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CHAPTER

5

Torsional and Bending Stresses in Machine Parts

5.1 Introduction

Sometimes machine parts are subjected to pure torsion or bending or combination of both torsion and bending stresses. We shall now discuss these stresses in detail in the following pages.

5.2 Torsional Shear Stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to ***torsion.*** The stress set up by torsion is known as ***torsional shear stress.*** It is zero at the centroidal axis and maximum at the outer surface.

Consider a shaft fixed at one end and subjected to a torque (T) at the other end as shown in Fig. 5.1. As a result of this torque, every cross-section of the shaft is subjected to torsional shear stress. We have discussed above that the

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torsional shear stress is zero at the centroidal axis and maximum at the outer surface. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l} \qquad \text{...}(i)$$

where
- τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,
- r = Radius of the shaft,
- T = Torque or twisting moment,
- J = Second moment of area of the section about its polar axis or polar moment of inertia,
- C = Modulus of rigidity for the shaft material,
- l = Length of the shaft, and
- θ = Angle of twist in radians on a length l.

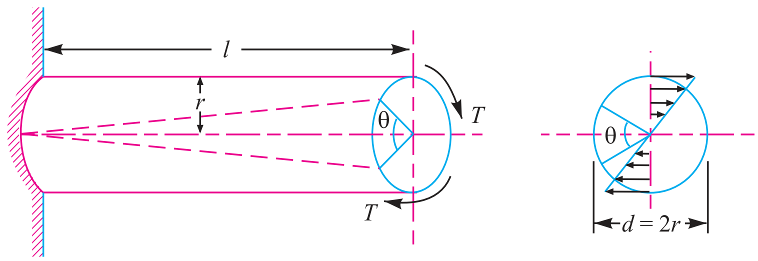

Fig. 5.1. Torsional shear stress.

The equation (*i*) is known as ***torsion equation***. It is based on the following assumptions:

1. The material of the shaft is uniform throughout.
2. The twist along the length of the shaft is uniform.
3. The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.
4. All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.
5. The maximum shear stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

Notes : 1. Since the torsional shear stress on any cross-section normal to the axis is directly proportional to the distance from the centre of the axis, therefore the torsional shear stress at a distance x from the centre of the shaft is given by

$$\frac{\tau_x}{x} = \frac{\tau}{r}$$

2. From equation (*i*), we know that

$$\frac{T}{J} = \frac{\tau}{r} \quad \text{or} \quad T = \tau \times \frac{J}{r}$$

For a solid shaft of diameter (d), the polar moment of inertia,

$$J = I_{XX} + I_{YY} = \frac{\pi}{64} \times d^4 + \frac{\pi}{64} \times d^4 = \frac{\pi}{32} \times d^4$$

$$\therefore \quad T = \tau \times \frac{\pi}{32} \times d^4 \times \frac{2}{d} = \frac{\pi}{16} \times \tau \times d^3$$

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In case of a hollow shaft with external diameter (d_o) and internal diameter (d_i), the polar moment of inertia,

$$J = \frac{\pi}{32}\,[(d_o)^4 - (d_i)^4] \text{ and } r = \frac{d_o}{2}$$

$$\therefore \quad T = \tau \times \frac{\pi}{32}[(d_o)^4 - (d_4)^4] \times \frac{2}{d_o} = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$$

$$= \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad \ldots \left(\text{Substituting, } k = \frac{d_i}{d_o}\right)$$

3. The expression ($C \times J$) is called ***torsional rigidity*** of the shaft.

4. The strength of the shaft means the maximum torque transmitted by it. Therefore, in order to design a shaft for strength, the above equations are used. The power transmitted by the shaft (in watts) is given by

$$P = \frac{2\,\pi\,N\,.\,T}{60} = T\,.\,\omega \qquad \ldots \left(\because\ \omega = \frac{2\,\pi\,N}{60}\right)$$

where T = Torque transmitted in N-m, and

ω = Angular speed in rad/s.

Example 5.1. *A shaft is transmitting 100 kW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.*

Solution. Given : P = 100 kW = 100 × 10^3 W ; N = 160 r.p.m ; $T_{max} = 1.25\ T_{mean}$; τ = 70 MPa = 70 N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$100 \times 10^3 = \frac{2\,\pi\,N\,.\,T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76\ T_{mean}$$

$$\therefore \quad T_{mean} = 100 \times 10^3/16.76 = 5966.6 \text{ N-m}$$

A Helicopter propeller shaft has to bear torsional, tensile, as well as bending stresses.

Note : This picture is given as additional information and is not a direct example of the current chapter.

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and maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque (T_{max}),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\, d^3$$

$\therefore$ $d^3 = 7458 \times 10^3 / 13.75 = 542.4 \times 10^3$ or $d = 81.5$ mm **Ans.**

Example 5.2. *A steel shaft 35 mm in diameter and 1.2 m long held rigidly at one end has a hand wheel 500 mm in diameter keyed to the other end. The modulus of rigidity of steel is 80 GPa.*

1. What load applied to tangent to the rim of the wheel produce a torsional shear of 60 MPa?

2. How many degrees will the wheel turn when this load is applied?

Solution. Given : d = 35 mm or r = 17.5 mm ; l = 1.2 m = 1200 mm ; D = 500 mm or R = 250 mm ; C = 80 GPa = 80 kN/mm^2 = 80 × 10^3 N/mm^2 ; τ = 60 MPa = 60 N/mm^2

1. Load applied to the tangent to the rim of the wheel

Let W = Load applied (in newton) to tangent to the rim of the wheel.

We know that torque applied to the hand wheel,

$$T = W.R = W \times 250 = 250\, W \text{ N-mm}$$

and polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} \times d^4 = \frac{\pi}{32} (35)^4 = 147.34 \times 10^3 \text{ mm}^4$$

We know that $\dfrac{T}{J} = \dfrac{\tau}{r}$

$$\therefore \quad \frac{250\, W}{147.34 \times 10^3} = \frac{60}{17.5} \quad \text{or} \quad W = \frac{60 \times 147.34 \times 10^3}{17.5 \times 250} = 2020 \text{ N} \quad \textbf{Ans.}$$

2. Number of degrees which the wheel will turn when load W = 2020 N is applied

Let θ = Required number of degrees.

We know that $\dfrac{T}{J} = \dfrac{C.\theta}{l}$

$$\therefore \quad \theta = \frac{T.l}{C.J} = \frac{250 \times 2020 \times 1200}{80 \times 10^3 \times 147.34 \times 10^3} = 0.05° \quad \textbf{Ans.}$$

Example 5.3. *A shaft is transmitting 97.5 kW at 180 r.p.m. If the allowable shear stress in the material is 60 MPa, find the suitable diameter for the shaft. The shaft is not to twist more that 1° in a length of 3 metres. Take C = 80 GPa.*

Solution. Given : P = 97.5 kW = 97.5 × 10^3 W ; N = 180 r.p.m. ; τ = 60 MPa = 60 N/mm^2 ; θ = 1° = π / 180 = 0.0174 rad ; l = 3 m = 3000 mm ; C = 80 GPa = 80 × 10^9 N/m^2 = 80 × 10^3 N/mm^2

Let T = Torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted by the shaft (P),

$$97.5 \times 10^3 = \frac{2\,\pi\, N.T}{60} = \frac{2\pi \times 180 \times T}{60} = 18.852\, T$$

$\therefore$ $T = 97.5 \times 10^3 / 18.852 = 5172$ N-m $= 5172 \times 10^3$ N-mm

Now let us find the diameter of the shaft based on the strength and stiffness.

1. *Considering strength of the shaft*

We know that the torque transmitted (T),

$$5172 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$$\therefore \quad d^3 = 5172 \times 10^3 / 11.78 = 439 \times 10^3 \text{ or } d = 76 \text{ mm} \quad \ldots(i)$$

2. *Considering stiffness of the shaft*

Polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} \times d^4 = 0.0982\, d^4$$

We know that

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{5172 \times 10^3}{0.0982\, d^4} = \frac{80 \times 10^3 \times 0.0174}{3000} \text{ or } \frac{52.7 \times 10^6}{d^4} = 0.464$$

$$\therefore \quad d^4 = 52.7 \times 10^6 / 0.464 = 113.6 \times 10^6 \text{ or } d = 103 \text{ mm} \quad \ldots(ii)$$

Taking larger of the two values, we shall provide d = 103 say 105 mm **Ans.**

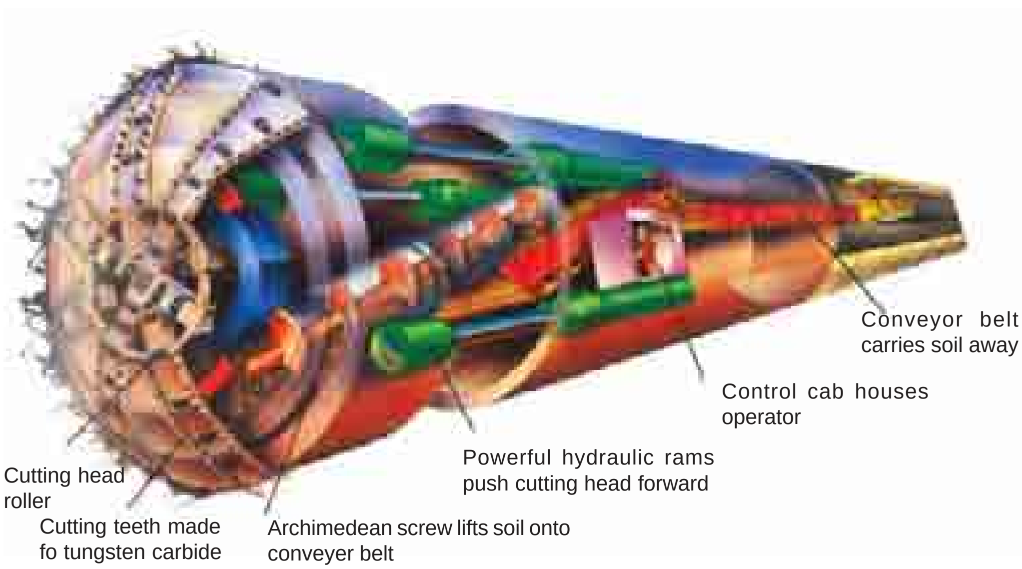
Example 5.4. *A hollow shaft is required to transmit 600 kW at 110 r.p.m., the maximum torque being 20% greater than the mean. The shear stress is not to exceed 63 MPa and twist in a length of 3 metres not to exceed 1.4 degrees. Find the external diameter of the shaft, if the internal diameter to the external diameter is 3/8. Take modulus of rigidity as 84 GPa.*

Solution. Given : P = 600 kW = 600 × 10^3 W ; N = 110 r.p.m. ; T_{max} = 1.2 T_{mean} ; τ = 63 MPa = 63 N/mm^2 ; l = 3 m = 3000 mm ; θ = 1.4 × π / 180 = 0.024 rad ; $k = d_i / d_o$ = 3/8 ; C = 84 GPa = 84 × 10^9 N/m^2 = 84 × 10^3 N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft,

d_o = External diameter of the shaft, and

d_i = Internal diameter of the shaft.



A tunnel-boring machine can cut through rock at up to one kilometre a month. Powerful hydraulic rams force the machine's cutting head fowards as the rock is cut away.

Note : This picture is given as additional information and is not a direct example of the current chapter.

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We know that power transmitted by the shaft (P),

$$600 \times 10^3 = \frac{2\,\pi\,N.T_{mean}}{60} = \frac{2\pi \times 110 \times T_{mean}}{60} = 11.52\,T_{mean}$$

$\therefore$ $T_{mean} = 600 \times 10^3/11.52 = 52 \times 10^3$ N-m $= 52 \times 10^6$ N-mm

and maximum torque transmitted by the shaft,

$$T_{max} = 1.2\,T_{mean} = 1.2 \times 52 \times 10^6 = 62.4 \times 10^6 \text{ N-mm}$$

Now let us find the diameter of the shaft considering strength and stiffness.

1. Considering strength of the shaft

We know that maximum torque transmitted by the shaft,

$$T_{max} = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

$$62.4 \times 10^6 = \frac{\pi}{16} \times 63 \times (d_o)^3 \left[1 - \left(\frac{3}{8}\right)^4\right] = 12.12\,(d_o)^3$$

$\therefore$ $(d_o)^3 = 62.4 \times 10^6/12.12 = 5.15 \times 10^6$ or $d_o = 172.7$ mm ...(*i*)

2. Considering stiffness of the shaft

We know that polar moment of inertia of a hollow circular section,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{32}(d_o)^4\left[1 - \left(\frac{d_i}{d_o}\right)^4\right]$$

$$= \frac{\pi}{32}(d_o)^4\,(1 - k^4) = \frac{\pi}{32}(d_o)^4\left[1 - \left(\frac{3}{8}\right)^4\right] = 0.0962\,(d_o)^4$$

We also know that

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{62.4 \times 10^6}{0.0962\,(d_o)^4} = \frac{84 \times 10^3 \times 0.024}{3000} \quad \text{or} \quad \frac{648.6 \times 10^6}{(d_o)^4} = 0.672$$

$\therefore$ $(d_o)^4 = 648.6 \times 10^6/0.672 = 964 \times 10^6$ or $d_o = 176.2$ mm ...(*ii*)

Taking larger of the two values, we shall provide

$$d_o = 176.2 \text{ say } 180 \text{ mm}$$ **Ans.**

5.3 Shafts in Series and Parallel

When two shafts of different diameters are connected together to form one shaft, it is then known as ***composite shaft.*** If the driving torque is applied at one end and the resisting torque at the other end, then the shafts are said to be connected in series as shown in Fig. 5.2 (*a*). In such cases, each shaft transmits the same torque and the total angle of twist is equal to the sum of the angle of twists of the two shafts.

Mathematically, total angle of twist,

$$\theta = \theta_1 + \theta_2 = \frac{T.l_1}{C_1 J_1} + \frac{T.l_2}{C_2 J_2}$$

If the shafts are made of the same material, then $C_1 = C_2 = C$.

$$\therefore \quad \theta = \frac{T.l_1}{CJ_1} + \frac{T.l_2}{CJ_2} = \frac{T}{C}\left[\frac{l_1}{J_1} + \frac{l_2}{J_2}\right]$$

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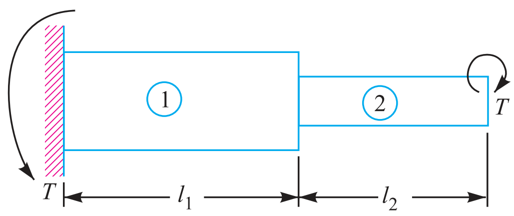
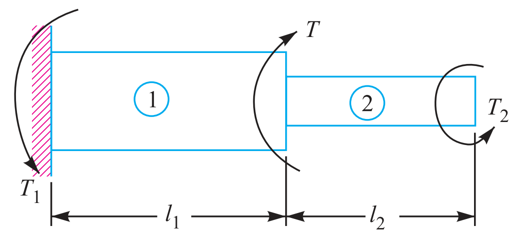


(*a*) Shafts in series.



(*b*) Shafts in parallel.

Fig. 5.2. Shafts in series and parallel.

When the driving torque (T) is applied at the junction of the two shafts, and the resisting torques T_1 and T_2 at the other ends of the shafts, then the shafts are said to be connected in parallel, as shown in Fig. 5.2 (*b*). In such cases, the angle of twist is same for both the shafts, *i.e.*

$$\theta_1 = \theta_2$$

or
$$\frac{T_1 l_1}{C_1 J_1} = \frac{T_2 l_2}{C_2 J_2} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{l_2}{l_1} \times \frac{C_1}{C_2} \times \frac{J_1}{J_2}$$

and
$$T = T_1 + T_2$$

If the shafts are made of the same material, then $C_1 = C_2$.

$$\therefore \quad \frac{T_1}{T_2} = \frac{l_2}{l_1} \times \frac{J_1}{J_2}$$

Example 5.5. *A steel shaft ABCD having a total length of 3.5 m consists of three lengths having different sections as follows:*

AB is hollow having outside and inside diameters of 100 mm and 62.5 mm respectively, and BC and CD are solid. BC has a diameter of 100 mm and CD has a diameter of 87.5 mm. If the angle of twist is the same for each section, determine the length of each section. Find the value of the applied torque and the total angle of twist, if the maximum shear stress in the hollow portion is 47.5 MPa and shear modulus, C = 82.5 GPa.

Solution. Given: $L = 3.5$ m ; $d_o = 100$ mm ; $d_i = 62.5$ mm ; $d_2 = 100$ mm ; $d_3 = 87.5$ mm ; $\tau = 47.5$ MPa $= 47.5$ N/mm^2 ; $C = 82.5$ GPa $= 82.5 \times 10^3$ N/mm^2

The shaft *ABCD* is shown in Fig. 5.3.

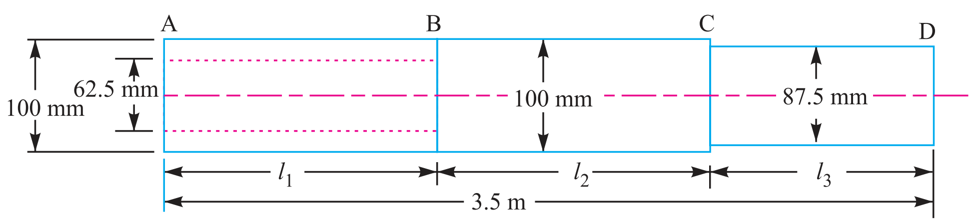


Fig. 5.3

Length of each section

Let l_1, l_2 and l_3 = Length of sections *AB*, *BC* and *CD* respectively.

We know that polar moment of inertia of the hollow shaft *AB*,

$$J_1 = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{32}\left[(100)^4 - (62.5)^4\right] = 8.32 \times 10^6 \text{ mm}^4$$

Polar moment of inertia of the solid shaft *BC*,

$$J_2 = \frac{\pi}{32}(d_2)^4 = \frac{\pi}{32}(100)^4 = 9.82 \times 10^6 \text{ mm}^4$$

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and polar moment of inertia of the solid shaft *CD*,

$$J_3 = \frac{\pi}{32}(d_3)^4 = \frac{\pi}{32}(87.5)^4 = 5.75 \times 10^6 \text{ mm}^4$$

We also know that angle of twist,

$$\theta = T \,.\, l / C \,.\, J$$

Assuming the torque *T* and shear modulus *C* to be same for all the sections, we have

Angle of twist for hollow shaft *AB*,

$$\theta_1 = T \,.\, l_1 / C \,.\, J_1$$

Similarly, angle of twist for solid shaft *BC*,

$$\theta_2 = T \,.\, l_2 / C \,.\, J_2$$

and angle of twist for solid shaft *CD*,

$$\theta_3 = T \,.\, l_3 / C \,.\, J_3$$

Machine part of a jet engine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Since the angle of twist is same for each section, therefore

$$\theta_1 = \theta_2$$

$$\frac{T \,.\, l_1}{C \,.\, J_1} = \frac{T \,.\, l_2}{C \,.\, J_2} \quad \text{or} \quad \frac{l_1}{l_2} = \frac{J_1}{J_2} = \frac{8.32 \times 10^6}{9.82 \times 10^6} = 0.847 \qquad ...(i)$$

Also $\theta_1 = \theta_3$

$$\frac{T \,.\, l_1}{C \,.\, J_1} = \frac{T \,.\, l_3}{C \,.\, J_3} \quad \text{or} \quad \frac{l_1}{l_3} = \frac{J_1}{J_3} = \frac{8.32 \times 10^6}{5.75 \times 10^6} = 1.447 \qquad ...(ii)$$

We know that $l_1 + l_2 + l_3 = L = 3.5 \text{ m} = 3500 \text{ mm}$

$$l_1\left(1 + \frac{l_2}{l_1} + \frac{l_3}{l_1}\right) = 3500$$

$$l_1\left(1 + \frac{1}{0.847} + \frac{1}{1.447}\right) = 3500$$

$$l_1 \times 2.8717 = 3500 \quad \text{or} \quad l_1 = 3500 / 2.8717 = 1218.8 \text{ mm} \ \textbf{Ans.}$$

From equation ***(i)***,

$$l_2 = l_1 / 0.847 = 1218.8 / 0.847 = 1439 \text{ mm} \ \textbf{Ans.}$$

and from equation ***(ii)***, $l_3 = l_1 / 1.447 = 1218.8 / 1.447 = 842.2 \text{ mm}$ **Ans.**

Value of the applied torque

We know that the maximum shear stress in the hollow portion,

$$\tau = 47.5 \text{ MPa} = 47.5 \text{ N/mm}^2$$

For a hollow shaft, the applied torque,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times 47.5 \left[\frac{(100)^4 - (62.5)^4}{100}\right]$$

$$= 7.9 \times 10^6 \text{ N-mm} = 7900 \text{ N-m} \ \textbf{Ans.}$$

Total angle of twist

When the shafts are connected in series, the total angle of twist is equal to the sum of angle of twists of the individual shafts. Mathematically, the total angle of twist,

$$\theta = \theta_1 + \theta_2 + \theta_3$$

$$= \frac{T \cdot l_1}{C \cdot J_1} + \frac{T \cdot l_2}{C \cdot J_2} + \frac{T \cdot l_3}{C \cdot J_3} = \frac{T}{C}\left[\frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}\right]$$

$$= \frac{7.9 \times 10^6}{82.5 \times 10^3}\left[\frac{1218.8}{8.32 \times 10^6} + \frac{1439}{9.82 \times 10^6} + \frac{842.2}{5.75 \times 10^6}\right]$$

$$= \frac{7.9 \times 10^6}{82.5 \times 10^3 \times 10^6}\,[146.5 + 146.5 + 146.5] = 0.042 \text{ rad}$$

$= 0.042 \times 180 / \pi = 2.406°$ **Ans.**

5.4 Bending Stress in Straight Beams

In engineering practice, the machine parts of structural members may be subjected to static or dynamic loads which cause bending stress in the sections besides other types of stresses such as tensile, compressive and shearing stresses.

Consider a straight beam subjected to a bending moment *M* as shown in Fig. 5.4. The following assumptions are usually made while deriving the bending formula.

1. The material of the beam is perfectly homogeneous (*i.e.* of the same material throughout) and isotropic (*i.e.* of equal elastic properties in all directions).
2. The material of the beam obeys Hooke's law.
3. The transverse sections (*i.e.* *BC* or *GH*) which were plane before bending, remain plane after bending also.
4. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.
5. The Young's modulus (*E*) is the same in tension and compression.
6. The loads are applied in the plane of bending.

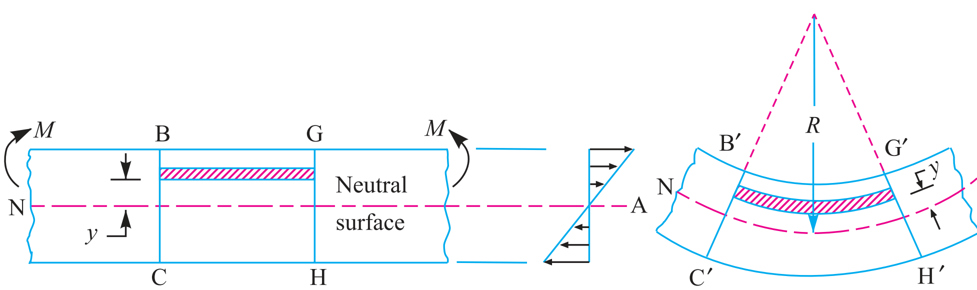

Fig. 5.4. Bending stress in straight beams.

A little consideration will show that when a beam is subjected to the bending moment, the fibres on the upper side of the beam will be shortened due to compression and those on the lower side will be elongated due to tension. It may be seen that somewhere between the top and bottom fibres there is a surface at which the fibres are neither shortened nor lengthened. Such a surface is called ***neutral surface***. The intersection of the neutral surface with any normal cross-section of the beam is known *as* ***neutral axis.*** The stress distribution of a beam is shown in Fig. 5.4. The bending equation is given by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

where M = Bending moment acting at the given section,

σ = Bending stress,

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I = Moment of inertia of the cross-section about the neutral axis,
y = Distance from the neutral axis to the extreme fibre,
E = Young's modulus of the material of the beam, and
R = Radius of curvature of the beam.

From the above equation, the bending stress is given by

$$\sigma = y \times \frac{E}{R}$$

Since E and R are constant, therefore within elastic limit, the stress at any point is directly proportional to y, *i.e.* the distance of the point from the neutral axis.

Also from the above equation, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I/y} = \frac{M}{Z}$$

The ratio I/y is known as ***section modulus*** and is denoted by Z.

Notes : 1. The neutral axis of a section always passes through its centroid.

2. In case of symmetrical sections such as circular, square or rectangular, the neutral axis passes through its geometrical centre and the distance of extreme fibre from the neutral axis is $y = d / 2$, where d is the diameter in case of circular section or depth in case of square or rectangular section.

3. In case of unsymmetrical sections such as L-section or T-section, the neutral axis does not pass through its geometrical centre. In such cases, first of all the centroid of the section is calculated and then the distance of the extreme fibres for both lower and upper side of the section is obtained. Out of these two values, the bigger value is used in bending equation.

Parts in a machine.

Table 5.1 (from pages 130 to 134) shows the properties of some common cross-sections.

This is the first revolver produced in a production line using interchangeable parts.

Note : This picture is given as additional information and is not a direct example of the current chapter.

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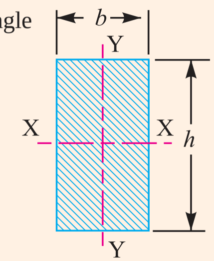
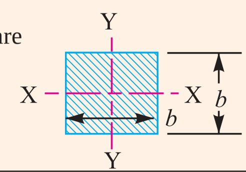
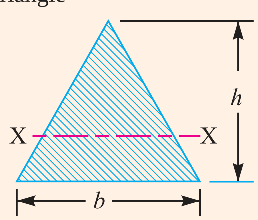
Table 5.1. Properties of commonly used cross-sections.

Section	Area (A)	Moment of inertia (I)	*Distance from the neutral axis to the extreme fibre (y)	Section modulus $\left[Z = \frac{I}{y}\right]$	Radius of gyration $\left[k = \sqrt{\frac{I}{A}}\right]$
1. Rectangle 	bh	$I_{xx} = \frac{b.h^3}{12}$ $I_{yy} = \frac{h.b^3}{12}$	$\frac{h}{2}$ $\frac{b}{2}$	$Z_{xx} = \frac{b.h^2}{6}$ $Z_{yy} = \frac{h.b^2}{6}$	$k_{xx} = 0.289\ h$ $k_{yy} = 0.289\ b$
2. Square 	b^2	$I_{xx} = I_{yy} = \frac{b^4}{12}$	$\frac{b}{2}$	$Z_{xx} = Z_{yy} = \frac{b^3}{6}$	$k_{xx} = k_{yy} = 0.289\ b$
3. Triangle 	$\frac{bh}{2}$	$I_{xx} = \frac{b.h^3}{36}$	$\frac{h}{3}$	$Z_{xx} = \frac{bh^2}{12}$	$k_{xx} = 0.2358\ h$

* The distances from the neutral axis to the bottom extreme fibre is taken into consideration.

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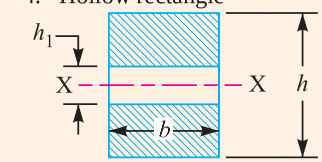
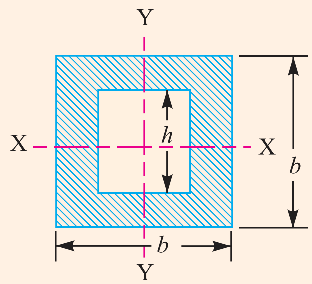
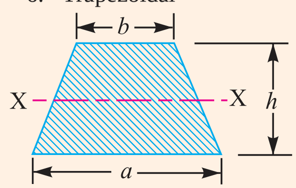
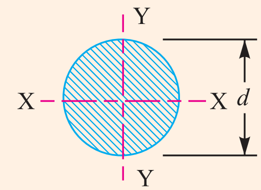
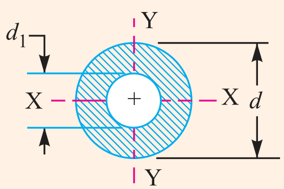
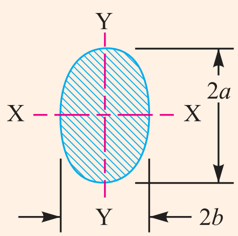
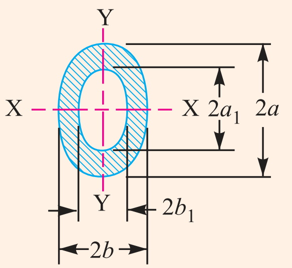
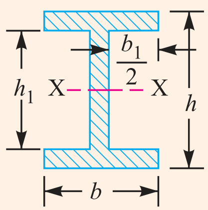
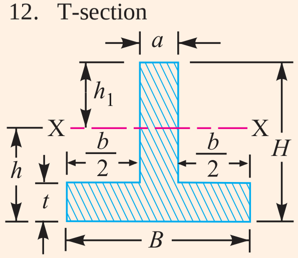
Section	*(A)*	*(I)*	*(y)*	$Z = \frac{I}{y}$	$k = \sqrt{\frac{I}{A}}$
4. Hollow rectangle 	$b\,(h - h_1)$	$I_{xx} = \frac{b}{12}(h^3 - h_1^3)$	$\frac{h}{2}$	$Z_{xx} = \frac{b}{6}\left(\frac{h^3 - h_1^3}{h}\right)$	$k_{xx} = 0.289\sqrt{\frac{h^3 - h_1^3}{h - h_1}}$
5. Hollow square 	$b^2 - h^2$	$I_{xx} = I_{yy} = \frac{b^4 - h^4}{12}$	$\frac{b}{2}$	$Z_{xx} = Z_{yy} = \frac{b^4 - h^4}{6b}$	$0.289\sqrt{b^2 + h^2}$
6. Trapezoidal 	$\frac{a + b}{2} \times h$	$I_{xx} = \frac{h^2\,(a^2 + 4ab + b^2)}{36\,(a + b)}$	$\frac{a + 2b}{3\,(a + b)} \times h$	$Z_{xx} = \frac{a^2 + 4ab + b^2}{12\,(a + 2b)}$	$\frac{0.236}{a+b}\sqrt{h\,(a^2 + 4ab + b^2)}$

Section	*(A)*	*(I)*	*(y)*	$Z = \frac{I}{y}$	$k = \sqrt{\frac{I}{A}}$
7. Circle 	$\frac{\pi}{4} \times d^2$	$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$	$\frac{d}{2}$	$Z_{xx} = Z_{yy} = \frac{\pi d^3}{32}$	$k_{xx} = k_{yy} = \frac{d}{2}$
8. Hollow circle 	$\frac{\pi}{4}(d^2 - d_1^2)$	$I_{xx} = I_{yy} = \frac{\pi}{64}(d^4 - d_1^4)$	$\frac{d}{2}$	$Z_{xx} = Z_{yy} = \frac{\pi}{32}\left(\frac{d^4 - d_1^4}{d}\right)$	$k_{xx} = k_{yy} = \frac{\sqrt{d^2 + d_1^2}}{4}$
9. Elliptical 	πab	$I_{xx} = \frac{\pi}{4} \times a^3 b$ $I_{yy} = \frac{\pi}{4} \times ab^3$	a b	$Z_{xx} = \frac{\pi}{4} \times a^2 b$ $Z_{yy} = \frac{\pi}{4} \times ab^2$	$k_{xx} = 0.5a$ $k_{yy} = 0.5b$

Section	(A)	(I)	(y)	$Z = \frac{I}{y}$	$k = \sqrt{\frac{I}{A}}$
10. Hollow elliptical 	$\pi (ab - a_1 b_1)$	$I_{xx} = \frac{\pi}{4} (ba^3 - b_1 a_1^3)$ $I_{yy} = \frac{\pi}{4} (ab^3 - a_1 b_1^3)$	a b	$Z_{xx} = \frac{\pi}{4a} (ba^3 - b_1 a_1^3)$ $Z_{yy} = \frac{\pi}{4b} (ab^3 - a_1 b_1^3)$	$k_{xx} = \frac{1}{2}\sqrt{\frac{ba^3 - b_1 a_1^3}{ab - a_1 b_1}}$ $k_{yy} = \frac{1}{2}\sqrt{\frac{ab^3 - a_1 b_1^3}{ab - a_1 b_1}}$
11. I-section 	$bh - b_1 h_1$	$I_{xx} = \frac{bh^3 - b_1 h_1^3}{12}$	$\frac{h}{2}$	$Z_{xx} = \frac{bh^3 - b_1 h_1^3}{6h}$	$k_{xx} = 0.289 \sqrt{\frac{bh^3 - b_1 h_1^3}{bh - b_1 h_1}}$
12. T-section 	$Bt + (H - t)\, a$	$I_{xx} = \frac{Bh^3 - b(h-t)^3 + ah_1^3}{3}$	$h = H - h_1$ $= \frac{aH^2 + bt^2}{2(aH + bt)}$	$Z_{xx} = \frac{2 I_{xx} (aH + bt)}{aH^2 + bt^2}$	$k_{xx} = \sqrt{\frac{I_{xx}}{Bt + (H - t)a}}$

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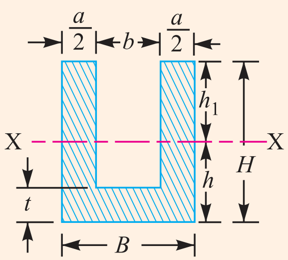
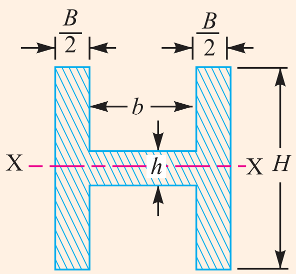
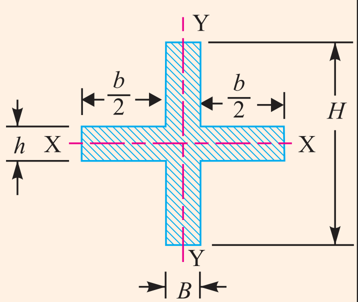
Section	*(A)*	*(I)*	*(y)*	$Z = \frac{I}{y}$	$k = \sqrt{\frac{I}{A}}$
13. Channel Section 	$Bt + (H - t)\,a$	$I_{xx} = \frac{Bh^3 - b(h-t)^3 + ah_1^3}{3}$	$h = H - h_1 = \frac{aH^2 + bt^2}{2(aH + bt)}$	$Z_{xx} = \frac{2I_{xx}(aH + bt)}{aH^2 + bt^2}$	$k_{xx} = \sqrt{\frac{I_{xx}}{Bt + (H - t)\,a}}$
14. H-Section 	$BH + bh$	$I_{xx} = \frac{BH^3 + bh^3}{12}$	$\frac{H}{2}$	$Z_{xx} = \frac{BH^3 + bh^3}{6H}$	$k_{xx} = 0.289\sqrt{\frac{BH^3 + bh^3}{BH + bh}}$
15. Cross-section 	$BH + bh$	$I_{xx} = \frac{Bh^3 + bh^3}{12}$	$\frac{H}{2}$	$Z_{xx} = \frac{BH^3 + bh^3}{6H}$	$k_{xx} = 0.289\sqrt{\frac{BH^3 + bh^3}{BH + bh}}$

Example 5.6. *A pump lever rocking shaft is shown in Fig. 5.5. The pump lever exerts forces of 25 kN and 35 kN concentrated at 150 mm and 200 mm from the left and right hand bearing respectively. Find the diameter of the central portion of the shaft, if the stress is not to exceed 100 MPa.*

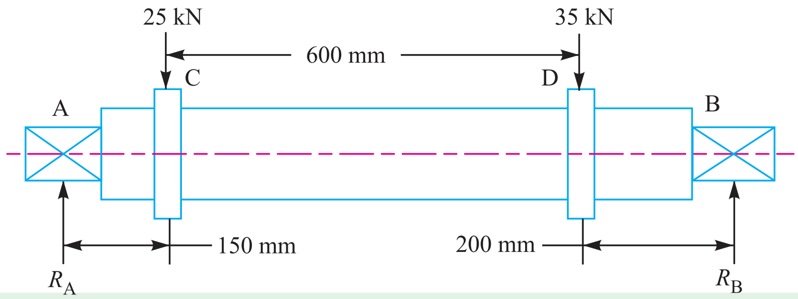


Fig. 5.5

Solution. Given : $\sigma_b = 100$ MPa = 100 N/mm^2

Let R_A and R_B = Reactions at *A* and *B* respectively.

Taking moments about *A*, we have

$$R_B \times 950 = 35 \times 750 + 25 \times 150 = 30\ 000$$

$$\therefore \quad R_B = 30\ 000 / 950 = 31.58 \text{ kN} = 31.58 \times 10^3 \text{ N}$$

and $$R_A = (25 + 35) - 31.58 = 28.42 \text{ kN} = 28.42 \times 10^3 \text{ N}$$

∴ Bending moment at *C*

$$= R_A \times 150 = 28.42 \times 10^3 \times 150 = 4.263 \times 10^6 \text{ N-mm}$$

and bending moment at $D = R_B \times 200 = 31.58 \times 10^3 \times 200 = 6.316 \times 10^6$ N-mm

We see that the maximum bending moment is at *D*, therefore maximum bending moment, $M = 6.316 \times 10^6$ N-mm.

Let *d* = Diameter of the shaft.

∴ Section modulus,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$$

We know that bending stress (σ_b),

$$100 = \frac{M}{Z} = \frac{6.316 \times 10^6}{0.0982\, d^3} = \frac{64.32 \times 10^6}{d^3}$$

The picture shows a method where sensors are used to measure torsion

Note : This picture is given as additional information and is not a direct example of the current chapter.

∴ $d^3 = 64.32 \times 10^6/100 = 643.2 \times 10^3$ or $d = 86.3$ say 90 mm **Ans.**

Example 5.7. *An axle 1 metre long supported in bearings at its ends carries a fly wheel weighing 30 kN at the centre. If the stress (bending) is not to exceed 60 MPa, find the diameter of the axle.*

Solution. Given : L = 1 m = 1000 mm ; W = 30 kN = 30 × 10^3 N ; σ_b = 60 MPa = 60 N/mm^2

The axle with a flywheel is shown in Fig. 5.6.

Let *d* = Diameter of the axle in mm.

∴ Section modulus,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$$

Maximum bending moment at the centre of the axle,

$$M = \frac{W.L}{4} = \frac{30 \times 10^3 \times 1000}{4} = 7.5 \times 10^6 \text{ N-mm}$$

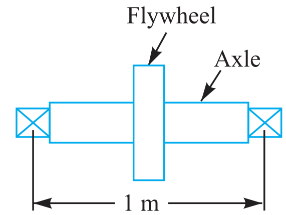


Fig. 5.6

We know that bending stress (σ_b),

$$60 = \frac{M}{Z} = \frac{7.5 \times 10^6}{0.0982\, d^3} = \frac{76.4 \times 10^6}{d^3}$$

∴ $d^3 = 76.4 \times 10^6/60 = 1.27 \times 10^6$ or $d = 108.3$ say 110 mm **Ans.**

Example 5.8. *A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam, if depth is twice that of width.*

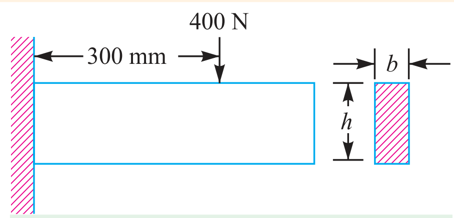


Fig. 5.7

Solution. Given: $W = 400$ N ; $L = 300$ mm ; $\sigma_b = 40$ MPa $= 40$ N/mm^2 ; $h = 2b$

The beam is shown in Fig. 5.7.

Let b = Width of the beam in mm, and

h = Depth of the beam in mm.

∴ Section modulus,

$$Z = \frac{b\,.\,h^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2\,b^3}{3} \text{ mm}^3$$

Maximum bending moment (at the fixed end),

$$M = W.L = 400 \times 300 = 120 \times 10^3 \text{ N-mm}$$

We know that bending stress (σ_b),

$$40 = \frac{M}{Z} = \frac{120 \times 10^3 \times 3}{2\,b^3} = \frac{180 \times 10^3}{b^3}$$

∴ $b^3 = 180 \times 10^3/40 = 4.5 \times 10^3$ or $b = 16.5$ mm **Ans.**

and $h = 2b = 2 \times 16.5 = 33$ mm **Ans.**

Example 5.9. *A cast iron pulley transmits 10 kW at 400 r.p.m. The diameter of the pulley is 1.2 metre and it has four straight arms of elliptical cross-section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa.*

Solution. Given : $P = 10$ kW $= 10 \times 10^3$ W ; $N = 400$ r.p.m ; $D = 1.2$ m $= 1200$ mm or $R = 600$ mm ; $\sigma_b = 15$ MPa $= 15$ N/mm^2

Let T = Torque transmitted by the pulley.

We know that the power transmitted by the pulley (P),

$$10 \times 10^3 = \frac{2\,\pi\,N\,.\,T}{60} = \frac{2\,\pi \times 400 \times T}{60} = 42\,T$$

∴ $T = 10 \times 10^3/42 = 238$ N-m $= 238 \times 10^3$ N-mm

Since the torque transmitted is the product of the tangential load and the radius of the pulley, therefore tangential load acting on the pulley

$$= \frac{T}{R} = \frac{238 \times 10^3}{600} = 396.7 \text{ N}$$

Since the pulley has four arms, therefore tangential load on each arm,

$$W = 396.7/4 = 99.2 \text{ N}$$

and maximum bending moment on the arm,

$$M = W \times R = 99.2 \times 600 = 59\,520 \text{ N-mm}$$

Let $2b$ = Minor axis in mm, and

$2a$ = Major axis in mm = $2 \times 2b = 4b$ ...(Given)

∴ Section modulus for an elliptical cross-section,

$$Z = \frac{\pi}{4} \times a^2 b = \frac{\pi}{4}(2b)^2 \times b = \pi\, b^3 \text{ mm}^3$$

We know that bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{59\,520}{\pi\, b^3} = \frac{18\,943}{b^3}$$

or $b^3 = 18\,943/15 = 1263$ or $b = 10.8$ mm

∴ Minor axis, $2b = 2 \times 10.8 = 21.6$ mm **Ans.**

and major axis, $2a = 2 \times 2b = 4 \times 10.8 = 43.2$ mm **Ans.**

5.5 Bending Stress in Curved Beams

We have seen in the previous article that for the straight beams, the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear. But in case of curved beams, the neutral axis of the cross-section is shifted towards the centre of curvature of the beam causing a non-linear (hyperbolic) distribution of stress, as shown in Fig. 5.8. It may be noted that the neutral axis lies between the centroidal axis and the centre of curvature and always occurs within the curved beams. The application of curved beam principle is used in crane hooks, chain links and frames of punches, presses, planers etc.

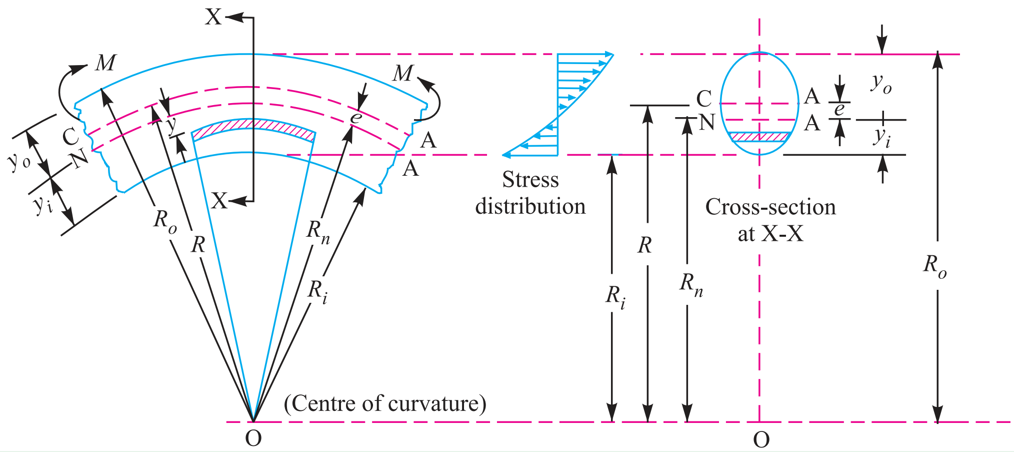


Fig. 5.8. Bending stress in a curved beam.

Consider a curved beam subjected to a bending moment M, as shown in Fig. 5.8. In finding the bending stress in curved beams, the same assumptions are used as for straight beams. The general expression for the bending stress (σ_b) in a curved beam at any fibre at a distance y from the neutral

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axis, is given by

$$\sigma_b = \frac{M}{A \cdot e}\left(\frac{y}{R_n - y}\right)$$

where M = Bending moment acting at the given section about the centroidal axis,

A = Area of cross-section,

e = Distance from the centroidal axis to the neutral axis = $R - R_n$,

R = Radius of curvature of the centroidal axis,

R_n = Radius of curvature of the neutral axis, and

y = Distance from the neutral axis to the fibre under consideration. It is positive for the distances towards the centre of curvature and negative for the distances away from the centre of curvature.

Notes : 1. The bending stress in the curved beam is zero at a point other than at the centroidal axis.

2. If the section is symmetrical such as a circle, rectangle, I-beam with equal flanges, then the maximum bending stress will always occur at the inside fibre.

3. If the section is unsymmetrical, then the maximum bending stress may occur at either the inside fibre or the outside fibre. The maximum bending stress at the inside fibre is given by

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i}$$

where y_i = Distance from the neutral axis to the inside fibre = $R_n - R_i$, and

R_i = Radius of curvature of the inside fibre.

The maximum bending stress at the outside fibre is given by

$$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o}$$

where y_o = Distance from the neutral axis to the outside fibre = $R_o - R_n$, and

R_o = Radius of curvature of the outside fibre.

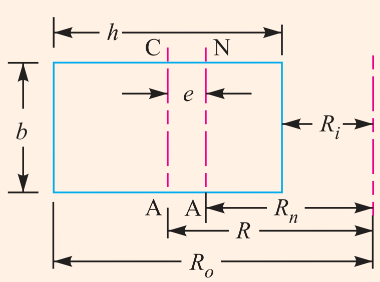
It may be noted that the bending stress at the inside fibre is ***tensile*** while the bending stress at the outside fibre is ***compressive.***

4. If the section has an axial load in addition to bending, then the axial or direct stress (σ_d) must be added algebraically to the bending stress, in order to obtain the resultant stress on the section. In other words,

Resultant stress, $\sigma = \sigma_d \pm \sigma_b$

The following table shows the values of R_n and R for various commonly used cross-sections in curved beams.

Table 5.2. Values of R_n and R for various commonly used cross-section in curved beams.

<table>
<tr><th>Section</th><th>Values of R_n and R</th></tr>
<tr><td>
</td><td>$$R_n = \frac{h}{\log_e\left(\frac{R_o}{R_i}\right)}$$

$$R = R_i + \frac{h}{2}$$</td></tr>
</table>

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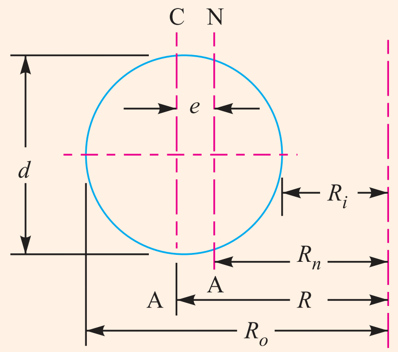
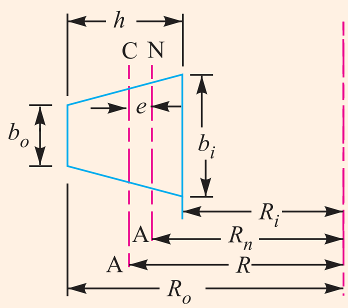
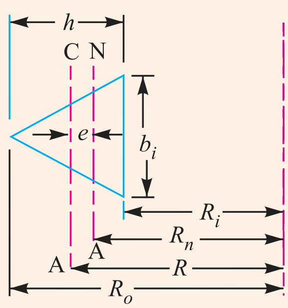
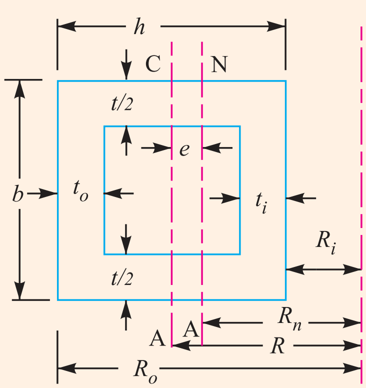
<table>
<tr><th>Section</th><th>Values of R_n and R</th></tr>
<tr><td>
</td><td>$R_n = \frac{\left[\sqrt{R_o} + \sqrt{R_i}\right]^2}{4}$
$R = R_i + \frac{d}{2}$</td></tr>
<tr><td>
</td><td>$R_n = \frac{\left(\frac{b_i + b_o}{2}\right)h}{\left(\frac{b_i R_o - b_o R_i}{h}\right)\log_e\left(\frac{R_o}{R_i}\right) - (b_i - b_o)}$
$R = R_i + \frac{h\,(b_i + 2b_o)}{3\,(b_i + b_o)}$</td></tr>
<tr><td>
</td><td>$R_n = \frac{\frac{1}{2} b_i \times h}{\frac{b_i R_o}{h}\log_e\left(\frac{R_o}{R_i}\right) - b_i}$
$R = R_i + \frac{h}{3}$</td></tr>
<tr><td>
</td><td>$R_n = \frac{(b-t)(t_i + t_o) + t.h}{b\left[\log_e\left(\frac{R_i + t_i}{R_i}\right) + \log_e\left(\frac{R_o}{R_o - t_o}\right)\right] + t.\log_e\left(\frac{R_o - t_o}{R_i + t_i}\right)}$
$R = R_i + \frac{\frac{1}{2}h^2.t + \frac{1}{2}t_i^2(b-t) + (b-t)\,t_o\,(h - \frac{1}{2}t_o)}{h.t + (b-t)(t_i + t_o)}$</td></tr>
</table>

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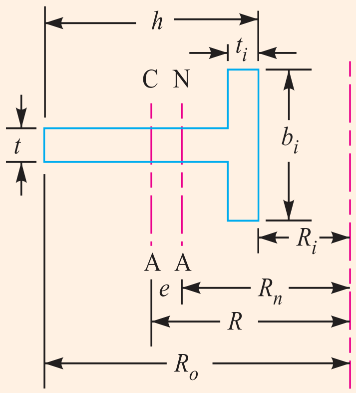
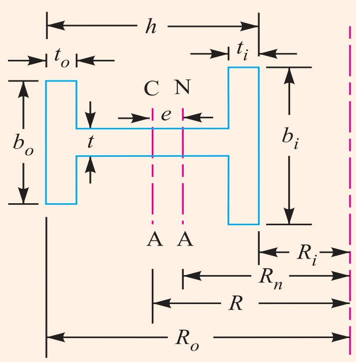
Section	*Values of R_n and R*
	$R_n = \dfrac{t_i(b_i - t) + t.h}{(b_i - t)\log_e\left(\frac{R_i + t_i}{R_i}\right) + t.\log_e\left(\frac{R_o}{R_i}\right)}$ $R = R_i + \dfrac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b_i - t)}{h.t + t_i(b_i - t)}$
	$R_n = \dfrac{t_i(b_i - t) + t_o(b_o - t) + t.h}{b_i\log_e\left(\frac{R_i + t_i}{R_i}\right) + t\log_e\left(\frac{R_o - t_o}{R_i + t_i}\right) + b_o\log_e\left(\frac{R_o}{R_o - t_o}\right)}$ $R = R_i + \dfrac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b_i - t) + (b_o - t)\,t_o\left(h - \frac{1}{2}t_o\right)}{t_i(b_i - t) + t_o(b_o - t) + t.h}$

Example 5.10. *The frame of a punch press is shown in Fig. 5.9. Find the stresses at the inner and outer surface at section X-X of the frame, if W = 5000 N.*

Solution. Given : W = 5000 N ; b_i = 18 mm ; b_o = 6 mm ; h = 40 mm ; R_i = 25 mm ; R_o = 25 + 40 = 65 mm

We know that area of section at X-X,

$$A = \frac{1}{2}(18 + 6)\,40 = 480 \text{ mm}^2$$

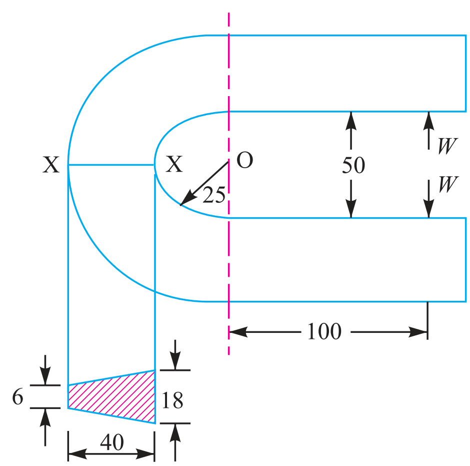
The various distances are shown in Fig. 5.10.

We know that radius of curvature of the neutral axis,

$$R_n = \frac{\left(\frac{b_i + b_o}{2}\right)h}{\left(\frac{b_i R_o - b_o R_i}{h}\right)\log_e\left(\frac{R_o}{R_i}\right) - (b_i - b_o)}$$

$$= \frac{\left(\frac{18 + 6}{2}\right) \times 40}{\left(\frac{18 \times 65 - 6 \times 25}{40}\right)\log_e\left(\frac{65}{25}\right) - (18 - 6)}$$

$$= \frac{480}{(25.5 \times 0.9555) - 12} = 38.83 \text{ mm}$$



Section at X-X

All dimensions in mm.

Fig. 5.9

Top

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{h\,(b_i + 2b_o)}{3\,(b_i + b_o)} = 25 + \frac{40\,(18 + 2 \times 6)}{3\,(18 + 6)} \text{ mm}$$

$$= 25 + 16.67 = 41.67 \text{ mm}$$

Distance between the centroidal axis and neutral axis,

$$e = R - R_n = 41.67 - 38.83 = 2.84 \text{ mm}$$

and the distance between the load and centroidal axis,

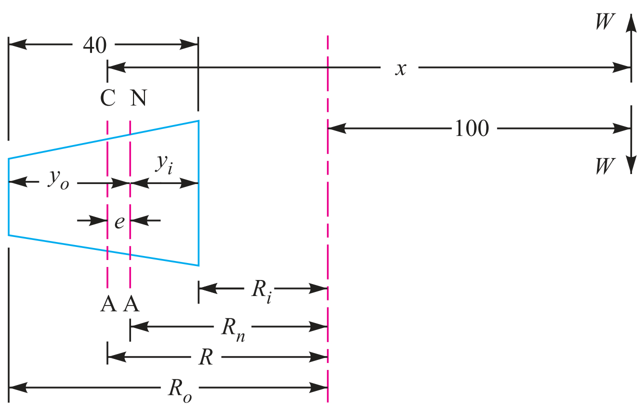
$$x = 100 + R = 100 + 41.67 = 141.67 \text{ mm}$$

∴ Bending moment about the centroidal axis,

$$M = W.x = 5000 \times 141.67 = 708\,350 \text{ N-mm}$$

The section at *X-X* is subjected to a direct tensile load of *W* = 5000 N and a bending moment of *M* = 708 350 N-mm. We know that direct tensile stress at section *X-X*,

$$\sigma_t = \frac{W}{A} = \frac{5000}{480} = 10.42 \text{ N/mm}^2 = 10.42 \text{ MPa}$$



All dimensions in mm.

Fig. 5.10

Distance from the neutral axis to the inner surface,

$$y_i = R_n - R_i = 38.83 - 25 = 13.83 \text{ mm}$$

Distance from the neutral axis to the outer surface,

$$y_o = R_o - R_n = 65 - 38.83 = 26.17 \text{ mm}$$

We know that maximum bending stress at the inner surface,

$$\sigma_{bi} = \frac{M\,.\,y_i}{A\,.\,e\,.\,R_i} = \frac{708\,350 \times 13.83}{480 \times 2.84 \times 25} = 287.4 \text{ N/mm}^2$$

$$= 287.4 \text{ MPa (tensile)}$$

and maximum bending stress at the outer surface,

$$\sigma_{b0} = \frac{M\,.\,y_o}{A\,.\,e\,.\,R_o} = \frac{708\,350 \times 26.17}{480 \times 2.84 \times 65} = 209.2 \text{ N/mm}^2$$

$$= 209.2 \text{ MPa (compressive)}$$

∴ Resultant stress on the inner surface

$$= \sigma_t + \sigma_{bi} = 10.42 + 287.4 = 297.82 \text{ MPa (tensile)}$$ **Ans.**

and resultant stress on the outer surface,

$$= \sigma_t - \sigma_{bo} = 10.42 - 209.2 = -198.78 \text{ MPa}$$

$$= 198.78 \text{ MPa (compressive)}$$ **Ans.**

A big crane hook

Example 5.11. *The crane hook carries a load of 20 kN as shown in Fig. 5.11. The section at X-X is rectangular whose horizontal side is 100 mm. Find the stresses in the inner and outer fibres at the given section.*

Solution. Given : $W = 20 \text{ kN} = 20 \times 10^3 \text{ N}$; $R_i = 50$ mm ; $R_o = 150$ mm ; $h = 100$ mm ; $b = 20$ mm

We know that area of section at *X-X*,

$$A = b.h = 20 \times 100 = 2000 \text{ mm}^2$$

The various distances are shown in Fig. 5.12.

We know that radius of curvature of the neutral axis,

$$R_n = \frac{h}{\log_e\left(\frac{R_o}{R_i}\right)} = \frac{100}{\log_e\left(\frac{150}{50}\right)} = \frac{100}{1.098} = 91.07 \text{ mm}$$

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{h}{2} = 50 + \frac{100}{2} = 100 \text{ mm}$$

∴ Distance between the centroidal axis and neutral axis,

$$e = R - R_n = 100 - 91.07 = 8.93 \text{ mm}$$

and distance between the load and the centroidal axis,

$$x = R = 100 \text{ mm}$$

∴ Bending moment about the centroidal axis,

$$M = W \times x = 20 \times 10^3 \times 100 = 2 \times 10^6 \text{ N-mm}$$

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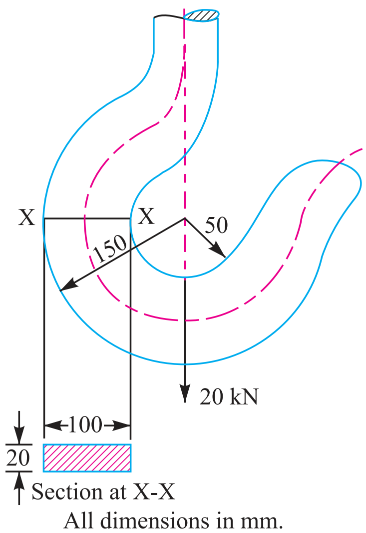
The section at X-X is subjected to a direct tensile load of $W = 20 \times 10^3$ N and a bending moment of $M = 2 \times 10^6$ N-mm. We know that direct tensile stress at section X-X,

$$\sigma_t = \frac{W}{A} = \frac{20 \times 10^3}{2000} = 10 \text{ N/mm}^2 = 10 \text{ MPa}$$



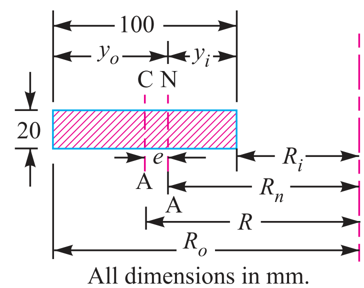


Fig. 5.11 **Fig. 5.12**

We know that the distance from the neutral axis to the inside fibre,

$$y_i = R_n - R_i = 91.07 - 50 = 41.07 \text{ mm}$$

and distance from the neutral axis to outside fibre,

$$y_o = R_o - R_n = 150 - 91.07 = 58.93 \text{ mm}$$

∴ Maximum bending stress at the inside fibre,

$$\sigma_{bi} = \frac{M \, . \, y_i}{A \, . \, e \, . \, R_i} = \frac{2 \times 10^6 \times 41.07}{2000 \times 8.93 \times 50} = 92 \text{ N/mm}^2 = 92 \text{ MPa (tensile)}$$

and maximum bending stress at the outside fibre,

$$\sigma_{bo} = \frac{M \, . \, y_o}{A \, . \, e \, . \, R_o} = \frac{2 \times 10^6 \times 58.93}{2000 \times 8.93 \times 150} = 44 \text{ N/mm}^2$$

$$= 44 \text{ MPa (compressive)}$$

∴ Resultant stress at the inside fibre

$$= \sigma_t + \sigma_{bi} = 10 + 92 = 102 \text{ MPa (tensile)}$$ **Ans.**

and resultant stress at the outside fibre

$$= \sigma_t - \sigma_{bo} = 10 - 44 = -34 \text{ MPa} = 34 \text{ MPa (compressive)}$$ **Ans.**

Example 5.12. *A C-clamp is subjected to a maximum load of W, as shown in Fig. 5.13. If the maximum tensile stress in the clamp is limited to 140 MPa, find the value of load W.*

Solution. Given : $\sigma_{t(max)} = 140$ MPa = 140 N/mm^2 ; $R_i = 25$ mm ; $R_o = 25 + 25 = 50$ mm ; $b_i = 19$ mm ; $t_i = 3$ mm ; $t = 3$ mm ; $h = 25$ mm

We know that area of section at X-X,

$$A = 3 \times 22 + 3 \times 19 = 123 \text{ mm}^2$$

Top

The various distances are shown in Fig. 5.14. We know that radius of curvature of the neutral axis,

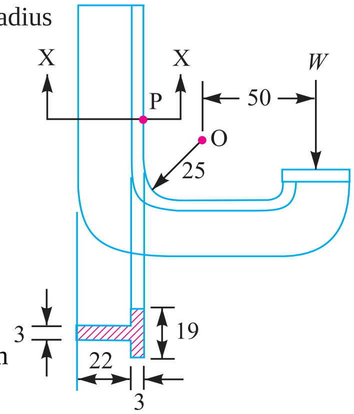


Fig. 5.13

$$R_n = \frac{t_i\,(b_i - t) + t\,.\,h}{(b_i - t)\log_e\left(\frac{R_i + t_i}{R_i}\right) + t\log_e\left(\frac{R_o}{R_i}\right)}$$

$$= \frac{3\,(19 - 3) + 3 \times 25}{(19 - 3)\log_e\left(\frac{25 + 3}{25}\right) + 3\log_e\left(\frac{50}{25}\right)}$$

$$= \frac{123}{16 \times 0.113 + 3 \times 0.693} = \frac{123}{3.887} = 31.64 \text{ mm}$$

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{\frac{1}{2}h^2\,.\,t + \frac{1}{2}t_i^2\,(b_i - t)}{h\,.\,t + t_i\,(b_i - t)}$$

$$= 25 + \frac{\frac{1}{2} \times 25^2 \times 3 + \frac{1}{2} \times 3^2\,(19 - 3)}{25 \times 3 + 3\,(19 - 3)} = 25 + \frac{937.5 + 72}{75 + 48}$$

$$= 25 + 8.2 = 33.2 \text{ mm}$$

Distance between the centroidal axis and neutral axis,

$$e = R - R_n = 33.2 - 31.64 = 1.56 \text{ mm}$$

and distance between the load W and the centroidal axis,

$$x = 50 + R = 50 + 33.2 = 83.2 \text{ mm}$$

∴ Bending moment about the centroidal axis,

$$M = W.x = W \times 83.2 = 83.2\,W \text{ N-mm}$$

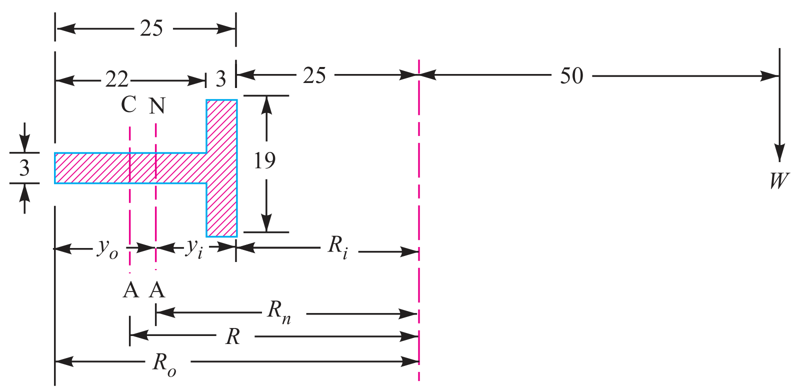


Fig. 5.14

The section at X-X is subjected to a direct tensile load of W and a bending moment of 83.2 W. The maximum tensile stress will occur at point P (*i.e.* at the inner fibre of the section).

Distance from the neutral axis to the point P,

$$y_i = R_n - R_i = 31.64 - 25 = 6.64 \text{ mm}$$

Direct tensile stress at section *X-X*,

$$\sigma_t = \frac{W}{A} = \frac{W}{123} = 0.008\ W\ \text{N/mm}^2$$

and maximum bending stress at point *P*,

$$\sigma_{bi} = \frac{M\ .\ y_i}{A\ .\ e\ .\ R_i} = \frac{83.2\ W \times 6.64}{123 \times 1.56 \times 25} = 0.115\ W\ \text{N/mm}^2$$

We know that the maximum tensile stress $\sigma_{t(max)}$,

$$140 = \sigma_t + \sigma_{bi} = 0.008\ W + 0.115\ W = 0.123\ W$$

$$\therefore \quad W = 140/0.123 = 1138\ \text{N}$$ **Ans.**

Note : We know that distance from the neutral axis to the outer fibre,

$$y_o = R_o - R_n = 50 - 31.64 = 18.36\ \text{mm}$$

∴ Maximum bending stress at the outer fibre,

$$\sigma_{bo} = \frac{M\ .\ y_o}{A\ .\ e\ .\ R_o} = \frac{83.2\ W \times 18.36}{123 \times 1.56 \times 50} = 0.16\ W$$

and maximum stress at the outer fibre,

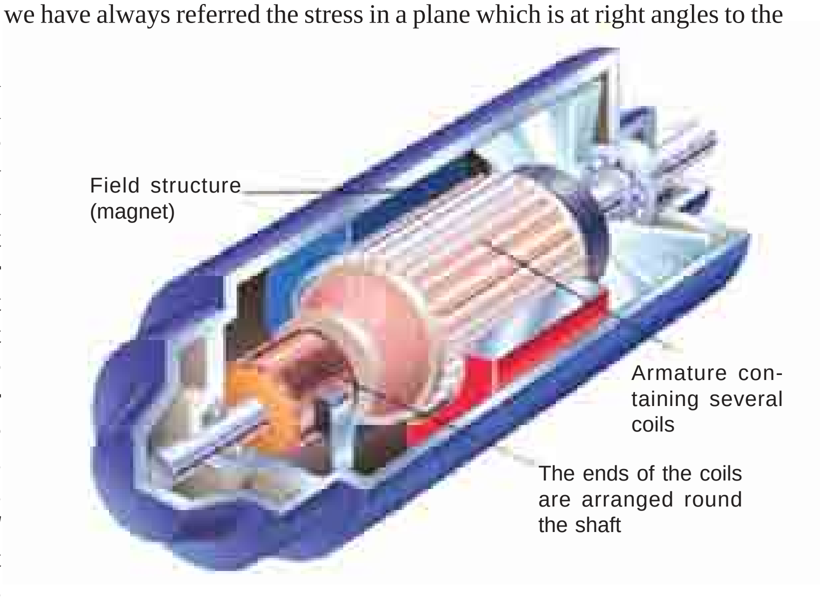
$$= \sigma_t - \sigma_{bo} = 0.008\ W - 0.16\ W = -\ 0.152\ W\ \text{N/mm}^2$$

$$= 0.152\ W\ \text{N/mm}^2\ \text{(compressive)}$$

From above we see that stress at the outer fibre is larger in this case than at the inner fibre, but this stress at outer fibre is compressive.

5.6 Principal Stresses and Principal Planes

In the previous chapter, we have discussed about the direct tensile and compressive stress as well as simple shear. Also we have always referred the stress in a plane which is at right angles to the line of action of the force. But it has been observed that at any point in a strained material, there are three planes, mutually perpendicular to each other which carry direct stresses only and no shear stress. It may be noted that out of these three direct stresses, one will be maximum and the other will be minimum. These perpendicular planes which have no shear stress are known as ***principal planes*** and the direct stresses along these planes are known as ***principal stresses***. The planes on which the maximum shear stress act are known as planes of maximum shear.



Big electric generators undergo high torsional stresses.

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5.7 Determination of Principal Stresses for a Member Subjected to Bi-axial Stress

When a member is subjected to bi-axial stress (*i.e.* direct stress in two mutually perpendicular planes accompanied by a simple shear stress), then the normal and shear stresses are obtained as discussed below:

Consider a rectangular body *ABCD* of uniform cross-sectional area and unit thickness subjected to normal stresses σ_1 and σ_2 as shown in Fig. 5.15 (*a*). In addition to these normal stresses, a shear stress τ also acts.

It has been shown in books on ***'Strength of Materials'*** that the normal stress across any oblique section such as *EF* inclined at an angle θ with the direction of σ_2, as shown in Fig. 5.15 (*a*), is given by

$$\sigma_t = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta \qquad ...(i)$$

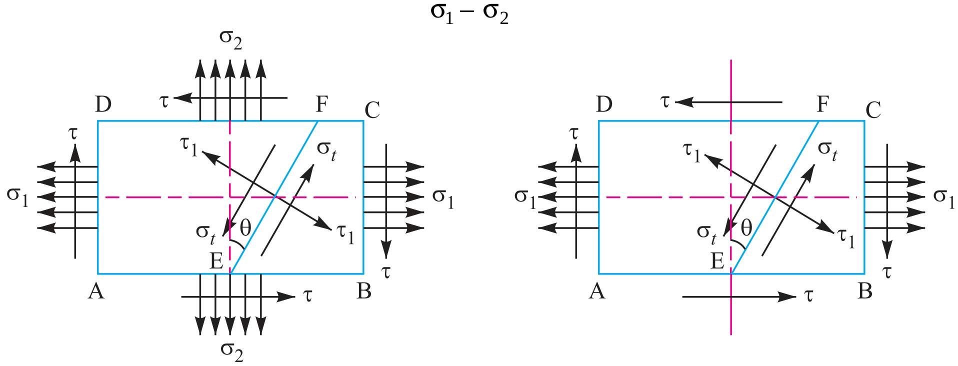
and tangential stress (*i.e.* shear stress) across the section *EF*,

$$\tau_1 = \frac{1}{2}(\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta \qquad ...(ii)$$

Since the planes of maximum and minimum normal stress (*i.e.* principal planes) have no shear stress, therefore the inclination of principal planes is obtained by equating $\tau_1 = 0$ in the above equation (*ii*), *i.e.*

$$\frac{1}{2}(\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta = 0$$

$$\therefore \qquad \tan 2\theta = \frac{2\,\tau}{\sigma_1 - \sigma_2} \qquad ...(iii)$$



(*a*) Direct stress in two mutually prependicular planes accompanied by a simple shear stress.

(*b*) Direct stress in one plane accompanied by a simple shear stress.

Fig. 5.15. Principal stresses for a member subjected to bi-axial stress.

We know that there are two principal planes at right angles to each other. Let θ_1 and θ_2 be the inclinations of these planes with the normal cross-section.

From Fig. 5.16, we find that

$$\sin 2\theta = \pm \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}}$$

Top

$$\therefore \quad \sin 2\theta_1 = + \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}}$$

and
$$\sin 2\theta_2 = - \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}}$$

Also
$$\cos 2\theta = \pm \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}}$$

$$\therefore \quad \cos 2\theta_1 = + \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}}$$

and
$$\cos 2\theta_2 = - \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}}$$

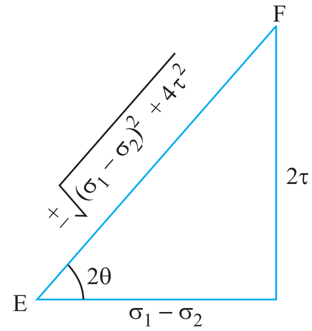


Fig. 5.16

The maximum and minimum principal stresses may now be obtained by substituting the values of sin 2θ and cos 2θ in equation ***(i)***.

∴ Maximum principal (or normal) stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2} \qquad ...(iv)$$

and minimum principal (or normal) stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2} \qquad ...(v)$$

The planes of maximum shear stress are at right angles to each other and are inclined at 45° to the principal planes. The maximum shear stress is given by ***one-half the algebraic difference between the principal stresses,*** *i.e.*

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2} \qquad ...(vi)$$

A Boring mill.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Notes: 1. When a member is subjected to direct stress in one plane accompanied by a simple shear stress as shown in Fig. 5.15 (*b*), then the principal stresses are obtained by substituting $\sigma_2 = 0$ in equation **(*iv*), (*v*)** and **(*vi*).**

$$\therefore \qquad \sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

and

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

2. In the above expression of σ_{t2}, the value of $\frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$ is more than $\frac{\sigma_1}{2}$. Therefore the nature of σ_{t2} will be opposite to that of σ_{t1}, *i.e.* if σ_{t1} is tensile then σ_{t2} will be compressive and *vice-versa*.

5.8 Application of Principal Stresses in Designing Machine Members

There are many cases in practice, in which machine members are subjected to combined stresses due to simultaneous action of either tensile or compressive stresses combined with shear stresses. In many shafts such as propeller shafts, C-frames etc., there are direct tensile or compressive stresses due to the external force and shear stress due to torsion, which acts normal to direct tensile or compressive stresses. The shafts like crank shafts, are subjected simultaneously to torsion and bending. In such cases, the maximum principal stresses, due to the combination of tensile or compressive stresses with shear stresses may be obtained.

The results obtained in the previous article may be written as follows:

1. Maximum tensile stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

2. Maximum compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right]$$

3. Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

where σ_t = Tensile stress due to direct load and bending,

σ_c = Compressive stress, and

τ = Shear stress due to torsion.

Notes : 1. When $\tau = 0$ as in the case of thin cylindrical shell subjected in internal fluid pressure, then

$$\sigma_{t\,(max)} = \sigma_t$$

2. When the shaft is subjected to an axial load (P) in addition to bending and twisting moments as in the propeller shafts of ship and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). This will give the resultant tensile stress or compressive stress (σ_t or σ_c) depending upon the type of axial load (*i.e.* pull or push).

Example 5.13. *A hollow shaft of 40 mm outer diameter and 25 mm inner diameter is subjected to a twisting moment of 120 N-m, simultaneously, it is subjected to an axial thrust of 10 kN and a bending moment of 80 N-m. Calculate the maximum compressive and shear stresses.*

Solution. Given: d_o = 40 mm ; d_i = 25 mm ; T = 120 N-m = 120 × 10^3 N-mm ; P = 10 kN = 10 × 10^3 N ; M = 80 N-m = 80 × 10^3 N-mm

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] = \frac{\pi}{4}\left[(40)^2 - (25)^2\right] = 766 \text{ mm}^2$$

∴ Direct compressive stress due to axial thrust,

$$\sigma_o = \frac{P}{A} = \frac{10 \times 10^3}{766} = 13.05 \text{ N/mm}^2 = 13.05 \text{ MPa}$$

Section modulus of the shaft,

$$Z = \frac{\pi}{32}\left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{32}\left[\frac{(40)^4 - (25)^4}{40}\right] = 5325 \text{ mm}^3$$

∴ Bending stress due to bending moment,

$$\sigma_b = \frac{M}{Z} = \frac{80 \times 10^3}{5325} = 15.02 \text{ N/mm}^2 = 15.02 \text{ MPa (compressive)}$$

and resultant compressive stress,

$$\sigma_c = \sigma_b + \sigma_o = 15.02 + 13.05 = 28.07 \text{ N/mm}^2 = 28.07 \text{ MPa}$$

We know that twisting moment (T),

$$120 \times 10^3 = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \left[\frac{(40)^4 - (25)^4}{40}\right] = 10\,650\ \tau$$

$$\therefore \quad \tau = 120 \times 10^3/10\,650 = 11.27 \text{ N/mm}^2 = 11.27 \text{ MPa}$$

Maximum compressive stress

We know that maximum compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right]$$

$$= \frac{28.07}{2} + \frac{1}{2}\left[\sqrt{(28.07)^2 + 4\,(11.27)^2}\right]$$

$$= 14.035 + 18 = 32.035 \text{ MPa}$$ **Ans.**

Maximum shear stress

We know that maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(28.07)^2 + 4(11.27)^2}\right] = 18 \text{ MPa}$$ **Ans.**

Example 5.14. *A shaft, as shown in Fig. 5.17, is subjected to a bending load of 3 kN, pure torque of 1000 N-m and an axial pulling force of 15 kN.*

Calculate the stresses at A and B.

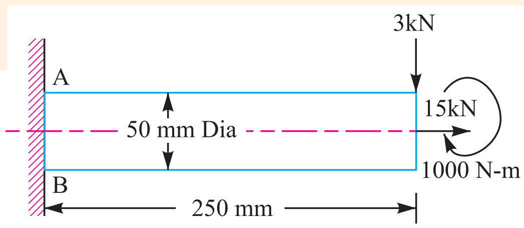


Fig. 5.17

Solution. Given : W = 3 kN = 3000 N ; T = 1000 N-m = 1×10^6 N-mm ; P = 15 kN = 15×10^3 N ; d = 50 mm; x = 250 mm

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4}(50)^2 = 1964 \text{ mm}^2$$

∴ Tensile stress due to axial pulling at points A and B,

$$\sigma_o = \frac{P}{A} = \frac{15 \times 10^3}{1964} = 7.64 \text{ N/mm}^2 = 7.64 \text{ MPa}$$

Bending moment at points A and B,

$$M = W.x = 3000 \times 250 = 750 \times 10^3 \text{ N-mm}$$

Section modulus for the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} (50)^3$$

$$= 12.27 \times 10^3 \text{ mm}^3$$

∴ Bending stress at points *A* and *B*,

$$\sigma_b = \frac{M}{Z} = \frac{750 \times 10^3}{12.27 \times 10^3}$$

$$= 61.1 \text{ N/mm}^2 = 61.1 \text{ MPa}$$

This bending stress is tensile at point *A* and compressive at point *B*.

∴ Resultant tensile stress at point *A*,

$$\sigma_A = \sigma_b + \sigma_o = 61.1 + 7.64$$

$$= 68.74 \text{ MPa}$$

and resultant compressive stress at point *B*,

$$\sigma_B = \sigma_b - \sigma_o = 61.1 - 7.64 = 53.46 \text{ MPa}$$

This picture shows a machine component inside a crane

Note : This picture is given as additional information and is not a direct example of the current chapter.

We know that the shear stress at points *A* and *B* due to the torque transmitted,

$$\tau = \frac{16\,T}{\pi\, d^3} = \frac{16 \times 1 \times 10^6}{\pi\,(50)^3} = 40.74 \text{ N/mm}^2 = 40.74 \text{ MPa} \qquad \ldots\left(\because\ T = \frac{\pi}{16} \times \tau \times d^3\right)$$

Stresses at point A

We know that maximum principal (or normal) stress at point *A*,

$$\sigma_{A(max)} = \frac{\sigma_A}{2} + \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right]$$

$$= \frac{68.74}{2} + \frac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$

$$= 34.37 + 53.3 = 87.67 \text{ MPa (tensile)}$$ **Ans.**

Minimum principal (or normal) stress at point *A*,

$$\sigma_{A(min)} = \frac{\sigma_A}{2} - \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = 34.37 - 53.3 = -18.93 \text{ MPa}$$

$$= 18.93 \text{ MPa (compressive)}$$ **Ans.**

and maximum shear stress at point *A*,

$$\tau_{A(max)} = \tfrac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$

$$= 53.3 \text{ MPa}$$ **Ans.**

Stresses at point B

We know that maximum principal (or normal) stress at point *B*,

$$\sigma_{B(max)} = \frac{\sigma_B}{2} + \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right]$$

$$= \frac{53.46}{2} + \frac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$

$$= 26.73 + 48.73 = 75.46 \text{ MPa (compressive)}$$ **Ans.**

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Minimum principal (or normal) stress at point *B*,

$$\sigma_{B(min)} = \frac{\sigma_B}{2} - \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\tau^2}\right]$$

$$= 26.73 - 48.73 = -22 \text{ MPa}$$

$$= 22 \text{ MPa (tensile)} \textbf{ Ans.}$$

and maximum shear stress at point *B*,

$$\tau_{B(max)} = \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$

$$= 48.73 \text{ MPa} \textbf{ Ans.}$$

Example 5.15. *An overhang crank with pin and shaft is shown in Fig. 5.18. A tangential load of 15 kN acts on the crank pin. Determine the maximum principal stress and the maximum shear stress at the centre of the crankshaft bearing.*

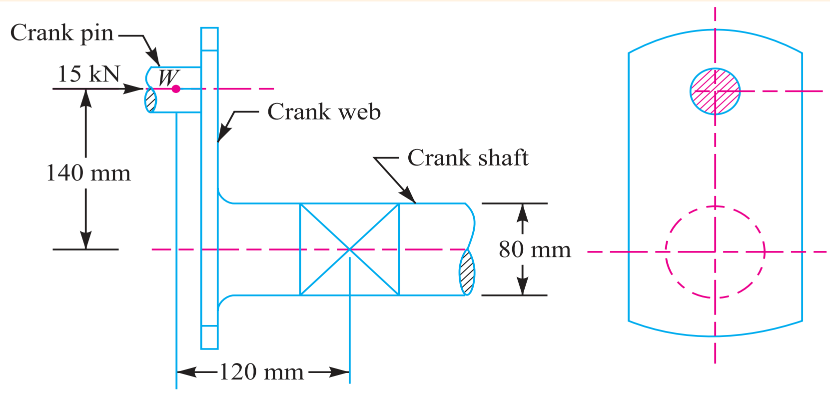

Fig. 5.18

Solution. Given : $W = 15$ kN $= 15 \times 10^3$ N ; $d = 80$ mm ; $y = 140$ mm ; $x = 120$ mm

Bending moment at the centre of the crankshaft bearing,

$$M = W \times x = 15 \times 10^3 \times 120 = 1.8 \times 10^6 \text{ N-mm}$$

and torque transmitted at the axis of the shaft,

$$T = W \times y = 15 \times 10^3 \times 140 = 2.1 \times 10^6 \text{ N-mm}$$

We know that bending stress due to the bending moment,

$$\sigma_b = \frac{M}{Z} = \frac{32\,M}{\pi\,d^3} \qquad \ldots\left(\because Z = \frac{\pi}{32} \times d^3\right)$$

$$= \frac{32 \times 1.8 \times 10^6}{\pi\,(80)^3} = 35.8 \text{ N/mm}^2 = 35.8 \text{ MPa}$$

and shear stress due to the torque transmitted,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16 \times 2.1 \times 10^6}{\pi\,(80)^3} = 20.9 \text{ N/mm}^2 = 20.9 \text{ MPa}$$

Maximum principal stress

We know that maximum principal stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

$$= \frac{35.8}{2} + \frac{1}{2}\left[\sqrt{(35.8)^2 + 4\,(20.9)^2}\right] \qquad \ldots (\text{Substituting } \sigma_t = \sigma_b)$$

$$= 17.9 + 27.5 = 45.4 \text{ MPa} \textbf{ Ans.}$$

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Maximum shear stress

We know that maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(35.8)^2 + 4\,(20.9)^2}\right]$$

$$= 27.5 \text{ MPa}$$ **Ans.**

5.9 Theories of Failure Under Static Load

It has already been discussed in the previous chapter that strength of machine members is based upon the mechanical properties of the materials used. Since these properties are usually determined from simple tension or compression tests, therefore, predicting failure in members subjected to uni-axial stress is both simple and straight-forward. But the problem of predicting the failure stresses for members subjected to bi-axial or tri-axial stresses is much more complicated. In fact, the problem is so complicated that a large number of different theories have been formulated. The principal theories of failure for a member subjected to bi-axial stress are as follows:

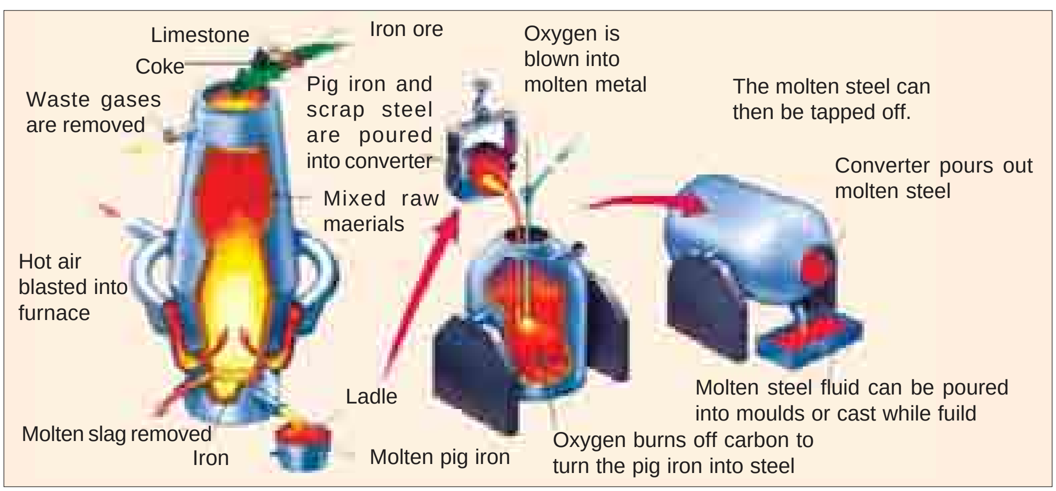
1. Maximum principal (or normal) stress theory (also known as Rankine's theory).
2. Maximum shear stress theory (also known as Guest's or Tresca's theory).
3. Maximum principal (or normal) strain theory (also known as Saint Venant theory).
4. Maximum strain energy theory (also known as Haigh's theory).
5. Maximum distortion energy theory (also known as Hencky and Von Mises theory).

Since ductile materials usually fail by yielding *i.e.* when permanent deformations occur in the material and brittle materials fail by fracture, therefore the limiting strength for these two classes of materials is normally measured by different mechanical properties. For ductile materials, the limiting strength is the stress at yield point as determined from simple tension test and it is, assumed to be equal in tension or compression. For brittle materials, the limiting strength is the ultimate stress in tension or compression.

5.10 Maximum Principal or Normal Stress Theory (Rankine's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test.

Since the limiting strength for ductile materials is yield point stress and for brittle materials (which do not have well defined yield point) the limiting strength is ultimate stress, therefore according



Pig iron is made from iron ore in a blast furnace. It is a brittle form of iron that contains 4-5 per cent carbon.

Note : This picture is given as additional information and is not a direct example of the current chapter.

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to the above theory, taking factor of safety (*F.S.*) into consideration, the maximum principal or normal stress (σ_{t1}) in a bi-axial stress system is given by

$$\sigma_{t1} = \frac{\sigma_{yt}}{F.S.}, \text{ for ductile materials}$$

$$= \frac{\sigma_u}{F.S.}, \text{ for brittle materials}$$

where σ_{yt} = Yield point stress in tension as determined from simple tension test, and

σ_u = Ultimate stress.

Since the maximum principal or normal stress theory is based on failure in tension or compression and ignores the possibility of failure due to shearing stress, therefore it is not used for ductile materials. However, for brittle materials which are relatively strong in shear but weak in tension or compression, this theory is generally used.

Note : The value of maximum principal stress (σ_{t1}) for a member subjected to bi-axial stress system may be determined as discussed in Art. 5.7.

5.11 Maximum Shear Stress Theory (Guest's or Tresca's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test. Mathematically,

$$\tau_{max} = \tau_{yt}/F.S. \quad \text{...(i)}$$

where τ_{max} = Maximum shear stress in a bi-axial stress system,

τ_{yt} = Shear stress at yield point as determined from simple tension test, and

$F.S.$ = Factor of safety.

Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, therefore the equation ***(i)*** may be written as

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.}$$

This theory is mostly used for designing members of ductile materials.

Note: The value of maximum shear stress in a bi-axial stress system (τ_{max}) may be determined as discussed in Art. 5.7.

5.12 Maximum Principal Strain Theory (Saint Venant's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain (*i.e.* strain at yield point) as determined from a simple tensile test. The maximum principal (or normal) strain in a bi-axial stress system is given by

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \cdot E}$$

∴ According to the above theory,

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \cdot E} = \varepsilon = \frac{\sigma_{yt}}{E \times F.S.} \quad \text{...(i)}$$

where σ_{t1} and σ_{t2} = Maximum and minimum principal stresses in a bi-axial stress system,

ε = Strain at yield point as determined from simple tension test,

$1/m$ = Poisson's ratio,

E = Young's modulus, and

$F.S.$ = Factor of safety.

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From equation *(i)*, we may write that

$$\sigma_{t1} - \frac{\sigma_{t2}}{m} = \frac{\sigma_{yt}}{F.S.}$$

This theory is not used, in general, because it only gives reliable results in particular cases.

5.13 Maximum Strain Energy Theory (Haigh's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the strain energy per unit volume in a bi-axial stress system reaches the limiting strain energy (*i.e.* strain energy at the yield point) per unit volume as determined from simple tension test.

This double-decker A 380 has a passenger capacity of 555. Its engines and parts should be robust which can bear high torsional and variable stresses.

We know that strain energy per unit volume in a bi-axial stress system,

$$U_1 = \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1}\times\sigma_{t2}}{m}\right]$$

and limiting strain energy per unit volume for yielding as determined from simple tension test,

$$U_2 = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

According to the above theory, $U_1 = U_2$.

$$\therefore \quad \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1}\times\sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

or

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1}\times\sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

This theory may be used for ductile materials.

5.14 Maximum Distortion Energy Theory (Hencky and Von Mises Theory)

According to this theory, the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (*i.e.* distortion energy at yield point) per unit volume as determined from a simple tension test. Mathematically, the maximum distortion energy theory for yielding is expressed as

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1}\times\sigma_{t2} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

This theory is mostly used for ductile materials in place of maximum strain energy theory.

Note: The maximum distortion energy is the difference between the total strain energy and the strain energy due to uniform stress.

Example 5.16. *The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN. Find the diameter of bolt required according to*

1. Maximum principal stress theory; 2. Maximum shear stress theory; 3. Maximum principal strain theory; 4. Maximum strain energy theory; and 5. Maximum distortion energy theory.

Take permissible tensile stress at elastic limit = 100 MPa and poisson's ratio = 0.3.

Solution. Given : P_{t1} = 10 kN ; P_s = 5 kN ; $\sigma_{t(el)}$ = 100 MPa = 100 N/mm^2 ; $1/m$ = 0.3

Let d = Diameter of the bolt in mm.

∴ Cross-sectional area of the bolt,

$$A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that axial tensile stress,

$$\sigma_1 = \frac{P_{t1}}{A} = \frac{10}{0.7854\, d^2} = \frac{12.73}{d^2} \text{ kN/mm}^2$$

and transverse shear stress,

$$\tau = \frac{P_s}{A} = \frac{5}{0.7854\, d^2} = \frac{6.365}{d^2} \text{ kN/mm}^2$$

1. *According to maximum principal stress theory*

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \ldots(\because \sigma_2 = 0)$$

$$= \frac{12.73}{2\,d^2} + \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} + \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 + \frac{1}{2}\sqrt{4+4}\right] = \frac{15.365}{d^2} \text{ kN/mm}^2 = \frac{15\,365}{d^2} \text{ N/mm}^2$$

According to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{t(el)} \quad \text{or} \quad \frac{15\,365}{d^2} = 100$$

∴ d^2 = 15 365/100 = 153.65 or d = 12.4 mm **Ans.**

2. *According to maximum shear stress theory*

We know that maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \ldots(\because \sigma_2 = 0)$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right] = \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{9}{d^2} \text{ kN/mm}^2 = \frac{9000}{d^2} \text{ N/mm}^2$$

According to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{t(el)}}{2} \quad \text{or} \quad \frac{9000}{d^2} = \frac{100}{2} = 50$$

∴ d^2 = 9000 / 50 = 180 or d = 13.42 mm **Ans.**

3. *According to maximum principal strain theory*

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \frac{15\,365}{d^2} \qquad \text{...(As calculated before)}$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{12.73}{2\,d^2} - \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} - \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 - \sqrt{2}\right] = \frac{-2.635}{d^2}\ \text{kN/mm}^2$$

$$= \frac{-2635}{d^2}\ \text{N/mm}^2$$

Front view of a jet engine. The rotors undergo high torsional and bending stresses.

We know that according to maximum principal strain theory,

$$\frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{mE} = \frac{\sigma_{t(el)}}{E} \quad \text{or} \quad \sigma_{t1} - \frac{\sigma_{t2}}{m} = \sigma_{t(el)}$$

$$\therefore \quad \frac{15\,365}{d^2} + \frac{2635 \times 0.3}{d^2} = 100 \quad \text{or} \quad \frac{16\,156}{d^2} = 100$$

$d^2 = 16\,156 / 100 = 161.56$ or $d = 12.7$ mm **Ans.**

4. *According to maximum strain energy theory*

We know that according to maximum strain energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-2635}{d^2} \times 0.3 = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{24.3 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{2430}{d^4} = 1 \quad \text{or} \quad \frac{26\,724}{d^4} = 1$$

$\therefore$ $d^4 = 26\,724$ or $d = 12.78$ mm **Ans.**

5. *According to maximum distortion energy theory*

According to maximum distortion energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-2635}{d^2} = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{80.97 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{8097}{d^4} = 1 \quad \text{or} \quad \frac{32\,391}{d^4} = 1$$

$\therefore$ $d^4 = 32\,391$ or $d = 13.4$ mm **Ans.**

Example 5.17. *A cylindrical shaft made of steel of yield strength 700 MPa is subjected to static loads consisting of bending moment 10 kN-m and a torsional moment 30 kN-m. Determine the diameter of the shaft using two different theories of failure, and assuming a factor of safety of 2. Take E = 210 GPa and poisson's ratio = 0.25.*

Solution. Given : σ_{yt} = 700 MPa = 700 N/mm^2 ; M = 10 kN-m = 10 × 10^6 N-mm ; T = 30 kN-m = 30 × 10^6 N-mm ; $F.S.$ = 2 ; E = 210 GPa = 210 × 10^3 N/mm^2 ; 1/m = 0.25

Let d = Diameter of the shaft in mm.

First of all, let us find the maximum and minimum principal stresses.

We know that section modulus of the shaft

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

∴ Bending (tensile) stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{10 \times 10^6}{0.0982\, d^3} = \frac{101.8 \times 10^6}{d^3} \text{ N/mm}^2$$

and shear stress due to torsional moment,

$$\tau = \frac{16\, T}{\pi\, d^3} = \frac{16 \times 30 \times 10^6}{\pi\, d^3} = \frac{152.8 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \text{...}(\because \sigma_2 = 0)$$

$$= \frac{101.8 \times 10^6}{2d^3} + \frac{1}{2}\left[\sqrt{\left(\frac{101.8 \times 10^6}{d^3}\right)^2 + 4\left(\frac{152.8 \times 10^6}{d^3}\right)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{1}{2} \times \frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{161 \times 10^6}{d^3} = \frac{211.9 \times 10^6}{d^3} \text{ N/mm}^2$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \text{...}(\because \sigma_2 = 0)$$

$$= \frac{50.9 \times 10^6}{d^3} - \frac{161 \times 10^6}{d^3} = \frac{-110.1 \times 10^6}{d^3} \text{ N/mm}^2$$

Let us now find out the diameter of shaft (d) by considering the maximum shear stress theory and maximum strain energy theory.

1. According to maximum shear stress theory

We know that maximum shear stress,

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{1}{2}\left[\frac{211.9 \times 10^6}{d^3} + \frac{110.1 \times 10^6}{d^3}\right] = \frac{161 \times 10^6}{d^3}$$

We also know that according to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{yt}}{2\, F.S.} \quad \text{or} \quad \frac{161 \times 10^6}{d^3} = \frac{700}{2 \times 2} = 175$$

∴ d^3 = 161 × 10^6 / 175 = 920 × 10^3 or d = 97.2 mm **Ans.**

Note: The value of maximum shear stress (τ_{max}) may also be obtained by using the relation,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{101.8\times10^6}{d^3}\right)^2 + 4\left(\frac{152.8\times10^6}{d^3}\right)^2}\right]$$

$$= \frac{1}{2}\times\frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{1}{2}\times\frac{10^6}{d^3}\times 322 = \frac{161\times10^6}{d^3}\ \text{N/mm}^2 \qquad \text{...(Same as before)}$$

2. *According to maximum strain energy theory*

We know that according to maximum strain energy theory,

$$\frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1}\times\sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

or

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1}\times\sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

$$\left[\frac{211.9\times10^6}{d^3}\right]^2 + \left[\frac{-110.1\times10^6}{d^3}\right]^2 - 2\times\frac{211.9\times10^6}{d^3}\times\frac{-110.1\times10^6}{d^3}\times 0.25 = \left(\frac{700}{2}\right)^2$$

or

$$\frac{44\,902\times10^{12}}{d^6} + \frac{12\,122\times10^{12}}{d^6} + \frac{11\,665\times10^{12}}{d^6} = 122\,500$$

$$\frac{68\,689\times10^{12}}{d^6} = 122\,500$$

∴ $d^6 = 68\,689 \times 10^{12}/122\,500 = 0.5607 \times 10^{12}$ or $d = 90.8$ mm **Ans.**

Example 5.18. *A mild steel shaft of 50 mm diameter is subjected to a bending moment of 2000 N-m and a torque T. If the yield point of the steel in tension is 200 MPa, find the maximum value of this torque without causing yielding of the shaft according to 1. the maximum principal stress; 2. the maximum shear stress; and 3. the maximum distortion strain energy theory of yielding.*

Solution. Given: $d = 50$ mm ; $M = 2000$ N-m $= 2 \times 10^6$ N-mm ; $\sigma_{yt} = 200$ MPa $= 200$ N/mm^2

Let T = Maximum torque without causing yielding of the shaft, in N-mm.

1. *According to maximum principal stress theory*

We know that section modulus of the shaft,

$$Z = \frac{\pi}{32}\times d^3 = \frac{\pi}{32}\,(50)^3 = 12\,273\ \text{mm}^3$$

∴ Bending stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{2\times10^6}{12\,273} = 163\ \text{N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16\,T}{\pi\,(50)^3} = 0.0407\times10^{-3}\,T\ \text{N/mm}^2$$

$$\ldots\left[\because T = \frac{\pi}{16}\times\tau\times d^3\right]$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} + \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407\times10^{-3}\,T)^2}\right]$$

$$= 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} \text{ N/mm}^2$$

Minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} - \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\,T)^2}\right]$$

$$= 81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} \text{ N/mm}^2$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\,T)^2}\right]$$

$$= \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} \text{ N/mm}^2$$

We know that according to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{yt} \qquad \text{...(Taking } F.S. = 1)$$

$$\therefore \quad 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} = 200$$

$$6642.5 + 1.65 + 10^{-9}\,T^2 = (200 - 81.5)^2 = 14\,042$$

$$T^2 = \frac{14\,042 - 6642.5}{1.65 \times 10^{-9}} = 4485 \times 10^9$$

or $T = 2118 \times 10^3$ N-mm = 2118 N-m **Ans.**

2. *According to maximum shear stress theory*

We know that according to maximum shear stress theory,

$$\tau_{max} = \tau_{yt} = \frac{\sigma_{yt}}{2}$$

$$\therefore \quad \sqrt{6642.5 + 1.65 \times 10^{-9} T^2} = \frac{200}{2} = 100$$

$$6642.5 + 1.65 \times 10^{-9}\,T^2 = (100)^2 = 10\,000$$

$$T^2 = \frac{10\,000 - 6642.5}{1.65 \times 10^{-9}} = 2035 \times 10^9$$

$\therefore$ $T = 1426 \times 10^3$ N-mm = 1426 N-m **Ans.**

3. *According to maximum distortion strain energy theory*

We know that according to maximum distortion strain energy theory

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \sigma_{t1} \times \sigma_{t2} = (\sigma_{yt})^2$$

$$\left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9} T^2}\right]^2 + \left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9} T^2}\right]^2$$

$$- \left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9} T^2}\right]\left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9} T^2}\right] = (200)^2$$

$$2\left[(81.5)^2 + 6642.5 + 1.65 \times 10^{-9}\,T^2\right] - \left[(81.5)^2 - 6642.5 + 1.65 \times 10^{-9}\,T^2\right] = (200)^2$$

$$(81.5)^2 + 3 \times 6642.5 + 3 \times 1.65 \times 10^{-9}\,T^2 = (200)^2$$

$$26\,570 + 4.95 \times 10^{-9}\,T^2 = 40\,000$$

$$T^2 = \frac{40\,000 - 26\,570}{4.95 \times 10^{-9}} = 2713 \times 10^9$$

$\therefore$ $T = 1647 \times 10^3$ N-mm = 1647 N-m **Ans.**

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5.15 Eccentric Loading - Direct and Bending Stresses Combined

An external load, whose line of action is parallel but does not coincide with the centroidal axis of the machine component, is known as an ***eccentric load***. The distance between the centroidal axis of the machine component and the eccentric load is called ***eccentricity*** and is generally denoted by *e*. The examples of eccentric loading, from the subject point of view, are *C*-clamps, punching machines, brackets, offset connecting links etc.

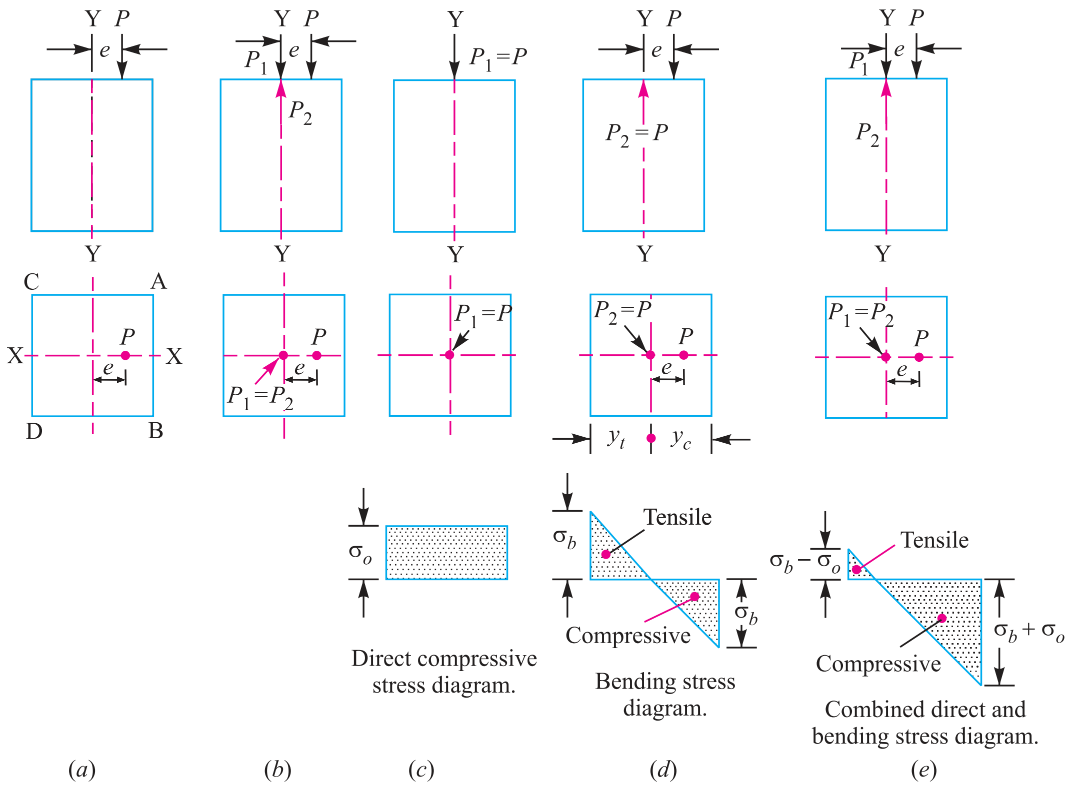


Fig. 5.19. Eccentric loading.

Consider a short prismatic bar subjected to a compressive load *P* acting at an eccentricity of *e* as shown in Fig. 5.19 (*a*).

Let us introduce two forces P_1 and P_2 along the centre line or neutral axis equal in magnitude to *P*, without altering the equilibrium of the bar as shown in Fig. 5.19 (*b*). A little consideration will show that the force P_1 will induce a direct compressive stress over the entire cross-section of the bar, as shown in Fig. 5.19 (*c*).

The magnitude of this direct compressive stress is given by

$$\sigma_o = \frac{P_1}{A} \text{ or } \frac{P}{A}, \text{ where } A \text{ is the cross-sectional area of the bar.}$$

The forces P_1 and P_2 will form a couple equal to $P \times e$ which will cause bending stress. This bending stress is compressive at the edge *AB* and tensile at the edge *CD*, as shown in Fig. 5.19 (*d*). The magnitude of bending stress at the edge *AB* is given by

$$\sigma_b = \frac{P \cdot e \cdot y_c}{I} \text{ (compressive)}$$

and bending stress at the edge *CD*,

$$\sigma_b = \frac{P \cdot e \cdot y_t}{I} \text{ (tensile)}$$

Top

where y_c and y_t = Distances of the extreme fibres on the compressive and tensile sides, from the neutral axis respectively, and

I = Second moment of area of the section about the neutral axis *i.e.* *Y*-axis.

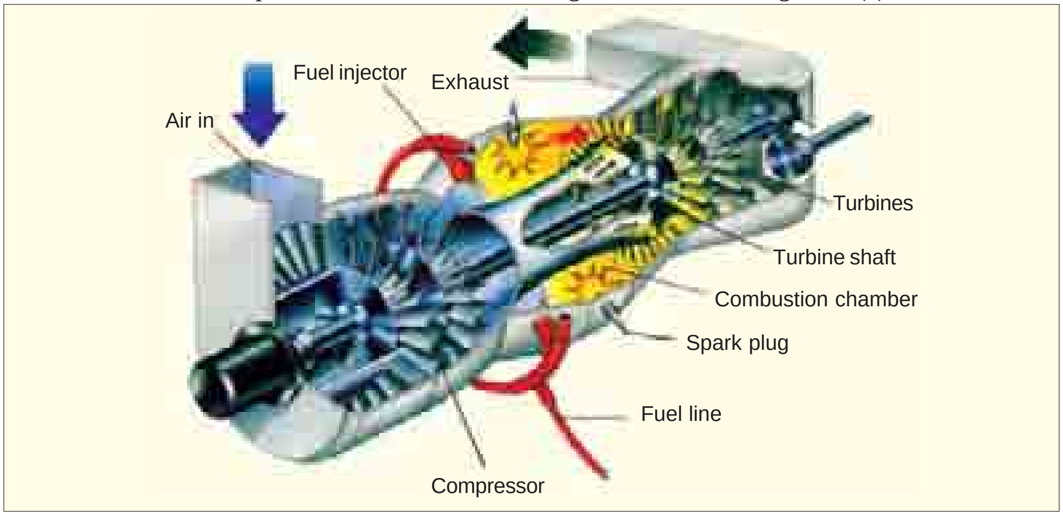
According to the principle of superposition, the maximum or the resultant compressive stress at the edge *AB*,

$$\sigma_c = \frac{P \cdot e \cdot y_c}{I} + \frac{P}{A} = {}^{*}\frac{M}{Z} + \frac{P}{A} = \sigma_b + \sigma_o$$

and the maximum or resultant tensile stress at the edge *CD*,

$$\sigma_t = \frac{P \cdot e \cdot y_t}{I} - \frac{P}{A} = \frac{M}{Z} - \frac{P}{A} = \sigma_b - \sigma_o$$

The resultant compressive and tensile stress diagram is shown in Fig. 5.19 (*e*).



In a gas-turbine system, a compressor forces air into a combustion chamber. There, it mixes with fuel. The mixture is ignited by a spark. Hot gases are produced when the fuel burns. They expand and drive a series of fan blades called a turbine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Notes: 1. When the member is subjected to a tensile load, then the above equations may be used by interchanging the subscripts *c* and *t*.

2. When the direct stress σ_o is greater than or equal to bending stress σ_b, then the compressive stress shall be present all over the cross-section.

3. When the direct stress σ_o is less than the bending stress σ_b, then the tensile stress will occur in the left hand portion of the cross-section and compressive stress on the right hand portion of the cross-section. In Fig. 5.19, the stress diagrams are drawn by taking σ_o less than σ_b.

In case the eccentric load acts with eccentricity about two axes, as shown in Fig. 5.20, then the total stress at the extreme fibre

$$= \frac{P}{A} \pm \frac{P \cdot e_x \cdot x}{I_{XX}} \pm \frac{P \cdot e_y \cdot y}{I_{YY}}$$

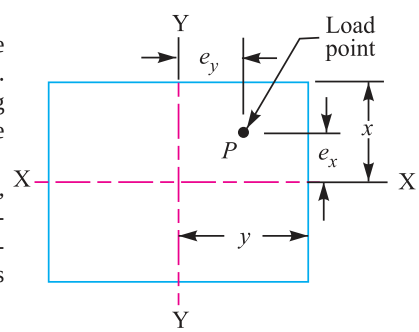


Fig. 5.20. Eccentric load with eccentricity about two axes.

* We know that bending moment, $M = P.e$ and section modulus, $Z = \dfrac{I}{y} = \dfrac{I}{y_c \text{ or } y_t}$

∴ Bending stress, $\sigma_b = M / Z$

Example 5.19. *A rectangular strut is 150 mm wide and 120 mm thick. It carries a load of 180 kN at an eccentricity of 10 mm in a plane bisecting the thickness as shown in Fig. 5.21. Find the maximum and minimum intensities of stress in the section.*

Solution. Given : b = 150 mm ; d = 120 mm ; P = 180 kN = 180×10^3 N ; e = 10 mm

We know that cross-sectional area of the strut,

$$A = b.d = 150 \times 120 = 18 \times 10^3 \text{ mm}^2$$

∴ Direct compressive stress,

$$\sigma_o = \frac{P}{A} = \frac{180 \times 10^3}{18 \times 10^3} = 10 \text{ N/mm}^2 = 10 \text{ MPa}$$

Section modulus for the strut,

$$Z = \frac{I_{YY}}{y} = \frac{d\,.\,b^3/12}{b/2} = \frac{d\,.\,b^2}{6} = \frac{120\,(150)^2}{6} = 450 \times 10^3 \text{ mm}^3$$

Bending moment, $M = P.e = 180 \times 10^3 \times 10 = 1.8 \times 10^6$ N-mm

∴ Bending stress, $\sigma_b = \dfrac{M}{Z} = \dfrac{1.8 \times 10^6}{450 \times 10^3} = 4 \text{ N/mm}^2 = 4 \text{ MPa}$

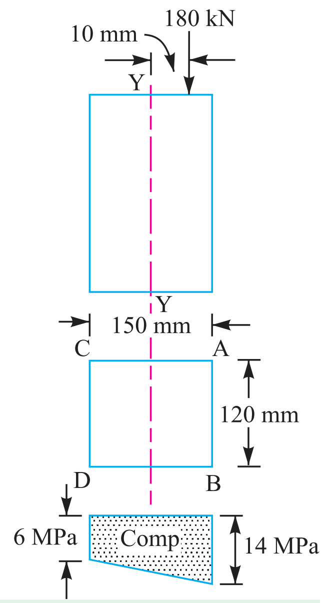


Fig. 5.21

Since σ_o is greater than σ_b, therefore the entire cross-section of the strut will be subjected to compressive stress. The maximum intensity of compressive stress will be at the edge *AB* and minimum at the edge *CD*.

∴ Maximum intensity of compressive stress at the edge *AB*

$$= \sigma_o + \sigma_b = 10 + 4 = 14 \text{ MPa} \quad \textbf{Ans.}$$

and minimum intensity of compressive stress at the edge *CD*

$$= \sigma_o - \sigma_b = 10 - 4 = 6 \text{ MPa} \quad \textbf{Ans.}$$

Example 5.20. *A hollow circular column of external diameter 250 mm and internal diameter 200 mm, carries a projecting bracket on which a load of 20 kN rests, as shown in Fig. 5.22. The centre of the load from the centre of the column is 500 mm. Find the stresses at the sides of the column.*

Solution. Given : D = 250 mm ; d = 200 mm ; P = 20 kN = 20×10^3 N ; e = 500 mm

We know that cross-sectional area of column,

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}[(250)^2 - (200)^2] = 17\ 674 \text{ mm}^2$$

∴ Direct compressive stress,

$$\sigma_o = \frac{P}{A} = \frac{20 \times 10^3}{17\ 674} = 1.13 \text{ N/mm}^2 = 1.13 \text{ MPa}$$

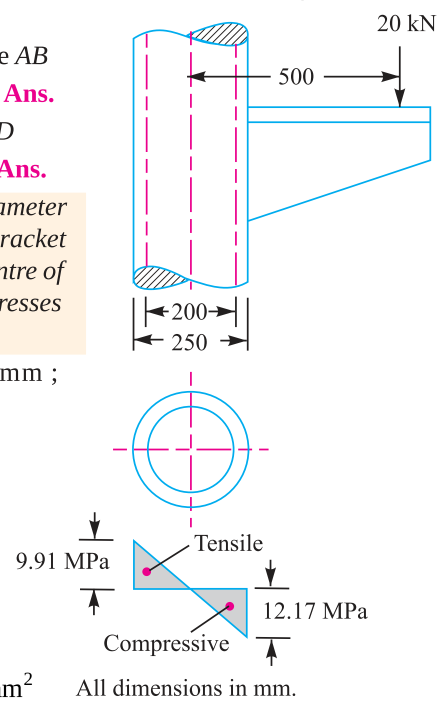


Fig. 5.22

Section modulus for the column,

$$Z = \frac{I}{y} = \frac{\frac{\pi}{64}\left[D^4 - d^4\right]}{D/2} = \frac{\frac{\pi}{64}\left[(250)^4 - (200)^4\right]}{250/2}$$

$= 905.8 \times 10^3 \text{ mm}^3$

Bending moment,

$M = P.e$

$= 20 \times 10^3 \times 500$

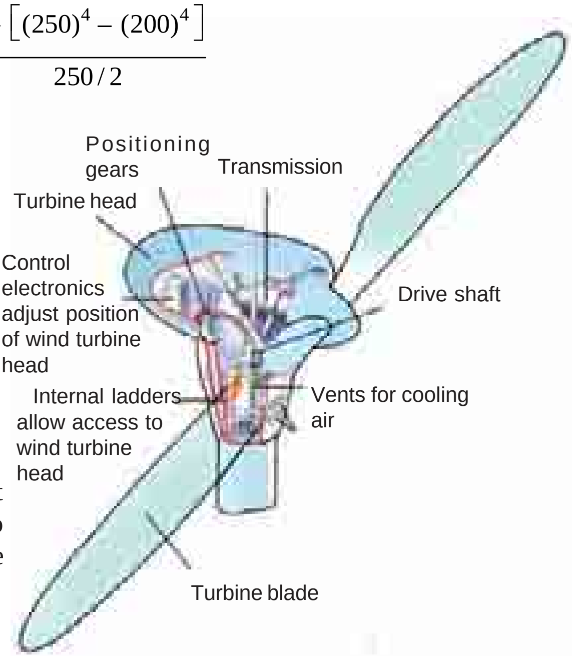
$= 10 \times 10^6$ N-mm

∴ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{10 \times 10^6}{905.8 \times 10^3}$$

$= 11.04 \text{ N/mm}^2$

$= 11.04$ MPa



Wind turbine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Since σ_o is less than σ_b, therefore right hand side of the column will be subjected to compressive stress and the left hand side of the column will be subjected to tensile stress.

∴ Maximum compressive stress,

$\sigma_c = \sigma_b + \sigma_o = 11.04 + 1.13$

$= 12.17$ MPa **Ans.**

and maximum tensile stress,

$\sigma_t = \sigma_b - \sigma_o = 11.04 - 1.13 = 9.91$ MPa **Ans.**

Example 5.21. *A masonry pier of width 4 m and thickness 3 m, supports a load of 30 kN as shown in Fig. 5.23. Find the stresses developed at each corner of the pier.*

Solution. Given: $b = 4$ m ; $d = 3$ m ; $P = 30$ kN ; $e_x = 0.5$ m ; $e_y = 1$ m

We know that cross-sectional area of the pier,

$A = b \times d = 4 \times 3 = 12 \text{ m}^2$

Moment of inertia of the pier about X-axis,

$$I_{XX} = \frac{b\,.\,d^3}{12} = \frac{4 \times 3^3}{12} = 9 \text{ m}^4$$

and moment of inertia of the pier about Y-axis,

$$I_{YY} = \frac{d\,.\,b^3}{12} = \frac{3 \times 4^3}{12} = 16 \text{ m}^4$$

Distance between X-axis and the corners A and B,

$x = 3 / 2 = 1.5$ m

Distance between Y-axis and the corners A and C,

$y = 4 / 2 = 2$ m

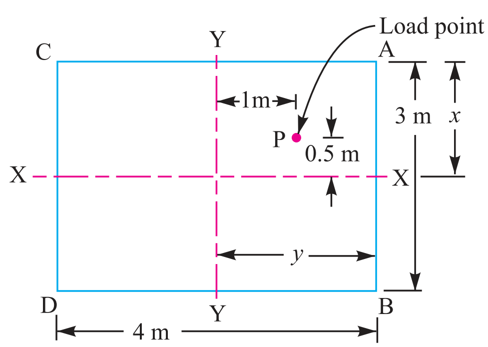


Fig. 5.23

We know that stress at corner A,

$$\sigma_A = \frac{P}{A} + \frac{P\,.\,e_x\,.\,x}{I_{XX}} + \frac{P\,.\,e_y\,.\,y}{I_{YY}} \quad \text{... [∵ At } A \text{, both } x \text{ and } y \text{ are +ve]}$$

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$$= \frac{30}{12} + \frac{30 \times 0.5 \times 1.5}{9} + \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 + 2.5 + 3.75 = 8.75 \text{ kN/m}^2$$ **Ans.**

Similarly stress at corner *B*,

$$\sigma_B = \frac{P}{A} + \frac{P \cdot e_x \cdot x}{I_{XX}} - \frac{P \cdot e_y \cdot y}{I_{YY}}$$ ... [∵ At *B*, *x* is +ve and *y* is –ve]

$$= \frac{30}{12} + \frac{30 \times 0.5 \times 1.5}{9} - \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 + 2.5 - 3.75 = 1.25 \text{ kN/m}^2$$ **Ans.**

Stress at corner *C*,

$$\sigma_C = \frac{P}{A} - \frac{P \cdot e_x \cdot x}{I_{XX}} + \frac{P \cdot e_y \cdot y}{I_{YY}}$$ ... [At *C*, *x* is –ve and *y* is +ve]

$$= \frac{30}{12} - \frac{30 \times 0.5 \times 1.5}{9} + \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 - 2.5 + 3.75 = 3.75 \text{ kN/m}^2$$ **Ans.**

and stress at corner *D*,

$$\sigma_D = \frac{P}{A} - \frac{P \cdot e_x \cdot x}{I_{XX}} - \frac{P \cdot e_y \cdot y}{I_{YY}}$$ ... [At *D*, both *x* and *y* are – ve]

$$= \frac{30}{12} - \frac{30 \times 0.5 \times 1.5}{9} - \frac{30 \times 1 \times 2}{16}$$

$$= 2.5 - 2.5 - 3.75 = -3.75 \text{ kN/m}^2 = 3.75 \text{ kN/m}^2 \text{ (tensile)}$$ **Ans.**

Example 5.22. *A mild steel link, as shown in Fig. 5.24 by full lines, transmits a pull of 80 kN. Find the dimensions b and t if b = 3t. Assume the permissible tensile stress as 70 MPa. If the original link is replaced by an unsymmetrical one, as shown by dotted lines in Fig. 5.24, having the same thickness t, find the depth b_1, using the same permissible stress as before.*

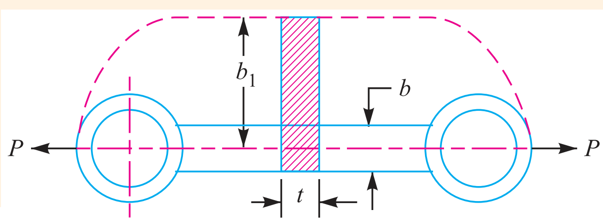

Fig. 5.24

Solution. Given : $P = 80$ kN $= 80 \times 10^3$ N ; $\sigma_t = 70$ MPa $= 70$ N/mm²

When the link is in the position shown by full lines in Fig. 5.24, the area of cross-section,

$$A = b \times t = 3\,t \times t = 3\,t^2$$ ...(∵ $b = 3t$)

We know that tensile load (*P*),

$$80 \times 10^3 = \sigma_t \times A = 70 \times 3\,t^2 = 210\,t^2$$

∴ $t^2 = 80 \times 10^3 / 210 = 381$ or $t = 19.5$ say 20 mm **Ans.**

and $b = 3\,t = 3 \times 20 = 60$ mm **Ans.**

When the link is in the position shown by dotted lines, it will be subjected to direct stress as well as bending stress. We know that area of cross-section,

$$A_1 = b_1 \times t$$

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∴ Direct tensile stress,

$$\sigma_o = \frac{P}{A} = \frac{P}{b_1 \times t}$$

and bending stress, $$\sigma_b = \frac{M}{Z} = \frac{P\,.\,e}{Z} = \frac{6\,P\,.\,e}{t\,(b_1)^2} \qquad \ldots \left(\because Z = \frac{t\,(b_1)^2}{6}\right)$$

∴ Total stress due to eccentric loading

$$= \sigma_b + \sigma_o = \frac{6\,P\,.\,e}{t\,(b_1)^2} + \frac{P}{b_1 \times t} = \frac{P}{t\,.\,b_1}\left(\frac{6\,e}{b_1} + 1\right)$$

Since the permissible tensile stress is the same as 70 N/mm², therefore

$$70 = \frac{80 \times 10^3}{20\,b_1}\left(\frac{6 \times b_1}{b_1 \times 2} + 1\right) = \frac{16 \times 10^3}{b_1} \qquad \ldots \left(\because \text{Eccentricity, } e = \frac{b_1}{2}\right)$$

∴ $b_1 = 16 \times 10^3 / 70 = 228.6$ say 230 mm **Ans.**

Example 5.23. *A cast-iron link, as shown in Fig. 5.25, is to carry a load of 20 kN. If the tensile and compressive stresses in the link are not to exceed 25 MPa and 80 MPa respectively, obtain the dimensions of the cross-section of the link at the middle of its length.*

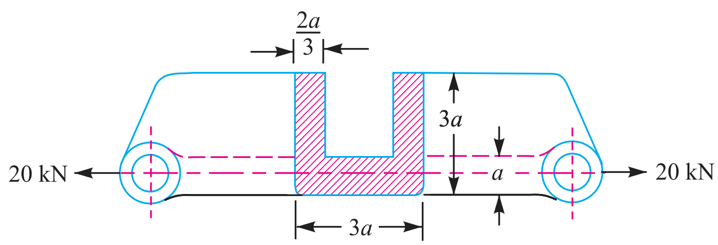

Fig. 5.25

Solution. Given : P = 20 kN = 20 × 10³ N ; $\sigma_{t(max)}$ = 25 MPa = 25 N/mm² ; $\sigma_{c(max)}$ = 80 MPa = 80 N/mm²

Since the link is subjected to eccentric loading, therefore there will be direct tensile stress as well as bending stress. The bending stress at the bottom of the link is tensile and in the upper portion is compressive.

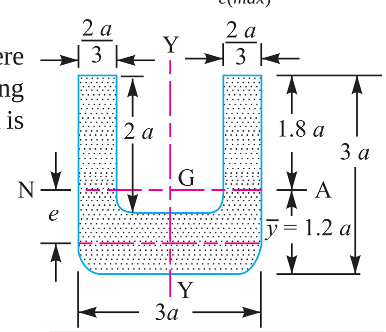

Fig. 5.26

We know that cross-sectional area of the link,

$$A = 3a \times a + 2 \times \frac{2a}{3} \times 2a$$

$$= 5.67\,a^2 \text{ mm}^2$$

∴ Direct tensile stress,

$$\sigma_o = \frac{P}{A} = \frac{20 \times 10^3}{5.67\,a^2} = \frac{3530}{a^2} \text{ N/mm}^2$$

Now let us find the position of centre of gravity (or neutral axis) in order to find the bending stresses.

Let $\bar{y}$ = Distance of neutral axis (N.A.) from the bottom of the link as shown in Fig. 5.26.

∴ $$\bar{y} = \frac{3a^2 \times \frac{a}{2} + 2 \times \frac{4a^2}{3} \times 2a}{5.67\,a^2} = 1.2\,a \text{ mm}$$

Moment of inertia about N.A.,

$$I = \left[\frac{3a \times a^3}{12} + 3a^2 (1.2a - 0.5a)^2\right] + 2\left[\frac{\frac{2}{3} a \times (2a)^3}{12} + \frac{4a^2}{3}(2a - 1.2a)^2\right]$$

$$= (0.25\, a^4 + 1.47\, a^4) + 2\,(0.44a^4 + 0.85\, a^4) = 4.3\, a^4 \text{ mm}^4$$

Distance of N.A. from the bottom of the link,

$$y_t = \bar{y} = 1.2\, a \text{ mm}$$

Distance of N.A. from the top of the link,

$$y_c = 3\,a - 1.2\,a = 1.8\,a \text{ mm}$$

Eccentricity of the load (*i.e.* distance of N.A. from the point of application of the load),

$$e = 1.2\,a - 0.5\,a = 0.7\,a \text{ mm}$$

We know that bending moment exerted on the section,

$$M = P.e = 20 \times 10^3 \times 0.7\,a = 14 \times 10^3\,a \text{ N-mm}$$

∴ Tensile stress in the bottom of the link,

$$\sigma_t = \frac{M}{Z_t} = \frac{M}{I / y_t} = \frac{M \,.\, y_t}{I} = \frac{14 \times 10^3\, a \times 1.2\, a}{4.3\, a^4} = \frac{3907}{a^2}$$

and compressive stress in the top of the link,

$$\sigma_c = \frac{M}{Z_c} = \frac{M}{I / y_c} = \frac{M \,.\, y_c}{I} = \frac{14 \times 10^3\, a \times 1.8\, a}{4.3\, a^4} = \frac{5860}{a^2}$$

We know that maximum tensile stress $[\sigma_{t\,(max)}]$,

$$25 = \sigma_t + \sigma_c = \frac{3907}{a^2} + \frac{5860}{a^2} = \frac{9767}{a^2}$$

∴ $a^2 = 9767 / 25 = 390.7$ or $a = 19.76$ mm ...(*i*)

and maximum compressive stress $[\sigma_{c(max)}]$,

$$80 = \sigma_c - \sigma_0 = \frac{5860}{a^2} - \frac{3530}{a^2} = \frac{2330}{a^2}$$

∴ $a^2 = 2330 / 80 = 29.12$ or $a = 5.4$ mm ...(*ii*)

We shall take the larger of the two values, *i.e.*

$$a = 19.76 \text{ mm}$$ **Ans.**

Example 5.24. *A horizontal pull P = 5 kN is exerted by the belting on one of the cast iron wall brackets which carry a factory shafting. At a point 75 mm from the wall, the bracket has a T-section as shown in Fig. 5.27. Calculate the maximum stresses in the flange and web of the bracket due to the pull.*

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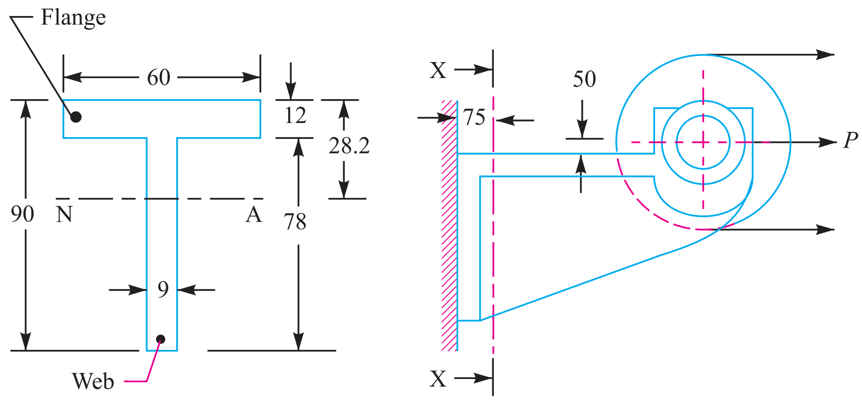


All dimensions in mm.

Fig. 5.27

Solution. Given : Horizontal pull, P = 5 kN = 5000 N

Since the section is subjected to eccentric loading, therefore there will be direct tensile stress as well as bending stress. The bending stress at the flange is tensile and in the web is compressive.

We know that cross-sectional area of the section,

$$A = 60 \times 12 + (90 - 12)9 = 720 + 702 = 1422 \text{ mm}^2$$

$$\therefore \text{ Direct tensile stress, } \sigma_0 = \frac{P}{A} = \frac{5000}{1422} = 3.51 \text{ N/mm}^2 = 3.51 \text{ MPa}$$

Now let us find the position of neutral axis in order to determine the bending stresses. The neutral axis passes through the centre of gravity of the section.

Let $\bar{y}$ = Distance of centre of gravity (*i.e.* neutral axis) from top of the flange.

$$\therefore \quad \bar{y} = \frac{60 \times 12 \times \frac{12}{2} + 78 \times 9\left(12 + \frac{78}{2}\right)}{720 + 702} = 28.2 \text{ mm}$$

Moment of inertia of the section about N.A.,

$$I = \left[\frac{60\,(12)^3}{12} + 720\,(28.2 - 6)^2\right] + \left[\frac{9\,(78)^3}{12} + 702\,(51 - 28.2)^2\right]$$

$$= (8640 + 354\,845) + (355\,914 + 364\,928) = 1\,084\,327 \text{ mm}^4$$

This picture shows a reconnoissance helicopter of air force. Its dark complexion absorbs light that falls on its surface. The flat and sharp edges deflect radar waves and they do not return back to the radar. These factors make it difficult to detect the helicopter.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Top

Distance of N.A. from the top of the flange,

$$y_t = \bar{y} = 28.2 \text{ mm}$$

Distance of N.A. from the bottom of the web,

$$y_c = 90 - 28.2 = 61.8 \text{ mm}$$

Distance of N.A. from the point of application of the load (*i.e.* eccentricity of the load),

$$e = 50 + 28.2 = 78.2 \text{ mm}$$

We know that bending moment exerted on the section,

$$M = P \times e = 5000 \times 78.2 = 391 \times 10^3 \text{ N-mm}$$

∴ Tensile stress in the flange,

$$\sigma_t = \frac{M}{Z_t} = \frac{M}{I / y_t} = \frac{M \cdot y_t}{I} = \frac{391 \times 10^3 \times 28.2}{1\,084\,327} = 10.17 \text{ N/mm}^2$$
$$= 10.17 \text{ MPa}$$

and compressive stress in the web,

$$\sigma_c = \frac{M}{Z_c} = \frac{M}{I / y_c} = \frac{M \cdot y_c}{I} = \frac{391 \times 10^3 \times 61.8}{1\,084\,327} = 22.28 \text{ N/mm}^2$$
$$= 22.28 \text{ MPa}$$

We know that maximum tensile stress in the flange,

$$\sigma_{t(max)} = \sigma_b + \sigma_o = \sigma_t + \sigma_o = 10.17 + 3.51 = 13.68 \text{ MPa}$$ **Ans.**

and maximum compressive stress in the flange,

$$\sigma_{c(max)} = \sigma_b - \sigma_o = \sigma_c - \sigma_o = 22.28 - 3.51 = 18.77 \text{ MPa}$$ **Ans.**

Example 5.25. *A mild steel bracket as shown in Fig. 5.28, is subjected to a pull of 6000 N acting at 45° to its horizontal axis. The bracket has a rectangular section whose depth is twice the thickness. Find the cross-sectional dimensions of the bracket, if the permissible stress in the material of the bracket is limited to 60 MPa.*

Solution. Given : $P = 6000$ N ; $\theta = 45°$; $\sigma = 60$ MPa $= 60$ N/mm^2

Let t = Thickness of the section in mm, and

b = Depth or width of the section $= 2\,t$ ...(Given)

We know that area of cross-section,

$$A = b \times t = 2\,t \times t = 2\,t^2 \text{ mm}^2$$

and section modulus,

$$Z = \frac{t \times b^2}{6} = \frac{t\,(2t)^2}{6} = \frac{4\,t^3}{6} \text{ mm}^3$$

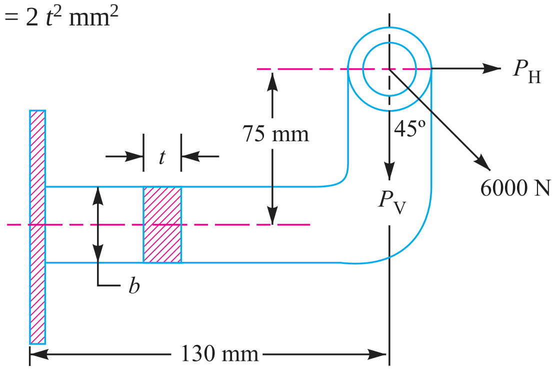


Fig. 5.28

Horizontal component of the load,

$$P_H = 6000 \cos 45° = 6000 \times 0.707 = 4242 \text{ N}$$

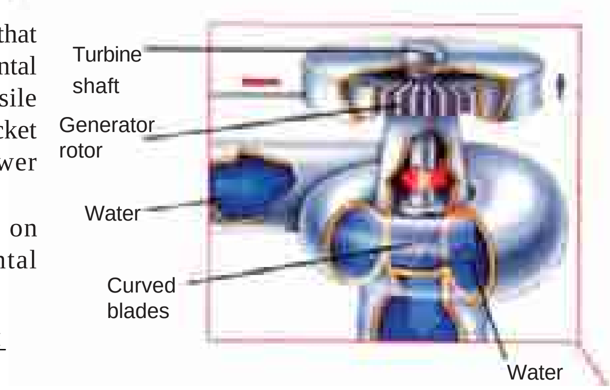
∴ Bending moment due to horizontal component of the load,

$$M_H = P_H \times 75 = 4242 \times 75 = 318\,150 \text{ N-mm}$$

A little consideration will show that the bending moment due to the horizontal component of the load induces tensile stress on the upper surface of the bracket and compressive stress on the lower surface of the bracket.

∴ Maximum bending stress on the upper surface due to horizontal component,

$$\sigma_{bH} = \frac{M_H}{Z} = \frac{318\,150 \times 6}{4\,t^3}$$



Schematic of a hydel turbine.

Note : This picture is given as additional information and is not a direct example of the current chapter.

$$= \frac{477\,225}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

Vertical component of the load,

$$P_V = 6000 \sin 45° = 6000 \times 0.707 = 4242 \text{ N}$$

∴ Direct stress due to vertical component,

$$\sigma_{oV} = \frac{P_V}{A} = \frac{4242}{2t^2} = \frac{2121}{t^2} \text{ N/mm}^2 \text{ (tensile)}$$

Bending moment due to vertical component of the load,

$$M_V = P_V \times 130 = 4242 \times 130 = 551\,460 \text{ N-mm}$$

This bending moment induces tensile stress on the upper surface and compressive stress on the lower surface of the bracket.

∴ Maximum bending stress on the upper surface due to vertical component,

$$\sigma_{bV} = \frac{M_V}{Z} = \frac{551\,460 \times 6}{4\,t^3} = \frac{827\,190}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

and total tensile stress on the upper surface of the bracket,

$$\sigma = \frac{477\,225}{t^3} + \frac{2121}{t^2} + \frac{827\,190}{t^3} = \frac{1\,304\,415}{t^3} + \frac{2121}{t^2}$$

Since the permissible stress (σ) is 60 N/mm^2, therefore

$$\frac{1\,304\,415}{t^3} + \frac{2121}{t^2} = 60 \quad \text{or} \quad \frac{21\,740}{t^3} + \frac{35.4}{t^2} = 1$$

∴ $t = 28.4$ mm **Ans.** ... (By hit and trial)

and $b = 2t = 2 \times 28.4 = 56.8$ mm **Ans.**

Example 5.26. *A C-clamp as shown in Fig. 5.29, carries a load P = 25 kN. The cross-section of the clamp at X-X is rectangular having width equal to twice thickness. Assuming that the clamp is made of steel casting with an allowable stress of 100 MPa, find its dimensions. Also determine the stresses at sections Y-Y and Z-Z.*

Solution. Given : $P = 25$ kN $= 25 \times 10^3$ N ; $\sigma_{t(max)} = 100$ MPa $= 100$ N/mm^2

Dimensions at X-X

Let t = Thickness of the section at *X-X* in mm, and

b = Width of the section at *X-X* in mm = $2t$ ...(Given)

We know that cross-sectional area at *X-X*,

$$A = b \times t = 2\,t \times t = 2\,t^2 \text{ mm}^2$$

∴ Direct tensile stress at *X-X*,

$$\sigma_o = \frac{P}{A} = \frac{25 \times 10^3}{2\,t^2} = \frac{12.5 \times 10^3}{t^3} \text{ N/mm}^2$$

Bending moment at *X-X* due to the load *P*,

$$M = P \times e = 25 \times 10^3 \times 140 = 3.5 \times 10^6 \text{ N-mm}$$

Section modulus, $Z = \dfrac{t\,.\,b^2}{6} = \dfrac{t\,(2t)^2}{6} = \dfrac{4\,t^3}{6} \text{ mm}^3$

...(∵ $b = 2t$)

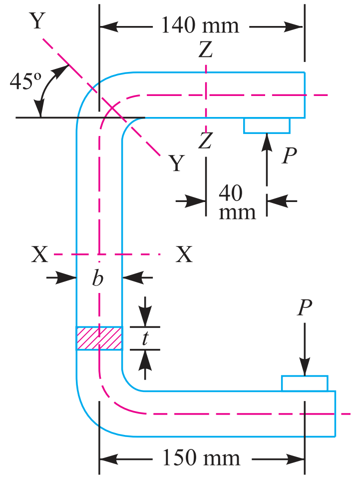


Fig. 5.29

∴ Bending stress at *X-X*,

$$\sigma_b = \frac{M}{Z} = \frac{3.5 \times 10^6 \times 6}{4\,t^3} = \frac{5.25 \times 10^6}{t^3} \text{ N/mm}^2 \text{ (tensile)}$$

We know that the maximum tensile stress [$\sigma_{t\,(max)}$],

$$100 = \sigma_o + \sigma_b = \frac{12.5 \times 10^3}{t^2} + \frac{5.25 \times 10^6}{t^3}$$

or $$\frac{125}{t^2} + \frac{52.5 \times 10^3}{t^3} - 1 = 0$$

∴ $t = 38.5$ mm **Ans.** ...(By hit and trial)

and $b = 2\,t = 2 \times 38.5 = 77$ mm **Ans.**

Stresses at section Y-Y

Since the cross-section of frame is uniform throughout, therefore cross-sectional area of the frame at section *Y-Y*,

$$A = b \sec 45° \times t = 77 \times 1.414 \times 38.5 = 4192 \text{ mm}^2$$

Component of the load perpendicular to the section

$$= P \cos 45° = 25 \times 10^3 \times 0.707 = 17\,675 \text{ N}$$

This component of the load produces uniform tensile stress over the section.

∴ Uniform tensile stress over the section,

$$\sigma = 17\,675 / 4192 = 4.2 \text{ N/mm}^2 = 4.2 \text{ MPa}$$

Component of the load parallel to the section

$$= P \sin 45° = 25 \times 10^3 \times 0.707 = 17\,675 \text{ N}$$

This component of the load produces uniform shear stress over the section.

∴ Uniform shear stress over the section,

$$\tau = 17\,675 / 4192 = 4.2 \text{ N/mm}^2 = 4.2 \text{ MPa}$$

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We know that section modulus,

$$Z = \frac{t\,(b \sec 45°)^2}{6} = \frac{38.5\,(77 \times 1.414)^2}{6} = 76 \times 10^3 \text{ mm}^3$$

Bending moment due to load (P) over the section Y-Y,

$$M = 25 \times 10^3 \times 140 = 3.5 \times 10^6 \text{ N-mm}$$

∴ Bending stress over the section,

$$\sigma_b = \frac{M}{Z} = \frac{3.5 \times 10^6}{76 \times 10^3} = 46 \text{ N/mm}^2 = 46 \text{ MPa}$$

Due to bending, maximum tensile stress at the inner corner and the maximum compressive stress at the outer corner is produced.

∴ Maximum tensile stress at the inner corner,

$$\sigma_t = \sigma_b + \sigma_o = 46 + 4.2 = 50.2 \text{ MPa}$$

and maximum compressive stress at the outer corner,

$$\sigma_c = \sigma_b - \sigma_o = 46 - 4.2 = 41.8 \text{ MPa}$$

Since the shear stress acts at right angles to the tensile and compressive stresses, therefore maximum principal stress (tensile) on the section Y-Y at the inner corner

$$= \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right] = \frac{50.2}{2} + \frac{1}{2}\left[\sqrt{(50.2)^2 + 4 \times (4.2)^2}\right] \text{ MPa}$$

$$= 25.1 + 25.4 = 50.5 \text{ MPa}$$ **Ans.**

and maximum principal stress (compressive) on section Y-Y at outer corner

$$= \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right] = \frac{41.8}{2} + \frac{1}{2}\left[\sqrt{(41.8)^2 + 4 \times (4.2)^2}\right] \text{ MPa}$$

$$= 20.9 + 21.3 = 42.2 \text{ MPa}$$ **Ans.**

Maximum shear stress $= \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(50.2)^2 + 4 \times (4.2)^2}\right] = 25.4 \text{ MPa}$ **Ans.**

Stresses at section Z-Z

We know that bending moment at section Z-Z,

$$= 25 \times 10^3 \times 40 = 1 \times 10^6 \text{ N-mm}$$

and section modulus,

$$Z = \frac{t\,.\,b^2}{6} = \frac{38.5\,(77)^2}{6} = 38 \times 10^3 \text{ mm}^3$$

∴ Bending stress at section Z-Z,

$$\sigma_b = \frac{M}{Z} = \frac{1 \times 10^6}{38 \times 10^3} = 26.3 \text{ N/mm}^2 = 26.3 \text{ MPa}$$ **Ans.**

The bending stress is tensile at the inner edge and compressive at the outer edge. The magnitude of both these stresses is 26.3 MPa. At the neutral axis, there is only transverse shear stress. The shear stress at the inner and outer edges will be zero.

We know that *maximum transverse shear stress,

$$\tau_{max} = 1.5 \times \text{Average shear stress} = 1.5 \times \frac{P}{b.t} = 1.5 \times \frac{25 \times 10^3}{77 \times 38.5}$$

$$= 12.65 \text{ N/mm}^2 = 12.65 \text{ MPa}$$ **Ans.**

* Refer Art. 5.16

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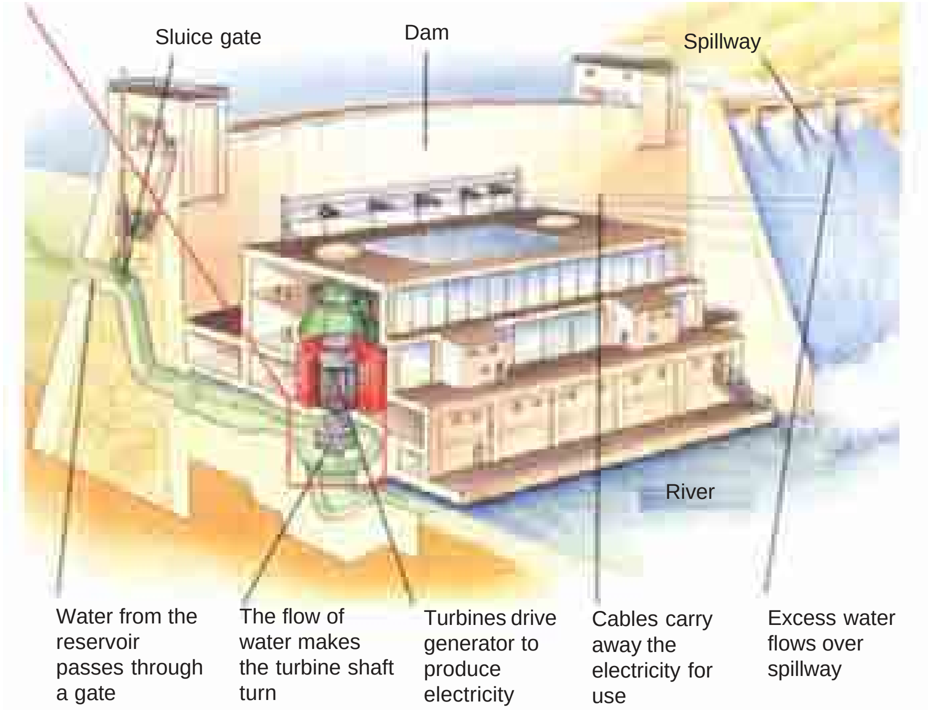


General layout of a hydroelectric plant.

Note : This picture is given as additional information and is not a direct example of the current chapter.

5.16 Shear Stresses in Beams

In the previous article, we have assumed that no shear force is acting on the section. But, in actual practice, when a beam is loaded, the shear force at a section always comes into play along with the bending moment. It has been observed that the effect of the shear stress, as compared to the bending stress, is quite negligible and is of not much importance. But, sometimes, the shear stress at a section is of much importance in the design. It may be noted that the shear stress in a beam is not uniformly distributed over the cross-section but varies from zero at the outer fibres to a maximum at the neutral surface as shown in Fig. 5.30 and Fig. 5.31.

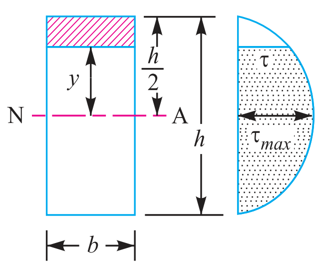


Fig. 5.30. Shear stress in a rectangular beam.

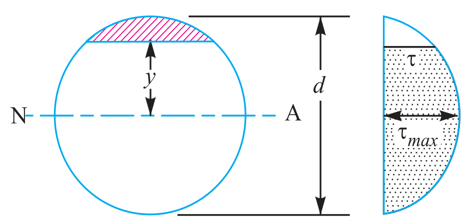


Fig. 5.31. Shear stress in a circular beam.

The shear stress at any section acts in a plane at right angle to the plane of the bending stress and its value is given by

$$\tau = \frac{F}{I \cdot b} \times A.\bar{y}$$

where F = Vertical shear force acting on the section,

I = Moment of inertia of the section about the neutral axis,

b = Width of the section under consideration,

A = Area of the beam above neutral axis, and

$\bar{y}$ = Distance between the C.G. of the area and the neutral axis.

The following values of maximum shear stress for different cross-section of beams may be noted :

1. For a beam of rectangular section, as shown in Fig. 5.30, the shear stress at a distance y from neutral axis is given by

$$\tau = \frac{F}{2I}\left(\frac{h^2}{4} - y^2\right) = \frac{3F}{2b\,.\,h^3}\,(h^2 - 4y^2) \qquad \ldots\left[\because I = \frac{b\,.\,h^3}{12}\right]$$

and maximum shear stress,

$$\tau_{max} = \frac{3F}{2b\,.\,h} \qquad \ldots\left(\text{Substituting } y = \frac{h}{2}\right)$$

$$= 1.5\,\tau_{(average)} \qquad \ldots\left[\because \tau_{(average)} = \frac{F}{\text{Area}} = \frac{F}{b.h}\right]$$

The distribution of stress is shown in Fig. 5.30.

2. For a beam of circular section as shown in Fig. 5.31, the shear stress at a distance y from neutral axis is given by

$$\tau = \frac{F}{3I}\left(\frac{d^2}{4} - y^2\right) = \frac{16\,F}{3\,\pi\,d^4}\,(d^2 - 4y^2)$$

and the maximum shear stress,

$$\tau_{max} = \frac{4F}{3 \times \frac{\pi}{4}\,d^2} \qquad \ldots\left(\text{Substituting } y = \frac{d}{2}\right)$$

$$= \frac{4}{3}\,\tau_{(average)} \qquad \ldots\left[\because \tau_{(average)} = \frac{F}{\text{Area}} = \frac{F}{\frac{\pi}{4}\,d^2}\right]$$

The distribution of stress is shown in Fig. 5.31.

3. For a beam of I-section as shown in Fig. 5.32, the maximum shear stress occurs at the neutral axis and is given by

$$\tau_{max} = \frac{F}{I\,.\,b}\left[\frac{B}{8}\,(H^2 - h^2) + \frac{b.h^2}{8}\right]$$

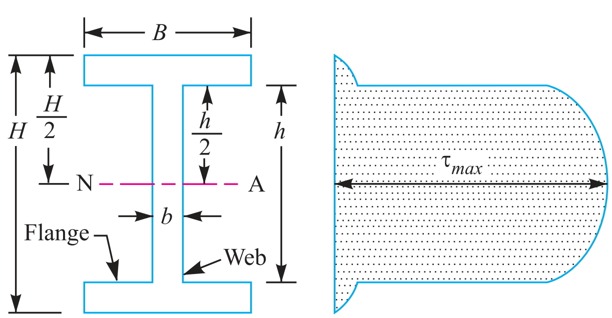


Fig. 5.32

Shear stress at the joint of the web and the flange

$$= \frac{F}{8I}(H^2 - h^2)$$

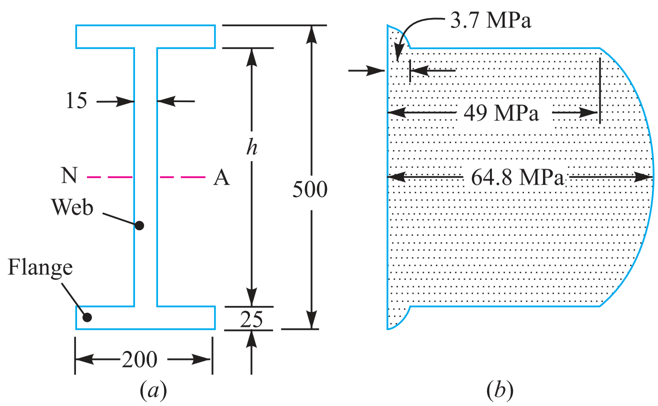
and shear stress at the junction of the top of the web and bottom of the flange

$$= \frac{F}{8I} \times \frac{B}{b}(H^2 - h^2)$$

The distribution of stress is shown in Fig. 5.32.

Example 5.27. *A beam of I-section 500 mm deep and 200 mm wide has flanges 25 mm thick and web 15 mm thick, as shown in Fig. 5.33 (a). It carries a shearing force of 400 kN. Find the maximum intensity of shear stress in the section, assuming the moment of inertia to be 645 × 10^6 mm^4. Also find the shear stress at the joint and at the junction of the top of the web and bottom of the flange.*

Solution. Given : H = 500 mm ; B = 200 mm ; h = 500 – 2 × 25 = 450 mm ; b = 15 mm ; F = 400 kN = 400 × 10^3 N ; I = 645 × 10^6 mm^4



All dimensions in mm.

Fig. 5.33

Maximum intensity of shear stress

We know that maximum intensity of shear stress,

$$\tau_{max} = \frac{F}{I\,.\,b}\left[\frac{B}{8}(H^2 - h^2) + \frac{b\,.\,h^2}{8}\right]$$

$$= \frac{400 \times 10^3}{645 \times 10^6 \times 15}\left[\frac{200}{8}(500^2 - 450^2) + \frac{15 \times 450^2}{8}\right] \text{N/mm}^2$$

$= 64.8$ N/mm^2 = 64.8 MPa **Ans.**

The maximum intensity of shear stress occurs at neutral axis.

Note : The maximum shear stress may also be obtained by using the following relation :

$$\tau_{max} = \frac{F\,.\,A\,.\,\bar{y}}{I\,.\,b}$$

We know that area of the section above neutral axis,

$$A = 200 \times 25 + \frac{450}{2} \times 15 = 8375 \text{ mm}^2$$

Distance between the centre of gravity of the area and neutral axis,

$$\bar{y} = \frac{200 \times 25\,(225 + 12.5) + 225 \times 15 \times 112.5}{8375} = 187 \text{ mm}$$

$$\therefore \qquad \tau_{max} = \frac{400 \times 10^3 \times 8375 \times 187}{645 \times 10^6 \times 15} = 64.8 \text{ N/mm}^2 = 64.8 \text{ MPa} \quad \textbf{Ans.}$$

Shear stress at the joint of the web and the flange

We know that shear stress at the joint of the web and the flange

$$= \frac{F}{8I}(H^2 - h^2) = \frac{400 \times 10^3}{8 \times 645 \times 10^6}\left[(500)^2 - (450)^2\right] \text{ N/mm}^2$$

$$= 3.7 \text{ N/mm}^2 = 3.7 \text{ MPa} \quad \textbf{Ans.}$$

Shear stress at the junction of the top of the web and bottom of the flange

We know that shear stress at junction of the top of the web and bottom of the flange

$$= \frac{F}{8I} \times \frac{B}{b}(H^2 - h^2) = \frac{400 \times 10^3}{8 \times 645 \times 10^6} \times \frac{200}{15}\left[(500)^2 - (450)^2\right] \text{ N/mm}^2$$

$$= 49 \text{ N/mm}^2 = 49 \text{ MPa} \quad \textbf{Ans.}$$

The stress distribution is shown in Fig. 5.33 (*b*)

EXERCISES

1. A steel shaft 50 mm diameter and 500 mm long is subjected to a twisting moment of 1100 N-m, the total angle of twist being 0.6°. Find the maximum shearing stress developed in the shzaft and modulus of rigidity. **[Ans. 44.8 MPa; 85.6 kN/m^2]**

2. A shaft is transmitting 100 kW at 180 r.p.m. If the allowable stress in the material is 60 MPa, find the suitable diameter for the shaft. The shaft is not to twist more than 1° in a length of 3 metres. Take C = 80 GPa. **[Ans. 105 mm]**

3. Design a suitable diameter for a circular shaft required to transmit 90 kW at 180 r.p.m. The shear stress in the shaft is not to exceed 70 MPa and the maximum torque exceeds the mean by 40%. Also find the angle of twist in a length of 2 metres. Take C = 90 GPa. **[Ans. 80 mm; 2.116°]**

4. Design a hollow shaft required to transmit 11.2 MW at a speed of 300 r.p.m. The maximum shear stress allowed in the shaft is 80 MPa and the ratio of the inner diameter to outer diameter is 3/4. **[Ans. 240 mm; 320 mm]**

5. Compare the weights of equal lengths of hollow shaft and solid shaft to transmit a given torque for the same maximum shear stress. The material for both the shafts is same and inside diameter is 2/3 of outside diameter in case of hollow shaft. **[Ans. 0.56]**

6. A spindle as shown in Fig. 5.34, is a part of an industrial brake and is loaded as shown. Each load P is equal to 4 kN and is applied at the mid point of its bearing. Find the diameter of the spindle, if the maximum bending stress is 120 MPa. **[Ans. 22 mm]**

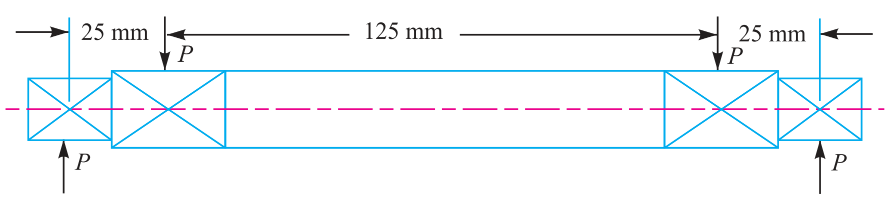


Fig. 5.34

7. A cast iron pulley transmits 20 kW at 300 r.p.m. The diameter of the pulley is 550 mm and has four straight arms of elliptical cross-section in which the major axis is twice the minor axis. Find the dimensions of the arm, if the allowable bending stress is 15 MPa. **[Ans. 60 mm; 30 mm]**

8. A shaft is supported in bearings, the distance between their centres being 1 metre. It carries a pulley in the centre and it weighs 1 kN. Find the diameter of the shaft, if the permissible bending stress for the shaft material is 40 MPa. **[Ans. 40 mm]**

9. A punch press, used for stamping sheet metal, has a punching capacity of 50 kN. The section of the frame is as shown in Fig. 5.35. Find the resultant stress at the inner and outer fibre of the section. **[Ans. 28.3 MPa (tensile); 17.7 MPa (compressive)]**

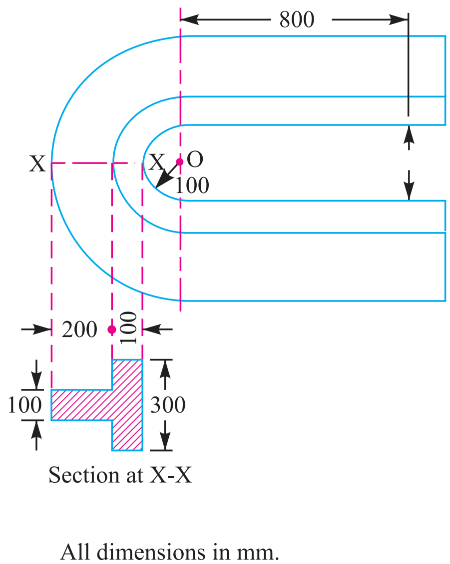


Fig. 5.35

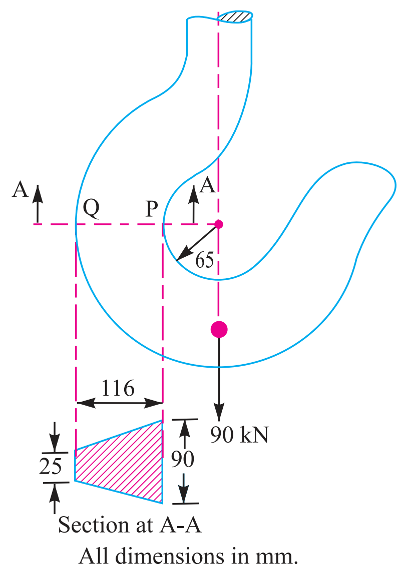


Fig. 5.36

10. A crane hook has a trapezoidal section at *A-A* as shown in Fig. 5.36. Find the maximum stress at points *P* and *Q*. **[Ans. 118 MPa (tensile); 62 MPa (compressive)]**

11. A rotating shaft of 16 mm diameter is made of plain carbon steel. It is subjected to axial load of 5000 N, a steady torque of 50 N-m and maximum bending moment of 75 N-m. Calculate the factor of safety available based on 1. Maximum normal stress theory; and 2. Maximum shear stress theory.

    Assume yield strength as 400 MPa for plain carbon steel. If all other data remaining same, what maximum yield strength of shaft material would be necessary using factor of safety of 1.686 and maximum distortion energy theory of failure. Comment on the result you get. **[Ans. 1.752; 400 MPa]**

12. A hand cranking lever, as shown in Fig. 5.37, is used to start a truck engine by applying a force *F* = 400 N. The material of the cranking lever is 30C8 for which yield strength = 320 MPa; Ultimate tensile strength = 500 MPa ; Young's modulus = 205 GPa ; Modulus of rigidity = 84 GPa and poisson's ratio = 0.3.

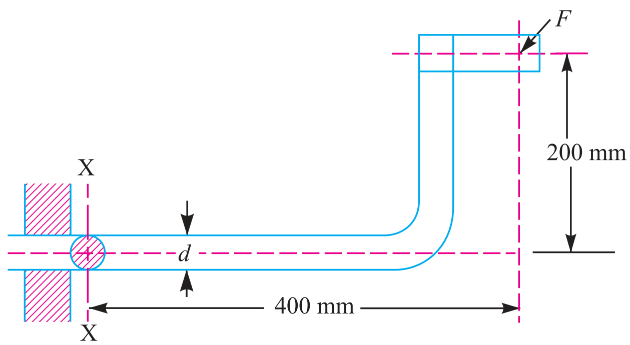


Fig. 5.37

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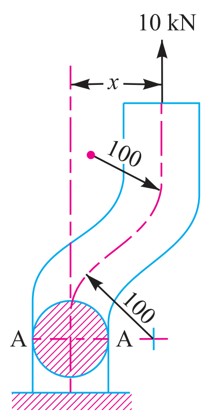
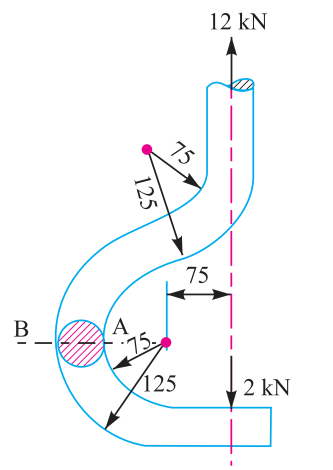
Assuming factor of safety to be 4 based on yield strength, design the diameter '*d*' of the lever at section *X-X* near the guide bush using : 1. Maximum distortion energy theory; and 2. Maximum shear stress theory. **[Ans. 28.2 mm; 28.34 mm]**

13. An offset bar is loaded as shown in Fig. 5.38. The weight of the bar may be neglected. Find the maximum offset (*i.e.*, the dimension *x*) if allowable stress in tension is limited to 70 MPa. **[Ans. 418 mm]**



All dimensions in mm.

Fig. 5.38

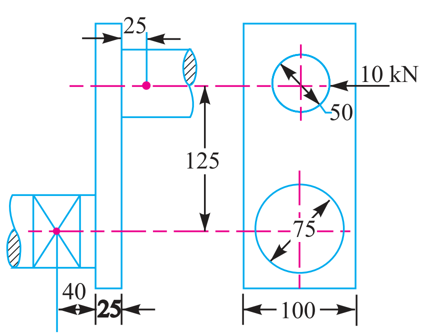


All dimensions in mm.

Fig. 5.39

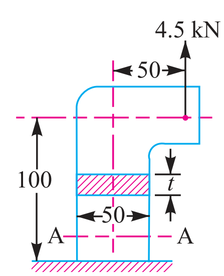
14. A crane hook made from a 50 mm diameter bar is shown in Fig. 5.39. Find the maximum tensile stress and specify its location. **[Ans. 35.72 MPa at A]**

15. An overhang crank, as shown in Fig. 5.40 carries a tangential load of 10 kN at the centre of the crankpin. Find the maximum principal stress and the maximum shear stress at the centre of the crank-shaft bearing. **[Ans. 29.45 MPa; 18.6 MPa]**



All dimensions in mm.

Fig. 5.40



All dimensions in mm.

Fig. 5.41

16. A steel bracket is subjected to a load of 4.5 kN, as shown in Fig. 5.41. Determine the required thickness of the section at A-A in order to limit the tensile stress to 70 MPa. **[Ans. 9 mm]**

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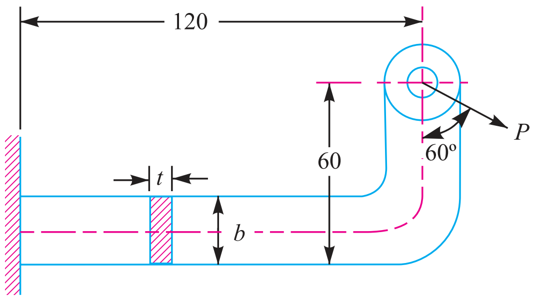
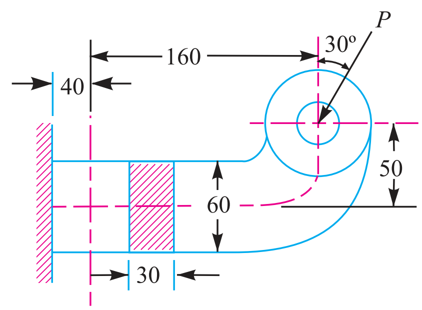
17. A wall bracket, as shown in Fig. 5.42, is subjected to a pull of P = 5 kN, at 60° to the vertical. The cross-section of bracket is rectangular having $b = 3t$. Determine the dimensions b and t if the stress in the material of the bracket is limited to 28 MPa. **[Ans. 75 mm; 25 mm]**



All dimensions in mm.

Fig. 5.42



All dimensions in mm.

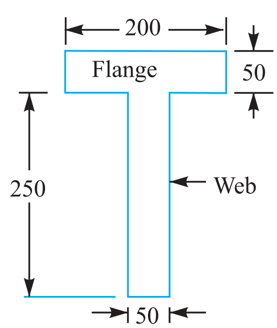
Fig. 5.43

18. A bracket, as shown in Fig. 5.43, is bolted to the framework of a machine which carries a load P. The cross-section at 40 mm from the fixed end is rectangular with dimensions, 60 mm × 30 mm. If the maximum stress is limited to 70 MPa, find the value of P.

[Ans. 3000 N]

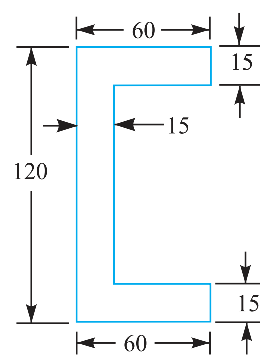
19. A T-section of a beam, as shown in Fig. 5.44, is subjected to a vertical shear force of 100 kN. Calculate the shear stress at the neutral axis and at the junction of the web and the flange. The moment of inertia at the neutral axis is 113.4×10^6 mm^4.

[Ans. 11.64 MPa; 11 MPa; 2.76 MPa]



All dimensions in mm.

Fig. 5.44



All dimensions in mm.

Fig. 5.45

20. A beam of channel section, as shown in Fig. 5.45, is subjected to a vertical shear force of 50 kN. Find the ratio of maximum and mean shear stresses. Also draw the distribution of shear stresses.

[Ans. 2.22]

QUESTIONS

1. Derive a relation for the shear stress developed in a shaft, when it is subjected to torsion.
2. State the assumptions made in deriving a bending formula.

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3. Prove the relation: $M/I = \sigma/y = E/R$

   where M = Bending moment; I = Moment of inertia; σ = Bending stress in a fibre at a distance y from the neutral axis; E = Young's modulus; and R = Radius of curvature.

4. Write the relations used for maximum stress when a machine member is subjected to tensile or compressive stresses along with shearing stresses.
5. Write short note on maximum shear stress theory *verses* maximum strain energy theory.
6. Distinguish clearly between direct stress and bending stress.
7. What is meant by eccentric loading and eccentricity?
8. Obtain a relation for the maximum and minimum stresses at the base of a symmetrical column, when it is subjected to

   (*a*) an eccentric load about one axis, and (*b*) an eccentric load about two axes.

OBJECTIVE TYPE QUESTIONS

1. When a machine member is subjected to torsion, the torsional shear stress set up in the member is
   (*a*) zero at both the centroidal axis and outer surface of the member
   (*b*) Maximum at both the centroidal axis and outer surface of the member
   (*c*) zero at the centroidal axis and maximum at the outer surface of the member
   (*d*) none of the above
2. The torsional shear stress on any cross-section normal to the axis is .... the distance from the centre of the axis.
   (*a*) directly proportional to (*b*) inversely proportional to
3. The neutral axis of a beam is subjected to
   (*a*) zero stress (*b*) maximum tensile stress
   (*c*) maximum compressive stress (*d*) maximum shear stress
4. At the neutral axis of a beam,
   (*a*) the layers are subjected to maximum bending stress
   (*b*) the layers are subjected to tension (*c*) the layers are subjected to compression
   (*d*) the layers do not undergo any strain
5. The bending stress in a curved beam is
   (*a*) zero at the centroidal axis (*b*) zero at the point other than centroidal axis
   (*c*) maximum at the neutral axis (*d*) none of the above
6. The maximum bending stress, in a curved beam having symmetrical section, always occur, at the
   (*a*) centroidal axis (*b*) neutral axis
   (*c*) inside fibre (*d*) outside fibre
7. If d = diameter of solid shaft and τ = permissible stress in shear for the shaft material, then torsional strength of shaft is written as
   (*a*) $\frac{\pi}{32} d^4 \tau$ (*b*) $d \log_e \tau$
   (*c*) $\frac{\pi}{16} d^3 \tau$ (*d*) $\frac{\pi}{32} d^3 \tau$
8. If d_i and d_o are the inner and outer diameters of a hollow shaft, then its polar moment of inertia is
   (*a*) $\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$ (*b*) $\frac{\pi}{32}\left[(d_o)^3 - (d_i)^3\right]$
   (*c*) $\frac{\pi}{32}\left[(d_o)^2 - (d_i)^2\right]$ (*d*) $\frac{\pi}{32}(d_o - d_i)$

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9. Two shafts under pure torsion are of identical length and identical weight and are made of same material. The shaft *A* is solid and the shaft *B* is hollow. We can say that
   (*a*) shaft *B* is better than shaft *A*
   (*b*) shaft *A* is better than shaft *B*
   (*c*) both the shafts are equally good
10. A solid shaft transmits a torque *T*. The allowable shear stress is τ. The diameter of the shaft is
   (*a*) $\sqrt[3]{\frac{16\,T}{\pi\,\tau}}$ (*b*) $\sqrt[3]{\frac{32\,T}{\pi\,\tau}}$
   (*c*) $\sqrt[3]{\frac{64\,T}{\pi\,\tau}}$ (*d*) $\sqrt[3]{\frac{16\,T}{\tau}}$
11. When a machine member is subjected to a tensile stress (σ_t) due to direct load or bending and a shear stress (τ) due to torsion, then the maximum shear stress induced in the member will be
   (*a*) $\frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$ (*b*) $\frac{1}{2}\left[\sqrt{(\sigma_t)^2 - 4\,\tau^2}\right]$
   (*c*) $\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$ (*d*) $(\sigma_t)^2 + 4\,\tau^2$
12. Rankine's theory is used for
   (*a*) brittle materials   (*b*) ductile materials
   (*c*) elastic materials   (*d*) plastic materials
13. Guest's theory is used for
   (*a*) brittle materials   (*b*) ductile materials
   (*c*) elastic materials   (*d*) plastic materials
14. At the neutral axis of a beam, the shear stress is
   (*a*) zero   (*b*) maximum
   (*c*) minimum
15. The maximum shear stress developed in a beam of rectangular section is .... the average shear stress.
   (*a*) equal to   (*b*) $\frac{4}{3}$ times
   (*c*) 1.5 times

ANSWERS

1. (*b*)	**2.** (*a*)	**3.** (*a*)	**4.** (*d*)	**5.** (*b*)
6. (*c*)	**7.** (*c*)	**8.** (*a*)	**9.** (*a*)	**10.** (*a*)
11. (*a*)	**12.** (*a*)	**13.** (*b*)	**14.** (*b*)	**15.** (*c*)

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C H A P T E R

6

Variable Stresses in Machine Parts

6.1 Introduction

We have discussed, in the previous chapter, the stresses due to static loading only. But only a few machine parts are subjected to static loading. Since many of the machine parts (such as axles, shafts, crankshafts, connecting rods, springs, pinion teeth etc.) are subjected to variable or alternating loads (also known as fluctuating or fatigue loads), therefore we shall discuss, in this chapter, the variable or alternating stresses.

6.2 Completely Reversed or Cyclic Stresses

Consider a rotating beam of circular cross-section and carrying a load *W*, as shown in Fig. 6.1. This load induces stresses in the beam which are cyclic in nature. A little consideration will show that the upper fibres of the beam (*i.e.* at point *A*) are under compressive stress and the lower fibres (*i.e.* at point *B*) are under tensile stress. After

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half a revolution, the point *B* occupies the position of point *A* and the point *A* occupies the position of point *B*. Thus the point *B* is now under compressive stress and the point *A* under tensile stress. The speed of variation of these stresses depends upon the speed of the beam.

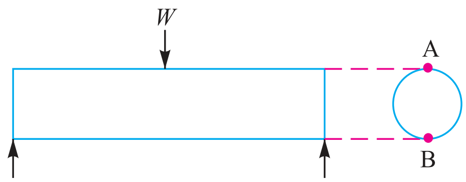


Fig. 6.1. Reversed or cyclic stresses.

From above we see that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or ***vice versa,*** are known as ***completely reversed*** or ***cyclic stresses.***

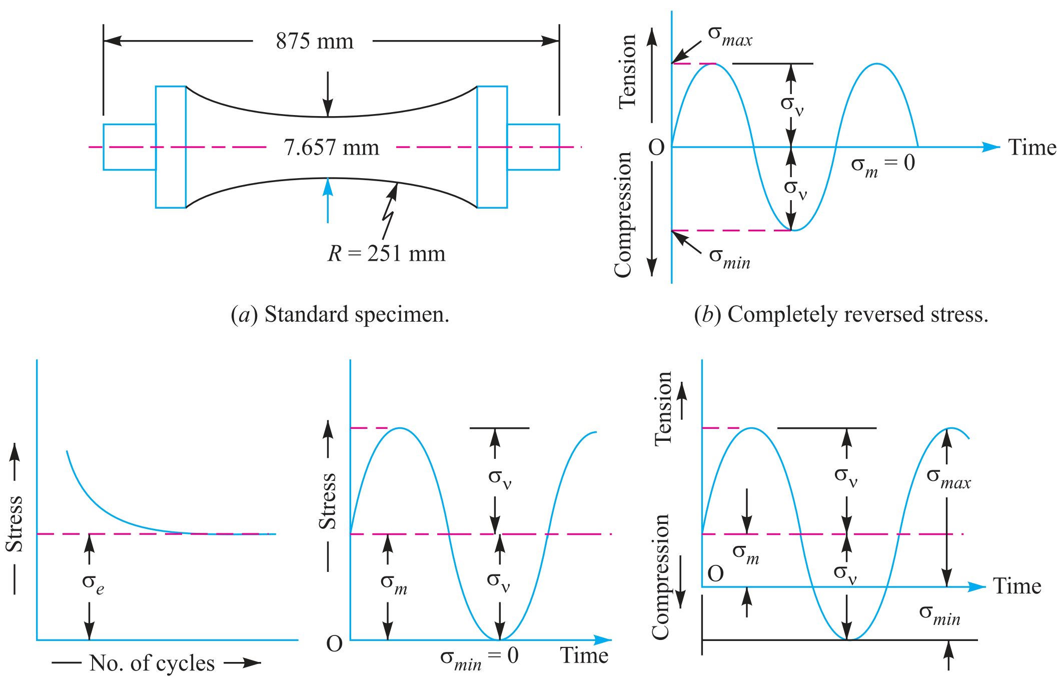
Notes: 1. The stresses which vary from a minimum value to a maximum value of the same nature, (*i.e.* tensile or compressive) are called ***fluctuating stresses.***

2. The stresses which vary from zero to a certain maximum value are called ***repeated stresses.***

3. The stresses which vary from a minimum value to a maximum value of the opposite nature (*i.e.* from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called ***alternating stresses.***

6.3 Fatigue and Endurance Limit

It has been found experimentally that when a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.



(*a*) Standard specimen. (*b*) Completely reversed stress.

(*c*) Endurance or fatigue limit. (*d*) Repeated stress. (*e*) Fluctuating stress.

Fig. 6.2. Time-stress diagrams.

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig. 6.2 (*a*), is rotated in a fatigue

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testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig. 6.2 (*b*). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Fig. 6.2 (*c*). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig. 6.2 (*c*), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as ***endurance*** or ***fatigue limit*** (σ_e). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 10^7 cycles).

A machine part is being turned on a Lathe.

It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term ***endurance strength*** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.

We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig. 6.2 (*b*). In actual practice, many machine members undergo different range of stress than the completely reversed stress.

The stress ***verses*** time diagram for fluctuating stress having values σ_{min} and σ_{max} is shown in Fig. 6.2 (*e*). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σ_v. The following relations are derived from Fig. 6.2 (*e*):

1. Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2. Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Note: For repeated loading, the stress varies from maximum to zero (*i.e.* $\sigma_{min} = 0$) in each cycle as shown in Fig. 6.2 (*d*).

$$\therefore \qquad \sigma_m = \sigma_v = \frac{\sigma_{max}}{2}$$

3. Stress ratio, $R = \dfrac{\sigma_{max}}{\sigma_{min}}$. For completely reversed stresses, $R = -1$ and for repeated stresses, $R = 0$. It may be noted that R cannot be greater than unity.

4. The following relation between endurance limit and stress ratio may be used

$$\sigma'_e = \frac{3\sigma_e}{2 - R}$$

where σ'_e = Endurance limit for any stress range represented by *R*.

σ_e = Endurance limit for completely reversed stresses, and

R = Stress ratio.

6.4 Effect of Loading on Endurance Limit—Load Factor

The endurance limit (σ_e) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let K_b = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

K_s = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

Shaft drive.

∴ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e.K_b = \sigma_e$ ...($\because K_b = 1$)

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e.K_a$

and endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e.K_s$

6.5 Effect of Surface Finish on Endurance Limit—Surface Finish Factor

When a machine member is subjected to variable loads, the endurance limit of the material for that member depends upon the surface conditions. Fig. 6.3 shows the values of surface finish factor for the various surface conditions and ultimate tensile strength.

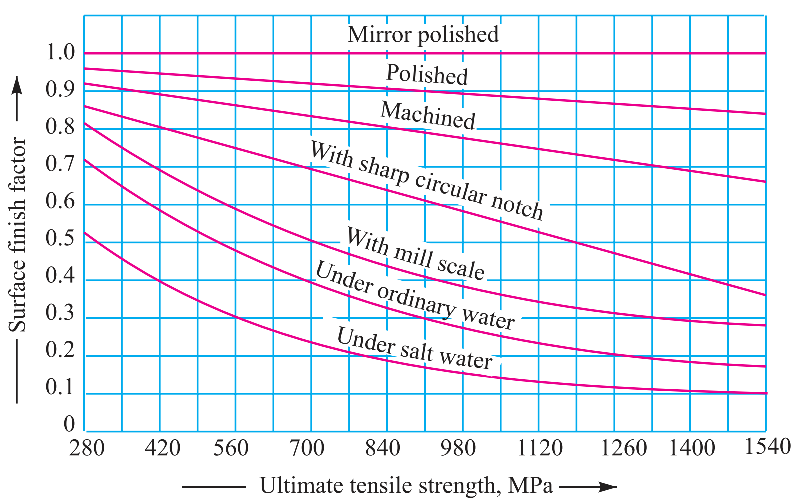

Fig. 6.3. Surface finish factor for various surface conditions.

When the surface finish factor is known, then the endurance limit for the material of the machine member may be obtained by multiplying the endurance limit and the surface finish factor. We see that

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for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for mirror polished material is maximum and it goes on reducing due to surface condition.

Let K_{sur} = Surface finish factor.

∴ Endurance limit,

$$\sigma_{e1} = \sigma_{eb}.K_{sur} = \sigma_e.K_b.K_{sur} = \sigma_e.K_{sur} \quad \text{...}(\because K_b = 1)$$

...(For reversed bending load)

$$= \sigma_{ea}.K_{sur} = \sigma_e.K_a.K_{sur} \quad \text{...(For reversed axial load)}$$

$$= \tau_e.K_{sur} = \sigma_e.K_s.K_{sur} \quad \text{...(For reversed torsional or shear load)}$$

Note : The surface finish factor for non-ferrous metals may be taken as unity.

6.6 Effect of Size on Endurance Limit—Size Factor

A little consideration will show that if the size of the standard specimen as shown in Fig. 6.2 (*a*) is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let K_{sz} = Size factor.

∴ Endurance limit,

$$\sigma_{e2} = \sigma_{e1} \times K_{sz} \quad \text{...(Considering surface finish factor also)}$$

$$= \sigma_{eb}.K_{sur}.K_{sz} = \sigma_e.K_b.K_{sur}.K_{sz} = \sigma_e.K_{sur}.K_{sz} \quad (\because K_b = 1)$$

$$= \sigma_{ea}.K_{sur}.K_{sz} = \sigma_e.K_a.K_{sur}.K_{sz} \quad \text{...(For reversed axial load)}$$

$$= \tau_e.K_{sur}.K_{sz} = \sigma_e.K_s.K_{sur}.K_{sz} \quad \text{... (For reversed torsional or shear load)}$$

Notes: 1. The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm.

2. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85.

3. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

6.7 Effect of Miscellaneous Factors on Endurance Limit

In addition to the surface finish factor (K_{sur}), size factor (K_{sz}) and load factors K_b, K_a and K_s, there are many other factors such as reliability factor (K_r), temperature factor (K_t), impact factor (K_i) etc. which has effect on the endurance limit of a material. Considering all these factors, the endurance limit may be determined by using the following expressions :

1. For the reversed bending load, endurance limit,

$$\sigma'_e = \sigma_{eb}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

2. For the reversed axial load, endurance limit,

$$\sigma'_e = \sigma_{ea}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

3. For the reversed torsional or shear load, endurance limit,

$$\sigma'_e = \tau_e.K_{sur}.K_{sz}.K_r.K_t.K_i$$

In solving problems, if the value of any of the above factors is not known, it may be taken as unity.

In addition to shear, tensile, compressive and torsional stresses, temperature can add its own stress (Ref. Chapter 4)

Note : This picture is given as additional information and is not a direct example of the current chapter.

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6.8 Relation Between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u). Fig. 6.4 shows the endurance limit of steel corresponding to ultimate tensile strength for different surface conditions. Following are some empirical relations commonly used in practice :

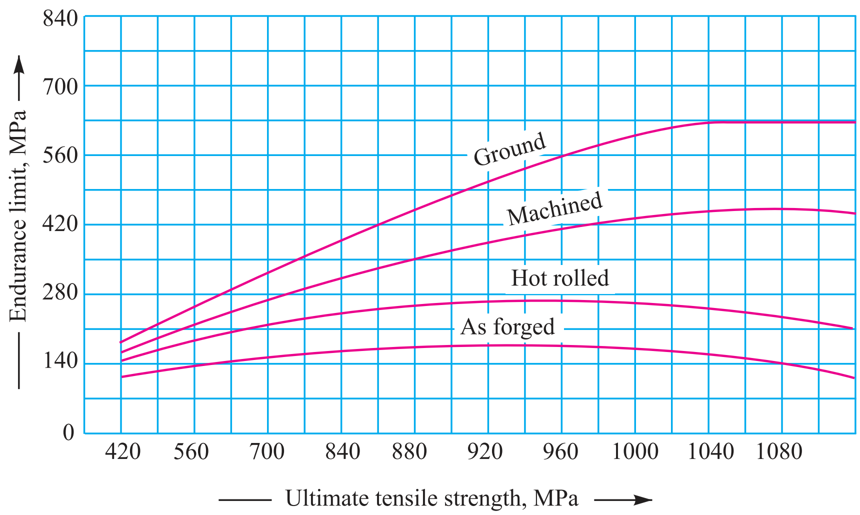


Fig. 6.4. Endurance limit of steel corresponding to ultimate tensile strength.

For steel, $\sigma_e = 0.5\ \sigma_u$;

For cast steel, $\sigma_e = 0.4\ \sigma_u$;

For cast iron, $\sigma_e = 0.35\ \sigma_u$;

For non-ferrous metals and alloys, $\sigma_e = 0.3\ \sigma_u$

6.9 Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for faliure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

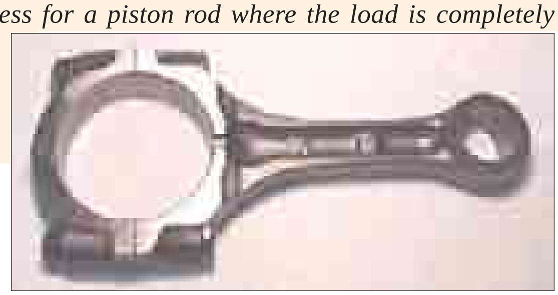
$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

Note: For steel, $\sigma_e = 0.8 \text{ to } 0.9\ \sigma_y$

where σ_e = Endurance limit stress for completely reversed stress cycle, and

σ_y = Yield point stress.

Example 6.1. *Determine the design stress for a piston rod where the load is completely reversed. The surface of the rod is ground and the surface finish factor is 0.9. There is no stress concentration. The load is predictable and the factor of safety is 2.*



Piston rod

Solution. Given : $K_{sur} = 0.9$; $F.S. = 2$

The piston rod is subjected to reversed axial loading. We know that for reversed axial loading, the load correction factor (K_a) is 0.8.

If σ_e is the endurance limit for reversed bending load, then endurance limit for reversed axial load,

$$\sigma_{ea} = \sigma_e \times K_a \times K_{sur} = \sigma_e \times 0.8 \times 0.9 = 0.72\ \sigma_e$$

We know that design stress,

$$\sigma_d = \frac{\sigma_{ea}}{F.S.} = \frac{0.72\ \sigma_e}{2} = 0.36\ \sigma_e \textbf{ Ans.}$$

6.10 Stress Concentration

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighbourhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration.*** It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc.

In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Fig. 6.5. A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the cross-section is changing, a re-distribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

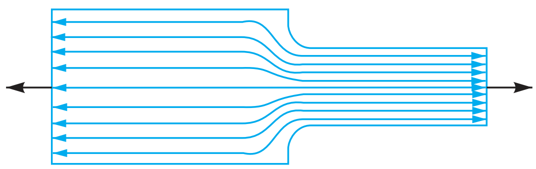


Fig. 6.5. Stress concentration.

6.11 Theoretical or Form Stress Concentration Factor

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area. Mathematically, theoretical or form stress concentration factor,

$$K_t = \frac{\text{Maximum stress}}{\text{Nominal stress}}$$

The value of K_t depends upon the material and geometry of the part.

Notes: 1. In static loading, stress concentration in ductile materials is not so serious as in brittle materials, because in ductile materials local deformation or yielding takes place which reduces the concentration. In brittle materials, cracks may appear at these local concentrations of stress which will increase the stress over the rest of the section. It is, therefore, necessary that in designing parts of brittle materials such as castings, care should be taken. In order to avoid failure due to stress concentration, fillets at the changes of section must be provided.

2. In cyclic loading, stress concentration in ductile materials is always serious because the ductility of the material is not effective in relieving the concentration of stress caused by cracks, flaws, surface roughness, or any sharp discontinuity in the geometrical form of the member. If the stress at any point in a member is above the endurance limit of the material, a crack may develop under the action of repeated load and the crack will lead to failure of the member.

6.12 Stress Concentration due to Holes and Notches

Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in Fig. 6.6 (*a*). We see from the stress-distribution that the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

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and the theoretical stress concentration factor,

$$K_t = \frac{\sigma_{max}}{\sigma} = \left(1 + \frac{2a}{b}\right)$$

When a/b is large, the ellipse approaches a crack transverse to the load and the value of K_t becomes very large. When a/b is small, the ellipse approaches a longitudinal slit [as shown in Fig. 6.6 (*b*)] and the increase in stress is small. When the hole is circular as shown in Fig. 6.6 (*c*), then $a/b = 1$ and the maximum stress is three times the nominal value.

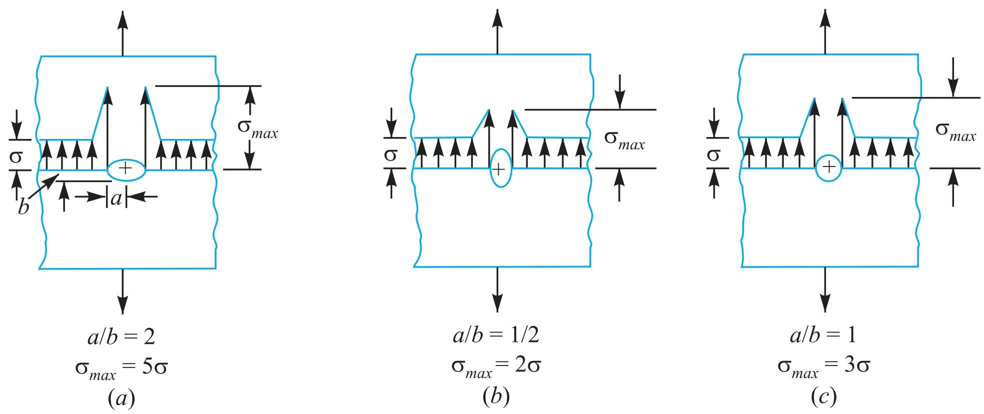

Fig. 6.6. Stress concentration due to holes.

The stress concentration in the notched tension member, as shown in Fig. 6.7, is influenced by the depth a of the notch and radius r at the bottom of the notch. The maximum stress, which applies to members having notches that are small in comparison with the width of the plate, may be obtained by the following equation,

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{r}\right)$$

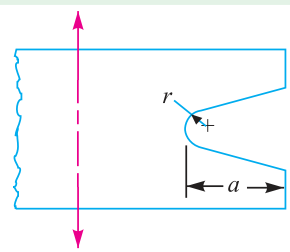

Fig. 6.7. Stress concentration due to notches.

6.13 Methods of Reducing Stress Concentration

We have already discussed in Art 6.10 that whenever there is a change in cross-section, such as shoulders, holes, notches or keyways and where there is an interference fit between a hub or bearing race and a shaft, then stress concentration results. The presence of

Crankshaft

stress concentration can not be totally eliminated but it may be reduced to some extent. A device or concept that is useful in assisting a design engineer to visualize the presence of stress concentration

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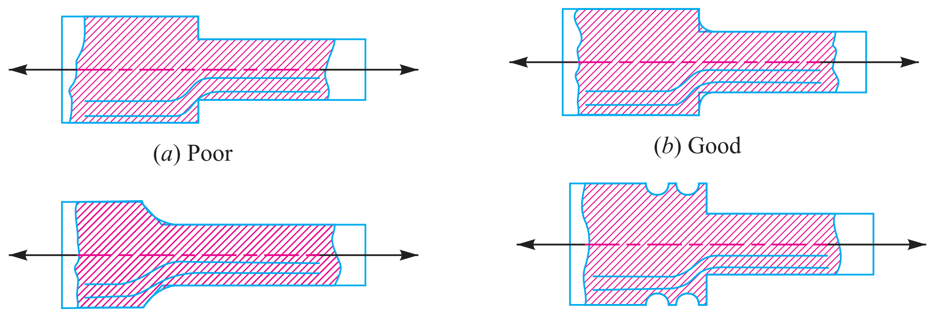
and how it may be mitigated is that of stress flow lines, as shown in Fig. 6.8. The mitigation of stress concentration means that the stress flow lines shall maintain their spacing as far as possible.

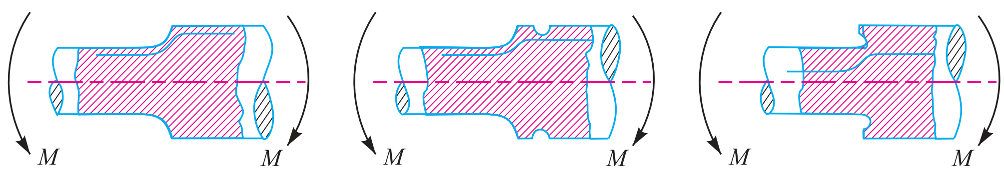


(*a*) Poor (*b*) Good (*c*) Preferred (*d*) Preferred

Fig. 6.8

In Fig. 6.8 (*a*) we see that stress lines tend to bunch up and cut very close to the sharp re-entrant corner. In order to improve the situation, fillets may be provided, as shown in Fig. 6.8 (*b*) and (*c*) to give more equally spaced flow lines.

Figs. 6.9 to 6.11 show the several ways of reducing the stress concentration in shafts and other cylindrical members with shoulders, holes and threads respectively. It may be noted that it is not practicable to use large radius fillets as in case of ball and roller bearing mountings. In such cases, notches may be cut as shown in Fig. 6.8 (*d*) and Fig. 6.9 (*b*) and (*c*).

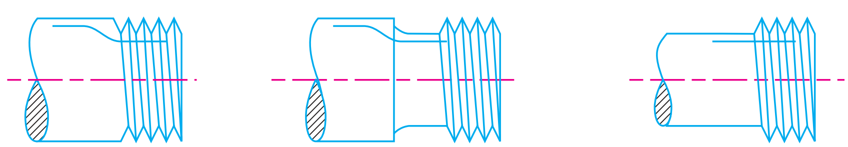


(*a*) Poor (*b*) Good (*c*) Preferred

Fig. 6.9. Methods of reducing stress concentration in cylindrical members with shoulders.

(*a*) Poor (*b*) Preferred

Fig. 6.10. Methods of reducing stress concentration in cylindrical members with holes.



(*a*) Poor (*b*) Good (*c*) Preferred

Fig. 6.11. Methods of reducing stress concentration in cylindrical members with holes.

The stress concentration effects of a press fit may be reduced by making more gradual transition from the rigid to the more flexible shaft. The various ways of reducing stress concentration for such cases are shown in Fig. 6.12 (*a*), (*b*) and (*c*).

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6.14 Factors to be Considered while Designing Machine Parts to Avoid Fatigue Failure

The following factors should be considered while designing machine parts to avoid fatigue failure:

1. The variation in the size of the component should be as gradual as possible.
2. The holes, notches and other stress raisers should be avoided.
3. The proper stress de-concentrators such as fillets and notches should be provided wherever necessary.

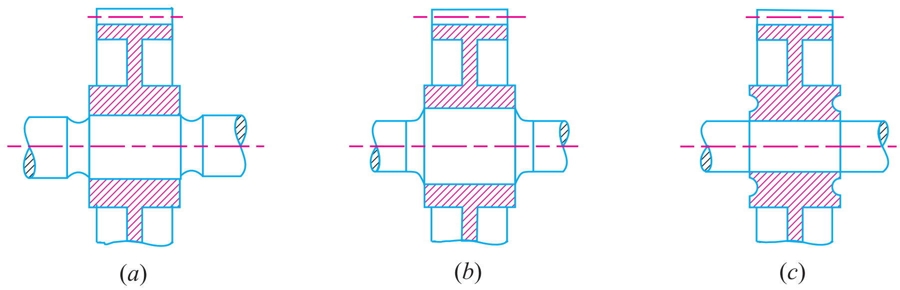

Fig. 6.12. Methods of reducing stress concentration of a press fit.

4. The parts should be protected from corrosive atmosphere.
5. A smooth finish of outer surface of the component increases the fatigue life.
6. The material with high fatigue strength should be selected.
7. The residual compressive stresses over the parts surface increases its fatigue strength.

6.15 Stress Concentration Factor for Various Machine Members

The following tables show the theoretical stress concentration factor for various types of members.

Table 6.1. Theoretical stress concentration factor (K_t) for a plate with hole (of diameter d) in tension.

$\frac{d}{b}$	0.05	0.1	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55
K_t	2.83	2.69	2.59	2.50	2.43	2.37	2.32	2.26	2.22	2.17	2.13

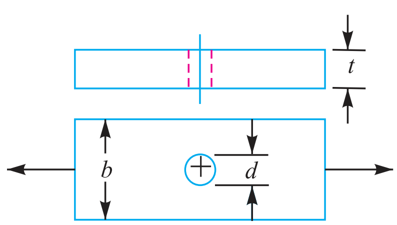

Fig. for Table 6.1

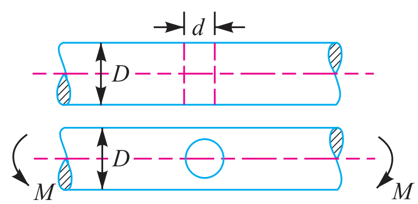

Fig. for Table 6.2

Table 6.2. Theoretical stress concentration factor (K_t) for a shaft with transverse hole (of diameter d) in bending.

$\frac{d}{D}$	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
K_t	2.70	2.52	2.33	2.26	2.20	2.11	2.03	1.96	1.92	1.90

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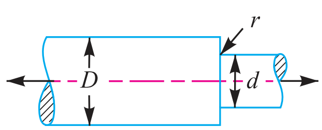
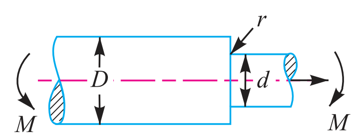
Table 6.3. Theoretical stress concentration factor (K_t) for stepped shaft with a shoulder fillet (of radius *r*) in tension.

D/d	Theoretical stress concentration factor (K_t) — r/d									
	0.08	*0.10*	*0.12*	*0.16*	*0.18*	*0.20*	*0.22*	*0.24*	*0.28*	*0.30*
1.01	1.27	1.24	1.21	1.17	1.16	1.15	1.15	1.14	1.13	1.13
1.02	1.38	1.34	1.30	1.26	1.24	1.23	1.22	1.21	1.19	1.19
1.05	1.53	1.46	1.42	1.36	1.34	1.32	1.30	1.28	1.26	1.25
1.10	1.65	1.56	1.50	1.43	1.39	1.37	1.34	1.33	1.30	1.28
1.15	1.73	1.63	1.56	1.46	1.43	1.40	1.37	1.35	1.32	1.31
1.20	1.82	1.68	1.62	1.51	1.47	1.44	1.41	1.38	1.35	1.34
1.50	2.03	1.84	1.80	1.66	1.60	1.56	1.53	1.50	1.46	1.44
2.00	2.14	1.94	1.89	1.74	1.68	1.64	1.59	1.56	1.50	1.47



$$A = \frac{\pi}{4} \times d^2$$

Fig. for Table 6.3



$$Z = \frac{\pi}{32} \times d^3$$

Fig. for Table 6.4

Table 6.4. Theoretical stress concentration factor (K_t) for a stepped shaft with a shoulder fillet (of radius *r*) in bending.

D/d	Theoretical stress concentration factor (K_t) — r/d									
	0.02	*0.04*	*0.08*	*0.10*	*0.12*	*0.16*	*0.20*	*0.24*	*0.28*	*0.30*
1.01	1.85	1.61	1.42	1.36	1.32	1.24	1.20	1.17	1.15	1.14
1.02	1.97	1.72	1.50	1.44	1.40	1.32	1.27	1.23	1.21	1.20
1.05	2.20	1.88	1.60	1.53	1.48	1.40	1.34	1.30	1.27	1.25
1.10	2.36	1.99	1.66	1.58	1.53	1.44	1.38	1.33	1.28	1.27
1.20	2.52	2.10	1.72	1.62	1.56	1.46	1.39	1.34	1.29	1.28
1.50	2.75	2.20	1.78	1.68	1.60	1.50	1.42	1.36	1.31	1.29
2.00	2.86	2.32	1.87	1.74	1.64	1.53	1.43	1.37	1.32	1.30
3.00	3.00	2.45	1.95	1.80	1.69	1.56	1.46	1.38	1.34	1.32
6.00	3.04	2.58	2.04	1.87	1.76	1.60	1.49	1.41	1.35	1.33

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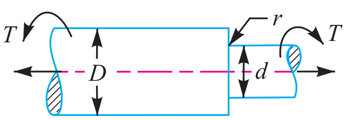
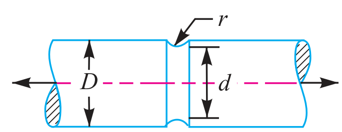
Table 6.5. Theoretical stress concentration factor (K_t) for a stepped shaft with a shoulder fillet (of radius r) in torsion.

$\frac{D}{d}$	Theoretical stress concentration factor (K_t)									
	r/d									
	0.02	*0.04*	*0.08*	*0.10*	*0.12*	*0.16*	*0.20*	*0.24*	*0.28*	*0.30*
1.09	1.54	1.32	1.19	1.16	1.15	1.12	1.11	1.10	1.09	1.09
1.20	1.98	1.67	1.40	1.33	1.28	1.22	1.18	1.15	1.13	1.13
1.33	2.14	1.79	1.48	1.41	1.35	1.28	1.22	1.19	1.17	1.16
2.00	2.27	1.84	1.53	1.46	1.40	1.32	1.26	1.22	1.19	1.18



$$\frac{J}{d/2} = \frac{\pi d^3}{16}$$

Fig. for Table 6.5



$$A = \frac{\pi}{4} \times d^2$$

Fig. for Table 6.6

Table 6.6. Theoretical stress concentration factor (K_t) for a grooved shaft in tension.

$\frac{D}{d}$	Theoretical stress concentration (K_t)									
	r/d									
	0.02	*0.04*	*0.08*	*0.10*	*0.12*	*0.16*	*0.20*	*0.24*	*0.28*	*0.30*
1.01	1.98	1.71	1.47	1.42	1.38	1.33	1.28	1.25	1.23	1.22
1.02	2.30	1.94	1.66	1.59	1.54	1.45	1.40	1.36	1.33	1.31
1.03	2.60	2.14	1.77	1.69	1.63	1.53	1.46	1.41	1.37	1.36
1.05	2.85	2.36	1.94	1.81	1.73	1.61	1.54	1.47	1.43	1.41
1.10	..	2.70	2.16	2.01	1.90	1.75	1.70	1.57	1.50	1.47
1.20	..	2.90	2.36	2.17	2.04	1.86	1.74	1.64	1.56	1.54
1.30	..	..	2.46	2.26	2.11	1.91	1.77	1.67	1.59	1.56
1.50	..	..	2.54	2.33	2.16	1.94	1.79	1.69	1.61	1.57
2.00	..	..	2.61	2.38	2.22	1.98	1.83	1.72	1.63	1.59
∞	..	..	2.69	2.44	2.26	2.03	1.86	1.74	1.65	1.61

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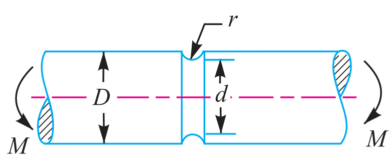
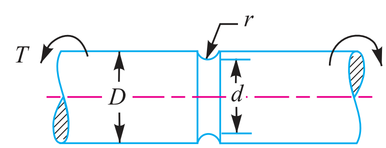
Table 6.7. Theoretical stress concentration factor (K_t) of a grooved shaft in bending.

$\frac{D}{d}$	Theoretical stress concentration factor (K_t)									
	r/d									
	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
1.01	1.74	1.68	1.47	1.41	1.38	1.32	1.27	1.23	1.22	1.20
1.02	2.28	1.89	1.64	1.53	1.48	1.40	1.34	1.30	1.26	1.25
1.03	2.46	2.04	1.68	1.61	1.55	1.47	1.40	1.35	1.31	1.28
1.05	2.75	2.22	1.80	1.70	1.63	1.53	1.46	1.40	1.35	1.33
1.12	3.20	2.50	1.97	1.83	1.75	1.62	1.52	1.45	1.38	1.34
1.30	3.40	2.70	2.04	1.91	1.82	1.67	1.57	1.48	1.42	1.38
1.50	3.48	2.74	2.11	1.95	1.84	1.69	1.58	1.49	1.43	1.40
2.00	3.55	2.78	2.14	1.97	1.86	1.71	1.59	1.55	1.44	1.41
∞	3.60	2.85	2.17	1.98	1.88	1.71	1.60	1.51	1.45	1.42



$$Z = \frac{\pi}{32} \times d^3$$

Fig. for Table 6.7



$$\frac{J}{d/2} = \frac{\pi d^3}{16}$$

Fig. for Table 6.8

Table 6.8. Theoretical stress concentration factor (K_t) for a grooved shaft in torsion.

$\frac{D}{d}$	Theoretical stress concentration factor (K_{ts})									
	r/d									
	0.02	0.04	0.08	0.10	0.12	0.16	0.20	0.24	0.28	0.30
1.01	1.50	1.03	1.22	1.20	1.18	1.16	1.13	1.12	1.12	1.12
1.02	1.62	1.45	1.31	1.27	1.23	1.20	1.18	1.16	1.15	1.16
1.05	1.88	1.61	1.40	1.35	1.32	1.26	1.22	1.20	1.18	1.17
1.10	2.05	1.73	1.47	1.41	1.37	1.31	1.26	1.24	1.21	1.20
1.20	2.26	1.83	1.53	1.46	1.41	1.34	1.27	1.25	1.22	1.21
1.30	2.32	1.89	1.55	1.48	1.43	1.35	1.30	1.26	—	—
2.00	2.40	1.93	1.58	1.50	1.45	1.36	1.31	1.26	—	—
∞	2.50	1.96	1.60	1.51	1.46	1.38	1.32	1.27	1.24	1.23

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Example 6.2. *Find the maximum stress induced in the following cases taking stress concentration into account:*

1. A rectangular plate 60 mm × 10 mm with a hole 12 diameter as shown in Fig. 6.13 (a) and subjected to a tensile load of 12 kN.

2. A stepped shaft as shown in Fig. 6.13 (b) and carrying a tensile load of 12 kN.

Stepped shaft

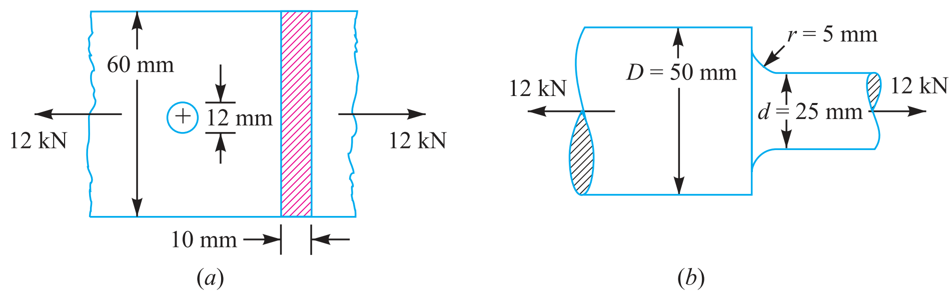


Fig. 6.13

Solution. Case 1. Given : b = 60 mm ; t = 10 mm ; d = 12 mm ; W = 12 kN = 12 × 10^3 N

We know that cross-sectional area of the plate,

$$A = (b - d)\, t = (60 - 12)\, 10 = 480 \text{ mm}^2$$

$$\therefore \quad \text{Nominal stress} = \frac{W}{A} = \frac{12 \times 10^3}{480} = 25 \text{ N/mm}^2 = 25 \text{ MPa}$$

Ratio of diameter of hole to width of plate,

$$\frac{d}{b} = \frac{12}{60} = 0.2$$

From Table 6.1, we find that for d / b = 0.2, theoretical stress concentration factor,

$$K_t = 2.5$$

$$\therefore \quad \text{Maximum stress} = K_t \times \text{Nominal stress} = 2.5 \times 25 = 62.5 \text{ MPa}$$ **Ans.**

Case 2. Given : D = 50 mm ; d = 25 mm ; r = 5 mm ; W = 12 kN = 12 × 10^3 N

We know that cross-sectional area for the stepped shaft,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(25)^2 = 491 \text{ mm}^2$$

$$\therefore \quad \text{Nominal stress} = \frac{W}{A} = \frac{12 \times 10^3}{491} = 24.4 \text{ N/mm}^2 = 24.4 \text{ MPa}$$

Ratio of maximum diameter to minimum diameter,

$$D/d = 50/25 = 2$$

Ratio of radius of fillet to minimum diameter,

$$r/d = 5/25 = 0.2$$

From Table 6.3, we find that for D/d = 2 and r/d = 0.2, theoretical stress concentration factor, K_t = 1.64.

$$\therefore \quad \text{Maximum stress} = K_t \times \text{Nominal stress} = 1.64 \times 24.4 = 40 \text{ MPa}$$ **Ans.**

6.16 Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

6.17 Notch Sensitivity

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term ***notch sensitivity*** is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material. Since the extensive data for estimating the notch sensitivity factor (q) is not available, therefore the curves, as shown in Fig. 6.14, may be used for determining the values of q for two steels.

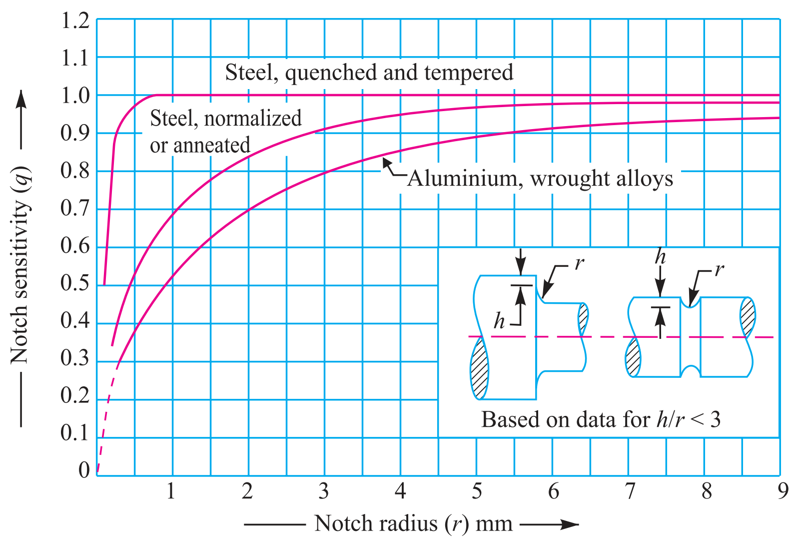


Fig. 6.14. Notch sensitivity.

When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

or $$K_f = 1 + q\,(K_t - 1) \qquad \text{...[For tensile or bending stress]}$$

and $$K_{fs} = 1 + q\,(K_{ts} - 1) \qquad \text{...[For shear stress]}$$

where K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.

Protective colour coatings are added to make components it corrosion resistant. Corrosion if not taken care can magnify other stresses.

Note : This picture is given as additional information and is not a direct example of the current chapter.

6.18 Combined Steady and Variable Stress

The failure points from fatigue tests made with different steels and combinations of mean and variable stresses are plotted in Fig. 6.15 as functions of variable stress (σ_v) and mean stress (σ_m). The most significant observation is that, in general, the failure point is little related to the mean stress when it is compressive but is very much a function of the mean stress when it is tensile. In practice, this means that fatigue failures are rare when the mean stress is compressive (or negative). Therefore, the greater emphasis must be given to the combination of a variable stress and a steady (or mean) tensile stress.

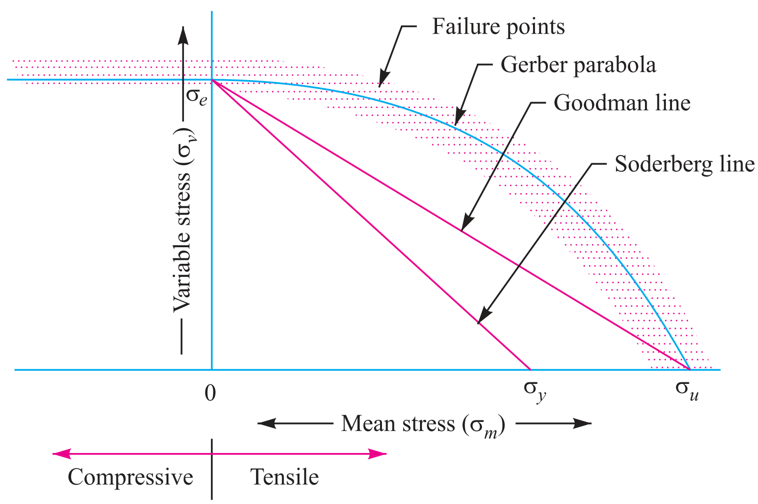


Fig. 6.15. Combined mean and variable stress.

There are several ways in which problems involving this combination of stresses may be solved, but the following are important from the subject point of view :

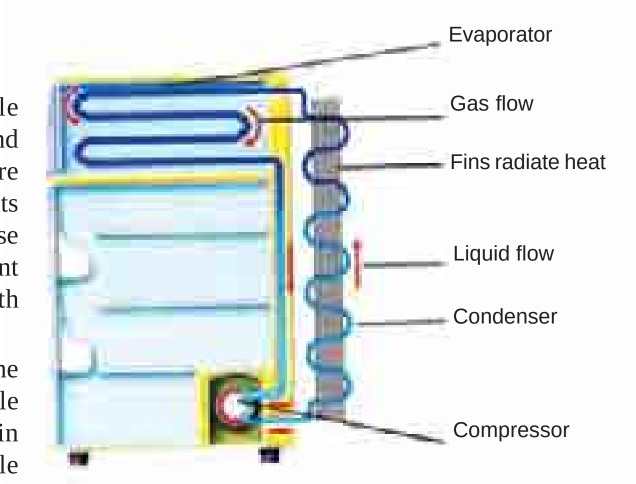
1. Gerber method, **2.** Goodman method, and **3.** Soderberg method.

We shall now discuss these methods, in detail, in the following pages.

6.19 Gerber Method for Combination of Stresses

The relationship between variable stress (σ_v) and mean stress (σ_m) for axial and bending loading for ductile materials are shown in Fig. 6.15. The point σ_e represents the fatigue strength corresponding to the case of complete reversal ($\sigma_m = 0$) and the point σ_u represents the static ultimate strength corresponding to $\sigma_v = 0$.

A parabolic curve drawn between the endurance limit (σ_e) and ultimate tensile strength (σ_u) was proposed by Gerber in 1874. Generally, the test data for ductile material fall closer to Gerber parabola as shown in Fig. 6.15, but because of scatter in the test points, a straight line relationship (*i.e.* Goodman line and Soderberg line) is usually preferred in designing machine parts.



Liquid refrigerant absorbs heat as it vaporizes inside the evaporator coil of a refrigerator. The heat is released when a compressor turns the refrigerant back to liquid.

Note : This picture is given as additional information and is not a direct example of the current chapter.

According to Gerber, variable stress,

$$\sigma_v = \sigma_e \left[\frac{1}{F.S.} - \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. \right]$$

or
$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. + \frac{\sigma_v}{\sigma_e} \quad ...(i)$$

where $F.S.$ = Factor of safety,

σ_m = Mean stress (tensile or compressive),

σ_u = Ultimate stress (tensile or compressive), and

σ_e = Endurance limit for reversal loading.

Considering the fatigue stress concentration factor (K_f), the equation (*i*) may be written as

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u} \right)^2 F.S. + \frac{\sigma_v \times K_f}{\sigma_e}$$

6.20 Goodman Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the ultimate strength (σ_u), as shown by line *AB* in Fig. 6.16, follows the suggestion of Goodman. A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

In Fig. 6.16, line *AB* connecting σ_e and

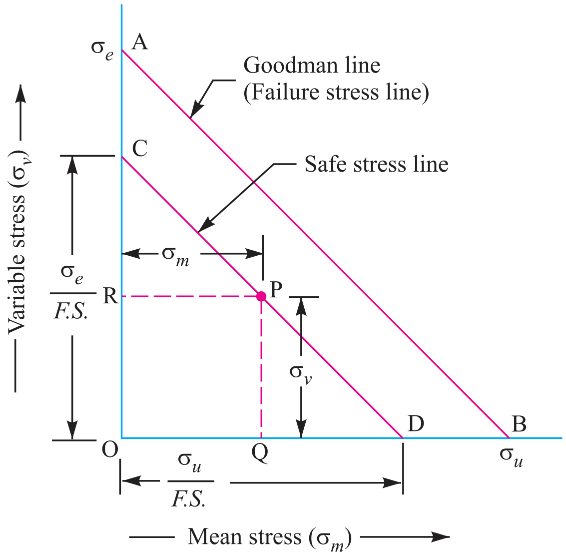


Fig. 6.16. Goodman method.

σ_u is called ***Goodman's failure stress line.*** If a suitable factor of safety (*F.S.*) is applied to endurance limit and ultimate strength, a safe stress line *CD* may be drawn parallel to the line *AB*. Let us consider a design point *P* on the line *CD*.

Now from similar triangles *COD* and *PQD*,

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD} \qquad ...(\because QD = OD - OQ)$$

$$\therefore \qquad \frac{{}^{*}\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_u / F.S.}$$

$$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_u / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u}\right]$$

or

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} \qquad ...(i)$$

This expression does not include the effect of stress concentration. It may be noted that for ductile materials, the stress concentration may be ignored under steady loads.

Since many machine and structural parts that are subjected to fatigue loads contain regions of high stress concentration, therefore equation (*i*) must be altered to include this effect. In such cases, the fatigue stress concentration factor (K_f) is used to multiply the variable stress (σ_v). The equation ***(i)*** may now be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad ...(ii)$$

where $F.S.$ = Factor of safety,

σ_m = Mean stress,

σ_u = Ultimate stress,

σ_v = Variable stress,

σ_e = Endurance limit for reversed loading, and

K_f = Fatigue stress concentration factor.

Considering the load factor, surface finish factor and size factor, the equation ***(ii)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}} \qquad ...(iii)$$

$$= \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}} \qquad ...(\because \sigma_{eb} = \sigma_e \times K_b \text{ and } K_b = 1)$$

where K_b = Load factor for reversed bending load,

K_{sur} = Surface finish factor, and

K_{sz} = Size factor.

---

* Here we have assumed the same factor of safety (*F.S.*) for the ultimate tensile strength (σ_u) and endurance limit (σ_e). In case the factor of safety relating to both these stresses is different, then the following relation may be used :

$$\frac{\sigma_v}{\sigma_e / (F.S.)_e} = 1 - \frac{\sigma_m}{\sigma_u / (F.S.)_u}$$

where $(F.S.)_e$ = Factor of safety relating to endurance limit, and

$(F.S.)_u$ = Factor of safety relating to ultimate tensile strength.

Notes : 1. The equation ***(iii)*** is applicable to ductile materials subjected to reversed bending loads (tensile or compressive). For brittle materials, the theoretical stress concentration factor (K_t) should be applied to the mean stress and fatigue stress concentration factor (K_f) to the variable stress. Thus for brittle materials, the equation ***(iii)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} \qquad ...(iv)$$

2. When a machine component is subjected to a load other than reversed bending, then the endurance limit for that type of loading should be taken into consideration. Thus for reversed axial loading (tensile or compressive), the equations ***(iii)*** and ***(iv)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}} \qquad \text{...(For ductile materials)}$$

and

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}} \qquad \text{...(For brittle materials)}$$

Similarly, for reversed torsional or shear loading,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \qquad \text{...(For ductile materials)}$$

and

$$\frac{1}{F.S.} = \frac{\tau_m \times K_{ts}}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \qquad \text{...(For brittle materials)}$$

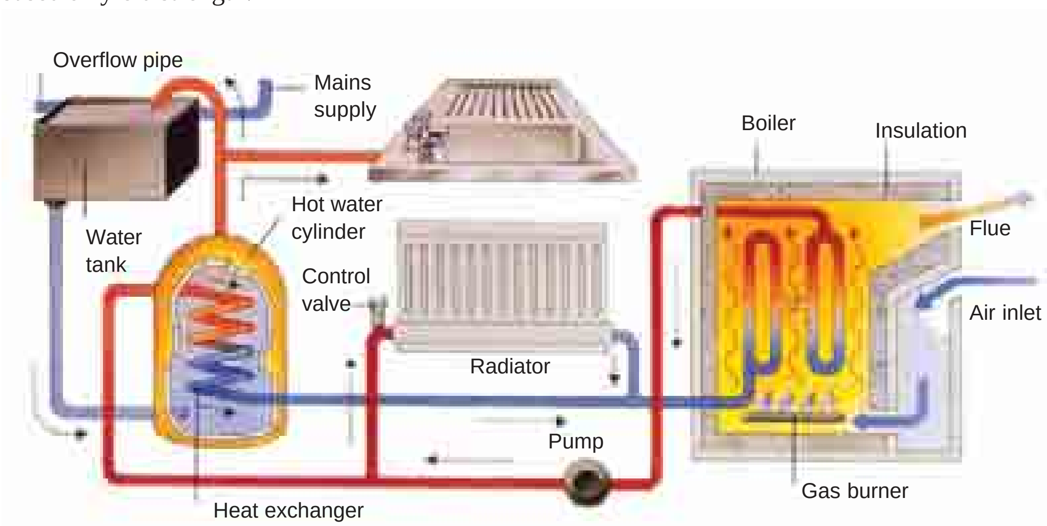
where suffix '*s*'denotes for shear.

For reversed torsional or shear loading, the values of ultimate shear strength (τ_u) and endurance shear strength (τ_e) may be taken as follows:

$$\tau_u = 0.8\ \sigma_u; \text{ and } \tau_e = 0.8\ \sigma_e$$

6.21 Soderberg Method for Combination of Stresses

A straight line connecting the endurance limit (σ_e) and the yield strength (σ_y), as shown by the line *AB* in Fig. 6.17, follows the suggestion of Soderberg line. This line is used when the design is based on yield strength.



In this central heating system, a furnace burns fuel to heat water in a boiler. A pump forces the hot water through pipes that connect to radiators in each room. Water from the boiler also heats the hot water cylinder. Cooled water returns to the boiler.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Proceeding in the same way as discussed in Art 6.20, the line *AB* connecting σ_e and σ_y, as shown in Fig. 6.17, is called ***Soderberg's failure stress line***. If a suitable factor of safety (*F.S.*) is applied to the endurance limit and yield strength, a safe stress line *CD* may be drawn parallel to the line *AB*. Let us consider a design point *P* on the line *CD*. Now from similar triangles *COD* and *PQD*,

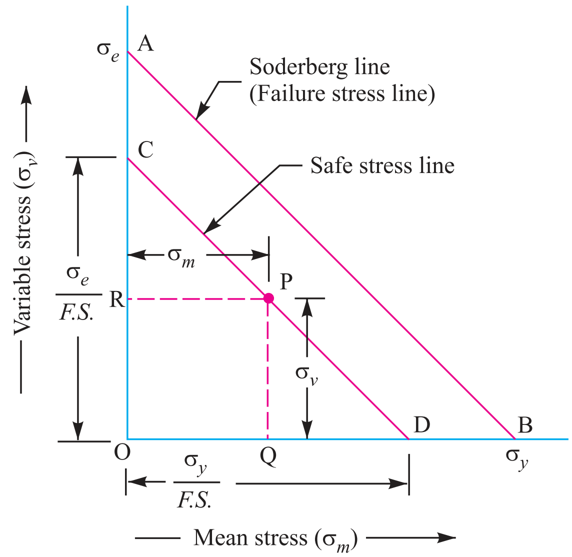


Fig. 6.17. Soderberg method.

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD}$$

$$= 1 - \frac{OQ}{OD} \qquad ...(\because \; QD = OD - OQ)$$

$$\therefore \quad \frac{\sigma_v}{\sigma_e / F.S.} = 1 - \frac{\sigma_m}{\sigma_y / F.S.}$$

or $$\sigma_v = \frac{\sigma_e}{F.S.}\left[1 - \frac{\sigma_m}{\sigma_y / F.S.}\right] = \sigma_e\left[\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_y}\right]$$

$$\therefore \quad \frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e} \qquad ...(i)$$

For machine parts subjected to fatigue loading, the fatigue stress concentration factor (K_f) should be applied to only variable stress (σ_v). Thus the equations ***(i)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad ...(ii)$$

Considering the load factor, surface finish factor and size factor, the equation ***(ii)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} \qquad ...(iii)$$

Since $\sigma_{eb} = \sigma_e \times K_b$ and $K_b = 1$ for reversed bending load, therefore $\sigma_{eb} = \sigma_e$ may be substituted in the above equation.

Notes: 1. The Soderberg method is particularly used for ductile materials. The equation ***(iii)*** is applicable to ductile materials subjected to reversed bending load (tensile or compressive).

2. When a machine component is subjected to reversed axial loading, then the equation ***(iii)*** may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

3. When a machine component is subjected to reversed shear loading, then equation ***(iii)*** may be written as

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

where K_{fs} is the fatigue stress concentration factor for reversed shear loading. The yield strength in shear (τ_y) may be taken as one-half the yield strength in reversed bending (σ_y).

Example 6.3. *A machine component is subjected to a flexural stress which fluctuates between + 300 MN/m² and – 150 MN/m². Determine the value of minimum ultimate strength according to 1. Gerber relation; 2. Modified Goodman relation; and 3. Soderberg relation.*

Take yield strength = 0.55 Ultimate strength; Endurance strength = 0.5 Ultimate strength; and factor of safety = 2.

Springs often undergo variable stresses.

Solution. Given : σ_1 = 300 MN/m² ; σ_2 = – 150 MN/m² ; $\sigma_y = 0.55\ \sigma_u$; $\sigma_e = 0.5\ \sigma_u$; *F.S.* = 2

Let σ_u = Minimum ultimate strength in MN/m².

We know that the mean or average stress,

$$\sigma_m = \frac{\sigma_1 + \sigma_2}{2} = \frac{300 + (-150)}{2} = 75 \text{ MN/m}^2$$

and variable stress,

$$\sigma_v = \frac{\sigma_1 - \sigma_2}{2} = \frac{300 - (-150)}{2} = 225 \text{ MN/m}^2$$

1. ***According to Gerber relation***

We know that according to Gerber relation,

$$\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S. + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{2} = \left(\frac{75}{\sigma_u}\right)^2 2 + \frac{225}{0.5\sigma_u} = \frac{11\,250}{(\sigma_u)^2} + \frac{450}{\sigma_u} = \frac{11\,250 + 450\ \sigma_u}{(\sigma_u)^2}$$

$$(\sigma_u)^2 = 22\,500 + 900\ \sigma_u$$

or $$(\sigma_u)^2 - 900\ \sigma_u - 22\,500 = 0$$

$\therefore$ $$\sigma_u = \frac{900 \pm \sqrt{(900)^2 + 4 \times 1 \times 22\,500}}{2 \times 1} = \frac{900 \pm 948.7}{2}$$

$$= 924.35 \text{ MN/m}^2$$ **Ans.** ...(Taking +ve sign)

2. ***According to modified Goodman relation***

We know that according to modified Goodman relation,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}$$

or $$\frac{1}{2} = \frac{75}{\sigma_u} + \frac{225}{0.5\ \sigma_u} = \frac{525}{\sigma_u}$$

$\therefore$ $$\sigma_u = 2 \times 525 = 1050 \text{ MN/m}^2$$ **Ans.**

3. ***According to Soderberg relation***

We know that according to Soderberg relation,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

or $$\frac{1}{2} = \frac{75}{0.55\ \sigma_u} + \frac{255}{0.5\ \sigma_u} = \frac{586.36}{\sigma_u}$$

$\therefore$ $$\sigma_u = 2 \times 586.36 = 1172.72 \text{ MN/m}^2$$ **Ans.**

Example 6.4. *A bar of circular cross-section is subjected to alternating tensile forces varying from a minimum of 200 kN to a maximum of 500 kN. It is to be manufactured of a material with an ultimate tensile strength of 900 MPa and an endurance limit of 700 MPa. Determine the diameter of bar using safety factors of 3.5 related to ultimate tensile strength and 4 related to endurance limit and a stress concentration factor of 1.65 for fatigue load. Use Goodman straight line as basis for design.*

Solution. Given : W_{min} = 200 kN ; W_{max} = 500 kN ; σ_u = 900 MPa = 900 N/mm^2 ; σ_e = 700 MPa = 700 N/mm^2 ; $(F.S.)_u$ = 3.5 ; $(F.S.)_e$ = 4 ; K_f = 1.65

Let d = Diameter of bar in mm.

$$\therefore \quad \text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that mean or average force,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{500 + 200}{2} = 350 \text{ kN} = 350 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{350 \times 10^3}{0.7854\, d^2} = \frac{446 \times 10^3}{d^2} \text{ N/mm}^2$$

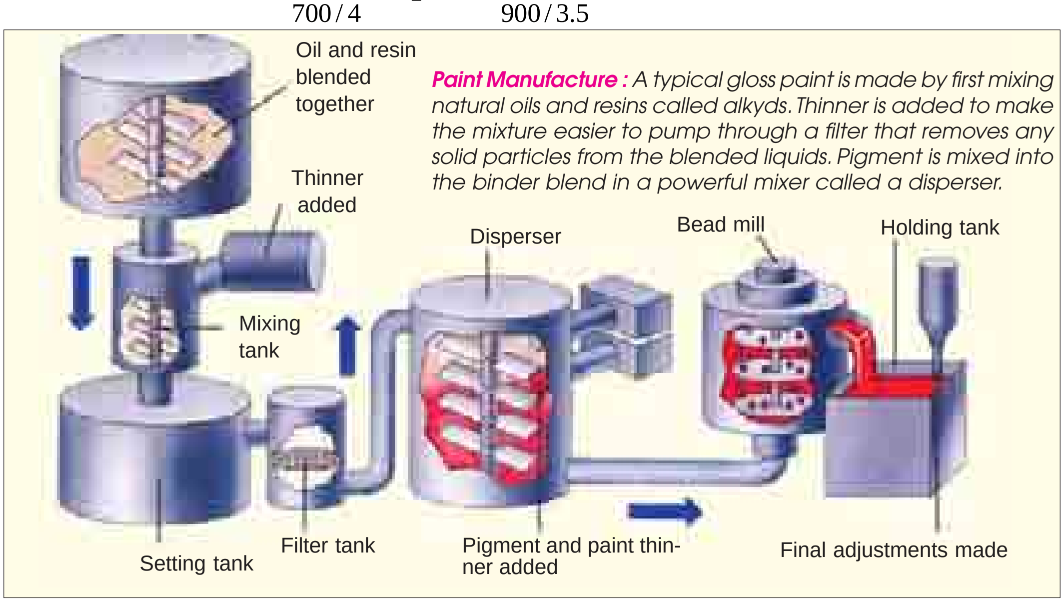
$$\text{Variable force, } W_v = \frac{W_{max} - W_{min}}{2} = \frac{500 - 200}{2} = 150 \text{ kN} = 150 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Variable stress, } \sigma_v = \frac{W_v}{A} = \frac{150 \times 10^3}{0.7854\, d^2} = \frac{191 \times 10^3}{d^2} \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{\sigma_v}{\sigma_e / (F.S.)_e} = 1 - \frac{\sigma_m . K_f}{\sigma_u / (F.S.)_u}$$

$$\frac{\dfrac{191 \times 10^3}{d^2}}{700/4} = 1 - \frac{\dfrac{446 \times 10^3}{d^2} \times 1.65}{900/3.5}$$



Paint Manufacture : *A typical gloss paint is made by first mixing natural oils and resins called alkyds. Thinner is added to make the mixture easier to pump through a filter that removes any solid particles from the blended liquids. Pigment is mixed into the binder blend in a powerful mixer called a disperser.*

Note : This picture is given as additional information and is not a direct example of the current chapter.

$$\frac{1100}{d^2} = 1 - \frac{2860}{d^2} \quad \text{or} \quad \frac{1100 + 2860}{d^2} = 1$$

∴ $d^2 = 3960$ or $d = 62.9$ say 63 mm **Ans.**

Example 6.5. *Determine the thickness of a 120 mm wide uniform plate for safe continuous operation if the plate is to be subjected to a tensile load that has a maximum value of 250 kN and a minimum value of 100 kN. The properties of the plate material are as follows:*

Endurance limit stress = 225 MPa, and Yield point stress = 300 MPa.

The factor of safety based on yield point may be taken as 1.5.

Solution. Given : b = 120 mm ; W_{max} = 250 kN; W_{min} = 100 kN ; σ_e = 225 MPa = 225 N/mm^2 ; σ_y = 300 MPa = 300 N/mm^2; *F.S.* = 1.5

Let t = Thickness of the plate in mm.

∴ Area, $A = b \times t = 120\, t$ mm^2

We know that mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{250 + 100}{2} = 175 \text{ kN} = 175 \times 10^3 \text{ N}$$

∴ Mean stress, $\sigma_m = \dfrac{W_m}{A} = \dfrac{175 \times 10^3}{120\,t}$ N/mm^2

Variable load, $W_v = \dfrac{W_{max} - W_{min}}{2} = \dfrac{250 - 100}{2} = 75 \text{ kN} = 75 \times 10^3$ N

∴ Variable stress, $\sigma_v = \dfrac{W_v}{A} = \dfrac{75 \times 10^3}{120\,t}$ N/mm^2

According to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{1.5} = \frac{175 \times 10^3}{120\,t \times 300} + \frac{75 \times 10^3}{120\,t \times 225} = \frac{4.86}{t} + \frac{2.78}{t} = \frac{7.64}{t}$$

∴ $t = 7.64 \times 1.5 = 11.46$ say 11.5 mm **Ans.**

Example 6.6. *Determine the diameter of a circular rod made of ductile material with a fatigue strength (complete stress reversal), σ_e = 265 MPa and a tensile yield strength of 350 MPa. The member is subjected to a varying axial load from W_{min} = – 300 × 10^3 N to W_{max} = 700 × 10^3 N and has a stress concentration factor = 1.8. Use factor of safety as 2.0.*

Solution. Given : σ_e = 265 MPa = 265 N/mm^2 ; σ_y = 350 MPa = 350 N/mm^2 ; W_{min} = – 300 × 10^3 N ; W_{max} = 700 × 10^3 N ; K_f = 1.8 ; *F.S.* = 2

Let d = Diameter of the circular rod in mm.

∴ Area, $A = \dfrac{\pi}{4} \times d^2 = 0.7854\, d^2$ mm^2

We know that the mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{700 \times 10^3 + (-300 \times 10^3)}{2} = 200 \times 10^3 \text{ N}$$

∴ Mean stress, $\sigma_m = \dfrac{W_m}{A} = \dfrac{200 \times 10^3}{0.7854\, d^2} = \dfrac{254.6 \times 10^3}{d^2}$ N/mm^2

Variable load, $W_v = \frac{W_{max} - W_{min}}{2} = \frac{700 \times 10^3 - (-300 \times 10^3)}{2} = 500 \times 10^3$ N

$\therefore$ Variable stress, $\sigma_v = \frac{W_v}{A} = \frac{500 \times 10^3}{0.7854\, d^2} = \frac{636.5 \times 10^3}{d^2}$ N/mm^2

We know that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e}$$

$$\frac{1}{2} = \frac{254.6 \times 10^3}{d^2 \times 350} + \frac{636.5 \times 10^3 \times 1.8}{d^2 \times 265} = \frac{727}{d^2} + \frac{4323}{d^2} = \frac{5050}{d^2}$$

$\therefore$ $d^2 = 5050 \times 2 = 10\,100$ or $d = 100.5$ mm **Ans.**

Example 6.7. *A steel rod is subjected to a reversed axial load of 180 kN. Find the diameter of the rod for a factor of safety of 2. Neglect column action. The material has an ultimate tensile strength of 1070 MPa and yield strength of 910 MPa. The endurance limit in reversed bending may be assumed to be one-half of the ultimate tensile strength. Other correction factors may be taken as follows:*

For axial loading = 0.7; For machined surface = 0.8 ; For size = 0.85 ; For stress concentration = 1.0.

Solution. Given : W_{max} = 180 kN ; W_{min} = – 180 kN ; $F.S.$ = 2 ; σ_u = 1070 MPa = 1070 N/mm^2; σ_y = 910 MPa = 910 N/mm^2 ; $\sigma_e = 0.5\, \sigma_u$; $K_a = 0.7$; $K_{sur} = 0.8$; $K_{sz} = 0.85$; $K_f = 1$

Let d = Diameter of the rod in mm.

$\therefore$ Area, $A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2$ mm^2

We know that the mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{180 + (-180)}{2} = 0$$

$\therefore$ Mean stress, $\sigma_m = \frac{W_m}{A} = 0$

Variable load, $W_v = \frac{W_{max} - W_{min}}{2} = \frac{180 - (-180)}{2} = 180 \text{ kN} = 180 \times 10^3$ N

$\therefore$ Variable stress, $\sigma_v = \frac{W_v}{A} = \frac{180 \times 10^3}{0.7854\, d^2} = \frac{229 \times 10^3}{d^2}$ N/mm^2

Endurance limit in reversed axial loading,

$$\sigma_{ea} = \sigma_e \times K_a = 0.5\, \sigma_u \times 0.7 = 0.35\, \sigma_u \quad \text{...}(\because \sigma_e = 0.5\, \sigma_u)$$

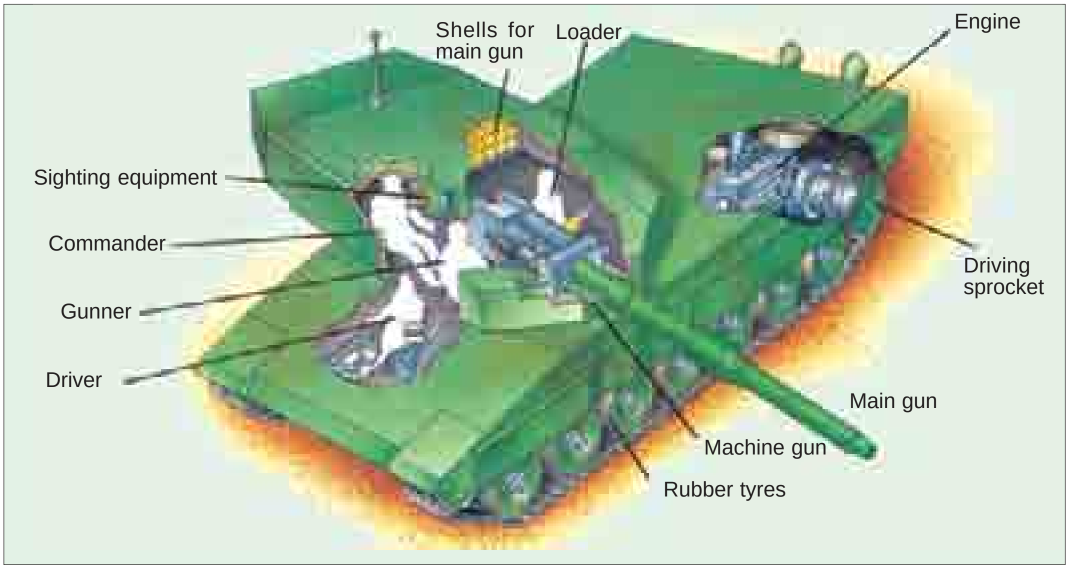
$$= 0.35 \times 1070 = 374.5 \text{ N/mm}^2$$

We know that according to Soderberg's formula for reversed axial loading,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

$$\frac{1}{2} = 0 + \frac{229 \times 10^3 \times 1}{d^2 \times 374.5 \times 0.8 \times 0.85} = \frac{900}{d^2}$$

$\therefore$ $d^2 = 900 \times 2 = 1800$ or $d = 42.4$ mm **Ans.**



Layout of a military tank.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Example 6.8. *A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maximum value of 50 kN. Determine the diameter of bar by taking a factor of safety of 1.5, size effect of 0.85, surface finish factor of 0.9. The material properties of bar are given by : ultimate strength of 650 MPa, yield strength of 500 MPa and endurance strength of 350 MPa.*

Solution. Given : l = 500 mm ; W_{min} = 20 kN = 20 × 10^3 N ; W_{max} = 50 kN = 50 × 10^3 N ; *F.S.* = 1.5 ; K_{sz} = 0.85 ; K_{sur} = 0.9 ; σ_u = 650 MPa = 650 N/mm^2 ; σ_y = 500 MPa = 500 N/mm^2 ; σ_e = 350 MPa = 350 N/mm^2

Let d = Diameter of the bar in mm.

We know that the maximum bending moment,

$$M_{max} = \frac{W_{max} \times l}{4} = \frac{50 \times 10^3 \times 500}{4} = 6250 \times 10^3 \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times l}{4} = \frac{20 \times 10^3 \times 500}{4} = 2550 \times 10^3 \text{ N-mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{6250 \times 10^3 + 2500 \times 10^3}{2} = 4375 \times 10^3 \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{6250 \times 10^3 - 2500 \times 10^3}{2} = 1875 \times 10^3 \text{ N-mm}$$

Section modulus of the bar,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\ d^3 \text{ mm}^3$$

∴ Mean or average bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{4375 \times 10^3}{0.0982\, d^3} = \frac{44.5 \times 10^6}{d^3} \text{ N/mm}^2$$

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and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{1875 \times 10^3}{0.0982\, d^3} = \frac{19.1 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 650} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \quad \text{...(Taking } K_f = 1)$$

$$= \frac{68 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{139 \times 10^3}{d^3}$$

$\therefore$ $d^3 = 139 \times 10^3 \times 1.5 = 209 \times 10^3$ or $d = 59.3$ mm

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 500} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \quad \text{...(Taking } K_f = 1)$$

$$= \frac{89 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{160 \times 10^3}{d^3}$$

$\therefore$ $d^3 = 160 \times 10^3 \times 1.5 = 240 \times 10^3$ or $d = 62.1$ mm

Taking larger of the two values, we have $d = 62.1$ mm **Ans.**

Example 6.9. *A 50 mm diameter shaft is made from carbon steel having ultimate tensile strength of 630 MPa. It is subjected to a torque which fluctuates between 2000 N-m to – 800 N-m. Using Soderberg method, calculate the factor of safety. Assume suitable values for any other data needed.*

Solution. Given : $d = 50$ mm ; $\sigma_u = 630$ MPa $= 630$ N/mm^2 ; $T_{max} = 2000$ N-m ; $T_{min} = -800$ N-m

We know that the mean or average torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{2000 + (-800)}{2} = 600 \text{ N-m} = 600 \times 10^3 \text{ N-mm}$$

$\therefore$ Mean or average shear stress,

$$\tau_m = \frac{16\, T_m}{\pi d^3} = \frac{16 \times 600 \times 10^3}{\pi (50)^3} = 24.4 \text{ N/mm}^2 \quad \text{...}\left(\because T = \frac{\pi}{16} \times \tau \times d^3\right)$$

Variable torque,

$$T_v = \frac{T_{max} - T_{min}}{2} = \frac{2000 - (-800)}{2} = 1400 \text{ N-m} = 1400 \times 10^3 \text{ N-mm}$$

$\therefore$ Variable shear stress, $\tau_v = \dfrac{16\, T_v}{\pi d^3} = \dfrac{16 \times 1400 \times 10^3}{\pi (50)^3} = 57 \text{ N/mm}^2$

Since the endurance limit in reversed bending (σ_e) is taken as one-half the ultimate tensile strength (*i.e.* $\sigma_e = 0.5\, \sigma_u$) and the endurance limit in shear (τ_e) is taken as 0.55 σ_e, therefore

$$\tau_e = 0.55\, \sigma_e = 0.55 \times 0.5\, \sigma_u = 0.275\, \sigma_u$$
$$= 0.275 \times 630 = 173.25 \text{ N/mm}^2$$

Assume the yield stress (σ_y) for carbon steel in reversed bending as 510 N/mm^2, surface finish factor (K_{sur}) as 0.87, size factor (K_{sz}) as 0.85 and fatigue stress concentration factor (K_{fs}) as 1.

Top

Army Tank

Note : This picture is given as additional information and is not a direct example of the current chapter.

Since the yield stress in shear (τ_y) for shear loading is taken as one-half the yield stress in reversed bending (σ_y), therefore

$$\tau_y = 0.5\ \sigma_y = 0.5 \times 510 = 255 \text{ N/mm}^2$$

Let $F.S.$ = Factor of safety.

We know that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} = \frac{24.4}{255} + \frac{57 \times 1}{173.25 \times 0.87 \times 0.85}$$

$$= 0.096 + 0.445 = 0.541$$

$\therefore$ $F.S.$ = 1 / 0.541 = 1.85 **Ans.**

Example 6.10. *A cantilever beam made of cold drawn carbon steel of circular cross-section as shown in Fig. 6.18, is subjected to a load which varies from – F to 3 F. Determine the maximum load that this member can withstand for an indefinite life using a factor of safety as 2. The theoretical stress concentration factor is 1.42 and the notch sensitivity is 0.9. Assume the following values :*

Ultimate stress = 550 MPa

Yield stress = 470 MPa

Endurance limit = 275 MPa

Size factor = 0.85

Surface finish factor = 0.89

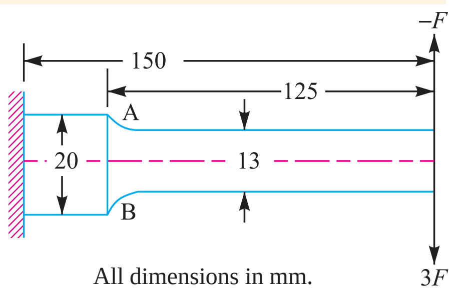

Fig. 6.18

Solution. Given : $W_{min} = -F$; $W_{max} = 3F$; $F.S. = 2$; $K_t = 1.42$; $q = 0.9$; $\sigma_u = 550$ MPa = 550 N/mm^2 ; $\sigma_y = 470$ MPa = 470 N/mm^2 ; $\sigma_e = 275$ MPa = 275 N/mm^2 ; $K_{sz} = 0.85$; $K_{sur} = 0.89$

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The beam as shown in Fig. 6.18 is subjected to a reversed bending load only. Since the point *A* at the change of cross section is critical, therefore we shall find the bending moment at point *A*.

We know that maximum bending moment at point *A*,

$$M_{max} = W_{max} \times 125 = 3F \times 125 = 375\ F \text{ N-mm}$$

and minimum bending moment at point *A*,

$$M_{min} = W_{min} \times 125 = -F \times 125 = -125\ F \text{ N-mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{375\ F + (-125\ F)}{2} = 125\ F \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{375\ F - (-125\ F)}{2} = 250\ F \text{ N-mm}$$

Section modulus,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(13)^3 = 215.7 \text{ mm}^3 \quad \ldots(\because d = 13 \text{ mm})$$

∴ Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{125\ F}{215.7} = 0.58\ F \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{250\ F}{215.7} = 1.16\ F \text{ N/mm}^2$$

Fatigue stress concentration factor, $K_f = 1 + q\ (K_t - 1) = 1 + 0.9\ (1.42 - 1) = 1.378$

We know that according to Goodman's formula

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{2} = \frac{0.58\ F}{550} + \frac{1.16\ F \times 1.378}{275 \times 0.89 \times 0.85}$$

$$= 0.001\ 05\ F + 0.007\ 68\ F = 0.008\ 73\ F$$

$$\therefore \quad F = \frac{1}{2 \times 0.008\ 73} = 57.3 \text{ N}$$

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{2} = \frac{0.58\ F}{470} + \frac{1.16\ F \times 1.378}{275 \times 0.89 \times 0.85}$$

$$= 0.001\ 23\ F + 0.007\ 68\ F = 0.008\ 91\ F$$

$$\therefore \quad F = \frac{1}{2 \times 0.008\ 91} = 56 \text{ N}$$

Taking larger of the two values, we have $F = 57.3$ N **Ans.**

Example 6.11. *A simply supported beam has a concentrated load at the centre which fluctuates from a value of P to 4 P. The span of the beam is 500 mm and its cross-section is circular with a diameter of 60 mm. Taking for the beam material an ultimate stress of 700 MPa, a yield stress of 500 MPa, endurance limit of 330 MPa for reversed bending, and a factor of safety of 1.3, calculate the maximum value of P. Take a size factor of 0.85 and a surface finish factor of 0.9.*

Solution. Given : $W_{min} = P$; $W_{max} = 4P$; $L = 500$ mm; $d = 60$ mm ; $\sigma_u = 700$ MPa $= 700$ N/mm^2 ; $\sigma_y = 500$ MPa $= 500$ N/mm^2 ; $\sigma_e = 330$ MPa $= 330$ N/mm^2 ; $F.S. = 1.3$; $K_{sz} = 0.85$; $K_{sur} = 0.9$

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We know that maximum bending moment,

$$M_{max} = \frac{W_{max} \times L}{4} = \frac{4P \times 500}{4} = 500\,P \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times L}{4} = \frac{P \times 500}{4} = 125\,P \text{ N-mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{500\,P + 125\,P}{2} = 312.5\,P \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{500\,P - 125\,P}{2} = 187.5\,P \text{ N-mm}$$

Section modulus, $Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(60)^3 = 21.21 \times 10^3 \text{ mm}^3$

∴ Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{312.5\,P}{21.21 \times 10^3} = 0.0147\,P \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{187.5\,P}{21.21 \times 10^3} = 0.0088\,P \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147\,P}{700} + \frac{0.0088\,P \times 1}{330 \times 0.9 \times 0.85} \quad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{21\,P}{10^6} + \frac{34.8\,P}{10^6} = \frac{55.8\,P}{10^6}$$

$$\therefore \quad P = \frac{1}{1.3} \times \frac{10^6}{55.8} = 13\,785 \text{ N} = 13.785 \text{ kN}$$

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147\,P}{500} + \frac{0.0088\,P \times 1}{330 \times 0.9 \times 0.85} = \frac{29.4\,P}{10^6} + \frac{34.8\,P}{10^6} = \frac{64.2\,P}{10^6}$$

$$\therefore \quad P = \frac{1}{1.3} \times \frac{10^6}{64.2} = 11\,982 \text{ N} = 11.982 \text{ kN}$$

From the above, we find that maximum value of P = 13.785 kN **Ans.**

6.22 Combined Variable Normal Stress and Variable Shear Stress

When a machine part is subjected to both variable normal stress and a variable shear stress; then it is designed by using the following two theories of combined stresses :

1. Maximum shear stress theory, and **2.** Maximum normal stress theory.

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We have discussed in Art. 6.21, that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}} \quad \text{...(For reversed bending load)}$$

Multiplying throughout by σ_y, we get

$$\frac{\sigma_y}{F.S.} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

The term on the right hand side of the above expression is known as ***equivalent normal stress*** due to reversed bending.

∴ Equivalent normal stress due to reversed bending,

$$\sigma_{neb} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}} \quad \text{...}(i)$$

Similarly, equivalent normal stress due to reversed axial loading,

$$\sigma_{nea} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fa}}{\sigma_{ea} \times K_{sur} \times K_{sz}} \quad \text{...}(ii)$$

and total equivalent normal stress,

$$\sigma_{ne} = \sigma_{neb} + \sigma_{nea} = \frac{\sigma_y}{F.S.} \quad \text{...}(iii)$$

We have also discussed in Art. 6.21, that for reversed torsional or shear loading,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

Multiplying throughout by τ_y, we get

$$\frac{\tau_y}{F.S.} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

The term on the right hand side of the above expression is known as ***equivalent shear stress.***

∴ Equivalent shear stress due to reversed torsional or shear loading,

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \quad \text{...}(iv)$$

The maximum shear stress theory is used in designing machine parts of ductile materials. According to this theory, maximum equivalent shear stress,

$$\tau_{es(max)} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2} = \frac{\tau_y}{F.S.}$$

The maximum normal stress theory is used in designing machine parts of brittle materials. According to this theory, maximum equivalent normal stress,

$$\sigma_{ne(max)} = \frac{1}{2}(\sigma_{ne}) + \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2} = \frac{\sigma_y}{F.S.}$$

Example 6.12. *A steel cantilever is 200 mm long. It is subjected to an axial load which varies from 150 N (compression) to 450 N (tension) and also a transverse load at its free end which varies from 80 N up to 120 N down. The cantilever is of circular cross-section. It is of diameter 2d for the first 50 mm and of diameter d for the remaining length. Determine its diameter taking a factor of safety of 2. Assume the following values :*

Yield stress	= *330 MPa*
Endurance limit in reversed loading	= *300 MPa*
Correction factors	= *0.7 in reversed axial loading*
	= *1.0 in reversed bending*

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Stress concentration factor	= *1.44 for bending*
	= *1.64 for axial loading*
Size effect factor	= *0.85*
Surface effect factor	= *0.90*
Notch sensitivity index	= *0.90*

Solution. Given : l = 200 mm; $W_{a(max)}$ = 450 N; $W_{a(min)}$ = – 150 N ; $W_{t(max)}$ = 120 N ; $W_{t(min)}$ = – 80 N; $F.S.$ = 2 ; σ_y = 330 MPa = 330 N/mm^2 ; σ_e = 300 MPa = 300 N/mm^2 ; K_a = 0.7; K_b = 1; K_{tb} = 1.44 ; K_{ta} = 1.64; K_{sz} = 0.85 ; K_{sur} = 0.90 ; q = 0.90

First of all, let us find the equivalent normal stress for point *A* which is critical as shown in Fig. 6.19. It is assumed that the equivalent normal stress at this point will be the algebraic sum of the equivalent normal stress due to axial loading and equivalent normal stress due to bending (*i.e.* due to transverse load acting at the free end).

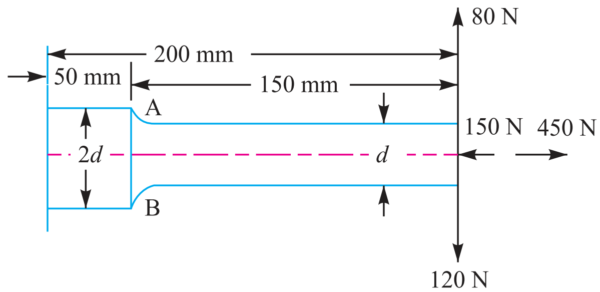

Fig. 6.19

Let us first consider the reversed axial loading. We know that mean or average axial load,

$$W_m = \frac{W_{a(max)} + W_{a(min)}}{2} = \frac{450 + (-150)}{2} = 150 \text{ N}$$

and variable axial load,

$$W_v = \frac{W_{a(max)} - W_{a(min)}}{2} = \frac{450 - (-150)}{2} = 300 \text{ N}$$

∴ Mean or average axial stress,

$$\sigma_m = \frac{W_m}{A} = \frac{150 \times 4}{\pi d^2} = \frac{191}{d^2} \text{ N/mm}^2 \qquad \ldots \left(\because A = \frac{\pi}{4} \times d^2\right)$$

and variable axial stress,

$$\sigma_v = \frac{W_v}{A} = \frac{300 \times 4}{\pi d^2} = \frac{382}{d^2} \text{ N/mm}^2$$

We know that fatigue stress concentration factor for reversed axial loading,

$$K_{fa} = 1 + q\,(K_{ta} - 1) = 1 + 0.9\,(1.64 - 1) = 1.576$$

and endurance limit stress for reversed axial loading,

$$\sigma_{ea} = \sigma_e \times K_a = 300 \times 0.7 = 210 \text{ N/mm}^2$$

We know that equivalent normal stress at point *A* due to axial loading,

$$\sigma_{nea} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fa}}{\sigma_{ea} \times K_{sur} \times K_{sz}} = \frac{191}{d^2} + \frac{382 \times 330 \times 1.576}{d^2 \times 210 \times 0.9 \times 0.85}$$

$$= \frac{191}{d^2} + \frac{1237}{d^2} = \frac{1428}{d^2} \text{ N/mm}^2$$

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Now let us consider the reversed bending due to transverse load. We know that mean or average bending load,

$$W_m = \frac{W_{t(max)} + W_{t(min)}}{2}$$

$$= \frac{120 + (-80)}{2} = 20 \text{ N}$$

and variable bending load,

$$W_v = \frac{W_{t(max)} - W_{t(min)}}{2}$$

$$= \frac{120 - (-80)}{2} = 100 \text{ N}$$

Machine transporter

∴ Mean bending moment at point *A*,

$$M_m = W_m (l - 50) = 20 (200 - 50) = 3000 \text{ N-mm}$$

and variable bending moment at point *A*,

$$M_v = W_v (l - 50) = 100 (200 - 50) = 15\,000 \text{ N-mm}$$

We know that section modulus,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

∴ Mean or average bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{3000}{0.0982\, d^3} = \frac{30\,550}{d^3} \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{15\,000}{0.0982\, d^3} = \frac{152\,750}{d^3} \text{ N/mm}^2$$

We know that fatigue stress concentration factor for reversed bending,

$$K_{fb} = 1 + q (K_{tb} - 1) = 1 + 0.9 (1.44 - 1) = 1.396$$

Since the correction factor for reversed bending load is 1 (*i.e.* $K_b = 1$), therefore the endurance limit for reversed bending load,

$$\sigma_{eb} = \sigma_e \,.\, K_b = \sigma_e = 300 \text{ N/mm}^2$$

We know that the equivalent normal stress at point *A* due to bending,

$$\sigma_{neb} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{30\,550}{d^3} + \frac{152\,750 \times 330 \times 1.396}{d^3 \times 300 \times 0.9 \times 0.85}$$

$$= \frac{30\,550}{d^3} + \frac{306\,618}{d^3} = \frac{337\,168}{d^3} \text{ N/mm}^2$$

∴ Total equivalent normal stress at point *A*,

$$\sigma_{ne} = \sigma_{neb} + \sigma_{nea} = \frac{337\,168}{d^3} + \frac{1428}{d^2} \text{ N/mm}^2 \quad ...(i)$$

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We know that equivalent normal stress at point A,

$$\sigma_{ne} = \frac{\sigma_y}{F.S.} = \frac{330}{2} = 165 \text{ N/mm}^2 \quad ...(ii)$$

Equating equations (i) and (ii), we have

$$\frac{337\,168}{d^3} + \frac{1428}{d^2} = 165 \quad \text{or} \quad 337\,168 + 1428\,d = 165\,d^3$$

$\therefore$ $236.1 + d = 0.116\,d^3$ or $d = 12.9$ mm **Ans.** ...(By hit and trial)

Example 6.13. *A hot rolled steel shaft is subjected to a torsional moment that varies from 330 N-m clockwise to 110 N-m counterclockwise and an applied bending moment at a critical section varies from 440 N-m to – 220 N-m. The shaft is of uniform cross-section and no keyway is present at the critical section. Determine the required shaft diameter. The material has an ultimate strength of 550 MN/m² and a yield strength of 410 MN/m². Take the endurance limit as half the ultimate strength, factor of safety of 2, size factor of 0.85 and a surface finish factor of 0.62.*

Solution. Given : $T_{max} = 330$ N-m (clockwise) ; $T_{min} = 110$ N-m (counterclockwise) = – 110 N-m (clockwise) ; $M_{max} = 440$ N-m ; $M_{min} = -220$ N-m ; $\sigma_u = 550$ MN/m² $= 550 \times 10^6$ N/m² ; $\sigma_y = 410$ MN/m² $= 410 \times 10^6$ N/m² ; $\sigma_e = \frac{1}{2}\sigma_u = 275 \times 10^6$ N/m² ; $F.S. = 2$; $K_{sz} = 0.85$; $K_{sur} = 0.62$

Let d = Required shaft diameter in metres.

We know that mean torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{330 + (-110)}{2} = 110 \text{ N-m}$$

and variable torque,

$$T_v = \frac{T_{max} - T_{min}}{2} = \frac{330 - (-110)}{2} = 220 \text{ N-m}$$

$\therefore$ Mean shear stress,

$$\tau_m = \frac{16\,T_m}{\pi\,d^3} = \frac{16 \times 110}{\pi\,d^3} = \frac{560}{d^3} \text{ N/m}^2$$

and variable shear stress,

$$\tau_v = \frac{16\,T_v}{\pi\,d^3} = \frac{16 \times 220}{\pi\,d^3} = \frac{1120}{d^3} \text{ N/m}^2$$

Since the endurance limit in shear (τ_e) is 0.55 σ_e, and yield strength in shear (τ_y) is 0.5 σ_y, therefore

$$\tau_e = 0.55 \times 275 \times 10^6 = 151.25 \times 10^6 \text{ N/m}^2$$

and

$$\tau_y = 0.5 \times 410 \times 10^6 = 205 \times 10^6 \text{ N/m}^2$$

We know that equivalent shear stress,

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y\ K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

$$= \frac{560}{d^3} + \frac{1120 \times 205 \times 10^6 \times 1}{d^3 \times 151.25 \times 10^6 \times 0.62 \times 0.85} \quad \text{...(Taking } K_{fs} = 1)$$

$$= \frac{560}{d^3} + \frac{2880}{d^3} = \frac{3440}{d^3} \text{ N/m}^2$$

Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{440 + (-220)}{2} = 110 \text{ N-m}$$

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and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{440 - (-220)}{2} = 330 \text{ N-m}$$

Section modulus, $Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ m}^3$

∴ Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{110}{0.0982\, d^3} = \frac{1120}{d^3} \text{ N/m}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{330}{0.0982\, d^3} = \frac{3360}{d^3} \text{ N/m}^2$$

Machine parts are often made of alloys to improve their mechanical properties.

Since there is no reversed axial loading, therefore the equivalent normal stress due to reversed bending load,

$$\sigma_{neb} = \sigma_{ne} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

$$= \frac{1120}{d^3} + \frac{3360 \times 410 \times 10^6 \times 1}{d^3 \times 275 \times 10^6 \times 0.62 \times 0.85}$$

...(Taking $K_{fb} = 1$ and $\sigma_{eb} = \sigma_e$)

$$= \frac{1120}{d^3} + \frac{9506}{d^3} = \frac{10\,626}{d^3} \text{ N/m}^2$$

We know that the maximum equivalent shear stress,

$$\tau_{es(max)} = \frac{\tau_y}{F.S.} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2}$$

$$\frac{205 \times 10^6}{2} = \frac{1}{2}\sqrt{\left(\frac{10\,625}{d^3}\right)^2 + 4\left(\frac{3440}{d^3}\right)^2}$$

$$205 \times 10^6 \times d^3 = \sqrt{113 \times 10^6 + 4 \times 11.84 \times 10^6} = 12.66 \times 10^3$$

$$\therefore \quad d^3 = \frac{12.66 \times 10^3}{205 \times 10^6} = \frac{0.0617}{10^3}$$

or $\quad d = \frac{0.395}{10} = 0.0395 \text{ m} = 39.5 \text{ say } 40 \text{ mm}$ **Ans.**

Example 6.14. *A pulley is keyed to a shaft midway between two bearings. The shaft is made of cold drawn steel for which the ultimate strength is 550 MPa and the yield strength is 400 MPa. The bending moment at the pulley varies from – 150 N-m to + 400 N-m as the torque on the shaft varies from – 50 N-m to + 150 N-m. Obtain the diameter of the shaft for an indefinite life. The stress concentration factors for the keyway at the pulley in bending and in torsion are 1.6 and 1.3 respectively. Take the following values:*

Factor of safety	*= 1.5*
Load correction factors	*= 1.0 in bending, and 0.6 in torsion*
Size effect factor	*= 0.85*
Surface effect factor	*= 0.88*

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Solution. Given : σ_u = 550 MPa = 550 N/mm^2 ; σ_y = 400 MPa = 400 N/mm^2 ; M_{min} = – 150 N-m; M_{max} = 400 N-m ; T_{min} = – 50 N-m ; T_{max} = 150 N-m ; K_{fb} = 1.6 ; K_{fs} = 1.3 ; *F.S.* = 1.5 ; K_b = 1 ; K_s = 0.6 ; K_{sz} = 0.85 ; K_{sur} = 0.88

Let d = Diameter of the shaft in mm.

First of all, let us find the equivalent normal stress due to bending.

We know that the mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{400 + (-150)}{2} = 125 \text{ N-m} = 125 \times 10^3 \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{400 - (-150)}{2} = 275 \text{ N-m} = 275 \times 10^3 \text{ N-mm}$$

Section modulus, $Z = \dfrac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$

∴ Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{125 \times 10^3}{0.0982\, d^3} = \frac{1273 \times 10^3}{d^3} \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{275 \times 10^3}{0.0982\, d^3} = \frac{2800 \times 10^3}{d^3} \text{ N/mm}^2$$

Assuming the endurance limit in reversed bending as one-half the ultimate strength and since the load correction factor for reversed bending is 1 (*i.e.* K_b = 1), therefore endurance limit in reversed bending,

$$\sigma_{eb} = \sigma_e = \frac{\sigma_u}{2} = \frac{550}{2} = 275 \text{ N/mm}^2$$

Since there is no reversed axial loading, therefore equivalent normal stress due to bending,

$$\sigma_{neb} = \sigma_{ne} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

$$= \frac{1273 \times 10^3}{d^3} + \frac{2800 \times 10^3 \times 400 \times 1.6}{d^3 \times 275 \times 0.88 \times 0.85}$$

$$= \frac{1273 \times 10^3}{d^3} + \frac{8712 \times 10^3}{d^3} = \frac{9985 \times 10^3}{d^3} \text{ N/mm}^2$$

Now let us find the equivalent shear stress due to torsional moment. We know that the mean torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{150 + (-50)}{2} = 50 \text{ N-m} = 50 \times 10^3 \text{ N-mm}$$

and variable torque, $T_v = \dfrac{T_{max} - T_{min}}{2} = \dfrac{150 - (-50)}{2} = 100 \text{ N-m} = 100 \times 10^3 \text{ N-mm}$

∴ Mean shear stress,

$$\tau_m = \frac{16\, T_m}{\pi d^3} = \frac{16 \times 50 \times 10^3}{\pi d^3} = \frac{255 \times 10^3}{d^3} \text{ N/mm}^2$$

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and variable shear stress,

$$\tau_v = \frac{16\,T_v}{\pi\,d^3} = \frac{16 \times 100 \times 10^3}{\pi\,d^3} = \frac{510 \times 10^3}{d^3} \text{ N/mm}^2$$

Endurance limit stress for reversed torsional or shear loading,

$$\tau_e = \sigma_e \times K_s = 275 \times 0.6 = 165 \text{ N/mm}^2$$

Assuming yield strength in shear,

$$\tau_y = 0.5\,\sigma_y = 0.5 \times 400 = 200 \text{ N/mm}^2$$

We know that equivalent shear stress,

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

$$= \frac{255 \times 10^3}{d^3} + \frac{510 \times 10^3 \times 200 \times 1.3}{d^3 \times 165 \times 0.88 \times 0.85}$$

$$= \frac{255 \times 10^3}{d^3} + \frac{1074 \times 10^3}{d^3} = \frac{1329 \times 10^3}{d^3} \text{ N/mm}^2$$

and maximum equivalent shear stress,

$$\tau_{es(max)} = \frac{\tau_y}{F.S.} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2}$$

$$\frac{200}{1.5} = \frac{1}{2}\sqrt{\left(\frac{9985 \times 10^3}{d^3}\right)^2 + 4\left(\frac{1329 \times 10^3}{d^3}\right)^2} = \frac{5165 \times 10^3}{d^3}$$

$$\therefore \quad d^3 = \frac{5165 \times 10^3 \times 1.5}{200} = 38\,740 \text{ or } d = 33.84 \text{ say } 35 \text{ mm}$$ **Ans.**

6.23 Application of Soderberg's Equation

We have seen in Art. 6.21 that according to Soderberg's equation,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e} \qquad \text{...}(i)$$

This equation may also be written as

$$\frac{1}{F.S.} = \frac{\sigma_m \times \sigma_e + \sigma_v \times \sigma_y \times K_f}{\sigma_y \times \sigma_e}$$

or

$$F.S. = \frac{\sigma_y \times \sigma_e}{\sigma_m \times \sigma_e + \sigma_v \times \sigma_y \times K_f} = \frac{\sigma_y}{\sigma_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \sigma_v} \qquad \text{...}(ii)$$

Since the factor of safety based on yield strength is the ratio of the yield point stress to the working or design stress, therefore from equation ***(ii)***, we may write

Working or design stress

$$= \sigma_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \sigma_v \qquad \text{...}(iii)$$

Let us now consider the use of Soderberg's equation to a ductile material under the following loading conditions.

1. *Axial loading*

In case of axial loading, we know that the mean or average stress,

$$\sigma_m = W_m / A$$

and variable stress, $\sigma_v = W_v / A$

where W_m = Mean or average load,

W_v = Variable load, and

A = Cross-sectional area.

The equation **(*iii*)** may now be written as follows :

Working or design stress,

$$= \frac{W_m}{A} + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \frac{W_v}{A} = \frac{W_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times W_v}{A}$$

$$\therefore \quad F.S. = \frac{\sigma_y \times A}{W_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times W_v}$$

A large disc- shaped electromagnet hangs from jib of this scrapyard crane. Steel and iron objects fly towards the magnet when the current is switched on. In this way, iron and steel can be separated for recycling.

Note : This picture is given as additional information and is not a direct example of the current chapter.

2. *Simple bending*

In case of simple bending, we know that the bending stress,

$$\sigma_b = \frac{M.y}{I} = \frac{M}{Z} \qquad \ldots \left(\because Z = \frac{I}{y}\right)$$

$\therefore$ Mean or average bending stress,

$$\sigma_m = M_m / Z$$

and variable bending stress,

$$\sigma_v = M_v / Z$$

where M_m = Mean bending moment,

M_v = Variable bending moment, and

Z = Section modulus.

The equation **(*iii*)** may now be written as follows :

Working or design bending stress,

$$\sigma_b = \frac{M_m}{Z} + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times \frac{M_v}{Z}$$

$$= \frac{M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v}{Z}$$

$$= \frac{32}{\pi d^3}\left[M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v\right] \qquad \ldots \left(\because \text{For circular shafts, } Z = \frac{\pi}{32} \times d^3\right)$$

$$\therefore \quad F.S. = \frac{\sigma_y}{\frac{32}{\pi d^3}\left[M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v\right]}$$

3. *Simple torsion of circular shafts*

In case of simple torsion, we know that the torque,

$$T = \frac{\pi}{16} \times \tau \times d^3 \text{ or } \tau = \frac{16\,T}{\pi\, d^3}$$

∴ Mean or average shear stress,

$$\tau_m = \frac{16\,T_m}{\pi\, d^3}$$

and variable shear stress, $\tau_v = \dfrac{16\,T_v}{\pi\, d^3}$

where T_m = Mean or average torque,

T_v = Variable torque, and

d = Diameter of the shaft.

The equation ***(iii)*** may now be written as follows :

Working or design shear stress,

$$\tau = \frac{16\,T_m}{\pi d^3} + \left(\frac{\tau_y}{\tau_e}\right) K_{fs} \times \frac{16\,T_v}{\pi d^3} = \frac{16}{\pi d^3}\left[T_m + \left(\frac{\tau_y}{\tau_e}\right) K_{fs} \times T_v\right]$$

$$\therefore \qquad F.S. = \frac{\tau_y}{\dfrac{16}{\pi d^3}\left[T_m + \left(\dfrac{\tau_y}{\tau_e}\right) K_{fs} \times T_v\right]}$$

where K_{fs} = Fatigue stress concentration factor for torsional or shear loading.

Note : For shafts made of ductile material, $\tau_y = 0.5\ \sigma_y$, and $\tau_e = 0.5\ \sigma_e$ may be taken.

4. *Combined bending and torsion of circular shafts*

In case of combined bending and torsion of circular shafts, the maximum shear stress theory may be used. According to this theory, maximum shear stress,

$$\tau_{max} = \frac{\tau_y}{F.S.} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{\left[\frac{32}{\pi d^3}\left\{M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v\right\}\right]^2 + 4\left[\frac{16}{\pi d^3}\left\{T_m + \left(\frac{\tau_y}{\tau_e}\right) K_{fs} \times T_v\right\}\right]^2}$$

$$= \frac{16}{\pi d^3}\sqrt{\left[M_m + \left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M_v\right]^2 + \left[T_m + \left(\frac{\tau_y}{\tau_e}\right) K_{fs} \times T_v\right]^2}$$

The majority of rotating shafts carry a steady torque and the loads remain fixed in space in both direction and magnitude. Thus during each revolution every fibre on the surface of the shaft undergoes a complete reversal of stress due to bending moment. Therefore for the usual case when $M_m = 0$, $M_v = M$, $T_m = T$ and $T_v = 0$, the above equation may be written as

$$\frac{\tau_y}{F.S.} = \frac{16}{\pi d^3}\sqrt{\left[\left(\frac{\sigma_y}{\sigma_e}\right) K_f \times M\right]^2 + T^2}$$

Note: The above relations apply to a solid shaft. For hollow shaft, the left hand side of the above equations must be multiplied by $(1 - k^4)$, where k is the ratio of inner diameter to outer diameter.

Example 6.15. *A centrifugal blower rotates at 600 r.p.m. A belt drive is used to connect the blower to a 15 kW and 1750 r.p.m. electric motor. The belt forces a torque of 250 N-m and a force of 2500 N on the shaft. Fig. 6.20 shows the location of bearings, the steps in the shaft and the plane in which the resultant belt force and torque act. The ratio of the journal diameter to the overhung shaft diameter is 1.2 and the radius of the fillet is 1/10th of overhung shaft diameter. Find the shaft diameter, journal diameter and radius of fillet to have a factor of safety 3. The blower shaft is to be machined from hot rolled steel having the following values of stresses:*

Endurance limit = 180 MPa; Yield point stress = 300 MPa; Ultimate tensile stress = 450 MPa.

Solution. Given: $^*N_B$ = 600 r.p.m. ; $^*P$ = 15 kW; $^*N_M$ = 1750 r.p.m. ; T = 250 N-m = 250 × 10^3 N-mm; F = 2500 N ; $F.S.$ = 3; σ_e = 180 MPa = 180 N/mm^2 ; σ_y = 300 MPa = 300 N/mm^2 ; σ_u = 450 MPa = 450 N/mm^2

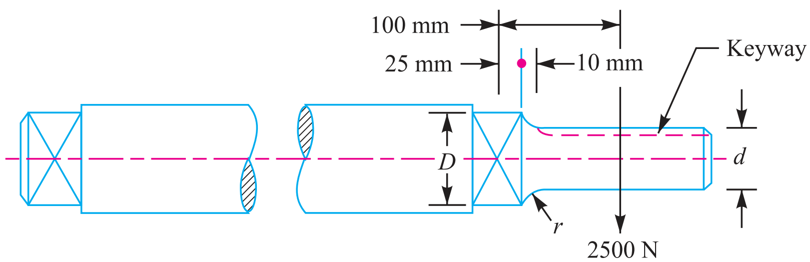

Fig. 6.20

Let D = Journal diameter,

d = Shaft diameter, and r = Fillet radius.

∴ Ratio of journal diameter to shaft diameter,

$$D/d = 1.2 \qquad \text{...(Given)}$$

and radius of the fillet, r = 1/10 × Shaft diameter (d) = 0.1 d

$$\therefore \quad r/d = 0.1 \qquad \text{...(Given)}$$

From Table 6.3, for D/d = 1.2 and r/d = 0.1, the theoretical stress concentration factor,

$$K_t = 1.62$$

The two points at which failure may occur are at the end of the keyway and at the shoulder fillet. The critical section will be the one with larger product of $K_f \times M$. Since the notch sensitivity factor q is dependent upon the unknown dimensions of the notch and since the curves for notch sensitivity factor (Fig. 6.14) are not applicable to keyways, therefore the product $K_t \times M$ shall be the basis of comparison for the two sections.

∴ Bending moment at the end of the keyway,

$$K_t \times M = 1.6 \times 2500\,[100 - (25 + 10)] = 260 \times 10^3 \text{ N-mm}$$

...(∵ K_t for key ways = 1.6)

and bending moment at the shoulder fillet,

$$K_t \times M = 1.62 \times 2500\,(100 - 25) = 303\ 750 \text{ N-mm}$$

Since $K_t \times M$ at the shoulder fillet is large, therefore considering the shoulder fillet as the critical section. We know that

$$\frac{\tau_y}{F.S.} = \frac{16}{\pi d^3}\sqrt{\left[\left(\frac{\sigma_y}{\sigma_e}\right)K_f \times M\right]^2 + T^2}$$

* Superfluous data

$$\frac{0.5 \times 300}{3} = \frac{16}{\pi d^3}\sqrt{\left[\left(\frac{300}{180} \times 303750\right)^2 + (250 \times 10^3)^2\right]}$$

... (Substituting, $\tau_y = 0.5\ \sigma_y$)

$$50 = \frac{16}{\pi d^3} \times 565 \times 10^3 = \frac{2877 \times 10^3}{d^3}$$

∴ $d^3 = 2877 \times 10^3/50 = 57\ 540$ or $d = 38.6$ say 40 mm **Ans.**

Note: Since r is known (because $r/d = 0.1$ or $r = 0.1d = 4$ mm), therefore from Fig. 6.14, the notch sensitivity factor (q) may be obtained. For $r = 4$ mm, we have $q = 0.93$.

∴ Fatigue stress concentration factor,

$$K_f = 1 + q\,(K_t - 1) = 1 + 0.93\,(1.62 - 1) = 1.58$$

Using this value of K_f instead of K_t, a new value of d may be calculated. We see that magnitudes of K_f and K_t are very close, therefore recalculation will not give any improvement in the results already obtained.

EXERCISES

1. A rectangular plate 50 mm × 10 mm with a hole 10 mm diameter is subjected to an axial load of 10 kN. Taking stress concentration into account, find the maximum stress induced. **[Ans. 50 MPa]**

2. A stepped shaft has maximum diameter 45 mm and minimum diameter 30 mm. The fillet radius is 6 mm. If the shaft is subjected to an axial load of 10 kN, find the maximum stress induced, taking stress concentration into account. **[Ans. 22 MPa]**

3. A leaf spring in an automobile is subjected to cyclic stresses. The average stress = 150 MPa; variable stress = 500 MPa; ultimate stress = 630 MPa; yield point stress = 350 MPa and endurance limit = 150 MPa. Estimate, under what factor of safety the spring is working, by Goodman and Soderberg formulae. **[Ans. 1.75, 1.3]**

4. Determine the design stress for bolts in a cylinder cover where the load is fluctuating due to gas pressure. The maximum load on the bolt is 50 kN and the minimum is 30 kN. The load is unpredictable and factor of safety is 3. The surface of the bolt is hot rolled and the surface finish factor is 0.9.

   During a simple tension test and rotating beam test on ductile materials (40 C 8 steel annealed), the following results were obtained :

   Diameter of specimen = 12.5 mm; Yield strength = 240 MPa; Ultimate strength = 450 MPa; Endurance limit = 180 MPa. **[Ans. 65.4 MPa]**

5. Determine the diameter of a tensile member of a circular cross-section. The following data is given :

   Maximum tensile load = 10 kN; Maximum compressive load = 5 kN; Ultimate tensile strength = 600 MPa; Yield point = 380 MPa; Endurance limit = 290 MPa; Factor of safety = 4; Stress concentration factor = 2.2 **[Ans. 24 mm]**

6. Determine the size of a piston rod subjected to a total load of having cyclic fluctuations from 15 kN in compression to 25 kN in tension. The endurance limit is 360 MPa and yield strength is 400 MPa. Take impact factor = 1.25, factor of safety = 1.5, surface finish factor = 0.88 and stress concentration factor = 2.25. **[Ans. 35.3 mm]**

7. A steel connecting rod is subjected to a completely reversed axial load of 160 kN. Suggest the suitable diameter of the rod using a factor of safety 2. The ultimate tensile strength of the material is 1100 MPa, and yield strength 930 MPa. Neglect column action and the effect of stress concentration. **[Ans. 30.4 mm]**

8. Find the diameter of a shaft made of 37 Mn 2 steel having the ultimate tensile strength as 600 MPa and yield stress as 440 MPa. The shaft is subjected to completely reversed axial load of 200 kN. Neglect stress concentration factor and assume surface finish factor as 0.8. The factor of safety may be taken as 1.5. **[Ans. 51.7 mm]**

9. Find the diameter of a shaft to transmit twisting moments varying from 800 N-m to 1600 N-m. The ultimate tensile strength for the material is 600 MPa and yield stress is 450 MPa. Assume the stress concentration factor = 1.2, surface finish factor = 0.8 and size factor = 0.85. **[Ans. 27.7 mm]**

10. A simply supported shaft between bearings carries a steady load of 10 kN at the centre. The length of shaft between bearings is 450 mm. Neglecting the effect of stress concentration, find the minimum diameter of shaft. Given that

Endurance limit = 600 MPa; surface finish factor = 0.87; size factor = 0.85; and factor of safety = 1.6. **[Ans. 35 mm]**

11. Determine the diameter of a circular rod made of ductile material with a fatigue strength (complete stress reversal) σ_e = 280 MPa and a tensile yield strength of 350 MPa. The member is subjected to a varying axial load from 700 kN to – 300 kN. Assume K_t = 1.8 and *F.S.* = 2. **[Ans. 80 mm]**

12. A cold drawn steel rod of circular cross-section is subjected to a variable bending moment of 565 N-m to 1130 N-m as the axial load varies from 4500 N to 13 500 N. The maximum bending moment occurs at the same instant that the axial load is maximum. Determine the required diameter of the rod for a factor of safety 2. Neglect any stress concentration and column effect. Assume the following values:

Ultimate strength	=	550 MPa
Yield strength	=	470 MPa
Size factor	=	0.85
Surface finish factor	=	0.89
Correction factors	=	1.0 for bending
	=	0.7 for axial load

The endurance limit in reversed bending may be taken as one-half the ultimate strength. **[Ans. 41 mm]**

13. A steel cantilever beam, as shown in Fig. 6.21, is subjected to a transverse load at its end that varies from 45 N up to 135 N down as the axial load varies from 110 N (compression) to 450 N (tension). Determine the required diameter at the change of section for infinite life using a factor of safety of 2. The strength properties are as follows:

Ultimate strength	=	550 MPa
Yield strength	=	470 MPa
Endurance limit	=	275 MPa

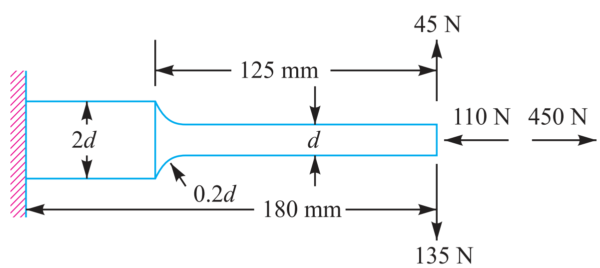

Fig. 6.21

The stress concentration factors for bending and axial loads are 1.44 and 1.63 respectively, at the change of cross-section. Take size factor = 0.85 and surface finish factor = 0.9. **[Ans. 12.5 mm]**

14. A steel shaft is subjected to completely reversed bending moment of 800 N-m and a cyclic twisting moment of 500 N-m which varies over a range of ± 40%. Determine the diameter of shaft if a reduction factor of 1.2 is applied to the variable component of bending stress and shearing stress. Assume

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(*a*) that the maximum bending and shearing stresses are in phase;

(*b*) that the tensile yield point is the limiting stress for steady state component;

(*c*) that the maximum shear strength theory can be applied; and

(*d*) that the Goodman relation is valid.

Take the following material properties:

Yield strength = 500 MPa ; Ultimate strength = 800 MPa ; Endurance limit = ± 400 MPa.

[Ans. 40 mm]

15. A pulley is keyed to a shaft midway between two anti-friction bearings. The bending moment at the pulley varies from – 170 N-m to 510 N-m and the torsional moment in the shaft varies from 55 N-m to 165 N-m. The frequency of the variation of the loads is the same as the shaft speed. The shaft is made of cold drawn steel having an ultimate strength of 540 MPa and a yield strength of 400 MPa. Determine the required diameter for an indefinite life. The stress concentration factor for the keyway in bending and torsion may be taken as 1.6 and 1.3 respectively. The factor of safety is 1.5. Take size factor = 0.85 and surface finish factor = 0.88. **[Ans. 36.5 mm]**

[**Hint.** Assume $\sigma_e = 0.5\ \sigma_u$; $\tau_y = 0.5\ \sigma_y$; $\tau_e = 0.55\ \sigma_e$]

QUESTIONS

1. Explain the following terms in connection with design of machine members subjected to variable loads:

(*a*) Endurance limit, (*b*) Size factor,

(*c*) Surface finish factor, and (*d*) Notch sensitivity.

2. What is meant by endurance strength of a material? How do the size and surface condition of a component and type of load affect such strength?

3. Write a note on the influence of various factors of the endurance limit of a ductile material.

4. What is meant by `stress concentration'? How do you take it into consideration in case of a component subjected to dynamic loading?

5. Illustrate how the stress concentration in a component can be reduced.

6. Explain how the factor of safety is determined under steady and varying loading by different methods.

7. Write Soderberg's equation and state its application to different type of loadings.

8. What information do you obtain from Soderberg diagram?

OBJECTIVE TYPE QUESTIONS

1. The stress which vary from a minimum value to a maximum value of the same nature (*i.e.* tensile or compressive) is called

(*a*) repeated stress (*b*) yield stress

(*c*) fluctuating stress (*d*) alternating stress

2. The endurance or fatigue limit is defined as the maximum value of the stress which a polished standard specimen can withstand without failure, for infinite number of cycles, when subjected to

(*a*) static load (*b*) dynamic load

(*c*) static as well as dynamic load (*d*) completely reversed load

3. Failure of a material is called fatigue when it fails

(*a*) at the elastic limit (*b*) below the elastic limit

(*c*) at the yield point (*d*) below the yield point

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4. The resistance to fatigue of a material is measured by
   (*a*) elastic limit (*b*) Young's modulus
   (*c*) ultimate tensile strength (*d*) endurance limit
5. The yield point in static loading is .... as compared to fatigue loading.
   (*a*) higher (*b*) lower (*c*) same
6. Factor of safety for fatigue loading is the ratio of
   (*a*) elastic limit to the working stress
   (*b*) Young's modulus to the ultimate tensile strength
   (*c*) endurance limit to the working stress
   (*d*) elastic limit to the yield point
7. When a material is subjected to fatigue loading, the ratio of the endurance limit to the ultimate tensile strength is
   (*a*) 0.20 (*b*) 0.35
   (*c*) 0.50 (*d*) 0.65
8. The ratio of endurance limit in shear to the endurance limit in flexure is
   (*a*) 0.25 (*b*) 0.40
   (*c*) 0.55 (*d*) 0.70
9. If the size of a standard specimen for a fatigue testing machine is increased, the endurance limit for the material will
   (*a*) have same value as that of standard specimen (*b*) increase (*c*) decrease
10. The residential compressive stress by way of surface treatment of a machine member subjected to fatigue loading
    (*a*) improves the fatigue life (*b*) deteriorates the fatigue life
    (*c*) does not affect the fatigue life (*d*) immediately fractures the specimen
11. The surface finish factor for a mirror polished material is
    (*a*) 0.45 (*b*) 0.65
    (*c*) 0.85 (*d*) 1
12. Stress concentration factor is defined as the ratio of
    (*a*) maximum stress to the endurance limit (*b*) nominal stress to the endurance limit
    (*c*) maximum stress to the nominal stress (*d*) nominal stress to the maximum stress
13. In static loading, stress concentration is more serious in
    (*a*) brittle materials (*b*) ductile materials
    (*c*) brittle as well as ductile materials (*d*) elastic materials
14. In cyclic loading, stress concentration is more serious in
    (*a*) brittle materials (*b*) ductile materials
    (*c*) brittle as well as ductile materials (*d*) elastic materials
15. The notch sensitivity q is expressed in terms of fatigue stress concentration factor K_f and theoretical stress concentration factor K_t, as

    (*a*) $\dfrac{K_f + 1}{K_t + 1}$ (*b*) $\dfrac{K_f - 1}{K_t - 1}$

    (*c*) $\dfrac{K_t + 1}{K_f + 1}$ (*d*) $\dfrac{K_t - 1}{K_f - 1}$

ANSWERS

1. (*c*)	**2.** (*d*)	**3.** (*d*)	**4.** (*d*)	**5.** (*a*)
6. (*c*)	**7.** (*c*)	**8.** (*c*)	**9.** (*c*)	**10.** (*a*)
11. (*d*)	**12.** (*c*)	**13.** (*a*)	**14.** (*b*)	**15.** (*b*)

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