12 Flow in Open Channels

12.0 INTRODUCTION

Flow in rivers, irrigation canals, drainage ditches and aqueducts are some examples for open channel flow. These flows occur with a free surface and the pressure over the surface is atmospheric. The surface actually represents the hydraulic grade line. In most cases water is the fluid encountered in open channel flow. While in closed conduits the flow is sustained by pressure difference, **the driving force in open channel flow is due to gravity, and is proportional to the bed slope**. The depth of flow is not restrained and this makes the analysis more complex. As most of the flow are large in scale and as viscosity of water is lower, Reynolds number are high. Hence the flow is generally turbulent. As seen in chapter 8 and 9, Froude number is the improtant parameter in the general study of open channel flow which is free surface flow. **The balance of gravity forces and surface friction forces controls the flow**. Changes in channel cross-section and changes in the slope cause changes and readjustments in the flow depth which may or not propagate upstream.

12.1.1 Characteristics of Open Channels

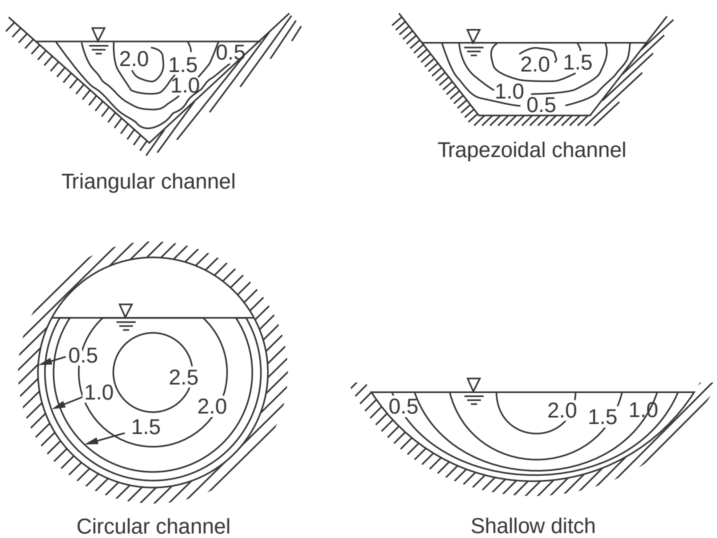


Figure 12.1.1 Velocity distribution in open channel sections

Open channels may have different cross-sections. Some of the simpler ones are **Trapezoidal, Triangular, Rectangular and Circular sections.** When the width is large, it is considered as wide flat.

Natural channels have very irregular sections and suitable approximations should be used for analysis. **The two main physical dimensions used in the analysis are the flow area, A and the wetted perimeter P. The ratio of flow area to the perimeter is defined as hydraulic radius, R_h** (alternately *m*) and is used in all analysis to take care of all types of sections. With this definition laminar flow is limited to Reynolds number up to 500.

This is different from hydraulic mean diameter used in the analysis of flow through conduits which is four times this value. **Hydraulic depth is another term defined as the ratio of flow area to top width.** This represents the average depth of the section. For analysis purposes the average velocity of flow is used and this equals, volume flow rate/area. But actually the flow velocity varies with the depth almost logarithmically, low near the wetted surface and increasing towards the free surface. However the velocity is not maximum at the surface. The maximum velocity occurs below the free surface. **For analysis,** as mentioned earlier, the **average velocity is used.** Some examples of velocity distribution is shown in Fig. 12.1.1. The maximum velocity occurs at about 0.2 times the depth from top. The average velocity occurs between 0.4 and 0.8 times the depth from top.

12.1.2 Classification of Open Channel Flow

The common classification is based on the rate of change of free surface depth. When the **depth and velocity remain constant along the length of flow** it is called **uniform flow**. For such flow the slope and area should be uniform. When the depth changes gradually, due to area or slope changes, it is defined as g**radually varying flow.** If the slope change rapidly or suddenly, then such flow is called **rapidly varying flow.** The slope of the free surface is governed by the way in which the slope and the area change. The value of **Froude number characterises the nature of the flow** in such situations.

12.2 UNIFORM FLOW: (ALSO CALLED FLOW AT NORMAL DEPTH)

This is the simplest and common type of flow and occurs when conditions are steady and slope is not steep. This is also non accelerating flow. Consider the control volume between sections 1 and 2 shown in Fig. 12.2.1 by the dotted line. For steady uniform incompressible flow the height of the water level and area are constant, the hence velocity is constant.

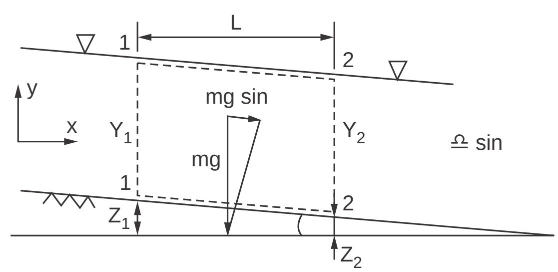


Figure 12.2.1 Flow at normal depth

(i) Continuity equation, $\rho_1 A_1V_1 = \rho_2 A_2V_2$ as $\rho_1 = \rho_2$ and $A_1 = A_2$, $V_1 = V_2$ or the velocity remains constant along the flow

(ii) Momentum equation. Bed slope is defined as the ratio of change in elevation over a length with the length. The bed slope $S_b(= \sin\theta)$ is small and pressure distribution is hydrostatic. As there is no change in the depth of flow or velocity (no acceleration), the momentum flux through the control surface is zero. As the pressure distribution is hydrostatic the net pressure force on the control volume is zero. Only body force due to gravity and friction forces/ on the wetted surfaces, F_f, act on the control volume.

$$-F_f + mg\sin\theta = 0$$

$$\therefore \quad F_f = mg\sin\theta = mg\,S_b \qquad (12.2.1)$$

$mg\sin\theta$ is the component of gravity force parallel to the flow and F_f is the friction force on the wetted surface.

(iii) Energy equation. The sum of potential and kinetic heads between sections 1 and 2 should be the same if there are no losses. Assuming a loss of head of h_L,

$$\frac{V_1^2}{2g} + y_1 + z_1 = \frac{V_2^2}{2g} + y_2 + z_2 + h_L$$

For the steady flow $V_1 = V_2$, $y_1 = y_2$ and for a length L,

$$\therefore \quad h_L = z_1 - z_2 = L\,S_b \qquad (12.2.2)$$

The head loss due to friction in steady flow between two sections equals the change in elevation of the bed. Defining specific energy, *E*, at a section by the sum $(V^2/2g) + y$, the specific energy is constant along this type of flow. The energy grade line, hydraulic grade line and the channel bed are all parallel.

12.3 CHEZY'S EQUATION FOR DISCHARGE

Considering the control volume shown in Fig. 12.2, the force balance yields frictional resistance over the wetted surface equals the component of the gravity forces along the surface, on the volume.

Gravity force component $= mg\,A\,L\sin\theta$

Frictional force $= \tau_w\,PL$

(tan θ = sin θ = θ in radians for small angles)

Equating the forces $\tau_w\,PL = \rho g\,AL\sin\theta \qquad (12.3.1)$

or $$\tau_w = \rho g\,(A/P)\sin\theta = \rho\,g\,R_h\,S_b \qquad (12.3.2)$$

This is the reason for defining hydraulic radius as A/P. For large values of Reynolds number the friction factor is independent of Reynolds number and wall shear stress is proportional to the dynamic pressure $\rho V^2/2g$ and is independent of viscosity. Hence

$\tau_w = K\rho\,V^2/2\,g_o$ where K is a constant of proportionality.

Substituting in equation 12.3.2,

$$K\frac{\rho V^2}{2\,g_o} = \rho\,g\,R_h\,S_b$$

Denoting $2gg_o/K$ by constant C,

$$V = C\sqrt{R_h S_b} \tag{12.3.3}$$

This equation published in 1775 is known as **Chezy's** equation and the constant C is known as Chezy's constant. (for R_h and S_b other symbols like m, i etc are also used).

Flow rate $$Q = \rho A V = C \rho A (R_h S_b)^{0.5} \tag{12.3.4}$$

It may be noted that Chezy coefficient C is not dimensionless. It has a dimension of (length$^{1/2}$/ time) and hence will have different numerical values in different systems of units.

Another method of deriving the equation is as below. As in the case of closed duct flow, the friction head loss is given by

$$h_L = f L V^2/ 2 gD,$$

Substituting $D = 4R_h$, and $h_L = L S_b$,

$$V = \left(\frac{8g}{f}\right)^{0.5} (R_h S_b)^{0.5}$$

Denoting constant $$C = (8g/f)^{0.5} \tag{12.3.5}$$

$$V = C (R_h S_b)^{0.5} \tag{12.3.6}$$

Example 12.1 ***Determine the flow rate of water through a rectangular channel 3 m wide with a flow depth of 1 m. The bed slope is 1 in 2500. f = 0.038.***

wetted perimeter, $P = 3 + (2 \times 1) = 5$ m, Area $= 3 \times 1 = 3$ m^2

$\therefore$ $R_h = 3/5 = 0.6$ m, $S_b = 1/2500$, $C = \left(\frac{8g}{f}\right)^{0.5} = \left(\frac{8 \times 9.81}{0.038}\right)^{0.5} = 45.455$

$\therefore$ $V = 45.445\ (0.6 \times 1/2500)^{0.5} = 0.704$ m/s,

$\therefore$ **Flow rate** $= 3 \times 1 \times 0.704 =$ **2.112 m^3/s**

12.4 DETERMINATION OF CHEZY'S CONSTANT

The Chezy's equation is simple but the determination of the constant C is rather involved. Several correlations have been suggested from experimental measurements for obtaining the value of Chezy's constant C. Out of these, three are more popular namely Bazins, Kutters and Mannings. As the first two correlations are more complex, Mannings correlation is generally used in designs.

12.4.1 Bazin's Equation for Chezy's Constant

The equation suggested by Bazin is given below.

$$C = 86.9/(1 + k/\sqrt{R_h}\ ) \tag{12.4.1}$$

where k is known as Bazins constant. The value varies from 0.11 for smooth cement surfaces to 3.17 for earthen channel in rough condition. For Brick lined channel the value is about 0.5. This correlation is independent of bed slope.

Example 12.2 *Calculate the value of the Chezy's constants using Bazins equation in the case of a rectangular channel 3 m wide and 1 m deep for the following conditions. For a bed slope of 1/2500 find the flow rate. The value of Bazins constant, k, are given below.*

(i) Smooth cement lining	:	*0.06*
(ii) Smooth brick	:	*0.16*
(iii) Rubble masonry	:	*0.46*
(iv) Earthen channel is ordinary condition	:	*1.303*
(v) Earthen channel in rough condition	:	*1.75*

$R_h = 3/5 = 0.6.$

Using the above values, the constant C is calculated by the equations below:

$$C = 86.9/(1 + k/\sqrt{0.6}),\ V = C\sqrt{0.6/2500}$$

These values of Bazins constant k can be taken as typical values

S. No.	Nature of Surface	Bazins Constant	Chezy's Constant	V, m/s	Q, m^3/s
1	Smooth cement lining	0.060	**80.65**	1.250	3.75
2	Smooth brick	0.160	**72.02**	1.116	3.35
3	Rubble masonry	0.460	**54.52**	0.847	2.53
4	Earthen channel in ordinary condition	1.303	**32.40**	0.502	1.51
5	Earthen channel in rough condition	1.750	**26.66**	0.413	1.24

12.4.2 Kutter's Equation for Chezy's Constant C

$$C = \frac{23 + (0.00155/S_b) + (1/N)}{1 + (23 + 0.00155/S_b)(N + R_h^{0.5})} \qquad (12.4.2)$$

Where N is Kutter's constant, the value of which varies from 0.011 for smooth cement surface to 0.08 for poorly maintained earthen channels.

Example 12.3 *Determine* ***the flow rate*** *for a rectangular channel 3 m wide and 1 m deep with a slope of 1/2500. Using the values of Kutters constant and conditions of surface given in the tabulation.*

The results are also tabulated. Equation (12.4.2) is used for the determination of C. Equation (12.3.6) is used to calculate V. R_h = 0.06 m, A = 3 m^2, S_b = 1/2500

S. No.	Type of surface	Kutters const, N	Chezy's Constant	V, m/s	Q, m^3/s
1	Smooth cement lining	0.110	85.25	1.32	**3.96**
2	Smooth concrete	0.013	71.53	1.11	**3.32**
3	Rough brick	0.015	61.52	0.95	**2.86**
4	Rubble masonry/Rough concrete	0.017	53.90	0.84	**2.51**
5	Clean earthen channel	0.018	50.74	0.79	**2.36**
6	Earthen channel after weathering	0.022	41.02	0.64	**1.91**
7	Earthen channel rough with bush	0.050	1714	0.27	**0.80**

It can be seen that as N increases the flow decreases for the same slope. The values of N are available for a few more conditions, particularly for earthen channels and natural streams. Note that in examples 2.2 and 2.3 there is good agreement between the results. Due to simplicity and availability of extensive experimental support Mannings correlation is more popularly used.

12.4.3 Manning's Equation for C

In 1890 Robert Manning proposed in place of the relation given in equation (12.3.5), $C = (8g/f)^{1/2}$ that

$$C = (1/N)\, R_h^{1/6} \tag{12.4.3}$$

where N is Mannings constant established by experiments for various types of surfaces. When combined with Chezy's equation (12.3.6), this leads to

$$V = (1/N)\, R_h^{2/3}\, S_b^{1/2} \tag{12.4.4}$$

The values of N is generally a small fraction varying from 0.011 to 0.06. Sometimes the reciprocal of N is also referred to as Mannings constant. In this case the value will be in the range 16 to 90.

Example 12.4 *For a rectangular channel 3 m wide and 1 m deep with a slope of 1 to 2500 determine the* ***values of Chezy's constant and also the flow rate****. The types of surface with Mannings constant are tabulated.*

The results are obtained using equation (12.4.3) and (12.4.4) and are presented in the tabulation.

$$R_h = 0.6\ m,\ A = 3\ m^2,\ S_b = 1/2500$$

S. No.	Surface type	N	C	V (m/s)	Q (m^3/s)
1	Smooth cement lined surface	0.011	**83.48**	1.29	**3.88**
2	Smooth concrete	0.013	**70.64**	1.09	**3.28**
3	Rough Brick	0.015	**61.23**	0.95	**2.85**
4	Rubble masonary/Rough concrete	0.017	**54.02**	0.84	**2.51**
5	Clean earthen channel	0.018	**51.02**	0.79	**2.37**
6	Earthen channel after weathering	0.022	**41.74**	0.65	**1.94**
7	Rough earthen channel with weeds	0.050	**18.37**	0.29	**0.85**

Comparing with the result of example 12.2 and 12.3 it may be noted that there is close agreement between the results except in the case of earthen channels. Values for some more types of surfaces are given in table 12.1.

Table 12.1 Values of Manning's coefficient, N

S. No.	Surface Type	N
1	Natural channels	
	Clean and straight	0.030 ± 0.005
	Sluggish with deep pools	0.040 ± 0.010
	Major rivers	0.035 ± 0.010
2	Flood plains	
	Pasture, farm land	0.035 ± 0.010
	Light brush	0.050 ± 0.020
	Heavy brush	0.075 ± 0.025
	Trees	0.150 ± 0.050
3	Excavated eathen channels	
	Clean	0.022 ± 0.004
	Gravelly	0.025 ± 0.005
	Weedy	0.030 ± 0.005
	Stoney, cobbles	0.035 ± 0.010
4	Lined channels	
	Glass	0.010 ± 0.002
	Brass	0.011 ± 0.002
	Steel, smooth	0.012 ± 0.002
	Steel, painted	0.014 ± 0.003
	Steel, riveted	0.015 ± 0.002
	Cast iron	0.013 ± 0.003
	Concrete, finished	0.012 ± 0.002
	Concrete, Rough	0.014 ± 0.002
	Planed wood	0.012 ± 0.002
	Clay tile	0.014 ± 0.003
	Asphalt	0.016 ± 0.003
	Corrugated metal	0.022 ± 0.005
	Rubble masonry	0.025 ± 0.005
	Brick work	0.015 ± 0.002

Example 12.5 ***Determine the slope*** *required for a flow of 1500 litre of water per second for a pipe of 2 m diameter flowing half full. Use Mannings equation. The value of Mannings constant is 0.015 for the rough concrete lining used.*

$$\text{Area} = \pi D^2/(4 \times 2),\ \text{Perimeter} = \pi D/2, \quad \therefore \quad R_h = D/4 = 2/4 = 0.5 \text{ m}$$

$$C = (1/N)R_h^{1/6} = (1/0.0015) \times 0.5^{1/6} = 59.4,$$

Velocity = Volume flow/area

$$V = 1.5 \times (4 \times 2/\pi \times 2 \times 2) = 0.955 \text{ m/s. } (1500l = 1.5 \text{ m}^3)$$

$$V^2 = C^2 R_h S_b$$

$$\therefore \quad S_b = V^2/C^2 R_h = 0.955^2/(59.4^2 \times 0.5) = 517 \times 10^{-6}$$

or 1 : 1934, $S_b = 1/1934$

12.5 ECONOMICAL CROSS-SECTION FOR OPEN CHANNELS

For a given flow rate and slope (determined by the ground slope) any one of several types of sections can be chosen. After a particular type of section, say rectangular, circular etc. is chosen, various alternatives are possible. A wider shallow section or a narrower deeper section may carry the same flow under the same slope. Hence there can exist a section which will involve minimum section in terms of cost of excavation, lining etc. This is illustrated in the problem Example 12.6, by a trial process, assuming depth and width. Derivations are given in example 12.7, 12.8 and 12.9 and solved problem Problem 12.7.

Example 12.6 *A smooth cement lined rectangular channel is proposed with a slope of 1/2500. Mannings coefficient for the surface is 0.011. For a total area of 4 m², assuming different ratios of depth to width* ***determine the flow rates****.*

The assumed depth to width ratios and corresponding values of depth d, width b, perimeter P, hydraulic radius R_h, velocity V, and flow rates are tabulated below.

$$V = \frac{1}{N} \cdot S_b^{1/2} R_h^{2/3} = \frac{1}{0.011} \cdot \left(\frac{1}{2500}\right)^{0.5} \cdot R_h^{2/3}$$

$$= 1.8782\ R_h^{2/3} \qquad (A)$$

$$Q = A \times V = 4 \times 1.8182\ R_h^{2/3}$$

$$= 7.2728\ R_h^{2/3} \qquad (B)$$

The results using equations (A) and (B) are tabulated below.

d : b	d, m	b, m	P, m	R_h, m	V, m/s	Q, m³/s
1 : 1.5	1.6330	2.4495	5.7155	0.6999	1.4333	**5.7331**
1 : 1.75	1.5119	2.6458	5.6695	0.7055	1.4409	**5.7636**
1 : 2	1.4142	2.8284	5.6569	0.7071	1.4431	**5.7723**
1 : 2.25	1.3333	3.0000	5.6667	0.7059	1.4414	**5.7658**
1 : 2.5	1.2649	3.1622	5.6920	0.7027	1.4371	**5.7484**

It is seen that the flow rate is maximum at $d/b = 0.5$ or $b = 2d$ for rectangular channel. It is to be noted that perimeter is minimum at this value. For the square section the perimeter is maximum at *bm* when the flow rate is minimum at 5.55 m³/s.

Thus in the case of rectangular channels, it appears that the economical section for a given area is the 1 : 2 section.

Example 12.7 *Derive an expression for the ratio of depth to width for open channel flow in the case of a rectangular section of a given area for economical conditions.*

This is achieved when the flow is maximum for a given area or the perimeter is minimum.

Consider a width b and depth d perimeter $P = 2d + b = 2d + (A/d)$ ...(*A*)

$$Q = AC\sqrt{\frac{A}{P}S_b}$$

As A, C and S_b are specified, only variable is P in this case and so $Q \propto P^{-0.5}$

i.e. ***Q* will be maximum when *P* is minimum and depth *d* is the independent variable.** Or

$\frac{dP}{dd}$ should be zero for such a condition, using equation *A*

$$\frac{dP}{dd} = -\frac{A}{d^2} + 2 = 0 \quad \text{(also } \frac{d^2P}{dd^2} \text{ is } -ve)$$

$$\therefore \quad A = 2d^2 = d \times b \quad \therefore \quad b = 2d$$

The **depth should be one half of the width for economical rectangular section.** In this case $R_h = d/2$ (see example 12.6 also)

Using Mannings equation the flow rate for the economical rectangular section is given in terms of the depth d as

$$Q = \frac{2^{1/3}}{N} d^{8/3} S_b^{1/2} = \frac{1.26}{N} (\text{depth})^{8/3} (\text{slope})^{1/2} \tag{12.4.5}$$

Example 12.8 *Derive an expression for the hydraulic radius for a trapezoidal channel section for maximum flow conditions. Assume uniform flow conditions. Also determine the optimum side slope.*

Refer Fig. Ex. 12.8 for variable names.

Let $a = \cot\theta$, Area $A = by + ay^2 = y(b + ay)$ (1)

$P = b + 2w = b + 2y(1 + a^2)^{1/2}$ (2)

Eliminating b using equation 1

$$b = \frac{A}{y} - ay$$

$$\therefore \quad P = \frac{A}{y} - ay + 2y(1 + a^2)^{1/2}$$

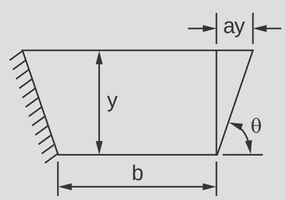


Figure Ex. 12.8

As $Q = C\sqrt{R_h S_b} = C\, S_b^{0.5} A^{0.5}/P^{0.5}$ (3)

and as A, C and S_b are constants and as $R_h = A/P$, Q is maximum when P is minimum. Also for a given channel θ is fixed and hence a is fixed, y being the independent variables, and P is to be minimized. Using equation (3) and equating (dp/dy) to zero.

$$\frac{dp}{dy} = -\frac{A}{y^2} - a + 2(1 + a^2)^{1/2} = 0 \quad \text{Solving for } A$$

$$A = y^2\,[2\,(1 + a^2)^{1/2} - a], \quad \text{From equation (3),}$$

$$P = \frac{A}{y} - ay + 2y(1 + a^2)^{1/2} = \frac{A}{y} + y\ [2(1 + a^2)^{1/2} - a]$$

Substituting for A, $\quad P = y[2(1 + a^2)^{1/2} - a] + y[2(1 + a^2)^{1/2} - a] = 2y[2\ (1 + a^2)^{1/2} - a]$

$$\therefore \qquad \mathbf{R_h} = \frac{A}{P} = \frac{\mathbf{y}}{\mathbf{2}}$$

In the case of a rectangle $a = 0 \quad \therefore \quad A = 2y^2$, $P = 4y$, and from equation 1, $b = 2y$.

In order to determine the optimum value of a (cot θ) for a given section of area A and depth y, using equation (3)

$$\frac{dP}{da} = -y + \frac{1}{2} \times 2a \times 2y(1 + a^2)^{-1/2} \quad \text{Rearranging} \quad 2a/(1 + a^2)^{1/2} = 1$$

Squaring both sides, and solving

$$a = \frac{1}{\sqrt{3}} = \cot\theta \quad \therefore \quad \theta = 60°$$

Hence the optimum condition for economical section for a given area and depth is $\theta = 60°$

This shows that the section is half of a regular hexagon, as it can be shown that base b = side length W

$$W^2 = y^2 + a^2y^2 = y^2(1 + a^2), \text{ as} \quad a = \cot 60 = 0.5774$$

$$W^2 = 1.333\ y^2 = (4/3)\ y^2 \quad \therefore \quad W = (2/\sqrt{3}\,)y$$

$$b = \frac{A}{y} - ay,\ A = y^2\ [2(1 + a^2)^{1/2} - a]$$

$$\therefore \qquad b = 2y[(1 + a^2)^{1/2} - a], \quad \therefore \quad b = \frac{2}{\sqrt{3}}y \quad \therefore \quad b = W$$

Example 12.9 *Derive an expression for the optimum angle for a triangular channel section of given area.*

Refer Fig. Ex. 12.9

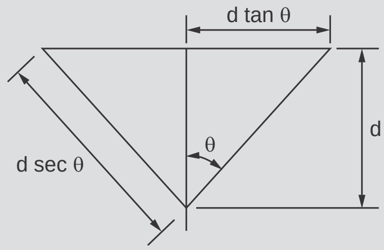


Figure Ex. 12.9

Area, $\quad A = d^2 \tan\theta \quad \therefore \quad d = \left(\frac{A}{\tan\theta}\right)^{0.5}$

Perimeter, $P = 2d \sec\theta$, Substituting for d

$$P = 2\sec\theta\left(\frac{A}{\tan\theta}\right)^{0.5}$$

$$= 2\left(\frac{A\sec^2\theta}{\tan\theta}\right)^{0.5} = 2\left(\frac{A(1+\tan^2\theta)}{\tan\theta}\right)^{0.5}$$

$$= 2\{A(\tan\theta + \cot\theta)\}^{0.5} \quad \text{As} \quad Q = AV = AC\sqrt{\frac{A}{P}S_b}$$

$\therefore$ For maximum of Q, P should be minimized, Considering θ as independent variable, and minimizing P^2,

$$\frac{dP}{d\theta} = \frac{d}{d\theta}[\tan\theta + \cot\theta] = \sec^2\theta - \text{cosec}^2\theta = 0$$

$$\therefore \quad \cos^2\theta = \sin^2\theta \quad \therefore \quad \theta = 45°, \text{ The optimum half angle is } 45°$$

$$d = (A/\tan 45)^{0.5} = A^{0.5} \quad \text{or} \quad A = d^2$$

$$\therefore \quad \mathbf{P} = 2d \sec 45 = 2\sqrt{2}\, d = \mathbf{2.828\ d}$$

$$\therefore \quad R_h = A/P = d/2^{3/2} = 0.353\ d$$

12.5.1 Examples 12.6 to 12.9 provide the optimum geometry for some sections. Table 12.2 gives the conditions for optimum geometry for various sections together with normal depth d and cross sectional area in terms of flow rate, Mannings coefficient and bed slope.

$$(Q/A) = R_h^{2/3} S_b^{1/2}/N, \text{ and } R_h = A/P, \quad \therefore \quad A = \left[\frac{QN}{S_b^{1/2}}\right]^{3/5} P^{2/5}$$

Table 12.2. Optimum geometry, normal depth and area when flow rate and bed slope are fixed.

Section	Optimum Geometry	Normal depth	Cross sectional area
Rectangular, width, b depth, d	**b = 2d**	$0.917\left[\frac{QN}{S_b^{1/2}}\right]^{3/8}$	$1.682\left[\frac{QN}{S_b^{1/2}}\right]^{3/4}$
Trapezoidal width, b depth, d	θ = 60° $\mathbf{b = 2d/\sqrt{3}}$	$0.968\left[\frac{QN}{S_b^{1/2}}\right]^{3/8}$	$1.622\left[\frac{QN}{S_b^{1/2}}\right]^{3/4}$
Circular, diameter, D depth, d	**D = 2d**	$1.00\left[\frac{QN}{S_b^{1/2}}\right]^{3/8}$	$1.583\left[\frac{QN}{S_b^{1/2}}\right]^{3/4}$
Wide flat, width b, depth d q, flow/m	**b>>d**	$1.00\left[\frac{qN}{S_b^{1/2}}\right]^{3/8}$	---

Example 12.10 *For a given slope of 1/2500 and flow rate of 4m^3/s,* ***determine the depth of flow and area of cross-section at optimum conditions*** *for (i) Rectangular, (ii) Trapezoidal, (iii) Triangular and (iv) Circular sections. Mannings coefficient, N = 0.011.*

Using equations in table 12.2, the values are calculated as below.

(i) Rectangular: Normal depth

$$d = 0.917\left(\frac{QN}{S_b^{1/2}}\right)^{3/8} = 0.917\left[\frac{4 \times 0.011}{(1/2500)^{1/2}}\right]^{3/8}$$

$$= 0.917\ (2.2)^{3/8} = 1.2347 \text{ m},$$

$\therefore$ $b = 2.4649$ m

$A = d \times b = 3.04$ m^2, Also $A = 1.682(2.2)^{0.75} = 3.04\ m^2$

(ii) Trapezoidal (Note: $QN/S_b^{1/2} = 2.2$ from previous calculation is used in the following calculations)

$$d = 0.968\ (2.2)^{3/8} = 1.301 \text{ m},\ b = 2 \times 1.301/\sqrt{3} = 1.502 \text{ m}$$

$\therefore$ $A = 1.622(2.2)^{0.75} = 2.93$ m^2

check (1.502 + 1.301 cot 60) 1.301 = **2.93 m^2**

(iii) Triangular: $d = 1.297\ (2.2)^{3/8} = 1.743$ m, top width = 2 × 1.743 = 3.468 m

$A = 3.039$ m^2, check: $1.682\ (2.2)^{0.75}$ = **3.04 m^2**

(iv) Circular: $d = 1.0(2.2)^{3/8} = 1.344$ m, Diameter $D = 2 \times 1.344 = 2.688$ m

$A = \pi\ 2.688^2/(2 \times 4) = 2.84$ m^2, check: $1.583\ (2.2)^{3/4}$ = **2.86 m^2**

Note that the minimum area is in the case of the circular section. Next comes trapezoidal one. The triangular and rectangular sections need the same areas, but the depth is more for the triangular section. The velocity will be maximum in the circular section.

Example 12.11 *Using the results of Example 12.10,* ***determine the flow velocity*** *in each case and check the same using Mannings equation.*

$$V = \frac{1}{N} R_h^{2/3} S_b^{1/2} = \frac{1}{0.011} \times \left(\frac{1}{2500}\right)^{0.5} R_h^{2/3} = 1.8182 \times R_h^{2/3}$$

The result are tabulated below: Flow rate = 4 m^3/s

Section	R_h	Area m^2	Velocity using Area m/s	Velocity using Manning's m/s	Fr
Rectangular	$(d/2) = 1.2347/2$	3.04	**1.32**	**1.3168**	0.38
Tapezoidal	$(d/2) = 1.301/2$	2.93	**1.37**	**1.3650**	0.42
Triangular	$(d/2\sqrt{2}) = (1.743/2\sqrt{2})$	3.04	**1.32**	**1.3166**	0.45
Circular	$(d/2) = (1.344/2)$	2.84	**1.41**	**1.3950**	0.44

Froude number is calculated on the basis of hydraulic depth = Area/top width

$$Fr = V/\sqrt{yg} \text{ where } y \text{ is the average depth given by, Area/top width.}$$

Trapezoidal $$y = \frac{A}{\text{Topwidth}} = \frac{d(b + d \sin 30)}{b + 2d \sin 30} = \frac{1.301(1.502 + 0.65)}{1.502 + 1.301} = 0.9991$$

Triangular $$y = \frac{3.04}{2 \times 1.743} = 0.872 \text{ m, circular } \quad y = \frac{2.84}{2.688} = 1.057 \text{ m}$$

Note : Velocity is maximum in the case of circular section.

Example 12.12 ***Determine the height of flow*** *in the case of a rectangular channel of 3 m width and slope of 1/2500 for a flow rate of 4 m³/s. Consider well finished concrete surface*

The value of Mannings coefficient for well finished concrete surface is 0.011.

Assume that the depth is d as $p = 3$ m

∴ $$A = b \times d = 3d,\ P = b + 2d = 3 + 2d,\ R_h = 3d/(3 + 2d)$$

$$V = \frac{1}{N} R_h^{2/3} S_b^{1/2},\ Q = AV = V \times 3d, \quad \text{Substituting the values,}$$

$$4 = \frac{1}{0.011} \times \left(\frac{1}{2500}\right)^{0.5} \left(\frac{3d}{3 + 2d}\right)^{2/3} \times 3d = 5.4545 \times d \times \left(\frac{3d}{3 + 2d}\right)^{2/3}$$

$$\left(\frac{d^{5/2}}{3 + 2d}\right) = 0.2093, \text{ Solving by trial } \mathbf{d = 1.021 \ m}$$

Check: $$R_h = (3 \times 1.021)/(3 + (2 \times 1.021)) = 0.6033 \text{ m}$$

$$Q = [(3 \times 1.021)/(0.011)] \times (1/2500)^{0.5} \times (0.6033)^{2/3} = 3.976 \text{ m}^3/\text{s}$$

Note: Determination of depth or width for specified values of flow rate, slope etc. involves polynomials of more than degree 2. Hence iterative working is necessary.

12.6 FLOW WITH VARYING SLOPES AND AREAS

It is found almost impossible to have open channels of uniform slope and or uniform area all over the length of the channel. When the terrain changes the slope has to change, sometimes gradually and sometimes steeply, and hence the flow area also should change. In such conditions the flow depth readjusts in some cases gradually and in some cases, suddenly. Certain situations like in spillways the energy in the flow has to be dissipated without damage to the surfaces. The study of such flows is an important and practical aspect of open channel flow.

12.6.1 Velocity of Wave Propagation in Open Surface Flow

The nature of readjustment of flow level due to slope change or sudden drop in bed level is found to depend on the velocity of propagation of any disturbance or wave velocity in the flow. In stagnant surface any disturbance will spread uniformly around the point of disturbance. An example is dropping of a small stone in a stagnant pool. In case water is flowing with a velocity V and if the wave velocity is c then if $V < c$, the disturbance will travel upstream at a lower speed. In case $V \geq c$ then disturbance cannot travel upstream and only the down steam flow will change. The change is mainly in the form of change in height of flow.

Referring to Fig. 12.6.1, let a disturbance be created by moving the vertical plate slightly along the x direction. This will cause a small ripple or wave and let its velocity or propagation be c. In order to facilitate analysis, the wave can be brought to rest by imposing a velocity c in the opposite direction to the wave movement.

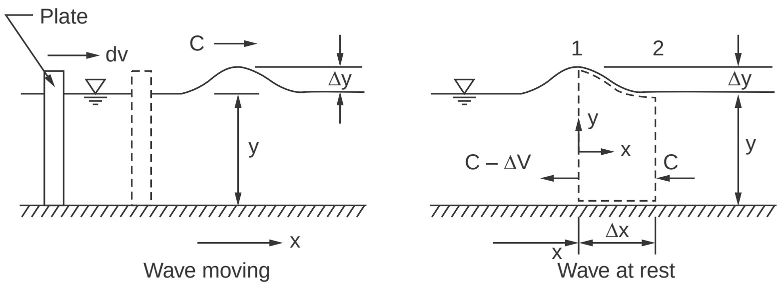


Figure 12.6.1 Wave celerity

The flow is assumed steady, incompressible and the section is constant across the x direction with a width b.

Considering sections 1 and 2 and applying continuity conditions *i.e.* flow is equal at sections 1 and 2

$$\rho\, b(y + \Delta y)\,(c - \Delta V) - \rho\, byc = 0 \quad \text{Solving}$$

$$\Delta V = c\,\frac{\Delta y}{y + \Delta y},$$

as Δy is small this can be approximately as $\Delta V = c\dfrac{\Delta y}{y}$ (*A*)

Applying momentum equation to the sections, as the pressure is hydrostatic at any section, the pressure force is given by (for unit width) $\rho\, g\, y^2/2$

At section 1, Force = $\rho\, g\, (y + \Delta y)^2/2$

At section 2, Force = $\rho g\, y^2/2$

Net pressure force between the sections equals the difference between these two, $\rho gy\, \Delta y + g\,\dfrac{\Delta y^2}{2}$

Neglecting the second order term, net force equals $\rho g\, y\, \Delta y$

The rate of change of momentum is dV × flow rate $\therefore$ $dV\, \rho cy$

Subsituting for dV from equation (*A*) and equating the force and change of momentum,

$$\rho\, gy\, \Delta y = c^2\, \rho\, \Delta y$$

$$c^2 = gy \quad \text{or} \quad \mathbf{c = \sqrt{gy}} \tag{12.6.1}$$

The wave velocity c is also called wave celerity.

The main assumption is that $\Delta y << y$ *i.e.* the wave height is small compared to the depth. The local depth at the crest of the wave is more compared to the trailing edge. This causes accumulation of water in the crest and as the wave travels further the height increases, finally leading to the breaking of the waves, which is seen near beaches. Note that the velocity calculated is that of surface wave and not that of propagation at depths.

Example 12.13 *Compare the celerity of waves for depths of 0.1, 0.5, 100, 1000 and 4000 m.*

Using equation (12.6.1), $c = \sqrt{gy}$, the results obtained are tabulated below :

depth, *m*	0.1	0.5	1	10	100	1000	4000
velocity, *m/s*	0.99	2.21	3.13	9.91	31.32	99.05	198.09

The depth of deep ocean is about 4 km. For this depth the disturbance will travel at 198 m/s which is about 713 km/hr. Such a disturbance can be produced by earthquake or volcanic activity. The resulting tidal waves may travel at this speed and cause heavy damage.

12.6.2 Froude Number

Froude number in connection with open surface flow is defined as $V/\sqrt{gl}$. In the case of open channel flow, the characteristic length, l, is the depth y and V is the flow velocity. Hence Froude number can be represented by the ratio, Flow velocity/wave velocity.

As already indicated, there are three possible flow situations namely $(V/c) < 1$, $(V/c) = 1$ and $(V/c) > 1$ or the Froude number for the flow is less than or equal to or greater than 1.

Case (i) If Fr < 1, then V < c and any disturbance can travel upstream. The downstream conditions can change the flow conditions upstream. Such a flow is called subcritical or **tranquil** flow. Only gradual changes occur in such a situation.

Case (ii) Fr = 1. The flow is called critical flow. Disturbances cannot travel upstream. A standing wave may generally result.

Case (iii) Fr > 1, such flows are called supercritical or rapid or shooting flows. Disturbances cannot travel upstream. Downstream conditions cannot be felt upstream. Changes occur only in the downstream flow. These are similar to subsonic, sonic and supersonic flows in the case of flow of compressible fluids where Match number is the governing factor also defined as V/c, where c is the sonic speed or velocity of propagation of small disturbance in the fluid.

12.6.3 Energy Equation for Steady Flow and Specific Energy

Assumptions in the case are

(*i*) steady incompressible and uniform flow

(*ii*) pressure distribution is hydrostatic

(*iii*) small bed slope, ($\sin\theta \approx \tan\theta \approx \theta$)

(*iv*) Shear work term negligible,

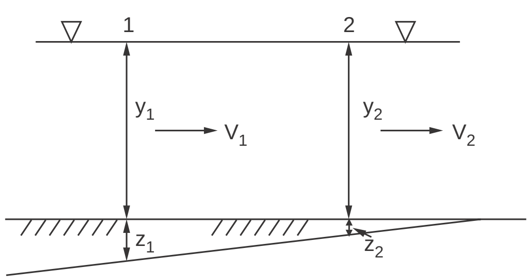


Figure 12.6.2

Considering sections 1 and 2 in the flow as shown in Figure 12.6.2, Bernoulli equation is written including head loss due friction h_L.

$$\frac{V_1^{\,2}}{2g} + y_1 + z_1 = \frac{V_2^{\,2}}{2g} + y_2 + z_2 + h_L \tag{12.6.1}$$

As in pipe flow the pressure loss in due to friction in the open channel flow. The head due to change in level namely ($z_1 - z_2$) equals the friction head h_L. The term $\mathbf{\frac{V^2}{2g}}$ **+ y is found to be an important parameter is open channel flow. This quantity is defined as specific energy or specific head. The symbol used is E.** The variation of depth and velocity for a given specific energy provides an idea about the type of flow.

To illustrate the idea the case of a rectangular section is analysed in the following sections.

For a flow rate Q and sectional area A,

$$V = \frac{Q}{A} \quad \text{and} \quad A = b \times y \text{ where } b \text{ is the width and } y \text{ is the depth.}$$

$$\therefore \qquad \frac{V^2}{2g} = \frac{Q^2}{2g\,b^2y^2}.$$

The specific energy can now be expressed as below

$$E = y + \frac{Q^2}{2g\,b^2y^2} \tag{12.6.2}$$

For unit width, $\quad q = Q/b$

$$\therefore \qquad E = y + \frac{q^2}{2gy^2} \tag{12.6.3}$$

It will be useful to investigate the variation of depth and velocity for a given flow rate. In this process the value of **minimum energy for a given flow** is found as follows.

Differentiating the equation (12.6.3) and equating the result to zero, we get

$$\frac{dE}{dy} = -\frac{q^2}{gy^3} + 1 = 0.$$

$$\therefore \qquad y^3 = \left(\frac{q^2}{g}\right) \quad \text{or} \quad \mathbf{y} = \left(\frac{\mathbf{q^2}}{\mathbf{g}}\right)^{\mathbf{1/3}} \tag{12.6.4}$$

The value of y for the minimum every for a given flow rate is formed as critical depty y_c.

$$\therefore \qquad y_c = \left(\frac{q^2}{g}\right)^{1/3} \tag{12.6.4(a)}$$

Substituting the value in the general equation (12.6.3).

$$\mathbf{E_{min}} = y_c + \frac{y_c^{\,3}}{2\,y_c^{\,2}} = \mathbf{3/2\ y_c.} \tag{12.6.5}$$

or

$$\mathbf{y_c} = \frac{2}{3}\,\mathbf{E_{min}}. \tag{12.6.5a}$$

From the definition of velocity, and eqn. 12.6.4*a* for unit width at the critical condition,

$$V_c^{\,2} = \frac{q^2}{y_c^{\,2}} = \frac{g\,.\,y_c^{\,3}}{y_c^{\,2}} = gy \quad \text{or} \quad \mathbf{V_c = \sqrt{gy}} \tag{12.6.6}$$

From the expression of wave velocity, $\sqrt{gy} = c$

$\therefore \quad V_c = c$, and $\dfrac{V_c}{c} = \dfrac{V_c}{\sqrt{gy}} = 1$ or Froude number is unity.

The flow rate at this condition is given by (for unit width)

$$q_{max} = V_c y_c = \sqrt{g y_c^{\,3}} \tag{12.6.7}$$

At this critical depth condition the following relations hold.

$$y_c = \left(\frac{q^2}{g}\right)^{1/3},\ V_c = \sqrt{g y_c}\,,\ E_c = \frac{3}{2}\,y_c,\ q_c = \sqrt{g y_c^{\,3}}$$

and

$$A_c = b y_c = b\left(\frac{q^2}{g}\right)^{1/3}$$

These relations specify only a single condition in the flow. To investigate the complete variation the general equation (12.6.3) is modified and written as

$$y^3 - Ey^2 + \frac{q^2}{2g} = 0. \tag{12.6.8}$$

This equation leads to two positive values of y for a given E and q. The third solution is negative and has no significance.

This means that **For a given value of E and q two combinations of depth and velocity exist. These are called alternate depths and velocities.** As two quantities are involved with depth of flow two separate plots are used to illustrate the flow variation. One plot is for a constant energy and the head variation with flow rate. This is shown in Figure 12.6.3 The other is for constant flow rate and head variation with energy. This is shown in Figure 12.6.4.

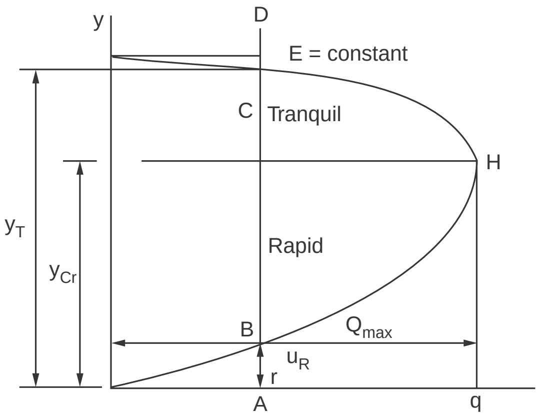


Figure 12.6.3 Variation of depth with specific energy for a given flow rate

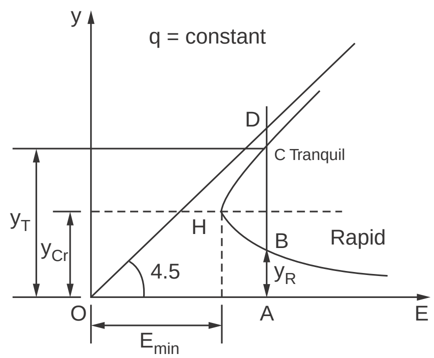


Figure 12.6.4 Variation of discharge with depth for a given specific energy

In figure 12.6.3 the variation of depth with flow rate is shown plotted for a constant value of E. Similar curves will result for other value of specific energy E. The resulting curve shows that at any flow less than q_{max}, there are two possible depths of flow, one greater than y_c and the other smaller than y_c. The condition y_c divides the flow into two regions namely tranuil and rapid flows. In the tranquil flow the depth is larger and the velocity is smaller. In the rapid flow the depth is smaller and the velocity is more. For example for the tranquil flow of the given q at A, the depth is AC and the dynamic head is CD. In the rapid flow region, the depth is AB and the dynamic head is BD. At q_{max} or the critical condition there is only one solution for the depth.

In the tranquil flow region, the Froude number is less than 1 and any disturbance downstream will be felt upstream and the flow upstream will be readjusted by the disturbing wave. The flow velocity V will be less than V_c.

In the rapid flow region, the Froude number is greater than one and down stream disturbances like changed slope will not be felt upstream. and $V > V_c$.

At the critical condition a standing wave will be generated.

Figure 12.6.4 is a plot of depth against specific energy for a given flow rate. At $q = 0$, $y = E$. This is represented by the line at 45°. A curve for a given flow rate is shown ploted. Here also very similar conditions are seen. For a given flow rate at any energy greater than E min, the flow can exist at two different combinations of depth and velocity. This is shown by the line $ABCD$. When the depth is equal to $y_R = AB$, the kinetic energy is given by BD. This is in the rapid flow region. When the depth is y_T (AC), the kinetic energy is CD. For different values of flow rate different curves will result.

12.6.4 Non Dimensional Representation of Specific Energy Curve

By one dimensional representation a single curve will result for all values of specific energy. The specific energy curve can be presented in a non dimensional form by dividing the terms of equation (12.6.3) by q_{max}.

$$\frac{E}{q_{max}^2} = \frac{y}{q_{max}^2} + \frac{1}{2gy^2}\left(\frac{q}{q_{max}}\right)^2$$ As $q_{max} = (g\, y_c^3)^{1/2}$, this reduces to

$$\frac{E}{gy_c^{\,3}} = \frac{y}{gy_c^{\,3}} + \frac{1}{2gy^2}\left(\frac{q}{q_{\max}}\right)^2$$

or
$$\left(\frac{q}{q_{\max}}\right)^2 = \frac{2E}{y_c}\left(\frac{y}{y_c}\right)^2 - 2\left(\frac{y}{y_c}\right)^3$$

For a rectangular channel E_{min} = (3/2) y_c

$$\therefore \qquad \left(\frac{q}{q_{\max}}\right)^2 = 3\left(\frac{y}{y_c}\right)^3 - 2\left(\frac{y}{y_c}\right)^3 \qquad (12.6.6)$$

This will result in a single curve for all values of E when q is plotted against y. Similarly for given values of q, the equation below will result in a single curve. This is obtained by dividing the general eqaution by y_c and then simplifying

$$\frac{E}{y_c} = \frac{y}{y_c} + \frac{1}{2}\left(\frac{y_c}{y}\right)^2 \qquad (12.6.7)$$

This will result in a single curve for all values of q when E is plotted against y.

Example 12.14 *Water flows in a rectangular channel at the rate of 3 m³/s per m width, the depth being 1.5 m, Determine whether the flow is subcritical or supercritical. Also determine the alternate depth and Froude numbers in both cases.*

Considering 1 m width

Velocity = 3/(1.5 × 1) = 2 m/s, Specific energy = ($V^2/2g$) + y = 1.7039 m.

$$Fr = \frac{V}{\sqrt{gy}} = \frac{2}{\sqrt{9.81 \times 1.5}} = 0.5214.\text{ The flow is subcritical}$$

Critical height is given by $y_c = (q^2/g)^{1/3} = 0.9717$ m

$E_{min} = (3/2) \times y_c = 1.4575$ m, $V_c = (g \times y_c)^{1/2} = 3.0874$ m/s

check: $q = V_c \times y_c = 3.0874 \times 0.9717 = 3$ m³/s/m

$$Fr = \frac{3.0874}{\sqrt{9.81 \times 0.9717}} = 1.0$$

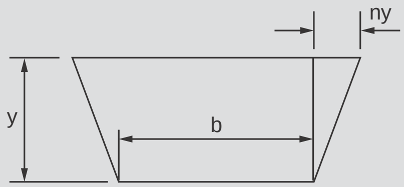
The alternate depth is obtained using equation (12.6.3), $y^3 - Ey^2 + q^2/2g = 0$

As q and E are known, solving by trial, y_{al} = 0.6643 m; ∴ V = 4.516 m/s

$$\textbf{Fr} = \frac{4.516}{\sqrt{9.81 \times 0.6643}} = \textbf{1.7961, The flow is supercritical}$$

check $$\frac{V^2}{2g} + y = \frac{4.516^2}{2 \times 9.81} + 0.6643 = 1.7038\text{ m.}$$

Example 12.15 ***Derive an expression for critical depth and critical velocity*** *of a trapezoidal channel in terms of specific energy E. Assume a bottom width b and side slope 1 : n.*



Area of section, $A = (b + ny)\, y$,

specific energy is given by

$$E = \frac{V^2}{2g} + y \quad \therefore \quad V = \sqrt{2g(E - y)}$$

Volume flow, $Q = A\,V = y(b + ny)\,\sqrt{2g(E - y)}$ ...(*A*)

For constant value of E, Q is maximum when $(dQ/dy) = 0$

Taking natural log on both sides of equation (*A*).

$$\ln Q = \ln y + \ln (b + ny) + (1/2) \ln (2g) + (1/2) \ln (E - y)$$

differentiating with respect to y,

$$\frac{1}{Q}\frac{dQ}{dy} = \frac{1}{y} + \frac{n}{b + ny} - \frac{1}{2(E - y)}$$, from section 12.6.3 when $\frac{dQ}{dy} = 0, y = y_c$

$$\therefore \quad \frac{1}{y_c} + \frac{n}{b + ny_c} - \frac{1}{2(E - y_c)} = 0$$

Rearranging after summing up and considering the numerator to be zero,

$$5ny_c^2 + (3b - 4nE)\, y_c - 2bE = 0 \qquad (B)$$

Solving for y_c

$$\mathbf{y_c} = \frac{-(3b - 4nE) \pm \sqrt{9b^2 - 24bnE + 16n^2E^2 + 40bnE}}{10n}$$

$$= \frac{(4nE - 3b) \pm \sqrt{16n^2E^2 + 16nEb + 9b^2}}{10n} \qquad (C)$$

This is a general solution. Chart solutions are available for various values of b and n.

In the case of triangle, $b = 0$ and $y_c = (4/5)\, E$ (*D*)

From equation (*B*) for a rectangular section as $n = 0$, $y_c = (2/3)\, E$, as was established earlier. Critical velocity is obtained by substituting y_c in the equation

$$\mathbf{V_c = \{2g[E - y_c]\}^{0.5}}$$

Example 12.16 ***Determine the critical depth*** *of a channel with trapezoidal cross-section with a flow of (1/3) m^3/s. The base width is 0.6 m. and the side slope is 45°*

In the case of sections other than rectangles, the hydraulic depth is given by Area/top width. In this case for critical flow, Hydraulic depth = y_c , i.e. A/top width

From section 12.6.3,

$$A_c = \left(\frac{b_{top}Q^2}{g}\right)^{1/3} \text{; also } A_c = y_c\,(b + 2y_c) = y_c\,(0.6 + 2y_c)$$

$Q = (1/3)\ m^3/s$, $b_{top} = 2y_c + b$, Substituting

$$y_c(b + 2y_c) = \left(\frac{(b+2y_c)\times Q^2}{g}\right)^{1/3} \quad \therefore \quad y_c^{\,3}\,(0.6 + 2y_c)^2 = \frac{Q^2}{g} = \left(\frac{1}{9\times 9.81}\right)$$

$$\therefore \qquad \mathbf{y_c}\,(0.6 + 2y_c)^{2/3} = \left(\frac{1}{9\times 9.81}\right)^{1/3} = \mathbf{0.2246\ m}$$

Solving by trial $\mathbf{y_c} = \mathbf{0.2244\ m}$

The actual depth is different from hydraulic depth. Let it be equal to y

$$\text{Area} = (y + b)\,y = (b + 2y_c)\,y_c, \text{ Solving } y = 0.265 \text{ m}$$

$$\mathbf{V_c} = (gy_c)^{0.5} = (9.81 \times 0.2244)^{0.5} = 1.4867 \text{ m/s}$$

check flow rate: 1.4837 × 0.265 (0.6 + 0.265) = 0.34 m^3/s

Example 12.17 *A rectangular channel 6 m wide is to carry a flow of 22.5 m^3/s. For depth of 3 m and 0.6 m* ***determine the slope required.*** *Also determine the* ***Froude number*** *and* ***alternate depth*** *for the specific energy conditions. Calculate the critical depth also. Take Mannings coefficient as 0.012.*

Critical depth is given by the equation (12.6.4*a*) (for rectangular section)

$$y_c = \left(\frac{Q^2}{gb^2}\right)^{1/3} = \left(\frac{22.5^2}{9.81\times 6^2}\right)^{1/3} = 1.1275 \text{ m}$$

Hence for a depth of 3 m, the flow is subcritical. For a depth of 0.6 m, the flow is supercritical.

Case (1) depth = 3 m, $\mathbf{R_h}$ = 6 × 3/(6 + 2 × 3) = **1.5 m**

To determine the slope, the Manning flow equation is used.

$$Q = A \times R_h^{2/3}\, S_b^{1/2}/N, \text{ Substituting the values,}$$

$$22.5 = (6 \times 3) \times \frac{1.5^{2/3}}{0.012}\, S_b^{1/2}, \text{ solving } \mathbf{S_b = 1/7631}$$

$$\text{Velocity} = 22.5/6 \times 3 = 1.25 \text{ m/s}$$

$$\therefore \qquad E = (V^2/2g) + y = \{1.25^2/(2 \times 9.81)\} + 3 = 3.0796 \text{ m}$$

Froude number: $1.25/\sqrt{3\times 9.81}$ = **0.2304**

To determine alternate depth, the equation used is (12.6.8)

$$y^3 - Ey^2 + (q^2/2g) = 0$$

Substituting values for E = 3.0796, q = (22.5/6) $m^3/s/m$

$y^3 = 3.0796\,y^2 + 0.7167 = 0$, solving by trial the alternate value of **y = 0.53 m**

V = 22.5/(6 × 0.53) = 7.07 m/s, **Fr = 7.075/$\sqrt{0.53 \times 9.81}$ = 3.1**

Specific energy calculated using this velocity is 3.081 m (check)

Case (2) 0.6 m depth, R_h = 6 × 0.6/(6 + 2 × 0.6) = 0.5 m

$$22.5 = (6 \times 0.6) \times (1/0.012)\,(0.5)^{2/3}\, S_b^{1/2}, \text{ solving } S_b = 1/70.5$$

$$\mathbf{Fr} = \frac{22.5}{6\times 0.6} \times \frac{1}{\sqrt{9.81\times 0.6}} = 2.58,$$

Alternate depth is obtained using

$$y^3 - Ey^2 + q^2/2g = 0, \quad \mathbf{E} = 0.6 + \left(\frac{22.5}{6 \times 0.6}\right)^2 \times \frac{1}{2 \times 9.81} = \mathbf{1.991\ m}$$

Substituting and solving by trial, alternate depth is 1.7595 m, V = 2.137 m/s, Fr = 0.513

At the critical flow condition, as calculated y_c = 1.1275 m

$$E = 1.1275 + \left(\frac{22.5}{6 \times 1.1275}\right)^2 \times \frac{1}{2 \times 9.81} = 1.691 \text{ m, which is } 1.5 \times y_c\text{, checks.}$$

12.7 EFFECT OF AREA CHANGE

When changes in flow area occurs over short distances like over a short bump or flow under a sluice gate, the effect of area change is more compared to frictional effects. Assuming channel bed to be horizontal, these cases can be analysed. As friction is neglected Bernoulli equation applies for steady flow conditions.

12.7.1 Flow Over a Bump

The flow along a horizontal rectangular channel of constant width, b, is considered. There is a bump in the channel bed as shown in Fig. 12.7.1. The height of the bump at location x is h, and the water depth is y measured from the bed level at the location 2. The flow is assumed to be uniform at each section. As the pressure on the free surface is the same at all locations, Bernoulli equation reduces to (taking bed level as datum)

$$\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + h = \text{constant over the bump.}$$

$$\frac{Q}{b} = V_1 y_1 = V_2 y_2, \text{ Substituting for } V_1 \text{ and } V_2.$$

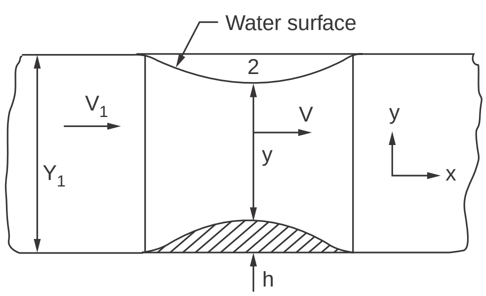


Figure 12.7.1 Flow over a bump

$$\frac{Q^2}{2gb^2 y_1^2} + y_1 = \frac{Q^2}{2gb^2 y^2} + y + h = \text{constant} \tag{12.7.1}$$

In order to determine the variation of y along the flow, the expression on the R.H.S. is differentiated with respect to x.

$$-\frac{Q^2}{2gb^2y^3}\frac{dy}{dx}+\frac{dy}{dx}+\frac{dh}{dx}=0, \quad \text{Solving for } (dy/dx),$$

$$\frac{dy}{dx}=\frac{(dh/dx)}{[(Q^2/gb^2y^3)-1]}=\frac{(dh/dx)}{[(V^2/gy)-1)}=\frac{(dh/dx)}{Fr^2-1} \qquad (12.7.2)$$

As dh/dx is specified, dy/dx can be determined. It is seen that the variation depends on the flow Froude number.

(1) There are two possible variations in the bed level, namely a positive bump and a negative bump or depression.

(2) There are two possible regimes of flow namely subcritical and supercritical.

The type of surface variations are listed below:

1. For subcritical flow + ve bump decreases the flow height, (similar to subsonic nozzle $M < 1$)

2. For subcritical flow – ve bump increases the flow height, (similar to subsonic diffuser $M < 1$)

3. For supercritical flow + ve bump increases the flow height, (similar to supersonic nozzle $M > 1$)

4. Supercritical flow – ve bump decreases the flow height, (similar to supersonic diffuser $M > 1$)

The case of $Fr = 1$ is more complex and other factors have to considered to determine the flow downstream.

Example 12.18 *In a rectangular channel, the flow height is 0.6 m and the flow velocity is 0.6 m/s. A smooth bump with a peak height of 0.06 m exists on the bed surface.* ***Determine the flow velocity and depth over the peak of the bump***.

Refer Fig. Ex. 12.18. Consider sections 1 and 2

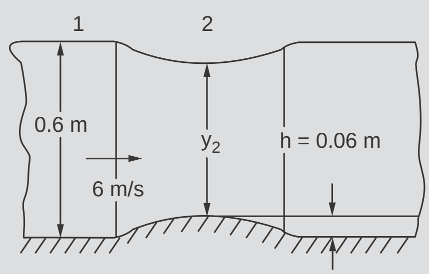


Figure Ex. 12.18

Under steady flow conditions, Bernoulli equation reduces to $\frac{V_1^2}{2g}+y_1=\frac{V_2^2}{2g}+y_2+h$, $V_2=(V_1y_1/y_2)$

Continuity equation is $\therefore \quad V_1y_1 = V_2y_2$

specific energy at section (1), $E_1 = \frac{V_1^2}{2g} + y_1 = \frac{0.6^2}{2 \times 9.81} + 0.6 = 0.61835$ m

specific energy at section 2, $E_2 = \frac{V_2^2}{2g} + y_2 = E_1 - h$

$= 0.61835 - 0.06 = 0.5583$ m,

substituting for V_2, $= \frac{(V_1 y_1)^2}{2g} \times \frac{1}{y_2^2} + y_2 = 0.5583$ m,

solving by trial, $\mathbf{y_2 = 0.5353}$ **m**. $\therefore$ $\mathbf{V_2 = 0.6725}$ **m/s**

change in surface level $= 0.5353 - 0.6 + 0.06 = -0.0047$ m

or 4.7 mm, decrease in level.

$$\mathbf{Fr} = \frac{0.6}{\sqrt{9.81 \times 0.6}} = \mathbf{0.2473}$$

In case $Fr = 1$ to occur at the bump, then $y_c = (q^2/g)^{1/3} = 0.3323$ m. The flow depth at section 2 should be 0.3323 m. The size of the bump corresponding to this can be worked out by the above procedure.

12.7.2 Flow Through Sluice Gate, from Stagnant Condition

The reservoir is assumed to be large or y_0 = constant, $V_0 \simeq 0$.

Applying Bernoulli equation between section 0 and 1

$$y_0 = \frac{V_1^2}{2g} + y_1,$$

Since $V = Q/A$ and $A = by$, $y_0 = \frac{Q^2}{2gb^2 y_1^2} + y_1$ (12.7.3)

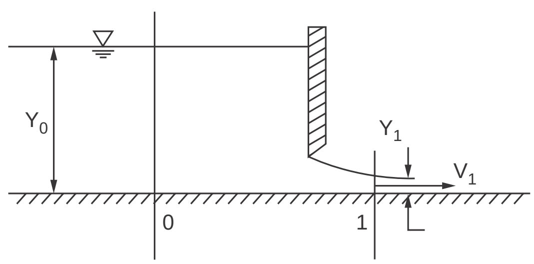


Figure 12.7.3 Flow through sluice gate

Rearranging

$$y_1^2 (y_0 - y_1) = Q^2/2gb^2 \quad (12.7.4)$$

The flow rate Q fixes the value of y_1 and as the equation is a quadratic, there can be two solutions. One will be in the subcritical flow and the other in supercritical flow region.

The same is shown plotted in non dimensional form in Fig. 12.7.4.

At maximum flow rate there is a single value for y_1.

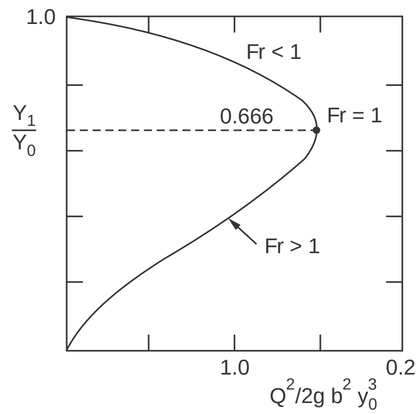


Figure 12.7.4

Taking the derivative of equation (12.7.4) with respect to y_1

$$\frac{d}{dy_1}\left[\frac{Q^2}{2gb^2}\right] = 0 \quad i.e., \quad 2y_1y_0 - 3y_1^2 = 0$$

$$\therefore \qquad y_1 = (2/3)\,y_0 \tag{12.7.5}$$

Substituting this value in flow rate term,

$$\frac{Q_{max}{}^2}{gb^2} = 2\left[\frac{2}{3}y_0\right]^2\left[y_0 - \frac{2}{3}y_0\right] = \frac{8}{27}\,y_0{}^3$$

Also $Q/b = q$, then $q_{max}{}^2/g = (8/27)\,y_0{}^3$ (12.7.6)

$$V_{max} = \frac{q_{max}}{A} = \frac{q_{max}}{1\times y_1},\ Fr^2 = \frac{V_{max}{}^2}{gy_1{}^8} = \frac{q_{max}{}^2}{gy_1{}^3} = \frac{8}{27}\,y_0{}^3\left(\frac{3}{2y_0}\right)^3 = 1$$

Maximum flow corresponds to $Fr = 1$

There is no correlation available relating y_1 and the gate opening.

12.7.3 Flow Under a Sluice Gate in a Channel

In this case water flows with velocity V_1 at section 1 and the level before the gate is y_1. The velocity at section 2 is V_2 and depth is y_2. Steady, incompressible uniform flow is assumed. Also specific energy at section 1 equals specific energy at section 2.

$$E_1 = \frac{V_1^2}{2g} + y_1 \text{ Replacing } V \text{ by } q, \text{ (flow/unit width)},$$

$$E_1 = \frac{q^2}{2gy_2{}^2} + y_2$$

The depth y_2 should be the alternative depth. As y_1 and V_1 are specified, y_2 is determined by trial using specific energy value. $V_2 = V_1y_1/y_2$ and so V_2 can be determined. In case velocity $V_1 = 0$, then the condition will give y_0. Maximum flow rate can be obtained by the condition that $y_2 = (2/3)\ y_0$. When the sluice is opened to obtain this maximum flow, the condition at section 1 will change. But y_0 will remain the same.

Example 12.19 *Water is let off from a large reservoir through a sluice gate. The water level in the dam above the level of the sluice is 6 m.* ***Calculate the flow rate*** *for values of y_1/y_0 = 0.8 and 0.4 (y_1 is the flow depth downstream). Also determine the* ***maximum flow rate*** *and the minimum depth of flow down stream.*

y_0 = 6 m, Considering unit width, From equation 12.7.3. and 12.6.3.

Case (i) $y_1 = 0.8 \times 6 = 4.8$ m, $y_0 = \dfrac{q^2}{2gy_1^2} + y_1$

$\therefore$ $q^2 = 2g\, y_1^2\, (y_0 - y_1) = 4.8^2\, (6 - 4.8) \times 2 \times 9.81 = 542\ .45$

$\therefore$ **q = 23.29 m³/s/m, V_1 = 23.29/4.8 = 4.852 m/s**

$Fr = V/\sqrt{gy} = 4.852/\sqrt{9.81 \times 4.8} = 0.7071$, (subcritical)

Case (ii) $y_1/y_0 = 0.4 \quad \therefore \quad y_1 = 6 \times 0.4 = 2.4$ m,

$q^2 = 2.4^2\, (6 - 2.4) \times 2 \times 9.81 = 406.84$

q = 20.17 m³/s/m, V = 20.17/2.4 = 8.404 m/s

$\text{Fr} = 8.404/\sqrt{9.81 \times 2.4} = 1.732$, (supercritical)

Maximum flow occurs at $y_1 = (2/3)\ y_0 = 4$ m

$\therefore$ $q^2 = 4^2(6 - 4) \times 2 \times 9.81 = 627.84$, **q = 25.06 m³/s/m**

V = 25.06/4 = **6.264**, m/s, $Fr = 6.264/\sqrt{9.81 \times 4} = 1$

Example 12.20 *Water flows at the upstream of a channel at 2m/s and the depth is 2 m, when the sluice is opened and flow is steady.* ***Determine the depth at the downstream side.*** *Also determine the* ***maximum flow rate*** *conditions i.e. depth velocity and flow rate downstream.*

Steady, incompressible uniform flow is assumed to prevail.

Specific energy upstream $= \dfrac{V_1^2}{2g} + y_1 = \dfrac{2^2}{2g} + 2 = 2.2039$ m

$Fr = 2/\sqrt{9.81 \times 2} = 0.452$ (subcritical)

In case the velocity upstream is negligible, this will be the value of y_0 *i.e.* y_0 = 2.2039 m

As the downstream flow is the same, specific energy remains constant. Expressing it in terms of q

$q^2/(2gy_2^2) + y_2 = 2.2039$

q = 2 × 2 = **4 m³/s/m**

This depth will be the alternate depth for the flow.

$y_2^3 - 2.2039\, y_2^2 + \dfrac{4^2}{2 \times 9.81} = 0$, Solving by trial $\mathbf{y_2 = 0.7485\ m}$

$$\mathbf{V_2} = 4/0.7485 = \mathbf{5.344\ m/s} \text{ (supercritical)}$$

Maximum flow will occur when

$$y_2 = (2/3)\, y_0 \quad \therefore \quad \mathbf{y_{max}} = 2.2039 \times (2/3) = \mathbf{1.4692}$$

As $$Fr = 1 \quad \mathbf{V_{max}} = \sqrt{g y_{max}} = \sqrt{1.4692 \times 9.81} = \mathbf{3.796\ m/s}$$

$$\mathbf{q_{max}} = 7.796 \times 1.469 = \mathbf{5.58\ m^3/s/m}$$

When the sluice position corresponds to this flow rate, the upstream condition at section 1 will change.

12.8 FLOW WITH GRADUALLY VARYING DEPTH

When open channel flow encounters a change in bed slope or is approaching normal depth, flow depth changes gradually. As the change is continuous, the analysis should take into consideration a differential control volume instead of sections upstream and downstream. Water depth and channel bed height are assumed to change slowly. The velocity at any section is assumed to be uniform. Refer Fig. 12.8.1.

In this case the energy grade line and free surface are not parallel. The slope of the bed is S_b. The slope of the energy grade line is S.

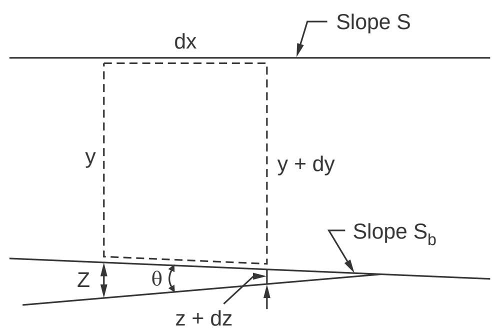


Figure 12.8.1 Flow with gradually varying depth

The specific energy flowing in at location x is $\frac{V^2}{2g} + y + z$,

The energy flow out at location $x + dx$ is

$$\frac{V^2}{2g} + d\left[\frac{V^2}{2g}\right] + y + dy + z + dz + dh_L$$

where dh_L is the head loss. The change in bed elevation can be expressed in terms of bed slope as $-\ S_b\, dx.dh_L = Sdx$ where S is the slope of the energy grade line. Taking the net flow and equating to the gravity drap,

$$d\left[\frac{V^2}{2g}\right] + dy = (\ S_b - S)\, dx \tag{12.8.1}$$

$$\frac{d}{dx}\left[\frac{V^2}{2g}\right]+\frac{dy}{dx}=S_b-S \tag{12.8.2}$$

expressing V in terms of flow q

$$\frac{d}{dx}\left[\frac{V^2}{2g}\right]=\frac{d}{dx}\left[\frac{q^2}{2gy^2}\right]=-2\,\frac{q^2}{2gy^3}\,\frac{dy}{dx}=-\frac{V^2}{gy}\,\frac{dy}{dx}=-Fr^2\,\frac{dy}{dx}$$

Substituting in 12.8.2.

$$\frac{dy}{dx}=\frac{S_b-S}{1-Fr^2} \tag{12.8.3}$$

The change in bed level dy along the flow direction for length dx is given by this equation. The gradient of the energy grade line can be obtained by assuming that loss is equal to the loss in steady uniform flow at normal depth. This is obtained from Manning's equation

$$\mathbf{S}=\frac{\mathbf{N^2V^2}}{\mathbf{R_h}^{4/3}} \tag{12.8.4}$$

Substituting for S in equation (12.8.3), results in a first order nonlinear ordinary differential equation that describes the variation of water surface profile. The sign of dy depends on the value of Froude number and the relative magnitudes of S and S_b

12.8.1 Classification of Surface Variations

The study is somewhat simplified when applied to a wide rectangular channel of depth y, when $R_h = y$. The equation (12.8.3) can be applied to this situation.

$$\frac{dy}{dx}=\frac{S_b-S}{1-Fr^2}=\frac{S_b[1-(S/S_b]}{1-Fr^2} \tag{12.8.5}$$

Using equation (12.8.4) we can show that, where y_n is the depth for normal flow

$$\frac{S}{S_b}=\left(\frac{y_n}{y}\right)^{10/3} \tag{12.8.6}$$

Also $$Fr=\left(\frac{y_c}{y}\right)^{3/2}$$

Substituting in (12.8.5).

$$\frac{dy}{dx}=S_b\,\frac{[1-(y_n/y)^{10/3}]}{[1-(y_c/y)^3]} \tag{12.8.7}$$

Also

On the basis of relative values of y_n and y_c the flow can be classified as mild slope, critical slope and steep slope types.

Then on the basis relative values of y, y_c and y_n for these types there can be three types of profiles.

In addition, horizontal bed slope and adverse bed slope type of flows will also lead to different shapes.

More often, the surface profiles are to be calculated by numerical methods. Now computer software's are also available to show the profiles graphically.

12.9 THE HYDRAULIC JUMP (RAPIDLY VARIED FLOW)

In subcritical flow due to any change in bed slope or cross-section, the disturbance produced will move upstream and downstream resulting in smooth adjustment of the flow depth. In supercritical or shooting flow such a disturbance cannot move upstream. The flow upstream will remain unchanged. The adjustment is sudden and can be only in the downstream side. **Change from supercritical to subcritical conditions thus cannot be smooth. Such a change occurs by increase of flow depth downstream. This is called Hydraulic jump.** The abrupt change involves loss of mechanical energy due to turbulent mixing. The heat produced does not significantly affect the temperature of the stream.

Hydraulic jump is used to dissipate mechanical energy into heat in various hydraulic structures. Fig. 12.9.1 shows a typical hydraulic jump. Horizontal bed condition is assumed.

Considering the control volume, the net momentum flow is equal to the net force (b = width).

Net momentum flow $= \rho\, Q\, (V_2 - V_1) = \rho\, V_1 y_1 b (V_2 - V_1)$ (12.9.1)

The pressure distribution in the flow depth is hydrostatic and acts at the centroids

$$F_1 = \rho\, gb\, y_1^2/2,\ F_2 = \rho\, gb\, y_2^2/2 \qquad (12.9.2)$$

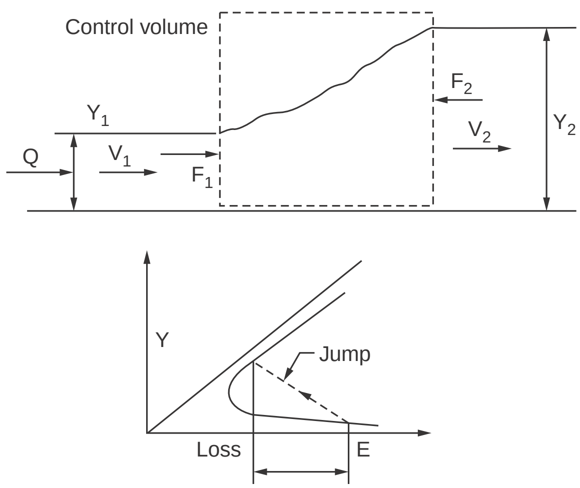


Figure 12.9.1 Hydraulic jump

$$\therefore \quad F_1 - F_2 = \rho\, gb\left(\frac{y_1^{\,2}}{2} - \frac{y_2^{\,2}}{2}\right) \quad \text{Equating}$$

$$\rho\, gb\left(\frac{y_1^{\,2}}{2} - \frac{y_2^{\,2}}{2}\right) = \rho\, bV_1 y_1 (V_2 - V_1) \quad \text{Rearranging}$$

$$(y_1^{\,2} - y_2^{\,2})/2 = \frac{V_1 y_1}{g}(V_2 - V_1) \tag{12.9.3}$$

Continiuity equation is $y_1\, b\, V_1 = y_2 b V_2 = Q$

Energy equation is

$$y_1 + \frac{V_1^{\,2}}{2g} = y_2 + \frac{V_2^{\,2}}{2g} + h_L \tag{12.9.4}$$

Wall shear is neglected. Bed is assumed to be horizontal. The loss is due of violent mixing. Using momentum and continuity equations, to eliminate V_2

$$(y_1^{\,2} - y_2^{\,2})/2 = \frac{V_1^{\,2} y_1}{g y_2}(y_1 - y_2)$$

or $$(1/2)(y_1 + y_2)(y_1 - y_2) = \frac{V_1^{\,2} y_1}{g y_2}(y_1 - y_2) \tag{12.9.5}$$

Cancelling $(y_1 - y_2)$ and multiplying by $(y_2/y_1^{\,2})$ and nothing $Fr_1 = V_1\sqrt{g y_1}$

$$\left(\frac{y_2}{y_1}\right)^2 + \left(\frac{y_2}{y_1}\right) - 2Fr_1^{\,2} = 0 \tag{12.9.6}$$

Solving for (y_2/y_1), from the quadratic equation above and nothing that – ve sign for the root is not possible

$$\mathbf{\frac{y_2}{y_1} = (1/2)\left[\sqrt{1 + 8Fr_1^{\,2}} - 1\right]} \tag{12.9.7}$$

The depth ratio is tabulated below for various values of upstream Froude number. As hydraulic jump is possible only in supercritical flow conditions, $Fr > 1$ alone is considered. Note that if $Fr_1 < 1$ then $y_2 < y_1$. This leads to increase in specific energy which is not possible see Fig. 12.9.1, *y Vs E* diagram.

Fr	1	1.5	2	2.5	3	3.5	4	4.5	5	10
$\mathbf{y_2/y_1}$	1.0	1.67	2.37	3.07	3.77	4.48	5.18	5.88	6.59	13.7

The depth downstream is always higher than that at the upstream. Depth upstream should be smaller than critical depth. Depth down stream will be higher than critical depth.

From Equation (12.9.5), loss of head can be directly determined as

$$h_L = (y_1 - y_2) + (V_1^{\,2} - V_2^{\,2})/2g$$

$$\frac{h_L}{y_1} = [1 - (y_2/y_1)] + \frac{Fr_1^2}{2}[1 - (y_1/y_2)^2] \tag{12.9.8}$$

(y_2/y_1) is obtained from Equation (12.9.7). This equation can be also simplified (using a rather long algebraic work) as

$$\mathbf{\frac{h_L}{y_1} = \frac{y_1}{4y_2}\left[\frac{y_2}{y_1} - 1\right]^3} \tag{12.9.9}$$

As h_L is positive (y_2/y_1) should be greater than one.

Expressing (y_2/y_1) in terms Froude number, the loss can be expressed as a fraction of specific energy at section 1.

$$\frac{h_L}{E_1} = \frac{(a-3)^3}{[8(a-1)(2+Fr_1^2)]} \quad \text{where } a = \sqrt{1 + 8\,Fr_1^2} \tag{12.9.10}$$

The ratio h_L/E_1 is shown tabulated below as a percentage for various values of Froude number.

Fr	1.5	2	3	4	5	6	8	10
h_L/E_1	2.2	9.1	25.7	39.1	49.1	56.4	66.4	72.7

It can be seen that for streaming flows with $Fr > 5$, a larger fraction of the mechanical energy is dissipated by eddies. This idea is used to dissipate the energy of water flowing over spillways. By proper design, energy may be dissipated without damage to the structure.

Hydraulic jump is similar to normal shock in compressible flow. In flow through supersonic nozzles, back pressure changes will not pass the supersonic region. In hydraulic jump also the disturbance downstream does not pass upstream. Pressure adjustment is automatic with normal shock and so also in hydraulic jump. Normal shock occurs in supersonic flow. Hydraulic jump occurs in supercritical flow. There is a pressure rise across normal shock. There is head increase in hydraulic jump.

Example 12.21 *In the flow through a sluice in a large reservoir, the velocity downstream is 5.33 m/s while the flow depth is 0.0563 m.* ***Determine the downstream conditions if a hydraulic jump*** *takes place downstream. Calculate the energy dissipated by eddies in the jump.*

$$Fr = V/\sqrt{gy} = 5.33/\sqrt{9.81 \times 0.0563} = 7.172$$

$$\frac{y_2}{y_1} = (1/2)\left(\sqrt{1 + 8\,Fr_1^2} - 1\right) = (1/2)\left(\sqrt{1 + 8 \times 7.172^2} - 1\right) = 9.655$$

$\therefore$ $\mathbf{y_2} = 9.655 \times 0.0563 = \mathbf{0.5436\ m}$

$\therefore$ $\mathbf{V_2} = \dfrac{0.0563}{0.5436} \times 5.33 = \mathbf{0.552\ m/s} \quad Fr_2 = 0.239$

$$\frac{h_L}{y_1} = \frac{y_1}{4y_2}\left[\frac{y_2}{y_1} - 1\right]^3 = \frac{1}{4} \times \frac{1}{9.655}[9.655 - 1]^3 = 16.78$$

∴ $h_L = 16.78 \times 0.053 =$ **0.945 m**

Specific energy at inlet $= \dfrac{V_1^2}{2g} + y_1 = \dfrac{5.33^2}{2 \times 9.81} + 0.0563 = 1.504$ m

∴ % **dissipation** = (0.945/1.504) × 100 = **62.82** %, check using

$$\frac{h_L}{E_1} = \frac{\left[\sqrt{1+8\times Fr_1^2} - 3\right]^3}{8\left[\left(\sqrt{1+8\times Fr_1^2} - 1\right)(Fr^2+2)\right]},$$

Substituting $Fr = 7.172$,

$$\frac{h_L}{E_1} = 0.6283$$

About 63% of mechanical energy is dissipated by the hydraulic jump. This can be shown on the specific energy diagram in Figure 12.9.1. The jump does not proceed along the specific energy curve, rather it jumps from 1 to 2 directly. The solutions for hydraulic jumps in other than rectangular channels is similar to that of rectangular channel. The phenomenon is similar. Froude number should be calculated using hydraulic depth.

12.10 FLOW OVER BROAD CRESTED WEIR

A broad crested weir consists of an obstruction in the form of raised portion of bed extending across the full width of the channel with a flat upper surface sufficiently broad in the direction of flow. Fig. 12.10.1 shows a broad crested weir with free fall. As the crest is broad, the flow surface becomes parallel to the crest. The upstream edge is well rounded to avoid losses.

The flow upstream will be subcritical and the downstream allows free fall. As there is no restraint downstream the flow will be maximum or the depth above the weir surface will be the critical depth y_c. Considering rectangular channel, expression for flow rate is derived

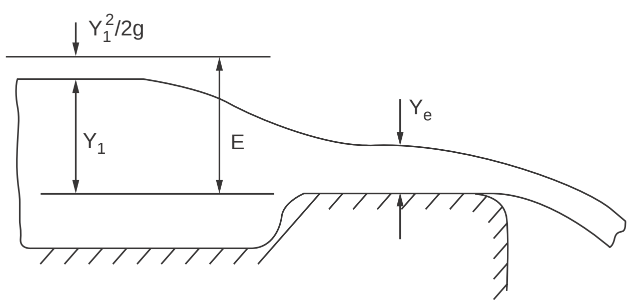


Figure 12.10.1 Broad crested weir

$y_c = (Q^2/gb^2)^{1/3} = (q^2/g)^{1/3}$

∴ $Q = b(gy_c^{\,3})^{0.5}$ or $q = (gy_c^{\,3})^{0.5}$

As $y_c = (2/3)E$, $\mathbf{Q} = b\,\{g \times (8/27)\,E^3\} = \mathbf{1.705\ bE^{3/2}}$ (12.10.1)

$$E = y_1 + (V_1^2/2g)$$

In the tranquil flow upstream $\mathbf{V_1}$ **is small.** In that case $\mathbf{Q = 1.705 \times b \times y^{3/2}}$

The measurement of h (over the crest of the weir) would be sufficient to determine the discharge. The height of the weir will not affect the flow over the crest. The upstream level will adjust as per the height of the weir.

In case the bed level downstream is equal to the bed level upstream, the level downstream will rise.

In this case though the level over the crest will decrease, it may not fall to the critical value y_c. The rate of flow is calculated using Bernoulli equation and continuity equation.

12.11 EFFECT OF LATERAL CONTRACTION

The channel width may be reduced keeping the bed horizontal. **If the upstream flow is subcritical, the depth in the reduced section will decrease. If the upstream flow is supercritical the level in the reduced section will increase.** Flumes with lateral contraction followed by expansion can be used for flow measurement. If the free surface in the section does not pass through critical depth, the arrangement is called **Venturi Flume,** similar to venturimeter.

The details are shown in Fig. 12.11.1

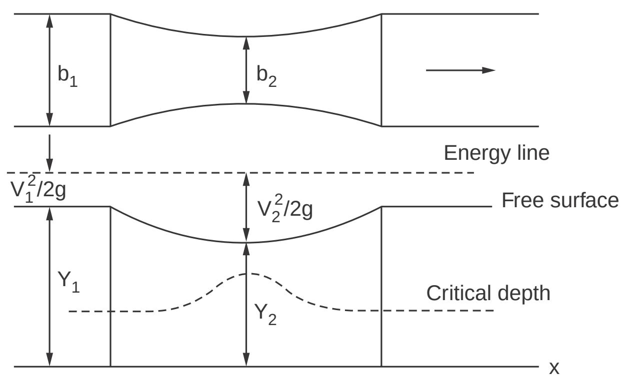


Figure 12.11.1 Lateral contraction

Using continuity and energy relations, it can be shown that

$$V_2 = \left[\frac{2gh}{1-(b_2y_2/b_1y_1)^2}\right]^{0.5} \quad \text{and} \quad Q = b_2y_2\left[\frac{2gh}{1-(b_2y_2/b_1y_1)^2}\right]^{0.5} \qquad (12.11.1)$$

The flow rate $Q = C_d A_2 V_2$ where C_d is the coefficient of discharge having values in the range 0.95 to 0.99. In case the upstream flow is subcritical, and if critical conditions occur at the throat, then, the flow rate is given by, where E is the specific energy and b is the channel width.

$$Q = 1.705\ b_2\ E^{3/2} \qquad (12.11.2)$$

In case the flow velocity upstream is small, then,

$$Q = 1.705\ b_2\ H^{3/2} \qquad (12.11.3)$$

where H is the difference between the level at the throat and upstream water level. In this case if the bed slope down stream is the same as in upstream, the flow will revert to subcritical condition downstream by means of a hydraulic jump. Such a flume is known as standing wave flume.

Example 12.22 *A venture flume is formed in a horizontal channel of 2 m width by constructing the width to 1.3 m and raising floor level in the constricted section by 0.2 m above that of the channel. If the difference in level between the throat and downstream is 25 mm and both upstream and downstream depths are 0.6 m,* ***determine the rate of flow****.*

In the case the downstream conditions are changed such that a standing wave forms after the throat, calculate the flow assuming upstream depth is still 0.6 m.

The flow rate is given by the equation [Refer equation 12.11.1]

$$Q = b_2 y_2 \left[\frac{2gh}{1-(b_2 y_2 / b_1 y_1)^2}\right]^{0.5}$$

$$b_1 = 2 \text{ m},\ b_2 = 0.9 \text{ m},\ y_2 = 0.6 - 0.2 - 0.025 = 0.375 \text{ m},\ h = 0.025 \text{ m},$$

$$\mathbf{Q} = 1.3 \times 0.375 \left[\frac{2 \times 9.81 \times 0.025}{1-(1.3 \times 0.375 / 2 \times 0.6)^2}\right]^{0.5} = \mathbf{0.3736\ m^3/s}$$

When standing wave forms consider the equation (12.11.2)

$Q = 1.705\ b_2\ E^{3/2}$,

E = the total head at construction which is 0.6 – 0.2 = 0.4 m

$\therefore \quad Q = 1.705 \times 1.3 \times 0.4^3 = 0.50607 \text{ m}^3/\text{s}$

The velocity head portion of energy upstream is neglected in this case.

The velocity upstream, $V_1 = 0.5607/(2 \times 0.6) = 0.4673$ m/s

$\therefore \quad V_1^{\,2}/2g = 0.011$ m,

Compared to 0.4 m, this is 2.8% and hence may be neglected.

SOLVED PROBLEMS

Problem 12.1 ***Determine the flow rate of water*** *in a rectangular channel of 3 m width when the depth of flow is 1 m. The bed slope is 1 in 2500. Friction factor f = 0.038.*

As friction factor is given Chezy' constant can be determined. Chezy's constant,

$$C = \sqrt{8g/f} = [8 \times 9.81/0.038]^{0.5} = \mathbf{45.45}$$

To determine the hydraulic depth,

$$A = 3 \times 1 = 3 \text{ m}^2,\ P = 3 + 1 + 1 = 5 \text{ m},$$

$$\therefore \quad R_h = 3/5 = 0.6, \text{ bed slope} = 1/2500$$

Using Chezy's equation, indicating velocity by, V

$$V = C\sqrt{R_h S_b} = 45.45\ \sqrt{0.6/2500} = 0.704 \text{ m/s},$$

Flow rate = VA = 0.704 × 3 = **2.11 m^3/s**

Froude number = $V/\sqrt{gy} = 0.704/\sqrt{9.81 \times 1} = 0.225$

So the flow is in the subcritical region.

Problem 12.2 *Analyse the flow in the channel of problem 12.1. Considering combination of depths of 0.5, 1, 2 and 2.5 m with bed slopes of 1/500, 1/1000, 1/1500, 1/2000 and 1/2500.*

The values calculated using Chezy's equation are tabulated below. Velocity V, m/s, Flow rate Q, m^3/s. Depth and R_h are given in m.

Slope		1/500		1/1000		1/1500		1/2000		1/2500	
Depth	**R_h**	**V**	**Q**	**V**	**Q**	**V**	**Q**	**V**	**Q**	**V**	**Q**
0.5	0.38	1.24	1.87	0.88	1.32	0.7	1.1	0.6	0.9	0.6	0.8
1.0	0.60	1.57	4.72	1.11	3.34	0.9	2.7	0.8	2.4	0.7	2.1
1.5	0.75	1.76	7.92	1.24	5.60	1.0	4.6	0.9	4.0	0.8	3.5
2.0	0.86	1.88	11.3	1.33	7.98	1.1	6.5	0.9	5.6	0.8	5.1
2.5	0.94	1.97	14.8	1.39	10.4	1.1	8.5	1.0	7.4	0.9	6.6

Note. 1. **As slope becomes less steep the velocity and flow rates decrease,** but not in direct proportion.

2. **As depth increases the flow increases more rapidly** because both velocity and area increase with depth.

Problem 12.3 *In problem 12.2 determine the Froude number in each of the cases.*

$Fr = V/\sqrt{gy}$, Depth and R_h are given in m.

Slope		1/500		1/1000		1/1500		1/2000		1/2500	
Depth	**R_h**	**V**	**Fr**	**V**	**Fr**	**V**	**Fr**	**V**	**Fr**	**V**	**Fr**
0.5	0.38	1.24	0.56	0.88	0.40	0.7	0.3	0.6	0.3	0.6	0.3
1.0	0.60	1.57	0.50	1.11	0.35	0.9	0.3	0.8	0.3	0.7	0.2
1.5	0.75	1.76	0.46	1.24	0.32	1.0	0.3	0.9	0.2	0.8	0.2
2.0	0.86	1.88	0.42	1.33	0.30	1.1	0.3	0.9	0.2	0.8	0.2
2.5	0.94	1.97	0.40	1.39	0.28	1.1	0.2	1.0	0.2	0.9	0.2

As depth increases for a given slope, velocity increases but Froude number decreases and flow is subcritical. As slope becomes steeper Froude number increases for the same depth due to velocity increase.

Problem 12.4 ***Determine the slope*** *with which a waste water pipe of 2 m diameter is to be laid for carrying water at the rate of 1500 l/s. The depth of flow is to be half the diameter. Assume CI pipe with Manning constant N = 0.05.*

Manning equation for discharge is

$$Q = \frac{A}{N} R_h^{2/3} S_b^{1/2}, \quad A = \frac{\pi \times 2 \times 2}{4 \times 2} = \frac{\pi}{2} \text{m}^2, P = \pi D/2 = \pi \text{ m}$$

$$\therefore \qquad R_h = (\pi/2)/\pi = 0.5 \text{ m}, \; Q = 1500 l/\text{s} = 1.5 \text{ m}^3/\text{s}$$

$$\therefore \qquad 1.5 = \frac{\pi}{2} \times \frac{1}{0.015} (0.5)^{2/3} (S_b)^{1/2} \quad \therefore \quad \mathbf{S_b = 1/1934}$$

The flow rate and depth uniquely define the slope. Similarity flow rate and slope uniquely define the depth, called normal depth.

Problem 12.5 ***Determine the maximum discharge*** *through a rectangular open channel of area 8 m ^{2} with a bed slope of 1/2000. Assume Mannings constant as 0.022.*

For rectangular section the optimum depth equals half the width and maximum discharge occurs for this condition.

$$A = y \times b = y \times 2y = 2y^2 = 8 \quad \therefore \quad y = 2, b = 4, \quad \therefore \quad R_h = 8/8 = 1 \text{ m}$$

$$\mathbf{Q} = \frac{A}{N} R_h^{2/3} S_b^{1/2} = \frac{8}{0.022} (1)^{2/3} \left(\frac{1}{2000}\right)^{0.5} = \mathbf{8.13 \; m^3/s}$$

This is maximum can be verified by calculating the flows for different depths like 1.8, 1.9, 2.1 and 2.2 for which the flows are 8.11, 8.12, 8.12 and 8.11 m^3/s-very near the optimum value but is still less.

Problem 12.6 ***Determine the economical cross-section*** *for an open channel of trapezoidal section with side slopes of 1 vertical to 2 horizontal, to carry 10 m^3/s, the bed slope being 1/2000. Assume Manning coefficient as 0.022.*

For economical section perimeter should be minimum. The condition for the same is (n-side slope) Refer example 12.8.

$$\frac{b + 2ny}{2} = y\sqrt{n^2 + 1}. \text{ Here } n = 2 \text{ and } b + 4y = 2y\sqrt{5},$$

$$b = y\,(\sqrt{20} - 4) = 0.4721\, y, A = (b + 2y)\, y = 2.4721\, y^2$$

Hydraulic mean depth (A/P) for economical section:

$$P = b + 2y\sqrt{n^2 + 1} = b + 2\,y\sqrt{5} = 4.9443y$$

$$\therefore \qquad R_h = A/P = 2.4721\, y^2/4.9443\, y = y/2$$

$$Q = 10 \text{ m}^3/\text{s}, N = 0.022, S_b = 1/2000$$

$$\therefore \qquad 10 = \frac{1}{0.022} \times 2.4721 y^2 \left(\frac{y}{2}\right)^{3/2} (S_b)^{1/2}$$

$$10 = \frac{1}{0.022} \times \frac{2.4721y^2}{2^{3/2}} (y)^{3/2} \left(\frac{1}{2000}\right)^{1/2} = 0.8884\, y^{7/2}$$

$\therefore$ **y = 1.9971 m, b = 0.9429 m** (check for flow)

Problem 12.7 ***Derive the expression for depth of flow*** *in a channel of circular section for maximum flow.*

Refer figure.

Let the flow depth be y, and Let the angle subtended be θ.

Wetted perimeter $P = 2\,R\,\theta$

Flow area $= (2\,\theta/2\,\pi)\,\pi R^2 - R \sin\theta\, R\cos\theta = R^2\theta - (R^2/2)\sin 2\theta$

$= (R^2/2)\,(2\theta - \sin 2\theta)$

Using Chezy's equation,

$$Q = AV = AC\sqrt{\frac{A}{P} S_b} = C\left[\frac{A^3}{P} S_b\right]^{0.5}$$

Q will be maximum if A^3/P is maximum. The condition is determined, using

$$\left[\frac{d(A^3/P)}{d\theta}\right] = 0 \quad i.e., \quad \frac{P \times 3 \times A^2 \dfrac{dA}{d\theta} - A^3 \dfrac{dP}{d\theta}}{P^2} = 0$$

or $$3P\frac{dA}{d\theta} = A\frac{dP}{d\theta}$$

$$P = 2\,R\,\theta, \quad \therefore \quad \frac{dP}{d\theta} = 2\,R,$$

$$A = (R^2/2)\,[2\theta - \sin 2\theta]$$

$$\frac{dA}{d\theta} = \frac{R^2}{2}\,(2 - 2\cos 2\theta), \text{ Substituting}$$

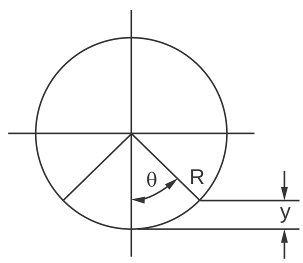


Figure P. 12.7

$$3 \times 2R\theta\,\frac{R^2}{2}\,(2 - 2\cos 2\theta) = \frac{R^2}{2}\,(2 - 2\sin 2\theta)\,2R$$

$$3\theta(1 - \cos 2\theta) = [\theta - (\sin 2\theta/2)]$$

$$2\theta - 3\theta\cos 2\theta + \frac{\sin 2\theta}{2} = 0$$

This is a transcendental equation to be solved by trial, θ = 2.69 radian or about 154°. (check using radian mode in the calculator)

The area for flow $= R^2\,\theta - \dfrac{R^2}{2}\sin 2\theta = 3.0818\,R^2$

Perimeter $= 2R\theta = 5.3756\,R \quad \therefore \quad R_h = 0.5733\,R$

Depth of flow $= R - R\cos\theta = \mathbf{R(1 - \cos\theta)} = 1.8988\,R$ or $0.9494\,D$

Note: This uses Chezy equation:

In case Manning equation is used, then the flow,

$$Q = \frac{A}{N}\left(\frac{A}{P}\right)^{3/2} S_b^{\,1/2} = \frac{1}{N}\frac{A^{5/2}}{P^{3/2}} S_b^{\,1/2}$$

$\dfrac{A^{5/2}}{P^{3/2}}$ or $\dfrac{A^5}{P^3}$ should be maximized with θ as independent variable

$$P^3 \times 5 \times A^4 \frac{dA}{d\theta} - A^5\, 3P^2 \frac{dP}{d\theta} = 0, \text{ Substituting,}$$

$$5P\frac{dA}{d\theta} - 3A\frac{dP}{d\theta} = 0 \quad \text{and} \quad \text{Substituting for } \frac{dA}{d\theta} \text{ and } \frac{dP}{d\theta}$$

$$5 \times 2R\theta \frac{R^2}{2}(2 - 2\cos 2\theta) = 3\frac{R^2}{2}(2\theta - \sin 2\theta) \times 2R$$

$$5\theta(2 - 2\cos 2\theta) - 3(2\theta - \sin 2\theta) = 0, \quad \text{or} \quad 4\theta - 10\theta\cos 2\theta + 3\sin 2\theta = 0$$

Solving, $\theta = 2.5007$ **radian or 143.28°**

In this case depth $R - R\cos\theta = R(1 - \cos\theta) = 1.802\,R$ or $0.901\,D$

Problem 12.8 Derive the condition for maximum velocity of flow in a channel of circular section.

Refer problem Problem 12.7.

$$A = \frac{R^2}{2}(2\,\theta - 2\sin 2\theta),\ P = 2R\theta,\ V = C\sqrt{R_h S_b},\ R_h = \frac{A}{P}$$

For V to be maximum, A/P should be maximized,

$$\frac{d(A/P)}{d\,\theta}\left(P\frac{dA}{d\theta} - A\frac{dP}{d\theta}\right)/P^2 = 0 \quad \therefore \quad P\frac{dA}{d\,\theta} = A\frac{dP}{d\theta} \qquad \text{(A)}$$

$$\frac{dA}{d\theta} = \frac{R^2}{2}(2 - 2\cos 2\theta),\ \frac{dP}{d\theta} = 2R, \text{ Substituting}$$

$$2R\theta\frac{R^2}{2}(2 - 2\cos 2\theta) = \frac{R^2}{2}(2\theta - 2\cos 2\theta)\,2R$$

$$2\theta(1 - \cos 2\theta) = 2\theta - \sin 2\theta$$

$$-2\theta\cos 2\theta + \sin 2\theta = 0 \quad \text{or} \quad \tan 2\theta = 2\theta$$

∴ Solving $\theta = 2.247$ radians, or 128.75°

Depth for maximum velocity y = R(1 – cos θ) $= 1.626\,R$ or $0.813\,D$

Note: This is using Chezy equation, In case Manning equation is used, the same result is obtained as in equation *A*.

Problem 12.9 ***Determine the maximum discharge*** *through a circular pipe of 2 m diameter with a bed slope of 1/1000. Also determine the depth for maximum velocity and the corresponding discharge Chezy's constant C = 60*

Adopting Chezy equation (Refer problem 12.7)

$$A = \frac{R^2}{2}(2\theta - 2\sin 2\theta),\ P = 2R\theta,\ R = 1,\ Q = 2.69 \text{ radians}$$

$$\therefore \qquad A = \frac{1}{2}[2 \times 2.69 - \sin(2 \times 2.69)] = 3.083 \text{ m}^2,$$

$$P = 2 \times 1 \times 2.69 = 5.38 \text{ m}$$

$$R_h = A/P = 3.083/5.38 = 0.573 \text{ m}$$

$$Q = AC\sqrt{R_h S_b} = 3.083 \times 60\sqrt{0.573 \times \left(\frac{1}{1000}\right)} = 4.43 \text{ m}^3/\text{s}$$

For maximum velocity: $\theta = 2.247$ radians

$$A = \frac{R^2}{2}(2\theta - 2\sin 2\theta) = \frac{1}{2}(2 \times 2.247 - \sin(2 \times 2.247))$$

$$= 2.7351 \text{ m}^2$$

$$P = 2R\theta = 2 \times 1 \times 2.247 = 4.494 \text{ m}$$

$$R_h = A/P = 2.7351/4.494 = 0.6086 \text{ m}$$

$$Q = 2.7351 \times 60 \times \sqrt{0.6086 \times \frac{1}{1000}} = 4.049 \text{ m}^3/\text{s}$$

In case of full area flow: $A = \pi \times 1^2 = \pi \text{m}^2$

$$P = \pi D \quad \therefore \quad R_h = \pi/\pi D = 1/2 = 0.5$$

$$\therefore \qquad \mathbf{Q} = \pi \times 60\sqrt{\frac{0.5}{1000}} = \mathbf{4.215\ m^3/s}$$

(**Note:** Compare the flows for maximum discharge, maximum velocity and full flow)

Problem 12.10 *In a rectangular open channel of 5 m width the flow rate is 10 m³/s and depth of flow is 1.0 m.* ***Determine the critical depth and the alternate depth.***

Flow velocity, $V = 10/5 \times 1 = 2$ m/s, Froude number $= V/\sqrt{gy} = 2/\sqrt{9.81 \times 1} = 0.64$

Flow is in the subcritical region.

$$\text{Specific energy} \qquad = \frac{Q^2}{2gA^2} + y = \frac{10^2}{2 \times 9.81 \times 5^2} + 1 = 1.2039 \text{ m}$$

$$\text{At critical condition, } \mathbf{y_c} = \left(\frac{Q^2}{gb^2}\right)^{1/3} = \left(\frac{10^2}{5^2 \times 9.81}\right)^{1/3} = \mathbf{0.7415\ m}$$

Velocity at this condition = 10/(5 × 0.745) = 2.6971 m/s

Froude number = $2.6971/\sqrt{9.81 \times 0.7415}$ = 1.0 (checks)

Minimum energy = (3/2) y_c = 1.11225 m

To determine the alternate depth,

$$\frac{V_2{}^2}{2g} + y_2 = 1.2039,\ V_2{}^2 = \left(\frac{Q}{A_2}\right)^2 = \frac{Q^2}{b^2 y^2}$$

$$\frac{Q^2}{2gb^2 y_2{}^2} + y_2 = 1.2039,\ \frac{10^2}{2 \times 9.81 \times 5^2}\frac{1}{y_2{}^2} + y_2 = 1.2039$$

$0.2039 + y_2{}^3 = 1.2039\, y_2{}^2$ or $y_2{}^3 - 1.2039\, y_2{}^2 + 0.2039 = 0$

Solving by trail y_2 = **0.565 m** and V_2 = 3.5398 m/s, *Fr* = 1.5036,

∴ Supercritical region. Check for energy :

∴ $(V^2/2g) + y = 1.2036$ (checks).

Problem 12.11 ***Calculate the bed slope of a trapezoidal channel*** *of bed width 6 m and horizontal to vertical side slope of 1:3. Water flows at the rate of 10 m³/s at a depth of 2 m. Assume Chezy's constant as 50.*

Discharge $Q = AC\sqrt{R_h S_b}$

$A = 2 \times [6 + (2/3)] = 13.3333$ m²,

$$P = 6 + \left(\sqrt{2^2} + (2/3)^2\right)^2 = 10.2164 \text{ m}$$

$R_h = A/P = 13.3333/10.2164 = 1.3051$ m

$10 = 13.3333 \times 50\sqrt{1.3051 \times S_b}$. Solving, $\mathbf{S_b = 1/5800}$

Problem 12.12 ***Calculate the water discharge through an open channel*** *shown in figure. Assume Chezy's constant as 60 and bed slope as 1 in 2000. Also calculate the Mannings constant for the flow.*

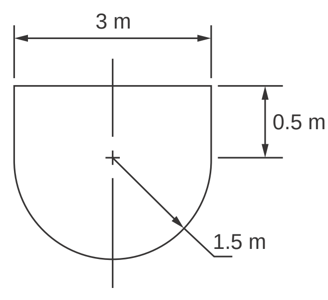


Figure P. 12.12

Discharge $Q = AC\sqrt{R_h S_b}$, $A = (3 \times 0.5) + \dfrac{\pi \times 1.5^2}{2}$

= 5.0343 m²

$P = 0.5 + 0.5 + (\pi \times 1.5) = 5.7123$ m,

$R_h = 5.0343/5.7123 = 0.8813$ m

$\boldsymbol{Q}$ = 5.0343 × 60 [0.8813/2000]$^{0.5}$ = **6.341 m³/s**

To determine Mannings constant

$$6.341 = \frac{5.0343}{N}\left[0.8813^{2/3}\left(\frac{1}{2000}\right)^{1/2}\right]$$

∴ $N = 0.0163$

Problem 12.13 *Using Bazins formula,* ***determine the discharge through*** *a rectangular ordinary earthen channel 2 m wide and 0.5 m deep with a slope of l in 2500. Assume Bazins constant k = 1.303. If Manning constant for this type is 0.025, determine and compare the flow.*

Discharge $Q = AC\sqrt{R_h S_b}$, $C = \dfrac{86.9}{1 + k/\sqrt{R_h}}$

$A = 2 \times 0.5 = 1\ \text{m}^2$, $P = 0.5 + 2 + 0.5 = 3$ m,

$R_h = A/P = 1/3 = 0.333$

$$C = \frac{86.9}{1 + 1.303/\sqrt{0.3333}} = 26.682$$

∴ $$\mathbf{Q} = 1 \times 26.682\sqrt{0.333 \times \left(\frac{1}{2500}\right)} = \mathbf{0.3081\ m^3/s}$$

By Mannings equation,

$$\mathbf{Q} = \frac{1}{0.025}(0.333)^{2/3}\left(\frac{1}{2500}\right)^{1/2} = \mathbf{0.385\ m^3/s}$$

Problem 12.14 *A rectangular open channel having 4 m depth and 8 m width is concrete lined with a bed slope of 1 in 2000. Use Kutter's formula and* ***calculate the discharge through*** *it. Kutters constant = 0.012*

Discharge $Q = AC\sqrt{R_h S_b}$

$A = 32\ \text{m}^2$, $P = 16$ m, $R_h = 2$ m, $S_b = 1/2000 = 0.0005$

$$C = \frac{23 + (0.00155/S_b) + \dfrac{1}{N}}{1 + (23 + (0.00155/S_b))\dfrac{N}{\sqrt{R_h}}}$$

$$= \frac{23 + (0.00155/0.0005) + (1/0.012)}{1 + (23 + (0.00155/0.0005))(0.012/\sqrt{2})} = 89.59$$

$$\mathbf{Q} = 32 \times 89.59\sqrt{2 \times 0.0005} = \mathbf{90.66\ m^3/s},$$

For concrete, taking **Manning constant** as 0.012

$$\mathbf{Q} = (32/0.012)2^{2/3}(0.0005)^{1/5} = \mathbf{94.65\ m^3/s}$$

Problem 12.15 ***Determine the slope for a V-shaped concrete*** *lined channel with total included angle of 80° and a depth of 3 m if the discharge is 10.7 m³/s. Manning constant N = 0.012*

$$\text{Area} = \frac{2 \times 3 \tan 40 \times 3}{2} = 7.5519 \text{ m}^2,$$

Wetted perimeter = 2 × 3/cos 40 = 7.8324 m

Hydraulic mean depth = $R_h = A/P$ = 7.552/7.8324 = 0.9642 m

$$Q = 10.7 = \frac{7.5519}{0.012}(0.9642)^{2/3} S_b^{1/2} \quad \therefore \mathbf{S_b = 1/3295}$$

Calculating the corresponding Chezy constant, C

$$10.7 = 7.5519 \times C\left(0.9642 \times \frac{1}{3295}\right)^{1/2} \quad \therefore C = 82.83$$

Problem 12.16 ***Estimate the discharge of water in*** *an open channel of trapezoidal section with bottom width of 1 m and side slope of 1:1 with a flow depth of 1 m. The bed slope is 1 in 2000. Use Manning formula with constant N = 0.05*

Discharge $Q = \frac{A}{N} \times R_h^{2/3} \times S_b^{1/2}$, $A = \{(1 + 3)/2\} \times 1 = 2 \text{ m}^2$;

$P = 1 + 2\sqrt{1^2 + 1^2} = 3.83$ m, $R_h = A/P = 2/3.83 = 0.5224$ m

∴ $\mathbf{Q} = (2/0.05) \times (0.5224)^{2/3} \times (1/2000)^{1/2} = 0.58 \text{ m}^3/\text{s} =$ **580 *l/s***

Problem 12.17 ***Determine the most economical cross-section*** *of a rectangular channel of width b and depth y to carry 1000 litres of water per second with a bed slope of 1 in 500. assume Mannings constant, N = 0.022*

Area = $A = b \times y$

For most economical cross-section

$b = 2y$, $A = 2y^2$, $R_h = y/2$

$Q = \frac{A}{N} R_h^{2/3} S_b^{1/2}$, flow rate = 1000 *l/s or* 1 m³/s

$$1 = \frac{2y^2}{0.022}\left(\frac{y}{2}\right)^{2/3}\left(\frac{1}{500}\right)^{1/2},$$

Solving depth **y = 0.7028 m, width b = 1.4056 m**

Problem 12.18 *The most economical cross-section of a trapezoidal open channel is 5 m². **Find the discharge in the channel** for a depth of flow of 0.5 m. Assume the Chezy constant C = 50 and bed slope as 1 in 1000.*

For most economical cross-section of the trapezoidal channel

$R_h = y/2 = 0.5/2 = 0.25$ m,

Discharge, $\mathbf{Q} = AC\sqrt{R_h S_b} = 5 \times 50 \sqrt{0.25/1000} =$ **3.95 m³/s**

Problem 12.19 ***Design the bed slope*** *for the most economical cross-section for a trapezoidal earthen open channel with a flow velocity of 2 m/s and discharge of 5 m³/s. The side slope vertical to horizontal may be taken as 1 in 2 and Chezy constant C = 50*

$$A = \frac{Q}{V} = \frac{5}{2} = 2.5 \text{ m}^2\text{, For most economical cross-section}$$

$$A = y(b + ny) = y(b + 2y),\ 2.5 = b \times y + 2y^2,\ b \times y = 2.5 - 2y \quad (1)$$

Also $R_h = y/2$

Perimeter $P = b + 2y\sqrt{n^2 + 1} = b + 2y\sqrt{5},\ R_h = y/2 = A/P = 2.5/(b + 2y\sqrt{5})$

$$b \times y + 4.472\,y^2 = 5 \quad (2)$$

Substituting equation (1) in equation (2)

$$2.5 - 2y^2 + 4.472y^2 = 5,\quad 2.472\,y^2 = 2.5,\quad y = \sqrt{\frac{2.5}{2.472}} = 1.006 \text{ m}$$

Substituting $y = 1.006$ in equation (1)

$$1.006\,b = 2.5 - 2 \times (1.006)^2 \quad b = 0.473 \text{ m}$$

Velocity, $V = C\sqrt{R_h S_b}$ *i.e.* $2 = 50\sqrt{\frac{1.006}{2} \times S_b}$ ∴ **Slope, S_b = 1/316**

Problem 12.20 *A rectangular channel of 5 m width discharges water at the rate of 1.5 m³/s into a 5 m wide apron with 1/3000 slope at a velocity of 5 m/s.* ***Determine the height of the hydraulic jump*** *and energy loss.*

Depth of water on the upstream side of the jump,

$$y = \frac{Q}{V_1 \times b} = \frac{1.5}{5 \times 5} = 0.06 \text{ m}$$

$$\therefore \quad Fr = \frac{5}{\sqrt{9.81 \times 0.06}} = 6.52\text{. Hence flow is supercritical.}$$

Hence hydraulic jump is possible. Depth of water on the downstream side of jump

$$y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + 2y_1V_1^2/g}$$

$$= -\frac{0.06}{2} + \sqrt{\frac{0.06^2}{4} + \frac{2 \times 0.06 \times 5^2}{9.81}} = 0.5238 \text{ m}$$

Height of hydraulic jump = $y_2 - y_1$ = 0.5238 – 0.06 = **0.4638 m**

$$V_2 = \frac{V_1 y_1}{y_2} = \frac{5 \times 0.06}{0.5238} = 0.5757 \text{ m/s}$$

$$\textbf{Energy loss} = E_1 - E_2 = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

$$= \left(0.06 + \frac{5^2}{2 \times 9.81}\right) - \left(0.5238 + \frac{0.5727^2}{2 \times 9.81}\right)$$

= **0.7937 m head of water.**

Problem 12.21 *Water is discharged at a velocity of 8 m/s with a depth of 0.7 m in a horizontal rectangular open channel of constant width when the sluice gate is opened upwards.* ***Determine the height of the hydraulic jump and the loss of energy.***

Depth of water on the downstream of the jump (modified equation 12.9.7)

$$\mathbf{y_2} = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2y_1V_1^2}{g}} = -\frac{0.7}{2} + \sqrt{\frac{0.7^2}{4} + \frac{2 \times 0.7 \times 8^2}{9.81}} = \mathbf{2.69\ m}$$

$$V_2 = \frac{V_1 h_1}{y_2} = \frac{8 \times 0.7}{2.69} = 2.082 \text{ m/s}$$

Energy loss = $\mathbf{E_1 - E_2}$

$$= \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right) = \left(0.7 + \frac{8^2}{2 \times 9.81}\right) - \left(2.69 + \frac{2.082^2}{2 \times 9.81}\right)$$

= 1.05 m head of water.

Problem 12.22. *A Wide channel of uniform rectangular section with a slope of 1/95 has a flow rate of 3.75 $m^3/s/m$. The Manning constant is 0.013. Suddenly the slope changes to 1/1420.* ***Determine the normal depths for*** *each case. Show that a hydraulic jump has to occur and calculate the downstream flow height.*

As the channel is said to be wide the hydraulic mean depth will be equal to the depth of flow.

The normal depth is obtained from the Manning equation

$$Q = \frac{A}{N} R_h^{2/3} S_b^{1/2}$$

Here $A = 1 \times y$, $R_h = y$, $S_b = 1/95$. Substituting the values

$$Q = \frac{S_b^{1/2}}{N} y^{5/3}$$

$\therefore$ $$3.75 = \frac{(1/95)^{1/2}}{0.013} y_1^{5/3}, \text{ Solving } \quad y_1 = \mathbf{0.6399,}$$

$V_1 = 3.75/0.6399 =$ **5.86 m/s, $\mathbf{Fr_1}$ = 2.339, Supercritical flow**

For slope 1/1420, Normal depth is

$$3.75 = \frac{(1/1420)^{1/2}}{0.013} y_2^{5/3}. \text{ Solving } \mathbf{y_2 = 1.4404\ m}$$

$$\mathbf{V_2} = 3.75/1.4404 = \mathbf{2.6035\ m/s}$$

$$\mathbf{Fr_2} = 2.6035/\sqrt{9.81 \times 1.4404} = \mathbf{0.6926} \quad \therefore \text{ Subcritical}$$

As flow is from supercritical to subcritical flow, hydraulic jump should occur. (equation 12.9.7)

$$y_2 = \frac{y_1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$$

$$= \frac{0.6399}{2}\left[-1 + \sqrt{1 + 8/2.339^2}\right] = 1.8208 \text{ m}$$

Problem 12.23 *A trapezoidal channel has a bed slope of 1/2500. The channel is to carry 2 m³/s.* ***Determine the optimum dimensions****. Side slope is 1:1. Chezy' constant = 50 $m^{1/2}\ s^{-1}$.*

The conditions for the most optimum section are

$$b = 2y\ [(n^2 + 1)^{0.5} - n] \text{ and } n = 1 \text{ As slope 1:1}$$

$$\therefore \quad b = 2y\ [\sqrt{2} - 1] = 0.8284y,\ A = (b + y)\ y = y^2 + 0.8284\ y^2 = 1.8284\ y^2$$

$$P = b + 2\sqrt{(y^2 + y^2)} = 0.8284y + 2.82y = 3.6568y$$

$$R_h = \frac{1.8284y^2}{3.6568y} = 0.5\ y$$

$$Q = 2 = 18284\ y^2 \times 50\sqrt{(0.5y)(1/2500)} = 1.29287y^{5/2}$$

Solving **y = 1.1907 m, b = 0.9863 m**

Problem 12.24 *A rectangular channel of 6 m width has a flow rate of 22.5 m³/s when the depth is 3 m.* ***Determine the alternate depth and the critical depth.***

Refer section 12.9

Velocity $V = 22.5/(6 \times 3) = 1.25$ m/s, $Fr = 1.25/\sqrt{3 \times 9.81} = 0.23$

Flow is subcritical.

$$\text{Specific energy} = \frac{V_2^2}{2g} + y = \frac{1.25^2}{2 \times 9.81} + 3 = 3.0796 \text{ m}$$

The specific energy is the same at the alternate depth.

$$\frac{V^2}{2g} + y_2 = 3.0796. \text{ Expressing } V_2 \text{ in terms of flow,}$$

$$\frac{Q^2}{2gb^2\ y_2^2} + y_2 = 3.0796. \text{ or } y_2^3 - 3.0796y^2 + 0.726 = 0. \text{ Solving by trial,}$$

$$\mathbf{y_2 = 0.5302\ m},\ V_2 = 7.07\text{ m},\ Fr = 7.07/\sqrt{9.81 \times 0.5302} = 3.33$$

Hence supercritical

To find the critical depth,

$$\mathbf{y_c} = \left(\frac{Q^2}{gb^2}\right)^{1/3} = \left(\frac{22.5^2}{9.81 \times 6^2}\right)^{1/3} = \mathbf{1.1275\ m}$$

$$E_{min} = (3/2)\,y_c = 1.691\text{ m},\ V_c = \frac{22.5}{6 \times 1.1275} = 3.32\text{ m/s},$$

$$Fr = 3.32/\sqrt{9.81 \times 1.1275} = 1$$

Problem 12.25 *Water flows across a broad crested weir in a rectangular channel 0.4 m wide. The depth of water upstream is 0.07 m and the crest of the weir is 40 mm above the channel bed.* ***Determine the fall in the surface level*** *and the discharge over the weir. Assume negligible velocity of approach.*

As there is free fall over the weir, the flow will be maximum. The discharge is given by the equation (12.10.1),

$$Q = 1.705\ b\ H^{3/2},$$

where H the upstream level above the crest in the channel.

Given $b = 0.4$ m, $H = 0.03$ m

$\therefore$ $$\mathbf{Q} = 1.705 \times 0.4 \times 0.03^{3/2} = \mathbf{3.54 \times 10^{-3}\ m^3/s}$$

As the flow is maximum the level above the crest should equal critical depth.

$$\mathbf{y_c} = (Q^2/gb^2)^{1/3} = \left[\left(\frac{3.54 \times 10^{-3}}{0.4}\right)^2 \frac{1}{9.81}\right]^{1/3} = \mathbf{0.020\ m}$$

$\therefore$ drop in depth $= 0.03 - 0.02 = 0.01$ m or 10 mm

Problem 12.26 *A venturi flume with level bed is 12 m wide and the depth of flow upstream is 1.5 m. The throat is 6 m wide. In case a standing wave forms downstream,* ***calculate the rate of flow of water,*** *correcting for the velocity of approach. Assume $C_d = 0.94$.*

Refer section 12.11

For venturi flume with standing wave downstream $Q = C_d 1.705\ b_2\ E^{3/2}$

where b_2 is the throat area and E is the specific energy. In this case the first assumption is E = y upstream.

$\therefore$ $$\mathbf{Q} = 0.94 \times 1.705 \times 6 \times 1.5^{3/2} = \mathbf{17.666\ m^3/s}$$

Velocity upstream $= 17.666/12 \times 1.5 = 0.981$ m/s

$\therefore$ $$E = 1.5 + 0.9814^2/(2 \times 9.81) = 1.549\text{ m}$$

Now using the corrected value of E

$$\mathbf{Q} = 0.94 \times 1.705 \times 6 \times 1.549^{3/2} = \mathbf{18.54\ m^3/s}$$

Further iteration can be made using this flow.

Problem 12.27 *A venturi flume is placed in a rectangular channel of 2 m width in which the throat width is 1.2 m. The upstream flow depth is 1 m and the depth at the throat is 0.9 m. Determine the flow rate. If a standing wave forms downstream of throat* ***determine the flow.*** *Assume the bed to be horizontal.*

In the first case the flow Q is given by equation (12.11.1)

$$Q = b_2 y_2 \left[\frac{2gh}{1-(b_2 y_2 / b_1 y_1)^2}\right]^{0.5}$$

where h is the difference in levels between upstream and throat

$$\mathbf{Q} = 1.2 \times 0.9 \left[\frac{2 \times 9.81 \times 0.1}{1-(1.2 \times 0.9 / 2 \times 1)^2}\right]^{0.5} = \mathbf{1.797\ m^3/s}$$

In case standing wave forms, considering upstream velocity. Specific energy

$E = 0.9 + (V^2/2g)$, As $V = 1.797/(1.2 \times 0.9) = 1.663$ m/s,

$E = 1.041$ m.

$Q = 1.705\ b_2\ E^{3/2} = 1.705 \times 1.2 \times 1.041^{1.5} = 2.17\ m^3/s$

In case upstream velocity is neglected, then

$Q = 1.705 \times 1.2 \times 1^{3/2} = 2.046\ m^3/s$.

Problem 12.28 *Water enters a channel at a velocity of 4 m/s. The Froude number is 1.4. Calculate the depth of flow after the jump. Also calculate the loss of specific energy.*

The depth of water after the jump is given by equation (12.9.7)

$$y_2 = \frac{y_1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right] \qquad (A)$$

To determine, $Fr = V/\sqrt{gy}$, $1.4 = 4/\sqrt{9.81}\sqrt{y}$, Solving for y, $y = 0.8321$ m

Substituting in equation (A),

$$y_2 = \frac{0.8321}{2}\left[-1+\sqrt{1+8 \times 1.4^2}\right]$$

$$= 1.2832 \text{ m}, V_2 = \frac{y_1 V_1}{y_2} = \frac{0.8321 \times 4}{1.2832} = 2.5939 \text{ m/s}$$

$$\textbf{Loss} = \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

$$= \left(0.8321 + \frac{4^2}{2 \times 9.81}\right) - \left(1.2832 + \frac{2.5939^2}{2 \times 9.81}\right)$$

$= 1.6476 - 1.6261 =$ **0.021 m head.**

REVIEW QUESTIONS

1. Define open channel flow.
2. State Chezy, Kutter and Manning formula for uniform flow through an open channel.
3. Distinguish between uniform and non uniform open channel flow.
4. Derive the condition for the most economical section of a rectangular channel.
5. Derive the condition for the best side slope of the most economical trapezoidal channel.
6. Prove that for a channel of circular section for flow to be maximum, the depth of flow, $h = 0.95 \times$ diameter of circular section.
7. Explain the terms (*i*) specific energy (*ii*) critical depth and (*iii*) Critical velocity.
8. Derive an expression for critical depth and critical velocity.
9. Explain the term hydraulic jump. Derive an expression for the downstream depth of hydraulic jump.
10. Show that in a rectangular open channel the critical depth is two thirds of specific energy.
11. Derive an expression for wave celerity.

OBJECTIVE QUESTIONS

O Q. 12.1 Define the following terms

(1) Froude number (2) Wetted perimeter (3) Hydraulic radius (4) Wave celerity (5) Subcritical, critical and supercritical flow (6) Specific energy (7) Alternate depths (8) Normal depth (9) Energy grade line (10) Hydraulic grade line (11) Manning roughness coefficient (12) Optimum cross section of channels (13) Critical depth (14) Hydraulic jump (15) Venturi flume (16) Broad crested weir (17) Standing wave (18) Bed slope.

O Q. 12.2 Fill in the blanks

(1) Depth of flow is the vertical distance between the free liquid surface to the __________

(2) The channel lining (side and base of the channel) which comes in direct contact with the liquid stream is called __________

(3) The hydraulic mean depth represents the ratio of the flow area to the __________

(4) The slope of the total energy line is referred to as __________

(5) The flow is laminar in an open channel if the Reynolds number is less than __________

(6) The flow is certainly turbulent in an open channel if the Reynolds number is greater than __________

(7) The Chezy coefficient C is a variable with its value depending on the flow __________ number and the __________

(8) The phenomenon of sudden increase in depth of flow in a channel is referred to as __________

(9) The energy loss through a hydraulic jump equals the difference between the __________ at the upstream and downstream sections.

(10) A strong hydraulic jump causes about __________ of energy dissipation.

Answers

(1) Lowest point of the channel section (2) Wetted perimeter (3) Wetted perimeter (4) Energy gradient (5) 500 (6) 2000 (7) Reynolds, Surface roughness (8) Hydraulic jump (9) Specific energies (10) 95%

O Q. 12.3 Fill in the blanks :

1. For a supercritical flow Froude number should be __________
2. For a given slope and flow rate in a channel, the depth is called __________
3. When there is a change in slope in supercritical flow __________ will form.
4. Across a hydraulic jump specific energy will __________
5. For a given specific energy as flow depth increases Froude number will __________
6. For a given specific energy if one flow depth is supercritical the other will be __________
7. The flow depth in subcritical flow will be __________ compared to the flow depth a critical flow.
8. For a given specific energy the two possible depths of flow are called __________
9. As roughness increases Mannings coefficient will __________
10. For a given area __________ section gives the maximum flow.
11. The flow depth __________ across hydraulic jump.

Answers

1. Higher than one 2. Normal depth 3. Hydraulic jump 4. Decrease 5. Decrease 6. Subcritical 7. Higher 8. Alternate depths 9. Increase 10. Semicircular 11. Increases

O Q. 12.4 State Correct or Incorrect

1. Hydraulic radius is the ratio of wetted perimeter to area or (P/A).
2. Hydraulic depth is the average depth (Area/Topwidth).
3. Chezy's constant is a dimensional constant.
4. Bazin formula does not relate chezy coefficient C to bed slope S_b.
5. The dimension of Chezy constant is $\sqrt{L} / T$.
6. Kutters formula considers bed slope.
7. Manning proposed that C varied as $R_h^{1/3}$.
8. Flow rate through open channels is inversely proportional to square root of bed slope ($S_b^{1/2}$).
9. The slope is unique for a given flow rate and depth of flow in a given rectangular channel.
10. As Mannings constant increases the flow will increases.
11. As roughness increases, Mannings constant will decrease.
12. Specific energy is the sum of kinetic head and flow depth.
13. As depth increases the wave velocity decreases.

Answers

Correct : 2, 3, 4, 5, 6, 9, 12 **Incorrect :** 1, 7, 8, 10, 11, 13

O Q. 12.5 State Correct or Incorrect :

1. At critical depth of flow Froude number should be equal to one.
2. If one depth of flow for a given specific energy is at supercritical condition the alternate depth also should be at supercritical condition.
3. Disturbance in supercritical flow will not be communicated upstream.
4. Across the hydraulic jump, specific energy remains constant.
5. At critical flow, the specific energy is minimum.
6. Hydraulic jump helps to dissipate energy without damage to surfaces/structures.
7. When supercritical flow meets a bump in the bed, the level at that point will decrease.
8. Hydraulic gradient line represents the depth of flow.

9. Energy gradient line represents specific energy.
10. As velocity drops hydraulic gradient line will rise.
11. The distance between energy gradient line and hydraulic line represents velocity head.
12. For optimum area in a rectangular channel, the depth should be twice the width.

Answers

Correct 1, 3, 5, 6, 8, 9, 10, 11 **Incorrect :** 2, 4, 7, 12

O Q. 12.6 State Correct or Incorrect

1. The included angle at optimum flow in a triangular channel is 45°.
2. In the case of circular section, the flow height for maximum flow is the same as per maximum velocity.
3. Optimum circular section for a given flow is a semicircle.
4. For a given area, maximum value for hydraulic mean depth will be for semi circular shape.
5. For maximum flow through a sluice gate the downstream depth should be 2/3 of upstream depth.
6. In subcritical flow level will increase for negative bump.
7. At minimum specific energy condition for flow through a rectangular channel, the kinetic head will be 0.5 times the flow depth.

Answers

Correct : 3, 4, 5, 7 **Incorrect :** 1, 2, 6

EXERCISE PROBLEMS

E12.1 A rectangular open channel has 5 m width and 1.5 m depth. The bed slope is 1:1000. Assuming Chezy constant $C = 50$, determine the flow rate. **(11.48 m^3/s)**

E12.2 A triangular open channel with 0.25 m depth and 60° angle conveys water. If the bed slope is 1:137 and Chezy constant $C = 52$, determine the flow rate. **(40 liters/s)**

E12.3 A semicircular open channel of diameter 1 m conveys water at the rate of 1.83 m^3/s. If the slope of the bed is 1:950, fine the Chezy constant.

E12.4 Determine the bed slope of a circular pipe that should carry 2.47 m^3/s of water at half full condition. Pipe diameter is 256 cm. Assume Chezy constant as 60. **(1:2500)**

E12.5 A rectangular channel with most economical cross-section carries 8000 l/s of water with an average velocity of 2 m/s. If Chezy constant is 65, determine it's cross-section and slope. **(2.8 × 1.4 m, 1:746)**

E12.6 The area of cross-section of a trapezoidal channel is 30 m^2. Find the base width and flow depth for most economical design if the slope of the bed is 1 in 1500. Side slope is 1 vertical 2 horizontal. **(h = 4.16 m, w = 5.138 m)**

E12.7 Determine the dimensions of a trapezoidal section for a discharge of 40 m^3/s with a bed slope of 1:2500 and Manning's constant $N = 1/50$. **(h = 3.86 m, w = 4.459 m)**

E12.8 Water flows at the rate of 5 m^3/s in a rectangular open channel of 3 m width. Assuming Manning's constant $N = 1/50$, calculate the slope required to maintain a depth of 2 m. **(1:736)**

E12.9 A rectangular channel of 5 m width carries water at the rate of 15 m^3/s. Calculate the critical depth and velocity. **(h_c = 0.972 m, V_c = 3.69 m/s)**

E12.10 Water flows through a rectangular open channel at the rate of 2 m^3/s. If the width of the channel is 2 m, what would be the critical depth of the channel ? If a standing wave is to be formed at a point where the upstream depth is 0.2 m , what would be the rise in water level. **(h_c = 0.467 m, 0.715 m)**

E12.11 In a pensive mood a boy throws a stone in a mountain stream 1.3 m deep. It is observed that the waves created do not travel upstream. Calculate the minimum velocity of the stream. **(3.571 m/s)**

E12.12 Water flows in a wide rectangular channel with a flow rate of 2.5 m^3/s per m width. If the specific energy is 2.2 m, determine the two depths of flow possible. **(2.12 m, 0.426 m)**

E12.13 Water flows over a smooth bump in a wide rectangular channel. The height of the bump at any location is $h(x)$. Neglecting energy losses, **show that the slope of the water surface dy/dx = – (dh/dx)/[1 – (u^2/gy)]** where u is the velocity and y is the depth at location x where the height of the bump is h.

E12.14 The measurement of the parameters of a small stream shows that $A = 26\ m^2$, $P = 16$ m and S_b = 1/3100. Determine the average shear stress on the wetted perimeter of the channel. **(5.14 N/m^2)**

E12.15 Determine the percentage reduction in flow in a rectangular channel if a thin partition in the middle divides it into two equal widths along the flow. **(23.71)**

E12.16 A trapezoidal channel with side slopes of 45° and bottom width of 8 m is to carry a flow of 20 m^3/s. The slope is 1/1796. Considering Mannings coefficient as 0.03, determine the width at the water line. **(12 m)**

E12.17 Compare the perimeter length for a (*i*) 90° triangle and (*ii*) square section to carry water at 2 m^3/s with a slope of 1/80.

E12.18 Water flows in a rectangular channel of width b and depth $b/3$. Considering same slope and Manning coefficient, determine the diameter of a circular channel that will carry the same flow when (*i*) half full (*ii*) Maximum flow condition and (*iii*) Maximum velocity condition.

E12.19 Determine the percentage reduction flow in an equilateral triangular section, flowing full if the top is closed, with water wetting the surface. **(23.7%)**

E12.20 Determine the critical depth in a rectangular channel 10 m wide when the flow rate is 200 m^3/s. **(3.44 m)**

E12.21 Show that for a hydraulic jump in a rectangular channel, the Froude numbers upstream and downstream are related by

$$F_{r2}^2 = \frac{8F_{r1}^2}{[(1+8F_{r1}^2)^{1/2} - 1]^3} \ .$$

E12.22 In a hydraulic jump in a spillway the upstream and downstream depths are 0.7 m and 3.6 m, determine the flow rate per m width of the spillway. Also calculate the Froude numbers. **(7.29 m^3/s, 3.97, 0.34)**

E12.23 Water flows over a broad crested weir with a height of 1.5 m above the bed. If the free surface well upstream is at 0.5 m above the weir surface. Determine the flow rate. Also determine the minimum depth above the weir. **(0.3395 m^3/s, 0.333 m)**

E12.24 In a horizontal rectangular channel there is a small bump of height 30 mm on the bed. At upstream the depth is 0.3 m and velocity is 0.5 m/s. Determine the height of flow above the bump and the flow speed at this section. **(0.267 m, 0.563 m/s)**

E12.25 Water flows under a sluice gate. Upstream the depth is 1.5 m and the velocity is 0.2 m/s. Assuming that specific energy remains constant determine the depth downstream, if the flow rate is 0.3 m^3/s/m. Also calculate the maximum flow rate. **(0.0563 m, 3.14 m^3/s/m)**

E12.26 Compare the flow rates of square and semicircular channels of 2 m water surface, when slope is 1/1000 and Manning coefficient N = 0.015. **(6.44, 2.09 m^3/s)**

E12.27 At the exit flow under a sluice gate the depth of flow was 56.3 mm and the velocity was 5.33 m/s. Calculate the downstream depth and head loss across the jump. **(0.543 m, 0.942 m)**

E12.28 In a venturi flume the bed is horizontal. The width reduces from 600 mm to 300 mm at the throat while the flow depth changes from 300 mm to 225 mm at the throat. Determine the flow rate. **(0.0883 m^3/s)**

E12.29 In a trapezoidal channel of 2.4 m bottom width and 45° side slope the flow rate is 7.1 m^3/s. with normal depth of flow of 1.2 m. Determine the bed slope. N = 0.022 **(1.93/1000)**

E12.30 In the above problem with the same conditions as mentioned if the flow rate is increases to 15 m^3/s, determine the normal depth. **(1.79 m)**

E12.31 A hydraulic jump occurs on a horizontal apron downstream from a spillway at a location where the depth is 0.9 m and speed is 25 m/s. Estimate the depth and velocity downstream. Also calculate the percentage loss of head. **(10.3 m, 2.18 m/s, 67.9%)**