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CHAPTER

29

Helical Gears

29.1 Introduction

A helical gear has teeth in form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gears. The helixes may be right handed on one gear and left handed on the other. The pitch surfaces are cylindrical as in spur gearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contact, as in spur gearing. This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with a high efficiency of transmission.

We have already discussed in Art. 28.4 that the helical gears may be of ***single helical type*** or ***double helical type.*** In case of single helical gears there is some axial thrust between the teeth, which is a disadvantage. In order to eliminate this axial thrust, double helical gears (*i.e.*

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herringbone gears) are used. It is equivalent to two single helical gears, in which equal and opposite thrusts are provided on each gear and the resulting axial thrust is zero.

29.2 Terms used in Helical Gears

The following terms in connection with helical gears, as shown in Fig. 29.1, are important from the subject point of view.

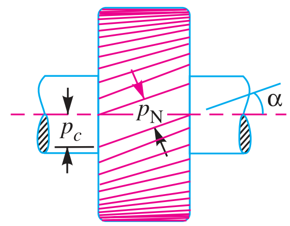

Fig. 29.1. Helical gear (nomenclature).

1. *Helix angle.* It is a constant angle made by the helices with the axis of rotation.

2. *Axial pitch.* It is the distance, parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by p_c. The axial pitch may also be defined as the circular pitch in the plane of rotation or the diametral plane.

3. *Normal pitch.* It is the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by p_N. The normal pitch may also be defined as the circular pitch in the normal plane which is a plane perpendicular to the teeth. Mathematically, normal pitch,

$$p_N = p_c \cos \alpha$$

Note : If the gears are cut by standard hobs, then the pitch (or module) and the pressure angle of the hob will apply in the normal plane. On the other hand, if the gears are cut by the Fellows gear-shaper method, the pitch and pressure angle of the cutter will apply to the plane of rotation. The relation between the normal pressure angle (ϕ_N) in the normal plane and the pressure angle (ϕ) in the diametral plane (or plane of rotation) is given by

$$\tan \phi_N = \tan \phi \times \cos \alpha$$

29.3 Face Width of Helical Gears

In order to have more than one pair of teeth in contact, the tooth displacement (*i.e.* the advancement of one end of tooth over the other end) or overlap should be atleast equal to the axial pitch, such that

$$\text{Overlap} = p_c = b \tan \alpha \qquad ...(i)$$

The normal tooth load (W_N) has two components ; one is tangential component (W_T) and the other axial component (W_A), as shown in Fig. 29.2. The axial or end thrust is given by

$$W_A = W_N \sin \alpha = W_T \tan \alpha \qquad ...(ii)$$

From equation (*i*), we see that as the helix angle increases, then the tooth overlap increases. But at the same time, the end thrust as given by equation (*ii*), also increases, which is undesirable. It is usually recommended that the overlap should be 15 percent of the circular pitch.

$$\therefore \qquad \text{Overlap} = b \tan \alpha = 1.15\, p_c$$

or
$$b = \frac{1.15\, p_c}{\tan \alpha} = \frac{1.15 \times \pi m}{\tan \alpha} \qquad ... (\because p_c = \pi m)$$

where b = Minimum face width, and

m = Module.

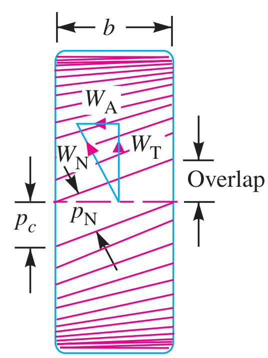

Fig. 29.2. Face width of helical gear.

Notes : 1. The maximum face width may be taken as 12.5 m to 20 m, where m is the module. In terms of pinion diameter (D_P), the face width should be 1.5 D_P to 2 D_P, although 2.5 D_P may be used.

2. In case of double helical or herringbone gears, the minimum face width is given by

$$b = \frac{2.3\, p_c}{\tan \alpha} = \frac{2.3 \times \pi m}{\tan \alpha}$$

The maximum face width ranges from 20 m to 30 m.

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3. In single helical gears, the helix angle ranges from 20° to 35°, while for double helical gears, it may be made upto 45°.

29.4 Formative or Equivalent Number of Teeth for Helical Gears

The formative or equivalent number of teeth for a helical gear may be defined as the number of teeth that can be generated on the surface of a cylinder having a radius equal to the radius of curvature at a point at the tip of the minor axis of an ellipse obtained by taking a section of the gear in the normal plane. Mathematically, formative or equivalent number of teeth on a helical gear,

$$T_E = T / \cos^3 \alpha$$

where T = Actual number of teeth on a helical gear, and

α = Helix angle.

29.5 Proportions for Helical Gears

Though the proportions for helical gears are not standardised, yet the following are recommended by American Gear Manufacturer's Association (AGMA).

Pressure angle in the plane of rotation,

ϕ = 15° to 25°

Helix angle, α = 20° to 45°

Addendum = 0.8 m (Maximum)

Dedendum = 1 m (Minimum)

Minimum total depth = 1.8 m

Minimum clearance = 0.2 m

Thickness of tooth = 1.5708 m

In helical gears, the teeth are inclined to the axis of the gear.

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29.6 Strength of Helical Gears

In helical gears, the contact between mating teeth is gradual, starting at one end and moving along the teeth so that at any instant the line of contact runs diagonally across the teeth. Therefore in order to find the strength of helical gears, a modified Lewis equation is used. It is given by

$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y'$$

where W_T = Tangential tooth load,

σ_o = Allowable static stress,

C_v = Velocity factor,

b = Face width,

m = Module, and

y' = Tooth form factor or Lewis factor corresponding to the formative or virtual or equivalent number of teeth.

Notes : 1. The value of velocity factor (C_v) may be taken as follows :

$$C_v = \frac{6}{6+v}, \text{ for peripheral velocities from 5 m / s to 10 m / s.}$$

$$= \frac{15}{15+v}, \text{ for peripheral velocities from 10 m / s to 20 m / s.}$$

$$= \frac{0.75}{0.75+\sqrt{v}}, \text{ for peripheral velocities greater than 20 m / s.}$$

$$= \frac{0.75}{1+v} + 0.25, \text{ for non-metallic gears.}$$

2. The dynamic tooth load on the helical gears is given by

$$W_D = W_T + \frac{21\, v\, (b.C \cos^2 \alpha + W_T) \cos \alpha}{21\, v + \sqrt{b.C \cos^2 \alpha + W_T}}$$

where v, b and C have usual meanings as discussed in spur gears.

3. The static tooth load or endurance strength of the tooth is given by

$$W_S = \sigma_e.b.\pi\, m.y'$$

4. The maximum or limiting wear tooth load for helical gears is given by

$$W_w = \frac{D_P .b.Q.K}{\cos^2 \alpha}$$

where D_P, b, Q and K have usual meanings as discussed in spur gears.

In this case, $$K = \frac{(\sigma_{es})^2 \sin \phi_N}{1.4}\left[\frac{1}{E_P} + \frac{1}{E_G}\right]$$

where ϕ_N = Normal pressure angle.

Example 29.1. *A pair of helical gears are to transmit 15 kW. The teeth are 20° stub in diametral plane and have a helix angle of 45°. The pinion runs at 10 000 r.p.m. and has 80 mm pitch diameter. The gear has 320 mm pitch diameter. If the gears are made of cast steel having allowable static strength of 100 MPa; determine a suitable module and face width from static strength considerations and check the gears for wear, given* σ_{es} *= 618 MPa.*

Solution. Given : P = 15 kW = 15 × 10^3 W; ϕ = 20° ; α = 45° ; N_P = 10 000 r.p.m. ; D_P = 80 mm = 0.08 m ; D_G = 320 mm = 0.32 m ; $\sigma_{OP} = \sigma_{OG}$ = 100 MPa = 100 N/mm^2 ; σ_{es} = 618 MPa = 618 N/mm^2

Module and face width

Let m = Module in mm, and

b = Face width in mm.

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Since both the pinion and gear are made of the same material (*i.e.* cast steel), therefore the pinion is weaker. Thus the design will be based upon the pinion.

We know that the torque transmitted by the pinion,

$$T = \frac{P \times 60}{2\pi N_P} = \frac{15 \times 10^3 \times 60}{2\pi \times 10\,000} = 14.32 \text{ N-m}$$

∴ *Tangential tooth load on the pinion,

$$W_T = \frac{T}{D_P/2} = \frac{14.32}{0.08/2} = 358 \text{ N}$$

We know that number of teeth on the pinion,

$$T_P = D_P / m = 80 / m$$

and formative or equivalent number of teeth for the pinion,

$$T_E = \frac{T_P}{\cos^3 \alpha} = \frac{80/m}{\cos^3 45°} = \frac{80/m}{(0.707)^3} = \frac{226.4}{m}$$

∴ Tooth form factor for the pinion for 20° stub teeth,

$$y'_P = 0.175 - \frac{0.841}{T_E} = 0.175 - \frac{0.841}{226.4/m} = 0.175 - 0.0037\, m$$

We know that peripheral velocity,

$$v = \frac{\pi D_P . N_P}{60} = \frac{\pi \times 0.08 \times 10\,000}{60} = 42 \text{ m/s}$$

∴ Velocity factor,

$$C_v = \frac{0.75}{0.75 + \sqrt{v}} = \frac{0.75}{0.75 + \sqrt{42}} = 0.104 \quad \text{...(∵ } v \text{ is greater than 20 m/s)}$$

Since the maximum face width (b) for helical gears may be taken as 12.5 m to 20 m, where m is the module, therefore let us take

$$b = 12.5\, m$$

We know that the tangential tooth load (W_T),

$$358 = (\sigma_{OP} . C_v)\, b.\pi\, m.y'_P$$
$$= (100 \times 0.104)\, 12.5\, m \times \pi\, m\, (0.175 - 0.0037\, m)$$
$$= 409\, m^2 (0.175 - 0.0037\, m) = 72\, m^2 - 1.5\, m^3$$

Solving this expression by hit and trial method, we find that

$$m = 2.3 \text{ say } 2.5 \text{ mm}$$ **Ans.**

and face width, $b = 12.5\, m = 12.5 \times 2.5 = 31.25$ say 32 mm **Ans.**

Checking the gears for wear

We know that velocity ratio,

$$V.R. = \frac{D_G}{D_P} = \frac{320}{80} = 4$$

∴ Ratio factor,

$$Q = \frac{2 \times V.R.}{V.R. + 1} = \frac{2 \times 4}{4 + 1} = 1.6$$

We know that $\tan \phi_N = \tan \phi \cos \alpha = \tan 20° \times \cos 45° = 0.2573$

∴ $\phi_N = 14.4°$

* The tangential tooth load on the pinion may also be obtained by using the relation,

$$W_T = \frac{P}{v}, \text{ where } v = \frac{\pi D_P . N_P}{60} \text{ (in m/s)}$$

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The picture shows double helical gears which are also called herringbone gears.

Since both the gears are made of the same material (*i.e.* cast steel), therefore let us take

$$E_P = E_G = 200 \text{ kN/mm}^2 = 200 \times 10^3 \text{ N/mm}^2$$

∴ Load stress factor,

$$K = \frac{(\sigma_{es})^2 \sin \phi_N}{1.4}\left(\frac{1}{E_P} + \frac{1}{E_G}\right)$$

$$= \frac{(618)^2 \sin 14.4°}{1.4}\left(\frac{1}{200 \times 10^3} + \frac{1}{200 \times 10^3}\right) = 0.678 \text{ N/mm}^2$$

We know that the maximum or limiting load for wear,

$$W_w = \frac{D_P . b . Q . K}{\cos^2 \alpha} = \frac{80 \times 32 \times 1.6 \times 0.678}{\cos^2 45°} = 5554 \text{ N}$$

Since the maximum load for wear is much more than the tangential load on the tooth, therefore the design is satisfactory from consideration of wear.

Example 29.2. *A helical cast steel gear with 30° helix angle has to transmit 35 kW at 1500 r.p.m. If the gear has 24 teeth, determine the necessary module, pitch diameter and face width for 20° full depth teeth. The static stress for cast steel may be taken as 56 MPa. The width of face may be taken as 3 times the normal pitch. What would be the end thrust on the gear? The tooth factor for 20° full depth involute gear may be taken as* $0.154 - \frac{0.912}{T_E}$, *where* T_E *represents the equivalent number of teeth.*

Solution. Given : α = 30° ; P = 35 kW = 35 × 10^3 W ; N = 1500 r.p.m. ; T_G = 24 ; ϕ = 20° ; σ_o = 56 MPa = 56 N/mm^2 ; b = 3 × Normal pitch = 3 p_N

Module

Let m = Module in mm, and

D_G = Pitch circle diameter of the gear in mm.

We know that torque transmitted by the gear,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{35 \times 10^3 \times 60}{2\,\pi \times 1500} = 223 \text{ N-m} = 223 \times 10^3 \text{ N-mm}$$

Formative or equivalent number of teeth,

$$T'_E = \frac{T_G}{\cos^3 \alpha} = \frac{24}{\cos^3 30°} = \frac{24}{(0.866)^3} = 37$$

∴ Tooth factor, $y' = 0.154 - \frac{0.912}{T_E} = 0.154 - \frac{0.912}{37} = 0.129$

We know that the tangential tooth load,

$$W_T = \frac{T}{D_G / 2} = \frac{2\,T}{D_G} = \frac{2\,T}{m \times T_G} \qquad \text{... } (\because D_G = m.T_G)$$

$$= \frac{2 \times 223 \times 10^3}{m \times 24} = \frac{18\,600}{m} \text{ N}$$

and peripheral velocity,

$$v = \frac{\pi\, D_G\, .N}{60} = \frac{\pi .m.\, T_G\, .N}{60} \text{ mm / s} \qquad \text{...}(D_G \text{ and } m \text{ are in mm})$$

$$= \frac{\pi \times m \times 24 \times 1500}{60} = 1885\, m \text{ mm / s} = 1.885\, m \text{ m / s}$$

Let us take velocity factor,

$$C_v = \frac{15}{15 + v} = \frac{15}{15 + 1.885\, m}$$

We know that tangential tooth load,

$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y' = (\sigma_o \times C_v)\, 3p_N \times \pi\, m \times y' \qquad \text{...}(\because b = 3\, p_N)$$

$$= (\sigma_o \times C_v)\, 3 \times p_c \cos \alpha \times \pi\, m \times y' \qquad \text{...}(\because p_N = p_c \cos \alpha)$$

$$= (\sigma_o \times C_v)\, 3\, \pi\, m \cos \alpha \times \pi\, m \times y' \qquad \text{...}(\because p_c = \pi\, m)$$

∴ $$\frac{18\,600}{m} = 56 \left(\frac{15}{15 + 1.885\, m} \right) 3\, \pi\, m \times \cos 30° \times \pi\, m \times 0.129$$

$$= \frac{2780\, m^2}{15 + 1.885\, m}$$

or $279\,000 + 35\,061\, m = 2780\, m^3$

Solving this equation by hit and trial method, we find that

m = 5.5 say 6 mm **Ans.**

Pitch diameter of the gear

We know that the pitch diameter of the gear,

$D_G = m \times T_G = 6 \times 24 = 144$ mm **Ans.**

Face width

It is given that the face width,

$$b = 3\, p_N = 3\, p_c \cos \alpha = 3 \times \pi\, m \cos \alpha$$

$$= 3 \times \pi \times 6 \cos 30° = 48.98 \text{ say } 50 \text{ mm}$$ **Ans.**

End thrust on the gear

We know that end thrust or axial load on the gear,

$$W_A = W_T \tan \alpha = \frac{18\,600}{m} \times \tan 30° = \frac{18\,600}{6} \times 0.577 = 1790 \text{ N}$$ **Ans.**

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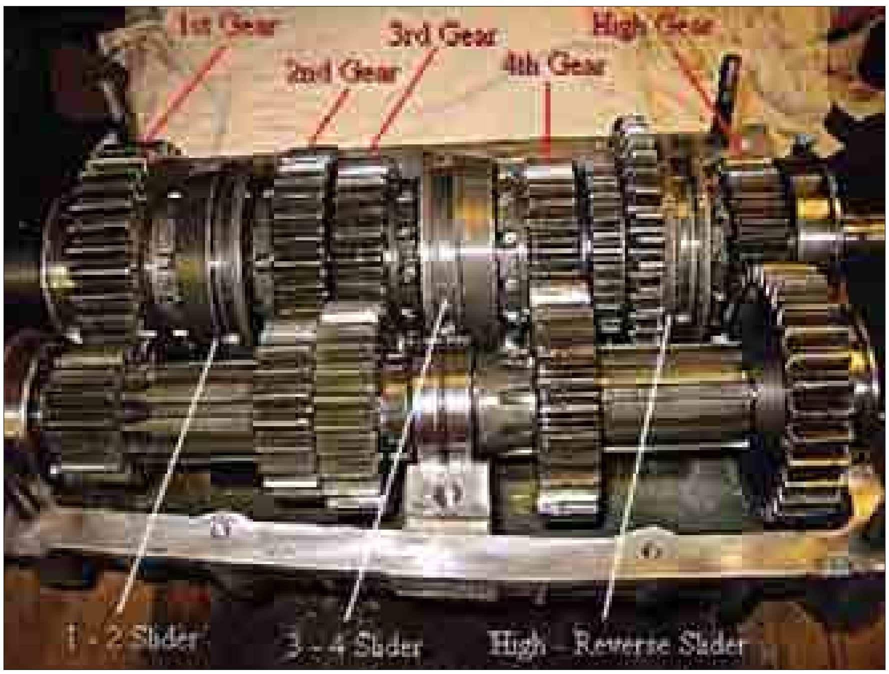
Example 29.3. *Design a pair of helical gears for transmitting 22 kW. The speed of the driver gear is 1800 r.p.m. and that of driven gear is 600 r.p.m. The helix angle is 30° and profile is corresponding to 20° full depth system. The driver gear has 24 teeth. Both the gears are made of cast steel with allowable static stress as 50 MPa. Assume the face width parallel to axis as 4 times the circular pitch and the overhang for each gear as 150 mm. The allowable shear stress for the shaft material may be taken as 50 MPa. The form factor may be taken as 0.154 – 0.912 / T_E, where T_E is the equivalent number of teeth. The velocity factor may be taken as $\frac{350}{350 + v}$, where v is pitch line velocity in m / min. The gears are required to be designed only against bending failure of the teeth under dynamic condition.*

Solution. Given : P = 22 kW = 22 × 10^3 W ; N_P = 1800 r.p.m.; N_G = 600 r.p.m. ; α = 30° ; ϕ = 20° ; T_P = 24 ; σ_o = 50 MPa = 50 N/mm^2 ; $b = 4\,p_c$; Overhang = 150 mm ; τ = 50 MPa = 50 N/mm^2



Gears inside a car

Design for the pinion and gear

We know that the torque transmitted by the pinion,

$$T = \frac{P \times 60}{2\,\pi\,N_P} = \frac{22 \times 10^3 \times 60}{2\,\pi \times 1800} = 116.7 \text{ N-m} = 116\ 700 \text{ N-mm}$$

Since both the pinion and gear are made of the same material (*i.e.* cast steel), therefore the pinion is weaker. Thus the design will be based upon the pinion. We know that formative or equivalent number of teeth,

$$T_E = \frac{T_P}{\cos^3\alpha} = \frac{24}{\cos^3 30°} = \frac{24}{(0.866)^3} = 37$$

∴ Form factor, $$y' = 0.154 - \frac{0.912}{T_E} = 0.154 - \frac{0.912}{37} = 0.129$$

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First of all let us find the module of teeth.

Let m = Module in mm, and

D_P = Pitch circle diameter of the pinion in mm.

We know that the tangential tooth load on the pinion,

$$W_T = \frac{T}{D_P/2} = \frac{2T}{D_P} = \frac{2T}{m \times T_P} \quad ...(\because D_P = m.T_P)$$

$$= \frac{2 \times 116\ 700}{m \times 24} = \frac{9725}{m} \text{ N}$$

and peripheral velocity, $v = \pi D_P.N_P = \pi m.T_P.N_P$

$= \pi m \times 24 \times 1800 = 135\ 735\ m$ mm / min $= 135.735\ m$ m / min

$$\therefore \text{ Velocity factor, } C_v = \frac{350}{350 + v} = \frac{350}{350 + 135.735\ m}$$

We also know that the tangential tooth load on the pinion,

$$W_T = (\sigma_o.C_v)\ b.\pi\ m.y' = (\sigma_o.C_v)\ 4\ p_c \times \pi\ m \times y' \quad ... (\because b = 4\ p_c)$$

$$= (\sigma_o.C_v)\ 4 \times \pi\ m \times \pi\ m \times y' \quad ... (\because p_c = \pi\ m)$$

$$\therefore \quad \frac{9725}{m} = 50\left(\frac{350}{350 + 135.735m}\right) 4 \times \pi^2\ m^2 \times 0.129 = \frac{89\ 126\ m^2}{350 + 135.735m}$$

$3.4 \times 10^6 + 1.32 \times 10^6\ m = 89\ 126\ m^3$

Solving this expression by hit and trial method, we find that

m = 4.75 mm say 6 mm **Ans.**

Helical gears.

We know that face width,

$b = 4\ p_c = 4\ \pi\ m = 4\ \pi \times 6 = 75.4$ say 76 mm **Ans.**

and pitch circle diameter of the pinion,

$D_P = m \times T_P = 6 \times 24 = 144$ mm **Ans.**

Since the velocity ratio is 1800 / 600 = 3, therefore number of teeth on the gear,

$T_G = 3\ T_P = 3 \times 24 = 72$

and pitch circle diameter of the gear,

$D_G = m \times T_G = 6 \times 72 = 432$ mm **Ans.**

Design for the pinion shaft

Let d_P = Diameter of the pinion shaft.

We know that the tangential load on the pinion,

$$W_T = \frac{9725}{m} = \frac{9725}{6} = 1621 \text{ N}$$

and the axial load of the pinion,

$$W_A = W_T \tan \alpha = 1621 \tan 30°$$
$$= 1621 \times 0.577 = 935 \text{ N}$$

Since the overhang for each gear is 150 mm, therefore bending moment on the pinion shaft due to the tangential load,

$$M_1 = W_T \times \text{Overhang} = 1621 \times 150 = 243\ 150 \text{ N-mm}$$

and bending moment on the pinion shaft due to the axial load,

$$M_2 = W_A \times \frac{D_P}{2} = 935 \times \frac{144}{2} = 67\,320 \text{ N-mm}$$

Since the bending moment due to the tangential load (*i.e.* M_1) and bending moment due to the axial load (*i.e.* M_2) are at right angles, therefore resultant bending moment on the pinion shaft,

$$M = \sqrt{(M_1)^2 + (M_2)^2} = \sqrt{(243\,150)^2 + (67\ 320)^2} = 252\,293 \text{ N-mm}$$

The pinion shaft is also subjected to a torque T = 116 700 N-mm, therefore equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(252\,293)^2 + (116\,700)^2} = 277\ 975 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$277\ 975 = \frac{\pi}{16} \times \tau\,(d_P)^3 = \frac{\pi}{16} \times 50\,(d_P)^3 = 9.82\,(d_P)^3$$

$\therefore$ $(d_P)^3 = 277\ 975 / 9.82 = 28\ 307$ or $d_P = 30.5$ say 35 mm **Ans.**

Let us now check for the principal shear stress.

We know that the shear stress induced,

$$\tau = \frac{16\,T_e}{\pi\,(d_P)^3} = \frac{16 \times 277\,975}{\pi (35)^3} = 33 \text{ N/mm}^2 = 33 \text{ MPa}$$

and direct stress due to axial load,

$$\sigma = \frac{W_A}{\frac{\pi}{4}\,(d_P)^2} = \frac{935}{\frac{\pi}{4}\,(35)^2} = 0.97 \text{ N/mm}^2 = 0.97 \text{ MPa}$$

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∴ Principal shear stress,

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(0.97)^2 + 4(33)^2}\right] = 33 \text{ MPa}$$

Since the principal shear stress is less than the permissible shear stress of 50 MPa, therefore the design is satisfactory.

We know that the diameter of the pinion hub

$= 1.8\, d_P = 1.8 \times 35 = 63$ mm **Ans.**

and length of the hub $= 1.25\, d_P = 1.25 \times 35 = 43.75$ say 44 mm

Since the length of the hub should not be less than the face width, therefore let us take length of the hub as 76 mm. **Ans.**

Note : Since the pitch circle diameter of the pinion is 144 mm, therefore the pinion should be provided with a web. Let us take the thickness of the web as 1.8 *m*, where *m* is the module.

∴ Thickness of the web $= 1.8\, m = 1.8 \times 6 = 10.8$ say 12 mm **Ans.**

Design for the gear shaft

Let d_G = Diameter of the gear shaft.

We have already calculated that the tangential load,

$$W_T = 1621 \text{ N}$$

and the axial load, $W_A = 935$ N

∴ Bending moment due to the tangential load,

$$M_1 = W_T \times \text{Overhang} = 1621 \times 150 = 243\ 150 \text{ N-mm}$$

and bending moment due to the axial load,

$$M_2 = W_A \times \frac{D_G}{2} = 935 \times \frac{432}{2} = 201\,960 \text{ N-mm}$$

∴ Resultant bending moment on the gear shaft,

$$M = \sqrt{(M_1)^2 + (M_2)^2} = \sqrt{(243\,150)^2 + (201\,960)^2} = 316\ 000 \text{ N-mm}$$

Since the velocity ratio is 3, therefore the gear shaft is subjected to a torque equal to 3 times the torque on the pinion shaft.

∴ Torque on the gear shaft,

$$T = \text{Torque on the pinion shaft} \times V.R.$$

$$= 116\ 700 \times 3 = 350\ 100 \text{ N-mm}$$

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(316\ 000)^2 + (350\ 100)^2} = 472\ 000 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$472\ 000 = \frac{\pi}{16} \times \tau \times (d_G)^3 = \frac{\pi}{16} \times 50\,(d_G)^3 = 9.82\,(d_G)^3$$

∴ $(d_G)^3 = 472\ 000 / 9.82 = 48\ 065$ or $d_G = 36.3$ say 40 mm **Ans.**

Let us now check for the principal shear stress.

We know that the shear stress induced,

$$\tau = \frac{16\, T_e}{\pi\,(d_G)^3} = \frac{16 \times 472\,000}{\pi\,(40)^3} = 37.6 \text{ N/mm}^2 = 37.6 \text{ MPa}$$

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and direct stress due to axial load,

$$\sigma = \frac{W_A}{\frac{\pi}{4}(d_G)^2} = \frac{935}{\frac{\pi}{4}(40)^2} = 0.744 \text{ N/mm}^2 = 0.744 \text{ MPa}$$

∴ Principal shear stress

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(0.744)^2 + 4\,(37.6)^2}\right] = 37.6 \text{ MPa}$$

Since the principal shear stress is less than the permissible shear stress of 50 MPa, therefore the design is satisfactory.

We know that the diameter of the gear hub

$$= 1.8\, d_G = 1.8 \times 40 = 72 \text{ mm} \quad \textbf{Ans.}$$

and length of the hub $= 1.25\, d_G = 1.25 \times 40 = 50$ mm

We shall take the length of the hub equal to the face width, *i.e.* 76 mm. **Ans.**

Since the pitch circle diameter of the gear is 432 mm, therefore the gear should be provided with four arms. The arms are designed in the similar way as discussed for spur gears.

Design for the gear arms

Let us assume that the cross-section of the arms is elliptical with major axis (a_1) equal to twice the minor axis (b_1). These dimensions refer to hub end.

∴ Section modulus of arms,

$$Z = \frac{\pi b_1 (a_1)^2}{32} = \frac{\pi (a_1)^3}{64} = 0.05\,(a_1)^3 \qquad \left(\because b_1 = \frac{a_1}{2}\right)$$

Since the arms are designed for the stalling load and it is taken as the design tangential load divided by the velocity factor, therefore

Stalling load,

$$W_S = \frac{W_T}{C_v} = 1621\left(\frac{350 + 135.735m}{350}\right)$$

$$= 1621\left(\frac{350 + 135.735 \times 6}{350}\right) = 5393 \text{ N}$$

∴ Maximum bending moment on each arm,

$$M = \frac{W_S}{n} \times \frac{D_G}{2} = \frac{5393}{4} \times \frac{432}{2} = 291\,222 \text{ N-mm}$$

We know that bending stress (σ_b),

$$42 = \frac{M}{Z} = \frac{291\,222}{0.05\,(a_1)^3} = \frac{5824 \times 10^3}{(a_1)^3} \qquad \text{... (Taking } \sigma_b = 42 \text{ N/mm}^2\text{)}$$

∴ $(a_1)^3 = 5824 \times 10^3 / 42 = 138.7 \times 10^3$ or $a_1 = 51.7$ say 54 mm **Ans.**

and $b_1 = a_1 / 2 = 54 / 2 = 27$ mm **Ans.**

Since the arms are tapered towards the rim and the taper is 1/16 mm per mm length of the arm (or radius of the gear), therefore

Major axis of the arm at the rim end,

$$a_2 = a_1 - \text{Taper} = a_1 - \frac{1}{16} \times \frac{D_G}{2}$$

$$= 54 - \frac{1}{16} \times \frac{432}{2} = 40 \text{ mm} \quad \textbf{Ans.}$$

and minor axis of the arm at the rim end,

$$b_2 = a_2 / 2 = 40 / 2 = 20 \text{ mm} \quad \textbf{Ans.}$$

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Design for the rim

The thickness of the rim for the pinion may be taken as 1.6 *m* to 1.9 *m*, where *m* is the module. Let us take thickness of the rim for pinion,

$$t_{RP} = 1.6\, m = 1.6 \times 6 = 9.6 \text{ say } 10 \text{ mm} \quad \textbf{Ans.}$$

The thickness of the rim for the gear (t_{RG}) is given by

$$t_{RG} = m\sqrt{\frac{T_G}{n}} = 6\sqrt{\frac{72}{4}} = 25.4 \text{ say } 26 \text{ mm} \quad \textbf{Ans.}$$

EXERCISES

1. A helical cast steel gear with 30° helix angle has to transmit 35 kW at 2000 r.p.m. If the gear has 25 teeth, find the necessary module, pitch diameters and face width for 20° full depth involute teeth. The static stress for cast steel may be taken as 100 MPa. The face width may be taken as 3 times the normal pitch. The tooth form factor is given by the expression $y' = 0.154 - 0.912/T_E$, where T_E represents the equivalent number of teeth. The velocity factor is given by $C_v = \frac{6}{6 + v}$, where v is the peripheral speed of the gear in m/s.

[Ans. 6 mm ; 150 mm ; 50 mm]

2. A pair of helical gears with 30° helix angle is used to transmit 15 kW at 10 000 r.p.m. of the pinion. The velocity ratio is 4 : 1. Both the gears are to be made of hardened steel of static strength 100 N/mm^2. The gears are 20° stub and the pinion is to have 24 teeth. The face width may be taken as 14 times the module. Find the module and face width from the standpoint of strength and check the gears for wear.

[Ans. 2 mm ; 28 mm]

Gears inside a car engine.

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3. A pair of helical gears consist of a 20 teeth pinion meshing with a 100 teeth gear. The pinion rotates at 720 r.p.m. The normal pressure angle is 20° while the helix angle is 25°. The face width is 40 mm and the normal module is 4 mm. The pinion as well as gear are made of steel having ultimate strength of 600 MPa and heat treated to a surface hardness of 300 B.H.N. The service factor and factor of safety are 1.5 and 2 respectively. Assume that the velocity factor accounts for the dynamic load and calculate the power transmitting capacity of the gears. **[Ans. 8.6 kW]**

4. A single stage helical gear reducer is to receive power from a 1440 r.p.m., 25 kW induction motor. The gear tooth profile is involute full depth with 20° normal pressure angle. The helix angle is 23°, number of teeth on pinion is 20 and the gear ratio is 3. Both the gears are made of steel with allowable beam stress of 90 MPa and hardness 250 B.H.N.

(*a*) Design the gears for 20% overload carrying capacity from standpoint of bending strength and wear.

(*b*) If the incremental dynamic load of 8 kN is estimated in tangential plane, what will be the safe power transmitted by the pair at the same speed?

QUESTIONS

1. What is a herringbone gear? Where they are used?

2. Explain the following terms used in helical gears :

(*a*) Helix angle; (*b*) normal pitch; and

(*c*) axial pitch.

3. Define formative or virtual number of teeth on a helical gear. Derive the expression used to obtain its value.

4. Write the expressions for static strength, limiting wear load and dynamic load for helical gears and explain the various terms used therein.

OBJECTIVE TYPE QUESTIONS

1. If T is the actual number of teeth on a helical gear and ϕ is the helix angle for the teeth, the formative number of teeth is written as

(*a*) $T \sec^3 \phi$ (*b*) $T \sec^2 \phi$

(*c*) $T/\sec^3\phi$ (*d*) $T \operatorname{cosec} \phi$

2. In helical gears, the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth, is called

(*a*) normal pitch (*b*) axial pitch

(*c*) diametral pitch (*d*) module

3. In helical gears, the right hand helices on one gear will mesh .... helices on the other gear.

(*a*) right hand (*b*) left hand

4. The helix angle for single helical gears ranges from

(*a*) 10° to 15° (*b*) 15° to 20°

(*c*) 20° to 35° (*d*) 35° to 50°

5. The helix angle for double helical gears may be made up to

(*a*) 45° (*b*) 60°

(*c*) 75° (*d*) 90°

ANSWERS

1. (*a*) **2.** (*a*) **3.** (*b*) **4.** (*c*) **5.** (*a*)

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CHAPTER

30

Bevel Gears

30.1 Introduction

The bevel gears are used for transmitting power at a constant velocity ratio between two shafts whose axes intersect at a certain angle. The pitch surfaces for the bevel gear are frustums of cones. The two pairs of cones in contact is shown in Fig. 30.1. The elements of the cones, as shown in Fig. 30.1 (*a*), intersect at the point of intersection of the axis of rotation. Since the radii of both the gears are proportional to their distances from the apex, therefore the cones may roll together without sliding. In Fig. 30.1 (*b*), the elements of both cones do not intersect at the point of shaft intersection. Consequently, there may be pure rolling at only one point of contact and there must be tangential sliding at all other points of contact. Therefore, these cones, cannot be used as pitch surfaces because it is impossible to have positive driving and sliding in the same direction at the same time. We, thus, conclude that the elements of bevel

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gear pitch cones and shaft axes must intersect at the same point.

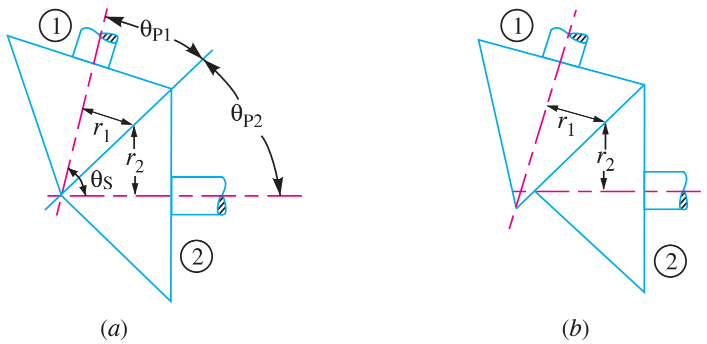


Fig. 30.1. Pitch surface for bevel gears.

The bevel gear is used to change the axis of rotational motion. By using gears of differing numbers of teeth, the speed of rotation can also be changed.

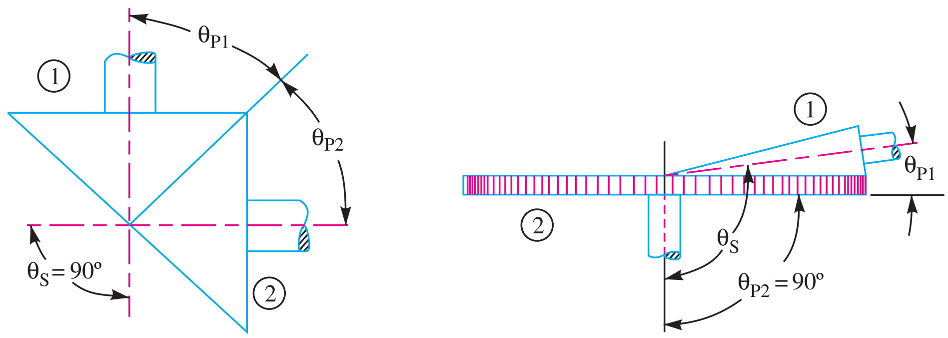
30.2 Classification of Bevel Gears

The bevel gears may be classified into the following types, depending upon the angles between the shafts and the pitch surfaces.

1. *Mitre gears.* When equal bevel gears (having equal teeth and equal pitch angles) connect two shafts whose axes intersect at right angle, as shown in Fig. 30.2 (*a*), then they are known as ***mitre gears.***

2. *Angular bevel gears.* When the bevel gears connect two shafts whose axes intersect at an angle other than a right angle, then they are known as ***angular bevel gears.***

3. *Crown bevel gears.* When the bevel gears connect two shafts whose axes intersect at an angle greater than a right angle and one of the bevel gears has a pitch angle of 90°, then it is known as a crown gear. The crown gear corresponds to a rack in spur gearing, as shown in Fig. 30.2 (*b*).



(*a*) Mitre gears. (*b*) Crown bevel gear.

Fig. 30.2. Classification of bevel gears.

4. *Internal bevel gears.* When the teeth on the bevel gear are cut on the inside of the pitch cone, then they are known as ***internal bevel gears.***

Note : The bevel gears may have straight or spiral teeth. It may be assumed, unless otherwise stated, that the bevel gear has straight teeth and the axes of the shafts intersect at right angle.

30.3 Terms used in Bevel Gears

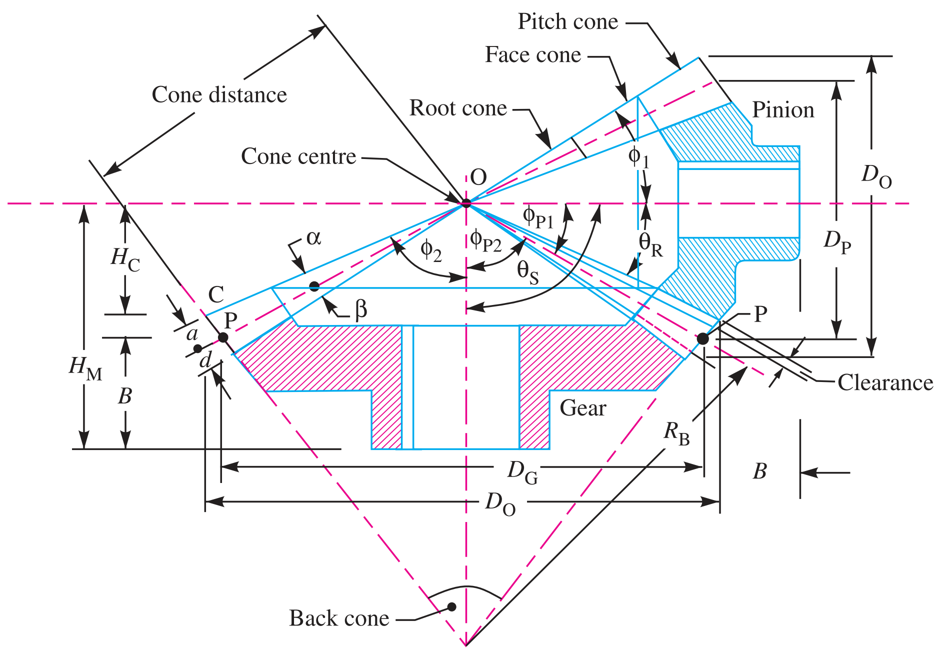


Fig. 30.3. Terms used in bevel gears.

A sectional view of two bevel gears in mesh is shown in Fig. 30.3. The following terms in connection with bevel gears are important from the subject point of view :

1. *Pitch cone.* It is a cone containing the pitch elements of the teeth.

2. *Cone centre.* It is the apex of the pitch cone. It may be defined as that point where the axes of two mating gears intersect each other.

3. *Pitch angle.* It is the angle made by the pitch line with the axis of the shaft. It is denoted by 'θ_P'.

4. *Cone distance.* It is the length of the pitch cone element. It is also called as a ***pitch cone radius***. It is denoted by '*OP*'. Mathematically, cone distance or pitch cone radius,

$$OP = \frac{\text{Pitch radius}}{\sin \theta_P} = \frac{D_P/2}{\sin \theta_{P1}} = \frac{D_G/2}{\sin \theta_{P2}}$$

5. *Addendum angle.* It is the angle subtended by the addendum of the tooth at the cone centre. It is denoted by 'α' Mathematically, addendum angle,

$$\alpha = \tan^{-1}\left(\frac{a}{OP}\right)$$

where a = Addendum, and OP = Cone distance.

6. *Dedendum angle.* It is the angle subtended by the dedendum of the tooth at the cone centre. It is denoted by 'β'. Mathematically, dedendum angle,

$$\beta = \tan^{-1}\left(\frac{d}{OP}\right)$$

where d = Dedendum, and OP = Cone distance.

7. *Face angle.* It is the angle subtended by the face of the tooth at the cone centre. It is denoted by 'ϕ'. The face angle is equal to the pitch angle *plus* addendum angle.

8. *Root angle.* It is the angle subtended by the root of the tooth at the cone centre. It is denoted by 'θ_R'. It is equal to the pitch angle *minus* dedendum angle.

9. *Back (or normal) cone.* It is an imaginary cone, perpendicular to the pitch cone at the end of the tooth.

10. Back cone distance. It is the length of the back cone. It is denoted by 'R_B'. It is also called back cone radius.

11. *Backing.* It is the distance of the pitch point (*P*) from the back of the boss, parallel to the pitch point of the gear. It is denoted by '*B*'.

12. *Crown height.* It is the distance of the crown point (*C*) from the cone centre (*O*), parallel to the axis of the gear. It is denoted by 'H_C'.

13. *Mounting height.* It is the distance of the back of the boss from the cone centre. It is denoted by 'H_M'.

14. *Pitch diameter.* It is the diameter of the largest pitch circle.

15. *Outside or addendum cone diameter.* It is the maximum diameter of the teeth of the gear. It is equal to the diameter of the blank from which the gear can be cut. Mathematically, outside diameter,

$$D_O = D_P + 2\,a \cos \theta_P$$

where D_P = Pitch circle diameter,

a = Addendum, and

θ_P = Pitch angle.

16. *Inside or dedendum cone diameter.* The inside or the dedendum cone diameter is given by

$$D_d = D_P - 2d \cos \theta_P$$

where D_d = Inside diameter, and

d = Dedendum.

30.4 Determination of Pitch Angle for Bevel Gears

Mitre gears

Consider a pair of bevel gears in mesh, as shown in Fig. 30.3.

Let θ_{P1} = Pitch angle for the pinion,

θ_{P2} = Pitch angle for the gear,

θ_S = Angle between the two shaft axes,

D_P = Pitch diameter of the pinion,

D_G = Pitch diameter of the gear, and

$V.R.$ = Velocity ratio $= \dfrac{D_G}{D_P} = \dfrac{T_G}{T_P} = \dfrac{N_P}{N_G}$

From Fig. 30.3, we find that

$$\theta_S = \theta_{P1} + \theta_{P2} \quad \text{or} \quad \theta_{P2} = \theta_S - \theta_{P1}$$

$$\therefore \quad \sin \theta_{P2} = \sin(\theta_S - \theta_{P1}) = \sin\theta_S \,.\, \cos\theta_{P1} - \cos\theta_S \,.\, \sin\theta_{P1} \quad \text{...(i)}$$

We know that cone distance,

$$OP = \frac{D_P/2}{\sin\theta_{P1}} = \frac{D_G/2}{\sin\theta_{P2}} \quad \text{or} \quad \frac{\sin\theta_{P2}}{\sin\theta_{P1}} = \frac{D_G}{D_P} = V.R.$$

$$\therefore \quad \sin\theta_{P2} = V.R. \times \sin\theta_{P1} \quad \text{...(ii)}$$

From equations *(i)* and *(ii)*, we have

$$V.R. \times \sin\theta_{P1} = \sin\theta_S \,.\, \cos\theta_{P1} - \cos\theta_S \,.\, \sin\theta_{P1}$$

Dividing throughout by $\cos\theta_{P1}$ we get

$$V.R. \tan\theta_{P1} = \sin\theta_S - \cos\theta_S . \tan\theta_{P1}$$

or

$$\tan\theta_{P1} = \frac{\sin\theta_S}{V.R + \cos\theta_S}$$

$$\therefore \quad \theta_{P1} = \tan^{-1}\left(\frac{\sin\theta_S}{V.R + \cos\theta_S}\right) \quad \text{...(iii)}$$

Similarly, we can find that

$$\tan\theta_{P2} = \frac{\sin\theta_S}{\dfrac{1}{V.R} + \cos\theta_S}$$

$$\therefore \quad \theta_{P2} = \tan^{-1}\left(\frac{\sin\theta_S}{\dfrac{1}{V.R} + \cos\theta_S}\right) \quad \text{...(iv)}$$

Note : When the angle between the shaft axes is 90° *i.e.* θ_S = 90°, then equations *(iii)* and *(iv)* may be written as

$$\theta_{P1} = \tan^{-1}\left(\frac{1}{V.R}\right) = \tan^{-1}\left(\frac{D_P}{D_G}\right) = \tan^{-1}\left(\frac{T_P}{T_G}\right) = \tan^{-1}\left(\frac{N_G}{N_P}\right)$$

and

$$\theta_{P2} = \tan^{-1}(V.R.) = \tan^{-1}\left(\frac{D_G}{D_P}\right) = \tan^{-1}\left(\frac{T_G}{T_P}\right) = \tan^{-1}\left(\frac{N_P}{N_G}\right)$$

30.5 Proportions for Bevel Gear

The proportions for the bevel gears may be taken as follows :

1. Addendum, $a = 1\,m$

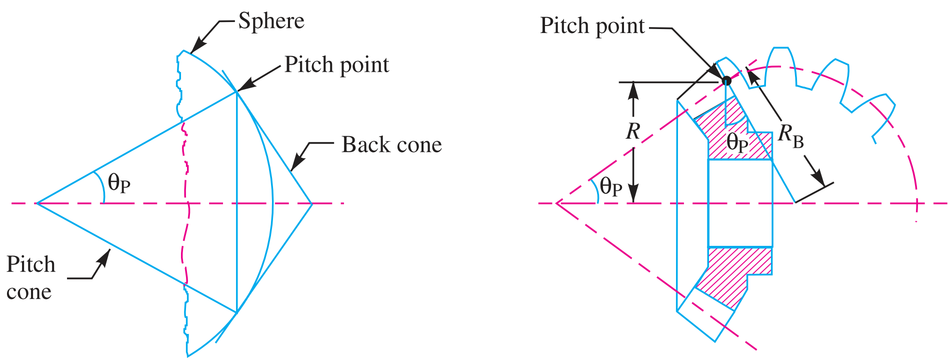
2. Dedendum, $d = 1.2\ m$
3. Clearance $= 0.2\ m$
4. Working depth $= 2\ m$
5. Thickness of tooth $= 1.5708\ m$

where m is the module.

Note : Since the bevel gears are not interchangeable, therefore these are designed in pairs.

30.6 Formative or Equivalent Number of Teeth for Bevel Gears – Tredgold's Approximation

We have already discussed that the involute teeth for a spur gear may be generated by the edge of a plane as it rolls on a base cylinder. A similar analysis for a bevel gear will show that a true section of the resulting involute lies on the surface of a sphere. But it is not possible to represent on a plane surface the exact profile of a bevel gear tooth lying on the surface of a sphere. Therefore, it is important to approximate the bevel gear tooth profiles as accurately as possible. The approximation (known as ***Tredgold's approximation***) is based upon the fact that a cone tangent to the sphere at the pitch point will closely approximate the surface of the sphere for a short distance either side of the pitch point, as shown in Fig. 30.4 (*a*). The cone (known as back cone) may be developed as a plane surface and spur gear teeth corresponding to the pitch and pressure angle of the bevel gear and the radius of the developed cone can be drawn. This procedure is shown in Fig. 30.4 (*b*).



(*a*) Back cone. (*b*) Development of back cone.

Fig. 30.4

Let θ_P = Pitch angle or half of the cone angle,

R = Pitch circle radius of the bevel pinion or gear, and

R_B = Back cone distance or equivalent pitch circle radius of spur pinion or gear.

Now from Fig. 30.4 (*b*), we find that

$$R_B = R \sec \theta_P$$

We know that the equivalent (or formative) number of teeth,

$$T_E = \frac{2\ R_B}{m} \quad \ldots \left(\because \text{Number of teeth} = \frac{\text{Pitch circle diameter}}{\text{Module}}\right)$$

$$= \frac{2\ R \sec \theta_P}{m} = T \sec \theta_P$$

where T = Actual number of teeth on the gear.

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Notes : 1. The action of bevel gears will be same as that of equivalent spur gears.

2. Since the equivalent number of teeth is always greater than the actual number of teeth, therefore a given pair of bevel gears will have a larger contact ratio. Thus, they will run more smoothly than a pair of spur gears with the same number of teeth.

30.7 Strength of Bevel Gears

The strength of a bevel gear tooth is obtained in a similar way as discussed in the previous articles. The modified form of the Lewis equation for the tangential tooth load is given as follows:

$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y' \left(\frac{L-b}{L}\right)$$

where σ_o = Allowable static stress,

C_v = Velocity factor,

$= \dfrac{3}{3+v}$, for teeth cut by form cutters,

$= \dfrac{6}{6+v}$, for teeth generated with precision machines,

v = Peripheral speed in m / s,

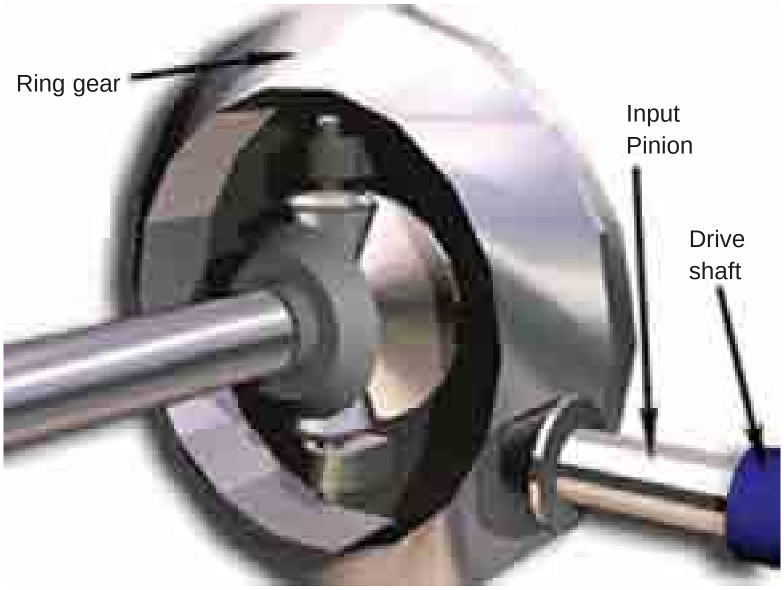
b = Face width,

m = Module,

y' = Tooth form factor (or Lewis factor) for the equivalent number of teeth,

L = Slant height of pitch cone (or cone distance),

$$= \sqrt{\left(\frac{D_G}{2}\right)^2 + \left(\frac{D_P}{2}\right)^2}$$



Hypoid bevel gears in a car differential

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D_G = Pitch diameter of the gear, and

D_P = Pitch diameter of the pinion.

Notes : 1. The factor $\left(\dfrac{L - b}{L}\right)$ may be called as ***bevel factor.***

2. For satisfactory operation of the bevel gears, the face width should be from 6.3 *m* to 9.5 *m*, where *m* is the module. Also the ratio *L* / *b* should not exceed 3. For this, the number of teeth in the pinion must not less than $\dfrac{48}{\sqrt{1 + (V.R.)^2}}$, where *V.R.* is the required velocity ratio.

3. The dynamic load for bevel gears may be obtained in the similar manner as discussed for spur gears.

4. The static tooth load or endurance strength of the tooth for bevel gears is given by

$$W_S = \sigma_e . b . \pi\, m . y' \left(\frac{L - b}{L}\right)$$

The value of flexural endurance limit (σ_e) may be taken from Table 28.8, in spur gears.

5. The maximum or limiting load for wear for bevel gears is given by

$$W_w = \frac{D_P . b . Q . K}{\cos \theta_{P1}}$$

where D_P, *b*, *Q* and *K* have usual meanings as discussed in spur gears except that *Q* is based on formative or equivalent number of teeth, such that

$$Q = \frac{2\, T_{EG}}{T_{EG} + T_{EP}}$$

30.8 Forces Acting on a Bevel Gear

Consider a bevel gear and pinion in mesh as shown in Fig. 30.5. The normal force (W_N) on the tooth is perpendicular to the tooth profile and thus makes an angle equal to the pressure angle (ϕ) to the pitch circle. Thus normal force can be resolved into two components, one is the tangential component (W_T) and the other is the radial component (W_R). The tangential component (*i.e.* the tangential tooth load) produces the bearing reactions while the radial component produces end thrust in the shafts. The magnitude of the tangential and radial components is as follows :

$$W_T = W_N \cos \phi, \text{ and } W_R = W_N \sin \phi = W_T \tan \phi \qquad \text{...}(i)$$

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These forces are considered to act at the mean radius (R_m). From the geometry of the Fig. 30.5, we find that

$$R_m = \left(L - \frac{b}{2}\right) \sin \theta_{P1} = \left(L - \frac{b}{2}\right) \frac{D_P}{2L} \qquad \ldots \left(\because \sin \theta_{P1} = \frac{D_P/2}{L}\right)$$

Now the radial force (W_R) acting at the mean radius may be further resolved into two components, W_{RH} and W_{RV}, in the axial and radial directions as shown in Fig. 30.5. Therefore the axial force acting on the pinion shaft,

$$W_{RH} = W_R \sin \theta_{P1} = W_T \tan \phi \,.\, \sin \theta_{P1} \qquad \ldots\text{[From equation } (i)\text{]}$$

and the radial force acting on the pinion shaft,

$$W_{RV} = W_R \cos \theta_{P1} = W_T \tan \phi . \cos \theta_{P1}$$

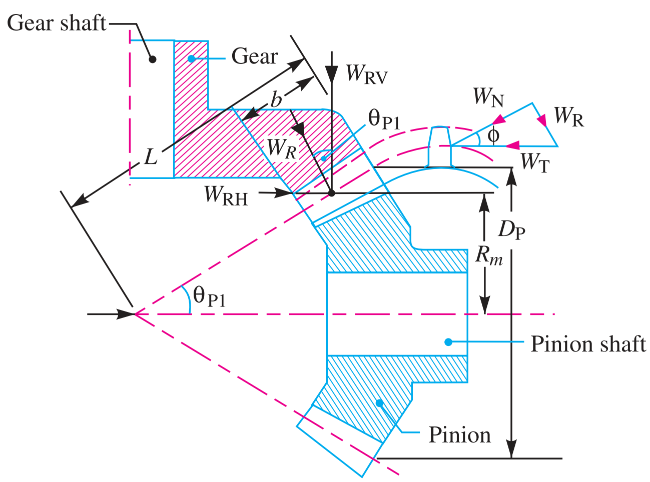

Fig. 30.5. Forces acting on a bevel gear.

A little consideration will show that the axial force on the pinion shaft is equal to the radial force on the gear shaft but their directions are opposite. Similarly, the radial force on the pinion shaft is equal to the axial force on the gear shaft, but act in opposite directions.

30.9 Design of a Shaft for Bevel Gears

In designing a pinion shaft, the following procedure may be adopted :

1. First of all, find the torque acting on the pinion. It is given by

$$T = \frac{P \times 60}{2 \pi N_P} \text{ N-m}$$

where P = Power transmitted in watts, and

N_P = Speed of the pinion in r.p.m.

2. Find the tangential force (W_T) acting at the mean radius (R_m) of the pinion. We know that

$$W_T = T / R_m$$

3. Now find the axial and radial forces (*i.e.* W_{RH} and W_{RV}) acting on the pinion shaft as discussed above.

4. Find resultant bending moment on the pinion shaft as follows :

The bending moment due to W_{RH} and W_{RV} is given by

$$M_1 = W_{RV} \times \text{Overhang} - W_{RH} \times R_m$$

and bending moment due to W_T,

$$M_2 = W_T \times \text{Overhang}$$

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∴ Resultant bending moment,

$$M = \sqrt{(M_1)^2 + (M_2)^2}$$

5. Since the shaft is subjected to twisting moment (T) and resultant bending moment (M), therefore equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2}$$

6. Now the diameter of the pinion shaft may be obtained by using the torsion equation. We know that

$$T_e = \frac{\pi}{16} \times \tau\,(d_P)^3$$

where d_P = Diameter of the pinion shaft, and

τ = Shear stress for the material of the pinion shaft.

7. The same procedure may be adopted to find the diameter of the gear shaft.

Example 30.1. *A 35 kW motor running at 1200 r.p.m. drives a compressor at 780 r.p.m. through a 90° bevel gearing arrangement. The pinion has 30 teeth. The pressure angle of teeth is* $14\,^1/_2°$. *The wheels are capable of withstanding a dynamic stress,*

$$\sigma_w = 140\left(\frac{280}{280 + v}\right) \text{ MPa, where } v \text{ is the pitch line speed in m / min.}$$

The form factor for teeth may be taken as $0.124 - \frac{0.686}{T_E}$, *where* T_E *is the number of teeth equivalent of a spur gear.*

The face width may be taken as $\frac{1}{4}$ *of the slant height of pitch cone. Determine for the pinion, the module pitch, face width, addendum, dedendum, outside diameter and slant height.*

High performance 2- and 3 -way bevel gear boxes

Solution : Given : P = 35 kW = 35 × 10^3 W ; N_P = 1200 r.p.m. ; N_G = 780 r.p.m. ; θ_S = 90° ; T_P = 30 ; ϕ = $14\,^1/_2°$; $b = L / 4$

Module and face width for the pinion

Let m = Module in mm,

b = Face width in mm

= L / 4, and ...(Given)

D_P = Pitch circle diameter of the pinion.

We know that velocity ratio,

$$V.R. = \frac{N_P}{N_G} = \frac{1200}{780} = 1.538$$

∴ Number of teeth on the gear,

$$T_G = V.R. \times T_P = 1.538 \times 30 = 46$$

Since the shafts are at right angles, therefore pitch angle for the pinion,

$$\theta_{P1} = \tan^{-1}\left(\frac{1}{V.R.}\right) = \tan^{-1}\left(\frac{1}{1.538}\right) = \tan^{-1}(0.65) = 33°$$

and pitch angle for the gear,

$$\theta_{P2} = 90° - 33° = 57°$$

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We know that formative number of teeth for pinion,

$$T_{EP} = T_P.\sec \theta_{P1} = 30 \times \sec 33^o = 35.8$$

and formative number of teeth for the gear,

$$T_{EG} = T_G.\sec \theta_{P2} = 46 \times \sec 57^o = 84.4$$

Tooth form factor for the pinion

$$y'_P = 0.124 - \frac{0.686}{T_{EP}} = 0.124 - \frac{0.686}{35.8} = 0.105$$

and tooth form factor for the gear,

$$y'_G = 0.124 - \frac{0.686}{T_{EG}} = 0.124 - \frac{0.686}{84.4} = 0.116$$

Since the allowable static stress (σ_o) for both the pinion and gear is same (*i.e.* 140 MPa or N/mm^2) and y'_P is less than y'_G, therefore the pinion is weaker. Thus the design should be based upon the pinion.

We know that the torque on the pinion,

$$T = \frac{P \times 60}{2\pi N_P} = \frac{35 \times 10^3 \times 60}{2\pi \times 1200} = 278.5 \text{ N-m} = 278\ 500 \text{ N-mm}$$

∴ Tangential load on the pinion,

$$W_T = \frac{2T}{D_P} = \frac{2T}{m.T_P} = \frac{2 \times 278\ 500}{m \times 30} = \frac{18\ 567}{m} \text{ N}$$

We know that pitch line velocity,

$$v = \frac{\pi D_P . N_P}{1000} = \frac{\pi m . T_P . N_P}{1000} = \frac{\pi m \times 30 \times 1200}{1000} \text{ m / min}$$

$$= 113.1\, m \text{ m / min}$$

∴ Allowable working stress,

$$\sigma_w = 140 \left(\frac{280}{280 + v}\right) = 140 \left(\frac{280}{280 + 113.1m}\right) \text{ MPa or N / mm}^2$$

We know that length of the pitch cone element or slant height of the pitch cone,

$$L = \frac{D_P}{2 \sin \theta_{P1}} = \frac{m \times T_P}{2 \sin \theta_{P1}} = \frac{m \times 30}{2 \sin 33^o} = 27.54\, m \text{ mm}$$

Since the face width (b) is 1/4th of the slant height of the pitch cone, therefore

$$b = \frac{L}{4} = \frac{27.54m}{4} = 6.885\, m \text{ mm}$$

We know that tangential load on the pinion,

$$W_T = (\sigma_{OP} \times C_v)\, b.\pi\, m.y'_P \left(\frac{L - b}{L}\right)$$

$$= \sigma_w.b.\pi\, m.y'_P \left(\frac{L - b}{L}\right) \qquad \ldots (\because\ \sigma_w = \sigma_{OP} \times C_v)$$

or

$$\frac{18\ 567}{m} = 140 \left(\frac{280}{280 + 113.1m}\right) 6.885\, m \times \pi\, m \times 0.105 \left(\frac{27.54m - 6.885m}{27.54m}\right)$$

$$= \frac{66\ 780\, m^2}{280 + 113.1m}$$

or

$$280 + 113.1\, m = 66\ 780\, m^2 \times \frac{m}{18\ 567} = 3.6\, m^3$$

Solving this expression by hit and trial method, we find that

$m = 6.6$ say 8 mm **Ans.**

and face width, $b = 6.885\,m = 6.885 \times 8 = 55$ mm **Ans.**

Addendum and dedendum for the pinion

We know that addendum,

$a = 1\,m = 1 \times 8 = 8$ mm **Ans.**

and dedendum, $d = 1.2\,m = 1.2 \times 8 = 9.6$ mm **Ans.**

Outside diameter for the pinion

We know that outside diameter for the pinion,

$$D_O = D_P + 2\,a\cos\theta_{P1} = m.T_P + 2\,a\cos\theta_{P1} \qquad \ldots (\because\ D_P = m\,.\,T_P)$$

$$= 8 \times 30 + 2 \times 8\cos 33^\circ = 253.4 \text{ mm } \textbf{Ans.}$$

Slant height

We know that slant height of the pitch cone,

$L = 27.54\,m = 27.54 \times 8 = 220.3$ mm **Ans.**

Example 30.2. *A pair of cast iron bevel gears connect two shafts at right angles. The pitch diameters of the pinion and gear are 80 mm and 100 mm respectively. The tooth profiles of the gears are of 14 $^1/_2$° composite form. The allowable static stress for both the gears is 55 MPa. If the pinion transmits 2.75 kW at 1100 r.p.m., find the module and number of teeth on each gear from the stand-point of strength and check the design from the standpoint of wear. Take surface endurance limit as 630 MPa and modulus of elasticity for cast iron as 84 kN/mm².*

Solution. Given : $\theta_S = 90^\circ$; $D_P = 80$ mm $= 0.08$ m ; $D_G = 100$ mm $= 0.1$ m ; $\phi = 14\frac{1}{2}^\circ$; $\sigma_{OP} = \sigma_{OG} = 55$ MPa $= 55$ N/mm² ; $P = 2.75$ kW $= 2750$ W ; $N_P = 1100$ r.p.m. ; $\sigma_{es} = 630$ MPa $= 630$ N/mm² ; $E_P = E_G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

Module

Let m = Module in mm.

Since the shafts are at right angles, therefore pitch angle for the pinion,

$$\theta_{P1} = \tan^{-1}\left(\frac{1}{V.R.}\right) = \tan^{-1}\left(\frac{D_P}{D_G}\right) = \tan^{-1}\left(\frac{80}{100}\right) = 38.66^\circ$$

and pitch angle for the gear,

$$\theta_{P2} = 90^\circ - 38.66^\circ = 51.34^\circ$$

We know that formative number of teeth for pinion,

$$T_{EP} = T_P\,.\sec\theta_{P1} = \frac{80}{m} \times \sec 38.66^\circ = \frac{102.4}{m} \qquad \ldots (\because\ T_P = D_P / m)$$

and formative number of teeth on the gear,

$$T_{EG} = T_G\,.\sec\theta_{P2} = \frac{100}{m} \times \sec 51.34^\circ = \frac{160}{m} \qquad \ldots (\because\ T_G = D_G / m)$$

Since both the gears are made of the same material, therefore pinion is the weaker. Thus the design should be based upon the pinion.

We know that tooth form factor for the pinion having 14 $^1/_2$° composite teeth,

$$y'_P = 0.124 - \frac{0.684}{T_{EP}} = 0.124 - \frac{0.684 \times m}{102.4}$$

$$= 0.124 - 0.006\,68\,m$$

and pitch line velocity,

$$v = \frac{\pi D_P\,.\,N_P}{60} = \frac{\pi \times 0.08 \times 1100}{60} = 4.6 \text{ m/s}$$

Taking velocity factor,

$$C_v = \frac{6}{6 + v} = \frac{6}{6 + 4.6} = 0.566$$

We know that length of the pitch cone element or slant height of the pitch cone,

$$*L = \sqrt{\left(\frac{D_G}{2}\right)^2 + \left(\frac{D_P}{2}\right)^2} = \sqrt{\left(\frac{100}{2}\right)^2 + \left(\frac{80}{2}\right)^2} = 64 \text{ mm}$$

Assuming the face width (b) as 1/3rd of the slant height of the pitch cone (L), therefore

$$b = L / 3 = 64 / 3 = 21.3 \text{ say } 22 \text{ mm}$$

We know that torque on the pinion,

$$T = \frac{P \times 60}{2\pi \times N_P} = \frac{2750 \times 60}{2\pi \times 1100} = 23.87 \text{ N-m} = 23\,870 \text{ N-mm}$$

∴ Tangential load on the pinion,

$$W_T = \frac{T}{D_P / 2} = \frac{23\,870}{80/2} = 597 \text{ N}$$

We also know that tangential load on the pinion,

$$W_T = (\sigma_{OP} \times C_v)\, b \times \pi\, m \times y'_P \left(\frac{L - b}{L}\right)$$

or

$$597 = (55 \times 0.566)\, 22 \times \pi\, m\, (0.124 - 0.00\,668\, m) \left(\frac{64 - 22}{64}\right)$$

$$= 1412\, m\, (0.124 - 0.006\,68\, m)$$

$$= 175\, m - 9.43\, m^2$$

Solving this expression by hit and trial method, we find that

$$m = 4.5 \text{ say } 5 \text{ mm} \textbf{ Ans.}$$

The bevel gear turbine

Number of teeth on each gear

We know that number of teeth on the pinion,

$$T_P = D_P / m = 80 / 5 = 16 \textbf{ Ans.}$$

and number of teeth on the gear,

$$T_G = D_G / m = 100 / 5 = 20 \textbf{ Ans.}$$

Checking the gears for wear

We know that the load-stress factor,

$$K = \frac{(\sigma_{es})^2 \sin \phi}{1.4} \left[\frac{1}{E_P} + \frac{1}{E_G}\right]$$

$$= \frac{(630)^2 \sin 14\,^1/_2\,°}{1.4} \left[\frac{1}{84 \times 10^3} + \frac{1}{84 \times 10^3}\right] = 1.687$$

and ratio factor, $Q = \dfrac{2\,T_{EG}}{T_{EG} + T_{EP}} = \dfrac{2 \times 160 / m}{160 / m + 102.4 / m} = 1.22$

* The length of the pitch cone element (L) may also obtained by using the relation

$$L = D_P / 2 \sin \theta_{P1}$$

∴ Maximum or limiting load for wear,

$$W_w = \frac{D_P \, . \, b \, . \, Q \, . \, K}{\cos \theta_{P1}} = \frac{80 \times 22 \times 1.22 \times 1.687}{\cos 38.66^\circ} = 4640 \text{ N}$$

Since the maximum load for wear is much more than the tangential load (W_T), therefore the design is satisfactory from the consideration of wear. **Ans.**

Example 30.3. *A pair of bevel gears connect two shafts at right angles and transmits 9 kW. Determine the required module and gear diameters for the following specifications :*

Particulars	*Pinion*	*Gear*
Number of teeth	*21*	*60*
Material	*Semi-steel*	*Grey cast iron*
Brinell hardness number	*200*	*160*
Allowable static stress	*85 MPa*	*55 MPa*
Speed	*1200 r.p.m.*	*420 r.p.m.*
Tooth profile	$14\frac{1}{2}^\circ$ *composite*	$14\frac{1}{2}^\circ$ *composite*

Check the gears for dynamic and wear loads.

Solution. Given : $\theta_S = 90^\circ$; $P = 9$ kW = 9000 W ; $T_P = 21$; $T_G = 60$; $\sigma_{OP} = 85$ MPa = 85 N/mm^2 ; $\sigma_{OG} = 55$ MPa = 55 N/mm^2 ; $N_P = 1200$ r.p.m. ; $N_G = 420$ r.p.m. ; $\phi = 14\,^1/_2{}^\circ$

Required module

Let m = Required module in mm.

Since the shafts are at right angles, therefore pitch angle for the pinion,

$$\theta_{P1} = \tan^{-1}\left(\frac{1}{V.R.}\right) = \tan^{-1}\left(\frac{T_P}{T_G}\right) = \tan^{-1}\left(\frac{21}{60}\right) = 19.3^\circ$$

and pitch angle for the gear,

$$\theta_{P2} = \theta_S - \theta_{P1} = 90^\circ - 19.3^\circ = 70.7^\circ$$

We know that formative number of teeth for the pinion,

$$T_{EP} = T_P \, . \, \sec \theta_{P1} = 21 \sec 19.3^\circ = 22.26$$

and formative number of teeth for the gear,

$$T_{EG} = T_G \, . \, \sec \theta_{P2} = 60 \sec 70.7^\circ = 181.5$$

We know that tooth form factor for the pinion,

$$y'_P = 0.124 - \frac{0.684}{T_{EP}} = 0.124 - \frac{0.684}{22.26} = 0.093$$

... (For $14\,^1/_2{}^\circ$ composite system)

and tooth form factor for the gear,

$$y'_G = 0.124 - \frac{0.684}{T_{EG}} = 0.124 - \frac{0.684}{181.5} = 0.12$$

∴ $\sigma_{OP} \times y'_P = 85 \times 0.093 = 7.905$

and $\sigma_{OG} \times y'_G = 55 \times 0.12 = 6.6$

Since the product $\sigma_{OG} \times y'_G$ is less than $\sigma_{OP} \times y'_P$, therefore the gear is weaker. Thus, the design should be based upon the gear.

We know that torque on the gear,

$$T = \frac{P \times 60}{2 \pi N_G} = \frac{9000 \times 60}{2 \pi \times 420} = 204.6 \text{ N-m} = 204\,600 \text{ N-mm}$$

∴ Tangential load on the gear,

$$W_T = \frac{T}{D_G/2} = \frac{2T}{m.T_G} = \frac{2 \times 204\ 600}{m \times 60} = \frac{6820}{m} \text{ N} \qquad \ldots (\because D_G = m.T_G)$$

We know that pitch line velocity,

$$v = \frac{\pi D_G . N_G}{60} = \frac{\pi\ m.T_G . N_G}{60} = \frac{\pi\ m \times 60 \times 420}{60} \text{ mm/s}$$

$$= 1320\ m \text{ mm/s} = 1.32\ m \text{ m/s}$$

Taking velocity factor,

$$C_v = \frac{6}{6+v} = \frac{6}{6+1.32m}$$

We know that length of pitch cone element,

$$*L = \frac{D_G}{2 \sin \theta_{P2}} = \frac{m.T_G}{2 \sin 70.7°} = \frac{m \times 60}{2 \times 0.9438} = 32\ m \text{ mm}$$

Assuming the face width (b) as 1/3rd of the length of the pitch cone element (L), therefore

$$b = \frac{L}{3} = \frac{32m}{3} = 10.67\ m \text{ mm}$$

We know that tangential load on the gear,

$$W_T = (\sigma_{OG} \times C_v)\ b.\pi\ m.y'_G \left(\frac{L-b}{L}\right)$$

$$\therefore \qquad \frac{6820}{m} = 55\left(\frac{6}{6+1.32m}\right) 10.67\ m \times \pi\ m \times 0.12 \left(\frac{32\ m - 10.67\ m}{32\ m}\right)$$

$$= \frac{885 m^2}{6+1.32m}$$

or $\qquad 40\ 920 + 9002\ m = 885\ m^3$

Solving this expression by hit and trial method, we find that

$m = 4.52$ say 5 mm **Ans.**

and $\qquad b = 10.67\ m = 10.67 \times 5 = 53.35$ say 54 mm **Ans.**

Gear diameters

We know that pitch diameter for the pinion,

$D_P = m.T_P = 5 \times 21 = 105$ mm **Ans.**

and pitch circle diameter for the gear,

$D_G = m.T_G = 5 \times 60 = 300$ mm **Ans.**

Check for dynamic load

We know that pitch line velocity,

$v = 1.32\ m = 1.32 \times 5 = 6.6$ m/s

and tangential tooth load on the gear,

$$W_T = \frac{6820}{m} = \frac{6820}{5} = 1364 \text{ N}$$

From Table 28.7, we find that tooth error action for first class commercial gears having module 5 mm is

$e = 0.055$ mm

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* The length of pitch cone element (L) may be obtained by using the following relation, *i.e.*

$$L = \sqrt{\left(\frac{D_G}{2}\right)^2 + \left(\frac{D_P}{2}\right)^2} = \sqrt{\left(\frac{m.T_G}{2}\right)^2 + \left(\frac{m.T_P}{2}\right)^2} = \frac{m}{2}\sqrt{(T_G)^2 + (T_P)^2}$$

Taking $K = 0.107$ for $14\,^1/_2°$ composite teeth, $E_P = 210 \times 10^3$ N/mm²; and $E_G = 84 \times 10^3$ N/mm², we have

Deformation or dynamic factor,

$$C = \frac{K.e}{\frac{1}{E_P} + \frac{1}{E_G}} = \frac{0.107 \times 0.055}{\frac{1}{210 \times 10^3} + \frac{1}{84 \times 10^3}} = 353 \text{ N / mm}$$

We know that dynamic load on the gear,

$$W_D = W_T + \frac{21\,v\,(b.C + W_T)}{21\,v + \sqrt{b.C + W_T}}$$

$$= 1364 + \frac{21 \times 6.6\,(54 \times 353 + 1364)}{21 \times 6.6 + \sqrt{54 \times 353 + 1364}}$$

$$= 1364 + 10\,054 = 11\,418 \text{ N}$$

From Table 28.8, we find that flexural endurance limit (σ_e) for the gear material which is grey cast iron having B.H.N. = 160, is

$$\sigma_e = 84 \text{ MPa} = 84 \text{ N/mm}^2$$

We know that the static tooth load or endurance strength of the tooth,

$$W_S = \sigma_e.b.\pi\, m.y'_G = 84 \times 54 \times \pi \times 5 \times 0.12 = 8552 \text{ N}$$

Since W_S is less that W_D, therefore the design is not satisfactory from the standpoint of dynamic load. We have already discussed in spur gears (Art. 28.20) that $W_S \geq 1.25\, W_D$ for steady loads. For a satisfactory design against dynamic load, let us take the precision gears having tooth error in action (e = 0.015 mm) for a module of 5 mm.

∴ Deformation or dynamic factor,

$$C = \frac{0.107 \times 0.015}{\frac{1}{210 \times 10^3} + \frac{1}{84 \times 10^3}} = 96 \text{ N/mm}$$

and dynamic load on the gear,

$$W_D = 1364 + \frac{21 \times 6.6\,(54 \times 96 + 1364)}{21 \times 6.6 + \sqrt{54 \times 96 + 1364}} = 5498 \text{ N}$$

From above we see that by taking precision gears, W_S is greater than W_D, therefore the design is satisfactory, from the standpoint of dynamic load.

Check for wear load

From Table 28.9, we find that for a gear of grey cast iron having B.H.N. = 160, the surface endurance limit is,

$$\sigma_{es} = 630 \text{ MPa} = 630 \text{ N/mm}^2$$

∴ Load-stress factor,

$$K = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left[\frac{1}{E_P} + \frac{1}{E_G}\right]$$

$$= \frac{(630)^2 \sin 14\tfrac{1}{2}°}{1.4}\left[\frac{1}{210 \times 10^3} + \frac{1}{84 \times 10^3}\right] = 1.18 \text{ N/mm}^2$$

and ratio factor,

$$Q = \frac{2\,T_{EG}}{T_{EG} + T_{EP}} = \frac{2 \times 181.5}{181.5 + 22.26} = 1.78$$

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We know that maximum or limiting load for wear,

$$W_w = D_P.b.Q.K = 105 \times 54 \times 1.78 \times 1.18 = 11\,910 \text{ N}$$

Since W_w is greater then W_D, therefore the design is satisfactory from the standpoint of wear.

Example 30.4. *A pair of 20º full depth involute teeth bevel gears connect two shafts at right angles having velocity ratio 3 : 1. The gear is made of cast steel having allowable static stress as 70 MPa and the pinion is of steel with allowable static stress as 100 MPa. The pinion transmits 37.5 kW at 750 r.p.m. Determine : 1. Module and face width; 2. Pitch diameters; and 3. Pinion shaft diameter.*

Assume tooth form factor, $y = 0.154 - \dfrac{0.912}{T_E}$, where T_E is the formative number of teeth, width = 1/3 rd the length of pitch cone, and pinion shaft overhangs by 150 mm.

Involute teeth bevel gear

Solution. Given : $\phi = 20º$; $\theta_S = 90º$; $V.R. = 3$; $\sigma_{OG} = 70$ MPa $= 70$ N/mm^2 ; $\sigma_{OP} = 100$ MPa $= 100$ N/mm^2 ; $P = 37.5$ kW $= 37\,500$ W ; $N_P = 750$ r.p.m. ; $b = L/3$; Overhang = 150 mm

Module and face width

Let m = Module in mm,

b = Face width in mm = $L/3$, ...(Given)

D_G = Pitch circle diameter of the gear in mm.

Since the shafts are at right angles, therefore pitch angle for the pinion,

$$\theta_{P1} = \tan^{-1}\left(\frac{1}{V.R.}\right) = \tan^{-1}\left(\frac{1}{3}\right) = 18.43º$$

and pitch angle for the gear,

$$\theta_{P2} = \theta_S - \theta_{P1} = 90º - 18.43º = 71.57º$$

Assuming number of teeth on the pinion (T_P) as 20, therefore number of teeth on the gear,

$$T_G = V.R. \times T_P = 3 \times 20 = 60 \qquad \ldots (\because V.R. = T_G / T_P)$$

We know that formative number of teeth for the pinion,

$$T_{EP} = T_P \,.\, \sec \theta_{P1} = 20 \times \sec 18.43º = 21.08$$

and formative number of teeth for the gear,

$$T_{EG} = T_G \,.\, \sec \theta_{P2} = 60 \sec 71.57º = 189.8$$

We know that tooth form factor for the pinion,

$$y'_P = 0.154 - \frac{0.912}{T_{EP}} = 0.154 - \frac{0.912}{21.08} = 0.111$$

and tooth form factor for the gear,

$$y'_G = 0.154 - \frac{0.912}{T_{EG}} = 0.154 - \frac{0.912}{189.8} = 0.149$$

$$\therefore \quad \sigma_{OP} \times y'_P = 100 \times 0.111 = 11.1$$

and

$$\sigma_{OG} \times y'_G = 70 \times 0.149 = 10.43$$

Top

Since the product $\sigma_{OG} \times y'_G$ is less than $\sigma_{OP} \times y'_P$, therefore the gear is weaker. Thus, the design should be based upon the gear and not the pinion.

We know that the torque on the gear,

$$T = \frac{P \times 60}{2\pi N_G} = \frac{P \times 60}{2\pi \times N_P / 3} \quad \text{... } (\because V.R. = N_P / N_G = 3)$$

$$= \frac{37\,500 \times 60}{2\pi \times 750/3} = 1432 \text{ N-m} = 1432 \times 10^3 \text{ N-mm}$$

∴ Tangential load on the gear,

$$W_T = \frac{2\,T}{D_G} = \frac{2T}{m.T_G} \quad \text{... } (\because D_G = m.T_G)$$

$$= \frac{2 \times 1432 \times 10^3}{m \times 60} = \frac{47.7 \times 10^3}{m} \text{ N}$$

We know that pitch line velocity,

$$v = \frac{\pi D_G . N_G}{60} = \frac{\pi m . T_G . N_P / 3}{60}$$

$$= \frac{\pi m \times 60 \times 750/3}{60} = 785.5\, m \text{ mm / s} = 0.7855\, m \text{ m / s}$$

Taking velocity factor,

$$C_v = \frac{3}{3 + v} = \frac{3}{3 + 0.7855\, m}$$

We know that length of the pitch cone element,

$$L = \frac{D_G}{2 \sin \theta_{P2}} = \frac{m.T_G}{2 \sin 71.57°} = \frac{m \times 60}{2 \times 0.9487} = 31.62\, m \text{ mm}$$

Since the face width (b) is 1/3rd of the length of the pitch cone element, therefore

$$b = \frac{L}{3} = \frac{31.62\, m}{3} = 10.54\, m \text{ mm}$$

We know that tangential load on the gear,

$$W_T = (\sigma_{OG} \times C_v)\, b.\pi\, m.y'_G \left(\frac{L - b}{L}\right)$$

Racks

$$\therefore \quad \frac{47.7 \times 10^3}{m} = 70\left(\frac{3}{3 + 0.7855\,m}\right) 10.54\,m \times \pi\,m \times 0.149 \left(\frac{31.62\,m - 10.54\,m}{31.62\,m}\right)$$

$$= \frac{691\,m^2}{3 + 0.7855\,m}$$

$$143\,100 + 37\,468\,m = 691\,m^3$$

Solving this expression by hit and trial method, we find that

$$m = 8.8 \text{ say } 10 \text{ mm} \quad \textbf{Ans.}$$

and $$b = 10.54\,m = 10.54 \times 10 = 105.4 \text{ mm} \quad \textbf{Ans.}$$

Pitch diameters

We know that pitch circle diameter of the larger wheel (*i.e.* gear),

$$D_G = m.T_G = 10 \times 60 = 600 \text{ mm} \quad \textbf{Ans.}$$

and pitch circle diameter of the smaller wheel (*i.e.* pinion),

$$D_P = m.T_P = 10 \times 20 = 200 \text{ mm} \quad \textbf{Ans.}$$

Pinion shaft diameter

Let d_P = Pinion shaft diameter.

We know that the torque on the pinion,

$$T = \frac{P \times 60}{2\pi \times N_P} = \frac{37\,500 \times 60}{2\pi \times 750} = 477.4 \text{ N-m} = 477\,400 \text{ N-mm}$$

and length of the pitch cone element,

$$L = 31.62\,m = 31.62 \times 10 = 316.2 \text{ mm}$$

∴ Mean radius of the pinion,

$$R_m = \left(L - \frac{b}{2}\right)\frac{D_P}{2\,L} = \left(316.2 - \frac{105.4}{2}\right)\frac{200}{2 \times 316.2} = 83.3 \text{ mm}$$

We know that tangential force acting at the mean radius,

$$W_T = \frac{T}{R_m} = \frac{477\,400}{83.3} = 5731 \text{ N}$$

Axial force acting on the pinion shaft,

$$W_{RH} = W_T \tan\phi . \sin\theta_{P1} = 5731 \times \tan 20^\circ \times \sin 18.43^\circ$$
$$= 5731 \times 0.364 \times 0.3161 = 659.4 \text{ N}$$

and radial force acting on the pinion shaft,

$$W_{RV} = W_T \tan\phi \,.\, \cos\theta_{P1} = 5731 \times \tan 20^\circ \times \cos 18.43^\circ$$
$$= 5731 \times 0.364 \times 0.9487 = 1979 \text{ N}$$

∴ Bending moment due to W_{RH} and W_{RV},

$$M_1 = W_{RV} \times \text{Overhang} - W_{RH} \times R_m$$
$$= 1979 \times 150 - 659.4 \times 83.3 = 241\,920 \text{ N-mm}$$

and bending moment due to W_T,

$$M_2 = W_T \times \text{Overhang} = 5731 \times 150 = 859\,650 \text{ N-mm}$$

∴ Resultant bending moment,

$$M = \sqrt{(M_1)^2 + (M_2)^2} = \sqrt{(241\,920)^2 + (859\,650)^2} = 893\,000 \text{ N-mm}$$

Since the shaft is subjected to twisting moment (T) and bending moment (M), therefore equivalent twisting moment,

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$$T_e = \sqrt{M^2 + T^2}$$

$$= \sqrt{(893\ 000)^2 + (477\ 400)^2}$$

$$= 1013 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1013 \times 10^3 = \frac{\pi}{16} \times \tau\, (d_P)^3$$

$$= \frac{\pi}{16} \times 45\, (d_P)^3 = 8.84\, (d_P)^3$$

... (Taking τ = 45 N/mm^2)

$\therefore$ $(d_P)^3 = 1013 \times 10^3 / 8.84 = 114.6 \times 10^3$

or d_P = 48.6 say 50 mm **Ans.**

Bevel gears

EXERCISES

1. A pair of straight bevel gears is required to transmit 10 kW at 500 r.p.m. from the motor shaft to another shaft at 250 r.p.m. The pinion has 24 teeth. The pressure angle is 20°. If the shaft axes are at right angles to each other, find the module, face width, addendum, outside diameter and slant height. The gears are capable of withstanding a static stress of 60 MPa. The tooth form factor may be taken as 0.154 – 0.912/T_E, where T_E is the equivalent number of teeth. Assume velocity factor as $\frac{4.5}{4.5 + v}$, where v the pitch line speed in m/s. The face width may be taken as $\frac{1}{4}$ of the slant height of the pitch cone. **[Ans. m = 8 mm ; b = 54 mm ; a = 8 mm ; D_O = 206.3 mm ; L = 214.4 mm]**

2. A 90° bevel gearing arrangement is to be employed to transmit 4 kW at 600 r.p.m. from the driving shaft to another shaft at 200 r.p.m. The pinion has 30 teeth. The pinion is made of cast steel having a static stress of 80 MPa and the gear is made of cast iron with a static stress of 55 MPa. The tooth profiles of the gears are of $14\frac{1}{2}$° composite form. The tooth form factor may be taken as y' = 0.124 – 0.684 / T_E, where T_E is the formative number of teeth and velocity factor, $C_v = \frac{3}{3 + v}$, where v is the pitch line speed in m/s.

   The face width may be taken as $^1/_3$ rd of the slant height of the pitch cone. Determine the module, face width and pitch diameters for the pinion and gears, from the standpoint of strength and check the design from the standpoint of wear. Take surface endurance limit as 630 MPa and modulus of elasticity for the material of gears is E_P = 200 kN/mm^2 and E_G = 80 kN/mm^2.

   [Ans. m = 4 mm ; b = 64 mm ; D_P = 120 mm ; D_G = 360 mm]

3. A pair of bevel gears is required to transmit 11 kW at 500 r.p.m. from the motor shaft to another shaft, the speed reduction being 3 : 1. The shafts are inclined at 60°. The pinion is to have 24 teeth with a pressure angle of 20° and is to be made of cast steel having a static stress of 80 MPa. The gear is to be made of cast iron with a static stress of 55 MPa. The tooth form factor may be taken as y = 0.154 – 0.912/T_E, where T_E is formative number of teeth. The velocity factor may be taken as $\frac{3}{3 + v}$, where v is the pitch line velocity in m/s. The face width may be taken as $^1/_4$ th of the slant height of the pitch cone. The mid-plane of the gear is 100 mm from the left hand bearing and 125 mm from the right hand bearing. The gear shaft is to be made of colled-rolled steel for which the allowable tensile stress may be taken as 80 MPa. Design the gears and the gear shaft.

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QUESTIONS

1. How the bevel gears are classified ? Explain with neat sketches.
2. Sketch neatly the working drawing of bevel gears in mesh.
3. For bevel gears, define the following :
   (*i*) Cone distance; (*ii*) Pitch angle; (*iii*) Face angle; (*iv*) Root angle; (*v*) Back cone distance; and (*vi*) Crown height.
4. What is Tredgold's approximation about the formative number of teeth on bevel gear?
5. What are the various forces acting on a bevel gear ?
6. Write the procedure for the design of a shaft for bevel gears.

OBJECTIVE TYPE QUESTIONS

1. When bevel gears having equal teeth and equal pitch angles connect two shafts whose axes intersect at right angle, then they are known as
   (*a*) angular bevel gears (*b*) crown bevel gears
   (*c*) internal bevel gears (*d*) mitre gears
2. The face angle of a bevel gear is equal to
   (*a*) pitch angle – addendum angle (*b*) pitch angle + addendum angle
   (*c*) pitch angle – dedendum angle (*d*) pitch angle + dedendum angle
3. The root angle of a bevel gear is equal to
   (*a*) pitch angle – addendum angle (*b*) pitch angle + addendum angle
   (*c*) pitch angle – dedendum angle (*d*) pitch angle + dedendum angle
4. If b denotes the face width and L denotes the cone distance, then the bevel factor is written as
   (*a*) b / L (b) $b / 2L$
   (c) $1 - 2\, b.L$ (*d*) $1 - b / L$
5. For a bevel gear having the pitch angle θ, the ratio of formative number of teeth (T_E) to actual number of teeth (T) is
   (*a*) $\frac{1}{\sin \theta}$ (*b*) $\frac{1}{\cos \theta}$
   (*c*) $\frac{1}{\tan \theta}$ (*d*) $\sin \theta \cos \theta$

ANSWERS

1. (*d*)	**2.** (*b*)	**3.** (*c*)	**4.** (*d*)	**5.** (*b*)