Contents

CHAPTER

25

Brakes

25.1 Introduction

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc. The energy absorbed by brakes is dissipated in the form of heat. This heat is dissipated in the surrounding air (or water which is circulated through the passages in the brake drum) so that excessive heating of the brake lining does not take place. The design or capacity of a brake depends upon the following factors :

1. The unit pressure between the braking surfaces,
2. The coefficient of friction between the braking surfaces,
3. The peripheral velocity of the brake drum,

Top

4. The projected area of the friction surfaces, and
5. The ability of the brake to dissipate heat equivalent to the energy being absorbed.

The major functional difference between a clutch and a brake is that a clutch is used to keep the driving and driven member moving together, whereas brakes are used to stop a moving member or to control its speed.

25.2 Energy Absorbed by a Brake

The energy absorbed by a brake depends upon the type of motion of the moving body. The motion of a body may be either pure translation or pure rotation or a combination of both translation and rotation. The energy corresponding to these motions is kinetic energy. Let us consider these motions as follows :

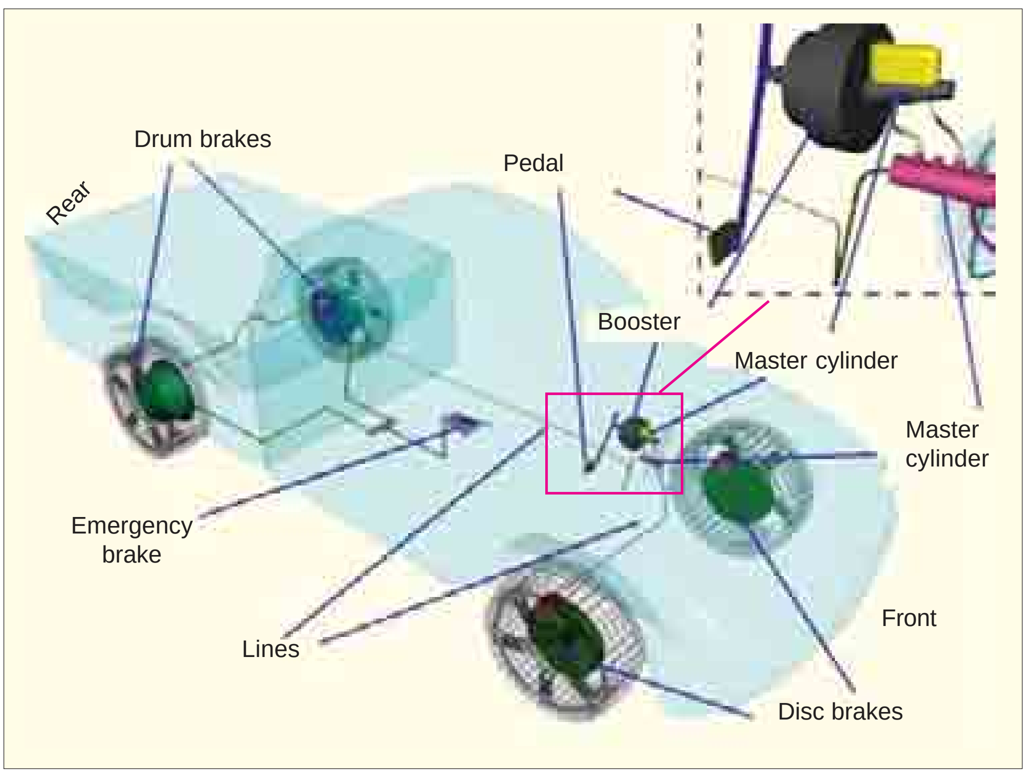
1. ***When the motion of the body is pure translation.*** Consider a body of mass (m) moving with a velocity v_1 m / s. Let its velocity is reduced to v_2 m / s by applying the brake. Therefore, the change in kinetic energy of the translating body or kinetic energy of translation,

$$E_1 = \frac{1}{2} m \left[(v_1)^2 - (v_2)^2 \right]$$

This energy must be absorbed by the brake. If the moving body is stopped after applying the brakes, then $v_2 = 0$, and

$$E_1 = \frac{1}{2} m (v_1)^2$$

2. ***When the motion of the body is pure rotation.*** Consider a body of mass moment of inertia I (about a given axis) is rotating about that axis with an angular velocity ω_1 rad / s. Let its angular velocity is reduced to ω_2 rad / s after applying the brake. Therefore, the change in kinetic energy of



Brake System Components

Note : This picture is given as additional information and is not a direct example of the current chapter.

the rotating body or kinetic energy of rotation,

$$E_2 = \frac{1}{2} I \left[(\omega_1)^2 - (\omega_2)^2 \right]$$

This energy must be absorbed by the brake. If the rotating body is stopped after applying the brakes, then $\omega_2 = 0$, and

$$E_2 = \frac{1}{2} I (\omega_1)^2$$

3. ***When the motion of the body is a combination of translation and rotation.*** Consider a body having both linear and angular motions, *e.g.* in the locomotive driving wheels and wheels of a moving car. In such cases, the total kinetic energy of the body is equal to the sum of the kinetic energies of translation and rotation.

∴ Total kinetic energy to be absorbed by the brake,

$$E = E_1 + E_2$$

Sometimes, the brake has to absorb the potential energy given up by objects being lowered by hoists, elevators etc. Consider a body of mass m is being lowered from a height h_1 to h_2 by applying the brake. Therefore the change in potential energy,

$$E_3 = m.g\,(h_1 - h_2)$$

If v_1 and v_2 m / s are the velocities of the mass before and after the brake is applied, then the change in potential energy is given by

$$E_3 = m.g \left(\frac{v_1 + v_2}{2} \right) t = m.g.v.t$$

where v = Mean velocity = $\frac{v_1 + v_2}{2}$, and

t = Time of brake application.

Thus, the total energy to be absorbed by the brake,

$$E = E_1 + E_2 + E_3$$

Let F_t = Tangential braking force or frictional force acting tangentially at the contact surface of the brake drum,

d = Diameter of the brake drum,

N_1 = Speed of the brake drum before the brake is applied,

N_2 = Speed of the brake drum after the brake is applied, and

N = Mean speed of the brake drum = $\frac{N_1 + N_2}{2}$

We know that the work done by the braking or frictional force in time t seconds

$$= F_t \times \pi\, d\, N \times t$$

Since the total energy to be absorbed by the brake must be equal to the wordone by the frictional force, therefore

$$E = F_t \times \pi\, d\, N \times t \quad \text{or} \quad F_t = \frac{E}{\pi\, d\, N.t}$$

The magnitude of F_t depends upon the final velocity (v_2) and on the braking time (t). Its value is maximum when $v_2 = 0$, *i.e.* when the load comes to rest finally.

We know that the torque which must be absorbed by the brake,

$$T = F_t \times r = F_t \times \frac{d}{2}$$

where r = Radius of the brake drum.

25.3 Heat to be Dissipated during Braking

The energy absorbed by the brake and transformed into heat must be dissipated to the surrounding air in order to avoid excessive temperature rise of the brake lining. The *temperature rise depends upon the mass of the brake drum, the braking time and the heat dissipation capacity of the brake. The highest permissible temperatures recommended for different brake lining materials are given as follows :

1. For leather, fibre and wood facing = 65 – 70°C
2. For asbestos and metal surfaces that are slightly lubricated = 90 – 105°C
3. For automobile brakes with asbestos block lining = 180 – 225°C

Since the energy absorbed (or heat generated) and the rate of wear of the brake lining at a particular speed are dependent on the normal pressure between the braking surfaces, therefore it is an important factor in the design of brakes. The permissible normal pressure between the braking surfaces depends upon the material of the brake lining, the coefficient of friction and the maximum rate at which the energy is to be absorbed. The energy absorbed or the heat generated is given by

$$E = H_g = \mu.R_N.v = \mu.p.A.v \text{ (in J/s or watts)} \quad \text{...}(i)$$

where μ = Coefficient of friction,

R_N = Normal force acting at the contact surfaces, in newtons,

p = Normal pressure between the braking surfaces in N/m^2,

A = Projected area of the contact surfaces in m^2, and

v = Peripheral velocity of the brake drum in m/s.

The heat generated may also be obtained by considering the amount of kinetic or potential energies which is being absorbed. In other words,

$$H_g = E_K + E_P$$

where E_K = Total kinetic energy absorbed, and

E_P = Total potential energy absorbed.

The heat dissipated (H_d) may be estimated by

$$H_d = C (t_1 - t_2) A_r \quad \text{...}(ii)$$

where C = Heat dissipation factor or coefficient of heat transfer in W /m^2 / °C

$t_1 - t_2$ = Temperature difference between the exposed radiating surface and the surrounding air in °C, and

A_r = Area of radiating surface in m^2.

The value of C may be of the order of 29.5 W / m^2 /°C for a temperature difference of 40°C and increase up to 44 W/m^2/°C for a temperature difference of 200°C.

The expressions for the heat dissipated are quite approximate and should serve only as an indication of the capacity of the brake to dissipate heat. The exact performance of the brake should be determined by test.

It has been found that 10 to 25 per cent of the heat generated is immediately dissipated to the surrounding air while the remaining heat is absorbed by the brake drum causing its temperature to rise. The rise in temperature of the brake drum is given by

$$\Delta t = \frac{H_g}{m.c} \quad \text{...}(iii)$$

where Δt = Temperature rise of the brake drum in °C,

* When the temperature increases, the coefficient of friction decreases which adversely affect the torque capacity of the brake. At high temperature, there is a rapid wear of friction lining, which reduces the life of lining. Therefore, the temperature rise should be kept within the permissible range.

Contents

H_g = Heat generated by the brake in joules,

m = Mass of the brake drum in kg, and

c = Specific heat for the material of the brake drum in J/kg °C.

In brakes, it is very difficult to precisely calculate the temperature rise. In preliminary design analysis, the product *p.v* is considered in place of temperature rise. The experience has also shown that if the product *p.v* is high, the rate of wear of brake lining will be high and the brake life will be low. Thus the value of *p.v* should be lower than the upper limit value for the brake lining to have reasonable wear life. The following table shows the recommended values of *p.v* as suggested by various designers for different types of service.

Table 25.1. Recommended values of *p.v*.

S.No.	*Type of service*	*Recommended value of p.v in N-m/m² of projected area per second*
1.	Continuous application of load as in lowering operations and poor dissipation of heat.	0.98×10^6
2.	Intermittent application of load with comparatively long periods of rest and poor dissipation of heat.	1.93×10^6
3.	For continuous application of load and good dissipation of heat as in an oil bath.	2.9×10^6

Example 25.1. *A vehicle of mass 1200 kg is moving down the hill at a slope of 1: 5 at 72 km / h. It is to be stopped in a distance of 50 m. If the diameter of the tyre is 600 mm, determine the average braking torque to be applied to stop the vehicle, neglecting all the frictional energy except for the brake. If the friction energy is momentarily stored in a 20 kg cast iron brake drum, What is average temperature rise of the drum? The specific heat for cast iron may be taken as 520 J / kg°C.*

Determine, also, the minimum coefficient of friction between the tyres and the road in order that the wheels do not skid, assuming that the weight is equally distributed among all the four wheels.

Solution. Given : m = 1200 kg ; Slope = 1: 5 ; v = 72 km / h = 20 m/s ; h = 50 m ; d = 600 mm or r = 300 mm = 0.3 m ; m_b = 20 kg ; c = 520 J / kg°C

Average braking torque to be applied to stop the vehicle

We know that kinetic energy of the vehicle,

$$E_K = \frac{1}{2} m.v^2 = \frac{1}{2} \times 1200\ (20)^2 = 240\ 000 \text{ N-m}$$

and potential energy of the vehicle,

$$E_P = m.g.h \times \text{Slope} = 1200 \times 9.81 \times 50 \times \frac{1}{5} = 117\ 720 \text{ N-m}$$

∴ Total energy of the vehicle or the energy to be absorbed by the brake,

$$E = E_K + E_P = 240\ 000 + 117\ 720 = 357\ 720 \text{ N-m}$$

Since the vehicle is to be stopped in a distance of 50 m, therefore tangential braking force required,

$$F_t = 357\ 720 / 50 = 7154.4 \text{ N}$$

We know that average braking torque to be applied to stop the vehicle,

$$T_B = F_t \times r = 7154.4 \times 0.3 = 2146.32 \text{ N-m}$$ **Ans.**

Top

Average temperature rise of the drum

Let Δt = Average temperature rise of the drum in °C.

We know that the heat absorbed by the brake drum,

H_g = Energy absorbed by the brake drum

= 357 720 N-m = 357 720 J ... (∵ 1 N-m = 1 J)

We also know that the heat absorbed by the brake drum (H_g),

$$357\ 720 = m_b \times c \times \Delta t = 20 \times 520 \times \Delta t = 10\ 400\ \Delta t$$

$$\therefore \quad \Delta t = 357\ 720 / 10\ 400 = 34.4°\text{C}$$ **Ans.**

Minimum coefficient of friction between the tyre and road

Let μ = Minimum coefficient of friction between the tyre and road, and

R_N = Normal force between the contact surface. This is equal to weight of the vehicle

$= m.g = 1200 \times 9.81 = 11\ 772$ N

We know that tangential braking force (F_t),

$$7154.4 = \mu.R_N = \mu \times 11\ 772$$

$$\therefore \quad \mu = 7154.4 / 11772 = 0.6$$ **Ans.**

25.4 Materials for Brake Lining

The material used for the brake lining should have the following characteristics :

1. It should have high coefficient of friction with minimum fading. In other words, the coefficient of friction should remain constant over the entire surface with change in temperature.

2. It should have low wear rate.

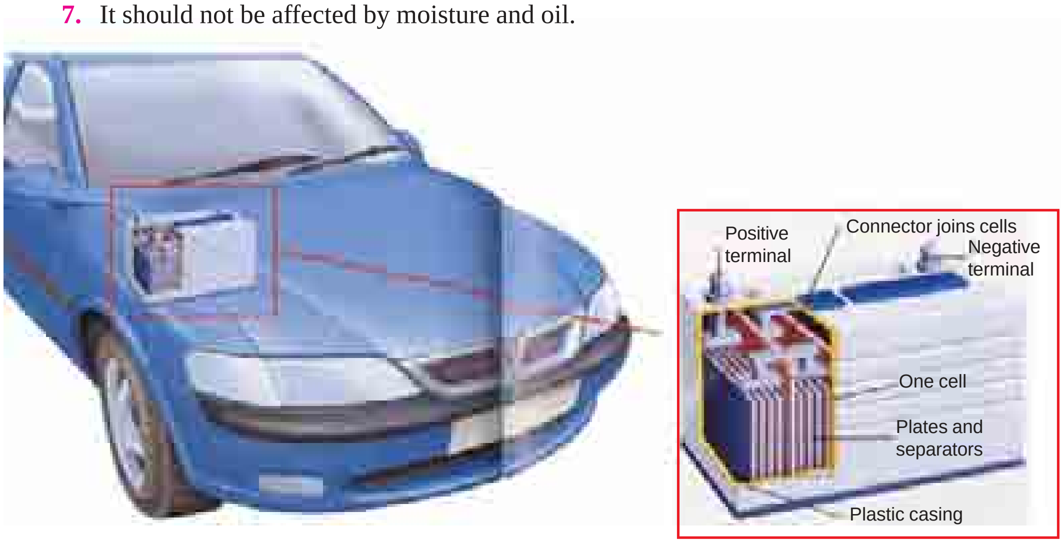
3. It should have high heat resistance.

4. It should have high heat dissipation capacity.

5. It should have low coefficient of thermal expansion.

6. It should have adequate mechanical strength.

7. It should not be affected by moisture and oil.



The rechargeable battery found in most cars is a combination of lead acid cells. A small dynamo, driven by the vehicle's engine, charges the battery whenever the engine is running.

Contents

The materials commonly used for facing or lining of brakes and their properties are shown in the following table.

Table 25.2. Properties of materials for brake lining.

Material for braking lining	*Coefficient of friction* (μ)			*Allowable pressure (p)*
	Dry	*Greasy*	*Lubricated*	*N/mm*^{2}
Cast iron on cast iron	0.15 – 0.2	0.06 – 0.10	0.05 – 0.10	1.0 – 1.75
Bronze on cast iron	–	0.05 – 0.10	0.05 – 0.10	0.56 – 0.84
Steel on cast iron	0.20 – 0.30	0.07 – 0.12	0.06 – 0.10	0.84 – 1.4
Wood on cast iron	0.20 – 0.35	0.08 – 0.12	–	0.40 – 0.62
Fibre on metal	–	0.10 – 0.20	–	0.07 – 0.28
Cork on metal	0.35	0.25 – 0.30	0.22 – 0.25	0.05 – 0.10
Leather on metal	0.3 – 0.5	0.15 – 0.20	0.12 – 0.15	0.07 – 0.28
Wire asbestos on metal	0.35 – 0.5	0.25 – 0.30	0.20 – 0.25	0.20 – 0.55
Asbestos blocks on metal	0.40 – 0.48	0.25 – 0.30	–	0.28 – 1.1
Asbestos on metal (Short action)	–	–	0.20 – 0.25	1.4 – 2.1
Metal on cast iron (Short action)	–	–	0.05 – 0.10	1.4 – 2.1

25.5 Types of Brakes

The brakes, according to the means used for transforming the energy by the braking element, are classified as :

1. Hydraulic brakes *e.g.* pumps or hydrodynamic brake and fluid agitator,
2. Electric brakes *e.g.* generators and eddy current brakes, and
3. Mechanical brakes.

Shoes of disk brakes of a racing car

The hydraulic and electric brakes cannot bring the member to rest and are mostly used where large amounts of energy are to be transformed while the brake is retarding the load such as in laboratory dynamometers, high way trucks and electric locomotives. These brakes are also used for retarding or controlling the speed of a vehicle for down-hill travel.

The mechanical brakes, according to the direction of acting force, may be divided into the following two groups :

Top

Contents

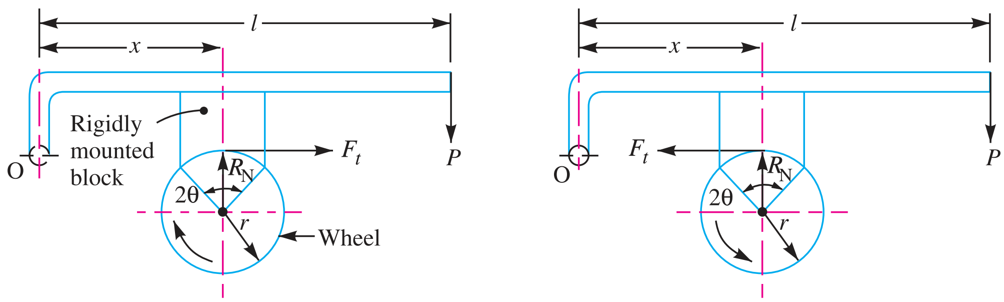
(*a*) ***Radial brakes.*** In these brakes, the force acting on the brake drum is in radial direction. The radial brakes may be sub-divided into ***external brakes*** and ***internal brakes.*** According to the shape of the friction element, these brakes may be block or shoe brakes and band brakes.

(*b*) ***Axial brakes.*** In these brakes, the force acting on the brake drum is in axial direction. The axial brakes may be disc brakes and cone brakes. The analysis of these brakes is similar to clutches.

Since we are concerned with only mechanical brakes, therefore, these are discussed in detail, in the following pages.

25.6 Single Block or Shoe Brake

A single block or shoe brake is shown in Fig. 25.1. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

Fig. 25.1. Single block brake. Line of action of tangential force passes through the fulcrum of the lever.

the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed as shown in Fig. 25.1. The other end of the lever is pivoted on a fixed fulcrum *O*.

Let P = Force applied at the end of the lever,

R_N = Normal force pressing the brake block on the wheel,

r = Radius of the wheel,

2θ = Angle of contact surface of the block,

μ = Coefficient of friction, and

F_t = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel.

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$$F_t = \mu . R_N \quad ...(i)$$

and the braking torque, $T_B = F_t . r = \mu \, R_N . r$ ...(*ii*)

Let us now consider the following three cases :

Case 1. When the line of action of tangential braking force (F_t) passes through the fulcrum *O* of the lever, and the brake wheel rotates clockwise as shown in Fig. 25.1 (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x = P \times l \quad \text{or} \quad R_N = \frac{P \times l}{x}$$

Top

Contents

∴ Braking torque, $T_B = \mu.R_N.r = \mu \times \frac{P.l}{x} \times r = \frac{\mu.P.l.r}{x}$

It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. 25.1 (*b*), then the braking torque is same, *i.e.*

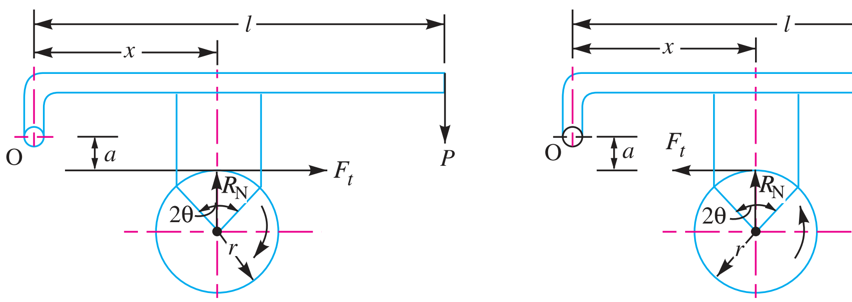
$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x}$$

Case 2. When the line of action of the tangential braking force (F_t) passes through a distance '*a*' below the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig. 25.2 (*a*), then for equilibrium, taking moments about the fulcrum *O*,

$$R_N \times x + F_t \times a = P.l$$

or $R_N \times x + \mu\, R_N \times a = P.l$ or $R_N = \frac{P.l}{x + \mu.a}$

and braking torque, $T_B = \mu\, R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

Fig. 25.2. Single block brake. Line of action of F_t passes below the fulcrum.

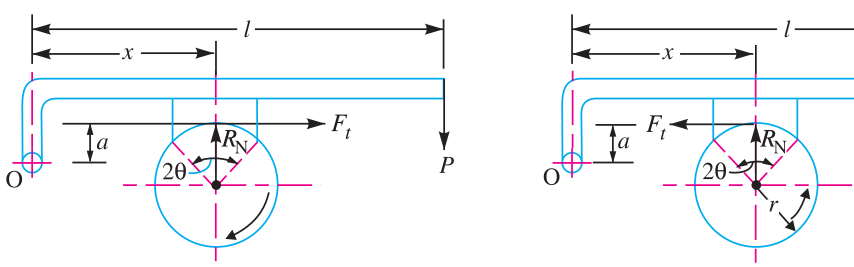
When the brake wheel rotates anticlockwise, as shown in Fig. 25.2 (*b*), then for equilibrium,

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \quad ...(i)$$

or $R_N (x - \mu.a) = P.l$ or $R_N = \frac{P.l}{x - \mu.a}$

and braking torque, $T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$

Case 3. When the line of action of the tangential braking force passes through a distance '*a*' above the fulcrum, and the brake wheel rotates clockwise as shown in Fig. 25.3 (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

Fig. 25.3. Single block brake. Line of action of F_t passes above the fulcrum.

Top

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \quad ...(ii)$$

or $R_N (x - \mu.a) = P.l$ or $R_N = \dfrac{P.l}{x - \mu.a}$

and braking torque, $T_B = \mu.R_N.r = \dfrac{\mu.P.l.r}{x - \mu.a}$

When the brake wheel rotates anticlockwise as shown in Fig. 25.3 (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x + F_t \times a = P.l$$

or $R_N \times x + \mu.R_N \times a = P.l$ or $R_N = \dfrac{P.l}{x + \mu.a}$

and braking torque, $T_B = \mu.R_N.r = \dfrac{\mu.P.l.r}{x + \mu.a}$

Notes: 1. From above we see that when the brake wheel rotates anticlockwise in case 2 [Fig. 25.2 (*b*)] and when it rotates clockwise in case 3 [Fig. 25.3 (*a*)], the equations ***(i)*** and ***(ii)*** are same, *i.e.*

$$R_N \times x = P.l + \mu.R_N.a$$

From this we see that the moment of frictional force ($\mu. R_N.a$) adds to the moment of force ($P.l$). In other words, the frictional force helps to apply the brake. Such type of brakes are said to be **self energizing brakes.** When the frictional force is great enough to apply the brake with no external force, then the brake is said to be **self-locking brake.**

From the above expression, we see that if $x \leq \mu.a$, then *P* will be negative or equal to zero. This means no external force is needed to apply the brake and hence the brake is self locking. Therefore the condition for the brake to be self locking is

$$x \leq \mu.a$$

The self-locking brake is used only in back-stop applications.

Shoe of a bicycle

2. The brake should be self-energizing and not the self-locking.

3. In order to avoid self-locking and to prevent the brake from grabbing, *x* is kept greater than $\mu.a$.

4. If A_b is the projected bearing area of the block or shoe, then the bearing pressure on the shoe,

$$p_b = R_N / A_b$$

We know that A_b = Width of shoe × Projected length of shoe = $w\,(2r \sin \theta)$

5. When a single block or shoe brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to heavy normal force (R_N) and produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake, as discussed in Art. 25.8, is used.

25.7 Pivoted Block or Shoe Brake

We have discussed in the previous article that when the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. But when the angle of contact is greater than 60°, then the unit pressure normal to the surface of contact is less at the ends than at the centre. In such cases, the block or shoe is pivoted to the lever as shown in Fig. 25.4, instead of being rigidly attached to the lever. This gives uniform wear of the brake lining in the direction of the applied force. The braking torque for a pivoted block or shoe brake (*i.e.* when $2\theta > 60°$) is given by

Contents

$$T_B = F_t \times r = \mu'.R_N.\ r$$

where μ' = Equivalent coefficient of friction = $\dfrac{4\mu \sin\theta}{2\,\theta + \sin 2\theta}$, and

μ = Actual coefficient of friction.

These brakes have more life and may provide a higher braking torque.

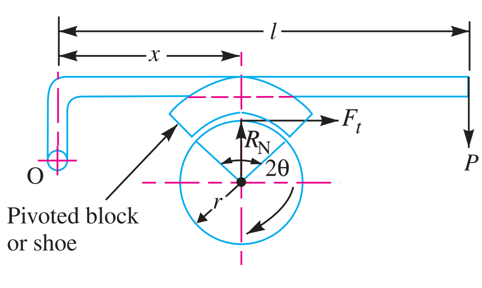
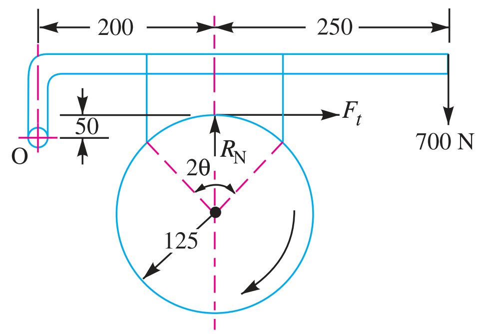


Fig. 25.4. Pivoted block or shoe brake.



All dimensions in mm.

Fig. 25.5

Example 25.2. *A single block brake is shown in Fig. 25.5. The diameter of the drum is 250 mm and the angle of contact is 90°. If the operating force of 700 N is applied at the end of a lever and the coefficient of friction between the drum and the lining is 0.35, determine the torque that may be transmitted by the block brake.*

Solution. Given : d = 250 mm or r = 125 mm ; $2\theta = 90° = \pi / 2$ rad ; P = 700 N ; μ = 0.35

Since the angle of contact is greater than 60° , therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\,\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 45°}{\pi/2 + \sin 90°} = 0.385$$

Let R_N = Normal force pressing the block to the brake drum, and

F_t = Tangential braking force = $\mu'.\ R_N$

Taking moments above the fulcrum O, we have

$$700\ (250 + 200) + F_t \times 50 = R_N \times 200 = \frac{F_t}{\mu'} \times 200 = \frac{F_t}{0.385} \times 200 = 520\ F_t$$

or $520\ F_t - 50\ F_t = 700 \times 450$ or $F_t = 700 \times 450 / 470 = 670$ N

We know that torque transmitted by the block brake,

$$T_B = F_t \times r = 670 \times 125 = 83\ 750 \text{ N-mm} = 83.75 \text{ N-m}$$ **Ans.**

Example 25.3. *Fig. 25.6 shows a brake shoe applied to a drum by a lever AB which is pivoted at a fixed point A and rigidly fixed to the shoe. The radius of the drum is 160 mm. The coefficient of friction of the brake lining is 0.3. If the drum rotates clockwise, find the braking torque due to the horizontal force of 600 N applied at B.*

Solution. Given : r = 160 mm = 0.16 m ; μ = 0.3 ; P = 600 N

Since the angle subtended by the shoe at the centre of the drum is 40°, therefore we need not to calculate the equivalent coefficient of friction (μ').

Let R_N = Normal force pressing the shoe on the drum, and

F_t = Tangential braking force = $\mu.R_N$

Top

Contents

Taking moments about point A,

$R_N \times 350 + F_t (200 - 160) = 600 (400 + 350)$

$\frac{F_t}{0.3} \times 350 + 40 F_t = 600 \times 750$

or $207 F_t = 450 \times 10^3$

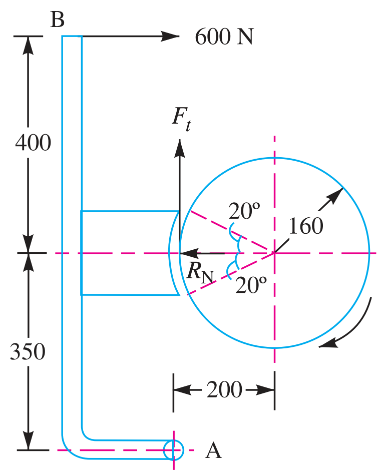
$\therefore F_t = 450 \times 10^3 / 1207 = 372.8$ N

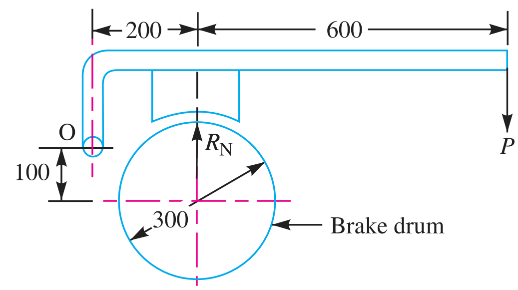
We know that braking torque,

$T_B = F_t \times r = 372.8 \times 0.16$

$= 59.65$ N-m **Ans.**

Brakes on a car wheel (inner side)





All dimensions in mm.

Fig. 25.6 **Fig. 25.7**

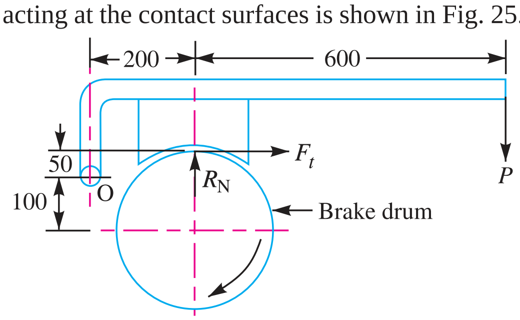
Example 25.4. *The block brake, as shown in Fig. 25.7, provides a braking torque of 360 N-m. The diameter of the brake drum is 300 mm. The coefficient of friction is 0.3. Find :*

1. *The force (P) to be applied at the end of the lever for the clockwise and counter clockwise rotation of the brake drum; and*
2. *The location of the pivot or fulcrum to make the brake self locking for the clockwise rotation of the brake drum.*

Solution. Given : T_B = 360 N-m = 360×10^3 N-mm ; d = 300 mm or r = 150 mm = 0.15 m ; μ = 0.3

1. *Force (P) for the clockwise and counter clockwise rotation of the brake drum*

For the clockwise rotation of the brake drum, the frictional force or the tangential force (F_t) acting at the contact surfaces is shown in Fig. 25.8.



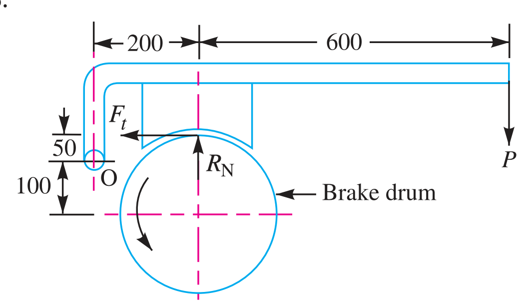


Fig. 25.8 **Fig. 25.9**

Top

Contents

We know that braking torque (T_B),

$$360 = F_t \times r = F_t \times 0.15 \quad \text{or} \quad F_t = 360 / 0.15 = 2400 \text{ N}$$

and normal force, $$R_N = F_t / \mu = 2400 / 0.3 = 8000 \text{ N}$$

Now taking moments about the fulcrum O, we have

$$P (600 + 200) + F_t \times 50 = R_N \times 200$$

$$P \times 800 + 2400 \times 50 = 8000 \times 200$$

$$P \times 800 = 8000 \times 200 - 2400 \times 50 = 1480 \times 10^3$$

$$\therefore \quad P = 1480 \times 10^3 / 800 = 1850 \text{ N}$$ **Ans.**

For the counter clockwise rotation of the drum, the frictional force or the tangential force (F_t) acting at the contact surfaces is shown in Fig. 25.9.

Taking moments about the fulcrum O, we have

$$P (600 + 200) = F_t \times 50 + R_N \times 200$$

$$P \times 800 = 2400 \times 50 + 8000 \times 200 = 1720 \times 10^3$$

$$\therefore \quad P = 1720 \times 10^3 / 800 = 2150 \text{ N}$$ **Ans.**

2. *Location of the pivot or fulcrum to make the brake self-locking*

The clockwise rotation of the brake drum is shown in Fig. 25.8. Let x be the distance of the pivot or fulcrum O from the line of action of the tangential force (F_t). Taking moments about the fulcrum O, we have

$$P (600 + 200) + F_t \times x - R_N \times 200 = 0$$

In order to make the brake self-locking, $F_t \times x$ must be equal to $R_N \times 200$ so that the force P is zero.

$$\therefore \quad F_t \times x = R_N \times 200$$

$$2400 \times x = 8000 \times 200 \quad \text{or} \quad x = 8000 \times 200 / 2400 = 667 \text{ mm}$$ **Ans.**

Example 25.5. *A rope drum of an elevator having 650 mm diameter is fitted with a brake drum of 1 m diameter. The brake drum is provided with four cast iron brake shoes each subtending an angle of 45°. The mass of the elevator when loaded is 2000 kg and moves with a speed of 2.5 m / s. The brake has a sufficient capacity to stop the elevator in 2.75 metres. Assuming the coefficient of friction between the brake drum and shoes as 0.2, find: 1. width of the shoe, if the allowable pressure on the brake shoe is limited to 0.3 N/mm²; and 2. heat generated in stopping the elevator.*

Solution. Given : d_e = 650 mm or r_e = 325 mm = 0.325 m ; d = 1 m or r = 0.5 m = 500 mm ; n = 4 ; 2 θ = 45° or θ = 22.5° ; m = 2000 kg ; v = 2.5 m / s ; h = 2.75 m ; μ = 0.2 ; p_b = 0.3 N/mm^2

1. *Width of the shoe*

Let w = Width of the shoe in mm.

First of all, let us find out the acceleration of the rope (a). We know that

$$v^2 - u^2 = 2\, a.h \quad \text{or} \quad (2.5)^2 - 0 = 2a \times 2.75 = 5.5a$$

$$\therefore \quad a = (2.5)^2 / 5.5 = 1.136 \text{ m/s}^2$$

and accelerating force = Mass × Acceleration = $m \times a$ = 2000 × 1.136 = 2272 N

∴ Total load acting on the rope while moving,

$$W = \text{Load on the elevator in newtons + Accelerating force}$$
$$= 2000 \times 9.81 + 2272 = 21\,892 \text{ N}$$

We know that torque acting on the shaft,

$$T = W \times r_e = 21\,892 \times 0.325 = 7115 \text{ N-m}$$

Top

Contents

∴ Tangential force acting on the drum

$$= \frac{T}{r} = \frac{7115}{0.5} = 14\ 230 \text{ N}$$

The brake drum is provided with four cast iron shoes, therefore tangential force acting on each shoe,

$$F_t = 14\ 230 / 4 = 3557.5 \text{ N}$$

Since the angle of contact of each shoe is 45°, therefore we need not to calculate the equivalent coefficient of friction (μ').

∴ Normal load on each shoe,

$$R_N = F_t / \mu = 3557.5 / 0.2 = 17\ 787.5 \text{ N}$$

We know that the projected bearing area of each shoe,

$$A_b = w\,(2r \sin\theta) = w\,(2 \times 500 \sin 22.5°) = 382.7\, w \text{ mm}^2$$

We also know that bearing pressure on the shoe (p_b),

$$0.3 = \frac{R_N}{A_b} = \frac{17\ 787.5}{382.7\ w} = \frac{46.5}{w}$$

∴ $w = 46.5 / 0.3 = 155$ mm **Ans.**

2. Heat generated in stopping the elevator

We know that heat generated in stopping the elevator

= Total energy absorbed by the brake

= Kinetic energy + Potential energy = $\frac{1}{2}\, m.v^2 + m.g.h$

$= \frac{1}{2} \times 2000\,(2.5)^2 + 2000 \times 9.81 \times 2.75 = 60\ 205$ N-m

= 60.205 kN-m = 60.205 kJ **Ans.**

25.8 Double Block or Shoe Brake

When a single block brake is applied to a rolling wheel, and additional load is thrown on the shaft bearings due to the normal force (R_N). This produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake as shown in Fig. 25.10, is used. It consists of two brake blocks applied at the opposite ends of a diameter of the wheel which eliminate or reduces the unbalanced force on the shaft. The brake is set by a spring which pulls the upper ends of the brake arms together. When a force *P* is applied to the bell crank lever, the spring is compressed and the brake is released. This type of brake is often used on electric cranes and the force *P* is produced by an electromagnet or solenoid. When the current is switched off, there is no force on the bell crank lever and the brake is engaged automatically due to the spring force and thus there will be no downward movement of the load.

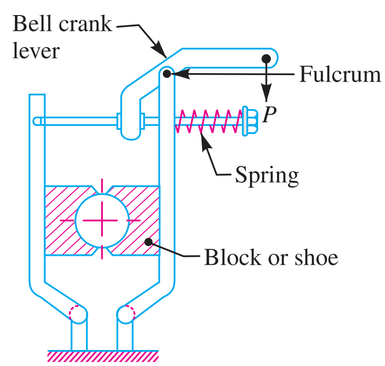


Fig. 25.10. Double block or shoe brake.

In a double block brake, the braking action is doubled by the use of two blocks and the two blocks may be operated practically by the same force which will operate one. In case of double block or shoe brake, the braking torque is given by

$$T_B = (F_{t1} + F_{t2})\, r$$

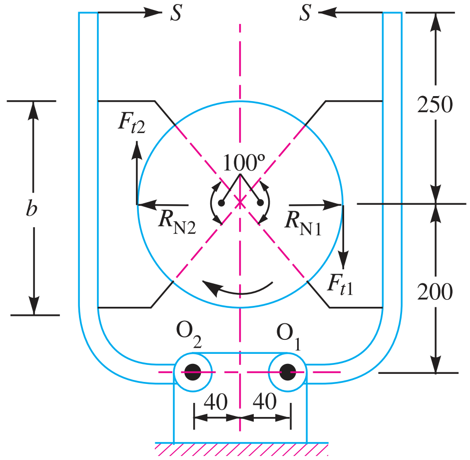
where F_{t1} and F_{t2} are the braking forces on the two blocks.

Example 25.6. *A double shoe brake, as shown in Fig. 25.11 is capable of absorbing a torque of 1400 N-m. The diameter of the brake drum is 350 mm and the angle of contact for each shoe is 100°. If the coefficient of friction between the brake drum and lining is 0.4; find : 1. the spring force*

Top

Contents

necessary to set the brake; and 2. the width of the brake shoes, if the bearing pressure on the lining material is not to exceed 0.3 N/mm².



All dimensions in mm.

Fig. 25.11

Solution. Given : T_B = 1400 N-m = 1400 × 10^3 N-mm ; d = 350 mm or r = 175 mm ; 2θ = 100° = 100 × π / 180 = 1.75 rad ; μ = 0.4 ; p_b = 0.3 N/mm²

1. ***Spring force necessary to set the brake***

Let S = Spring force necessary to set the brake,

R_{N1} and F_{t1} = Normal reaction and the braking force on the right hand side shoe, and

R_{N2} and F_{t2} = Corresponding values on the left hand side shoe.

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.4 \times \sin 50^\circ}{1.75 + \sin 100^\circ} = 0.45$$

Taking moments about the fulcrum O_1, we have

$$S \times 450 = R_{N1} \times 200 + F_{t1}\,(175 - 40) = \frac{F_{t1}}{0.45} \times 200 + F_{t1} \times 135 = 579.4\, F_{t1}$$

...(Substituting $R_{N1} = F_{t1} / \mu'$)

∴ $F_{t1} = S \times 450 / 579.4 = 0.776\, S$

(A)

(B)

Train braking system : ***(A)*** *Flexible hose carries the brakepipe between car;* ***(B)*** *Brake hydraulic cylinder and the associated hardware.*

Top

(C) (D)

Train braking system : **(C)** *Shoe of the train brake* **(D)** *Overview of train brake*

Again taking moments about O_2, we have

$$S \times 450 + F_{t2}(175 - 40) = R_{N2} \times 200 = \frac{F_{t2}}{0.45} \times 200 = 444.4\ F_{t2}$$

...(Substituting $R_{N2} = F_{t2}/\mu'$)

$444.4\ F_{t2} - 135\ F_{t2} = S \times 450$ or $309.4\ F_{t2} = S \times 450$

$\therefore \quad F_{t2} = S \times 450 / 309.4 = 1.454\ S$

We know that torque capacity of the brake (T_B),

$$1400 \times 10^3 = (F_{t1} + F_{t2})\ r = (0.776\ S + 1.454\ S)\ 175 = 390.25\ S$$

$\therefore \quad S = 1400 \times 10^3 / 390.25 = 3587$ N **Ans.**

2. *Width of the brake shoes*

Let b = Width of the brake shoes in mm.

We know that projected bearing area for one shoe,

$$A_b = b\,(2r \sin\theta) = b\,(2 \times 175 \sin 50°) = 268\ b \text{ mm}^2$$

$\therefore$ Normal force on the right hand side of the shoe,

$$R_{N1} = \frac{F_{t1}}{\mu'} = \frac{0.776 \times S}{0.45} = \frac{0.776 \times 3587}{0.45} = 6186 \text{ N}$$

and normal force on the left hand side of the shoe,

$$R_{N2} = \frac{F_{t2}}{\mu'} = \frac{1.454 \times S}{0.45} = \frac{1.454 \times 3587}{0.45} = 11\,590 \text{ N}$$

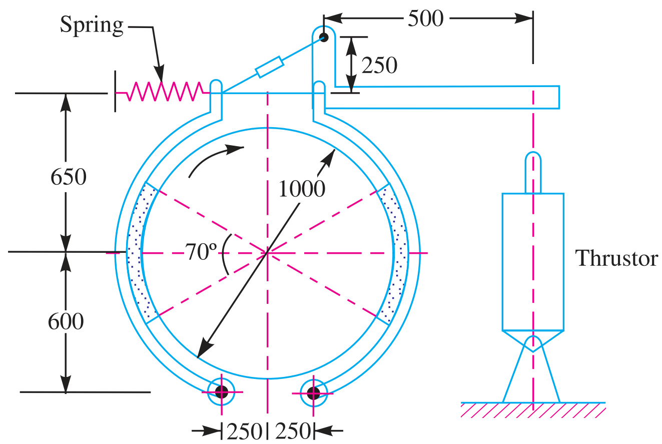
We see that the maximum normal force is on the left hand side of the shoe. Therefore we shall design the shoe for the maximum normal force *i.e.* R_{N2}.

We know that the bearing pressure on the lining material (p_b),

$$0.3 = \frac{R_{N2}}{A_b} = \frac{11\,590}{268\,b} = \frac{43.25}{b}$$

$\therefore \quad b = 43.25 / 0.3 = 144.2$ mm **Ans.**

Example 25.7. *A spring closed thrustor operated double shoe brake is to be designed for a maximum torque capacity of 3000 N-m. The brake drum diameter is not to exceed 1 metre and the shoes are to be lined with Ferrodo having a coefficient of friction 0.3. The other dimensions are as shown in Fig. 25.12.*



All dimensions in mm.

Fig. 25.12

1. *Find the spring force necessary to set the brake.*
2. *If the permissible stress of the spring material is 500 MPa, determine the dimensions of the coil assuming spring index to be 6. The maximum spring force is to be 1.3 times the spring force required during braking. There are eight active coils. Specify the length of the spring in the closed position of the brake. Modulus of rigidity is 80 kN / mm^2.*
3. *Find the width of the brake shoes if the bearing pressure on the lining material is not to exceed 0.5 N/mm^2.*
4. *Calculate the force required to be exerted by the thrustor to release the brake.*

Solution. Given : T_B = 3000 N-m = 3 × 10^6 N-mm ; d = 1 m or r = 0.5 m = 500 mm ; μ = 0.3 ; 2θ = 70° = 70 × π / 180 = 1.22 rad

1. Spring force necessary to set the brake

Let S = Spring force necessary to set the brake,

R_{N1} and F_{t1} = Normal reaction and the braking force on the right hand side shoe,

and R_{N2} and F_{t2} = Corresponding values for the left hand side shoe.

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.3 \times \sin 35°}{1.22 + \sin 70°} = 0.32$$

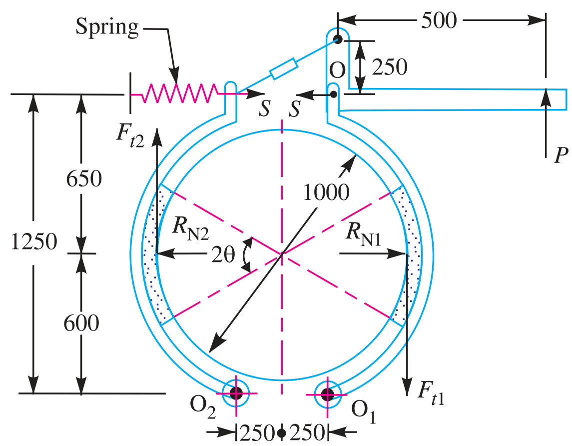
Taking moments about the fulcrum O_1 (Fig. 25.13), we have

$$S \times 1250 = R_{N1} \times 600 + F_{t1} (500 - 250)$$

$$= \frac{F_{t1}}{0.32} \times 600 + 250\ F_{t1} = 2125\ F_{t1} \qquad ...(\because\ R_{N1} = F_{t1} / \mu')$$

$\therefore$ $F_{t1} = S \times 1250 / 2125 = 0.59\ S$ N

Contents



All dimensions in mm.

Fig. 25.13

Again taking moments about the fulcrum O_2, we have

$$S \times 1250 + F_{t2}\,(500 - 250) = R_{N2} \times 600 = \frac{F_{t2}}{0.32} \times 600 = 1875\,F_{t2} \qquad ...(\because R_{N2} = F_{t2}/\mu')$$

or $\quad 1875\,F_{t2} - 250\,F_{t2} = S \times 1250 \quad$ or $\quad 1625\,F_{t2} = S \times 1250$

$\therefore \quad F_{t2} = S \times 1250\ /\ 1625 = 0.77\,S$ N

We know that torque capacity of the brake (T_B),

$$3 \times 10^6 = (F_{t1} + F_{t2})\,r = (0.59\,S + 0.77\,S)\,500 = 680\,S$$

$\therefore \quad S = 3 \times 10^6 / 680 = 4412$ N **Ans.**

2. *Dimensions of the spring coil*

Given : $\tau = 500$ MPa $= 500$ N/mm^2 ; $C = D/d = 6$; $n = 8$; $G = 80$ kN/mm^2 $= 80 \times 10^3$ N/mm^2

Let $\quad D$ = Mean diameter of the spring, and

$\quad d$ = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

Since the maximum spring force is 1.3 times the spring force required during braking, therefore maximum spring force,

$$W_S = 1.3\,S = 1.3 \times 4412 = 5736 \text{ N}$$

We know that the shear stress induced in the spring (τ),

$$500 = \frac{K \times 8 W_S . C}{\pi d^2} = \frac{1.2525 \times 8 \times 5736 \times 6}{\pi d^2} = \frac{109\ 754}{d^2}$$

$\therefore \quad d^2 = 109\ 754 / 500 = 219.5 \quad$ or $\quad d = 14.8$ say 15 mm **Ans.**

and $\quad D = C.d = 6 \times 15 = 90$ mm **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8 W_S . C^3 . n}{G . d} = \frac{8 \times 5736 \times 6^3 \times 8}{80 \times 10^3 \times 15} = 66 \text{ mm}$$

Top

The length of the spring in the closed position of the brake will be its free length. Assuming that the ends of the coil are squared and ground, therefore total number of coils,

$$n' = n + 2 = 8 + 2 = 10$$

∴ Free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta$$

$$= 10 \times 15 + 66 + 0.15 \times 66 = 226 \text{ mm}$$ **Ans.**

3. *Width of the brake shoes*

Let b = Width of the brake shoes in mm, and

p_b = Bearing pressure on the lining material of the shoes.

$= 0.5 \text{ N/mm}^2$ ...(Given)

We know that projected bearing area for one shoe,

$$A_b = b\,(2r.\sin\theta) = b\,(2 \times 500 \sin 35°) = 574\, b \text{ mm}^2$$

We know that normal force on the right hand side of the shoe,

$$R_{N1} = \frac{F_{t1}}{\mu'} = \frac{0.59\,S}{0.32} = \frac{0.59 \times 4412}{0.32} = 8135 \text{ N}$$

and normal force on the left hand side of the shoe,

$$R_{N2} = \frac{F_{t2}}{\mu'} = \frac{0.77\,S}{0.32} = \frac{0.77 \times 4412}{0.32} = 10\,616 \text{ N}$$

We see that the maximum normal force is on the left hand side of the shoe. Therefore we shall design the shoe for the maximum normal force *i.e.* R_{N2}.

We know that bearing pressure on the lining material (p_b),

$$0.5 = \frac{R_{N2}}{A_b} = \frac{10\,616}{574\,b} = \frac{18.5}{b}$$

∴ $b = 18.5 / 0.5 = 37$ mm **Ans.**

4. *Force required to be exerted by the thrustor to release the brake*

Let P = Force required to be exerted by the thrustor to release the brake.

Taking moments about the fulcrum of the lever O, we have

$$P \times 500 + R_{N1} \times 650 = F_{t1}\,(500 - 250) + F_{t2}\,(500 + 250) + R_{N2} \times 650$$

$$P \times 500 + 8135 \times 650 = 0.59 \times 4412 + 250 + 0.77 \times 4412 \times 750 + 10\,616 \times 650$$

...(Substituting $F_{t1} = 0.59\,S$ and $F_{t2} = 0.77\,S$)

$$P \times 500 + 5.288 \times 10^6 = 0.65 \times 10^6 + 2.55 \times 10^6 + 6.9 \times 10^6 = 10.1 \times 10^6$$

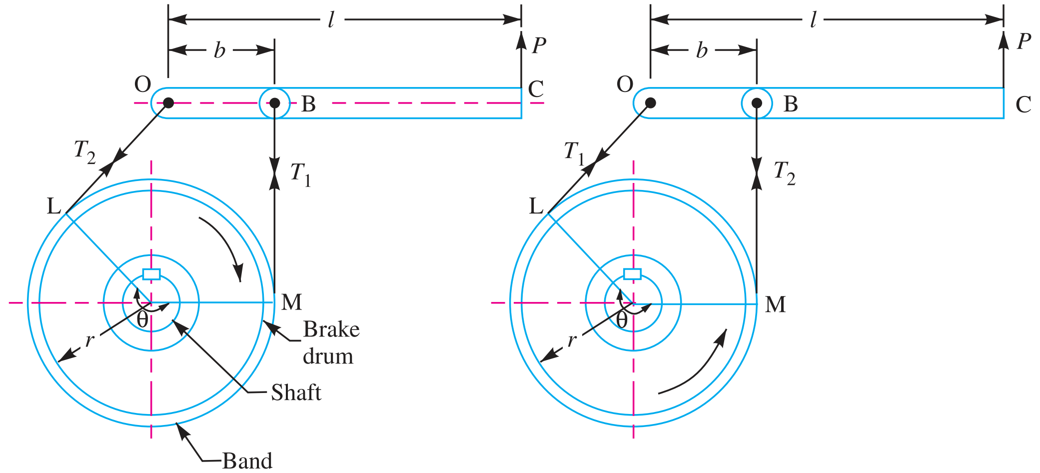
$$\therefore \quad P = \frac{10.1 \times 10^6 - 5.288 \times 10^6}{500} = 9624 \text{ N}$$ **Ans.**

25.9 Simple Band Brake

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Fig. 25.14, is called a ***simple band brake*** in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum.

Top

When a force P is applied to the lever at C, the lever turns about the fulcrum pin O and tightens the band on the drum and hence the brakes are applied. The friction between the band and the drum provides the braking force. The force P on the lever at C may be determined as discussed below :



(*a*) Clockwise rotation of drum. (*b*) Anticlockwise rotation of drum.

Fig. 25.14. Simple band brake.

Let T_1 = Tension in the tight side of the band,

T_2 = Tension in the slack side of the band,

θ = Angle of lap (or embrace) of the band on the drum,

μ = Coefficient of friction between the band and the drum,

r = Radius of the drum,

t = Thickness of the band, and

r_e = Effective radius of the drum = $r + t / 2$.

We know that limiting ratio of the tensions is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad 2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta$$

and braking force on the drum

$$= T_1 - T_2$$

∴ Braking torque on the drum,

$$T_B = (T_1 - T_2)\, r \quad \text{...(Neglecting thickness of band)}$$

$$= (T_1 - T_2)\, r_e \quad \text{...(Considering thickness of band)}$$

Now considering the equilibrium of the lever OBC. It may be noted that when the drum rotates in the clockwise direction as shown in Fig. 25.14 (*a*), the end of the band attached to the fulcrum O will be slack with tension T_2 and end of the band attached to B will be tight with tension T_1. On the other hand, when the drum rotates in the anticlockwise direction as shown in Fig. 25.14 (*b*), the tensions in the band will reverse, *i.e.* the end of the band attached to the fulcrum O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. Now taking moments about the fulcrum O, we have

$$P.l = T_1.b \quad \text{...(for clockwise rotation of the drum)}$$

and

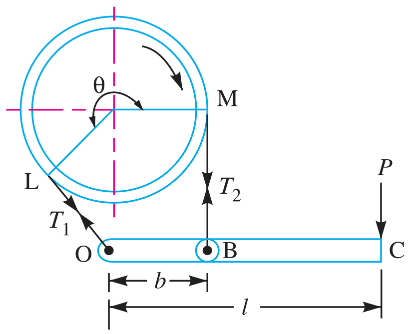
$$P.l = T_2.b \quad \text{...(for anticlockwise rotation of the drum)}$$

where l = Length of the lever from the fulcrum (OC), and

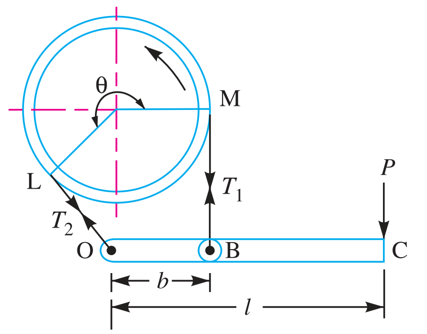
b = Perpendicular distance from O to the line of action of T_1 or T_2.

Notes: 1. When the brake band is attached to the lever, as shown in Fig. 25.14 (***a***) and (*b*), then the force (P) must act in the upward direction in order to tighten the band on the drum.

2. Sometimes the brake band is attached to the lever as shown in Fig. 25.15 (*a*) and (*b*), then the force (P) must act in the downward direction in order to tighten the band. In this case, for clockwise rotation of the drum, the end of the band attached to the fulcrum O will be tight with tension T_1 and band of the band attached to B will be slack with tension T_2. The tensions T_1 and T_2 will reverse for anticlockwise rotation of the drum.



(*a*) Clockwise rotation of drum.



(*b*) Anticlockwise rotation of drum.

Fig. 25.15. Simple band brake.

3. If the permissible tensile stress (σ_t) for the material of the band is known, then maximum tension in the band is given by

$$T_1 = \sigma_t \times w \times t$$

where w = Width of the band, and

t = Thickness of the band.

4. The width of band (w) should not exceed 150 mm for drum diameter (d) greater than 1 metre and 100 mm for drum diameter less than 1 metre. The band thickness (t) may also be obtained by using the empirical relation *i.e.* $t = 0.005\ d$

For brakes of hand operated winches, the steel bands of the following sizes are usually used :

Width of band (w) in mm	25 – 40	40 – 60	80	100	140 – 200
Thickness of band (t) in mm	3	3 – 4	4 – 6	4 – 7	6 – 10

Example 25.8. *A simple band brake operates on a drum of 600 mm in diameter that is running at 200 r.p.m. The coefficient of friction is 0.25. The brake band has a contact of 270°, one end is fastened to a fixed pin and the other end to the brake arm 125 mm from the fixed pin. The straight brake arm is 750 mm long and placed perpendicular to the diameter that bisects the angle of contact.*

(a) What is the pull necessary on the end of the brake arm to stop the wheel if 35 kW is being absorbed ? What is the direction for this minimum pull ?

(b) What width of steel band of 2.5 mm thick is required for this brake if the maximum tensile stress is not to exceed 50 MPa ?

Solution. Given : d = 600 mm or r = 300 mm ; N = 200 r.p.m. ; μ = 0.25 ; θ = 270° = 270 × π/180 = 4.713 rad ; Power = 35 kW = 35 × 10^3 W ; t = 2.5 mm ; σ_t = 50 MPa = 50 N/mm^2

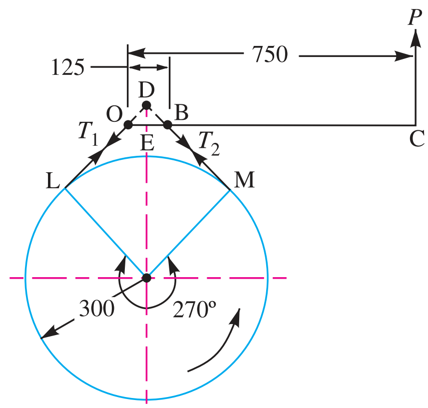
(a) Pull necessary on the end of the brake arm to stop the wheel

Let P = Pull necessary on the end of the brake arm to stop the wheel.

Band brake

Bands of a brake shown separately

The simple band brake is shown in Fig. 25.16. Since one end of the band is attached to the fixed pin O, therefore the pull P on the end of the brake arm will act upward and when the wheel rotates anticlockwise, the end of the band attached to O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. First of all, let us find the tensions T_1 and T_2. We know that



All dimensions in mm.

Fig. 25.16

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 4.713 = 1.178$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = 1.178 / 2.3 = 0.5123$$

or $$\frac{T_1}{T_2} = 3.25 \qquad \text{...}(i)$$

...(Taking antilog of 0.5123)

Let T_B = Braking torque.

We know that power absorbed,

$$35 \times 10^3 = \frac{2\pi N.T_B}{60} = \frac{2\pi \times 200 \times T_B}{60} = 21\ T_B$$

$$\therefore \quad T_B = 35 \times 10^3 / 21 = 1667 \text{ N-m} = 1667 \times 10^3 \text{ N-mm}$$

We also know that braking torque (T_B),

$$1667 \times 10^3 = (T_1 - T_2)\, r = (T_1 - T_2)\, 300$$

$$\therefore \quad T_1 - T_2 = 1667 \times 10^3 / 300 = 5557 \text{ N} \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*), we find that

$$T_1 = 8027 \text{ N} \;;\; \text{and } T_2 = 2470 \text{ N}$$

Now taking moments about O, we have

$$P \times 750 = T_2 \times {}^*OD = T_2 \times 62.5\sqrt{2} = 2470 \times 88.4 = 218\ 348$$

$$\therefore \quad P = 218\ 348 / 750 = 291 \text{ N} \quad \textbf{Ans.}$$

* OD = Perpendicular distance from O to the line of action of tension T_2.

$OE = EB = OB / 2 = 125 / 2 = 62.5$ mm, and $\angle DOE = 45°$

$\therefore \quad OD = OE \sec 45° = 62.5\sqrt{2}$ mm

(b) Width of steel band

Let w = Width of steel band in mm.

We know that maximum tension in the band (T_1),

$$8027 = \sigma_t \times w \times t = 50 \times w \times 2.5 = 125\, w$$

$$\therefore \quad w = 8027/125 = 64.2 \text{ mm}$$ **Ans.**

Example 25.9. *A band brake acts on the $\frac{3}{4}$th of circumference of a drum of 450 mm diameter which is keyed to the shaft. The band brake provides a braking torque of 225 N-m. One end of the band is attached to a fulcrum pin of the lever and the other end to a pin 100 mm from the fulcrum. If the operating force applied at 500 mm from the fulcrum and the coefficient of friction is 0.25, find the operating force when the drum rotates in the anticlockwise direction.*

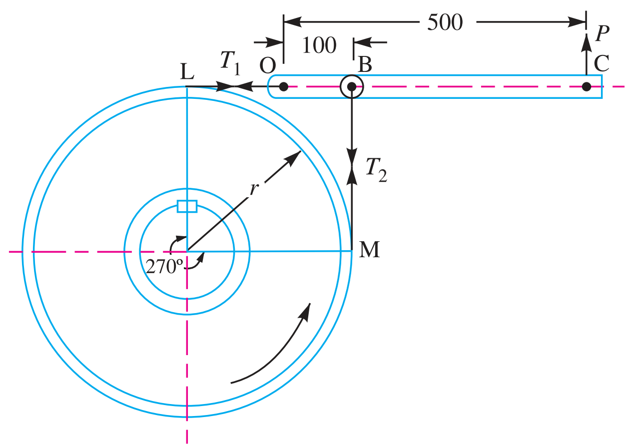
If the brake lever and pins are to be made of mild steel having permissible stresses for tension and crushing as 70 MPa and for shear 56 MPa, design the shaft, key, lever and pins. The bearing pressure between the pin and the lever may be taken as 8 N/mm².

Solution. Given : d = 450 mm or r = 225 mm ; T_B = 225 N-m = 225 × 10^3 N-mm ; OB = 100 mm ; l = 500 mm ; μ = 0.25 ; $\sigma_t = \sigma_c$ = 70 MPa = 70 N/mm² ; τ = 56 MPa = 56 N/mm² ; p_b = 8 N/mm²

Operating force

Let P = Operating force.

The band brake is shown in Fig. 25.17. Since one end of the band is attached to the fulcrum at O, therefore the operating force P will act upward and when the drum rotates anticlockwise, the end of the band attached to O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. First of all, let us find the tensions T_1 and T_2.



All dimensions in mm.

Fig. 25.17

We know that angle of wrap,

$$\theta = \frac{3}{4}\text{th of circumference} = \frac{3}{4} \times 360° = 270°$$

$$= 270 \times \frac{\pi}{180} = 4.713 \text{ rad}$$

Contents

and $$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 4.713 = 1.178$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{1.178}{2.3} = 0.5123 \quad \text{or} \quad \frac{T_1}{T_2} = 3.25 \quad \text{...(i)}$$

...(Taking antilog of 0.5123)


Drums for band brakes.

We know that braking torque (T_B),

$$225 \times 10^3 = (T_1 - T_2)\, r = (T_1 - T_2)\, 225$$

$$\therefore \quad T_1 - T_2 = 225 \times 10^3 / 225 = 1000 \text{ N} \quad \text{...(ii)}$$

From equations **(*i*)** and **(*ii*),** we have

$$T_1 = 1444 \text{ N and } T_2 = 444 \text{ N}$$

Taking moments about the fulcrum O, we have

$$P \times 500 = T_2 \times 100 = 444 \times 100 = 44\,400$$

$$\therefore \quad P = 44\,400 / 500 = 88.8 \text{ N}$$ **Ans.**

Design of shaft

Let d_s = Diameter of the shaft in mm.

Since the shaft has to transmit torque equal to the braking torque (T_B), therefore

$$225 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_s)^3 = \frac{\pi}{16} \times 56\,(d_s)^3 = 11\,(d_s)^3$$

$$\therefore \quad (d_s)^3 = 225 \times 10^3/11 = 20.45 \times 10^3 \text{ or } d_s = 27.3 \text{ say } 30 \text{ mm}$$ **Ans.**

Design of key

The standard dimensions of the key for a 30 mm diameter shaft are as follows :

Width of key, w = 10 mm **Ans.**

Thickness of key, t = 8 mm **Ans.**

Let l = Length of key.

Considering the key in shearing, we have braking torque (T_B),

$$225 \times 10^3 = l \times w \times \tau \times \frac{d_s}{2} = l \times 10 \times 56 \times \frac{30}{2} = 8400\, l$$

$$\therefore \quad l = 225 \times 10^3 / 8400 = 27 \text{ mm}$$

Now considering the key in crushing, we have braking torque (T_B),

$$225 \times 10^3 = l \times \frac{t}{2} \times \sigma_c \times \frac{d_s}{2} = l \times \frac{8}{2} \times 70 \times \frac{30}{2} = 4200\, l$$

$$\therefore \quad l = 225 \times 10^3 / 4200 = 54 \text{ mm}$$

Taking larger of two values, we have l = 54 mm **Ans.**

Design of lever

Let t_1 = Thickness of the lever in mm, and

B = Width of the lever in mm.

The lever is considered as a cantilever supported at the fulcrum O. The effect of T_2 on the lever for determining the bending moment on the lever is neglected. This error is on the safer side.

∴ Maximum bending moment at O due to the force P,

$$M = P \times l = 88.8 \times 500 = 44\,400 \text{ N-m}$$

Section modulus,

$$Z = \frac{1}{6}\, t_1.B^2 = \frac{1}{6}\, t_1\, (2t_1)^2 = 0.67\, (t_1)^3 \text{ mm}^3 \quad \text{...(Assuming } B = 2t_1)$$

Top

Contents

We know that the bending stress (σ_t),

$$70 = \frac{M}{Z} = \frac{44\ 400}{0.67\ (t_1)^3} = \frac{66\ 300}{(t_1)^3}$$

$\therefore$ $(t_1)^3 = 66\ 300 / 70 = 947$ or $t_1 = 9.82$ say 10 mm **Ans.**

and $B = 2\ t_1 = 2 \times 10 = 20$ mm **Ans.**

Design of pins

Let d_1 = Diameter of the pins at O and B, and

l_1 = Length of the pins at O and $B = 1.25\ d_1$ ...(Assume)

The pins at O and B are designed for the maximum tension in the band (*i.e.* $T_1 = 1444$ N),

Considering bearing of the pins at O and B, we have maximum tension (T_1),

$$1444 = d_1.l_1.p_b = d_1 \times 1.25\ d_1 \times 8 = 10\ (d_1)^2$$

$\therefore$ $(d_1)^2 = 1444 / 10 = 144.4$ or $d_1 = 12$ mm **Ans.**

and $l_1 = 1.25\ d_1 = 1.25 \times 12 = 15$ mm **Ans.**

Let us now check the pin for induced shearing stress. Since the pin is in double shear, therefore maximum tension (T_1),

$$1444 = 2 \times \frac{\pi}{4}\ (d_1)^2\ \tau = 2 \times \frac{\pi}{4}\ (12)^2\ \tau = 226\ \tau$$

$\therefore$ $\tau = 1444 / 226 = 6.4\ \text{N/mm}^2 = 6.4$ MPa

This induced stress is quite within permissible limits.

The pin may be checked for induced bending stress. We know that maximum bending moment,

$$M = \frac{5}{24} \times W.l_1 = \frac{5}{24} \times 1444 \times 15 = 4513\ \text{N-mm}$$

... (Here $W = T_1 = 1444$ N)

and section modulus, $$Z = \frac{\pi}{32}\ (d_1)^3 = \frac{\pi}{32}\ (12)^3 = 170\ \text{mm}^3$$

$\therefore$ Bending stress induced

$$= \frac{M}{Z} = \frac{4513}{170} = 26.5\ \text{N-mm}^2 = 26.5\ \text{MPa}$$

This induced bending stress is within safe limits of 70 MPa.

The lever has an eye hole for the pin and connectors at band have forked end.

Thickness of each eye,

$$t_2 = \frac{l_1}{2} = \frac{15}{2} = 7.5\ \text{mm}$$

Outer diameter of the eye,

$$D = 2d_1 = 2 \times 12 = 24\ \text{mm}$$

A clearance of 1.5 mm is provided on either side of the lever in the fork.

A brass bush of 3 mm thickness may be provided in the eye of the lever.

$\therefore$ Diameter of hole in the lever

$$= d_1 + 2 \times 3 = 12 + 6 = 18\ \text{mm}$$

Top

The boss is made at pin joints whose outer diameter is taken equal to twice the diameter of the pin and length equal to length of the pin.

The inner diameter of the boss is equal to diameter of hole in the lever.

∴ Outer diameter of boss

$= 2\, d_1 = 2 \times 12 = 24$ mm

and length of boss $= l_1 = 15$ mm

Let us now check the bending stress induced in the lever at the fulcrum. The section of the lever at the fulcrum is shown in Fig. 25.18.

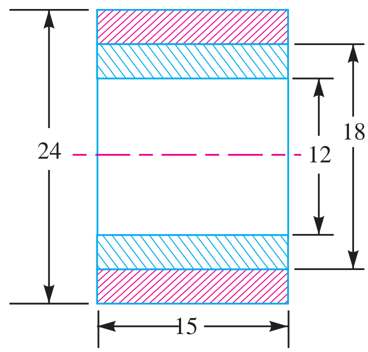


Fig. 25.18

We know that maximum bending moment at the fulcrum,

$$M = P.l = 88.8 \times 500$$
$$= 44\,400 \text{ N-mm}$$

and section modulus,

$$Z = \frac{\frac{1}{12} \times 15\left[(24)^3 - (18)^3\right]}{24/2}$$
$$= 833 \text{ mm}^3$$

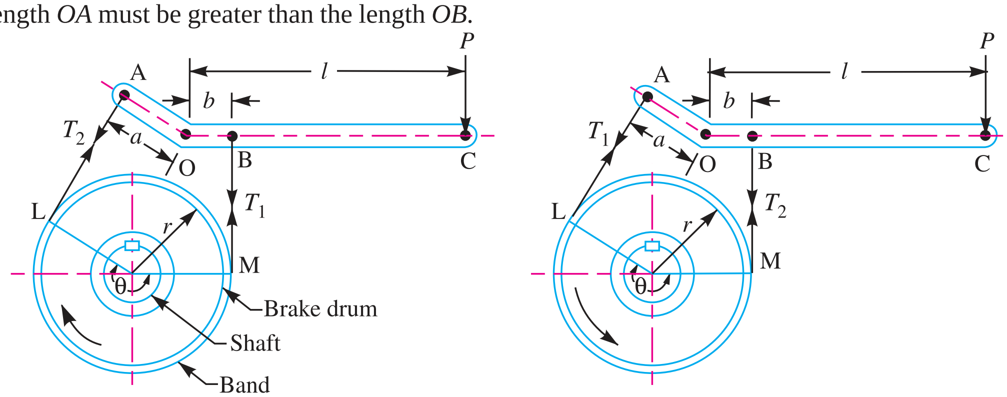
∴ Bending stress induced

$$= \frac{M}{Z} = \frac{44\,400}{833} = 53.3 \text{ N/mm}^2$$
$$= 53.3 \text{ MPa}$$

This induced stress is within safe limits of 70 MPa.

25.10 Differential Band Brake

In a differential band brake, as shown in Fig. 25.19, the ends of the band are joined at *A* and *B* to a lever *AOC* pivoted on a fixed pin or fulcrum *O*. It may be noted that for the band to tighten, the length *OA* must be greater than the length *OB*.



(*a*) Clockwise rotation of the drum. (*b*) Anticlockwise rotation of the drum.

Fig. 25.19. Differenctial band brake.

The braking torque on the drum may be obtained in the similar way as discussed in simple band brake. Now considering the equilibrium of the lever *AOC*. It may be noted that when the drum rotates in the clockwise direction, as shown in Fig. 25.19 (*a*), the end of the band attached to *A* will be slack with tension T_2 and end of the band attached to *B* will be tight with tension T_1. On the other hand, when the drum rotates in the anticlockwise direction, as shown in Fig. 25.19 (*b*), the end of the band attached to *A* will be tight with tension T_1 and end of the band attached to *B* will be slack with tension T_2. Now taking moments about the fulcrum *O*, we have

$$P.l + T_1.b = T_2.a \qquad \text{...(for clockwise rotation of the drum)}$$

or $$P.l = T_2.a - T_1.b \qquad \text{...}(i)$$

and $$P.l + T_2.b = T_1.a \qquad \text{...(for anticlockwise rotation of the drum)}$$

or $$P.l = T_1.a - T_2.b \qquad \text{...}(ii)$$

We have discussed in block brakes (Art. 25.6), that when the frictional force helps to apply the brake, it is said to be self energizing brake. In case of differential band brake, we see from equations **(*i*)** and **(*ii*)** that the moment $T_1.b$ and $T_2.b$ helps in applying the brake (because it adds to the moment $P.l$) for the clockwise and anticlockwise rotation of the drum respectively.

We have also discussed that when the force P is negative or zero, then brake is self locking. Thus for differential band brake and for clockwise rotation of the drum, the condition for self- locking is

$$T_2.a \le T_1.b \quad \text{or} \quad T_2/T_1 \le b / a$$

and for anticlockwise rotation of the drum, the condition for self-locking is

$$T_1.a \le T_2.b \quad \text{or} \quad T_1/T_2 \le b / a$$

Notes: 1. The condition for self-locking may also be written as follows. For clockwise rotation of the drum,

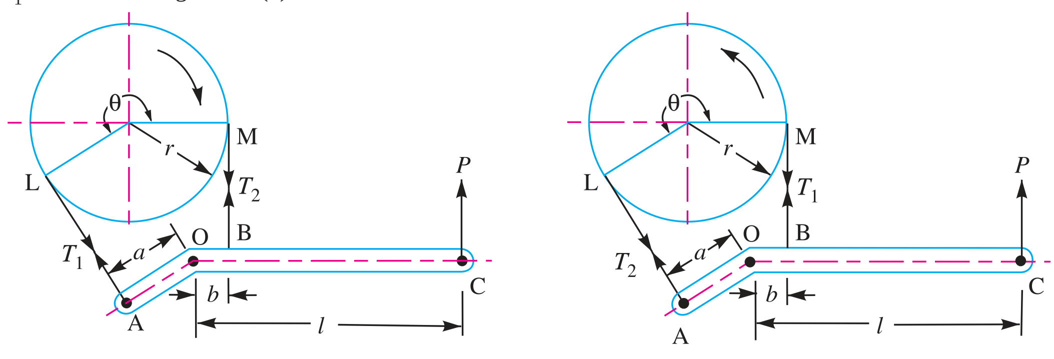
$$T_1.b \ge T_2.a \quad \text{or} \quad T_1/T_2 \ge a / b$$

and for anticlockwise rotation of the drum,

$$T_2.b \ge T_1.a \quad \text{or} \quad T_2/T_1 \ge a/b$$

2. When in Fig. 25.19 (*a*) and (*b*), the length *OB* is greater than *OA*, then the force P must act in the upward direction in order to apply the brake. The tensions in the band, *i.e.* T_1 and T_2 will remain unchanged.

3. Sometimes, the band brake is attached to the lever as shown in Fig. 25.20 (*a*) and (*b*). In such cases, when *OA* is greater than *OB*, the force (P) must act upwards. When the drum rotates in the clockwise direction, the end of the band attached to *A* will be tight with tension T_1 and the end of the band attached to *B* will be slack with tension T_2, as shown in Fig. 25.20 (*a*). When the drum rotates in the anticlockwise direction, the end of the band attached to *A* will be slack with tension T_2 and the end of the band attached to *B* will be tight with tension T_1, as shown in Fig. 25.20 (*b*).



(*a*) Clockwise rotation of the drum. (*b*) Anticlockwise rotation of the drum.

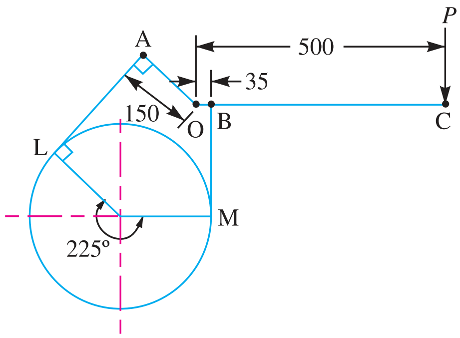
Fig. 25.20. Differential band brake.

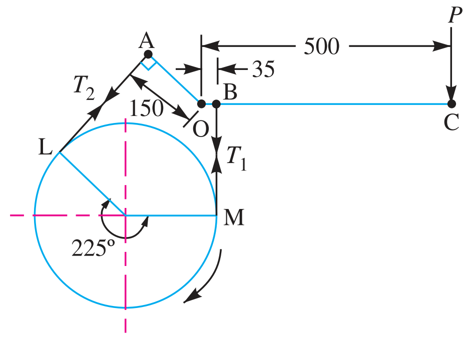
4. When in Fig. 25.20 (*a*) and (*b*), the length *OB* is greater than *OA*, then the force (P) must act downward in order to apply the brake. The position of tensions T_1 and T_2 will remain unchanged.

Example 25.10. *A differential band brake, as shown in Fig. 25.21, has an angle of contact of 225°. The band has a compressed woven lining and bears against a cast iron drum of 350 mm diameter. The brake is to sustain a torque of 350 N-m and the coefficient of friction between the band and the drum is 0.3. Find : 1. the necessary force (P) for the clockwise and anticlockwise rotation of the drum; and 2. The value of 'OA' for the brake to be self locking, when the drum rotates clockwise.*

Contents

Solution. Given : θ = 225° = 225 × π / 180 = 3.93 rad ; d = 350 mm or r = 175 mm ; T = 350 N-m = 350 × 10^3 N-mm ; μ = 0.3





All dimensions in mm.

Fig. 25.21 **Fig. 25.22**

1. *Necessary force (P) for the clockwise and anticlockwise rotation of the drum*

When the drum rotates in the clockwise direction, the end of the band attached to A will be slack with tension T_2 and the end of the band attached to B will be tight with tension T_1, as shown in Fig. 25.22. First of all, let us find the values of tensions T_1 and T_2.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.93 = 1.179$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{1.179}{2.3} = 0.5126$$

or

$$\frac{T_1}{T_2} = 3.256 \quad \text{...(Taking antilog of 0.5126)} \quad \text{...}(i)$$

and braking torque (T_B),

$$350 \times 10^3 = (T_1 - T_2)\, r = (T_1 - T_2)\, 175$$

$$\therefore \quad T_1 - T_2 = 350 \times 10^3 / 175 = 2000 \text{ N} \quad \text{...}(ii)$$

Another picture of car brake shoes

Top

Contents

From equations **(*i*)** and **(*ii*)**, we find that

$$T_1 = 2886.5 \text{ N} ; \text{ and } T_2 = 886.5 \text{ N}$$

Now taking moments about the fulcrum *O*, we have

$$P \times 500 = T_2 \times 150 - T_1 \times 35$$

or $$P \times 500 = 886.5 \times 150 - 2886.5 \times 35 = 31\,947.5$$

$\therefore$ $$P = 31\,947.5 / 500 = 64 \text{ N} \quad \textbf{Ans.}$$

When the drum rotates in the anticlockwise direction, the end of the band attached to *A* will be tight with tension T_1 and end of the band attached to *B* will be slack with tension T_2, as shown in Fig. 25.23. Taking moments about the fulcrum *O*, we have

$$P \times 500 = T_1 \times 150 - T_2 \times 35$$

or $$P \times 500 = 2886.5 \times 150 - 886.5 \times 35 = 401\,947.5$$

$\therefore$ $$P = 401\,947.5/500 = 804 \text{ N} \quad \textbf{Ans.}$$

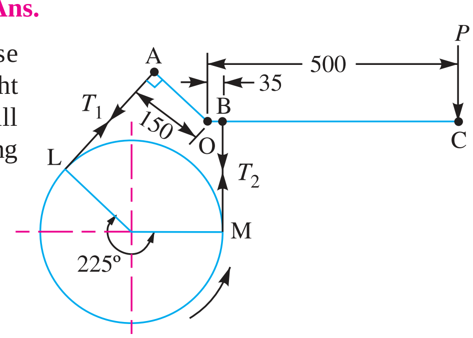


Fig. 25.23

2. *Valve of 'OA' for the brake to be self locking, when the drum rotates clockwise*

The clockwise rotation of the drum is shown in Fig. 25.22.

For clockwise rotation of the drum, we know that

$$P \times 500 + T_1 \times OB = T_2 \times OA$$

or $$P \times 500 = T_2 \times OA - T_1 \times OB$$

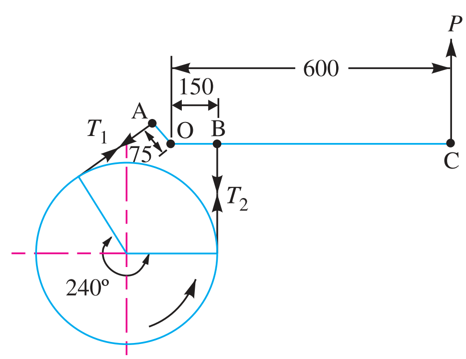
For the brake to be self-locking, *P* must be equal to zero or

$$T_2 \times OA = T_1 \times OB$$

or $$OA = \frac{T_1 \times OB}{T_2} = \frac{2886.5 \times 35}{886.5} = 114 \text{ mm} \quad \textbf{Ans.}$$

Example 25.11. *A differential band brake, as shown in Fig. 25.24, has a drum diameter of 600 mm and the angle of contact is 240°. The brake band is 5 mm thick and 100 mm wide. The coefficient of friction between the band and the drum is 0.3. If the band is subjected to a stress of 50 MPa, find :*

1. *The least force required at the end of a 600 mm lever, and*
2. *The torque applied to the brake drum shaft.*



All dimensions in mm.

Fig. 25.24

Solution. Given : d = 600 mm or r = 300 mm = 0.3 m ; θ = 240° = 240 × π / 180 = 4.2 rad ; t = 5 mm ; w = 100 mm ; μ = 0.3; σ_t = 50 MPa = 50 N/mm^2

1. *Least force required at the end of a lever*

Let $$P = \text{Least force required at the end of the lever.}$$

Top

Since the length *OB* is greater than *OA*, therefore the force at the end of the lever (*P*) must act in the upward direction. When the drum rotates anticlockwise, the end of the band attached to *A* will be tight with tension T_1 and the end of the band attached to *B* will be slack with tension T_2. First of all, let us find the values of tensions T_1 and T_2. We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 4.2 = 1.26$$

$$\log \left(\frac{T_1}{T_2}\right) = \frac{1.26}{2.3} = 0.5478 \quad \text{or} \quad \frac{T_1}{T_2} = 3.53 \quad \text{... (Taking antilog of 0.5478)} \quad ...(i)$$

We know that maximum tension in the band,

$$T_1 = \text{Stress} \times \text{Area of band} = \sigma_t \times t \times w$$
$$= 50 \times 5 \times 100 = 25\,000 \text{ N}$$

and $$T_2 = T_1 / 3.53 = 25\,000 / 3.53 = 7082 \text{ N} \quad \text{...[From equation } (i)]$$

Now taking moments about the fulcrum *O*, we have

$$P \times 600 + T_1 \times 75 = T_2 \times 150$$

$$\therefore \quad P = \frac{T_2 \times 150 - T_1 \times 75}{600} = \frac{7082 \times 150 - 25\,000 \times 75}{600} = -1355 \text{ N}$$

$$= 1355 \text{ N (in magnitude)} \quad \textbf{Ans.}$$

Since *P* is negative, therefore the brake is self-locking.

2. Torque applied to the brake drum shaft

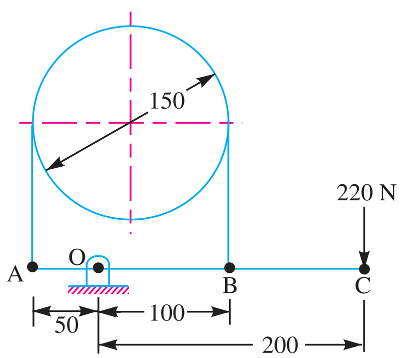
We know that torque applied to the brake drum shaft,

$$T_B = (T_1 - T_2)\, r = (25\,000 - 7082)\, 0.3 = 5375 \text{ N-m} \quad \textbf{Ans.}$$

Example 25.12. *A differential band brake has a force of 220 N applied at the end of a lever as shown in Fig. 25.25. The coefficient of friction between the band and the drum is 0.4. The angle of lap is 180°. Find :*

1. The maximum and minimum force in the band, when a clockwise torque of 450 N-m is applied to the drum; and

2. The maximum torque that the brake may sustain for counter clockwise rotation of the drum.



All dimensions in mm.

Fig. 25.25

Solution. Given : $P = 220$ N ; $\mu = 0.4$; $\theta = 180° = \pi$ rad ; $d = 150$ mm or $r = 75$ mm $= 0.075$ m

1. Maximum and minimum force in the band

Let T_1 = Maximum force in the band,

T_2 = Minimum force in the band,

and T_B = Torque applied to the drum = 450 N-m ...(Given)

In a differential band brake, when *OB* is greater than *OA* and the clockwise torque (T_B) is applied to the drum, then the maximum force (T_1) will be in the band attached to *A* and the minimum force (T_2) will be in the band attached to *B*, as shown in Fig. 25.26.

We know that braking torque (T_B),

$$450 = (T_1 - T_2)\, r = (T_1 - T_2)\, 0.075$$

$$\therefore \quad T_1 - T_2 = 450 / 0.075 = 6000 \text{ N}$$

or $$T_1 = (T_2 + 6000) \text{ N} \quad ...(i)$$

Now taking moments about the pivot O, we have

$$220 \times 200 + T_1 \times 50 = T_2 \times 100$$

$$44\,000 + (T_2 + 6000)\,50 = T_2 \times 100$$

$$44\,000 + 50\,T_2 + 300\,000 = T_2 \times 100$$

or $T_2 = 6880$ N **Ans.**

and $T_1 = T_2 + 6000$

$= 6880 + 6000 = 12\,880$ N **Ans.** ...[From equation (*i*)]

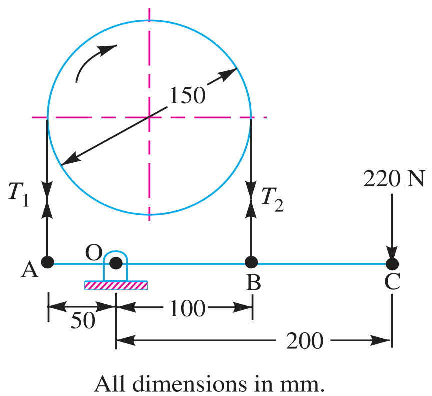


Fig. 25.26

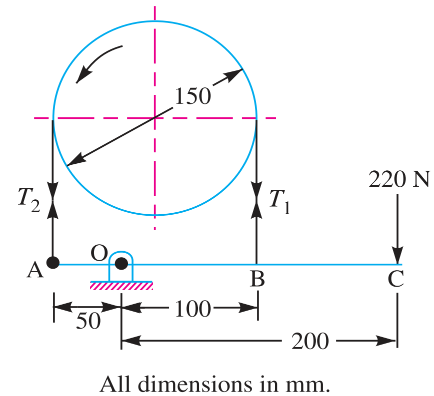


Fig. 25.27

2. *Maximum torque that the brake may sustain for counter clockwise rotation of the drum.*

When the drum rotates in the counter clockwise direction, the maximum force (T_1) will be in the band attached to B and the minimum force (T_2) will be in the band attached to A, as shown in Fig. 25.27. We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.4 \times \pi = 1.257$$

$$\log \left(\frac{T_1}{T_2}\right) = \frac{1.257}{2.3} = 0.5465$$

$\therefore$ $\dfrac{T_1}{T_2} = 3.52$ ...(Taking antilog of 0.5465) ...(*i*)

Now taking moments about the pivot O, we have

$220 \times 200 + T_2 \times 50 = T_1 \times 100 = 3.52\,T_2 \times 100 = 352\,T_2$ ...[From equation (*i*)]

or $44\,000 = 352\,T_2 - 50\,T_2 = 302\,T_2$

$\therefore$ $T_2 = 44\,000 / 302 = 146$ N

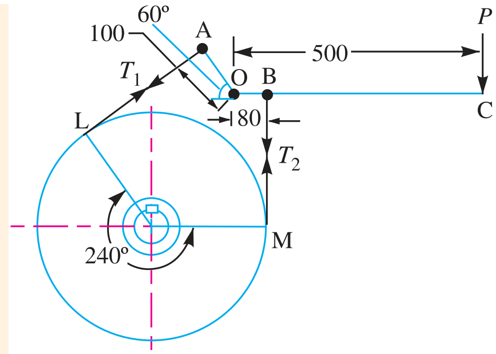
and $T_1 = 3.52\,T_2 = 3.52 \times 146 = 514$ N

We know that the maximum torque that the brake may sustain,

$$T_B = (T_1 - T_2)\,r = (514 - 146)\,0.075 = 27.6 \text{ N-m}$$ **Ans.**

Example 25.13. *A differential band brake is operated by a lever of length 500 mm. The brake drum has a diameter of 500 mm and the maximum torque on the drum is 1000 N-m. The band brake embraces 2/3rd of the circumference. One end of the band is attached to a pin 100 mm from the fulcrum and the other end to another pin 80 mm from the fulcrum and on the other side of it when the operating force is also acting. If the band brake is lined with asbestos fabric having a coefficient of friction 0.3, find the operating force required.*

Design the steel band, shaft, key, lever and fulcrum pin. The permissible stresses may be taken as 70 MPa in tension, 50 MPa in shear and 20 MPa in bearing. The bearing pressure for the brake lining should not exceed 0.2 N/mm².



All dimensions in mm.

Fig. 25.28

Solution. Given : l = 500 mm ; d = 500 mm or r = 250 mm ; T_B = 1000 N-m = 1 × 10^6 N-mm ; OA = 100 mm ; OB = 80 mm ; μ = 0.3 ; σ_t = 70 MPa = 70 N/mm² ; τ = 50 MPa = 50 N/mm² ; σ_b = 20 MPa = 20 N/mm² ; p_b = 0.2 N/mm²

Operating force

Let P = Operating force

The differential band brake is shown in Fig. 25.28. Since $OA > OB$, therefore the operating force (P) will act downward. When the drum rotates anticlockwise, the end of the band attached to A will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. First of all, let us find the values of tensions T_1 and T_2.

We know that angle of wrap,

$$\theta = \frac{2}{3} \text{rd of circumference}$$

$$= \frac{2}{3} \times 360° = 240° = 240 \times \frac{\pi}{180} = 4.19 \text{ rad}$$

and $$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 4.19 = 1.257$$

$\therefore$ $$\log \left(\frac{T_1}{T_2}\right) = \frac{1.257}{2.3} = 0.5465 \quad \text{or} \quad \frac{T_1}{T_2} = 3.52$$ ...(Taking antilog of 0.5465) ...(*i*)

We know that the braking torque (T_B),

$$1 \times 10^6 = (T_1 - T_2)\, r = (T_1 - T_2)\, 250$$

$\therefore$ $$T_1 - T_2 = 1 \times 10^6 / 250 = 4000 \text{ N}$$ ...(*ii*)

From equations (*i*) and (*ii*), we have

$$T_1 = 5587 \text{ N, and } T_2 = 1587 \text{ N}$$

Now taking moments about the fulcrum O, we have

$$P \times 500 = T_1 \times 100 - T_2 \times 80$$

$$= 5587 \times 100 - 1587 \times 80$$

$$= 431\,740$$

$\therefore$ $$P = 431\,740 / 500 = 863.5 \text{ N}$$ **Ans.**

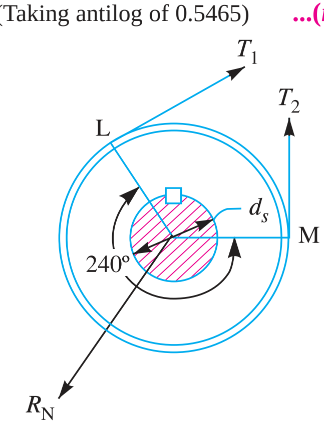


Fig. 25.29

Design for steel band

Let t = Thickness of band in mm, and

b = Width of band in mm.

We know that length of contact of band, as shown in Fig. 25.29,

$$= \pi d \times \frac{240}{360} = \pi \times 500 \times \frac{240}{360} \text{ mm} = 1047 \text{ mm}$$

∴ Area of contact of the band,

$$A_b = \text{Length} \times \text{Width of band} = 1047\, b \text{ mm}^2$$

We know that normal force acting on the band,

$$R_N = \frac{T_1 - T_2}{\mu} = \frac{5587 - 1587}{0.3} = 13\ 333 \text{ N}$$

We also know that normal force on the band (R_N),

$$13\ 333 = p_b \times A_b = 0.2 \times 1047\, b = 209.4\, b$$

$$\therefore \quad b = 13\ 333/209.4 = 63.7 \text{ say } 64 \text{ mm} \quad \textbf{Ans.}$$

and cross-sectional area of the band,

$$A = b \times t = 64\, t \text{ mm}^2$$

∴ Tensile strength of the band

$$= A \times \sigma_t = 64\, t \times 70 = 4480\, t \text{ N}$$

Since the band has to withstand a maximum tension (T_1) equal to 5587 N, therefore

$$4480\, t = 5587 \quad \text{or} \quad t = 5587 / 4480 = 1.25 \text{ mm} \quad \textbf{Ans.}$$

Design of shaft

Let d_s = Diameter of the shaft in mm.

Since the shaft has to transmit torque equal to the braking torque (T_B), therefore

$$1 \times 10^6 = \frac{\pi}{16} \times \tau\, (d_s)^3 = \frac{\pi}{16} \times 50\, (d_s)^3 = 9.82\, (d_s)^3$$

$$\therefore \quad (d_s)^3 = 1 \times 10^6 / 9.82 = 101\ 833 \quad \text{or} \quad d_s = 46.7 \text{ say } 50 \text{ mm} \quad \textbf{Ans.}$$

Design of key

The standard dimensions of the key for a 50 mm diameter shaft are as follows :

Width of key, w = 16 mm

Thickness of key, t_1 = 10 mm

Let l = Length of the key.

Considering shearing of the key, we have braking torque (T_B),

$$1 \times 10^6 = l \times w \times \tau \times \frac{d_s}{2} = l \times 16 \times 50 \times \frac{50}{2} = 20 \times 10^3\, l$$

$$\therefore \quad l = 1 \times 10^6/20 \times 10^3 = 50 \text{ mm} \quad \textbf{Ans.}$$

Note : The dimensions of the key may also be obtained from the following relations :

$$w = \frac{d_s}{4} + 3 \text{ mm}; \quad \text{and} \quad t_1 = \frac{w}{2}$$

Design for lever

Let t_2 = Thickness of lever in mm, and

B = Width of the lever in mm.

It is assumed that the lever extends up to the centre of the fulcrum. This assumption results in a slightly stronger lever. Neglecting the effect of T_2 on the lever, the maximum bending moment at the centre of the fulcrum,

$$M = P \times l = 863.5 \times 500 = 431\ 750 \text{ N-mm}$$

and section modulus, $Z = \frac{1}{6} t_2.B^2 = \frac{1}{6} t_2 \ (2t_2)^2 = 0.67 \ (t_2)^3$ ...(Assuming $B = 2t_2$)

We know that bending tensile stress (σ_t),

$$70 = \frac{M}{Z} = \frac{431\ 750}{0.67\ (t_2)^3} = \frac{644\ 400}{(t_2)^3}$$

$\therefore$ $(t_2)^3 = 644\ 400 / 70 = 9206$ or $t_2 = 21$ mm **Ans.**

and $B = 2\ t_2 = 2 \times 21 = 42$ mm **Ans.**

Design for fulcrum pin

Let d_1 = Diameter of the fulcrum pin, and

l_1 = Length of the fulcrum pin.

First of all, let us find the resultant force acting on the pin. Resolving the three forces T_1, T_2 and P into their vertical and horizontal components, as shown in Fig. 25.30.

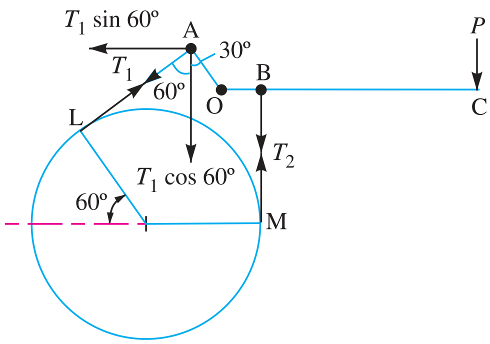
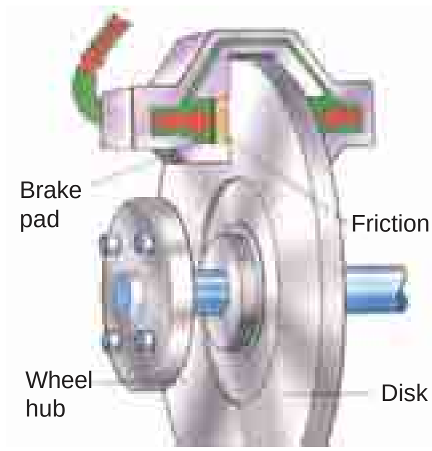


Fig. 25.30



Another type brake disc

We know that sum of vertical components,

$$\Sigma V = T_1 \cos 60° + T_2 + P = 5587 \times \frac{1}{2} + 1587 + 863.5 = 5244 \text{ N}$$

and sum of horizontal components,

$$\Sigma H = T_1 \sin 60° = 5587 \times 0.866 = 4838 \text{ N}$$

$\therefore$ Resultant force acting on pin,

$$R_P = \sqrt{(\Sigma V)^2 + (\Sigma H)^2} = \sqrt{(5244)^2 + (4838)^2} = 7135 \text{ N}$$

Considering bearing of the pin, we have resultant force on the pin (R_P),

$$7\ 135 = d_1.l_1.\sigma_b = d_1 \times 1.25\ d_1 \times 20 = 25\ (d_1)^2$$ ...(Assuming $l_1 = 1.25\ d_1$)

$\therefore$ $(d_1)^2 = 7135 / 25 = 285.4$ or $d_1 = 16.9$ say 18 mm **Ans.**

and $l_1 = 1.25\ d_1 = 1.25 \times 18 = 22.5$ mm **Ans.**

Let us now check the pin for induced shear stress. Since the pin is in double shear, therefore resultant force on the pin (R_P),

$$7135 = 2 \times \frac{\pi}{4}\ (d_1)^2\ \tau = 2 \times \frac{\pi}{4}\ (18)^2\ \tau = 509\ \tau$$

$\therefore$ $\tau = 7135 / 509 = 14 \text{ N/mm}^2$

This induced shear stress is within permissible limits.

The pin may be checked for induced bending stress. We know that maximum bending moment,

$$M = \frac{5}{24} \times W.l_1 = \frac{5}{24} \times 7135 \times 22.5 = 33\,445 \text{ N-mm}$$

...(Here $W = R_P = 7135$ N)

and section modulus, $Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(18)^3 = 573 \text{ mm}^3$

∴ Bending stress induced

$$= \frac{M}{Z} = \frac{33\,445}{573} = 58.4 \text{ N/mm}^2$$

This induced bending stress in the pin is within safe limit of 70 N/mm^2.

The lever has an eye hole for the pin and connectors at band have forked end. A brass bush of 3 mm thickness may be provided in the eye of the lever. Therefore, diameter of hole in the lever

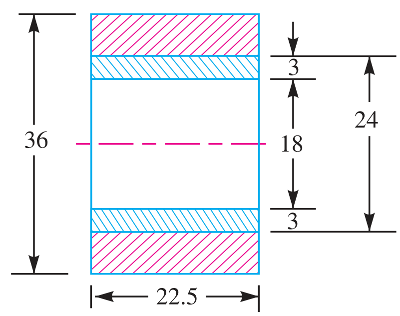
$$= d_1 + 2 \times 3 = 18 + 6 = 24 \text{ mm}$$

The boss is made at the pin joints whose outer diameter is taken equal to twice the diameter of pin and length equal to the length of pin. The inner diameter of boss is equal to the diameter of hole in the lever.

∴ Outer diameter of boss

$$= 2\,d_1 = 2 \times 18 = 36 \text{ mm}$$

and length of boss = 22.5 mm



All dimensions in mm.

Fig. 25.31

Let us now check the induced bending stress in the lever at the fulcrum. The section of the lever at the fulcrum is shown in Fig. 25.31. We know that maximum bending moment at the fulcrum,

$$M = P \times l = 863.5 \times 500 = 431\,750 \text{ N-mm}$$

and section modulus,

$$Z = \frac{\frac{1}{12} \times 22.5\left[(36)^3 - (24)^3\right]}{36/2} = 3420 \text{ mm}^3$$

∴ Bending stress induced in the lever

$$= \frac{M}{Z} = \frac{431\,750}{3420} = 126 \text{ N/mm}^2$$

Since the induced bending stress is more than the permissible value of 70 N/mm^2, therefore the diameter of pin is required to be increased. Let us take

Diameter of pin, $d_1 = 22$ mm **Ans.**

∴ Length of pin, $l_1 = 1.25\,d_1 = 1.25 \times 22 = 27.5$ say 28 mm **Ans.**

Diameter of hole in the lever

$$= d_1 + 2 \times 3 = 22 + 6 = 28 \text{ mm}$$

Contents

Outer diameter of boss

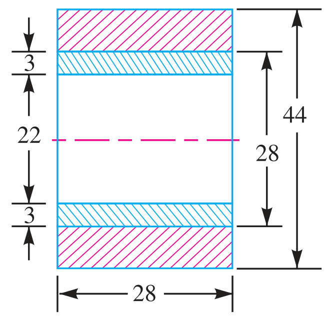
$= 2\, d_1 = 2 \times 22 = 44$ mm

= Outer diameter of eye

and thickness of each eye $= l_1 / 2 = 28 / 2 = 14$ mm

A clearance of 1.5 mm is provided on either side of the lever in the fork.

The new section of the lever at the fulcrum will be as shown in Fig. 25.32.



All dimensions in mm.

Fig. 25.32

∴ Section modulus,

$$Z = \frac{\frac{1}{12} \times 28 \left[(44)^3 - (28)^3\right]}{44/2} = 6706 \text{ mm}^2$$

and induced bending stress $= \dfrac{431\ 750}{6706} = 64.4$ N/mm²

This induced bending stress is within permissible limits.

25.11 Band and Block Brake

The band brake may be lined with blocks of wood or other material, as shown in Fig. 25.33 (*a*). The friction between the blocks and the drum provides braking action. Let there are '*n*' number of blocks, each subtending an angle 2 θ at the centre and the drum rotates in anticlockwise direction.

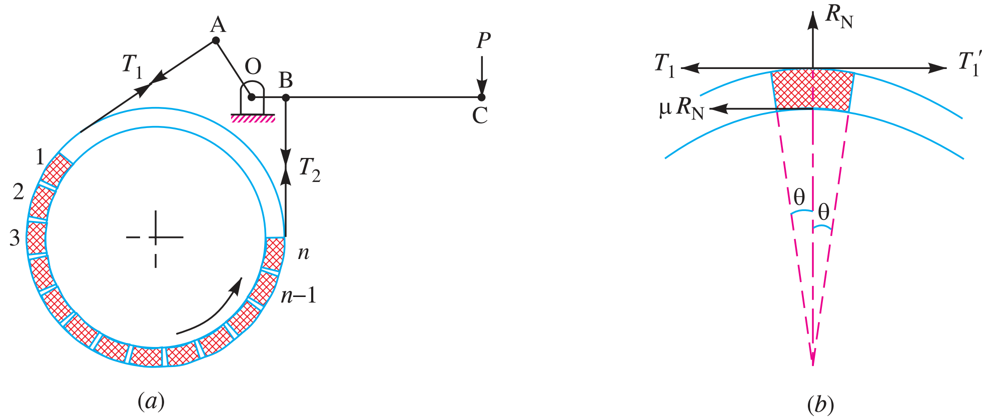


Fig. 25.33. Band and block brake.

Let T_1 = Tension in the tight side,

T_2 = Tension in the slack side,

μ = Coefficient of friction between the blocks and drum,

T_1' = Tension in the band between the first and second block,

T_2', T_3' etc. = Tensions in the band between the second and third block, between the third and fourth block etc.

Consider one of the blocks (say first block) as shown in Fig. 25.33 (*b*). This is in equilibrium under the action of the following forces :

1. Tension in the tight side (T_1),
2. Tension in the slack side (T_1') or tension in the band between the first and second block,
3. Normal reaction of the drum on the block (R_N), and
4. The force of friction (μ.R_N).

Top

Contents

Resolving the forces radially, we have

$$(T_1 + T_1')\sin\theta = R_N \quad ...(i)$$

Resolving the forces tangentially, we have

$$(T_1 - T_1')\cos\theta = \mu.R_N \quad ...(ii)$$

Dividing equation ***(ii)*** by ***(i)***, we have

$$\frac{(T_1 - T_1')\cos\theta}{(T_1 + T_1')\sin\theta} = \frac{\mu.R_N}{R_N}$$

or

$$(T_1 - T_1') = \mu\tan\theta\,(T_1 + T_1')$$

$$\therefore \quad \frac{T_1}{T_1'} = \frac{1+\mu\tan\theta}{1-\mu\tan\theta}$$

Similarly it can be proved for each of the blocks that

$$\frac{T_1'}{T_2'} = \frac{T_2'}{T_3'} = \frac{T_3'}{T_4'} = ... = \frac{T_{n-1}}{T_2} = \frac{1+\mu\tan\theta}{1-\mu\tan\theta}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{T_1}{T_1'} \times \frac{T_1'}{T_2'} \times \frac{T_2'}{T_3'} \times ... \times \frac{T_{n-1}}{T_2} = \left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)^n \quad ...(iii)$$

Braking torque on the drum of effective radius r_e,

$$T_B = (T_1 - T_2)\,r_e$$

$$= (T_1 - T_2)\,r \quad \text{...(Neglecting thickness of band)}$$

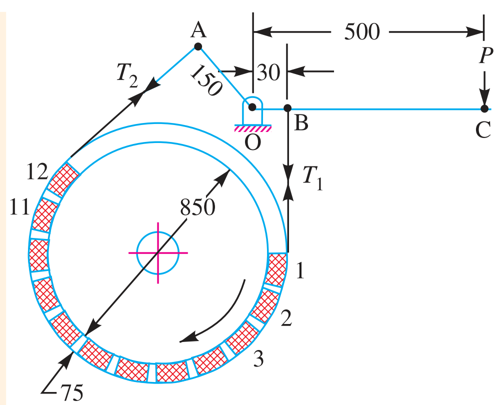
Note: For the first block, the tension in the tight side is T_1 and in the slack side is T_1' and for the second block, the tension in the tight side is T_1' and in the slack side is T_2'. Similarly for the third block, the tension in the tight side is T_2' and in the slack side is T_3' and so on. For the last block, the tension in the tight side is T_{n-1} and in the slack side is T_2.

Example 25.14. *In the band and block brake shown in Fig. 25.34, the band is lined with 12 blocks each of which subtends an angle of 15° at the centre of the rotating drum. The thickness of the blocks is 75 mm and the diameter of the drum is 850 mm. If, when the brake is inaction, the greatest and least tensions in the brake strap are T_1 and T_2, show that*

$$\frac{T_1}{T_2} = \left(\frac{l+\mu\tan 7\,^1/_2{}^\circ}{l-\mu\tan 7\,^1/_2{}^\circ}\right)^{12}$$

where μ is the coefficient of friction for the blocks.

With the lever arrangement as shown in Fig. 25.34, find the least force required at C for the blocks to absorb 225 kW at 240 r.p.m. The coefficient of friction between the band and blocks is 0.4.



All dimensions in mm.

Fig. 25.34

Solution. Given : $n = 12$; $2\theta = 15°$ or $\theta = 7\,^1/_2{}°$; $t = 75$ mm = 0.075 m ; $d = 850$ mm = 0.85 m ; Power = 225 kW = 225×10^3 W ; $N = 240$ r.p.m. ; $\mu = 0.4$

Top

Contents

Since $OA > OB$, therefore the force at C must act downward. Also, the drum rotates clockwise, therefore the end of the band attached to A will be slack with tension T_2 (least tension) and the end of the band attached to B will be tight with tension T_1 (greatest tension).

Consider one of the blocks (say first block) as shown is Fig. 25.35. This is in equilibrium under the action of the following four forces :

1. Tension in the tight side (T_1),
2. Tension in the slack side (T_1') or the tension in the band between the first and second block,
3. Normal reaction of the drum on the block (R_N), and
4. The force of friction ($\mu.R_N$).

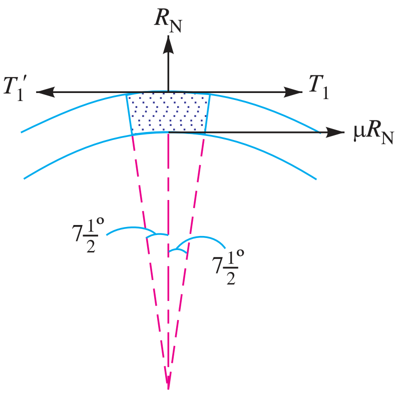


Fig. 25.35

Car wheels are made of alloys to bear high stresses and fatigue.

Resolving the forces radially, we have

$$(T_1 + T_1') \sin 7\tfrac{1}{2}° = R_N \quad \text{...}(i)$$

Resolving the forces tangentially, we have

$$(T_1 - T_1') \cos 7\tfrac{1}{2}° = \mu.R_N \quad \text{...}(ii)$$

Dividing equation (*ii*) by (*i*), we have

$$\frac{(T_1 - T_1') \cos 7\tfrac{1}{2}°}{(T_1 + T_1') \sin 7\tfrac{1}{2}°} = \mu \quad \text{or} \quad \frac{T_1 - T_1'}{T_1 + T_1'} = \mu \tan 7\tfrac{1}{2}°$$

$$\therefore \quad \frac{T_1}{T_1'} = \frac{1 + \mu \tan 7\tfrac{1}{2}°}{1 - \mu \tan 7\tfrac{1}{2}°}$$

Similarly, for the other blocks, the ratio of tensions $\frac{T_1'}{T_2'} = \frac{T_2'}{T_3'}$ etc., remains constant. Therefore for 12 blocks having greatest tension T_1 and least tension T_2 is

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7\tfrac{1}{2}°}{1 - \mu \tan 7\tfrac{1}{2}°}\right)^{12}$$

Least force required at C

Let P = Least force required at C.

We know that diameter of band,

$$D = d + 2\,t = 0.85 + 2 \times 0.075 = 1 \text{ m}$$

Top

Contents

and power absorbed $= \dfrac{(T_1 - T_2)\pi D.N}{60}$

$\therefore \quad T_1 - T_2 = \dfrac{\text{Power} \times 60}{\pi DN} = \dfrac{225 \times 10^3 \times 60}{\pi \times 1 \times 240} = 17\ 900 \text{ N}$ ...(*iii*)

We have proved that

$$\frac{T_1}{T_2} = \left(\frac{1 + \mu \tan 7\frac{1}{2}°}{1 - \mu \tan 7\frac{1}{2}°}\right)^{12} = \left(\frac{1 + 0.4 \times 0.1317}{1 - 0.4 \times 0.1317}\right)^{12} = 3.55 \quad ...(iv)$$

From equations (*iii*) and (*iv*), we find that

$T_1 = 24\ 920$ N ; and $T_2 = 7020$ N

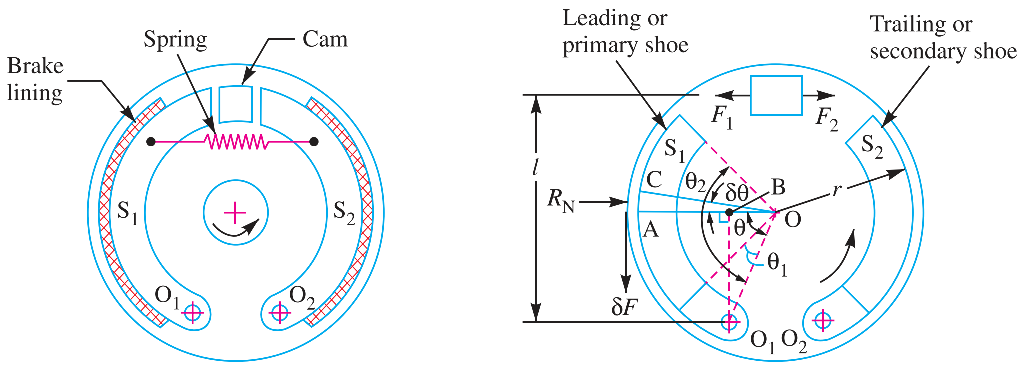
Now taking moments about O, we have

$P \times 500 = T_2 \times 150 - T_1 \times 30 = 7020 \times 150 - 24\ 920 \times 30 = 305\ 400$

$\therefore \quad P = 305\ 400 / 500 = 610.8$ N **Ans.**

25.12 Internal Expanding Brake

An internal expanding brake consists of two shoes S_1 and S_2 as shown in Fig. 25.36 (*a*). The outer surface of the shoes are lined with some friction material (usually with Ferodo) to increase the coefficient of friction and to prevent wearing away of the metal. Each shoe is pivoted at one end about a fixed fulcrum O_1 and O_2 and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. The shoes are normally held in off position by a spring as shown in Fig. 25.36 (*a*). The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in motor cars and light trucks.



(*a*) Internal expanding brake. (*b*) Forces on an internal expanding brake.

Fig. 25.36

We shall now consider the forces acting on such a brake, when the drum rotates in the anticlockwise direction as shown in Fig. 25.36 (*b*). It may be noted that for the anticlockwise direction, the left hand shoe is known as ***leading* or *primary shoe*** while the right hand shoe is known as ***trailing* or *secondary shoe.***

Let r = Internal radius of the wheel rim.

b = Width of the brake lining.

p_1 = Maximum intensity of normal pressure,

p_N = Normal pressure,

F_1 = Force exerted by the cam on the leading shoe, and

F_2 = Force exerted by the cam on the trailing shoe.

Top

Consider a small element of the brake lining AC subtending an angle $\delta\theta$ at the centre. Let OA makes an angle θ with OO_1 as shown in Fig. 25.36 (*b*). It is assumed that the pressure distribution on the shoe is nearly uniform, however the friction lining wears out more at the free end. Since the shoe turns about O_1, therefore the rate of wear of the shoe lining at A will be proportional to the radial displacement of that point. The rate of wear of the shoe lining varies directly as the perpendicular distance from O_1 to OA, *i.e.* O_1B. From the geometry of the figure,

Inside view of a truck disk brake

$$O_1B = OO_1 \sin\theta$$

and normal pressure at A, $p_N \propto \sin\theta$ or $p_N = p_1 \sin\theta$

∴ Normal force acting on the element,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$

$$= p_N (b \,.\, r \,.\, \delta\theta) = p_1 \sin\theta\, (b \,.\, r \,.\, \delta\theta)$$

and braking or friction force on the element,

$$\delta F = \mu.\delta R_N = \mu\, p_1 \sin\theta\, (b \,.\, r \,.\, \delta\theta)$$

∴ Braking torque due to the element about O,

$$\delta T_B = \delta F \,.\, r = \mu\, p_1 \sin\theta\, (b \,.\, r \,.\, \delta\theta)\, r = \mu\, p_1\, b\, r^2 (\sin\theta.\delta\theta)$$

and total braking torque about O for whole of one shoe,

$$T_B = \mu\, p_1\, b\, r^2 \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = \mu\, p_1\, b\, r^2 \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1\, b\, r^2 (\cos\theta_1 - \cos\theta_2)$$

Moment of normal force δR_N of the element about the fulcrum O_1,

$$\delta M_N = \delta R_N \times O_1B = \delta R_N (OO_1 \sin\theta)$$

$$= p_1 \sin\theta\, (b \,.\, r \,.\, \delta\theta)\, (OO_1 \sin\theta) = p_1 \sin^2\theta\, (b \,.\, r \,.\, \delta\theta)\, OO_1$$

Total moment of normal forces about the fulcrum O_1,

$$M_N = \int_{\theta_1}^{\theta_2} p_1 \sin^2\theta\, (b \,.\, r\, \delta\theta)\, OO_1 = p_1 \,.\, b \,.\, r \,.\, OO_1 \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta$$

$$= p_1 . b . r . OO_1 \int_{\theta_1}^{\theta_2} \tfrac{1}{2}(1 - \cos 2\theta)\, d\theta \qquad \ldots\left[\because \sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta)\right]$$

$$= \tfrac{1}{2}\, p_1 \,.\, b \,.\, r \,.\, OO_1 \left[\theta - \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

Contents

$$= \frac{1}{2}\, p_1 \,.\, b \,.\, r \,.\, OO_1 \left[\theta_2 - \frac{\sin 2\theta_2}{2} - \theta_1 + \frac{\sin 2\theta_1}{2}\right]$$

$$= \frac{1}{2}\, p_1 \,.\, b \,.\, r \,.\, OO_1 \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$

Moment of frictional force δF about the fulcrum O_1,

$$\delta M_F = \delta F \times AB = \delta F\,(r - OO_1 \cos\theta) \qquad ...(\because AB = r - OO_1 \cos\theta)$$

$$= \mu \,.\, p_1 \sin\theta\,(b \,.\, r \,.\, \delta\theta)\,(r - OO_1 \cos\theta)$$

$$= \mu \,.\, p_1 \,.\, b \,.\, r\,(r \sin\theta - OO_1 \sin\theta\cos\theta)\,\delta\theta$$

$$= \mu \,.\, p_1 \,.\, b \,.\, r\left(r\sin\theta - \frac{OO_1}{2}\sin 2\theta\right)\delta\theta \qquad ...(\because 2\sin\theta\cos\theta = \sin 2\theta)$$

$\therefore$ Total moment of frictional force about the fulcrum O_1,

$$M_F = \mu \,.\, p_1 \,.\, b \,.\, r \int_{\theta_1}^{\theta_2}\left(r\sin\theta - \frac{OO_1}{2}\sin 2\theta\right)d\theta$$

$$= \mu \,.\, p_1 \,.\, b \,.\, r\left[-r\cos\theta + \frac{OO_1}{4}\cos 2\theta\right]_{\theta_1}^{\theta_2}$$

$$= \mu \,.\, p_1 \,.\, b \,.\, r\left[-r\cos\theta_2 + \frac{OO_1}{4}\cos 2\theta_2 + r\cos\theta_1 - \frac{OO_1}{4}\cos 2\theta_1\right]$$

$$= \mu \,.\, p_1 \,.\, b \,.\, r\left[r\,(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$

Now for leading shoe, taking moments about the fulcrum O_1,

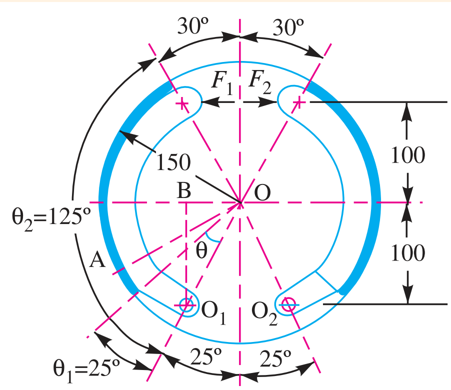
$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

Note: If $M_F > M_N$, then the brake becomes self locking.

Example 25.15. *Fig. 25.37 shows the arrangement of two brake shoes which act on the internal surface of a cylindrical brake drum. The braking force F_1 and F_2 are applied as shown and each shoe pivots on its fulcrum O_1 and O_2. The width of the brake lining is 35 mm. The intensity of pressure at any point A is 0.4 sin θ N/mm², where θ is measured as shown from either pivot. The coefficient of friction is 0.4. Determine the braking torque and the magnitude of the forces F_1 and F_2.*



All dimensions in mm.

Fig. 25.37

Top

Solution. Given : b = 35 mm ; μ = 0.4 ; r = 150 mm ; l = 200 mm ; θ_1 = 25° ; θ_2 = 125°

Since the intensity of normal pressure at any point is 0.4 sin θ N/mm², therefore maximum intensity of normal pressure,

$$p_1 = 0.4 \text{ N/mm}^2$$

We know that the braking torque for one shoe,

$$= \mu \cdot p_1 \cdot b \cdot r^2 (\cos\theta_1 - \cos\theta_2)$$
$$= 0.4 \times 0.4 \times 35 (150)^2 (\cos 25° - \cos 125°)$$
$$= 126\,000 (0.9063 + 0.5736) = 186\,470 \text{ N-mm}$$

∴ Total braking torque for two shoes,

$$T_B = 2 \times 186\,470 = 372\,940 \text{ N-mm}$$

Magnitude of the forces F_1 and F_2

From the geometry of the figure, we find that

$$OO_1 = \frac{O_1B}{\cos 25°} = \frac{100}{0.9063} = 110.3 \text{ mm}$$

$$\theta_1 = 25° = 25 \times \pi / 180 = 0.436 \text{ rad}$$

and $\theta_2 = 125° = 125 \times \pi / 180 = 2.18 \text{ rad}$

We know that the total moment of normal forces about the fulcrum O_1,

$$M_N = \tfrac{1}{2} p_1 \cdot b \cdot r \cdot OO_1 \left[(\theta_2 - \theta_1) + \tfrac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$
$$= \tfrac{1}{2} \times 0.4 \times 35 \times 150 \times 110.3 \left[(2.18 - 0.436) + \tfrac{1}{2}(\sin 50° - \sin 250°)\right]$$
$$= 115\,815 \left[1.744 + \frac{1}{2}(0.766 + 0.9397)\right] = 300\,754 \text{ N-mm}$$

and total moment of friction force about the fulcrum O_1,

$$M_F = \mu \cdot p_1 \cdot b \cdot r \left[r(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$
$$= 0.4 \times 0.4 \times 35 \times 150 \left[150(\cos 25° - \cos 125°) + \frac{110.3}{4}(\cos 250° - \cos 50°)\right]$$
$$= 840 [150 (0.9063 + 0.5736) + 27.6 (-0.342 - 0.6428)]$$
$$= 840 (222 - 27) = 163\,800 \text{ N-mm}$$

For the leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

or $F_1 \times 200 = 300\,754 - 163\,800 = 136\,954$

∴ $F_1 = 136\,954 / 200 = 685$ N **Ans.**

For the trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

or $F_2 \times 200 = 300\,754 + 163\,800 = 464\,554$

∴ $F_2 = 464\,554 / 200 = 2323$ N **Ans.**

EXERCISES

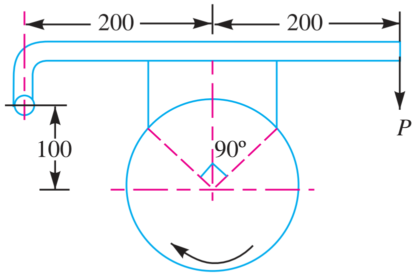
1. A flywheel of mass 100 kg and radius of gyration 350 mm is rotating at 720 r.p.m. It is brought to rest by means of a brake. The mass of the brake drum assembly is 5 kg. The brake drum is made of cast iron FG 260 having specific heat 460 J / kg°C. Assuming that the total heat generated is absorbed by the brake drum only, calculate the temperature rise.

[Ans. 15.14°C]

Contents

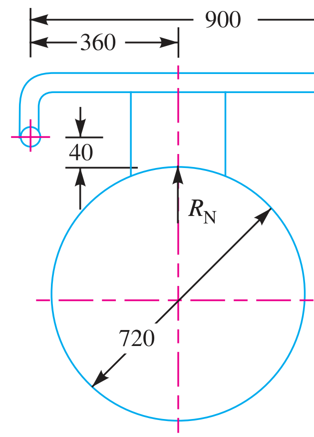
2. A single block brake, as shown in Fig. 25.38, has the drum diameter 250 mm. The angle of contact is 90° and the coefficient of friction between the drum and the lining is 0.35. If the torque transmitted by the brake is 70 N-m, find the force *P* required to operate the brake.

[Ans. 700 N]



All dimensions in mm.

Fig. 25.38



All dimensions in mm.

Fig. 25.39

3. A single block brake, as shown in Fig. 25.39, has a drum diameter of 720 mm. If the brake sustains 225 N-m torque at 500 r.p.m.; find :

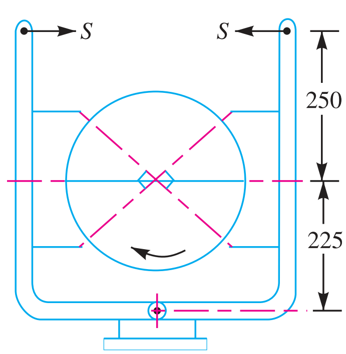
(*a*) the required force (*P*) to apply the brake for clockwise rotation of the drum;

(*b*) the required force (*P*) to apply the brake for counter clockwise rotation of the drum;

(*c*) the location of the fulcrum to make the brake self-locking for clockwise rotation of the drum; and

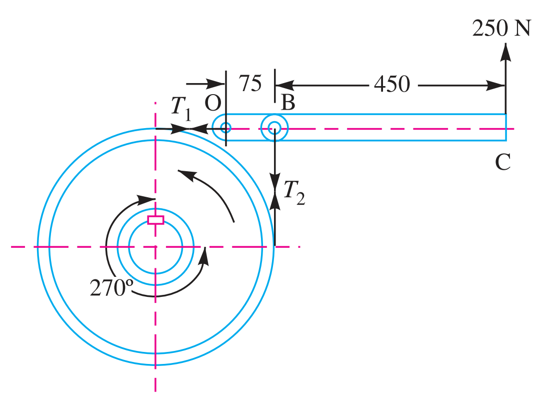
The coefficient of friction may be taken as 0.3. **[Ans. 805.4 N ; 861 N; 1.2 m ; 11.78 kW]**

4. The layout and dimensions of a double shoe brake is shown in Fig. 25.40. The diameter of the brake drum is 300 mm and the contact angle for each shoe is 90°. If the coefficient of friction for the brake lining and the drum is 0.4, find the spring force necessary to transmit a torque of 30 N-m. Also determine the width of the brake shoes, if the bearing pressure on the lining material is not to exceed 0.28 N/mm^2. **[Ans. 99.1 N ; 5 mm]**



All dimensions in mm.

Fig. 25.40



All dimensions in mm.

Fig. 25.41

Top

5. The drum of a simple band brake is 450 mm. The band embraces 3/4th of the circumference of the drum. One end of the band is attached to the fulcrum pin and the other end is attached to a pin *B* as shown in Fig. 25.41. The band is to be lined with asbestos fabric having a coefficient of friction 0.3. The allowable bearing pressure for the brake lining is 0.21 N/mm². Design the band shaft, key, lever and fulcrum pin. The material of these parts is mild steel having permissible stresses as follows :

$$\sigma_t = \sigma_c = 70 \text{ MPa, and } \tau = 56 \text{ MPa}$$

6. A band brake as shown in Fig. 25.42, is required to balance a torque of 980 N-m at the drum shaft. The drum is to be made of 400 mm diameter and is keyed to the shaft. The band is to be lined with ferodo lining having a coefficient of friction 0.25. The maximum pressure between the lining and drum is 0.5 N/mm². Design the steel band, shaft, key on the shaft, brake lever and fulcrum pin. The permissible stresses for the steel to be used for the shaft, key, band lever and pin are 70 MPa in tension and compression and 56 MPa in shear.

7. A differential band brake is shown in Fig. 25.43. The diameter of the drum is 800 mm. The coefficient of friction between the band and the drum is 0.3 and the angle of embrace is 240°. When a force of 600 N

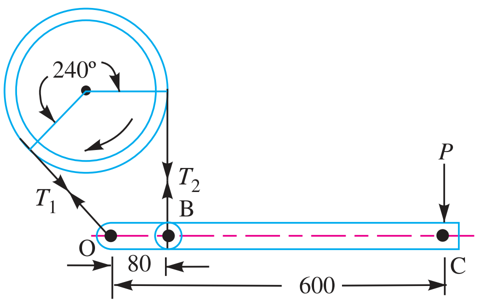


Fig. 25.42

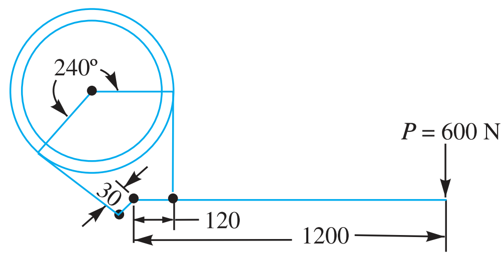


Fig. 25.43

is applied at the free end of the lever, find for the clockwise and anticlockwise rotation of the drum: 1. the maximum and minimum forces in the band; and 2. the torque which can be applied by the brake.

[Ans. 176 kN, 50 kN, 50.4 kN-m ; 6.46 kN, 1.835 kN, 1.85 kN-m]

8. In a band and block brake, the band is lined with 14 blocks, each of which subtends an angle of 20° at the drum centre. One end of the band is attached to the fulcrum of the brake lever and the other to a pin 150 mm from the fulcrum. Find the force required at the end of the lever 1 metre long from the fulcrum to give a torque of 4 kN-m. The diameter of the brake drum is 1 metre and the coefficient of friction between the blocks and the drum is 0.25. **[Ans. 1692 N]**

QUESTIONS

1. How does the function of a brake differ from that of a clutch ?
2. A weight is brought to rest by applying brakes to the hoisting drum driven by an electric motor. How will you estimate the total energy absorbed by the brake ?
3. What are the thermal considerations in brake design ?
4. What is the significance of *pV* value in brake design ?
5. What are the materials used for brake linings.
6. Discuss the different types of brakes giving atleast one practical application for each.
7. List the important factors upon which the capacity of a brake depends.
8. What is a self-energizing brake ? When a brake becomes self-locking.

Contents

9. What is back stop action in band brakes ? Explain the condition for it.

10. Describe with the help of a neat sketch the principle of operation of an internal expanding shoe brake. Derive the expression for the braking torque.

Truck suspension system : *Front Pivot ball suspension soaks up the bumps and provides unmatched adjustability. Chrome 8 mm CVA joints give added strength.*

Note : This picture is given as additional information and is not a direct example of the current chapter.

OBJECTIVE TYPE QUESTIONS

1. A brake commonly used in railway trains is

(*a*) shoe brake (*b*) band brake

(*c*) band and block brake (*d*) internal expanding brake

2. A brake commonly used in motor cars is

(*a*) shoe brake (*b*) band brake

(*c*) band and block brake (*d*) internal expanding brake

3. The material used for brake lining should have .... coefficient of friction.

(*a*) low (*b*) high

4. When the frictional force helps to apply the brake, then the brake is said to be

(*a*) self-energizing brake (*b*) self-locking brake

5. For a band brake, the width of the band for a drum diameter greater than 1 m, should not exceed

(*a*) 150 mm (*b*) 200 mm

(*c*) 250 mm (*d*) 300 mm

ANSWERS

1. (*a*) **2.** (*d*) **3.** (*b*) **4.** (*a*) **5.** (*a*)

Top

Contents

CHAPTER

26

Sliding Contact Bearings

26.1 Introduction

A bearing is a machine element which support another moving machine element (known as journal). It permits a relative motion between the contact surfaces of the members, while carrying the load. A little consideration will show that due to the relative motion between the contact surfaces, a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are in direct contact, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid (known as lubricant) may be provided. The lubricant used to separate the journal and bearing is usually a mineral oil refined from petroleum, but vegetable oils, silicon oils, greases etc., may be used.

26.2 Classification of Bearings

Though the bearings may be classified in many ways, yet the following are important from the subject point of view:

Top

Contents

Roller Bearing

1. *Depending upon the direction of load to be supported.* The bearings under this group are classified as:

(*a*) Radial bearings, and **(*b*)** Thrust bearings.

In ***radial bearings***, the load acts perpendicular to the direction of motion of the moving element as shown in Fig. 26.1 (*a*) and (*b*).

In ***thrust bearings,*** the load acts along the axis of rotation as shown in Fig. 26.1 (*c*).

Note : These bearings may move in either of the directions as shown in Fig. 26.1.

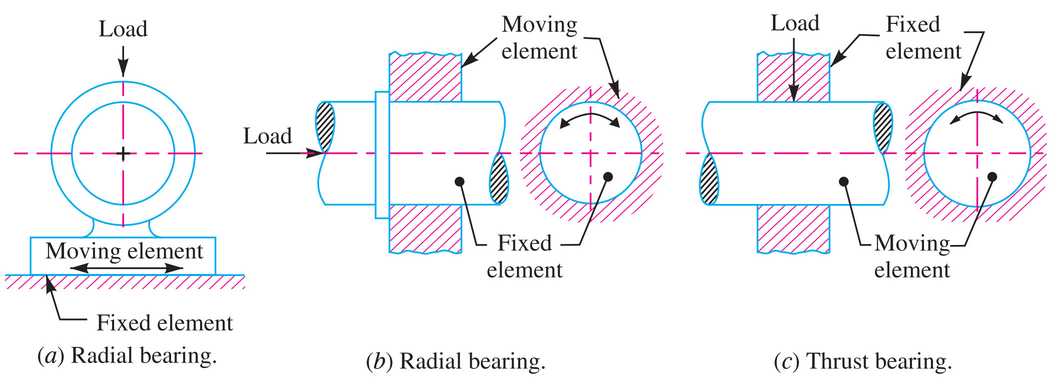


Fig. 26.1. Radial and thrust bearings.

2. *Depending upon the nature of contact.* The bearings under this group are classified as :

(*a*) Sliding contact bearings, and **(*b*)** Rolling contact bearings.

In ***sliding contact bearings,*** as shown in Fig. 26.2 (*a*), the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as ***plain bearings.***

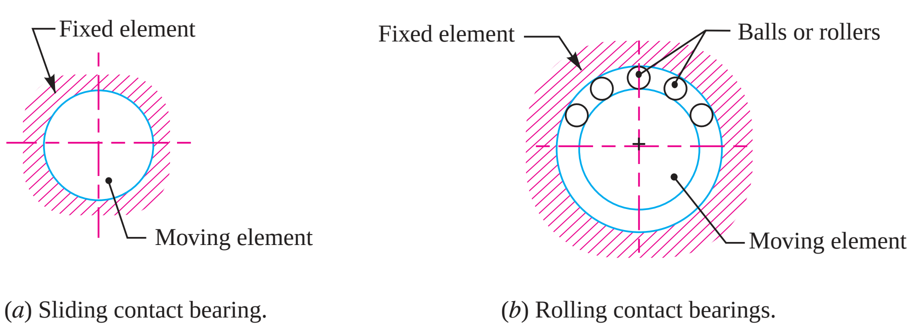


Fig. 26.2. Sliding and rolling contact bearings.

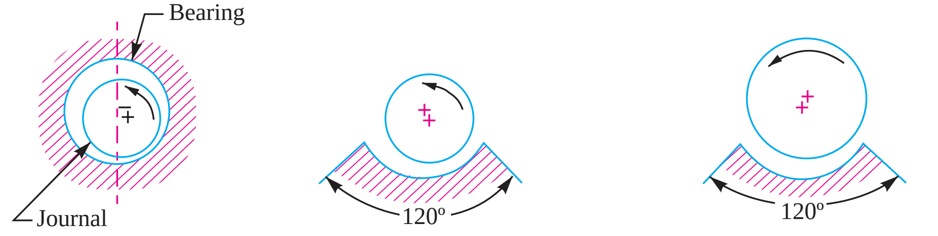
Top

Contents

In ***rolling contact bearings,*** as shown in Fig. 26.2 (*b*), the steel balls or rollers, are interposed between the moving and fixed elements. The balls offer rolling friction at two points for each ball or roller.

26.3 Types of Sliding Contact Bearings

The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads, as shown in Fig. 26.1 (*a*), may be called ***slipper*** or ***guide bearings.*** Such type of bearings are usually found in cross-head of steam engines.



(*a*) Full journal bearing. (*b*) Partial journal bearing. (*c*) Fitted journal bearing.

Fig. 26.3. Journal or sleeve bearings.

The sliding contact bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as ***journal*** or ***sleeve bearings.*** When the angle of contact of the bearing with the journal is 360° as shown in Fig. 26.3 (*a*), then the bearing is called a ***full journal bearing.*** This type of bearing is commonly used in industrial machinery to accommodate bearing loads in any radial direction.

When the angle of contact of the bearing with the journal is 120°, as shown in Fig. 26.3 (*b*), then the bearing is said to be ***partial journal bearing.*** This type of bearing has less friction than full journal bearing, but it can be used only where the load is always in one direction. The most common application of the partial journal bearings is found in rail road car axles. The full and partial journal bearings may be called as ***clearance bearings*** because the diameter of the journal is less than that of bearing.

Sliding contact bearings are used in steam engines

Top

When a partial journal bearing has no clearance *i.e*. the diameters of the journal and bearing are equal, then the bearing is called a ***fitted bearing,*** as shown in Fig. 26.3 (*c*).

The sliding contact bearings, according to the thickness of layer of the lubricant between the bearing and the journal, may also be classified as follows :

1. ***Thick film bearings.*** The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as ***hydrodynamic lubricated bearings.***
2. ***Thin film bearings.*** The thin film bearings are those in which, although lubricant is present, the working surfaces partially contact each other atleast part of the time. Such type of bearings are also called ***boundary lubricated bearings.***
3. ***Zero film bearings.*** The zero film bearings are those which operate without any lubricant present.
4. ***Hydrostatic or externally pressurized lubricated bearings.*** The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members.

26.4 Hydrodynamic Lubricated Bearings

Hydrodynamic Lubricated Bearings

We have already discussed that in hydrodynamic lubricated bearings, there is a thick film of lubricant between the journal and the bearing. A little consideration will show that when the bearing is supplied with sufficient lubricant, a pressure is build up in the clearance space when the journal is rotating about an axis that is eccentric with the bearing axis. The load can be supported by this fluid pressure without any actual contact between the journal and bearing. The load carrying ability of a hydrodynamic bearing arises simply because a viscous fluid resists being pushed around. Under the proper conditions, this resistance to motion will develop a pressure distribution in the lubricant film that can support a useful load. The load supporting pressure in hydrodynamic bearings arises from either

1. the flow of a viscous fluid in a converging channel (known as ***wedge film lubrication***), or
2. the resistance of a viscous fluid to being squeezed out from between approaching surfaces (known as ***squeeze film lubrication***).

26.5 Assumptions in Hydrodynamic Lubricated Bearings

The following are the basic assumptions used in the theory of hydrodynamic lubricated bearings:

1. The lubricant obeys Newton's law of viscous flow.
2. The pressure is assumed to be constant throughout the film thickness.
3. The lubricant is assumed to be incompressible.
4. The viscosity is assumed to be constant throughout the film.
5. The flow is one dimensional, *i.e.* the side leakage is neglected.

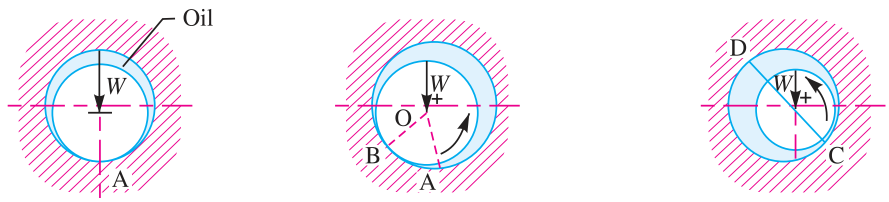
26.6 Important Factors for the Formation of Thick Oil Film in Hydrodynamic Lubricated Bearings

According to Reynolds, the following factors are essential for the formation of a thick film of oil in hydrodynamic lubricated bearings :

1. A continuous supply of oil.
2. A relative motion between the two surfaces in a direction approximately tangential to the surfaces.
3. The ability of one of the surfaces to take up a small inclination to the other surface in the direction of the relative motion.
4. The line of action of resultant oil pressure must coincide with the line of action of the external load between the surfaces.

26.7 Wedge Film Journal Bearings

The load carrying ability of a wedge-film journal bearing results when the journal and/or the bearing rotates relative to the load. The most common case is that of a steady load, a fixed (non-rotating) bearing and a rotating journal. Fig. 26.4 (*a*) shows a journal at rest with metal to metal contact at *A* on the line of action of the supported load. When the journal rotates slowly in the anticlockwise direction, as shown in Fig. 26.4 (*b*), the point of contact will move to *B*, so that the angle *AOB* is the angle of sliding friction of the surfaces in contact at *B*. In the absence of a lubricant, there will be dry metal to metal friction. If a lubricant is present in the clearance space of the bearing and journal, then a thin absorbed film of the lubricant may partly separate the surface, but a continuous fluid film completely separating the surfaces will not exist because of slow speed.



(*a*) At rest. (*b*) Slow speed. (*c*) High speed.

Fig. 26.4. Wedge film journal bearing.

When the speed of the journal is increased, a continuous fluid film is established as in Fig. 26.4 (*c*). The centre of the journal has moved so that the minimum film thickness is at *C*. It may be noted that from *D* to *C* in the direction of motion, the film is continually narrowing and hence is a converging film. The curved converging film may be considered as a wedge shaped film of a slipper bearing wrapped around the journal. A little consideration will show that from *C* to *D* in the direction of rotation, as shown in Fig. 26.4 (*c*), the film is diverging and cannot give rise to a positive pressure or a supporting action.

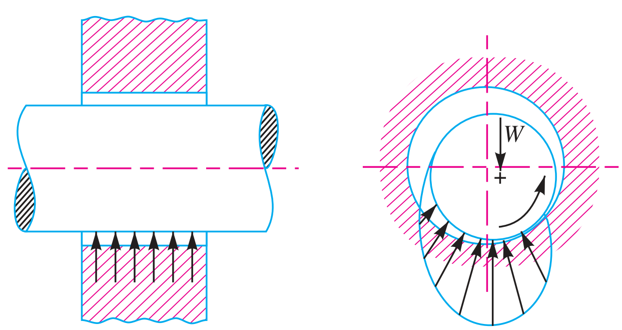


Fig. 26.5. Variation of pressure in the converging film.

Fig. 26.5 shows the two views of the bearing shown in Fig. 26.4 (*c*), with the variation of pressure in the converging film. Actually, because of side leakage, the angle of contact on which pressure acts is less than 180°.

26.8 Squeeze Film Journal Bearing

We have seen in the previous article that in a wedge film journal bearing, the bearing carries a steady load and the journal rotates relative to the bearing. But in certain cases, the bearings oscillate or rotate so slowly that the wedge film cannot provide a satisfactory film thickness. If the load is uniform or varying in magnitude while acting in a constant direction, this becomes a thin film or possibly a zero film problem. But if the load reverses its direction, the squeeze film may develop sufficient capacity to carry the dynamic loads without contact between the journal and the bearing. Such bearings are known as ***squeeze film journal bearing.***

Journal bearing

26.9 Properties of Sliding Contact Bearing Materials

When the journal and the bearings are having proper lubrication *i.e.* there is a film of clean, non-corrosive lubricant in between, separating the two surfaces in contact, the only requirement of the bearing material is that they should have sufficient strength and rigidity. However, the conditions under which bearings must operate in service are generally far from ideal and thus the other properties as discussed below must be considered in selecting the best material.

1. *Compressive strength.* The maximum bearing pressure is considerably greater than the average pressure obtained by dividing the load to the projected area. Therefore the bearing material should have high compressive strength to withstand this maximum pressure so as to prevent extrusion or other permanent deformation of the bearing.

2. *Fatigue strength.* The bearing material should have sufficient fatigue strength so that it can withstand repeated loads without developing surface fatigue cracks. It is of major importance in aircraft and automotive engines.

3. *Comformability.* It is the ability of the bearing material to accommodate shaft deflections and bearing inaccuracies by plastic deformation (or creep) without excessive wear and heating.

4. *Embeddability.* It is the ability of bearing material to accommodate (or embed) small particles of dust, grit etc., without scoring the material of the journal.

5. *Bondability.* Many high capacity bearings are made by bonding one or more thin layers of a bearing material to a high strength steel shell. Thus, the strength of the bond *i.e.* bondability is an important consideration in selecting bearing material.

Contents

Marine bearings

6. *Corrosion resistance.* The bearing material should not corrode away under the action of lubricating oil. This property is of particular importance in internal-combustion engines where the same oil is used to lubricate the cylinder walls and bearings. In the cylinder, the lubricating oil comes into contact with hot cylinder walls and may oxidise and collect carbon deposits from the walls.

7. *Thermal conductivity.* The bearing material should be of high thermal conductivity so as to permit the rapid removal of the heat generated by friction.

8. *Thermal expansion.* The bearing material should be of low coefficient of thermal expansion, so that when the bearing operates over a wide range of temperature, there is no undue change in the clearance.

All these properties as discussed above are, however, difficult to find in any particular bearing material. The various materials are used in practice, depending upon the requirement of the actual service conditions. The choice of material for any application must represent a compromise. The following table shows the comparison of some of the properties of more common metallic bearing materials.

Table 26.1. Properties of metallic bearing materials.

Bearing material	*Fatigue strength*	*Comfor-mability*	*Embed-dability*	*Anti scoring*	*Corrosion resistance*	*Thermal conductivity*
Tin base babbit	Poor	Good	Excellent	Excellent	Excellent	Poor
Lead base babbit	Poor to fair	Good	Good	Good to excellent	Fair to good	Poor
Lead bronze	Fair	Poor	Poor	Poor	Good	Fair
Copper lead	Fair	Poor	Poor to fair	Poor to fair	Poor to fair	Fair to good
Aluminium	Good	Poor to fair	Poor	Good	Excellent	Fair
Silver	Excellent	Almost none	Poor	Poor	Excellent	Excellent
Silver lead deposited	Excellent	Excellent	Poor	Fair to good	Excellent	Excellent

26.10 Materials used for Sliding Contact Bearings

The materials commonly used for sliding contact bearings are discussed below :

1. *Babbit metal.* The tin base and lead base babbits are widely used as a bearing material, because they satisfy most requirements for general applications. The babbits are recommended where the maximum bearing pressure (on projected area) is not over 7 to 14 N/mm^2. When applied in

Top

automobiles, the babbit is generally used as a thin layer, 0.05 mm to 0.15 mm thick, bonded to an insert or steel shell. The composition of the babbit metals is as follows :

Tin base babbits : Tin 90% ; Copper 4.5% ; Antimony 5% ; Lead 0.5%.

Lead base babbits : Lead 84% ; Tin 6% ; Anitmony 9.5% ; Copper 0.5%.

2. *Bronzes.* The bronzes (alloys of copper, tin and zinc) are generally used in the form of machined bushes pressed into the shell. The bush may be in one or two pieces. The bronzes commonly used for bearing material are gun metal and phosphor bronzes.

The ***gun metal*** (Copper 88% ; Tin 10% ; Zinc 2%) is used for high grade bearings subjected to high pressures (not more than 10 N/mm^2 of projected area) and high speeds.

The ***phosphor bronze*** (Copper 80% ; Tin 10% ; Lead 9% ; Phosphorus 1%) is used for bearings subjected to very high pressures (not more than 14 N/mm^2 of projected area) and speeds.

3. *Cast iron.* The cast iron bearings are usually used with steel journals. Such type of bearings are fairly successful where lubrication is adequate and the pressure is limited to 3.5 N/mm^2 and speed to 40 metres per minute.

4. *Silver.* The silver and silver lead bearings are mostly used in aircraft engines where the fatigue strength is the most important consideration.

5. *Non-metallic bearings.* The various non-metallic bearings are made of carbon-graphite, rubber, wood and plastics. The **carbon-graphite bearings** are self lubricating, dimensionally stable over a wide range of operating conditions, chemically inert and can operate at higher temperatures than other bearings. Such type of bearings are used in food processing and other equipment where contamination by oil or grease must be prohibited. These bearings are also used in applications where the shaft speed is too low to maintain a hydrodynamic oil film.

The ***soft rubber bearings*** are used with water or other low viscosity lubricants, particularly where sand or other large particles are present. In addition to the high degree of embeddability and comformability, the rubber bearings are excellent for absorbing shock loads and vibrations. The rubber bearings are used mainly on marine propeller shafts, hydraulic turbines and pumps.

The ***wood bearings*** are used in many applications where low cost, cleanliness, inattention to lubrication and anti-seizing are important.

Industrial bearings.

Contents

The commonly used plastic material for bearings is ***Nylon*** and ***Teflon.*** These materials have many characteristics desirable in bearing materials and both can be used dry *i.e*. as a zero film bearing. The Nylon is stronger, harder and more resistant to abrasive wear. It is used for applications in which these properties are important *e.g*. elevator bearings, cams in telephone dials etc. The Teflon is rapidly replacing Nylon as a wear surface or liner for journal and other sliding bearings because of the following properties:

1. It has lower coefficient of friction, about 0.04 (dry) as compared to 0.15 for Nylon.

2. It can be used at higher temperatures up to about 315°C as compared to 120°C for Nylon.

3. It is dimensionally stable because it does not absorb moisture, and

4. It is practically chemically inert.

26.11 Lubricants

The lubricants are used in bearings to reduce friction between the rubbing surfaces and to carry away the heat generated by friction. It also protects the bearing against corrosion. All lubricants are classified into the following three groups :

1. Liquid, **2.** Semi-liquid, and **3.** Solid.

The ***liquid lubricants*** usually used in bearings are mineral oils and synthetic oils. The mineral oils are most commonly used because of their cheapness and stability. The liquid lubricants are usually preferred where they may be retained.

A grease is a ***semi-liquid lubricant*** having higher viscosity than oils. The greases are employed where slow speed and heavy pressure exist and where oil drip from the bearing is undesirable. The ***solid lubricants*** are useful in reducing friction where oil films cannot be maintained because of pressures or temperatures. They should be softer than materials being lubricated. A graphite is the most common of the solid lubricants either alone or mixed with oil or grease.

Wherever moving and rotating parts are present proper lubrication is essential to protect the moving parts from wear and tear and reduce friction.

26.12 Properties of Lubricants

1. *Viscosity*. It is the measure of degree of fluidity of a liquid. It is a physical property by virtue of which an oil is able to form, retain and offer resistance to shearing a buffer film-under heat and pressure. The greater the heat and pressure, the greater viscosity is required of a lubricant to prevent thinning and squeezing out of the film.

The fundamental meaning of viscosity may be understood by considering a flat plate moving under a force P parallel to a stationary plate, the two plates being separated by a thin film of a fluid lubricant of thickness h, as shown in Fig. 26.6. The particles of the lubricant adhere strongly to the moving and stationary plates. The motion is accompanied by a linear slip or shear between the particles throughout the entire height (h) of the film thickness. If A is the area of the plate in contact with the lubricant, then the unit shear stress is given by

$$\tau = P / A$$

Top

According to Newton's law of viscous flow, the magnitude of this shear stress varies directly with the velocity gradient (dV / dy). It is assumed that

(*a*) the lubricant completely fills the space between the two surfaces,

(*b*) the velocity of the lubricant at each surface is same as that of the surface, and

(*c*) any flow of the lubricant perpendicular to the velocity of the plate is negligible.

$$\therefore \quad \tau = \frac{P}{A} \propto \frac{dV}{dy} \quad \text{or} \quad \tau = Z \times \frac{dV}{dy}$$

where Z is a constant of proportionality and is known as ***absolute viscosity*** (or simply viscosity) of the lubricant.

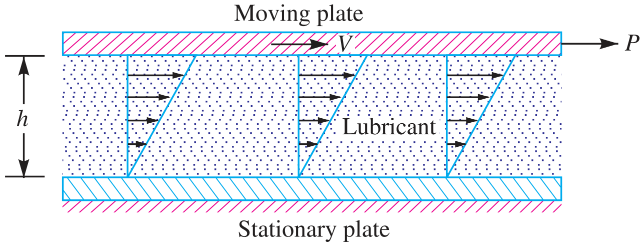


Fig. 26.6. Viscosity.

When the thickness of the fluid lubricant is small which is the case for bearings, then the velocity gradient is very nearly constant as shown in Fig. 26.6, so that

$$\frac{dV}{dy} = \frac{V}{y} = \frac{V}{h}$$

$$\therefore \quad \tau = Z \times \frac{V}{h} \quad \text{or} \quad Z = \tau \times \frac{h}{V}$$

When τ is in N/m^2, h is in metres and V is in m/s, then the unit of absolute viscosity is given by

$$Z = \tau \times \frac{h}{V} = \frac{\text{N}}{\text{m}^2} \times \frac{\text{m}}{\text{m/s}} = \text{N-s/m}^2$$

However, the common practice is to express the absolute viscosity in mass units, such that

$$1 \text{ N-s/m}^2 = \frac{1 \text{ kg-m}}{\text{s}^2} \times \frac{\text{s}}{\text{m}^2} = 1 \text{ kg/m-s} \qquad \ldots (\because \; 1 \text{ N} = 1 \text{ kg-m/s}^2)$$

Thus the unit of absolute viscosity in S.I. units is kg / m-s.

The viscocity of the lubricant is measured by Saybolt universal viscometer. It determines the time required for a standard volume of oil at a certain temperature to flow under a certain head through a tube of standard diameter and length. The time so determined in seconds is the Saybolt universal viscosity. In order to convert Saybolt universal viscosity in seconds to absolute viscosity (in kg / m-s), the following formula may be used:

$$Z = \text{Sp. gr. of oil} \left(0.000\,22\,S - \frac{0.18}{S}\right) \text{kg/m-s} \qquad \ldots(i)$$

where Z = Absolute viscosity at temperature t in kg / m-s, and

S = Saybolt universal viscosity in seconds.

The variation of absolute viscosity with temperature for commonly used lubricating oils is shown in Table 26.2 on the next page.

2. *Oiliness.* It is a joint property of the lubricant and the bearing surfaces in contact. It is a measure of the lubricating qualities under boundary conditions where base metal to metal is prevented only by absorbed film. There is no absolute measure of oiliness.

Contents

Table 26.2. Absolute viscosity of commonly used lubricating oils.

S. No.	*Type of oil*	*Absolute viscosity in kg / m-s at temperature in °C*											
		30	*35*	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	*80*	*90*
1.	SAE 10	0.05	0.036	0.027	0.0245	0.021	0.017	0.014	0.012	0.011	0.009	0.008	0.005
2.	SAE 20	0.069	0.055	0.042	0.034	0.027	0.023	0.020	0.017	0.014	0.011	0.010	0.0075
3.	SAE 30	0.13	0.10	0.078	0.057	0.048	0.040	0.034	0.027	0.022	0.019	0.016	0.010
4.	SAE 40	0.21	0.17	0.12	0.096	0.78	0.06	0.046	0.04	0.034	0.027	0.022	0.013
5.	SAE 50	0.30	0.25	0.20	0.17	0.12	0.09	0.076	0.06	0.05	0.038	0.034	0.020
6.	SAE 60	0.45	0.32	0.27	0.20	0.16	0.12	0.09	0.072	0.057	0.046	0.040	0.025
7.	SAE 70	1.0	0.69	0.45	0.31	0.21	0.165	0.12	0.087	0.067	0.052	0.043	0.033

Note : We see from the above table that the viscosity of oil decreases when its temperature increases.

Top

3. *Density.* This property has no relation to lubricating value but is useful in changing the kinematic viscosity to absolute viscosity. Mathematically

$$\text{Absolute viscosity} = \rho \times \text{Kinematic viscosity (in m}^2\text{/s)}$$

where ρ = Density of the lubricating oil.

The density of most of the oils at 15.5°C varies from 860 to 950 kg / m³ (the average value may be taken as 900 kg / m³). The density at any other temperature (t) may be obtained from the following relation, *i.e.*

$$\rho_t = \rho_{15.5} - 0.000\,657\,t$$

where $\rho_{15.5}$ = Density of oil at 15.5° C.

4. *Viscosity index.* The term viscosity index is used to denote the degree of variation of viscosity with temperature.

5. *Flash point.* It is the lowest temperature at which an oil gives off sufficient vapour to support a momentary flash without actually setting fire to the oil when a flame is brought within 6 mm at the surface of the oil.

6. *Fire point.* It is the temperature at which an oil gives off sufficient vapour to burn it continuously when ignited.

7. *Pour point or freezing point.* It is the temperature at which an oil will cease to flow when cooled.

26.13 Terms used in Hydrodynamic Journal Bearing

A hydrodynamic journal bearing is shown in Fig. 26.7, in which O is the centre of the journal and O' is the centre of the bearing.

Let D = Diameter of the bearing,

d = Diameter of the journal, and

l = Length of the bearing.

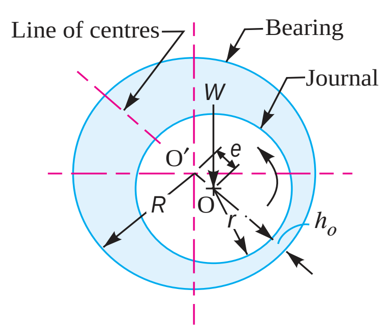


Fig. 26.7. Hydrodynamic journal bearing.

The following terms used in hydrodynamic journal bearing are important from the subject point of view :

1. *Diametral clearance.* It the difference between the diameters of the bearing and the journal. Mathematically, diametral clearance,

$$c = D - d$$

Note : The diametral clearance (c) in a bearing should be small enough to produce the necessary velocity gradient, so that the pressure built up will support the load. Also the small clearance has the advantage of decreasing side leakage. However, the allowance must be made for manufacturing tolerances in the journal and bushing. A commonly used clearance in industrial machines is 0.025 mm per cm of journal diameter.

2. *Radial clearance.* It is the difference between the radii of the bearing and the journal. Mathematically, radial clearance,

$$c_1 = R - r = \frac{D - d}{2} = \frac{c}{2}$$

3. *Diametral clearance ratio.* It is the ratio of the diametral clearance to the diameter of the journal. Mathematically, diametral clearance ratio

$$= \frac{c}{d} = \frac{D - d}{d}$$

4. *Eccentricity.* It is the radial distance between the centre (*O*) of the bearing and the displaced centre (*O′*) of the bearing under load. It is denoted by *e*.

5. *Minimum oil film thickness.* It is the minimum distance between the bearing and the journal, under complete lubrication condition. It is denoted by h_0 and occurs at the line of centres as shown in Fig. 26.7. Its value may be assumed as *c* / 4.

6. *Attitude or eccentricity ratio.* It is the ratio of the eccentricity to the radial clearance. Mathematically, attitude or eccentricity ratio,

$$\varepsilon = \frac{e}{c_1} = \frac{c_1 - h_0}{c_1} = 1 - \frac{h_0}{c_1} = 1 - \frac{2h_0}{c} \quad \ldots (\because \; c_1 = c/2)$$

7. *Short and long bearing.* If the ratio of the length to the diameter of the journal (*i.e.* *l* / *d*) is less than 1, then the bearing is said to be **short bearing.** On the other hand, if *l* / *d* is greater than 1, then the bearing is known as ***long bearing.***

Axle bearings

Notes : 1. When the length of the journal (*l*) is equal to the diameter of the journal (*d*), then the bearing is called ***square bearing.***

2. Because of the side leakage of the lubricant from the bearing, the pressure in the film is atmospheric at the ends of the bearing. The average pressure will be higher for a long bearing than for a short or square bearing. Therefore, from the stand point of side leakage, a bearing with a large *l* / *d* ratio is preferable. However, space requirements, manufacturing, tolerances and shaft deflections are better met with a short bearing. The value of *l* / *d* may be taken as 1 to 2 for general industrial machinery. In crank shaft bearings, the *l* / *d* ratio is frequently less than 1.

26.14 Bearing Characteristic Number and Bearing Modulus for Journal Bearings

The coefficient of friction in design of bearings is of great importance, because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiments that the coefficient of friction for a full lubricated journal bearing is a function of three variables, *i.e.*

(*i*) $\dfrac{ZN}{p}$; (*ii*) $\dfrac{d}{c}$; and (*iii*) $\dfrac{l}{d}$

Therefore the coefficient of friction may be expressed as

$$\mu = \phi\left(\frac{ZN}{p}, \frac{d}{c}, \frac{l}{d}\right)$$

where
μ = Coefficient of friction,
ϕ = A functional relationship,
Z = Absolute viscosity of the lubricant, in kg / m-s,
N = Speed of the journal in r.p.m.,
p = Bearing pressure on the projected bearing area in N/mm^2,
= Load on the journal ÷ $l \times d$
d = Diameter of the journal,
l = Length of the bearing, and
c = Diametral clearance.

The factor *ZN* / *p* is termed as ***bearing characteristic number*** and is a dimensionless number. The variation of coefficient of friction with the operating values of bearing characteristic number (*ZN* / *p*) as obtained by McKee brothers (S.A. McKee and T.R. McKee) in an actual test of friction is shown in Fig. 26.8. The factor *ZN/p* helps to predict the performance of a bearing.

The part of the curve *PQ* represents the region of thick film lubrication. Between *Q* and *R*, the viscosity (*Z*) or the speed (*N*) are so low, or the pressure (*p*) is so great that their combination *ZN* / *p* will reduce the film thickness so that partial metal to metal contact will result. The thin film or boundary lubrication or imperfect lubrication exists between *R* and *S* on the curve. This is the region where the viscosity of the lubricant ceases to be a measure of friction characteristics but the oiliness of the lubricant is effective in preventing complete metal to metal contact and seizure of the parts.

Clutch bearing

It may be noted that the part *PQ* of the curve represents stable operating conditions, since from any point of stability, a decrease in viscosity (*Z*) will reduce *ZN* / *p*. This will result in a decrease in coefficient of friction (μ) followed by a lowering of bearing temperature that will raise the viscosity (*Z*).

From Fig. 26.8, we see that the minimum amount of friction occurs at *A* and at this point the value of *ZN* / *p* is known as ***bearing modulus*** which is denoted by *K*. The bearing should not be operated at this value of bearing modulus, because a slight decrease in speed or slight increase in pressure will break the oil film and make the journal to operate with metal to metal contact. This will result in high friction, wear and heating. In order to prevent such conditions, the bearing should be designed for a value of *ZN* / *p* at least three times the minimum value of bearing modulus (*K*). If the bearing is subjected to large fluctuations of load and heavy impacts, the value of *ZN* / *p* = 15 *K* may be used.

From above, it is concluded that when the value of *ZN* / *p* is greater than *K*, then the bearing will operate with thick film lubrication or under hydrodynamic conditions. On the other hand, when the value of *ZN* / *p* is less than *K*, then the oil film will rupture and there is a metal to metal contact.

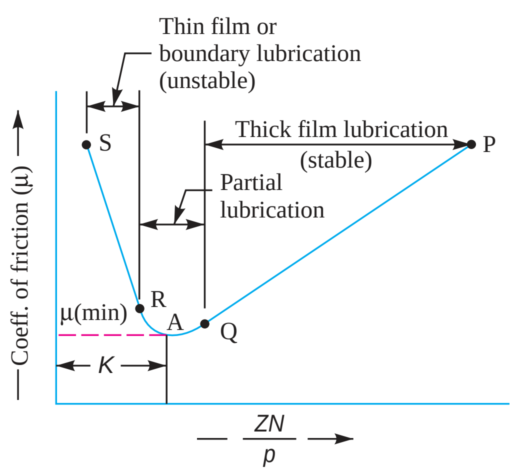


Fig. 26.8. Variation of coefficient of friction with *ZN*/*p*.

26.15 Coefficient of Friction for Journal Bearings

In order to determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data, may be used.

Contents

*Coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k \quad \text{... (when Z is in kg / m-s and } p \text{ is in N / mm}^2\text{)}$$

where Z, N, p, d and c have usual meanings as discussed in previous article, and

k = Factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (*i.e.* l / d).

= 0.002 for l / d ratios of 0.75 to 2.8.

The operating values of ZN / p should be compared with values given in Table 26.3 to ensure safe margin between operating conditions and the point of film breakdown.

Table 26.3. Design values for journal bearings.

Machinery	*Bearing*	*Maximum bearing pressure (p) in N/mm²*	*Operating values*			
			Absolute Viscosity (Z) in kg/m-s	*ZN/p Z in kg/m-s p in N/mm²*	$\frac{c}{d}$	$\frac{l}{d}$
Automobile and air-craft engines	Main Crank pin Wrist pin	5.6 – 12 10.5 – 24.5 16 – 35	0.007 0.008 0.008	2.1 1.4 1.12	—	0.8 – 1.8 0.7 – 1.4 1.5 – 2.2
Four stroke-Gas and oil engines	Main Crank pin Wrist pin	5 – 8.5 9.8 – 12.6 12.6 – 15.4	0.02 0.04 0.065	2.8 1.4 0.7	0.001	0.6 – 2 0.6 – 1.5 1.5 – 2
Two stroke-Gas and oil engines	Main Crank pin Wrist pin	3.5 – 5.6 7 – 10.5 8.4 – 12.6	0.02 0.04 0.065	3.5 1.8 1.4	0.001	0.6 – 2 0.6 – 1.5 1.5 – 2
Marine steam engines	Main Crank pin Wrist pin	3.5 4.2 10.5	0.03 0.04 0.05	2.8 2.1 1.4	0.001	0.7 – 1.5 0.7 – 1.2 1.2 – 1.7
Stationary, slow speed steam engines	Main Crank pin Wrist pin	2.8 10.5 12.6	0.06 0.08 0.06	2.8 0.84 0.7	0.001	1 – 2 0.9 – 1.3 1.2 – 1.5
Stationary, high speed steam engine	Main Crank pin Wrist pin	1.75 4.2 12.6	0.015 0.030 0.025	3.5 0.84 0.7	0.001	1.5 – 3 0.9 – 1.5 13 – 1.7
Reciprocating pumps and compressors	Main Crank pin Wrist pin	1.75 4.2 7.0	0.03 0.05 0.08	4.2 2.8 1.4	0.001	1 – 2.2 0.9 – 1.7 1.5 – 2.0
Steam locomotives	Driving axle Crank pin Wrist pin	3.85 14 28	0.10 0.04 0.03	4.2 0.7 0.7	0.001	1.6 – 1.8 0.7 – 1.1 0.8 – 1.3

* This is the equation of a straight line portion in the region of thick film lubrication (*i.e.* line *PQ*) as shown in Fig. 26.8.

Top

Contents

Machinery	*Bearing*	*Maximum bearing pressure (p) in N/mm²*	*Operating values*			
			Absolute Viscosity (Z) in kg/m-s	*ZN/p Z in kg/m-s p in N/mm²*	$\frac{c}{d}$	$\frac{l}{d}$
Railway cars	Axle	3.5	0.1	7	0.001	1.8 – 2
Steam turbines	Main	0.7 – 2	0.002 – 0.016	14	0.001	1 – 2
Generators, motors, centrifugal pumps	Rotor	0.7 – 1.4	0.025	28	0.0013	1 – 2
Transmission shafts	Light, fixed Self -aligning Heavy	0.175 1.05 1.05	0.025- 0.060	7 2.1 2.1	0.001	2 – 3 2.5 – 4 2– 3
Machine tools	Main	2.1	0.04	0.14	0.001	1–4
Punching and shearing machines	Main Crank pin	28 56	0.10	—	0.001	1–2
Rolling Mills	Main	21	0.05	1.4	0.0015	1–1.5

26.16 Critical Pressure of the Journal Bearing

The pressure at which the oil film breaks down so that metal to metal contact begins, is known as **critical pressure** or the **minimum operating pressure** of the bearing. It may be obtained by the following empirical relation, *i.e.*

Critical pressure or minimum operating pressure,

$$p = \frac{ZN}{4.75 \times 10^6}\left(\frac{d}{c}\right)^2\left(\frac{l}{d+l}\right) \text{ N/mm}^2 \qquad \text{...(when Z is in kg / m-s)}$$

26.17 Sommerfeld Number

The Sommerfeld number is also a dimensionless parameter used extensively in the design of journal bearings. Mathematically,

$$\text{Sommerfeld number} = \frac{ZN}{p}\left(\frac{d}{c}\right)^2$$

For design purposes, its value is taken as follows :

$$\frac{ZN}{p}\left(\frac{d}{c}\right)^2 = 14.3 \times 10^6 \qquad \text{... (when } Z \text{ is in kg / m-s and } p \text{ is in N / mm}^2)$$

26.18 Heat Generated in a Journal Bearing

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing,

$$Q_g = \mu.W.V \text{ N-m/s or J/s or watts} \qquad \text{...}(i)$$

where μ = Coefficient of friction,

W = Load on the bearing in N,

Top

Contents

= Pressure on the bearing in N/mm^2 × Projected area of the bearing in mm^2 = $p\ (l \times d)$,

V = Rubbing velocity in m/s = $\frac{\pi d.N}{60}$, d is in metres, and

N = Speed of the journal in r.p.m.

After the thermal equilibrium has been reached, heat will be dissipated at the outer surface of the bearing at the same rate at which it is generated in the oil film. The amount of heat dissipated will depend upon the temperature difference, size and mass of the radiating surface and on the amount of air flowing around the bearing. However, for the convenience in bearing design, the actual heat dissipating area may be expressed in terms of the projected area of the journal.

Heat dissipated by the bearing,

$$Q_d = C.A\ (t_b - t_a) \text{ J/s or W} \quad \ldots (\because 1 \text{ J/s} = 1 \text{ W}) \quad \ldots(ii)$$

where C = Heat dissipation coefficient in W/m^2/°C,

A = Projected area of the bearing in m^2 = $l \times d$,

t_b = Temperature of the bearing surface in °C, and

t_a = Temperature of the surrounding air in °C.

The value of C have been determined experimentally by O. Lasche. The values depend upon the type of bearing, its ventilation and the temperature difference. The average values of C (in W/m^2/°C), for journal bearings may be taken as follows :

For unventilated bearings (Still air)

= 140 to 420 W/m^2/°C

For well ventilated bearings

= 490 to 1400 W/m^2/°C

It has been shown by experiments that the temperature of the bearing (t_b) is approximately mid-way between the temperature of the oil film (t_0) and the temperature of the outside air (t_a). In other words,

$$t_b - t_a = \frac{1}{2}(t_0 - t_a)$$

Notes : 1. For well designed bearing, the temperature of the oil film should not be more than 60°C, otherwise the viscosity of the oil decreases rapidly and the operation of the bearing is found to suffer. The temperature of the oil film is often called as the ***operating temperature*** of the bearing.

2. In case the temperature of the oil film is higher, then the bearing is cooled by circulating water through coils built in the bearing.

3. The mass of the oil to remove the heat generated at the bearing may be obtained by equating the heat generated to the heat taken away by the oil. We know that the heat taken away by the oil,

$$Q_t = m.S.t \text{ J/s or watts}$$

where m = Mass of the oil in kg / s,

S = Specific heat of the oil. Its value may be taken as 1840 to 2100 J / kg / °C,

t = Difference between outlet and inlet temperature of the oil in °C.

26.19 Design Procedure for Journal Bearing

The following procedure may be adopted in designing journal bearings, when the bearing load, the diameter and the speed of the shaft are known.

Top

Contents

1. Determine the bearing length by choosing a ratio of l / d from Table 26.3.
2. Check the bearing pressure, $p = W / l.d$ from Table 26.3 for probable satisfactory value.
3. Assume a lubricant from Table 26.2 and its operating temperature (t_0). This temperature should be between 26.5°C and 60°C with 82°C as a maximum for high temperature installations such as steam turbines.
4. Determine the operating value of ZN / p for the assumed bearing temperature and check this value with corresponding values in Table 26.3, to determine the possibility of maintaining fluid film operation.
5. Assume a clearance ratio c / d from Table 26.3.
6. Determine the coefficient of friction (μ) by using the relation as discussed in Art. 26.15.
7. Determine the heat generated by using the relation as discussed in Art. 26.18.
8. Determine the heat dissipated by using the relation as discussed in Art. 26.18.
9. Determine the thermal equilibrium to see that the heat dissipated becomes atleast equal to the heat generated. In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water.

Journal bearings are used in helicopters, primarily in the main rotor axis and in the landing gear for fixed wing aircraft.

Example 26.1. *Design a journal bearing for a centrifugal pump from the following data :*

Load on the journal = 20 000 N; Speed of the journal = 900 r.p.m.; Type of oil is SAE 10, for which the absolute viscosity at 55°C = 0.017 kg / m-s; Ambient temperature of oil = 15.5°C ; Maximum bearing pressure for the pump = 1.5 N / mm^2.

Calculate also mass of the lubricating oil required for artificial cooling, if rise of temperature of oil be limited to 10°C. Heat dissipation coefficient = 1232 W/m^2*/°C.*

Solution. Given : W = 20 000 N ; N = 900 r.p.m. ; t_0 = 55°C ; Z = 0.017 kg/m-s ; t_a = 15.5°C ; p = 1.5 N/mm^2 ; t = 10°C ; C = 1232 W/m^2/°C

The journal bearing is designed as discussed in the following steps :

1. First of all, let us find the length of the journal (l). Assume the diameter of the journal (d) as 100 mm. From Table 26.3, we find that the ratio of l / d for centrifugal pumps varies from 1 to 2. Let us take l / d = 1.6.

$$\therefore \qquad l = 1.6\,d = 1.6 \times 100 = 160 \text{ mm}$$ **Ans.**

2. We know that bearing pressure,

$$p = \frac{W}{l.d} = \frac{20\,000}{160 \times 100} = 1.25$$

Since the given bearing pressure for the pump is 1.5 N/mm^2, therefore the above value of p is safe and hence the dimensions of l and d are safe.

3.
$$\frac{Z.N}{p} = \frac{0.017 \times 900}{1.25} = 12.24$$

From Table 26.3, we find that the operating value of

$$\frac{Z.N}{p} = 28$$

We have discussed in Art. 26.14, that the minimum value of the bearing modulus at which the oil film will break is given by

Top

$$3\,K = \frac{ZN}{p}$$

∴ Bearing modulus at the minimum point of friction,

$$K = \frac{1}{3}\left(\frac{Z.N}{p}\right) = \frac{1}{3} \times 28 = 9.33$$

Since the calculated value of bearing characteristic number $\left(\frac{Z.N}{p} = 12.24\right)$ is more than 9.33, therefore the bearing will operate under hydrodynamic conditions.

4. From Table 26.3, we find that for centrifugal pumps, the clearance ratio (c/d)

$$= 0.0013$$

5. We know that coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k = \frac{33}{10^8} \times 12.24 \times \frac{1}{0.0013} + 0.002$$

$$= 0.0031 + 0.002 = 0.0051 \qquad \text{... [From Art. 26.13, } k = 0.002]$$

6. Heat generated,

$$Q_g = \mu\, W\, V = \mu\, W \left(\frac{\pi\, d.N}{60}\right) \text{W} \qquad \text{...}\left(\because V = \frac{\pi\, d.N}{60}\right)$$

$$= 0.0051 \times 20\,000 \left(\frac{\pi \times 0.1 \times 900}{60}\right) = 480.7 \text{ W}$$

... (d is taken in metres)

7. Heat dissipated,

$$Q_d = C.A\,(t_b - t_a) = C.l.d\,(t_b - t_a) \text{ W} \qquad \text{... } (\because A = l \times d)$$

We know that

$$(t_b - t_a) = \tfrac{1}{2}\,(t_0 - t_a) = \tfrac{1}{2}\,(55° - 15.5°) = 19.75°\text{C}$$

$$\therefore \qquad Q_d = 1232 \times 0.16 \times 0.1 \times 19.75 = 389.3 \text{ W}$$

... (l and d are taken in metres)

We see that the heat generated is greater than the heat dissipated which indicates that the bearing is warming up. Therefore, either the bearing should be redesigned by taking t_0 = 63°C or the bearing should be cooled artificially.

We know that the amount of artificial cooling required

$$= \text{Heat generated} - \text{Heat dissipated} = Q_g - Q_d$$

$$= 480.7 - 389.3 = 91.4 \text{ W}$$

Mass of lubricating oil required for artificial cooling

Let m = Mass of the lubricating oil required for artificial cooling in kg / s.

We know that the heat taken away by the oil,

$$Q_t = m.S.t = m \times 1900 \times 10 = 19\,000\, m \text{ W}$$

... [∵ Specific heat of oil (S) = 1840 to 2100 J/kg/°C]

Equating this to the amount of artificial cooling required, we have

$$19\,000\, m = 91.4$$

$$\therefore \qquad m = 91.4 / 19\,000 = 0.0048 \text{ kg / s} = 0.288 \text{ kg / min}$$ **Ans.**

Example 26.2. *The load on the journal bearing is 150 kN due to turbine shaft of 300 mm diameter running at 1800 r.p.m. Determine the following :*

Contents

1. Length of the bearing if the allowable bearing pressure is 1.6 N/mm², and

2. Amount of heat to be removed by the lubricant per minute if the bearing temperature is 60°C and viscosity of the oil at 60°C is 0.02 kg/m-s and the bearing clearance is 0.25 mm.

Axle bearing

Solution. Given : $W = 150$ kN $= 150 \times 10^3$ N ; $d = 300$ mm $= 0.3$ m ; $N = 1800$ r.p.m. ; $p = 1.6$ N/mm² ; $Z = 0.02$ kg / m-s ; $c = 0.25$ mm

1. Length of the bearing

Let l = Length of the bearing in mm.

We know that projected bearing area,

$$A = l \times d = l \times 300 = 300\, l \text{ mm}^2$$

and allowable bearing pressure (p),

$$1.6 = \frac{W}{A} = \frac{150 \times 10^3}{300\, l} = \frac{500}{l}$$

$\therefore$ $l = 500 / 1.6 = 312.5$ mm **Ans.**

2. Amount of heat to be removed by the lubricant

We know that coefficient of friction for the bearing,

$$\mu = \frac{33}{10^8}\left(\frac{Z.N}{p}\right)\left(\frac{d}{c}\right) + k = \frac{33}{10^8}\left(\frac{0.02 \times 1800}{1.6}\right)\left(\frac{300}{0.25}\right) + 0.002$$

$$= 0.009 + 0.002 = 0.011$$

Rubbing velocity,

$$V = \frac{\pi\, d.N}{60} = \frac{\pi \times 0.3 \times 1800}{60} = 28.3 \text{ m/s}$$

$\therefore$ Amount of heat to be removed by the lubricant,

$$Q_g = \mu.W.V = 0.011 \times 150 \times 10^3 \times 28.3 = 46\,695 \text{ J/s or W}$$

$$= 46.695 \text{ kW} \quad \textbf{Ans.} \qquad \text{... (1 J/s = 1 W)}$$

Example 26.3. *A full journal bearing of 50 mm diameter and 100 mm long has a bearing pressure of 1.4 N/mm². The speed of the journal is 900 r.p.m. and the ratio of journal diameter to the diametral clearance is 1000. The bearing is lubricated with oil whose absolute viscosity at the operating temperature of 75°C may be taken as 0.011 kg/m-s. The room temperature is 35°C. Find : 1. The amount of artificial cooling required, and 2. The mass of the lubricating oil required, if the difference between the outlet and inlet temperature of the oil is 10°C. Take specific heat of the oil as 1850 J / kg / °C.*

Solution. Given : $d = 50$ mm $= 0.05$ m ; $l = 100$ mm $= 0.1$ m; $p = 1.4$ N/mm² ; $N = 900$ r.p.m. ; $d / c = 1000$; $Z = 0.011$ kg / m-s ; $t_0 = 75$°C ; $t_a = 35$°C ; $t = 10$°C ; $S = 1850$ J/kg / °C

1. Amount of artificial cooling required

We know that the coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k = \frac{33}{10^8}\left(\frac{0.011 \times 900}{1.4}\right)(1000) + 0.002$$

$$= 0.002\,33 + 0.002 = 0.004\,33$$

Load on the bearing,

$$W = p \times d.l = 1.4 \times 50 \times 100 = 7000 \text{ N}$$

Top

and rubbing velocity,

$$V = \frac{\pi d.N}{60} = \frac{\pi \times 0.05 \times 900}{60} = 2.36 \text{ m/s}$$

∴ Heat generated,

$$Q_g = \mu.W.V = 0.004\,33 \times 7000 \times 2.36 = 71.5 \text{ J/s}$$

Let t_b = Temperature of the bearing surface.

We know that

$$(t_b - t_a) = \frac{1}{2}(t_0 - t_a) = \frac{1}{2}(75 - 35) = 20°\text{C}$$

Since the value of heat dissipation coefficient (C) for unventilated bearing varies from 140 to 420 W/m^2/°C, therefore let us take

$$C = 280 \text{ W/m}^2/°\text{ C}$$

We know that heat dissipated,

$$Q_d = C.A\,(t_b - t_a) = C.l.d\,(t_b - t_a)$$
$$= 280 \times 0.05 \times 0.1 \times 20 = 28 \text{ W} = 28 \text{ J/s}$$

∴ Amount of artificial cooling required

$$= \text{Heat generated} - \text{Heat dissipated} = Q_g - Q_d$$
$$= 71.5 - 28 = 43.5 \text{ J/s or W}$$ **Ans.**

2. Mass of the lubricating oil required

Let m = Mass of the lubricating oil required in kg / s.

We know that heat taken away by the oil,

$$Q_t = m.S.t = m \times 1850 \times 10 = 18\,500\, m \text{ J/s}$$

Since the heat generated at the bearing is taken away by the lubricating oil, therefore equating

$$Q_g = Q_t \quad \text{or} \quad 71.5 = 18\,500\, m$$

∴ $m = 71.5 / 18\,500 = 0.003\,86 \text{ kg / s} = 0.23 \text{ kg / min}$ **Ans.**

Example 26.4. *A 150 mm diameter shaft supporting a load of 10 kN has a speed of 1500 r.p.m. The shaft runs in a bearing whose length is 1.5 times the shaft diameter. If the diametral clearance of the bearing is 0.15 mm and the absolute viscosity of the oil at the operating temperature is 0.011 kg/m-s, find the power wasted in friction.*

Solution. Given : d = 150 mm = 0.15 m ; W = 10 kN = 10 000 N ; N = 1500 r.p.m. ; $l = 1.5\,d$; c = 0.15 mm ; Z = 0.011 kg/m-s

We know that length of bearing,

$$l = 1.5\,d = 1.5 \times 150 = 225 \text{ mm}$$

∴ Bearing pressure,

$$p = \frac{W}{A} = \frac{W}{l.d} = \frac{10\,000}{225 \times 150} = 0.296 \text{ N/mm}^2$$

We know that coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k = \frac{33}{10^8}\left(\frac{0.011 \times 1500}{0.296}\right)\left(\frac{150}{0.15}\right) + 0.002$$
$$= 0.018 + 0.002 = 0.02$$

and rubbing velocity,

$$V = \frac{\pi d.N}{60} = \frac{\pi \times 0.15 \times 1500}{60} = 11.78 \text{ m/s}$$

We know that heat generated due to friction,

$$Q_g = \mu.W.V = 0.02 \times 10\,000 \times 11.78 = 2356 \text{ W}$$

∴ Power wasted in friction

$$= Q_g = 2356 \text{ W} = 2.356 \text{ kW} \textbf{ Ans.}$$

Example 26.5. *A 80 mm long journal bearing supports a load of 2800 N on a 50 mm diameter shaft. The bearing has a radial clearance of 0.05 mm and the viscosity of the oil is 0.021 kg / m-s at the operating temperature. If the bearing is capable of dissipating 80 J/s, determine the maximum safe speed.*

Solution. Given : l = 80 mm ; W = 2800 N ; d = 50 mm ; = 0.05 m ; c / 2 = 0.05 mm or c = 0.1 mm ; Z = 0.021 kg/m-s ; Q_d = 80 J/s

Let N = Maximum safe speed in r.p.m.

We know that bearing pressure,

$$p = \frac{W}{l.d} = \frac{2800}{80 \times 50} = 0.7 \text{ N/mm}^2$$

and coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + 0.002 = \frac{33}{10^8}\left(\frac{0.021\,N}{0.7}\right)\left(\frac{50}{0.1}\right) + 0.002$$

$$= \frac{495\,N}{10^8} + 0.002$$

Front hub-assembly bearing

∴ Heat generated,

$$Q_g = \mu.W.V = \mu.W\left(\frac{\pi\,d\,N}{60}\right) \text{J/s}$$

$$= \left(\frac{495\,N}{10^8} + 0.002\right) 2800\left(\frac{\pi \times 0.05\,N}{60}\right)$$

$$= \frac{3628\,N^2}{10^8} + 0.014\,66\,N$$

Equating the heat generated to the heat dissipated, we have

$$\frac{3628\,N^2}{10^8} + 0.014\,66\,N = 80$$

Contents

or $\quad N^2 + 404\,N - 2.2 \times 10^6 = 0$

$$\therefore \quad N = \frac{-404 \pm \sqrt{(404)^2 + 4 \times 2.2 \times 10^6}}{2}$$

$$= \frac{-404 \pm 2994}{2} = 1295 \text{ r.p.m} \quad \textbf{Ans.} \qquad \text{... (Taking +ve sign)}$$

Example 26.6. *A journal bearing 60 mm is diameter and 90 mm long runs at 450 r.p.m. The oil used for hydrodynamic lubrication has absolute viscosity of 0.06 kg / m-s. If the diametral clearance is 0.1 mm, find the safe load on the bearing.*

Solution. Given : d = 60 mm = 0.06 m ; l = 90 mm = 0.09 m ; N = 450 r.p.m. ; Z = 0.06 kg / m-s ; c = 0.1 mm

First of all, let us find the bearing pressure (p) by using Sommerfeld number. We know that

$$\frac{Z\,N}{p}\left(\frac{d}{c}\right)^2 = 14.3 \times 10^6$$

$$\frac{0.06 \times 450}{p}\left(\frac{60}{0.1}\right)^2 = 14.3 \times 10^6 \quad \text{or} \quad \frac{9.72 \times 10^6}{p} = 14.3 \times 10^6$$

$$\therefore \quad p = 9.72 \times 10^6 / 14.3 \times 10^6 = 0.68 \text{ N/mm}^2$$

We know that safe load on the bearing,

$$W = p.A = p.l.d = 0.68 \times 90 \times 60 = 3672 \text{ N} \quad \textbf{Ans.}$$

26.20 Solid Journal Bearing

A solid bearing, as shown in Fig. 26.9, is the simplest form of journal bearing. It is simply a block of cast iron with a hole for a shaft providing running fit. The lower portion of the block is extended to form a base plate or sole with two holes to receive bolts for fastening it to the frame. An oil hole is drilled at the top for lubrication. The main disadvantages of this bearing are

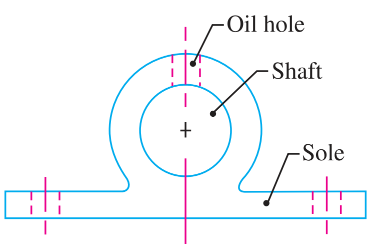


Fig. 26.9. Solid journal bearing.

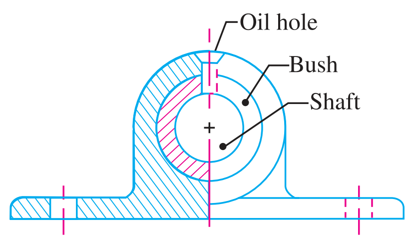


Fig. 26.10. Bushed bearing.

1. There is no provision for adjustment in case of wear, and
2. The shaft must be passed into the bearing axially, *i.e.* endwise.

Since there is no provision for wear adjustment, therefore this type of bearing is used when the shaft speed is not very high and the shaft carries light loads only.

26.21 Bushed Bearing

A bushed bearing, as shown in Fig. 26.10, is an improved solid bearing in which a bush of brass or gun metal is provided. The outside of the bush is a driving fit in the hole of the casting whereas the inside is a running fit for the shaft. When the bush gets worn out, it can be easily replaced. In small bearings, the frictional force itself holds the bush in position, but for shafts transmitting high power, grub screws are used for the prevention of rotation and sliding of the bush.

Top

Bronze bushed bearing assemblies

26.22 Split Bearing or Plummer Block

A split-bearing is used for shafts running at high speeds and carrying heavy loads. A split-bearing, as shown in Fig. 26.11, consists of a cast iron base (also called block or pedestal), gunmetal or phosphor bronze brasses, bushes or steps made in two-halves and a cast iron cap. The two halves of the brasses are held together by a cap or cover by means of mild steel bolts and nuts. Sometimes thin shims are introduced between the cap and the base to provide an adjustment for wear. When the bottom wears out, one or two shims are removed and then the cap is tightened by means of bolts.

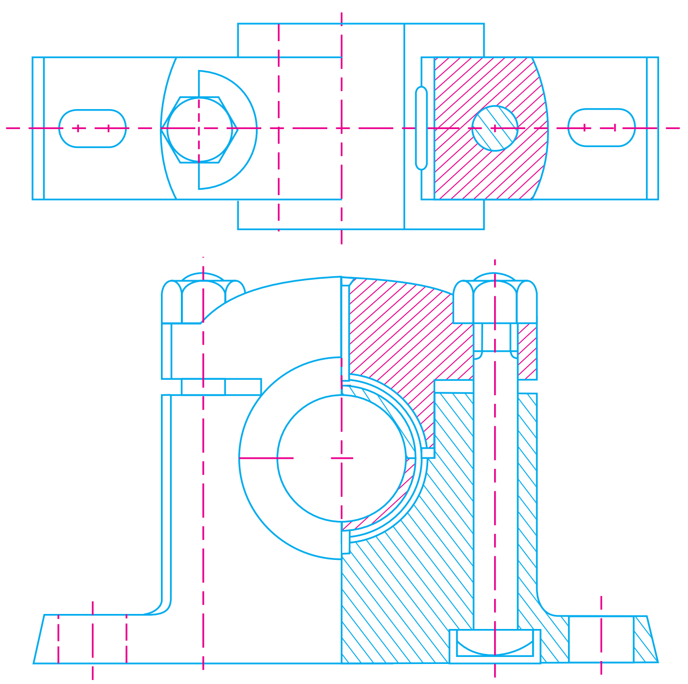


Fig. 26.11. Split bearing or plummer block.

Contents

The brasses are provided with collars or flanges on either side in order to prevent its axial movement. To prevent its rotation along with the shaft, the following four methods are usually used in practice.

1. The sungs are provided at the sides as shown in Fig. 26.12 (*a*).
2. A sung is provided at the top, which fits inside the cap as shown in Fig. 26.12 (*b*). The oil hole is drilled through the sung.
3. The steps are made rectangular on the outside and they are made to fit inside a corresponding hole, as shown in Fig. 26.12 (*c*).
4. The steps are made octagonal on the outside and they are made to fit inside a corresponding hole, as shown in Fig. 26.12 (*d*).

The split bearing must be lubricated properly.

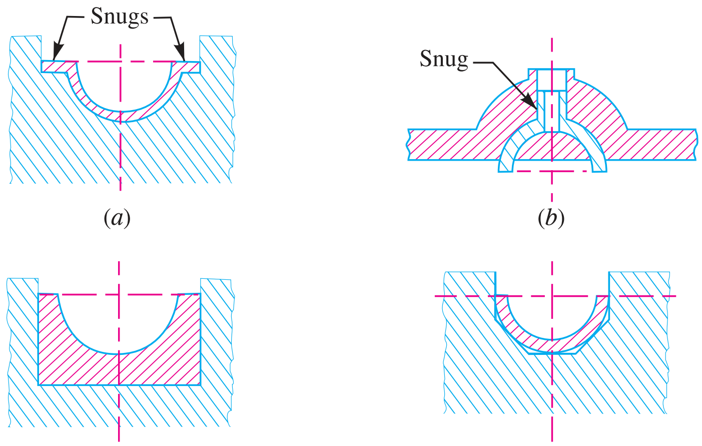


Fig. 26.12. Methods of preventing rotation of brasses.

26.23 Design of Bearing Caps and Bolts

When a split bearing is used, the bearing cap is tightened on the top. The load is usually carried by the bearing and not the cap, but in some cases *e.g.* split connecting rod ends in double acting steam engines, a considerable load comes on the cap of the bearing. Therefore, the cap and the holding down bolts must be designed for full load.

The cap is generally regarded as a simply supported beam, supported by holding down bolts and loaded at the centre as shown in Fig. 26.13.

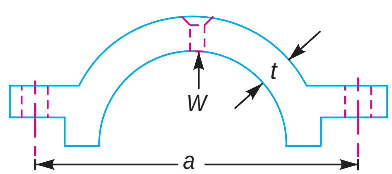


Fig. 26.13. Bearing cap.

Let W = Load supported at the centre,

α = Distance between centres of holding down bolts,

l = Length of the bearing, and

t = Thickness of the cap.

We know that maximum bending moment at the centre,

$$M = W.a / 4$$

and the section modulus of the cap,

$$Z = l.t^2 / 6$$

Top

∴ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{W.a}{4} \times \frac{6}{l.t^2} = \frac{3W.a}{2\,l.t^2}$$

and

$$t = \sqrt{\frac{3\,W.a}{2\sigma_b.l}}$$

Note : When an oil hole is provided in the cap, then the diameter of the hole should be subtracted from the length of the bearing.

The cap of the bearing should also be investigated for the stiffness. We know that for a simply supported beam loaded at the centre, the deflection,

$$\delta = \frac{W.a^3}{48\,E.I} = \frac{W.a^3}{48\,E \times \frac{l.t^3}{12}} = \frac{W.a^3}{4\,E.l\,t^3} \qquad ...\left(\because I = \frac{l.t^3}{12}\right)$$

$$\therefore \qquad t = 0.63\,a\left[\frac{W}{E.l.\delta}\right]^{1/3}$$

The deflection of the cap should be limited to about 0.025 mm.

In order to design the holding down bolts, the load on each bolt is taken 33% higher than the normal load on each bolt. In other words, load on each bolt is taken $\frac{4\,W}{3n}$, where n is the number of bolts used for holding down the cap.

Let d_c = Core diameter of the bolt, and

σ_t = Tensile stress for the material of the bolt.

$$\therefore \qquad \frac{\pi}{4}(d_c)^2\,\sigma_t = \frac{4}{3} \times \frac{W}{n}$$

From this expression, the core diameter (d_c) may be calculated. After finding the core diameter, the size of the bolt is fixed.

26.24 Oil Groves

A self-locking nut used in bearing assemblies.

The oil grooves are cut into the plain bearing surfaces to assist in the distribution of the oil between the rubbing surfaces. It prevents squeezing of the oil film from heavily loaded low speed journals and bearings. The tendency to squeeze out oil is greater in low speed than in high speed bearings, because the oil has greater wedging action at high speeds. At low speeds, the journal rests upon a given area of oil film for a longer period of time, tending to squeeze out the oil over the area of greatest pressure. The grooves function as oil reservoirs which holds and distributes the oil especially during starting or at very low speeds. The oil grooves are cut at right angles to the line of the load. The circumferential and diagonal grooves should be avoided, if possible. The effectiveness of the oil grooves is greatly enhanced if the edges of grooves are chamfered. The shallow and narrow grooves with chamfered edges distributes the oil more evenly. A chamfered edge should always be provided at the parting line of the bearing.

Example 26.7. *A wall bracket supports a plummer block for 80 mm diameter shaft. The length of bearing is 120 mm. The cap of bearing is fastened by means of four bolts, two on each side of the shaft. The cap is to withstand a load of 16.5 kN. The distance between the centre lines of the bolts is*

150 mm. Determine the thickness of the bearing cap and the diameter of the bolts. Assume safe stresses in tension for the material of the cap, which is cast iron, as 15 MPa and for bolts as 35 MPa. Also check the deflection of the bearing cap taking E = 110 kN / mm².

Solution : Given : $d = 80$ mm ; $l = 120$ mm ; $n = 4$; $W = 16.5$ kN $= 16.5 \times 10^3$ N ; $a = 150$ mm ; $\sigma_b = 15$ MPa $= 15$ N/mm^2; $\sigma_t = 35$ MPa $= 35$ N/mm^2 ; $E = 110$ kN/mm$^2 = 110 \times 10^3$ N/mm^2

Thickness of the bearing cap

We know that thickness of the bearing cap,

$$t = \sqrt{\frac{3\,W.a}{2\,\sigma_b.l}} = \sqrt{\frac{3 \times 16.5 \times 10^3 \times 150}{2 \times 15 \times 120}} = \sqrt{2062.5}$$

$$= 45.4 \text{ say } 46 \text{ mm } \textbf{Ans.}$$

Diameter of the bolts

Let d_c = Core diameter of the bolts.

We know that

$$\frac{\pi}{4}(d_c)^2\,\sigma_t = \frac{4}{3} \times \frac{W}{n}$$

or

$$\frac{\pi}{4}(d_c)^2\,35 = \frac{4}{3} \times \frac{16.5 \times 10^3}{4} = 5.5 \times 10^3$$

$$\therefore \quad (d_c)^2 = \frac{5.5 \times 10^3 \times 4}{\pi \times 35} = 200 \quad \text{or} \quad d_c = 14.2 \text{ mm } \textbf{Ans.}$$

Deflection of the cap

We know that deflection of the cap,

$$\delta = \frac{W.a^3}{4\,E.I.t^3} = \frac{16.5 \times 10^3\,(150)^3}{4 \times 110 \times 10^3 \times 120\,(46)^3} = 0.0108 \text{ mm } \textbf{Ans.}$$

Since the limited value of the deflection is 0.025 mm, therefore the above value of deflection is within limits.

26.25 Thrust Bearings

A thrust bearing is used to guide or support the shaft which is subjected to a load along the axis of the shaft. Such type of bearings are mainly used in turbines and propeller shafts. The thrust bearings are of the following two types :

1. Foot step or pivot bearings, and **2.** Collar bearings.

In a ***foot step*** or ***pivot bearing,*** the loaded shaft is vertical and the end of the shaft rests within the bearing. In case of ***collar bearing,*** the shaft continues through the bearing. The shaft may be vertical or horizontal with single collar or many collars. We shall now discuss the design aspects of these bearings in the following articles.

Footstep bearing

26.26 Footstep or Pivot Bearings

A simple type of footstep bearing, suitable for a slow running and lightly loaded shaft, is shown in Fig. 26.14. If the shaft is not of steel, its end

Contents

must be fitted with a steel face. The shaft is guided in a gunmetal bush, pressed into the pedestal and prevented from turning by means of a pin.

Since the wear is proportional to the velocity of the rubbing surface, which (*i.e.* rubbing velocity) increases with the distance from the axis (*i.e.* radius) of the bearing, therefore the wear will be different at different radii. Due to this wear, the distribution of pressure over the bearing surface is not

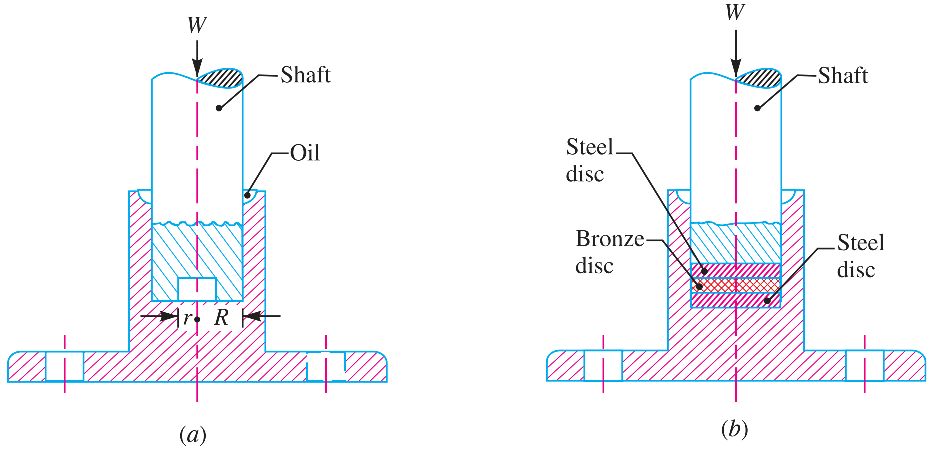


Fig. 26.14. Footstep or pivot bearings.

uniform. It may be noted that the wear is maximum at the outer radius and zero at the centre. In order to compensate for end wear, the following two methods are employed.

1. The shaft is counter-bored at the end, as shown in Fig. 26.14 (*a*).

2. The shaft is supported on a pile of discs. It is usual practice to provide alternate discs of different materials such as steel and bronze, as shown in Fig. 26.14 (*b*), so that the next disc comes into play, if one disc seizes due to improper lubrication.

It may be noted that a footstep bearing is difficult to lubricate as the oil is being thrown outwards from the centre by centrifugal force.

In designing, it is assumed that the pressure is uniformly distributed throughout the bearing surface.

Let W = Load transmitted over the bearing surface,

R = Radius of the bearing surface (or shaft),

A = Cross-sectional area of the bearing surface,

p = Bearing pressure per unit area of the bearing surface between rubbing surfaces,

μ = Coefficient of friction, and

N = Speed of the shaft in r.p.m.

When the pressure in uniformly distributed over the bearing area, then

$$p = \frac{W}{A} = \frac{W}{\pi R^2}$$

and the total frictional torque,

$$T = \frac{2}{3} \mu.W.R$$

∴ Power lost in friction,

$$P = \frac{2\pi N.T}{60} \text{ watts} \quad \text{... (} T \text{ being in N-m)}$$

Top

Contents

Notes : 1. When the counter-boring of the shaft is considered, then the bearing pressure,

$$p = \frac{W}{\pi(R^2 - r^2)}, \quad \text{where } r = \text{Radius of counter-bore,}$$

and the total frictional torque,

$$T = \frac{2}{3}\mu.W\left(\frac{R^3 - r^3}{R^2 - r^2}\right)$$

2. The allowable bearing pressure (p) for the footstep bearings may be taken as follows :

(*a*) For rubbing speeds (V) from 15 to 60 m/min, the bearing pressure should be such that $p.V \le 42$, when p is in N/mm^2 and V in m/min.

(*b*) For rubbing speeds over 60 m/min., the pressure should not exceed 0.7 N/mm^2.

(*c*) For intermittent service, the bearing pressure may be taken as 10.5 N/mm^2.

(*d*) For very slow speeds, the bearing pressure may be taken as high as 14 N/mm^2.

3. The coefficient of friction for the footstep bearing may be taken as 0.015.

26.27 Collar Bearings

We have already discussed that in a collar bearing, the shaft continues through the bearing. The shaft may be vertical or horizontal, with single collar or many collars. A simple multicollar bearing for horizontal shaft is shown in Fig. 26.15. The collars are either integral parts of the shaft or rigidly fastened to it. The outer diameter of the collar is usually taken as 1.4 to 1.8 times the inner diameter of the collar (*i.e*. diameter of the shaft). The thickness of the collar is kept as one-sixth diameter of the shaft and clearance between collars as one-third diameter of the shaft. In designing collar bearings, it is assumed that the pressure is uniformly distributed over the bearing surface.

Collar bearings

Let
W = Load transmitted over the bearing surface,
n = Number of collars,
R = Outer radius of the collar,
r = Inner radius of the collar,
A = Cross-sectional area of the bearing surface = $n\,\pi\,(R^2 - r^2)$,
p = Bearing pressure per unit area of the bearing surface, between rubbing surfaces,
μ = Coefficient of friction, and
N = Speed of the shaft in r.p.m.

When the pressure is uniformly distributed over the bearing surface, then bearing pressure,

$$p = \frac{W}{A} = \frac{W}{n.\pi\,(R^2 - r^2)}$$

and the total frictional torque,

$$T = \frac{2}{3}\mu.\,W\left(\frac{R^3 - r^3}{R^2 - r^2}\right)$$

Top

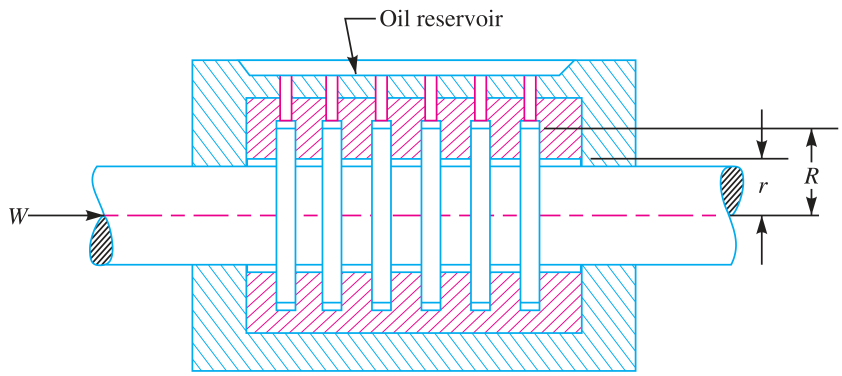


Fig. 26.15. Collar bearing.

∴ Power lost in friction,

$$P = \frac{2\pi\, N.T}{60} \text{ watts} \qquad \text{... (when } T \text{ is in N-m)}$$

Notes : 1. The coefficient of friction for the collar bearings may be taken as 0.03 to 0.05.

2. The bearing pressure for a single collar and water cooled multi-collared bearings may be taken same as for footstep bearings.

Example 26.8. *A footstep bearing supports a shaft of 150 mm diameter which is counter-bored at the end with a hole diameter of 50 mm. If the bearing pressure is limited to 0.8 N/mm² and the speed is 100 r.p.m.; find : 1. The load to be supported; 2. The power lost in friction; and 3. The heat generated at the bearing.*

Assume coefficient of friction = 0.015.

Solution. Given : D = 150 mm or R = 75 mm ; d = 50 mm or r = 25 mm ; p = 0.8 N/mm² ; N = 100 r.p.m. ; μ = 0.015

1. Load to be supported

Let W = Load to be supported.

Assuming that the pressure is uniformly distributed over the bearing surface, therefore bearing pressure (p),

$$0.8 = \frac{W}{\pi (R^2 - r^2)} = \frac{W}{\pi [(75)^2 - (25)^2]} = \frac{W}{15\,710}$$

∴ $W = 0.8 \times 15\,710 = 12\,568$ N **Ans.**

2. Power lost in friction

We know that total frictional torque,

$$T = \frac{2}{3} \mu.W \left(\frac{R^3 - r^3}{R^2 - r^2} \right)$$

$$= \frac{2}{3} \times 0.015 \times 12\,568 \left[\frac{(75)^3 - (25)^3}{(75)^2 - (25)^2} \right] \text{ N-mm}$$

$$= 125.68 \times 81.25 = 10\,212 \text{ N-mm} = 10.212 \text{ N-m}$$

∴ Power lost in friction,

$$P = \frac{2\,\pi\,N\,T}{60} = \frac{2\,\pi \times 100 \times 10.212}{60} = 107 \text{ W} = 0.107 \text{ kW}$$ **Ans.**

3. *Heat generated at the bearing*

We know that heat generated at the bearing

$$= \text{Power lost in friction} = 0.107 \text{ kW or kJ / s}$$

$$= 0.107 \times 60 = 6.42 \text{ kJ/min } \textbf{Ans.}$$

Example 26.9. *The thrust of propeller shaft is absorbed by 6 collars. The rubbing surfaces of these collars have outer diameter 300 mm and inner diameter 200 mm. If the shaft runs at 120 r.p.m., the bearing pressure amounts to 0.4 N/mm². The coefficient of friction may be taken as 0.05. Assuming that the pressure is uniformly distributed, determine the power absorbed by the collars.*

Solution. Given : $n = 6$; $D = 300$ mm or $R = 150$ mm ; $d = 200$ mm or $r = 100$ mm ; $N = 120$ r.p.m. ; $p = 0.4$ N/mm^2 ; $\mu = 0.05$

First of all, let us find the thrust on the shaft (W). Since the pressure is uniformly distributed over the bearing surface, therefore bearing pressure (p),

$$0.4 = \frac{W}{n\,\pi\,(R^2 - r^2)} = \frac{W}{6\pi\,[(150)^2 - (100)^2]} = \frac{W}{235\,650}$$

$$\therefore \quad W = 0.4 \times 235\,650 = 94\,260 \text{ N}$$

We know that total frictional torque,

$$T = \frac{2}{3}\,\mu.W\left(\frac{R^3 - r^3}{R^2 - r^2}\right) = \frac{2}{3} \times 0.05 \times 94260\left[\frac{(150)^3 - (100)^3}{(150)^2 - (100)^2}\right] \text{N-mm}$$

$$= 597\,000 \text{ N-mm} = 597 \text{ N-m}$$

∴ Power absorbed by the collars,

$$P = \frac{2\,\pi.N\,.T}{60} = \frac{2\,\pi \times 120 \times 597}{60} = 7503 \text{ W} = 7.503 \text{ kW} \quad \textbf{Ans.}$$

Example 26.10. *The thrust of propeller shaft in a marine engine is taken up by a number of collars integral with the shaft which is 300 mm is diameter. The thrust on the shaft is 200 kN and the speed is 75 r.p.m. Taking μ constant and equal to 0.05 and assuming the bearing pressure as uniform and equal to 0.3 N/mm², find : 1. Number of collars required, 2. Power lost in friction, and 3. Heat generated at the bearing in kJ/min.*

Industrial bearings.

Solution. Given : $d = 300$ mm or $r = 150$ mm ; $W = 200$ kN $= 200 \times 10^3$ N ; $N = 75$ r.p.m. ; $\mu = 0.05$; $p = 0.3$ N/mm^2

1. *Number of collars required*

Let n = Number of collars required.

Since the outer diameter of the collar (D) is taken as 1.4 to 1.8 times the diameter of shaft (d), therefore let us take

$$D = 1.4\,d = 1.4 \times 300 = 420 \text{ mm} \quad \text{or} \quad R = 210 \text{ mm}$$

We know that the bearing pressure (p),

$$0.3 = \frac{W}{n\,\pi\,(R^2 - r^2)} = \frac{200 \times 10^3}{n\,\pi\,[(210)^2 - (150)^2]} = \frac{2.947}{n}$$

$$\therefore \quad n = 2.947 / 0.3 = 9.8 \text{ say } 10 \textbf{ Ans.}$$

2. Power lost in friction

We know that total frictional torque,

$$T = \frac{2}{3}\mu W \left(\frac{R^3 - r^3}{R^2 - r^2}\right) = \frac{2}{3} \times 0.05 \times 200 \times 10^3 \left[\frac{(210)^3 - (150)^3}{(210)^2 - (150)^2}\right] \text{ N-mm}$$

$= 1817 \times 10^3$ N-mm = 1817 N-m

∴ Power lost in friction,

$$P = \frac{2\,\pi N.T}{60} = \frac{2\,\pi \times 75 \times 1817}{60} = 14\ 270\ \text{W} = 14.27\ \text{kW} \quad \textbf{Ans.}$$

3. Heat generated at the bearing

We know that heat generated at the bearing

= Power lost in friction = 14.27 kW or kJ/s

= 14.27 × 60 = 856.2 kJ/min **Ans.**

EXERCISES

1. The main bearing of a steam engine is 100 mm in diameter and 175 mm long. The bearing supports a load of 28 kN at 250 r.p.m. If the ratio of the diametral clearance to the diameter is 0.001 and the absolute viscosity of the lubricating oil is 0.015 kg/m-s, find : 1. The coefficient of friction ; and 2. The heat generated at the bearing due to friction.

   [Ans. 0.002 77 ; 101.5 J/s]

2. A journal bearing is proposed for a steam engine. The load on the journal is 3 kN, diameter 50 mm, length 75 mm, speed 1600 r.p.m., diametral clearance 0.001 mm, ambient temperature 15.5°C. Oil SAE 10 is used and the film temperature is 60°C. Determine the heat generated and heat dissipated.

   Take absolute viscosity of SAE10 at 60°C = 0.014 kg/m-s. **[Ans. 141.3 J/s ; 25 J/s]**

3. A 100 mm long and 60 mm diameter journal bearing supports a load of 2500 N at 600 r.p.m. If the room temperature is 20°C, what should be the viscosity of oil to limit the bearing surface temperature to 60°C? The diametral clearance is 0.06 mm and the energy dissipation coefficient based on projected area of bearing is 210 W/m^2/°C. **[Ans. 0.0183 kg/m-s]**

4. A tentative design of a journal bearing results in a diameter of 75 mm and a length of 125 mm for supporting a load of 20 kN. The shaft runs at 1000 r.p.m. The bearing surface temperature is not to exceed 75°C in a room temperature of 35°C. The oil used has an absolute viscosity of 0.01 kg/m-s at the operating temperature. Determine the amount of artificial cooling required in watts. Assume d/c = 1000. **[Ans. 146 W]**

5. A journal bearing is to be designed for a centrifugal pump for the following data :

   Load on the journal = 12 kN ; Diameter of the journal = 75 mm ; Speed = 1440 r.p.m ; Atmospheric temperature of the oil = 16°C ; Operating temperature of the oil = 60°C; Absolute viscosity of oil at 60°C = 0.023 kg/m-s.

   Give a systematic design of the bearing.

6. Design a journal bearing for a centrifugal pump running at 1440 r.p.m. The diameter of the journal is 100 mm and load on each bearing is 20 kN. The factor ZN/p may be taken as 28 for centrifugal pump bearings. The bearing is running at 75°C temperature and the atmosphere temperaturic is 30°C. The energy dissipation coefficient is 875 W/m^2/°C. Take diametral clearance as 0.1 mm.

7. Design a suitable journal bearing for a centrifugal pump from the following available data :

   Load on the bearing = 13.5 kN; Diameter of the journal = 80 mm; Speed = 1440 r.p.m.; Bearing characterisitic number at the working temperature (75°C) = 30 ; Permissible bearing pressure intensity

Contents

= 0.7 N/mm^2 to 1.4 N/mm^2; Average atmospheric temperature = 30°C.

Calculate the cooling requirements, if any.

8. A journal bearing with a diameter of 200 mm and length 150 mm carries a load of 20 kN, when the journal speed is 150 r.p.m. The diametral clearance ratio is 0.0015.

If possible, the bearing is to operate at 35°C ambient temperature without external cooling with a maximum oil temperature of 90°C. If external cooling is required, it is to be as little as possible to minimise the required oil flow rate and heat exchanger size.

1. What type of oil do you recommend ?
2. Will the bearing operate without external cooling?
3. If the bearing operates without external cooling, determine the operating oil temperature?
4. If the bearing operates with external cooling, determine the amount of oil in kg/min required to carry away the excess heat generated over heat dissipated, when the oil temperature rises from 85°C to 90°C, when passing through the bearing.

QUESTIONS

1. What are journal bearings? Give a classification of these bearings.

2. What is meant by hydrodynamic lubrication?

3. List the basic assumptions used in the theory of hydrodynamic lubrication.

4. Explain wedge film and squeeze film journal bearings.

5. Enumerate the factors that influence most the formation and maintenance of the thick oil film in hydrodynamic bearings.

6. Make sketches to show the pressure distribution in a journal bearing with thick film lubrication in axial and along the circumference.

7. List the important physical characteristics of a good bearing material.

8. What are the commonly used materials for sliding contact bearings?

9. Write short note on the lubricants used in sliding contact bearings.

10. Explain the following terms as applied to journal bearings :

(*a*) Bearing characteristic number ; and (*b*) Bearing modulus.

11. What are the various terms used in journal bearings analysis and design? Give their definitions in brief.

12. Explain with reference to a neat plot the importance of the bearing characteristic curve.

13. What is the procedure followed in designing a journal bearing?

14. Explain with sketches the working of different types of thrust bearing.

OBJECTIVE TYPE QUESTIONS

1. In a full journal bearing, the angle of contact of the bearing with the journal is

(*a*) 120° (*b*) 180°

(*c*) 270° (*d*) 360°

2. A sliding bearing which can support steady loads without any relative motion between the journal and the bearing is called

(*a*) zero film bearing (*b*) boundary lubricated bearing

(*c*) hydrodynamic lubricated bearing (*d*) hydrostatic lubricated bearing

3. In a boundary lubricated bearing, there is a .... of lubricant between the journal and the bearing.

(*a*) thick film (*b*) thin film

4. When a shaft rotates in anticlockwise direction at slow speed in a bearing, then it will

(*a*) have contact at the lowest point of bearing

(*b*) move towards right of the bearing making metal to metal contact

(*c*) move towards left of the bearing making metal to metal contact

(*d*) move towards right of the bearing making no metal to metal contact

5. The property of a bearing material which has the ability to accommodate small particles of dust, grit etc., without scoring the material of the journal, is called

(*a*) bondability (*b*) embeddability

(*c*) comformability (*d*) fatigue strength

6. Teflon is used for bearings because of

(*a*) low coefficient of friction (*b*) better heat dissipation

(*c*) smaller space consideration (*d*) all of these

7. When the bearing is subjected to large fluctuations of load and heavy impacts, the bearing characteristic number should be .... the bearing modulus.

(*a*) 5 times (*b*) 10 times

(*c*) 15 times (*d*) 20 times

8. When the length of the journal is equal to the diameter of the journal, then the bearing is said to be a

(*a*) short bearing (*b*) long bearing

(*c*) medium bearing (*d*) square bearing

9. If Z = Absolute viscosity of the lubricant in kg/m-s, N = Speed of the journal in r.p.m., and p = Bearing pressure in N/mm^2, then the bearing characteristic number is

(*a*) $\frac{Z\ N}{p}$ (*b*) $\frac{Z\ p}{N}$

(*c*) $\frac{Z}{p\ N}$ (*d*) $\frac{p\ N}{Z}$

10. In thrust bearings, the load acts

(*a*) along the axis of rotation (*b*) parallel to the axis of rotation

(*c*) perpendicular to the axis of rotation (*d*) in any direction

ANSWERS

1. (*d*)	**2.** (*d*)	**3.** (*b*)	**4.** (*c*)	**5.** (*b*)
6. (*a*)	**7.** (*c*)	**8.** (*d*)	**9.** (*a*)	**10.** (*a*)