} = \frac{2D \times 250}{60} = 13.1\ D$$

$$\therefore \qquad D = 26.8 / 13.1 = 2.04 \text{ m} \quad \textbf{Ans.}$$

2. *Mass of the flywheel rim*

Let m = Mass of the flywheel rim in kg.

We know that angular speed of the flywheel rim,

$$\omega = \frac{2\ \pi\ N}{60} = \frac{2\ \pi \times 250}{60} = 25.14 \text{ rad / s}$$

and coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = 0.03$$

We know that maximum fluctuation of energy (ΔE),

$$98\ 555 = m.R^2.\omega^2.C_S = m\left(\frac{2.04}{2}\right)^2 (25.14)^2\ 0.03 = 19.73\ m$$

$$\therefore \qquad m = 98\ 555 / 19.73 = 4995 \text{ kg} \quad \textbf{Ans.}$$

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* The approximate value of maximum fluctuation of energy may be obtained as discussed below :

Workdone per cycle = 90 000 N-mm ...(as calculated above)

Workdone per stroke = 90 000 / 4 = 22 500 N-m ...(∵ of four stroke engine)

and workdone during power stroke = 120 000 N-m

∴ Maximum fluctuation of energy,

$$\Delta E = 120\ 000 - 22\ 500 = 97\ 500 \text{ N-m}$$

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3. *Cross-sectional dimensions of the rim*

Let t = Depth or thickness of the rim in metres, and

b = Width of the rim in metres = $2\,t$ ...(Assume)

∴ Cross-sectional area of the rim,

$$A = b.t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim (m),

$$4995 = A \times \pi D \times \rho = 2\,t^2 \times \pi \times 2.04 \times 7220 = 92\,556\,t^2$$

∴ t^2 = 4995 / 92 556 = 0.054 or t = 0.232 say 0.235 m = 235 mm **Ans.**

and $b = 2\,t = 2 \times 235 = 470$ mm **Ans.**

4. *Diameter and length of hub*

Let d = Diameter of the hub,

d_1 = Diameter of the shaft, and

l = Length of the hub.

Since the maximum torque on the shaft is twice the mean torque, therefore maximum torque acting on the shaft,

$$T_{max} = 2 \times T_{mean} = 2 \times 7161 = 14\,322 \text{ N-m} = 14\,322 \times 10^3 \text{ N-mm}$$

We know that the maximum torque acting on the shaft (T_{max}),

$$14\,322 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_1)^3 = \frac{\pi}{16} \times 40\,(d_1)^3 = 7.855\,(d_1)^3$$

...(Taking τ = 40 MPa = 40 N/mm^2)

∴ $(d_1)^3 = 14\,322 \times 10^3 / 7.855 = 1823 \times 10^3$

or d_1 = 122 say 125 m **Ans.**

The diameter of the hub is made equal to twice the diameter of shaft and length of hub is equal to width of the rim.

∴ $d = 2\,d_1 = 2 \times 125 = 250$ mm = 0.25 m

and $l = b = 470$ mm = 0.47 mm **Ans.**

5. *Cross-sectional dimensions of the elliptical arms*

Let a_1 = Major axis,

b_1 = Minor axis = $0.5\,a_1$ ...(Assume)

n = Number of arms = 6 ...(Assume)

σ_b = Bending stress for the material of arms = 15 MPa = 15 N/mm^2 ...(Assume)

We know that the maximum bending moment in the arm at the hub end, which is assumed as cantilever is given by

$$M = \frac{T}{R.n}(R - r) = \frac{T}{D\,.\,n}(D - d) = \frac{14\,322}{2.04 \times 6}(2.04 - 0.25) \text{ N-m}$$

$$= 2094.5 \text{ N-m} = 2094.5 \times 10^3 \text{ N-mm}$$

and section modulus for the cross-section of the arm,

$$Z = \frac{\pi}{32} \times b_1\,(a_1)^2 = \frac{\pi}{32} \times 0.5\,a_1\,(a_1)^2 = 0.05\,(a_1)^3$$

We know that the bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{2094.5 \times 10^3}{0.05(a_1)^3} = \frac{41\,890 \times 10^3}{(a_1)^3}$$

$\therefore$ $(a_1)^3 = 41\,890 \times 10^3 / 15 = 2793 \times 10^3$ or $a_1 = 140$ mm **Ans.**

and $b_1 = 0.5\, a_1 = 0.5 \times 140 = 70$ mm **Ans.**

6. *Dimensions of key*

The standard dimensions of rectangular sunk key for a shaft of diameter 125 mm are as follows:

Width of key, $w = 36$ mm **Ans.**

and thickness of key $= 20$ mm **Ans.**

The length of key (L) is obtained by considering the failure of key in shearing.

We know that the maximum torque transmitted by the shaft (T_{max}),

$$14\,322 \times 10^3 = L \times w \times \tau \times \frac{d_1}{2} = L \times 36 \times 40 \times \frac{125}{2} = 90 \times 10^3\, L$$

$\therefore$ $L = 14\,322 \times 10^3 / 90 \times 10^3 = 159$ say 160 mm **Ans.**

Let us now check the total stress in the rim which should not be greater than 15 MPa. We know that total stress in the rim,

$$\sigma = \rho.v^2 \left(0.75 + \frac{4.935\, R}{n^2 \,.\, t}\right)$$

$$= 7220\,(26.8)^2 \left[0.75 + \frac{4.935\,(2.04 / 2)}{6^2 \times 0.235}\right] \text{N/m}^2$$

$$= 5.18 \times 10^6\,(0.75 + 0.595) = 6.97 \times 10^6 \text{ N/m}^2 = 6.97 \text{ MPa}$$

Since it is less than 15 MPa, therefore the design is safe.

Example 22.11. *A single cylinder double acting steam engine delivers 185 kW at 100 r.p.m. The maximum fluctuation of energy per revolution is 15 per cent of the energy developed per revolution. The speed variation is limited to 1 per cent either way from the mean. The mean diameter of the rim is 2.4 m. Design and draw two views of the flywheel.*

Solution. Given : $P = 185$ kW $= 185 \times 10^3$ W ; $N = 100$ r.p.m ; $\Delta E = 15\%\, E = 0.15\, E$; $D = 2.4$ m or $R = 1.2$ m

1. *Mass of the flywheel rim*

Let m = Mass of the flywheel rim in kg.

We know that the workdone or energy developed per revolution,

$$E = \frac{P \times 60}{N} = \frac{185 \times 10^3 \times 60}{100} = 111\,000 \text{ N-m}$$

$\therefore$ Maximum fluctuation of energy,

$$\Delta E = 0.15\, E = 0.15 \times 111\,000 = 16\,650 \text{ N-m}$$

Since the speed variation is 1% either way from the mean, therefore the total fluctuation of speed,

$$N_1 - N_2 = 2\% \text{ of mean speed} = 0.02\, N$$

and coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = 0.02$$

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Velocity of the flywheel,

$$v = \frac{\pi D.N}{60} = \frac{\pi \times 2.4 \times 100}{60} = 12.57 \text{ m/s}$$

We know that the maximum fluctuation of energy (ΔE),

$$16\,650 = m.v^2.C_S = m\,(12.57)^2\,0.02 = 3.16\,m$$

$$\therefore \quad m = 16\,650 / 3.16 = 5270 \text{ kg} \quad \textbf{Ans.}$$

2. Cross-sectional dimensions of the flywheel rim

Let t = Thickness of the flywheel rim in metres, and

b = Width of the flywheel rim in metres = $2\,t$ ...(Assume)

∴ Cross-sectional area of the rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim (m),

$$5270 = A \times \pi D \times \rho = 2\,t^2 \times \pi \times 2.4 \times 7200 = 108\,588\,t^2$$

...(Taking ρ = 7200 kg / m^3)

$$\therefore \quad t^2 = 5270 / 108\,588 = 0.0485 \quad \text{or} \quad t = 0.22 \text{ m} = 220 \text{ mm} \quad \textbf{Ans.}$$

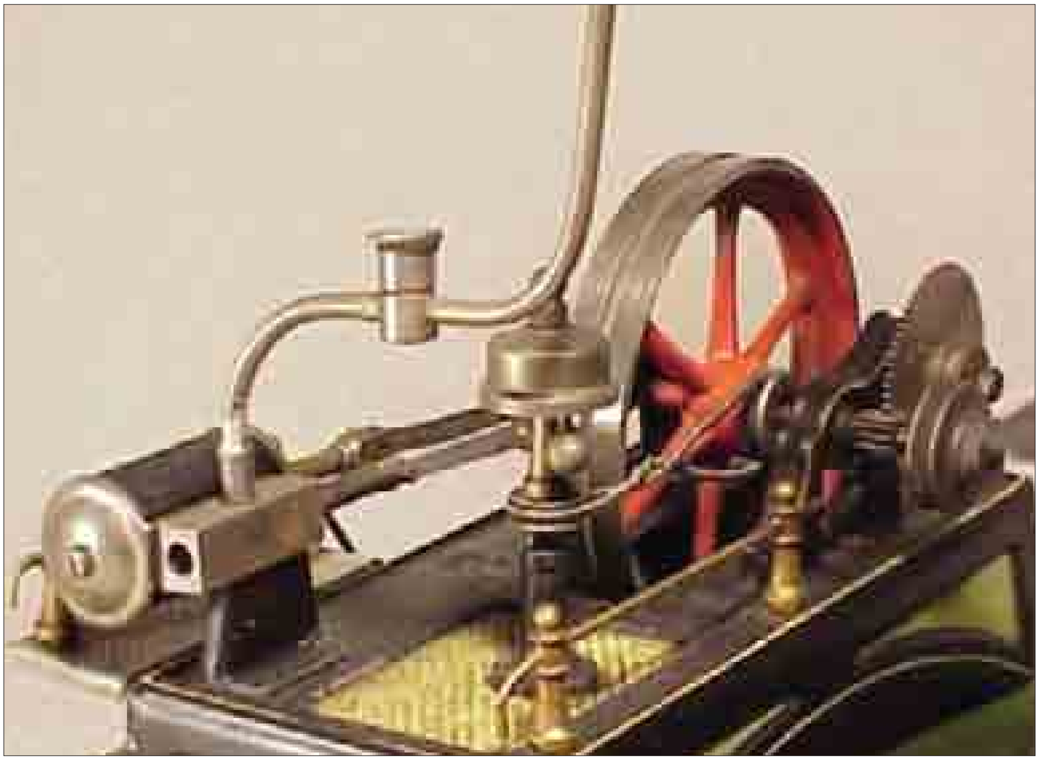
and $b = 2\,t = 2 \times 220 = 440$ mm **Ans.**

3. Diameter and length of hub

Let d = Diameter of the hub,

d_1 = Diameter of the shaft, and

l = Length of the hub,



Steam engine in a Laboratory

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We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\,\pi\,N} = \frac{185 \times 10^3 \times 60}{2\,\pi \times 100} = 17\ 664 \text{ N-m}$$

Assuming that the maximum torque transmitted (T_{max}) by the shaft is twice the mean torque, therefore

$$T_{max} = 2 \times T_{mean} = 2 \times 17\ 664 = 35\ 328 \text{ N-m} = 35.328 \times 10^6 \text{ N-mm}$$

We also know that maximum torque transmitted by the shaft (T_{max}),

$$35.328 \times 10^6 = \frac{\pi}{16} \times \tau\,(d_1)^3 = \frac{\pi}{16} \times 40\,(d_1)^3 = 7.855\,(d_1)^3$$

...(Assuming τ = 40 MPa = 40 N/mm^2)

$\therefore$ $(d_1)^3 = 35.328 \times 10^6 / 7.855 = 4.5 \times 10^6$ or $d_1 = 165$ mm **Ans.**

The diameter of the hub (d) is made equal to twice the diameter of the shaft (d_1) and length of the hub (l) is equal to the width of the rim (b).

$\therefore$ $d = 2\,d_1 = 2 \times 165 = 330$ mm ; and $l = b = 440$ mm **Ans.**

4. Cross-sectional dimensions of the elliptical arms

Let a_1 = Major axis,

b_1 = Minor axis = $0.5\,a_1$ ...(Assume)

n = Number of arms = 6 ...(Assume)

σ_b = Bending stress for the material of the arms = 14 MPa = 14 N/mm^2 ...(Assume)

We know that the maximum bending moment in the arm at the hub end which is assumed as cantilever is given by

$$M = \frac{T}{R.n}(R - r) = \frac{T}{D\,.\,n}(D - d) = \frac{35\ 328}{2.4 \times 6}(2.4 - 0.33) \text{ N-m}$$

$$= 5078 \text{ N-m} = 5078 \times 10^3 \text{ N-mm}$$

...(d is taken in metres)

and section modulus for the cross-section of the arm,

$$Z = \frac{\pi}{32}\,b_1\,(a_1)^2 = \frac{\pi}{32} \times 0.5\,a_1\,(a_1)^2 = 0.05\,(a_1)^3$$

We know that the bending stress (σ_b),

$$14 = \frac{M}{Z} = \frac{5078 \times 10^3}{0.05\,(a_1)^3} = \frac{101\ 560 \times 10^3}{(a_1)^3}$$

$\therefore$ $(a_1)^3 = 101\ 560 \times 10^3 / 14 = 7254 \times 10^3$

or a_1 = 193.6 say 200 mm **Ans.**

and $b_1 = 0.5\,a_1 = 0.5 \times 200 = 100$ mm **Ans.**

5. Dimensions of key

The standard dimensions of rectangular sunk key for a shaft of 165 mm diameter are as follows:

Width of key, w = 45 mm **Ans.**

and thickness of key = 25 mm **Ans.**

The length of key (L) is obtained by considering the failure of key in shearing.

We know that the maximum torque transmitted by the shaft (T_{max}),

$$35.328 \times 10^6 = L \times w \times \tau \times \frac{d_1}{2} = L \times 45 \times 40 \times \frac{165}{2} = 148\ 500\,L$$

$\therefore$ $L = 35.328 \times 10^6 / 148\ 500 = 238$ mm **Ans.**

Let us now check the total stress in the rim which should not be greater than 14 MPa. We know that the total stress in the rim,

$$= \rho . v^2 \left(0.75 + \frac{4.935\, R}{n^2 .\, t}\right)$$

$$= 7200\,(12.57)^2 \left[0.75 + \frac{4.935 \times 1.2}{6^2 \times 0.22}\right] \text{N/m}^2$$

$$= 1.14 \times 10^6\,(0.75 + 0.75) = 1.71 \times 10^6 \text{ N/m}^2 = 1.71 \text{ MPa}$$

Since it is less than 14 MPa, therefore the design is safe.

Example 22.12. *A punching press pierces 35 holes per minute in a plate using 10 kN-m of energy per hole during each revolution. Each piercing takes 40 per cent of the time needed to make one revolution. The punch receives power through a gear reduction unit which in turn is fed by a motor driven belt pulley 800 mm diameter and turning at 210 r.p.m. Find the power of the electric motor if overall efficiency of the transmission unit is 80 per cent. Design a cast iron flywheel to be used with the punching machine for a coefficient of steadiness of 5, if the space considerations limit the maximum diameter to 1.3 m.*

Allowable shear stress in the shaft material = 50 MPa

Allowable tensile stress for cast iron = 4 MPa

Density of cast iron = 7200 kg / m^3

Solution. Given : No. of holes = 35 per min ; Energy per hole = 10 kN-m = 10 000 N-m ; d = 800 mm = 0.8 m ; N = 210 r.p.m. ; η = 80% = 0.8 ; $1/C_S$ = 5 or C_S = 1/5 = 0.2 ; D_{max} = 1.3 m ; τ = 50 MPa = 50 N/mm^2 ; σ_t = 4 MPa = 4 N/mm^2 ; ρ = 7200 kg / m^3

Power of the electric motor

We know that energy used for piercing holes per minute

= No. of holes pierced × Energy used per hole

= 35 × 10 000 = 350 000 N-m / min

∴ Power needed for the electric motor,

$$P = \frac{\text{Energy used per minute}}{60 \times \eta} = \frac{350\,000}{60 \times 0.8} = 7292 \text{ W} = 7.292 \text{ kW} \quad \textbf{Ans.}$$

Design of cast iron flywheel

First of all, let us find the maximum fluctuation of energy.

Since the overall efficiency of the transmission unit is 80%, therefore total energy to be supplied during each revolution,

$$E_T = \frac{10\,000}{0.8} = 12\,500 \text{ N-m}$$

We know that velocity of the belt,

$$v = \pi\, d.N = \pi \times 0.8 \times 210 = 528 \text{ m/min}$$

∴ Net tension or pull acting on the belt

$$= \frac{P \times 60}{v} = \frac{7292 \times 60}{528} = 828.6 \text{ N}$$

Since each piercing takes 40 per cent of the time needed to make one revolution, therefore time required to punch a hole

= 0.4 / 35 = 0.0114 min

and the distance moved by the belt during punching a hole

= Velocity of the belt × Time required to punch a hole

= 528 × 0.0114 = 6.03 m

∴ Energy supplied by the belt during punching a hole,

$$E_B = \text{Net tension} \times \text{Distance travelled by belt}$$
$$= 828.6 \times 6.03 = 4996 \text{ N-m}$$

Thus energy to be supplied by the flywheel for punching during each revolution or maximum fluctuation of energy,

$$\Delta E = E_T - E_B = 12\,500 - 4996 = 7504 \text{ N-m}$$

1. *Mass of the flywheel*

Let m = Mass of the flywheel rim.

Since space considerations limit the maximum diameter of the flywheel as 1.3 m ; therefore let us take the mean diameter of the flywheel,

$$D = 1.2 \text{ m or } R = 0.6 \text{ m}$$

We know that angular velocity

$$\omega = \frac{2\,\pi \times N}{60} = \frac{2\,\pi \times 210}{60} = 22 \text{ rad / s}$$

We also know that the maximum fluctuation of energy (ΔE),

$$7504 = m.R^2.\omega^2.C_S = m\,(0.6)^2\,(22)^2\,0.2 = 34.85\,m$$

∴ $m = 7504 / 34.85 = 215.3$ kg **Ans.**

2. *Cross-sectional dimensions of the flywheel rim*

Let t = Thickness of the flywheel rim in metres, and

b = Width of the flywheel rim in metres = $2\,t$ ...(Assume)

∴ Cross-sectional area of the rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim (m),

$$215.3 = A \times \pi\,D \times \rho = 2\,t^2 \times \pi \times 1.2 \times 7200 = 54.3 \times 10^3\,t^2$$

∴ $t^2 = 215.3 / 54.3 \times 10^3 = 0.003\,96$

or $t = 0.063$ say 0.065 m = 65 mm **Ans.**

and $b = 2\,t = 2 \times 65 = 130$ mm **Ans**

3. *Diameter and length of hub*

Let d = Diameter of the hub,

d_1 = Diameter of the shaft, and

l = Length of the hub.

First of all, let us find the diameter of the shaft (d_1). We know that the mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\,\pi\,N} = \frac{7292 \times 60}{2\,\pi \times 210} = 331.5 \text{ N-m}$$

Assuming that the maximum torque transmitted by the shaft is twice the mean torque, therefore maximum torque transmitted by the shaft,

$$T_{max} = 2 \times T_{mean} = 2 \times 331.5 = 663 \text{ N-m} = 663 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted by the shaft (T_{max}),

$$663 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_1)^3 = \frac{\pi}{16} \times 50\,(d_1)^3 = 9.82\,(d_1)^3$$

∴ $(d_1)^3 = 663 \times 10^3 / 9.82 = 67.5 \times 10^3$

or $d_1 = 40.7$ say 45 mm **Ans.**

The diameter of the hub (d) is made equal to twice the diameter of the shaft (d_1) and length of hub (l) is equal to the width of the rim (b).

$\therefore \quad d = 2\,d_1 = 2 \times 45 = 90$ mm = 0.09 m and $l = b = 130$ mm **Ans.**

4. Cross-sectional dimensions of the elliptical cast iron arms

Let a_1 = Major axis,

b_1 = Minor axis = 0.5 a_1 ...(Assume)

n = Number of arms = 6 ... (Assume)

We know that the maximum bending moment in the arm at the hub end, which is assumed as cantilever is given by

$$M = \frac{T}{R\,.\,n}(R - r) = \frac{T}{D.n}(D - d) = \frac{663}{1.2 \times 6}(1.2 - 0.09) \text{ N-m}$$

$$= 102.2 \text{ N-m} = 102\,200 \text{ N-mm}$$

and section modulus for the cross-section of the arms,

$$Z = \frac{\pi}{32} \times b_1 (a_1)^2 = \frac{\pi}{32} \times 0.5\,a_1 (a_1)^2 = 0.05\,(a_1)^3$$

We know that bending stress (σ_t),

$$4 = \frac{M}{Z} = \frac{102\,200}{0.05\,(a_1)^3} = \frac{2044 \times 10^3}{(a_1)^3}$$

$\therefore \quad (a_1)^3 = 2044 \times 10^3 / 4 = 511 \times 10^3$ or $a_1 = 80$ mm **Ans.**

and $\quad b_1 = 0.5\,a_1 = 0.5 \times 80 = 40$ mm **Ans.**

5. Dimensions of key

The standard dimensions of rectangular sunk key for a shaft of diameter 45 mm are as follows:

Width of key, w = 16 mm **Ans.**

and thickness of key = 10 mm **Ans.**

The length of key (L) is obtained by considering the failure of key in shearing.

We know that maximum torque transmitted by the shaft (T_{max}),

$$663 \times 10^3 = L \times w \times \tau \times \frac{d_1}{2} = L \times 16 \times 50 \times \frac{45}{2} = 18 \times 10^3\,L$$

$\therefore \quad L = 663 \times 10^3/18 \times 10^3 = 36.8$ say 38 mm **Ans.**

Let us now check the total stress in the rim which should not be greater than 4 MPa.

We know that the velocity of the rim,

$$v = \frac{\pi D \times N}{60} = \frac{\pi \times 1.2 \times 210}{60} = 13.2 \text{ m/s}$$

$\therefore$ Total stress in the rim,

$$\sigma = \rho.v^2 \left(0.75 + \frac{4.935\,R}{n^2\,.\,t}\right) = 7200\,(13.2)^2 \left[0.75 + \frac{4.935 \times 0.6}{6^2 \times 0.065}\right]$$

$$= 1.25 \times 10^6\,(0.75 + 1.26) = 2.5 \times 10^6 \text{ N/m}^2 = 2.5 \text{ MPa}$$

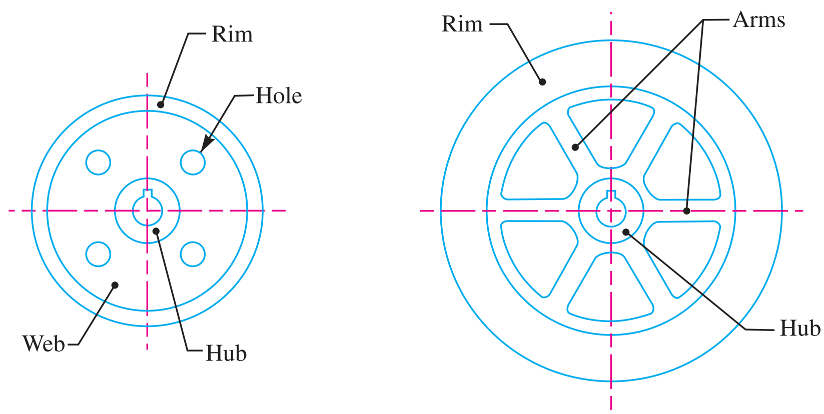
Since it is less than 4 MPa, therefore the design is safe.

22.11 Construction of Flywheels

The flywheels of smaller size (upto 600 mm diameter) are casted in one piece. The rim and hub are joined together by means of web as shown in Fig. 22.19 (a). The holes in the web may be made for handling purposes.

In case the flywheel is of larger size (upto 2.5 metre diameter), the arms are made instead of web, as shown in Fig. 22.19 (*b*). The number of arms depends upon the size of flywheel and its speed of rotation. But the flywheels above 2.5 metre diameter are usually casted in two piece. Such a flywheel is known as ***split flywheel.*** A ***split flywheel*** has the advantage of relieving the shrinkage stresses in the arms due to unequal rate of cooling of casting. A flywheel made in two halves should be spilt at the arms rather than between the arms, in order to obtain better strength of the joint. The two halves of the flywheel are connected by means of bolts through the hub, as shown in Fig. 22.20. The two halves are also joined at the rim by means of cotter joint (as shown in Fig. 22.20) or shrink links (as shown in Fig. 22.21). The width or depth of the shrink link is taken as 1.25 to 1.35 times the thickness of link. The slot in the rim into which the link is inserted is made slightly larger than the size of link.

Flywheel with web (no spokes)



(*a*) Flywheel with web.

(*b*) Flywheel with arms.

Fig. 22.19

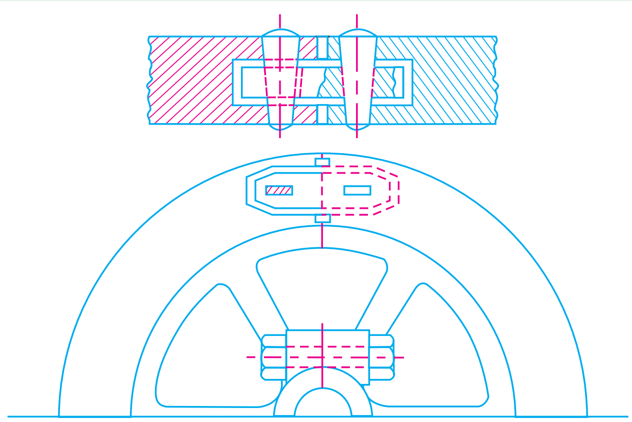


Fig. 22.20. Split flywheel.

Fig. 22.21. Shrink links.

The relative strength of a rim joint and the solid rim are given in the following table.

Table 22.3 Relative strength of a rim joint and the solid rim.

S.No.	*Type of construction*	*Relative strength*
1.	Solid rim.	1.00
2.	Flanged joint, bolted, rim parted between arms.	0.25
3.	Flanged joint, bolted, rim parted on an arm.	0.50
4.	Shrink link joint.	0.60
5.	Cotter or anchor joints.	0.70

Example 22.13. *A split type flywheel has outside diameter of the rim 1.80 m, inside diameter 1.35 m and the width 300 mm. the two halves of the wheel are connected by four bolts through the hub and near the rim joining the split arms and also by four shrink links on the rim. The speed is 250 r.p.m. and a turning moment of 15 kN-m is to be transmitted by the rim. Determine:*

1. *The diameter of the bolts at the hub and near the rim, σ_{tb} = 35 MPa.*

2. *The cross-sectional dimensions of the rectangular shrink links at the rim, σ_{tl} = 40 MPa ; $w = 1.25\ h$.*

3. *The cross-sectional dimensions of the elliptical arms at the hub and rim if the wheel has six arms, σ_{ta} = 15 MPa, minor axis being 0.5 times the major axis and the diameter of shaft being 150 mm.*

Assume density of the material of the flywheel as 7200 kg / m^3.

Solution. Given : D_0 = 1.8 m ; D_i = 1.35 m ; b = 300 mm = 0.3 m ; N = 250 r.p.m. ; T = 15 kN-m = 15 000 N-m ; σ_{tb} = 35 MPa = 35 N/mm^2 ; σ_{tl} = 40 MPa = 40 N/mm^2 ; $w = 1.25\ h$; $n = 6$; $b_1 = 0.5\ a_1$; σ_{ta} = 15 MPa = 15 N / mm^2 ; d_1 = 150 mm; ρ = 7200 kg / m^3.

1. Diameter of the bolts at the hub and near the rim

Let d_c = Core diameter of the bolts in mm.

We know that mean diameter of the rim,

$$D = \frac{D_o + D_i}{2} = \frac{1.8 + 1.35}{2} = 1.575 \text{ m}$$

and thickness of the rim,

$$t = \frac{D_o - D_i}{2} = \frac{1.8 - 1.35}{2} = 0.225 \text{ m}$$

Peripheral speed of the flywheel,

$$v = \frac{\pi D . N}{60} = \frac{\pi \times 1.575 \times 250}{60} = 20.6 \text{ m / s}$$

We know that centrifugal stress (or tensile stress) at the rim,

$$\sigma_t = \rho \times v^2 = 7200\ (20.6)^2 = 3.1 \times 10^6 \text{ N/m}^2 = 3.1 \text{ N/mm}^2$$

Cross-sectional area of the rim,

$$A = b \times t = 0.3 \times 0.225 = 0.0675 \text{ m}^2$$

∴ Maximum tensile force acting on the rim

$$= \sigma_t \times A = 3.1 \times 10^6 \times 0.0675 = 209\ 250 \text{ N} \qquad \text{...}(i)$$

We know that tensile strength of the four bolts

$$= \frac{\pi}{4}(d_c)^2 \sigma_{tb} \times \text{No. of bolts} = \frac{\pi}{4}(d_c)^2\, 35 \times 4 = 110\,(d_c)^2 \qquad ...(ii)$$

Since the bolts are made as strong as the rim joint, therefore from equations ***(i)*** and ***(ii)***, we have

$$(d_c)^2 = 209\,250 / 110 = 1903 \quad \text{or} \quad d_c = 43.6 \text{ mm}$$

The standard size of the bolt is M 56 with d_c = 48.65 mm **Ans.**

2. Cross-sectional dimensions of rectangular shrink links at the rim

Let h = Depth of the link in mm, and

w = Width of the link in mm = 1.25 h ...(Given)

∴ Cross-sectional area of each link,

$$A_l = w \times h = 1.25\, h^2 \text{ mm}^2$$

We know that the maximum tensile force on half the rim

$$= 2 \times \sigma_t \text{ for rim} \times \text{Cross-sectional area of rim}$$

$$= 2 \times 3.1 \times 10^6 \times 0.0675 = 418\,500 \text{ N} \qquad ...(iii)$$

and tensile strength of the four shrink links

$$= \sigma_{tl} \times A_l \times 4 = 40 \times 1.25\, h^2 \times 4 = 200\, h^2 \qquad ...(iv)$$

From equations ***(iii)*** and ***(iv)***, we have

$$h^2 = 418\,500 / 200 = 2092.5 \quad \text{or} \quad h = 45.7 \text{ say } 46 \text{ mm} \quad \textbf{Ans.}$$

and $w = 1.25\, h = 1.25 \times 46 = 57.5$ say 58 mm **Ans.**

3. Cross-sectional dimensions of the elliptical arms

Let a_1 = Major axis,

b_1 = Minor axis = 0.5 a_1 ...(Given)

n = Number of arms = 6 ...(Given)

Since the diameter of shaft (d_1) is 150 mm and the diameter of hub (d) is taken equal to twice the diameter of shaft, therefore

$$d = 2\, d_1 = 2 \times 150 = 300 \text{ mm} = 0.3 \text{ m}$$

We know that maximum bending moment on arms at the hub end,

$$M = \frac{T}{R.n}(R - r) = \frac{T}{D.n}(D - d) = \frac{15000}{1.575 \times 6}(1.575 - 0.3)$$

$$= 2024 \text{ N-m} = 2024 \times 10^3 \text{ N-mm}$$

Section modulus, $Z = \frac{\pi}{32} \times b_1 (a_1)^2 = \frac{\pi}{32} \times 0.5\, a_1 (a_1)^2 = 0.05\,(a_1)^3$

We know that bending stress for arms (σ_{ta}),

$$15 = \frac{M}{Z} = \frac{2024 \times 10^3}{0.05\,(a_1)^3} = \frac{40.5 \times 10^6}{(a_1)^3}$$

∴ $(a_1)^3 = 40.5 \times 10^6 / 15 = 2.7 \times 10^6$ or $a_1 = 139.3$ say 140 mm **Ans.**

and $b_1 = 0.5\, a_1 = 0.5 \times 140 = 70$ mm **Ans.**

EXERCISES

1. The turning moment diagram for a multicylinder engine has been drawn to a scale of 1 mm = 1000 N-m and 1 mm = 6º. The areas above and below the mean turning moment line taken in order are 530, 330, 380, 470, 180, 360, 350 and 280 sq.mm.
   For the engine, find the diameter of the flywheel. The mean r.p.m is 150 and the total fluctuation of speed must not exceed 3.5% of the mean.

Determine a suitable cross-sectional area of the rim of the flywheel, assuming the total energy of the flywheel to be $\frac{15}{14}$ that of the rim. The peripheral velocity of the flywheel is 15 m/s.

2. A machine has to carry out punching operation at the rate of 10 holes/min. It does 6 N-m of work per sq mm of the sheared area in cutting 25 mm diameter holes in 20 mm thick plates. A flywheel is fitted to the machine shaft which is driven by a constant torque. The fluctuation of speed is between 180 and 200 r.p.m. Actual punching takes 1.5 seconds. Frictional losses are equivalent to 1/6 of the workdone during punching. Find:

(*a*) Power required to drive the punching machine, and

(*b*) Mass of the flywheel, if radius of gyration of the wheel is 450 mm.

3. The turning moment diagram for an engine is drawn to the following scales:

1 mm = 3100 N-m ; 1 mm = 1.6º

The areas of the loops above and below the mean torque line taken in order are: 77, 219, 588, 522, 97, 116, 1200 and 1105 mm^2.

The mean speed of the engine is 300 r.p.m. and the permissible fluctuation in speed is ± 2 per cent of mean speed. The stress in the material of the rim is not to exceed 4.9 MPa and density of its material is 7200 kg/m^3. Assuming that the rim stores $\frac{15}{16}$ of the energy that is stored by the flywheel, estimate

(*a*) Diameter of rim; and (*b*) Area of cross-section of rim.

4. A single cylinder internal combustion engine working on the four stroke cycle develops 75 kW at 360 r.p.m. The fluctuation of energy can be assumed to be 0.9 times the energy developed per cycle. If the fluctuation of speed is not to exceed 1 per cent and the maximum centrifugal stress in the flywheel is to be 5.5 MPa, estimate the mean diameter and the cross-sectional area of the rim. The material of the rim has a density of 7200 kg / m^3. **[Ans. 1.464 m ; 0.09 m^2]**

5. Design a cast iron flywheel for a four stroke cycle engine to develop 110 kW at 150 r.p.m. The work done in the power stroke is 1.3 times the average work done during the whole cycle. Take the mean diameter of the flywheel as 3 metres. The total fluctuation of speed is limited to 5 per cent of the mean speed. The material density is 7250 kg / m^3. The permissible shear stress for the shaft material is 40 MPa and flexural stress for the arms of the flywheel is 20 MPa.

6. A punching press is required to punch 40 mm diameter holes in a plate of 15 mm thickness at the rate of 30 holes per minute. It requires 6 N-m of energy per mm^2 of sheared area. Determine the moment of inertia of the flywheel if the punching takes one-tenth of a second and the r.p.m. of the flywheel varies from 160 to 140.

7. A punch press is fitted with a flywheel capable of furnishing 3000 N-m of energy during quarter of a revolution near the bottom dead centre while blanking a hole on sheet metal. The maximum speed of the flywheel during the operation is 200 r.p.m. and the speed decreases by 10% during the cutting stroke. The mean radius of the rim is 900 mm. Calculate the approximate mass of the flywheel rim assuming that it contributes 90% of the energy requirements.

8. A punching machine makes 24 working strokes per minute and is capable of punching 30 mm diameter holes in 20 mm thick steel plates having an ultimate shear strength of 350 MPa. The punching operation takes place during $\frac{1}{10}$ th of a revolution of the crankshaft. Find the power required for the driving motor, assuming a mechanical efficiency of 76%. Determine suitable dimensions for the rim cross-section of the flywheel, which revolves at 9 times the speed of crankshaft. The permissible coefficient of fluctuation of speed is 0.4.

The flywheel is to be made of cast iron having a safe tensile stress of 6 MPa and density 7250 kg/m^3. The diameter of the flywheel must not exceed 1.05 m owing to space restrictions. The hub and spokes may be assumed to provide 5% of the rotational inertia of the wheel. Check for the centrifugal stress induced in the rim.

9. Design completely the flywheel, shaft and the key for securing the flywheel to the shaft, for a punching machine having a capacity of producing 30 holes of 20 mm diameter per minute in steel plate 16 mm thickness. The ultimate shear stress for the material of the plate is 360 MPa. The actual punching operation estimated to last for a period of 36º rotation of the punching machine crankshaft. This crank shaft is powered by a flywheel shaft through a reduction gearing having a ratio 1 : 8. Assume that the mechanical efficiency of the punching machine is 80% and during the actual punching operation the flywheel speed is reduced by a maximum of 10%. The diameter of flywheel is restricted to 0.75 m due to space limitations.

10. A cast iron wheel of mean diameter 3 metre has six arms of elliptical section. The energy to be stored in it is 560 kN-m when rotating at 120 r.p.m. The speed of the mean diameter is 18 m/s. Calculate the following:

    (*a*)Assuming that the whole energy is stored in the rim, find the cross-section, if the width is 300 mm.

    (*b*)Find the cross-section of the arms near the boss on the assumption that their resistance to bending is equal to the torsional resistance of the shaft which is 130 mm in diameter.

    The maximum shear stress in the shaft is to be within 63 MPa and the tensile stress 16 MPa. Assume the minor axis of the ellipse to be 0.65 major axis.

11. A cast iron flywheel is to be designed for a single cylinder double acting steam engine which delivers 150 kW at 80 r.p.m. The maximum fluctuation of energy per revolution is 10%. The total fluctuation of the speed is 4 per cent of the mean speed. If the mean diameter of the flywheel rim is 2.4 metres, determine the following :

    (*a*) Cross-sectional dimensions of the rim, assuming that the hub and spokes provide 5% of the rotational inertia of the wheel. The density of cast iron is 7200 kg/m^3 and tensile stress 16 MPa.

    Take width of rim equal to twice of thickness.

    (*b*) Dimensions of hub and rectangular sunk key. The shear stress for the material of shaft and key is 40 MPa.

    (*c*) Cross-sectional dimensions of the elliptical arms assuming major axis as twice of minor axis and number of arms equal to six.

12. Design a cast iron flywheel having six arms for a four stroke engine developing 120 kW at 150 r.p.m. The mean diameter of the flywheel may be taken as 3 metres. The fluctuation of speed is 2.5% of mean speed. The workdone during the working stroke is 1.3 times the average workdone during the whole cycle. Assume allowable shear stress for the shaft and key as 40 MPa and tensile stress for cast iron as 20 MPa. The following proportions for the rim and elliptical arms may be taken:

    (*a*) Width of rim = 2 × Thickness of rim

    (*b*) Major axis = 2 × Minor axis.

13. A multi-cylinder engine is to run at a speed of 500 r.p.m. On drawing the crank effort diagram to scale 1 mm = 2500 N-m and 1 mm = 3º, the areas above and below the mean torque line are in sq mm as below:

    + 160, – 172, + 168, – 191, + 197, – 162

    The speed is to be kept within ± 1% of the mean speed of the engine. Design a suitable rim type C.I. flywheel for the above engine. Assume rim width as twice the thickness and the overhang of the flywheel from the centre of the nearest bearing as 1.2 metres. The permissible stresses for the rim in tension is 6 MPa and those for shaft and key in shear are 42 MPa. The allowable stress for the arm is 14 MPa. Sketch a dimensioned end view of the flywheel.

14. An engine runs at a constant load at a speed of 480 r.p.m. The crank effort diagram is drawn to a scale 1 mm = 200 N-m torque and 1 mm = 3.6º crank angle. The areas of the diagram above and below the mean torque line in sq mm are in the following order:

+ 110, – 132, + 153, – 166, + 197, – 162

Design the flywheel if the total fluctuation of speed is not to exceed 10 r.p.m. and the centrifugal stress in the rim is not to exceed 5 MPa. You may assume that the rim breadth is approximately 2.5 times the rim thickness and 90% of the moment of inertia is due to the rim. The density of the material of the flywheel is 7250 kg/m^3.

Make a sketch of the flywheel giving the dimensions of the rim, the mean diameter of the rim and other estimated dimensions of spokes, hub etc.

15. A four stroke oil engine developing 75 kW at 300 r.p.m is to have the total fluctuation of speed limited to 5%. Two identical flywheels are to be designed. The workdone during the power stroke is found to be 1.3 times the average workdone during the whole cycle. The turning moment diagram can be approximated as a triangle during the power stroke. Assume that the hoop stress in the flywheel and the bending stress in the arms should not exceed 25 MPa. The shear stress in the key and shaft material should not exceed 40 MPa. Give a complete design of the flywheel. Assume four arms of elliptical cross-section with the ratio of axes 1 : 2. Design should necessarily include (*i*) moment of inertia of the flywheel, (*ii*) flywheel rim dimensions, (*iii*) arm dimensions, and (*iv*) flywheel boss and key dimensions and sketch showing two views of the flywheel with all the dimensions.

QUESTIONS

1. What is the main function of a flywheel in an engine?
2. In what way does a flywheel differ from that of a governor? Illustrate your answer with suitable examples.
3. Explain why flywheels are used in punching machines. Does the mounting of a flywheel reduce the stress induced in the shafts.
4. Define 'coefficient of fluctuation of speed' and 'coefficient of steadiness'.
5. What do you understand by 'fluctuation of energy' and 'maximum fluctuation of energy'.
6. Define 'coefficient of fluctuation of energy'.
7. Discuss the various types of stresses induced in a flywheel rim.
8. Explain the procedure for determining the size and mass of a flywheel with the help of a turning moment diagram.
9. Discuss the procedure for determining the cross-sectional dimensions of arms of a flywheel.
10. State the construction of flywheels.

OBJECTIVE TYPE QUESTIONS

1. The maximum fluctuation of speed is the

(*a*) difference of minimum fluctuation of speed and the mean speed

(*b*) difference of the maximum and minimum speeds

(*c*) sum of the maximum and minimum speeds

(*d*) variations of speed above and below the mean resisting torque line

2. The coefficient of fluctuation of speed is the .... of maximum fluctuation of speed and the mean speed.

(*a*) product (*b*) ratio

(*c*) sum (*d*) difference

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3. In a turning moment diagram, the variations of energy above and below the mean resisting torque line is called

(*a*) fluctuation of energy (*b*) maximum fluctuation of energy

(*c*) coefficient of fluctuation of energy (*d*) none of these

4. If E = Mean kinetic energy of the flywheel, C_S = Coefficient of fluctuation of speed and ΔE = Maximum fluctuation of energy, then

(*a*) $\Delta E = E / C_S$ (*b*) $\Delta E = E^2 \times C_S$

(*c*) $\Delta E = E \times C_S$ (*d*) $\Delta E = 2\,E \times C_S$

5. The ratio of the maximum fluctuation of energy to the .... is called coefficient of fluctuation of energy.

(*a*) minimum fluctuation of energy (*b*) workdone per cycle

6. Due to the centrifugal force acting on the rim, the flywheel arms will be subjected to

(*a*) tensile stress (*b*) compressive stress

(*c*) shear stress (*d*) none of these

7. The tensile stress in the flywheel rim due to the centrifugal force acting on the rim is given by

(*a*) $\dfrac{\rho . v^2}{4}$ (*b*) $\dfrac{\rho . v^2}{2}$

(*c*) $\dfrac{3\rho . v^2}{4}$ (*d*) $\rho . v^2$

where ρ = Density of the flywheel material, and

v = Linear velocity of the flywheel.

8. The cross-section of the flywheel arms is usually

(*a*) elliptical (*b*) rectangular

(*c*) *I*-section (*d*) L-section

9. In order to find the maximum bending moment on the arms, it is assumed as a

(*a*) simply supported beam carrying a uniformly distributed load over the arm

(*b*) fixed at both ends (*i.e.* at the hub and at the free end of the rim) and carrying a uniformly distributed load over the arm.

(*c*) cantilever beam fixed at the hub and carrying a concentrated load at the free end of the rim

(*d*) none of the above

10. The diameter of the hub of the flywheel is usually taken

(*a*) equal to the diameter of the shaft (*b*) twice the diameter of the shaft

(*c*) three times the diameter of the shaft (*d*) four times the diameter of the shaft

ANSWERS

1. (*b*)	**2.** (*b*)	**3.** (*a*)	**4.** (*d*)	**5.** (*b*)
6. (*a*)	**7.** (*d*)	**8.** (*a*)	**9.** (*c*)	**10.** (*b*)

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