1 Physical Properties of Fluids

1.0 INTRODUCTION

The flow of ideal non-viscous fluids was extensively studied and mathematical theories were developed during the last century. The field of study was called as 'Hydrodynamics'. However the results of mathematical analysis could not be applied directly to the flow of real fluids. Experiments with water flow resulted in the formulation of empirical equations applicable to engineering designs. The field was called Hydraulics. Due to the development of industries there arose a need for the study of fluids other than water. Theories like boundary layer theory were developed which could be applied to all types of real fluids, under various conditions of flow. The combination of experiments, the mathematical analysis of hydrodynamics and the new theories is known as 'Fluid Mechanics'. **Fluid Mechanics encompasses the study of all types of fluids under static, kinematic and dynamic conditions.**

The study of properties of fluids is basic for the understanding of flow or static condition of fluids. The important properties are **density, viscosity, surface tension, bulk modulus and vapour pressure.** Viscosity causes resistance to flow. Surface tension leads to capillary effects. Bulk modulus is involved in the propagation of disturbances like sound waves in fluids. Vapour pressure can cause flow disturbances due to evaporation at locations of low pressure. It plays an important role in cavitation studies in fluid machinery.

In this chapter various properties of fluids are discussed in detail, with stress on their effect on flow. Fairly elaborate treatment is attempted due to their importance in engineering applications. The basic laws used in the discussions are :

(*i*) Newton's laws of motion,

(*ii*) Laws of conservation of mass and energy,

(*iii*) Laws of Thermodynamics, and

(*iv*) Newton's law of viscosity.

A fluid is defined as a material which will continue to deform with the application of shear force however small the force may be.

1.1 THREE PHASES OF MATTER

Generally matter exists in three phases namely (*i*) Solid (*ii*) Liquid and (*iii*) Gas (includes vapour). The last two together are also called by the common term **fluids**.

In solids atoms/molecules are closely spaced and the attractive (cohesive) forces between atoms/molecules is high. The shape is maintained by the cohesive forces binding the atoms. When an external force is applied on a solid component, slight rearrangement in atomic positions balances the force. Depending upon the nature of force the solid may elongate or shorten or bend. When the applied force is removed the atoms move back to the original position and the former shape is regained. Only when the forces exceed a certain value (yield), a small deformation called plastic deformation will be retained as the atoms are unable to move to their original positions. When the force exceeds a still higher value (ultimate), the cohesive forces are not adequate to resist the applied force and the component will break.

In liquids the inter molecular distances are longer and the cohesive forces are of smaller in magnitude. The molecules are not bound rigidly as in solids and can move randomly. However, the cohesive forces are large enough to hold the molecules together below a free surface that forms in the container. Liquids will continue to deform when a shear or tangential force is applied. The deformation continues as long as the force exists. In fluids the **rate of deformation** controls the force (not deformation as in solids). More popularly it is stated that a fluid (liquid) cannot withstand applied shear force and will continue to deform. When at rest liquids will assume the shape of the container forming a free surface at the top.

In gases the distance between molecules is much larger compared to atomic dimensions and the cohesive force between atoms/molecules is low. So gas molecules move freely and fill the full volume of the container. If the container is open the molecules will diffuse to the outside. Gases also cannot withstand shear. The **rate of deformation** is proportional to the applied force as in the case of liquids.

Liquids and gases together are classified as fluids. Vapour is gaseous state near the evaporation temperature. The state in which a material exists depends on the pressure and temperature. For example, steel at atmospheric temperature exists in the solid state. At higher temperatures it can be liquefied. At still higher temperatures it will exist as a vapour.

A fourth state of matter is its existence as charged particles or ions known as plasma. This is encountered in MHD power generation. This phase is not considered in the text.

1.2 COMPRESSIBLE AND INCOMPRESSIBLE FLUIDS

If the density of a fluid varies significantly due to moderate changes in pressure or temperature, then the fluid is called compressible fluid. Generally gases and vapours under normal conditions can be classified as compressible fluids. In these phases the distance between atoms or molecules is large and cohesive forces are small. So increase in pressure or temperature will change the density by a significant value.

If the change in density of a fluid is small due to changes in temperature and or pressure, then the fluid is called incompressible fluid. All liquids are classified under this category.

When the change in pressure and temperature is small, gases and vapours are treated as incompressible fluids. For certain applications like propagation of pressure disturbances, liquids should be considered as compressible.

In this chapter some of the properties relevant to fluid mechanics are discussed with a view to bring out their influence on the design and operation of fluid machinery and equipments.

1.3 DIMENSIONS AND UNITS

It is necessary to distinguish clearly between the terms "Units" and "Dimensions". The word "dimension" is used to describe basic concepts like mass, length, time, temperature and force. "Large mass, long distance, high temperature" does not mean much in terms of visualising the quantity. Dimension merely describes the concept and does not provide any method for the quantitative expression of the same. Units are the means of expressing the value of the dimension quantitatively or numerically The term "second" for example is used to quantify time. "Ten seconds elapsed between starting and ending of an act" is the way of expressing the elapsed time in numerical form. The value of dimension should be expressed in terms of units before any quantitative assessment can be made.

There are three widely used systems of units in the world. These are (1) British or English system (it is not in official use now in Briton) (2) Metric system and (3) SI system (System International d'Unites or International System of Units). India has passed through the first two systems in that order and has now adopted the SI system of units.

The basic units required in Fluid Mechanics are for mass, length, time and temperature. These are kilogram (**kg**), metre (**m**), second (**s**) and kelvin (**K**). The unit of force is defined using Newton's second law of motion which states that applied force is proportional to the time rate of change of momentum of the body on which the force acts.

For a given mass **m**, subjected to the action of a force **F**, resulting in an acceleration **a**, Newton's law can be written in the form

$$\mathbf{F = (1/g_o)\ m\ a} \tag{1.3.1}$$

where g_o is a dimensional constant whose numerical value and units depend on those selected for **force, F, mass, m, and acceleration, a**. The unit of force is newton (N) in the SI system.

One newton is defined as the force which acting on a mass of one kilogram will produce an acceleration of 1 m/s^2. This leads to the relation

$$\mathbf{1\ N = (1/g_o) \times 1\ kg \times 1\ m/s^2} \tag{1.3.2}$$

Hence
$$\mathbf{g_o = 1\ kg\ m/N\ s^2} \tag{1.3.3}$$

The numerical value of g_o is unity (1) in the SI system and this is found advantageous in numerical calculations. However this constant should necessarily be used to obtain dimensional homogeneity in equations.

In metric system the unit of force is kg$_f$ defined as the force acted on one kg mass by standard gravitational acceleration taken as 9.81 m/s^2. The value of g_o is 9.81 kg m/kg$_f$s^2.

In the English system the unit of force is lb$_f$ defined as the force on one lb mass due to standard gravitational acceleration of 32.2 ft/s^2.

The value of g_o is 32.2 ft lb/lb$_f$s^2.

Some of the units used in this text are listed in the table below:

Quantity	Unit symbol	Derived units
mass	kg	ton (tonne) = 1000 kg
time	s	min (60s), hr (3600s)
length	m	mm, cm, km
temperature	K, (273 + °C)	°C
force	N (newton)	kN, MN (10^6 N)
energy, work, heat	Nm, J	kJ, MJ, kNm
power	W = (Nm/s, J/s)	kW, MW
pressure	N/m^2, (pascal, pa)	kPa, MPa, bar (10^5Pa)

Conversion constants between the metric and SI system of units are tabulated elsewhere in the text.

1.4 CONTINUUM

As gas molecules are far apart from each other and as there is empty space between molecules doubt arises as to whether a gas volume can be considered as a continuous matter like a solid for situations similar to application of forces.

Under normal pressure and temperature levels, gases are considered as a continuum (*i.e.,* as if no empty spaces exist between atoms). The test for continuum is to measure properties like density by sampling at different locations and also reducing the sampling volume to low levels. If the property is constant irrespective of the location and size of sample volume, then the gas body can be considered as a continuum for purposes of mechanics (application of force, consideration of acceleration, velocity etc.) and for the gas volume to be considered as a single body or entity. This is a very important test for the application of all laws of mechanics to a gas volume as a whole. When the pressure is extremely low, and when there are only few molecules in a cubic metre of volume, then the laws of mechanics should be applied to the molecules as entities and not to the gas body as a whole. In this text, only systems satisfying continuum requirements are discussed.

1.5 DEFINITION OF SOME COMMON TERMINOLOGY

***Density (mass density)*:** The mass per unit volume is defined as density. The unit used is kg/m^3. The measurement is simple in the case of solids and liquids. In the case of gases and vapours it is rather involved. The symbol used is ρ. The characteristic equation for gases provides a means to estimate the density from the measurement of pressure, temperature and volume.

***Specific Volume*:** The volume occupied by unit mass is called the specific volume of the material. The symbol used is v, the unit being m^3/kg. Specific volume is the reciprocal of density.

In the case of solids and liquids, the change in density or specific volume with changes in pressure and temperature is rather small, whereas in the case of gases and vapours, density will change significantly due to changes in pressure and/or temperature.

***Weight Density or Specific Weight*:** The force due to gravity on the mass in unit volume is defined as Weight Density or Specific Weight. The unit used is N/m^3. The symbol used is γ. At a location where g is the local acceleration due to gravity,

$$\textbf{Specific weight, } \gamma = g\,\rho \qquad (1.5.1)$$

In the above equation direct substitution of dimensions will show apparent non-homogeneity as the dimensions on the LHS and RHS will not be the same. On the LHS the dimension will be N/m^3 but on the RHS it is kg/m^2 s^2. The use of g_o will clear this anomaly. As seen in section 1.1, g_o = 1 kg m/N s^2. The RHS of the equation 1.3.1 when divided by g_o will lead to perfect dimensional homogeneity. The equation should preferably be written as,

$$\textbf{Specific weight, } \gamma = (g/g_o)\,\rho \qquad (1.5.2)$$

Since newton (N) is defined as the force required to accelerate 1 kg of mass by 1/s^2, it can also be expressed as kg.m/s^2. Density can also be expressed as Ns^2/m^4 (as kg = Ns^2/m). Beam balances compare the mass while spring balances compare the weights. The mass is the same (invariant) irrespective of location but the weight will vary according to the local gravitational constant. Density will be invariant while specific weight will vary with variations in gravitational acceleration.

***Specific Gravity or Relative Density*:** The ratio of the density of the fluid to the density of water—usually 1000 kg/m^3 at a standard condition—is defined as Specific Gravity or Relative Density δ of fluids. This is a ratio and hence no dimension or unit is involved.

Example 1.1. *The weight of an object measured on ground level where g_e = 9.81 m/s^2 is 35,000 N. Calculate its weight at the following locations (i) Moon, g_m = 1.62 m/s^2 (ii) Sun, g_s = 274.68 m/s^2 (iii) Mercury, g_{me} = 3.53 m/s^2 (iv) Jupiter, g_j = 26.0 m/s^2 (v) Saturn, g_{sa} = 11.2 m/s^2 and (vi) Venus, g_v = 8.54 m/s^2.*

Mass of the object, m_e = weight × (g_o/g) = 35,000 × (1/9.81) = 3567.8 kg

Weight of the object on a planet, $p = m_e \times (g_p/g_o)$ where m_e is the mass on earth, g_p is gravity on the planet and g_o has the usual meaning, force conversion constant.

Hence the weight of the given object on,

(*i*)	Moon	=	3567.8 × 1.62	= 5,780 N
(*ii*)	Sun	=	3567.8 × 274.68	= 9,80,000 N
(*iii*)	Mercury	=	3567.8 × 3.53	= 12,594 N
(*iv*)	Jupiter	=	3567.8 × 26.0	= 92,762 N
(*v*)	Saturn	=	3567.8 × 11.2	= 39,959 N
(*vi*)	Venus	=	3567.8 × 8.54	= 30,469 N

Note that the mass is constant whereas the weight varies directly with the gravitational constant. Also note that the ratio of weights will be the same as the ratio of gravity values.

1.6 VAPOUR AND GAS

When a liquid is heated under a constant pressure, first its temperature rises to the boiling point (defined as saturation temperature). Then the liquid begins to change its phase to the gaseous condition, with molecules escaping from the surface due to higher thermal energy level. **When the gas phase is in contact with the liquid or its temperature is near the saturation condition it is termed as vapour.**

Vapour is in gaseous condition but it does not follow the gas laws. Its specific heats will vary significantly. Moderate changes in temperature may change its phase to the liquid state.

When the temperature is well above the saturation temperature, vapour begins to behave as a gas. It will also obey the characteristic equation for gases. Then the specific heat will be nearly constant.

1.7 CHARACTERISTIC EQUATION FOR GASES

The characteristic equation for gases can be derived from Boyle's law and Charles' law. Boyle's law states that at constant temperature the volume of a gas body will vary inversely with pressure. Charles' law states that at constant pressure, the temperature will vary inversely with volume. Combining these two, the characteristic equation for a system containing **m** kg of a gas can be obtained as

$$\mathbf{PV = mRT} \tag{1.7.1}$$

This equation when applied to a given system leads to the relation 1.7.2 applicable for all equilibrium conditions irrespective of the process between the states.

$$\mathbf{(P_1V_1/T_1) = (P_2V_2/T_2) = (P_3\,V_3/T_3) = (PV/T) = Constant} \tag{1.7.2}$$

In the SI system, the units to be used in the equation are Pressure, $P \rightarrow$ N/m^2, volume, $V \rightarrow$ m^3, mass, m $\rightarrow$ kg, temperature, $T \rightarrow$ K and gas constant, $R \rightarrow$ Nm/kgK or J/kgK (Note: K = (273 + °C), J = Nm).

This equation defines the equilibrium state for any gas body. For a specified gas body with mass **m,** if two properties like **P, V** are specified then the third property **T** is automatically specified by this equation. The equation can also be written as,

$$\mathbf{Pv = RT} \tag{1.7.3}$$

where $v = V$/m or specific volume. **The value for R for air is 287 J/kgK.**

Application of Avagadro's hypothesis leads to the definition of a new volume measure called molal volume. This is the volume occupied by the molecular mass of any gas at standard temperature and pressure. This volume as per the above hypothesis will be the same for all gases at any given temperature and pressure. Denoting this volume as V_m and the pressure as P and the temperature as T,

For a gas a, $$\mathbf{PV_m = M_a\,R_a\,T} \tag{1.7.4}$$

For a gas b, $$\mathbf{PV_m = M_b\,R_b\,T} \tag{1.7.5}$$

As P, T and V_m are the same in both cases.

$$\mathbf{M_aR_a = M_bR_b = M \times R = Constant} \tag{1.7.6}$$

The product **M × R** is called **Universal gas constant** and is denoted by the symbol $\mathfrak{R}$. Its numerical value in SI system is **8314 J/kg mole K**. For any gas the value of gas constant R is obtained by dividing universal gas constant by the molecular mass in kg of that gas. The gas constant R for any gas (in the SI system, J/kg K) can be calculated using,

$$\mathbf{R = 8314/M} \tag{1.7.7}$$

The characteristic equation for gases can be applied for all gases with slight approximations, and for practical calculations this equation is used in all cases.

Example 1.2. *A balloon is filled with 6 kg of hydrogen at 2 bar and 20°C.* ***What will be the diameter of the balloon*** *when it reaches an altitude where the pressure and temperature are 0.2 bar and –60° C. Assume that the pressure and temperature inside are the same as that at the outside at this altitude.*

The characteristic equation for gases $PV = mRT$ is used to calculate the initial volume,

$V_1 = [(m\,RT_1)/P_1]$, For hydrogen, molecular mass = 2, and so

$R_H = 8314/2 = 4157$ J/kgK, $\therefore$ $V_1 = 6 \times 4157 \times (273 + 20)/2 \times 10^5 = 36.54$ m^3

Using the general gas equation the volume after the balloon has reached the altitude, V_2 is calculated. $[(P_1V_1)/T_1] = [(P_2V_2)/T_2]$

$[(2 \times 10^5 \times 36.54)/(273+20)] = [(0.2) \times 10^5 \times V_2)/(273 - 60)]$ solving,

$V_2 =$ **265.63 m^3**, Considering the shape of the balloon as a sphere of radius r,

Volume = $(4/3)\,\pi\, r^3 = 265.63$ m^3, solving

Radius, r = 3.99 m and **diameter of the balloon = 7.98 m**

(The pressure inside the balloon should be slightly higher to overcome the stress in the wall material)

1.8 VISCOSITY

A fluid is defined as a material which will continue to deform with the application of a shear force. However, different fluids deform at different rates when the same shear stress (force/area) is applied.

Viscosity is that property of a real fluid by virtue of which it offers resistance to shear force. Referring to Fig. 1.8.1, it may be noted that a force is required to move one layer of fluid over another.

For a given fluid the force required varies directly as the rate of deformation. As the rate of deformation increases the force required also increases. This is shown in Fig. 1.8.1 (*i*).

The force required to cause the same rate of movement depends on the nature of the fluid. The resistance offered for the same rate of deformation varies directly as the viscosity of the fluid. As viscosity increases the force required to cause the same rate of deformation increases. This is shown in Fig. 1.8.1 (*ii*).

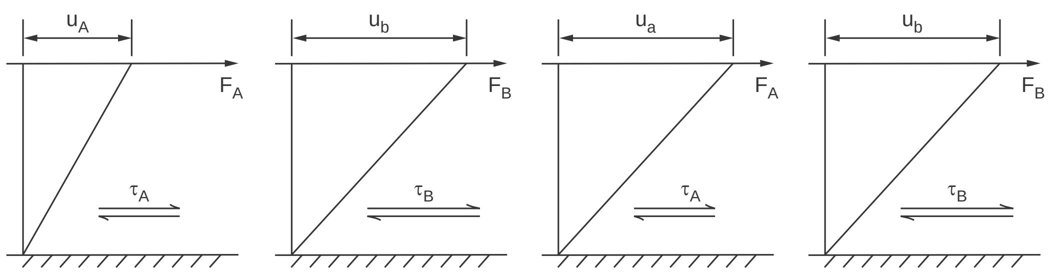
Newton's law of viscosity states that the shear force to be applied for a deformation rate of (du/dy) over an area A is given by,

$$\mathbf{F = \mu A (du/dy)} \tag{1.8.1}$$

or

$$\mathbf{(F/A) = \tau = \mu (du/dy) = \mu (u/y)} \tag{1.8.2}$$

where F is the applied force in N, A is area in m^2, du/dy is the velocity gradient (or rate of deformation), 1/s, perpendicular to flow direction, here assumed linear, and μ is the proportionality constant defined as the **dynamic or absolute viscosity** of the fluid.



$u_b > u_a$, $F_b > F_a$ $\mu_a = \mu_b$
(*i*) same fluid

$u_a = u_b$, $\mu_a < \mu_b$, $F_b > F_a$
(*ii*) same velocity

Figure 1.8.1 Concept of viscosity

The dimensions for dynamic viscosity μ can be obtained from the definition as Ns/m^2 or kg/ms. The first dimension set is more advantageously used in engineering problems. However, if the dimension of N is substituted, then the second dimension set, more popularly used by scientists can be obtained. The numerical value in both cases will be the same.

$$N = \text{kg m/s}^2 \ ; \ \mu = (\text{kg m/s}^2)\ (\text{s/m}^2) = \text{kg/ms}$$

The popular unit for viscosity is Poise named in honour of Poiseuille.

$$\textbf{Poise = 0.1 Ns/m}^2 \qquad (1.8.3)$$

Centipoise (cP) is also used more frequently as,

$$\textbf{cP = 0.001 Ns/m}^2 \qquad (1.8.3a)$$

For water the viscosity at 20°C is nearly 1 cP. The ratio of dynamic viscosity to the density is defined as kinematic viscosity, ν, having a dimension of m^2/s. Later it will be seen to relate to momentum transfer. Because of this kinematic viscosity is also called momentum diffusivity. The popular unit used is stokes (in honour of the scientist Stokes). Centistoke is also often used.

$$\textbf{1 stoke = 1 cm}^2\textbf{/s = 10}^{-4}\ \textbf{m}^2\textbf{/s} \qquad (1.8.3b)$$

Of all the fluid properties, viscosity plays a very important role in fluid flow problems. The velocity distribution in flow, the flow resistance etc. are directly controlled by viscosity. In the study of fluid statics (*i.e.,* when fluid is at rest), viscosity and shear force are not generally involved. In this chapter problems are worked assuming linear variation of velocity in the fluid filling the clearance space between surfaces with relative movement.

Example 1.3. *The space between two large inclined parallel planes is 6mm and is filled with a fluid. The planes are inclined at 30° to the horizontal. A small thin square plate of 100 mm side slides freely down parallel and midway between the inclined planes with a constant velocity of 3 m/s due to its weight of 2N.* ***Determine the viscosity of the fluid.***

The vertical force of 2 N due to the weight of the plate can be resolved along and perpendicular to the inclined plane. The force along the inclined plane is equal to the drag force on both sides of the plane due to the viscosity of the oil.

Force due to the weight of the sliding plane along the direction of motion

= 2 sin 30 = 1N

Viscous force, F = (A × 2) × μ × (du/dy) (both sides of plate). Substituting the values,

$$1 = \mu \times [(0.1 \times 0.1 \times 2)] \times [(3 - 0)/6/(2 \times 1000)\}]$$

Solving for viscosity, **μ = 0.05 Ns/m² or 0.5 Poise**

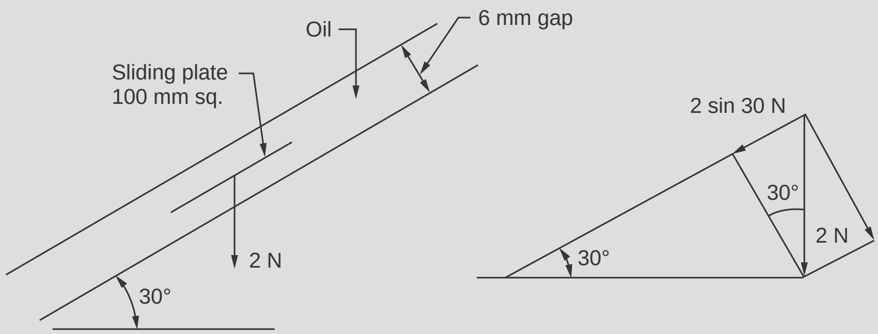


Figure Ex. 1.3

Example 1.4. *The velocity of the fluid filling a hollow cylinder of radius 0.1 m varies as u = 10 [1 – (r/0.1)²] m/s along the radius r. The viscosity of the fluid is 0.018 Ns/m². For 2 m length of the cylinder,* ***determine the shear stress and shear force over cylindrical layers of fluid*** *at r = 0 (centre line), 0.02, 0.04, 0.06 0.08 and 0.1 m (wall surface.)*

Shear stress = μ (du/dy) or μ (du/dr), $u = 10\ [1 - (r/0.1)^2]$ m/s

$\therefore \quad du/dr = 10\ (-2r/0.1^2) = -2000\ r$

The – ve sign indicates that the force acts in a direction opposite to the direction of velocity, u.
Shear stress = 0.018 × 2000 r = 36 rN/m²

Shear force over 2 m length = shear stress × area over 2m

$$= 36r \times 2\pi r L = 72\ \pi r^2 \times 2 = 144\ \pi r^2$$

The calculated values are tabulated below:

Radius, m	Shear stress, N/m²	Shear force, N	Velocity, m/s
0.00	0.00	0.00	0.00
0.02	0.72	0.18	9.60
0.04	1.44	0.72	8.40
0.06	2.16	1.63	6.40
0.08	2.88	2.90	3.60
0.10	3.60	4.52	0.00

Example 1.5. *The 8 mm gap between two large vertical parallel plane surfaces is filled with a liquid of dynamic viscosity 2 × 10⁻² Ns/m². A thin sheet of 1 mm thickness and 150 mm × 150 mm size, when dropped vertically between the two plates attains a steady velocity of 4 m/s.* ***Determine weight of the plate.*** *Assume that the plate moves centrally.*

$$F = \tau\,(A \times 2) = \mu \times (du/dy)\,(A \times 2) = \text{weight of the plate.}$$

Substituting the values, $dy = [(8 - 1)/(2 \times 1000)]$ m and du = 4 m/s

$$F = 2 \times 10^{-2}\ [4/\{(8 - 1)/(2 \times 1000)\}]\ [0.15 \times 0.15 \times 2] = \textbf{1.02 N (weight of the plate)}$$

Example 1.6. ***Determine the resistance offered to the downward sliding*** *of a shaft of 400 mm dia and 0.1 m length by the oil film between the shaft and a bearing of ID 402 mm. The kinematic viscosity is 2.4×10^{-4} m²/s and density is 900 kg/m³. The shaft is to move centrally and axially at a constant velocity of 0.1 m/s.*

Force, F opposing the movement of the shaft = shear stress × area

$$F = \mu\ (du/dy)\ (\ \pi \times D \times L\ )$$

$$\mu = 2.4 \times 10^{-4} \times 900 \text{ Ns/m}^2,\ du = 0.1 \text{ m/s},\ L = 0.1 \text{ m},\ D = 0.4 \text{ m}$$

$$dy = (402 - 400)/(2 \times 1000)\text{m, Substituting,}$$

$$F = 2.4 \times 10^{-4} \times 900 \times \{(0.1 - 0)/[(402 - 400)/\ (2 \times 1000)]\}\ (\ \pi \times 0.4 \times 0.1) = \mathbf{2714\ N}$$

1.8.1 Newtonian and Non Newtonian Fluids

An ideal fluid has zero viscosity. Shear force is not involved in its deformation. An ideal fluid has to be also incompressible. Shear stress is zero irrespective of the value of du/dy. Bernoulli equation can be used to analyse the flow.

Real fluids having viscosity are divided into two groups namely Newtonian and non Newtonian fluids. In Newtonian fluids a linear relationship exists between the magnitude of the applied shear stress and the resulting rate of deformation. It means that the proportionality parameter (in equation 1.8.2, $\tau = \mu\ (du/dy)$), viscosity, μ is constant in the case of Newtonian fluids (other conditions and parameters remaining the same). The viscosity at any given temperature and pressure is constant for a Newtonian fluid and is independent of the rate of deformation. The characteristics is shown plotted in Fig. 1.8.2. Two different plots are shown as different authors use different representations.

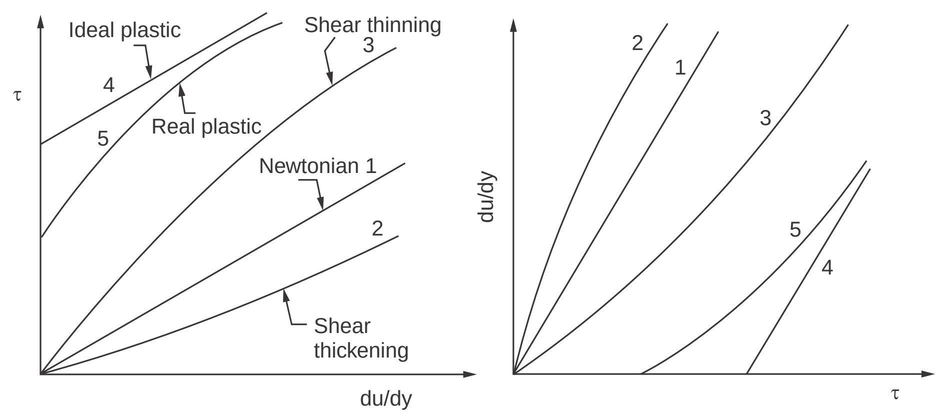


Figure 1.8.2 Rheological behaviour of fluids

Non Newtonian fluids can be further classified as simple non Newtonian, ideal plastic and shear thinning, shear thickening and real plastic fluids. In non Newtonian fluids the viscosity will vary with variation in the rate of deformation. Linear relationship between shear stress and rate of deformation (du/dy) does not exist. In plastics, up to a certain value of applied shear stress there is no flow. After this limit it has a constant viscosity at any given temperature. In shear thickening materials, the viscosity will increase with (du/dy) deformation rate. In shear thinning materials viscosity will decrease with du/dy. Paint, tooth paste, printers ink are some examples for different behaviours. These are also shown in Fig. 1.8.2. Many other behaviours have been observed which are more specialised in nature. The main topic of study in this text will involve only Newtonian fluids.

1.8.2 Viscosity and Momentum Transfer

In the flow of liquids and gases molecules are free to move from one layer to another. When the velocity in the layers are different as in viscous flow, the molecules moving from the layer at lower speed to the layer at higher speed have to be accelerated. Similarly the molecules moving from the layer at higher velocity to a layer at a lower velocity carry with them a higher value of momentum and these are to be slowed down. Thus the molecules diffusing across layers transport a net momentum introducing a shear stress between the layers. The force will be zero if both layers move at the same speed or if the fluid is at rest.

When cohesive forces exist between atoms or molecules these forces have to be overcome, for relative motion between layers. A shear force is to be exerted to cause fluids to flow.

Viscous forces can be considered as the sum of these two, namely, the force due to momentum transfer and the force for overcoming cohesion. In the case of liquids, the viscous forces are due more to the breaking of cohesive forces than due to momentum transfer (as molecular velocities are low). In the case of gases viscous forces are more due to momentum transfer as distance between molecules is larger and velocities are higher.

1.8.3 Effect of Temperature on Viscosity

When temperature increases the distance between molecules increases and the cohesive force decreases. So, viscosity of liquids decrease when temperature increases.

In the case of gases, the contribution to viscosity is more due to momentum transfer. As temperature increases, more molecules cross over with higher momentum differences. Hence, in the case of gases, viscosity increases with temperature.

1.8.4 Significance of Kinematic Viscosity

Kinematic viscosity, $\nu = \mu/\rho$, The unit in SI system is m^2/s.

$$(Ns/m^2)\ (m^3/\ kg) = [(kg.m/s^2)\ (s/m^2)]\ [m^3/kg] = m^2/s$$

Popularly used unit is stoke (cm^2/s) = 10^{-4} m^2/s named in honour of Stokes.

Centi stoke is also popular = 10^{-6} m^2/s.

Kinematic viscosity represents momentum diffusivity. It may be explained by modifying equation 1.8.2

$$\boldsymbol{\tau = \mu\ (du/dy) = (\mu/\rho) \times \{d\ (\rho u/dy)\} = \nu \times \{d\ (\rho u/dy)\}} \tag{1.8.4}$$

$d\ (\rho u/dy)$ represents momentum flux in the y direction.

So, $(\mu/\rho) = \nu$ kinematic viscosity gives the rate of momentum flux or momentum diffusivity.

With increase in temperature kinematic viscosity decreases in the case of liquids and increases in the case of gases. For liquids and gases absolute (dynamic) viscosity is not influenced significantly by pressure. But kinematic viscosity of gases is influenced by pressure due to change in density. In gas flow it is better to use absolute viscosity and density, rather than tabulated values of kinematic viscosity, which is usually for 1 atm.

1.8.5 Measurement of Viscosity of Fluids

1.8.5.1 Using Flow Through Orifices

In viscosity determination using Saybolt or Redwood viscometers, the time for the flow through a standard orifice, of a fixed quantity of the liquid kept in a cup of specified dimensions is measured in seconds and the viscosity is expressed as Saybolt seconds or Redwood seconds. The time is converted to poise by empirical equations. These are the popular instruments for industrial use. The procedure is simple and a quick assessment is possible. However for design purposes viscosity should be expressed in the standard units of Ns/m^2.

1.8.5.2 Rotating Cylinder Method

The fluid is filled in the interspace between two cylinders. The outer cylinder is rotated keeping the inner cylinder stationary and the reaction torque on the inner cylinder is measured using a torsion spring. Knowing the length, diameter, film thickness, rpm and the torque, the value of viscosity can be calculated. Refer Example 1.7.

Example 1.7. *In a test set up as in figure to measure viscosity, the cylinder supported by a torsion spring is 20 cm in dia and 20 cm long. A sleeve surrounding the cylinder rotates at 900 rpm and the torque measured is 0.2 Nm. If the film thickness between the cylinder and sleeve is 0.15 mm,* ***determine the viscosity of the oil.***

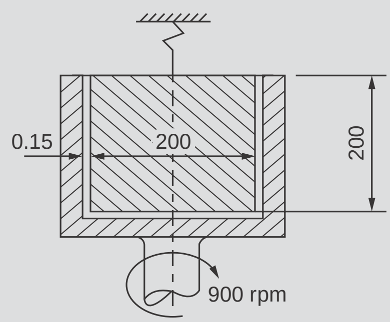


Figure Ex. 1.7 Viscosity test setup

The total torque is given by the sum of the torque due to the shear forces on the cylindrical surface and that on the bottom surface.

Torque due to shear on the cylindrical surface (eqn 1.9.1*a*),
$T_s = \mu\pi^2 NLR^3/15\ h$,

Torque on bottom surface (eqn 1.9.3),
$T_b = \mu\pi^2 NR^4/60\ h$

Where h is the clearance between the sleeve and cylinder and also base and bottom. In this case both are assumed to be equal. Total torque is the sum of values given by the above equations. In case the clearances are different then h_1 and h_2 should be used.

Total torque = $(\mu\ \pi^2 NR^3/\ 15.\text{h})\ \{L + (R/4)\}$, substituting,

$$0.2 = [(\mu \times \pi^2\ 900 \times 0.1^3)/(15 \times 0.0015)] \times [0.2 + (0.1/4)]$$

Solving for viscosity, $\boldsymbol{\mu = 0.00225\ Ns/m^2}$ **or 2.25 cP.**

This situation is similar to that in a Foot Step bearing.

1.8.5.3 Capillary Tube Method

The time for the flow of a given quantity under a constant head (pressure) through a tube of known diameter d, and length L is measured or the pressure causing flow is maintained constant and the flow rate is measured.

$$\Delta \mathbf{P} = \mathbf{(32\ \mu\ VL)/d^2} \quad (1.8.5)$$

This equation is known as Hagen-Poiseuille equation. The viscosity can be calculated using the flow rate and the diameter. Volume flow per second, $Q = (\pi d^2/4)\,V$. Q is experimentally measured using the apparatus. The head causing flow is known. Hence μ can be calculated.

1.8.5.4 Falling Sphere Method

A small polished steel ball is allowed to fall freely through the liquid column. The ball will reach a uniform velocity after some distance. At this condition, gravity force will equal the viscous drag. The velocity is measured by timing a constant distance of fall.

$$\mu = \mathbf{2r^2g\ (\rho_1 - \rho_2)/9V} \quad (1.8.6)$$

(μ will be in poise. 1 poise = 0.1 Ns/m^2)

where r is the radius of the ball, V is the terminal velocity (constant velocity), ρ_1 and ρ_2 are the densities of the ball and the liquid. This equation is known as **Stokes equation.**

Example 1.8. *Oil flows at the rate of 3 l / s through a pipe of 50 mm diameter. The pressure difference across a length of 15 m of the pipe is 6 kPa.* ***Determine the viscosity of oil flowing through the pipe.***

Using Hagen-Poiesuille equation-1.8.5 , $\Delta P = (32\ \mu u L)/d^2$

$u = Q/(\pi d^2/4) = 3 \times 10^{-3}/(\pi \times 0.05^2/4) = 1.53$ m/s

$\mu = \Delta P \times d^2/32uL = (6000 \times 0.05^2)/(32 \times 1.53 \times 15) =$ **0.0204 Ns m²**

Example 1.9. *A steel ball of 2 mm dia and density 8000 kg/m³ dropped into a column of oil of specific gravity 0.80 attains a terminal velocity of 2mm/s.* ***Determine the viscosity of the oil****.*
Using Stokes equation, 1.8.6

$\mu = 2r^2g\ (\rho_1 - \rho_2)/9u$

$= 2 \times (0.002/2)^2 \times 9.81 \times (8000 - 800)/(9 \times 0.002) = 7.85$ **Ns/m².**

1.9 APPLICATION OF VISCOSITY CONCEPT

1.9.1 Viscous Torque and Power—Rotating Shafts

Refer Figure 1.9.1

Shear stress, $\tau = \mu\ (du/dy) = \mu\ (u/y)$, as linearity is assumed

$u = \pi DN/60$, $y = h$, clearance in m

$\tau = \mu\ (\pi DN/60h)$, Tangential force $= \tau \times A$, $A = \pi DL$

Torque, T = tangential force $\times D/2 = \mu\ (\pi DN/60h)\ (\pi DL)\ (D/2)$

substituting $$T = \mu\ \pi^2 NLD^3/\ 120\ h \quad (1.9.1)$$

If radius is used, $$T = \mu\pi^2 NLR^3/15\ h \quad (1.9.1a)$$

As power, $P = 2\pi NT/60$,

$$P = \mu\pi^3 N^2 LR^3/450\ h \quad (1.9.2)$$

For equations 1.9.1 and 1.9.2, proper units are listed below:

L, R, D, h should be in meter and *N* in rpm. Viscosity μ should be in Ns/m^2 (or Pas). The torque will be obtained in Nm and the power calculated will be in *W*.

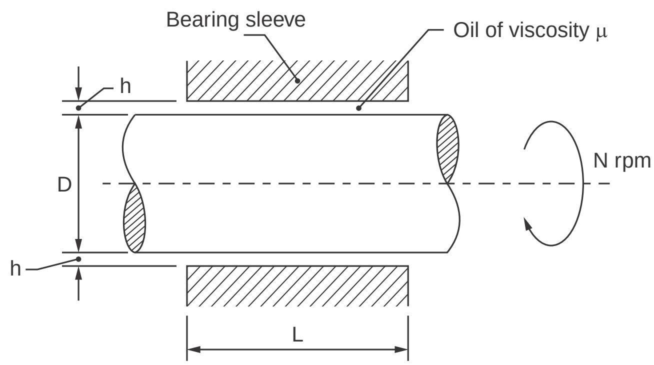


Figure 1.9.1 Rotating Shaft in Bearing

Note: Clearance h is also the oil film thickness in bearings. End effects are neglected. Linear velocity variation is assumed. Axial location is assumed.

Example 1.10. *Determine the power required to run a 300 mm dia shaft at 400 rpm in journals with uniform oil thickness of 1 mm. Two bearings of 300 mm width are used to support the shaft. The dynamic viscosity of oil is 0.03 Pas. (Pas = (N/m²) × s).*

Shear stress on the shaft surface = $\tau = \mu\ (du/dy) = \mu(u/y)$

$$u = \pi DN/60 = \pi \times 0.3 \times 400/60 = 6.28 \text{ m/s}$$

$$\tau = 0.03\ \{(6.28 - 0)/\ 0.001\} = 188.4 \text{ N/m}^2$$

Surface area of the two bearings, $A = 2\ \pi\ DL$

Force on shaft surface $= \tau \times A = 188.4 \times (2 \times \pi \times 0.3 \times 0.3) = 106.6\ N$

Torque $= 106.6 \times 0.15 = 15.995$ Nm

Power required $= 2\ \pi\ NT/60 = 2 \times \pi \times 400 \times 15.995/60 =$ **670 W.**

(check using eqn. 1.9.2, $P = \mu\ \pi^3\ N^2LR^3/450\ h = 669.74$ W)

1.9.2 Viscous Torque—Disk Rotating Over a Parallel Plate

Refer Figure 1.9.2.

Consider an annular strip of radius **r** and width **dr** shown in Figure 1.9.2. The force on the strip is given by,

$$F = A\mu\ (du/dy) = A\ \mu\ (u/y)$$

(as y is small linear velocity variation can be assumed)

$$u = 2\ \pi rN/60,\ y = h,\ A = 2\pi r\ dr$$

Torque = Force × radius, substituting the above values

torque dT on the strip is, $dT = 2\pi r\ dr\ \mu(2\pi rN/60h)r$

$$dT = 2\pi r.dr.\mu.\ 2\pi rN.r/60.h = [\mu\pi^2N/15.h]r^3dr$$

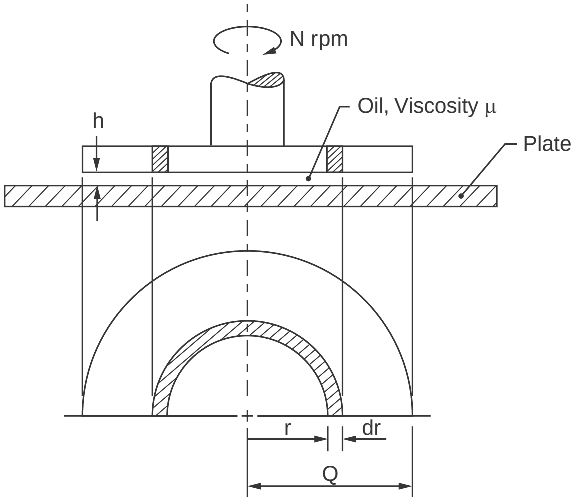


Figure 1.9.2 Rotating disk

Integrating the expression from centre to edge *i.e.*, 0 to R,

$$\mathbf{T = \mu\pi^2 NR^4/60\ h} \tag{1.9.3}$$

If diameter is used, $R^4 = (1/16)D^4$

$$\mathbf{T = \mu\pi^2 ND^4/960\ h} \tag{1.9.3a}$$

The power required, $P = 2\pi NT/60$

$$\mathbf{P = \mu\pi^3 N^2 R^4/1800\ h} \tag{1.9.4}$$

use R in metre, N in rpm and μ in Ns/m^2 or Pa s.

For an annular area like a collar the integration limits are $\mathbf{R_o}$ and $\mathbf{R_i}$ and the torque is given by

$$\mathbf{T = \mu\pi^2 N(R_o^{\ 4} - R_i^{\ 4})/60\ h} \tag{1.9.5}$$

Power, $$\mathbf{P = \mu\pi^3 N^2(R_o^{\ 4} - R_i^{\ 4})/1800\ h} \tag{1.9.6}$$

Example 1.11. Determine the oil film thickness *between the plates of a collar bearing of 0.2 m ID and 0.3 m OD transmitting power, if 50 W was required to overcome viscous friction while running at 700 rpm. The oil used has a viscosity of 30 cP.*

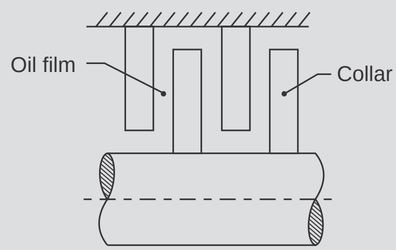


Figure Ex. 1.11

Power = $2\pi NT/60$ W, substituting the given values,

$50 = 2\pi \times 700 \times T/60$, Solving torque,

T = 0.682 Nm

This is a situation where an annular surface rotates over a flat surface. Hence, using equation 1.9.5, Torque, $T = \mu\pi^2 N\,(R_O^{\ 4} - R_i^{\ 4})/60.\text{h}$

μ = 30 cP = 30 × .0001 Ns/m^2, substituting the values,

$0.682 = (30 \times 0.0001) \times \pi^2 \times 700 \times (0.15^4 - 0.1^4)/60 \times h$

$\therefore$ **h = 0.000206m = 0.206 mm**

1.9.3 Viscous Torque—Cone in a Conical Support

Considering a small element between radius r and $r + dr$, as shown in figure 1.9.3. The surface width of the element in contact with oil is

$$dx = dr/\sin\theta$$

The surface area should be calculated with respect to centre O as shown in figure—the point where the normal to the surface meets the axis—or the centre of rotation, the length OA being $r/\cos\theta$.

Hence contact surface area = $2\pi r.dr/\sin\theta.\cos\theta$.

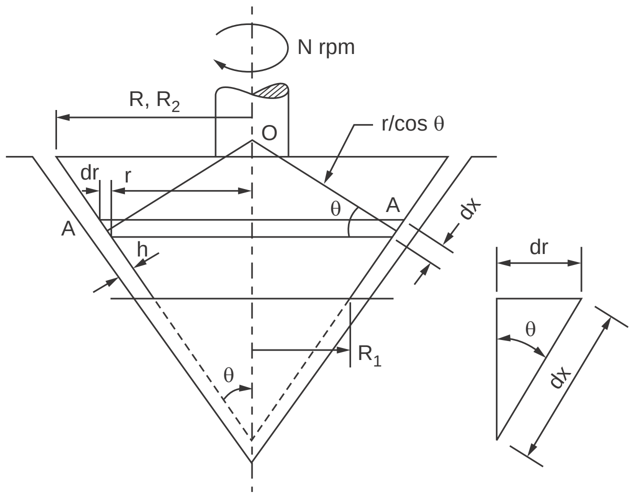


Figure 1.9.3 Rotating cone or conical bearing

The velocity along the surface is $(2\pi rN/60).\cos\theta$ and the film thickness is h.

$$F = A\mu\,(du/dy) = \{(2\pi r./\sin\theta.\cos\theta)\}\,\mu(2\pi rN.\cos\theta/60)\,(1/h)$$

$$F = (\pi^2\mu Nr^2dr)/(15.h.\sin\theta), \qquad \text{Torque} = F.r$$

Torque on element, $dT = \pi^2\mu Nr^2dr.r/15.h.\sin\theta = (\pi\mu N/15\text{ h }\sin\theta)r^3\,dr$

Integrating between $r = 0$ and $r = R$

$$\mathbf{T = \pi^2\,\mu NR^4/60.h\ \sin\theta} \tag{1.9.7}$$

Using μ in Ns/m^2, h and R in metre the torque will be in N.m. When semicone angle θ = 90°, this reduces to the expression for the disk—equation 1.9.3. For contact only between R_1 and R_2.

$$\mathbf{T = \mu\pi^2\,N(R_2^{\,4} - R_1^{\,4})/60.h.\ \sin\theta} \tag{1.9.8}$$

Power required, $$\mathbf{P = 2\pi NT/60 = \mu^3N^2[R_2^{\,4} - R_1^{\,4}]/1800\ h\ \sin\theta} \tag{1.9.9}$$

Exmaple 1.12. ***Determine the power required*** *to overcome viscous friction for a shaft running at 700 rpm fitted with a conical bearing. The inner and outer radius of the conical bearing are 0.3 m and 0.5 m. The height of the cone is 0.3 m. The 1.5 mm uniform clearance between the bearing and support is filled with oil of viscosity 0.02 Ns/m².*

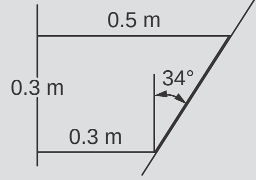


Figure Ex. 1.12

Equation 1.9.8 is applicable in this case.

$$\tan\theta = (0.5 - 0.3)/0.3 = 0.667, \quad \therefore \quad \theta = 34°$$

$T = \pi^2\mu\ N\ (R_o^{\,4} - R_i^{\,4})/\ 60.\ h.\sin\theta$, substituting the values

$$T = \pi^2 \times 0.03 \times 700 \times (0.5^4 - 0.3^4)/60 \times 0.0015 \times \sin 34 = 149.36 \text{ Nm}$$

Power required $= 2\pi NT/60 = 2\pi \times 700 \times 149.36/60 =$ **10948 W**

Check using equation 1.9.9 also,

$$P = \mu \times \pi^3 \times 700^2 \times [0.5^4 - 0.3^4]/[1800 \times 0.0015 \times \sin 34] = 10948 \text{ W.}$$

Note the high value of viscosity

1.10 SURFACE TENSION

Many of us would have seen the demonstration of a needle being supported on water surface without it being wetted. This is due to the surface tension of water.

All liquids exhibit a free surface known as meniscus when in contact with vapour or gas. Liquid molecules exhibit cohesive forces binding them with each other. The molecules below the surface are generally free to move within the liquid and they move at random. When they reach the surface they reach a dead end in the sense that no molecules are present in great numbers above the surface to attract or pull them out of the surface. So they stop and return back into the liquid. A thin layer of few atomic thickness at the surface formed by the cohesive bond between atoms slows down and sends back the molecules reaching the surface. This cohesive bond exhibits a tensile strength for the surface layer and this is known as surface tension. Force is found necessary to stretch the surface.

Surface tension may also be defined as the work in Nm/m^2 or N/m required to create unit surface of the liquid. The work is actually required for pulling up the molecules with lower energy from below, to form the surface.

Another definition for surface tension is the force required to keep unit length of the surface film in equilibrium (N/m). The formation of bubbles, droplets and free jets are due to the surface tension of the liquid.

1.10.1 Surface Tension Effect on Solid-Liquid Interface

In liquids cohesive forces between molecules lead to surface tension. The formation of droplets is a direct effect of this phenomenon. So also the formation of a free jet, when liquid flows out of an orifice or opening like a tap. The pressure inside the droplets or jet is higher due to the surface tension.

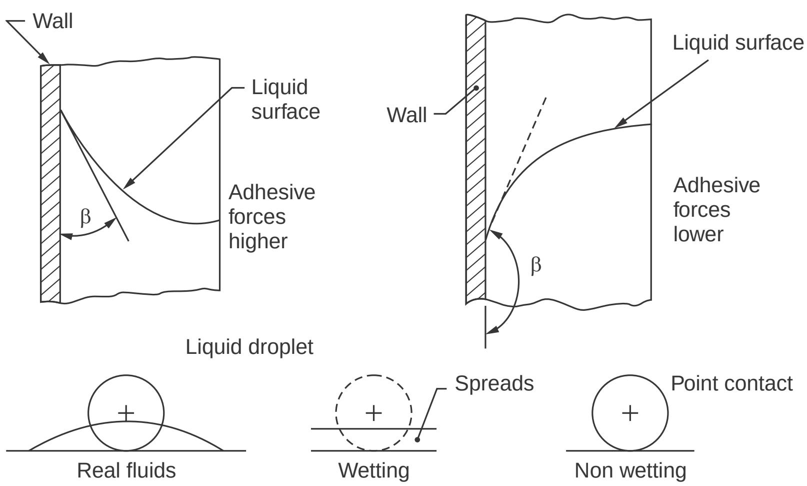


Figure 1.10.1 Surface tension effect at solid-liquid interface

Liquids also exhibit adhesive forces when they come in contact with other solid or liquid surfaces. At the interface this leads to the liquid surface being moved up or down forming a curved surface. When the adhesive forces are higher the contact surface is lifted up forming a concave surface. Oils, water etc. exhibit such behaviour. These are said to be surface wetting. When the adhesive forces are lower, the contact surface is lowered at the interface and a convex surface results as in the case of mercury. Such liquids are called nonwetting. These are shown in Fig. 1.10.1.

The angle of contact "β" defines the concavity or convexity of the liquid surface. It can be shown that if the surface tension at the solid liquid interface (due to adhesive forces) is σ_{s1} and if the surface tension in the liquid (due to cohesive forces) is σ_{11} then

$$\cos \beta = [(2\sigma_{s1}/\sigma_{11}) - 1] \qquad (1.10.1)$$

At the surface this contact angle will be maintained due to molecular equilibrium. The result of this phenomenon is capillary action at the solid liquid interface. The curved surface creates a pressure differential across the free surface and causes the liquid level to be raised or lowered until static equilibrium is reached.

Example 1.13. ***Determine the surface tension*** *acting on the surface of a vertical thin plate of 1m length when it is lifted vertically from a liquid using a force of 0.3N.*

Two contact lines form at the surface and hence, Force = 2 × 1 × Surface tension

0.3 = 2 × 1 × Surface tension. Solving, Surface tension, **σ = 0.15 N/m.**

1.10.2 Capillary Rise or Depression

Refer Figure 1.10.2.

Let D be the diameter of the tube and β is the contact angle. The surface tension forces acting around the circumference of the tube = $\pi \times D \times \sigma$.

The vertical component of this force = $\pi \times D \times \sigma \times \cos\beta$

This is balanced by the fluid column of height, h, the specific weight of liquid being γ.

Equating, $h \times \gamma \times A = \pi \times D \times \sigma \cos\beta$, $A = \pi D^2/4$ and so

$$\mathbf{h = (4\pi \times D \times \sigma \times \cos\beta)/(\gamma\pi D^2) = (4\sigma \times \cos\beta)/\rho g D} \qquad (1.10.2)$$

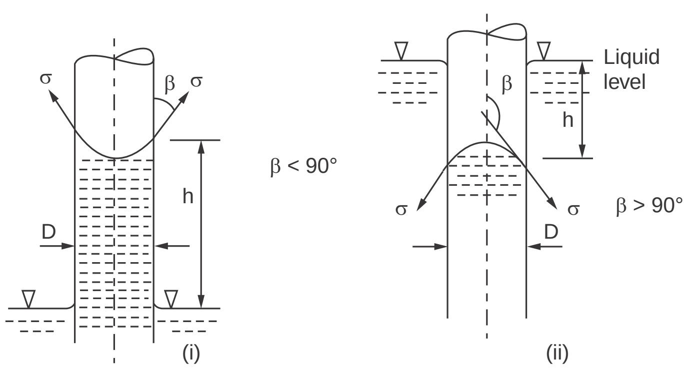


Figure 1.10.2 Surface tension, (*i*) capillary rise (*ii*) depression

This equation provides the means for calculating the capillary rise or depression. The sign of cos β depending on $\beta > 90$ or otherwise determines the capillary rise or depression.

Example 1.14. ***Determine the capillary depression*** *of mercury in a 2 mm ID glass tube. Assume $\sigma = 0.5$ N/m and $\beta = 130°$.*

Specific weight of mercury, $\gamma = 13600 \times 9.81$ N/m^3

Using eqn. 1.10.2, $h = (4\ \sigma \times \cos\beta)/\rho g/D$

$= (4 \times 0.5 \times \cos 130)/(13600 \times 9.81 \times 0.002)$

$= -4.82 \times 10^{-3}$ m = **– 4.82 mm**

Example1.15. *In a closed end single tube manometer, the height of mercury column above the mercury well shows 757 mm against the atmospheric pressure. The ID of the tube is 2 mm. The contact angle is 135°.* ***Determine the actual height*** *representing the atmospheric pressure if surface tension is 0.48 N/m. The space above the column may be considered as vacuum.*

Actual height of mercury column = Mercury column height + Capillary depression

Specific weight of mercury $= \rho g = 13600 \times 9.81$ N/m^3

Capillary depression, $\mathbf{h} = (4\ \sigma \times \cos\beta)/\gamma D$

$= (4 \times 0.48 \times \cos 135)/(0.002 \times 13600 \times 9.81)$

$= -5.09 \times 10^{-3}$m **= – 5.09 mm (depression)**

Corrected height of mercury column = 757 + 5.09 = **762.09 mm**

1.10.3 Pressure Difference Caused by Surface Tension on a Doubly Curved Surface

Consider the small doubly curved element with radius r_1 and included angle $d\phi$ in one direction and radius r_2 and $d\theta$ in the perpendicular direction referred to the normal at its center.

For equilibrium the components of the surface tension forces along the normal should be equal to the pressure difference.

The sides are $r_1\, d\phi$ and $r_2\, d\theta$ long. Components are $\sigma r_1 \sin(d\theta/2)$ from θ direction sides and $\sigma r_2 \sin(d\phi/2)$ from the ϕ direction sides.

$$2\sigma r_1 d\phi \sin(d\theta/2) + 2\sigma r_2\, d\theta \sin(d\phi/2) = (p_i - p_o) r_1 r_2\, d\theta d\phi$$

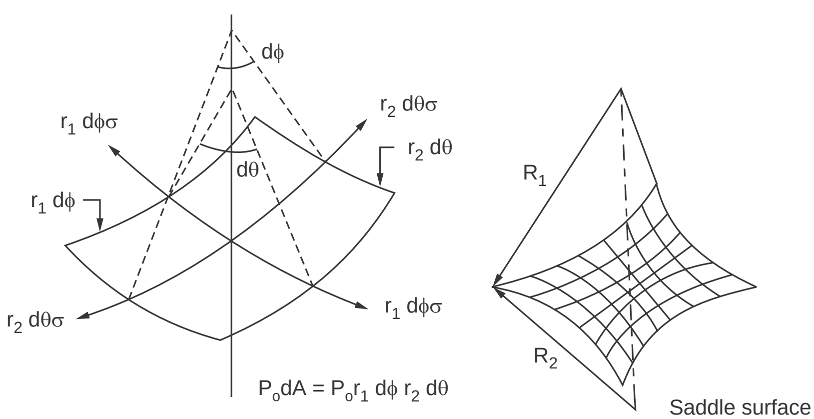


Figure 1.10.3 Pressure difference, doubly curved surface

For small values of angles, sin θ = θ, in radians. Cancelling the common terms

$$\sigma\,[r_1 + r_2] = (p_i - p_o) \times r_1 r_2. \text{ Rearranging,} \tag{1.10.3}$$

$$\mathbf{(p_i - p_o) = [(1/r_1) + (1/r_2)] \times \sigma}$$

For a spherical surface, $r_1 = r_2 = R$

So, $$\mathbf{(p_i - p_o) = 2\sigma/R} \tag{1.10.4}$$

where R is the radius of the sphere.

For cylindrical shapes one radius is infinite, and so

$$(\mathbf{p_i - p_o}) = \sigma/\mathbf{R} \tag{1.10.4a}$$

These equations give the pressure difference between inside and outside of droplets and free jets of liquids due to surface tension. The pressure inside air bubbles will be higher compared to the outside pressure. The pressure inside a free jet will be higher compared to the outside.

The pressure difference can be made zero for a doubly curved surface if the curvature is like that of a saddle (one positive and the other negative). This situation can be seen in the jet formed in tap flow where internal pressure cannot be maintained.

1.10.4 Pressure Inside a Droplet and a Free Jet

Refer Figure 1.10.4.

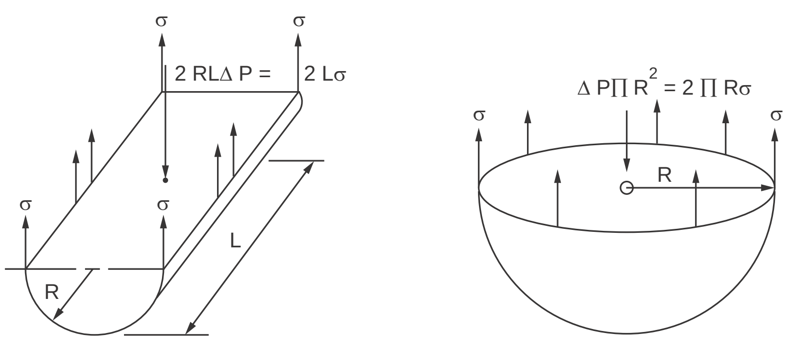


Figure 1.10.4 Surface tension effects on bubbles and free jets

Considering the sphere as two halves or hemispheres of diameter D and considering the equilibrium of these halves,

Pressure forces = Surface tension forces, $(p_i - p_o)(\pi D^2/4) = \sigma \times \pi \times D$

$$\mathbf{(p_i - p_o) = 4(\sigma/D) = 2(\sigma/R)} \tag{1.10.5}$$

Considering a cylinder of length L and diameter D and considering its equilibrium, taking two halves of the cylinder.

pressure force = $DL(p_i - p_o)$, surface tension force = $2\sigma L$

$$\mathbf{(p_i - p_o) = 2\,(\sigma/D) = (\sigma/R)} \tag{1.10.6}$$

Example 1.16. ***Determine the pressure difference*** *across a nozzle if diesel is sprayed through it with an average diameter of 0.03mm.* The surface tension is 0.04N/m.

The spray is of cylindrical shape

$$\mathbf{P} = \sigma/R = 0.04/(0.03 \times 10^{-3}/2) = 2666.67\ \text{N/m}^2 = \mathbf{2.67\ kpa}$$

Example 1.17. ***Calculate the surface tension*** *if the pressure difference between the inside and outside of a soap bubble of 3mm dia is 18 N/m².*

Referring equation 1.10.5, $\Delta P = 4\sigma/D$

Surface tension, $\sigma = \Delta P \times D/4 = 18 \times (0.003/4) = \mathbf{0.0135\ N/m}$

1.11 COMPRESSIBILITY AND BULK MODULUS

Bulk modulus, E_v is defined as the ratio of the change in pressure to the rate of change of volume due to the change in pressure. It can also be expressed in terms of change of density.

$$\mathbf{E_v = -\,dp/(dv/v) = dp/(d\rho/\rho)} \quad (1.11.1)$$

where **dp** is the change in pressure causing a change in volume **dv** when the original volume was **v**. The unit is the same as that of pressure, obviously. Note that $dv/v = -\,d\rho/\rho$.

The negative sign indicates that if dp is positive then dv is negative and vice versa, so that the bulk modulus is always positive (N/m²). The symbol used in this text for bulk modulus is E_v (K is more popularly used).

This definition can be applied to liquids as such, without any modifications. In the case of gases, the value of compressibility will depend on the process law for the change of volume and will be different for different processes.

The bulk modulus for liquids depends on both pressure and temperature. The value increases with pressure as dv will be lower at higher pressures for the same value of dp. With temperature the bulk modulus of liquids generally increases, reaches a maximum and then decreases. For water the maximum is at about 50°C. The value is in the range of 2000 MN/m² or 2000×10^6 N/m² or about 20,000 atm. Bulk modulus influences the velocity of sound in the medium, which equals $(g_o \times E_v/\rho)^{0.5}$.

Example 1.18. ***Determine the bulk modulus of*** *a liquid whose volume decreases by 4% for an increase in pressure of 500 × 10⁵ pa. Also* ***determine the velocity of sound*** *in the medium if the density is 1000 kg/m³.*

Bulk modulus is defined as $E_v = -\,dp/(dv/v)$, substituting the values,

$$\boldsymbol{E_v} = (-\,500 \times 10^5)/(-4/100) = \mathbf{1.25 \times 10^9\ N/m^2}$$

Velocity of sound c is defined as $= (g_o \times E_v/\rho)^{0.5}$

$\therefore$ $$\mathbf{c} = [1 \times 1.25 \times 10^9/100]^{0.05} = \mathbf{1118\ m/s.}$$

Example 1.19. *The pressure of water in a power press cylinder is released from 990 bar to 1 bar isothermally. If the average value of bulk modulus for water in this range is 2430 × 10⁶ N/m².* ***What will be the percentage increase in specific volume?***

The definition of bulk modulus, $E_v = -\,dp/(dv/v)$ is used to obtain the solution. Macroscopically the above equation can be modified as

$$E_v = -\,\{P_1 - P_2\}\{(v_2 - v_1)/v_1\}, \text{ Rearranging,}$$

Change in specific volume $= (v_2 - v_1)/v_1 = -(P_2 - P_1)/E_v$

$= (990 \times 10^5 - 1 \times 10^5)/2430 \times 10^6 = 0.0407$

% change in specific volume = 4.07%

Example 1.20. *Density of sea water at the surface was measured as 1040 kg/m^3 at an atmospheric pressure of 1 bar. At certain depth in water, the density was found to be 1055 kg/m^3.* ***Determine the pressure at that point.*** *The bulk modulus is 2290 × 10^6 N/m^2.*

Bulk modulus, $E_v = -dp/(dv/v) = -(P_2 - P_1)/[(v_2 - v_1)v_1]$

As $v = 1/\rho$, $-(P_2 - P_1) = E_v \times [\{1/\rho_2) - (1/\rho_1)\}/(1/\rho_1)]$

$= E_v \times [(\rho_1 - \rho_2)/\rho_2]$

$P_2 = P_1 - E_v \times [(\rho_1 - \rho_2)/\rho_2] = 1 \times 10^5 - 2290 \times 10^6 \{(1040 - 1055)/1055\}$

$=$ **32.659 × 10^6 N/m^2 or about 326.59 bar.**

1.11.1 Expressions for the Compressibility of Gases

The expression for compressibility of gases for different processes can be obtained using the definition, namely, **compressibility = – dp/(dv/v)**. In the case of gases the variation of volume, **dv**, with variation in pressure, **dp**, will depend on the process used. The relationship between these can be obtained using the characteristic gas equation and the equation describing the process.

Process equation for gases can be written in the following general form

$$\mathbf{Pv^n = constant} \tag{1.11.2}$$

where n can take values from 0 to ∞. If $n = 0$, then P = constant or the process is a constant pressure process. If $n = \infty$, then v = constant and the process is constant volume process. These are not of immediate interest in calculating compressibility. If $dp = 0$, compressibility is zero and if $dv = 0$, compressibility is infinite.

The processes of practical interest are for values of $n = 1$ to $n = c_p/c_v$ (the ratio of specific heats, denoted as k). The value $n = 1$ means Pv = constant or isothermal process and $n = c_p/c_v = k$ means isentropic process.

Using the equation Pv^n = constant and differentiating the same,

$$\mathbf{nPv^{(n-1)}dv + v^n dp = 0} \tag{1.11.3}$$

rearranging and using the definition of E_v,

$$\mathbf{E_v = -dp/(dv/v) = n \times P} \tag{1.11.4}$$

Hence compressibility of gas varies as the product $n \times P$.

For isothermal process, $n = 1$, compressibility $= P$.

For isentropic process, compressibility $= k \times P$.

For constant pressure and constant volume processes compressibility values are zero and ∞ respectively.

In the case of gases the velocity of propagation of sound is assumed to be isentropic. From the definition of velocity of sound as $[g_o \times E_v/\rho]^{0.5}$ it can be shown that

$$\mathbf{c = [g_o \times k\, P/\rho]^{0.5} = [g_o \times k \times R \times T]^{0.5}} \tag{1.11.5}$$

It may be noted that for a given gas the velocity of sound depends only on the temperature. As an exercise the velocity of sound at 27°C for air, oxygen, nitrogen and hydrogen may be calculated as 347.6 m/s, 330.3 m/s, 353.1 m/s and 1321.3 m/s.

1.12 VAPOUR PRESSURE

Liquids exhibit a free surface in the container whereas vapours and gases fill the full volume. Liquid molecules have higher cohesive forces and are bound to each other. In the gaseous state the binding forces are minimal.

Molecules constantly escape out of a liquid surface and an equal number constantly enter the surface when there is no energy addition. The number of molecules escaping from the surface or re-entering will depend upon the temperature.

Under equilibrium conditions these molecules above the free surface exert a certain pressure. This pressure is known as vapour pressure corresponding to the temperature. As the temperature increases, more molecules will leave and re-enter the surface and so the vapour pressure increases with temperature. All liquids exhibit this phenomenon. Sublimating solids also exhibit this phenomenon.

The vapour pressure is also known as saturation pressure corresponding to the temperature. The temperature corresponding to the pressure is known as saturation temperature. If liquid is in contact with vapour both will be at the same temperature and under this condition these phases will be in equilibrium unless energy transaction takes place.

The vapour pressure data for water and refrigerants are available in tabular form. The vapour pressure increases with the temperature. For all liquids there exists a pressure above which there is no observable difference between the two phases. This pressure is known as critical pressure. Liquid will begin to boil if the pressure falls to the level of vapour pressure corresponding to that temperature. Such boiling leads to the phenomenon known as cavitation in pumps and turbines. In pumps it is usually at the suction side and in turbines it is usually at the exit end.

1.12.1 Partial Pressure

In a mixture of gases the total pressure P will equal the sum of pressures exerted by each of the components if that component alone occupies the full volume at that temperature. The pressure exerted by each component is known as its partial pressure.

$$\mathbf{P = p_1 + p_2 + p_3 + \ldots} \tag{1.12.1}$$

where $p_1 = (m_1R_1T)/V$; $p_2 = (m_2R_2T)/V$ in which T and V are the common temperature and volume.

For example air is a mixture of various gases as well as some water vapour. The atmospheric pressure is nothing but the sum of the pressures exerted by each of these components. Of special interest in this case is the partial pressure of water vapour. This topic is studied under Psychrometry. The various properties like specific heat, gas constant etc. of the mixture can be determined from the composition.

$$\mathbf{c_m = \Sigma (c_i \times m_i)/\Sigma m_i} \tag{1.12.2}$$

where c_m is the specific heat of the mixture and c_i and m_i are the specific heat and the mass of component i in the mixture.

SOLVED PROBLEMS

Problem 1.1. *A liquid with kinematic viscosity of 3 centi stokes and specific weight 9 kN/m³ fills the space between a large stationary plate and a parallel plate of 475 mm square, the film thickness being 1 mm. If the smaller plate is to be pulled with uniform velocity of 4 m/s,* ***determine the force required*** *if the liquid film is maintained all through.*

The force required (eqn 1.8.2), $\tau \times A = A \times \mu \times (du/dy)$, where τ is shear stress, and μ is dynamic viscosity. In this problem kinematic viscosity and specific weight are given.

Stoke = 10^{-4} m²/s. Density = specific weight/g. So, $\mu = 0.03 \times 10^{-4} \times 9000/9.81$ Ns/m²

Force = $[0.03 \times 10^{-4} \times 9000/9.81] \times (4.0/0.001) \times 0.475 \times 0.475$ = **2.484 N.**

Problem 1.2. *A small thin plane surface is pulled through the liquid filled space between two large horizontal planes in the parallel direction. Show that the force required will be minimum if the plate is located midway between the planes.*

Let the velocity of the small plane be u, and the distance between the large planes be h.

Let the small plane be located at a distance of y from the bottom plane. Assume linear variation of velocity and unit area. Refer Fig. P 1.2.

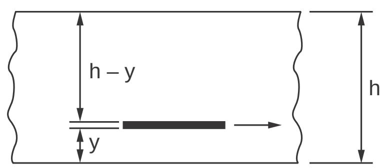


Figure P.1.2 Problem model

Velocity gradient on the bottom surface = u/y

Velocity gradient on the top surface = $u/(h - y)$,

Considering unit area,

Force on the bottom surface = $\mu \times (u/y)$, Force on the top surface = $\mu \times u/(h - y)$

Total force to pull the plane = $\mu \times u \times \{(1/y) + [1/(h - y)]\}$ ...(A)

To obtain the condition for minimisation of the force the variation of force with respect to y should be zero, or $dF/dy = 0$, Differentiating the expression A,

$$dF/dy = \mu \times u \, \{(-1/y^2) + [1/(h - y)^2]\}, \text{ Equating to zero}$$

$$y^2 = (h - y)^2 \text{ or } y = h/2$$

or the plane should be located at the mid gap position for the force to be minimum.

The force required for different location of the plate is calculated using the following data and tabulated below.

$$\mu = 0.014 \text{ Ns/m}^2, \; u = 5 \text{ m/s}, \; h = 0.1 \text{ m}.$$

Equation A is used in the calculation.

Model calculation is given for $y = 0.002$ m.

$$F = 0.014 \times 5 \times \{(1/0.002) + [1/0.01 - 0.002)]\} = 43.75 \text{ N/m}^2$$

Note that the minimum occurs at mid position

Distance, y mm	2	3	4	5	6
Force, N/m²	43.75	33.33	29.17	28.00	29.17

Problem 1.3. *A small plane is pulled along the centre plane of the oil filled space between two large horizontal planes with a velocity u and the force was measured as F. The viscosity of the oil was μ_1. If a lighter oil of viscosity μ_2 fills the gap what should be the location of the plate for the force to be the same when pulled with the same velocity u.*

If the plane is located centrally in the case where the oil is lighter the force will be smaller.

So the plane should now be located away from the central plane. Let it be located at a distance, y from the lower plane as shown in Fig. P1.2 :

Case 1: The velocity gradient is equal on both sides = $u/(h/2) = 2 \times u/h$

Total force = $\mu_1 \times \{(2u/h) + (2u/h)\}$ = $\mathbf{4 \times \mu_1 \times u/h}$

Case 2: Velocity gradient on the top surface = $u/(h–y)$

Velocity gradient on the bottom surface = u/y

Total force = $\mu_2 \times u \times \{(1/y) + [1/(h - y)]\} = \mu_2 \times u \times \{h/[y \times (h - y)]\}$

Equating and solving, $(\mu_2/\mu_1) = 4 \times y \times (h - y)/h^2$ = $\mathbf{4[y/h] \times [1 - (y/h)]}$

Solve for (y/h). A quadratic equation.

Problem 1.4. *A large thin plate is pulled through a narrow gap filled with a fluid of viscosity μ on the upper side and a fluid of viscosity $c\mu$ on the lower side. Derive an expression for the location of the plate in the gap for the total force to be minimum.*

The force will not be minimum if the plate is centrally located as the viscosity are not equal. Let the plate be located at a distance of y from the lower surface on the side where the viscosity is $c\mu$. Let the gap size be h, the total force for unit area will be

$$F = c\mu \times (u/y) + \mu \times u/(h - y) = \mu \times u\ \{(c/y) + [1/(h - y)]\}$$

At the minimum conditions the slope *i.e.*, the derivative dF/dy should be zero.

$$dF/dy = \mu \times u\ \{[1/(h - y)^2] - [c/y^2]\}, \text{ Equating to zero yields, } y^2 = c \times (h - y)^2$$

Taking the root,

$$\sqrt{c} \times (h - y) = y \quad \text{or} \quad y = (h \times \sqrt{c})/(1 + \sqrt{c}) = h/[1 + (1/\sqrt{c})]$$

Consider the following values for the variables and calculate the force for different locations of the plate.

$$u = 5 \text{ m/s}, \ \mu = 0.014 \text{ N/m}^2, \ h = 4 \text{ mm} \quad \text{and} \quad c = 0.49 \quad \text{or} \quad \sqrt{c} = 0.7$$

For optimum conditions

$$y = (0.004 \times 0.7)/(1 + 0.7) = 0.001647 \text{ m}$$

Using $F = 5 \times 0.014 \times \{(0.49/y) + [1/(0.004 - y)]\}$, the force for various locations is calculated and tabulated below:

y, mm	1.0	1.5	1.65	2.0	2.5
Force, N/m^2	57.63	50.87	50.58	52.15	60.39

Problem 1.5. *A hydraulic lift shaft of 450 mm dia moves in a cylinder of 451 mm dia with the length of engagement of 3 m. The interface is filled with oil of kinematic viscosity of 2.4×10^{-4} m^2/s and density of 900 kg/m^3. Determine the uniform velocity of movement of the shaft if the drag resistance was 300 N.*

The force can be determined assuming that the sliding is between the developed surfaces, the area being $\pi \times D \times L$, $\mu = \rho v = 2.4 \times 10^{-4} \times 900 = 0.216$ Ns/m^2,

Clearance $= (D_o - D_i)/2 = 0.5$ mm. Using equations 1.8.1 and 1.8.2

Drag resistance $= 300 = \mu \times 0.45 \times 3 \times 0.216 \times (u / 0.0005)$

Solving for u, velocity = **0.16374 m/s.**

Problem 1.6. *A shaft of 145 mm dia runs in journals with a uniform oil film thickness of 0.5 mm. Two bearings of 20 cm width are used. The viscosity of the oil is 19 cP.* ***Determine the speed if the power absorbed is 15 W.***

The equation that can be used is, 1.9.2 *i.e.*, (n is used to denote rpm)

$$P = [\mu \pi^3 n^2 L R^3 / 450\ h]$$

The solution can be obtained from basics also. Adopting the second method,

$$\tau = \mu\ (du/dy) = \mu\ (u/y),\ \mu = 19 \text{ cP} = 0.019 \text{ Ns/m}^2,$$

$$y = 0.5 \text{ mm} = 0.0005 \text{ m, let the rpm be } n$$

$$u = \pi Dn / 60 = \pi \times 0.145 \times n/60 = 7.592 \times 10^{-3} \times n$$

$$\tau = 0.019\ (7.592 \times 10^{-3} \times n/0.0005) = 0.2885 \times n \text{ N/m}^2,$$

$$A = 2 \times \pi DL = 0.182 \text{ m}^2, \text{ Force F} = \text{A} \times \tau = 0.2885 \times n \times 0.182 = 0.0525 \times n,$$

Torque = force × radius,

$$T = 0.0525 \times n \times 0.145/2 = 3.806 \times 10^{-3} \times n \text{ Nm}$$

Power, $P = 2\pi nT/60 = 15 = 2 \times \pi \times n\ 3.806 \times 10^{-3} \times n / 60$

Solving, speed, n = **194 rpm.** (Check using the equation 1.9.2)

$$15 = [0.019 \times \pi^3 \times n^2\ (2 \times 0.20) \times 0.0725^3 / (450 \times 0.0005)]$$

Solving speed, **n = 194 rpm.**

Problem 1.7. *A circular disc of 0.3 m dia rotates over a large stationary plate with 1 mm thick fluid film between them.* ***Determine the viscosity of the fluid*** *if the torque required to rotate the disc at 300 rpm was 0.1 Nm.*

The equation to be used is 1.9.3, (n denoting rpm)

Torque $T = (\mu \times \pi^2 \times n \times R^4)/(60 \times h)$, ($h$ – clearance),

$n = 300$ rpm, $R = 0.15$ m, $h = 0.001$ m, Substituting the values,

$$0.1 = \mu \times \pi^2 \times 300 \times 0.15^4 / (60 \times 0.001), \text{ Solving for } \mu$$

Viscosity $\boldsymbol{\mu = 4 \times 10^{-3}}$ **Ns/m^2 or 4 cP.**

(care should be taken to use radius value, check from basics.)

Problem 1.8. ***Determine the viscous drag torque and power*** *absorbed on one surface of a collar bearing of 0.2 m ID and 0.3 m OD with an oil film thickness of 1 mm and a viscosity of 30 cP if it rotates at 500 rpm.*

The equation applicable is 1.9.5. $T = \mu \times \pi^2 \times n \times (R_o^4 - R_i^4)/60 \times h$

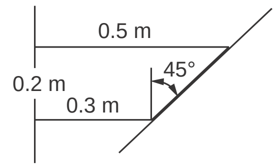
$$\mu = 30 \times 0.001 \text{ Ns/m}^2,\ n = 500 \text{ rpm},\ R_o = 0.15 \text{ m},\ R_i = 0.1 \text{ m},\ h = 0.002 \text{ m}$$

substituting the values

$$\mathbf{T} = 30 \times 0.001 \times \pi^2 \times 500 \times \{0.15^4 - 0.1^4\}/\{(60 \times 0.002)\} = \mathbf{0.5012\ Nm}$$

$$\mathbf{P} = 2\pi nT/60 = 2 \times \pi \times 500 \times 0.5012/60 = \mathbf{26.243\ W.}$$

Problem 1.9. *A conical bearing of outer radius 0.5 m and inner radius 0.3 m and height 0.2 m runs on a conical support with a uniform clearance between surfaces. Oil with viscosity of 30 cP is used. The support is rotated at 500 rpm.* ***Determine the clearance*** *if the power required was 1500 W.*



The angle θ is determined using the difference in radius and the length.

$$\tan\theta = (0.5 - 0.3)/0.2 = 1.0;\ \text{So } \theta = 45°.$$

Using equation 1.9.9 *i.e.*,

$$P = \pi^3 \times \mu \times n^2 \times (R_2^4 - R_1^4)/1800 \times h \times \sin\theta$$

$$(\mu = 30 \text{ cP} = 0.03 \text{ Ns/m}^2,\ n = 500 \text{ rpm},\ R_2 = 0.5 \text{ m},\ R_1 = 0.3 \text{ m})$$

$$1500 = \pi^3 \times 0.03 \times 500^2 \times (0.5^4 - 0.3^4)/1800 \times h \times \sin 45°$$

Solving for clearance, $\mathbf{h = 6.626 \times 10^{-3}}$ **m or 6.63 mm**

Problem 1.10. If the variation of velocity with distance from the surface, y is given by $u = 10\, y^{0.5}$ *whre u is in m/s and y is in m in a flow field up to y = 0.08 m,* **determine the wall shear stress** *and the shear stress at y = 0.04 and 0.08 m from the surface.*

$$u = 10y^{0.5},\ (du/dy) = 5/y^{0.5}.$$

The substitution $y = 0$ in the above will give division by zero error. It has to be approximated as $(u_2 - u_1)/(y_2 - y_1)$ for near zero values of y.

Considering layers $y = 0$ and $y = 10^{-6}$, the velocities are 0.0 and 0.01 m/s

(using $u = 10\, y^{0.5}$), the difference in y value is 10^{-6}.

So $(u_2 - u_1)/(y_2 - y_1) = 0.01/10^{-6} = 10000$,

At the wall, $(du/dy) = 10000$, $\tau = \mu\,(du/dy) = \mathbf{10000 \times \mu}$

At $\mathbf{y = 0.04,}$ $(du/dy) = 5/0.04^{0.5} = 25$, $\tau = \mathbf{25 \times \mu}$

At $\mathbf{y = 0.08,}$ $(du/dy) = 5/0.08^{0.5} = 17.68$, $\tau = \mathbf{17.68 \times \mu}$

In this case the clearance considered is large and so the assumption of linear velocity variation may lead to larger error. The concept that the torque along the radius should be constant can be used to determine the torque more accurately.

Problem 1.11. *A hollow cylinder of 12 cm ID filled with fluid of viscosity 14 cP rotates at 600 rpm. A shaft of diameter 4 cm is placed centrally inside.* ***Determine the shear stress*** *on the shaft wall.*

The hollow cylinder rotates while the shaft is stationary. Shear stress is first calculated at the hollow cylinder wall (Assume 1 m length).

Solution is obtained from basics. Linear velocity variation is assumed. Clearance,

$$h = 0.04 \text{ m},\ \mu = 14 \times 0.001 = 0.014 \text{ N/m}^2$$

At the inside wall of the hollow cylinder,

$$u = 2\,\pi Rn/60 = 3.77 \text{ m/s}$$

$$(du/dr) = u/h = 3.77/0.04 = 94.25/\text{s},\ \tau = \mu\,(du/dr)$$

$$= 0.014 \times 94.25 = 1.32 \text{ N/m}^2$$

$$F = \pi \times D \times L \times \tau = \pi \times 0.12 \times 1 \times 1.31 = 0.498 \text{ N}$$

$$\text{torque} = F \times R = 0.498 \times 0.06 = 29.86 \times 10^{-3} \text{ Nm}$$

Torque at all radii should be the same. At mid radius $R = 0.04$ m, the velocity gradient is obtained by using this concept.

$$29.86\ 10^{-3} = \left.\frac{du}{dr}\right|_{0.04} \times 0.014 \times \pi \times 0.08 \times 1 \times 0.04,$$

Solving, $$\left.\frac{du}{dr}\right|_{0.04} = 212.06/\text{s},$$

This can be checked using equation, (see problem 1.13)

$$\left.\frac{du}{dr}\right|_{R_1} = \left.\frac{du}{dr}\right|_{R_2} \times (R_2^{\ 2}/R_1^{\ 2}) \text{ at } 0.04,\ \left.\frac{du}{dr}\right|_{0.04} \times 25 \times 0.06^2/0.04^2 = 212.06/\text{s}$$

The velocity gradient at the shaft surface = $94.25 \times 0.06^2/0.02^2 = 848.25$/s

Shear stress at the shaft wall = 848.25×0.014 = **11.88 N/m².**

Problem 1.12. *The velocity along the radius of a pipe of 0.1 m radius varies as $u = 10 \times [1 - (r/0.1)^2]$ m/s. The viscosity of the fluid is 0.02 Ns/m². **Determine the shear stress** and the shear force over the surface at r = 0.05 and r = 0.1 m.*

$$\tau = \mu\,(du/dr),\ u = 10 \times [1 - (r/0.1)^2],$$

$$du/dr = -\ 10 \times (2 \times r/0.1^2) = -\ 2000\ r$$

(the –ve sign indicates that the force acts opposite to the flow direction.)

$$\tau = 0.02 \times (-\ 2000) \times r = -\ 40\ r, \text{ Shear force } F = 2\pi rL\tau, \text{ Considering } L = 1$$

At r = 0.05, $\tau = -\ 2$ N/m², F = 0.628 N

At r = 0.1, $\tau = -\ 4$ N/m², F = 2.513 N.

Problem 1.13. *A sleeve surrounds a shaft with the space between them filled with a fluid. Assuming that when the sleeve rotates velocity gradient exists only at the sleeve surface and when the shaft rotates velocity gradient exists only at the shaft surface, determine the ratio of these velocity gradients.*

The torque required for the rotation will be the same in both cases. Using general notations,

$$\tau_i\,[2\pi\,r_i \times L] \times r_i = \tau_o\,[2\pi\,r_o \times L] \times r_o$$

$$\tau_i = \mu\,(du/dr)_{ri},\ \tau_o = \mu\,(du/dr)_{ro}$$

Substituting in the previous expression and solving

$$(du/dr)_i = (du/dr)_o \times [r_o^{\ 2}/r_i^{\ 2}]$$

This will plot as a second degree curve. When the gap is large % error will be high if linear variation is assumed.

Problem 1.14. *Derive an expression for the force required for axial movement of a shaft through a taper bearing as shown in figure. The diameter of the shaft is D m and the length is L m. The clearance at the ends are* t_1*m and* t_2*m. The oil has a viscosity of* μ *and the shaft moves axially at a velocity u.*

In this case the clearance varies along the length and so the velocity gradient will vary along the length. Hence the shear stress also will vary along the length. The total force required can be determined by integrating the elemental force over a differential length dX. The clearance, t at location X is obtained, assuming $t_1 > t_2$,

$$t = t_1 - (t_1 - t_2) \times (X/L) = \{(t_1 \times L) - (t_1 - t_2)\,X\}/L$$

$$du/dy = u/t = u \times L/\{(t_1 \times L) - (t_1 - t_2) \times X\}$$

The velocity gradient at this location is u/t, assumed linear.

$$\tau = \mu\,(du/dy),\ dF = \tau\,dA = \tau \times \pi \times D \times dX, \text{ substituting}$$

$$dF = [\{L \times \mu \times u \times \pi \times D\}] \times [dX/\{(t_1 \times L) - (t_1 - t_2) \times X\}]$$

Integrating between the limits $X = 0$ to $X = L$

$$\mathbf{F = [\{\pi \times D \times u \times L \times \mu\}/\{t_1 - t_2\}] \times [\ln(t_1/t_2)]}$$

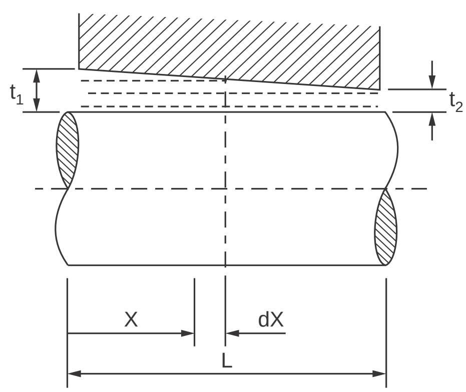


Figure P.1.14

Problem 1.15. *The clearance between the shaft of 100 mm dia and the bearing varies from 0.2 mm to 0.1 mm over a length of 0.3 m. The viscosity of the oil filling the clearance is* 4.8×10^{-2} *Ns/m*^{2}*. The axial velocity of the shaft is 0.6 m/s.* ***Determine the force required.***

Using the equation derived in the previous problem as given below and substituting the values $F = [\{\pi \times D \times u \times L \times \mu\}/\{t_1 - t_2\}]\ [\ln(t_1/t_2)]$

$\mathbf{F} = [\{\pi \times 0.1 \times 0.6 \times 0.3 \times 4.8 \times 10^{-2}\}/\{0.0002 - 0.0001\}] \times [\ln(0.0002/0.0001)] = \mathbf{18.814\ N}$

If the clearance was uniform, $F = \pi \times D \times L \times u \times \mu/t$

For $t = 0.2$ mm, $F_{0.2} = 13.572$ N, For $t = 0.1$ mm, $F_{0.1} = 27.143$ N

The arithmetic average is 20.36 N, while the logarithmic average is what is determined in this problem, 18.814 N.

Problem 1.16. ***Derive an expression for the torque required*** *to overcome the viscous resistance when a circular shaft of diameter D rotating at N rpm in a bearing with the clearance t varying uniformly from t_1 m at one end to t_2m at the other end. The distance between the ends is L m. The oil has a viscosity of μ.*

In this case the clearance varies along the length and so the velocity gradient (du/dr) will vary along the length. Hence the shear stress and the torque also will vary along the length. The total torque required can be determined by integrating the elemental torque over a differential length dX.

The clearance, t at location X is obtained, assuming $t_1 > t_2$,

$$t = t_1 - (t_1 - t_2) \times (X/L) = \{(t_1 \times L) - (t_1 - t_2) \times X\}/L$$

The velocity gradient at this location X is u/t, as linear profile is assumed.

$$\therefore \quad du/dy = u/t = u \times L/\{(t_1 \times L) - (t_1 - t_2) \times X\}$$

$$\tau = \mu\,(du/dy),\ dF = \tau\, dA = \tau \times \pi \times D \times dX,\ \text{ substituting}$$

$$dF = [\{L \times \mu \times u \times \pi \times D\}] \times [dX/\{(t_1 \times L) - (t_1 - t_2) \times X\}]$$

Torque = $dF \times (d/2)$ and $u = (\pi\, DN)/60$. Substituting and Integrating between the limits

$X = 0$ to $X = L$, Torque $= \mathbf{[\{\pi^2 \times D^3 \times L \times N \times \mu\}/\ \{120(t_1 - t_2)\}] \times [\ln\ (t_1/t_2)]}$

Power = $2\pi NT/60$, hence

$$\mathbf{P = [\{\pi^3 \times D^3 \times L \times N^2 \times \mu\}/\{3600(t_1 - t_2)\}] \times [\ln\ (t_1/t_2)].}$$

Problem 1.17 *The clearance between the shaft of 100 mm dia and the bearing varies from 0.2 mm to 0.1 mm over a length of 0.3 m. The viscosity of the oil filling the clearance is 7.1 × 10^{-2} Pa.s (Ns/m^2). The shaft runs at 600 rpm.* ***Determine the torque and power required.***

Using the equations derived in the previous problem as given below and substituting the values

$$T = [\{\pi^2 \times D^3 \times L \times N \times \mu\}/\{120(t_1 - t_2)\}] \times [\ln(t_1/t_2)]$$

$$P = [\{\pi^3 \times D^3 \times L \times N^2 \times \mu\}/\{3600(t_1 - t_2)\}] \times [\ln(t_1/t_2)]$$

$$\mathbf{T} = [\{\pi^2 \times 0.1^3 \times 0.3 \times 600 \times 7.1 \times 10^{-2}\}/\{120(0.0002 - 0.0001)\}] \times [\ln\ (0.0002/0.0001)]$$

$$= \mathbf{7.29\ Nm.}$$

$$\mathbf{P} = [\{\pi^3 \times 0.1^3 \times 0.3 \times 600^2 \times 7.1 \times 10^{-2}\}/\{3600(0.0002 - 0.0001)\}] \times [\ln\ (0.0002/0.0001)]$$

$$= \mathbf{457.8\ W.}$$

Check: $P = 2\pi \times 600 \times 7.29/60 = 458\text{W}$.

Problem 1.18. ***Determine the capillary depression*** *of mercury in a 4 mm ID glass tube. Assume surface tension as 0.45 N/m and β =115°.*

The specific weight of mercury = 13550 × 9.81 N/m^3, Equating the surface force and the pressure force, $[h \times \gamma \times \pi D^2/4] = [\pi \times D \times \sigma \times \cos\beta]$, Solving for h,

$$h = \{4 \times \sigma \times \cos\beta\}/\{\gamma \times D\} = [4 \times 0.45 \times \cos 115]/[13550 \times 9.81 \times 0.004]$$

$$= \mathbf{-\ 1.431 \times 10^{-3}\ m \quad or \quad -\ 1.431\ mm,\ (depression)}$$

Problem 1.19. *A ring 200 mm mean dia is to be separated from water surface as shown in figure. The force required at the time of separation was 0.1005 N.* ***Determine the surface tension of water.***

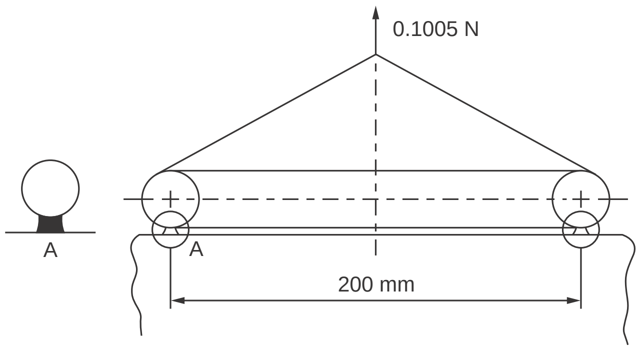


Figure P.1.19

The total length of contact just before lifting from the surface will be twice the circumference or $2\pi D$. The force will equal the product of surface tension and the length of contact.

$\sigma \times 2 \times \pi \times 0.2 = 0.1005$ N. Solving $\boldsymbol{\sigma = 0.08}$ **N/m**

The surface tension of a liquid can be measured using this principle provided the fluid wets the surface.

Problem 1.20. *A thin plate 1 m wide is slowly lifted vertically from a liquid with a surface tension of 0.1 N/m. Determine what force will be required to overcome the surface tension. Assume* $\beta = 0$.

The total length of contact just before separation from the surface will be twice the width of the plate or $2L$. The force will equal the product of surface tension and the length of contact.

$$\mathbf{F = 2 \times 1 \times 0.1 = 0.02\ N.}$$

Problem 1.21. *Diesel injection nozzle sprays fuel with an average diameter of 0.0254 mm. The surface tension is 0.0365 N/m. Determine the pressure difference between the inside and outside of the nozzle. Also determine the pressure difference if the droplet size is reduced to 10 µm.*

A droplet forms at the mouth of the nozzle. The pressure inside the droplet will be higher compared to that at outside.

The equation applicable is $(P_i - P_o) = 2\sigma/R$.

So $(P_i - P_o) = \{2 \times 0.0365 \times 2\}/\{0.0254 \times 10^{-3}\} = \mathbf{5748\ N/m^2 = 5.748\ kN/m^2}$

When the droplet size is reduced to 10 µm the pressure difference is

$$(P_i - P_o) = \{2 \times 0.0365 \times 2\}/\{10 \times 10^{-6}\} = \mathbf{14600\ N/m^2 = 14.6\ kN/m^2.}$$

Problem 1.22. *A glass tube of 8 mm ID is immersed in a liquid at 20°C. The specific weight of the liquid is 20601 N/m³. The contact angle is 60°. Surface tension is 0.15 N/m.* ***Calculate the capillary rise*** *and* ***also the radius of curvature of the meniscus****.*

Capillary rise, $h = \{4 \times \sigma \times \cos\beta\}/\{\gamma \times D\} = \{4 \times 0.15 \times \cos 60\}/\{20601 \times 0.008\}$

$= \mathbf{1.82 \times 10^{-3}\ m}$ **or 1.82 mm.**

The meniscus is a doubly curved surface with **equal radius** as the section is circular. (using equation 1.10.3)

$$(P_i - P_o) = \sigma \times \{(1/R_1) + (1/R_2)\} = 2\,\sigma/R$$

$$R = 2\sigma/(P_i - P_o),\ (P_i - P_o) = \text{specific weight} \times h$$

So, $R = [2 \times 0.15]/\ [1.82 \times 10^{-3} \times 2060] = \mathbf{8 \times 10^{-3}\ m}$ **or 8 mm.**

Problem 1.23. *A mercury column is used to measure the atmospheric pressure. The height of column above the mercury well surface is 762 mm. The tube is 3 mm in dia. The contact angle is 140°.* ***Determine the true pressure in mm*** *of mercury if surface tension is 0.51 N/m. The space above the column may be considered as vacuum.*

In this case capillary depression is involved and so the true pressure = mercury column + capillary depression.

The specific weight of mercury = 13550 × 9.81 N/m^3, equating forces,

$$[h \times \gamma \times \pi D^2/4] = [\pi \times D \times \sigma \times \cos\beta].$$

So $h = \{4 \times \sigma \times \cos\beta\}/\{\gamma \times D\}$

$$h = (4 \times 0.51) \times \cos 140]/[13550 \times 9.81 \times 0.003]$$

$= \mathbf{-3.92 \times 10^{-3}\ m}$ **or – 3.92 mm, (depression)**

Hence actual pressure indicated = 762 + 3.92 = **765.92 mm** of mercury.

Problem 1.24. ***Calculate the pressure difference*** *between the inside and outside of a soap bubble of 2.5 mm dia if the surface tension is 0.022 N/m.*

The pressure difference in the case of a sphere is given by, equation 1.10.5

$$(P_i - P_o) = 2\sigma/R = \{2 \times 0.022\}/\{0.0025\} = \mathbf{17.5\ N/m^2}.$$

Problem 1.25. *A hollow cylinder of 150 mm OD with its weight equal to the buoyant forces is to be kept floating vertically in a liquid with a surface tension of 0.45 N/m^2. The contact angle is 60°. Determine the additional force required due to surface tension.*

In this case a capillary rise will occur and this requires an additional force to keep the cylinder floating.

Capillary rise, $h = \{4 \times \sigma \times \cos\beta\}/\{\gamma \times D\}$.

As $(P_i - P_o) = h \times \text{specific weight},\ (P_i - P_o) = \{4 \times \sigma \times \cos\beta\}/D$

$$(P_i - P_o) = \{4 \times 0.45 \times \cos 60\}/\{0.15\} = 6.0\ \text{N/m}^2$$

$$\text{Force} = \text{Area} \times (P_i - P_o) = \{\pi \times 0.015^2/4\} \times 6 = 0.106\ \text{N}$$

As the immersion leads to additional buoyant force the force required to kept the cylinder floating will be double this value.

So the additional force = 2 × 0.106 = **0.212 N.**

Problem 1.26. *The volume of liquid in a rigid piston—cylinder arrangement is 2000 cc. Initially the pressure is 10 bar. The piston diameter is 100 mm. Determine the distance through which the piston has to move so that the pressure will increase to 200 bar. The temperature remains constant. The average value of bulk modulus for the liquid is 2430 × 10^{-6} N/m^2.*

By definition—refer eqn 1.11.1

$$E_v = -\,dP/(dv/v) = -\,(P_2 - P_1)/[(v_2 - v_1)/v_1]$$

So $\quad 2430 \times 10^6 = -190 \times 10^5/(dv/0.002)$, Solving,

$$dv = -\,0.002 \times 190 \times 10^5/2430 \times 10^6 = 15.64 \times 10^{-6}\ \text{m}^3$$

Piston movement, $L = dv/\text{area}$

$$L = dv \times 4/\pi D^2 = 15.64 \times 10^{-6} \times 4/\pi \times 0.1^2 = 1.991 \times 10^{-3}\ \text{m} = \mathbf{1.991\ mm}$$

(the piston-cylinder arrangement is assumed to be rigid so that there is no expansion of the container)

Problem 1.27. *The pressure of water increases with depth in the ocean. At the surface, the density was measured as 1015 kg/m³. The atmospheric pressure is 1.01 bar. At a certain depth, the pressure is 880 bar.* ***Determine the density of sea water at the depth.*** *The average value of bulk modulus is 2330 × 10⁶ N/m².*

The density will increase due to the pressure increase.

Bulk modulus is defined in eqn 1.11.1 as $E_v = -\,dP/(dv/v) = -\,(P_2 - P_1)/[v_2 - v_1)/v_1]$,

$$[(v_2 - v_1)/v_1] = -\,(P_2 - P_1)/E_v = -\,[880 \times 10^5 - 1.01 \times 10^5]/2330 \times 10^6 = -0.03772$$

$v_1 = 1/1015\ \text{m}^3/\text{kg}$, substituting the values in

$$v_2 = [v_1 \times \{-\,(P_2 - P_1)/E_v\}] + v_1,$$

$$v_2 = [-\,0.03772 \times (1/1015)] + (1/1015) = 9.48059 \times 10^{-4}\ \text{m}^3/\text{kg}$$

Density $= 1/(9.48059 \times 10^{-4}\ \text{m}^3/\text{kg}) =$ **1054.79 kg/m³** an increase of 4%.

The density increases by 4.0% due to the increase in pressure.

$[(v_2 - v_1)/v_1]$ also equals $[(\rho_1 - \rho_2)/\rho_2] = [(P_2 - P_1)/E_v]$

Use of this equation should also give the same answer.

Problem 1.28. *A diesel fuel pump of 10 mm ID is to deliver against a pressure of 200 bar. The fuel volume in the barrel at the time of closure is 1.5 cc. Assuming rigid barrel determine the plunger movement before delivery begins. The bulk modulus of the fuel is 1100 × 10⁶ N/m².*

By definition—eqn 1.11.1—the bulk modulus is $E_v = -\,dP/(dv/v)$,

$$1100 \times 10^6 = -\,200 \times 10^5/(dv/1.5 \times 10^{-6}),\ \text{Solving}\ dv = -\,2.77 \times 10^{-8}\ \text{m}^3$$

Plunger movement $= dv/\text{area} = -\,2.77 \times 10^{-8} \times 4/(\pi \times 0.0015^2)$

$$= 3.47 \times 10^{-4}\ \text{m} = \mathbf{0.347\ mm}$$

(the pressure rise will also be affected by the expansion of the pipe line).

OBJECTIVE QUESTIONS

O Q 1.1 Fill in the blanks with suitable words:

1. Cohesive forces between molecules/atoms are highest in the _________ phase.
2. When the applied load is released solids _________.
3. Solids _________ applied shear while liquids _________.

4. In solids _________ is proportional to the applied stress.
5. The mobility of atoms is least in _________.
6. The distance between atoms is least in _________.
7. _________ have specific shape that does not change by itself.
8. When heated the atoms in solids _________.
9. In some solids molecules come out when heated. The phenomenon is called _________.
10. For solids the proportionality limit between deformation and stress is called _________.

Answers

(1) Solid (2) regain their original shape (3) resist, continue to deform (4) deformation (5) Solids (6) Solids (7) Solids (8) vibrate more (9) Sublimation (10) Elastic limit.

O Q 1.2 Fill in the blanks with suitable words:

1. Fluids cannot withstand _________.
2. Fluids _________ to deform when a shear force is applied.
3. The atoms/molecules are _________ to move in fluids.
4. In liquids _________ is proportional to shear stress.
5. The difference between liquids and gases is _________.
6. Liquids form a _________ when in a container.
7. Gases _________ the container.
8. The distance between molecules is highest in _________.
9. Cohesive faces between atoms is least in _________.
10. Vapour is the gaseous state of matter when the temperature is near the _________.

Answers

(1) shear force (2) continue (3) free (4) rate of deformation (5) that the atomic molecular spacing is much larger in gas and atoms move all over the container filling it (6) free surface (7) completely fill (8) gases (9) gases (10) saturation conditions (Boiling conditions)

O Q 1.3 Fill in the blanks with suitable words:

1. The three phases of matter are _________ .
2. A special state of matter at very high temperatures is _________.
3. Density is defined as _________.
4. Specific weight is defined as _________.
.5. Specific gravity is defined as _________.
6. A fluid is defined as _________.
7. A liquid is defined as _________.
8. A vapour is defined as _________.
9. A gas is defined as _________.
10. A mole is defined as _________.

Answers

(1) Solid, liquid & gas (2) plasma (3) mass per unit volume (4) force due to gravity on mass in unit volume (5) ratio of mass of substance/mass of water at 10°C per unit volume (6) Material which cannot resist shear stress or material which will continuously deform under applied shear stress (7) A material which will exhibit a free surface in a container (8) gaseous state

very near the formation temperature at that pressure (9) material with low cohesive force with large distance between molecules which will occupy the full volume of the container (10) Molecular mass of a substance.

O Q 1.4 Fill in the blanks with suitable words:

1. Bulk modulus is defined as _________.
2. Bulk modulus of gases depend on _________.
3. Bulk modulus of liquid will _________ with pressure.
4. Liquids have _________bulk modulus.
5. Unit of bulk modulus is the same as that of _________.
6. The concept of bulk modulus is used in the analysis of _________ propagation in the medium.
7. Viscosity is defined as _________.
8. Kinematic viscosity is defined as _________.
9. An ideal fluid is defined as _________.
10. A Newtonian fluid is defined as one having _________.

Answers

(1) $-dp/(dv/v)$ (2) the process of change (3) increase (4) high (5) pressure (6) sound (7) $\mu = \tau/(du/dy)$ the proportionality constant between shear stress and velocity gradient (8) μ/ρ (9) one with no viscosity or compressibility (10) A constant viscosity irrespective of the velocity gradient.

O Q 1.5 Fill in the blanks with suitable words:

1. A non Newtonian fluid is defined as _________.
2. An ideal plastic is defined as _________.
3. A thixotropic fluid is defined as _________.
4. Surface tension is defined as _________.
5. Vapour pressure is defined as _________.
6. Surface tension is due to _________ forces.
7. Capillary rise is caused by _________ forces.
8. Capillary rise is when _________ forces predominate.
9. Capillary depression is when _________ forces predominate.
10. Droplet formation and free circular jet formation is due to _________.

Answers

(1) a fluid whose viscosity varies with the velocity gradient (2) a material which requires a definite shear to cause the first deformation but then the stress is proportional to the velocity gradient (3) A substance whose viscosity increases with increase in velocity gradient (4) Work required to create a unit area of free surface in a liquid/force required to keep unit length of free surface in equilibrium (5) The pressure over the fluid due to the vapour over a liquid under equilibrium conditions of temperature (6) Cohesive (7) Adhesive forces (8) Adhesive (9) Cohesive (10) Surface tension

O Q 1.6 Fill in the blanks with "increasing " or "decreasing" or "remains constant":

1. When gravitational force increases specific weight _________.
2. When gravitational force decreases specific weight _________.
3. When gravitational force increases density _________.
4. When gravitational force decreases density _________.

5. The specific gravity _________ when density increases.
6. As molecular weight of a gas increases its gas constant _________.
7. The product of gas constant and molecular weight _________.
8. At constant temperature, the pressure exerted by a gas in a container _________ when the volume increases.
9. Bulk modulus of liquids _________ with increase in pressure at constant temperature.
10. At constant pressure the bulk modulus of liquids (*a*) _________ and then (*b*) _________ with increase in temperature.

Answers

Increases 1, 5, 9, 10a **Decrease 2, 6, 8, 10b** **Remains constant 3, 4, 7**

O Q 1.7 Fill in the blanks with "increases", "decreases" or "remains constant"

1. Viscosity of liquids _________ with increase of temperature.
2. Viscosity of gases _________ with increase of temperature.
3. As tube diameter decreases the capillary rise _________.
4. As tube diameter increases the capillary rise _________.
5. As the diameter of a bubble increases the pressure difference between inside and outside _________.
6. As the diameter decreases the pressure difference between inside and outside of a free jet _________.
7. At a given temperature the vapour pressure for a liquid _________.
8. As temperature increases, the vapour pressure _________.
9. The vapour pressure over a liquid _________ when other gases are present in addition to the vapour.
10. As cohesive force _________ compared to adhesive forces, the capillary will rise.

Answers

Increases 2, 3, 6, 8 **Decreases 1, 4, 5, 10** **Remains constant 7, 9**

O Q 1.8 Indicate whether the statements are correct or incorrect.

1. Density is the ratio of mass of unit volume of liquid to the mass of unit volume of water.
2. In the gas equation temperature should be used in Kelvin scale.
3. Specific weight is the mass of unit volume.
4. The cohesive forces are highest in gases.
5. The shear force in solid is proportional to the deformation.
6. In fluids the shear force is proportional to the rate of deformation.
7. Newtonian fluid is one whose viscosity will increase directly with rate of deformation.
8. The vapour pressure will vary with temperature.
9. Ideal fluid has zero viscosity and is incompressible.
10. Gases can be treated as incompressible when small changes in pressure and temperature are involved.

Answers

Correct 2, 5, 6, 8, 9, 10 Incorrect 1, 3, 4, 7

O Q 1.9 Indicate whether the statements are correct or incorrect.

1. Specific weight of a body will vary from place to place.
2. Mass is measured by a spring balance.
3. Specific weight is measured by a spring balance.
4. The weight of man will be lower on the moon.
5. The weight of a man will be higher in Jupiter.
6. Dynamic viscosity is a measure of momentum diffusivity.
7. Viscosity of liquids increases with temperature.
8. Viscosity of gases increases with temperature.
9. Higher the surface tension higher will be the pressure inside a bubble.
10. The head indicated by a mercury manometer is lower than the actual value.
11. The head indicated by a water manometer is lower than the actual value.

Answers

Correct 1, 3, 4, 5, 8, 9, 10 Incorrect 2, 6, 7, 11

O Q 1.10 The following refer either to viscosity effects or surface tension effects. Classify them accordingly.

1. Capillary rise
2. Drag
3. Liquid bubble
4. Free jet
5. Heating of lubricating oil in bearings
6. Free surface of liquids
7. Momentum transfer
8. Gas flow

Answers

Surface tension 1, 3, 4, 6 Viscosity 2, 5, 7, 8

O Q 1.11 Choose the correct answer.

1. The mass of an object is 10 kg. The gravitational acceleration at a location is 5 m/s^2. The specific weight is
   (*a*) 2 N (*b*) 15 N (*c*) 5 N (*d*) 50 N
2. The dynamic viscosity is 1.2×10^{-4} Ns/m^2. The density is 600 kg/m^3. The kinematic viscosity in m^2/s is
   (*a*) 72×10^{-3} (*b*) 20×10^{-8} (*c*) 7.2×10^{3} (*d*) 70×10^{6}
3. The velocity gradient is 1000/s. The viscosity is 1.2×10^{-4} Ns/m^2. The shear stress is
   (*a*) 1.2×10^{-1} N/m (*b*) 1.2×10^{-7} N/m^2 (*c*) 1.2×10^{2} N/m^2 (*d*) 1.2×10^{-10} N/m^2
4. The velocity distribution in a flow through a tube is given by $u = (-10/\mu)(0.01 - r^2)$. The pipe radius $R = 0.1$ m. The shear stress at the wall in N/m^2 is
   (*a*) $10/\mu$ (*b*) 0 (*c*) 2μ (*d*) $2/\mu$
5. The excess pressure in a droplet of 0.002 m dia a fluid with surface tension of 0.01 N/m is
   (*a*) 10 (*b*) 20 (*c*) 4π (*d*) $0.00004\ \pi$

Answers

(1) *d* (2) *b* (3) *a* (4) *d* (5) *b*

O Q 1.12 Match the pairs.

	Set A	Set B
(*i*)	1. density	A. N/m
	2. surface tension	B. N/m^3
	3. kinematic viscosity	C. kg/m^3
	4. specific weight	D. m^2/s
(*ii*)	1. dynamic viscosity	A. surface tension
	2. capillary rise	B. momentum transfer
	3. kinematic viscosity	C. liquid
	4. free surface	D. shear stress
(*iii*)	1. droplet formation	A. zero viscosity
	2. weight	B. constant viscosity
	3. ideal fluid	C. gravitational acceleration
	4. Newtonian fluid	D. surface tension

Answers

(*i*) 1-C, 2-A, 3-D, 4-B (*ii*) 1-D, 2-A, 3-B, 4-C (*iii*) 1-D, 2-C, 3-A, 4-B

REVIEW QUESTIONS

1. Differentiate between the three states of matter.
2. Distinguish between compressible and incompressible fluids.
3. Distinguish between vapour and gas.
4. Explain the concept of “Continuum”.
5. Define density, specific volume, weight density and specific gravity.
6. Define “Compressibility” and “Bulk Modulus”.
7. State the characteristic equation for gases and explain its significance.
8. Derive the general expression for compressibility of gases.
9. Define the term viscosity and explain the significance of the same.
10. Distinguish between Newtonian and non Newtonian Fluids.
11. Explain from microscopic point of view the concept of viscosity and momentum transfer. Explain how viscosity of liquids and gases behave with temperature.
12. Define kinematic viscosity and explain the significance of the same.
13. Derive an expression for the torque and power required to overcome the viscous drag for a shaft running at a particular rpm.
14. Derive an expression for the torque required to rotate a collar bearing (disc over a parallel plate).
15. Derive an expression for the torque required to rotate a conical bearing.
16. Describe some methods to determine the viscosity of a fluid.
17. Explain the concepts of (*i*) vapour pressure (*ii*) partial pressure and (*iii*) surface tension.
18. Explain how liquid surface behaves by itself and when it is in contact with other surfaces.
19. Derive an expression for the capillary rise or depression, given the value of the contact angle β and the density and surface tension of the liquid.
20. Derive an expression for the pressure difference caused by surface tension on a doubly curved surface.
21. Derive expressions from basics for the pressure inside a droplet and a free jet.

EXERCISE PROBLEMS

E1.1. Determine the density, specific weight and specific volume of air if the specific gravity (with water as reference fluid) is 0.011614. **(11.614 kg/m^3, 113.94 N/m^3, 0.0861 m^3/kg)**

E1.2. A liquid with kinematic viscosity of 2.7 centistokes fills the space between a large stationary plate and a parallel plate of 500 mm square, the film thickness being 1 mm. If the force required to pull the smaller plate with a uniform velocity of 3 m/s was 1.734 N, determine specific weight of the liquid. Assume that the liquid film is maintained all over. **(8.4 kN/m^3)**

E1.3. Two large plates are 6 mm apart and the space in-between in filled with a fluid. A plate of 1 mm thickness and 10 cm square is pulled parallel to the planes and midway between them with a velocity of 2 m/s. Assume linear velocity profile on either side. The force required was 0.32 N. Determine the viscosity of the fluid. **(2×10^{-2} Ns/m^2)**

E1.4. Two large vertical plane parallel surfaces are 5 mm apart and the space between them is filled with a fluid. A thin plate of 12.5 cm square falls freely between the planes along the central plane and reaches a steady velocity of 2 m/s. Determine the weight of the plate if the viscosity of the fluid filling the space is 0.02 Ns/m^2. **(0.5 N)**

E1.5. Two large planes are parallel to each other and are inclined at 30° to the horizontal with the space between them filled with a fluid of viscosity 20 cp. A small thin plate of 0.125 m square slides parallel and midway between the planes and reaches a constant velocity of 2 m/s. The weight of the plate is 1 N. Determine the distance between the plates. **(5 mm)**

E1.6. A hydraulic lift shaft of 500 mm dia moves in a cylindrical sleeve the length of engagement being 2 m. The interface is filled with oil of kinematic viscosity of 2.4×10^{-4} m^2/s and density of 888 kg/m^3 . The drag resistance when the shaft moves at 0.2 m/s is 267.81 N. Determine the *ID* of the cylinder.

E1.7 A shaft of 150 mm dia rotates in bearings with a uniform oil film of thickness 0.8 mm. Two bearings of 15 cm width are used. The viscosity of the oil is 22 cP. Determine the torque if the speed is 210 rpm. **(10.58 Nm)**

E1.8 A circular disc rotates over a large stationary plate with a 2 mm thick fluid film between them, the viscosity of the fluid being 40 cp. The torque required to rotate the disc at 200 rpm was 0.069 Nm. Determine the diameter of the disc. **(200 mm)**

E1.9 The torque to overcome viscous drag of the oil film of viscosity of 28 cp in collar bearing of 0.16 m *ID* and 0.28 m *OD* running at 600 rpm was 0.79 Nm. Determine the film thickness. **(1.2 mm)**

E1.10 A conical bearing of outer radius 0.4 m and inner radius 0.2 m and height 0.2 m runs on a conical support with a clearance of 1 mm all around. The support is rotated at 600 rpm. Determine the viscosity of the oil used if the torque required was 21.326 Nm. Also determine the power dissipated (Fig. 1.9.3). **(20.0 cP, 1340 W)**

E1.11 If $u = 10\,y^{1.5}$ where u is in m/s and y is in m in a flow field up to $y = 0.08$ m, determine the wall shear stress and the shear stress at $y = 0.04$ and 0.08.

E1.12 Determine the pressure difference between two points 10 m apart in flow of oil of viscosity 13.98 cp in a pipe, pipe of 40 mm diameter, the flow velocity being 1.8 m/s. **(5 kN/m^2)**

E1.13 The viscosity of an oil of density 820 kg/m^3 is 30.7 poise. What will be the terminal velocity of a steel ball of density 7800 kg/m^3 and dia 1.1 mm when dropped in the oil? **(90 mm/min)**

E1.14 As shown in figure, a block of mass M slides on a horizontal table on oil film of thickness h and viscosity μ. The mass m causes the movement. Derive an expression for the viscous force on the block when it moves at a velocity u. Also obtain an expression for the maximum speed of the block. **($F = \mu u A/h$, $u_{max} = mgh/\mu A$)**

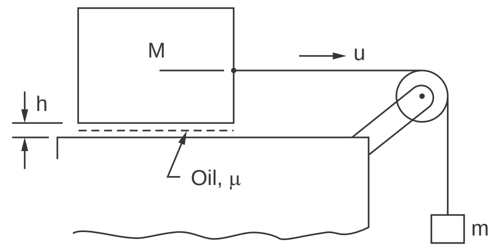


Figure E.1.13

E1.15 A viscous clutch as shown in figure transmits torque. Derive an expression for the torque and power transmitted. **($T = \pi\mu(\omega_1 - \omega_2) R^4/2a$, $P = \pi\mu\omega_2(\omega_1 - \omega_2) R^4/2a$)**

E1.16 Derive an expression for the torque in the case of a spherical bearing as shown in figure, in terms of h, r, ω, α and μ.

$$T = \frac{2\pi\mu\omega R^4}{h}\left[\frac{\cos^2\alpha}{3} - \cos\alpha + \frac{2}{3}\right]$$

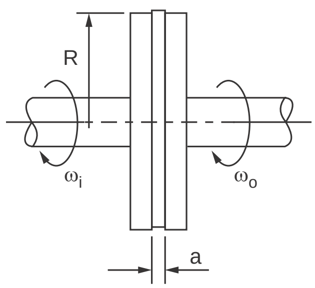


Figure E.1.15

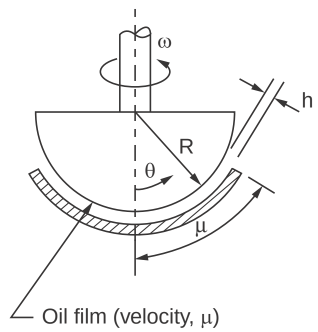


Figure E.1.16

E1.17 Calculate the shear stress due to fluid flow at the plate and at 10 mm above it if the velocity distribution along y direction is $u = 2y - 2y^3 + y^4$. $\mu = 0.001$ Ns/m^2.

(2×10^{-3} N/m^2, 1.94 N/m^2)

E1.18 The capillary depression of mercury in a 3.25 mm *ID* glass tube was found as 2.99 mm. Determine the value of surface tension. $\beta = 129°$. **(0.511 N/m)**

E1.19 In order to separate a ring of 160 mm mean dia from water surface the force required just at the point of separation was 0.0732 N. Determine the surface tension of water at that temperature. **(0.0728 N/m)**

E1.20 In manometers an error in measurement will occur when a small bore tube is used. Capillary rise adds to the column height and capillary depression reduces the column height. The height of water column (at 20°C) in a tube of 8 mm *ID* is 12 mm, out of this 3.17 mm is due to capillary action. Determine the value of surface tension. $\beta = 0$. **(0.0728 N/m)**

E1.21 In order to lift a thin plate 1 m width slowly and vertically from a liquid, a force of 0.2 N was required at the instant of separation to overcome surface tension forces. Assume $\beta = 0$. Determine the value of surface tension. **(0.1 N/m)**

E1.22 Diesel injection nozzle sprays fuel of surface tension 0.0365 N/m. The pressure difference between the inside and outside of the nozzle was 2.874 kN/m^2. Determine the droplet size.

(0.0254 mm)

E1.23 Determine the droplet size if the pressure difference is increased to 7300 N/m^3 in the nozzle of a diesel engine. Assume the value of surface tension as 0.0365 N/m. **(10 μm)**

E1.24 A glass tube of 8 mm *ID* is immersed in a liquid at 20°C. The specific weight of the liquid is 20601 N/m^3. The contact angle is 60°. The capillary rise was 1.82 mm. Determine the value of surface tension and also the radius of curvature of the meniscus. **(0.15 N/m, 8 mm)**

E1.25 The actual atmospheric pressure was 765.92 mm of mercury column. Determine the height of column above the mercury well in a Fortins barometer using a tube of 3 mm dia. The contact angle is 140°. The surface tension is 0.51 N/m. The space above the column may be considered as vacuum. **(762 mm)**

E1.26 If the pressure difference between the inside and outside of a soap bubble of 2.5 mm dia is 17.6 N/m^2, determine the value of surface tension of the soap solution. **(0.022 N/m)**

E1.27 An additional force of 0.212 N was required to keep a cylinder of 150 mm OD with weight equal to the buoyant forces, floating in a liquid with contact angle β = 60° due to surface tension effects. Determine the value of surface tension. **(0.45 N/m)**

E1.28 Show that the capillary rise in an annulus is given by $2\sigma \cos \beta/\gamma (r_o - r_i)$, where r_o and r_i are the radii and σ is the surface tension, γ is the specific weight and β is the contact angle.

E1.29 In case a capillary of diameter 3×10^{-6} m is used, determine the capillary rise in water. σ = 0.0735 N/m. **(10 m)**

E1.30 Bubbles are to be blown using a glass tube of 2 mm diameter immersed in oil to a depth of 10 mm. The specific gravity of oil is 0.96. If the surface tension of the oil is 0.0389 N/m. Determine the pressure inside the bubble at formation. **(172 N/m^2)**

E1.31 When 1000 cc of water is heated in a cylindrical vessel of 100 mm diameter from 20°C to 50°C the increase in the water level was 0.76 mm. Determine the coefficient of linear expansion for the vessel material. For water the coefficient of cubical expansion is 2.1×10^{-4} m^3/m^3 per °C. **(3.6×10^{-6} m/m per °C)**

E1.32 When the pressure of water in a press cylinder is released from 1000×10^5 N/m^2 to 1 bar, there was a 4.11 percentage increase in specific volume while the temperature remained constant. Determine the average value of bulk modulus for water in this range. **(2430×10^6 N/m^2)**

E133 The pressure of water increases with depth in the ocean. At the surface, the density was measured as 1024.5 kg/m^3. The atmospheric pressure is 1.01 bar. At a certain depth where the pressure was 900 bar the density was measured as 1065.43 kg/m^3. Determine the average value of bulk modulus. **(2340×10^6 N/m^2)**

E1.34 When water was heated in a rigid vessel the pressure rise was 14.49×10^6 N/m^2. Assuming that the vessel volume did not increase due to the increase in temperature or due to the stress induced, determine the percentage change in density. Assume $E_v = 2300 \times 10^6$ N/m^2. **(0.63%)**

E1.35 Due to an increase in pressure the volume of a liquid increases by 2.7%. Determine the pressure increase. The bulk modulus of the liquid is 37.04×10^9 N/m^2. **(1000 bar)**

E1.36 Determine the diameter of a spherical balloon at an altitude where pressure and temperature are 0.1 bar and –50°C, if 5.65 kg of hydrogen was charged into the balloon at ground level where the pressure and temperature were 1 bar and 30°C. **(10 m)**