CHAPTER

31

Worm Gears

31.1 Introduction

The worm gears are widely used for transmitting power at high velocity ratios between non-intersecting shafts that are generally, but not necessarily, at right angles. It can give velocity ratios as high as 300 : 1 or more in a single step in a minimum of space, but it has a lower efficiency. The worm gearing is mostly used as a speed reducer, which consists of worm and a worm wheel or gear. The worm (which is the driving member) is usually of a cylindrical form having threads of the same shape as that of an involute rack. The threads of the worm may be left handed or right handed and single or multiple threads. The worm wheel or gear (which is the driven member) is similar to a helical gear with a face curved to conform to the shape of the worm. The worm is generally made of steel while the worm gear is made of bronze or cast iron for light service.

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The worm gearing is classified as non-interchangeable, because a worm wheel cut with a hob of one diameter will not operate satisfactorily with a worm of different diameter, even if the thread pitch is same.

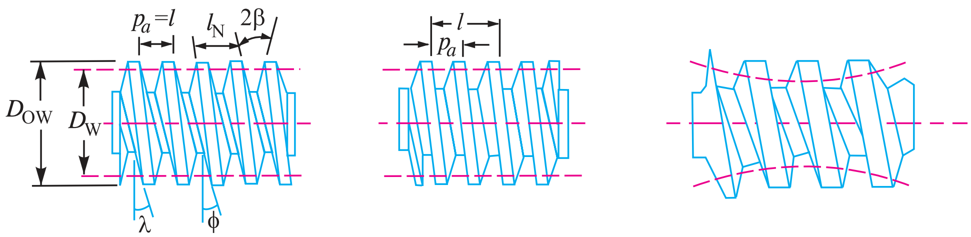
31.2 Types of Worms

The following are the two types of worms :

1. Cylindrical or straight worm, and
2. Cone or double enveloping worm.

The ***cylindrical*** or ***straight worm***, as shown in Fig. 31.1 (*a*), is most commonly used. The shape of the thread is involute helicoid of pressure angle 14 ½° for single and double threaded worms and 20° for triple and quadruple threaded worms. The worm threads are cut by a straight sided milling cutter having its diameter not less than the outside diameter of worm or greater than 1.25 times the outside diameter of worm.

The ***cone*** or ***double enveloping worm***, as shown in Fig. 31.1 (*b*), is used to some extent, but it requires extremely accurate alignment.



Single threaded. Double threaded.

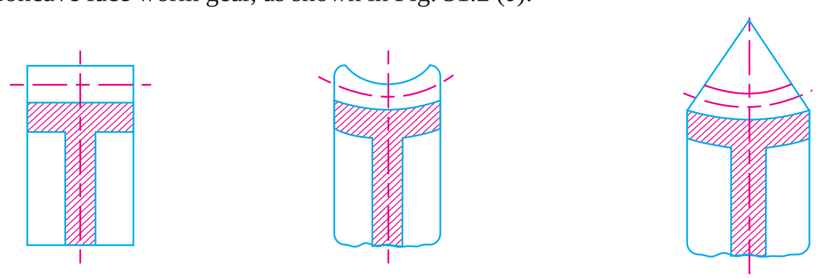
(*a*) Cylindrical or straight worm. (*b*) Cone or double enveloping worm.

Fig. 31.1. Types of worms.

31.3 Types of Worm Gears

The following three types of worm gears are important from the subject point of view :

1. Straight face worm gear, as shown in Fig. 31.2 (*a*),
2. Hobbed straight face worm gear, as shown in Fig. 31.2 (*b*), and
3. Concave face worm gear, as shown in Fig. 31.2 (*c*).



(*a*) Straight face. (*b*) Hobbed straight face. (*c*) Concave face.

Fig. 31.2. Types of worms gears.

The ***straight face worm gear*** is like a helical gear in which the straight teeth are cut with a form cutter. Since it has only point contact with the worm thread, therefore it is used for light service.

The ***hobbed straight face worm gear*** is also used for light service but its teeth are cut with a hob, after which the outer surface is turned.

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The ***concave face worm gear*** is the accepted standard form and is used for all heavy service and general industrial uses. The teeth of this gear are cut with a hob of the same pitch diameter as the mating worm to increase the contact area.

Worm gear is used mostly where the power source operates at a high speed and output is at a slow speed with high torque. It is also used in some cars and trucks.

31.4 Terms used in Worm Gearing

The worm and worm gear in mesh is shown in Fig. 31.3.

The following terms, in connection with the worm gearing, are important from the subject point of view :

1. *Axial pitch.* It is also known as ***linear pitch*** of a worm. It is the distance measured axially (*i.e.* parallel to the axis of worm) from a point on one thread to the corresponding point on the adjacent thread on the worm, as shown in Fig. 31.3. It may be noted that the axial pitch (p_a) of a worm is equal to the circular pitch (p_c) of the mating worm gear, when the shafts are at right angles.

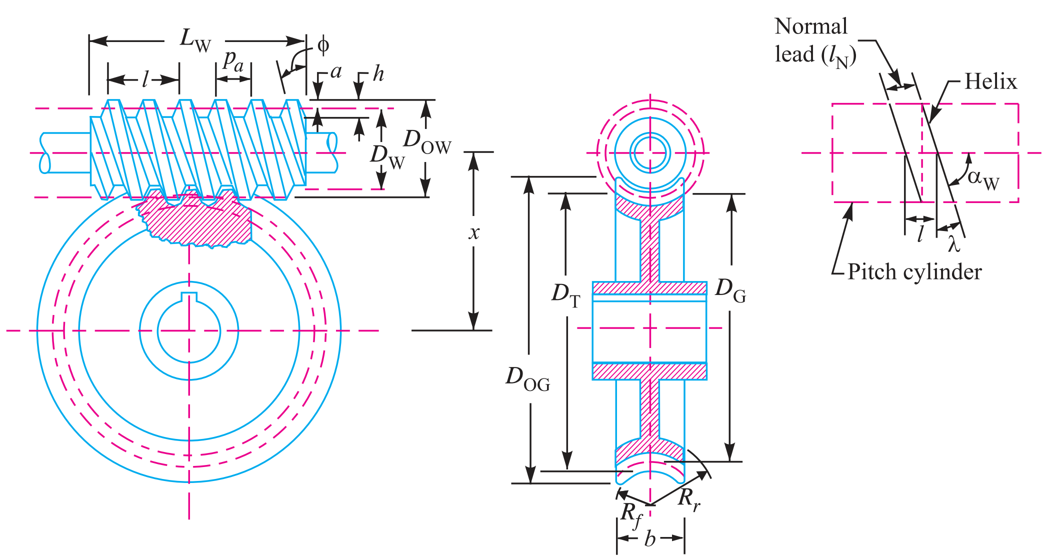

Fig. 31.3 . Worm and Worm gear.

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2. *Lead.* It is the linear distance through which a point on a thread moves ahead in one revolution of the worm. For single start threads, lead is equal to the axial pitch, but for multiple start threads, lead is equal to the product of axial pitch and number of starts. Mathematically,

Lead, $l = p_a \cdot n$

where p_a = Axial pitch ; and n = Number of starts.

3. *Lead angle.* It is the angle between the tangent to the thread helix on the pitch cylinder and the plane normal to the axis of the worm. It is denoted by λ.

A little consideration will show that if one complete turn of a worm thread be imagined to be unwound from the body of the worm, it will form an inclined plane whose base is equal to the pitch circumference of the worm and altitude equal to lead of the worm, as shown in Fig. 31.4.

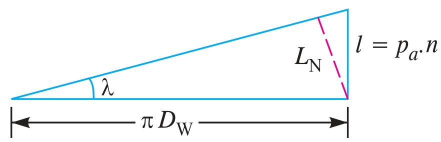


Fig. 31.4. Development of a helix thread.

From the geometry of the figure, we find that

$$\tan \lambda = \frac{\text{Lead of the worm}}{\text{Pitch circumference of the worm}}$$

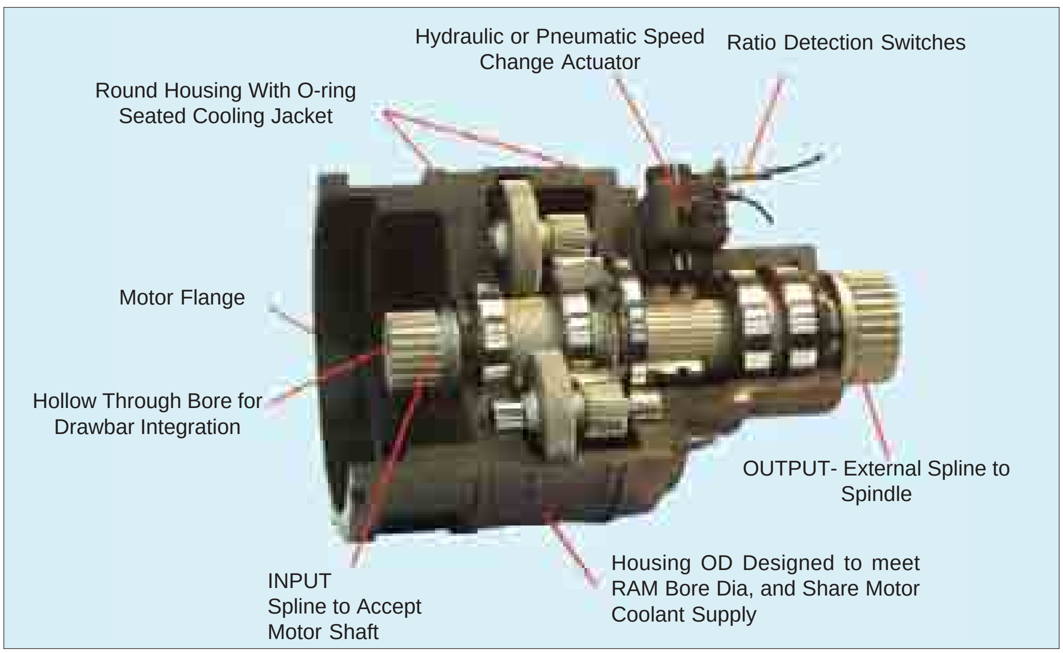
$$= \frac{l}{\pi D_W} = \frac{p_a \cdot n}{\pi D_W} \qquad \ldots (\because l = p_a \cdot n)$$

$$= \frac{p_c \cdot n}{\pi D_W} = \frac{\pi\, m \cdot n}{\pi D_W} = \frac{m \cdot n}{D_W} \qquad \ldots (\because p_a = p_c \text{ ; and } p_c = \pi m)$$

where m = Module, and

D_W = Pitch circle diameter of worm.

The lead angle (λ) may vary from 9° to 45°. It has been shown by F.A. Halsey that a lead angle less than 9° results in rapid wear and the safe value of λ is 12½°.



Model of sun and planet gears.

For a compact design, the lead angle may be determined by the following relation, *i.e.*

$$\tan \lambda = \left(\frac{N_G}{N_W}\right)^{1/3},$$

where N_G is the speed of the worm gear and N_W is the speed of the worm.

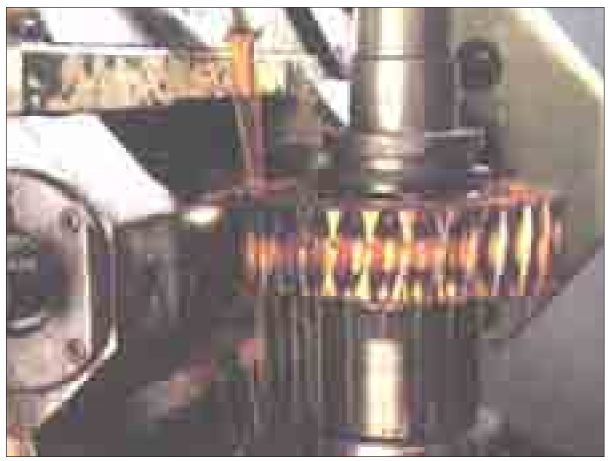
4. *Tooth pressure angle.* It is measured in a plane containing the axis of the worm and is equal to one-half the thread profile angle as shown in Fig. 31.3.

The following table shows the recommended values of lead angle (λ) and tooth pressure angle (φ).

Table 31.1. Recommended values of lead angle and pressure angle.

Lead angle (λ) in degrees	0 – 16	16 – 25	25 – 35	35 – 45
Pressure angle(φ) in degrees	14½	20	25	30

For automotive applications, the pressure angle of 30° is recommended to obtain a high efficiency and to permit overhauling.



Worm gear teeth generation on gear hobbing machine.

5. *Normal pitch.* It is the distance measured along the normal to the threads between two corresponding points on two adjacent threads of the worm. Mathematically,

Normal pitch, $p_N = p_a . \cos \lambda$

Note. The term normal pitch is used for a worm having single start threads. In case of a worm having multiple start threads, the term normal lead (l_N) is used, such that

$$l_N = l . \cos \lambda$$

6. *Helix angle.* It is the angle between the tangent to the thread helix on the pitch cylinder and the axis of the worm. It is denoted by α_W, in Fig. 31.3. The worm helix angle is the complement of worm lead angle, *i.e.*

$$\alpha_W + \lambda = 90°$$

It may be noted that the helix angle on the worm is generally quite large and that on the worm gear is very small. Thus, it is usual to specify the lead angle (λ) on the worm and helix angle (α_G) on the worm gear. These two angles are equal for a 90° shaft angle.

7. *Velocity ratio.* It is the ratio of the speed of worm (N_W) in r.p.m. to the speed of the worm gear (N_G) in r.p.m. Mathematically, velocity ratio,

$$V.R. = \frac{N_W}{N_G}$$

Let l = Lead of the worm, and

D_G = Pitch circle diameter of the worm gear.

We know that linear velocity of the worm,

$$v_W = \frac{l . N_W}{60}$$

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and linear velocity of the worm gear,

$$v_G = \frac{\pi \, D_G \; N_G}{60}$$

Since the linear velocity of the worm and worm gear are equal, therefore

$$\frac{l \, . N_W}{60} = \frac{\pi \, D_G \, . N_G}{60} \quad \text{or} \quad \frac{N_W}{N_G} = \frac{\pi \, D_G}{l}$$

We know that pitch circle diameter of the worm gear,

$$D_G = m \, . \, T_G$$

where m is the module and T_G is the number of teeth on the worm gear.

$$\therefore \qquad V.R. = \frac{N_W}{N_G} = \frac{\pi \, D_G}{l} = \frac{\pi \, m . T_G}{l}$$

$$= \frac{p_c \, . T_G}{l} = \frac{p_a \, . T_G}{p_a \, . n} = \frac{T_G}{n} \qquad \ldots (\because p_c = \pi \, m = p_a \, ; \text{ and } l = p_a \, . \, n)$$

where n = Number of starts of the worm.

From above, we see that velocity ratio may also be defined as the ratio of number of teeth on the worm gear to the number of starts of the worm.

The following table shows the number of starts to be used on the worm for the different velocity ratios :

Table 31.2. Number of starts to be used on the worm for different velocity ratios.

Velocity ratio (*V.R.*)	36 and above	12 to 36	8 to 12	6 to 12	4 to 10
Number of starts or threads on the worm ($n = T_w$)	Single	Double	Triple	Quadruple	Sextuple

31.5 Proportions for Worms

The following table shows the various porportions for worms in terms of the axial or circular pitch (p_c) in mm.

Table 31.3. Proportions for worm.

S. No.	*Particulars*	*Single and double threaded worms*	*Triple and quadruple threaded worms*
1.	Normal pressure angle (φ)	14½°	20°
2.	Pitch circle diameter for worms integral with the shaft	$2.35 \, p_c$ + 10 mm	$2.35 \, p_c$ + 10 mm
3.	Pitch circle diameter for worms bored to fit over the shaft	$2.4 \, p_c$ + 28 mm	$2.4 \, p_c$ + 28 mm
4.	Maximum bore for shaft	p_c + 13.5 mm	p_c + 13.5 mm
5.	Hub diameter	$1.66 \, p_c$ + 25 mm	$1.726 \, p_c$ + 25 mm
6.	Face length (L_W)	$p_c \, (4.5 + 0.02 \, T_W)$	$p_c \, (4.5 + 0.02 \, T_W)$
7.	Depth of tooth (h)	$0.686 \, p_c$	$0.623 \, p_c$
8.	Addendum (a)	$0.318 \, p_c$	$0.286 \, p_c$

Notes: 1. The pitch circle diameter of the worm (D_W) in terms of the centre distance between the shafts (x) may be taken as follows :

$$D_W = \frac{(x)^{0.875}}{1.416} \qquad \ldots (\text{when } x \text{ is in mm})$$

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2. The pitch circle diameter of the worm (D_W) may also be taken as

$$D_W = 3\,p_c, \text{ where } p_c \text{ is the axial or circular pitch.}$$

3. The face length (or length of the threaded portion) of the worm should be increased by 25 to 30 mm for the feed marks produced by the vibrating grinding wheel as it leaves the thread root.

31.6 Proportions for Worm Gear

The following table shows the various proportions for worm gears in terms of circular pitch (p_c) in mm.

Table 31.4. Proportions for worm gear.

S. No.	*Particulars*	*Single and double threads*	*Triple and quadruple threads*
1.	Normal pressure angle (ϕ)	14½°	20°
2.	Outside diameter (D_{OG})	$D_G + 1.0135\,p_c$	$D_G + 0.8903\,p_c$
3.	Throat diameter (D_T)	$D_G + 0.636\,p_c$	$D_G + 0.572\,p_c$
4.	Face width (b)	$2.38\,p_c + 6.5$ mm	$2.15\,p_c + 5$ mm
5.	Radius of gear face (R_f)	$0.882\,p_c + 14$ mm	$0.914\,p_c + 14$ mm
6.	Radius of gear rim (R_r)	$2.2\,p_c + 14$ mm	$2.1\,p_c + 14$ mm

31.7 Efficiency of Worm Gearing

The efficiency of worm gearing may be defined as the ratio of work done by the worm gear to the work done by the worm.

Mathematically, the efficiency of worm gearing is given by

$$\eta = \frac{\tan \lambda\,(\cos \phi - \mu \tan \lambda)}{\cos \phi \tan \lambda + \mu} \quad \text{...}(i)$$

where ϕ = Normal pressure angle,

μ = Coefficient of friction, and

λ = Lead angle.

The efficiency is maximum, when

$$\tan \lambda = \sqrt{1 + \mu^2} - \mu$$

In order to find the approximate value of the efficiency, assuming square threads, the following relation may be used :

$$\text{Efficiency, } \eta = \frac{\tan \lambda\,(1 - \mu \tan \lambda)}{\tan \lambda + \mu}$$

$$= \frac{1 - \mu \tan \lambda}{1 + \mu / \tan \lambda}$$

$$= \frac{\tan \lambda}{\tan (\lambda + \phi_1)}$$

...(Substituting in equation (*i*), $\phi = 0$, for square threads)

where ϕ_1 = Angle of friction, such that $\tan \phi_1 = \mu$.

A gear-cutting machine is used to cut gears.

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The coefficient of friction varies with the speed, reaching a minimum value of 0.015 at a rubbing speed $\left(v_r = \frac{\pi D_W . N_W}{\cos \lambda}\right)$ between 100 and 165 m/min. For a speed below 10 m/min, take μ = 0.015. The following empirical relations may be used to find the value of μ, *i.e.*

$$\mu = \frac{0.275}{(v_r)^{0.25}}, \text{ for rubbing speeds between 12 and 180 m/min}$$

$$= 0.025 + \frac{v_r}{18000} \text{ for rubbing speed more than 180 m/min}$$

Note : If the efficiency of worm gearing is less than 50%, then the worm gearing is said to be ***self locking,*** *i.e.* it cannot be driven by applying a torque to the wheel. This property of self locking is desirable in some applications such as hoisting machinery.

Hardened and ground worm shaft and worm wheel pair

Example 31.1. *A triple threaded worm has teeth of 6 mm module and pitch circle diameter of 50 mm. If the worm gear has 30 teeth of 14½° and the coefficient of friction of the worm gearing is 0.05, find 1. the lead angle of the worm, 2. velocity ratio, 3. centre distance, and 4. efficiency of the worm gearing.*

Solution. Given : n = 3 ; m = 6 ; D_W = 50 mm ; T_G = 30 ; ϕ = 14.5° ; μ = 0.05.

1. Lead angle of the worm

Let λ = Lead angle of the worm.

We know that $\tan \lambda = \frac{m.n}{D_W} = \frac{6 \times 3}{50} = 0.36$

$\therefore$ $\lambda = \tan^{-1}(0.36) = 19.8°$ **Ans.**

2. Velocity ratio

We know that velocity ratio,

$$V.R. = T_G / n = 30 / 3 = 10 \textbf{ Ans.}$$

3. Centre distance

We know that pitch circle diameter of the worm gear

$$D_G = m.T_G = 6 \times 30 = 180 \text{ mm}$$

$\therefore$ Centre distance,

$$x = \frac{D_W + D_G}{2} = \frac{50 + 180}{2} = 115 \text{ mm} \textbf{ Ans.}$$

4. Efficiency of the worm gearing

We know that efficiency of the worm gearing.

$$\eta = \frac{\tan \lambda (\cos \phi - \mu \tan \lambda)}{\cos \phi . \tan \lambda + \mu}$$

$$= \frac{\tan 19.8° (\cos 14.5° - 0.05 \times \tan 19.8°)}{\cos 14.5° \times \tan 19.8° + 0.05}$$

$$= \frac{0.36 (0.9681 - 0.05 \times 0.36)}{0.9681 \times 0.36 + 0.05} = \frac{0.342}{0.3985} = 0.858 \text{ or } 85.8\% \textbf{ Ans.}$$

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Note : The approximate value of the efficiency assuming square threads is

$$\eta = \frac{1 - \mu \tan \lambda}{1 + \mu / \tan \lambda} = \frac{1 - 0.05 \times 0.36}{1 + 0.05/0.36} = \frac{0.982}{1.139} = 0.86 \text{ or } 86\% \quad \textbf{Ans.}$$

31.8 Strength of Worm Gear Teeth

In finding the tooth size and strength, it is safe to assume that the teeth of worm gear are always weaker than the threads of the worm. In worm gearing, two or more teeth are usually in contact, but due to uncertainty of load distribution among themselves it is assumed that the load is transmitted by one tooth only. We know that according to Lewis equation,

$$W_T = (\sigma_o \,.\, C_v)\, b \,.\, \pi\, m \,.\, y$$

where W_T = Permissible tangential tooth load or beam strength of gear tooth,

σ_o = Allowable static stress,

C_v = Velocity factor,

b = Face width,

m = Module, and

y = Tooth form factor or Lewis factor.

Notes : 1. The velocity factor is given by

$$C_v = \frac{6}{6 + v}, \text{ where } v \text{ is the peripheral velocity of the worm gear in m/s.}$$

2. The tooth form factor or Lewis factor (y) may be obtained in the similar manner as discussed in spur gears (Art. 28.17), *i.e.*

$$y = 0.124 - \frac{0.684}{T_G}, \text{ for } 14\tfrac{1}{2}° \text{ involute teeth.}$$

$$= 0.154 - \frac{0.912}{T_G}, \text{ for } 20° \text{ involute teeth.}$$

3. The dynamic tooth load on the worm gear is given by

$$W_D = \frac{W_T}{C_v} = W_T \left(\frac{6 + v}{6}\right)$$

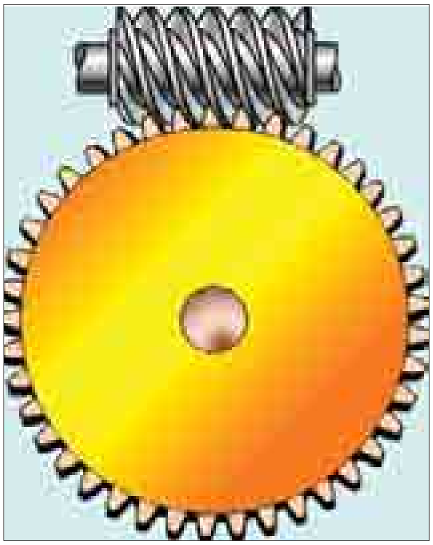
where W_T = Actual tangential load on the tooth.

The dynamic load need not to be calculated because it is not so severe due to the sliding action between the worm and worm gear.

4. The static tooth load or endurance strength of the tooth (W_S) may also be obtained in the similar manner as discussed in spur gears (Art. 28.20), *i.e.*

$$W_S = \sigma_e . b\, \pi\, m . y$$

where σ_e = Flexural endurance limit. Its value may be taken as 84 MPa for cast iron and 168 MPa for phosphor bronze gears.



Worm gear assembly.

31.9 Wear Tooth Load for Worm Gear

The limiting or maximum load for wear (W_W) is given by

$$W_W = D_G \,.\, b \,.\, K$$

where D_G = Pitch circle diameter of the worm gear,

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b = Face width of the worm gear, and

K = Load stress factor (also known as material combination factor).

The load stress factor depends upon the combination of materials used for the worm and worm gear. The following table shows the values of load stress factor for different combination of worm and worm gear materials.

Table 31.5. Values of load stress factor (*K*).

S.No.	*Material*		*Load stress factor (K) N/mm²*
	Worm	*Worm gear*	
1.	Steel (B.H.N. 250)	Phosphor bronze	0.415
2.	Hardened steel	Cast iron	0.345
3.	Hardened steel	Phosphor bronze	0.550
4.	Hardened steel	Chilled phosphor bronze	0.830
5.	Hardened steel	Antimony bronze	0.830
6.	Cast iron	Phosphor bronze	1.035

Note : The value of K given in the above table are suitable for lead angles upto 10°. For lead angles between 10° and 25°, the values of K should be increased by 25 per cent and for lead angles greater than 25°, increase the value of K by 50 per cent.

31.10 Thermal Rating of Worm Gearing

In the worm gearing, the heat generated due to the work lost in friction must be dissipated in order to avoid over heating of the drive and lubricating oil. The quantity of heat generated (Q_g) is given by

$$Q_g = \text{Power lost in friction in watts} = P(1 - \eta) \quad \text{...}(i)$$

where P = Power transmitted in watts, and

η = Efficiency of the worm gearing.

The heat generated must be dissipated through the lubricating oil to the gear box housing and then to the atmosphere. The heat dissipating capacity depends upon the following factors :

1. Area of the housing (A),

2. Temperature difference between the housing surface and surrounding air ($t_2 - t_1$), and

3. Conductivity of the material (K).

Mathematically, the heat dissipating capacity,

$$Q_d = A(t_2 - t_1) K \quad \text{...}(ii)$$

From equations (*i*) and (*ii*), we can find the temperature difference ($t_2 - t_1$). The average value of K may be taken as 378 W/m²/°C.

Notes : 1. The maximum temperature ($t_2 - t_1$) should not exceed 27 to 38°C.

2. The maximum temperature of the lubricant should not exceed 60°C.

3. According to AGMA recommendations, the limiting input power of a plain worm gear unit from the standpoint of heat dissipation, for worm gear speeds upto 2000 r.p.m., may be checked from the following relation, *i.e.*

$$P = \frac{3650\, x^{1.7}}{V.R. + 5}$$

where P = Permissible input power in kW,

x = Centre distance in metres, and

$V.R.$ = Velocity ratio or transmission ratio.

31.11 Forces Acting on Worm Gears

When the worm gearing is transmitting power, the forces acting on the worm are similar to those on a power screw. Fig. 31.5 shows the forces acting on the worm. It may be noted that the forces on a worm gear are equal in magnitude to that of worm, but opposite in direction to those shown in Fig. 31.5.

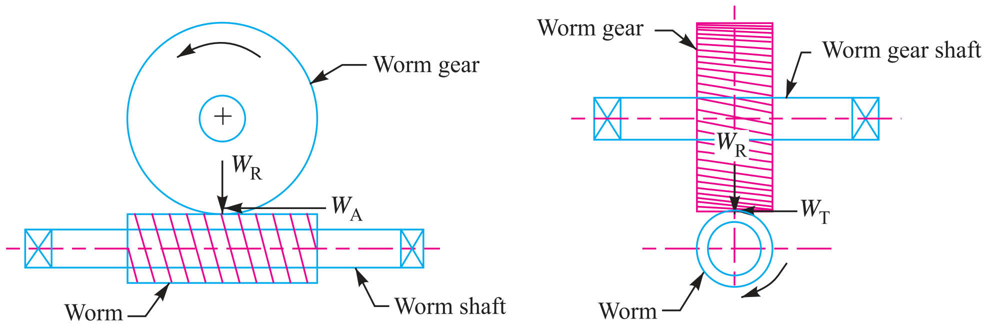

Fig. 31.5. Forces acting on worm teeth.

The various forces acting on the worm may be determined as follows :

1. Tangential force on the worm,

$$W_T = \frac{2 \times \text{Torque on worm}}{\text{Pitch circle diameter of worm } (D_W)}$$

$$= \text{Axial force or thrust on the worm gear}$$

The tangential force (W_T) on the worm produces a twisting moment of magnitude ($W_T \times D_W / 2$) and bends the worm in the horizontal plane.

2. Axial force or thrust on the worm,

$$W_A = W_T / \tan \lambda = \text{Tangential force on the worm gear}$$

$$= \frac{2 \times \text{Torque on the worm gear}}{\text{Pitch circle diameter of worm gear}(D_G)}$$

The axial force on the worm tends to move the worm axially, induces an axial load on the bearings and bends the worm in a vertical plane with a bending moment of magnitude ($W_A \times D_W / 2$).

3. Radial or separating force on the worm,

$$W_R = W_A \,.\, \tan \phi = \text{Radial or separating force on the worm gear}$$

The radial or separating force tends to force the worm and worm gear out of mesh. This force also bends the worm in the vertical plane.

Example 31.2. *A worm drive transmits 15 kW at 2000 r.p.m. to a machine carriage at 75 r.p.m. The worm is triple threaded and has 65 mm pitch diameter. The worm gear has 90 teeth of 6 mm module. The tooth form is to be 20° full depth involute. The coefficient of friction between the mating teeth may be taken as 0.10. Calculate : 1. tangential force acting on the worm ; 2. axial thrust and separating force on worm; and 3. efficiency of the worm drive.*

Solution. Given : P = 15 kW = 15 × 10^3 W ; N_W = 2000 r.p.m. ; N_G = 75 r.p.m. ; n = 3 ; D_W = 65 mm ; T_G = 90 ; m = 6 mm ; ϕ = 20° ; μ = 0.10

1. ***Tangential force acting on the worm***

We know that the torque transmitted by the worm

$$= \frac{P \times 60}{2 \pi N_W} = \frac{15 \times 10^3 \times 60}{2 \pi \times 2000} = 71.6 \text{ N-m} = 71\,600 \text{ N-mm}$$

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∴ Tangential force acting on the worm,

$$W_T = \frac{\text{Torque on worm}}{\text{Radius of worm}} = \frac{71\,600}{65/2} = 2203 \text{ N} \quad \textbf{Ans.}$$

2. Axial thrust and separating force on worm

Let λ = Lead angle.

We know that $\tan \lambda = \frac{m \,.\, n}{D_W} = \frac{6 \times 3}{65} = 0.277$

or $\lambda = \tan^{-1}(0.277) = 15.5°$

∴ Axial thrust on the worm,

$$W_A = W_T / \tan \lambda = 2203 / 0.277 = 7953 \text{ N} \quad \textbf{Ans.}$$

and separating force on the worm

$$W_R = W_A \,.\, \tan \phi = 7953 \times \tan 20° = 7953 \times 0.364 = 2895 \text{ N} \quad \textbf{Ans.}$$

3. Efficiency of the worm drive

We know that efficiency of the worm drive,

$$\eta = \frac{\tan \lambda\,(\cos\phi - \mu \,.\, \tan \lambda)}{\cos\phi \,.\tan \lambda + \mu}$$

$$= \frac{\tan 15.5°\,(\cos 20° - 0.10 \times \tan 15.5°)}{\cos 20° \times \tan 15.5° + 0.10}$$

$$= \frac{0.277\,(0.9397 - 0.10 \times 0.277)}{0.9397 \times 0.277 + 0.10} = \frac{0.2526}{0.3603} = 0.701 \text{ or } 70.1\% \quad \textbf{Ans.}$$

31.12 Design of Worm Gearing

In designing a worm and worm gear, the quantities like the power transmitted, speed, velocity ratio and the centre distance between the shafts are usually given and the quantities such as lead angle, lead and number of threads on the worm are to be determined. In order to determine the satisfactory combination of lead angle, lead and centre distance, the following method may be used:

From Fig. 31.6 we find that the centre distance,

$$x = \frac{D_W + D_G}{2}$$

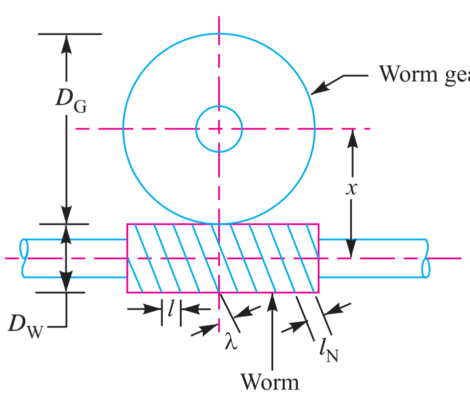

Fig. 31.6. Worm and worm gear.

Worm gear boxes are noted for reliable power transmission.

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The centre distance may be expressed in terms of the axial lead (l), lead angle (λ) and velocity ratio ($V.R.$), as follows :

$$x = \frac{l}{2\pi}(\cot\lambda + V.R.)$$

In terms of normal lead ($l_N = l\cos\lambda$), the above expression may be written as :

$$x = \frac{l_N}{2\pi}\left(\frac{1}{\sin\lambda} + \frac{V.R.}{\cos\lambda}\right)$$

or $$\frac{x}{l_N} = \frac{1}{2\pi}\left(\frac{1}{\sin\lambda} + \frac{V.R.}{\cos\lambda}\right) \quad \text{...}(i)$$

Since the velocity ratio ($V.R.$) is usually given, therefore the equation (*i*) contains three variables *i.e.* x, l_N and λ. The right hand side of the above expression may be calculated for various values of velocity ratios and the curves are plotted as shown in Fig. 31.7. The lowest point on each of the curves gives the lead angle which corresponds to the minimum value of x / l_N. This minimum value represents the minimum centre distance that can be used with a given lead or inversely the maximum lead that can be used with a given centre distance. Now by using Table 31.2 and standard modules, we can determine the combination of lead angle, lead, centre distance and diameters for the given design specifications.

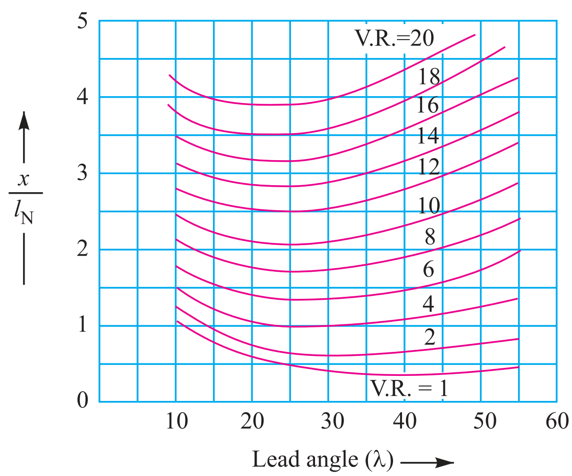


Fig. 31.7. Worm gear design curves.

Note : The lowest point on the curve may be determined mathematically by differentiating the equation (*i*) with respect to λ and equating to zero, *i.e.*

$$\frac{(V.R.)\sin^3\lambda - \cos^3\lambda}{\sin^2\lambda.\cos^2\lambda} = 0 \quad \text{or} \quad V.R. = \cot^3\lambda$$

Example 31.3. *Design 20° involute worm and gear to transmit 10 kW with worm rotating at 1400 r.p.m. and to obtain a speed reduction of 12 : 1. The distance between the shafts is 225 mm.*

Solution. Given : $\phi = 20°$; $P = 10$ kW = 10 000 W ; $N_W = 1400$ r.p.m. ; $V.R. = 12$; $x = 225$ mm

The worm and gear is designed as discussed below :

1. *Design of worm*

Let l_N = Normal lead, and

λ = Lead angle.

Top

Worm gear of a steering mechanism in an automobile.

We have discussed in Art. 31.12 that the value of x / l_N will be minimum corresponding to

$$\cot^3 \lambda = V.R. = 12 \quad \text{or} \quad \cot \lambda = 2.29$$

$$\therefore \quad \lambda = 23.6°$$

We know that

$$\frac{x}{l_N} = \frac{1}{2\pi}\left(\frac{1}{\sin \lambda} + \frac{V.R.}{\cos \lambda}\right)$$

$$\frac{225}{l_N} = \frac{1}{2\pi}\left(\frac{1}{\sin 23.6°} + \frac{12}{\cos 23.6°}\right) = \frac{1}{2\pi}(2.5 + 13.1) = 2.5$$

$$\therefore \quad l_N = 225 / 2.5 = 90 \text{ mm}$$

and axial lead, $l = l_N / \cos \lambda = 90 / \cos 23.6° = 98.2$ mm

From Table 31.2, we find that for a velocity ratio of 12, the number of starts or threads on the worm,

$$n = T_W = 4$$

∴ Axial pitch of the threads on the worm,

$$p_a = l / 4 = 98.2 / 4 = 24.55 \text{ mm}$$

$$\therefore \quad m = p_a / \pi = 24.55 / \pi = 7.8 \text{ mm}$$

Let us take the standard value of module, $m = 8$ mm

∴ Axial pitch of the threads on the worm,

$$p_a = \pi m = p \times 8 = 25.136 \text{ mm}$$ **Ans.**

Axial lead of the threads on the worm,

$$l = p_a \cdot n = 25.136 \times 4 = 100.544 \text{ mm}$$ **Ans.**

and normal lead of the threads on the worm,

$$l_N = l \cos \lambda = 100.544 \cos 23.6° = 92 \text{ mm}$$ **Ans.**

We know that the centre distance,

$$x = \frac{l_N}{2\pi}\left(\frac{1}{\sin \lambda} + \frac{V.R.}{\cos \lambda}\right) = \frac{92}{2\pi}\left(\frac{1}{\sin 23.6°} + \frac{12}{\cos 23.6°}\right)$$

$$= 14.64 (2.5 + 13.1) = 230 \text{ mm}$$ **Ans.**

Let D_W = Pitch circle diameter of the worm.

We know that

$$\tan \lambda = \frac{l}{\pi D_W}$$

$$\therefore \quad D_W = \frac{l}{\pi \tan \lambda} = \frac{100.544}{\pi \tan 23.6°} = 73.24 \text{ mm}$$ **Ans.**

Since the velocity ratio is 12 and the worm has quadruple threads (*i.e.* $n = T_W = 4$), therefore number of teeth on the worm gear,

$$T_G = 12 \times 4 = 48$$

From Table 31.3, we find that the face length of the worm or the length of threaded portion is

$$L_W = p_c (4.5 + 0.02\, T_W)$$

$$= 25.136 (4.5 + 0.02 \times 4) = 115 \text{ mm} \qquad ...(\because p_c = p_a)$$

This length should be increased by 25 to 30 mm for the feed marks produced by the vibrating grinding wheel as it leaves the thread root. Therefore let us take

$$L_W = 140 \text{ mm} \quad \textbf{Ans.}$$

We know that depth of tooth,

$$h = 0.623\, p_c = 0.623 \times 25.136 = 15.66 \text{ mm} \quad \textbf{Ans.}$$

...(From Table 31.3)

and addendum, $a = 0.286\, p_c = 0.286 \times 25.136 = 7.2$ mm **Ans.**

∴ Outside diameter of worm,

$$D_{OW} = D_W + 2a = 73.24 + 2 \times 7.2 = 87.64 \text{ mm} \quad \textbf{Ans.}$$

2. *Design of worm gear*

We know that pitch circle diameter of the worm gear,

$$D_G = m \,.\, T_G = 8 \times 48 = 384 \text{ mm} = 0.384 \text{ m} \quad \textbf{Ans.}$$

From Table 31.4, we find that outside diameter of worm gear,

$$D_{OG} = D_G + 0.8903\, p_c = 384 + 0.8903 \times 25.136 = 406.4 \text{ mm} \quad \textbf{Ans.}$$

Throat diameter,

$$D_T = D_G + 0.572\, p_c = 384 + 0.572 \times 25.136 = 398.4 \text{ mm} \quad \textbf{Ans.}$$

and face width, $b = 2.15\, p_c + 5 \text{ mm} = 2.15 \times 25.136 + 5 = 59$ mm **Ans.**

Let us now check the designed worm gearing from the standpoint of tangential load, dynamic load, static load or endurance strength, wear load and heat dissipation.

(a) Check for the tangential load

Let N_G = Speed of the worm gear in r.p.m.

We know that velocity ratio of the drive,

$$V.R. = \frac{N_W}{N_G} \quad \text{or} \quad N_G = \frac{N_W}{V.R.} = \frac{1400}{12} = 116.7 \text{ r.p.m}$$

∴ Torque transmitted,

$$T = \frac{P \times 60}{2\pi N_G} = \frac{10\,000 \times 60}{2\pi \times 116.7} = 818.2 \text{ N-m}$$

and tangential load acting on the gear,

$$W_T = \frac{2 \times \text{Torque}}{D_G} = \frac{2 \times 818.2}{0.384} = 4260 \text{ N}$$

We know that pitch line or peripheral velocity of the worm gear,

$$v = \frac{\pi \,.\, D_G \,.\, N_G}{60} = \frac{\pi \times 0.384 \times 116.7}{60} = 2.35 \text{ m/s}$$

∴ Velocity factor,

$$C_v = \frac{6}{6 + v} = \frac{6}{6 + 2.35} = 0.72$$

Gears are usually enclosed in boxes to protect them from environmental pollution and provide them proper lubrication.

and tooth form factor for 20° involute teeth,

$$y = 0.154 - \frac{0.912}{T_G} = 0.154 - \frac{0.912}{48} = 0.135$$

Since the worm gear is generally made of phosphor bronze, therefore taking the allowable static stress for phosphor bronze, σ_o = 84 MPa or N/mm^2.

We know that the designed tangential load,

$$W_T = (\sigma_o \cdot C_v)\, b.\, \pi\, m \cdot y = (84 \times 0.72)\, 59 \times \pi \times 8 \times 0.135 \text{ N}$$
$$= 12\ 110 \text{ N}$$

Since this is more than the tangential load acting on the gear (*i.e.* 4260 N), therefore the design is safe from the standpoint of tangential load.

(b) Check for dynamic load

We know that the dynamic load,

$$W_D = W_T / C_v = 12\ 110 / 0.72 = 16\ 820 \text{ N}$$

Since this is more than W_T = 4260 N, therefore the design is safe from the standpoint of dynamic load.

(c) Check for static load or endurance strength

We know that the flexural endurance limit for phosphor bronze is

$$\sigma_e = 168 \text{ MPa or N/mm}^2$$

∴ Static load or endurance strength,

$$W_S = \sigma_e \cdot b.\, \pi\, m \cdot y = 168 \times 59 \times \pi \times 8 \times 0.135 = 33\ 635 \text{ N}$$

Since this is much more than W_T = 4260 N, therefore the design is safe from the standpoint of static load or endurance strength.

(d) Check for wear

Assuming the material for worm as hardened steel, therefore from Table 31.5, we find that for hardened steel worm and phosphor bronze worm gear, the value of load stress factor,

$$K = 0.55 \text{ N/mm}^2$$

∴ Limiting or maximum load for wear,

$$W_W = D_G \cdot b \cdot K = 384 \times 59 \times 0.55 = 12\ 461 \text{ N}$$

Since this is more than $W_T = 4260$ N, therefore the design is safe from the standpoint of wear.

(e) Check for heat dissipation

First of all, let us find the efficiency of the worm gearing (η).

We know that rubbing velocity,

$$v_r = \frac{\pi D_W \cdot N_W}{\cos \lambda} = \frac{\pi \times 0.07324 \times 1400}{\cos 23.6°} = 351.6 \text{ m/min}$$

...(D_W is taken in metres)

∴ Coefficient of friction,

$$\mu = 0.025 + \frac{v_r}{18\ 000} = 0.025 + \frac{351.6}{18\ 000} = 0.0445$$

...(∴ v_r is greater than 180 m/min)

and angle of friction, $\phi_1 = \tan^{-1} \mu = \tan^{-1} (0.0445) = 2.548°$

We know that efficiency,

$$\eta = \frac{\tan \lambda}{\tan (\lambda + \phi_1)} = \frac{\tan 23.6°}{\tan (23.6 + 2.548)} = \frac{0.4369}{0.4909} = 0.89 \text{ or } 89\%$$

Assuming 25 per cent overload, heat generated,

$$Q_g = 1.25\, P\, (1 - \eta) = 1.25 \times 10\ 000\ (1 - 0.89) = 1375 \text{ W}$$

We know that projected area of the worm,

$$A_W = \frac{\pi}{4} (D_W)^2 = \frac{\pi}{4} (73.24)^2 = 4214 \text{ mm}^2$$

and projected area of the worm gear,

$$A_G = \frac{\pi}{4} (D_G)^2 = \frac{\pi}{4} (384)^2 = 115\ 827 \text{ mm}^2$$

∴ Total projected area of worm and worm gear,

$$A = A_W + A_G = 4214 + 115\ 827 = 120\ 041 \text{ mm}^2$$

$$= 120\ 041 \times 10^{-6} \text{ m}^2$$

We know that heat dissipating capacity,

$$Q_d = A\, (t_2 - t_1)\, K = 120\ 041 \times 10^{-6}\, (t_2 - t_1)\, 378 = 45.4\, (t_2 - t_1)$$

The heat generated must be dissipated in order to avoid over heating of the drive, therefore equating $Q_g = Q_d$, we have

$$t_2 - t_1 = 1375 / 45.4 = 30.3°\text{C}$$

Since this temperature difference $(t_2 - t_1)$ is within safe limits of 27 to 38°C, therefore the design is safe from the standpoint of heat.

3. Design of worm shaft

Let d_W = Diameter of worm shaft.

We know that torque acting on the worm gear shaft,

$$T_{gear} = \frac{1.25\, P \times 60}{2\, \pi\, N_G} = \frac{1.25 \times 10\,000 \times 60}{2\, \pi \times 116.7} = 1023 \text{ N-m}$$

$$= 1023 \times 10^3 \text{ N-mm}$$

...(Taking 25% overload)

∴ Torque acting on the worm shaft,

$$T_{worm} = \frac{T_{gear}}{V.R. \times \eta} = \frac{1023}{12 \times 0.89} = 96 \text{ N-m} = 96 \times 10^3 \text{ N-mm}$$

Differential inside an automobile.

We know that tangential force on the worm,

$$W_T = \text{Axial force on the worm gear}$$

$$= \frac{2 \times T_{worm}}{D_W} = \frac{2 \times 96 \times 10^3}{73.24} = 2622 \text{ N}$$

Axial force on the worm,

$$W_A = \text{Tangential force on the worm gear}$$

$$= \frac{2 \times T_{gear}}{D_G} = \frac{2 \times 1023 \times 10^3}{384} = 5328 \text{ N}$$

and radial or separating force on the worm

$$W_R = \text{Radial or separating force on the worm gear}$$

$$= W_A \cdot \tan \phi = 5328 \times \tan 20° = 1940 \text{ N}$$

Let us take the distance between the bearings of the worm shaft (x_1) equal to the diameter of the worm gear (D_G), *i.e.*

$$x_1 = D_G = 384 \text{ mm}$$

∴ Bending moment due to the radial force (W_R) in the vertical plane

$$= \frac{W_R \times x_1}{4} = \frac{1940 \times 384}{4} = 186\,240 \text{ N-mm}$$

and bending moment due to axial force (W_A) in the vertical plane

$$= \frac{W_A \times D_W}{4} = \frac{5328 \times 73.24}{4} = 97\,556 \text{ N-mm}$$

∴ Total bending moment in the vertical plane,

$$M_1 = 186\,240 + 97\,556 = 283\,796 \text{ N-mm}$$

We know that bending moment due to tangential force (W_T) in the horizontal plane,

$$M_2 = \frac{W_T \times D_G}{4} = \frac{2622 \times 384}{4} = 251\,712 \text{ N-mm}$$

∴ Resultant bending moment on the worm shaft,

$$M_{worm} = \sqrt{(M_1)^2 + (M_2)^2} = \sqrt{(283\,796)^2 + (251\,712)^2} = 379\,340 \text{ N-mm}$$

We know that equivalent twisting moment on the worm shaft,

$$T_{ew} = \sqrt{(T_{worm})^2 + (M_{worm})^2} = \sqrt{(96 \times 10^3)^2 + (379\ 340)^2} \text{ N-mm}$$

$$= 391\ 300 \text{ N-mm}$$

We also know that equivalent twisting moment (T_{ew}),

$$391\ 300 = \frac{\pi}{16} \times \tau\ (d_W)^3 = \frac{\pi}{16} \times 50\ (d_W)^3 = 9.82\ (d_W)^3$$

...(Taking τ = 50 MPa or N/mm^2)

$\therefore$ $(d_W)^3$ = 391 300 / 9.82 = 39 850 or d_W = 34.2 say 35 mm **Ans.**

Let us now check the maximum shear stress induced.

We know that the actual shear stress,

$$\tau = \frac{16\ T_{ew}}{\pi\ (d_W)^3} = \frac{16 \times 391\ 300}{\pi\ (35)^3} = 46.5 \text{ N/mm}^2$$

and direct compressive stress on the shaft due to the axial force,

$$\sigma_c = \frac{W_A}{\frac{\pi}{4}\ (d_W)^2} = \frac{5328}{\frac{\pi}{4}\ (35)^2} = 5.54 \text{ N/mm}^2$$

$\therefore$ Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\ \tau^2} = \frac{1}{2}\sqrt{(5.54)^2 + 4\ (46.5)^2} = 46.6 \text{ MPa}$$

Since the maximum shear stress induced is less than 50 MPa (assumed), therefore the design of worm shaft is satisfactory.

4. Design of worm gear shaft

Let d_G = Diameter of worm gear shaft.

We have calculated above that the axial force on the worm gear

= 2622 N

Tangential force on the worm gear

= 5328 N

and radial or separating force on the worm gear

= 1940 N

We know that bending moment due to the axial force on the worm gear

$$= \frac{\text{Axial force} \times D_G}{4} = \frac{2622 \times 384}{4} = 251\ 712 \text{ N-mm}$$

The bending moment due to the axial force will be in the vertical plane.

Let us take the distance between the bearings of the worm gear shaft (x_2) as 250 mm.

$\therefore$ Bending moment due to the radial force on the worm gear

$$= \frac{\text{Radial force} \times x_2}{4} = \frac{1940 \times 250}{4} = 121\ 250 \text{ N-mm}$$

The bending moment due to the radial force will also be in the vertical plane.

$\therefore$ Total bending moment in the vertical plane

$$M_3 = 251\ 712 + 121\ 250 = 372\ 962 \text{ N-mm}$$

We know that the bending moment due to the tangential force in the horizontal plane

$$M_4 = \frac{\text{Tangential force} \times x_2}{4} = \frac{5328 \times 250}{4} = 333\ 000 \text{ N-mm}$$

∴ Resultant bending moment on the worm gear shaft,

$$M_{gear} = \sqrt{(M_3)^2 + (M_4)^2} = \sqrt{(372\ 962)^2 + (333\ 000)^2} \text{ N-mm}$$

$$= 500 \times 10^3 \text{ N-mm}$$

We have already calculated that the torque acting on the worm gear shaft,

$$T_{gear} = 1023 \times 10^3 \text{ N-mm}$$

∴ Equivalent twisting moment on the worm gear shaft,

$$T_{eg} = \sqrt{(T_{gear})^2 + (M_{gear})^2} = \sqrt{(1023 \times 10^3)^2 + (500 \times 10^3)^2} \text{ N-mm}$$

$$= 1.14 \times 10^6 \text{ N-mm}$$

We know that equivalent twisting moment (T_{eg}),

$$1.14 \times 10^6 = \frac{\pi}{16} \times \tau\,(d_G)^3 = \frac{\pi}{16} \times 50\,(d_G)^3 = 9.82\,(d_G)^3$$

∴ $(d_G)^3 = 1.14 \times 10^6 / 9.82 = 109 \times 10^3$

or $d_G = 48.8$ say 50 mm **Ans.**

Let us now check the maximum shear stress induced.

We know that actual shear stress,

$$\tau = \frac{16\,T_{eg}}{\pi\,(d_G)^3} = \frac{16 \times 1.14 \times 10^6}{\pi\,(50)^3} = 46.4 \text{ N/mm}^2 = 46.4 \text{ MPa}$$

and direct compressive stress on the shaft due to the axial force,

$$\sigma_c = \frac{\text{Axial force}}{\frac{\pi}{4}(d_G)^2} = \frac{2622}{\frac{\pi}{4}(50)^2} = 1.33 \text{ N/mm}^3 = 1.33 \text{ MPa}$$

∴ Maximum shear stress,

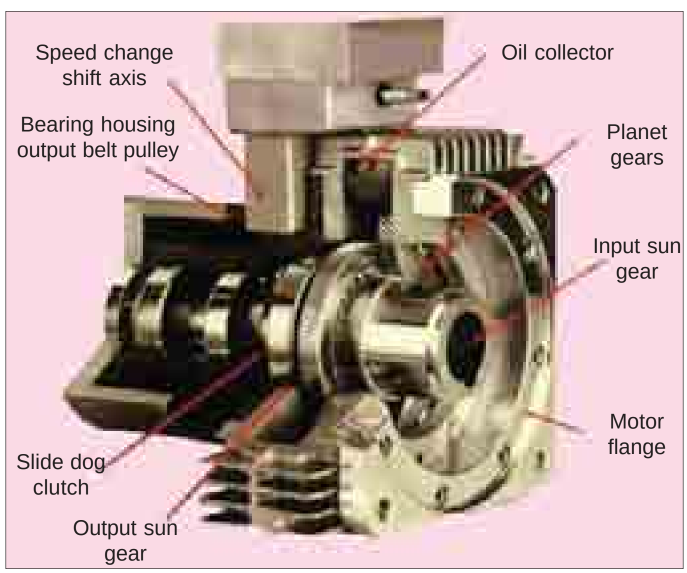
$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{(1.33)^2 + 4\,(46.4)^2} = 46.4 \text{ MPa}$$

Since the maximum shear stress induced is less than 50 MPa (assumed), therefore the design for worm gear shaft is satisfactory.

Example 31.4. *A speed reducer unit is to be designed for an input of 1.1 kW with a transmission ratio 27. The speed of the hardened steel worm is 1440 r.p.m. The worm wheel is to be made of phosphor bronze. The tooth form is to be 20° involute.*

Solution. Given : $P = 1.1$ kW $= 1100$ W ; $V.R. = 27$; $N_W = 1440$ r.p.m. ; $\phi = 20°$

A speed reducer unit (*i.e.*, worm and worm gear) may be designed as discussed below.



Sun and Planet gears.

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Since the centre distance between the shafts is not known, therefore let us assume that for this size unit, the centre distance (x) = 100 mm.

We know that pitch circle diameter of the worm,

$$D_W = \frac{(x)^{0.875}}{1.416} = \frac{(100)^{0.875}}{1.416} = 39.7 \text{ say } 40 \text{ mm}$$

∴ Pitch circle diameter of the worm gear,

$$D_G = 2x - D_W = 2 \times 100 - 40 = 160 \text{ mm}$$

From Table 31.2, we find that for the transmission ratio of 27, we shall use double start worms.

∴ Number of teeth on the worm gear,

$$T_G = 2 \times 27 = 54$$

We know that the axial pitch of the threads on the worm (p_a) is equal to circular pitch of teeth on the worm gear (p_c).

$$\therefore \quad p_a = p_c = \frac{\pi\, D_G}{T_G} = \frac{\pi \times 160}{54} = 9.3 \text{ mm}$$

and module,

$$m = \frac{p_c}{\pi} = \frac{9.3}{\pi} = 2.963 \text{ say } 3 \text{ mm}$$

∴ Actual circular pitch,

$$p_c = \pi\, m = \pi \times 3 = 9.426 \text{ mm}$$

Actual pitch circle diameter of the worm gear,

$$D_G = \frac{p_c\,.T_G}{\pi} = \frac{9.426 \times 54}{\pi} = 162 \text{ mm} \quad \textbf{Ans.}$$

and actual pitch circle diameter of the worm,

$$D_W = 2x - D_G = 2 \times 100 - 162 = 38 \text{ mm} \quad \textbf{Ans.}$$

The face width of the worm gear (b) may be taken as 0.73 times the pitch circle diameter of worm (D_W).

$$\therefore \quad b = 0.73\, D_W = 0.73 \times 38 = 27.7 \text{ say } 28 \text{ mm}$$

Let us now check the design from the standpoint of tangential load, dynamic load, static load or endurance strength, wear load and heat dissipation.

1. *Check for the tangential load*

Let N_G = Speed of the worm gear in r.p.m.

We know that velocity ratio of the drive,

$$V.R. = \frac{N_W}{N_G} \text{ or } N_G = \frac{N_W}{V.R.} = \frac{1440}{27} = 53.3 \text{ r.p.m}$$

∴ Peripheral velocity of the worm gear,

$$v = \frac{\pi\, D_G\, .\, N_G}{60} = \frac{\pi \times 0.162 \times 53.3}{60} = 0.452 \text{ m/s}$$

... (D_G is taken in metres)

and velocity factor,

$$C_v = \frac{6}{6 + v} = \frac{6}{6 + 0.452} = 0.93$$

We know that for 20° involute teeth, the tooth form factor,

$$y = 0.154 - \frac{0.912}{T_G} = 0.154 - \frac{0.912}{54} = 0.137$$

Top

From Table 31.4, we find that allowable static stress for phosphor bronze is

$$\sigma_o = 84 \text{ MPa or N/mm}^2$$

∴ Tangential load transmitted,

$$W_T = (\sigma_o \,.\, C_v)\, b \,.\, \pi\, m \,.\, y = (84 \times 0.93)\, 28 \times \pi \times 3 \times 0.137 \text{ N}$$
$$= 2825 \text{ N}$$

and power transmitted due to the tangential load,

$$P = W_T \times v = 2825 \times 0.452 = 1277 \text{ W} = 1.277 \text{ kW}$$

Since this power is more than the given power to be transmitted (1.1 kW), therefore the design is safe from the standpoint of tangential load.

2. Check for the dynamic load

We know that the dynamic load,

$$W_D = W_T / C_v = 2825 / 0.93 = 3038 \text{ N}$$

and power transmitted due to the dynamic laod,

$$P = W_D \times v = 3038 \times 0.452 = 1373 \text{ W} = 1.373 \text{ kW}$$

Since this power is more than the given power to be transmitted, therefore the design is safe from the standpoint of dynamic load.

3. Check for the static load or endurance strength

From Table 31.8, we find that the flexural endurance limit for phosphor bronze is

$$\sigma_e = 168 \text{ MPa or N/mm}^2$$

∴ Static load or endurance strength,

$$W_S = \sigma_e \,.\, b.\, \pi\, m \,.\, y = 168 \times 28 \times \pi \times 3 \times 0.137 = 6075 \text{ N}$$

and power transmitted due to the static load,

$$P = W_S \times v = 6075 \times 0.452 = 2746 \text{ W} = 2.746 \text{ kW}$$

Since this power is more than the power to be transmitted (1.1 kW), therefore the design is safe from the standpoint of static load.

4. Check for the wear load

From Table 31.5, we find that the load stress factor for hardened steel worm and phosphor bronze worm gear is

$$K = 0.55 \text{ N/mm}^2$$

∴ Limiting or maximum load for wear,

$$W_W = D_G \,.\, b \,.\, K = 162 \times 28 \times 0.55 = 2495 \text{ N}$$

and power transmitted due to the wear load,

$$P = W_W \times v = 2495 \times 0.452 = 1128 \text{ W} = 1.128 \text{ kW}$$

Since this power is more than the given power to be transmitted (1.1 kW), therefore the design is safe from the standpoint of wear.

5. Check for the heat dissipation

We know that permissible input power,

$$P = \frac{3650\,(x)^{1.7}}{V.R + 5} = \frac{3650\,(0.1)^{1.7}}{27 + 5} = 2.27 \text{ kW} \qquad \text{... (}x\text{ is taken in metres)}$$

Since this power is more than the given power to be transmitted (1.1 kW), therefore the design is safe from the standpoint of heat dissipation.

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EXERCISES

1. A double threaded worm drive is required for power transmission between two shafts having their axes at right angles to each other. The worm has 14½° involute teeth. The centre distance is approximately 200 mm. If the axial pitch of the worm is 30 mm and lead angle is 23°, find 1. lead; 2. pitch circle diameters of worm and worm gear; 3. helix angle of the worm; and 4. efficiency of the drive if the coefficient of friction is 0.05. **[Ans. 60 mm ; 45 mm ; 355 mm ; 67° ; 87.4%]**

The worm in its place. One can also see the two cubic worm bearing blocks and the big gear.

2. A double threaded worm drive has an axial pitch of 25 mm and a pitch circle diameter of 70 mm. The torque on the worm gear shaft is 1400 N-m. The pitch circle diameter of the worm gear is 250 mm and the tooth pressure angle is 25°. Find : 1. tangential force on the worm gear, 2. torque on the worm shaft, 3. separating force on the worm, 4. velocity ratio, and 5. efficiency of the drive, if the coefficient of friction between the worm thread and gear teeth is 0.04.
[Ans. 11.2 kN ; 88.97 N-m ; 5220 N ; 82.9%]
3. Design a speed reducer unit of worm and worm wheel for an input of 1 kW with a transmission ratio of 25. The speed of the worm is 1600 r.p.m. The worm is made of hardened steel and wheel of phosphor bronze for which the material combination factor is 0.7 N/mm^2. The static stress for the wheel material is 56 MPa. The worm is made of double start and the centre distance between the axes of the worm and wheel is 120 mm. The tooth form is to be 14½° involute. Check the design for strength, wear and heat dissipation.
4. Design worm and gear speed reducer to transmit 22 kW at a speed of 1440 r.p.m. The desired velocity ratio is 24 : 1. An efficiency of atleast 85% is desired. Assume that the worm is made of hardened steel and the gear of phosphor bronze.

QUESTIONS

1. Discuss, with neat sketches, the various types of worms and worm gears.
2. Define the following terms used in worm gearing :
(*a*) Lead; (*b*) Lead angle; (*c*) Normal pitch; and (*d*) Helix angle.
3. What are the various forces acting on worm and worm gears ?
4. Write the expression for centre distance in terms of axial lead, lead angle and velocity ratio.

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OBJECTIVE TYPE QUESTIONS

1. The worm gears are widely used for transmitting power at .... velocity ratios between non-intersecting shafts.
   (*a*) high (*b*) low
2. In worm gears, the angle between the tangent to the thread helix on the pitch cylinder and the plane normal to the axis of worm is called
   (*a*) pressure angle (*b*) lead angle
   (*c*) helix angle (*d*) friction angle
3. The normal lead, in a worm having multiple start threads, is given by
   (*a*) $l_N = l / \cos \lambda$ (*b*) $l_N = l \cdot \cos \lambda$
   (*c*) $l_N = l$ (*d*) $l_N = l \tan$
   where l_N = Normal lead,
   l = Lead, and
   λ = Lead angle.
4. The number of starts on the worm for a velocity ratio of 40 should be
   (*a*) single (*b*) double
   (*c*) triple (*d*) quadruple
5. The axial thrust on the worm (W_A) is given by
   (*a*) $W_A = W_T \cdot \tan \phi$ (*b*) $W_A = W_T / \tan \phi$
   (*c*) $W_A = W_T \cdot \tan \lambda$ (*d*) $W_A = W_T / \tan \lambda$
   where W_T = Tangential force acting on the worm,
   ϕ = Pressure angle, and
   λ = Lead angle.

ANSWERS

1. (*a*)	**2.** (*b*)	**3.** (*b*)	**4.** (*a*)	**5.** (*d*)

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CHAPTER

32

Internal Combustion Engine Parts

32.1 Introduction

As the name implies, the internal combustion engines (briefly written as I. C. engines) are those engines in which the combustion of fuel takes place inside the engine cylinder. The I.C. engines use either petrol or diesel as their fuel. In petrol engines (also called ***spark ignition engines*** or ***S.I engines***), the correct proportion of air and petrol is mixed in the carburettor and fed to engine cylinder where it is ignited by means of a spark produced at the spark plug. In diesel engines (also called ***compression ignition engines*** or ***C.I engines***), only air is supplied to the engine cylinder during suction stroke and it is compressed to a very high pressure, thereby raising its temperature from 600°C to 1000°C. The desired quantity of fuel (diesel) is now injected into the engine cylinder in the form of a very fine spray and gets ignited when comes in contact with the hot air.

The operating cycle of an I.C. engine may be completed either by the two strokes or four strokes of the

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piston. Thus, an engine which requires two strokes of the piston or one complete revolution of the crankshaft to complete the cycle, is known as ***two stroke engine.*** An engine which requires four strokes of the piston or two complete revolutions of the crankshaft to complete the cycle, is known as ***four stroke engine.***

The two stroke petrol engines are generally employed in very light vehicles such as scooters, motor cycles and three wheelers. The two stroke diesel engines are generally employed in marine propulsion.

The four stroke petrol engines are generally employed in light vehicles such as cars, jeeps and also in aeroplanes. The four stroke diesel engines are generally employed in heavy duty vehicles such as buses, trucks, tractors, diesel locomotive and in the earth moving machinery.

32.2 Principal Parts of an Engine

The principal parts of an I.C engine, as shown in Fig. 32.1 are as follows :

1. Cylinder and cylinder liner, 2. Piston, piston rings and piston pin or gudgeon pin, 3. Connecting rod with small and big end bearing, 4. Crank, crankshaft and crank pin, and 5. Valve gear mechanism.

The design of the above mentioned principal parts are discussed, in detail, in the following pages.

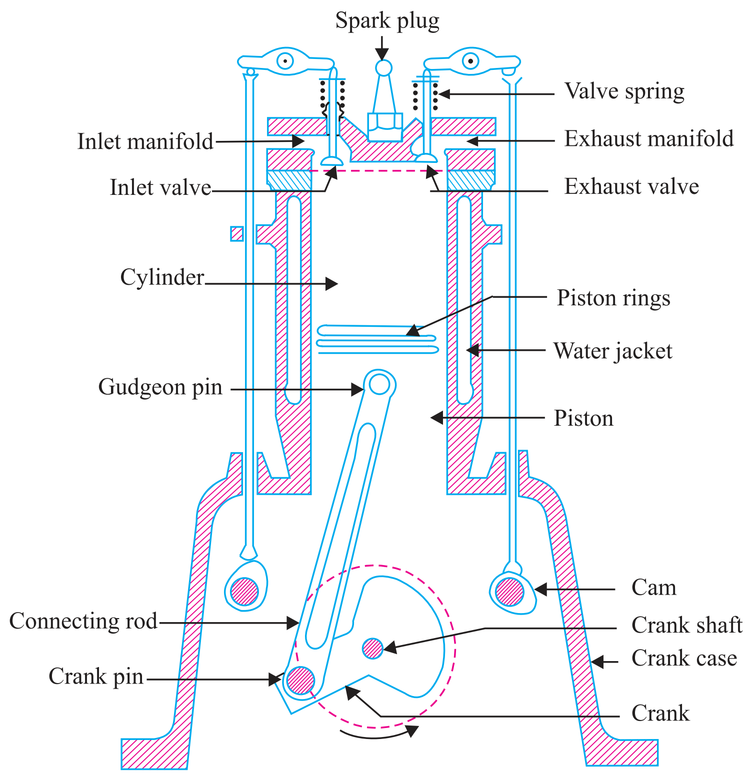

Fig. 32.1. Internal combustion engine parts.

32.3 Cylinder and Cylinder Liner

The function of a cylinder is to retain the working fluid and to guide the piston. The cylinders are usually made of cast iron or cast steel. Since the cylinder has to withstand high temperature due to the combustion of fuel, therefore, some arrangement must be provided to cool the cylinder. The single cylinder engines (such as scooters and motorcycles) are generally air cooled. They are provided with fins around the cylinder. The multi-cylinder engines (such as of cars) are provided with water jackets around the cylinders to cool it. In smaller engines. the cylinder, water jacket and the frame are

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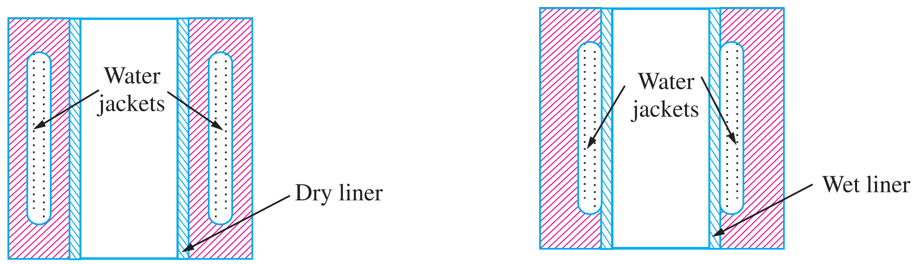
made as one piece, but for all the larger engines, these parts are manufactured separately. The cylinders are provided with cylinder liners so that in case of wear, they can be easily replaced. The cylinder liners are of the following two types :

1. Dry liner, and **2.** Wet liner.



(*a*) Dry liner. (*b*) Wet liner.

Fig. 32.2. Dry and wet liner.

A cylinder liner which does not have any direct contact with the engine cooling water, is known as ***dry liner,*** as shown in Fig. 32.2 (*a*). A cylinder liner which have its outer surface in direct contact with the engine cooling water, is known as ***wet liner,*** as shown in Fig. 32.2 (*b*).

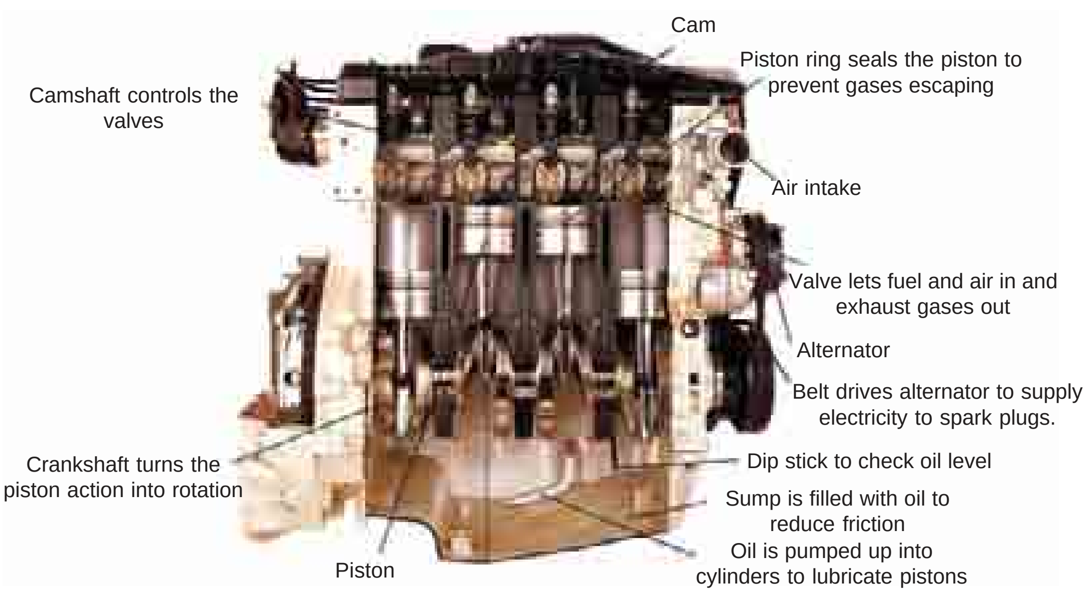
The cylinder liners are made from good quality close grained cast iron (*i.e.* pearlitic cast iron), nickel cast iron, nickel chromium cast iron. In some cases, nickel chromium cast steel with molybdenum may be used. The inner surface of the liner should be properly heat-treated in order to obtain a hard surface to reduce wear.

32.4 Design of a Cylinder

In designing a cylinder for an I. C. engine, it is required to determine the following values :

1. *Thickness of the cylinder wall.* The cylinder wall is subjected to gas pressure and the piston side thrust. The gas pressure produces the following two types of stresses :

(*a*) Longitudinal stress, and **(*b*)** Circumferential stress.



The above picture shows crankshaft, pistons and cylinder of a 4-stroke petrol engine.

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Since these two stressess act at right angles to each other, therefore, the net stress in each direction is reduced.

The piston side thrust tends to bend the cylinder wall, but the stress in the wall due to side thrust is very small and hence it may be neglected.

Let D_0 = Outside diameter of the cylinder in mm,

D = Inside diameter of the cylinder in mm,

p = Maximum pressure inside the engine cylinder in N/mm^2,

t = Thickness of the cylinder wall in mm, and

$1/m$ = Poisson's ratio. It is usually taken as 0.25.

The apparent longitudinal stress is given by

$$\sigma_l = \frac{\text{Force}}{\text{Area}} = \frac{\frac{\pi}{4} \times D^2 \times p}{\frac{\pi}{4}[(D_0)^2 - D^2]} = \frac{D^2 \cdot p}{(D_0)^2 - D^2}$$

and the apparent circumferential stresss is given by

$$\sigma_c = \frac{\text{Force}}{\text{Area}} = \frac{D \times l \times p}{2t \times l} = \frac{D \times p}{2t}$$

... (where l is the length of the cylinder and area is the projected area)

$\therefore$ Net longitudinal stress $= \sigma_l - \dfrac{\sigma_c}{m}$

and net circumferential stress $= \sigma_c - \dfrac{\sigma_l}{m}$

The thickness of a cylinder wall (t) is usually obtained by using a thin cylindrical formula, *i.e.*,

$$t = \frac{p \times D}{2\sigma_c} + C$$

where p = Maximum pressure inside the cylinder in N/mm^2,

D = Inside diameter of the cylinder or cylinder bore in mm,

σ_c = Permissible circumferential or hoop stress for the cylinder material in MPa or N/mm^2. Its value may be taken from 35 MPa to 100 MPa depending upon the size and material of the cylinder.

C = Allowance for reboring.

The allowance for reboring (C) depending upon the cylinder bore (D) for I. C. engines is given in the following table :

Table 32.1. Allowance for reboring for I. C. engine cylinders.

D (mm)	75	100	150	200	250	300	350	400	450	500
C (mm)	1.5	2.4	4.0	6.3	8.0	9.5	11.0	12.5	12.5	12.5

The thickness of the cylinder wall usually varies from 4.5 mm to 25 mm or more depending upon the size of the cylinder. The thickness of the cylinder wall (t) may also be obtained from the following empirical relation, *i.e.*

$$t = 0.045\,D + 1.6 \text{ mm}$$

The other empirical relations are as follows :

Thickness of the dry liner

$$= 0.03\,D \text{ to } 0.035\,D$$

Thickness of the water jacket wall

= 0.032 D + 1.6 mm or t / 3 m for bigger cylinders and 3t /4 for smaller cylinders

Water space between the outer cylinder wall and inner jacket wall

= 10 mm for a 75 mm cylinder to 75 mm for a 750 mm cylinder

or 0.08 D + 6.5 mm

2. *Bore and length of the cylinder.* The bore (*i.e.* inner diameter) and length of the cylinder may be determined as discussed below :

Let p_m = Indicated mean effective pressure in N/mm^2,

D = Cylinder bore in mm,

A = Cross-sectional area of the cylinder in mm^2,

= $\pi D^2/4$

l = Length of stroke in metres,

N = Speed of the engine in r.p.m., and

n = Number of working strokes per min

= N, for two stroke engine

= N/2, for four stroke engine.

We know that the power produced inside the engine cylinder, *i.e.* indicated power,

$$I.P. = \frac{p_m \times l \times A \times n}{60} \text{ watts}$$

From this expression, the bore (D) and length of stroke (l) is determined. The length of stroke is generally taken as 1.25 D to 2D.

Since there is a clearance on both sides of the cylinder, therefore length of the cylinder is taken as 15 percent greater than the length of stroke. In other words,

Length of the cylinder, L = 1.15 × Length of stroke = 1.15 l

Notes : (*a*) If the power developed at the crankshaft, *i.e.* brake power (*B. P.*) and the mechanical efficiency (η_m) of the engine is known, then

$$I.P. = \frac{B.P.}{\eta_m}$$

(*b*) The maximum gas pressure (p) may be taken as 9 to 10 times the mean effective pressure (p_m).

3. *Cylinder flange and studs.* The cylinders are cast integral with the upper half of the crank-case or they are attached to the crankcase by means of a flange with studs or bolts and nuts. The cylinder flange is integral with the cylinder and should be made thicker than the cylinder wall. The flange thickness should be taken as 1.2 t to 1.4 t, where t is the thickness of cylinder wall.

The diameter of the studs or bolts may be obtained by equating the gas load due to the maximum pressure in the cylinder to the resisting force offered by all the studs or bolts. Mathematically,

$$\frac{\pi}{4} \times D^2 . p = n_s \times \frac{\pi}{4}(d_c)^2 \sigma_t$$

where D = Cylinder bore in mm,

p = Maximum pressure in N/mm^2,

n_s = Number of studs. It may be taken as 0.01 D + 4 to 0.02 D + 4

d_c = Core or minor diameter, *i.e.* diameter at the root of the thread in mm,

σ_t = Allowable tensile stress for the material of studs or bolts in MPa or N/mm^2. It may be taken as 35 to 70 MPa.

The nominal or major diameter of the stud or bolt (d) usually lies between 0.75 t_f to t_f, where t_f is the thickness of flange. In no case, a stud or bolt less than 16 mm diameter should be used.

The distance of the flange from the centre of the hole for the stud or bolt should not be less than d + 6 mm and not more than 1.5 d, where d is the nominal diameter of the stud or bolt.

In order to make a leak proof joint, the pitch of the studs or bolts should lie between $19\sqrt{d}$ to $28.5\sqrt{d}$, where d is in mm.

4. *Cylinder head.* Usually, a separate cylinder head or cover is provided with most of the engines. It is, usually, made of box type section of considerable depth to accommodate ports for air and gas passages, inlet valve, exhaust valve and spark plug (in case of petrol engines) or atomiser at the centre of the cover (in case of diesel engines).

The cylinder head may be approximately taken as a flat circular plate whose thickness (t_h) may be determined from the following relation :

$$t_h = D\sqrt{\frac{C \cdot p}{\sigma_c}}$$

where D = Cylinder bore in mm,

p = Maximum pressure inside the cylinder in N/mm^2,

σ_c = Allowable circumferential stress in MPa or N/mm^2. It may be taken as 30 to 50 MPa, and

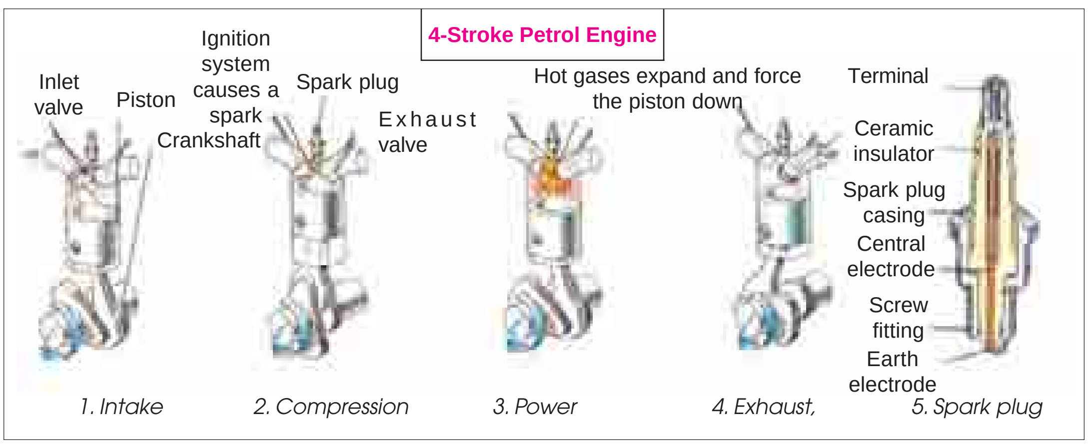
C = Constant whose value is taken as 0.1.

The studs or bolts are screwed up tightly alongwith a metal gasket or asbestos packing to provide a leak proof joint between the cylinder and cylinder head. The tightness of the joint also depends upon the pitch of the bolts or studs, which should lie between $19\sqrt{d}$ to $28.5\sqrt{d}$. The pitch circle diameter (D_p) is usually taken as D + 3d. The studs or bolts are designed in the same way as discussed above.

Example 32.1. *A four stroke diesel engine has the following specifications :*

Brake power = 5 kW ; Speed = 1200 r.p.m. ; Indicated mean effective pressure = 0.35 N / mm^2 ; Mechanical efficiency = 80 %.

Determine : 1. bore and length of the cylinder ; 2. thickness of the cylinder head ; and 3. size of studs for the cylinder head.



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Solution. Given: $B.P.$ = 5kW = 5000 W ; N = 1200 r.p.m. or $n = N/2 = 600$; $p_m = 0.35$ N/mm^2; η_m = 80% = 0.8

1. *Bore and length of cylinder*

Let D = Bore of the cylinder in mm,

A = Cross-sectional area of the cylinder = $\frac{\pi}{4} \times D^2$ mm^2

l = Length of the stroke in m.

= 1.5 D mm = 1.5 D / 1000 m ....(Assume)

We know that the indicated power,

$$I.P = B.P./\eta_m = 5000 / 0.8 = 6250 \text{ W}$$

We also know that the indicated power ($I.P.$),

$$6250 = \frac{p_m . l . A . n}{60} = \frac{0.35 \times 1.5 D \times \pi D^2 \times 600}{60 \times 1000 \times 4} = 4.12 \times 10^{-3} D^3$$

...($\because$ For four stroke engine, $n = N/2$)

$\therefore$ $D^3 = 6250 / 4.12 \times 10^{-3} = 1517 \times 10^3$ or D = 115 mm **Ans.**

and $l = 1.5 D = 1.5 \times 115 = 172.5$ mm

Taking a clearance on both sides of the cylinder equal to 15% of the stroke, therefore length of the cylinder,

$$L = 1.15\, l = 1.15 \times 172.5 = 198 \text{ say } 200 \text{ mm}$$ **Ans.**

2. *Thickness of the cylinder head*

Since the maximum pressure (p) in the engine cylinder is taken as 9 to 10 times the mean effective pressure (p_m), therefore let us take

$$p = 9\, p_m = 9 \times 0.35 = 3.15 \text{ N/mm}^2$$

We know that thickness of the cyclinder head,

$$t_h = D\sqrt{\frac{C.p}{\sigma_t}} = 115\sqrt{\frac{0.1 \times 3.15}{42}} = 9.96 \text{ say } 10 \text{ mm}$$ **Ans.**

...(Taking C = 0.1 and σ_t = 42 MPa = 42 N/mm^2)

3. *Size of studs for the cylinder head*

Let d = Nominal diameter of the stud in mm,

d_c = Core diameter of the stud in mm. It is usually taken as 0.84 d.

σ_t = Tensile stress for the material of the stud which is usually nickel steel.

n_s = Number of studs.

We know that the force acting on the cylinder head (or on the studs)

$$= \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4}(115)^2 3.15 = 32\ 702 \text{ N} \quad ...(i)$$

The number of studs (n_s) are usually taken between 0.01 D + 4 (*i.e.* 0.01 × 115 + 4 = 5.15) and 0.02 D + 4 (*i.e.* 0.02 × 115 + 4 = 6.3). Let us take n_s = 6.

We know that resisting force offered by all the studs

$$= n_s \times \frac{\pi}{4}(d_c)^2\, \sigma_t = 6 \times \frac{\pi}{4}(0.84d)^2\, 65 = 216\, d^2 \text{N} \quad ...(ii)$$

...(Taking σ_t = 65 MPa = 65 N/mm^2)

From equations (*i*) and (*ii*),

$$d^2 = 32\ 702 / 216 = 151 \text{ or } d = 12.3 \text{ say } 14 \text{ mm}$$

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The pitch circle diameter of the studs (D_p) is taken $D + 3d$.

$\therefore \quad D_p = 115 + 3 \times 14 = 157$ mm

We know that pitch of the studs

$$= \frac{\pi \times D_p}{n_s} = \frac{\pi \times 157}{6} = 82.2 \text{ mm}$$

We know that for a leak-proof joint, the pitch of the studs should lie between $19\sqrt{d}$ to $28.5\sqrt{d}$, where d is the nominal diameter of the stud.

$\therefore$ Minimum pitch of the studs

$$= 19\sqrt{d} = 19\sqrt{14} = 71.1 \text{ mm}$$

and maximum pitch of the studs

$$= 28.5\sqrt{d} = 28.5\sqrt{14} = 106.6 \text{ mm}$$

Since the pitch of the studs obtained above (*i.e.* 82.2 mm) lies within 71.1 mm and 106.6 mm, therefore, size of the stud (d) calculated above is satisfactory.

$\therefore \quad d = 14$ mm **Ans.**

32.5 Piston

The piston is a disc which reciprocates within a cylinder. It is either moved by the fluid or it moves the fluid which enters the cylinder. The main function of the piston of an internal combustion engine is to receive the impulse from the expanding gas and to transmit the energy to the crankshaft through the connecting rod. The piston must also disperse a large amount of heat from the combustion chamber to the cylinder walls.

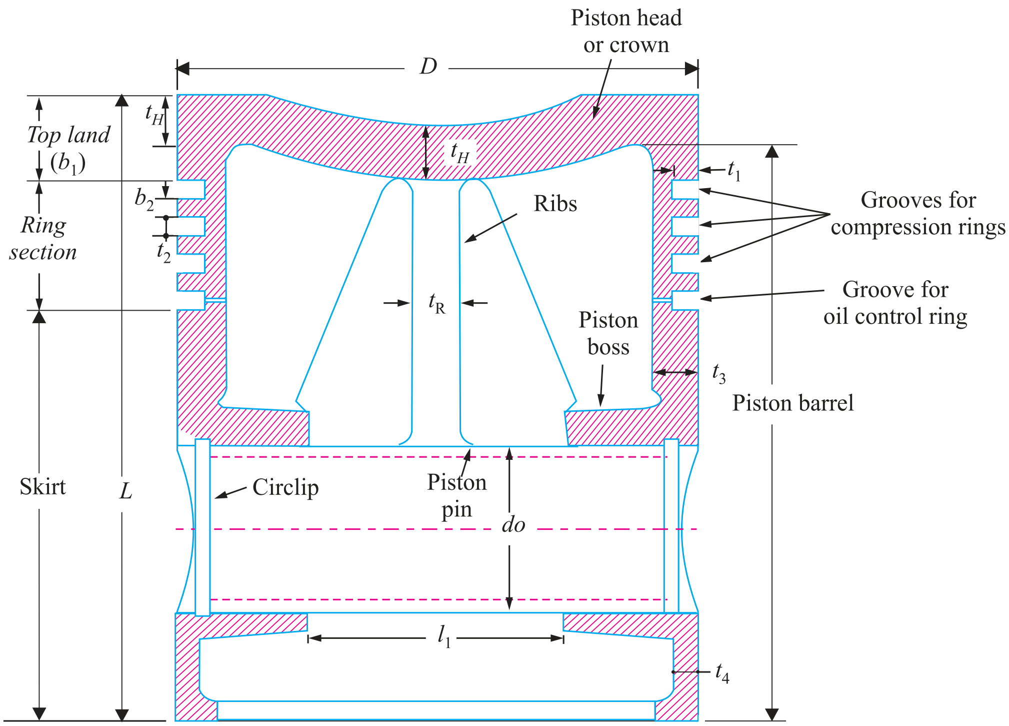

Fig. 32.3. Piston for I.C. engines (Trunk type).

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The piston of internal combustion engines are usually of trunk type as shown in Fig. 32.3. Such pistons are open at one end and consists of the following parts :

1. *Head or crown.* The piston head or crown may be flat, convex or concave depending upon the design of combustion chamber. It withstands the pressure of gas in the cylinder.

2. *Piston rings.* The piston rings are used to seal the cyliner in order to prevent leakage of the gas past the piston.

3. *Skirt.* The skirt acts as a bearing for the side thrust of the connecting rod on the walls of cylinder.

4. *Piston pin.* It is also called ***gudgeon pin*** or ***wrist pin.*** It is used to connect the piston to the connecting rod.

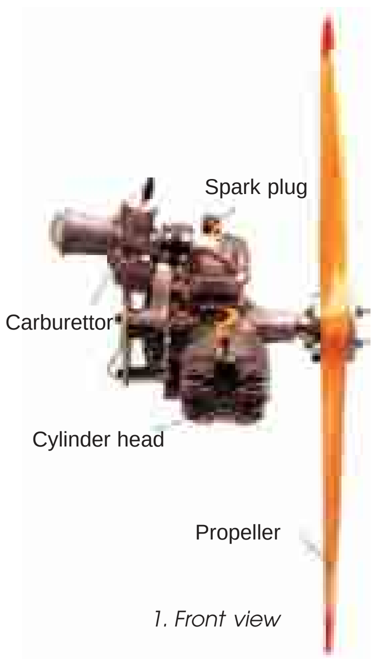
32.6 Design Considerations for a Piston

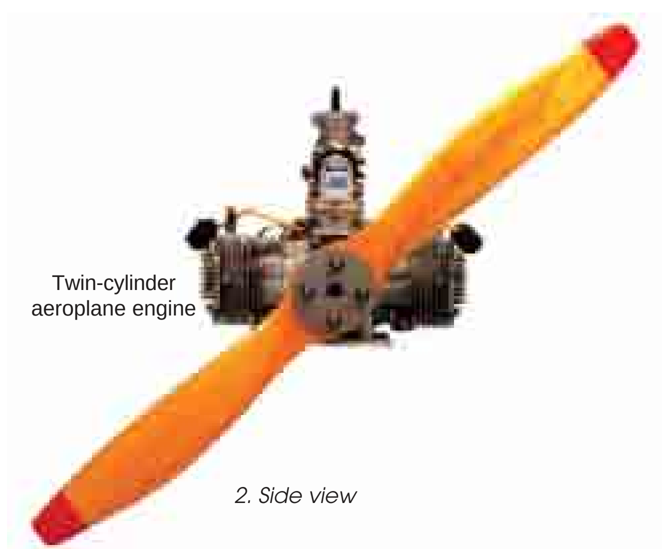
In designing a piston for I.C. engine, the following points should be taken into consideration :

1. It should have enormous strength to withstand the high gas pressure and inertia forces.
2. It should have minimum mass to minimise the inertia forces.
3. It should form an effective gas and oil sealing of the cylinder.
4. It should provide sufficient bearing area to prevent undue wear.
5. It should disprese the heat of combustion quickly to the cylinder walls.
6. It should have high speed reciprocation without noise.
7. It should be of sufficient rigid construction to withstand thermal and mechanical distortion.
8. It should have sufficient support for the piston pin.

32.7 Material for Pistons

The most commonly used materials for pistons of I.C. engines are cast iron, cast aluminium, forged aluminium, cast steel and forged steel. The cast iron pistons are used for moderately rated





Twin cylinder airplane engine of 1930s.

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engines with piston speeds below 6 m / s and aluminium alloy pistons are used for highly rated engines running at higher piston sppeds. It may be noted that

1. Since the *coefficient of thermal expansion for aluminium is about 2.5 times that of cast iron, therefore, a greater clearance must be provided between the piston and the cylinder wall (than with cast iron piston) in order to prevent siezing of the piston when engine runs continuously under heavy loads. But if excessive clearance is allowed, then the piston will develop ***'piston slap'*** while it is cold and this tendency increases with wear. The less clearance between the piston and the cylinder wall will lead to siezing of piston.

2. Since the aluminium alloys used for pistons have high **heat conductivity (nearly four times that of cast iron), therefore, these pistons ensure high rate of heat transfer and thus keeps down the maximum temperature difference between the centre and edges of the piston head or crown.

Notes: (*a*) For a cast iron piston, the temperature at the centre of the piston head (T_C) is about 425°C to 450°C under full load conditions and the temperature at the edges of the piston head (T_E) is about 200°C to 225°C.

(*b*) For aluminium alloy pistons, T_C is about 260°C to 290°C and T_E is about 185°C to 215°C.

3. Since the aluminium alloys are about ***three times lighter than cast iron, therfore, its mechanical strength is good at low temperatures, but they lose their strength (about 50%) at temperatures above 325°C. Sometimes, the pistons of aluminium alloys are coated with aluminium oxide by an electrical method.

32.8 Piston Head or Crown

The piston head or crown is designed keeping in view the following two main considerations, *i.e.*

1. It should have adequate strength to withstand the straining action due to pressure of explosion inside the engine cylinder, and

2. It should dissipate the heat of combustion to the cylinder walls as quickly as possible.

On the basis of first consideration of straining action, the thickness of the piston head is determined by treating it as a flat circular plate of uniform thickness, fixed at the outer edges and subjected to a uniformly distributed load due to the gas pressure over the entire cross-section.

The thickness of the piston head (t_H), according to Grashoff's formula is given by

$$t_H = \sqrt{\frac{3p.D^2}{16\sigma_t}} \text{ (in mm)} \qquad ...(i)$$

where p = Maximum gas pressure or explosion pressure in N/mm^2,

D = Cylinder bore or outside diameter of the piston in mm, and

σ_t = Permissible bending (tensile) stress for the material of the piston in MPa or N/mm^2. It may be taken as 35 to 40 MPa for grey cast iron, 50 to 90 MPa for nickel cast iron and aluminium alloy and 60 to 100 MPa for forged steel.

On the basis of second consideration of heat transfer, the thickness of the piston head should be such that the heat absorbed by the piston due combustion of fuel is quickly transferred to the cylinder walls. Treating the piston head as a flat ciucular plate, its thickness is given by

$$t_H = \frac{H}{12.56k(T_C - T_E)} \text{ (in mm)} \qquad ...(ii)$$

* The coefficient of thermal expansion for aluminium is 0.24×10^{-6} m / °C and for cast iron it is 0.1×10^{-6} m / °C.

** The heat conductivity for aluminium is 174.75 W/m/°C and for cast iron it is 46.6 W/m /°C.

*** The density of aluminium is 2700 kg / m^3 and for cast iron it is 7200 kg / m^3.

where H = Heat flowing through the piston head in kJ/s or watts,

k = Heat conductivity factor in W/m/°C. Its value is 46.6 W/m/°C for grey cast iron, 51.25 W/m/°C for steel and 174.75 W/m/°C for aluminium alloys.

T_C = Temperture at the centre of the piston head in °C, and

T_E = Temperature at the edges of the piston head in °C.

The temperature difference ($T_C - T_E$) may be taken as 220°C for cast iron and 75°C for aluminium.

The heat flowing through the positon head (H) may be determined by the following expression, *i.e.*,

$$H = C \times HCV \times m \times B.P. \text{ (in kW)}$$

where C = Constant representing that portion of the heat supplied to the engine which is absorbed by the piston. Its value is usually taken as 0.05.

HCV = Higher calorific value of the fuel in kJ/kg. It may be taken as 45×10^3 kJ/kg for diesel and 47×10^3 kJ/ kg for petrol,

m = Mass of the fuel used in kg per brake power per second, and

$B.P.$ = Brake power of the engine per cylinder

Notes : 1. The thickness of the piston head (t_H) is calculated by using equations ***(i)*** and ***(ii)*** and larger of the two values obtained should be adopted.

2. When t_H is 6 mm or less, then no ribs are required to strengthen the piston head against gas loads. But when t_H is greater then 6 mm, then a suitable number of ribs at the centre line of the boss extending around the skirt should be provided to distribute the side thrust from the connecting rod and thus to prevent distortion of the skirt. The thickness of the ribs may be takes as $t_H / 3$ to $t_H / 2$.

3. For engines having length of stroke to cylinder bore (L / D) ratio upto 1.5, a cup is provided in the top of the piston head with a radius equal to 0.7 D. This is done to provide a space for combustion chamber.

32.9 Piston Rings

The piston rings are used to impart the necessary radial pressure to maintain the seal between the piston and the cylinder bore. These are usually made of grey cast iron or alloy cast iron because of their good wearing properties and also they retain spring characteristics even at high temperatures. The piston rings are of the following two types :

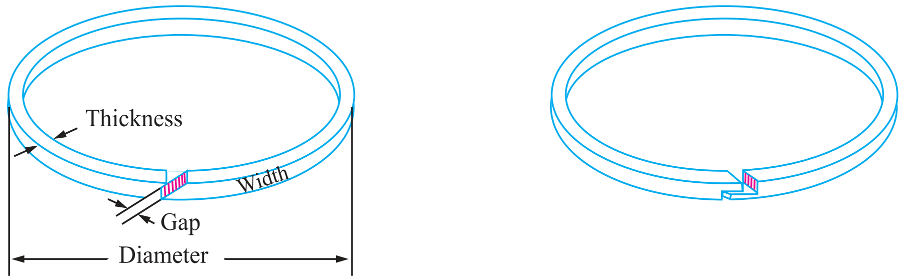
1. Compression rings or pressure rings, and

2. Oil control rings or oil scraper.

The ***compression rings*** *or* ***pressure rings*** are inserted in the grooves at the top portion of the piston and may be three to seven in number. These rings also transfer heat from the piston to the cylinder liner and absorb some part of the piston fluctuation due to the side thrust.

The ***oil control rings*** or ***oil scrapers*** are provided below the compression rings. These rings provide proper lubrication to the liner by allowing sufficient oil to move up during upward stroke and at the same time scraps the lubricating oil from the surface of the liner in order to minimise the flow of the oil to the combustion chamber.

The compression rings are usually made of rectangular cross-section and the diameter of the ring is slightly larger than the cylinder bore. A part of the ring is cut- off in order to permit it to go into the cylinder against the liner wall. The diagonal cut or step cut ends, as shown in Fig. 32.4 (a) and (b) respectively, may be used. The gap between the ends should be sufficiently large when the ring is put cold so that even at the highest temperature, the ends do not touch each other when the ring expands, otherwise there might be buckling of the ring.



(*a*) Diagonal cut. (*b*) Step cut.

Fig. 32.4. Piston rings.

The radial thickness (t_1) of the ring may be obtained by considering the radial pressure between the cylinder wall and the ring. From bending stress consideration in the ring, the radial thickness is given by

$$t_1 = D\sqrt{\frac{3p_w}{\sigma_t}}$$

where D = Cylinder bore in mm,

p_w = Pressure of gas on the cylinder wall in N/mm². Its value is limited from 0.025 N/mm² to 0.042 N/mm², and

σ_t = Allowable bending (tensile) stress in MPa. Its value may be taken from 85 MPa to 110 MPa for cast iron rings.

The axial thickness (t_2) of the rings may be taken as 0.7 t_1 to t_1.

The minimum axial thickness (t_2) may also be obtained from the following empirical relation:

$$t_2 = \frac{D}{10 n_R}$$

where n_R = Number of rings.

The width of the top land (*i.e.* the distance from the top of the piston to the first ring groove) is made larger than other ring lands to protect the top ring from high temperature conditions existing at the top of the piston,

∴ Width of top land,

$$b_1 = t_H \text{ to } 1.2\, t_H$$

The width of other ring lands (*i.e.* the distance between the ring grooves) in the piston may be made equal to or slightly less than the axial thickness of the ring (t_2).

∴ Width of other ring lands,

$$b_2 = 0.75\, t_2 \text{ to } t_2$$

The depth of the ring grooves should be more than the depth of the ring so that the ring does not take any piston side thrust.

The gap between the free ends of the ring is given by 3.5 t_1 to 4 t_1. The gap, when the ring is in the cylinder, should be 0.002 D to 0.004 D.

32.10 Piston Barrel

It is a cylindrical portion of the piston. The maximum thickness (t_3) of the piston barrel may be obtained from the following empirical relation :

$$t_3 = 0.03\, D + b + 4.5 \text{ mm}$$

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where b = Radial depth of piston ring groove which is taken as 0.4 mm larger than the radial thickness of the piston ring (t_1)

$= t_1 + 0.4$ mm

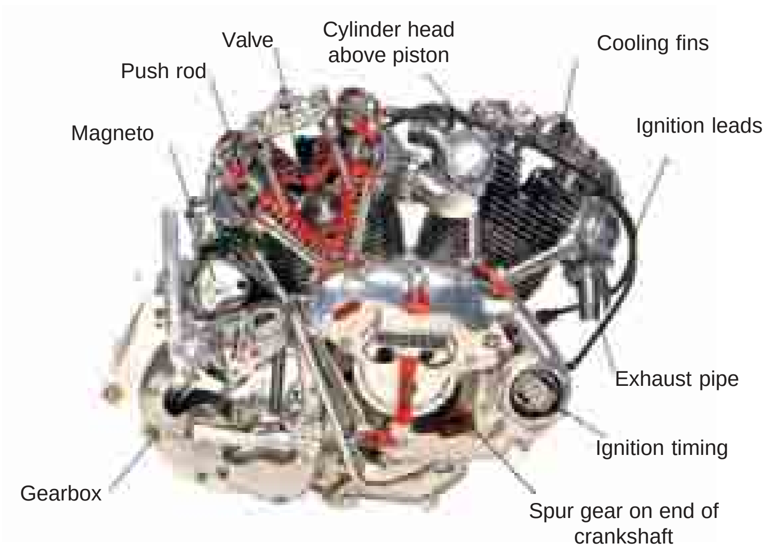
Thus, the above relation may be written as

$$t_3 = 0.03\,D + t_1 + 4.9 \text{ mm}$$

The piston wall thickness (t_4) towards the open end is decreased and should be taken as 0.25 t_3 to 0.35 t_3.

32.11 Piston Skirt

The portion of the piston below the ring section is known as ***piston skirt.*** In acts as a bearing for the side thrust of the connecting rod. The length of the piston skirt should be such that the bearing pressure on the piston barrel due to the side thrust does not exceed 0.25 N/mm^2 of the projected area for low speed engines and 0.5 N/mm^2 for high speed engines. It may be noted that the maximum thrust will be during the expansion stroke. The side thrust (R) on the cylinder liner is usually taken as 1/10 of the maximum gas load on the piston.



1000 cc twin -cylinder motorcycle engine.

We know that maximum gas load on the piston,

$$P = p \times \frac{\pi D^2}{4}$$

∴ Maximum side thrust on the cylinder,

$$R = P/10 = 0.1\,p \times \frac{\pi D^2}{4} \quad \text{...}(i)$$

where p = Maximum gas pressure in N/mm^2, and

D = Cylinder bore in mm.

The side thrust (R) is also given by

R = Bearing pressure × Projected bearing area of the piston skirt

$= p_b \times D \times l$

where l = Length of the piston skirt in mm. ...(*ii*)

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From equations *(i)* and *(ii)*, the length of the piston skirt (l) is determined. In actual practice, the length of the piston skirt is taken as 0.65 to 0.8 times the cylinder bore. Now the total length of the piston (L) is given by

$$L = \text{Length of skirt + Length of ring section + Top land}$$

The length of the piston usually varies between D and 1.5 D. It may be noted that a longer piston provides better bearing surface for quiet running of the engine, but it should not be made unnecessarily long as it will increase its own mass and thus the inertia forces.

32.12 Piston Pin

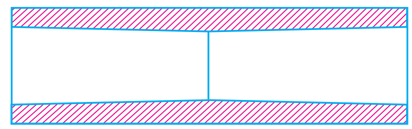


Fig.32.5. Piston pin.

The piston pin (also called gudgeon pin or wrist pin) is used to connect the piston and the connecting rod. It is usually made hollow and tapered on the inside, the smallest inside diameter being at the centre of the pin, as shown in Fig. 32.5. The piston pin passes through the bosses provided on the inside of the piston skirt and the bush of the small end of the connecting rod. The centre of piston pin should be 0.02 D to 0.04 D above the centre of the skirt, in order to off-set the turning effect of the friction and to obtain uniform distribution of pressure between the piston and the cylinder liner.

The material used for the piston pin is usually case hardened steel alloy containing nickel, chromium, molybdenum or vanadium having tensile strength from 710 MPa to 910 MPa.

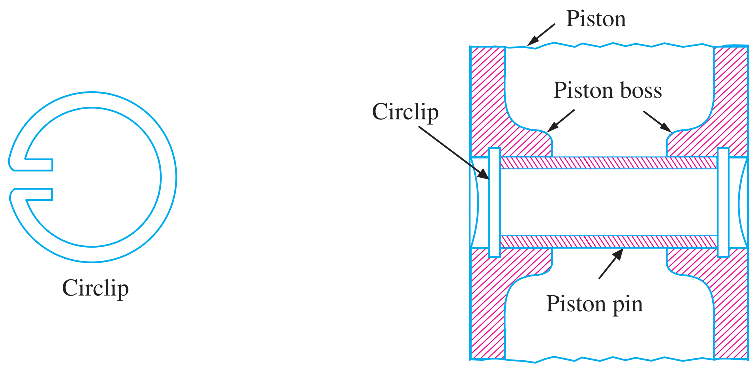


Fig. 32.6. Full floating type piston pin.

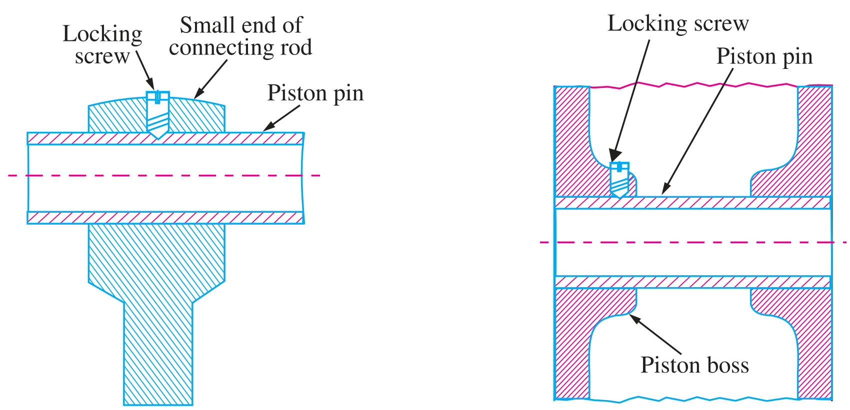
The connection between the piston pin and the small end of the connecting rod may be made either ***full floating type*** or ***semi-floating type.*** In the full floating type, the piston pin is free to turn both in the *piston bosses and the bush of the small end of the connecting rod. The end movements of the piston pin should be secured by means of spring circlips, as shown in Fig. 32.6, in order to prevent the pin from touching and scoring the cylinder liner.

In the semi-floating type, the piston pin is either free to turn in the piston bosses and rigidly secured to the small end of the connecting rod, or it is free to turn in the bush of the small end of the connecting rod and is rigidly secured in the piston bosses by means of a screw, as shown in Fig. 32.7

The piston pin should be designed for the maximum gas load or the inertia force of the piston, whichever is larger. The bearing area of the piston pin should be about equally divided between the piston pin bosses and the connecting rod bushing. Thus, the length of the pin in the connecting rod bushing will be about 0.45 of the cylinder bore or piston diameter (D), allowing for the end clearance

* The mean diameter of the piston bosses is made 1.4 d_0 for cast iron pistons and 1.5 d_0 for aluminium pistons, where d_0 is the outside diameter of the piston pin. The piston bosses are usually tapered, increasing the diameter towards the piston wall.

of the pin etc. The outside diameter of the piston pin (d_0) is determined by equating the load on the piston due to gas pressure (p) and the load on the piston pin due to bearing pressure (p_{b1}) at the small end of the connecting rod bushing.



(*a*) Piston pin secured to the small end of the connecting rod.

(*b*) Piston pin secured to the boss of the piston.

Fig. 32.7. Semi-floating type piston pin.

Let d_0 = Outside diameter of the piston pin in mm

l_1 = Length of the piston pin in the bush of the small end of the connecting rod in mm. Its value is usually taken as 0.45 *D*.

p_{b1} = Bearing pressure at the small end of the connecting rod bushing in N/mm^2. Its value for the bronze bushing may be taken as 25 N/mm^2.

We know that load on the piston due to gas pressure or gas load

$$= \frac{\pi D^2}{4} \times p \qquad ..(i)$$

and load on the piston pin due to bearing pressure or bearing load

$$= \text{Bearing pressure} \times \text{Bearing area} = p_{b1} \times d_0 \times l_1 \qquad ...(ii)$$

From equations (*i*) and (*ii*), the outside diameter of the piston pin (d_0) may be obtained.

The piston pin may be checked in bending by assuming the gas load to be uniformly distributed over the length l_1 with supports at the centre of the bosses at the two ends. From Fig. 32.8, we find that the length between the supports,

$$l_2 = l_1 + \frac{D - l_1}{2} = \frac{l_1 + D}{2}$$

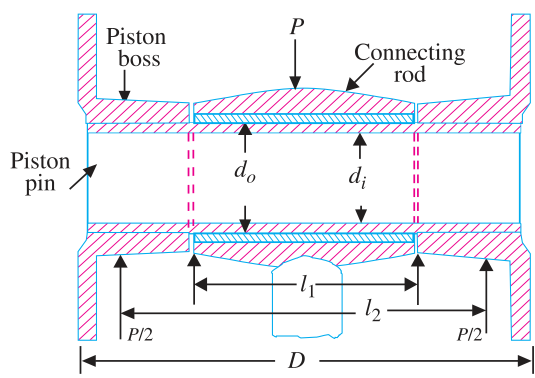


Fig. 32.8

Now maximum bending moment at the centre of the pin,

$$M = \frac{P}{2} \times \frac{l_2}{2} - \frac{P}{l_1} \times \frac{l_1}{2} \times \frac{l_1}{4}$$

$$= \frac{P}{2} \times \frac{l_2}{2} - \frac{P}{2} \times \frac{l_1}{4}$$

$$= \frac{P}{2}\left(\frac{l_1 + D}{2 \times 2}\right) - \frac{P}{2} \times \frac{l_1}{4}$$

$$= \frac{P.\,l_1}{8} + \frac{P.D}{8} - \frac{P.\,l_1}{8} = \frac{P.D}{8} \qquad ...(iii)$$

We have already discussed that the piston pin is made hollow. Let d_0 and d_i be the outside and inside diameters of the piston pin. We know that the section modulus,

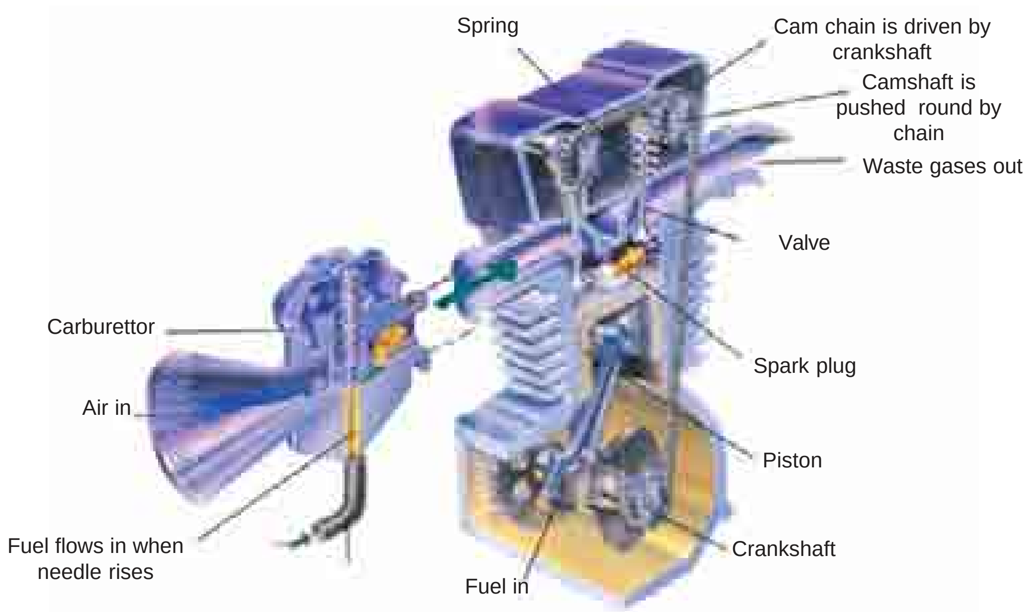
$$Z = \frac{\pi}{32}\left[\frac{(d_0)^4 - (d_i)^4}{d_0}\right]$$

We know that maximum bending moment,

$$M = Z \times \sigma_b = \frac{\pi}{32}\left[\frac{(d_0)^4 - (d_i)^4}{d_0}\right]\sigma_b$$

where σ_b = Allowable bending stress for the material of the piston pin. It is usually taken as 84 MPa for case hardened carbon steel and 140 MPa for heat treated alloy steel.

Assuming $d_i = 0.6\ d_0$, the induced bending stress in the piston pin may be checked.



Another view of a single cylinder 4-stroke petrol engine.

Example 32.2. *Design a cast iron piston for a single acting four stroke engine for the following data:*

Cylinder bore = 100 mm ; Stroke = 125 mm ; Maximum gas pressure = 5 N/mm² ; Indicated mean effective pressure = 0.75 N/mm² ; Mechanical efficiency = 80% ; Fuel consumption = 0.15 kg per brake power per hour ; Higher calorific value of fuel = 42 × 10³ kJ/kg ; Speed = 2000 r.p.m.

Any other data required for the design may be assumed.

Solution. Given : D = 100 mm ; L = 125 mm = 0.125 m ; p = 5 N/mm² ; p_m = 0.75 N/mm²; η_m = 80% = 0.8 ; m = 0.15 kg / BP / h = 41.7 × 10^{-6} kg / BP / s; HCV = 42 × 10^3 kJ / kg ; N = 2000 r.p.m.

The dimensions for various components of the piston are determined as follows :

1. *Piston head or crown*

The thickness of the piston head or crown is determined on the basis of strength as well as on the basis of heat dissipation and the larger of the two values is adopted.

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We know that the thickness of piston head on the basis of strength,

$$t_H = \sqrt{\frac{3p.D^2}{16\,\sigma_t}} = \sqrt{\frac{3\times 5(100)^2}{16\times 38}} = 15.7 \text{ say } 16 \text{ mm}$$

...(Taking σ_t for cast iron = 38 MPa = 38 N/mm^2)

Since the engine is a four stroke engine, therefore, the number of working strokes per minute,

$$n = N / 2 = 2000 / 2 = 1000$$

and cross-sectional area of the cylinder,

$$A = \frac{\pi\ D^2}{4} = \frac{\pi\ (100)^2}{4} = 7855 \text{ mm}^2$$

We know that indicated power,

$$IP = \frac{p_m.L.A.n}{60} = \frac{0.75\times 0.125\times 7855\times 1000}{60} = 12\ 270 \text{ W}$$

$$= 12.27 \text{ kW}$$

∴ Brake power, $BP = IP \times \eta_m = 12.27 \times 0.8 = 9.8$ kW ...($\because \eta_m = BP / IP$)

We know that the heat flowing through the piston head,

$$H = C \times HCV \times m \times BP$$

$$= 0.05 \times 42 \times 10^3 \times 41.7 \times 10^{-6} \times 9.8 = 0.86 \text{ kW} = 860 \text{ W}$$

....(Taking C = 0.05)

∴Thickness of the piston head on the basis of heat dissipation,

$$t_H = \frac{H}{12.56\,k\,(T_C - T_E)} = \frac{860}{12.56\times 46.6\times 220} = 0.0067 \text{ m} = 6.7 \text{ mm}$$

...($\because$ For cast iron , k = 46.6 W/m/°C, and $T_C - T_E$ = 220°C)

Taking the larger of the two values, we shall adopt

$$t_H = 16 \text{ mm}$$ **Ans.**

Since the ratio of L / D is 1.25, therefore a cup in the top of the piston head with a radius equal to 0.7 D (*i.e.* 70 mm) is provided.

2. Radial ribs

The radial ribs may be four in number. The thickness of the ribs varies from t_H / 3 to t_H / 2.

∴ Thickness of the ribs, t_R = 16 / 3 to 16 / 2 = 5.33 to 8 mm

Let us adopt t_R = 7 mm **Ans.**

3. Piston rings

Let us assume that there are total four rings (*i.e.* n_r = 4) out of which three are compression rings and one is an oil ring.

We know that the radial thickness of the piston rings,

$$t_1 = D\sqrt{\frac{3p_w}{\sigma_t}} = 100\sqrt{\frac{3\times 0.035}{90}} = 3.4 \text{ mm}$$

...(Taking p_w = 0.035 N/mm^2, and σ_t = 90 MPa)

and axial thickness of the piston rings

$$t_2 = 0.7\,t_1 \text{ to } t_1 = 0.7 \times 3.4 \text{ to } 3.4 \text{ mm} = 2.38 \text{ to } 3.4 \text{ mm}$$

Let us adopt t_2 = 3 mm

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We also know that the minimum axial thickness of the pistion ring,

$$t_2 = \frac{D}{10\ n_r} = \frac{100}{10 \times 4} = 2.5 \text{ mm}$$

Thus the axial thickness of the piston ring as already calculated (*i.e.* t_2 = 3 mm)is satisfactory. **Ans.**

The distance from the top of the piston to the first ring groove, *i.e.* the width of the top land,

$$b_1 = t_H \text{ to } 1.2\ t_H = 16 \text{ to } 1.2 \times 16 \text{ mm} = 16 \text{ to } 19.2 \text{ mm}$$

and width of other ring lands,

$$b_2 = 0.75\ t_2 \text{ to } t_2 = 0.75 \times 3 \text{ to } 3 \text{ mm} = 2.25 \text{ to } 3 \text{ mm}$$

Let us adopt b_1 = 18 mm ; and b_2 = 2.5 mm **Ans.**

We know that the gap between the free ends of the ring,

$$G_1 = 3.5\ t_1 \text{ to } 4\ t_1 = 3.5 \times 3.4 \text{ to } 4 \times 3.4 \text{ mm} = 11.9 \text{ to } 13.6 \text{ mm}$$

and the gap when the ring is in the cylinder,

$$G_2 = 0.002\ D \text{ to } 0.004\ D = 0.002 \times 100 \text{ to } 0.004 \times 100 \text{ mm}$$
$$= 0.2 \text{ to } 0.4 \text{ mm}$$

Let us adopt G_1 = 12.8 mm ; and G_2 = 0.3 mm **Ans.**

4. *Piston barrel*

Since the radial depth of the piston ring grooves (b) is about 0.4 mm more than the radial thickness of the piston rings (t_1), therefore,

$$b = t_1 + 0.4 = 3.4 + 0.4 = 3.8 \text{ mm}$$

We know that the maximum thickness of barrel,

$$t_3 = 0.03\ D + b + 4.5 \text{ mm} = 0.03 \times 100 + 3.8 + 4.5 = 11.3 \text{ mm}$$

and piston wall thickness towards the open end,

$$t_4 = 0.25\ t_3 \text{ to } 0.35\ t_3 = 0.25 \times 11.3 \text{ to } 0.35 \times 11.3 = 2.8 \text{ to } 3.9 \text{ mm}$$

Let us adopt t_4 = 3.4 mm

5. *Piston skirt*

Let l = Length of the skirt in mm.

We know that the maximum side thrust on the cylinder due to gas pressure (p),

$$R = \mu \times \frac{\pi D^2}{4} \times p = 0.1 \times \frac{\pi (100)^2}{4} \times 5 = 3928 \text{ N}$$

...(Taking μ = 0.1)

We also know that the side thrust due to bearing pressure on the piston barrel (p_b),

$$R = p_b \times D \times l = 0.45 \times 100 \times l = 45\ l \text{ N}$$

...(Taking p_b = 0.45 N/mm^2)

From above, we find that

$$45\ l = 3928 \text{ or } l = 3928 / 45 = 87.3 \text{ say } 90 \text{ mm}$$ **Ans.**

∴ Total length of the piston ,

$$L = \text{Length of the skirt + Length of the ring section + Top land}$$
$$= l + (4\ t_2 + 3b_2) + b_1$$
$$= 90 + (4 \times 3 + 3 \times 3) + 18 = 129 \text{ say } 130 \text{ mm}$$ **Ans.**

6. *Piston pin*

Let d_0 = Outside diameter of the pin in mm,

l_1 = Length of pin in the bush of the small end of the connecting rod in mm, and

p_{b1} = Bearing pressure at the small end of the connecting rod bushing in N/mm^2. It value for bronze bushing is taken as 25 N/mm^2.

We know that load on the pin due to bearing pressure

$$= \text{Bearing pressure} \times \text{Bearing area} = p_{b1} \times d_0 \times l_1$$

$$= 25 \times d_0 \times 0.45 \times 100 = 1125\, d_0 \text{ N} \quad \text{...(Taking } l_1 = 0.45\, D)$$

We also know that maximum load on the piston due to gas pressure or maximum gas load

$$= \frac{\pi D^2}{4} \times p = \frac{\pi (100)^2}{4} \times 5 = 39\,275 \text{ N}$$

From above, we find that

$$1125\, d_0 = 39\,275 \quad \text{or} \quad d_0 = 39\,275 / 1125 = 34.9 \text{ say } 35 \text{ mm}$$ **Ans.**

The inside diameter of the pin (d_i) is usually taken as 0.6 d_0.

$\therefore$ $d_i = 0.6 \times 35 = 21$ mm **Ans.**

Let the piston pin be made of heat treated alloy steel for which the bending stress (σ_b) may be taken as 140 MPa. Now let us check the induced bending stress in the pin.

We know that maximum bending moment at the centre of the pin,

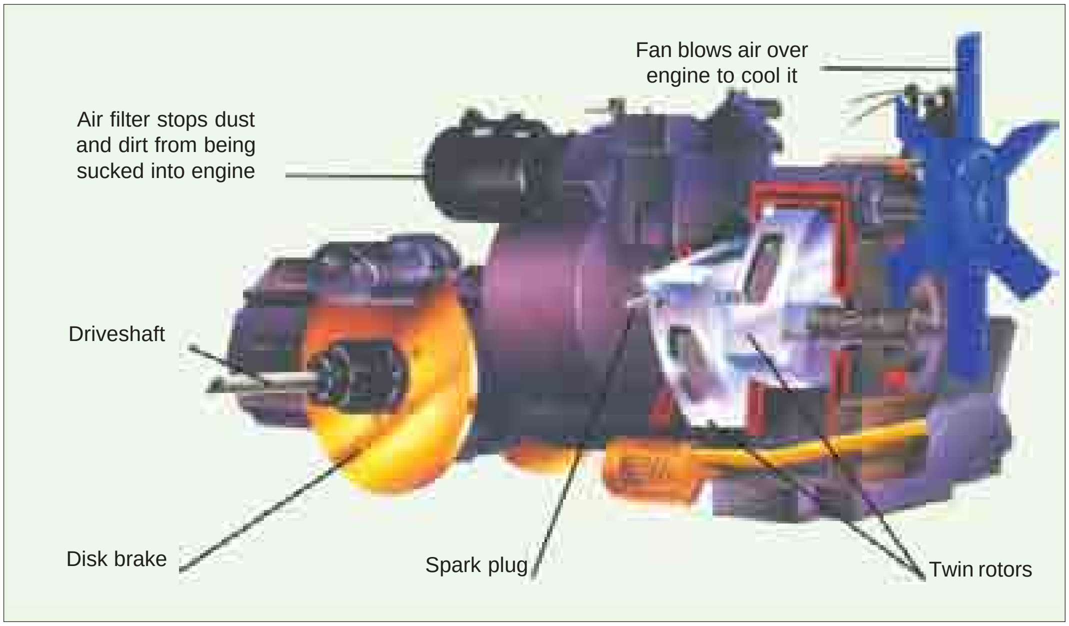
$$M = \frac{P.D}{8} = \frac{39\,275 \times 100}{8} = 491 \times 10^3 \text{ N-mm}$$

We also know that maximum bending moment (M),

$$491 \times 10^3 = \frac{\pi}{32}\left[\frac{(d_0)^4 - (d_i)^4}{d_0}\right]\sigma_b = \frac{\pi}{32}\left[\frac{(35)^4 - (21)^4}{35}\right]\sigma_b = 3664\,\sigma_b$$

$\therefore$ $\sigma_b = 491 \times 10^3 / 3664 = 134$ N/mm^2 or MPa

Since the induced bending stress in the pin is less than the permissible value of 140 MPa (*i.e.* 140 N/mm^2), therefore, the dimensions for the pin as calculated above (*i.e.* d_0 = 35 mm and d_i = 21 mm) are satisfactory.



German engineer Fleix Wankel (1902-88) built a rotary engine in 1957. A triangular piston turns inside a chamber through the combustion cycle.

32.13 Connecting Rod

The connecting rod is the intermediate member between the piston and the crankshaft. Its primary function is to transmit the push and pull from the piston pin to the crankpin and thus convert the reciprocating motion of the piston into the rotary motion of the crank. The usual form of the connecting rod in internal combustion engines is shown in Fig. 32.9. It consists of a long shank, a small end and a big end. The cross-section of the shank may be rectangular, circular, tubular, *I*-section or *H*-section. Generally circular section is used for low speed engines while *I*-section is preferred for high speed engines.

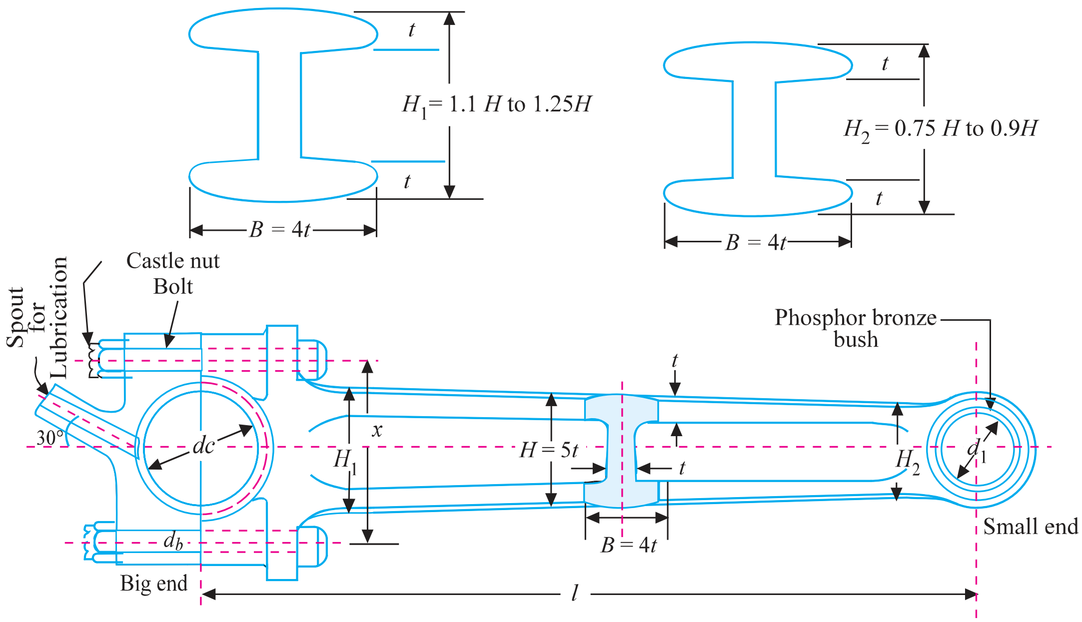


Fig. 32.9. Connecting rod.

The *length of the connecting rod (l) depends upon the ratio of l / r, where r is the radius of crank. It may be noted that the smaller length will decrease the ratio l / r. This increases the angularity of the connecting rod which increases the side thrust of the piston against the cylinder liner which in turn increases the wear of the liner. The larger length of the connecting rod will increase the ratio l / r. This decreases the angularity of the connecting rod and thus decreases the side thrust and the resulting wear of the cylinder. But the larger length of the connecting rod increases the overall height of the engine. Hence, a compromise is made and the ratio l / r is generally kept as 4 to 5.

The small end of the connecting rod is usually made in the form of an eye and is provided with a bush of phosphor bronze. It is connected to the piston by means of a piston pin.

The big end of the connecting rod is usually made split (in two **halves) so that it can be mounted easily on the crankpin bearing shells. The split cap is fastened to the big end with two cap bolts. The bearing shells of the big end are made of steel, brass or bronze with a thin lining (about 0.75 mm) of white metal or babbit metal. The wear of the big end bearing is allowed for by inserting thin metallic strips (known as ***shims***) about 0.04 mm thick between the cap and the fixed half of the connecting rod. As the wear takes place, one or more strips are removed and the bearing is trued up.

* It is the distance between the centres of small end and big end of the connecting rod.

** One half is fixed with the connecting rod and the other half (known as cap) is fastened with two cap bolts.

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The connecting rods are usually manufactured by drop forging process and it should have adequate strength, stiffness and minimum weight. The material mostly used for connecting rods varies from mild carbon steels (having 0.35 to 0.45 percent carbon) to alloy steels (chrome-nickel or chrome-molybdenum steels). The carbon steel having 0.35 percent carbon has an ultimate tensile strength of about 650 MPa when properly heat treated and a carbon steel with 0.45 percent carbon has a ultimate tensile strength of 750 MPa. These steels are used for connecting rods of industrial engines. The alloy steels have an ultimate tensile strength of about 1050 MPa and are used for connecting rods of aeroengines and automobile engines.

The bearings at the two ends of the connecting rod are either splash lubricated or pressure lubricated. The big end bearing is usually splash lubricated while the small end bearing is pressure lubricated. In the **splash lubrication system**, the cap at the big end is provided with a dipper or spout and set at an angle in such a way that when the connecting rod moves downward, the spout will dip into the lubricating oil contained in the sump. The oil is forced up the spout and then to the big end bearing. Now when the connecting rod moves upward, a splash of oil is produced by the spout. This splashed up lubricant find its way into the small end bearing through the widely chamfered holes provided on the upper surface of the small end.

In the **pressure lubricating system,** the lubricating oil is fed under pressure to the big end bearing through the holes drilled in crankshaft, crankwebs and crank pin. From the big end bearing, the oil is fed to small end bearing through a fine hole drilled in the shank of the connecting rod. In some cases, the small end bearing is lubricated by the oil scrapped from the walls of the cyinder liner by the oil scraper rings.

32.14 Forces Acting on the Connecting Rod

The various forces acting on the connecting rod are as follows :

1. Force on the piston due to gas pressure and inertia of the reciprocating parts,
2. Force due to inertia of the connecting rod or inertia bending forces,
3. Force due to friction of the piston rings and of the piston, and
4. Force due to friction of the piston pin bearing and the crankpin bearing.

We shall now derive the expressions for the forces acting on a vertical engine, as discussed below.

1. Force on the piston due to gas pressure and inertia of reciprocating parts

Consider a connecting rod *PC* as shown in Fig. 32.10.

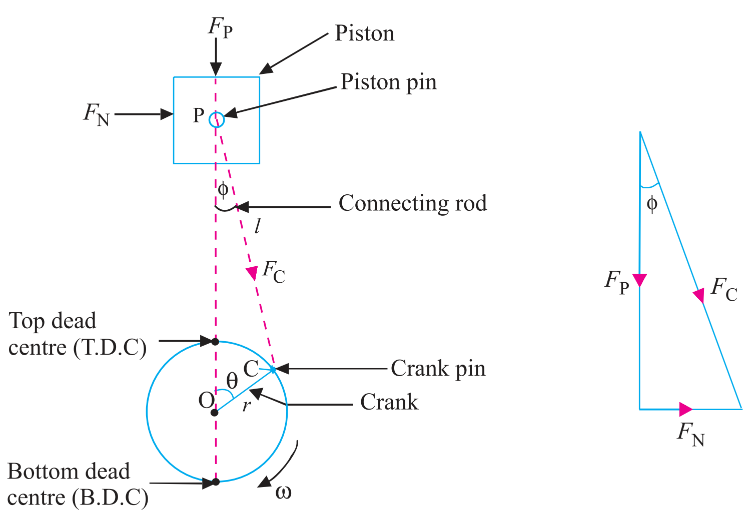

Fig. 32.10. Forces on the connecting rod.

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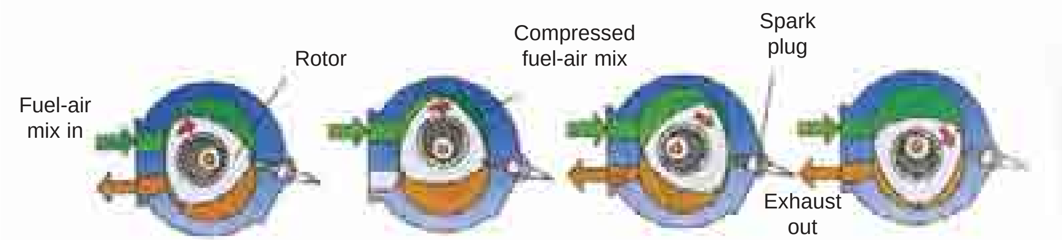

1. Induction : turning rotor sucks in mixture of petrol and air. **2. Compression:** Fuel-air mixture is compressed as rotor carriers it round. **3. Ignition:** Compressed fuel-air mixture is ignited by the spark plug. **4. Exhaust :** the rotor continues to turn and pushed out waste gases.

Let p = Maximum pressure of gas,

D = Diameter of piston,

A = Cross-section area of piston $= \dfrac{\pi D^2}{4}$,

m_R = Mass of reciprocating parts,

= Mass of piston, gudgeon pin etc. $+ \dfrac{1}{3}$ rd mass of connecting rod,

ω = Angular speed of crank,

ϕ = Angle of inclination of the connecting rod with the line of stroke,

θ = Angle of inclination of the crank from top dead centre,

r = Radius of crank,

l = Length of connecting rod, and

n = Ratio of length of connecting rod to radius of crank = l / r.

We know that the force on the piston due to pressure of gas,

$$F_L = \text{Pressure} \times \text{Area} = p \,.\, A = p \times \pi D^2/4$$

and inertia force of reciprocating parts,

$$F_I = \text{Mass} \times {}^*\text{Acceleration} = m_R \,.\, \omega^2 \,.\, r \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

It may be noted that the inertia force of reciprocating parts opposes the force on the piston when it moves during its downward stroke (*i. e.* when the piston moves from the top dead centre to bottom dead centre). On the other hand, the inertia force of the reciprocating parts helps the force on the piston when it moves from the bottom dead centre to top dead centre.

∴ Net force acting on the piston or piston pin (or gudgeon pin or wrist pin),

$$F_P = \text{Force due to gas pressure} \mp \text{Inertia force}$$

$$= F_L \mp F_I$$

The –ve sign is used when piston moves from TDC to BDC and +ve sign is used when piston moves from BDC to TDC.

When weight of the reciprocating parts ($W_R = m_R \,.\, g$) is to be taken into consideration, then

$$F_P = F_L \mp F_1 \pm W_R$$

---

* Acceleration of reciprocating parts $= \omega^2 \cdot r \left(\cos\theta + \dfrac{\cos 2\theta}{n}\right)$

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The force F_P gives rise to a force F_C in the connecting rod and a thrust F_N on the sides of the cylinder walls. From Fig. 32.10, we see that force in the connecting rod at any instant,

$$F_C = \frac{F_P}{\cos\phi} = \frac{{}^{*}F_P}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}}$$

The force in the connecting rod will be maximum when the crank and the connecting rod are perpendicular to each other (*i.e.* when $\theta = 90°$). But at this position, the gas pressure would be decreased considerably. ***Thus, for all practical purposes, the force in the connecting rod (F_C) is taken equal to the maximum force on the piston due to pressure of gas (F_L)***, neglecting piston inertia effects.

2. *Force due to inertia of the connecting rod or inertia bending forces*

Consider a connecting rod *PC* and a crank *OC* rotating with uniform angular velocity ω rad / s. In order to find the accleration of various points on the connecting rod, draw the Klien's acceleration diagram *CQNO* as shown in Fig. 32.11 (*a*). *CO* represents the acceleration of *C* towards *O* and *NO* represents the accleration of *P* towards *O*. The acceleration of other points such as *D*, *E*, *F* and *G* etc., on the connecting rod *PC* may be found by drawing horizontal lines from these points to intresect *CN* at *d*, *e*, *f*, and *g* respectively. Now *dO*, *eO*, *fO* and *gO* respresents the acceleration of *D*, *E*, *F* and *G* all towards *O*. The inertia force acting on each point will be as follows:

Inertia force at $C = m \times \omega^2 \times CO$

Inertia force at $D = m \times \omega^2 \times dO$

Inertia force at $E = m \times \omega^2 \times eO$, and so on.

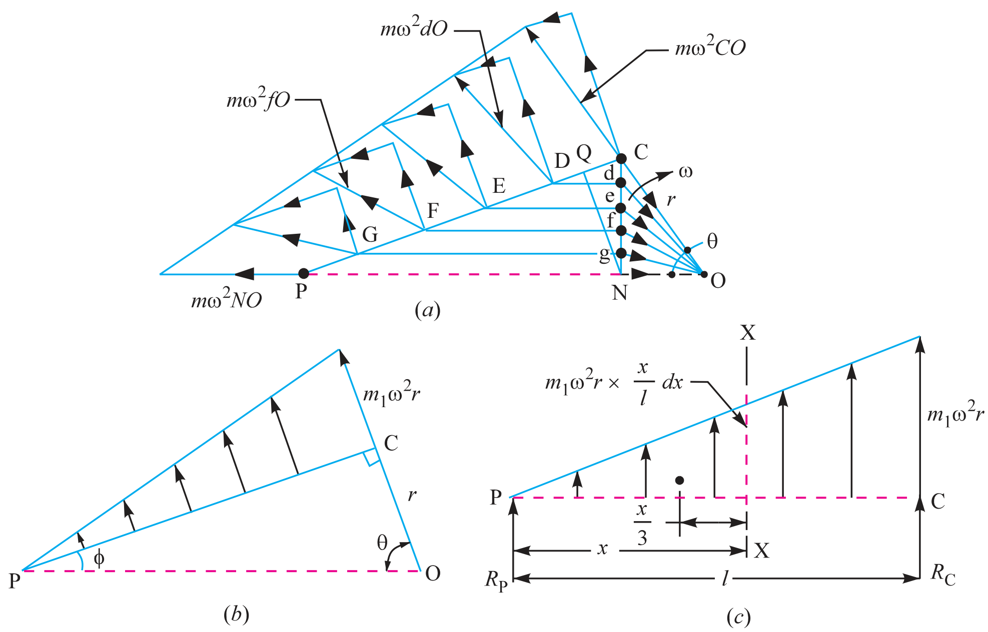

Fig. 32.11. Inertia bending forces.

The inertia forces will be opposite to the direction of acceleration or centrifugal forces. The inertia forces can be resolved into two components, one parallel to the connecting rod and the other perpendicular to rod. The parallel (or longitudinal) components adds up algebraically to the force

* For derivation, please refer ot Authors' popular book on **'Theory of Machines'.**

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acting on the connecting rod (F_C) and produces thrust on the pins. The perpendicular (or transverse) components produces bending action (also called whipping action) and the stress induced in the connecting rod is called ***whipping stress.***

It may be noted that the perpendicular components will be maximum, when the crank and connecting rod are at right angles to each other.

The variation of the inertia force on the connecting rod is linear and is like a simply supported beam of variable loading as shown in Fig. 32.11 (*b*) and (*c*). Assuming that the connecting rod is of uniform cross-section and has mass m_1 kg per unit length, therefore,

Inertia force per unit length at the crankpin

$$= m_1 \times \omega^2 r$$

and inertia force per unit length at the piston pin

$$= 0$$

Inertia force due to small element of length dx at a distance x from the piston pin P,

$$dF_1 = m_1 \times \omega^2 r \times \frac{x}{l} \times dx$$

∴ Resultant inertia force,

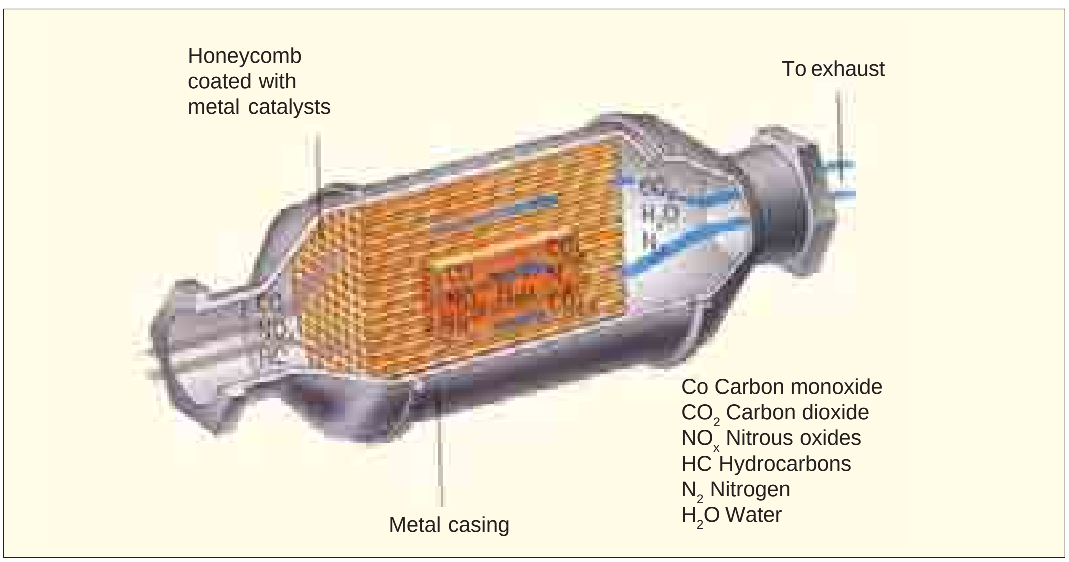
$$F_I = \int_0^l m_1 \times \omega^2 r \times \frac{x}{l} \times dx = \frac{m_1 \times \omega^2 r}{l} \left[\frac{x^2}{2} \right]_0^l$$

$$= \frac{m_1 . l}{2} \times \omega^2 r = \frac{m}{2} \times \omega^2 r \qquad \text{...(Substituting } m_1 \cdot l = m)$$

This resultant inertia force acts at a distance of $2l / 3$ from the piston pin P.

Since it has been assumed that $\frac{1}{3}$ rd mass of the connecting rod is concentrated at piston pin P (*i.e.* small end of connecting rod) and $\frac{2}{3}$ rd at the crankpin (*i.e.* big end of connecting rod), therefore, the reaction at these two ends will be in the same proportion. *i.e.*

$$R_P = \frac{1}{3} F_I, \text{ and } R_C = \frac{2}{3} F_I$$



Emissions of an automobile.

Top

Now the bending moment acting on the rod at section $X - X$ at a distance x from P,

$$M_X = R_P \times x - {}^*m_1 \times \omega^2 r \times \frac{x}{l} \times \frac{1}{2} x \times \frac{x}{3}$$

$$= \frac{1}{3} F_I \times x - \frac{m_1 . l}{2} \times \omega^2 r \times \frac{x^3}{3 l^2}$$

...(Multiplying and dividing the latter expression by l)

$$= \frac{F_I \times x}{3} - F_I \times \frac{x^3}{3l^2} = \frac{F_I}{3}\left(x - \frac{x^3}{l^2}\right) \quad \text{...(i)}$$

For maximum bending moment, differentiate M_X with respect to x and equate to zero, *i.e.*

$$\frac{d_{MX}}{dx} = 0 \quad \text{or} \quad \frac{F_I}{3}\left[1 - \frac{3x^2}{l^2}\right] = 0$$

$$\therefore \quad 1 - \frac{3x^2}{l^2} = 0 \quad \text{or} \quad 3x^2 = l^2 \text{ or } x = \frac{l}{\sqrt{3}}$$

Maximum bending moment,

$$M_{max} = \frac{F_I}{3}\left[\frac{l}{\sqrt{3}} - \frac{\left(\frac{l}{\sqrt{3}}\right)^3}{l^2}\right] \quad \text{...[From equation (i)]}$$

$$= \frac{F_I}{3}\left[\frac{l}{\sqrt{3}} - \frac{l}{3\sqrt{3}}\right] = \frac{F_I \times l}{3\sqrt{3}} \times \frac{2}{3} = \frac{2F_I \times l}{9\sqrt{3}}$$

$$= 2 \times \frac{m}{2} \times \omega^2 r \times \frac{l}{9\sqrt{3}} = m \times \omega^2 r \times \frac{l}{9\sqrt{3}}$$

and the maximum bending stress, due to inertia of the connecting rod,

$$\sigma_{max} = \frac{M_{max}}{Z}$$

where Z = Section modulus.

From above we see that the maximum bending moment varies as the square of speed, therefore, the bending stress due to high speed will be dangerous. It may be noted that the maximum axial force and the maximum bending stress do not occur simultaneously. In an I.C. engine, the maximum gas load occurs close to top dead centre whereas the maximum bending stress occurs when the crank angle θ = 65° to 70° from top dead centre. The pressure of gas falls suddenly as the piston moves from dead centre. ***Thus the general practice is to design a connecting rod by assuming the force in the connecting rod (F_C) equal to the maximum force due to pressure (F_L), neglecting piston inertia effects and then checked for bending stress due to inertia force (i.e. whipping stress).***

3. Force due to friction of piston rings and of the piston

The frictional force (F) of the piston rings may be determined by using the following expression :

$$F = \pi D \cdot t_R \cdot n_R \cdot p_R \cdot \mu$$

where D = Cylinder bore,

t_R = Axial width of rings,

* B.M. due to variable force from $\left(0 \text{ to } m_1\ \omega^2 r \times \frac{x}{l}\right)$ is equal to the area of triangle multiplied by the distance of C.G. from $X - X \left(i.e. \frac{x}{3}\right)$.

n_R = Number of rings,

p_R = Pressure of rings (0.025 to 0.04 N/mm^2), and

μ = Coefficient of friction (about 0.1).

Since the frictional force of the piston rings is usually very small, therefore, it may be neglected.

The friction of the piston is produced by the normal component of the piston pressure which varies from 3 to 10 percent of the piston pressure. If the coefficient of friction is about 0.05 to 0.06, then the frictional force due to piston will be about 0.5 to 0.6 of the piston pressure, which is very low. Thus, the frictional force due to piston is also neglected.

4. Force due to friction of the piston pin bearing and crankpin bearing

The force due to friction of the piston pin bearing and crankpin bearing, is to bend the connecting rod and to increase the compressive stress on the connecting rod due to the direct load. Thus, the maximum compressive stress in the connecting rod will be

$\sigma_{c\,(max)}$ = Direct compressive stress + Maximum bending or whipping stress due to inertia bending stress

32.15 Design of Connecting Rod

In designing a connecting rod, the following dimensions are required to be determined :

1. Dimensions of cross-section of the connecting rod,
2. Dimensions of the crankpin at the big end and the piston pin at the small end,
3. Size of bolts for securing the big end cap, and
4. Thickness of the big end cap.

The procedure adopted in determining the above mentioned dimensions is discussed as below :

This experimental car burns hydrogen fuel in an ordinary piston engine. Its exhaust gases cause no pollution, because they contain only water vapour.

1. *Dimensions of cross-section of the connecting rod*

A connecting rod is a machine member which is subjected to alternating direct compressive and tensile forces. Since the compressive forces are much higher than the tensile forces, therefore, the cross-section of the connecting rod is designed as a strut and the Rankine's formula is used.

A connecting rod, as shown in Fig. 32.12, subjected to an axial load W may buckle with X-axis as neutral axis (*i.e.* in the plane of motion of the connecting rod) or Y-axis as neutral axis (*i.e.* in the plane perpendicular to the plane of motion). The connecting rod is considered like both ends hinged for buckling about X-axis and both ends fixed for buckling about Y-axis.

A connecting rod should be equally strong in buckling about both the axes.

Let A = Cross-sectional area of the connecting rod,

l = Length of the connecting rod,

σ_c = Compressive yield stress,

W_B = Buckling load,

I_{xx} and I_{yy} = Moment of inertia of the section about X-axis and Y-axis respectively, and

k_{xx} and k_{yy} = Radius of gyration of the section about X-axis and Y-axis respectively.

According to Rankine's formula,

$$W_B \text{ about } X-\text{axis} = \frac{\sigma_c . A}{1 + a\left(\frac{L}{k_{xx}}\right)^2} = \frac{\sigma_c . A}{1 + a\left(\frac{l}{k_{xx}}\right)^2} \quad \text{...}(\because \text{ For both ends hinged, } L = l)$$

and

$$W_B \text{ about } Y-\text{axis} = \frac{\sigma_c . A}{1 + a\left(\frac{L}{k_{yy}}\right)^2} = \frac{\sigma_c . A}{1 + a\left(\frac{l}{2\,k_{yy}}\right)^2} \quad \text{...}[\because \text{ For both ends fixed, } L = \frac{l}{2}]$$

where L = Equivalent length of the connecting rod, and

a = Constant

= 1 / 7500, for mild steel

= 1 / 9000, for wrought iron

= 1 / 1600, for cast iron

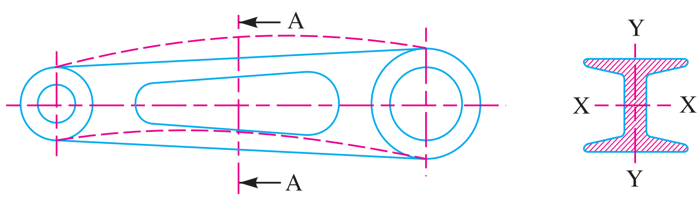


Fig. 32.12. Buckling of connecting rod.

In order to thave a connecting rod equally strong in buckling about both the axes, the buckling loads must be equal, *i.e.*

$$\frac{\sigma_c . A}{1 + a\left(\frac{l}{k_{xx}}\right)^2} = \frac{\sigma_c . A}{1 + a\left(\frac{l}{2\,k_{yy}}\right)^2} \quad \text{or} \quad \left(\frac{l}{k_{xx}}\right)^2 = \left(\frac{l}{2\,k_{yy}}\right)^2$$

$$\therefore \quad k_{xx}^2 = 4\,k_{yy}^2 \quad \text{or} \quad I_{xx} = 4\,I_{yy} \quad \text{...}(\because I = A \cdot k^2)$$

This shows that the connecting rod is four times strong in buckling about *Y*-axis than about *X*-axis. If $I_{xx} > 4\, I_{yy}$, then buckling will occur about *Y*- axis and if $I_{xx} < 4\, I_{yy}$, buckling will occur about *X*-axis. In actual practice, I_{xx} is kept slightly less than $4\, I_{yy}$. It is usually taken between 3 and 3.5 and the connecting rod is designed for bucking about *X*-axis. The design will always be satisfactory for buckling about *Y*-axis.

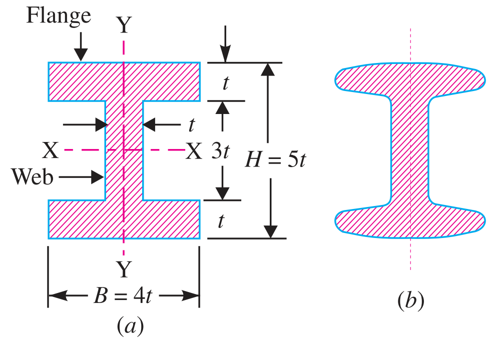


Fig. 32.13. *I*-section of connecting rod.

The most suitable section for the connecting rod is *I*-section with the proportions as shown in Fig. 32.13 (*a*).

Let thickness of the flange and web of the section = *t*

Width of the section, $B = 4\, t$

and depth or height of the section,

$$H = 5t$$

From Fig. 32.13 (*a*), we find that area of the section,

$$A = 2\,(4\,t \times t) + 3\,t \times t = 11\,t^2$$

Moment of inertia of the section about *X*-axis,

$$I_{xx} = \frac{1}{12}\left[4\,t\,(5t)^3 - 3t\,(3t)^3\right] = \frac{419}{12}t^4$$

and moment of inertia of the section about *Y*-axis,

$$I_{yy} = \left[2 \times \frac{1}{12}t \times (4t)^3 + \frac{1}{12}(3t)\,t^3\right] = \frac{131}{12}t^4$$

$$\therefore \qquad \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

Since the value of $\frac{I_{xx}}{I_{yy}}$ lies between 3 and 3.5, therefore, *I*-section chosen is quite satisfactory.

After deciding the proportions for *I*-section of the connecting rod, its dimensions are determined by considering the buckling of the rod about *X*-axis (assuming both ends hinged) and applying the Rankine's formula. We know that buckling load,

$$W_B = \frac{\sigma_c . A}{1 + a\left(\frac{L}{k_{xx}}\right)^2}$$

The buckling load (W_B) may be calculated by using the following relation, *i.e.*

$$W_B = \text{Max. gas force} \times \text{Factor of safety}$$

The factor of safety may be taken as 5 to 6.

Notes : (*a*) The *I*-section of the connecting rod is used due to its lightness and to keep the inertia forces as low as possible specially in case of high speed engines. It can also withstand high gas pressure.

(*b*) Sometimes a connecting rod may have rectangular section. For slow speed engines, circular section may be used.

(*c*) Since connecting rod is manufactured by forging, therefore the sharp corner of *I*-section are rounded off as shown in Fig. 32.13 (*b*) for easy removal of the section from dies.

The dimensions $B = 4\,t$ and $H = 5\,t$, as obtained above by applying the Rankine's formula, are at the middle of the connecting rod. The width of the section (B) is kept constant throughout the length of the connecting rod, but the depth or height varies. The depth near the small end (or piston end) is taken as $H_1 = 0.75\,H$ to $0.9H$ and the depth near the big end (or crank end) is taken $H_2 = 1.1H$ to $1.25H$.

2. *Dimensions of the crankpin at the big end and the piston pin at the small end*

Since the dimensions of the crankpin at the big end and the piston pin (also known as gudgeon pin or wrist pin) at the small end are limited, therefore, fairly high bearing pressures have to be allowed at the bearings of these two pins.

The crankpin at the big end has removable precision bearing shells of brass or bronze or steel with a thin lining (1 mm or less) of bearing metal (such as tin, lead, babbit, copper, lead) on the inner surface of the shell. The allowable bearing pressure on the crankpin depends upon many factors such as material of the bearing, viscosity of the lubricating oil, method of lubrication and the space limitations.The value of bearing pressure may be taken as 7 N/mm^2 to 12.5 N/mm^2 depending upon the material and method of lubrication used.

Engine of a motorcyle.

The piston pin bearing is usually a phosphor bronze bush of about 3 mm thickness and the allowable bearing pressure may be taken as 10.5 N/mm^2 to 15 N/mm^2.

Since the maximum load to be carried by the crankpin and piston pin bearings is the maximum force in the connecting rod (F_C), therefore the dimensions for these two pins are determined for the maximum force in the connecting rod (F_C) which is taken equal to the maximum force on the piston due to gas pressure (F_L) neglecting the inertia forces.

We know that maximum gas force,

$$F_L = \frac{\pi\, D^2}{4} \times p \qquad ...(i)$$

where D = Cylinder bore or piston diameter in mm, and

p = Maximum gas pressure in N/mm^2

Now the dimensions of the crankpin and piston pin are determined as discussed below :

Let d_c = Diameter of the crank pin in mm,

l_c = Length of the crank pin in mm,

p_{bc} = Allowable bearing pressure in N/mm^2, and

d_p, l_p and p_{bp} = Corresponding values for the piston pin,

We know that load on the crank pin

= Projected area × Bearing pressure

$$= d_c \cdot l_c \cdot p_{bc} \qquad ...(ii)$$

Similarly, load on the piston pin

$$= d_p \cdot l_p \cdot p_{bp} \qquad ...(iii)$$

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Equating equations (*i*) and (*ii*), we have

$$F_L = d_c \cdot l_c \cdot p_{bc}$$

Taking $l_c = 1.25\, d_c$ to $1.5\, d_c$, the value of d_c and l_c are determined from the above expression.

Again, equating equations (*i*) and (*iii*), we have

$$F_L = d_p \cdot l_p \cdot p_{bp}$$

Taking $l_p = 1.5\, d_p$ to $2\, d_p$, the value of d_p and l_p are determined from the above expression.

3. ***Size of bolts for securing the big end cap***

The bolts and the big end cap are subjected to tensile force which corresponds to the inertia force of the reciprocating parts at the top dead centre on the exhaust stroke. We know that inertia force of the reciprocating parts,

$$\therefore \quad F_I = m_R . \omega^2 . r \left(\cos\theta + \frac{\cos 2\theta}{l/r} \right)$$

We also know that at the top dead centre, the angle of inclination of the crank with the line of stroke, $\theta = 0$

$$\therefore \quad F_I = m_R . \omega^2 . r \left(1 + \frac{r}{l} \right)$$

where m_R = Mass of the reciprocating parts in kg,

ω = Angular speed of the engine in rad / s,

r = Radius of the crank in metres, and

l = Length of the connecting rod in metres.

The bolts may be made of high carbon steel or nickel alloy steel. Since the bolts are under repeated stresses but not alternating stresses, therefore, a factor of safety may be taken as 6.

Let d_{cb} = Core diameter of the bolt in mm,

σ_t = Allowable tensile stress for the material of the bolts in MPa, and

n_b = Number of bolts. Generally two bolts are used.

$\therefore$ Force on the bolts

$$= \frac{\pi}{4} (d_{cb})^2 \sigma_t \times n_b$$

Top

Equating the inertia force to the force on the bolts, we have

$$F_I = \frac{\pi}{4}(d_{cb})^2 \sigma_t \times n_b$$

From this expression, d_{cb} is obtained. The nominal or major diameter (d_b) of the bolt is given by

$$d_b = \frac{d_{cb}}{0.84}$$

4. *Thickness of the big end cap*

The thickness of the big end cap (t_c) may be determined by treating the cap as a beam freely supported at the cap bolt centres and loaded by the inertia force at the top dead centre on the exhaust stroke (*i.e.* F_I when $\theta = 0$). This load is assumed to act in between the uniformly distributed load and the centrally concentrated load. Therefore, the maximum bending moment acting on the cap will be taken as

$$M_C = {}^{*}\frac{F_I \times x}{6}$$

where x = Distance between the bolt centres.

= Dia. of crankpin or big end bearing (d_c) + 2 × Thickness of bearing liner (3 mm) + Clearance (3 mm)

Let b_c = Width of the cap in mm. It is equal to the length of the crankpin or big end bearing (l_c), and

σ_b = Allowable bending stress for the material of the cap in MPa.

We know that section modulus for the cap,

$$Z_C = \frac{b_c (t_c)^2}{6}$$

∴ Bending stress,

$$\sigma_b = \frac{M_C}{Z_C} = \frac{F_I \times x}{6} \times \frac{6}{b_c (t_c)^2} = \frac{F_I \times x}{b_c (t_c)^2}$$

From this expression, the value of t_c is obtained.

Note: The design of connecting rod should be checked for whipping stress (*i.e.* bending stress due to inertia force on the connecting rod).

Example 32.3. *Design a connecting rod for an I.C. engine running at 1800 r.p.m. and developing a maximum pressure of 3.15 N/mm^2. The diameter of the piston is 100 mm ; mass of the reciprocating parts per cylinder 2.25 kg; length of connecting rod 380 mm; stroke of piston 190 mm and compression ratio 6 : 1. Take a factor of safety of 6 for the design. Take length to diameter ratio for big end bearing as 1.3 and small end bearing as 2 and the corresponding bearing pressures as 10 N/mm^2 and 15 N/mm^2. The density of material of the rod may be taken as 8000 kg/m^3 and the allowable stress in the bolts as 60 N/mm^2 and in cap as 80 N/mm^2. The rod is to be of I-section for which you can choose your own proportions.*

Draw a neat dimensioned sketch showing provision for lubrication. Use Rankine formula for which the numerator constant may be taken as 320 N/mm^2 and the denominator constant 1 / 7500.

---

* We know that the maximum bending moment for a simply or freely supported beam with a uniformly distributed load of F_I over a length x between the supports (In this case, x is the distance between the cap bolt centres) is $\frac{F_I \times x}{8}$. When the load F_I is assumed to act at the centre of the freely supported beam, then the maximum bending moment is $\frac{F_I \times x}{4}$. Thus the maximum bending moment in between these two bending moments $\left(i.e. \frac{F_I \times x}{8} \text{ and } \frac{F_I \times x}{4}\right)$ is $\frac{F_I \times x}{6}$.

Solution. Given : N = 1800 r.p.m. ; p = 3.15 N/mm^2 ; D = 100 mm ; m_R = 2.25 kg ; l = 380 mm = 0.38 m ; Stroke = 190 mm ; *Compression ratio = 6 : 1 ; $F. S.$ = 6.

The connecting rod is designed as discussed below :

1. *Dimension of I- section of the connecting rod*

Let us consider an I-section of the connecting rod, as shown in Fig. 32.14 (a), with the following proportions :

Flange and web thickness of the section = t

Width of the section, $B = 4t$

and depth or height of the section,

$$H = 5t$$

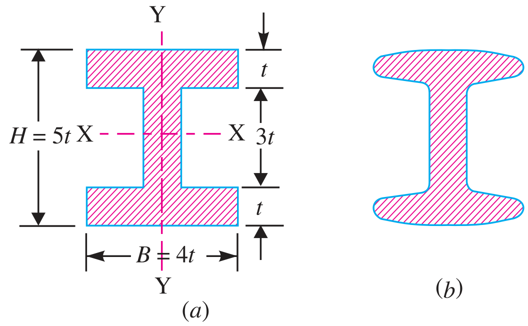


Fig. 32.14

First of all, let us find whether the section chosen is satisfactory or not.

We have already discussed that the connecting rod is considered like both ends hinged for buckling about X-axis and both ends fixed for buckling about Y-axis. The connecting rod should be equally strong in buckling about both the axes. We know that in order to have a connecting rod equally strong about both the axes,

$$I_{xx} = 4\, I_{yy}$$

where I_{xx} = Moment of inertia of the section about X-axis, and

I_{yy} = Moment of inertia of the section about Y-axis.

In actual practice, I_{xx} is kept slightly less than 4 I_{yy}. It is usually taken between 3 and 3.5 and the connecting rod is designed for buckling about X-axis.

Now, for the section as shown in Fig. 32.14 (a), area of the section,

$$A = 2\,(4\,t \times t\,) + 3t \times t = 11\,t^2$$

$$I_{xx} = \frac{1}{12}\left[4t(5t)^3 - 3t \times (3t)^3\right] = \frac{419}{12}t^4$$

and
$$I_{yy} = 2 \times \frac{1}{12} \times t(4t)^3 + \frac{1}{12} \times 3t \times t^3 = \frac{131}{12}t^4$$

$$\therefore \quad \frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

Since $\dfrac{I_{xx}}{I_{yy}} = 3.2,$ therefore the section chosen in quite satisfactory.

Now let us find the dimensions of this I-section. Since the connecting rod is designed by taking the force on the connecting rod (F_C) equal to the maximum force on the piston (F_L) due to gas pressure, therefore,

$$F_C = F_L = \frac{\pi\, D^2}{4} \times p = \frac{\pi (100)^2}{4} \times 3.15 = 24\ 740\ \text{N}$$

We know that the connecting rod is designed for buckling about X-axis ($i.e.$ in the plane of motion of the connecting rod) assuming both ends hinged. Since a factor of safety is given as 6, therefore the buckling load,

$$W_B = F_C \times F.\ S. = 24\ 740 \times 6 = 148\ 440\ \text{N}$$

* Superfluous data

We know that radius of gyration of the section about *X*-axis,

$$k_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{419t^4}{12} \times \frac{1}{11t^2}} = 1.78\ t$$

Length of crank,

$$r = \frac{\text{Stroke of piston}}{2} = \frac{190}{2} = 95 \text{ mm}$$

Length of the connecting rod,

$$l = 380 \text{ mm} \qquad \text{...(Given)}$$

∴ Equivalent length of the connecting rod for both ends hinged,

$$L = l = 380 \text{ mm}$$

Now according to Rankine's formula, we know that buckling load (W_B),

$$148\ 440 = \frac{\sigma_c . A}{1 + a\left(\frac{L}{k_{xx}}\right)^2} = \frac{320 \times 11\ t^2}{1 + \frac{1}{7500}\left(\frac{380}{1.78\ t}\right)^2}$$

... (It is given that σ_c = 320 MPa or N/mm^2 and a = 1 / 7500)

$$\frac{148\ 440}{320} = \frac{11\ t^2}{1 + \frac{6.1}{t^2}} = \frac{11\ t^4}{t^2 + 6.1}$$

$$464\ (t^2 + 6.1) = 11\ t^4$$

or $$t^4 - 42.2\ t^2 - 257.3 = 0$$

∴ $$t^2 = \frac{42.2 \pm \sqrt{(42.2)^2 + 4 \times 257.3}}{2} = \frac{42.2 \pm 53}{2} = 47.6$$

... (Taking +ve sign)

or $$t = 6.9 \text{ say } 7 \text{ mm}$$

Thus, the dimensions of *I*-section of the connecting rod are :

Thickness of flange and web of the section

$= t = 7$ mm **Ans.**

Width of the section, $B = 4\ t = 4 \times 7 = 28$ mm **Ans.**

and depth or height of the section,

$H = 5\ t = 5 \times 7 = 35$ mm **Ans.**

Piston and connecting rod.

These dimensions are at the middle of the connecting rod. The width (*B*) is kept constant throughout the length of the rod, but the depth (*H*) varies. The depth near the big end or crank end is kept as 1.1*H* to 1.25*H* and the depth near the small end or piston end is kept as 0.75*H* to 0.9*H*. Let us take

Depth near the big end,

$$H_1 = 1.2H = 1.2 \times 35 = 42 \text{ mm}$$

and depth near the small end,

$$H_2 = 0.85H = 0.85 \times 35 = 29.75 \text{ say } 30 \text{ mm}$$

∴ Dimensions of the section near the big end

= 42 mm × 28 mm **Ans.**

and dimensions of the section near the small end

= 30 mm × 28 mm **Ans.**

Since the connecting rod is manufactured by forging, therefore the sharp corners of *I*-section are rounded off, as shown in Fig. 32.14 (*b*), for easy removal of the section from the dies.

2. Dimensions of the crankpin or the big end bearing and piston pin or small end bearing

Let d_c = Diameter of the crankpin or big end bearing,

l_c = length of the crankpin or big end bearing = 1.3 d_c ...(Given)

p_{bc} = Bearing pressure = 10 N/mm^2 ...(Given)

We know that load on the crankpin or big end bearing

= Projected area × Bearing pressure

$$= d_c . l_c . p_{bc} = d_c \times 1.3\, d_c \times 10 = 13\,(d_c)^2$$

Since the crankpin or the big end bearing is designed for the maximum gas force (F_L), therefore, equating the load on the crankpin or big end bearing to the maximum gas force, *i.e.*

$$13\,(d_c)^2 = F_L = 24\,740 \text{ N}$$

∴ $(d_c)^2 = 24\,740 / 13 = 1903$ or $d_c = 43.6$ say 44 mm **Ans.**

and $l_c = 1.3\, d_c = 1.3 \times 44 = 57.2$ say 58 mm **Ans.**

The big end has removable precision bearing shells of brass or bronze or steel with a thin lining (1mm or less) of bearing metal such as babbit.

Again, let d_p = Diameter of the piston pin or small end bearing,

l_p = Length of the piston pin or small end bearing = 2d_p ...(Given)

p_{bp} = Bearing pressure = 15 N/mm^2 ..(Given)

We know that the load on the piston pin or small end bearing

= Project area × Bearing pressure

$$= d_p . l_p . p_{bp} = d_p \times 2\, d_p \times 15 = 30\,(d_p)^2$$

Since the piston pin or the small end bearing is designed for the maximum gas force (F_L), therefore, equating the load on the piston pin or the small end bearing to the maximum gas force,

i.e.

$$30\,(d_p)^2 = 24\,740 \text{ N}$$

∴ $(d_p)^2 = 24\,740 / 30 = 825$ or $d_p = 28.7$ say 29 mm **Ans.**

and $l_p = 2\, d_p = 2 \times 29 = 58$ mm **Ans.**

The small end bearing is usually a phosphor bronze bush of about 3 mm thickness.

3. *Size of bolts for securing the big end cap*

Let d_{cb} = Core diameter of the bolts,

σ_t = Allowable tensile stress for the material of the bolts

= 60 N/mm^2 ...(Given)

and n_b = Number of bolts. Generally two bolts are used.

We know that force on the bolts

$$= \frac{\pi}{4}(d_{cb})^2 \sigma_t \times n_b = \frac{\pi}{4}(d_{cb})^2 \; 60 \times 2 = 94.26 \; (d_{cb})^2$$

The bolts and the big end cap are subjected to tensile force which corresponds to the inertia force of the reciprocating parts at the top dead centre on the exhaust stroke. We know that inertia force of the reciprocating parts,

$$F_I = m_R \,.\, \omega^2 \,.\, r \left(\cos\theta + \frac{\cos 2\theta}{l / r} \right)$$

We also know that at top dead centre on the exhaust stroke, θ = 0.

$$\therefore \quad F_I = m_R \,.\, \omega^2 \,.\, r \left(1 + \frac{r}{l}\right) = 2.25 \left(\frac{2\pi \times 1800}{60}\right)^2 0.095 \left(1 + \frac{0.095}{0.38}\right) \text{N}$$

$$= 9490 \text{ N}$$

Equating the inertia force to the force on the bolts, we have

$$9490 = 94.26 \; (d_{cb})^2 \quad \text{or} \quad (d_{cb})^2 = 9490 / 94.26 = 100.7$$

$$\therefore \quad d_{cb} = 10.03 \text{ mm}$$

and nominal diameter of the bolt,

$$d_b = \frac{d_{cb}}{0.84} = \frac{10.03}{0.84} = 11.94$$

say 12 mm **Ans.**

4. *Thickness of the big end cap*

Let t_c = Thickness of the big end cap,

b_c = Width of the big end cap. It is taken equal to the length of the crankpin or big end bearing (l_c)

= 58 mm (calculated above)

σ_b = Allowable bending stress for the material of the cap

= 80 N/mm^2 ...(Given)

The big end cap is designed as a beam freely supported at the cap bolt centres and loaded by the inertia force at the top dead centre on the exhaust stroke (*i.e.* F_I when θ = 0). Since the load is assumed to act in between the uniformly distributed load and the centrally concentrated load, therefore, maximum bending moment is taken as

$$M_C = \frac{F_I \times x}{6}$$

where x = Distance between the bolt centres

= Dia. of crank pin or big end bearing + 2 × Thickness of bearing liner + Nominal dia. of bolt + Clearance

$= (d_c + 2 \times 3 + d_b + 3)$ mm = 44 + 6 + 12 + 3 = 65 mm

∴ Maximum bending moment acting on the cap,

$$M_C = \frac{F_I \times x}{6} = \frac{9490 \times 65}{6} = 102\ 810 \text{ N-mm}$$

Section modulus for the cap

$$Z_C = \frac{b_c (t_c)^2}{6} = \frac{58 (t_c)^2}{6} = 9.7\ (t_c)^2$$

We know that bending stress (σ_b),

$$80 = \frac{M_C}{Z_C} = \frac{102\ 810}{9.7\ (t_c)^2} = \frac{10\ 600}{(t_c)^2}$$

∴ $(t_c)^2 = 10\ 600 / 80 = 132.5$ or $t_c = 11.5$ mm **Ans.**

Let us now check the design for the induced bending stress due to inertia bending forces on the connecting rod (*i.e.* whipping stress).

We know that mass of the connecting rod per metre length,

m_1 = Volume × density = Area × length × density

$= A \times l \times \rho = 11t^2 \times l \times \rho$ ...($\because A = 11t^2$)

$= 11(0.007)^2\ (0.38)\ 8000 = 1.64$ kg

...[$\because \rho = 8\ 000$ kg /m^3 (given)]

∴ Maximum bending moment,

$$M_{max} = m\ .\ \omega^2\ .\ r \times \frac{l}{9\sqrt{3}} = m_1\ .\ \omega^2\ .\ r \times \frac{l^2}{9\sqrt{3}} \quad ...(\because m = m_1 \cdot l)$$

$$= 1.64 \left(\frac{2\pi \times 1800}{60}\right)^2 (0.095) \frac{(0.38)^2}{9\sqrt{3}} = 51.3 \text{ N-m}$$

= 51 300 N-mm

and section modulus,

$$Z_{xx} = \frac{I_{xx}}{5t/2} = \frac{419\ t^4}{12} \times \frac{2}{5\ t} = 13.97\ t^3 = 13.97 \times 7^3 = 4792 \text{ mm}^3$$

∴ Maximum bending stress (induced) due to inertia bending forces or whipping stress,

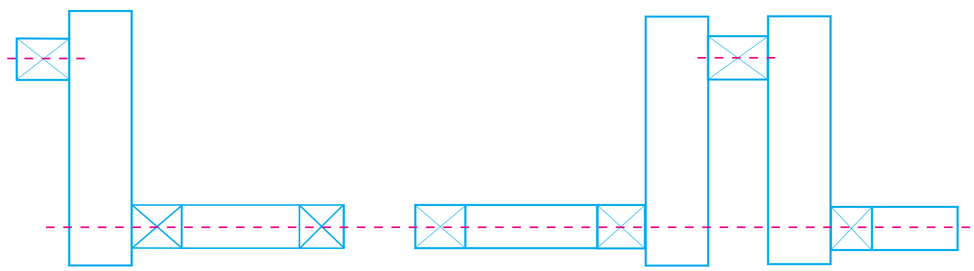
$$\sigma_{b(max)} = \frac{M_{max}}{Z_{xx}} = \frac{51\ 300}{4792} = 10.7 \text{ N/mm}^2$$

Since the maximum bending stress induced is less than the allowable bending stress of 80 N/mm^2, therefore the design is safe.

32.16 Crankshaft

A crankshaft (*i.e.* a shaft with a crank) is used to convert reciprocating motion of the piston into rotatory motion or vice versa. The crankshaft consists of the shaft parts which revolve in the main bearings, the crankpins to which the big ends of the connecting rod are connected, the crank arms or webs (also called cheeks) which connect the crankpins and the shaft parts. The crankshaft, depending upon the position of crank, may be divided into the following two types :

1. Side crankshaft or overhung crankshaft, as shown in Fig. 32.15 (*a*), and
2. Centre crankshaft, as shown in Fig. 32. 15 (*b*).



(*a*) Side Crankshaft.

(*a*) Centre Crankshaft.

Fig. 32.15. Types of crankshafts.

The crankshaft, depending upon the number of cranks in the shaft, may also be classfied as single throw or multi-throw crankshafts. A crankhaft with only one side crank or centre crank is called a ***single throw crankshaft*** whereas the crankshaft with two side cranks, one on each end or with two or more centre cranks is known as ***multi-throw crankshaft.***

The side crankshafts are used for medium and large size horizontal engines.

32.17 Material and manufacture of Crankshafts

The crankshafts are subjected to shock and fatigue loads. Thus material of the crankshaft should be tough and fatigue resistant. The crankshafts are generally made of carbon steel, special steel or special cast iron.

In industrial engines, the crankshafts are commonly made from carbon steel such as 40 C 8, 55 C 8 and 60 C 4. In transport engines, manganese steel such as 20 Mn 2, 27 Mn 2 and 37 Mn 2 are generally used for the making of crankshaft. In aero engines, nickel chromium steel such as 35 Ni 1 Cr 60 and 40 Ni 2 Cr 1 Mo 28 are extensively used for the crankshaft.

The crankshafts are made by drop forging or casting process but the former method is more common. The surface of the crankpin is hardened by case carburizing, nitriding or induction hardening.

32.18 Bearing Pressures and Stresses in Crankshaft

The bearing pressures are very important in the design of crankshafts. The *maximum permissible bearing pressure depends upon the maximum gas pressure, journal velocity, amount and method of lubrication and change of direction of bearing pressure.

The following two types of stresses are induced in the crankshaft.

1. Bending stress ; and **2.** Shear stress due to torsional moment on the shaft.

* The values of maximum permissible bearing pressures for different types of engines are given in Chapter 26, Table 26.3.

Most crankshaft failures are caused by a progressive fracture due to repeated bending or reversed torsional stresses. Thus the crankshaft is under fatigue loading and, therefore, its design should be based upon endurance limit. Since the failure of a crankshaft is likely to cause a serious engine destruction and neither all the forces nor all the stresses acting on the crankshaft can be determined accurately, therefore a high factor of safety from 3 to 4, based on the endurance limit, is used.

The following table shows the allowable bending and shear stresses for some commonly used materials for crankshafts :

Table 32.2. Allowable bending and shear stresses.

Material	*Endurance limit in MPa*		*Allowable stress in MPa*	
	Bending	*Shear*	*Bending*	*Shear*
Chrome nickel	525	290	130 to 175	72.5 to 97
Carbon steel and cast steel	225	124	56 to 75	31 to 42
Alloy cast iron	140	140	35 to 47	35 to 47

32.19 Design Procedure for Crankshaft

The following procedure may be adopted for designing a crankshaft.

1. First of all, find the magnitude of the various loads on the crankshaft.

2. Determine the distances between the supports and their position with respect to the loads.

3. For the sake of simplicity and also for safety, the shaft is considered to be supported at the centres of the bearings and all the forces and reactions to be acting at these points. The distances between the supports depend on the length of the bearings, which in turn depend on the diameter of the shaft because of the allowable bearing pressures.

4. The thickness of the cheeks or webs is assumed to be from 0.4 d_s to 0.6 d_s, where d_s is the diameter of the shaft. It may also be taken as 0.22D to 0.32 D, where D is the bore of cylinder in mm.

5. Now calculate the distances between the supports.

6. Assuming the allowable bending and shear stresses, determine the main dimensions of the crankshaft.

Notes: 1. The crankshaft must be designed or checked for at least two crank positions. Firstly, when the crankshaft is subjected to maximum bending moment and secondly when the crankshaft is subjected to maximum twisting moment or torque.

2. The additional moment due to weight of flywheel, belt tension and other forces must be considered.

3. It is assumed that the effect of bending moment does not exceed two bearings between which a force is considered.

32.20 Design of Centre Crankshaft

We shall design the centre crankshaft by considering the two crank possitions, *i.e.* when the crank is at dead centre (or when the crankshaft is subjected to maximum bending moment) and when the crank is at angle at which the twisting moment is maximum. These two cases are discussed in detail as below :

1. *When the crank is at dead centre*. At this position of the crank, the maximum gas pressure on the piston will transmit maximum force on the crankpin in the plane of the crank causing only bending of the shaft. The crankpin as well as ends of the crankshaft will be only subjected to bending moment. Thus, when the crank is at the dead centre, the bending moment on the shaft is maximum and the twisting moment is zero.

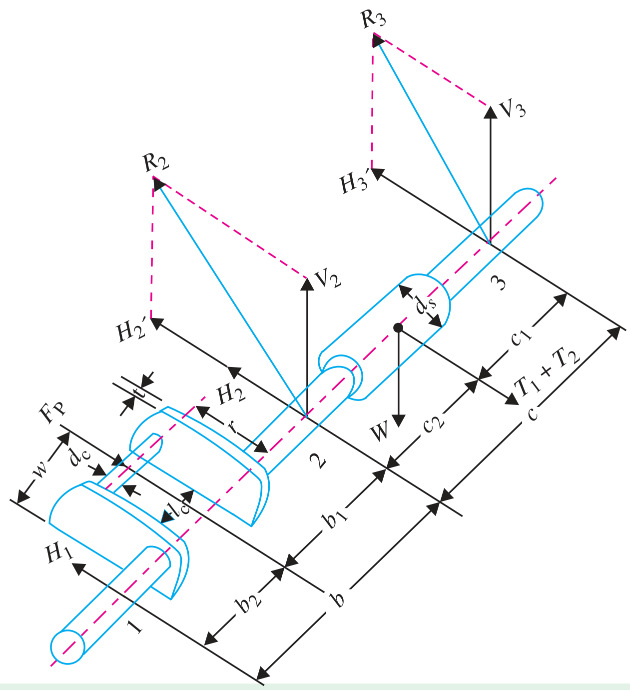


Fig. 32.16. Centre crankshaft at dead centre.

Consider a single throw three bearing crankshaft as shown in Fig. 32.16.

Let D = Piston diameter or cylinder bore in mm,

p = Maximum intensity of pressure on the piston in N/mm^2,

W = Weight of the flywheel acting downwards in N, and

*$T_1 + T_2$ = Resultant belt tension or pull acting horizontally in N.

The thrust in the connecting rod will be equal to the gas load on the piston (F_P). We know that gas load on the piston,

$$F_P = \frac{\pi}{4} \times D^2 \times p$$

Due to this piston gas load (F_P) acting horizontally, there will be two horizontal reactions H_1 and H_2 at bearings 1 and 2 respectively, such that

$$H_1 = \frac{F_P \times b_1}{b}; \quad \text{and} \quad H_2 = \frac{F_P \times b_2}{b}$$

Due to the weight of the flywheel (W) acting downwards, there will be two vertical reactions V_2 and V_3 at bearings 2 and 3 respectively, such that

$$V_2 = \frac{W \times c_1}{c}; \quad \text{and} \quad V_3 = \frac{W \times c_2}{c}$$

Now due to the resultant belt tension ($T_1 + T_2$), acting horizontally, there will be two horizontal reactions H_2' and H_3' at bearings 2 and 3 respectively, such that

$$H_2' = \frac{(T_1 + T_2)\, c_1}{c}; \quad \text{and} \quad H_3' = \frac{(T_1 + T_2)\, c_2}{c}$$

The resultant force at bearing 2 is given by

$$R_2 = \sqrt{(H_2 + H_2')^2 + (V_2)^2}$$

* T_1 is the belt tension in the tight side and T_2 is the belt tension in the slack side.

and the resultant force at bearing 3 is given by

$$R_3 = \sqrt{(H_3)^2 + (V_3)^2}$$

Now the various parts of the centre crankshaft are designed for bending only, as discussed below:

(a) Design of crankpin

Let d_c = Diameter of the crankpin in mm,

l_c = Length of the crankpin in mm,

σ_b = Allowable bending stress for the crankpin in N/mm².

We know that bending moment at the centre of the crankpin,

$$M_C = H_1 \cdot b_2 \qquad ...(i)$$

We also know that

$$M_C = \frac{\pi}{32}(d_c)^3 \sigma_b \qquad ...(ii)$$

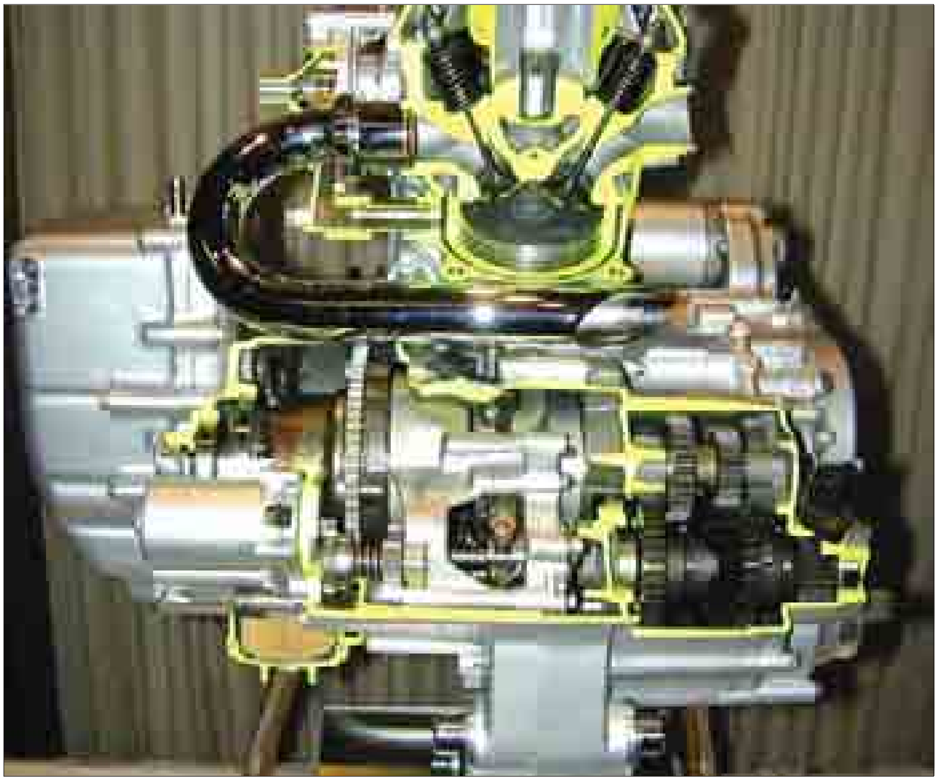
From equations (*i*) and (*ii*), diameter of the crankpin is determined. The length of the crankpin is given by

$$l_c = \frac{F_P}{d_c . p_b}$$

where p_b = Permissible bearing pressure in N/mm².

(b) Design of left hand crank web

The crank web is designed for eccentric loading. There will be two stresses acting on the crank web, one is direct compressive stress and the other is bending stress due to piston gas load (F_P).



Water cooled 4-cycle diesel engine

The thickness (t) of the crank web is given empirically as

$$t = 0.4\, d_s \text{ to } 0.6\, d_s$$
$$= 0.22D \text{ to } 0.32D$$
$$= 0.65\, d_c + 6.35 \text{ mm}$$

where d_s = Shaft diameter in mm,

D = Bore diameter in mm, and

d_c = Crankpin diameter in mm,

The width of crank web (w) is taken as

$$w = 1.125\, d_c + 12.7 \text{ mm}$$

We know that maximum bending moment on the crank web,

$$M = H_1\left(b_2 - \frac{l_c}{2} - \frac{t}{2}\right)$$

and section modulus,
$$Z = \frac{1}{6} \times w.\, t^2$$

$\therefore$ Bending stress,
$$\sigma_b = \frac{M}{Z} = \frac{6H_1\left(b_2 - \frac{l_c}{2} - \frac{t}{2}\right)}{w\,.\,t^2}$$

and direct compressive stress on the crank web,

$$\sigma_c = \frac{H_1}{w\,.\,t}$$

$\therefore$ Total stress on the crank web

$$= \text{Bending stress} + \text{Direct stress} = \sigma_b + \sigma_c$$
$$= \frac{6H_1\left(b_2 - \frac{l_c}{2} - \frac{t}{2}\right)}{w\,.\,t^2} + \frac{H_1}{w\,.\,t}$$

This total stress should be less than the permissible bending stress.

(c) Design of right hand crank web

The dimensions of the right hand crank web (*i.e.* thickness and width) are made equal to left hand crank web from the balancing point of view.

(d) Design of shaft under the flywheel

Let d_s = Diameter of shaft in mm.

We know that bending moment due to the weight of flywheel,

$$M_W = V_3\,.\,c_1$$

and bending moment due to belt tension,

$$M_T = H_3'\,.\,c_1$$

These two bending moments act at right angles to each other. Therefore, the resultant bending moment at the flywheel location,

$$M_S = \sqrt{(M_W)^2 + (M_T)^2} = \sqrt{(V_3 \cdot c_1)^2 + (H_3 \cdot c_1)^2} \quad \text{... } (i)$$

We also know that the bending moment at the shaft,

$$M_S = \frac{\pi}{32}(d_s)^3\, \sigma_b \quad \text{... } (ii)$$

where σ_b = Allowable bending stress in N/mm².

From equations ***(i)*** and ***(ii)***, we may determine the shaft diameter (d_s).

2. When the crank is at an angle of maximum twisting moment

The twisting moment on the crankshaft will be maximum when the tangential force on the crank (F_T) is maximum. The maximum value of tangential force lies when the crank is at angle of 25° to 30° from the dead centre for a constant volume combustion engines (*i.e.*, petrol engines) and 30° to 40° for constant pressure combustion engines (*i.e.*, diesel engines).

Consider a position of the crank at an angle of maximum twisting moment as shown in Fig. 32.17 (*a*). If p' is the intensity of pressure on the piston at this instant, then the piston gas load at this position of crank,

$$F_P = \frac{\pi}{4} \times D^2 \times p'$$

and thrust on the connecting rod,

$$F_Q = \frac{F_P}{\cos \phi}$$

where ϕ = Angle of inclination of the connecting rod with the line of stroke *PO*.

The *thrust in the connecting rod (F_Q) may be divided into two components, one perpendicular to the crank and the other along the crank. The component of F_Q perpendicular to the crank is the tangential force (F_T) and the component of F_Q along the crank is the radial force (F_R) which produces thrust on the crankshaft bearings. From Fig. 32.17 (*b*), we find that

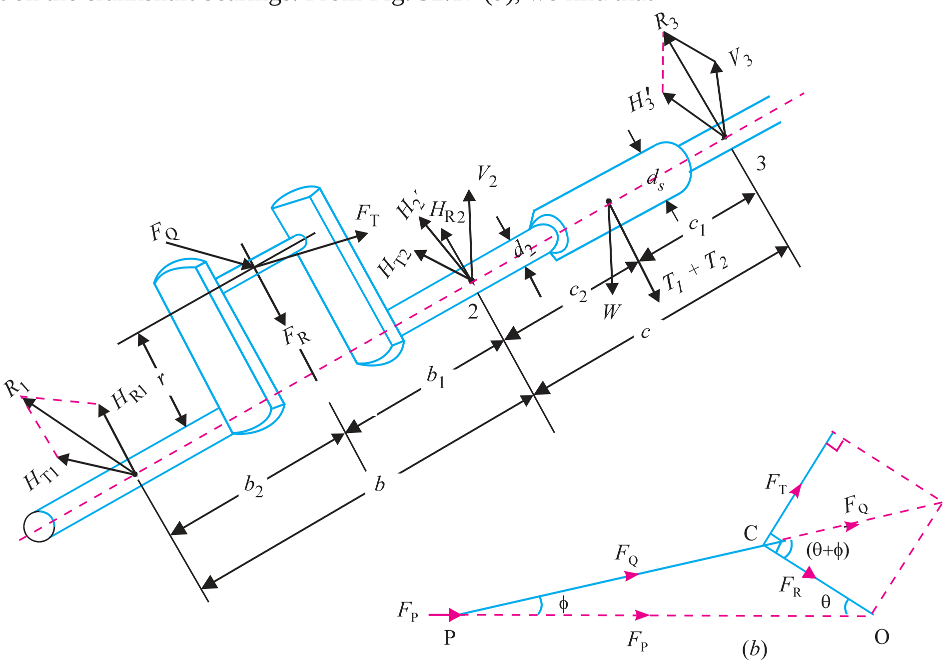


Fig. 32.17. (*a*) Crank at an angle of maximum twisting moment. (*b*) Forces acting on the crank.

$$F_T = F_Q \sin(\theta + \phi)$$

and

$$F_R = F_Q \cos(\theta + \phi)$$

It may be noted that the tangential force will cause twisting of the crankpin and shaft while the radial force will cause bending of the shaft.

* For further details, see Author's popular book on **'Theory of Machines'.**

Due to the tangential force (F_T), there will be two reactions at bearings 1 and 2, such that

$$H_{T1} = \frac{F_T \times b_1}{b}; \quad \text{and} \quad H_{T2} = \frac{F_T \times b_2}{b}$$

Due to the radial force (F_R), there will be two reactions at the bearings 1 and 2, such that

$$H_{R1} = \frac{F_R \times b_1}{b}; \quad \text{and} \quad H_{R2} = \frac{F_R \times b_2}{b}$$

Pull-start motor in an automobile

The reactions at the bearings 2 and 3, due to the flywheel weight (W) and resultant belt pull ($T_1 + T_2$) will be same as discussed earlier.

Now the various parts of the crankshaft are designed as discussed below :

(a) Design of crankpin

Let d_c = Diameter of the crankpin in mm.

We know that bending moment at the centre of the crankpin,

$$M_C = H_{R1} \times b_2$$

and twisting moment on the crankpin,

$$T_C = H_{T1} \times r$$

∴ Equivalent twisting moment on the crankpin,

$$T_e = \sqrt{(M_C)^2 + (T_C)^2} = \sqrt{(H_{R1} \times b_2)^2 + (H_{T1} \times r)^2} \quad ...(i)$$

We also know that twisting moment on the crankpin,

$$T_e = \frac{\pi}{16}(d_c)^3 \tau \quad ...(ii)$$

where τ = Allowable shear stress in the crankpin.

From equations ***(i)*** and ***(ii)***, the diameter of the crankpin is determined.

(b) Design of shaft under the flywheel

Let d_s = Diameter of the shaft in mm.

We know that bending moment on the shaft,

$$M_S = R_3 \times c_1$$

and twisting moment on the shaft,

$$T_S = F_T \times r$$

∴ Equivalent twisting moment on the shaft,

$$T_e = \sqrt{(M_S)^2 + (T_S)^2} = \sqrt{(R_3 \times c_1)^2 + (F_T \times r)^2} \quad \text{... (i)}$$

We also know that equivalent twisting moment on the shaft,

$$T_e = \frac{\pi}{16}(d_s)^3 \tau \quad \text{... (ii)}$$

where τ = Allowable shear stress in the shaft.

From equations ***(i)*** and ***(ii)***, the diameter of the shaft is determined.

(c) Design of shaft at the juncture of right hand crank arm

Let d_{s1} = Diameter of the shaft at the juncture of right hand crank arm.

We know that bending moment at the juncture of the right hand crank arm,

$$M_{S1} = R_1\left(b_2 + \frac{l_c}{2} + \frac{t}{2}\right) - F_Q\left(\frac{l_c}{2} + \frac{t}{2}\right)$$

and the twisting moment at the juncture of the right hand crank arm,

$$T_{S1} = F_T \times r$$

∴ Equivalent twisting moment at the juncture of the right hand crank arm,

$$T_e = \sqrt{(M_{S1})^2 + (T_{S1})^2} \quad \text{... (i)}$$

We also know that equivalent twisting moment,

$$T_e = \frac{\pi}{16}(d_{s1})^3 \tau \quad \text{... (ii)}$$

where τ = Allowable shear stress in the shaft.

From equations ***(i)*** and ***(ii)***, the diameter of the shaft at the juncture of the right hand crank arm is determined.

(d) Design of right hand crank web

The right hand crank web is subjected to the following stresses:

(i) Bending stresses in two planes normal to each other, due to the radial and tangential components of F_Q,

(ii) Direct compressive stress due to F_R, and

(iii) Torsional stress.

The bending moment due to the radial component of F_Q is given by,

$$M_R = H_{R2}\left(b_1 - \frac{l_c}{2} - \frac{t}{2}\right) \quad \text{... (i)}$$

We also know that $M_R = \sigma_{bR} \times Z = \sigma_{bR} \times \frac{1}{6} \times w.t^2$ ... ***(ii)*** where σ_{bR} = Bending stress in the radial direction, and

Z = Section modulus = $\frac{1}{6} \times w \cdot t^2$

From equations (*i*) and (*ii*), the value of bending stress σ_{bR} is determined.

The bending moment due to the tangential component of F_Q is maximum at the juncture of crank and shaft. It is given by

$$M_T = F_T \left[r - \frac{d_{s1}}{2} \right] \quad \text{... (iii)}$$

where d_{s1} = Shaft diameter at juncture of right hand crank arm, *i.e.* at bearing 2.

We also know that $M_T = \sigma_{bT} \times Z = \sigma_{bT} \times \frac{1}{6} \times t.w^2$ ... (*iv*)

where σ_{bT} = Bending stress in tangential direction.

From equations (*iii*) and (*iv*), the value of bending stress σ_{bT} is determined.

The direct compressive stress is given by,

$$\sigma_d = \frac{F_R}{2w.t}$$

The maximum compressive stress (σ_c) will occur at the upper left corner of the cross-section of the crank.

$\therefore$ $\sigma_c = \sigma_{bR} + \sigma_{bT} + \sigma_d$

Now, the twisting moment on the arm,

$$T = H_{T1}\left(b_2 + \frac{l_c}{2}\right) - F_T \times \frac{l_c}{2} = H_{T2}\left(b_1 - \frac{l_c}{2}\right)$$

We know that shear stress on the arm,

$$\tau = \frac{T}{Z_P} = \frac{4.5\,T}{w.t^2}$$

where Z_P = Polar section modulus = $\frac{w.t^2}{4.5}$

$\therefore$ Maximum or total combined stress,

$$(\sigma_c)_{max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2}$$

Snow blower on a railway track

The value of $(\sigma_c)_{max}$ should be within safe limits. If it exceeds the safe value, then the dimension *w* may be increased because it does not affect other dimensions.

(e) Design of left hand crank web

Since the left hand crank web is not stressed to the extent as the right hand crank web, therefore, the dimensions for the left hand crank web may be made same as for right hand crank web.

(f) Design of crankshaft bearings

The bearing 2 is the most heavily loaded and should be checked for the safe bearing pressure.

We know that the total reaction at the bearing 2,

$$R_2 = \frac{F_P}{2} + \frac{W}{2} + \frac{T_1 + T_2}{2}$$

$$\therefore \text{ Total bearing pressure } = \frac{R_2}{l_2 \cdot d_{s1}}$$

where l_2 = Length of bearing 2.

32.21 Side or Overhung Crankshaft

The side or overhung crankshafts are used for medium size and large horizontal engines. Its main advantage is that it requires only two bearings in either the single or two crank construction. The design procedure for the side or overhung crankshaft is same as that for centre crankshaft. Let us now design the side crankshaft by considering the two crank positions, *i.e.* when the crank is at dead centre (or when the crankshaft is subjected to maximum bending moment) and when the crank is at an angle at which the twisting moment is maximum. These two cases are discussed in detail as below:

1. *When the crank is at dead centre.* Consider a side crankshaft at dead centre with its loads and distances of their application, as shown in Fig. 32.18.

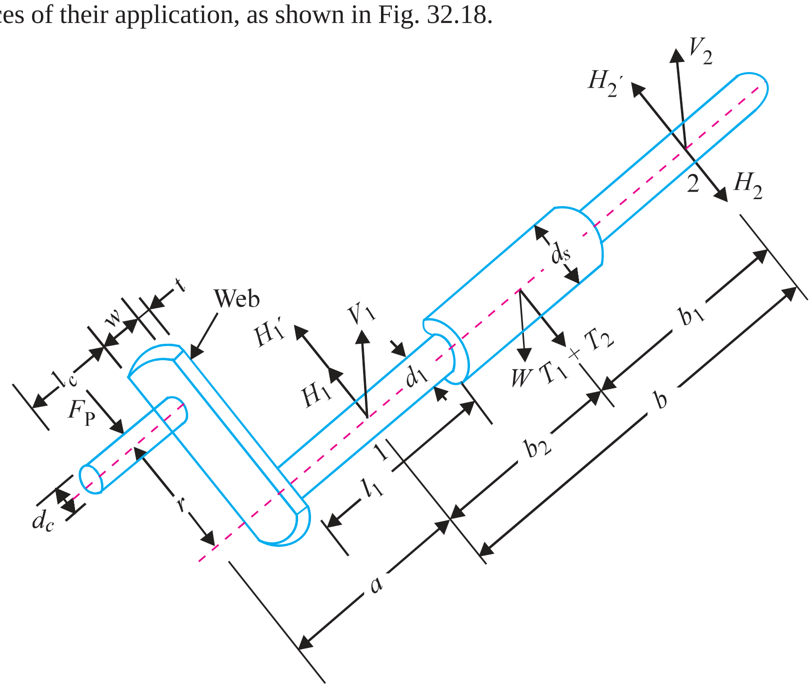


Fig. 32.18. Side crankshaft at dead centre.

Let D = Piston diameter or cylinder bore in mm,

p = Maximum intensity of pressure on the piston in N/mm^2,

W = Weight of the flywheel acting downwards in N, and

$T_1 + T_2$ = Resultant belt tension or pull acting horizontally in N.

We know that gas load on the piston,

$$F_P = \frac{\pi}{4} \times D^2 \times p$$

Due to this piston gas load (F_P) acting horizontally, there will be two horizontal reactions H_1 and H_2 at bearings 1 and 2 respectively, such that

$$H_1 = \frac{F_P (a + b)}{b}\ ; \quad \text{and} \quad H_2 = \frac{F_P \times a}{b}$$

Due to the weight of the flywheel (W) acting downwards, there will be two vertical reactions V_1 and V_2 at bearings l and 2 respectively, such that

$$V_1 = \frac{W . b_1}{b}\ ; \quad \text{and} \quad V_2 = \frac{W . b_2}{b}$$

Now due to the resultant belt tension ($T_1 + T_2$) acting horizontally, there will be two horizontal reactions H_1' and H_2' at bearings 1 and 2 respectively, such that

$$H_1' = \frac{(T_1 + T_2) b_1}{b}\ ; \quad \text{and} \quad H_2' = \frac{(T_1 + T_2) b_2}{b}$$

The various parts of the side crankshaft, when the crank is at dead centre, are now designed as discussed below:

(a) Design of crankpin. The dimensions of the crankpin are obtained by considering the crankpin in bearing and then checked for bending stress.

Let d_c = Diameter of the crankpin in mm,

l_c = Length of the crankpin in mm, and

p_b = Safe bearing pressure on the pin in N/mm^2. It may be between 9.8 to 12.6 N/mm^2.

We know that $F_P = d_c \,.\, l_c \,.\, p_b$

From this expression, the values of d_c and l_c may be obtained. The length of crankpin is usually from 0.6 to 1.5 times the diameter of pin.

The crankpin is now checked for bending stress. If it is assumed that the crankpin acts as a cantilever and the load on the crankpin is uniformly distributed, then maximum bending moment will be $\frac{F_P \times l_c}{2}$. But in actual practice, the bearing pressure on the crankpin is not uniformly distributed and may, therefore, give a greater value of bending moment ranging between $\frac{F_P \times l_c}{2}$ and $F_P \times l_c$. So, a mean value of bending moment, *i.e.* $\frac{3}{4} F_P \times l_c$ may be assumed.

Close-up view of an automobile piston

∴ Maximum bending moment at the crankpin,

$$M = \frac{3}{4} F_P \times l_c \quad \text{... (Neglecting pin collar thickness)}$$

Section modulus for the crankpin,

$$Z = \frac{\pi}{32}(d_c)^3$$

∴ Bending stress induced,

$$\sigma_b = M / Z$$

This induced bending stress should be within the permissible limits.

(b) Design of bearings. The bending moment at the centre of the bearing 1 is given by

$$M = F_P\,(0.75\, l_c + t + 0.5\, l_1) \quad \text{...(}i\text{)}$$

where l_c = Length of the crankpin,

t = Thickness of the crank web = 0.45 d_c to 0.75 d_c, and

l_1 = Length of the bearing = 1.5 d_c to 2 d_c.

We also know that

$$M = \frac{\pi}{32}(d_1)^3 \sigma_b \quad \text{...(}ii\text{)}$$

From equations (*i*) and (*ii*), the diameter of the bearing 1 may be determined.

Note : The bearing 2 is also made of the same diameter. The length of the bearings are found on the basis of allowable bearing pressures and the maximum reactions at the bearings.

(c) Design of crank web. When the crank is at dead centre, the crank web is subjected to a bending moment and to a direct compressive stress.

We know that bending moment on the crank web,

$$M = F_P\,(0.75\, l_c + 0.5\, t)$$

and section modulus,

$$Z = \frac{1}{6} \times w\,.\,t^2$$

∴ Bending stress,

$$\sigma_b = \frac{M}{Z}$$

We also know that direct compressive stress,

$$\sigma_d = \frac{F_P}{w\,.\,t}$$

∴ Total stress on the crank web,

$$\sigma_T = \sigma_b + \sigma_d$$

This total stress should be less than the permissible limits.

(d) Design of shaft under the flywheel. The total bending moment at the flywheel location will be the resultant of horizontal bending moment due to the gas load and belt pull and the vertical bending moment due to the flywheel weight.

Let d_s = Diameter of shaft under the flywheel.

We know that horizontal bending moment at the flywheel location due to piston gas load,

$$M_1 = F_P\,(a + b_2) - H_1\,.b_2 = H_2\,.b_1$$

Top

and horizontal bending moment at the flywheel location due to belt pull,

$$M_2 = H_1'.b_2 = H_2'.b_1 = \frac{(T_1 + T_2)\, b_1.b_2}{b}$$

∴ Total horizontal bending moment,

$$M_H = M_1 + M_2$$

We know that vertical bending moment due to flywheel weight,

$$M_V = V_1.b_2 = V_2.b_1 = \frac{W b_1 b_2}{b}$$

∴ Resultant bending moment,

$$M_R = \sqrt{(M_H)^2 + (M_V)^2} \quad \text{...}(i)$$

We also know that

$$M_R = \frac{\pi}{32}(d_s)^3 \sigma_b \quad \text{...}(ii)$$

From equations *(i)* and *(ii)*, the diameter of shaft (d_s) may determined.

2. ***When the crank is at an angle of maximum twisting moment.*** Consider a position of the crank at an angle of maximum twisting moment as shown in Fig. 32.19. We have already discussed in the design of a centre crankshaft that the thrust in the connecting rod (F_Q) gives rise to the tangential force (F_T) and the radial force (F_R).

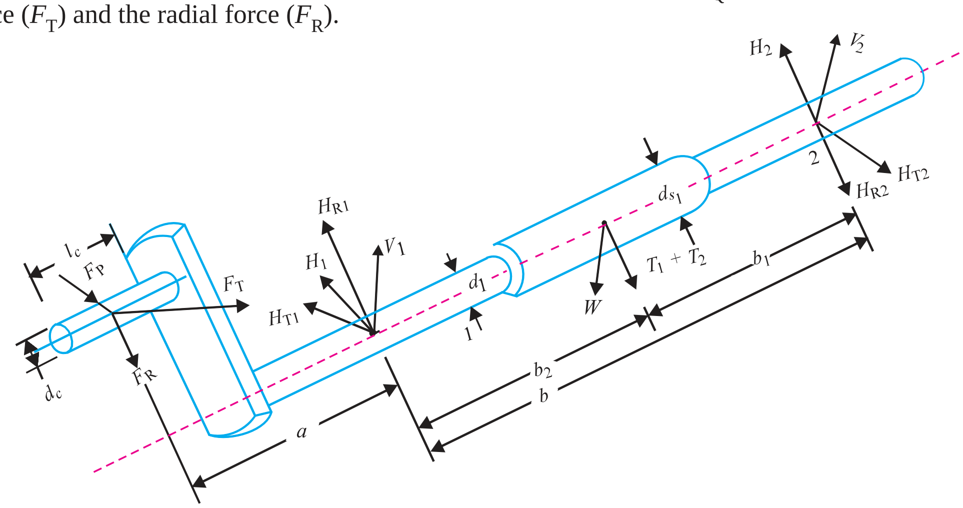


Fig. 32.19. Crank at an angle of maximum twisting moment.

Due to the tangential force (F_T), there will be two reactions at the bearings 1 and 2, such that

$$H_{T1} = \frac{F_T(a+b)}{b}; \quad \text{and} \quad H_{T2} = \frac{F_T \times a}{b}$$

Due to the radial force (F_R), there will be two reactions at the bearings 1 and 2, such that

$$H_{R1} = \frac{F_R(a+b)}{b}; \quad \text{and} \quad H_{R2} = \frac{F_R \times a}{b}$$

The reactions at the bearings 1 and 2 due to the flywheel weight (W) and resultant belt pull ($T_1 + T_2$) will be same as discussed earlier.

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crank web. The most critical section is where the web joins the shaft. This section is subjected to the following stresses :

(i) Bending stress due to the tangential force F_T ;

(*ii*) Bending stress due to the radial force F_R ;

(*iii*) Direct compressive stress due to the radial force F_R ; and

(*iv*) Shear stress due to the twisting moment of F_T.

We know that bending moment due to the tangential force,

$$M_{bT} = F_T\left(r - \frac{d_1}{2}\right)$$

where d_1 = Diameter of the bearing 1.

Diesel, petrol and steam engines have crank shaft

∴ Bending stress due to the tangential force,

$$\sigma_{bT} = \frac{M_{bT}}{Z} = \frac{6M_{bT}}{t.w^2} \qquad \ldots(\because Z = \frac{1}{6} \times t \cdot w^2) \ldots(i)$$

We know that bending moment due to the radial force,

$$M_{bR} = F_R\,(0.75\, l_c + 0.5\, t)$$

∴ Bending stress due to the radial force,

$$\sigma_{bR} = \frac{M_{bR}}{Z} = \frac{6\,M_{bR}}{w.t^2} \qquad \ldots(\text{Here } Z = \frac{1}{6} \times w \cdot t^2) \ldots(ii)$$

We know that direct compressive stress,

$$\sigma_d = \frac{F_R}{w.t} \qquad \ldots(iii)$$

∴ Total compressive stress,

$$\sigma_c = \sigma_{bT} + \sigma_{bR} + \sigma_d \qquad \ldots(iv)$$

We know that twisting moment due to the tangential force,

$$T = F_T\,(0.75\, l_c + 0.5\, t)$$

∴ Shear stress, $\tau = \dfrac{T}{Z_P} = \dfrac{4.5T}{w.t^2}$

where Z_P = Polar section modulus = $\dfrac{w.t^2}{4.5}$

Now the total or maximum stress is given by

$$\sigma_{max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2} \quad ...(v)$$

This total maximum stress should be less than the maximum allowable stress.

(b) Design of shaft at the junction of crank

Let d_{s1} = Diameter of the shaft at the junction of the crank.

We know that bending moment at the junction of the crank,

$$M = F_Q (0.75 l_c + t)$$

and twisting moment on the shaft

$$T = F_T \times r$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} \quad ...(i)$$

We also know that equivalent twisting moment,

$$T_e = \frac{\pi}{16}(d_{s1})^3 \tau \quad ...(ii)$$

From equations ***(i)*** and ***(ii)***, the diameter of the shaft at the junction of the crank (d_{s1}) may be determined.

(c) Design of shaft under the flywheel

Let d_s = Diameter of shaft under the flywheel.

The resultant bending moment (M_R) acting on the shaft is obtained in the similar way as discussed for dead centre position.

We know that horizontal bending moment acting on the shaft due to piston gas load,

$$M_1 = F_P(a + b_2) - \left[\sqrt{(H_{R1})^2 + (H_{T1})^2}\right] b_2$$

and horizontal bending moment at the flywheel location due to belt pull,

$$M_2 = H_1'.b_2 = H_2'.b_1 = \frac{(T_1 + T_2)\, b_1.b_2}{b}$$

∴ Total horizontal bending moment,

$$M_H = M_1 + M_2$$

Vertical bending moment due to the flywheel weight,

$$M_V = V_1 \cdot b_2 = V_2 \cdot b_1 = \frac{W\, b_1\, b_2}{b}$$

∴ Resultant bending moment,

$$M_R = \sqrt{(M_H)^2 + (M_V)^2}$$

We know that twisting moment on the shaft,

$$T = F_T \times r$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{(M_R)^2 + T^2} \quad ...(i)$$

We also know that equivalent twisting moment,

$$T_e = \frac{\pi}{16}(d_s)^3 \tau \quad ...(ii)$$

From equations ***(i)*** and ***(ii)***, the diameter of shaft under the flywheel (d_s) may be determined.

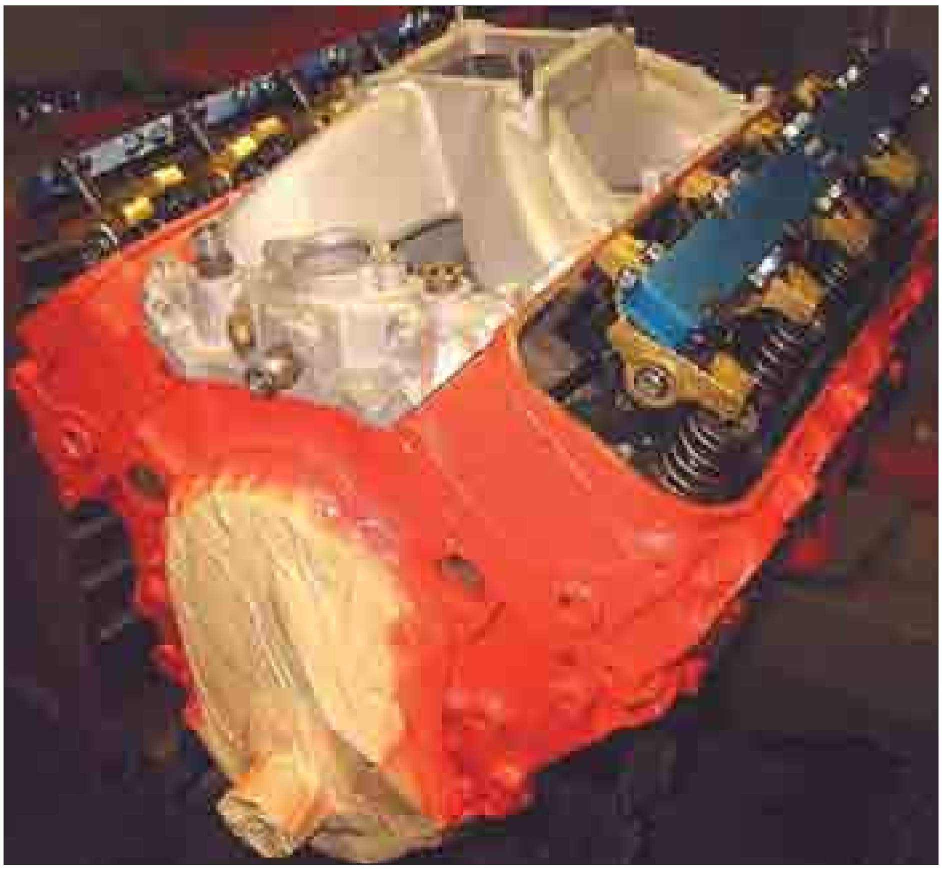
Example 32.4. *Design a plain carbon steel centre crankshaft for a single acting four stroke single cylinder engine for the following data:*

Bore = 400 mm ; Stroke = 600 mm ; Engine speed = 200 r.p.m. ; Mean effective pressure = 0.5 N/mm^2; Maximum combustion pressure = 2.5 N/mm^2; Weight of flywheel used as a pulley = 50 kN; Total belt pull = 6.5 kN.

When the crank has turned through 35° from the top dead centre, the pressure on the piston is 1N/mm^2 and the torque on the crank is maximum. The ratio of the connecting rod length to the crank radius is 5. Assume any other data required for the design.

Solution. Given : D = 400 mm ; L = 600 mm or r = 300 mm ; p_m = 0.5 N/mm^2 ; p = 2.5 N/mm^2 ; W = 50 kN ; $T_1 + T_2$ = 6.5 kN ; θ = 35° ; p' = 1N/mm^2 ; l / r = 5

We shall design the crankshaft for the two positions of the crank, *i.e.* firstly when the crank is at the dead centre ; and secondly when the crank is at an angle of maximum twisting moment.



Part of a car engine

1. *Design of the crankshaft when the crank is at the dead centre* (See Fig. 32.18)

We know that the piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} (400)^2 2.5 = 314\,200 \text{ N} = 314.2 \text{ kN}$$

Assume that the distance (b) between the bearings 1 and 2 is equal to twice the piston diameter (D).

$\therefore \quad b = 2D = 2 \times 400 = 800$ mm and $$b_1 = b_2 = \frac{b}{2} = \frac{800}{2} = 400 \text{ mm}$$

We know that due to the piston gas load, there will be two horizontal reactions H_1 and H_2 at bearings 1 and 2 respectively, such that

$$H_1 = \frac{F_P \times b_1}{b} = \frac{314.2 \times 400}{800} = 157.1 \text{ kN}$$

and $$H_2 = \frac{F_P \times b_2}{b} = \frac{314.2 \times 400}{800} = 157.1 \text{ kN}$$

Assume that the length of the main bearings to be equal, *i.e.*, $c_1 = c_2 = c / 2$. We know that due to the weight of the flywheel acting downwards, there will be two vertical reactions V_2 and V_3 at bearings 2 and 3 respectively, such that

$$V_2 = \frac{W \times c_1}{c} = \frac{W \times c/2}{c} = \frac{W}{2} = \frac{50}{2} = 25 \text{ kN}$$

and $$V_3 = \frac{W \times c_2}{c} = \frac{W \times c/2}{c} = \frac{W}{2} = \frac{50}{2} = 25 \text{ kN}$$

Due to the resultant belt tension $(T_1 + T_2)$ acting horizontally, there will be two horizontal reactions H_2' and H_3' respectively, such that

$$H_2' = \frac{(T_1 + T_2)\, c_1}{c} = \frac{(T_1 + T_2)\, c/2}{c} = \frac{T_1 + T_2}{2} = \frac{6.5}{2} = 3.25 \text{ kN}$$

and $$H_3' = \frac{(T_1 + T_2)\, c_2}{c} = \frac{(T_1 + T_2)\, c/2}{c} = \frac{T_1 + T_2}{2} = \frac{6.5}{2} = 3.25 \text{ kN}$$

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crankpin

Let d_c = Diameter of the crankpin in mm ;

l_c = Length of the crankpin in mm ; and

σ_b = Allowable bending stress for the crankpin. It may be assumed as 75 MPa or N/mm^2.

We know that the bending moment at the centre of the crankpin,

$$M_C = H_1 \cdot b_2 = 157.1 \times 400 = 62\,840 \text{ kN-mm} \quad ...(i)$$

We also know that

$$M_C = \frac{\pi}{32}(d_c)^3 \sigma_b = \frac{\pi}{32}(d_c)^3 75 = 7.364 (d_c)^3 \text{ N-mm}$$

$$= 7.364 \times 10^{-3}\,(d_c)^3 \text{ kN-mm} \quad ...(ii)$$

Equating equations (*i*) and (*ii*), we have

$$(d_c)^3 = 62\,840 / 7.364 \times 10^{-3} = 8.53 \times 10^6$$

or $$d_c = 204.35 \text{ say } 205 \text{ mm}$$ **Ans.**

We know that length of the crankpin,

$$l_c = \frac{F_P}{d_c \cdot p_b} = \frac{314.2 \times 10^3}{205 \times 10} = 153.3 \text{ say } 155 \text{ mm}$$ **Ans.**

...(Taking $p_b = 10$ N/mm^2)

(b) Design of left hand crank web

We know that thickness of the crank web,

$$t = 0.65\, d_c + 6.35 \text{ mm}$$

$$= 0.65 \times 205 + 6.35 = 139.6 \text{ say } 140 \text{ mm}$$ **Ans.**

and width of the crank web, $w = 1.125\, d_c + 12.7$ mm

$$= 1.125 \times 205 + 12.7 = 243.3 \text{ say } 245 \text{ mm} \quad \textbf{Ans.}$$

We know that maximum bending moment on the crank web,

$$M = H_1 \left(b_2 - \frac{l_c}{2} - \frac{t}{2} \right)$$

$$= 157.1 \left(400 - \frac{155}{2} - \frac{140}{2} \right) = 39\ 668 \text{ kN-mm}$$

Section modulus, $Z = \frac{1}{6} \times w.t^2 = \frac{1}{6} \times 245\,(140)^2 = 800 \times 10^3 \text{ mm}^3$

∴ Bending stress, $\sigma_b = \frac{M}{Z} = \frac{39\ 668}{800 \times 10^3} = 49.6 \times 10^{-3} \text{ kN/mm}^2 = 49.6 \text{ N/mm}^2$

We know that direct compressive stress on the crank web,

$$\sigma_c = \frac{H_1}{w.t} = \frac{157.1}{245 \times 140} = 4.58 \times 10^{-3} \text{ kN/mm}^2 = 4.58 \text{ N/mm}^2$$

∴ Total stress on the crank web

$$= \sigma_b + \sigma_c = 49.6 + 4.58 = 54.18 \text{ N/mm}^2 \text{ or MPa}$$

Since the total stress on the crank web is less than the allowable bending stress of 75 MPa, therefore, the design of the left hand crank web is safe.

(c) Design of right hand crank web

From the balancing point of view, the dimensions of the right hand crank web (*i.e.* thickness and width) are made equal to the dimensions of the left hand crank web.

(d) Design of shaft under the flywheel

Let d_s = Diameter of the shaft in mm.

Since the lengths of the main bearings are equal, therefore

$$l_1 = l_2 = l_3 = 2\left(\frac{b}{2} - \frac{l_c}{2} - t \right) = 2\left(400 - \frac{155}{2} - 140 \right) = 365 \text{ mm}$$

Assuming width of the flywheel as 300 mm, we have

$$c = 365 + 300 = 665 \text{ mm}$$

Hydrostatic transmission inside a tractor engine

Contents

Allowing space for gearing and clearance, let us take c = 800 mm.

$$\therefore \quad c_1 = c_2 = \frac{c}{2} = \frac{800}{2} = 400 \text{ mm}$$

We know that bending moment due to the weight of flywheel,

$$M_W = V_3 \cdot c_1 = 25 \times 400 = 10\,000 \text{ kN-mm} = 10 \times 10^6 \text{ N-mm}$$

and bending moment due to the belt pull,

$$M_T = H_3' \cdot c_1 = 3.25 \times 400 = 1300 \text{ kN-mm} = 1.3 \times 10^6 \text{ N-mm}$$

∴ Resultant bending moment on the shaft,

$$M_S = \sqrt{(M_W)^2 + (M_T)^2} = \sqrt{(10 \times 10^6)^2 + (1.3 \times 10^6)^2}$$

$$= 10.08 \times 10^6 \text{ N-mm}$$

We also know that bending moment on the shaft (M_S),

$$10.08 \times 10^6 = \frac{\pi}{32}(d_s)^3 \sigma_b = \frac{\pi}{32}(d_s)^3 42 = 4.12\,(d_s)^3$$

$$\therefore \quad (d_s)^3 = 10.08 \times 10^6 / 4.12 = 2.45 \times 10^6 \text{ or } d_s = 134.7 \text{ say } 135 \text{ mm}$$ **Ans.**

2. *Design of the crankshaft when the crank is at an angle of maximum twisting moment*

We know that piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p' = \frac{\pi}{4}(400)^2 1 = 125\,680 \text{ N} = 125.68 \text{ kN}$$

In order to find the thrust in the connecting rod (F_Q), we should first find out the angle of inclination of the connecting rod with the line of stroke (*i.e.* angle ϕ). We know that

$$\sin \phi = \frac{\sin \theta}{l / r} = \frac{\sin 35°}{5} = 0.1147$$

$$\therefore \quad \phi = \sin^{-1}(0.1147) = 6.58°$$

We know that thrust in the connecting rod,

$$F_Q = \frac{F_P}{\cos \phi} = \frac{125.68}{\cos 6.58°} = \frac{125.68}{0.9934} = 126.5 \text{ kN}$$

Tangential force acting on the crankshaft,

$$F_T = F_Q \sin(\theta + \phi) = 126.5 \sin(35° + 6.58°) = 84 \text{ kN}$$

and radial force,

$$F_R = F_Q \cos(\theta + \phi) = 126.5 \cos(35° + 6.58°) = 94.6 \text{ kN}$$

Due to the tangential force (F_T), there will be two reactions at bearings 1 and 2, such that

$$H_{T1} = \frac{F_T \times b_1}{b} = \frac{84 \times 400}{800} = 42 \text{ kN}$$

and

$$H_{T2} = \frac{F_T \times b_2}{b} = \frac{84 \times 400}{800} = 42 \text{ kN}$$

Due to the radial force (F_R), there will be two reactions at bearings 1 and 2, such that

$$H_{R1} = \frac{F_R \times b_1}{b} = \frac{94.6 \times 400}{800} = 47.3 \text{ kN}$$

$$H_{R2} = \frac{F_R \times b_2}{b} = \frac{94.6 \times 400}{800} = 47.3 \text{ kN}$$

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crankpin

Let d_c = Diameter of crankpin in mm.

Top

We know that the bending moment at the centre of the crankpin,

$$M_C = H_{R1} \times b_2 = 47.3 \times 400 = 18\,920 \text{ kN-mm}$$

and twisting moment on the crankpin,

$$T_C = H_{T1} \times r = 42 \times 300 = 12\,600 \text{ kN-mm}$$

∴ Equivalent twisting moment on the crankpin,

$$T_e = \sqrt{(M_C)^2 + (T_C)^2} = \sqrt{(18\,920)^2 + (12\,600)^2}$$

$$= 22\,740 \text{ kN-mm} = 22.74 \times 10^6 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$22.74 \times 10^6 = \frac{\pi}{16}(d_c)^3 \tau = \frac{\pi}{16}(d_c)^3 35 = 6.873\,(d_c)^3$$

...(Taking τ = 35 MPa or N/mm^2)

∴ $(d_c)^3 = 22.74 \times 10^6 / 6.873 = 3.3 \times 10^6$ or $d_c = 149$ mm

Since this value of crankpin diameter (*i.e.* d_c = 149 mm) is less than the already calculated value of d_c = 205 mm, therefore, we shall take d_c = 205 mm. **Ans.**

(b) Design of shaft under the flywheel

Let d_s = Diameter of the shaft in mm.

The resulting bending moment on the shaft will be same as calculated eariler, *i.e.*

$$M_S = 10.08 \times 10^6 \text{ N-mm}$$

and twisting moment on the shaft,

$$T_S = F_T \times r = 84 \times 300 = 25\,200 \text{ kN-mm} = 25.2 \times 10^6 \text{ N-mm}$$

∴ Equivalent twisting moment on shaft,

$$T_e = \sqrt{(M_S)^2 + (T_S)^2}$$

$$= \sqrt{(10.08 \times 10^6)^2 + (25.2 \times 10^6)^2} = 27.14 \times 10^6 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$27.14 \times 10^6 = \frac{\pi}{16}(d_s)^3 \tau = \frac{\pi}{16}(135)^3 \tau = 483\,156\,\tau$$

∴ $\tau = 27.14 \times 10^6 / 483\,156 = 56.17$ N/mm^2

From above, we see that by taking the already calculated value of *ds* = 135 mm, the induced shear stress is more than the allowable shear stress of 31 to 42 MPa. Hence, the value of d_s is calculated by taking τ = 35 MPa or N/mm^2 in the above equation, *i.e.*

$$27.14 \times 10^6 = \frac{\pi}{16}(d_s)^3 35 = 6.873\,(d_s)^3$$

∴ $(d_s)^3 = 27.14 \times 10^6 / 6.873 = 3.95 \times 10^6$ or d_s = 158 say 160 mm **Ans.**

(c) Design of shaft at the juncture of right hand crank arm

Let d_{s1} = Diameter of the shaft at the juncture of the right hand crank arm.

We know that the resultant force at the bearing 1,

$$R_1 = \sqrt{(H_{T1})^2 + (H_{R1})^2} = \sqrt{(42)^2 + (47.3)^2} = 63.3 \text{ kN}$$

∴ Bending moment at the juncture of the right hand crank arm,

$$M_{S1} = R_1\left(b_2 + \frac{l_c}{2} + \frac{t}{2}\right) - F_Q\left(\frac{l_c}{2} + \frac{t}{2}\right)$$

$$= 63.3\left(400 + \frac{155}{2} + \frac{140}{2}\right) - 126.5\left(\frac{155}{2} + \frac{140}{2}\right)$$

$$= 34.7 \times 10^3 - 18.7 \times 10^3 = 16 \times 10^3 \text{ kN-mm} = 16 \times 10^6 \text{ N-mm}$$

and twisting moment at the juncture of the right hand crank arm,

$$T_{S1} = F_T \times r = 84 \times 300 = 25\,200 \text{ kN-mm} = 25.2 \times 10^6 \text{ N-mm}$$

∴ Equivalent twisting moment at the juncture of the right hand crank arm,

$$T_e = \sqrt{(M_{S1})^2 + (T_{S1})^2}$$

$$= \sqrt{(16 \times 10^6)^2 + (25.2 \times 10^6)^2} = 29.85 \times 10^6 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$29.85 \times 10^6 = \frac{\pi}{16}(d_{s1})^3 \tau = \frac{\pi}{16}(d_{s1})^3 42 = 8.25\,(d_{s1})^3$$

...(Taking τ = 42 MPa or N/mm^2)

∴ $(d_{s1})^3 = 29.85 \times 10^6 / 8.25 = 3.62 \times 10^6$ or d_{s1} = 153.5 say 155 mm **Ans.**

(d) Design of right hand crank web

Let σ_{bR} = Bending stress in the radial direction ; and

σ_{bT} = Bending stress in the tangential direction.

We also know that bending moment due to the radial component of F_Q,

$$M_R = H_{R2}\left(b_1 - \frac{l_c}{2} - \frac{t}{2}\right) = 47.3\left(400 - \frac{155}{2} - \frac{140}{2}\right) \text{kN-mm}$$

$$= 11.94 \times 10^3 \text{ kN-mm} = 11.94 \times 10^6 \text{ N-mm} \quad \text{...}(i)$$

We also know that bending moment,

$$M_R = \sigma_{bR} \times Z = \sigma_{bR} \times \frac{1}{6} \times w.t^2 \quad \text{... }\left(\because Z = \frac{1}{6} \times w.t^2\right)$$

$$11.94 \times 10^6 = \sigma_{bR} \times \frac{1}{6} \times 245\,(140)^2 = 800 \times 10^3 \sigma_{bR}$$

∴ $\sigma_{bR} = 11.94 \times 10^6 / 800 \times 10^3 = 14.9$ N/mm^2 or MPa

We know that bending moment due to the tangential component of F_Q,

$$M_T = F_T\left(r - \frac{d_{s1}}{2}\right) = 84\left(300 - \frac{155}{2}\right) = 18\,690 \text{ kN-mm}$$

$$= 18.69 \times 10^6 \text{ N-mm}$$

We also know that bending moment,

$$M_T = \sigma_{bT} \times Z = \sigma_{bT} \times \frac{1}{6} \times t.w^2 \quad \text{... }\left(\because Z = \frac{1}{6} \times t.w^2\right)$$

$$18.69 \times 10^6 = \sigma_{bT} \times \frac{1}{6} \times 140\,(245)^2 = 1.4 \times 10^6 \sigma_{bT}$$

∴ $\sigma_{bT} = 18.69 \times 10^6 / 1.4 \times 10^6 = 13.35$ N/mm^2 or MPa

Direct compressive stress,

$$\sigma_b = \frac{F_R}{2w \cdot t} = \frac{94.6}{2 \times 245 \times 140} = 1.38 \times 10^{-3} \text{ kN/mm}^2 = 1.38 \text{ N/mm}^2$$

and total compressive stress,

$$\sigma_c = \sigma_{bR} + \sigma_{bT} + \sigma_d$$
$$= 14.9 + 13.35 + 1.38 = 29.63 \text{ N/mm}^2 \text{ or MPa}$$

We know that twisting moment on the arm,

$$T = H_{T2}\left(b_1 - \frac{l_c}{2}\right) = 42\left(400 - \frac{155}{2}\right) = 13\ 545 \text{ kN-mm}$$
$$= 13.545 \times 10^6 \text{ N-mm}$$

Piston and piston rod

and shear stress on the arm,

$$\tau = \frac{T}{Z_P} = \frac{4.5T}{w.t^2} = \frac{4.5 \times 13.545 \times 10^6}{245\ (140)^2} = 12.7 \text{ N/mm}^2 \text{ or MPa}$$

We know that total or maximum combined stress,

$$(\sigma_c)_{max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\tau^2}$$
$$= \frac{29.63}{2} + \frac{1}{2}\sqrt{(29.63)^2 + 4\ (12.7)^2} = 14.815 + 19.5 = 34.315 \text{ MPa}$$

Since the maximum combined stress is within the safe limits, therefore, the dimension w = 245 mm is accepted.

(e) Design of left hand crank web

The dimensions for the left hand crank web may be made same as for right hand crank web.

(f) Design of crankshaft bearings

Since the bearing 2 is the most heavily loaded, therefore, only this bearing should be checked for bearing pressure.

We know that the total reaction at bearing 2,

$$R_2 = \frac{F_P}{2} + \frac{W}{2} + \frac{T_1 + T_2}{2} = \frac{314.2}{2} + \frac{50}{2} + \frac{6.5}{2} = 185.35 \text{ kN} = 185\ 350 \text{ N}$$

∴ Total bearing pressure

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$$= \frac{R_2}{l_2 \cdot d_{s1}} = \frac{185\ 350}{365 \times 155} = 3.276 \text{ N/mm}^2$$

Since this bearing pressure is less than the safe limit of 5 to 8 N/mm^2, therefore, the design is safe.

Example 32.5. *Design a side or overhung crankshaft for a 250 mm × 300 mm gas engine. The weight of the flywheel is 30 kN and the explosion pressure is 2.1 N/mm^2. The gas pressure at the maximum torque is 0.9 N/mm^2, when the crank angle is 35° from I. D. C. The connecting rod is 4.5 times the crank radius.*

Solution. Given : D = 250 mm ; L = 300 mm or $r = L / 2 = 300 / 2$ = 150 mm ; W = 30 kN = 30 × 10^3 N ; p = 2.1 N/mm^2, p' = 0.9 N/mm^2 ; $l = 4.5\ r$ or $l / r = 4.5$

We shall design the crankshaft for the two positions of the crank, *i.e.* firstly when the crank is at the dead centre and secondly when the crank is at an angle of maximum twisting moment.


Valve guides of an IC engine

1. *Design of crankshaft when the crank is at the dead centre* (See Fig. 32.18)

We know that piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p$$

$$= \frac{\pi}{4}(250)^2 2.1 = 103 \times 10^3 \text{ N}$$

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crankpin

Let d_c = Diameter of the crankpin in mm, and

l_c = Length of the crankpin = 0.8 d_c ...(Assume)

Considering the crankpin in bearing, we have

$$F_P = d_c . l_c . p_b$$

$$103 \times 10^3 = d_c \times 0.8\ d_c \times 10 = 8\ (d_c)^2 \quad \text{...(Taking } p_b = 10 \text{ N/mm}^2\text{)}$$

$$\therefore \quad (d_c)^2 = 103 \times 10^3 / 8 = 12\ 875 \text{ or } d_c = 113.4 \text{ say } 115 \text{ mm}$$

and $l_c = 0.8\ d_c = 0.8 \times 115 = 92$ mm

Let us now check the induced bending stress in the crankpin.

We know that bending moment at the crankpin,

$$M = \frac{3}{4} F_P \times l_c = \frac{3}{4} \times 103 \times 10^3 \times 92 = 7107 \times 10^3 \text{ N-mm}$$

and section modulus of the crankpin,

$$Z = \frac{\pi}{32}(d_c)^3 = \frac{\pi}{32}(115)^3 = 149 \times 10^3 \text{mm}^3$$

∴ Bending stress induced

$$= \frac{M}{Z} = \frac{7107 \times 10^3}{149 \times 10^3} = 47.7 \text{ N/mm}^2 \text{ or MPa}$$

Since the induced bending stress is within the permissible limits of 60 MPa, therefore, design of crankpin is safe.

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(b) Design of bearings

Let d_1 = Diameter of the bearing 1.

Let us take thickness of the crank web,

$$t = 0.6\, d_c = 0.6 \times 115 = 69 \text{ or } 70 \text{ mm}$$

and length of the bearing, $l_1 = 1.7\, d_c = 1.7 \times 115 = 195.5$ say 200 mm

We know that bending moment at the centre of the bearing 1,

$$M = F_P\,(0.75 l_c + t + 0.5\, l_1)$$
$$= 103 \times 10^3\,(0.75 \times 92 + 70 + 0.5 \times 200) = 24.6 \times 10^6 \text{ N-mm}$$

We also know that bending moment (M),

$$24.6 \times 10^6 = \frac{\pi}{32}(d_1)^3 \sigma_b = \frac{\pi}{32}(d_1)^3 60 = 5.9\,(d_1)^3$$

...(Taking σ_b = 60 MPa or N/mm^2)

∴ $(d_1)^3 = 24.6 \times 10^6 / 5.9 = 4.2 \times 10^6$ or $d_1 = 161.3$ mm say 162 mm **Ans.**

(c) Design of crank web

Let w = Width of the crank web in mm.

We know that bending moment on the crank web,

$$M = F_P\,(0.75 l_c + 0.5\, t)$$
$$= 103 \times 10^3\,(0.75 \times 92 + 0.5 \times 70) = 10.7 \times 10^6 \text{ N-mm}$$

and section modulus, $$Z = \frac{1}{6} \times w \cdot t^2 = \frac{1}{6} \times w (70)^2 = 817\, w \text{ mm}^3$$

∴ Bending stress, $$\sigma_b = \frac{M}{Z} = \frac{10.7 \times 10^6}{817\, w} = \frac{13 \times 10^3}{w} \text{ N/mm}^2$$

and direct compressive stress,

$$\sigma_b = \frac{F_P}{w.t} = \frac{103 \times 10^3}{w \times 70} = \frac{1.47 \times 10^3}{w} \text{ N/mm}^2$$

We know that total stress on the crank web,

$$\sigma_T = \sigma_b + \sigma_d = \frac{13 \times 10^3}{w} + \frac{1.47 \times 10^3}{w} = \frac{14.47 \times 10^3}{w} \text{ N/mm}^2$$

The total stress should not exceed the permissible limit of 60 MPa or N/mm^2.

∴ $$60 = \frac{14.47 \times 10^3}{w} \text{ or } w = \frac{14.47 \times 10^3}{60} = 241 \text{ say } 245 \text{ mm}$$ **Ans.**

(d) Design of shaft under the flywheel.

Let d_s = Diameter of shaft under the flywheel.

First of all, let us find the horizontal and vertical reactions at bearings 1 and 2. Assume that the width of flywheel is 250 mm and $l_1 = l_2 = 200$ mm.

Allowing for certain clearance, the distance

$$b = 250 + \frac{l_1}{2} + \frac{l_2}{2} + \text{clearance}$$
$$= 250 + \frac{200}{2} + \frac{200}{2} + 20 = 470 \text{ mm}$$

and $$a = 0.75\, l_c + t + 0.5\, l_1$$
$$= 0.75 \times 92 + 70 + 0.5 \times 200 = 239 \text{ mm}$$

We know that the horizontal reactions H_1 and H_2 at bearings 1 and 2, due to the piston gas load (F_P) are

$$H_1 = \frac{F_P(a+b)}{b} = \frac{103 \times 10^3 (239 + 470)}{470} = 155.4 \times 10^3 \text{ N}$$

and $$H_2 = \frac{F_P \times a}{b} = \frac{103 \times 10^3 \times 239}{470} = 52.4 \times 10^3 \text{ N}$$

Assuming $b_1 = b_2 = b / 2$, the vertical reactions V_1 and V_2 at bearings 1 and 2 due to the weight of the flywheel are

$$V_1 = \frac{W \cdot b_1}{b} = \frac{W \times b/2}{b} = \frac{W}{2} = \frac{30 \times 10^3}{2} = 15 \times 10^3 \text{ N}$$

and $$V_2 = \frac{W \cdot b_2}{b} = \frac{W \times b/2}{b} = \frac{W}{2} = \frac{30 \times 10^3}{2} = 15 \times 10^3 \text{ N}$$

Since there is no belt tension, therefore the horizontal reactions due to the belt tension are neglected.

We know that horizontal bending moment at the flywheel location due to piston gas load.

$$M_1 = F_P(a + b_2) - H_1 \cdot b_2$$

$$= 103 \times 10^3 \left(239 + \frac{470}{2}\right) - 155.4 \times 10^3 \times \frac{470}{2} \qquad \ldots\left(\because\ b_2 = \frac{b}{2}\right)$$

$$= 48.8 \times 10^6 - 36.5 \times 10^6 = 12.3 \times 10^6 \text{ N-mm}$$

Since there is no belt pull, therefore, there will be no horizontal bending moment due to the belt pull, *i.e.* $M_2 = 0$.

∴ Total horizontal bending moment,

$$M_H = M_1 + M_2 = M_1 = 12.3 \times 10^6 \text{ N-mm}$$

We know that vertical bending moment due to the flywheel weight,

$$M_V = \frac{W . b_1 . b_2}{b} = \frac{W \times b \times b}{2 \times 2 \times b} = \frac{W \times b}{4}$$

$$= \frac{30 \times 10^3 \times 470}{4} = 3.525 \times 10^6 \text{ N-mm}$$

Inside view of a car engine

∴ Resultant bending moment,

$$M_R = \sqrt{(M_H)^2 + (M_V)^2} = \sqrt{(12.3 \times 10^6)^2 + (3.525 \times 10^6)^2}$$
$$= 12.8 \times 10^6 \text{ N-mm}$$

We know that bending moment (M_R),

$$12.8 \times 10^6 = \frac{\pi}{32}(d_s)^3 \sigma_b = \frac{\pi}{32}(d_s)^3\ 60 = 5.9(d_s)^3$$

∴ $(d_s)^3 = 12.8 \times 10^6 / 5.9 = 2.17 \times 10^6$ or $d_s = 129$ mm

Actually d_s should be more than d_1. Therefore let us take

$d_s = 200$ mm **Ans.**

2. *Design of crankshaft when the crank is at an angtle of maximum twisting moment*

We know that piston gas load,

$$F_P = \frac{\pi}{4} \times D^2 \times p' = \frac{\pi}{4}(250)^2\ 0.9 = 44\ 200\ \text{N}$$

In order to find the thrust in the connecting rod (F_Q), we should first find out the angle of inclination of the connecting rod with the line of storke (*i.e.* angle ϕ). We know that

$$\sin\phi = \frac{\sin\theta}{l/r} = \frac{\sin 35°}{4.5} = 0.1275$$

∴ $\phi = \sin^{-1}(0.1275) = 7.32°$

We know that thrust in the connecting rod,

$$F_Q = \frac{F_P}{\cos\phi} = \frac{44\ 200}{\cos 7.32°} = \frac{44\ 200}{0.9918} = 44\ 565\ \text{N}$$

Tangential force acting on the crankshaft,

$$F_T = F_Q \sin(\theta + \phi) = 44\ 565 \sin(35° + 7.32°) = 30 \times 10^3 \text{ N}$$

and radial force, $F_R = F_Q \cos(\theta + \phi) = 44\ 565 \cos(35° + 7.32°) = 33 \times 10^3$ N

Due to the tangential force (F_T), there will be two reactions at the bearings 1 and 2, such that

$$H_{T1} = \frac{F_T(a+b)}{b} = \frac{30 \times 10^3(239 + 470)}{470} = 45 \times 10^3\ \text{N}$$

and $$H_{T2} = \frac{F_T \times a}{b} = \frac{30 \times 10^3 \times 239}{470} = 15.3 \times 10^3\ \text{N}$$

Due to the radial force (F_R), there will be two reactions at the bearings 1 and 2, such that

$$H_{R1} = \frac{F_R(a+b)}{b} = \frac{33 \times 10^3 \times (239 + 470)}{470} = 49.8 \times 10^3\ \text{N}$$

and $$H_{R2} = \frac{F_R \times a}{b} = \frac{33 \times 10^3 \times 239}{470} = 16.8 \times 10^3\ \text{N}$$

Now the various parts of the crankshaft are designed as discussed below:

(a) Design of crank web

We know that bending moment due to the tangential force,

$$M_{bT} = F_T\left(r - \frac{d_1}{2}\right) = 30 \times 10^3\left(150 - \frac{180}{2}\right) = 1.8 \times 10^6\ \text{N-mm}$$

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∴ Bending stress due to the tangential force,

$$\sigma_{bT} = \frac{M_{bT}}{Z} = \frac{6M_{bT}}{t.w^2} = \frac{6 \times 1.8 \times 10^6}{70\,(245)^2} \quad \ldots \left(\because Z = \frac{1}{6} \times t.w^2\right)$$

$$= 2.6 \text{ N/mm}^2 \text{ or MPa}$$

Bending moment due to the radial force,

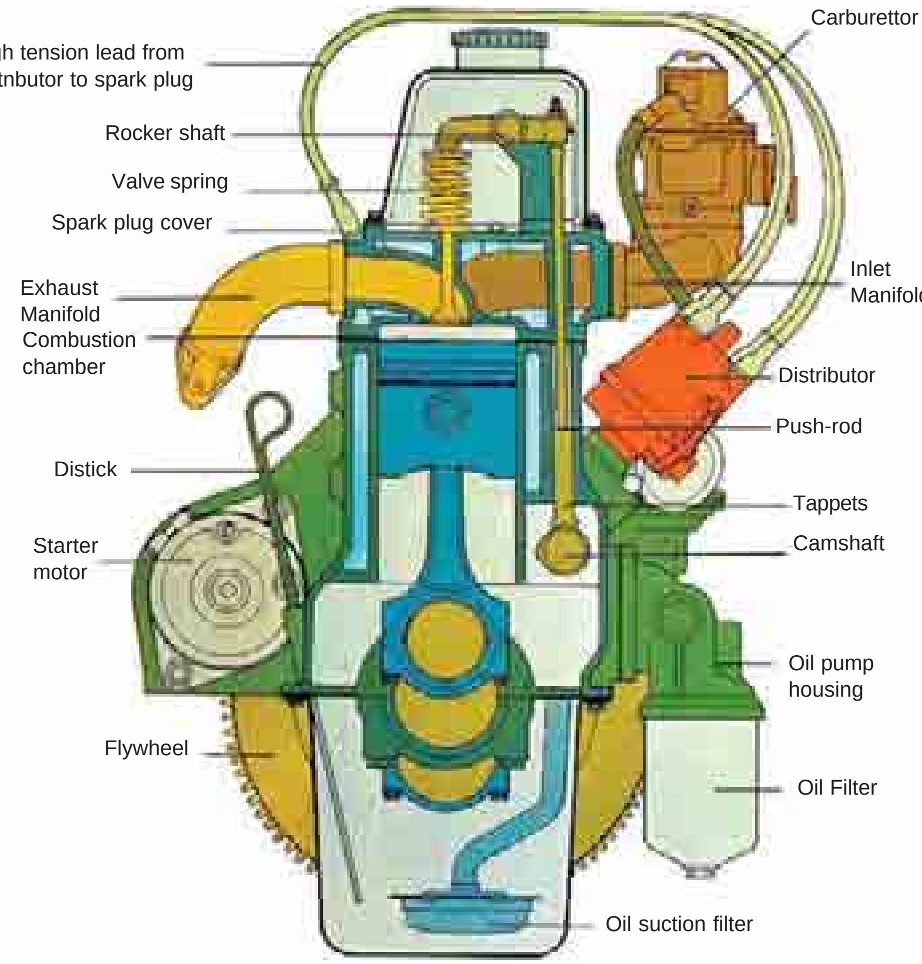
$$M_{bR} = F_R\,(0.75\, l_c + 0.5\, t)$$

$$= 33 \times 10^3\,(0.75 \times 92 + 0.5 \times 70) = 3.43 \times 10^6 \text{ N-mm}$$

∴ Bending stress due to the radial force,

$$\sigma_{bR} = \frac{M_{bR}}{Z} = \frac{6\, M_{bR}}{w.t^2} \quad \ldots \left(\because Z = \frac{1}{6} \times w.t^2\right)$$

$$= \frac{6 \times 3.43 \times 10^6}{245\,(70)^2} = 17.1 \text{ N/mm}^2 \text{ or MPa}$$



Schematic of a 4 cylinder IC engine

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We know that direct compressive stress,

$$\sigma_d = \frac{F_R}{w \cdot t} = \frac{33 \times 10^3}{245 \times 70} = 1.9 \text{ N/mm}^2 \text{ or MPa}$$

∴ Total compressive stress,

$$\sigma_c = \sigma_{bT} + \sigma_{bR} + \sigma_d = 2.6 + 17.1 + 1.9 = 21.6 \text{ MPa}$$

We know that twisting moment due to the tangential force,

$$T = F_T (0.75\, l_c + 0.5\, t)$$
$$= 30 \times 10^3 (0.75 \times 92 + 0.5 \times 70) = 3.12 \times 10^6 \text{ N-mm}$$

∴ Shear stress,

$$\tau = \frac{T}{Z_P} = \frac{4.5\, T}{w \cdot t^2} = \frac{4.5 \times 3.12 \times 10^6}{245\,(70)^2} \quad \ldots\left[\because Z_P = \frac{w \cdot t^2}{4.5}\right]$$
$$= 11.7 \text{ N/mm}^2 \text{ or MPa}$$

We know that total or maximum stress,

$$\sigma_{max} = \frac{\sigma_c}{2} + \frac{1}{2}\sqrt{(\sigma_c)^2 + 4\,\tau^2} = \frac{21.6}{2} + \frac{1}{2}\sqrt{(21.6)^2 + 4(11.7)^2}$$
$$= 10.8 + 15.9 = 26.7 \text{ MPa}$$

Since this stress is less than the permissible value of 60 MPa, therefore, the design is safe.

(b) Design of shaft at the junction of crank

Let d_{s1} = Diameter of shaft at the junction of crank.

We know that bending moment at the junction of crank,

$$M = F_Q (0.75 l_c + t) = 44\,565\,(0.75 \times 92 + 70) = 6.2 \times 10^6 \text{ N-mm}$$

and twisting moment, $T = F_T \times r = 30 \times 10^3 \times 150 = 4.5 \times 10^6$ N-mm

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(6.2 \times 10^6)^2 + (4.5 \times 10^6)^2} = 7.66 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$7.66 \times 10^6 = \frac{\pi}{16}(d_{s1})^3 \tau = \frac{\pi}{16}(180)^3 \tau = 1.14 \times 10^6 \tau \quad \ldots(\text{Taking } d_{s1} = d_1)$$

∴ $\tau = 7.66 \times 10^6 / 1.14 \times 10^6 = 6.72 \text{ N/mm}^2$ or MPa

Since the induced shear stress is less than the permissible limit of 30 to 40 MPa, therefore, the design is safe.

(c) Design of shaft under the flywheel

Let d_s = Diameter of shaft under the flywheel.

We know that horizontal bending moment acting on the shaft due to piston gas load,

$$M_H = F_P\,(a + b_2) - \left[\sqrt{(H_{R1})^2 + (H_{T1})^2}\right] b_2$$
$$= 44\,200\left(239 + \frac{470}{2}\right) - \left[\sqrt{(49.8 \times 10^3)^2 + (45 \times 10^3)^2}\right]\frac{470}{2}$$
$$= 20.95 \times 10^6 - 15.77 \times 10^6 = 5.18 \times 10^6 \text{ N-mm}$$

and bending moment due to the flywheel weight

$$M_V = \frac{W.b_1.b_2}{b} = \frac{30 \times 10^3 \times 235 \times 235}{470} = 3.53 \times 10^6 \text{ N-mm}$$

...($b_1 = b_2 = b / 2 = 470 / 2 = 235$ mm)

∴ Resultant bending moment,

$$M_R = \sqrt{(M_H)^2 + (M_V)^2} = \sqrt{(5.18 \times 10^6)^2 + (3.53 \times 10^6)^2}$$

$$= 6.27 \times 10^6 \text{ N-mm}$$

We know that twisting moment on the shaft,

$$T = F_T \times r = 30 \times 10^3 \times 150 = 4.5 \times 10^6 \text{ N-mm}$$

∴ Equivalent twisting moment,

$$T_e = \sqrt{(M_R)^2 + T^2} = \sqrt{(6.27 \times 10^6)^2 + (4.5 \times 10^6)^2}$$

$$= 7.72 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$7.72 \times 10^6 = \frac{\pi}{16}(d_s)^3 \tau = \frac{\pi}{16}(d_s)^3\ 30 = 5.9\,(d_s)^3 \quad \text{...(Taking } \tau = 30 \text{ MPa)}$$

∴ $(d_s)^3 = 7.72 \times 10^6 / 5.9 = 1.31 \times 10^6$ or $d_s = 109$ mm

Actually, d_s should be more than d_1. Therefore let us take

$d_s = 200$ mm **Ans.**

32.22 Valve Gear Mechanism

The valve gear mechanism of an I.C. engine consists of those parts which actuate the inlet and exhaust valves at the required time with respect to the position of piston and crankshaft.Fig .32.20 (*a*) shows the valve gear arrangement for vertical engines. The main components of the mechanism are valves, rocker arm, * valve springs, **push rod, ***cam and camshaft.

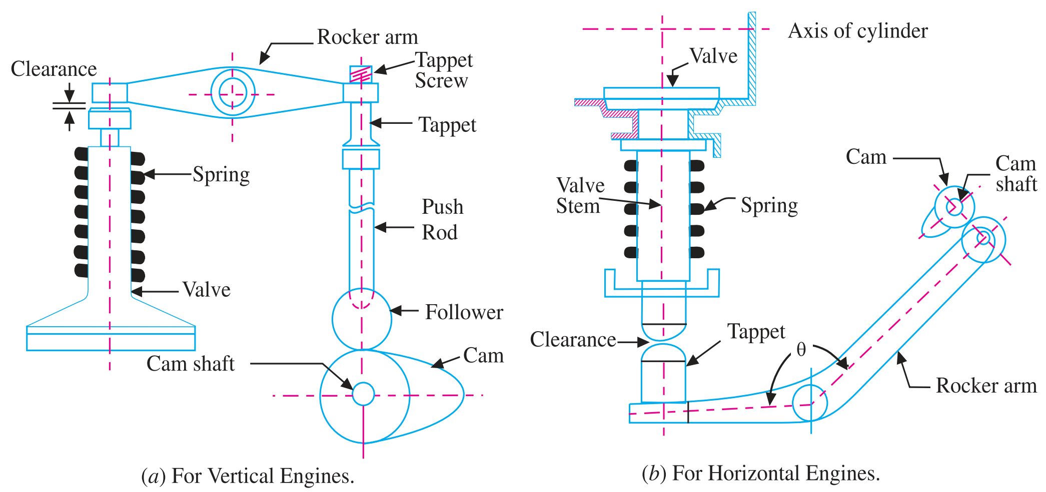


Fig. 32.20. Valve gear mechanism.

* For the design of springs, refer Chapter 23.

** For the design of push rod, refer Chapter 16 (Art. 16.14).

*** For the design of cams, refer to Authors' popular book on **'Theory of Machines'.**

The fuel is admitted to the engine by the inlet valve and the burnt gases are escaped through the exhaust valve. In vertical engines, the cam moving on the rotating camshaft pushes the cam follower and push rod upwards, thereby transmitting the cam action to rocker arm. The camshaft is rotated by the toothed belt from the crankshaft. The rocker arm is pivoted at its centre by a fulcrum pin. When one end of the rocker arm is pushed up by the push rod, the other end moves downward. This pushes down the valve stem causing the valve to move down, thereby opening the port. When the cam follower moves over the circular portion of cam, the pushing action of the rocker arm on the valve is released and the valve returns to its seat and closes it by the action of the valve spring.

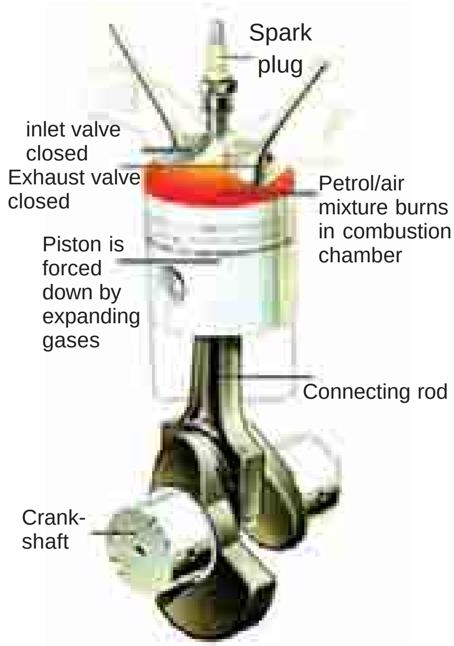
In some of the modern engines, the camshaft is located at cylinder head level. In such cases, the push rod is eliminated and the roller type cam follower is made part of the rocker arm. Such an arrangement for the horizontal engines is shown in Fig. 32.20 (*b*).

32.23 Valves

The valves used in internal combustion engines are of the following three types ;

1. Poppet or mushroom valve ; **2.** Sleeve valve ; **3.** Rotary valve.

Out of these three valves, poppet valve, as shown in Fig. 32.21, is very frequently used. It consists of head, face and stem. The head and face of the valve is separated by a small margin, to aviod sharp edge of the valve and also to provide provision for the regrinding of the face. The face angle generally varies from 30° to 45°. The lower part of the stem is provided with a groove in which spring retainer lock is installed.



piston and crankshaft

Since both the inlet and exhaust valves are subjected to high temperatures of 1930°C to 2200°C during the power stroke, therefore, it is necessary that the material of the valves should withstand these temperatures. Thus the material of the valves must have good heat conduction, heat resistance, corrosion resistance, wear resistance and shock resistance. It may be noted that the temperature at the inlet valve is less as compared to exhaust valve. Thus, the inlet valve is generally made of nickel chromium alloy steel and the exhaust valve (which is subjected to very high temperature of exhaust gases) is made from silchrome steel which is a special alloy of silicon and chromium.

In designing a valve, it is required to determine the following dimensions:

(a) Size of the valve port

Let a_p = Area of the port,

v_p = Mean velocity of gas flowing through the port,

a = Area of the piston, and

v = Mean velocity of the piston.

We know that $a_p . v_p = a.v$

$$\therefore \quad a_p = \frac{a.v}{v_p}$$

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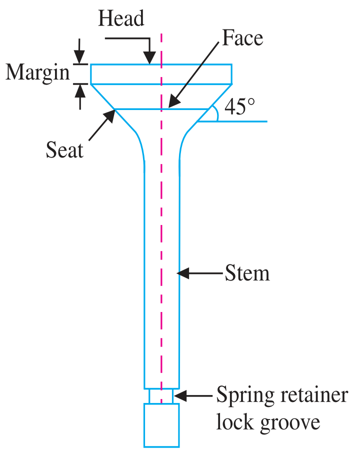

Fig. 32.21. Poppet or mushroom valve.

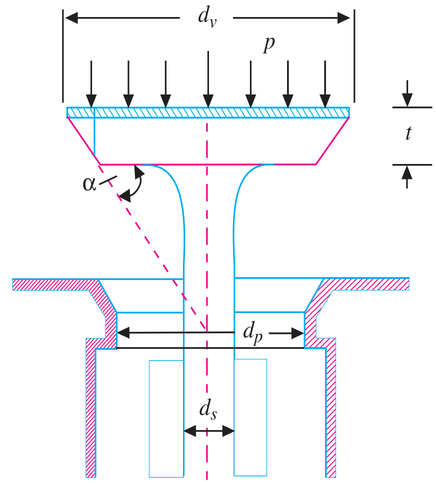

Fig. 32.22. Conical poppet valve in the port.

The mean velocity of the gas (v_p) may be taken from the following table.

Table 32.3. Mean velocity of the gas (v_p)

Type of engine	*Mean velocity of the gas* (v_p) m/s	
	Inlet valve	*Exhaust valve*
Low speed	33 – 40	40 – 50
High speed	80 – 90	90 – 100

Sometimes, inlet port is made 20 to 40 precent larger than exhaust port for better cylinder charging.

(b) Thickness of the valve disc

The thickness of the valve disc (t), as shown in Fig. 32.22, may be determined empirically from the following relation, *i.e.*

$$t = k.d_p \sqrt{\frac{p}{\sigma_b}}$$

where k = Constant = 0.42 for steel and 0.54 for cast iron,

d_p = Diameter of the port in mm,

p = Maximum gas pressure in N/mm^2, and

σ_b = Permissible bending stress in MPa or N/mm^2

= 50 to 60 MPa for carbon steel and 100 to 120 MPa for alloy steel.

(c) Maximum lift of the valve

h = Lift of the valve.

The lift of the valve may be obtained by equating the area across the valve seat to the area of the port. For a conical valve, as shown in Fig. 32.22, we have

$$\pi\, d_p \cdot h \cos\alpha = \frac{\pi}{4}(d_p)^2 \quad \text{or} \quad h = \frac{d_p}{4\cos\alpha}$$

where α = Angle at which the valve seat is tapered = 30° to 45°.

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In case of flat headed valve, the lift of valve is given by

$$h = \frac{d_p}{4} \quad \text{...(In this case, } \alpha = 0°\text{)}$$

The valve seats usually have the same angle as the valve seating surface. But it is preferable to make the angle of valve seat 1/2° to 1° larger than the valve angle as shown in Fig. 32.23. This results in more effective seat.

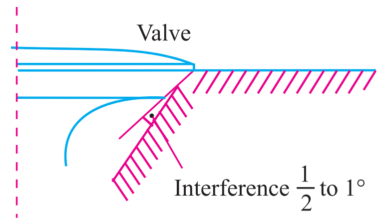

Fig. 32.23. Valve interference angle.

(d) Valve stem diameter

The valve stem diameter (d_s) is given by

$$d_s = \frac{d_p}{8} + 6.35 \text{ mm to } \frac{d_p}{8} + 11 \text{ mm}$$

Note: The valve is subjected to spring force which is taken as concentrated load at the centre. Due to this spring force (F_s), the stress in the valve (σ_t) is given by

$$\sigma_t = \frac{1.4 F_s}{t^2}\left(1 - \frac{2d_s}{3d_p}\right)$$

Example 32.6. *The conical valve of an I.C. engine is 60 mm in diameter and is subjected to a maximum gas pressure of 4 N/mm². The safe stress in bending for the valve material is 46 MPa. The valve is made of steel for which k = 0.42. The angle at which the valve disc seat is tapered is 30°.*

Determine : 1. thickness of the valve head ; 2. stem diameter ; and 3. maximum lift of the valve.

Solution. Given : $d_p = 60$ mm ; $p = 4$ N/mm² ; $\sigma_b = 46$ MPa = 46 N/mm² ; $k = 0.42$; $\alpha = 30°$

1. Thickness of the valve head

We know that thickness of the valve head,

$$t = k \cdot d_p \sqrt{\frac{p}{\sigma_b}} = 0.42 \times 60 \sqrt{\frac{4}{46}} = 7.43 \text{ say } 7.5 \text{ mm } \textbf{Ans.}$$

2. Stem diameter

We know that stem diameter,

$$d_s = \frac{d_p}{8} + 6.35 = \frac{60}{8} + 6.35 = 13.85 \text{ say } 14 \text{ mm } \textbf{Ans.}$$

3. Maximum lift of the valve

We know that maximum lift of the valve,

$$h = \frac{d_p}{4 \cos \alpha} = \frac{60}{4 \cos 30°} = \frac{60}{4 \times 0.866} = 17.32 \text{ say } 17.4 \text{ mm } \textbf{Ans.}$$

32.24 Rocker Arm

The * rocker arm is used to actuate the inlet and exhaust valves motion as directed by the cam and follower. It may be made of cast iron, cast steel, or malleable iron. In order to reduce inertia of the rocker arm, an *I*-section is used for the high speed engines and it may be rectangular section for low speed engines. In four stroke engines, the rocker arms for the exhaust valve is the most heavily loaded. Though the force required to operate the inlet valve is relatively small, yet it is usual practice to make the rocker

Roller followers in an engine rocker mechanism

* The rocker arm has also been discussed in Chapter 15 on Levers (Refer Art. 15.9).

arm for the inlet valve of the same dimensions as that for exhaust valve. A typical rocker arm for operating the exhaust valve is shown in Fig. 32.24. The lever ratio a / b is generally decided by considering the space available for rocker arm. For moderate and low speed engines, a / b is equal to one. For high speed engines, the ratio a / b is taken as 1/ 1.3. The various forces acting on the rocker arm of exhaust valve are the gas load, spring force and force due to valve acceleration.

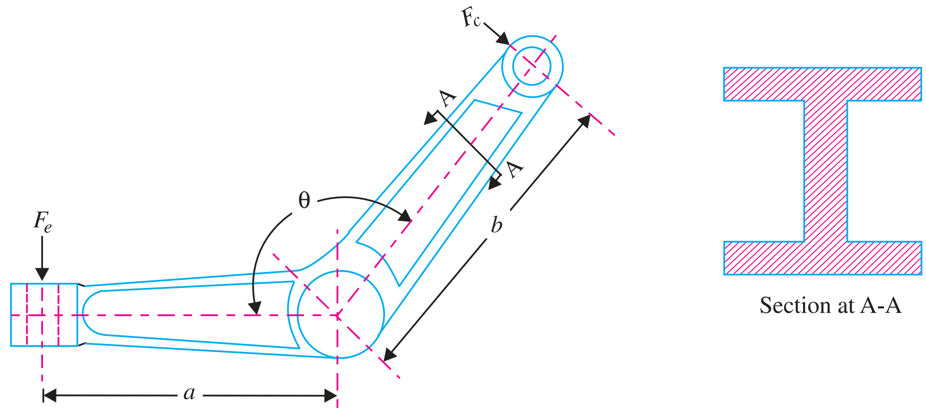

Fig. 32.24. Rocker arm for exhaust valve.

Let m_v = Mass of the valve,

d_v = Diameter of the valve head,

h = Lift of the valve,

a = Acceleration of the valve,

p_c = Cylinder pressure or back pressure when the exhust valve opens, and

p_s = Maximum suction pressure.

We know that gas load,

P = Area of valve × Cylinder pressure when the exhaust valve opens

$$= \frac{\pi}{4} (d_v)^2 \, p_c$$

Spring force, F_s = Area of valve × Maximum suction pressure

$$= \frac{\pi}{4} (d_v)^2 \, p_s$$

and force due to valve acceleration,

F_{va} = Mass of valve × Accleration of valve

$= m_v \times a$

∴ Maximum load on the rocker arm for exhaust valve,

$$F_e = P + F_s + F_{va}$$

It may be noted that maximum load on the rocker arm for inlet valve is

$$F_i = F_s + F_{va}$$

Since the maximum load on the rocker arm for exhaust valve is more than that of inlet valve, therefore, the rocker arm must be designed on the basis of maximum load on the rocker arm for exhaust valve, as discussed below :

1. *Design for fulcrum pin.* The load acting on the fulcrum pin is the total reaction (R_F) at the fulcrum point.

Let d_1 = Diameter of the fulcrum pin, and

l_1 = Length of the fulcrum pin.

Considering the bearing of the fulcrum pin. We know that load on the fulcrum pin,

$$R_F = d_1 \cdot l_1 \cdot p_b$$

The ratio of l_1 / d_1 is taken as 1.25 and the bearing pressure (p_b) for ordinary lubrication is taken from 3.5 to 6 N / mm^2 and it may go upto 10.5 N/mm^2 for forced lubrication.

The pin should be checked for the induced shear stress.

The thickness of the phosphor bronze bush may be taken from 2 to 4 mm. The outside diameter of the boss at the fulcrum is usually taken twice the diameter of the fulcrum pin.

2. *Design for forked end.* The forked end of the rocker arm carries a roller by means of a pin. For uniform wear, the roller should revolve in the eyes. The load acting on the roller pin is F_c.

Let d_2 = Diameter of the roller pin, and

l_2 = Length of the roller pin.

Consiering the bearing of the roller pin. We know that load on the roller pin,

$$F_c = d_2 \cdot l_2 \cdot p_b$$

The ratio of l_2 / d_2 may be taken as 1.25. The roller pin should be checked for induced shear stesss.

The roller pin is fixed in eye and the thickness of each eye is taken as half the length of the roller pin.

$\therefore$ Thickness of each eye $= l_2 / 2$

The radial thickness of eye (t_3) is taken as $d_1 / 2$. Therefore overall diameter of the eye,

$$D_1 = 2\, d_1$$

The outer diameter of the roller is taken slightly larger (atleast 3 mm more) than the outer diameter of the eye.

A clearance of 1.5 mm between the roller and the fork on either side of the roller is provided.

3. *Design for rocker arm cross-section.* The rocker arm may be treated as a simply supported beam and loaded at the fulcrum point. We have already discussed that the rocker arm is generally of *I*-section but for low speed engines, it can be of rectangular section. Due to the load on the valve, the rocker arm is subjected to bending moment.

Let l = Effective length of each rocker arm, and

σ_b = Permissible bending stress.

We know that bending moment on the rocker arm,

$$M = F_e \times l \qquad ...(i)$$

We also know that bending moment,

$$M = \sigma_b \times Z \qquad ...(ii)$$

where Z = Section modulus.

From equations (*i*) and (*ii*), the value of Z is obtained and thus the dimensions of the section are determined.

4. *Design for tappet.* The tappet end of the rocker arm is made circular to receive the tappet which is a stud with a lock nut. The compressive load acting on the tappet is the maximum load on the rocker arm for the exhaust valve (F_e).

Let d_c = Core diameter of the tappet, and

σ_c = Permissible compressive stress for the material of the tappet which is made of mild steel. It may be taken as 50 MPa.

We know that load on the tappet,

$$F_e = \frac{\pi}{4} (d_c)^2 \sigma_c$$

From this expression, the core diameter of the tappet is determined. The outer or nominal diameter of the tappet (d_n) is given as

$$d_n = d_c / 0.84$$

The diameter of the circular end of the rocker arm (D_3) and its depth (t_4) is taken as twice the nominal diameter of the tappet (d_n), *i.e.*

$$D_3 = 2\, d_n \; ; \text{ and } \; t_4 = 2\, d_n$$

5. *Design for valve spring.* The valve spring is used to provide sufficient force during the valve lifting process in order to overcome the inertia of valve gear and to keep it with the cam without bouncing. The spring is generally made from plain carbon spring steel. The total load for which the spring is designed is equal to the sum of initial load and load at full lift.

Let W_1 = Initial load on the spring

= Force on the valve tending to draw it into the cylinder on suction stroke,

W_2 = Load at full lift

= Full lift × Stiffness of spring

∴ Total load on the spring,

$$W = W_1 + W_2$$

Note : Here we are only interested in calculating the total load on the spring. The design of the valve spring is done in the similar ways as discussed for compression springs in Chapter 23 on Springs.

Example 32.7. *Design a rocker arm, and its bearings, tappet, roller and valve spring for the exhaust valve of a four stroke I.C. engine from the following data:*

Diameter of the valve head = 80 mm; Lift of the valve = 25 mm; Mass of associated parts with the valve = 0.4 kg ; Angle of action of camshaft = 110° ; R. P. M. of the crankshaft = 1500.

From the probable indicator diagram, it has been observed that the greatest back pressure when the exhaust valve opens is 0.4 N/mm^2 and the greatest suction pressure is 0.02 N/mm^2 below atmosphere.

The rocker arm is to be of I-section and the effective length of each arm may be taken as 180 mm ; the angle between the two arms being 135°.

The motion of the valve may be assumed S.H.M., without dwell in fully open position.

Choose your own materials and suitable values for the stresses.

Draw fully dimensioned sketches of the valve gear.

Solution. Given : d_v = 80 mm ; h = 25 mm ; or r = 25 / 2 = 12.5 mm = 0.0125 m ; m = 0.4 kg ; α = 110° ; N = 1500 r.p.m. ; p_c = 0.4 N/mm^2 ; p_s = 0.02 N/mm^2 ; l = 180 mm ; θ = 135°

A rocker arm for operating the exhaust valve is shown in Fig. 32.25.

First of all, let us find the various forces acting on the rocker arm of the exhaust valve.

We know that gas load on the valve,

$$P_1 = \frac{\pi}{4}(d_v)^2\, p_c = \frac{\pi}{4}(80)^2\; 0.4 = 2011 \text{ N}$$

Weight of associated parts with the valve,

$$w = m \cdot g = 0.4 \times 9.8 = 3.92 \text{ N}$$

∴ Total load on the valve,

$$P = P_1 + w = 2011 + 3.92 = 2014.92 \text{ N} \quad \text{...(i)}$$

Initial spring force considering weight of the valve,

$$F_s = \frac{\pi}{4}(d_v)^2\, p_s - w = \frac{\pi}{4}(80)^2\; 0.02 - 3.92 = 96.6 \text{ N} \quad \text{...(ii)}$$

The force due to valve acceleration (F_a) may be obtained as discussed below :

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We know that speed of camshaft

$$= \frac{N}{2} = \frac{1500}{2} = 750 \text{ r.p.m.}$$

and angle turned by the camshaft per second

$$= \frac{750}{60} \times 360 = 4500 \text{ deg / s}$$

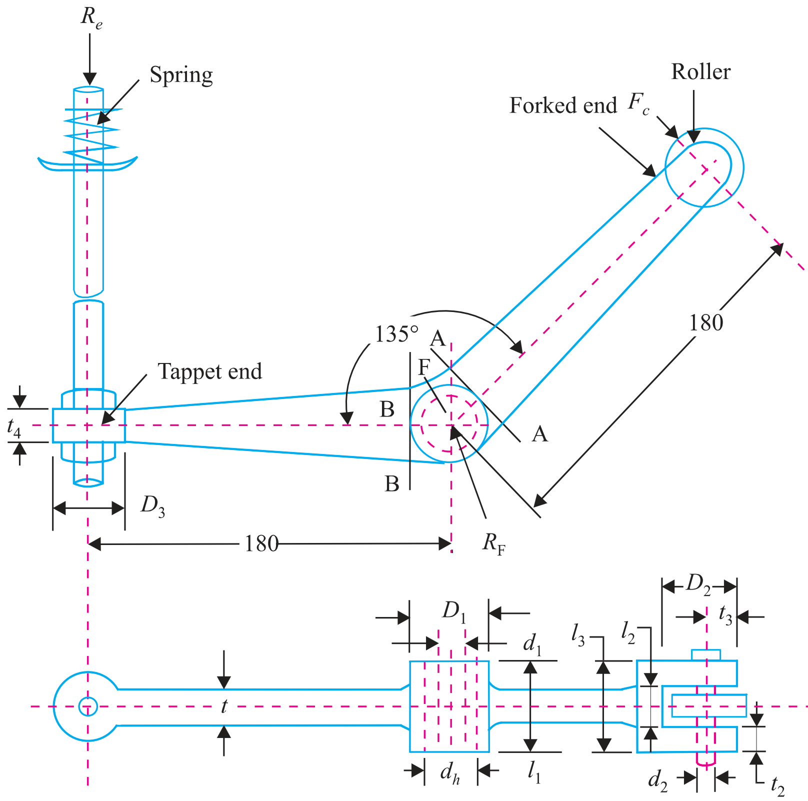

Fig. 32.25

∴ Time taken for the valve to open and close,

$$t = \frac{\text{Angle of action of cam}}{\text{Angle turned by camshaft}} = \frac{110}{4500} = 0.024 \text{ s}$$

We know that maximum acceleration of the valve

$$a = \omega^2 \cdot r = \left(\frac{2\pi}{t}\right)^2 r = \left(\frac{2\pi}{0.024}\right)^2 0.0125 = 857 \text{ m / s}^2 \quad \ldots \left(\because \omega = \frac{2\pi}{t}\right)$$

∴ Force due to valve acceleration, considering the weight of the valve,

$$F_a = m \cdot a + w = 0.4 \times 857 + 3.92 = 346.72 \text{ N} \quad \ldots (iii)$$

and maximum load on the rocker arm for exhaust valve,

$$F_e = P + F_s + F_a = 2014.92 + 96.6 + 346.72 = 2458.24 \text{ say } 2460 \text{ N}$$

Since the length of the two arms of the rocker are equal, therefore, the load at the two ends of the arm are equal, *i.e.* $F_e = F_c = 2460$ N.

Top

Front view of a racing car

We know that reaction at the fulcrum pin *F*,

$$R_F = \sqrt{(F_e)^2 + (F_c)^2 - 2\,F_e \times F_c \times \cos\theta}$$

$$= \sqrt{(2460)^2 + (2460)^2 - 2 \times 2460 \times 2460 \times \cos 135^\circ} = 4545 \text{ N}$$

Let us now design the various parts of the rocker arm.

1. *Design of fulcrum pin*

Let d_1 = Diameter of the fulcrum pin, and

l_1 = Length of the fulcrum pin = $1.25\, d_1$ ...(Assume)

Considering the bearing of the fulcrum pin. We know that load on the fulcrum pin (R_F),

$$4545 = d_1 \times l_1 \times p_b = d_1 \times 1.25\, d_1 \times 5 = 6.25\,(d_1)^2$$

...(For ordinary lubrication, p_b is taken as 5 N/mm²)

$\therefore$ $(d_1)^2 = 4545 / 6.25 = 727$ or $d_1 = 26.97$ say 30 mm **Ans.**

and $l_1 = 1.25\, d_1 = 1.25 \times 30 = 37.5$ mm **Ans.**

Now let us check the average shear stress induced in the pin. Since the pin is in double shear, therefore, load on the fulcrum pin (R_F),

$$4545 = 2 \times \frac{\pi}{4}(d_1)^2 \tau = 2 \times \frac{\pi}{4}(30)^2 \tau = 1414\,\tau$$

$\therefore$ $\tau = 4545 / 1414 = 3.2$ N/mm² or MPa

This induced shear stress is quite safe.

Now external diameter of the boss,

$$D_1 = 2d_1 = 2 \times 30 = 60 \text{ mm}$$

Assuming a phosphor bronze bush of 3 mm thick, the internal diameter of the hole in the lever,

$$d_h = d_1 + 2 \times 3 = 30 + 6 = 36 \text{ mm}$$

Let us now check the induced bending stress for the section of the boss at the fulcrum which is shown in Fig. 32.26.

Contents

Bending moment at this section,

$$M = F_e \times l = 2460 \times 180 = 443 \times 10^3 \text{ N-mm}$$

Section modulus,

$$Z = \frac{\frac{1}{12} \times 37.5\,[(60)^3 - (36)^3]}{60/2} = 17\,640 \text{ mm}^3$$

∴ Induced bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{443 \times 10^3}{17\,640} = 25.1 \text{ N/mm}^2 \text{ or MPa}$$

The induced bending stress is quite safe.

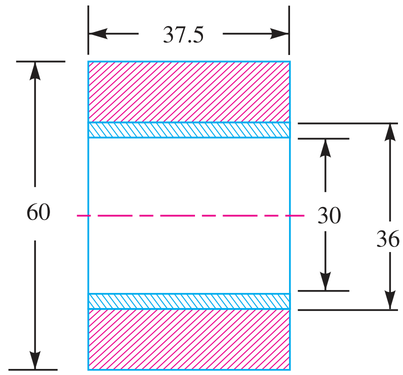

Fig. 32.26

2. *Design for forked end*

Let d_2 = Diameter of the roller pin,

and

l_2 = Length of the roller pin

$= 1.25\, d_1$ ...(Assume)

Considering bearing of the roller pin. We know that load on the roller pin (F_c),

$$2460 = d_2 \times l_2 \times p_b = d_2 \times 1.25\, d_2 \times 7 = 8.75\,(d_2)^2$$

... (Taking $p_b = 7$ N / mm^2)

∴ $(d_2)^2 = 2460 / 8.75 = 281$ or $d_2 = 16.76$ say 18 mm **Ans.**

and $l_2 = 1.25\, d_2 = 1.25 \times 18 = 22.5$ say 24 mm **Ans.**

Let us now check the roller pin for induced shearing stress. Since the pin is in double shear, therefore, load on the roller pin (F_c),

$$2460 = 2 \times \frac{\pi}{4}\,(d_2)^2\,\tau = 2 \times \frac{\pi}{4}\,(18)^2\,\tau = 509\,\tau$$

∴ $\tau = 2460 / 509 = 4.83$ N/mm^2 or MPa

This induced shear stress is quite safe.

The roller pin is fixed in the eye and thickenss of each eye is taken as one-half the length of the roller pin.

∴ Thickness of each eye,

$$t_2 = \frac{l_2}{2} = \frac{24}{2} = 12 \text{ mm}$$

Let us now theck the induced bending stress in the roller pin. The pin is neither simply supported in fork nor rigidly fixed at the end. Therefore, the common practice is to assume the load distrubution as shown in Fig. 32.27.

The maximum bending moment will occur at Y–Y. Neglecting the effect of clearance, we have

Maximum bending moment at $Y - Y$,

$$M = \frac{F_c}{2}\left(\frac{l_2}{2} + \frac{t_2}{3}\right) - \frac{F_c}{2} \times \frac{l_2}{4}$$

$$= \frac{F_c}{2}\left(\frac{l_2}{2} + \frac{l_2}{6}\right) - \frac{F_c}{2} \times \frac{l_2}{4} \qquad ...(\because\ t_2 = l_2 / 2)$$

$$= \frac{5}{24} \times F_c \times l_2 = \frac{5}{24} \times 2460 \times 24$$

$= 12\,300$ N–mm

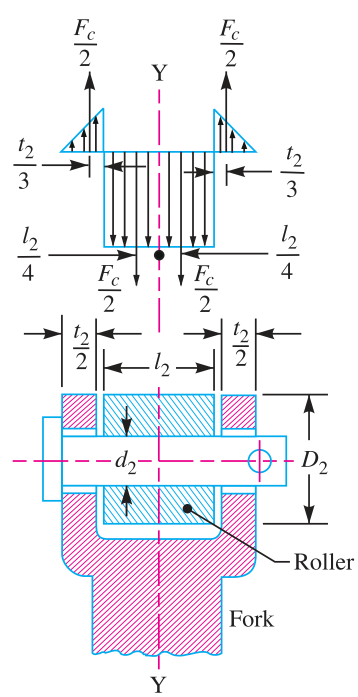

Fig. 32.27

Top

and section modulus of the pin,

$$Z = \frac{\pi}{32}(d_2)^3 = \frac{\pi}{32}(18)^3 = 573 \text{ mm}^3$$

∴ Bending stress induced in the pin

$$= \frac{M}{Z} = \frac{12\,300}{573} = 21.5 \text{ N/mm}^2 \text{ or MPa}$$

This bending stress induced in the pin is within permissible limits.

Since the radial thickness of eye (t_3) is taken as $d_2 / 2$, therefore, overall diameter of the eye,

$$D_2 = 2\, d_2 = 2 \times 18 = 36 \text{ mm}$$

The outer diameter of the roller is taken slightly larger (atleast 3 mm more) than the outer diameter of the eye.

In the present case, 42 mm outer diameter of the roller will be sufficient.

Providing a clearance of 1.5 mm between the roller and the fork on either side of the roller, we have

$$l_3 = l_2 + 2 \times \frac{t_2}{2} + 2 \times 1.5$$

$$= 24 + 2 \times \frac{12}{2} + 3 = 39 \text{ mm}$$

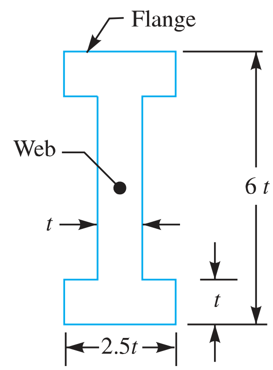

Fig. 32.28

3. *Design for rocker arm cross-section*

The cross-section of the roker arm is obtained by considering the bending of the sections just near the boss of fulcrum on both sides, such as section *A* – *A* and *B* – *B*.

We know that maximum bending moment at *A* – *A* and *B* – *B*.

$$M = 2460\left(180 - \frac{60}{2}\right) = 369 \times 10^3 \text{ N-mm}$$

The rocker arm is of *I*-section. Let us assume the proportions as shown in Fig. 32.28. We know that section modulus,

$$Z = \frac{\frac{1}{12}\left[2.5t\,(6t)^3 - 1.5t\,(4t)^3\right]}{6t/2} = \frac{37\,t^4}{3\,t} = 12.33\,t^3$$

∴ Bending stress (σ_b),

$$70 = \frac{M}{Z} = \frac{369 \times 10^3}{12.33 t^3} = \frac{29.93 \times 10^3}{t^3}$$

$$t^3 = 29.93 \times 10^3 / 70 = 427.6 \quad \text{or} \quad t = 7.5 \text{ say } 8 \text{ mm}$$

∴ Width of flange = 2.5 *t* = 2.5 × 8 = 20 mm **Ans.**

Depth of web = 4 *t* = 4 × 8 = 32 mm **Ans.**

and depth of the section = 6 *t* = 6 × 8 = 48 mm **Ans.**

Normally thickness of the flange and web is constant throughout, whereas the width and depth is tapered.

4. *Design for tappet screw*

The adjustable tappet screw carries a compressive load of F_e = 2460 N. Assuming the screw is made of mild steel for which the compressive stress (σ_c) may be taken as 50 MPa.

Let d_c = Core diameter of the tappet screw.

We know that the load on the tappet screw (F_e),

$$2460 = \frac{\pi}{4}(d_c)^2 \sigma_c = \frac{\pi}{4}(d_c)^2 50 = 39.3\,(d_c)^2$$

$\therefore$ $(d_c)^2 = 2460 / 39.3 = 62.6$ or $d_c = 7.9$ say 8 mm

and outer or nominal diameter of the screw,

$$d = \frac{d_c}{0.84} = \frac{8}{0.84} = 9.52 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

We shall use 10 mm stud and it is provided with a lock nut. The diameter of the circular end of the arm (D_3) and its depth (t_4) is taken as twice the diameter of stud.

$\therefore$ $D_3 = 2 \times 10 = 20$ mm **Ans.**

and $t_4 = 2 \times 10 = 20$ mm **Ans.**

5. Design for valve spring

First of all, let us find the total load on the valve spring.

We know that initial load on the spring,

W_1 = Initial spring force (F_s) = 96.6 N ...(Already calculated)

and load at full lift,

W_2 = Full valve lift × Stiffiness of spring (s)

= 25 × 10 = 250 N ...(Assuming s = 10 N/mm)

$\therefore$ Total load on the spring,

$$W = W_1 + W_2 = 96.6 + 250 = 346.6 \text{ N}$$

Now let us find the various dimensions for the valve spring, as discussed below:

(a) Mean diameter of spring coil

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 8-1}{4\times 8-4} + \frac{0.615}{8} = 1.184$$

...(Assuming $C = D/d = 8$)

and maximum shear stress (τ),

$$420 = K \times \frac{8WC}{\pi d^2} = 1.184 \times \frac{8 \times 346.6 \times 8}{\pi d^2} = \frac{8360}{d^2}$$

...(Assuming τ = 420 MPa or N/mm^2)

$\therefore$ $d^2 = 8360 / 420 = 19.9$ or $d = 4.46$ mm

The standard size of the wire is SWG 7 having diameter (d) = 4.47 mm. **Ans.** (See Table 22.2).

$\therefore$ Mean diameter of the spring coil,

$$D = C \cdot d = 8 \times 4.47 = 35.76 \text{ mm } \textbf{Ans.}$$

and outer diameter of the spring coil,

$$D_o = D + d = 35.76 + 4.47 = 40.23 \text{ mm } \textbf{Ans.}$$

(b) Number of turns of the coil

Let n = Number of active turns of the coil.

We know that maximum compression of the spring,

$$\delta = \frac{8\, W \cdot C^3 \cdot n}{G \cdot d} \quad \text{or} \quad \frac{\delta}{W} = \frac{8\, C^3 \cdot n}{G \cdot d}$$

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Power-brake mechanism of an automobile

Since the stiffness of the springs, $s = W / \delta = 10$ N/mm, therefore, $\delta / W = 1/10$. Taking $G = 84 \times 10^3$ MPa or N/mm², we have

$$\frac{1}{10} = \frac{8 \times 8^3 \times n}{84 \times 10^3 \times 4.47} = \frac{10.9\, n}{10^3}$$

$$\therefore \quad n = 10^3 / 10.9 \times 10 = 9.17 \text{ say } 10$$

For squared and ground ends, the total number of the turns,

$$n' = n + 2 = 10 + 2 = 12 \text{ **Ans.**}$$

(c) Free length of the spring

Since the compression produced under $W_2 = 250$ N is 25 mm (*i.e.* equal to full valve lift), therefore, maximum compression produced (δ_{max}) under the maximum load of $W = 346.6$ N is

$$\delta_{max} = \frac{25}{250} \times 346.6 = 34.66 \text{ mm}$$

We know that free length of the spring,

$$L_F = n' \cdot d + \delta_{max} + 0.15\, \delta_{max}$$

$$= 12 \times 4.47 + 34.66 + 0.15 \times 34.66 = 93.5 \text{ mm} \text{ **Ans.**}$$

(d) Pitch of the coil

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{93.5}{12 - 1} = 8.5 \text{ mm} \text{ **Ans.**}$$

Example 32. 8. *Design the various components of the valve gear mechanism for a horizontal diesel engine for the following data:*

Bore = 140 mm ; Stroke = 270 mm ; Power = 8.25 kW ; Speed = 475 r.p.m. ; Maximum gas pressure = 3.5 N/mm²

Top

The valve opens 33º before outer dead cerntre and closes 1º after inner dead centre. It opens and closes with constant acceleration and decleration for each half of the lift. The length of the rocker arm on either side of the fulcrum is 150 mm and the included angle is 160º. The weight of the valve is 3 N.

Solution. Given : D = 140 mm = 0.14 m ; L = 270 mm = 0.27 m ; Power = 8.25 kW = 8250 W ; N = 475 r.p.m ; p = 3.5 N/mm^2 ; l = 150 mm = 0.15 m ; θ = 160° ; w = 3 N

First of all, let us find out dimensions of the valve as discussed below :

Size of the valve port

Let d_p = Diameter of the valve port, and

a_p = Area of the valve port = $\frac{\pi}{4}(d_p)^2$

We know that area of the piston,

$$a = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.14)^2 = 0.0154\,\text{m}^2$$

and mean velocity of the piston,

$$v = \frac{2\,L\,N}{60} = \frac{2 \times 0.27 \times 475}{60} = 4.275 \text{ m/s}$$

From Table 32.3, let us take the mean velocity of the gas through the port (v_p) as 40 m/s.

We know that $a_p . v_p = a.v$

$$\frac{\pi}{4}(d_p)^2\, 40 = 0.0154 \times 4.275 \quad \text{or} \quad 31.42\,(d_p)^2 = 0.0658$$

$$\therefore \quad (d_p)^2 = 0.0658 / 31.42 = 2.09 \times 10^{-3} \quad \text{or} \quad d_p = 0.045 \text{ m} = 45 \text{ mm}$$ **Ans.**

Maximum lift of the valve

We know that maximum lift of the valve,

$$h = \frac{d_p}{4\cos\alpha} = \frac{45}{4\cos 45^\circ} = 15.9 \text{ say } 16 \text{ mm}$$ **Ans.**

...(Taking α = 45º)

Thickness of the valve head

We know that thickness of valve head,

$$t = k \cdot d_p \sqrt{\frac{p}{\sigma_b}} = 0.42 \times 45 \sqrt{\frac{3.5}{56}} = 4.72 \text{ mm}$$ **Ans.**

...(Taking k = 0.42 and σ_b = 56 MPa)

Valve stem diameter

We know that valve stem diameter,

$$d_s = \frac{d_p}{8} + 6.35 \text{ mm} = \frac{45}{8} + 6.35 = 11.97 \text{ say } 12 \text{ mm}$$ **Ans.**

Valve head diameter

The projected width of the valve seat, for a seat angle of 45º, may be empirically taken as 0.05 d_p to 0.07 d_p. Let us take width of the valve seat as 0.06 d_p *i.e.* 0.06 × 45 = 2.7 mm.

∴ Valve head diameter, $d_v = d_p + 2 \times 2.7 = 45 + 5.4 = 50.4$ say 51 mm **Ans.**

Now let us calculate the various forces acting on the rocker arm of exhaust valve.

We know that gas load on the valve,

$$P_1 = \frac{\pi}{4}(d_v)^2 p_c = \frac{\pi}{4}(51)^2 0.4 = 817 \text{ N} \qquad \text{...(Taking } p_c = 0.4 \text{ N/mm}^2\text{)}$$

Total load on the valve, considering the weight of the valve,

$$P = P_1 + w = 817 + 3 = 820 \text{ N}$$

Initial spring force, considering the weight of the valve,

$$F_s = \frac{\pi}{4}(d_v)^2 p_s - w = \frac{\pi}{4}(51)^2 0.025 - 3 = 48 \text{ N}$$

...(Taking p_s = 0.025 N/mm^2)

The force due to acceleration (F_a) may be obtained as discussed below :

We know that total angle of crank for which the valve remains open

$$= 33 + 180 + 1 = 214°$$

Since the engine is a four stroke engine, therefore the camshaft angle for which the valve remains open

$$= 214 / 2 = 107°$$

Now, when the camshaft turns through 107 / 2 = 53.5°, the valve lifts by a distance of 16 mm. It may be noted that the half of this period is occupied by constant acceleration and half by constant decleration. The same process occurs when the valve closes. Therefore, the period for constant acceleration is equal to camshaft rotation of 53.5 / 2 = 26.75 ° and during this time, the valve lifts through a distance of 8 mm.

We know that speed of camshaft

$$= \frac{N}{2} = \frac{475}{2} = 237.5 \text{ r.p.m.}$$

∴ Angle turned by the camshaft per second

$$= \frac{237.5}{60} \times 360 = 1425 \text{ deg / s}$$

and time taken by the camshaft for constant acceleration,

$$t = \frac{26.75}{1425} = 0.0188 \text{ s}$$

Let a = Acceleration of the valve.

We know that $s = u \,.\, t + \frac{1}{2} a.t^2$ ... (Equation of motion)

$$8 = 0 \times t + \frac{1}{2} a\,(0.0188)^2 = 1.767 \times 10^{-4} a \qquad \text{...}(\because u = o)$$

∴ $a = 8 / 1.767 \times 10^{-4} = 45\,274 \text{ mm / s}^2 = 45.274 \text{ m / s}^2$

and force due to valve acceleration, considering the weight of the valve,

$$F_a = m \cdot a + w = \frac{3}{9.81} \times 45.274 + 3 = 16.84 \text{ N} \qquad \text{... } (\because m = w/g)$$

We know that the maximum load on the rocker arm for exhaust valve,

$$F_e = P + F_s + F_a = 820 + 48 + 16.84 = 884.84 \text{ say } 885 \text{ N}$$

Since the length of the two arms of the rocker are equal, therefore, load at the two ends of the arm are equal, *i.e.* $F_e = F_c = 885$ N.

We know that reaction at the fulcrum pin *F*,

$$R_F = \sqrt{(F_e)^2 + (F_c)^2 - 2F_e \times F_c \times \cos\theta}$$

$$= \sqrt{(885)^2 + (885)^2 - 2 \times 885 \times 885 \times \cos 160^\circ} = 1743 \text{ N}$$

The rocker arm is shown in Fig. 32.29. We shall now design the various parts of rocker arm as discussed below:

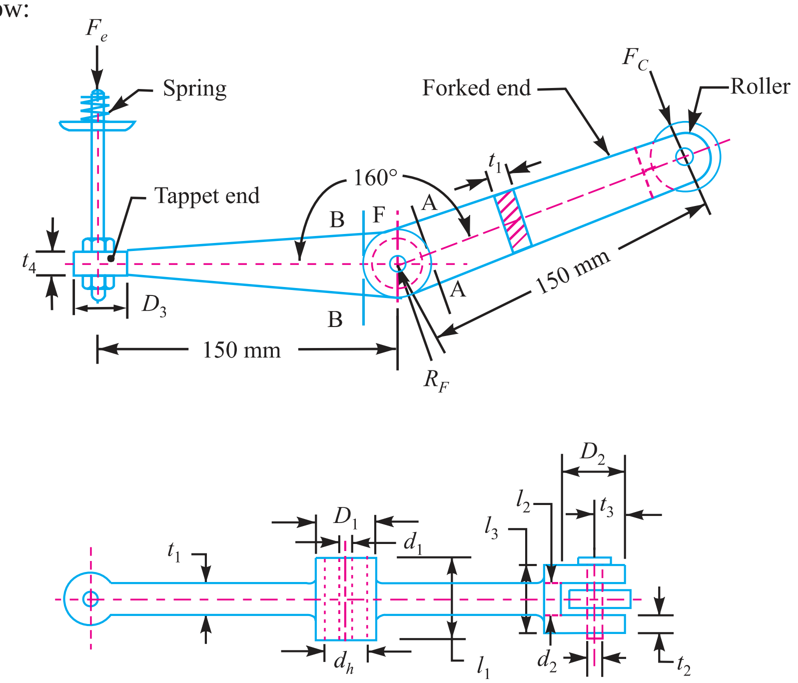

Fig. 32.29

1. *Design of fulcrum pin*

Let d_1 = Diameter of the fulcrum pin, and

l_1 = Length of the fulcrum pin = 1.25 d_1 ... (Assume)

Considering the bearing of the fulcrum pin. We know that load on the fulcrum pin (R_F),

$$1743 = d_1 \times l_1 \times p_b = d_1 \times 1.25\, d_1 \times 5 = 6.25\,(d_1)^2$$

... (For ordinary lubrication, p_b is taken as 5 N/mm^2)

∴ $(d_1)^2 = 1743 / 6.25 = 279$ or $d_1 = 16.7$ say 17 mm

and $l_1 = 1.25\, d_1 = 1.25 \times 17 = 21.25$ say 22 mm

Now let us check the average shear stress induced in the pin. Since the pin is in double shear, therefore, load on the fulcrum pin (R_F),

$$1743 = 2 \times \frac{\pi}{4}(d_1)^2 \tau = 2 \times \frac{\pi}{4}(17)^2 \tau = 454\,\tau$$

∴ $\tau = 1743 / 454 = 3.84$ N/mm^2 or MPa

This induced shear stress is quite safe.

Now external diameter of the boss,

$$D_1 = 2d_1 = 2 \times 17 = 34 \text{ mm}$$

Assuming a phosphor bronze bush of 3 mm thick, the internal diameter of the hole in the lever,

$$d_h = d_1 + 2 \times 3 = 17 + 6 = 23 \text{ mm}$$

Now, let us check the induced bending stress for the section of the boss at the fulcrum which is shown in Fig. 32.30.

Bending moment at this section,

$$M = F_e \times l = 885 \times 150 \text{ N-mm}$$
$$= 132\ 750 \text{ N-mm}$$

Section modulus,

$$Z = \frac{\frac{1}{12} \times 22\,[(34)^3 - (23)^3]}{34/2} = 2927 \text{ mm}^3$$

∴ Induced bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{132\ 750}{2927} = 45.3 \text{ N/mm}^2 \text{ or MPa}$$

The induced bending stress is quite safe.

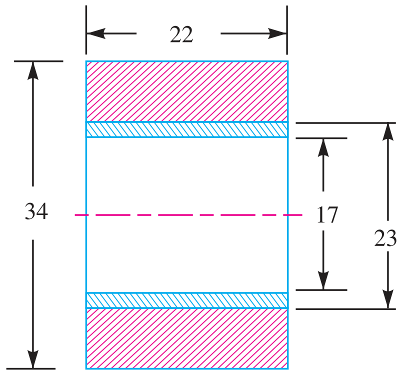

Fig. 32.30

2. *Design for forked end*

Let d_2 = Diameter of the roller pin, and

l_2 = Length of the roller pin = 1.25 d_2 ...(Assume)

Considering bearing of the roller pin. We know that load on the roller pin (F_c),

$$885 = d_2 \times l_2 \times p_b = d_2 \times 1.25\, d_2 \times 7 = 8.75\, (d_2)^2$$

...(Taking p_b = 7 N/mm^2)

∴ $(d_2)^2 = 885 / 8.75 = 101.14$ or $d_2 = 10.06$ say 11 mm **Ans.**

and $l_2 = 1.25\, d_2 = 1.25 \times 11 = 13.75$ say 14 mm **Ans.**

Power transmission gears in an automobile engine

Let us now check the roller pin for induced shearing stress. Since the pin is in double shear, therefore, load on the roller pin (F_c),

$$885 = 2 \times \frac{\pi}{4}(d_2)^2 \tau = 2 \times \frac{\pi}{4}(11)^2 \tau = 190\ \tau$$

$\therefore \quad \tau = 885 / 190 = 4.66$ N/mm^2 or MPa

This induced shear stress is quite safe.

The roller pin is fixed in the eye and thickness of each eye is taken as one-half the length of the roller pin.

$\therefore$ Thickness of each eye,

$$t_2 = \frac{l_2}{2} = \frac{14}{2} = 7 \text{ mm}$$

Let us now check the induced bending stress in the roller pin. The pin is neither simply supported in fork nor rigidly fixed at the end. Therefore, the common practice is to assume the load distribution as shown in Fig. 32.31.

The maximum bending moment will occur at *Y– Y*. Neglecting the effect of clearance, we have

Maximum bending moment at *Y–Y*,

$$M = \frac{F_c}{2}\left(\frac{l_2}{2} + \frac{t_2}{3}\right) - \frac{F_c}{2} \times \frac{l_2}{4}$$

$$= \frac{F_c}{2}\left(\frac{l_2}{2} + \frac{l_2}{6}\right) - \frac{F_c}{2} \times \frac{l_2}{4} \quad \ldots(\because t_2 = l_2/2)$$

$$= \frac{5}{24} \times F_c \times l_2$$

$$= \frac{5}{24} \times 885 \times 14 = 2581 \text{ N-mm}$$

and section modulus of the pin,

$$Z = \frac{\pi}{32}(d_2)^3 = \frac{\pi}{32}(11)^3 = 131 \text{ mm}^3$$

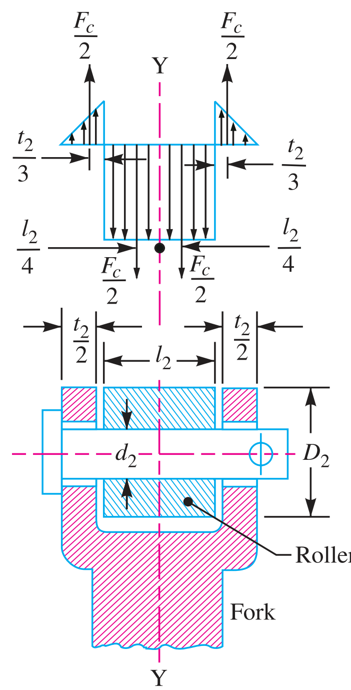

Fig. 32.31

$\therefore$ Bending stress induced in the pin

$$= \frac{M}{Z} = \frac{2581}{131} = 19.7 \text{ N/mm}^2 \text{ or MPa}$$

This bending stress induced in the pin is within permissible limits.

Since the radial thickness of eye (t_3) is taken as $d_2 / 2$, therefore, overall diameter of the eye,

$$D_2 = 2\, d_2 = 2 \times 11 = 22 \text{ mm}$$

The outer diameter of the roller is taken slightly larger (at least 3 mm more) than the outer diameter of the eye. In the present case, 28 mm outer diameter of the roller will be sufficient.

Providing a clearance of 1.5 mm between the roller and the fork on either side of the roller, we have

$$l_3 = l_2 + 2 \times \frac{t_2}{2} + 2 \times 1.5 = 14 + 2 \times \frac{7}{2} + 3 = 24 \text{ mm}$$

3. *Design for rocker arm cross-section*

Since the engine is a slow speed engine, therefore, a rectangular section may be selected for the rocker arm. The cross-section of the rocker arm is obtained by considering the bending of the sections just near the boss of fulcrum on both sides, such as section *A–A* and *B– B*.

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Let t_1 = Thickness of the rocker arm which is uniform throughout.

B = Width or depth of the rocker arm which varies from boss diameter of fulcrum to outside diameter of the eye (for the forked end side) and from boss diameter of fulcrum to thickness t_3 (for the tappet or stud end side).

Now bending moment on section $A - A$ and $B - B$,

$$M = 885\left(150 - \frac{34}{2}\right) = 117\ 705 \text{ N-mm}$$

and section modulus at $A - A$ and $B - B$,

$$Z = \frac{1}{6} \times t_1 \cdot B^2 = \frac{1}{6} \times t_1\ (D_1)^2 = \frac{1}{6} \times t_1\ (34)^2 = 193\ t_1$$

...(At sections A–A and B–B, $B = D$)

We know that bending stress (σ_b),

$$70 = \frac{M}{Z} = \frac{117\ 705}{193\ t_1} \qquad \text{...(Taking } \sigma_b = 70 \text{ MPa or N/mm}^2\text{)}$$

∴ $t_1 = 117\ 705 / 193 \times 70 = 8.7$ say 10 mm **Ans.**

4. *Design for tappet screw*

The adjustable tappet screw carries a compressive load of $F_e = 885$ N. Assuming the screw to be made of mild steel for which the compressive stress (σ_c) may be taken as 50 MPa.

Let d_c = Core diameter of the tappet screw.

We know that load on the tappet screw (F_e),

$$885 = \frac{\pi}{4}\ (d_c{}^2)\ \sigma_c = \frac{\pi}{4}\ (d_c)^2\ 50 = 39.3(d_c)^2$$

∴ $(d_c)^2 = 885 / 39.3 = 22.5$ or $d_c = 4.74$ say 5 mm **Ans.**

and outer or nominal diameter of the screw,

$$d = \frac{d_c}{0.84} = \frac{5}{0.8} = 6.25 \text{ say } 6.5 \text{ mm}$$ **Ans.**

We shall use 6.5 mm stud and it is provided with a lock nut. The diameter of the circular end of the arm (D_3) and its depth (t_4) is taken as twice the diameter of stud.

∴ $D_3 = 2 \times 6.5 = 13$ mm **Ans.**

and $t_4 = 2 \times 6.5 = 13$ mm **Ans.**

5. *Design for valve spring*

First of all, let us find the total load on the valve spring.

We know that intial load on the spring,

W_1 = Initial spring force (F_s) = 48 N ...(Already calculated)

and load at full lift, W_2 = Full valve lift × Stiffness of spring (s)

$= 16 \times 8 = 128$ N ...(Taking $s = 8$ N/mm)

∴ Total load on the spring,

$$W = W_1 + W_2 = 48 + 128 = 176 \text{ N}$$

Now let us find the various dimensions for the valve spring as discussed below:

(a) Mean diameter of the spring coil

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

Top

Inside view of an automobile

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

...(Assuming $C = D/d = 6$)

and maximum shear stress (τ),

$$420 = K \times \frac{8\,WC}{\pi d^2} = 1.2525 \times \frac{8 \times 176 \times 6}{\pi d^2} = \frac{3368}{d^2}$$

∴ $d^2 = 3368 / 420 = 8.02$ or $d = 2.83$ mm

The standard size of the wire is SWG 11 having a diameter (d) = 2.946 mm **Ans.** (see Table 22.2)

∴ Mean diameter of spring coil,

$$D = C \cdot d = 6 \times 2.946 = 17.676 \text{ mm} \quad \textbf{Ans.}$$

and outer diameter of the spring coil,

$$D_o = D + d = 17.676 + 2.946 = 20.622 \text{ mm} \quad \textbf{Ans.}$$

(b) Number of turns of the coil

Let n = Number of turns of the coil,

We know that maximum compression of the spring.

$$\delta = \frac{8\,W \cdot C^3 \cdot n}{G \cdot d} \quad \text{or} \quad \frac{\delta}{W} = \frac{8\,C^3 \cdot n}{G \cdot d}$$

Since the stiffness of the spring, $s = W / \delta = 8$ N / mm, therefore $\delta / W = 1 / 8$. Taking $G = 84 \times 10^3$ MPa or N/mm^2, we have

$$\frac{1}{8} = \frac{8 \times 6^3 \times n}{84 \times 10^3 \times 2.946} = \frac{6.98\, n}{10^3}$$

$$\therefore \quad n = 10^3 / 8 \times 6.98 = 17.9 \text{ say } 18$$

For squared and ground ends, the total number of turns,

$$n' = n + 2 = 18 + 2 = 20$$ **Ans.**

(c) *Free length of the spring*

Since the compression produced under $W_2 = 128$ N is 16 mm, therefore, maximum compression produced under the maximum load of $W = 176$ N is

$$\delta_{max} = \frac{16}{128} \times 176 = 22 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.\, d + \delta_{max} + 0.15\, \delta_{max}$$
$$= 20 \times 2.946 + 22 + 0.15 \times 22 = 84.22 \text{ say } 85 \text{ mm}$$ **Ans.**

(d) *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{85}{20 - 1} = 4.47 \text{ mm}$$ **Ans.**

Design of cam

The cam is forged as one piece with the camshaft. It is designed as discussed below :

The diameter of camshaft (D') is taken empirically as

$$D' = 0.16 \times \text{Cylinder bore} + 12.7 \text{ mm}$$
$$= 0.16 \times 140 + 12.7 = 35.1 \text{ say } 36 \text{ mm}$$

The base circle diameter is about 3 mm greater than the camshaft diameter.

$\therefore$ Base circle diameter $= 36 + 3 = 39$ say 40 mm

The width of cam is taken equal to the width of roller, *i.e.* 14 mm.

The width of cam (w') is also taken empirically as

$$w' = 0.09 \times \text{Cylinder bore} + 6 \text{ mm} = 0.09 \times 140 + 6 = 18.6 \text{ mm}$$

Let us take the width of cam as 18 mm.

Now the *cam is drawn according to the procedure given below :

First of all, the displacement diagram, as shown in Fig. 32.32, is drawn as discussed in the following steps :

1. Draw a horizontal line *ANM* such that *AN* represents the angular displacement when valve opens (*i.e.* 53.5º) to some suitable scale. The line *NM* represents the angular displacement of the cam when valve closes (*i.e.* 53.5º).
2. Divide *AN* and *NM* into any number of equal even parts (say six).
3. Draw vertical lines through points 0, 1, 2, 3 etc. equal to the lift of valve *i.e.* 16 mm.
4. Divide the vertical lines 3 – *f* and 3′ – *f* ′ into six equal parts as shown by points *a, b, c* ... and *a*′, *b*′, *c*′ .... in Fig. 32.32.
5. Since the valve moves with equal uniform acceleration and decleration for each half of the lift, therefore, valve displacement diagram for opening and closing of valve consists of double parabola.

* For complete details, refer Authors' popular book on **'Theory of Machines'.**

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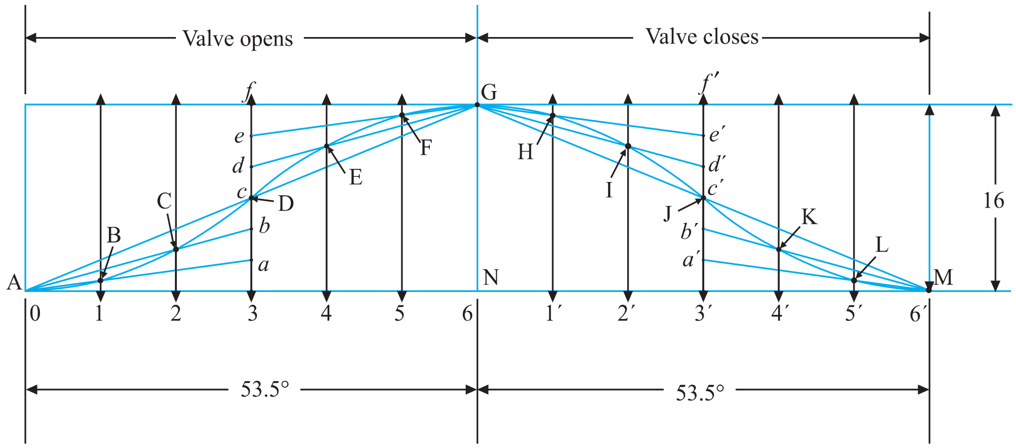

Fig. 32.32. Displacement diagram.

6. Join *Aa*, *Ab* , *Ac* intersecting the vertical lines through 1, 2, 3 at *B*, *C*, *D* respectively.
7. Join the points *B*, *C*, *D* with a smooth curve. This is the required parabola for the half of valve opening. Similarly other curves may be drawn as shown in Fig. 32.32.
8. The curve *A*, *B*, *C*, ..., *G*, *K*, *L*, *M* is the required displacement diagram.

Gears keyed to camshafts

Now the profile of the cam, as shown in Fig. 32.32, is drawn as discussed in the following steps:

1. Draw a base circle with centre *O* and diameter equal 40 mm (radius = 40/2 = 20 mm)
2. Draw a prime circle with centre *O* and radius, *OA*= Min. radius of cam + $\frac{1}{2}$ Diameter of roller = 20 + $\frac{1}{2}$ × 28

$$= 20 + 14 = 34 \text{ mm}$$

3. Draw angle *AOG* = 53.5º to represent opening of valve and angle *GOM* = 53.5º to represent closing of valve.
4. Divide the angular displacement of the cam during opening and closing of the valve (*i.e.* angle *AOG* and *GOM*) into same number of equal even parts as in displacement diagram.
5. Join the points 1, 2, 3, etc. with the centre *O* and produce the lines beyond prime circle as shown in Fig. 32.33.
6. Set off points 1*B*, 2*C*, 3*D*, etc. equal to the displacements from displacement diagram.
7. Join the points *A*, *B*, *C*, ...*L*, *M*, *A*. The curve drawn through these points is known as ***pitch curve.***
8. From the points *A*, *B*, *C*, ...*K*, *L*, draw circles of radius equal to the radius of the roller.

Top

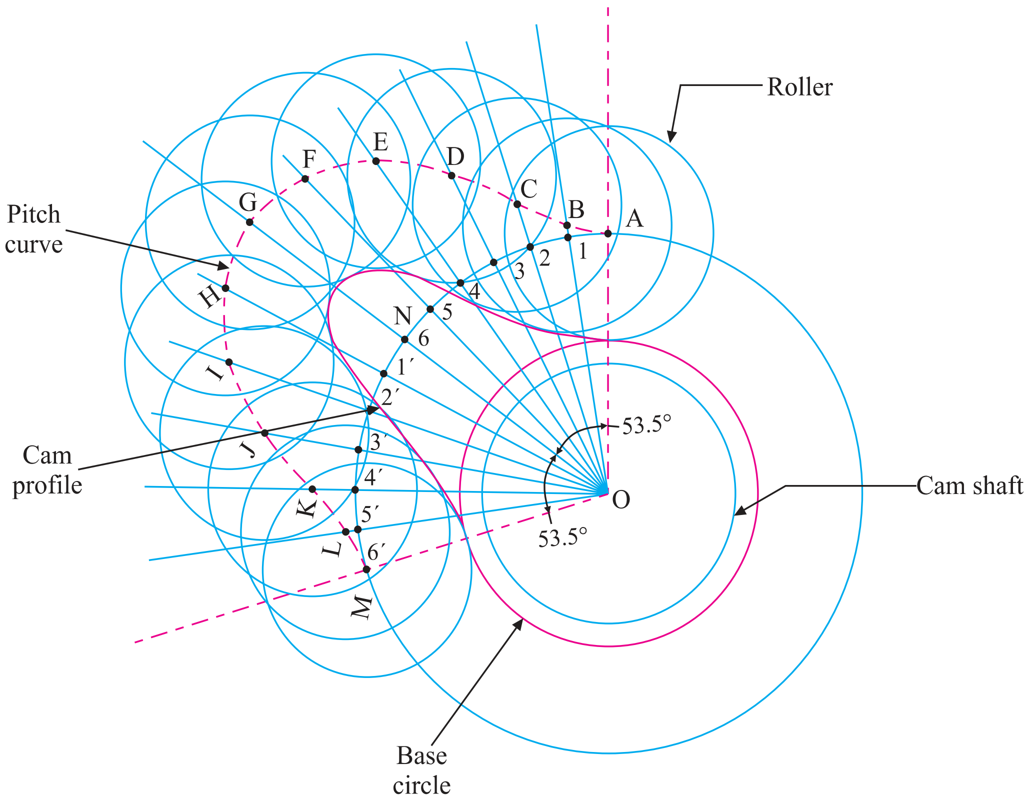

Fig. 32.33

9. Join the bottoms of the circle with a smooth curve as shown in Fig. 32.33. The is the required profile of cam.

EXERCISES

1. A four stroke internal combustion engine has the following specifications:
   Brake power = 7.5 kW; Speed = 1000 r.p.m.; Indicated mean effective pressure = 0.35 N/mm^2;Maximum gas pressure = 3.5 N/mm^2; Mechanical efficiency = 80 %.
   Determine: 1. The dimesions of the cylinder, if the length of stroke is 1.4 times the bore of the cylinder; 2. Wall thickness of the cylinder, if the hoop stress is 35 MPa; 3. Thickness of the cylinder head and the size of studs when the permissible stresses for the cylinder head and stud materials are 45 MPa and 65 MPa respectively.
2. Design a cast iron trunk type piston for a single acting four stroke engine developing 75 kW per cylinder when running at 600 r.p.m. The other avialable data is as follows:
   Maximum gas pressure = 4.8 N/mm^2; Indicated mean effective pressure = 0.65 N/mm^2; Mechanical efficiency = 95%; Radius of crank = 110 mm; Fuel consumption = 0.3 kg/BP/hr; Calorific value of fuel (higher) = 44 × 10^{3}kJ/kg; Difference of temperatures at the centre and edges of the piston head = 200ºC; Allowable stress for the material of the piston = 33.5 MPa; Allowable stress for the material of the piston rings and gudgeon pin = 80 MPa; Allowable bearing pressure on the piston barrel = 0.4 N/mm^2 and allowable bearing pressure on the gudgeon pin = 17 N/mm^2.
3. Design a piston for a four stroke diesel engine consuming 0.3 kg of fuel per kW of power per hour and produces a brake mean effective pressure of the 0.7 N/mm^2. The maximum gas pressure inside the cylinder is 5 N/mm^2 at a speed of 3500 r.p.m. The cylinder diameter is required to be 300 mm with stroke 1.5 times the diameter. The piston may have 4 compression rings and an oil ring. The following data can be used for design:

Higher calorific value of fuel = 46×10^3kJ/kg; Temperature at the piston centre = 700 K; Tempressure at the piston edge = 475 K; Heat conductivity factor = 46.6 W/m/K; Heat conducted through top = 5% of heat produced; Permissible tensile strength for the material of piston = 27 N/mm^2; Pressure between rings and piston = 0.04 N/mm^2; Permissible tensile stress in rings = 80 N/mm^2; Permissible Pressure on piston barrel = 0.4 N/mm^2; Permissible pressure on piston pin = 15 N/mm^2; Permissible stress in piston pin = 85 N/mm^2.

Any other data required for the design may be assumed.

4. Determine the dimensions of an *I*-section connecting rod for a petrol engine from the following data:

Diameter of the piston = 110 mm; Mass of the reciprocating parts = 2 kg; Length of the connecting rod from centre to centre = 325 mm; Stroke length = 150 mm; R.P.M. = 1500 with possible overspeed of 2500; Compression ratio = 4 : 1; Maximum explosion pressure = 2.5 N/mm^2.

5. The following particulars refer to a four stroke cycle diesel engine:

Cylinder bore = 150 mm; Stroke = 187.5 mm; R.P.M. = 1200; Maximum gas pressure = 5.6 N/mm^2; Mass of reciprocating parts = 1.75 kg.

1. The dimensions of an *I*-section connecting rod of forged steel with an elastic limit compressive stress of 350 MPa. The ratio of the length of connecting rod to the length of crank is 4 and the factor of safety may be taken as 5;

2. The wrist pin and crankpin dimensions on the basis of bearing pressures of 10 N/mm^2 and 6.5 N/mm^2 of the projected area respectively; and

3. The dimensions of the small and big ends of the connecting rods, including the size of the securing bolts of the crankpin end. Assume that the allowable stress in the bolts, is not to exceed 35 N/mm^2.

Draw dimensioned sketches of the connecting rod showing the provisions for lubrication.

6. A connecting rod is required to be designed for a high speed, four stroke I.C. engine. The following data are available.

Diameter of piston = 88 mm; Mass of reciprocating parts = 1.6 kg; Length of connecting rod (centre to centre) = 300 mm; Stroke = 125 mm; R.P.M. = 2200 (when developing 50 kW); Possible overspeed = 3000 r.p.m.; Compression ratio = 6.8 : 1 (approximately); Probale maximum explosion pressure (assumed shortly after dead centre, say at about 3°) = 3.5 N/mm^2.

Draw fully dimensioned drawings of the connecting rod showing the provision for the lubrication.

7. Design a plain carbon steel centre crankshaft for a single acting four stroke, single cylinder engine for the following data:

Piston diameter = 250 mm; Stroke = 400 mm; Maximum combustion pressure = 2.5 N/mm^2; Weight of the flywheel = 16 kN; Total belt pull = 3 N; Length of connecting rod = 950 mm.

When the crank has turned through 30° from top dead centre, the pressure on the piston is 1 N/mm^2 and the torque on the crank is maximum.

Any other data required for the design may be assumed.

8. Design a side crankshaft for a 500 mm × 600 mm gas engine. The weight of the flywheel is 80 kN and the explosion pressure is 2.5 N/mm^2. The gas pressure at maximum torque is 0.9 N/mm^2 when the crank angle. is 30º. The connecting rod is 4.5 times the crank radius.

Any other data required for the design may be assumed.

9. Design a rocker arm of *I*-section made of cast steel for operating an exhaust valve of a gas engine. The effective length of the rocker arm is 250 mm and the angle between the arm is 135°. The exhaust valve is 80 mm in diameter and the gas pressure when the valve begins to open is 0.4 N/mm^2. The greatest suction pressure is 0.03 N/mm^2 below atmospheric. The initial load may be assumed as 0.05 N/mm^2 of valve area and the valve inertia and friction losses as 120 N. The ultimate strength of cast steel is 750 MPa. The allowable bearing pressure is 8 N/mm^2 and the permissible stress in the material is 72 MPa.

10. Design the various components of a valve gear mechanism for a horizontal diesel engine having the following specifications:

Brake power = 10 kW; Bore = 140 mm; Stroke = 270 mm; Speed = 500 r.p.m. and maximum gas pressure = 3.5 N/mm^2.

The valve open 30° before top dead centre and closes 2° after bottom dead centre. It opens and closes with constant acceleration and deceleration for each half of the lift. The length of the rocker arm on either side of the fulcrum is 150 mm and the included angle is 135°. The mass of the valve is 0.3 kg.

QUESTIONS

1. Explain the various types of cylinder liners.
2. Discuss the design of piston for an internal combustion engine.
3. State the function of the following for an internal combustion engine piston:
   (*a*) Ribs ; (*b*) Piston rings ; (*c*) Piston skirt ; and (*d*) Piston pin
4. What is the function of a connecting rod of an internal combustion engine?
5. Explain the various stresses induced in the connecting rod.
6. Under what force, the big end bolts and caps are designed?
7. Explain the various types of crankshafts.
8. At what angle of the crank, the twisting moment is maximum in the crankshaft?
9. What are the methods and materials used in the manufacture of crankshafts?
10. Sketch a valve gear mechanism of an internal combustion engine and label its various parts.
11. Discuss the materials commonly used for making the valve of an I. C. engine.
12. Why the area of the inlet valve port is made larger than the area of exhaust valve port?

Transmission mechanism in a truck engine

OBJECTIVE TYPE QUESTIONS

1. The cylinders are usually made of
   (*a*) cast iron or cast steel (*b*) aluminium
   (*c*) stainless steel (*d*) copper
2. The length of the cylinder is usually taken as
   (*a*) equal to the length of piston (*b*) equal to the length of stroke
   (*c*) equal to the cylinder bore (*d*) 1.5 times the length of stroke
3. The skirt of piston
   (*a*) is used to withstand the pressure of gas in the cylinder
   (*b*) acts as a bearing for the side thrust of the connecting rod
   (*c*) is used to seal the cylinder in order to prevent leakage of the gas past the piston
   (*d*) none of the above
4. The side thrust on the cylinder liner is usually taken as .... of the maximum gas load on the piston.
   (*a*) 1/5 (*b*) 1/8
   (*c*) 1/10 (*d*) 1/5
5. The length of the piston usually varies between
   (*a*) D and 1.5 D (*b*) 1.5 D and 2 D
   (*c*) 2D and 2.5 D (*d*) 2.5 D and 3 D
   where D = Diameter of the piston.
6. In designing a connecting rod, it is considered like .... for buckling about X-axis.
   (*a*) both ends fixed
   (*b*) both ends hinged
   (*c*) one end fixed and the other end hinged
   (*d*) one end fixed and the other end free
7. Which of the following statement is wrong for a connecting rod?
   (*a*) The connecting rod will be equally strong in buckling about X-axis, if $I_{xx} = 4\ I_{yy}$.
   (*b*) If $I_{xx} > 4\ I_{yy}$,,the buckling will occur about Y-axis.
   (c) If $I_{xx} < 4I_{yy}$,the buckling will occur about X-axis.
   (*d*) The most suitable section for the connecting rod is T-section.
8. The crankshaft in an internal combustion engine
   (*a*) is a disc which reciprocates in a cylinder
   (*b*) is used to retain the working fluid and to guide the piston
   (*c*) converts reciprocating motion of the piston into rotary motion and vice versa
   (*d*) none of the above
9. The rocker arm is used to actuate the inlet and exhaust valves motion as directed by the
   (*a*) cam and follower (*b*) crank
   (*c*) crankshaft (*d*) none of these
10. For high speed engines, a rocker arm of........... should be used.
   (*a*) rectangular section (*b*) I-section
   (*c*) T-section (*d*) circular

ANSWERS

1. (*a*)	**2.** (*d*)	**3.** (*b*)	**4.** (*c*)	**5.** (*a*)
6. (*b*)	**7.** (*d*)	**8.** (*c*)	**9.** (*a*)	**10.** (*b*)