13 Dynamics of Fluid Flow

13.0 INTRODUCTION

In chapter 3 the forces exerted by static fluid on the containment surfaces was discussed. In this chapter the forces exerted by fluid particles on the surfaces over which they flow, is discussed. In case the surfaces cause a change in the magnitude and direction of the velocity of the fluid particles, the fluid particles exert a force on the surface. In turn the surfaces exert an equal and opposite force on the fluid particles. **The force exerted by moving fluid particles on the surface is called dynamic force.** Dynamic force always involves a change in the magnitude and direction of the velocity of the fluid. Forces due to viscous resistance is excluded in the discussions in this chapter to reduce complexity in the analysis.

13.1 IMPULSE MOMENTUM PRINCIPLE

When applied to a single body Newtons second law can be started as "The sum of forces on the body equals the rate of change of momentum of the body in the direction of the force. In equation from (F and V are in the same direction)

$$\Sigma F = \frac{d\,(mV)}{dt} \tag{13.1.1}$$

This can also be written as

$$\Sigma F\,dt = d\,(mV) \qquad (13.1.1\ a)$$

where m is the mass of the body and V is the velocity of the body and t is the time. This also means the impulse **Fdt equals the change in momentum of the body during the time dt.**

When applied to control volume, through which the fluid is flowing, the principle can be stated as "The sum of forces on the fluid equals the difference between the momentum flowing in and momentum flowing out and the change in momentum of the fluid inside the control volume under steady flow condition the last term vanishes. So the forces in the fluid is given by

$$\Sigma F = \frac{d\,(mV)_{out}}{dt} - \frac{d\,(mV)_{in}}{dt} \tag{13.1.2}$$

In other words, the net force on the fluid mass is equal to the net rate of out flow of momentum across the control surface.

This can also be written as

$$\Sigma F = \rho_2\, Q_2 V_2 - \rho_1 Q_1 V_1 \tag{13.1.3}$$

If the fluid is in compressible, then

$$\Sigma F = \rho Q\, (\Delta V) \tag{13.1.3a}$$

In this case Δ**V should be taken as the vectorial addition of V_1 and V_2 and the force will be in the direction of the resultant of V_1 and V_2 .**

In case the forces in the cartesian co-ordinate directions is required, the equation in scalar form is written as

$$\Sigma F_x = \rho Q\, \Delta u \tag{13.1.4}$$

$$\Sigma F_y = \rho Q\, \Delta v \tag{13.1.4a}$$

$$\Sigma F_z = \rho Q\, \Delta w \tag{13.1.4b}$$

where *u, v* and *w* are the components of velocity in the *x, y* and *z* directions.

When calculating the momentum flowing in or out, if the velocity over the section is not uniform a correction has to applied. The correction factor β is given by

$$\beta = \frac{1}{AV^2} \int_A u^2 dA \tag{13.1.5}$$

where *V* is the average velocity.

13.1.1 Forces Exerted on Pressure Conduits

Consider the reducer section shown in Figure 13.1.1 (*a*). The free body diagram is given in Figure 13.1.1 (*b*) :

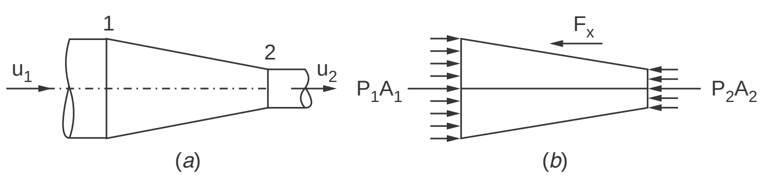

Figure 13.1.1

Assuming ideal fluid flow,

$$\Sigma F_x = P_1 A_1 - P_2 A_2 - F_x = \rho Q\, (u_2 - u_1) \tag{13.1.6}$$

or the force on the fluid is given by

$$F_x = P_1 A_1 - P_2 A_2 - \rho Q\, (u_2 - u_1) \tag{13.1.6a}$$

This force is the force exerted by the reducer on the fluid in the *x* direction. This force acts towards the left as assumed in the figure. The negative sign in the LHS of 13.1.6 is due to this assumption. The numerical value will show the actual sign. The force exerted by the fluid on the reducer will be equal and opposite to this force F_x. The plus and minus signs used in the equations depend in the + ve or – ve directions of the co-ordinate system along which the force is assumed to act.

In case both the magnitude and direction of the velocity is changed by a reducer bend then the force exerted by the bend on the fluid, the turning angle being θ, (suffix *F* indicates as on the fluid)

$$F_{xF} = P_1A_1 - P_2A_2 \cos \theta - \dot{m}\ (V_2 \cos \theta - V_1) \tag{13.1.7}$$

$$F_{yF} = P_2A_2\ \sin \theta + \dot{m} V_2 \sin \theta \tag{13.1.8}$$

The forces on the bend will be equal and opposite to these forces.

Example 13.1 *A reducer in the horizontal plane has an inlet area of 0.02 m² and the outlet area is 0.01 m². The velocity at the inlet is 4 m/s. The pressures are 40 kPa at inlet and 10 kPa of outlet.* ***Determine the force exerted by the reducer on the fluid.***

As the flow is in the horizental plane body forces are neglected. Refer Figure 13.1.1

As $A_1/A_2 = 2, \quad V_2 = 2V_1 = 8$ m/s

Using equation (13.1.6*a*)

$$F_x = P_1A_1 - P_2A_2 - m\,(u_2 - u_1)$$

$$m = 4 \times 0.01 \times 1000 = 40 \text{ kg/s}$$

$$F_x = 40 \times 10^3 \times 0.02 - 10 \times 10^3 \times 0.01 - 40\,(8 - 4)$$

= **540 N on the fluid towards left.**

On the reducer 540 N along positive *x* direction.

Example 13.2 *A 45° bend in the horizontal plane is shown in figure. The inlet area is 1.2 m² and the outlet area is 0.6 m². The velocity of water at inlet is 12 m/s. The pressures at inlet and cutlet are 40 and 30 kPa respectively.* **Calculate the** *magnitude and direction of the* **resultant force** *on the bend.*

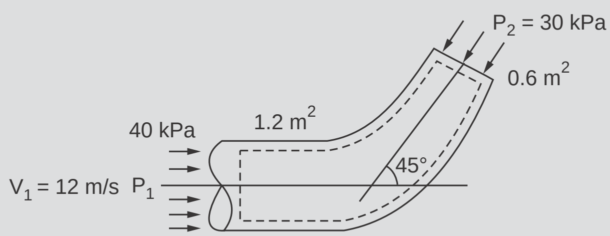

Figure Ex. 13.2

For convenience the control volume should be chosen such that the inlet and outlet areas are normal to the velocities at these sections. In this case the force on the bend is required. It is convenient to calculate the forces in the *x* and *y* directions separately.

$$u_1 = 12 \text{ m/s} \quad \therefore \quad u_2 = 24 \text{ m/s}$$

Mass flow $= 12 \times 1.2 \times 1000 = 14.4 \times 10^3$ kg/s

Using equations (13.1.7) and (13.1.8)

$$F_x = P_1A_1 - P_2A_2 \cos \theta - \dot{m}\ (V_2 \cos \theta - V_1)$$

$$F_y = P_2A_2\ \sin \theta + \dot{m}\ V_2 \sin \theta$$

Substituting the values, Force m the fluid is

$$\mathbf{F_x} = 40 \times 10^3 \times 1.2 - 30 \times 10^3 \times 0.6 \times \cos 45 - 14.4 \times 10^3 (24 \cos 45 - 12)$$

$$= \mathbf{- 36.3\ kN.}\quad \textbf{in the –ve } \boldsymbol{x} \text{ direction}$$

$$\mathbf{F_y} = 30 \times 10^3 \times 0.6 \times \sin 45 + 14.4 \times 10^3 \times 24 \times \sin 25$$

$$= \mathbf{257.1\ kN}\quad \textbf{in the +ve y direction.}$$

The forces on the bend will be 36.3 kN along x and 257.1 kN downwards.

The resultant is $\sqrt{257.1^2 + (-36.3)^2}$ = **259.65 N.**

The direction is $\theta = \tan^{-1} \dfrac{36.3}{257.1} = \mathbf{8.04°}$

with the negative y direction

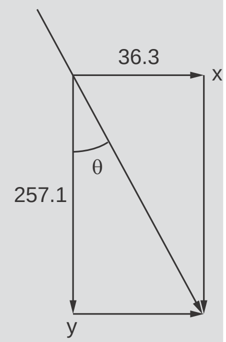

Figure Ex. 13.2

13.1.2 FORCE EXERTED ON A STATIONARY VANE OR BLADE

In the case of turbomachines fluid passes over blades and in this context, the force on a vane due to the fluid flowing over it is discussed. In turbomachines the blades are in motion. To start the analysis force on stationary vane is considered. Here the direction of the velocity is changed. There is negligible change in the magnitude. In the case considered pressure forces are equal both at inlet and outlet. The flow is assumed to occur in the horizental plane.

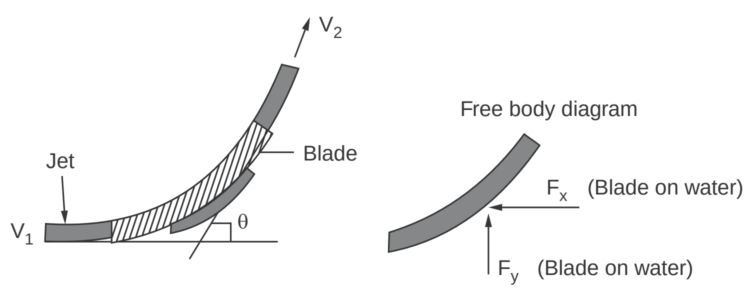

Figure 13.1.2

Force along x direction by the blade on fluid, with the assumed direction : Assuming $V_2 = V_1$ as no other energy transfer occurs,

$$-F_x = \dot{m}\ (V_{2u} - V_1) = \dot{m}\ (V_2 \cos\theta - V_1) = \dot{m}\ (V_1 \cos\theta - V_1) \qquad (13.1.8)$$

$$F_y = \dot{m}\ (V_{2y} - V_{1y}) = \dot{m}\ V_1 \sin\theta \qquad (13.1.9)$$

Example 13.3 *A blade turns the jet of diameter 3 cm at a velocity of 20 m/s by 60°.* ***Determine the force exerted by the blade*** *on the fluid.*

Rate of flow $\dot{m} = \dfrac{\pi \times 0.03^2}{4} \times 20 \times 1000 = 14.14$ kg/s

$$-F_x = 14.14\ (20 \cos 60 - 20) = -141.4 \text{ N}$$

or $\mathbf{F_x}$ **= 141.4 N.** in the assumed direction

$\mathbf{F_y}$ = 14.14 (20 × sin 60) = **244.9 N**

The forces on the blade are 141.4 N along x direction and 244.9 in the –ve y direction.

$$\textbf{Resultant} = (244.9^2 + 141.4^2)^{0.5} = \mathbf{282.8\ N}$$

$$\boldsymbol{\theta} = \tan^{-1}\frac{141.4}{244.9} = \mathbf{30°}$$

30° with the negative y direction as in figure.

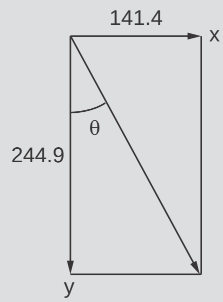

Figure Ex. 13.3

13.2 ABSOLUTE AND RELATIVE VELOCITY RELATIONS

In order to determine the force on moving blades and the energy transfer between the blades and the fluid the relative velocity between the fluid and the blade becomes an important factor. The blade may move in a direction at an angle to the velocity of the fluid. **The relative velocity of a body is its velocity relative to a second body which may in turn be in motion relative to the earth.**

The absolute velocity V of the first body, is the vector sum of its velocity relative to the second body v, and the absolute velocity of the latter, u

Vectorially $\mathbf{V = u + v}$

This is easily determined by vector diagram called as velocity triangle. Some possible diagrams are shown in Figure 13.2.1.

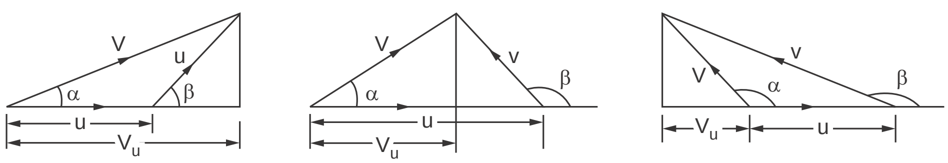

Figure 13.2.1 Sample Velocity diagrams

Some of the general relations are

$$V \sin \alpha = v \sin \beta \tag{13.2.1}$$

$$V_u = V \cos \alpha = u + v \cos \beta \tag{13.2.2}$$

V_u is the component of the absolute velocity of the first body in the direction of the velocity u of the second body.

13.3 FORCE ON A MOVING VANE OR BLADE

The force on a single moving vane is rarely met with. But this forms the basis for the calculation of force and torque on a series of moving vanes fixed on a rotor. There are two main differences between the action of the fluid on a stationary vane and a moving vane in the direction of the fluid motion. In the case of the moving vane it is necessary to consider both the absolute and relative velocities. The other difference is that the amount of fluid that strikes a moving vane at any time interval is different from that which strikes the stationary vane. If a jet of area A with a velocity V_1 strikes a stationary vane, the mass impinging per unit time on the vane equals $8AV_1$ kg/s. But when the vane moves away from the direction of the jet with a velocity u, then the mass of water striking the vane equals $\rho A(V_1 - u)$ kg/s. $(V - u)$ is the relative velocity between the jet and the vane. This can be realised when the consider the velocity of the vane to be equal to that of the jet. In this case no water will strike the vane, obviously. Consider the flow as shown in figure 13.3.1.

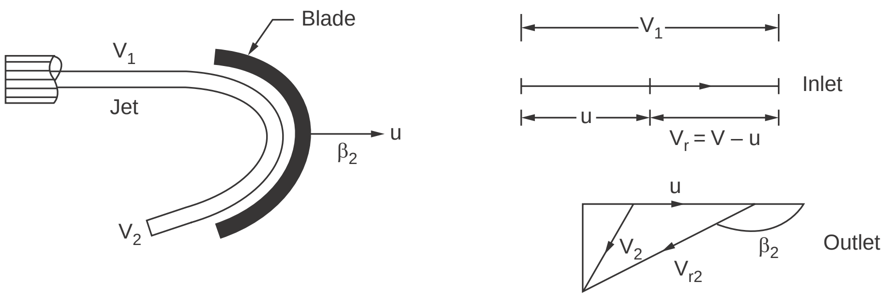

Figure 13.3.1

The velocity diagram with as inlet and outlet are shown in the figure. Considering the force on the fluid in the direction of blade velocity (can be considered as x direction)

$$F_u = \rho A\,(V_1 - u)\,(V_{u_2} - V_{u_1}) \tag{13.3.1}$$

$$V_{u2} = (V_{r_2}\cos\beta_2 - u), \text{ denoting } V_r \text{ as relative velocity}$$

$$\therefore \quad F_u = \rho A\,(V_1 - u)\,(V_{r_2}\cos\beta_2 - u - V_{u_1}) \tag{13.3.2}$$

In the case shown, $V_{u_1} = V$ itself

It is possible that $(V_{r_2}\cos\beta_2 - u)$ or V_{u_2} is negative depending upon the relative values of u and V_r *i.e.* $u > V_{r_2}\cos\beta_2$.

It is to be noted that the vane angle at the inlet should be in the direction of the relative velocity of the water when it touches the vane. Otherwise loss will occur due to the jet hitting the vane at an angle and then turning the follow on the vane surface.

It was assumed that the relative velocity at inlet and at outlet are equal as no work was done by the vane on the fluid. In case of friction, $V_{r_2} = cV_{r_1}$ where c is a fraction. In case the vane moves at a direction different from that of the jet velocity say at an angle α, then force on the fluid on the vane will be at an angle.

In such a case, $F_x = \rho A\,(V_1\cos\alpha_{1i} - V_{r_2}\cos\beta_2 - u)\,(V_1\cos\alpha_1 - u)$

$$= \rho A\,(V_1\cos\alpha_1 - V_2\cos\alpha_2)$$

As it was already mentioned, a single moving vane is not of practical importance when a series of vanes fixed on the periphery of a well is struck by the jet, then the mass of fluid striking the when will be ρAV itself.

Work or energy transfer between the fluid and the water will be $F \times u$.

Example 13.5 *A 4 cm diameter water jet with a velocity of 35 m/s impinges on a single vane moving in the same direction at a velocity of 20 m/s. The jet enters the vane tangentially along the x direction. The vane deflects the jet by 150°.* ***Calculate the force exerted by the water on the vane.***

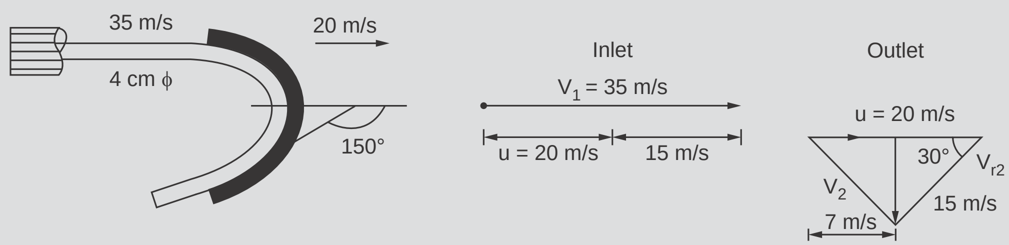

Figure Ex. 13.5

The relative velocity is given by

$$V_r = 35 - 20 = 15 \text{ m/s.}$$

$$V_{u_1} = 30 \text{ m/s itself in the direction of blade velocity.}$$

From exit velocity triangle

$$V_{u_2} = u - V_{r_2} \cos 30 = 20 - 15 \cos 30 = 7 \text{ m/s}$$

This is in the same direction as V_{u_1}

$\therefore$ $$\Delta V_u = 35 - 7 = 28 \text{ m/s.}$$

$$\mathbf{F_u} = \frac{1000 \times 0.04^2 \times \pi}{4} \times (35 - 20)(28) = \mathbf{527.8\ N.}$$

Energy transfer rate $= \mathbf{F \times u} = 527.8 \times 20 = \mathbf{10556\ Nm/s\ or\ W.}$

$$\mathbf{F_y} = \frac{1000 \times 0.04^2 \times \pi}{4} \times (35 - 20) \times (15 \sin 30 - 0) = \mathbf{141.37\ N}$$

(Note $V_2 \sin \alpha_2 = V_{r_2} \sin \beta_2$).

In case series of vanes have been used,

$$\mathbf{F_x} = \frac{1000 \times 0.04^2 \times \pi}{4} \times 35 \times 28 = \mathbf{1231.5\ N}$$

Energy transfer $= 1231.5 \times 20 = \mathbf{24630\ W}$

In case there is friction for the flow over the blade, $V_{r_2} = k V_{r_1}$

In case the water jet direction and blade velocity direction are at an angle α_1,then at the inlet $V_u \neq V_1$ but will be $V_u = V_1 \cos \alpha_1$. This is illustated by the following example.

Example 13.6 *A water jet 20 mm in diameter and having a velocity of 90 m/s strikes series of moving blades in a wheel. The direction of the jet makes 20° with the direction of movement of the blade. The blade angle at inlet is 35°. If the jet should enter the blade without striking, what should be the blade velocity. If the outlet angle of the blade is 30°,* ***determine the force on the blade.*** *Assume that there is no friction involved in the flow over the blade.*

This problem has to be solved using the velocity diagram.

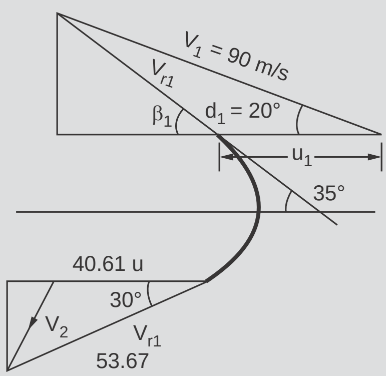

Figure Ex. 13.6

$$u = V_1 \cos \alpha_1 - V_{r_1} \cos \beta_1$$

$$V_1 \sin \alpha_1 = V_{r_1} \sin \beta_1$$

$\therefore$ $$V_{r_1} = \frac{V_1 \sin \alpha_1}{\sin \beta_1} = \frac{90 \times \sin 20}{\sin 35} = 53.67 \text{ m/s}$$

$\therefore$ $$u = 90 \cos 20 - 53.67 \cos 35 = 40.61 \text{ m/s}$$

$$V_{u_1} = V_1 \cos \alpha_1 = 90 \times \cos 20 = 84.57 \text{ m/s}$$

$$V_{u_2} = V_{r_2} \cos \beta_2 - u = 53.67 \times \cos 30 - 40.61$$

$$= 5.87 \text{ m/s (opposite direction to } V_{u_1})$$

$\therefore$ $$\Delta V_u = 84.57 + 5.87 = 90.44 \text{ m/s}$$

Series of blades : Mass flow $= \dfrac{\pi \times 0.02^2}{4} \times 1000 \times 90 = 28.274$ kg/s

$\therefore$ **Force F_x** $= 28.274 \times 90.44 =$ **2557 N**

Energy transfer rate $= 2557 \times 40.61 =$ **103845 Nm/s or W**

Energy in the jet $= \dfrac{mV_1^2}{2} = \dfrac{28.274 \times 90^2}{2} =$ **1145097 W**

F_y $= (90 \sin 20 - 53.67 \sin 30)\, 28.274 =$ **111.6 N**

13.4 TORQUE ON ROTATING WHEEL

Blades or vanes **may be fixed at the periphery** of the wheel in which case the radius at which fluid enters will be the same as at fluid exit. There are cases where the blades are **fixed at the sides of a disc** such that the radius at which the fluid enters the vane will be different from the radius at which it exits. The former type is known **axial blading** and the later is known **as radial blading.** In the former case the blade velocity will be constant and in the latter case the blade velocity will very with radius. Thus the force on the blade will very with the radius and the previous method cannot be used to find the fluid force on the blade. In this case the **moment of momentum theorem is used to determine the torque on the wheel.** The theorem states that **torque on the wheel equals the rate of change of moment of momentum of the fluid as it flows over the blades.** Thus it is necessary to determine the moment of momentum at the inlet and outlet to determine the torque. Torque can be produced only by the velocity component along the periphery.

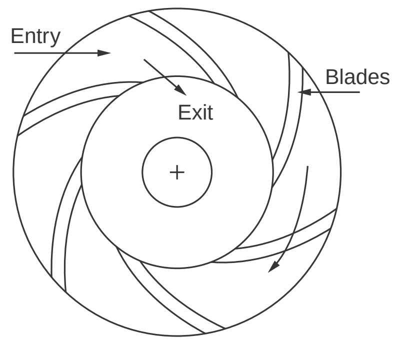

Figure 13.4.1 Radial blading

Chapter 13

The components of the velocity in the tangential direction are V_{u_1} and V_{u_2} equal to V_1 cos α_1 and V_2 cos α_2. Momentum at entry = $\dot{m} V_1 \cos \alpha_1$. Moment of momentum of entry = $\dot{m} V_1 \cos \alpha_1 \times r_1$

Similarly moment of momentum at exit = $\dot{m} V_2 \cos \alpha_2 \times r_2$

$$T = \dot{m}\ (r_1V_1 \cos \alpha_1 - r_2V_2 \cos \alpha_2)$$

$$\text{Power} = \omega T. \quad \omega = \frac{2\pi N}{60}$$

Substituting : $\dfrac{2\pi r_1 N}{60} = u_1$ tangential velocity at entry

$\dfrac{2\pi r_2 N}{60} = u_2$ tangential at exit

$$\therefore \quad P = \dot{m}(V_{u_1}u_1 - V_{u_2}u_2).$$

where V_{u_1} and V_{u_2} are the components of the absolute velocities of the fluid in the tangential direction. In this case the direction of blade velocity is the tangential direction to the wheel on which the blades are fixed.

Example 13.7 *Blades are fixed in a disc with outer and inner diameters of 0.8 m and 0.4 m. The disc rotates at 390 rpm. The flow rate through blades is 4000 kg/s. The inlet angle of the blade is 80°. The blade width is 0.25 m.*

If the flow at outlet is radial, determine ***the blade outlet angle.*** *Determine the angle at which the water should flow for smooth entry. Determine* ***the torque exerted*** *and the* ***power resulting therefrom.***

$$u_1 = \frac{\pi DN}{60} = \frac{\pi \times 0.8 \times 390}{60} = 16.34 \text{ m/s}$$

$$\therefore \quad u_2 = u_1 \times \frac{D_2}{D_1} = \frac{16.34 \times 0.4}{0.8} = 8.17 \text{ m/s}$$

From continuity $Q = \pi D b V_f$

where V_f is the flow velocity along the radius.

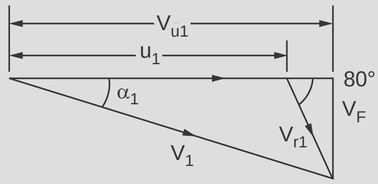

Figure Ex. 13.7a

$$\frac{4000}{1000} = \pi \times 0.8 \times 0.25 \times V_f$$

$$\therefore \quad V_f = 6.37 \text{ m/s.}$$

$$V_{u_1} = u_1 + V_f/\tan 80$$

$$= 16.34 + \frac{6.37}{\tan 80} = 16.52 \text{ m/s}$$

$$\frac{V_{f_1}}{V_{u_1}} = \tan \alpha_1, \quad \frac{6.37}{16.52} = \tan \alpha_1, \quad \alpha_1 = \mathbf{21.09°}$$

The jet should be inclined at this angle to the periphery of the wheel

$$\therefore \quad V_1 \tan \alpha_1 = V_{f_1} \quad \therefore \quad \mathbf{V_1} = \frac{6.37}{\tan 21.09} = \mathbf{16.51 \text{ m/s.}}$$

As the blade width is constant, the flow velocity at exit is

$$4 = \pi \times 0.4 \times 0.25 \times V_{f_2}$$

$$\therefore \quad V_{f_2} = 12.73 \text{ m/s}$$

$$\therefore \quad \tan \beta_2 = \frac{12.13}{8.17}$$

$$\therefore \quad \beta_2 = \mathbf{57.3°}$$

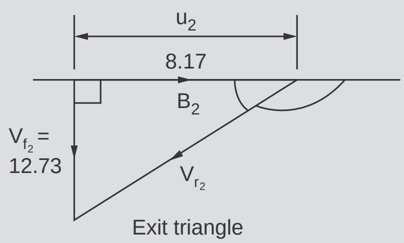

Figure Ex. 13.7b

As exit is radial, $V_{w_2} = 0$ as $V_{f_2} = V_2$

$$\mathbf{T} = \dot{m}\,(r_1 V_{w_1} - 0) = 4000 \times \frac{0.8}{2} \times 16.52 = \mathbf{26432 \text{ mN.}}$$

$$\mathbf{P} = \omega T = \frac{2\pi N}{60} \times \frac{26432}{1000}$$

$$= \mathbf{1079.5 \text{ kW.}} \text{ Also equal to } \dot{m}\, V_{w_1} V_{u_1} \text{ (check)}$$

We can also determine V_{r_1} and V_{r_2} if required.

SOLVED PROBLEMS

Problem 13.1 *A pipe line of 150 mm ID branches into two pipes which delivers the water at atmospheric pressure. The diameter of the branch 1 which is at 30° anti clockwise to the pipe axis is 75 mm. and the velocity at outlet is 12 m/s. The branch 2 is at 15° with the pipe centre line in the clockwise direction has a diameter of 100 mm. The outlet velocity is 12 m/s. The pipes lie in a horizental plane.* ***Determine the magnitude and direction of the forces on the pipes.***

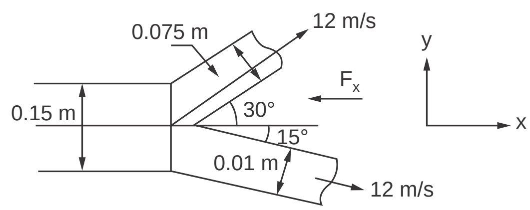

Figure P. 13.1

The flow rates in the pipes are

Branch 1 : $\dfrac{\pi \times 0.075^2}{4} \times 12 \times 1000 = 53$ kg/s

Branch 2 $= \dfrac{\pi \times 0.1^2 \times 12 \times 1000}{4} = 94$ kg/s

Flow in the pipe $= 94 + 53 = 147$ kg/s

Velocity in the pipe : $V_1 = 0.147 \Big/ \dfrac{\pi\, 0.15^2}{4} = 8.333$ m/s

To determine the pressure in the pipe

$$\frac{P_1}{p} = \frac{P_0}{S} + \frac{V_2^2}{2} - \frac{V_1^2}{2},$$

Assuming zero gauge pressure at exit.

$$P_1 = \rho \left[\frac{V_2^2 - V_1^2}{2}\right] = 1000 \left[\frac{12^2 - 8.33^2}{2}\right]$$

$$= 37.3 \times 10^3 \text{ N/m}^2$$

The x directional force assuming it to act in the – ve x direction

$$\mathbf{F_x} = 37.3 \times 10^3 \times \frac{\pi \times 0.15^2}{4} - 94 \times 12 \cos 15 - 53 \times 12 \cos 30 + 147 \times 8.333$$

$$= \mathbf{242\ N}$$

To determine F_y : Assuming to act in the + ve y direction

$$\mathbf{F_y} = 53 \times 12 \sin 30 - 94 \times 12 \sin 15$$
$$= \mathbf{26\ N}$$

Problem 13.2 *A jet 30 mm diameter with velocity of 10 m/s strikes a vertical plate in the normal direction.* ***Determine the force on the plate if*** *(i) The plate is stationary (ii) If it moves with a velocity of 4 m/s towards the jet and (iii) If the plate moves away from the plate at a velocity of 4 m/s.*

Case (*i*) The total x directional velocity is lost.

$\therefore$ $$F = \dot{m}V, \quad \dot{m} = \rho AV$$

$\therefore$ $$\mathbf{F} = \frac{\pi \times 0.03^2}{4} \times 10 \times 10 \times 1000 = \mathbf{70.7\ N}$$

Case (*ii*) $$\dot{m} = \rho A(V_r), \quad V_r = V + u = 14$$

$\therefore$ $$\mathbf{F} = \frac{\pi \times 0.032}{4} \times 14 \times 1000 \times 10 = \mathbf{99\ N}$$

Case (*iii*) $$F\ V_r = V - u = 6 \text{ m/s}$$

$$\mathbf{F} = \frac{\pi \times 0.03^2}{4} \times 6 \times 1000 \times 10 = \mathbf{42.4\ N}$$

Problem 13.3 *A jet of water at a velocity of 100 m/s strikes a series of moving vanes fixed at the periphery of a wheel, 5 at the rate of kg/s.*

The jet is inclined at 20° to the direction of motion of the vane. The blade speed is 50 m/s. The water leaves the blades at an angle of 130° to the direction of motion.

Calculate the blade angles at the forces on the wheel in the axial and tangential direction.

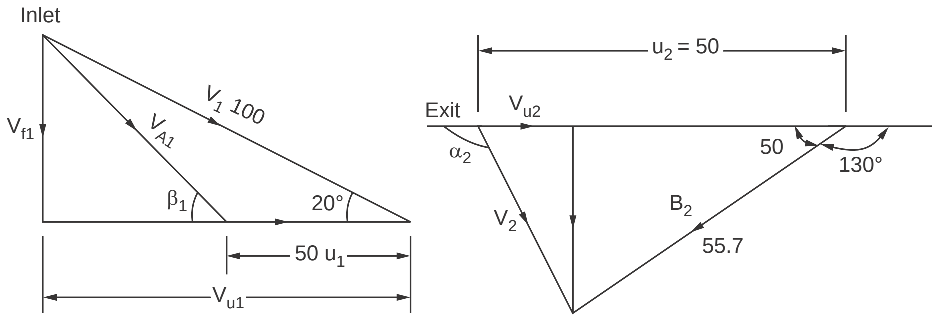

Figure P. 13.3

$$\tan \beta_1 = \frac{V_1 \sin \alpha_1}{V_1 \cos \alpha_1 - u} = \frac{100 \times \sin 20}{100 \cos 20 - 50}$$

Blade angle at inlet $\therefore$ β_1 = **37 .88°**

$$\sin \beta_1 = \frac{V_1 \sin \alpha_1}{V_{r_1}}$$

$$\therefore \quad V_{r_1} = \frac{100 \sin 20}{\sin 37.88} = 55.7 \text{ m/s}$$

In this type of blade fixing

$$V_{r_2} = V_{r_1} \quad \text{and} \quad u_2 = u_1$$

Referring to the exit triangle

$$V_{r_2} \cos 50 < u = 50$$

Hence this shape

$$V_{r_2} \cos 50 = 35.8. \quad \therefore \quad Vu_2 = 50 - 35.8$$

$$= \mathbf{14\ .2\ m/s} \text{ in the same direction as } V_{u_1}$$

$\therefore$ Tangential force $= 500 \times (V_{u_1} - V_{u_2})$

$$V_{u_1} = 100 \cos 20 = 93.97 \text{ m/s}$$

$\therefore$ **Tangential force** $= 5\ (93.97 - 14.2) =$ **3488 N**

Axial force $\mathbf{F} = \dot{m}\,[V_1 \sin \alpha - V_{r_1} \sin \beta_2]$

$$= 5\ [100 \sin 20 - 55.7\ .\ \sin 50] = \mathbf{-\,8.5\ N}$$

Problem 13.4 *Water jet at the rate of 10 kg/s strikes the series of moving blades at a velocity of 50 m/s. The blade angles with respect to the direction of motion are 35° and 140°. If the peripheral speed is 25 m/s,* ***determine the inclination of the jet*** *so that water enters the blades without shock. Also* ***calculate the power developed and the efficiency*** *of the system. Assume blades an mounting on the periphery of the wheel.*

In this type of mounting u remains the same so also relative velocity. β_1, V_1 and u are known :

Refer figure

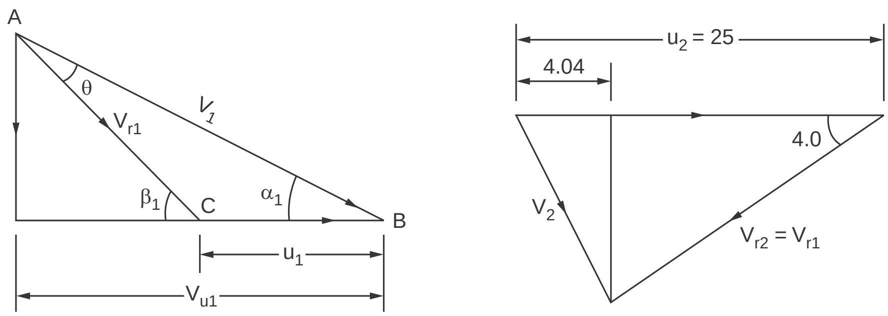

Figure P. 13.4

$$\frac{V_1}{\sin(180-\beta_1)} = \frac{u}{\sin\theta}$$

$$\therefore \quad \frac{50}{\sin(180-35)} = \frac{25}{\sin\theta}$$

Solving $\theta = 16.7°. \quad \therefore \quad \alpha_1 = 180 - (180 - 35) - 16.7 = \mathbf{18.3°}$

Direction of the jet is 18.3° to the direction of motion.

$$V_{u_1} = 50 \times \cos 18.3 = 47.47 \text{ m/s},$$

$$V_{r_1} = \frac{50 \sin 18.3}{\sin 35} = 27.37 \text{ m/s}$$

$$\beta_2 = (180 - 140) = 40°, \quad V_{r_2} \cos 40 = 20.96 < 25\ (u)$$

$\therefore$ The shape of the exit triangle will be as in figure

$$V_{u_2} = u - V_{r_2} \cos\beta_2 = 25 - 20.96 = 4.04 \text{ m/s}$$

Tangential force $= m\,(V_{u_1} - V_{u_2}) = 10\,(47.47 - 4.04) = \mathbf{434.3\ N}$

Power $= F \times u = 434.3 \times 25 = \mathbf{10.86 \times 10^3\ W}$

Energy in jet $= \dfrac{10 \times 50^2}{2} = 12.5 \times 10^3\ W$

$$\therefore \quad \eta = \frac{10.86 \times 10^3}{12.5 \times 10^3} = 0.8686 \text{ or } \mathbf{86.86\ \%}$$

Problem 13.5 *Curved vanes fixed on a wheel on the surface receive water at angle of 20° to the tangent of the wheel. The inner and outer diameter of the wheel are 0.9 and 1.6 m respectively.*

The speed of rotation of the wheel is 7 revolutions per second. The velocity of water at entry is 75 m/s. The water leaves the blades with an absolute velocity of 21 m/s at an angle of 120° with the wheel tangent at outlet. The flow rate is 400 kg/s. ***Determine the blade angles*** *for shockless entry and exit.* ***Determine the torque and power.*** *A also determine the radial force.*

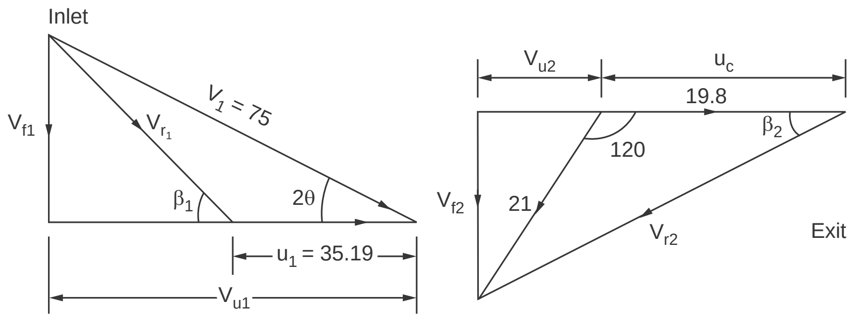

Figure P. 13.5

Blade velocity $u_1 = \pi dN = \pi \times 1.6 \times 7 = 35.19$ m/s

$$u_2 = \frac{9}{16} \times 35.19 = 19.8 \text{ m/s}$$

$$\tan \beta_1 = \frac{V_1 \sin \alpha_1}{V_1 \cos \alpha_1 - u} = \frac{75 \times \sin 20}{75 \times \cos 20 - 35.19}$$

Solving $\beta_1 = \mathbf{36°}$

$$\tan \beta_2 = \frac{21 \sin 60}{19.8 + 21 \cos 60}$$

Solving $\beta_2 = \mathbf{30.97°}$

$\mathbf{T} = \dot{m}\,[V_{u_1} r_1 + V_{u_2} r_2]$ (in this case, V_{u_2} is in the opposite direction)

$\therefore$ $\Delta V_w = V_{u_1} + V_{u_2}$

$= 400\ [0.8 \times 75 \cos 20 + 0.45 \times 21 \cos 60] =$ **24443 Nm**

Power $= 24443 \times \omega = 24443 \times 2\pi \times 7 = 1075042\ W$

or Ω **1075 kW.**

Power in the jet $= \dfrac{75^2}{2} \times 400 = 1125000$ W or **1125 kW**

$$\eta = \frac{1075}{1125} = 0.955 \text{ or } \mathbf{95.5\%}$$

Radial force $= 400\ (75 \sin 20 - 21 \sin 60) =$ **2986 N.**

Problem 13.6 *A jet of water with a velocity of 30 m/s impinges on a series of vanes moving at 12 m/s at 30 to the direction of motion. The vane angle at outlet is 162° to the direction of motion.* ***Complete (i) the vane angle at inlet*** *for shockless entry and (ii)* ***the efficiency of power transmission.***

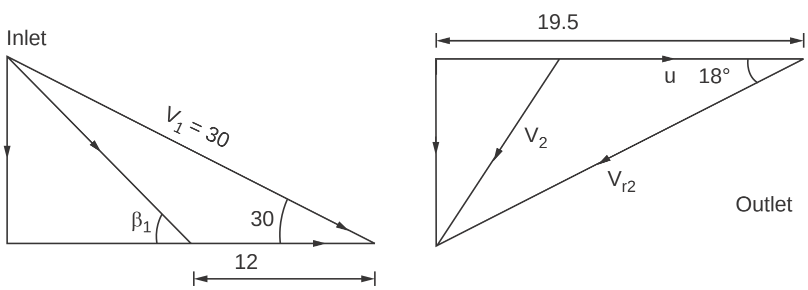

Figure P. 13.6

$$\tan \beta_1 = \frac{V_1 \sin \alpha_1}{V_1 \cos \alpha_1 - u} = \frac{30 \sin 30}{30 \cos 30 - 12} = 1.073$$

$\therefore$ $\beta_1 = \mathbf{47°}$

$$\sin \beta_1 = \frac{30 \sin 30}{V_{r_1}}$$

$$\therefore \qquad V_{r_1} = \frac{30 \sin 30}{\sin \beta_1} = 20.5 \text{ m/s} = V_{r_2}$$

$V_{r_2} \cos \beta_2 > u_1 \;\; \therefore$ hence the shape of the triangle.

$$V_{u_1} = 30 \cos 30 = 25.98 \text{ m/s}$$

$$V_{u_2} = 20.5 \cos 18 - 12 = 7.5 \text{ m/s}$$

Assuming unit mass flow rate :

$$\mathbf{P} = u\,[V_{w_1} + V_{w_2}] = 12\,[25.98 + 7.5] = \mathbf{401.76 \text{ W/kg/s}}$$

Energy in the jet $= \dfrac{30^2}{2} = 450\ W.$

$$\therefore \qquad \eta = \frac{401.76}{450} = \mathbf{0.893} \text{ or } 89.3\%$$

EXERCISE QUESTIONS

E 13.1 Derive the linear momentum equation using the control volume approach and determine the force exerted by the fluid flowing through a pipe bend.

E 13.2 Derive the expression for the force exerted by a water jet on a plate moving in the same direction of the jet with a velocity less than that of the jet.

E 13.3 A horizontal *Y* is shown in figure. Determine the *x* and *y* components of the force exerted in the pipe.

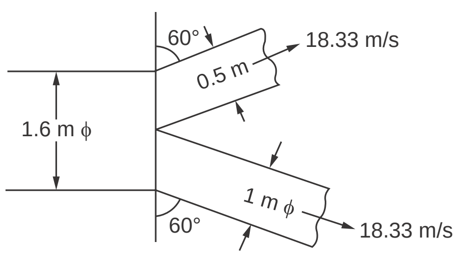

Figure E. 13.3

E 13.4 A nozzle of 5 cm diameter is fixed at the end of a pipe of 15 cm diameter with water flowing in the pipe at a velocity of 3 m/s. The jet discharges into the air. Determine the force exerted in the nozzle.

E 13.5 Water flows through a right angled reducer bend with inlet diameter of 60 cm and exit diameter of 40 cm. The entrance velocity is 6 m/s. If the bend lies on a horizontal plane, determine the magnitude and direction of the force on the bend.

E 13.6 A jet of 5 cm diameter enters a blade in the x direction with a velocity of 60 m/s. The blade angle at inlet is 0°. The outlet angle is 120° with x direction. If the blade moves with a velocity of 25 m/s along the x direction, determine the forces in the x and y directions. Also determine the energy transfer rate.

E 13.7 A jet of water 6 cm dia has a velocity of 30 m/s. If it impinges on a curved vane which turns the jet by 90° determine forces on the vane if the vane moves in the direction of the jet at a velocity of 14 m/s.

E 13.8 A series of vanes is acted upon by a 7.5 cm water jet having a velocity of 30 m/s. $\alpha_1 = \beta_1 = 0°$. If the force acting on the vane in the direction of the jet is 900 N determine the angle by which the jet is turned by the vane. The vane velocity is 15 m/s.

E 13.9 A 5 cm^2 area water jet impinges on a series of vanes as shown in figure. The absolute velocities and their directions are indicated on the figure. What is the power transmitted? Also determine the blade speed and blade inlet angle.

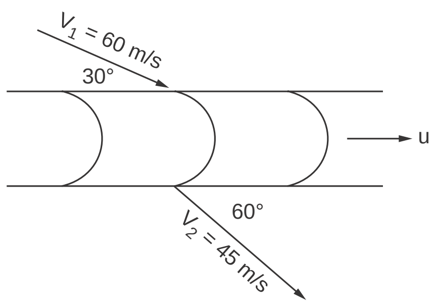

Figure E. 13.9

E 13.10 A water jet with a velocity of 60 m/s enters a series of curved vanes at an angle of 20° to the direction of blade movement. The peripheral speed of the disc on which the blades are mounted is 25 m/s. Calculate the vane inlet angle. If at the exit the component of absolute velocity along the direction of motion is zero, determine the outlet blade angle. Assume shockless enters and exit.