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CHAPTER

23

Springs

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23.1 Introduction

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows :

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.
2. To apply forces, as in brakes, clutches and spring-loaded valves.
3. To control motion by maintaining contact between two elements as in cams and followers.
4. To measure forces, as in spring balances and engine indicators.
5. To store energy, as in watches, toys, etc.

23.2 Types of Springs

Though there are many types of the springs, yet the following, according to their shape, are important from the subject point of view.

1. *Helical springs.* The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are ***compression helical spring*** as shown in Fig. 23.1 (*a*) and ***tension helical spring*** as shown in Fig. 23.1 (*b*).

(*a*) Compression helical spring. (*b*) Tension helical spring.

Fig. 23.1. Helical springs.

The helical springs are said to be ***closely coiled*** when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small, it is usually less than 10°. The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring.

In ***open coiled helical springs,*** the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open coiled helical springs are limited, therefore our discussion shall confine to closely coiled helical springs only.

The helical springs have the following advantages:

(*a*) These are easy to manufacture.

(*b*) These are available in wide range.

(*c*) These are reliable.

(*d*) These have constant spring rate.

(*e*) Their performance can be predicted more accurately.

(*f*) Their characteristics can be varied by changing dimensions.

2. *Conical and volute springs.* The conical and volute springs, as shown in Fig. 23.2, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired. The conical spring, as shown in Fig. 23.2 (*a*), is wound with a uniform pitch whereas the volute springs, as shown in Fig. 23.2 (*b*), are wound in the form of paraboloid with constant pitch

(*a*) Conical spring. (*b*) Volute spring.

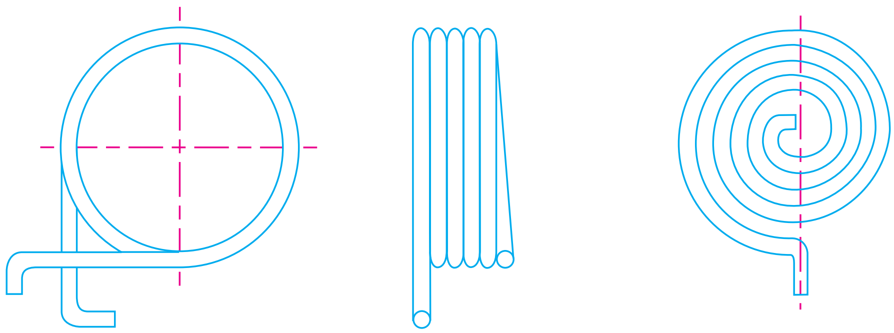
Fig. 23.2. Conical and volute springs.

and lead angles. The springs may be made either partially or completely telescoping. In either case, the number of active coils gradually decreases. The decreasing number of coils results in an increasing spring rate. This characteristic is sometimes utilised in vibration problems where springs are used to support a body that has a varying mass.

The major stresses produced in conical and volute springs are also shear stresses due to twisting.

3. *Torsion springs.* These springs may be of ***helical*** or ***spiral*** type as shown in Fig. 23.3. The **helical type** may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms. The **spiral type** is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks.

The major stresses produced in torsion springs are tensile and compressive due to bending.



(*a*) Helical torsion spring. (*b*) Spiral torsion spring.

Fig. 23.3. Torsion springs.

4. *Laminated or leaf springs.* The laminated or leaf spring (also known as ***flat spring*** or ***carriage spring***) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts, as shown in Fig. 23.4. These are mostly used in automobiles.

The major stresses produced in leaf springs are tensile and compressive stresses.

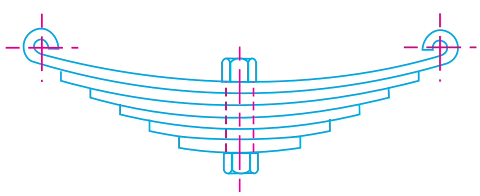


Fig. 23.4. Laminated or leaf springs.

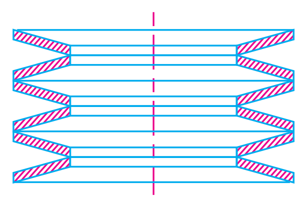


Fig. 23.5. Disc or bellevile springs.

5. *Disc or bellevile springs.* These springs consist of a number of conical discs held together against slipping by a central bolt or tube as shown in Fig. 23.5. These springs are used in applications where high spring rates and compact spring units are required.

The major stresses produced in disc or bellevile springs are tensile and compressive stresses.

6. Special purpose springs. These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

23.3 Material for Helical Springs

The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant. It largely depends upon the service for which they are used *i.e.* severe service, average service or light service.

Severe service means rapid continuous loading where the ratio of minimum to maximum load (or stress) is one-half or less, as in automotive valve springs.

Average service includes the same stress range as in severe service but with only intermittent operation, as in engine governor springs and automobile suspension springs.

Light service includes springs subjected to loads that are static or very infrequently varied, as in safety valve springs.

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 per cent carbon and 0.60 to 1.0 per cent manganese. Music wire is used for small springs. Non-ferrous materials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance and corrosion resistance.

Table 23.1 shows the values of allowable shear stress, modulus of rigidity and modulus of elasticity for various materials used for springs.

The helical springs are either cold formed or hot formed depending upon the size of the wire. Wires of small sizes (less than 10 mm diameter) are usually wound cold whereas larger size wires are wound hot. The strength of the wires varies with size, smaller size wires have greater strength and less ductility, due to the greater degree of cold working.

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Table 23.1. Values of allowable shear stress, Modulus of elasticity and Modulus of rigidity for various spring materials.

Material	*Allowable shear stress (τ) MPa*			*Modulus of rigidity (G) kN/m²*	*Modulus of elasticity (E) kN/mm²*
	Severe service	*Average service*	*Light service*		
1. Carbon steel					
(*a*) Upto to 2.125 mm dia.	420	525	651	80	210
(*b*) 2.125 to 4.625 mm	385	483	595	80	210
(*c*) 4.625 to 8.00 mm	336	420	525	80	210
(*d*) 8.00 to 13.25 mm	294	364	455	80	210
(*e*) 13.25 to 24.25 mm	252	315	392	80	210
(*f*) 24.25 to 38.00 mm	224	280	350	80	210
2. Music wire	392	490	612	80	210
3. Oil tempered wire	336	420	525	80	210
4. Hard-drawn spring wire	280	350	437.5	80	210
5. Stainless-steel wire	280	350	437.5	70	196
6. Monel metal	196	245	306	44	105
7. Phosphor bronze	196	245	306	44	105
8. Brass	140	175	219	35	100

23.4 Standard Size of Spring Wire

The standard size of spring wire may be selected from the following table :

Table 23.2. Standard wire gauge (SWG) number and corresponding diameter of spring wire.

SWG	*Diameter (mm)*	*SWG*	*Diameter (mm)*	*SWG*	*Diameter (mm)*	*SWG*	*Diameter (mm)*
7/0	12.70	7	4.470	20	0.914	33	0.2540
6/0	11.785	8	4.064	21	0.813	34	0.2337
5/0	10.973	9	3.658	22	0.711	35	0.2134
4/0	10.160	10	3.251	23	0.610	36	0.1930
3/0	9.490	11	2.946	24	0.559	37	0.1727
2/0	8.839	12	2.642	25	0.508	38	0.1524
0	8.229	13	2.337	26	0.457	39	0.1321
1	7.620	14	2.032	27	0.4166	40	0.1219
2	7.010	15	1.829	28	0.3759	41	0.1118
3	6.401	16	1.626	29	0.3454	42	0.1016
4	5.893	17	1.422	30	0.3150	43	0.0914
5	5.385	18	1.219	31	0.2946	44	0.0813
6	4.877	19	1.016	32	0.2743	45	0.0711

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23.5 Terms used in Compression Springs

The following terms used in connection with compression springs are important from the subject point of view.

1. *Solid length.* When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be ***solid***. The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

$$L_S = n'.d$$

where n' = Total number of coils, and

d = Diameter of the wire.

2. *Free length.* The free length of a compression spring, as shown in Fig. 23.6, is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed). Mathematically,

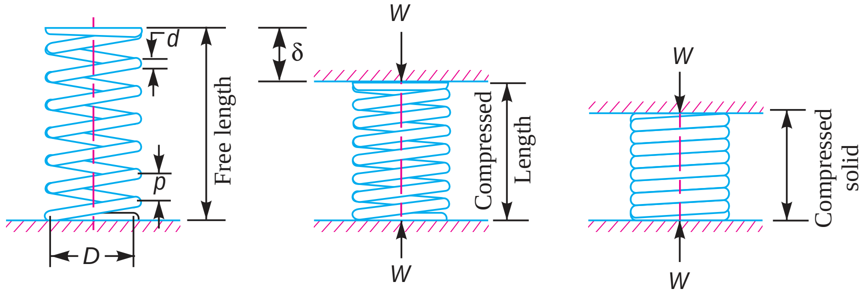

Fig. 23.6. Compression spring nomenclature.

Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$$= n'.d + \delta_{max} + 0.15\, \delta_{max}$$

The following relation may also be used to find the free length of the spring, *i.e.*

$$L_F = n'.d + \delta_{max} + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

3. *Spring index.* The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index, $C = D / d$

where D = Mean diameter of the coil, and

d = Diameter of the wire.

4. *Spring rate.* The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$

where W = Load, and

δ = Deflection of the spring.

* In actual practice, the compression springs are seldom designed to close up under the maximum working load and for this purpose a clearance (or clash allowance) is provided between the adjacent coils to prevent closing of the coils during service. It may be taken as 15 per cent of the maximum deflection.

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5. *Pitch*. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil, $$p = \frac{\text{Free length}}{n' - 1}$$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

Pitch of the coil, $$p = \frac{L_F - L_S}{n'} + d$$

where L_F = Free length of the spring,

L_S = Solid length of the spring,

n' = Total number of coils, and

d = Diameter of the wire.

In choosing the pitch of the coils, the following points should be noted :

(*a*) The pitch of the coils should be such that if the spring is accidently or carelessly compressed, the stress does not increase the yield point stress in torsion.

(*b*) The spring should not close up before the maximum service load is reached.

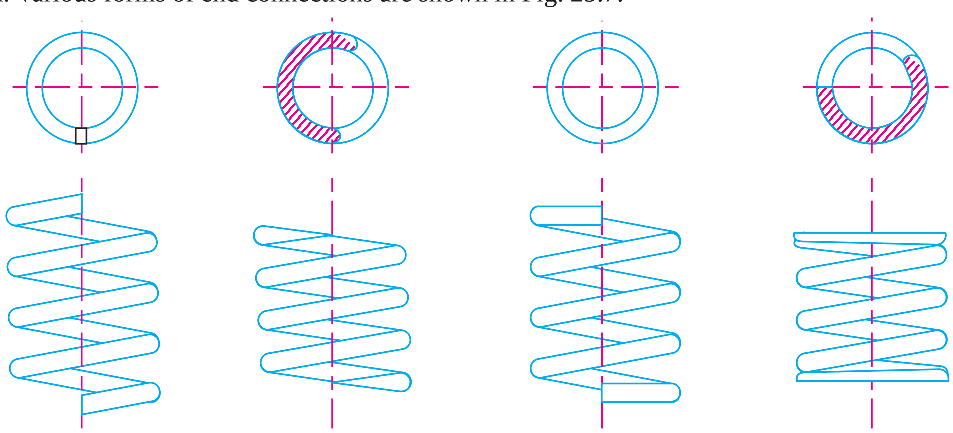
Note : In designing a tension spring (See Example 23.8), the minimum gap between two coils when the spring is in the free state is taken as 1 mm. Thus the free length of the spring,

$$L_F = n.d + (n - 1)$$

and pitch of the coil, $$p = \frac{L_F}{n - 1}$$

23.6 End Connections for Compression Helical Springs

The end connections for compression helical springs are suitably formed in order to apply the load. Various forms of end connections are shown in Fig. 23.7.



(*a*) Plain ends. (*b*) Ground ends. (*c*) Squared ends. (*d*) Squared and ground ends.

Fig 23.7. End connections for compression helical spring.

In all springs, the end coils produce an eccentric application of the load, increasing the stress on one side of the spring. Under certain conditions, especially where the number of coils is small, this effect must be taken into account. The nearest approach to an axial load is secured by squared and ground ends, where the end turns are squared and then ground perpendicular to the helix axis. It may be noted that part of the coil which is in contact with the seat does not contribute to spring action and hence are termed as ***inactive coils***. The turns which impart spring action are known as ***active turns.*** As the load increases, the number of inactive coils also increases due to seating of the end coils and the amount of increase varies from 0.5 to 1 turn at the usual working loads. The following table shows the total number of turns, solid length and free length for different types of end connections.

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Table 23.3. Total number of turns, solid length and free length for different types of end connections.

Type of end	*Total number of turns (n')*	*Solid length*	*Free length*
1. Plain ends	n	$(n + 1)\, d$	$p \times n + d$
2. Ground ends	n	$n \times d$	$p \times n$
3. Squared ends	$n + 2$	$(n + 3)\, d$	$p \times n + 3d$
4. Squared and ground ends	$n + 2$	$(n + 2)\, d$	$p \times n + 2d$

where n = Number of active turns,

p = Pitch of the coils, and

d = Diameter of the spring wire.

Tension helical spring

23.7 End Connections for Tension Helical Springs

The tensile springs are provided with hooks or loops as shown in Fig. 23.8. These loops may be made by turning whole coil or half of the coil. In a tension spring, large stress concentration is produced at the loop or other attaching device of tension spring.

The main disadvantage of tension spring is the failure of the spring when the wire breaks. A compression spring used for carrying a tensile load is shown in Fig. 23.9.

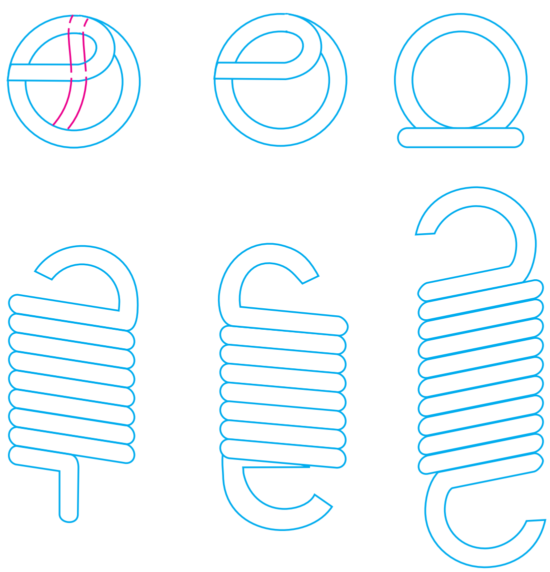

Fig. 23.8. End connection for tension helical springs.

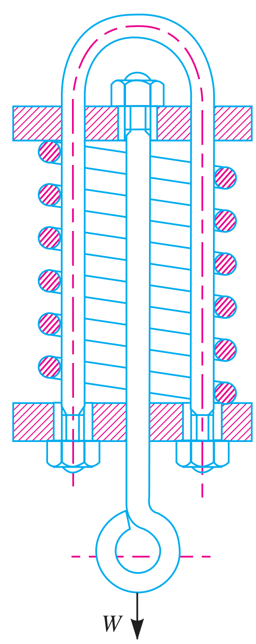

Fig. 23.9. Compression spring for carrying tensile load.

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Note : The total number of turns of a tension helical spring must be equal to the number of turns (n) between the points where the loops start plus the equivalent turns for the loops. It has been found experimentally that half turn should be added for each loop. Thus for a spring having loops on both ends, the total number of active turns,

$$n' = n + 1$$

23.8 Stresses in Helical Springs of Circular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown in Fig. 23.10 (a).

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

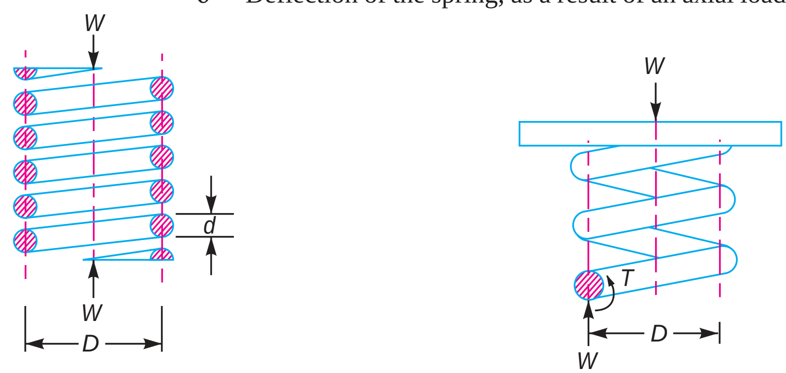
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,

C = Spring index = D/d,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W.



(a) Axially loaded helical spring.

(b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Fig. 23.10

Now consider a part of the compression spring as shown in Fig. 23.10 (b). The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire.

A little consideration will show that part of the spring, as shown in Fig. 23.10 (b), is in equilibrium under the action of two forces W and the twisting moment T. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad \tau_1 = \frac{8 W.D}{\pi\, d^3} \qquad \text{...}(i)$$

The torsional shear stress diagram is shown in Fig. 23.11 (a).

In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire :

1. Direct shear stress due to the load W, and
2. Stress due to curvature of wire.

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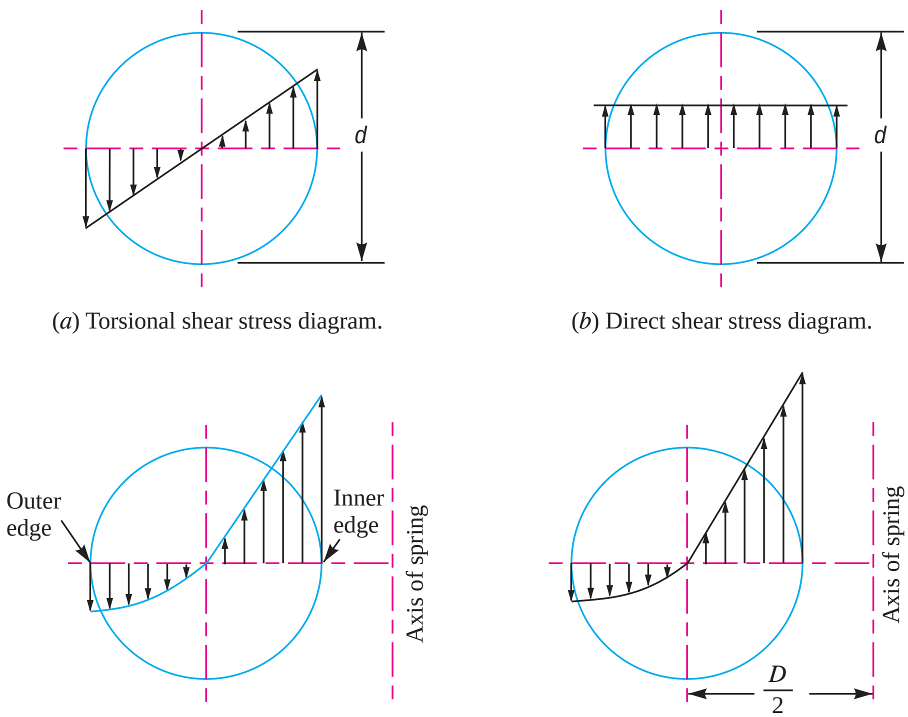
We know that direct shear stress due to the load W,

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi d^2} \qquad \text{...(ii)}$$

The direct shear stress diagram is shown in Fig. 23.11 (*b*) and the resultant diagram of torsional shear stress and direct shear stress is shown in Fig. 23.11 (*c*).



(*a*) Torsional shear stress diagram.

(*b*) Direct shear stress diagram.

(*c*) Resultant torsional shear and direct shear stress diagram.

(*d*) Resultant torsional shear, direct shear and curvature shear stress diagram.

Fig. 23.11. Superposition of stresses in a helical spring.

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

The ***positive*** sign is used for the inner edge of the wire and ***negative*** sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

$$= \text{Torsional shear stress} + \text{Direct shear stress}$$

$$= \frac{8W.D}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8W.D}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

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$$= \frac{8\,W.D}{\pi\,d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\,d^3} \qquad ...(iii)$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \dfrac{1}{2C}$

From the above equation, it can be observed that the effect of direct shear $\left(\dfrac{8\,WD}{\pi\,d^3} \times \dfrac{1}{2C}\right)$ is appreciable for springs of small spring index C. Also we have neglected the effect of wire curvature in equation ***(iii)***. It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected, because yielding of the material will relieve the stresses.

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. 23.11 (*d*).

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} = K \times \frac{8\,W.C}{\pi\,d^2} \qquad ...(iv)$$

where $K = \dfrac{4C - 1}{4C - 4} + \dfrac{0.615}{C}$

The values of K for a given spring index (C) may be obtained from the graph as shown in Fig. 23.12.

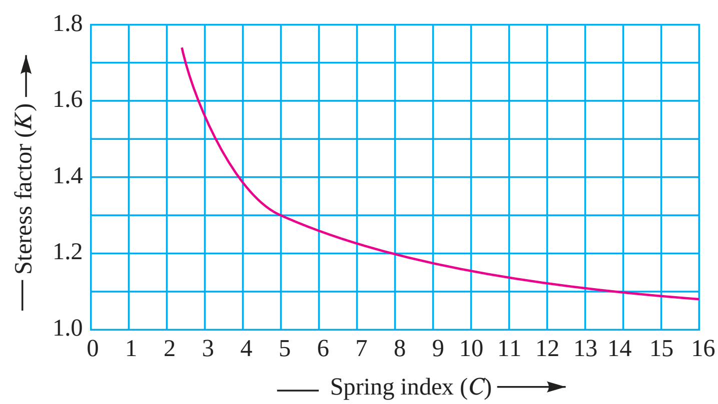

Fig. 23.12. Wahl's stress factor for helical springs.

We see from Fig. 23.12 that Wahl's stress factor increases very rapidly as the spring index decreases. The spring mostly used in machinery have spring index above 3.

Note: The Wahl's stress factor (K) may be considered as composed of two sub-factors, K_S and K_C, such that

$$K = K_S \times K_C$$

where K_S = Stress factor due to shear, and

K_C = Stress concentration factor due to curvature.

23.9 Deflection of Helical Springs of Circular Wire

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that

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Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi\, D \times n$$

Let θ = Angular deflection of the wire when acted upon by the torque T.

∴ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad \text{...}(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \qquad \text{...}\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$$= \frac{\pi}{32} \times d^4, \text{ } d \text{ being the diameter of spring wire.}$$

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi\, D.n}{\frac{\pi}{32} \times d^4\, G} = \frac{16\, W.D^2.n}{G.d^4} \qquad \text{...}(ii)$$

Substituting this value of θ in equation (i), we have

$$\delta = \frac{16\, W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\, W.D^3.n}{G.d^4} = \frac{8\, W.C^3.n}{G.d} \qquad \text{... } (\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\, D^3.n} = \frac{G.d}{8\, C^3.n} = \text{constant}$$

23.10 Eccentric Loading of Springs

Sometimes, the load on the springs does not coincide with the axis of the spring, *i.e.* the spring is subjected to an eccentric load. In such cases, not only the safe load for the spring reduces, the stiffness of the spring is also affected. The eccentric load on the spring increases the stress on one side of the spring and decreases on the other side. When the load is offset by a distance e from the spring axis, then the safe load on the spring may be obtained by multiplying the axial load by the factor $\dfrac{D}{2\, e + D}$, where D is the mean diameter of the spring.

23.11 Buckling of Compression Springs

It has been found experimentally that when the free length of the spring (L_F) is more than four times the mean or pitch diameter (D), then the spring behaves like a column and may fail by buckling at a comparatively low load as shown in Fig. 23.13. The critical axial load (W_{cr}) that causes buckling may be calculated by using the following relation, *i.e.*

$$W_{cr} = k \times K_B \times L_F$$

where k = Spring rate or stiffness of the spring = W/δ,

L_F = Free length of the spring, and

K_B = Buckling factor depending upon the ratio L_F / D.

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The buckling factor (K_B) for the hinged end and built-in end springs may be taken from the following table.

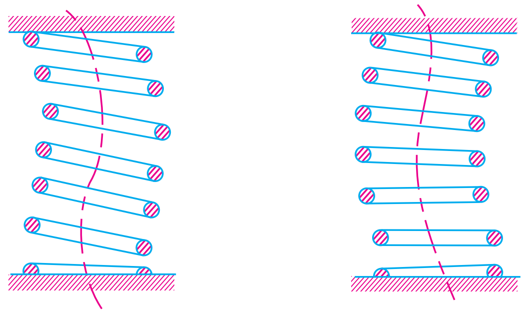

Fig. 23.13. Buckling of compression springs.

Table 23.4. Values of buckling factor (K_B).

L_F/D	*Hinged end spring*	*Built-in end spring*	L_F/D	*Hinged end spring*	*Built-in end spring*
1	0.72	0.72	5	0.11	0.53
2	0.63	0.71	6	0.07	0.38
3	0.38	0.68	7	0.05	0.26
4	0.20	0.63	8	0.04	0.19

It may be noted that a ***hinged end spring*** is one which is supported on pivots at both ends as in case of springs having plain ends where as a ***built-in end spring*** is one in which a squared and ground end spring is compressed between two rigid and parallel flat plates.

It order to avoid the buckling of spring, it is either mounted on a central rod or located on a tube. When the spring is located on a tube, the clearance between the tube walls and the spring should be kept as small as possible, but it must be sufficient to allow for increase in spring diameter during compression.

In railway coaches strongs springs are used for suspension.

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23.12 Surge in Springs

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end. This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called ***surge.***

It has been found that the natural frequency of spring should be atleast twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2.n}\sqrt{\frac{6\,G.g}{\rho}} \text{ cycles/s}$$

where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.

2. By using springs of high natural frequency.

3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Example 23.1. *A compression coil spring made of an alloy steel is having the following specifications :*

Mean diameter of coil = 50 mm ; Wire diameter = 5 mm ; Number of active coils = 20.

If this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.

Solution. Given : D = 50 mm ; d = 5 mm ; *n = 20 ; W = 500 N

We know that the spring index,

$$C = \frac{D}{d} = \frac{50}{5} = 10$$

∴ Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 10} = 1.05$$

and maximum shear stress (neglecting the effect of wire curvature),

$$\tau = K_S \times \frac{8W.D}{\pi\, d^3} = 1.05 \times \frac{8 \times 500 \times 50}{\pi \times 5^3} = 534.7 \text{ N/mm}^2$$

$= 534.7$ MPa **Ans.**

* Superfluous data.

Example 23.2. *A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn.*

Solution. Given : d = 6 mm ; D_o = 75 mm ; τ = 350 MPa = 350 N/mm² ; G = 84 kN/mm² = 84 × 10³ N/mm²

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

∴ Spring index,
$$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let W = Axial load, and

δ / n = Deflection per active turn.

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\,W.D}{\pi\, d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

∴ $W = 350 / 0.848 = 412.7$ N **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm}$$ **Ans.**

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\, d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\,W$$

∴ $W = 350 / 0.913 = 383.4$ N **Ans.**

and deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm}$$ **Ans.**

Example 23.3. *Design a spring for a balance to measure 0 to 1000 N over a scale of length 80 mm. The spring is to be enclosed in a casing of 25 mm diameter. The approximate number of turns is 30. The modulus of rigidity is 85 kN/mm². Also calculate the maximum shear stress induced.*

Solution. Given : W = 1000 N ; δ = 80 mm ; n = 30 ; G = 85 kN/mm² = 85 × 10³ N/mm²

Design of spring

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire, and

C = Spring index = D/d.

Since the spring is to be enclosed in a casing of 25 mm diameter, therefore the outer diameter of the spring coil ($D_o = D + d$) should be less than 25 mm.

We know that deflection of the spring (δ),

$$80 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 1000 \times C^3 \times 30}{85 \times 10^3 \times d} = \frac{240\,C^3}{85\,d}$$

$$\therefore \quad \frac{C^3}{d} = \frac{80 \times 85}{240} = 28.3$$

Let us assume that d = 4 mm. Therefore

$C^3 = 28.3\,d = 28.3 \times 4 = 113.2$ or $C = 4.84$

and $D = C.d = 4.84 \times 4 = 19.36$ mm **Ans.**

We know that outer diameter of the spring coil,

$$D_o = D + d = 19.36 + 4 = 23.36 \text{ mm} \quad \textbf{Ans.}$$

Since the value of D_o = 23.36 mm is less than the casing diameter of 25 mm, therefore the assumed dimension, d = 4 mm is correct.

Maximum shear stress induced

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 4.84 - 1}{4 \times 4.84 - 4} + \frac{0.615}{4.84} = 1.322$$

$\therefore$ Maximum shear stress induced,

$$\tau = K \times \frac{8\,W.C}{\pi\,d^2} = 1.322 \times \frac{8 \times 1000 \times 4.84}{\pi \times 4^2}$$

$$= 1018.2 \text{ N/mm}^2 = 1018.2 \text{ MPa} \quad \textbf{Ans.}$$

Example 23.4. *A mechanism used in printing machinery consists of a tension spring assembled with a preload of 30 N. The wire diameter of spring is 2 mm with a spring index of 6. The spring has 18 active coils. The spring wire is hard drawn and oil tempered having following material properties:*

Design shear stress = 680 MPa

Modulus of rigidity = 80 kN/mm²

Determine : 1. the initial torsional shear stress in the wire; 2. spring rate; and 3. the force to cause the body of the spring to its yield strength.

Tension springs are widely used in printing machines.

Solution. Given : W_i = 30 N ; d = 2 mm ; $C = D/d = 6$; $n = 18$; τ = 680 MPa = 680 N/mm² ; G = 80 kN/mm² = 80×10^3 N/mm²

1. *Initial torsional shear stress in the wire*

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

∴ Initial torsional shear stress in the wire,

$$\tau_i = K \times \frac{8\, W_i \times C}{\pi\, d^2} = 1.2525 \times \frac{8 \times 30 \times 6}{\pi \times 2^2} = 143.5 \text{ N/mm}^2$$

$= 143.5$ MPa **Ans.**

2. *Spring rate*

We know that spring rate (or stiffness of the spring),

$$= \frac{G.d}{8\, C^3.n} = \frac{80 \times 10^3 \times 2}{8 \times 6^3 \times 18} = 5.144 \text{ N/mm}$$ **Ans.**

3. *Force to cause the body of the spring to its yield strength*

Let W = Force to cause the body of the spring to its yield strength.

We know that design or maximum shear stress (τ),

$$680 = K \times \frac{8\, W.C}{\pi\, d^2} = 1.2525 \times \frac{8W \times 6}{\pi \times 2^2} = 4.78\, W$$

∴ $W = 680 / 4.78 = 142.25$ N **Ans.**

Example 23.5. *Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5.*

The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN/mm².

Take Wahl's factor, $K = \dfrac{4C - 1}{4C - 4} + \dfrac{0.615}{C}$, *where C = Spring index.*

Solution. Given : $W = 1000$ N ; $\delta = 25$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress (τ),

$$420 = K \times \frac{8\, W.C}{\pi\, d^2} = 1.31 \times \frac{8 \times 1000 \times 5}{\pi\, d^2} = \frac{16\,677}{d^2}$$

∴ $d^2 = 16\,677 / 420 = 39.7$ or $d = 6.3$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3 having diameter (d) = 6.401 mm.

∴ Mean diameter of the spring coil,

$D = C.d = 5\, d = 5 \times 6.401 = 32.005$ mm **Ans.** ... ($\because C = D/d = 5$)

and outer diameter of the spring coil,

$D_o = D + d = 32.005 + 6.401 = 38.406$ mm **Ans.**

2. *Number of turns of the coils*

Let n = Number of active turns of the coils.

We know that compression of the spring (δ),

$$25 = \frac{8W.C^3.n}{G.d} = \frac{8 \times 1000\ (5)^3 n}{84 \times 10^3 \times 6.401} = 1.86\ n$$

$\therefore$ $n = 25 / 1.86 = 13.44$ say 14 **Ans.**

For squared and ground ends, the total number of turns,

$$n' = n + 2 = 14 + 2 = 16 \textbf{ Ans.}$$

3. *Free length of the spring*

We know that free length of the spring

$$= n'.d + \delta + 0.15\ \delta = 16 \times 6.401 + 25 + 0.15 \times 25$$

$$= 131.2 \text{ mm} \textbf{ Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm} \textbf{ Ans.}$$

Example 23.6. *Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, G = 84 kN/mm^2.*

Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.

Solution. Given : W_1 = 2250 N ; W_2 = 2750 N ; δ = 6 mm ; $C = D/d = 5$; τ = 420 MPa = 420 N/mm^2 ; G = 84 kN/mm^2 = 84 × 10^3 N/mm^2

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil for a maximum load of W_2 = 2750 N, and

d = Diameter of the spring wire.

We know that twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2750 \times \frac{5d}{2} = 6875\ d \qquad ...\left(\because C = \frac{D}{d} = 5\right)$$

We also know that twisting moment (T),

$$6875\ d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\ d^3$$

$\therefore$ $d^2 = 6875 / 82.48 = 83.35$ or $d = 9.13$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3/0 having diameter (d) = 9.49 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = 5d = 5 \times 9.49 = 47.45 \text{ mm} \textbf{ Ans.}$$

We know that outer diameter of the spring coil,

$$D_o = D + d = 47.45 + 9.49 = 56.94 \text{ mm} \textbf{ Ans.}$$

and inner diameter of the spring coil,

$$D_i = D - d = 47.45 - 9.49 = 37.96 \text{ mm} \textbf{ Ans.}$$

2. *Number of turns of the spring coil*

Let n = Number of active turns.

It is given that the axial deflection (δ) for the load range from 2250 N to 2750 N (*i.e.* for W = 500 N) is 6 mm.

We know that the deflection of the spring (δ),

$$6 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 500\,(5)^3\,n}{84 \times 10^3 \times 9.49} = 0.63\,n$$

$\therefore \quad n = 6 / 0.63 = 9.5$ say 10 **Ans.**

For squared and ground ends, the total number of turns,

$$n' = 10 + 2 = 12 \text{ **Ans.**}$$

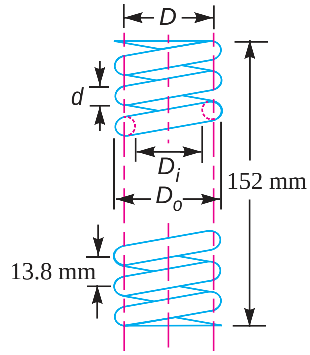

Fig. 23.14

3. ***Free length of the spring***

Since the compression produced under 500 N is 6 mm, therefore maximum compression produced under the maximum load of 2750 N is

$$\delta_{max} = \frac{6}{500} \times 2750 = 33 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 12 \times 9.49 + 33 + 0.15 \times 33$$
$$= 151.83 \text{ say } 152 \text{ mm}$$ **Ans.**

4. ***Pitch of the coil***

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{152}{12 - 1} = 13.73 \text{ say } 13.8 \text{ mm}$$ **Ans.**

The spring is shown in Fig. 23.14.

Example 23.7. *Design and draw a valve spring of a petrol engine for the following operating conditions :*

Spring load when the valve is open = 400 N
Spring load when the valve is closed = 250 N
Maximum inside diameter of spring = 25 mm
Length of the spring when the valve is open = 40 mm
Length of the spring when the valve is closed = 50 mm
Maximum permissible shear stress = 400 MPa

Solution. Given : W_1 = 400 N ; W_2 = 250 N ; D_i = 25 mm ; l_1 = 40 mm ; l_2 = 50 mm ; τ = 400 MPa = 400 N/mm^2

Petrol engine.

1. ***Mean diameter of the spring coil***

Let d = Diameter of the spring wire in mm, and

D = Mean diameter of the spring coil

= Inside dia. of spring + Dia. of spring wire = $(25 + d)$ mm

Since the diameter of the spring wire is obtained for the maximum spring load (W_1), therefore maximum twisting moment on the spring,

$$T = W_1 \times \frac{D}{2} = 400\left(\frac{25+d}{2}\right) = (5000 + 200\ d) \text{ N-mm}$$

We know that maximum twisting moment (T),

$$(5000 + 200\ d) = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 400 \times d^3 = 78.55\ d^3$$

Solving this equation by hit and trial method, we find that $d = 4.2$ mm.

From Table 23.2, we find that standard size of wire is *SWG* 7 having $d = 4.47$ mm.

Now let us find the diameter of the spring wire by taking Wahl's stress factor (K) into consideration.

We know that spring index,

$$C = \frac{D}{d} = \frac{25 + 4.47}{4.47} = 6.6 \qquad \text{... } (\because D = 25 + d)$$

∴ Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 6.6 - 1}{4 \times 6.6 - 4} + \frac{0.615}{6.6} = 1.227$$

We know that the maximum shear stress (τ),

$$400 = K \times \frac{8\ W_1 . C}{\pi\ d^2} = 1.227 \times \frac{8 \times 400 \times 6.6}{\pi\ d^2} = \frac{8248}{d^2}$$

$$\therefore \quad d^2 = 8248 / 400 = 20.62 \quad \text{or} \quad d = 4.54 \text{ mm}$$

Taking larger of the two values, we have

$$d = 4.54 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 6 having diameter (d) = 4.877 mm.

∴ Mean diameter of the spring coil

$$D = 25 + d = 25 + 4.877 = 29.877 \text{ mm}$$ **Ans.**

and outer diameter of the spring coil,

$$D_o = D + d = 29.877 + 4.877 = 34.754 \text{ mm}$$ **Ans.**

2. Number of turns of the coil

Let n = Number of active turns of the coil.

We are given that the compression of the spring caused by a load of $(W_1 - W_2)$, *i.e.* 400 – 250 = 150 N is $l_2 - l_1$, *i.e.* 50 – 40 = 10 mm. In other words, the deflection (δ) of the spring is 10 mm for a load (W) of 150 N

We know that the deflection of the spring (δ),

$$10 = \frac{8\ W . D^3 . n}{G . d^4} = \frac{8 \times 150\ (29.877)^3\ n}{80 \times 10^3\ (4.877)^4} = 0.707\ n$$

... (Taking $G = 80 \times 10^3$ N/mm²)

$$\therefore \quad n = 10 / 0.707 = 14.2 \text{ say } 15$$ **Ans.**

Taking the ends of the springs as squared and ground, the total number of turns of the spring,

$$n' = 15 + 2 = 17$$ **Ans.**

3. Free length of the spring

Since the deflection for 150 N of load is 10 mm, therefore the maximum deflection for the maximum load of 400 N is

$$\delta_{max} = \frac{10}{150} \times 400 = 26.67 \text{ mm}$$

An automobile suspension and shock-absorber. The two links with green ends are turnbuckles.

∴ Free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 17 \times 4.877 + 26.67 + 0.15 \times 26.67 = 113.58 \text{ mm} \quad \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{113.58}{17 - 1} = 7.1 \text{ mm} \quad \textbf{Ans.}$$

Example 23.8. *Design a helical spring for a spring loaded safety valve (Ramsbottom safety valve) for the following conditions :*

Diameter of valve seat = 65 mm ; Operating pressure = 0.7 N/mm^2; Maximum pressure when the valve blows off freely = 0.75 N/mm^2; Maximum lift of the valve when the pressure rises from 0.7 to 0.75 N/mm^2 = 3.5 mm ; Maximum allowable stress = 550 MPa ; Modulus of rigidity = 84 kN/mm^2; Spring index = 6.

Draw a neat sketch of the free spring showing the main dimensions.

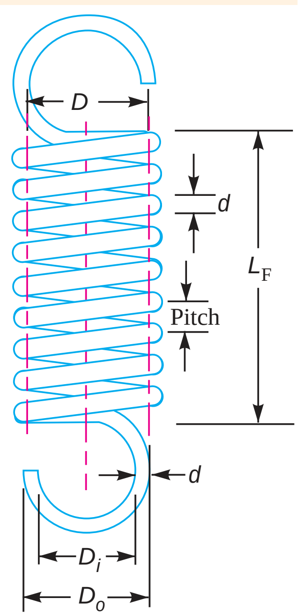

Fig. 23.15

Solution. Given : D_1 = 65 mm ; p_1 = 0.7 N/mm^2; p_2 = 0.75 N/mm^2; δ = 3.5 mm ; τ = 550 MPa = 550 N/mm^2; G = 84 kN/mm^2 = 84 × 10^3 N/mm^2 ; C = 6

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

Since the safety valve is a Ramsbottom safety valve, therefore the spring will be under tension. We know that initial tensile force acting on the spring (*i.e.* before the valve lifts),

$$W_1 = \frac{\pi}{4}(D_1)^2\,p_1 = \frac{\pi}{4}(65)^2\,0.7 = 2323 \text{ N}$$

and maximum tensile force acting on the spring (*i.e.* when the valve blows off freely),

$$W_2 = \frac{\pi}{4}(D_1)^2\, p_2 = \frac{\pi}{4}(65)^2\, 0.75 = 2489 \text{ N}$$

∴ Force which produces the deflection of 3.5 mm,

$$W = W_2 - W_1 = 2489 - 2323 = 166 \text{ N}$$

Since the diameter of the spring wire is obtained for the maximum spring load (W_2), therefore maximum twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2489 \times \frac{6\,d}{2} = 7467\,d \qquad \ldots (\because C = D/d = 6)$$

We know that maximum twisting moment (T),

$$7467\,d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 550 \times d^3 = 108\,d^3$$

$$\therefore \quad d^2 = 7467 / 108 = 69.14 \quad \text{or} \quad d = 8.3 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 2/0 having diameter (d) = 8.839 mm **Ans.**

∴ Mean diameter of the coil,

$$D = 6\,d = 6 \times 8.839 = 53.034 \text{ mm} \quad \textbf{Ans.}$$

Outside diameter of the coil,

$$D_o = D + d = 53.034 + 8.839 = 61.873 \text{ mm} \quad \textbf{Ans.}$$

and inside diameter of the coil,

$$D_i = D - d = 53.034 - 8.839 = 44.195 \text{ mm} \quad \textbf{Ans.}$$

2. Number of turns of the coil

Let n = Number of active turns of the coil.

We know that the deflection of the spring (δ),

$$3.5 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 166 \times 6^3 \times n}{84 \times 10^3 \times 8.839} = 0.386\,n$$

$$\therefore \quad n = 3.5 / 0.386 = 9.06 \text{ say } 10 \quad \textbf{Ans.}$$

For a spring having loop on both ends, the total number of turns,

$$n' = n + 1 = 10 + 1 = 11 \quad \textbf{Ans.}$$

3. Free length of the spring

Taking the least gap between the adjacent coils as 1 mm when the spring is in free state, the free length of the tension spring,

$$L_F = n.d + (n - 1)\,1 = 10 \times 8.839 + (10 - 1)\,1 = 97.39 \text{ mm} \quad \textbf{Ans.}$$

4. Pitch of the coil

We know that pitch of the coil

$$= \frac{\text{Free length}}{n - 1} = \frac{97.39}{10 - 1} = 10.82 \text{ mm} \quad \textbf{Ans.}$$

The tension spring is shown in Fig. 23.15.

Example 23.9. *A safety valve of 60 mm diameter is to blow off at a pressure of 1.2 N/mm². It is held on its seat by a close coiled helical spring. The maximum lift of the valve is 10 mm. Design a suitable compression spring of spring index 5 and providing an initial compression of 35 mm. The maximum shear stress in the material of the wire is limited to 500 MPa. The modulus of rigidity for the spring material is 80 kN/mm². Calculate : 1. Diameter of the spring wire, 2. Mean coil diameter, 3. Number of active turns, and 4. Pitch of the coil.*

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Take Wahl's factor, $K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$, *where C is the spring index.*

Solution. Given : Valve dia. = 60 mm ; Max. pressure = 1.2 N/mm^2 ; δ_2 = 10 mm ; C = 5 ; δ_1 = 35 mm ; τ = 500 MPa = 500 N/mm^2 ; G = 80 kN/mm^2 = 80 × 10^3 N/mm^2

1. Diameter of the spring wire

Let d = Diameter of the spring wire.

We know that the maximum load acting on the valve when it just begins to blow off,

$$W_1 = \text{Area of the valve} \times \text{Max. pressure}$$

$$= \frac{\pi}{4}(60)^2\ 1.2 = 3394 \text{ N}$$

and maximum compression of the spring,

$$\delta_{max} = \delta_1 + \delta_2 = 35 + 10 = 45 \text{ mm}$$

Since a load of 3394 N keeps the valve on its seat by providing initial compression of 35 mm, therefore the maximum load on the spring when the valve is oepn (*i.e.* for maximum compression of 45 mm),

$$W = \frac{3394}{35} \times 45 = 4364 \text{ N}$$

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times5-1}{4\times5-4} + \frac{0.615}{5} = 1.31$$

We also know that the maximum shear stress (τ),

$$500 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.31 \times \frac{8\times4364\times5}{\pi\,d^2} = \frac{72\ 780}{d^2}$$

$$\therefore \quad d^2 = 72\ 780 / 500 = 145.6 \quad \text{or} \quad d = 12.06 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 7/0 having diameter (d) = 12.7 mm. **Ans.**

2. Mean coil diameter

Let D = Mean coil diameter.

We know that the spring index,

$$C = D/d \quad \text{or} \quad D = C.d = 5 \times 12.7 = 63.5 \text{ mm}$$ **Ans.**

3. Number of active turns

Let n = Number of active turns.

We know that the maximum compression of the spring (δ),

$$45 = \frac{8\,W.C^3.n}{G.d} = \frac{8\times4364\times5^3\times n}{80\times10^3\times12.7} = 4.3\,n$$

$$\therefore \quad n = 45 / 4.3 = 10.5 \text{ say } 11$$ **Ans.**

Taking the ends of the coil as squared and ground, the total number of turns,

$$n' = n + 2 = 11 + 2 = 13$$ **Ans.**

Note : The valve of n may also be calculated by using

$$\delta_1 = \frac{8\,W_1.C^3.n}{G.d}$$

$$35 = \frac{8\times3394\times5^3\times n}{80\times10^3\times12.7} = 3.34\,n \quad \text{or} \quad n = 35/3.34 = 10.5 \text{ say } 11$$

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4. *Pitch of the coil*

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max} = 13 \times 12.7 + 45 + 0.15 \times 45$$
$$= 216.85 \text{ mm } \textbf{Ans}$$

$$\therefore \qquad \text{Pitch of the coil} = \frac{\text{Free length}}{n' - 1} = \frac{216.85}{13 - 1} = 18.1 \text{ mm} \quad \textbf{Ans.}$$

Example 23.10. *In a spring loaded governor as shown in Fig. 23.16, the balls are attached to the vertical arms of the bell crank lever, the horizontal arms of which lift the sleeve against the pressure exerted by a spring. The mass of each ball is 2.97 kg and the lengths of the vertical and horizontal arms of the bell crank lever are 150 mm and 112.5 mm respectively. The extreme radii of rotation of the balls are 100 mm and 150 mm and the governor sleeve begins to lift at 240 r.p.m. and reaches the highest position with a 7.5 percent increase of speed when effects of friction are neglected. Design a suitable close coiled round section spring for the governor.*

Assume permissible stress in spring steel as 420 MPa, modulus of rigidity 84 kN/mm² and spring index 8. Allowance must be made for stress concentration, factor of which is given by

$$\frac{4C - 1}{4C - 4} + \frac{0.615}{C}, \text{ where } C \text{ is the spring index.}$$

Solution. Given : $m = 2.97$ kg ; $x = 150$ mm $= 0.15$ m ; $y = 112.5$ mm $= 0.1125$ m ; $r_2 = 100$ mm $= 0.1$ m ; $r_1 = 150$ mm $= 0.15$ m ; $N_2 = 240$ r.p.m. ; $\tau = 420$ MPa $= 420$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm² ; $C = 8$

The spring loaded governor, as shown in Fig. 23.16, is a *Hartnell type governor. First of all, let us find the compression of the spring.

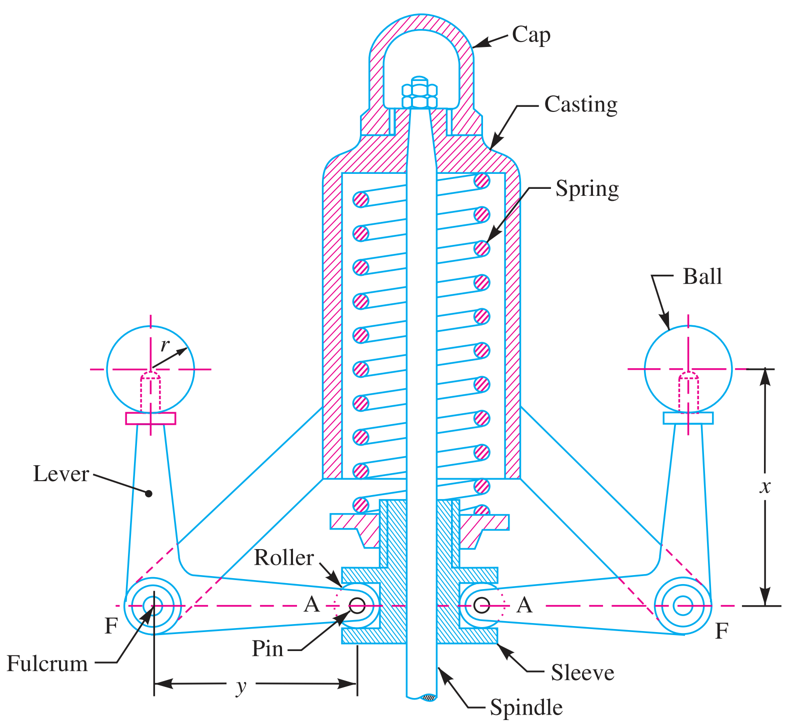

Fig. 23.16

* For further details, see authors' popular book on **'Theory of Machines'.**

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We know that minimum angular speed at which the governor sleeve begins to lift,

$$\omega_2 = \frac{2\,\pi\,N_2}{60} = \frac{2\,\pi \times 240}{60} = 25.14 \text{ rad/s}$$

Since the increase in speed is 7.5%, therefore maximum speed,

$$\omega_1 = \omega_2 + \frac{7.5}{100} \times \omega_2 = 25.14 + \frac{7.5}{100} \times 25.14 = 27 \text{ rad/s}$$

The position of the balls and the lever arms at the maximum and minimum speeds is shown in Fig. 23.17 (*a*) and (*b*) respectively.

Let F_{C1} = Centrifugal force at the maximum speed, and

F_{C2} = Centrifugal force at the minimum speed.

We know that the spring force at the maximum speed (ω_1),

$$S_1 = 2\,F_{C1} \times \frac{x}{y} = 2\,m\,(\omega_1)^2\,r_1 \times \frac{x}{y} = 2 \times 2.97\,(27)^2\;0.15 \times \frac{0.15}{0.1125} = 866 \text{ N}$$

Similarly, the spring force at the minimum speed ω_2,

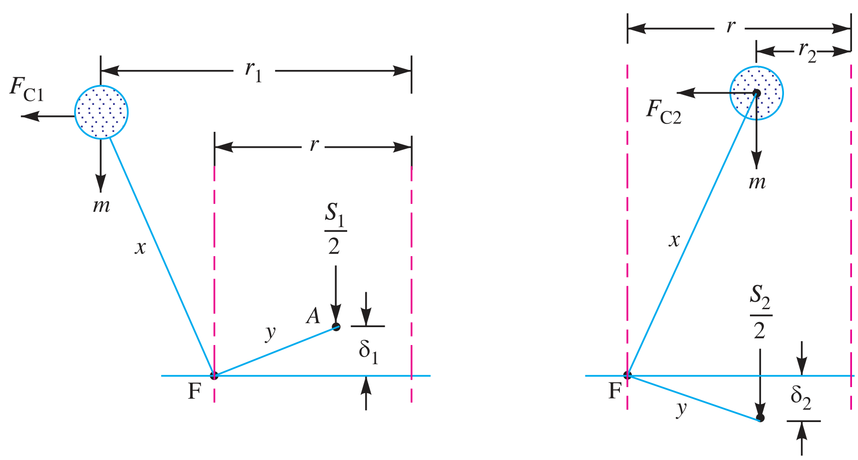
$$S_2 = 2\,F_{C2} \times \frac{x}{y} = 2m\,(\omega_2)^2\,r_2 \times \frac{x}{y} = 2 \times 2.97\,(25.14)^2\;0.1 \times \frac{0.15}{0.1125} = 500 \text{ N}$$

Since the compression of the spring will be equal to the lift of the sleeve, therefore compression of the spring,

$$\delta = \delta_1 + \delta_2 = (r_1 - r)\,\frac{y}{x} + (r - r_2)\,\frac{y}{x} = (r_1 - r_2)\,\frac{y}{x}$$

$$= (0.15 - 0.1)\,\frac{0.1125}{0.15} = 0.0375 \text{ m} = 37.5 \text{ mm}$$

This compression of the spring is due to the spring force of $(S_1 - S_2)$ *i.e.* $(866 - 500) = 366$ N.



(*a*) Maximum position. (*b*) Minimum position.

Fig. 23.17

1. *Diameter of the spring wire*

Let d = Diameter of the spring wire in mm.

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8} = 1.184$$

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We also know that maximum shear stress (τ),

$$420 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.184 \times \frac{8 \times 866 \times 8}{\pi\,d^2} = \frac{20\,885}{d^2}$$

... (Substituting $W = S_1$, the maximum spring force)

$\therefore$ $d^2 = 20\,885 / 420 = 49.7$ or $d = 7.05$ mm

From Table 23.2, we shall take the standard wire of size *SWG* 1 having diameter (d) = 7.62 mm **Ans.**

2. Mean diameter of the spring coil

Let D = Mean diameter of the spring coil.

We know that the spring index,

$$C = D/d \quad \text{or} \quad D = C.d = 8 \times 7.62 = 60.96 \text{ mm } \textbf{Ans.}$$

3. Number of turns of the coil

Let n = Number of active turns of the coil.

We know that compression of the spring (δ),

$$37.5 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 366 \times 8^3 \times n}{84 \times 10^3 \times 7.62} = 2.34\,n$$

... (Substituting $W = S_1 - S_2$)

$\therefore$ $n = 37.5 / 2.34 = 16$ **Ans.**

and total number of turns using squared and ground ends,

$$n' = n + 2 = 16 + 2 = 18$$

4. Free length of the coil

Since the compression produced under a force of 366 N is 37.5 mm, therefore maximum compression produced under the maximum load of 866 N is,

$$\delta_{max} = \frac{37.5}{366} \times 866 = 88.73 \text{ mm}$$

We know that free length of the coil,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 18 \times 7.62 + 88.73 + 0.15 \times 88.73 = 239.2 \text{ mm } \textbf{Ans.}$$

5. Pitch of the coil

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{239.2}{18 - 1} = 14.07 \text{ mm } \textbf{Ans.}$$

Example 23.11. *A single plate clutch is to be designed for a vehicle. Both sides of the plate are to be effective. The clutch transmits 30 kW at a speed of 3000 r.p.m. and should cater for an over load of 20%. The intensity of pressure on the friction surface should not exceed 0.085 N/mm^2 and the surface speed at the mean radius should be limited to 2300 m / min. The outside diameter of the surfaces may be assumed as 1.3 times the inside diameter and the coefficient of friction for the surfaces may be taken as 0.3. If the axial thrust is to be provided by six springs of about 25 mm mean coil diameter, design the springs selecting wire from the following gauges :*

SWG	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Dia. (mm)	*5.893*	*5.385*	*4.877*	*4.470*	*4.064*	*3.658*	*3.251*	*2.946*	*2.642*

Safe shear stress is limited to 420 MPa and modulus of rigidity is 84 kN/mm^2.

Solution. Given : $P = 30$ kW $= 30 \times 10^3$ W ; $N = 3000$ r.p.m. ; $p = 0.085$ N/mm^2 ; $v = 2300$ m/min ; $d_1 = 1.3\,d_2$ or $r_1 = 1.3\,r_2$; $\mu = 0.3$; No. of springs = 6 ; $D = 25$ mm ; $\tau = 420$ MPa = 420 N/mm^2 ; $G = 84$ kN/mm^2 = 84×10^3 N/mm^2

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First of all, let us find the maximum load on each spring. We know that the mean torque transmitted by the clutch,

$$T_{mean} = \frac{P \times 60}{2\,\pi\,N} = \frac{30 \times 10^3 \times 60}{2\,\pi \times 3000} = 95.5 \text{ N-m}$$

Since an overload of 20% is allowed, therefore maximum torque to which the clutch should be designed is given by

$$T_{max} = 1.2\,T_{mean} = 1.2 \times 95.5 = 114.6 \text{ N-m} = 114\,600 \text{ N-mm} \qquad ...(i)$$

Let r_1 and r_2 be the outside and inside radii of the friction surfaces. Since maximum intensity of pressure is at the inner radius, therefore for uniform wear,

$$*p \times r_2 = C \text{ (a constant) or } C = 0.085\, r_2$$

We know that the axial thrust transmitted,

$$W = C \times 2\pi\,(r_1 - r_2) \qquad ...(ii)$$

Since both sides of the plate are effective, therefore maximum torque transmitted,

$$T_{max} = \tfrac{1}{2}\,\mu \times W\,(r_1 + r_2)\,2 = 2\pi\,\mu.C\,[(r_1)^2 - (r_2)^2] \quad \text{... [From equation } (ii)]$$

$$114\,600 = 2\pi \times 0.3 \times 0.085\, r_2\,[(1.3\, r_2)^2 - (r_2)^2] = 0.11\,(r_2)^3$$

$\therefore$
$$(r_2)^3 = 114\,600 / 0.11 = 1.04 \times 10^6 \quad \text{or} \quad r_2 = 101.4 \text{ say } 102 \text{ mm}$$

and
$$r_1 = 1.3\, r_2 = 1.3 \times 102 = 132.6 \text{ say } 133 \text{ mm}$$

$\therefore$ Mean radius,

$$r = \frac{r_1 + r_2}{2} = \frac{133 + 102}{2} = 117.5 \text{ mm} = 0.1175 \text{ m}$$

We know that surface speed at the mean radius,

$$v = 2\,\pi\,r\,N = 2\,\pi \times 0.1175 \times 3000 = 2215 \text{ m/min}$$

Since the surface speed as obtained above is less than the permissible value of 2300 m/min, therefore the radii of the friction surface are safe.

We know that axial thrust,

$$W = C \times 2\pi\,(r_1 - r_2) = 0.085\, r_2 \times 2\pi\,(r_1 - r_2) \qquad \text{... } (\because C = 0.085\, r_2)$$
$$= 0.085 \times 102 \times 2\pi\,(133 - 102) = 1689 \text{ N}$$

Since this axial thrust is to be provided by six springs, therefore maximum load on each spring,

$$W_1 = \frac{1689}{6} = 281.5 \text{ N}$$

1. *Diameter of the spring wire*

Let d = Diameter of the spring wire.

We know that the maximum torque transmitted,

$$T = W_1 \times \frac{D}{2} = 281.5 \times \frac{25}{2} = 3518.75 \text{ N-mm}$$

We also know that the maximum torque transmitted (T),

$$3518.75 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\, d^3$$

$\therefore$
$$d^3 = 3518.75 / 82.48 = 42.66 \quad \text{or} \quad d = 3.494 \text{ mm}$$

Let us now find out the diameter of the spring wire by taking the stress factor (K) into consideration.

We know that the spring index,

$$C = \frac{D}{d} = \frac{25}{3.494} = 7.155$$

* Please refer Chapter 24 on Clutches.

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and Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 7.155 - 1}{4 \times 7.155 - 4} + \frac{0.615}{7.155} = 1.21$$

We know that the maximum shear stress (τ),

$$420 = K \times \frac{8\,W_1.D}{\pi\,d^3} = 1.21 \times \frac{8 \times 281.5 \times 25}{\pi\,d^3} = \frac{21\,681}{d^3}$$

$\therefore$ $d^3 = 21\,681 / 420 = 51.6$ or $d = 3.72$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 8 having diameter (d) = 4.064 mm. **Ans.**

Outer diameter of the spring,

$$D_o = D + d = 25 + 4.064 = 29.064 \text{ mm} \quad \textbf{Ans.}$$

and inner diameter of the spring,

$$D_i = D - d = 25 - 4.064 = 20.936 \text{ mm} \quad \textbf{Ans.}$$

2. Free length of the spring

Let us assume the active number of coils (n) = 8. Therefore compression produced by an axial thrust of 281.5 N per spring,

$$\delta = \frac{8\,W_1.D^3.n}{G.d^4} = \frac{8 \times 281.5\,(25)^3\,8}{84 \times 10^3\,(4.064)^4} = 12.285 \text{ mm}$$

For square and ground ends, the total number of turns of the coil,

$$n' = n + 2 = 8 + 2 = 10$$

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta = 10 \times 4.064 + 12.285 + 0.15 \times 12.285 \text{ mm}$$
$$= 54.77 \text{ mm} \quad \textbf{Ans.}$$

3. Pitch of the coil

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{54.77}{10 - 1} = 6.08 \text{ mm} \quad \textbf{Ans.}$$

23.13 Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta \qquad ...(i)$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} \quad \text{or} \quad W = \frac{\pi\,d^3.\tau}{8\,K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times \pi\,d^3.\tau}{8\,K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi\,\tau.D^2.n}{K.d.G}$$

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Substituting the values of W and δ in equation (*i*), we have

$$U = \frac{1}{2} \times \frac{\pi\, d^3 . \tau}{8\, K . D} \times \frac{\pi\, \tau . D^2 . n}{K . d . G}$$

$$= \frac{\tau^2}{4\, K^2 . G} (\pi\, D . n) \left(\frac{\pi}{4} \times d^2 \right) = \frac{\tau^2}{4\, K^2 . G} \times V$$

where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

$$= (\pi\, D . n) \left(\frac{\pi}{4} \times d^2 \right)$$

Note : When a load (say P) falls on a spring through a height h, then the energy absorbed in a spring is given by

$$U = P\,(h + \delta) = \frac{1}{2}\, W . \delta$$

where W = Equivalent static load *i.e.* the gradually applied load which shall produce the same effect as by the falling load P, and

δ = Deflection produced in the spring.

Another view of an automobile shock-absorber

Example 23.12. *Find the maximum shear stress and deflection induced in a helical spring of the following specifications, if it has to absorb 1000 N-m of energy.*

Mean diameter of spring = 100 mm ; Diameter of steel wire, used for making the spring = 20 mm; Number of coils = 30 ; Modulus of rigidity of steel = 85 kN/mm².

Solution. Given : U = 1000 N-m ; D = 100 mm = 0.1 m ; d = 20 mm = 0.02 m ; n = 30 ; G = 85 kN/mm² = 85 × 10⁹ N/m²

Maximum shear stress induced

Let τ = Maximum shear stress induced.

We know that spring index,

$$C = \frac{D}{d} = \frac{0.1}{0.02} = 5$$

∴ Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

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Volume of spring wire,

$$V = (\pi\, D.n)\left(\frac{\pi}{4} \times d^2\right) = (\pi \times 0.1 \times 30)\left[\frac{\pi}{4}\,(0.02)^2\right] \text{m}^3$$

$$= 0.002\,96 \text{ m}^3$$

We know that energy absorbed in the spring (U),

$$1000 = \frac{\tau^2}{4K^2.G} \times V = \frac{\tau^2}{4\,(1.31)^2\;85 \times 10^9} \times 0.00296 = \frac{5\,\tau^2}{10^{15}}$$

$$\therefore \qquad \tau^2 = 1000 \times 10^{15} / 5 = 200 \times 10^{15}$$

or $\tau = 447.2 \times 10^6$ N/m^2 = 447.2 MPa **Ans.**

Deflection produced in the spring

We know that deflection produced in the spring,

$$\delta = \frac{\pi\,\tau.D^2\,n}{K.d.G} = \frac{\pi \times 447.2 \times 10^6\;(0.1)^2\;30}{1.31 \times 0.02 \times 85 \times 10^9} = 0.1893 \text{ m}$$

$$= 189.3 \text{ mm}$$ **Ans.**

Example 23.13. *A closely coiled helical spring is made of 10 mm diameter steel wire, the coil consisting of 10 complete turns with a mean diameter of 120 mm. The spring carries an axial pull of 200 N. Determine the shear stress induced in the spring neglecting the effect of stress concentration. Determine also the deflection in the spring, its stiffness and strain energy stored by it if the modulus of rigidity of the material is 80 kN/mm².*

Solution. Given : d = 10 mm ; n = 10 ; D = 120 mm ; W = 200 N ; G = 80 kN/mm^2 = 80 × 10^3 N/mm^2

Shear stress induced in the spring neglecting the effect of stress concentration

We know that shear stress induced in the spring neglecting the effect of stress concentration is,

$$\tau = \frac{8\,W.D}{\pi\,d^3}\left(1 + \frac{d}{2D}\right) = \frac{8 \times 200 \times 120}{\pi\,(10)^3}\left[1 + \frac{10}{2 \times 120}\right] \text{N/mm}^2$$

$= 61.1 \times 1.04 = 63.54$ N/mm^2 = 63.54 MPa **Ans.**

Deflection in the spring

We know that deflection in the spring,

$$\delta = \frac{8\,W.D^3 n}{G.d^4} = \frac{8 \times 200\,(120)^3\;10}{80 \times 10^3\;(10)^4} = 34.56 \text{ mm}$$ **Ans.**

Stiffness of the spring

We know that stiffness of the spring

$$= \frac{W}{\delta} = \frac{200}{34.56} = 5.8 \text{ N/mm}$$

Strain energy stored in the spring

We know that strain energy stored in the spring,

$$U = \frac{1}{2}\,W.\delta = \frac{1}{2} \times 200 \times 34.56 = 3456 \text{ N-mm} = 3.456 \text{ N-m}$$ **Ans.**

Example 23.14. *At the bottom of a mine shaft, a group of 10 identical close coiled helical springs are set in parallel to absorb the shock caused by the falling of the cage in case of a failure. The loaded cage weighs 75 kN, while the counter weight has a weight of 15 kN. If the loaded cage falls through a height of 50 metres from rest, find the maximum stress induced in each spring if it is made of 50 mm diameter steel rod. The spring index is 6 and the number of active turns in each spring is 20. Modulus of rigidity, G = 80 kN/mm².*

Solution. Given : No. of springs = 10 ; W_1 = 75 kN = 75 000 N ; W_2 = 15 kN = 15 000 N ; h = 50 m = 50 000 mm ; d = 50 mm ; C = 6 ; n = 20 ; G = 80 kN/mm^2 = 80 × 10^3 N/mm^2

We know that net weight of the falling load,

$$P = W_1 - W_2 = 75\ 000 - 15\ 000 = 60\ 000 \text{ N}$$

Let W = The equivalent static (or gradually applied) load on each spring which can produce the same effect as by the falling load P.

We know that compression produced in each spring,

$$\delta = \frac{8\,W.C^3.n}{G.d} = \frac{8W \times 6^3 \times 20}{80 \times 10^3 \times 50} = 0.008\ 64\ W \text{ mm}$$

Since the work done by the falling load is equal to the energy stored in the helical springs which are 10 in number, therefore,

$$P\,(h + \delta) = \frac{1}{2} W \times \delta \times 10$$

$$60\ 000\ (50\ 000 + 0.008\ 64\ W) = \frac{1}{2} W \times 0.008\ 64 W \times 10$$

$$3 \times 10^9 + 518.4\ W = 0.0432\ W^2$$

or $\quad W^2 - 12\ 000\ W - 69.4 \times 10^9 = 0$

$$\therefore \quad W = \frac{12\ 000 \pm \sqrt{(12\ 000)^2 + 4 \times 1 \times 69.4 \times 10^9}}{2} = \frac{12\ 000 \pm 527\ 000}{2}$$

$$= 269\ 500 \text{ N} \quad \text{... (Taking +ve sign)}$$

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.25$$

and maximum stress induced in each spring,

$$\tau = K \times \frac{8W.C}{\pi\, d^2} = 1.25 \times \frac{8 \times 269\ 500 \times 6}{\pi\ (50)^2} = 2058.6 \text{ N/mm}^2$$

$$= 2058.6 \text{ MPa}$$ **Ans.**

Example 23.15. *A rail wagon of mass 20 tonnes is moving with a velocity of 2 m/s. It is brought to rest by two buffers with springs of 300 mm diameter. The maximum deflection of springs is 250 mm. The allowable shear stress in the spring material is 600 MPa. Design the spring for the buffers.*

Buffers have springs inside to absorb shock.

Solution. Given : m = 20 t = 20 000 kg ; v = 2 m/s ; D = 300 mm ; δ = 250 mm ; τ = 600 MPa = 600 N/mm^2

1. *Diameter of the spring wire*

Let d = Diameter of the spring wire.

We know that kinetic energy of the wagon

$$= \frac{1}{2} m\, v^2 = \frac{1}{2} \times 20\ 000\ (2)^2 = 40\ 000 \text{ N-m} = 40 \times 10^6 \text{ N-mm} \quad ...(i)$$

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Let W be the equivalent load which when applied gradually on each spring causes a deflection of 250 mm. Since there are two springs, therefore

Energy stored in the springs

$$= \frac{1}{2} \times W.\delta \times 2 = W.\delta = W \times 250 = 250\ W \text{ N-mm} \quad \text{...}(ii)$$

From equations (*i*) and (*ii*), we have

$$\therefore \quad W = 40 \times 10^6 / 250 = 160 \times 10^3 \text{ N}$$

We know that torque transmitted by the spring,

$$T = W \times \frac{D}{2} = 160 \times 10^3 \times \frac{300}{2} = 24 \times 10^6 \text{ N-mm}$$

We also know that torque transmitted by the spring (T),

$$24 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 600 \times d^3 = 117.8\ d^3$$

$$\therefore \quad d^3 = 24 \times 10^6 / 117.8 = 203.7 \times 10^3 \quad \text{or} \quad d = 58.8 \text{ say } 60 \text{ mm}$$ **Ans.**

2. Number of turns of the spring coil

Let n = Number of active turns of the spring coil.

We know that the deflection of the spring (δ),

$$250 = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times 160 \times 10^3\ (300)^3\ n}{84 \times 10^3\ (60)^4} = 31.7\ n$$

... (Taking G = 84 MPa = 84×10^3 N/mm^2)

$$\therefore \quad n = 250 / 31.7 = 7.88 \text{ say } 8$$ **Ans.**

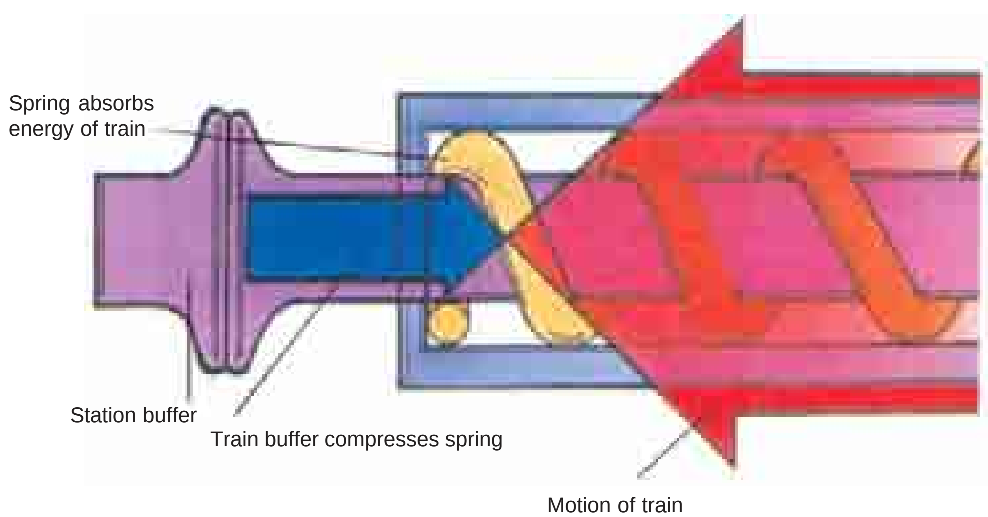
Assuming square and ground ends, total number of turns,

$$n' = n + 2 = 8 + 2 = 10$$ **Ans.**

3. Free length of the spring

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta = 10 \times 60 + 250 + 0.15 \times 250 = 887.5 \text{ mm}$$ **Ans.**



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4. Pitch of the coil

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{887.5}{10 - 1} = 98.6 \text{ mm} \quad \textbf{Ans.}$$

23.14 Stress and Deflection in Helical Springs of Non-circular Wire

The helical springs may be made of non-circular wire such as rectangular or square wire, in order to provide greater resilience in a given space. However these springs have the following main disadvantages :

1. The quality of material used for springs is not so good.
2. The shape of the wire does not remain square or rectangular while forming helix, resulting in trapezoidal cross-sections. It reduces the energy absorbing capacity of the spring.
3. The stress distribution is not as favourable as for circular wires. But this effect is negligible where loading is of static nature.

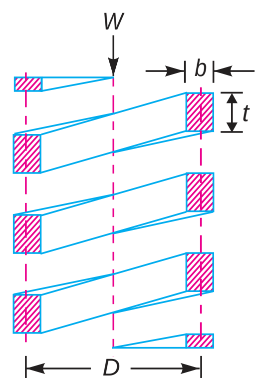

Fig. 23.18. Spring of rectangular wire.

For springs made of rectangular wire, as shown in Fig. 23.18, the maximum shear stress is given by

$$\tau = K \times \frac{W.D\,(1.5\,t + 0.9\,b)}{b^2.t^2}$$

This expression is applicable when the longer side (*i.e.* $t > b$) is parallel to the axis of the spring. But when the shorter side (*i.e.* $t < b$) is parallel to the axis of the spring, then maximum shear stress,

$$\tau = K \times \frac{W.D\,(1.5\,b + 0.9\,t)}{b^2.t^2}$$

and deflection of the spring,

$$\delta = \frac{2.45\,W.D^3.n}{G.b^3\,(t - 0.56\,b)}$$

For springs made of square wire, the dimensions b and t are equal. Therefore, the maximum shear stress is given by

$$\tau = K \times \frac{2.4\,W.D}{b^3}$$

and deflection of the spring,

$$\delta = \frac{5.568\,W.D^3.n}{G.b^4} = \frac{5.568\,W.C^3.n}{G.b} \qquad \ldots\left(\because C = \frac{D}{b}\right)$$

where b = Side of the square.

Note : In the above expressions,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}, \text{ and } C = \frac{D}{b}$$

Example 23.16. *A loaded narrow-gauge car of mass 1800 kg and moving at a velocity 72 m/min., is brought to rest by a bumper consisting of two helical steel springs of square section. The mean diameter of the coil is six times the side of the square section. In bringing the car to rest, the springs are to be compressed 200 mm. Assuming the allowable shear stress as 365 MPa and spring index of 6, find :*

1. Maximum load on each spring, 2. Side of the square section of the wire, 3. Mean diameter of coils, and 4. Number of active coils.

Take modulus of rigidity as 80 kN/mm^2.

Solution. Given : m = 1800 kg ; v = 72 m/min = 1.2 m/s ; δ = 200 mm ; τ = 365 MPa = 365 N/mm^2 ; C = 6 ; G = 80 kN/mm^2 = 80 × 10^3 N/mm^2

1. ***Maximum load on each spring,***

Let W = Maximum load on each spring.

We know that kinetic energy of the car

$$= \frac{1}{2}\, m.v^2 = \frac{1}{2} \times 1800\,(1.2)^2 = 1296 \text{ N-m} = 1296 \times 10^3 \text{ N-mm}$$

This energy is absorbed in the two springs when compressed to 200 mm. If the springs are loaded gradually from 0 to W, then

$$\left(\frac{0 + W}{2}\right) 2 \times 200 = 1296 \times 10^3$$

$\therefore$ W = 1296 × 10^3 / 200 = 6480 N **Ans.**

2. ***Side of the square section of the wire***

Let b = Side of the square section of the wire, and

D = Mean diameter of the coil = 6 b ... ($\because$ $C = D/b = 6$)

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stres (τ),

$$365 = K \times \frac{2.4\, W.D}{b^3} = 1.2525 \times \frac{2.4 \times 6480 \times 6\, b}{b^3} = \frac{116\,870}{b^2}$$

$\therefore$ b^2 = 116 870 / 365 = 320 or b = 17.89 say 18 mm **Ans.**

3. ***Mean diameter of the coil***

We know that mean diameter of the coil,

D = 6 b = 6 × 18 = 108 mm **Ans.**

4. ***Number of active coils***

Let n = Number of active coils.

We know that the deflection of the spring (δ),

$$200 = \frac{5.568\, W.C^3.n}{G.b} = \frac{5.568 \times 6480 \times 6^3 \times n}{80 \times 10^3 \times 18} = 5.4\, n$$

$\therefore$ n = 200 / 5.4 = 37 **Ans.**

23.15 Helical Springs Subjected to Fatigue Loading

The helical springs subjected to fatigue loading are designed by using the *Soderberg line method. The spring materials are usually tested for torsional endurance strength under a repeated stress that varies from zero to a maximum. Since the springs are ordinarily loaded in one direction only (the load in springs is never reversed in nature), therefore a modified Soderberg diagram is used for springs, as shown in Fig. 23.19.

The endurance limit for reversed loading is shown at point A where the mean shear stress is equal to τ_e / 2 and the variable shear stress is also equal to τ_e / 2. A line drawn from A to B (the yield point in shear, τ_y) gives the Soderberg's failure stress line. If a suitable factor of safety ($F.S.$) is applied to the yield strength (τ_y), a safe stress line CD may be drawn parallel to the line AB, as shown in Fig. 23.19. Consider a design point P on the line CD. Now the value of factor of safety may be obtained as discussed below :

* We have discussed the Soderberg method for completely reversed stresses in Chapter 6.

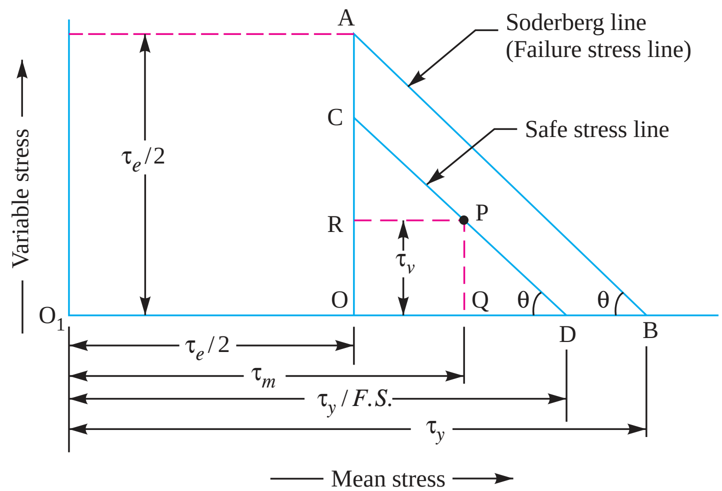

Fig. 23.19. Modified Soderberg method for helical springs.

From similar triangles *PQD* and *AOB*, we have

$$\frac{PQ}{QD} = \frac{OA}{OB} \quad \text{or} \quad \frac{PQ}{O_1D - O_1Q} = \frac{OA}{O_1B - O_1O}$$

$$\frac{\tau_v}{\dfrac{\tau_y}{F.S.} - \tau_m} = \frac{\tau_e/2}{\tau_y - \dfrac{\tau_e}{2}} = \frac{\tau_e}{2\tau_y - \tau_e}$$

or
$$2\,\tau_v\,.\,\tau_y - \tau_v.\,\tau_e = \frac{\tau_e\,.\,\tau_y}{F.S.} - \tau_m.\tau_e$$

∴
$$\frac{\tau_e\,.\,\tau_y}{F.S.} = 2\,\tau_v.\tau_y - \tau_v.\tau_e + \tau_m.\tau_e$$

Dividing both sides by $\tau_e.\tau_y$ and rearranging, we have

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e} \qquad ...(i)$$

Notes : 1. From equation (*i*), the expression for the factor of safety (*F.S.*) may be written as

$$F.S. = \frac{\tau_y}{\tau_m - \tau_v + \dfrac{2\,\tau_v.\tau_y}{\tau_e}}$$

2. The value of mean shear stress (τ_m) is calculated by using the shear stress factor (K_S), while the variable shear stress is calculated by using the full value of the Wahl's factor (*K*). Thus

Mean shear stress,

$$\tau_m = K_s \times \frac{8\,W_m \times D}{\pi d^3}$$

where
$$K_S = 1 + \frac{1}{2C}; \text{ and } W_m = \frac{W_{max} + W_{min}}{2}$$

and variable shear stress,
$$\tau_v = K \times \frac{8\,W_v \times D}{\pi\, d^3}$$

where
$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}; \text{ and } W_v = \frac{W_{max} - W_{min}}{2}$$

Example 23.17. *A helical compression spring made of oil tempered carbon steel, is subjected to a load which varies from 400 N to 1000 N. The spring index is 6 and the design factor of safety is 1.25. If the yield stress in shear is 770 MPa and endurance stress in shear is 350 MPa, find : 1. Size of the spring wire, 2. Diameters of the spring, 3. Number of turns of the spring, and 4. Free length of the spring.*

The compression of the spring at the maximum load is 30 mm. The modulus of rigidity for the spring material may be taken as 80 kN/mm^2.

Solution. Given : W_{min} = 400 N ; W_{max} = 1000 N ; C = 6 ; $F.S.$ = 1.25 ; τ_y = 770 MPa = 770 N/mm^2 ; τ_e = 350 MPa = 350 N/mm^2 ; δ = 30 mm ; G = 80 kN/mm^2 = 80 × 10^3 N/mm^2

1. Size of the spring wire

Let d = Diameter of the spring wire, and

D = Mean diameter of the spring = $C.d = 6\,d$ ... ($\because D/d = C = 6$)

We know that the mean load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{1000 + 400}{2} = 700 \text{ N}$$

and variable load,

$$W_v = \frac{W_{max} - W_{min}}{2} = \frac{1000 - 400}{2} = 300 \text{ N}$$

Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 6} = 1.083$$

Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

We know that mean shear stress,

$$\tau_m = K_S \times \frac{8\,W_m \times D}{\pi\,d^3} = 1.083 \times \frac{8 \times 700 \times 6\,d}{\pi\,d^3} = \frac{11\,582}{d^2} \text{ N/mm}^2$$

and variable shear stress,

$$\tau_v = K \times \frac{8\,W_v \times D}{\pi\,d^3} = 1.2525 \times \frac{8 \times 300 \times 6\,d}{\pi\,d^3} = \frac{5740}{d^2} \text{ N/mm}^2$$

We know that

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\,\tau_v}{\tau_e}$$

$$\frac{1}{1.25} = \frac{\frac{11\,582}{d^2} - \frac{5740}{d^2}}{770} + \frac{2 \times \frac{5740}{d^2}}{350} = \frac{7.6}{d^2} + \frac{32.8}{d^2} = \frac{40.4}{d^2}$$

$\therefore$ d^2 = 1.25 × 40.4 = 50.5 or d = 7.1 mm **Ans.**

2. Diameters of the spring

We know that mean diameter of the spring,

$D = C.d$ = 6 × 7.1 = 42.6 mm **Ans.**

Outer diameter of the spring,

$D_o = D + d$ = 42.6 + 7.1 = 49.7 mm **Ans.**

and inner diameter of the spring,

$D_i = D - d$ = 42.6 – 7.1 = 35.5 mm **Ans.**

3. Number of turns of the spring

Let n = Number of active turns of the spring.

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We know that deflection of the spring (δ),

$$30 = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times 1000\ (42.6)^3\ n}{80 \times 10^3\ (7.1)^4} = 3.04\,n$$

$\therefore \quad n = 30 / 3.04 = 9.87$ say 10 **Ans.**

Assuming the ends of the spring to be squared and ground, the total number of turns of the spring,

$$n' = n + 2 = 10 + 2 = 12$$ **Ans.**

4. Free length of the spring

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta = 12 \times 7.1 + 30 + 0.15 \times 30 \text{ mm}$$

$$= 119.7 \text{ say } 120 \text{ mm}$$ **Ans.**

23.16 Springs in Series

Consider two springs connected in series as shown in Fig. 23.20.

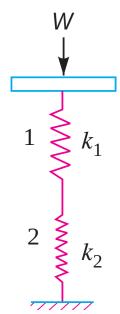
Let W = Load carried by the springs,

δ_1 = Deflection of spring 1,

δ_2 = Deflection of spring 2,

k_1 = Stiffness of spring 1 = W / δ_1, and

k_2 = Stiffness of spring 2 = W / δ_2



Springs in series.
Fig. 23.20

A little consideration will show that when the springs are connected in series, then the total deflection produced by the springs is equal to the sum of the deflections of the individual springs.

∴ Total deflection of the springs,

$$\delta = \delta_1 + \delta_2$$

or
$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

∴
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

where k = Combined stiffness of the springs.

23.17 Springs in Parallel

Consider two springs connected in parallel as shown in Fig 23.21.

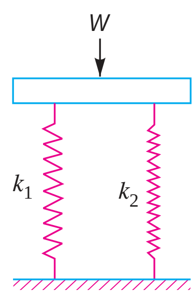
Let W = Load carried by the springs,

W_1 = Load shared by spring 1,

W_2 = Load shared by spring 2,

k_1 = Stiffness of spring 1, and

k_2 = Stiffness of spring 2.



Springs in parallel.
Fig. 23.21

A little consideration will show that when the springs are connected in parallel, then the total deflection produced by the springs is same as the deflection of the individual springs.

We know that $W = W_1 + W_2$

or $\delta.k = \delta.k_1 + \delta.k_2$

∴ $k = k_1 + k_2$

where k = Combined stiffness of the springs, and

δ = Deflection produced.

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Example 23.18. *A close coiled helical compression spring of 12 active coils has a spring stiffness of k. It is cut into two springs having 5 and 7 turns. Determine the spring stiffnesses of resulting springs.*

Solution. Given : $n = 12$; $n_1 = 5$; $n_2 = 7$

We know that the deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} \quad \text{or} \quad \frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n}$$

Since G, D and d are constant, therefore substituting

$$\frac{G.d^4}{8\,D^3} = X\text{, a constant, we have } \frac{W}{\delta} = k = \frac{X}{n}$$

or $$X = k.n = 12\,k$$

The spring is cut into two springs with $n_1 = 5$ and $n_2 = 7$.

Let k_1 = Stiffness of spring having 5 turns, and

k_2 = Stiffness of spring having 7 turns.

$\therefore$ $$k_1 = \frac{X}{n_1} = \frac{12\,k}{5} = 2.4\,k \text{ **Ans.**}$$

and $$k_2 = \frac{X}{n_2} = \frac{12\,k}{7} = 1.7\,k \text{ **Ans.**}$$

23.18 Concentric or Composite Springs

A concentric or composite spring is used for one of the following purposes :

1. To obtain greater spring force within a given space.

2. To insure the operation of a mechanism in the event of failure of one of the springs.

The concentric springs for the above two purposes may have two or more springs and have the same free lengths as shown in Fig. 23.22 (*a*) and are compressed equally. Such springs are used in automobile clutches, valve springs in aircraft, heavy duty diesel engines and rail-road car suspension systems.

A car shock absorber.

Sometimes concentric springs are used to obtain a spring force which does not increase in a direct relation to the deflection but increases faster. Such springs are made of different lengths as shown in Fig. 23.22 (*b*). The shorter spring begins to act only after the longer spring is compressed to a certain amount. These springs are used in governors of variable speed engines to take care of the variable centrifugal force.

The adjacent coils of the concentric spring are wound in opposite directions to eliminate any tendency to bind.

If the same material is used, the concentric springs are designed for the same stress. In order to get the same stress factor (K), it is desirable to have the same spring index (C).

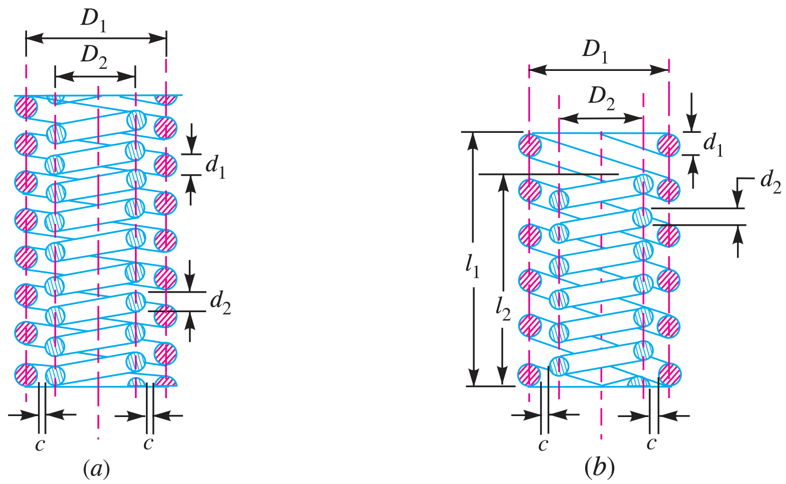

Fig. 23.22. Concentric springs.

Consider a concentric spring as shown in Fig. 23.22 (a).

Let W = Axial load,

W_1 = Load shared by outer spring,

W_2 = Load shared by inner spring,

d_1 = Diameter of spring wire of outer spring,

d_2 = Diameter of spring wire of inner spring,

D_1 = Mean diameter of outer spring,

D_2 = Mean diameter of inner spring,

δ_1 = Deflection of outer spring,

δ_2 = Deflection of inner spring,

n_1 = Number of active turns of outer spring, and

n_2 = Number of active turns of inner spring.

Assuming that both the springs are made of same material, then the maximum shear stress induced in both the springs is approximately same, *i.e.*

$$\tau_1 = \tau_2$$

$$\frac{8\,W_1 . D_1 . K_1}{\pi\,(d_1)^3} = \frac{8\,W_2 . D_2 . K_2}{\pi\,(d_2)^3}$$

When stress factor, $K_1 = K_2$, then

$$\frac{W_1 . D_1}{(d_1)^3} = \frac{W_2 . D_2}{(d_2)^3} \quad ...(i)$$

If both the springs are effective throughout their working range, then their free length and deflection are equal, *i.e.*

$$\delta_1 = \delta_2$$

or
$$\frac{8W_1\,(D_1)^3\,n_1}{(d_1)^4\,G} = \frac{8W_2\,(D_2)^3\,n_2}{(d_2)^4\,G} \quad \text{or} \quad \frac{W_1\,(D_1)^3\,n_1}{(d_1)^4} = \frac{W_2\,(D_2)^3\,n_2}{(d_2)^4} \quad ...(ii)$$

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When both the springs are compressed until the adjacent coils meet, then the solid length of both the springs is equal, *i.e.*

$$n_1.d_1 = n_2.d_2$$

∴ The equation ***(ii)*** may be written as

$$\frac{W_1 (D_1)^3}{(d_1)^5} = \frac{W_2 (D_2)^3}{(d_2)^5} \qquad ...(iii)$$

Now dividing equation ***(iii)*** by equation ***(i)***, we have

$$\frac{(D_1)^2}{(d_1)^2} = \frac{(D_2)^2}{(d_2)^2} \quad \text{or} \quad \frac{D_1}{d_1} = \frac{D_2}{d_2} = C, \text{ the spring index} \qquad ...(iv)$$

i.e. the springs should be designed in such a way that the spring index for both the springs is same.

From equations ***(i)*** and ***(iv)***, we have

$$\frac{W_1}{(d_1)^2} = \frac{W_2}{(d_2)^2} \quad \text{or} \quad \frac{W_1}{W_2} = \frac{(d_1)^2}{(d_2)^2} \qquad ..(v)$$

From Fig. 23.22 (*a*), we find that the radial clearance between the two springs,

$$*c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Usually, the radial clearance between the two springs is taken as $\frac{d_1 - d_2}{2}$.

$$\therefore \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$$

or

$$\frac{D_1 - D_2}{2} = d_1 \qquad ...(vi)$$

From equation ***(iv)***, we find that

$$D_1 = C.d_1, \text{ and } D_2 = C.d_2$$

Substituting the values of D_1 and D_2 in equation ***(vi)***, we have

$$\frac{C.d_1 - C.d_2}{2} = d_1 \quad \text{or} \quad C.d_1 - 2\,d_1 = C.d_2$$

$$\therefore \quad d_1 (C - 2) = C.d_2 \quad \text{or} \quad \frac{d_1}{d_2} = \frac{C}{C-2} \qquad ...(vii)$$

Example 23.19. *A concentric spring for an aircraft engine valve is to exert a maximum force of 5000 N under an axial deflection of 40 mm. Both the springs have same free length, same solid length and are subjected to equal maximum shear stress of 850 MPa. If the spring index for both the springs is 6, find (a) the load shared by each spring, (b) the main dimensions of both the springs, and (c) the number of active coils in each spring.*

Assume G = 80 kN/mm² and diametral clearance to be equal to the difference between the wire diameters.

Solution. Given : W = 5000 N ; δ = 40 mm ; $\tau_1 = \tau_2$ = 850 MPa = 850 N/mm² ; C = 6 ; G = 80 kN/mm² = 80 × 10³ N/mm²

The concentric spring is shown in Fig. 23.22 (*a*).

(a) Load shared by each spring

Let W_1 and W_2 = Load shared by outer and inner spring respectively,

d_1 and d_2 = Diameter of spring wires for outer and inner springs respectively, and

D_1 and D_2 = Mean diameter of the outer and inner springs respectively.

* The net clearance between the two springs is given by

$$2\,c = (D_1 - D_2) - (d_1 + d_2)$$

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Since the diametral clearance is equal to the difference between the wire diameters, therefore

$$(D_1 - D_2) - (d_1 + d_2) = d_1 - d_2$$

or $$D_1 - D_2 = 2\,d_1$$

We know that $D_1 = C.d_1$, and $D_2 = C.d_2$

$$\therefore \quad C.d_1 - C.d_2 = 2\,d_1$$

or $$\frac{d_1}{d_2} = \frac{C}{C-2} = \frac{6}{6-2} = 1.5 \qquad ...(i)$$

We also know that $$\frac{W_1}{W_2} = \left(\frac{d_1}{d_2}\right)^2 = (1.5)^2 = 2.25 \qquad ...(ii)$$

and $$W_1 + W_2 = W = 5000 \text{ N} \qquad ...(iii)$$

From equations (*ii*) and (*iii*), we find that

$$W_1 = 3462 \text{ N, and } W_2 = 1538 \text{ N } \textbf{Ans.}$$

(b) Main dimensions of both the springs

We know that Wahl's stress factor for both the springs,

$$K_1 = K_2 = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 6-1}{4\times 6-4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stress induced in the outer spring (τ_1),

$$850 = K_1 \times \frac{8\,W_1.C}{\pi(d_1)^2} = 1.2525 \times \frac{8\times 3462\times 6}{\pi(d_1)^2} = \frac{66\,243}{(d_1)^2}$$

$$\therefore \quad (d_1)^2 = 66\,243 / 850 = 78 \text{ or } d_1 = 8.83 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

and $$D_1 = C.d_1 = 6\,d_1 = 6 \times 10 = 60 \text{ mm } \textbf{Ans.}$$

Similarly, maximum shear stress induced in the inner spring (τ_2),

$$850 = K_2 \times \frac{8W_2.C}{\pi(d_2)^2} = 1.2525 \times \frac{8\times 1538\times 6}{\pi(d_2)^2} = \frac{29\,428}{(d_2)^2}$$

$$\therefore \quad (d_2)^2 = 29\,428 / 850 = 34.6 \text{ or } {}^*d_2 = 5.88 \text{ say } 6 \text{ mm } \textbf{Ans.}$$

and $$D_2 = C.d_2 = 6 \times 6 = 36 \text{ mm } \textbf{Ans.}$$

(c) Number of active coils in each spring

Let n_1 and n_2 = Number of active coils of the outer and inner spring respectively.

We know that the axial deflection for the outer spring (δ),

$$40 = \frac{8\,W_1.C^3.n_1}{G.d_1} = \frac{8\times 3462\times 6^3\times n_1}{80\times 10^3\times 10} = 7.48\,n_1$$

$$\therefore \quad n_1 = 40 / 7.48 = 5.35 \text{ say } 6 \textbf{ Ans.}$$

Assuming square and ground ends for the spring, the total number of turns of the outer spring,

$$n_1' = 6 + 2 = 8$$

∴ Solid length of the outer spring,

$$L_{S1} = n_1'\,.\,d_1 = 8 \times 10 = 80 \text{ mm}$$

Let n_2' be the total number of turns of the inner spring. Since both the springs have the same solid length, therefore,

$$n_2'.d_2 = n_1'.d_1$$

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* The value of d_2 may also be obtained from equation (*i*), *i.e.*

$$\frac{d_1}{d_2} = 1.5 \quad \text{or} \quad d_2 = \frac{d_1}{1.5} = \frac{8.83}{1.5} = 5.887 \text{ say } 6 \text{ mm}$$

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or $$n_2' = \frac{n_1'.d_1}{d_2} = \frac{8 \times 10}{6} = 13.3 \text{ say } 14$$

and $n_2 = 14 - 2 = 12$ **Ans.** ... ($\because n_2' = n_2 + 2$)

Since both the springs have the same free length, therefore

Free length of outer spring

= Free length of inner spring

= $L_{S1} + \delta + 0.15\,\delta = 80 + 40 + 0.15 \times 40 = 126$ mm **Ans.**

Other dimensions of the springs are as follows:

Outer diameter of the outer spring

= $D_1 + d_1 = 60 + 10 = 70$ mm **Ans.**

Inner diameter of the outer spring

= $D_1 - d_1 = 60 - 10 = 50$ mm **Ans.**

Outer diameter of the inner spring

= $D_2 + d_2 = 36 + 6 = 42$ mm **Ans.**

Inner diameter of the inner spring

= $D_2 - d_2 = 36 - 6 = 30$ mm **Ans.**

Shock absorbers

Example 23.20. *A composite spring has two closed coil helical springs as shown in Fig. 23.22 (b). The outer spring is 15 mm larger than the inner spring. The outer spring has 10 coils of mean diameter 40 mm and wire diameter 5mm. The inner spring has 8 coils of mean diameter 30 mm and wire diameter 4 mm. When the spring is subjected to an axial load of 400 N, find 1. compression of each spring, 2. load shared by each spring, and 3. shear stress induced in each spring. The modulus of rigidity may be taken as 84 kN/mm^2.*

Solution. Given : $\delta_1 = l_1 - l_2 = 15$ mm ; $n_1 = 10$; $D_1 = 40$ mm ; $d_1 = 5$ mm ; $n_2 = 8$; $D_2 = 30$ mm ; $d_2 = 4$ mm ; $W = 400$ N ; $G = 84$ kN/mm^2 = 84×10^3 N/mm^2

1. Compression of each spring

Since the outer spring is 15 mm larger than the inner spring, therefore the inner spring will not take any load till the outer spring is compressed by 15 mm. After this, both the springs are compressed together. Let P_1 be the load on the outer spring to compress it by 15 mm.

We know that compression of the spring (δ),

$$15 = \frac{8\,P_1\,(D_1)^3\,n_1}{G\,(d_1)^4} = \frac{8\,P_1\,(40)^3\,10}{84 \times 10^3 \times 5^4} = 0.0975\,P_1$$

$\therefore$ $P_1 = 15 / 0.0975 = 154$ N

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Now the remaining load *i.e.* $W - P_1 = 400 - 154 = 246$ N is taken together by both the springs.

Let δ_2 = Further compression of the outer spring or the total compression of the inner spring.

Since for compressing the outer spring by 15 mm, the load required is 154 N, therefore the additional load required by the outer spring to compress it by δ_2 mm is given by

$$P_2 = \frac{P_1}{\delta_1} \times \delta_2 = \frac{154}{15} \times \delta_2 = 10.27\ \delta_2$$

Let W_2 = Load taken by the inner spring to compress it by δ_2 mm.

We know that
$$\delta_2 = \frac{8\ W_2\ (D_2)^3\ n_2}{G\ (d_2)^4} = \frac{8\ W_2\ (30)^3\ 8}{84 \times 10^3 \times 4^4} = 0.08\ W_2$$

$\therefore$
$$W_2 = \delta_2 / 0.08 = 12.5\ \delta_2$$

and
$$P_2 + W_2 = W - P_1 = 400 - 154 = 246 \text{ N}$$

or
$$10.27\ \delta_2 + 12.5\ \delta_2 = 246 \quad \text{or} \quad \delta_2 = 246 / 22.77 = 10.8 \text{ mm}$$ **Ans.**

$\therefore$ Total compression of the outer spring

$$= \delta_1 + \delta_2 = 15 + 10.8 = 25.8 \text{ mm}$$ **Ans.**

2. Load shared by each spring

We know that the load shared by the outer spring,

$$W_1 = P_1 + P_2 = 154 + 10.27\ \delta_2 = 154 + 10.27 \times 10.8 = 265 \text{ N}$$ **Ans.**

and load shared by the inner spring,

$$W_2 = 12.5\ \delta_2 = 12.5 \times 10.8 = 135 \text{ N}$$ **Ans.**

Note : The load shared by the inner spring is also given by

$$W_2 = W - W_1 = 400 - 265 = 135 \text{ N}$$ **Ans.**

3. Shear stress induced in each spring

We know that the spring index of the outer spring,

$$C_1 = \frac{D_1}{d_1} = \frac{40}{5} = 8$$

and spring index of the inner spring,

$$C_2 = \frac{D_2}{d_2} = \frac{30}{4} = 7.5$$

$\therefore$ Wahl's stress factor for the outer spring,

$$K_1 = \frac{4C_1 - 1}{4C_1 - 4} + \frac{0.615}{C_1} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8} = 1.184$$

and Wahl's stress factor for the inner spring,

$$K_2 = \frac{4C_2 - 1}{4C_2 - 4} + \frac{0.615}{C_2} = \frac{4 \times 7.5 - 1}{4 \times 7.5 - 4} + \frac{0.615}{7.5} = 1.197$$

We know that shear stress induced in the outer spring,

$$\tau_1 = K_1 \times \frac{8\ W_1.D_1}{\pi\ (d_1)^3} = 1.184 \times \frac{8 \times 265 \times 40}{\pi \times 5^3} = 255.6 \text{ N/mm}^2$$

$$= 255.6 \text{ MPa}$$ **Ans.**

and shear stress induced in the inner spring,

$$\tau_2 = K_2 \times \frac{8\ W_2.D_2}{\pi\ (d_2)^3} = 1.197 \times \frac{8 \times 135 \times 30}{\pi \times 4^3} = 192.86 \text{ N/mm}^2$$

$$= 192.86 \text{ MPa}$$ **Ans.**

23.19 Helical Torsion Springs

The helical torsion springs as shown in Fig. 23.23, may be made from round, rectangular or square wire. These are wound in a similar manner as helical compression or tension springs but the ends are shaped to transmit torque. The primary stress in helical torsion springs is bending stress whereas in compression or tension springs, the stresses are torsional shear stresses. The helical torsion springs are widely used for transmitting small torques as in door hinges, brush holders in electric motors, automobile starters etc.

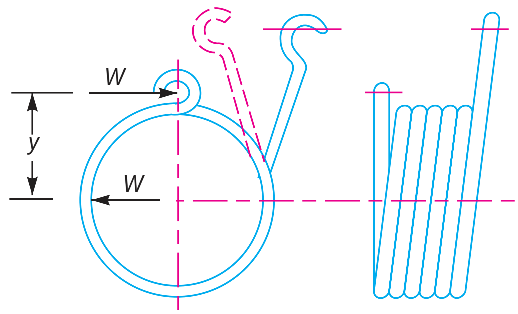

Fig. 23.23. Helical torsion spring.

A little consideration will show that the radius of curvature of the coils changes when the twisting moment is applied to the spring. Thus, the wire is under pure bending. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = K \times \frac{32\,W.y}{\pi\,d^3}$$

where K = Wahl's stress factor = $\dfrac{4C^2 - C - 1}{4C^2 - 4C}$,

C = Spring index,

M = Bending moment = $W \times y$,

W = Load acting on the spring,

y = Distance of load from the spring axis, and

d = Diameter of spring wire.

and total angle of twist or angular deflection,

$$^*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi\, d^4/64} = \frac{64\,M.D.n}{E.d^4}$$

where l = Length of the wire = $\pi.D.n$,

E = Young's modulus,

I = Moment of inertia = $\dfrac{\pi}{64} \times d^4$,

D = Diameter of the spring, and

n = Number of turns.

and deflection, $\delta = \theta \times y = \dfrac{64\,M.D.n}{E.d^4} \times y$

When the spring is made of rectangular wire having width b and thickness t, then

$$\sigma_b = K \times \frac{6\,M}{t.b^2} = K \times \frac{6\,W \times y}{t.b^2}$$

where $K = \dfrac{3C^2 - C - 0.8}{3C^2 - 3C}$

---

* We know that $M / I = E / R$, where R is the radius of curvature.

$\therefore \quad R = \dfrac{E.I}{M}$ or $\dfrac{l}{\theta} = \dfrac{E.I}{M}$ or $\theta = \dfrac{M.l}{E.I}$ $\quad ...\left(\because R = \dfrac{l}{\theta}\right)$

Angular deflection, $\theta = \dfrac{12\,\pi M.D.n}{E.t.b^3}$; and $\delta = \theta.y = \dfrac{12\,\pi M.D.n}{E.t.b^3} \times y$

In case the spring is made of square wire with each side equal to b, then substituting $t = b$, in the above relation, we have

$$\sigma_b = K \times \frac{6\,M}{b^3} = K \times \frac{6W \times y}{b^3}$$

$$\theta = \frac{12\,\pi M.D.n}{E.b^4}; \quad \text{and} \quad \delta = \frac{12\,\pi M.D.n}{E.b^4} \times y$$

Note : Since the diameter of the spring D reduces as the coils wind up under the applied load, therefore a clearance must be provided when the spring wire is to be wound round a mandrel. A small clearance must also be provided between the adjacent coils in order to prevent sliding friction.

Example 23.21. *A helical torsion spring of mean diameter 60 mm is made of a round wire of 6 mm diameter. If a torque of 6 N-m is applied on the spring, find the bending stress induced and the angular deflection of the spring in degrees. The spring index is 10 and modulus of elasticity for the spring material is 200 kN/mm^2. The number of effective turns may be taken as 5.5.*

Solution. Given : D = 60 mm ; d = 6 mm ; M = 6 N-m = 6000 N-mm ; C = 10 ; E = 200 kN/mm^2 = 200 × 10^3 N/mm^2 ; n = 5.5

Bending stress induced

We know that Wahl's stress factor for a spring made of round wire,

$$K = \frac{4C^2 - C - 1}{4C^2 - 4C} = \frac{4 \times 10^2 - 10 - 1}{4 \times 10^2 - 4 \times 10} = 1.08$$

∴ Bending stress induced,

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = 1.08 \times \frac{32 \times 6000}{\pi \times 6^3} = 305.5 \text{ N/mm}^2 \text{ or MPa} \quad \textbf{Ans.}$$

Angular deflection of the spring

We know that the angular deflection of the spring (in radians),

$$\theta = \frac{64\,M.D.n}{E.d^4} = \frac{64 \times 6000 \times 60 \times 5.5}{200 \times 10^3 \times 6^4} = 0.49 \text{ rad}$$

$$= 0.49 \times \frac{180}{\pi} = 28° \quad \textbf{Ans.}$$

23.20 Flat Spiral Spring

A flat spring is a long thin strip of elastic material wound like a spiral as shown in Fig. 23.24. These springs are frequently used in watches and gramophones etc.

When the outer or inner end of this type of spring is wound up in such a way that there is a tendency in the increase of number of spirals of the spring, the strain energy is stored into its spirals. This energy is utilised in any useful way while the spirals open out slowly. Usually the inner end of spring is clamped to an arbor while the outer end may be pinned or clamped. Since the radius of curvature of every spiral decreases when the spring is wound up, therefore the material of the spring is in a state of pure bending.

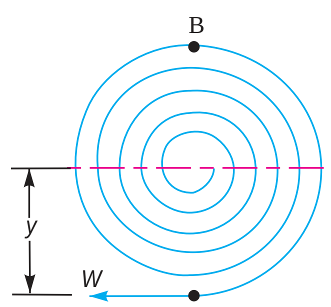

Fig. 23.24. Flat spiral spring.

Let W = Force applied at the outer end A of the spring,

y = Distance of centre of gravity of the spring from A,

l = Length of strip forming the spring,

b = Width of strip,
t = Thickness of strip,
I = Moment of inertia of the spring section = $b.t^3/12$, and
Z = Section modulus of the spring section = $b.t^2/6$

$$...\left(\because Z = \frac{I}{y} = \frac{b.t^3}{12 \times t/2} = \frac{b.t^2}{6}\right)$$

When the end A of the spring is pulled up by a force W, then the bending moment on the spring, at a distance y from the line of action of W is given by

$$M = W \times y$$

The greatest bending moment occurs in the spring at B which is at a maximum distance from the application of W.

∴ Bending moment at B,

$$M_B = M_{max} = W \times 2y = 2W.y = 2M$$

∴ Maximum bending stress induced in the spring material,

$$\sigma_b = \frac{M_{max}}{Z} = \frac{2W \times y}{b.t^2/6} = \frac{12W.y}{b.t^2} = \frac{12M}{b.t^2}$$

Flat spiral spring of a mechanical clock.

Assuming that both ends of the spring are clamped, the angular deflection (in radians) of the spring is given by

$$\theta = \frac{M.l}{E.I} = \frac{12\ M.l}{E.b.t^3} \qquad ...\left(\because I = \frac{b.t^3}{12}\right)$$

and the deflection,

$$\delta = \theta \times y = \frac{M.l.y}{E.I}$$

$$= \frac{12\ M.l.y}{E.b.t^3} = \frac{12W.y^2.l}{E.b.t^3} = \frac{\sigma_b.y.l}{E.t} \qquad ...\left(\because \sigma_b = \frac{12W.y}{b.t^2}\right)$$

The strain energy stored in the spring

$$= \frac{1}{2}M.\theta = \frac{1}{2}M \times \frac{M.l}{E.I} = \frac{1}{2} \times \frac{M^2.l}{E.I}$$

$$= \frac{1}{2} \times \frac{W^2.y^2.l}{E \times bt^3/12} = \frac{6\ W^2.y^2.l}{E.b.t^3}$$

$$= \frac{6\ W^2.y^2.l}{E.b.t^3} \times \frac{24bt}{24bt} = \frac{144\ W^2y^2}{Eb^2t^4} \times \frac{b\,t\,l}{24}$$

... (Multiplying the numerator and denominator by 24 bt)

$$= \frac{(\sigma_b)^2}{24\ E} \times b\,t\,l = \frac{(\sigma_b)^2}{24\ E} \times \text{Volume of the spring}$$

Example 23.22. *A spiral spring is made of a flat strip 6 mm wide and 0.25 mm thick. The length of the strip is 2.5 metres. Assuming the maximum stress of 800 MPa to occur at the point of greatest bending moment, calculate the bending moment, the number of turns to wind up the spring and the strain energy stored in the spring. Take E = 200 kN/mm².*

Solution. Given : b = 6 mm ; t = 0.25 mm ; l = 2.5 m = 2500 mm ; τ = 800 MPa = 800 N/mm² ; E = 200 kN/mm² = 200 × 10³ N/mm²

Contents

Bending moment in the spring

Let M = Bending moment in the spring.

We know that the maximum bending stress in the spring material (σ_b),

$$800 = \frac{12\ M}{b.t^2} = \frac{12\ M}{6\ (0.25)^2} = 32\ M$$

$$\therefore \quad M = 800 / 32 = 25 \text{ N-mm} \quad \textbf{Ans.}$$

Number of turns to wind up the spring

We know that the angular deflection of the spring,

$$\theta = \frac{12\ M.l}{E.b.t^3} = \frac{12 \times 25 \times 2500}{200 \times 10^3 \times 6\ (0.25)^3} = 40 \text{ rad}$$

Since one turn of the spring is equal to 2π radians, therefore number of turns to wind up the spring

$$= 40 / 2\pi = 6.36 \text{ turns} \quad \textbf{Ans.}$$

Strain energy stored in the spring

We know that strain energy stored in the spring

$$= \frac{1}{2} M.\theta = \frac{1}{2} \times 24 \times 40 = 480 \text{ N-mm} \quad \textbf{Ans.}$$

23.21 Leaf Springs

Leaf springs (also known as **flat springs**) are made out of flat plates. The advantage of leaf spring over helical spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural member in addition to energy absorbing device. Thus the leaf springs may carry lateral loads, brake torque, driving torque etc., in addition to shocks.

Consider a single plate fixed at one end and loaded at the other end as shown in Fig. 23.25. This plate may be used as a flat spring.

Let t = Thickness of plate,

b = Width of plate, and

L = Length of plate or distance of the load W from the cantilever end.

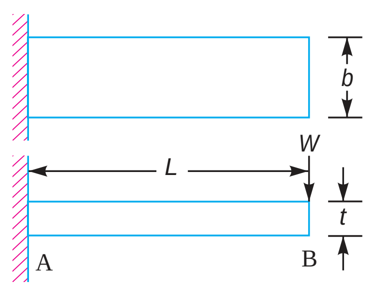

Fig. 23.25. Flat spring (cantilever type).

We know that the maximum bending moment at the cantilever end A,

$$M = W.L$$

and section modulus,

$$Z = \frac{I}{y} = \frac{b\ t^3/12}{t/2} = \frac{1}{6} \times b.t^2$$

$\therefore$ Bending stress in such a spring,

$$\sigma = \frac{M}{Z} = \frac{W.L}{\frac{1}{6} \times b.t^2} = \frac{6\ W.L}{b.t^2} \quad \text{...}(i)$$

We know that the maximum deflection for a cantilever with concentrated load at the free end is given by

$$\delta = \frac{W.L^3}{3E.I} = \frac{W.L^3}{3E \times b.t^3/12} = \frac{4\ W.L^3}{E.b.t^3} \quad \text{...}(ii)$$

$$= \frac{2\ \sigma.L^2}{3\ E.t} \quad \text{...}\left(\because \sigma = \frac{6W.L}{b.t^2}\right)$$

Top

It may be noted that due to bending moment, top fibres will be in tension and the bottom fibres are in compression, but the shear stress is zero at the extreme fibres and maximum at the centre, as shown in Fig. 23.26. Hence for analysis, both stresses need not to be taken into account simultaneously. We shall consider the bending stress only.

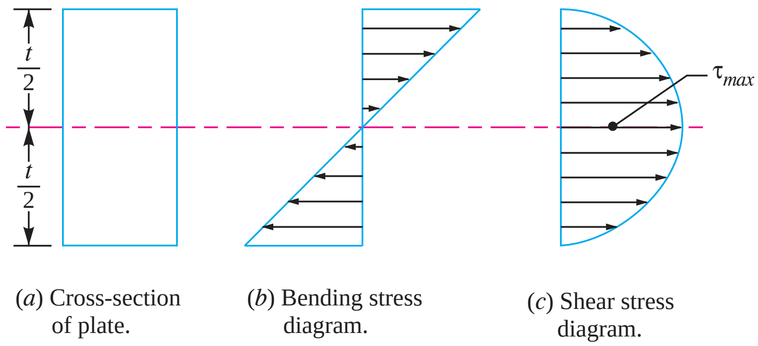

Fig. 23.26

If the spring is not of cantilever type but it is like a simply supported beam, with length $2L$ and load $2W$ in the centre, as shown in Fig. 23.27, then

Maximum bending moment in the centre,

$$M = W.L$$

Section modulus, $Z = b.t^2 / 6$

$$\therefore \text{ Bending stress, } \quad \sigma = \frac{M}{Z} = \frac{W.L}{b.t^2/6} = \frac{6\,W.L}{b.t^2}$$

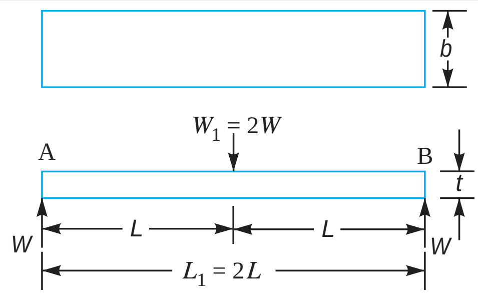

Fig. 23.27. Flat spring (simply supported beam type).

We know that maximum deflection of a simply supported beam loaded in the centre is given by

Leaf spring

$$\delta = \frac{W_1\,(L_1)^3}{48\,E.I} = \frac{(2W)\,(2L)^3}{48\,E.I} = \frac{W.L^3}{3\,E.I}$$

...($\because$ In this case, $W_1 = 2W$, and $L_1 = 2L$)

Contents

From above we see that a spring such as automobile spring (semi-elliptical spring) with length 2L and loaded in the centre by a load 2W, may be treated as a double cantilever.

If the plate of cantilever is cut into a series of n strips of width b and these are placed as shown in Fig. 23.28, then equations (*i*) and (*ii*) may be written as

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad ...(iii)$$

and
$$\delta = \frac{4\,W.L^3}{n.E.b.t^3} = \frac{2\,\sigma.L^2}{3\,E.t} \qquad ...(iv)$$

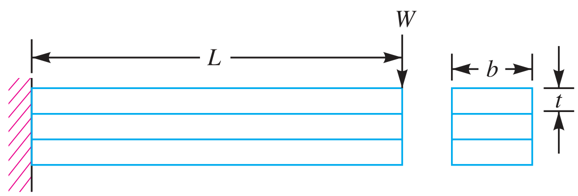

Fig. 23.28

The above relations give the stress and deflection of a leaf spring of uniform cross-section. The stress at such a spring is maximum at the support.

If a triangular plate is used as shown in Fig. 23.29 (*a*), the stress will be uniform throughout. If this triangular plate is cut into strips of uniform width and placed one below the other, as shown in Fig. 23.29 (*b*) to form a graduated or laminated leaf spring, then

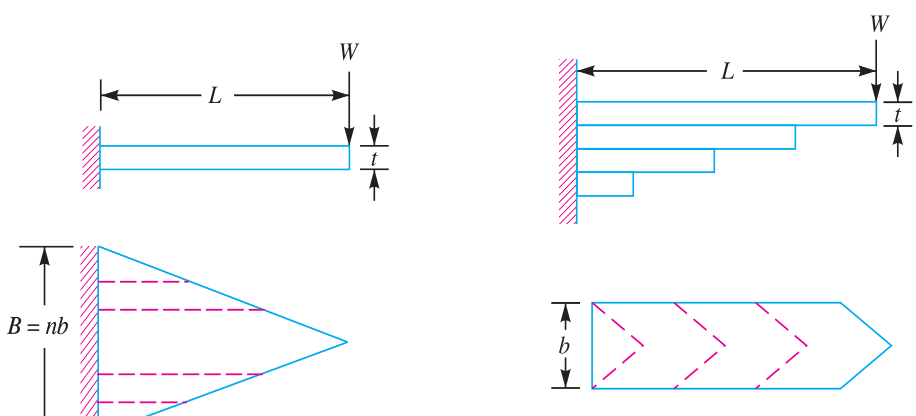

Fig. 23.29. Laminated leaf spring.

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad ...(v)$$

and
$$\delta = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{\sigma.L^2}{E.t} \qquad ...(vi)$$

where n = Number of graduated leaves.

A little consideration will show that by the above arrangement, the spring becomes compact so that the space occupied by the spring is considerably reduced.

Top

When bending stress alone is considered, the graduated leaves may have zero width at the loaded end. But sufficient metal must be provided to support the shear. Therefore, it becomes necessary to have one or more leaves of uniform cross-section extending clear to the end. We see from equations **(*iv*)** and **(*vi*)** that for the same deflection, the stress in the uniform cross-section leaves (*i.e.* full length leaves) is 50% greater than in the graduated leaves, assuming that each spring element deflects according to its own elastic curve. If the suffixes $_F$ and $_G$ are used to indicate the full length (or uniform cross-section) and graduated leaves, then

$$\sigma_F = \frac{3}{2}\sigma_G$$

$$\frac{6 W_F.L}{n_F.b.t^2} = \frac{3}{2}\left[\frac{6\, W_G.L}{n_G.b.t^2}\right] \quad \text{or} \quad \frac{W_F}{n_F} = \frac{3}{2}\times\frac{W_G}{n_G}$$

$$\therefore \quad \frac{W_F}{W_G} = \frac{3\,n_F}{2\,n_G} \qquad \text{...(vii)}$$

Adding 1 to both sides, we have

$$\frac{W_F}{W_G} + 1 = \frac{3\,n_F}{2\,n_G} + 1 \quad \text{or} \quad \frac{W_F + W_G}{W_G} = \frac{3\,n_F + 2\,n_G}{2\,n_G}$$

$$\therefore \quad W_G = \left(\frac{2\,n_G}{3\,n_F + 2\,n_G}\right)(W_F + W_G) = \left(\frac{2\,n_G}{3\,n_F + 2\,n_G}\right) W \qquad \text{...(viii)}$$

where W = Total load on the spring = $W_G + W_F$

W_G = Load taken up by graduated leaves, and

W_F = Load taken up by full length leaves.

From equation **(*vii*)**, we may write

$$\frac{W_G}{W_F} = \frac{2\,n_G}{3\,n_F}$$

or
$$\frac{W_G}{W_F} + 1 = \frac{2\,n_G}{3\,n_F} + 1 \qquad \text{... (Adding 1 to both sides)}$$

$$\frac{W_G + W_F}{W_F} = \frac{2\,n_G + 3\,n_F}{3\,n_F}$$

$$\therefore \quad W_F = \left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)(W_G + W_F) = \left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right) W \qquad \text{...(ix)}$$

$\therefore$ Bending stress for full length leaves,

$$\sigma_F = \frac{6\,W_F.L}{n_F.b\,t^2} = \frac{6\,L}{n_F.b.t^2}\left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right) W = \frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

Since $\sigma_F = \frac{3}{2}\sigma_G$, therefore

$$\sigma_G = \frac{2}{3}\sigma_F = \frac{2}{3}\times\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)} = \frac{12\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

The deflection in full length and graduated leaves is given by equation **(*iv*)**, *i.e.*

$$\delta = \frac{2\,\sigma_F \times L^2}{3\,E.t} = \frac{2\,L^2}{3\,E.t}\left[\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}\right] = \frac{12\,W.L^3}{E.b.t^3\,(2\,n_G + 3\,n_F)}$$

23.22 Construction of Leaf Spring

A leaf spring commonly used in automobiles is of semi-elliptical form as shown in Fig. 23.30.

It is built up of a number of plates (known as leaves). The leaves are usually given an initial curvature or cambered so that they will tend to straighten under the load. The leaves are held together by means of a band shrunk around them at the centre or by a bolt passing through the centre. Since the band exerts a stiffening and strengthening effect, therefore the effective length of the spring for bending will be overall length of the spring *minus* width of band. In case of a centre bolt, two-third distance between centres of *U*-bolt should be subtracted from the overall length of the spring in order to find effective length. The spring is clamped to the axle housing by means of *U*-bolts.

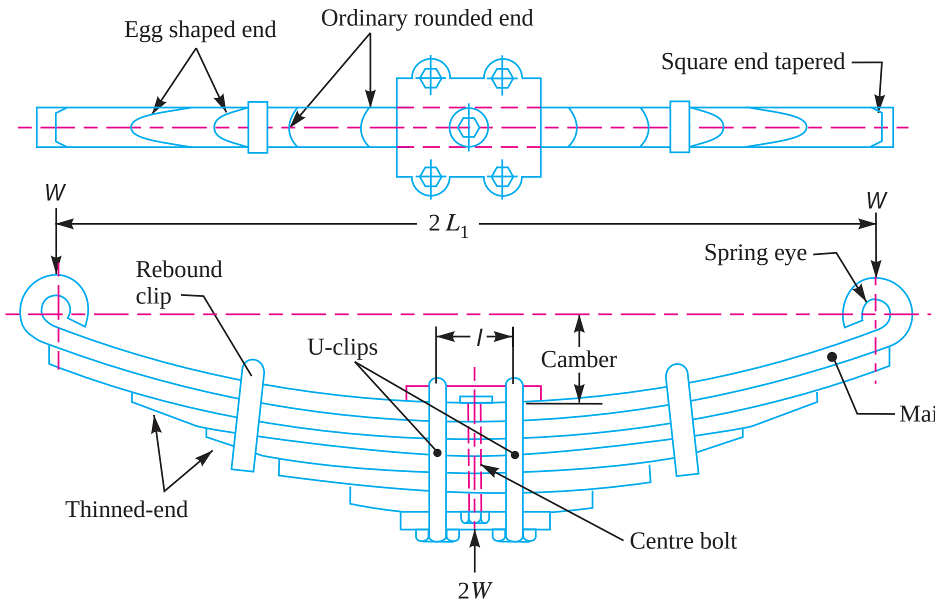

Fig. 23.30. Semi-elliptical leaf spring.

The longest leaf known as ***main leaf*** or ***master leaf*** has its ends formed in the shape of an eye through which the bolts are passed to secure the spring to its supports. Usually the eyes, through which the spring is attached to the hanger or shackle, are provided with bushings of some antifriction material such as bronze or rubber. The other leaves of the spring are known as ***graduated leaves.*** In order to prevent digging in the adjacent leaves, the ends of the graduated leaves are trimmed in various forms as shown in Fig. 23.30. Since the master leaf has to with stand vertical bending loads as well as loads due to sideways of the vehicle and twisting, therefore due to the presence of stresses caused by these loads, it is usual to provide two full length leaves and the rest graduated leaves as shown in Fig. 23.30.

Rebound clips are located at intermediate positions in the length of the spring, so that the graduated leaves also share the stresses induced in the full length leaves when the spring rebounds.

Leaf spring fatigue testing system.

23.23 Equalised Stress in Spring Leaves (Nipping)

We have already discussed that the stress in the full length leaves is 50% greater than the stress in the graduated leaves. In order to utilise the material to the best advantage, all the leaves should be equally stressed. This condition may be obtained in the following two ways :

1. By making the full length leaves of smaller thickness than the graduated leaves. In this way, the full length leaves will induce smaller bending stress due to small distance from the neutral axis to the edge of the leaf.

2. By giving a greater radius of curvature to the full length leaves than graduated leaves, as shown in Fig. 23.31, before the leaves are assembled to form a spring. By doing so, a gap or clearance will be left between the leaves. This initial gap, as shown by *C* in Fig. 23.31, is called ***nip.*** When the central bolt, holding the various leaves together, is tightened, the full length leaf will bend back as shown dotted in Fig. 23.31 and have an initial stress in a direction opposite to that of the normal load. The graduated

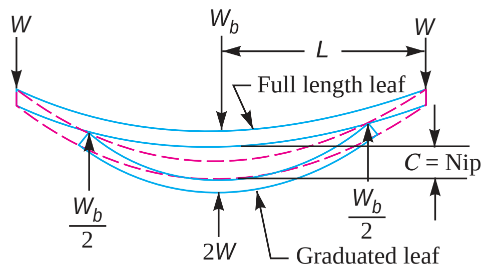

Fig. 23.31

leaves will have an initial stress in the same direction as that of the normal load. When the load is gradually applied to the spring, the full length leaf is first relieved of this initial stress and then stressed in opposite direction. Consequently, the full length leaf will be stressed less than the graduated leaf. The initial gap between the leaves may be adjusted so that under maximum load condition the stress in all the leaves is equal, or if desired, the full length leaves may have the lower stress. This is desirable in automobile springs in which full length leaves are designed for lower stress because the full length leaves carry additional loads caused by the swaying of the car, twisting and in some cases due to driving the car through the rear springs. Let us now find the value of initial gap or nip *C*.

Consider that under maximum load conditions, the stress in all the leaves is equal. Then at maximum load, the total deflection of the graduated leaves will exceed the deflection of the full length leaves by an amount equal to the initial gap *C*. In other words,

$$\delta_G = \delta_F + C$$

$$\therefore \quad C = \delta_G - \delta_F = \frac{6\,W_G \,.\, L^3}{n_G\, E.b.t^3} - \frac{4\,W_F.L^3}{n_F.E.b.t^3} \qquad ...(i)$$

Since the stresses are equal, therefore

$$\sigma_G = \sigma_F$$

$$\frac{6\,W_G.L}{n_G.b.t^2} = \frac{6\,W_F.L}{n_F.b.t^2} \quad \text{or} \quad \frac{W_G}{n_G} = \frac{W_F}{n_F}$$

$$\therefore \quad W_G = \frac{n_G}{n_F} \times W_F = \frac{n_G}{n} \times W$$

and
$$W_F = \frac{n_F}{n_G} \times W_G = \frac{n_F}{n} \times W$$

Substituting the values of W_G and W_F in equation (*i*), we have

$$C = \frac{6W.L^3}{n.E.b.t^3} - \frac{4W.L^3}{n.E.b.t^3} = \frac{2W.L^3}{n.E.b.t^3} \qquad ...(ii)$$

The load on the clip bolts (W_b) required to close the gap is determined by the fact that the gap is equal to the initial deflections of full length and graduated leaves.

$$\therefore \quad C = \delta_F + \delta_G$$

$$\frac{2W.L^3}{n.E.b.t^3} = \frac{4L^3}{n_F.E.b.t^3} \times \frac{W_b}{2} + \frac{6\,L^3}{n_G.E.b.t^3} \times \frac{W_b}{2}$$

or
$$\frac{W}{n} = \frac{W_b}{n_F} + \frac{3\,W_b}{2\,n_G} = \frac{2\,n_G.W_b + 3\,n_F.W_b}{2\,n_F.n_G} = \frac{W_b\,(2\,n_G + 3\,n_F)}{2\,n_F.n_G}$$

$$\therefore \quad W_b = \frac{2\,n_F.n_G.W}{n\,(2\,n_G + 3\,n_F)} \qquad ...(iii)$$

The final stress in spring leaves will be the stress in the full length leaves due to the applied load ***minus*** the initial stress.

$$\therefore \text{ Final stress,} \quad \sigma = \frac{6\,W_F.L}{n_F.b.t^2} - \frac{6\,L}{n_F.b.t^2} \times \frac{W_b}{2} = \frac{6\,L}{n_F.b.t^2}\left(W_F - \frac{W_b}{2}\right)$$

$$= \frac{6\,L}{n_F.b.t^2}\left[\frac{3n_F}{2n_G + 3n_F} \times W - \frac{n_F.n_G.W}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6W.L}{b.t^2}\left[\frac{3}{2n_G + 3n_F} - \frac{n_G}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6\,W.L}{b.t^2}\left[\frac{3n - n_G}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6\,W.L}{b.t^2}\left[\frac{3(n_F + n_G) - n_G}{n\,(2n_G + 3n_F)}\right] = \frac{6\,W.L}{n.b.t^2} \qquad ...(iv)$$

... (Substituting $n = n_F + n_G$)

Notes : 1. The final stress in the leaves is also equal to the stress in graduated leaves due to the applied load *plus* the initial stress.

2. The deflection in the spring due to the applied load is same as without initial stress.

23.24 Length of Leaf Spring Leaves

The length of the leaf spring leaves may be obtained as discussed below :

Let $2L_1$ = Length of span or overall length of the spring,

l = Width of band or distance between centres of *U*-bolts. It is the ineffective length of the spring,

n_F = Number of full length leaves,

n_G = Number of graduated leaves, and

n = Total number of leaves = $n_F + n_G$.

We have already discussed that the effective length of the spring,

$$2L = 2L_1 - l \qquad ...(\text{When band is used})$$

$$= 2L_1 - \frac{2}{3}\,l \qquad ...(\text{When } U\text{-bolts are used})$$

It may be noted that when there is only one full length leaf (*i.e.* master leaf only), then the number of leaves to be cut will be n and when there are two full length leaves (including one master leaf), then the number of leaves to be cut will be $(n - 1)$. If a leaf spring has two full length leaves, then the length of leaves is obtained as follows :

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$$\text{Length of smallest leaf} = \frac{\text{Effective length}}{n-1} + \text{Ineffective length}$$

$$\text{Length of next leaf} = \frac{\text{Effective length}}{n-1} \times 2 + \text{Ineffective length}$$

Similarly, length of $(n-1)$th leaf

$$= \frac{\text{Effective length}}{n-1} \times (n-1) + \text{Ineffective length}$$

The nth leaf will be the master leaf and it is of full length. Since the master leaf has eyes on both sides, therefore

$$\text{Length of master leaf} = 2 L_1 + \pi (d + t) \times 2$$

where d = Inside diameter of eye, and

t = Thickness of master leaf.

The approximate relation between the radius of curvature (R) and the camber (y) of the spring is given by

$$R = \frac{(L_1)^2}{2y}$$

The exact relation is given by

$$y (2R + y) = (L_1)^2$$

where L_1 = Half span of the spring.

Note : The maximum deflection (δ) of the spring is equal to camber (y) of the spring.

23.25 Standard Sizes of Automobile Suspension Springs

Following are the standard sizes for the automobile suspension springs:

1. Standard nominal widths are : 32, 40*, 45, 50*, 55, 60*, 65, 70*, 75, 80, 90, 100 and 125 mm. (Dimensions marked* are the preferred widths)

2. Standard nominal thicknesses are : 3.2, 4.5, 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11, 12, 14 and 16 mm.

3. At the eye, the following bore diameters are recommended :
19, 20, 22, 23, 25, 27, 28, 30, 32, 35, 38, 50 and 55 mm.

4. Dimensions for the centre bolts, if employed, shall be as given in the following table.

Table 23.5. Dimensions for centre bolts.

Width of leaves in mm	*Dia. of centre bolt in mm*	*Dia. of head in mm*	*Length of bolt head in mm*
Upto and including 65	8 or 10	12 or 15	10 or 11
Above 65	12 or 16	17 or 20	11

5. Minimum clip sections and the corresponding sizes of rivets and bolts used with the clips shall be as given in the following table (See Fig. 23.32).

Table 23.6. Dimensions of clip, rivet and bolts.

Spring width (B) in mm	*Clip section ($b \times t$) in mm × mm*	*Dia. of rivet (d_1) in mm*	*Dia. of bolt (d_2) in mm*
Under 50	20 × 4	6	6
50, 55 and 60	25 × 5	8	8
65, 70, 75 and 80	25 × 6	10	8
90, 100 and 125	32 × 6	10	10

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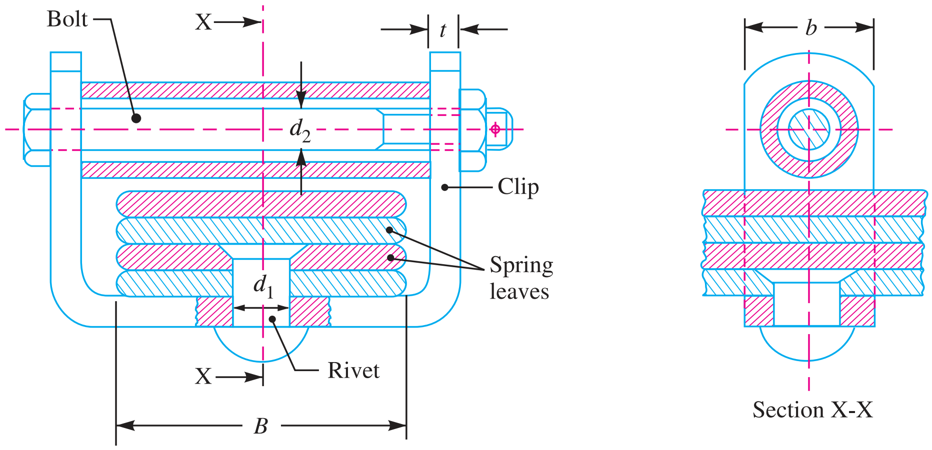

Fig. 23.32. Spring clip.

Notes : 1. For springs of width below 65 mm, one rivet of 6, 8 or 10 mm may be used. For springs of width above 65 mm, two rivets of 6 or 8 mm or one rivet of 10 mm may be used.

2. For further details, the following Indian Standards may be referred :

(*a*) IS : 9484 – 1980 (Reaffirmed 1990) on 'Specification for centre bolts for leaf springs'.

(*b*) IS : 9574 – 1989 (Reaffirmed 1994) on 'Leaf springs assembly-Clips-Specification'.

23.26 Materials for Leaf Springs

The material used for leaf springs is usually a plain carbon steel having 0.90 to 1.0% carbon. The leaves are heat treated after the forming process. The heat treatment of spring steel produces greater strength and therefore greater load capacity, greater range of deflection and better fatigue properties.

According to Indian standards, the recommended materials are :

1. For automobiles : 50 Cr 1, 50 Cr 1 V 23, and 55 Si 2 Mn 90 all used in hardened and tempered state.
2. For rail road springs : C 55 (water-hardened), C 75 (oil-hardened), 40 Si 2 Mn 90 (water-hardened) and 55 Si 2 Mn 90 (oil-hardened).
3. The physical properties of some of these materials are given in the following table. All values are for oil quenched condition and for single heat only.

Table 23.7. Physical properties of materials commonly used for leaf springs.

Material	*Condition*	*Ultimate tensile strength (MPa)*	*Tensile yield strength (MPa)*	*Brinell hardness number*
50 Cr 1	Hardened	1680 – 2200	1540 – 1750	461 – 601
50 Cr 1 V 23	and	1900 – 2200	1680 – 1890	534 – 601
55 Si 2 Mn 90	tempered	1820 – 2060	1680 – 1920	534 – 601

Note : For further details, Indian Standard [IS : 3431 – 1982 (Reaffirmed 1992)] on 'Specification for steel for the manufacture of volute, helical and laminated springs for automotive suspension' may be referred.

Example 23.23. *Design a leaf spring for the following specifications :*

Total load = 140 kN ; Number of springs supporting the load = 4 ; Maximum number of leaves = 10; Span of the spring = 1000 mm ; Permissible deflection = 80 mm.

Take Young's modulus, E = 200 kN/mm^2 and allowable stress in spring material as 600 MPa.

Solution. Given : Total load = 140 kN ; No. of springs = 4; n = 10 ; $2L$ = 1000 mm or L = 500 mm ; δ = 80 mm ; E = 200 kN/mm^2 = 200 × 10^3 N/mm^2 ; σ = 600 MPa = 600 N/mm^2

We know that load on each spring,

$$2W = \frac{\text{Total load}}{\text{No. of springs}} = \frac{140}{4} = 35 \text{ kN}$$

$$\therefore \quad W = 35 / 2 = 17.5 \text{ kN} = 17\,500 \text{ N}$$

Let t = Thickness of the leaves, and

b = Width of the leaves.

We know that bending stress (σ),

$$600 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 17\,500 \times 500}{n.b.t^2} = \frac{52.5 \times 10^6}{n.b.t^2}$$

$$\therefore \quad n.b.t^2 = 52.5 \times 10^6 / 600 = 87.5 \times 10^3 \qquad ...(i)$$

and deflection of the spring (δ),

$$80 = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{6 \times 17\,500\,(500)^3}{n \times 200 \times 10^3 \times b \times t^3} = \frac{65.6 \times 10^6}{n.b.t^3}$$

$$\therefore \quad n.b.t^3 = 65.6 \times 10^6 / 80 = 0.82 \times 10^6 \qquad ...(ii)$$

Dividing equation (*ii*) by equation (*i*), we have

$$\frac{n.b.t^3}{n.b.t^2} = \frac{0.82 \times 10^6}{87.5 \times 10^3} \quad \text{or} \quad t = 9.37 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

Now from equation (*i*), we have

$$b = \frac{87.5 \times 10^3}{n.t^2} = \frac{87.5 \times 10^3}{10\,(10)^2} = 87.5 \text{ mm}$$

and from equation (*ii*), we have

$$b = \frac{0.82 \times 10^6}{n.t^3} = \frac{0.82 \times 10^6}{10\,(10)^3} = 82 \text{ mm}$$

Taking larger of the two values, we have width of leaves,

$$b = 87.5 \text{ say } 90 \text{ mm } \textbf{Ans.}$$

Example 23.24. *A truck spring has 12 number of leaves, two of which are full length leaves. The spring supports are 1.05 m apart and the central band is 85 mm wide. The central load is to be 5.4 kN with a permissible stress of 280 MPa. Determine the thickness and width of the steel spring leaves. The ratio of the total depth to the width of the spring is 3. Also determine the deflection of the spring.*

Solution. Given : n = 12 ; n_F = 2 ; $2L_1$ = 1.05 m = 1050 mm ; l = 85 mm ; $2W$ = 5.4 kN = 5400 N or W = 2700 N ; σ_F = 280 MPa = 280 N/mm^2

Thickness and width of the spring leaves

Let t = Thickness of the leaves, and

b = Width of the leaves.

Since it is given that the ratio of the total depth of the spring ($n \times t$) and width of the spring (b) is 3, therefore

$$\frac{n \times t}{b} = 3 \quad \text{or} \quad b = n \times t / 3 = 12 \times t / 3 = 4\,t$$

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1050 - 85 = 965 \text{ mm}$$

$$\therefore \quad L = 965 / 2 = 482.5 \text{ mm}$$

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and number of graduated leaves,

$$n_G = n - n_F = 12 - 2 = 10$$

Assuming that the leaves are not initially stressed, therefore maximum stress or bending stress for full length leaves (σ_F),

$$280 = \frac{18\, W.L}{b.t^2\ (2n_G + 3n_F)} = \frac{18 \times 2700 \times 482.5}{4\, t \times t^2\, (2 \times 10 + 3 \times 2)} = \frac{225\,476}{t^3}$$

$\therefore$ $t^3 = 225\,476 / 280 = 805.3$ or $t = 9.3$ say 10 mm **Ans.**

and $b = 4\,t = 4 \times 10 = 40$ mm **Ans.**

Deflection of the spring

We know that deflection of the spring,

$$\delta = \frac{12\, W.L^3}{E.b.t^3\ (2n_G + 3n_F)}$$

$$= \frac{12 \times 2700 \times (482.5)^3}{210 \times 10^3 \times 40 \times 10^3\ (2 \times 10 + 3 \times 2)} \text{ mm}$$

$= 16.7$ mm **Ans.** ... (Taking $E = 210 \times 10^3$ N/mm^2)

Example 23.25. *A locomotive semi-elliptical laminated spring has an overall length of 1 m and sustains a load of 70 kN at its centre. The spring has 3 full length leaves and 15 graduated leaves with a central band of 100 mm width. All the leaves are to be stressed to 400 MPa, when fully loaded. The ratio of the total spring depth to that of width is 2. E = 210 kN/mm². Determine :*

1. The thickness and width of the leaves.

2. The initial gap that should be provided between the full length and graduated leaves before the band load is applied.

3. The load exerted on the band after the spring is assembled.

Solution. Given : $2L_1 = 1$ m $= 1000$ mm ; $2W = 70$ kN or $W = 35$ kN $= 35 \times 10^3$ N ; $n_F = 3$; $n_G = 15$; $l = 100$ mm ; $\sigma = 400$ MPa $= 400$ N/mm^2 ; $E = 210$ kN/mm$^2 = 210 \times 10^3$ N/mm^2

1. Thickness and width of leaves

Let t = Thickness of leaves, and

b = Width of leaves.

We know that the total number of leaves,

$$n = n_F + n_G = 3 + 15 = 18$$

Since it is given that ratio of the total spring depth ($n \times t$) and width of leaves is 2, therefore

$$\frac{n \times t}{b} = 2 \quad \text{or} \quad b = n \times t / 2 = 18 \times t / 2 = 9\,t$$

We know that the effective length of the leaves,

$$2L = 2L_1 - l = 1000 - 100 = 900 \text{ mm} \quad \text{or} \quad L = 900 / 2 = 450 \text{ mm}$$

Since all the leaves are equally stressed, therefore final stress (σ),

$$400 = \frac{6\, W.L}{n.b.t^2} = \frac{6 \times 35 \times 10^3 \times 450}{18 \times 9\, t \times t^2} = \frac{583 \times 10^3}{t^3}$$

$\therefore$ $t^3 = 583 \times 10^3 / 400 = 1458$ or $t = 11.34$ say 12 mm **Ans.**

and $b = 9\,t = 9 \times 12 = 108$ mm **Ans.**

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2. Initial gap

We know that the initial gap (C) that should be provided between the full length and graduated leaves before the band load is applied, is given by

$$C = \frac{2\,W.L^3}{n.E.b.t^3} = \frac{2 \times 35 \times 10^3\ (450)^3}{18 \times 210 \times 10^3 \times 108\ (12)^3} = 9.04 \text{ mm} \quad \textbf{Ans.}$$

3. Load exerted on the band after the spring is assembled

We know that the load exerted on the band after the spring is assembled,

$$W_b = \frac{2\,n_F.n_G.W}{n\,(2n_G + 3n_F)} = \frac{2 \times 3 \times 15 \times 35 \times 10^3}{18\ (2 \times 15 + 3 \times 3)} = 4487 \text{ N} \quad \textbf{Ans.}$$

Example 23.26. *A semi-elliptical laminated vehicle spring to carry a load of 6000 N is to consist of seven leaves 65 mm wide, two of the leaves extending the full length of the spring. The spring is to be 1.1 m in length and attached to the axle by two U-bolts 80 mm apart. The bolts hold the central portion of the spring so rigidly that they may be considered equivalent to a band having a width equal to the distance between the bolts. Assume a design stress for spring material as 350 MPa. Determine :*

1. Thickness of leaves, 2. Deflection of spring, 3. Diameter of eye, 4. Length of leaves, and 5. Radius to which leaves should be initially bent.

Sketch the semi-elliptical leaf-spring arrangement.

The standard thickness of leaves are : 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11 etc. in mm.

Solution. Given : $2W = 6000$ N or $W = 3000$ N ; $n = 7$; $b = 65$ mm ; $n_F = 2$; $2L_1 = 1.1$ m $= 1100$ mm or $L_1 = 550$ mm ; $l = 80$ mm ; $\sigma = 350$ MPa $= 350$ N/mm^2

1. Thickness of leaves

Let t = Thickness of leaves.

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1100 - 80 = 1020 \text{ mm}$$

$$\therefore \quad L = 1020 / 2 = 510 \text{ mm}$$

and number of graduated leaves,

$$n_G = n - n_F = 7 - 2 = 5$$

Assuming that the leaves are not initially stressed, the maximum stress (σ_F),

$$350 = \frac{18\,W.L}{b.t^2\ (2n_G + 3n_F)} = \frac{18 \times 3000 \times 510}{65 \times t^2\ (2 \times 5 + 3 \times 2)} = \frac{26\,480}{t^2} \quad \text{... } (\sigma_F = \sigma)$$

$$\therefore \quad t^2 = 26\,480 / 350 = 75.66 \quad \text{or} \quad t = 8.7 \text{ say } 9 \text{ mm} \quad \textbf{Ans.}$$

2. Deflection of spring

We know that deflection of spring,

$$\delta = \frac{12\,W.L^3}{E.b.t^3\ (2n_G + 3n_F)} = \frac{12 \times 3000\ (510)^3}{210 \times 10^3 \times 65 \times 9^3\ (2 \times 5 + 3 \times 2)}$$

$$= 30 \text{ mm} \quad \textbf{Ans.} \quad \text{... (Taking } E = 210 \times 10^3 \text{ N/mm}^2)$$

3. Diameter of eye

The inner diameter of eye is obtained by considering the pin in the eye in bearing, because the inner diameter of the eye is equal to the diameter of the pin.

Let d = Inner diameter of the eye or diameter of the pin,

l_1 = Length of the pin which is equal to the width of the eye or leaf (*i.e.* b) = 65 mm ...(Given)

p_b = Bearing pressure on the pin which may be taken as 8 N/mm^2.

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We know that the load on pin (W),

$$3000 = d \times l_1 \times p_b$$
$$= d \times 65 \times 8 = 520\, d$$
$$\therefore \quad d = 3000 / 520$$
$$= 5.77 \text{ say } 6 \text{ mm}$$

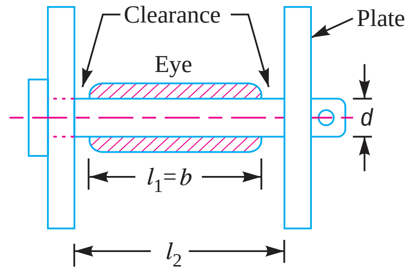

Fig. 23.33

Let us now consider the bending of the pin. Since there is a clearance of about 2 mm between the shackle (or plate) and eye as shown in Fig. 23.33, therefore length of the pin under bending,

$$l_2 = l_1 + 2 \times 2 = 65 + 4 = 69 \text{ mm}$$

Leaf spring.

Maximum bending moment on the pin,

$$M = \frac{W \times l_2}{4} = \frac{3000 \times 69}{4} = 51\,750 \text{ N-mm}$$

and section modulus, $\quad Z = \dfrac{\pi}{32} \times d^3 = 0.0982\, d^3$

We know that bending stress (σ_b),

$$80 = \frac{M}{Z} = \frac{51\,750}{0.0982\, d^3} = \frac{527 \times 10^3}{d^3} \quad \text{... (Taking } \sigma_b = 80 \text{ N/mm}^2\text{)}$$

$$\therefore \quad d^3 = 527 \times 10^3 / 80 = 6587 \quad \text{or} \quad d = 18.7 \text{ say } 20 \text{ mm}$$ **Ans.**

We shall take the inner diameter of eye or diameter of pin (d) as 20 mm **Ans.**

Let us now check the pin for induced shear stress. Since the pin is in double shear, therefore load on the pin (W),

$$3000 = 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (20)^2\ \tau = 628.4\ \tau$$

$$\therefore \quad \tau = 3000 / 628.4 = 4.77 \text{ N/mm}^2\text{, which is safe.}$$

4. Length of leaves

We know that ineffective length of the spring

$$= l = 80 \text{ mm} \quad \text{... } (\because U\text{-bolts are considered equivalent to a band})$$

$\therefore$ Length of the smallest leaf $= \dfrac{\text{Effective length}}{n-1} + \text{Ineffective length}$

$$= \frac{1020}{7-1} + 80 = 250 \text{ mm} \quad \textbf{Ans.}$$

Length of the 2nd leaf $= \dfrac{1020}{7-1} \times 2 + 80 = 420$ mm **Ans.**

Length of the 3rd leaf $= \dfrac{1020}{7-1} \times 3 + 80 = 590$ mm **Ans.**

Length of the 4th leaf $= \dfrac{1020}{7-1} \times 4 + 80 = 760$ mm **Ans.**

Length of the 5th leaf $= \dfrac{1020}{7-1} \times 5 + 80 = 930$ mm **Ans.**

Length of the 6th leaf $= \dfrac{1020}{7-1} \times 6 + 80 = 1100$ mm **Ans.**

The 6th and 7th leaves are full length leaves and the 7th leaf (*i.e.* the top leaf) will act as a master leaf.

We know that length of the master leaf

$$= 2L_1 + \pi\,(d + t)\,2 = 1100 + \pi\,(20 + 9)2 = 1282.2 \text{ mm} \quad \textbf{Ans.}$$

5. ***Radius to which the leaves should be initially bent***

Let R = Radius to which the leaves should be initially bent, and

y = Camber of the spring.

We know that

$$y\,(2R - y) = (L_1)^2$$

$$30(2R - 30) = (550)^2 \quad \text{or} \quad 2R - 30 = (550)^2/30 = 10\,083 \qquad \ldots (\because y = \delta)$$

$$\therefore \quad R = \frac{10\,083 + 30}{2} = 5056.5 \text{ mm} \quad \textbf{Ans.}$$

EXERCISES

1. Design a compression helical spring to carry a load of 500 N with a deflection of 25 mm. The spring index may be taken as 8. Assume the following values for the spring material:

   Permissible shear stress = 350 MPa

   Modulus of rigidity = 84 kN/mm^2

   Wahl's factor $= \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$, where C = spring index.

   [Ans. d = 5.893 mm ; D = 47.144 mm ; n = 6]

2. A helical valve spring is to be designed for an operating load range of approximately 90 to 135 N. The deflection of the spring for the load range is 7.5 mm. Assume a spring index of 10. Permissible shear stress for the material of the spring = 480 MPa and its modulus of rigidity = 80 kN/mm^2. Design the spring.

   Take Wahl's factor $= \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$, C being the spring index.

   [Ans. d = 2.74 mm ; D = 27.4 mm ; n = 6]

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3. Design a helical spring for a spring loaded safety valve for the following conditions :

Operating pressure = 1 N/mm^2

Maximum pressure when the valve blows off freely

= 1.075 N/mm^2

Maximum lift of the valve when the pressure is 1.075 N/mm^2

= 6 mm

Diameter of valve seat = 100 mm

Maximum shear stress = 400 MPa

Modulus of rigidity = 86 kN/mm^2

Spring index = 5.5 **[Ans. d = 17.2 mm ; D = 94.6 mm ; n = 12]**

4. A vertical spring loaded valve is required for a compressed air receiver. The valve is to start opening at a pressure of 1 N/mm^2 gauge and must be fully open with a lift of 4 mm at a pressure of 1.2 N/mm^2 gauge. The diameter of the port is 25 mm. Assume the allowable shear stress in steel as 480 MPa and shear modulus as 80 kN/mm^2.

Design a suitable close coiled round section helical spring having squared ground ends. Also specify initial compression and free length of the spring.

[Ans. d = 7 mm ; D = 42 mm ; n = 13]

5. A spring controlled lever is shown in Fig. 23.34. The spring is to be inserted with an initial compression to produce a force equal to 125 N between the right hand end of the lever and the stop. When the maximum force at A reaches to a value of 200 N, the end of the lever moves downward by 25 mm.

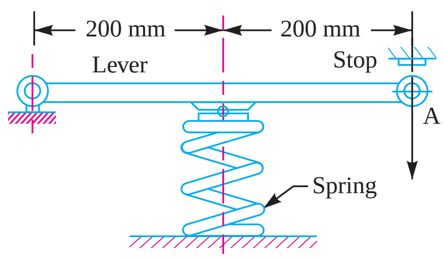

Fig. 23.34

Assuming a spring index as 8, determine: 1. spring rate, 2 size of wire, 3. outside diameter of the spring, 4. number of active coils, and 5. free length, assuming squared and ground ends.

The allowable shear stress may be taken as 420 MPa and G = 80 kN/mm^2.

[Ans. 0.33 N/mm ; 3.4 mm ; 27.2 mm; 10 ; 77 mm]

6. It is desired to design a valve spring of I.C. engine for the following details :

(*a*) Spring load when valve is closed = 80 N

(*b*) Spring load when valve is open = 100 N

(*c*) Space constraints for the fitment of spring are :

Inside guide bush diameter = 24 mm

Outside recess diameter = 36 mm

(*d*) Valve lift = 5 mm

(*e*) Spring steel has the following properties:

Maximum permissible shear stress = 350 MPa

Modulus of rigidity = 84 kN/mm^2

Find : 1. Wire diameter; 2. Spring index; 3. Total number of coils; 4. Solid length of spring; 5. Free

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length of spring; 6. Pitch of the coil when additional 15 percent of the working deflection is used to avoid complete closing of coils.

[Ans. 2.9 mm ; 10.345 ; 9 ; 26.1 mm ; 54.85 mm ; 7 mm]

7. A circular cam 200 mm in diameter rotates off centre with an eccentricity of 25 mm and operates the roller follower that is carried by the arm as shown in Fig. 23.35.

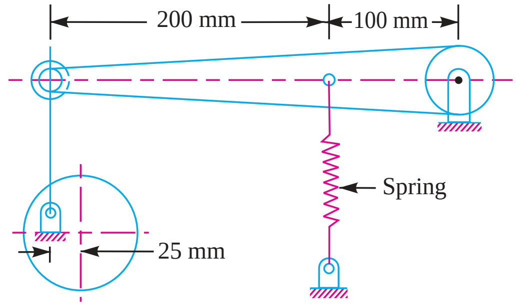

Fig. 23.35

The roller follower is held against the cam by means of an extension spring. Assuming that the force between the follower and the cam is approximately 250 N at the low position and 400 N at the high position.

If the spring index is 7, find the diameter of wire, outside diameter of spring and the number of active coils. The maximum shear stress may be taken as 280 MPa. Use G = 80 kN/mm^2.

8. The following data relate to a single plate friction clutch whose both sides are effective:

Power transmitted = 35 kW

Speed = 1000 r.p.m.

Permissible uniform pressure on lining material = 0.07 N/mm^2

$\frac{\text{Outer radius}}{\text{Inner radius}}$ of plate = 1.5

Coefficient of friction of lining material = 0.3

Number of springs = 6

Spring index = 6

Stress concentration factor in spring

$$= \frac{4C-1}{4C-4} + \frac{0.615}{C}, \quad C = \text{Spring index}$$

Permissible stress in spring steel = 420 MPa

Modulus of rigidity of spring steel = 84 kN/mm^2

Compression of spring to keep it engaged = 12.5 mm

Design and give a sketch of any one of the springs in the uncompressed position. The springs are of round section close coiled helical type and are situated at mean radius of the plate.

[Ans. d = 5.65 mm ; D = 33.9 mm ; n = 6]

9. A railway wagon weighing 50 kN and moving with a speed of 8 km per hour has to be stopped by four buffer springs in which the maximum compression allowed is 220 mm. Find the number of turns in each spring of mean diameter 150 mm. The diameter of spring wire is 25 mm. Take G = 84 kN/mm^2.

[Ans. 8 turns]

10. The bumper springs of a railway carriage are to be made of square section wire. The ratio of mean diameter of spring to the side of wire is nearly equal to 6. Two such springs are required to bring to rest a carriage weighing 20 kN moving with a velocity of 1.5 m/s with a maximum deflection of 200 mm. Design the spring if the allowable shear stress is not to exced 300 MPa and G = 84 kN/mm^2.

For square wire, $\tau = \dfrac{2.4\, W.D}{a^3}$; and $\delta = \dfrac{5.59\, W.D^3.n}{G.a^4}$

where a = Side of the wire section,

D = Mean diameter,

n = Number of coils, and

W = Load on each spring.

[Ans. a = 26.3 mm ; D = 157.8 mm ; n = 33]

11. A load of 2 kN is dropped axially on a close coiled helical spring, from a height of 250 mm. The spring has 20 effective turns, and it is made of 25 mm diameter wire. The spring index is 8. Find the maximum shear stress induced in the spring and the amount of compression produced. The modulus of rigidity for the material of the spring wire is 84 kN/mm^2.

[Ans. 287 MPa; 290 mm]

12. A helical compression spring made of oil tempered carbon steel, is subjected to a load which varies from 600 N to 1600 N. The spring index is 6 and the design factor of safety is 1.43. If the yield shear stress is 700 MPa and the endurance stress is 350 MPa, find the size of the spring wire and mean diameter of the spring coil. **[Ans. 10 mm ; 60 mm]**

13. A helical spring B is placed inside the coils of a second helical spring A, having the same number of coils and free length. The springs are made of the same material. The composite spring is compressed by an axial load of 2300 N which is shared between them. The mean diameters of the spring A and B are 100 mm and 70 mm respectively and wire diameters are 13 mm and 8 mm respectively. Find the load taken and the maximum stress in each spring.

[Ans. W_A = 1670 N ; W_B = 630 N ; σ_A = 230 MPa ; σ_B = 256 MPa]

14. Design a concentric spring for an air craft engine valve to exert a maximum force of 5000 N under a deflection of 40 mm. Both the springs have same free length, solid length and are subjected to equal maximum shear stress of 850 MPa. The spring index for both the springs is 6.

[Ans. d_1 = 8 mm ; d_2 = 6 mm ; n = 4]

15. The free end of a torsional spring deflects through 90° when subjected to a torque of 4 N-m. The spring index is 6. Determine the coil wire diameter and number of turns with the following data :

Modulus of rigidity = 80 GPa ; Modulus of elasticity = 200 GPa; Allowable stress = 500 MPa.

[Ans. 5 mm ; 26]

16. A flat spiral steel spring is to give a maximum torque of 1500 N-mm for a maximum stress of 1000 MPa. Find the thickness and length of the spring to give three complete turns of motion, when the stress decreases from 1000 to zero. The width of the spring strip is 12 mm. The Young's modulus for the material of the strip is 200 kN/mm^2. **[Ans. 1.225 mm ; 4.6 m]**

17. A semi-elliptical spring has ten leaves in all, with the two full length leaves extending 625 mm. It is 62.5 mm wide and 6.25 mm thick. Design a helical spring with mean diameter of coil 100 mm which will have approximately the same induced stress and deflection for any load. The Young's modulus for the material of the semi-elliptical spring may be taken as 200 kN/mm^2 and modulus of rigidity for the material of helical spring is 80 kN/mm^2.

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18. A carriage spring 800 mm long is required to carry a proof load of 5000 N at the centre. The spring is made of plates 80 mm wide and 7.5 mm thick. If the maximum permissible stress for the material of the plates is not to exceed 190 MPa, determine :

1. The number of plates required, 2. The deflection of the spring, and 3. The radius to which the plates must be initially bent.

The modulus of elasticity may be taken as 205 kN/mm^2. **[Ans. 6 ; 23 mm ; 3.5 m]**

19. A semi-elliptical laminated spring 900 mm long and 55 mm wide is held together at the centre by a band 50 mm wide. If the thickness of each leaf is 5 mm, find the number of leaves required to carry a load of 4500 N. Assume a maximum working stress of 490 MPa.

If the two of these leaves extend the full length of the spring, find the deflection of the spring. The Young's modulus for the spring material may be taken as 210 kN/mm^2.

[Ans. 9 ; 71.8 mm]

20. A semi-elliptical laminated spring is made of 50 mm wide and 3 mm thick plates. The length between the supports is 650 mm and the width of the band is 60 mm. The spring has two full length leaves and five graduated leaves. If the spring carries a central load of 1600 N, find :

1. Maximum stress in full length and graduated leaves for an initial condition of no stress in the leaves.
2. The maximum stress if the initial stress is provided to cause equal stress when loaded.
3. The deflection in parts (1) and (2). **[Ans. 590 MPa ; 390 MPa ; 450 MPa ; 54 mm]**

QUESTIONS

1. What is the function of a spring? In which type of spring the behaviour is non-linear?

2. Classify springs according to their shapes. Draw neat sketches indicating in each case whether stresses are induced by bending or by torsion.

3. Discuss the materials and practical applications for the various types of springs.

4. The extension springs are in considerably less use than the compression springs. Why?

5. Explain the following terms of the spring :

(*i*) Free length; (*ii*) Solid height;
(*iii*) Spring rate; (*iv*) Active and inactive coils;
(*v*) Spring index; and (*vi*) Stress factor.

6. Explain what you understand by A.M. Wahl's factor and state its importance in the design of helical springs?

7. Explain one method of avoiding the tendency of a compression spring to buckle.

8. A compression spring of spring constant K is cut into two springs having equal number of turns and the two springs are then used in parallel. What is the resulting spring constant of the combination? How does the load carrying capacity of the resulting combination compare with that of the original spring?

9. Prove that in a spring, using two concentric coil springs made of same material, having same length and compressed equally by an axial load, the loads shared by the two springs are directly proportional to the square of the diameters of the wires of the two springs.

10. What do you understand by full length and graduated leaves of a leaf spring? Write the expression for determining the stress and deflection in full length and graduated leaves.

11. What is nipping in a leaf spring? Discuss its role. List the materials commonly used for the manufacture of the leaf springs.

12. Explain the utility of the centre bolt, *U*-clamp, rebound clip and camber in a leaf spring.

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OBJECTIVE TYPE QUESTIONS

1. A spring used to absorb shocks and vibrations is
   (a) closely-coiled helical spring
   (b) open-coiled helical spring
   (c) conical spring
   (d) torsion spring
2. The spring mostly used in gramophones is
   (a) helical spring
   (b) conical spring
   (c) laminated spring
   (d) flat spiral spring
3. Which of the following spring is used in a mechanical wrist watch?
   (a) Helical compression spring
   (b) Spiral spring
   (c) Torsion spring
   (d) Bellevile spring
4. When a helical compression spring is subjected to an axial compressive load, the stress induced in the wire is
   (a) tensile stress
   (b) compressive stress
   (c) shear stress
   (d) bending stress
5. In a close coiled helical spring, the spring index is given by D/d where D and d are the mean coil diameter and wire diameter respectively. For considering the effect of curvature, the Wahl's stress factor K is given by
   (a) $\dfrac{4C-1}{4C+4}+\dfrac{0.615}{C}$
   (b) $\dfrac{4C-1}{4C-4}+\dfrac{0.615}{C}$
   (c) $\dfrac{4C+1}{4C-4}-\dfrac{0.615}{C}$
   (d) $\dfrac{4C+1}{4C+4}-\dfrac{0.615}{C}$
6. When helical compression spring is cut into halves, the stiffness of the resulting spring will be
   (a) same
   (b) double
   (c) one-half
   (d) one-fourth
7. Two close coiled helical springs with stiffness k_1 and k_2 respectively are conected in series. The stiffness of an equivalent spring is given by
   (a) $\dfrac{k_1 \cdot k_2}{k_1 + k_2}$
   (b) $\dfrac{k_1 - k_2}{k_1 + k_2}$
   (c) $\dfrac{k_1 + k_2}{k_1 \cdot k_2}$
   (d) $\dfrac{k_1 - k_2}{k_1 \cdot k_2}$
8. When two concentric coil springs made of the same material, having same length and compressed equally by an axial load, the load shared by the two springs will be .... to the square of the diameters of the wires of the two springs.
   (a) directly proportional
   (b) inversely proportional
   (c) equal to
9. A leaf spring in automobiles is used
   (a) to apply forces
   (b) to measure forces
   (c) to absorb shocks
   (d) to store strain energy
10. In leaf springs, the longest leaf is known as
   (a) lower leaf
   (b) master leaf
   (c) upper leaf
   (d) none of these

ANSWERS

1. (e)	2. (d)	3. (c)	4. (c)	5. (b)
6. (b)	7. (a)	8. (a)	9. (c)	10. (b)

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C H A P T E R

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Clutches

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6. Considerations in Designing a Friction Clutch.
7. Types of Friction Clutches.
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9. Design of a Disc or Plate Clutch.
10. Multiple Disc Clutch.
11. Cone Clutch.
12. Design of a Cone Clutch.
13. Centrifugal Clutch.
14. Design of a Centrifugal Clutch.

24.1 Introduction

A clutch is a machine member used to connect a driving shaft to a driven shaft so that the driven shaft may be started or stopped at will, without stopping the driving shaft. The use of a clutch is mostly found in automobiles. A little consideration will show that in order to change gears or to stop the vehicle, it is required that the driven shaft should stop, but the engine should continue to run. It is, therefore, necessary that the driven shaft should be disengaged from the driving shaft. The engagement and disengagement of the shafts is obtained by means of a clutch which is operated by a lever.

24.2 Types of Clutches

Following are the two main types of clutches commonly used in engineering practice :

1. Positive clutches, and **2.** Friction clutches.

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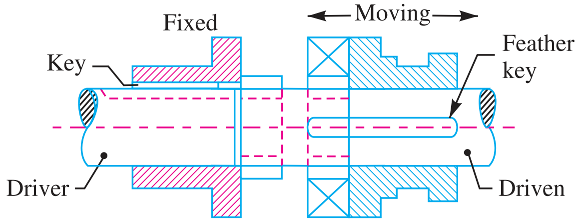
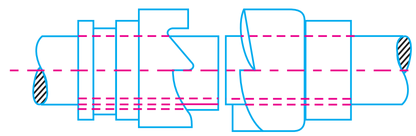
We shall now discuss these clutches in the following pages.

24.3 Positive Clutches

The positive clutches are used when a positive drive is required. The simplest type of a positive clutch is a ***jaw*** or ***claw clutch.*** The jaw clutch permits one shaft to drive another through a direct contact of interlocking jaws. It consists of two halves, one of which is permanently fastened to the



(*a*) Square jaw clutch.



(*b*) Spiral jaw clutch.

Fig. 24.1. Jaw clutches.

driving shaft by a sunk key. The other half of the clutch is movable and it is free to slide axially on the driven shaft, but it is prevented from turning relatively to its shaft by means of feather key. The jaws of the clutch may be of square type as shown in Fig. 24.1 (*a*) or of spiral type as shown in Fig. 24.1 (*b*).

A square jaw type is used where engagement and disengagement in motion and under load is not necessary. This type of clutch will transmit power in either direction of rotation. The spiral jaws may be left-hand or right-hand, because power transmitted by them is in one direction only. This type of clutch is occasionally used where the clutch must be engaged and disengaged while in motion. The use of jaw clutches are frequently applied to sprocket wheels, gears and pulleys. In such a case, the non-sliding part is made integral with the hub.

24.4 Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces. In automobiles, friction clutch is used to connect the engine to the drive shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually bring the driven shaft up to proper speed. The proper alignment of the bearing must be maintained and it should be located as close to the clutch as possible. It may be noted that :

1. The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.
2. The heat of friction should be rapidly *dissipated and tendency to grab should be at a minimum.
3. The surfaces should be backed by a material stiff enough to ensure a reasonably uniform distribution of pressure.

24.5 Material for Friction Surfaces

The material used for lining of friction surfaces of a clutch should have the following characteristics :

* During operation of a clutch, most of the work done against frictional forces opposing the motion is liberated as heat at the interface. It has been found that at the actual point of contact, the temperature as high as 1000°C is reached for a very short duration (*i.e.* for 0.0001 second). Due to this, the temperature of the contact surfaces will increase and may destroy the clutch.

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1. It should have a high and uniform coefficient of friction.
2. It should not be affected by moisture and oil.
3. It should have the ability to withstand high temperatures caused by slippage.
4. It should have high heat conductivity.
5. It should have high resistance to wear and scoring.

The materials commonly used for lining of friction surfaces and their important properties are shown in the following table.

Table 24.1. Properties of materials commonly used for lining of friction surfaces.

Material of friction surfaces	*Operating condition*	*Coefficient of friction*	*Maximum operating temperature (°C)*	*Maximum pressure (N/mm^2)*
Cast iron on cast iron or steel	dry	0.15 – 0.20	250 – 300	0.25– 0.4
Cast iron on cast iron or steel	In oil	0.06	250 – 300	0.6 – 0.8
Hardened steel on Hardened steel	In oil	0.08	250	0.8 – 0.8
Bronze on cast iron or steel	In oil	0.05	150	0.4
Pressed asbestos on cast iron or steel	dry	0.3	150 – 250	0.2 – 0.3
Powder metal on cast iron or steel	dry	0.4	550	0.3
Powder metal on cast iron or steel	In oil	0.1	550	0.8

24.6 Considerations in Designing a Friction Clutch

The following considerations must be kept in mind while designing a friction clutch.

1. The suitable material forming the contact surfaces should be selected.
2. The moving parts of the clutch should have low weight in order to minimise the inertia load, especially in high speed service.
3. The clutch should not require any external force to maintain contact of the friction surfaces.
4. The provision for taking up wear of the contact surfaces must be provided.
5. The clutch should have provision for facilitating repairs.
6. The clutch should have provision for carrying away the heat generated at the contact surfaces.
7. The projecting parts of the clutch should be covered by guard.

24.7 Types of Friction Clutches

Though there are many types of friction clutches, yet the following are important from the subject point of view :

1. Disc or plate clutches (single disc or multiple disc clutch),
2. Cone clutches, and
3. Centrifugal clutches.

We shall now discuss these clutches, in detail, in the following pages.

Note : The disc and cone clutches are known as ***axial friction clutches,*** while the centrifugal clutch is called ***radial friction clutch.***

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24.8 Single Disc or Plate Clutch

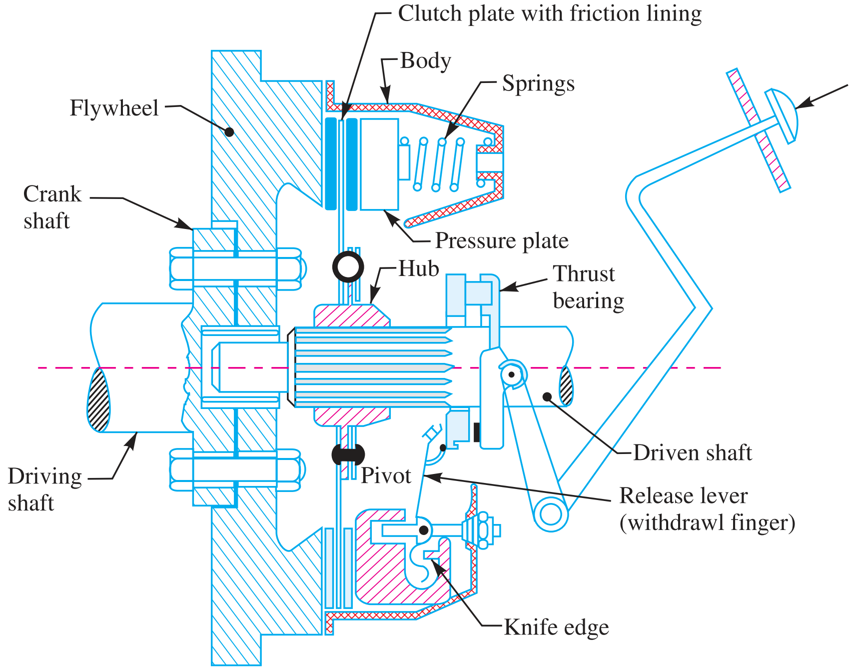

Fig. 24.2. Single disc or plate clutch.

A single disc or plate clutch, as shown in Fig 24.2, consists of a clutch plate whose both sides are faced with a frictional material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine crankshaft or the driving shaft. The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body. The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing. The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed.

When the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot and the pressure plate moves away from the flywheel by the knife edges, thereby compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel and the driven shaft becomes stationary. On the other hand, when the foot is taken off from the clutch pedal, the thrust bearing moves back by the levers. This allows the springs to extend and thus the pressure plate pushes the clutch plate back towards the flywheel.

When a car hits an object and decelerates quickly the objects are thrown forward as they continue to move forwards due to inertia.

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The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place. If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.

24.9 Design of a Disc or Plate Clutch

Consider two friction surfaces maintained in contact by an axial thrust (W) as shown in Fig. 24.3 (*a*).

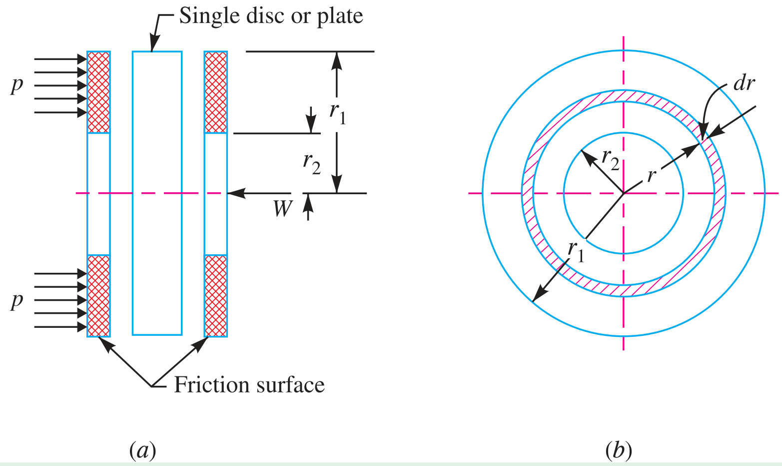

Fig. 24.3. Forces on a disc clutch.

Let T = Torque transmitted by the clutch,

p = Intensity of axial pressure with which the contact surfaces are held together,

r_1 and r_2 = External and internal radii of friction faces,

r = Mean radius of the friction face, and

μ = Coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in Fig. 24.3 (*b*).

We know that area of the contact surface or friction surface

$$= 2\pi\, r.dr$$

∴ Normal or axial force on the ring,

$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\pi\, r.dr$$

and the frictional force on the ring acting tangentially at radius r,

$$F_r = \mu \times \delta W = \mu.p \times 2\pi\, r.dr$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p \times 2\pi\, r.dr \times r = 2\,\pi\,\mu\, p.\, r^2.dr$$

We shall now consider the following two cases :

1. When there is a uniform pressure, and
2. When there is a uniform axial wear.

1. *Considering uniform pressure*. When the pressure is uniformly distributed over the entire area of the friction face as shown in Fig. 24.3 (*a*), then the intensity of pressure,

$$p = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]}$$

where W = Axial thrust with which the friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\pi\,\mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total friction torque.

∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\,\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\,\mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right] = 2\pi\,\mu \times \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

... (Substituting the value of p)

$$= \frac{2}{3}\,\mu.\,W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu\,.W.R$$

where $R = \dfrac{2}{3}\left[\dfrac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$ = Mean radius of the friction surface.

2. *Considering uniform axial wear.* The basic principle in designing machine parts that are subjected to wear due to sliding friction is that the normal wear is proportional to the work of friction. The work of friction is proportional to the product of normal pressure (p) and the sliding velocity (V). Therefore,

Normal wear ∝ Work of friction ∝ $p.V$

or $p.V = K$ (a constant) or $p = K/V$ ...(*i*)

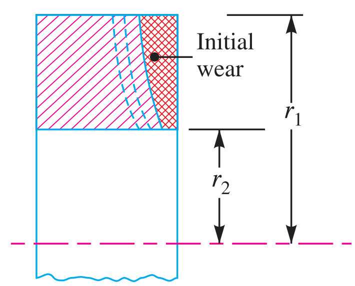

Fig. 24.4. Uniform axial wear.

It may be noted that when the friction surface is new, there is a uniform pressure distribution over the entire contact surface. This pressure will wear most rapidly where the sliding velocity is maximum and this will reduce the pressure between the friction surfaces. This wearing-in process continues until the product $p.V$ is constant over the entire surface. After this, the wear will be uniform as shown in Fig. 24.4.

Let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$p.r = C$ (a constant) or $p = C / r$ ...(*ii*)

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi\,r.dr = 2\pi C.dr$$

∴ Total force acing on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi\,C\,dr = 2\pi\,C\,[r]_{r_2}^{r_1} = 2\pi\,C\,(r_1 - r_2)$$

or $$C = \frac{W}{2\pi\,(r_1 - r_2)}$$

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We know that the frictional torque acting on the ring,

$$T_r = 2\pi\,\mu.p.r^2.dr = 2\pi\,\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr \qquad ...(\because p = C/r)$$

∴ Total frictional torque acting on the friction surface (or on the clutch),

$$T = \int_{r_2}^{r_1} 2\pi\,\mu\, C.r.dr = 2\pi\,\mu\, C\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu\,.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right] = \pi\,\mu.C\,[(r_1)^2 - (r_2)^2]$$

$$= \pi\mu \times \frac{W}{2\pi\,(r_1 - r_2)}\,[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W\,(r_1 + r_2) = \mu.W.R$$

where $R = \dfrac{r_1 + r_2}{2}$ = Mean radius of the friction surface.

Notes : 1. In general, total frictional torque acting on the friction surfaces (or on the clutch) is given by

$$T = n.\mu.W.R$$

where n = Number of pairs of friction (or contact) surfaces, and

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \text{... (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \qquad \text{... (For uniform wear)}$$

2. For a single disc or plate clutch, normally both sides of the disc are effective. Therefore a single disc clutch has two pairs of surfaces in contact (*i.e.* $n = 2$).

3. Since the intensity of pressure is maximum at the inner radius (r_2) of the friction or contact surface, therefore equation ***(ii)*** may be written as

$$p_{max} \times r_2 = C \qquad \text{or} \qquad p_{max} = C / r_2$$

4. Since the intensity of pressure is minimum at the outer radius (r_1) of the friction or contact surface, therefore equation ***(ii)*** may be written as

$$p_{min} \times r_1 = C \qquad \text{or} \qquad p_{min} = C / r_1$$

5. The average pressure (p_{av}) on the friction or contact surface is given by

$$p_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]}$$

6. In case of a new clutch, the intensity of pressure is approximately uniform, but in an old clutch, the uniform wear theory is more approximate.

7. The uniform pressure theory gives a higher friction torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

A twin disk clutch

24.10 Multiple Disc Clutch

A multiple disc clutch, as shown in Fig. 24.5, may be used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion (except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.

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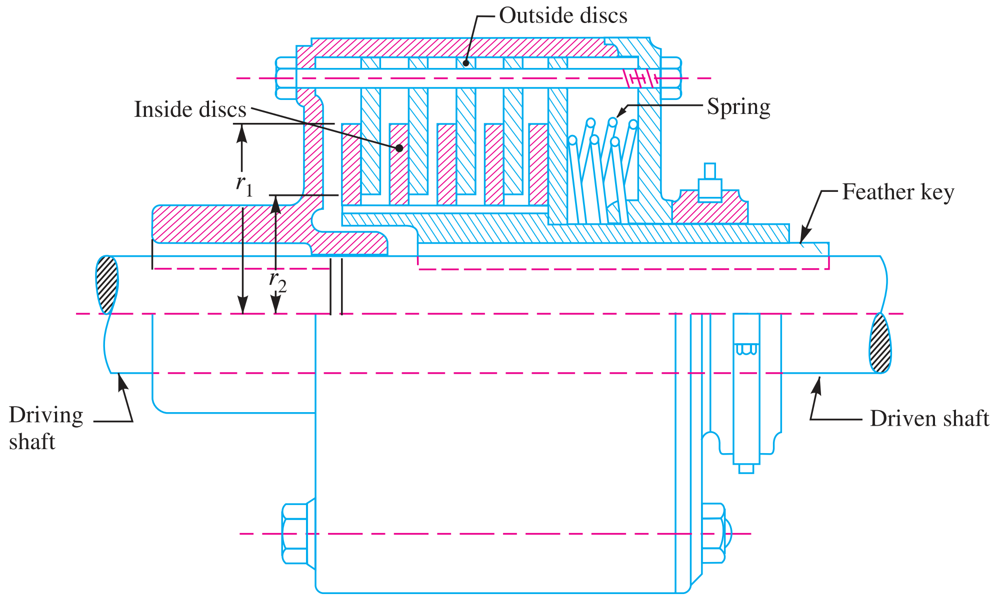

Fig. 24.5. Multiple disc clutch.

Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

∴ Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R$$

where R = Mean radius of friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{... (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{... (For uniform wear)}$$

Example 24.1. *Determine the maximum, minimum and average pressure in a plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.*

Solution. Given : W = 4 kN = 4000 N ; r_2 = 50 mm ; r_1 = 100 mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\, p_{max}$$

We also know that total force on the contact surface (W),

$$4000 = 2\pi C\,(r_1 - r_2) = 2\pi \times 50\, p_{max}\,(100 - 50) = 15\,710\, p_{max}$$

∴ p_{max} = 4000 / 15 710 = 0.2546 N/mm^2 **Ans.**

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), therefore,

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\, p_{min}$$

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We know that the total force on the contact surface (W),

$$4000 = 2\pi C\,(r_1 - r_2) = 2\pi \times 100\, p_{min}\,(100 - 50) = 31\,420\, p_{min}$$

$$\therefore \quad p_{min} = 4000 / 31\,420 = 0.1273 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surface}} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

$$= \frac{4000}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Example 24.2. *A plate clutch having a single driving plate with contact surfaces on each side is required to transmit 110 kW at 1250 r.p.m. The outer diameter of the contact surfaces is to be 300 mm. The coefficient of friction is 0.4.*

(a) Assuming a uniform pressure of 0.17 N/mm²; determine the inner diameter of the friction surfaces.

(b) Assuming the same dimensions and the same total axial thrust, determine the maximum torque that can be transmitted and the maximum intensity of pressure when uniform wear conditions have been reached.

Solution. Given : $P = 110$ kW $= 110 \times 10^3$ W ; $N = 1250$ r.p.m. ; $d_1 = 300$ mm or $r_1 = 150$ mm ; $\mu = 0.4$; $p = 0.17$ N/mm²

(a) Inner diameter of the friction surfaces

Let d_2 = Inner diameter of the contact or friction surfaces, and

r_2 = Inner radius of the contact or friction surfaces.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{110 \times 10^3 \times 60}{2\,\pi \times 1250} = 840 \text{ N-m}$$

$$= 840 \times 10^3 \text{ N-mm}$$

Axial thrust with which the contact surfaces are held together,

$$W = \text{Pressure} \times \text{Area} = p \times \pi\,[(r_1)^2 - (r_2)^2]$$

$$= 0.17 \times \pi\,[(150)^2 - (r_2)^2] = 0.534\,[(150)^2 - (r_2)^2] \qquad \ldots(i)$$

and mean radius of the contact surface for uniform pressure conditions,

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\left[\frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2}\right]$$

$\therefore$ Torque transmitted by the clutch (T),

$$840 \times 10^3 = n.\mu.W.R$$

$$= 2 \times 0.4 \times 0.534\,[(150)^2 - (r_2)^2] \times \frac{2}{3}\left[\frac{(150)^3 - (r_2)^3}{(150)^2 - (r_2)^2}\right] \qquad \ldots(\because n = 2)$$

$$= 0.285\,[(150)^3 - (r_2)^3]$$

or $\quad (150)^3 - (r_2)^3 = 840 \times 10^3 / 0.285 = 2.95 \times 10^6$

$\therefore \quad (r_2)^3 = (150)^3 - 2.95 \times 10^6 = 0.425 \times 10^6$ or $r_2 = 75$ mm

and $\quad d_2 = 2r_2 = 2 \times 75 = 150$ mm **Ans.**

(b) Maximum torque transmitted

We know that the axial thrust,

$$W = 0.534\,[(150)^2 - (r_2)^2] \qquad \text{... [From equation } (i)]$$

$$= 0.534\,[(150)^2 - (75)^2] = 9011 \text{ N}$$

and mean radius of the contact surfaces for uniform wear conditions,

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 75}{2} = 112.5 \text{ mm}$$

∴ Maximum torque transmitted,

$$T = n.\mu.W.R = 2 \times 0.4 \times 9011 \times 112.5 = 811 \times 10^3 \text{ N-mm}$$

$$= 811 \text{ N-m} \quad \textbf{Ans.}$$

Maximum intensity of pressure

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \qquad \text{or} \qquad C = p_{max} \times 75 \text{ N/mm}$$

We know that the axial thrust (W),

$$9011 = 2\,\pi\,C\,(r_1 - r_2) = 2\pi \times p_{max} \times 75\,(150 - 75) = 35\,347\,p_{max}$$

$$\therefore \quad p_{max} = 9011 / 35\,347 = 0.255 \text{ N/mm}^2 \qquad \textbf{Ans.}$$

Example 24.3. *A single plate clutch, effective on both sides, is required to transmit 25 kW at 3000 r.p.m. Determine the outer and inner diameters of frictional surface if the coefficient of friction is 0.255, ratio of diameters is 1.25 and the maximum pressure is not to exceed 0.1 N/mm². Also, determine the axial thrust to be provided by springs. Assume the theory of uniform wear.*

Solution. Given : $n = 2$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 3000$ r.p.m. ; $\mu = 0.255$; $d_1 / d_2 = 1.25$ or $r_1 / r_2 = 1.25$; $p_{max} = 0.1$ N/mm²

Outer and inner diameters of frictional surface

Let d_1 and d_2 = Outer and inner diameters (in mm) of frictional surface, and

r_1 and r_2 = Corresponding radii (in mm) of frictional surface.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{25 \times 10^3 \times 60}{2\,\pi \times 3000} = 79.6 \text{ N-m} = 79\,600 \text{ N-mm}$$

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore.

$$p_{max} \times r_2 = C$$

or

$$C = 0.1\,r_2 \text{ N/mm}$$

and normal or axial load acting on the friction surface,

$$W = 2\pi\,C\,(r_1 - r_2) = 2\pi \times 0.1\,r_2\,(1.25\,r_2 - r_2)$$

$$= 0.157\,(r_2)^2 \qquad \text{... } (\because r_1 / r_2 = 1.25)$$

We know that mean radius of the frictional surface (for uniform wear),

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\,r_2 + r_2}{2} = 1.125\,r_2$$

and the torque transmitted (T),

$$79\,600 = n.\mu.W.R = 2 \times 0.255 \times 0.157\,(r_2)^2\,1.125\,r_2 = 0.09\,(r_2)^3$$

$$\therefore \quad (r_2)^3 = 79.6 \times 10^3 / 0.09 = 884 \times 10^3 \quad \text{or} \quad r_2 = 96 \text{ mm}$$

and

$$r_1 = 1.25\,r_2 = 1.25 \times 96 = 120 \text{ mm}$$

∴ Outer diameter of frictional surface,

$$d_1 = 2r_1 = 2 \times 120 = 240 \text{ mm} \quad \textbf{Ans.}$$

and inner diameter of frictional surface,

$$d_2 = 2r_2 = 2 \times 96 = 192 \text{ mm} \quad \textbf{Ans.}$$

Axial thrust to be provided by springs

We know that axial thrust to be provided by springs,

$$W = 2\pi\, C\,(r_1 - r_2) = 2\pi \times 0.1\, r_2\,(1.25\, r_2 - r_2)$$
$$= 0.157\,(r_2)^2 = 0.157\,(96)^2 = 1447 \text{ N} \quad \textbf{Ans.}$$

Example 24.4. *A dry single plate clutch is to be designed for an automotive vehicle whose engine is rated to give 100 kW at 2400 r.p.m. and maximum torque 500 N-m. The outer radius of the friction plate is 25% more than the inner radius. The intensity of pressure between the plate is not to exceed 0.07 N/mm². The coefficient of friction may be assumed equal to 0.3. The helical springs required by this clutch to provide axial force necessary to engage the clutch are eight. If each spring has stiffness equal to 40 N/mm, determine the dimensions of the friction plate and initial compression in the springs.*

Solution. Given : P = 100 kW = 100×10^3 W ; *N = 2400 r.p.m. ; T = 500 N-m = 500×10^3 N-mm ; p = 0.07 N/mm² ; μ = 0.3 ; No. of springs = 8 ; Stiffness/spring = 40 N/mm

Dimensions of the friction plate

Let r_1 = Outer radius of the friction plate, and

r_2 = Inner radius of the friction plate.

Since the outer radius of the friction plate is 25% more than the inner radius, therefore

$$r_1 = 1.25\, r_2$$

For uniform wear conditions, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p.r_2 = C \quad \text{or} \quad C = 0.07\, r_2 \text{ N/mm}$$

and axial load acting on the friction plate,

$$W = 2\pi\, C\,(r_1 - r_2) = 2\pi \times 0.07\, r_2\,(1.25\, r_2 - r_2) = 0.11\,(r_2)^2 \text{ N} \quad ...(i)$$

We know that mean radius of the friction plate, for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25 r_2 + r_2}{2} = 1.125\, r_2$$

∴ Torque transmitted (T),

$$500 \times 10^3 = n.\mu.W.R = 2 \times 0.3 \times 0.11\,(r_2)^2\, 1.125\, r_2 = 0.074\,(r_2)^3 \quad ...(\because n = 2)$$

$$(r_2)^3 = 500 \times 10^3 / 0.074 = 6757 \times 10^3 \quad \text{or} \quad r_2 = 190 \text{ mm} \quad \textbf{Ans.}$$

and $$r_1 = 1.25\, r_2 = 1.25 \times 190 = 237.5 \text{ mm} \quad \textbf{Ans.}$$

Initial compression in the springs

We know that total stiffness of the springs,

$$s = \text{Stiffness per spring} \times \text{No. of springs} = 40 \times 8 = 320 \text{ N/mm}$$

Axial force required to engage the clutch,

$$W = 0.11\,(r_2)^2 = 0.11\,(190)^2 = 3970 \text{ N} \quad \text{... [From equation } (i)]$$

∴ Initial compression in the springs

$$= W / s = 3970 / 320 = 12.4 \text{ mm} \quad \textbf{Ans.}$$

* Superfluous data

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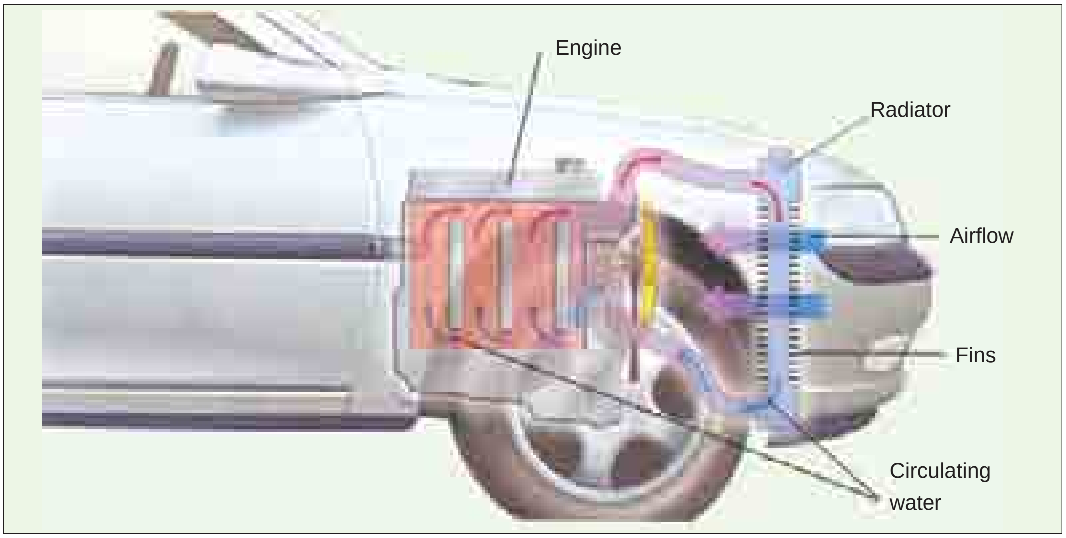

In car cooling system a pump circulates water through the engine and through the pipes of the radiator.

Example 24.5. *A single dry plate clutch is to be designed to transmit 7.5 kW at 900 r.p.m. Find :*

1. *Diameter of the shaft,*
2. *Mean radius and face width of the friction lining assuming the ratio of the mean radius to the face width as 4,*
3. *Outer and inner radii of the clutch plate, and*
4. *Dimensions of the spring, assuming that the number of springs are 6 and spring index = 6. The allowable shear stress for the spring wire may be taken as 420 MPa.*

Solution. Given : P = 7.5 kW = 7500 W ; N = 900 r.p.m. ; r/b = 4 ; No. of springs = 6 ; $C = D/d = 6$; τ = 420 MPa = 420 N/mm^2

1. *Diameter of the shaft*

Let d_s = Diameter of the shaft, and

τ_1 = Shear stress for the shaft material. It may be assumed as 40 N/mm^2.

We know that the torque transmitted,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{7500 \times 60}{2\,\pi \times 900} = 79.6 \text{ N-m} = 79\ 600 \text{ N-mm} \qquad ...(i)$$

We also know that the torque transmitted (T),

$$79\ 600 = \frac{\pi}{16} \times \tau_1\,(d_s)^3 = \frac{\pi}{16} \times 40\,(d_s)^3 = 7.855\,(d_s)^3$$

$$\therefore \quad (d_s)^3 = 79\ 600 / 7.855 = 10\ 134 \quad \text{or} \quad d_s = 21.6 \text{ say } 25 \text{ mm} \quad \textbf{Ans.}$$

2. *Mean radius and face width of the friction lining*

Let R = Mean radius of the friction lining, and

b = Face width of the friction lining = $R/4$ ... (Given)

We know that the area of the friction faces,

$$A = 2\pi\,R.b$$

$\therefore$ Normal or the axial force acting on the friction faces,

$$W = A \times p = 2\pi\,R.b.p$$

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and torque transmitted, $T = \mu\ W.R.n = \mu\ (2\pi\ Rb.p)\ R.n$

$$= \mu \left(2\,\pi R \times \frac{R}{4} \times p \right) R.n = \frac{\pi}{2} \times \mu.R^3.p.n \qquad \text{...}(ii)$$

Assuming the intensity of pressure (p) as 0.07 N/mm^2 and coefficient of friction (μ) as 0.25, we have from equations **(*i*)** and **(*ii*)**,

$$79\ 600 = \frac{\pi}{2} \times 0.25 \times R^3 \times 0.07 \times 2 = 0.055\ R^3$$

... ($\because$ n = 2, for both sides of plate effective)

$\therefore$ $R^3 = 79\ 600 / 0.055 = 1.45 \times 10^6$ or $R = 113.2$ say 114 mm **Ans.**

and $b = R / 4 = 114 / 4 = 28.5$ mm **Ans.**

3. Outer and inner radii of the clutch plate

Let r_1 and r_2 = Outer and inner radii of the clutch plate respectively.

Since the face width (or radial width) of the plate is equal to the difference of the outer and inner radii, therefore,

$$b = r_1 - r_2 \quad \text{or} \quad r_1 - r_2 = 28.5 \text{ mm} \qquad \text{...}(iii)$$

We know that for uniform wear, mean radius of the clutch plate,

$$R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R = 2 \times 114 = 228 \text{ mm} \qquad \text{...}(iv)$$

From equations **(*iii*)**, and **(*iv*)**, we find that

$r_1 = 128.25$ mm and $r_2 = 99.75$ mm **Ans.**

4. Dimensions of the spring

Let D = Mean diameter of the spring, and

d = Diameter of the spring wire.

We know that the axial force on the friction faces,

$$W = 2\pi\ R.b.p = 2\pi \times 114 \times 28.5 \times 0.07 = 1429.2 \text{ N}$$

In order to allow for adjustment and for maximum engine torque, the spring is designed for an overload of 25%.

$\therefore$ Total load on the springs

$$= 1.25\ W = 1.25 \times 1429.2 = 1786.5 \text{ N}$$

Since there are 6 springs, therefore maximum load on each spring,

$$W_s = 1786.5 / 6 = 297.75 \text{ N}$$

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

We also know that maximum shear stress induced in the wire (τ),

$$420 = K \times \frac{8\ W_s\ C}{\pi\ d^2} = 1.2525 \times \frac{8 \times 297.75 \times 6}{\pi\ d^2} = \frac{5697}{d^2}$$

$\therefore$ $d^2 = 5697 / 420 = 13.56$ or $d = 3.68$ mm

We shall take a standard wire of size *SWG* 8 having diameter (d) = 4.064 mm **Ans.**

and mean diameter of the spring,

$D = C.d = 6 \times 4.064 = 24.384$ say 24.4 mm **Ans.**

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Let us assume that the spring has 4 active turns (*i.e.* $n = 4$). Therefore compression of the spring,

$$\delta = \frac{8\, W_s . C^3 . n}{G.d} = \frac{8 \times 297.75 \times 6^3 \times 4}{84 \times 10^3 \times 4.064} = 6.03 \text{ mm}$$

... (Taking $G = 84 \times 10^3$ N/mm^2)

Assuming squared and ground ends, total number of turns,

$$n' = n + 2 = 4 + 2 = 6$$

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta$$

$$= 6 \times 4.064 + 6.03 + 0.15 \times 6.03 = 31.32 \text{ mm}$$ **Ans.**

and pitch of the coils $= \dfrac{L_F}{n' - 1} = \dfrac{31.32}{6 - 1} = 6.264$ mm **Ans.**

Example 24.6. *Design a single plate automobile clutch to transmit a maximum torque of 250 N-m at 2000 r.p.m. The outside diameter of the clutch is 250 mm and the clutch is engaged at 55 km/h. Find : 1. the number of revolutions of the clutch slip during engagement; and 2. heat to be dissipated by the clutch for each engagement.*

The following additional data is available:

Engine torque during engagement = 100 N-m; Mass of the automobile = 1500 kg; Diameter of the automobile wheel = 0.7 m; Moment of inertia of combined engine rotating parts, flywheel and input side of the clutch = 1 kg-m^2; Gear reduction ratio at differential = 5; Torque at rear wheels available for accelerating automobile = 175 N-m; Coefficient of friction for the clutch material = 0.3; Permissible pressure = 0.13 N/mm^2.

Solution. Given : T = 250 N-m = 250 × 10^3 N-mm ; N = 2000 r.p.m. ; d_1 = 250 mm or r_1 = 125 mm ; V = 55 km/h = 15.3 m/s ; T_e = 100 N-m ; m = 1500 kg ; D_w = 0.7 m or R_w = 0.35 m ; I = 1 kg-m^2 ; T_a = 175 N-m ; Gear ratio = 5 ; μ = 0.3 ; p = 0.13 N/mm^2

1. Number of revolutions of the clutch slip during engagement

First of all, let us find the inside radius of the clutch (r_2). We know that, for uniform wear, mean radius of the clutch,

$$R = \frac{r_1 + r_2}{2} = \frac{125 + r_2}{2} = 62.5 + 0.5\, r_2$$

and axial force on the clutch,

$$W = p.\pi\, [(r_1)^2 - (r_2)^2] = 0.13 \times \pi\, [(125)^2 - (r_2)^2]$$

We know that the torque transmitted (T),

$$250 \times 10^3 = n.\mu.W.R = 2 \times 0.3 \times 0.13\, \pi\, [(125)^2 - (r_2)^2]\, [62.5 + 0.5\, r_2]$$

$$= 0.245\, [\, 976.56 \times 10^3 + 7812.5\, r_2 - 62.5\, (r_2)^2 - 0.5\, (r_2)^3]$$

Solving by hit and trial, we find that

$$r_2 = 70 \text{ mm}$$

We know that angular velocity of the engine,

$$\omega_e = 2\pi N / 60 = 2\pi \times 2000 / 60 = 210 \text{ rad / s}$$

and angular velocity of the wheel,

$$\omega_W = \frac{\text{Velocity of wheel}}{\text{Radius of wheel}} = \frac{V}{R_w} = \frac{15.3}{0.35} = 43.7 \text{ rad/s}$$

Since the gear ratio is 5, therefore angular velocity of the clutch follower shaft,

$$\omega_0 = \omega_W \times 5 = 43.7 \times 5 = 218.5 \text{ rad / s}$$

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We know that angular acceleration of the engine during the clutch slip period of the clutch,

$$\alpha_e = \frac{T_e - T}{I} = \frac{100 - 250}{1} = -150 \text{ rad/s}^2$$

Let a = Linear acceleration of the automobile.

We know that accelerating force on the automobile,

$$F_a = \frac{T_a}{R} = \frac{175}{0.35} = 500 \text{ N}$$

We also know that accelerating force (F_a),

$$500 = m.a = 1500 \times a \quad \text{or} \quad a = 500 / 1500 = 0.33 \text{ m/s}^2$$

∴ Angular acceleration of the clutch output,

$$\alpha_0 = \frac{\text{Acceleration} \times \text{Gear ratio}}{\text{Radius of wheel}} = \frac{0.33 \times 5}{0.35} = 4.7 \text{ rad/s}^2$$

We know that clutch slip period,

$$\Delta t = \frac{\omega_0 - \omega_e}{\alpha_0 - \alpha_e} = \frac{218.5 - 210}{4.7 - (-150)} = 0.055 \text{ s}$$

Angle through which the input side of the clutch rotates during engagement time (Δt) is

$$\theta_e = \omega_e \times \Delta t + \frac{1}{2} \alpha_e (\Delta t)^2$$

$$= 210 \times 0.055 + \frac{1}{2} (-150)(0.055)^2 = 11.32 \text{ rad}$$

and angle through which the output side of the clutch rotates during engagement time (Δt) is

$$\theta_0 = \omega_0 \times \Delta t + \frac{1}{2} \alpha_0 (\Delta t)^2$$

$$= 218.5 \times 0.055 + \frac{1}{2} \times 4.7 (0.055)^2 = 12 \text{ rad}$$

∴ Angle of clutch slip,

$$\theta = \theta_0 - \theta_e = 12 - 11.32 = 0.68 \text{ rad}$$

We know that number of revolutions of the clutch slip during engagement

$$= \frac{\theta}{2\pi} = \frac{0.68}{2\pi} = 0.11 \text{ revolutions}$$ **Ans.**

Heat to be dissipated by the clutch for each engagement

We know that heat to be dissipated by the clutch for each engagement

$$= T.\theta = 250 \times 0.68 = 170 \text{ J}$$ **Ans.**

Example 24.7. *A multiple disc clutch has five plates having four pairs of active friction surfaces. If the intensity of pressure is not to exceed 0.127 N/mm², find the power transmitted at 500 r.p.m. The outer and inner radii of friction surfaces are 125 mm and 75 mm respectively. Assume uniform wear and take coefficient of friction = 0.3.*

A twin-disk clutch

Solution. Given : $n_1 + n_2 = 5$; $n = 4$; $p = 0.127$ N/mm² ; $N = 500$ r.p.m. ; $r_1 = 125$ mm ; $r_2 = 75$ mm ; $\mu = 0.3$

We know that for uniform wear, $p.r = C$ (a constant). Since the intensity of pressure is maximum at the inner radius (r_2), therefore,

$$p.r_2 = C \quad \text{or} \quad C = 0.127 \times 75 = 9.525 \text{ N/mm}$$

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and axial force required to engage the clutch,

$$W = 2\pi C\,(r_1 - r_2) = 2\pi \times 9.525\,(125 - 75) = 2993 \text{ N}$$

Mean radius of the friction surfaces,

$$R = \frac{r_1 + r_2}{2} = \frac{125 + 75}{2} = 100 \text{ mm} = 0.1 \text{ m}$$

We know that the torque transmitted,

$$T = n.\mu.W.R = 4 \times 0.3 \times 2993 \times 0.1 = 359 \text{ N-m}$$

$\therefore$ Power transmitted, $$P = \frac{T \times 2\,\pi\,N}{60} = \frac{359 \times 2\,\pi \times 500}{60} = 18\,800 \text{ W} = 18.8 \text{ kW}$$ **Ans.**

Example 24.8. *A multi-disc clutch has three discs on the driving shaft and two on the driven shaft. The inside diameter of the contact surface is 120 mm. The maximum pressure between the surface is limited to 0.1 N/mm². Design the clutch for transmitting 25 kW at 1575 r.p.m. Assume uniform wear condition and coefficient of friction as 0.3.*

Solution. Given : $n_1 = 3$; $n_2 = 2$; $d_2 = 120$ mm or $r_2 = 60$ mm ; $p_{max} = 0.1$ N/mm² ; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m. ; $\mu = 0.3$

Let r_1 = Outside radius of the contact surface.

We know that the torque transmitted,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{25 \times 10^3 \times 60}{2\,\pi \times 1575} = 151.6 \text{ N-m} = 151\,600 \text{ N-mm}$$

For uniform wear, we know that $p.r = C$. Since the intensity of pressure is maximum at the inner radius (r_2), therefore,

$$p_{max} \times r_2 = C \qquad \text{or} \qquad C = 0.1 \times 60 = 6 \text{ N/mm}$$

We know that the axial force on each friction surface,

$$W = 2\pi C\,(r_1 - r_2) = 2\pi \times 6(r_1 - 60) = 37.7\,(r_1 - 60) \qquad \text{...}(i)$$

For uniform wear, mean radius of the contact surface,

$$R = \frac{r_1 + r_2}{2} = \frac{r_1 + 60}{2} = 0.5\,r_1 + 30$$

We know that number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1 = 3 + 2 - 1 = 4$$

$\therefore$ Torque transmitted (T),

$$151\,600 = n.\mu.W.R = 4 \times 0.3 \times 37.7\,(r_1 - 60)\,(0.5\,r_1 + 30)$$

... [Substituting the value of W from equation (*i*)]

$$= 22.62\,(r_1)^2 - 81\,432$$

$$\therefore \qquad (r_1)^2 = \frac{151\,600 + 81\,432}{22.62} = 10\,302$$

or $r_1 = 101.5$ mm **Ans.**

Example 24.9. *A multiple disc clutch, steel on bronze, is to transmit 4.5 kW at 750 r.p.m. The inner radius of the contact is 40 mm and outer radius of the contact is 70 mm. The clutch operates in oil with an expected coefficient of 0.1. The average allowable pressure is 0.35 N/mm². Find : 1. the total number of steel and bronze discs; 2. the actual axial force required; 3. the actual average pressure; and 4. the actual maximum pressure.*

Solution. Given : $P = 4.5$ kW $= 4500$ W ; $N = 750$ r.p.m. ; $r_2 = 40$ mm ; $r_1 = 70$ mm ; $\mu = 0.1$; $p_{av} = 0.35$ N/mm²

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1. *Total number of steel and bronze discs*

Let n = Number of pairs of contact surfaces.

We know that the torque transmitted by the clutch,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{4500 \times 60}{2\,\pi \times 750} = 57.3 \text{ N-m} = 57\,300 \text{ N-mm}$$

For uniform wear, mean radius of the contact surfaces,

$$R = \frac{r_1 + r_2}{2} = \frac{70 + 40}{2} = 55 \text{ mm}$$

and average axial force required,

$$W = p_{av} \times \pi\,[(r_1)^2 - (r_2)^2] = 0.35 \times \pi\,[(70)^2 - (40)^2] = 3630 \text{ N}$$

We also know that the torque transmitted (T),

$$57\,300 = n.\mu.W.R = n \times 0.1 \times 3630 \times 55 = 19\,965\, n$$

$$\therefore \quad n = 57\,300 / 19\,965 = 2.87$$

Since the number of pairs of contact surfaces must be even, therefore we shall use 4 pairs of contact surfaces with 3 steel discs and 2 bronze discs (because the number of pairs of contact surfaces is one less than the total number of discs). **Ans.**

2. *Actual axial force required*

Let W' = Actual axial force required.

Since the actual number of pairs of contact surfaces is 4, therefore actual torque developed by the clutch for one pair of contact surface,

$$T' = \frac{T}{n} = \frac{57\,300}{4} = 14\,325 \text{ N-mm}$$

We know that torque developed for one pair of contact surface (T'),

$$14\,325 = \mu.W'.R = 0.1 \times W' \times 55 = 5.5\,W'$$

$$\therefore \quad W' = 14\,325 / 5.5 = 2604.5 \text{ N} \quad \textbf{Ans.}$$

3. *Actual average pressure*

We know that the actual average pressure,

$$p'_{av} = \frac{W'}{\pi[(r_1)^2 - (r_2)^2]} = \frac{2604.5}{\pi\,[(70)^2 - (40)^2]} = 0.25 \text{ N/mm}^2 \quad \textbf{Ans.}$$

4. *Actual maximum pressure*

Let p_{max} = Actual maximum pressure.

For uniform wear, $p.r = C$. Since the intensity of pressure is maximum at the inner radius, therefore,

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 40\,p_{max} \text{ N/mm}$$

We know that the actual axial force (W'),

$$2604.5 = 2\pi C\,(r_1 - r_2) = 2\pi \times 40\,p_{max}\,(70 - 40) = 7541\,p_{max}$$

$$\therefore \quad p_{max} = 2604.5 / 7541 = 0.345 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Example 24.10. *A plate clutch has three discs on the driving shaft and two discs on the driven shaft, providing four pairs of contact surfaces. The outside diameter of the contact surfaces is 240 mm and inside diameter 120 mm. Assuming uniform pressure and μ = 0.3, find the total spring load pressing the plates together to transmit 25 kW at 1575 r.p.m.*

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If there are 6 springs each of stiffness 13 kN/m and each of the contact surfaces has worn away by 1.25 mm, find the maximum power that can be transmitted, assuming uniform wear.

Solution. Given : $n_1 = 3$; $n_2 = 2$; $n = 4$; $d_1 = 240$ mm or $r_1 = 120$ mm ; $d_2 = 120$ mm or $r_2 = 60$ mm ; $\mu = 0.3$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m.

Total spring load

Let $\qquad W$ = Total spring load.

We know that the torque transmitted,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{25 \times 10^3 \times 60}{2\,\pi \times 1575} = 151.5 \text{ N-m}$$
$$= 151.5 \times 10^3 \text{ N-mm}$$

Mean radius of the contact surface, for uniform pressure,

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3}\left[\frac{(120)^3 - (60)^3}{(120)^2 - (60)^2}\right] = 93.3 \text{ mm}$$

and torque transmitted (T),

$$151.5 \times 10^3 = n.\mu.W.R = 4 \times 0.3 \times W \times 93.3 = 112\,W$$

$\therefore \qquad W = 151.5 \times 10^3 / 112 = 1353$ N **Ans.**

Maximum power transmitted

Given : No. of springs = 6

$\therefore$ Contact surfaces of the spring = 8

Wear on each contact surface = 1.25 mm

$\therefore \qquad$ Total wear $= 8 \times 1.25 = 10$ mm $= 0.01$ m

Stiffness of each spring $= 13$ kN/m $= 13 \times 10^3$ N/m

$\therefore$ Reduction in spring force

= Total wear × Stiffness per spring × No. of springs

$= 0.01 \times 13 \times 10^3 \times 6 = 780$ N

and new axial load, $\qquad W = 1353 - 780 = 573$ N

We know that mean radius of the contact surfaces for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{120 + 60}{2} = 90 \text{ mm} = 0.09 \text{ m}$$

and torque transmitted, $\qquad T = n\,.\,\mu\,W\,.\,R = 4 \times 0.3 \times 573 \times 0.09 = 62$ N-m

$\therefore$ Power transmitted, $\qquad P = \dfrac{T \times 2\pi\,N}{60} = \dfrac{62 \times 2\pi \times 1575}{60} = 10\,227 \text{ W} = 10.227 \text{ kW}$ **Ans.**

24.11 Cone Clutch

A cone clutch, as shown in Fig. 24.6, was extensively used in automobiles, but now-a-days it has been replaced completely by the disc clutch. It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven. The driven member resting on the feather key in the driven shaft, may be shifted along the shaft by a forked lever provided at B, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member is lined with some material like wood, leather, cork or asbestos etc. The material of the clutch faces (*i.e.* contact surfaces) depends upon the allowable normal pressure and the coefficient of friction.

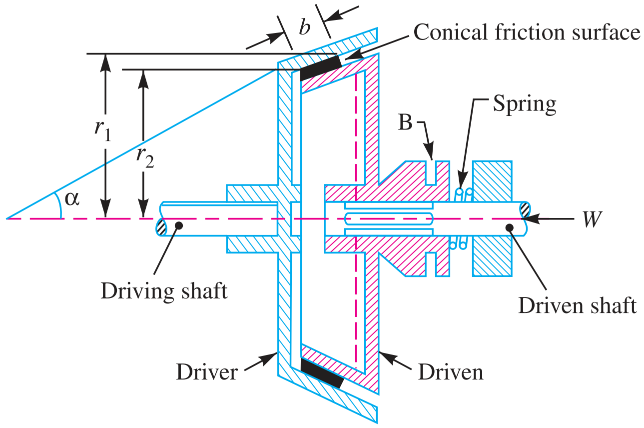


Fig. 24.6. Cone clutch.

24.12 Design of a Cone Clutch

Consider a pair of friction surfaces of a cone clutch as shown in Fig. 24.7. A little consideration will show that the area of contact of a pair of friction surface is a frustrum of a cone.

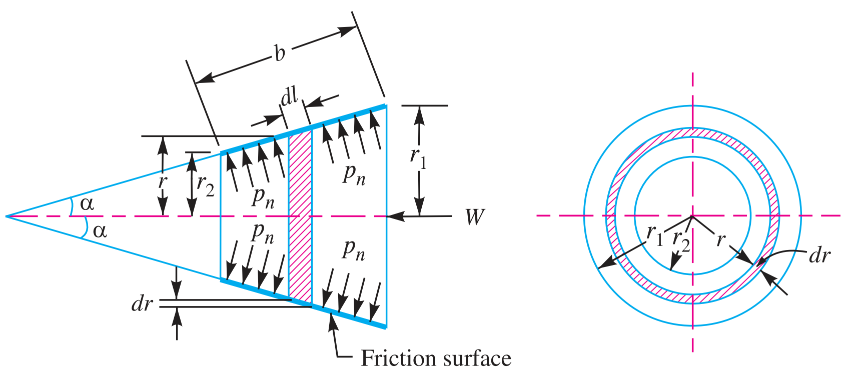


Fig. 24.7. Friction surfaces as a frustrum of a cone.

Let p_n = Intensity of pressure with which the conical friction surfaces are held together (*i.e.* normal pressure between the contact surfaces),

r_1 = Outer radius of friction surface,

r_2 = Inner radius of friction surface,

R = Mean radius of friction surface = $\dfrac{r_1 + r_2}{2}$,

α = Semi-angle of the cone (also called face angle of the cone) or angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between the contact surfaces, and

b = Width of the friction surfaces (also known as face width or cone face).

Consider a small ring of radius r and thickness dr as shown in Fig. 24.7. Let dl is the length of ring of the friction surface, such that,

$$dl = dr \operatorname{cosec} \alpha$$

$\therefore$ Area of ring $= 2\pi\, r.\, dl = 2\pi\, r.dr \operatorname{cosec} \alpha$

We shall now consider the following two cases :

1. When there is a uniform pressure, and

2. When there is a uniform wear.

1. Considering uniform pressure

We know that the normal force acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_n \times 2\pi\, r.dr \operatorname{cosec} \alpha$$

and the axial force acting on the ring,

$$\delta W = \text{Horizontal component of } \delta W_n \text{ (i.e. in the direction of } W)$$

$$= \delta W_n \times \sin \alpha = p_n \times 2\pi\, r.dr \operatorname{cosec} \alpha \times \sin \alpha = 2\pi \times p_n.r.dr$$

$\therefore$ Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi \times p_n.r.dr = 2\pi\ p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\, p_n\, [(r_1)^2 - (r_2)^2]$$

and $$p_n = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]} \quad ...(i)$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n \times 2\pi r.dr \operatorname{cosec} \alpha$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_n \times 2\pi r.dr \operatorname{cosec} \alpha \times r$$

$$= 2\pi\, \mu.p_n \operatorname{cosec} \alpha.r^2\, dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

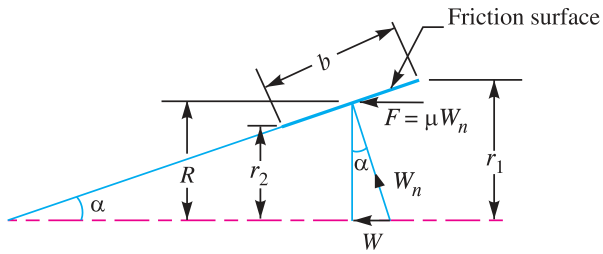
$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n. \operatorname{cosec} \alpha.r^2\ dr = 2\pi\ \mu.p_n\ \operatorname{cosec} \alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\ \mu.p_n \operatorname{cosec} \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

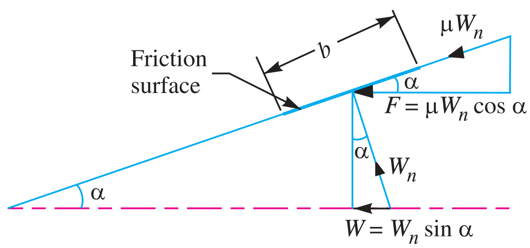
Substituting the value of p_n from equation (*i*), we get

$$T = 2\pi\,\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \text{cosec}\ \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W\ \text{cosec}\ \alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \text{...}(ii)$$



(*a*) For steady operation of the clutch.



(*b*) During engagement of the clutch.

Fig. 24.8. Forces on a friction surface.

2. Considering uniform wear

In Fig. 24.7, let p_r be the normal intensity of pressure at a distance r from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$$\therefore \qquad p_r.r = C \text{ (a constant)} \quad \text{or} \quad p_r = C/r$$

We know that the normal force acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_r \times 2\pi r.dr\ \text{cosec}\ \alpha$$

and the axial force acting on the ring,

$$\delta W = \delta W_n \times \sin\alpha = p_r \times 2\pi\ r.dr\ \text{cosec}\ \alpha \times \sin\alpha$$

$$= 2\pi \times p_r.r\ dr$$

$$= 2\pi \times \frac{C}{r} \times r.\ dr = 2\pi\ C.dr \qquad \text{...}\left(\because\ p_r = \frac{C}{r}\right)$$

$\therefore$ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi\ C.dr = 2\pi\ C\ [r]_{r_2}^{r_1} = 2\pi\ C\ (r_1 - r_2)$$

or

$$C = \frac{W}{2\pi\ (r_1 - r_2)} \qquad \text{...}(iii)$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_r \times 2\pi\ r.dr\ \text{cosec}\ \alpha$$

A mammoth caterpillar dump truck for use in quarries and open-cast mines.

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_r \times 2\pi\, r.dr \operatorname{cosec} \alpha \times r$$

$$= \mu \times \frac{C}{r} \times 2\pi\, r.dr \operatorname{cosec} \alpha \times r = 2\pi\, \mu.C \operatorname{cosec} \alpha \times r\, dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

∴ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\, \mu.C \operatorname{cosec} \alpha \times r\, dr = 2\pi\mu.C \operatorname{cosec} \alpha \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\, \mu.C \operatorname{cosec} \alpha \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

Substituting the value of C from equation ***(iii)***, we have

$$T = 2\pi\mu \times \frac{W}{2\pi\,(r_1 - r_2)} \times \operatorname{cosec} \alpha \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \mu.W \operatorname{cosec} \alpha \left[\frac{r_1 + r_2}{2}\right] = \mu\, WR \operatorname{cosec} \alpha \qquad ...(iv)$$

where $R = \dfrac{r_1 + r_2}{2}$ = Mean radius of friction surface.

Since the normal force acting on the friction surface, $W_n = W \operatorname{cosec} \alpha$, therefore the equation ***(iv)*** may be written as

$$T = \mu\, W_n\, R \qquad ...(v)$$

The forces on a friction surface, for steady operation of the clutch and after the clutch is engaged, is shown in Fig. 24.8 (*a*) and (*b*) respectively.

From Fig. 24.8 (*a*), we find that

$$r_1 - r_2 = b \sin \alpha \qquad \text{and} \qquad R = \frac{r_1 + r_2}{2} \qquad \text{or} \qquad r_1 + r_2 = 2R$$

∴ From equation (*i*), normal pressure acting on the friction surface,

$$p_n = \frac{W}{\pi [(r_1)^2 - (r_2)^2]} = \frac{W}{\pi (r_1 + r_2)(r_1 - r_2)} = \frac{W}{2\pi\, R.b \sin \alpha}$$

or $$W = p_n \times 2\pi\, R.b \sin \alpha = W_n \sin \alpha$$

where W_n = Normal load acting on the friction surface = $p_n \times 2\pi R.b$

Now the equation (*iv*) may be written as

$$T = \mu\, (p_n \times 2\pi\, R.\, b \sin \alpha)\, R \operatorname{cosec} \alpha = 2\pi\, \mu.p_n\, R^2.b$$

The following points may be noted for a cone clutch :

1. The above equations are valid for steady operation of the clutch and after the clutch is engaged.

2. If the clutch is engaged when one member is stationary and the other rotating (*i.e.* during engagement of the clutch) as shown in Fig. 24.8 (*b*), then the cone faces will tend to slide on each other due to the presence of relative motion. Thus an additional force (of magnitude $\mu.W_n \cos \alpha$) acts on the clutch which resists the engagement, and the axial force required for engaging the clutch increases.

∴ Axial force required for engaging the clutch,

$$W_e = W + \mu.W_n \cos \alpha = W_n.\sin \alpha + \mu\, W_n \cos \alpha$$
$$= W_n (\sin \alpha + \mu \cos \alpha)$$

It has been found experimentally that the term ($\mu\, W_n.\cos \alpha$) is only 25 percent effective.

∴ $$W_e = W_n \sin \alpha + 0.25\, \mu\, W_n \cos \alpha = W_n (\sin \alpha + 0.25\, \mu \cos \alpha)$$

3. Under steady operation of the clutch, a decrease in the semi-cone angle (α) increases the torque produced by the clutch (T) and reduces the axial force (W). During engaging period, the axial force required for engaging the clutch (W_e) increases under the influence of friction as the angle α decreases. The value of α can not be decreased much because smaller semi-cone angle (α) requires larger axial force for its disengagement.

If the clutch is to be designed for free disengagement, the value of tan α must be greater than μ. In case the value of tan α is less than μ, the clutch will not disengage itself and axial force required to disengage the clutch is given by

$$W_d = W_n (\mu \cos \alpha - \sin \alpha)$$

Example 24.11. *The contact surfaces in a cone clutch have an effective diameter of 80 mm. The semi-angle of the cone is 15° and coefficient of friction is 0.3. Find the torque required to produce slipping of the clutch, if the axial force applied is 200 N. The clutch is employed to connect an electric motor, running uniformly at 900 r.p.m. with a flywheel which is initially stationary. The flywheel has a mass of 14 kg and its radius of gyration is 160 mm. Calculate the time required for the flywheel to attain full-speed and also the energy lost in slipping of the clutch.*

Solution. Given : D = 80 mm or R = 40 mm ; α = 15° ; μ = 0.3 ; W = 200 N ; N = 900 r.p.m. or ω = 2π × 900/60 = 94.26 rad/s ; m = 14 kg ; k = 160 mm = 0.16 m

Torque required to produce slipping of the clutch

We know that the torque required to produce slipping of the clutch,

$$T = \mu\, WR \operatorname{cosec} \alpha = 0.3 \times 200 \times 40 \operatorname{cosec} 15° = 9273 \text{ N-mm}$$
$$= 9.273 \text{ N-m}$$ **Ans.**

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Time required for the flywheel to attain full-speed

Let t = Time required for the flywheel to attain full speed from the stationary position, and

α = Angular acceleration of the flywheel.

We know that mass moment of inertia of the flywheel,

$$I = m.k^2 = 14\,(0.16)^2 = 0.3584 \text{ kg-m}^2$$

We also know that the torque (T),

$$9.273 = I \times \alpha = 0.3584\,\alpha$$

$$\therefore \quad \alpha = 9.273 / 0.3584 = 25.87 \text{ rad / s}^2$$

and angular speed (ω),

$$94.26 = \omega_0 + \alpha.t = 0 + 25.87 \times t = 25.87\,t \qquad \text{... } (\because\ \omega_0 = 0)$$

$$\therefore \quad t = 94.26 / 25.87 = 3.64 \text{ s}$$ **Ans.**

Energy lost in slipping of the clutch

We know that angular displacement,

$$\theta = \text{Average angular speed} \times \text{time} = \frac{\omega_0 + \omega}{2} \times t$$

$$= \frac{0 + 94.26}{2} \times 3.64 = 171.6 \text{ rad}$$

$\therefore$ Energy lost in slipping of the clutch,

$$= T.\theta = 9.273 \times 171.6 = 1591 \text{ N-m}$$ **Ans.**

Example 24.12. *An engine developing 45 kW at 1000 r.p.m. is fitted with a cone clutch built inside the flywheel. The cone has a face angle of 12.5° and a maximum mean diameter of 500 mm. The coefficient of friction is 0.2. The normal pressure on the clutch face is not to exceed 0.1 N/mm². Determine : 1. the face width required, and 2. the axial spring force necessary to engage the clutch.*

Solution. Given : P = 45 kW = 45 × 10^3 W ; N = 1000 r.p.m. ; α = 12.5° ; D = 500 mm or R = 250 mm ; μ = 0.2 ; p_n = 0.1 N/mm²

1. *Face width*

Let b = Face width of the clutch in mm.

We know that torque developed by the clutch,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{45 \times 10^3 \times 60}{2\,\pi \times 1000} = 430 \text{ N-m} = 430 \times 10^3 \text{ N-mm}$$

We also know that torque developed by the clutch (T),

$$430 \times 10^3 = 2\pi.\,\mu.\,p_n.\,R^2.b = 2\pi \times 0.2 \times 0.1\,(250)^2 b = 7855\,b$$

$$\therefore \quad b = 430 \times 10^3 / 7855 = 54.7 \text{ say } 55 \text{ mm}$$ **Ans.**

2. *Axial spring force necessary to engage the clutch*

We know that the normal force acting on the contact surfaces,

$$W_n = p_n \times 2\pi R.b = 0.1 \times 2\pi \times 250 \times 55 = 8640 \text{ N}$$

$\therefore$ Axial spring force necessary to engage the clutch,

$$W_e = W_n\,(\sin\alpha + 0.25\,\mu\cos\alpha)$$

$$= 8640\,(\sin 12.5° + 0.25 \times 0.2 \cos 12.5°) = 2290 \text{ N}$$ **Ans.**

Example 24.13. *Determine the principal dimensions of a cone clutch faced with leather to transmit 30 kW at 750 r.p.m. from an electric motor to an air compressor. Sketch a sectional front view of the clutch and provide the main dimensions on the sketch.*

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Assume : semi-angle of the cone = $12\frac{1}{2}°$; μ = 0.2 ; mean diameter of cone = 6 to 10 d where d is the diameter of shaft; allowable normal pressure for leather and cast iron = 0.075 to 0.1 N/mm²; load factor = 1.75 and mean diameter to face width ratio = 6.

Solution. Given : P = 30 kW = 30 × 10³ W ; N = 750 r.p.m. ; $\alpha = 12\frac{1}{2}°$; μ = 0.2 ; D = 6 to 10 d ; p_n = 0.075 to 0.1 N/mm² ; K_L = 1.75 ; D / b = 6

First of all, let us find the diameter of shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi\ N} \times K_L = \frac{30 \times 10^3 \times 60}{2\ \pi \times 750} \times 1.75 = 668.4 \text{ N-m}$$

$$= 668.4 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$668.4 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3 \qquad \text{... (Taking } \tau = 42 \text{ N/mm}^2\text{)}$$

$$\therefore \quad d^3 = 668.4 \times 10^3 / 8.25 = 81 \times 10^3 \quad \text{or} \quad d = 43.3 \text{ say } 50 \text{ mm } \textbf{Ans.}$$

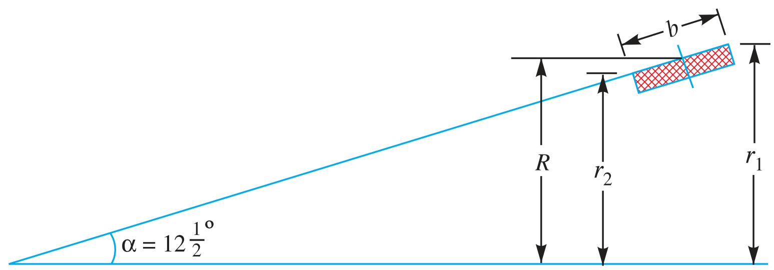

Fig. 24.9

Now let us find the principal dimensions of a cone clutch.

Let D = Mean diameter of the clutch,

R = Mean radius of the clutch, and

b = Face width of the clutch.

Since the allowable normal pressure (p_n) for leather and cast iron is 0.075 to 0.1 N/mm², therefore let us take p_n = 0.1 N/mm².

We know that the torque developed by the clutch (T),

$$668.4 \times 10^3 = 2\ \pi\ \mu .\ p_n .\ R^2 . b = 2\pi \times 0.2 \times 0.1 \times R^2 \times \frac{R}{3} = 0.042\ R^3$$

$$\text{... } (\because\ D / b = 6 \text{ or } 2R / b = 6 \text{ or } R / b = 3)$$

$$\therefore \quad R^3 = 668.4 \times 10^3 / 0.042 = 15.9 \times 10^6 \quad \text{or} \quad R = 250 \text{ mm}$$

and $D = 2R = 2 \times 250 = 500$ mm **Ans.**

Since this calculated value of the mean diameter of the clutch (D) is equal to 10 d and the given value of D is 6 to 10d, therefore the calculated value of D is safe.

We know that face width of the clutch,

$$b = D / 6 = 500 / 6 = 83.3 \text{ mm } \textbf{Ans.}$$

From Fig. 24.9, we find that outer radius of the clutch,

$$r_1 = R + \frac{b}{2} \sin \alpha = 250 + \frac{83.3}{2} \sin 12\frac{1}{2}° = 259 \text{ mm } \textbf{Ans.}$$

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and inner radius of the clutch,

$$r_2 = R - \frac{b}{2}\sin\alpha = 250 - \frac{83.3}{2}\sin 12\frac{1}{2}° = 241 \text{ mm} \quad \textbf{Ans.}$$

24.13 Centrifugal Clutch

Centrifugal clutch with three discs and four steel float plates.

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. 24.10. The outer surface of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The weight of the shoe, when revolving causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder and enables more torque to be transmitted.

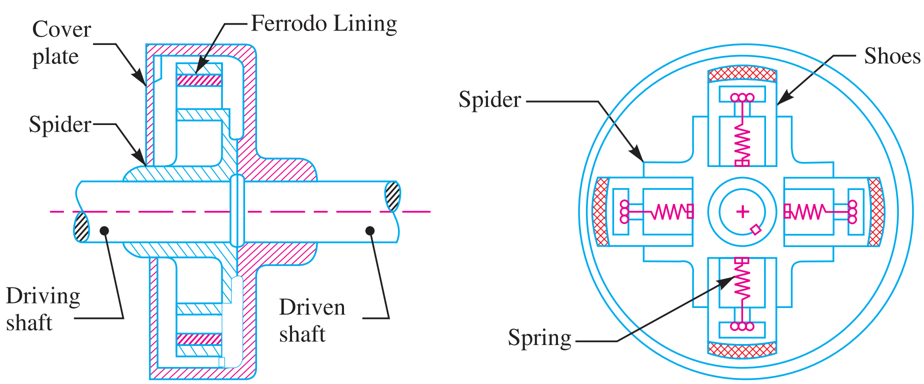

Fig. 24.10. Centrifugal clutch.

24.14 Design of a Centrifugal Clutch

In designing a centrifugal clutch, it is required to determine the weight of the shoe, size of the shoe and dimensions of the spring. The following procedure may be adopted for the design of a centrifugal clutch.

1. *Mass of the shoes*

Consider one shoe of a centrifugal clutch as shown in Fig. 24.11.

Let m = Mass of each shoe,

n = Number of shoes,

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r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad / s $= 2\pi N / 60$ rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

μ = Coefficient of friction between the shoe and rim.

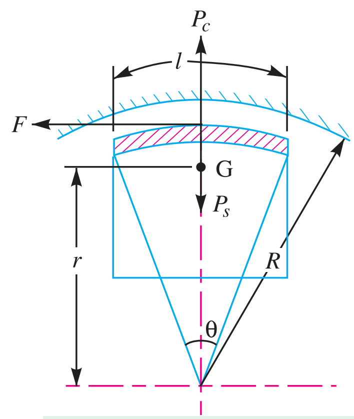

Fig. 24.11. Forces on a shoe of a centrifugal clucth.

We know that the centrifugal force acting on each shoe at the running speed,

$$^*P_c = m.\omega^2.r$$

Since the speed at which the engagement begins to take place is generally taken as 3/4th of the running speed, therefore the inward force on each shoe exerted by the spring is given by

$$P_s = m\,(\omega_1)^2\, r = m\left(\frac{3}{4}\omega\right)^2 r = \frac{9}{16}\, m.\omega^2 \cdot r$$

$\therefore$ Net outward radial force (*i.e.* centrifugal force) with which the shoe presses against the rim at the running speed

$$= P_c - P_s = m.\omega^2.r - \frac{9}{16}\, m.\omega^2.r = \frac{7}{16}\, m.\omega^2.r$$

and the frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s)$$

$\therefore$ Frictional torque acting on each shoe

$$= F \times R = \mu\,(P_c - P_s)\, R$$

and total frictional torque transmitted,

$$T = \mu\,(P_c - P_s)\, R \times n = n.F.R$$

From this expression, the mass of the shoes (m) may be evaluated.

2. Size of the shoes

Let l = Contact length of the shoes,

b = Width of the shoes,

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley,

θ = Angle subtended by the shoes at the centre of the spider in radians, and

p = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, it may be taken as 0.1 N/mm^2.

We know that $\theta = \dfrac{l}{R}$ or $l = \theta.R = \dfrac{\pi}{3} R$ ...(Assuming $\theta = 60° = \pi / 3$ rad)

* The radial clearance between the shoe and the rim is about 1.5 mm. Since this clearance is small as compared to r, therefore it is neglected for design purposes. If, however, the radial clearance is given, then the operating radius of the mass centre of the shoe from the axis of the clutch,

$r_1 = r + c$, where c is the radial clearance,

Then $P_c = m.\omega^2\, r_1$ and $P_s = m\,(\omega_1)^2\, r_1$

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∴ Area of contact of the shoe

$$= l \,.\, b$$

and the force with which the shoe presses against the rim

$$= A \times p = l.b.p$$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$, therefore

$$l.b.p = P_c - P_s$$

From this expression, the width of shoe (b) may be obtained.

3. *Dimensions of the spring*

We have discussed above that the load on the spring is given by

$$P_s = \frac{9}{16} \times m.\omega^2.r$$

The dimensions of the spring may be obtained as usual.

Example 24.14. *A centrifugal clutch is to be designed to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine: 1. mass of the shoes, and 2. size of the shoes.*

Solution. Given : P = 15 kW = 15 × 10^3 W ; N = 900 r.p.m. ; n = 4 ; R = 150 mm = 0.15 m ; μ = 0.25

1. *Mass of the shoes*

Let m = Mass of the shoes.

We know that the angular running speed,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 900}{60} = 94.26 \text{ rad/s}$$

Since the speed at which the engagement begins is 3/4 th of the running speed, therefore angular speed at which engagement begins is

$$\omega_1 = \frac{3}{4}\,\omega = \frac{3}{4} \times 94.26 = 70.7 \text{ rad/s}$$

Assuming that the centre of gravity of the shoe lies at a distance of 120 mm (30 mm less than R) from the centre of the spider, *i.e.*

$$r = 120 \text{ mm} = 0.12 \text{ m}$$

We know that the centrifugal force acting on each shoe,

$$P_c = m.\omega^2.r = m\,(94.26)^2\; 0.12 = 1066\; m \text{ N}$$

and the inward force on each shoe exerted by the spring *i.e.* the centrifugal force at the engagement speed, ω_1,

$$P_s = m(\omega_1)^2\, r = m\,(70.7)^2\; 0.12 = 600\; m \text{ N}$$

We know that the torque transmitted at the running speed,

$$T = \frac{P \times 60}{2\pi\, N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 900} = 159 \text{ N-m}$$

We also know that the torque transmitted (T),

$$159 = \mu\,(P_c - P_s)\, R \times n = 0.25\,(1066\, m - 600\, m)\; 0.15 \times 4 = 70\, m$$

$$\therefore \quad m = 159/70 = 2.27 \text{ kg} \qquad \textbf{Ans.}$$

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2. ***Size of the shoes***

Let l = Contact length of shoes in mm, and

b = Width of the shoes in mm.

Assuming that the arc of contact of the shoes subtend an angle of $\theta = 60°$ or $\pi / 3$ radians, at the centre of the spider, therefore

$$l = \theta.R = \frac{\pi}{3} \times 150 = 157 \text{ mm}$$

Area of contact of the shoes

$$A = l.b = 157\, b \text{ mm}^2$$

Assuming that the intensity of pressure (p) exerted on the shoes is 0.1 N/mm^2, therefore force with which the shoe presses against the rim

$$= A.p = 157b \times 0.1 = 15.7\, b \text{ N} \quad \text{...}(i)$$

We also know that the force with which the shoe presses against the rim

$$= P_c - P_s = 1066\, m - 600\, m = 466\, m$$

$$= 466 \times 2.27 = 1058 \text{ N} \quad \text{...}(ii)$$

From equations (*i*) and (*ii*), we find that

$$b = 1058 / 15.7 = 67.4 \text{ mm}$$ **Ans.**

Special trailers are made to carry very long loads. The longest load ever moved was gas storage vessel, 83.8 m long.

EXERCISES

1. A single disc clutch with both sides of the disc effective is used to transmit 10 kW power at 900 r.p.m. The axial pressure is limited to 0.085 N/mm^2. If the external diameter of the friction lining is 1.25 times the internal diameter, find the required dimensions of the friction lining and the axial force exerted by the springs. Assume uniform wear conditions. The coefficient of friction may be taken as 0.3.

[Ans. 132.5 mm ; 106 mm ; 1500 N]

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2. A single plate clutch with both sides of the plate effective is required to transmit 25 kW at 1600 r.p.m. The outer diameter of the plate is limited to 300 mm and the intensity of pressure between the plates not to exceed 0.07 N/mm^2. Assuming uniform wear and coefficient of friction 0.3, find the inner diameter of the plates and the axial force necessary to engage the clutch.

[Ans. 90 mm ; 2375 N]

3. Give a complete design analysis of a single plate clutch, with both sides effective, of a vehicle to transmit 22 kW at a speed of 2800 r.p.m. allowing for 25% overload. The pressure intensity is not to exceed 0.08 N/mm^2 and the surface speed at the mean radius is not to exceed 2000 m/min. Take coefficient of friction for the surfaces as 0.35 and the outside diameter of the surfaces is to be 1.5 times the inside diameter. The axial thrust is to be provided by 6 springs of about 24 mm coil diameter. For spring material, the safe shear stress is to be limited to 420 MPa and the modulus of rigidity may be taken as 80 kN/mm^2. **[Ans. 120 mm ; 80 mm ; 3.658 mm]**

4. A multiple disc clutch has three discs on the driving shaft and two on the driven shaft, providing four pairs of contact surfaces. The outer diameter of the contact surfaces is 250 mm and the inner diameter is 150 mm. Determine the maximum axial intensity of pressure between the discs for transmitting 18.75 kW at 500 r.p.m. Assume uniform wear and coefficient of friction as 0.3.

5. A multiple disc clutch employs 3 steel and 2 bronze discs having outer diameter 300 mm and inner diameter 200 mm. For a coefficient of friction of 0.22, find the axial pressure and the power transmitted at 750 r.p.m., if the normal unit pressure is 0.13 N/mm^2.

   Also find the axial pressure of the unit normal pressure, if this clutch transmits 22 kW at 1500 r.p.m. **[Ans. 5105 N ; 44.11 kW ; 0.0324 N/mm^2]**

6. A multiple disc clutch has radial width of the friction material as 1/5th of the maximum radius. The coefficient of friction is 0.25. Find the total number of discs required to transmit 60 kW at 3000 r.p.m. The maximum diameter of the clutch is 250 mm and the axial force is limited to 600 N. Also find the mean unit pressure on each contact surface. **[Ans. 13 ; 0.034 N/mm^2]**

7. An engine developing 22 kW at 1000 r.p.m. is fitted with a cone clutch having mean diameter of 300 mm. The cone has a face angle of 12°. If the normal pressure on the clutch face is not to exceed 0.07 N/mm^2 and the coefficient of friction is 0.2, determine :

   (*a*) the face width of the clutch, and

   (*b*) the axial spring force necessary to engage the clutch.

   [Ans. 106 mm ; 1796 N]

8. A cone clutch is to be designed to transmit 7.5 kW at 900 r.p.m. The cone has a face angle of 12°. The width of the face is half of the mean radius and the normal pressure between the contact faces is not to exceed 0.09 N/mm^2. Assuming uniform wear and the coefficient of friction between the contact faces as 0.2, find the main dimensions of the clutch and the axial force required to engage the clutch.

   [Ans. R = 112.4 mm ; b = 56.2 mm ; r_1 = 118.2 mm ; r_2 = 106.6 mm ; W_e = 917 N]

9. A soft cone clutch has a cone pitch angle of 10°, mean diameter of 300 mm and a face width of 100 mm. If the coefficient of friction is 0.2 and has an average pressure of 0.07 N/mm^2 for a speed of 500 r.p.m., find : (*a*) the force required to engage the clutch; and (*b*) the power that can be transmitted. Assume uniform wear. **[Ans. 1470 N ; 10.4 kW]**

10. A cone clutch is mounted on a shaft which transmits power at 225 r.p.m. The small diameter of the cone is 230 mm, the cone face is 50 mm and the cone face makes an angle of 15° with the horizontal. Determine the axial force necessary to engage the clutch to transmit 4.5 kW if the coefficient of friction of the contact surfaces is 0.25. What is the maximum pressure on the contact surfaces assuming uniform wear? **[Ans. 2414 N ; 0.216 N/mm^2]**

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11. A soft surface cone clutch transmits a torque of 200 N-m at 1250 r.p.m. The larger diameter of the clutch is 350 mm. The cone pitch angle is 7.5° and the face width is 65 mm. If the coefficient of friction is 0.2, find :

1. the axial force required to transmit the torque;
2. the axial force required to engage the clutch;
3. the average normal pressure on the contact surfaces when the maximum torque is being transmitted; and
4. the maximum normal pressure assuming uniform wear.

[Ans. 764 N ; 1057 N ; 0.084 N/mm^2 ; 0.086 N/mm^2]

12. A centrifugal friction clutch has a driving member consisting of a spider carrying four shoes which are kept from contact with the clutch case by means of flat springs until increase of centrifugal force overcomes the resistance of the springs and the power is transmitted by the friction between the shoes and the case.

Determine the necessary mass and size of each shoe if 22.5 kW is to be transmitted at 750 r.p.m. with engagement beginning at 75% of the running speed. The inside diameter of the drum is 300 mm and the radial distance of the centre of gravity of each shoe from the shaft axis is 125 mm. Assume μ = 0.25. **[Ans. 5.66 kg ; *l* = 157.1 mm ; *b* = 120 mm]**

Clutches, brakes, steering and transmission need to be carefully designed to ensure the efficiency and safety of an automobile

QUESTIONS

1. What is a clutch? Discuss the various types of clutches giving at least one practical application for each.

2. Why a positive clutch is used? Describe, with the help of a neat sketch, the working of a jaw or claw clutch.

3. Name the different types of clutches. Describe with the help of neat sketches the working principles of two different types of friction clutches.

4. What are the materials used for lining of friction surfaces?

5. Why it is necessary to dissipate the heat generated when clutches operate?

6. Establish a formula for the frictional torque transmitted by a cone clutch.

7. Describe, with the help of a neat sketch, a centrifugal clutch and deduce an expression for the total frictional torque transmitted. How the shoes and springs are designed for such a clutch?

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OBJECTIVE TYPE QUESTIONS

1. A jaw clutch is essentially a

   (*a*) positive action clutch (*b*) cone clutch

   (*c*) friction clutch (*d*) disc clutch

2. The material used for lining of friction surfaces of a clutch should have .... coefficient of friction.

   (*a*) low (*b*) high

3. The torque developed by a disc clutch is given by

   (*a*) $T = 0.25\ \mu.W.R$ (*b*) $T = 0.5\ \mu.W.R$

   (*c*) $T = 0.75\ \mu.W.R$ (*d*) $T = \mu.W.R$

   where W = Axial force with which the friction surfaces are held together ;

   μ = Coefficient of friction ; and

   R = Mean radius of friction surfaces.

4. In case of a multiple disc clutch, if n_1 are the number of discs on the driving shaft and n_2 are the number of the discs on the driven shaft, then the number of pairs of contact surfaces will be

   (*a*) $n_1 + n_2$ (*b*) $n_1 + n_2 - 1$

   (*c*) $n_1 + n_2 + 1$ (*d*) none of these

5. The cone clutches have become obsolete because of

   (*a*) small cone angles (*b*) exposure to dirt and dust

   (*c*) difficulty in disengaging (*d*) all of these

6. The axial force (W_e) required for engaging a cone clutch is given by

   (*a*) $W_n \sin \alpha$ (*b*) $W_n (\sin \alpha + \mu \cos \alpha)$

   (*c*) $W_n (\sin \alpha + 0.25\ \mu \cos \alpha)$ (*d*) none of these

   where W_n = Normal force acting on the contact surfaces,

   α = Face angle of the cone, and

   μ = Coefficient of friction.

7. In a centrifugal clutch, the force with which the shoe presses against the driven member is the .... of the centrifugal force and the spring force.

   (*a*) difference (*b*) sum

ANSWERS

1. (*a*)	**2.** (*b*)	**3.** (*d*)	**4.** (*b*)	**5.** (*d*)
6. (*c*)	**7.** (*a*)			

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