## 15 <br> Rotodynamic Pumps

### 15.0 INTRODUCTION

Liquids have to be moved from one location to another and one level to another in domestic, agricultural and industrial spheres. The liquid is more often water in the domestic and agriculture spheres. In industries chemicals, petroleum products and in some cases slurries have to be moved, by pumping. Three types of pumps are in use.
(1) Rotodynamic pumps which move the fluid by dynamic action of imparting momentum to the fluid using mechanical energy. (2) Reciprocating pumps which first trap the liquid in a cylinder by suction and then push the liquid against pressure. (3)Rotary positive displacement pumps which also trap the liquid in a volume and push the same out against pressure.

Reciprocating pumps are limited by the low speed of operation required and small volumes it can handle.

Rotary positive displacement pumps are limited by lower pressures of operation and small volumes these can handle. Gear, vane and lobe pumps are of these type. Rotodynamic pumps i.e. centrifugal and axial flow pumps can be operated at high speeds often directly coupled to electric motors. These can handle from small volumes to very large volumes. These pumps can handle corrosive and viscous, fluids and even slurries. The overall efficiency is high in the case of these pumps. Hence these are found to be the most popular pumps in use. Rotodynamic pumps can be of radial flow, mixed flow and axial flow types according to the flow direction. Radial flow or purely centrifugal pumps generally handle lower volumes at higher pressures. Mixed flow pumps handle comparatively larger volumes at medium range of pressures. Axial flow pumps can handle very large volumes, but the pressure against which these pumps operate is limited. The overall efficiency of the three types are nearly the same.

### 15.1 CENTRIFUGAL PUMPS

These are so called because energy is imparted to the fluid by centrifugal action of moving blades from the inner radius to the outer radius. The main components of centrifugal pumps are (1) the impeller, (2) the casing and (3) the drive shaft with gland and packing.

Additionally suction pipe with one way valve (foot valve) and delivery pipe with delivery valve completes the system.

The liquid enters the eye of the impeller axially due to the suction created by the impeller motion. The impeller blades guide the fluid and impart momentum to the fluid, which increases the total head (or pressure) of the fluid, causing the fluid to flow out. The fluid comes out at a high velocity which is not directly usable. The casing can be of simple volute type or a diffuser can be used as desired. The volute is a spiral casing of gradually increasing cross section. A part of the kinetic energy in the fluid is converted to pressure in the casing.

Figure 15.1 .1 shows a sectional view of the centrifugal pump.


Figure 15.1.1 Volute type centrifugal pump.
Gland and packing or so called stuffing box is used to reduce leakage along the drive shaft. By the use of the volute only a small fraction of the kinetic head can be recovered as useful static head. A diffuser can diffuse the flow more efficiently and recover kinetic head as useful static head. A view of such arrangement is shown in figure 15.1.2. Diffuser pump are also called as turbine pumps as these resembles Francis turbine with flow direction reversed.


Figure 15.1.2 Diffuser pump.

### 15.1.1 Impeller

The impeller consists of a disc with blades mounted perpendicularly on its surface. The blades may of three different orientations. These are (i) Radial, (ii) Backward curved, and (iii) Forward curved. Backward and forward refers to the direction of motion of the disc periphery. Of these the most popular one is the backward curved type, due to its desirable characteristics, which reference to the static head developed and power variation with flow rate. This will be discussed in detail later in this chapter.

A simple disc with blades mounted perpendicularly on it is called open impeller. If another disc is used to cover the blades, this type is called shrouded impeller. This is more popular with water pumps. Open impellers are well adopted for use with dirty or water containing solids. The third type is just the blades spreading out from the shaft. These are used to pump slurries. Impellers may be of cast iron or bronzes or steel or special alloys as required by the application. In order to maintain constant radial velocity, the width of the impeller will be wider at entrance and narrower at the exit. This may be also noted from figure 15.1.1.

The blades are generally cast integral with the disc. Recently even plastic material is used for the impeller. To start delivery of the fluid the casing and impeller should be filled with the fluid without any air pockets. This is called priming. If air is present the there will be only compression and no delivery of fluid. In order to release any air entrained an air valve is generally provided The one way foot value keeps the suction line and the pump casing filled with water.

### 15.1.2 Classification

As already mentioned, centrifugal pumps may be classified in several ways. On the basis of speed as low speed, medium speed and high speed pumps. On the basis of direction of flow of fluid, the classification is radial flow, mixed flow and radial flow. On the basis of head pumps may be classified as low head ( 10 m and below), medium head ( $10-50 \mathrm{~m}$ ) and high head pumps. Single entry type and double entry type is another classification. Double entry pumps have blades on both sides of the impeller disc. This leads to reduction in axial thrust and increase in flow for the same speed and diameter. Figure 15.1.3 illustrates the same. When the head required is high and which cannot be developed by a single impeller, multi staging is used. In deep well submersible pumps the diameter is limited by the diameter of the bore well casing. In this case multi stage pump becomes a must. In multi stage pumps several impellers are mounted on the same shaft and the outlet flow of one impeller is led to the inlet of the next impeller and so on. The total head developed equals the sum of heads developed by all the stages.


Figure 15.1.3 Single and double entry pumps

Pumps may also be operated in parallel to obtain large volumes of flow. The characteristics under series and parallel operations is discussed later in the chapter. The classification may also be based on the specific speed of the pump. In chapter 9 the dimensionless parameters have been derived in the case of hydraulic machines. The same is also repeated in example 15.1. The expression for the dimensionless specific speed is given in equation 15.1.1.

$$
\begin{equation*}
N_{s}=\frac{N \sqrt{Q}}{(g H)^{3 / 4}} \tag{15.1.1}
\end{equation*}
$$

More often dimensional specific speed is used in practise. In this case

$$
\begin{equation*}
N_{s}=\frac{N \sqrt{Q}}{H^{3 / 4}} \tag{15.1.1a}
\end{equation*}
$$

The units used are : $N$ in rpm, $Q$ in $\mathrm{m}^{3} / \mathrm{s}$, and $H$ in meter.
Typical values are given in table 15.1
Table 15.1 Specific speed classification of pumps.

| Flow <br> direction | speed | Dimensional <br> specific speed | Non Dimensional <br> specific speed |
| :--- | :--- | :---: | :---: |
|  | Low | $10-30$ | $1.8-5.4$ |
|  | Medium | $30-50$ | $5.4-9.0$ |
|  | High | $50-80$ | $9.0-14.0$ |
| Mixed flow |  | $80-160$ | $14-29$ |
| Axial flow |  | $100-450$ | $18-81$ |

The best efficiency is obtained for the various types of pumps in this range of specific speeds indicated.

### 15.2. PRESSURE DEVELOPED BY THE IMPELLER

The general arrangement of a centrigugal pump system is shown in Figure 15.2.1.
$H_{s}$-Suction level above water level.
$H_{d}$-Delivery level above the impeller outlet.
$h_{f d}, h_{f s}$-frictionless $\dot{m}, m$.
$V_{s^{\prime}}, V_{d}$-pipe velocities.
Applying Bernoullis equation between the water level and pump suction,

$$
\begin{align*}
& \frac{P_{a}}{\gamma}+H_{s}+h_{f s}+\frac{V_{s}^{2}}{2 g}=\frac{P_{s}}{\gamma}  \tag{15.2.1}\\
\therefore \quad & \frac{P_{s}}{\gamma}+\frac{P_{a}}{\gamma} H_{s}+h_{f s}+\frac{V_{s}^{2}}{2 g} \tag{15.2.2}
\end{align*}
$$

Similarly applying Bernoulli's theorem between the pump
 delivery and the delivery at the tank,

$$
\begin{gather*}
\frac{P_{d}}{\gamma}+\frac{V_{d}^{2}}{2 g}=\frac{P_{a}}{\gamma}+H_{d}+h_{f d}+\frac{V_{d}^{2}}{2 g}  \tag{15.2.3}\\
\frac{P_{d}}{\gamma}=\frac{P_{a}}{\gamma}+H_{d}+h_{f d} \tag{15.2.3a}
\end{gather*}
$$

where $P_{d}$ is the pressure at the pump delivery. From 15.2.2 and 15.2.3a

$$
\begin{align*}
\frac{P_{d}}{\gamma}-\frac{P_{s}}{\gamma} & =\frac{P_{a}}{\pi}+H_{d}+h_{f d}-\frac{P_{a}}{\gamma}+\frac{V_{s}^{2}}{2 g}+H_{s}+h_{f s} \\
& =H_{d}+H_{s}+h_{f}+\frac{V_{s}^{2}}{2 g}=H_{e}+\frac{V_{s}^{2}}{2 g} \tag{15.2.4}
\end{align*}
$$

where $H_{e}$ is the effective head.

### 15.2.1 Manometric Head

The official code defines the head on the pump as the difference in total energy heads at the suction and delivery flanges. This head is defined as manometric head.

The total energy at suction inlet (expressed as head of fluid)

$$
\frac{P_{s}}{\gamma}+\frac{V_{s}^{2}}{2 g}+Z_{s}
$$

where $Z_{s}$ is the height of suction gauge from datum.
The total energy at the delivery of the pump

$$
=\frac{P_{d}}{\gamma}+\frac{V_{d}^{2}}{2 g}+Z_{d}
$$

$Z_{2}$ is the height of delivery gauge from datum.
$\therefore \quad$ The difference in total energy is defined as $H_{m}$

$$
=\left(\frac{P_{d}}{\gamma}-\frac{P_{s}}{\gamma}\right)+\frac{V_{d}^{2}-V_{s}^{2}}{2 g}+\left(Z_{d}-Z_{s}\right)
$$

From equation 15.2.4,

$$
\frac{P_{d}}{\gamma}-\frac{P_{s}}{\gamma}=H_{e}+\frac{V_{s}{ }^{2}}{2 g}
$$

Substituting

$$
\begin{equation*}
H_{m}=H_{e}+\frac{V_{d}^{2}}{2 g}+\left(Z_{d}-Z_{s}\right) \tag{15.2.5}
\end{equation*}
$$

As $\left(Z_{d}-Z_{s}\right)$ is small and $\frac{V_{d}{ }^{2}}{2 g}$ is also small as the gauges are fixed as close as possible.
$\therefore \quad H_{m}=$ Static head + all losses.

### 15.3 ENERGY TRANSFER BY IMPELLER

The energy transfer is given by Euler Turbine equation applied to work absorbing machines,

$$
W=-\left(u_{1} V_{u 1}-u_{2} V_{u 2}\right)=\left(u_{2} V_{u 2}-u_{1} V_{u 1}\right)
$$

This can be expressed as ideal head imparted as

$$
\begin{equation*}
H_{\text {ideal }}=\frac{u_{2} V_{u 2}-u_{1} V_{u 1}}{g} \tag{15.3.1}
\end{equation*}
$$

The velocity diagrams at inlet and outlet of a backward curved vaned impeller is shown in figure 15.3.1. The inlet whirl is generally zero. There is no guide vanes at inlet to impart whirl. So the inlet triangle is right angled.

$$
\begin{align*}
V_{1} & =V_{f 1} \text { and are radial } \\
\tan \beta_{1} & =\frac{V_{1}}{u_{1}} \text { or } \frac{V_{f}}{u_{1}} \\
V_{u 1} & =0 \\
\therefore \quad H_{\text {ideal }} & =\frac{u_{2} V_{u 2}}{g} \quad \tag{15.3.2}
\end{align*}
$$

From the outlet triangle,

$$
\begin{align*}
u_{2}=\pi D_{2} N / 60, \\
V_{u 2}=u_{2}-\frac{V_{f 2}}{\tan \beta_{2}} \\
\therefore \quad H_{\text {ideal }}=\frac{u_{2}}{g}\left[u_{2}-\frac{V_{f 2}}{\tan \beta_{2}}\right] \tag{15.3.3}
\end{align*}
$$

Manometric efficiency is defined as the ratio of manometric head and ideal head.


Figure 15.3.1 Velocity triangles for backward curved bladed pump.

$$
\begin{align*}
& \qquad \begin{aligned}
& \eta_{m}=\frac{H_{m} \times g}{u_{2}\left(u_{2}-V_{f 2} / \tan \beta_{2}\right)} \\
& H_{m}=\text { Static head }+ \text { all losses (for practical purposes) } \\
& \text { Mechanical efficiency }=\eta_{\text {mech }}=\frac{\text { Energy transferred to the fluid }}{\text { Work input }} \\
&=\frac{\left(u_{2} V_{u 2}\right) Q \rho}{\text { power input }} \\
& \text { Overall efficiency }=\eta_{o}=\frac{\text { Static head } \times Q \times \rho \times g}{\text { Power input }}
\end{aligned}
\end{align*}
$$

There are always some leakage of fluid after being imparted energy by the impeller.
Volumetric efficiency $=\frac{\text { Volume delivered }}{\text { Volume passing through impeller }}$
Thus

$$
\begin{equation*}
\eta_{\mathrm{o}}=\eta_{\mathrm{m}} \mathbf{m}_{\text {mech }} \cdot \eta_{\mathrm{vol}}{ }^{-} \tag{15.3.6}
\end{equation*}
$$

$\frac{V_{d}{ }^{2}}{2 g}$ is not really useful as output of the pump. Hence the useful amount of energy transfer (as head, is taken as $\left(H_{a}\right)$

$$
H_{a}=\frac{u_{2} V_{u 2}}{g}-\frac{V_{d}{ }^{2}}{2 g}
$$

By algebraic manipulation, this can be obtained as

$$
\begin{equation*}
H_{a}=\left(u_{2}{ }^{2}-V_{f}^{2} \operatorname{cosec}^{2} \beta_{2}\right) / 2 g \tag{15.3.7}
\end{equation*}
$$

### 15.3.1 Slip and Slip Factor

In the analysis it is assumed that all the fluid between two blade passages have the same velocity (both magnitude of direction). Actually at the leading edge the pressure is higher and velocity is lower. On the trailing edge the pressure is lower and the velocity is higher. This leads to a circulation over the blades. Causing a non uniform velocity distribution. The average angle at which the fluid leaves the blade is less than the blades angle. The result is a reduction in the exit whirl velocity $V_{u 2}$. This is illustrated in the following figure. The solid lines represent the velocity diagram without slip. The angle $\beta_{2}$ is the blade angle. The dotted lines represent the velocity diagram after slip. The angle $\beta_{2}{ }^{\prime}<\beta_{2}$. It may be seen that $V_{u 2}{ }^{\prime}<V_{u 2}$. The ratio $V_{u 2}{ }^{\prime} / V_{u 2}$ is known as slip factor. The result of the slip is that the energy transfer to the fluid is less than the theoretical


Figure 15.3.2 Velocity triangle with slip value.

$$
\begin{equation*}
H_{t h}=\sigma_{s} \cdot \frac{u_{2} V_{u 2}}{g} \tag{15.3.8}
\end{equation*}
$$

where $\sigma_{s}$ is the slip coefficient or slip factor.

### 15.3.3 Losses in Centrifugal Pumps

Mainly there are three specific losses which can be separately calculated. These are


Figure 15.3.3 Losses in pump
(i) Mechanical friction losses between the fixed and rotating parts in the bearings and gland and packing.
(ii) Disc friction loss between the impeller surfaces and the fluid.
(iii) Leakage and recirculation losses. The recirculation is along the clearance between the impeller and the casing due to the pressure difference between the hub and tip of the impeller. The various losses are indicated in figure 15.3.3.

### 15.3.4 Effect of Outlet Blade Angle

There are three possible orientation of the blade at the outlet. These are : forward curved, radial and Backward curved arrangements. The velocity triangles for the three arrangements are shown in Figure 15.3.4. In the case of forward curved blading $V_{u 2}>u_{2}$ and $V_{2}$ is larger comparatively. In the case of radial blades $V_{u 2}=u_{2}$. In the case of backward curved blading, $V_{u 2}<u_{2}$.


Figure 15.3.4 Different blade arrangements
The head-flow rate curves are shown in Figure 15.3.5. The theoretical head variation can be expressed as


Figure 15.3.5 Head variation

$$
\mathbf{H}_{\mathrm{th}}=\mathbf{k}_{1}-\mathbf{k}_{2} \cdot \cot \beta_{2} \mathbf{Q}
$$

where $k_{1}$ and $k_{2}$ are constants and $\beta_{2}$ is the outlet blade angle. cot $\beta_{2}$ becomes negative for forward curved blading. So head increases with flow rate. For radial blading cost $\beta_{2}=0$, and hence the head is constant with flow rate. In the case of backward curved blading, the head decreases with flow rate.

The rising characteristics of the forward curved blading leads to increase of power input with increase of $Q$. The power curve is not self limiting and damage to motor is possible. The forward curved blading is rarely used.

The backward curved blading leads to self limiting power characteristics and reduced losses in the exit kinetic energy.

So the backward curved blading is almost universally used. The radial blading also leads to rising power characteristics and it is used only in small sizes.

### 15.4 PUMP CHARACTERISTICS

We have seen that the theoretical head

$$
\begin{aligned}
H_{t h} & =\frac{u_{2} V_{u 2}}{g} \text { and } V_{u 2}=V_{f 2} \cot \beta_{2} \\
V_{f 2} & =\frac{Q}{A}, \text { where } A \text { is the circumferential area. } \\
u_{2} & =\pi D N .
\end{aligned}
$$

Substituting these relations in the general equation. We can write

$$
H_{t h}=\pi^{2} D^{2} N^{2}-\left(\frac{\pi D N}{A} \cdot \cot \beta_{2}\right) Q .
$$

For a given pump, $D, A, \beta_{2}$ and $N$ are fixed. So at constant speed we can write

$$
\begin{equation*}
\mathbf{H}_{\mathrm{th}}=\mathbf{k}_{1}-\mathbf{k}_{2} \mathbf{Q} \tag{15.4.1}
\end{equation*}
$$

where $k_{1}$ and $k_{2}$ are constants and

$$
k_{1}=\pi^{2} D^{2} N^{2} \text { and } k_{2}=\left(\frac{\pi D N}{A} \cdot \cot \beta_{2}\right)
$$

Hence at constant speed this leads to a drooping linear characteristics for backward curved blading. This is shown by curve 1 in Figure 15.4.1.

The slip causes drop in the head, which can be written as $\sigma V_{u 2} u_{2} / g$. As flow increases this loss also increases. Curve 2 shown the head after slip. The flow will enter without shock only at the design flow rate. At other flow rates, the water will enter with shock causing losses. This lose can be expressed as

$$
\mathbf{h}_{\text {shock }}=\mathbf{k}_{3}\left(\mathbf{Q}_{\mathrm{th}}-\mathbf{Q}\right)^{2}
$$

The reduced head after shock losses is shown in curve 5 . The shock losses with flow rate is shown by curve 3 . The mechanical losses can be represented by $\mathbf{h}_{f}=\mathbf{k}_{4} \mathbf{Q}^{2}$. The variation is
shown by curve 4 . With variation of speed the head characteristic is shifted near paralley with the curve 5 shown in Figure 15.4.1.


Figure 15.4.1 Characteristics of a centrifugal pump
The characteristic of a centrigugal pump at constant speed is shown in Figure 15.4.2. It may be noted that the power increases and decreases after the rated capacity. In this way the pump is self limiting in power and the choice of the motor is made easy. The distance between the brake power and water power curves gives the losses.


Figure 15.4.2 Centrifugal pump characteristics at constant speed
The pump characteristics at various speeds including efficiency contours in shown in Figure 15.4.3. Such a plot helps in the development of a pump, particularly in specifying the head and flow rates.


Figure 15.4.3 Pump charateristics at various speeds

### 15.5 OPERATION OF PUMPS IN SERIES AND PARALLEL

Pumps are chosen for particular requirement. The requirements are not constant as per example the pressure required for flow through a piping system. As flow increases, the pressure required increases. In the case of the pump as flow increases, the head decreases. The operating condition will be the meeting point of the two curves representing the variation of head required by the system and the variation of head of the pump. This is shown in Figure 15.5.1. The operating condition decides about the capacity of the pump or selection of the


Figure 15.5.1 Pump-load characteristics pump.

If in a certain setup, there is a need for increased load, either a completely new pump may be chosen. This may be costlier as well as complete revamping of the setup. An additional pump can be the alternate choice. If the head requirement increases the old pump and the new pump can operate in series. In case more flow is required the old pump and the new pump will operate in parallel. There are also additional advantages in two pump operation. When the
load is low one of the pump can operate with a higher efficiency when the load increases then the second pump can be switched on thus improving part load efficiency. The characteristics of parallel operation is depicted in Figure 15.5.2.


Figure 15.5.2 Pumps in parallel
The original requirement was $Q_{1}$ at $H_{1}$. Pump 1 could satisfy the same and operating point is at 1 . When the flow requirement and the system characteristic is changed such that $Q_{2}$ is required at head $H_{1}$, then two pumps of similar characteristics can satisfy the requirement. Providing a flow volume of $Q_{2}$ as head $H_{1}$. It is not necessary that similar pumps should be used. Suitable control system for switching on the second pump should be used in such a case.

When the head requirement is changed with flow volume being the same, then the pumps should work in series. The characteristics are shown in Figure 15.5.3.


Figure 15.5.3 Pumps in series
The flow requirement is $Q$. Originally head requirement was $H_{1}$ met by the first pump alone. The new requirement is flow rate $Q$ and head $H_{2}$. This can be met by adding in series the pump 2 , which meets this requirement. It is also possible to meet changes in both head and flow requirements by the use of two pumps. Suitable control system should be installed for such purposes.

### 15.6 SPECIFIC SPEED AND SIGNIFICANCE

Some of the dimensionless parameters pertaining to pumps have been derived in the chapter on Dimensional analysis. These are derived from basics below :

1. Flow coefficient :

$$
\begin{array}{ll} 
& Q \propto V_{f} A \propto u D b \propto u D D \propto D N D D \propto N D^{3} \\
\therefore & \frac{\mathbf{Q}}{\mathbf{N D}^{3}}=\mathbf{c o n s t a n t} \tag{15.6.1}
\end{array}
$$

For similar machine and also the same machine. In the case of same machine $D$ is constant.

$$
\therefore \quad \frac{Q}{N}=\text { constant or } \frac{Q_{1}}{N_{1}}=\frac{Q_{2}}{N_{2}} \text {, unit quantity }
$$

2. Head parameter :

$$
\begin{align*}
& H \propto u^{2} / g \propto D^{2} N^{2} / g \\
\therefore & \frac{\mathbf{g H}}{\mathbf{N}^{2} \mathbf{D}^{2}}=\mathrm{constant} \tag{15.6.2}
\end{align*}
$$

The head parameter is constant for similar machines. For the same machine

$$
\frac{H_{1}}{N_{1}{ }^{2}}=\frac{H_{2}}{N_{2}{ }^{2}}, \text { unit head }
$$

3. Power parameter :

Multiplying the two parameters,

$$
\begin{equation*}
\frac{g H}{N^{2} D^{2}} \cdot \frac{Q}{N D^{3}}=\frac{\rho Q g H}{\rho N^{3} D^{5}}=\frac{P}{\rho N^{3} D^{5}} \tag{15.6.3}
\end{equation*}
$$

4. Specific speed :

Specific speed $\quad=\frac{\sqrt{\text { Flow parameter }}}{3 / 4 \sqrt{\text { Head parameter }}}$

$$
=\frac{\sqrt{Q}}{N^{1 / 2} D^{1.5}} \cdot \frac{N^{1.5} D^{1.5}}{(g H)^{3 / 4}}
$$

$$
\begin{equation*}
N_{s}=\frac{N \sqrt{Q}}{(g H)^{3 / 4}} \tag{15.6.4}
\end{equation*}
$$

This quantity is known as the specific speed of pumps. This is dimensionless. In practise $N_{s}=\frac{N \sqrt{Q}}{H^{3 / 4}}$ is in usage. One definitions for the specific speed is the speed at which the pump will operate delivering unit flow under unit head.

Actually the significance of the specific speed is its indication of the flow direction, width etc. of the impeller. This is illustrated in Figure 15.6.1. It is seen that different types of pumps have best efficiency at different specific speeds.


Figure 15.6.1 Efficiency-specific speed and impeller shape relations

### 15.7 CAVITATION

What is cavitation and where and why it occurs has been discussed in the chapter on turbines. In the case of pumps, the pressure is lowest at the inlet and cavitation damage occurs at the inlet. For cavitation to occur the pressure at the location should be near the vapour pressure at the location.

Applying the energy equation between sump surface and the pump suction,

$$
\begin{equation*}
\frac{P_{s}}{\gamma}+\frac{V_{s}^{2}}{2 g}+Z=\frac{P_{a}}{\gamma}-h_{f s} \tag{15.7.1}
\end{equation*}
$$

where $Z$ is the height from sump surface and pump suction. The other terms have their usual significance. The term $h_{f s}$ should include all losses in the suction line.

Net Positive Suction Head (NPSH) is defined as the available total suction head at the pump inlet above the head corresponding to the vapour pressure at that temperature.

$$
\begin{equation*}
N P S H=\frac{P_{s}}{\gamma}+\frac{V_{s}^{2}}{2 g}-\frac{P_{v}}{\gamma} \tag{15.7.2}
\end{equation*}
$$

where $P_{v}$ is the vapour pressure.
From 15.7.1,

$$
\begin{equation*}
N P S H=\frac{P_{a}}{\gamma}-\frac{P_{v}}{\gamma}-Z-h_{f s} \tag{15.7.3}
\end{equation*}
$$

Thoma cavitation parameter is defined by

$$
\sigma=\frac{(N P S H)}{H}=\frac{\left(P_{A} / \gamma\right)-\left(P_{v} / \gamma\right)-Z-h_{f s}}{H}
$$

At cavitation conditions,

$$
\begin{array}{ll} 
& \sigma=\sigma_{c} \text { and } \frac{P_{s}}{\gamma}=\frac{P_{v}}{\gamma} \\
\therefore & \sigma_{c}=\frac{\left(P_{a} / \gamma\right)-\left(P_{v} / \gamma\right)-Z-h_{f s}}{H} \tag{15.7.4}
\end{array}
$$

The height of suction, the frictional losses in the suction line play an important role for avoiding cavitation at a location. When pumps designed for one location is used at another location, atmospheric pressure plays a role in the onset of cavitation. Some authors use the term "suction specific speed, ' $n s$ ". Where $H$ in the general equation is substituted by NPSH. One correlation for critical cavitation parameter for pumps is given as

$$
\begin{equation*}
\sigma_{c}=\left(\frac{n_{s}}{175}\right)^{4 / 3} \tag{15.7.5}
\end{equation*}
$$

These equations depend upon the units used and should be applied with caution.

### 15.8 AXIAL FLOW PUMP

A sectional view of axial flow pump is shown in Figure 15.8.1.
The flow in these machines is purely axial and axial velocity is constant at all radii. The blade velocity varies with radius and so the velocity diagrams and blade angles will be different at different radii. Twisted blade or airfoil sections are used for the blading. Guide vanes are situated behind the impeller to direct the flow axially without whirl. In large pumps inlet guide vanes may also be used. Such pumps are also called as propeller pumps. The head developed per stage is small, but due to increased flow area, large volumes can be handled.


Figure 15.8.1 Axial flow pums
A comparison of values of parameters is given in table 15.8.1.
Table 15.8.1 Comparative values of parameter for different types of pumps

|  |  |  | Centrifugal |  |
| :--- | :---: | :---: | :---: | :---: |
| Type of pump | Axial | Mixed flow | Diffuser pumps | Volute pumps |
| Flow ratio | $0.25-0.6$ | 0.3 | 0.15 | 0.2 |
| $\left(V_{f} / \sqrt{2 g H}\right)$ |  |  |  |  |
| Speed ratio | $2-2.7$ | 1.35 | $0.9-1.05$ | $1-1.2$ |
| $\frac{u_{2}}{\sqrt{2 g H}}$ |  |  |  |  |
| Specific speed | $150-800$ | $85-175$ | $15-20$ | $20-90$ |
| $N \sqrt{Q} / H^{3 / 4}$ |  |  |  |  |

The whirl at inlet is zero. The velocity triangles are given in Figure 15.8.2.
$V_{a}$ is constant at all sections both at inlet and outlet. $u$ varies with radius. Hence $\beta_{1}$ and $\beta_{2}$ will vary with radius.
$H_{t h}=\frac{u_{2} V_{u 2}}{g}$ as in the case of centrifugal pumps. All other efficiencies are similar to the contrifugal pump.

The angle turned by the fluid during the flow over the blades is about $10-15^{\circ}$. Hence whirl imparted per


Figure 15.8.2 Velocity trianglesaxial pump
stage is small. The number of blades is limited as in the case of Kaplan turbine ranging between 2 and 8 . The hub to tip ratio is in the range 0.3 to 0.6 . Generally the blades are fixed. In rare designs the blades are rotated as in the case of Kaplan turbine by suitable governing mechanism.

### 15.9 POWER TRANSMITTING SYSTEMS

Ordinarily power is transmitted by mechanical means like gear drive or belt drive. In the case of gear drive there is a rigid connection between the driving and driven shafts. The shocks and vibrations are passed on from one side to the other which is not desirable. Also gear drives can not provide a stepless variation of speeds. In certain cases where the driven machine has a large inertia, the driving prime mover like electric motor will not be able to provide a large starting torque. Instead of the mechanical connection if fluids can be used for such drives, high inertia can be met. Also shock loads and vibration will not be passed on. Smooth speed variation is also possible. The power transmitting systems offer these advantages.

There are two types power transmitting devices. These are (i) Fluid coupling and (ii) Torque converter or torque multiplier.

### 15.9.1 Fluid Coupling

A sectional view of a fluid coupling is shown in figure 15.9.1.


Figure 15.9.1 Fluid coupling
In this device the driving and driven shafts are not rigidly connected. The drive shaft carries a pump with radial vanes and the driven shaft carries a turbine runner. Both of these are enclosed in a casing filled with oil of suitable viscosity. The pump accelerates the oil by imparting energy to it. The oil is directed suitably to hit the turbine vanes where the energy is absorbed and the oil is decelerated. The decelerated oil now enters the pump and the cycle is repeated. There is no flow of fluid to or from the outside. The oil transfers the energy from the
drive shaft to the driven shaft. As there is no mechanical connection between the shafts, stock loads or vibration will not be passed on from one to the other. The turbine will start rotating only after a certain level of energy picked up by the oil from the pump.

Thus the prime mover can pick up speed with lower starting torque before the power is transmitted. In this way heavy devices like power plat blowers can be started with motors with lower starting torque. The pump and turbine can not rotate at the same speeds. In case these do run at the same speed, there can be no circulation of oil between them as the centrifugal heads of the pump and turbine are equal, and no energy will be transferred from one to the other. The ratio of difference in speeds to the driver speed is known as slips, $S$.

$$
\begin{equation*}
S=\frac{\omega_{p}-\omega_{T}}{\omega_{p}} \tag{15.9.1}
\end{equation*}
$$

where $\omega_{p}$ is the pump speed and $\omega_{T}$ is the turbine speed. The variation of slip with pump speed is shown in figure 15.9.2.

As shown up to the pump speed $\omega_{p s}$ the turbine will not run and slip is $100 \%$. As the driver speed increases slip rapidly decreases and at the operating conditions reaches values of about 2 to $5 \%$.

The efficiency of transmission

$$
=\frac{\tau_{t} \cdot N_{t}}{\tau_{p} N_{p}}
$$



Figure 15.9.2 Slip variation with pump speed

In the absence of mechanical friction $\tau_{t}=\tau_{p}$

$$
\begin{align*}
& \text { So, } \\
& \begin{array}{l}
\eta=\frac{N_{t}}{N_{p}} \\
\text { As slip, } \\
\\
S=\frac{N_{p}-N_{t}}{N_{p}}=1-\frac{N_{t}}{N_{p}} \\
\eta=(1-S)
\end{array}
\end{align*}
$$

### 15.9.2 Torque Converter

In the case of fluid coupling the torque on the driver and driven members are equal. The application is for direct drives of machines. But there are cases where the torque required at the driven member should be more than the torque on the driver. Of course the speeds in this case will be in the reverse ratio. Such an application is in automobiles where this is achieved in steps by varying the gear ratios. The desirable characteristics is a stepless variation of torque. This is shown in figure 15.9.3. The torque converter is thus superior to the gear train with few gear ratios. A sectional view of torque converter is shown in figure 15.9.4. Torque


Figure 15.9.3 Torque variation in torque converter and gear train
converter consists of three elements namely pump impeller, a turbine runner and a fixed guide wheel as shown in figure 15.9.4. The pump is connected to the drive shaft. The guide vanes are fixed. The turbine runner is connected to the driven shaft. All the three are enclosed in a casing filled with oil. The oil passing through the pump impeller receives energy. Then it passes to the turbine runner where energy is extracted from the oil to turn the shaft. Then the oil passes to the stationary guide vanes where the direction is changed. This introduces a reactive torque on the pump which increases the torque to be transmitted. The shape and size and direction of the guide vanes controls the increase in torque. More than three elements have also been used in advanced type of torque converters. It may be noted that the speed ratio will be the inverse of torque ratio. The efficiency is found to be highest at speed ratio of about 0.6 .


Figure 15.9.4 Torque converter

## SOLVED EXAMPLES

Problem 15.1. The following details refer to a centrifugal pump. Outer diameter : 30 cm . Eye diameter : 15 cm . Blade angle at inlet : $30^{\circ}$. Blade angle at outlet: $25^{\circ}$. Speed 1450 rpm . The flow velocity remains constant. The whirl at inlet is zero. Determine the work done per $\boldsymbol{k g}$. If the manometric efficiency is $82 \%$, determine the working head. If width at outlet is 2 $c m$, determine the power $\eta_{o}=76 \%$.

$$
\begin{aligned}
& u_{1}=\frac{\pi \times 0.3 \times 1450}{60}=22.78 \mathrm{~m} / \mathrm{s} \\
& u_{2}=11.39 \mathrm{~m} / \mathrm{s} .
\end{aligned}
$$

From inlet velocity diagram.

$$
\begin{aligned}
V_{f 1} & =u_{1} \tan \beta_{1} \\
& =11.39 \times \tan 30=6.58 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

From the outlet velocity diagram,


Figure P. 15.1(a)

$$
V_{u 2}=u_{1}-\frac{V_{f 2}}{\tan \beta_{2}}=22.78-\frac{6.58}{\tan 25}=8.69 \mathrm{~m} / \mathrm{s}
$$

Work done per kg $\quad=u_{2} V_{u 2}=22.78 \times 8.69$

$$
=197.7 \mathrm{Nm} / \mathrm{kg} / \mathrm{s}
$$

$$
\eta_{m}=0.82=\frac{g H}{197.7}
$$

$$
\therefore \quad H=16.52 \mathrm{~m}
$$

Flow rate $=\pi \times 0.3 \times 0.02 \times 6.58=0.124 \mathrm{~m}^{3} / \mathrm{s}$
Power $=\frac{0.124 \times 10^{3} \times 9.81 \times 16.52}{0.76 \times 10^{3}}=\mathbf{2 6 . 4 5} \mathbf{k W}$.


Figure P. 15.1(b)

Problem 15.2 A homologus model of a centrifugal pump runs at 600 rpm against a head of 8 m , the power required being 5 kW . If the prototype 5 times the model size is to develop a head of 40 m determine its speed, discharge and power. The overall efficiency of the model is 0.8 while that of the prototype is 0.85 .

$$
\begin{equation*}
Q \propto D^{2} H^{1 / 2}\left(\text { as } Q=A V_{f}, \mathrm{~A} \propto D b, b \propto D, V_{f} \propto u \propto \sqrt{H}\right) \tag{1}
\end{equation*}
$$

$$
\begin{align*}
u & \propto D N \propto \sqrt{H} \quad \therefore \frac{N D}{\sqrt{H}}=\text { const. }  \tag{2}\\
Q_{m} & =\frac{P_{m} \times \eta_{m}}{\rho g H_{m}}=\frac{5 \times 10^{3} \times 0.8}{10^{3} \times 9.81 \times 8}=0.05097 \mathrm{~m}^{3} / \mathrm{s}
\end{align*}
$$

From (1)

$$
\mathbf{Q}_{\mathbf{p}}=Q_{m} \cdot \frac{D_{p}{ }^{2}}{D_{m}{ }^{2}} \cdot \frac{H_{p}{ }^{1 / 2}}{H_{m}{ }^{1 / 2}}
$$

$$
\begin{aligned}
& =0.05097 \times 5^{2} \cdot\left(\frac{40}{8}\right)^{1 / 2}=2.8492 \mathbf{~ m}^{3} / \mathrm{s} \\
\mathbf{N}_{\mathbf{p}} & =N_{m} \cdot\left(\frac{H_{p}}{H_{m}}\right)^{1 / 2} \cdot \frac{D_{m}}{D_{p}}=600.5^{1 / 2} \cdot \frac{1}{5}=\mathbf{2 6 8 . 3 2} \mathbf{~ r p m} \\
\text { Power } & =\frac{2.8492 \times 9.81 \times 40 \times 10^{3}}{0.85 \times 10^{3}}=\mathbf{1 3 1 5 . 3} \mathbf{k W} .
\end{aligned}
$$

From (2)

Problem 15.3 The diameter and width of a contrifugal pump impeller are 50 cm and 2.5 cm . The pump runs at 1200 rpm . The suction head is 6 m and the delivery head is 40 m . The frictional drop in suction is 2 m and in the delivery 8 m . The blade angle at out let is $30^{\circ}$. The manometric efficiency is $80 \%$ and the overall efficiency is $75 \%$. Determine the power required to drive the pump. Also calculate the pressures at the suction and delivery side of the pump.

Inlet swirl is assumed as zero.
Total head against the pump is

$$
\begin{aligned}
40+6+2+8 & =56 \mathrm{~m} . \\
u_{2} & =\pi \times 0.5 \times 1200 / 60=31.42 \mathrm{~m} / \mathrm{s} \\
\eta_{m} & =\frac{g H}{u_{2} V_{u 2}}=0.8 \\
\therefore \quad \frac{9.81 \times 56}{31.42 \times V_{u 2}} & =0.8, \text { solving } V_{u 2}=21.86 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

To calculate $V_{f}$, the velocity triangle is used.

$$
\begin{aligned}
& \tan \beta_{2}=\frac{V_{f 2}}{u_{2}-V_{u 2}} \\
& \therefore \quad V_{f 2}=\tan 30(31.42-21.86)=5.52 \mathrm{~m} / \mathrm{s} \\
& \text { Flow rate } \quad=\pi D_{2} b_{2} V_{f 2}=\pi \times 0.5 \times 0.15 \times 5.52 \\
& =0.13006 \mathrm{~m}^{3} / \mathrm{s} \\
& \therefore \quad \text { Power }=\frac{0.13006 \times 10^{3} \times 9.81 \times 56}{0.75 \times 10^{3}}=\mathbf{9 5 . 3} \mathbf{k W}
\end{aligned}
$$

Considering the water level and the suction level as 1 and 2

$$
\begin{aligned}
\frac{P_{1}}{\gamma}+0+0 & =\frac{P_{2}}{\gamma}+Z+\frac{V_{2}^{2}}{2 g}+\text { losses } \\
10 & =\frac{P_{2}}{\gamma}+6+\frac{5.52^{2}}{2 \times 9.81}+2, \text { solving, }
\end{aligned}
$$

$$
\frac{\mathbf{P}_{2}}{\gamma}=0.447 \mathrm{~m} \text { absolute (vacuum) }
$$

Consider suction side and delivery side, as 2 and 3

$$
\begin{aligned}
\frac{P_{2}}{\gamma}+\frac{V_{2}{ }^{2}}{2 g}+\frac{u_{2} V_{u 2}}{g} & =\frac{P_{3}}{\gamma}+\frac{V_{3}{ }^{2}}{2 g} \\
V_{3} & =\sqrt{21.86^{2}+5.52^{2}}=22.55 \mathrm{~m} / \mathrm{s} \\
\frac{P_{3}}{\gamma} & =0.447+\frac{5.52^{2}}{2 \times 9.81}+\frac{31.42 \times 21.86}{9.81}-\frac{22.55^{2}}{2 \times 9.81}=40.1 \mathrm{~m} \text { absolute }
\end{aligned}
$$

Problem 15.4 It is proposed to design a homologous model for a centrifugal pump. The prototype pump is to run at 600 rpm and develop 30 m head the flow rate being $1 \mathrm{~m}^{3} / \mathrm{s}$. The model of $1 / 4$ scale is to run at 1450 rpm. Determine the head developed discharge and power required for the model. Overall efficiency $=80 \%$.

In this case the speeds and diameter ratios are specified.

$$
\begin{array}{lll} 
& Q=A V_{f}, A=\pi D b, b \propto D, & \therefore A \propto D^{2} \\
& V_{f} \propto u \propto D N \propto \sqrt{H} & \\
\therefore & \mathbf{Q} \propto \mathbf{D}^{3} \mathbf{N} & \\
\text { Also } & u \propto \sqrt{H} \propto D N & \\
\therefore & \mathbf{Q} \propto \mathbf{D}^{2} \mathbf{H}^{1 / 2} & \\
& \mathbf{P} \propto \mathbf{Q} \mathbf{H} & \\
\text { As } & u \propto D N \propto \sqrt{H} & \\
\therefore & \mathbf{H} \propto \mathbf{N}^{2} \mathbf{D}^{2} & \tag{4}
\end{array}
$$

Using (1)

$$
\mathbf{Q}_{\mathrm{m}}=Q_{p}\left(\frac{D_{m}}{D_{p}}\right)^{3} \cdot \frac{N_{m}}{N_{p}}=1 \times\left(\frac{1}{4}\right)^{3} \times \frac{1450}{600}=0.03776 \mathrm{~m}^{3} / \mathrm{s}
$$

Using (4)

$$
\mathbf{H}_{\mathrm{m}}=H_{p}\left(\frac{D_{m}}{D_{p}}\right)^{2} \cdot\left(\frac{N_{m}}{N_{p}}\right)^{2}=30 \times\left(\frac{1}{4}\right)^{2} \cdot\left(\frac{1450}{600}\right)^{2}=10.95 \mathrm{~m}
$$

Using (3)

$$
\mathbf{P}_{\mathrm{m}}=P_{p} \frac{Q_{m}}{Q_{p}} \cdot \frac{H_{m}}{H_{p}}=367.9 \times \frac{0.03776}{1} \cdot \frac{10.95}{30}=5.07 \mathrm{~kW}
$$

Check :

$$
P_{m}=\frac{0.03776 \times 10^{3} \times 9.81 \times 10.95}{0.8}=5.07 \mathbf{k W}
$$

Problem 15.5 A centrifugal pump has been designed to run at 950 rpm delivering 0.4 $\mathrm{m}^{3} / \mathrm{s}$ against a head of 16 m . If the pump is to be coupled to a motor of rated speed 1450 rpm . Calculate the discharge, head and power input. Assume that the overall efficiency is 0.82 remains constant.

For a given pump, diameter, blade angles and physical parameters remain the same.
Hence, we can derive the following relations. (Similar to unit quantities).

$$
\begin{array}{rlll} 
& Q & =A V_{f}, A \text { is constant } & \therefore Q \propto V_{f} \\
V_{f} & \propto u \text { and } u \propto N & \therefore Q \propto N \text { or } Q / N=\text { constant } \\
\therefore & \frac{Q_{2}}{Q_{1}} & =\frac{N_{2}}{N_{1}} &  \tag{1}\\
\end{array}
$$

For centrifugal pump, $H \propto u^{2} \propto N^{2}$

$$
\begin{array}{ll}
\therefore & \frac{H}{N^{2}}=\mathrm{constant} \\
\therefore & \frac{H_{2}}{H_{1}}=\left(\frac{N_{2}}{N_{1}}\right)^{2} \\
\text { Power } & \propto Q H \propto N N^{2} \propto N^{3} \\
\therefore & \frac{P_{2}}{P_{1}}=\left(\frac{N_{2}}{N_{1}}\right)^{3} \tag{3}
\end{array}
$$

Using the equation (1), (2), (3)

$$
\begin{aligned}
& P_{1}=\frac{1000 \times 0.4 \times 9.81 \times 16}{1000 \times 0.82}=76.57 \mathrm{~kW} \\
& \mathbf{Q}_{2}=Q_{1} \cdot \frac{N_{2}}{N_{1}}=0.4 \times \frac{1450}{950}=0.61 \mathrm{~m}^{3} / \mathbf{s} \\
& \mathbf{H}_{2}=H_{1} \cdot\left(\frac{N_{2}}{N_{1}}\right)^{2}=16 \times\left(\frac{1450}{950}\right)^{2}=\mathbf{3 7 . 2 7} \mathbf{~ m} \\
& \mathbf{P}_{2}=76.57 \times\left(\frac{1450}{950}\right)^{3}=\mathbf{2 7 2} \mathbf{~ k W}
\end{aligned}
$$

Check :

$$
\mathbf{P}_{2}=\frac{1000 \times 0.61 \times 37.27 \times 9.81}{1000 \times 0.82}=272 \mathbf{k W}
$$

Problem 15.6 A centrifugal pump running at 1450 rpm has an impeller diameter of 0.4 $m$. The backward curved blade outlet angle is $30^{\circ}$ to the tangent. The flow velocity at outlet is 10 $\mathrm{m} / \mathrm{s}$. Determine the static head through which water will be lifted. In case a diffuser reduces the outlet velocity to $40 \%$ of the velocity at the impeller outlet, what will be the increase in the static head.

The whirl at inlet is assumed as zero.
The velocity diagram at outlet is shown.

$$
u_{2}=\frac{\pi D N}{60}=\frac{\pi \times 0.4 \times 1450}{60}=30.37 \mathrm{~m} / \mathrm{s}
$$

From velocity triangle

$$
V_{u 2}=u_{2}-V_{f 2} / \tan \beta_{2}=30.27-\frac{10}{\tan 30}=\mathbf{1 3 . 0 5} \mathbf{~ m} / \mathbf{s}
$$

Total head developed by the impeller

$$
=\frac{u_{2} V_{u 2}}{g}=\frac{30.37 \times 13.05}{9.81}=40.4 \mathrm{~m}
$$

Absolute velocity at exit $=\left(V_{f}{ }^{2}+V_{u 2}{ }^{2}\right)^{0.5}$

$$
\begin{aligned}
& =\left(10^{2}+13.05^{2}\right)^{0.5} \\
& =16.44 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Kinetic head

$$
=\frac{V_{2}{ }^{2}}{2 g}=\frac{16.44^{2}}{2 \times 9.81}=13.77
$$



Figure P. 15.6

If diffuser is not used, static lift

$$
=40.4-13.77=\mathbf{2 6 . 6 3} \mathbf{~ m}
$$

The diffuser outlet velocity $=0.4 \times 16.44=6.576 \mathrm{~m} / \mathrm{s}$
Kinetic head at outlet $\quad=6.576^{2} / 2 \times 9.81=2.2 \mathrm{~m}$
With diffuser use, the static lift $=40.4-2.2=\mathbf{3 8 . 2} \mathbf{~ m}$
Increase in static head $\quad=38.2-26.63 \mathrm{~m}=\mathbf{1 1 . 5 7} \mathbf{~ m}$
Problem 15.7 A form stage centrifugal pump running at 600 rpm is to deliver $1 \mathrm{~m}^{3} / \mathrm{s}$ of water against a manometric head of 80 m . The vanes are curved back at $40^{\circ}$ to the tangent at outer periphery. The velocity of flow is $25 \%$ of the peripheral velocity at outlet. The hydraulic losses are 30\% of the velocity head at the outlet of the impeller. Determine the diameter of the impeller and the manometric efficiency.

$$
u_{2}=\frac{\pi D_{2} N}{60}=\frac{\pi \times D_{2} \times 600}{60}=31.42 D_{2} \mathrm{~m} / \mathrm{s}
$$

Velocity of flow at outlet $=\frac{1}{4} \times 31.42=7.85 D_{2} \mathrm{~m} / \mathrm{s}$

$$
\begin{aligned}
V_{u 2} & =31.42 D_{2}-\frac{7.85 D_{2}}{\tan 40}=2.06 D_{2} \mathrm{~m} / \mathrm{s} \\
V_{2} & =\left[V_{u 2}{ }^{2}+V_{f 2}{ }^{2}\right]^{0.5}=\left[22.06^{2} D_{2}{ }^{2}+7.85^{2} D_{2}{ }^{2}\right]^{0.5} \\
& =23.41 D_{2} \mathrm{~m} / \mathrm{s} \\
\frac{0.3 V_{2}{ }^{2}}{2 g} & =8.383 D_{2}{ }^{2}
\end{aligned}
$$

Energy imparted to impeller

$$
\begin{aligned}
& =\frac{u_{2} V_{u 2}}{g}=\frac{31.42 \times 22.06}{9.81} D_{2}^{2}=70.655 D_{2}^{2} \\
70.655 D_{2}^{2} & =\text { manometric head per stage + losses. } \\
& =(80 / 4)+8.838 D_{2}^{2} \\
\therefore \quad 62.272 D_{2}^{2} & =20 \\
\therefore \quad \text { Solving } & \mathbf{D}_{2}=\mathbf{0 . 5 7} \mathbf{~ m} \\
\eta_{m} & =\frac{20}{70.655 \times 0.57^{2}}=\mathbf{0 . 8 7 1 2} \text { or } \mathbf{8 7 . 1 2 \%}
\end{aligned}
$$

Problem 15.8 A centrifugal pump works at 900 rpm and is required to work against a head of 30 m . The blades are curved back at $25^{\circ}$ to the tangent at outlet. The flow velocity is 2.5 $\mathrm{m} / \mathrm{s}$. Determine the diameter (i) If all the kinetic energy is lost (ii) It the velocity is reduced to $\mathbf{5 0 \%}$ converting kinetic energy to pressure energy.

Energy imparted to the impeller $=u\left(u-\frac{2.5}{\tan 25}\right) / g$

Kinetic head at exit

$$
=\frac{V_{2}^{2}}{2 g}=\left\{\left(u-\frac{2.5}{\tan 25}\right)^{2}+2.5^{2}\right\} / 2 g
$$

The difference is the head against which the pump works (simplyfying)

$$
2\left(u^{2}-5.361 u\right)-\left(u^{2}+5.361^{2}-2 \times 5.361 u+6.25\right)=30 \times 2 \times 9.81
$$

This reduces to

$$
\begin{aligned}
u^{2} & =623.59 \quad \therefore \quad u=24.97 \mathrm{~m} / \mathrm{s} \\
u & =\frac{\pi D N}{60}, 24.97=\frac{\pi \times D \times 900}{60}, \\
\mathbf{D} & =\mathbf{0 . 5 3} \mathbf{~ m}
\end{aligned}
$$

Solving
In case the velocity at exit is reduced to half its value,

$$
\frac{u(u-5.361 u)}{g}-\frac{(u-5.361)^{2}+2.5^{2}}{8 g}=30
$$

This reduces to

Solving

$$
\begin{gathered}
7 u^{2}-32.166 u-2392.35=0 \\
u=20.93 \mathrm{~m} / \mathrm{s}
\end{gathered}
$$

This leads to $\mathbf{D}=0.44 \mathrm{~m}$.
Problem 15.9 The dimensionless specific speed of a centrifugal pump is 0.06. Static head is 30 m . Flow rate is $50 \mathrm{l} / \mathrm{s}$. The suction and delivery pipes are each of 15 cm diameter. The friction factor is 0.02 . Total length is 55 m other losses equal 4 times the velocity head in the pipe. The vanes are forward curved at $120^{\circ}$. The width is one tenth of the diameter. There is a $6 \%$ reduction in flow area due to the blade thickness. The manometric efficiency is $80 \%$. Determine the impeller diameter. Inlet is radial.

Frictional head is calculated first. Velocity in the pipe

$$
\begin{aligned}
\text { Total loss of head } \quad & =\frac{f l V^{2}}{2 g D}+\frac{4 V^{2}}{2 g} \\
& =\frac{0.02 \times 55 \times 2.83^{2}}{2 \times 9.81 \times 0.15}+\frac{4 \times 2.83^{2}}{2 \times 9.81}=4.63 \mathrm{~m}
\end{aligned}
$$

Total head against which pump operates $=34.63 \mathrm{~m}$
Speed is calculated from specific speed $N_{s}=N \sqrt{Q} /(g H)^{3 / 4}$

$$
N=\frac{0.06 \times(9.81 \times 34.63)^{3 / 4}}{0.05^{1 / 2}}=21.23 \mathrm{rps}
$$

Flow velocity is determined :
Flow area $=\pi \times D \times \frac{D}{10} \times 0.94=0.2953 D^{2}$

$$
\begin{aligned}
& V_{f 2}=\frac{0.05}{0.2953 D^{2}}=0.1693 / D^{2} \\
u_{2} & =\pi D N=21.23 \times \pi \times D=66.7 D \\
\eta_{m} & =0.8=\frac{9.81 \times 34.63}{66.7 D \times V_{u 2}} \\
\therefore \quad & V_{u 2}=\frac{6.367}{D}
\end{aligned}
$$



Figure P. 15.9 Outlet velocity diagram
(forwerd curved)

From velocity diagram,

$$
\tan 60=\frac{V_{f 2}}{V_{u 2}-u_{2}}=\frac{0.1693}{D^{2}} \cdot \frac{1}{\left(\frac{6.367}{D}-66.7 D\right)}
$$

Rearranging, $115.52 D^{3}-11.028 D+0.1693=0$
Solving,

$$
\mathrm{D}=0.3 \mathrm{~m}
$$

Problem 15.10 The head developed by a centrifugal pump running at 900 rpm is 27 m . The flow velocity is $3 \mathrm{~m} / \mathrm{s}$. The blade angle at exit is $45^{\circ}$. Determine the impeller diameter.

The head developed $\quad=\frac{u_{2} V_{u 2}}{g}=27 \mathrm{~m}$

$$
V_{u 2}=u_{2}-\frac{V_{f}}{\tan 45}=u_{2}-3
$$

$$
\begin{array}{rlrl}
\therefore & u_{2} & \times\left(u_{2}-3\right)=27 \times 9.81 \\
& u_{2}^{2}-3 u_{2}-264.87 & =0 \\
u_{2} & =17.84(\text { or }-14.84, \text { trivial }) \\
& \frac{\pi D N}{60} & =17.84, N=900 \\
& & & \\
\therefore & & =\frac{17.84 \times 60}{\pi \times 900}=\mathbf{0 . 3 9 7} \mathbf{~ m} \text { or } \mathbf{3 7 9} \mathbf{~ m m .}
\end{array}
$$

Problem 15.11 A radial vaned centrifugal compressor delivers $0.3 \mathrm{~m}^{3} / \mathrm{s}$ against a head of 20 m . The flow velocity is constant at $3 \mathrm{~m} / \mathrm{s}$. The manometric efficiency is $80 \%$. If the width is $1 / 10$ th of the diameter Calculate the diameter, width and speed. The eye diameter is 0.5 of outer diameter. Calculate the dimensions of inlet.

Assume zero whirl at inlet. The velocity diagram at outlet is shown

$$
\begin{array}{rlrl} 
& & \eta_{m} & =\frac{g H}{u_{2} V_{u 2}} \text { here } u_{2}=V_{u 2} \\
& \therefore & u_{2}{ }^{2} & =\frac{g H}{\eta_{m}}=\frac{9.81 \times 20}{0.8}=245.25 \\
\therefore & & u & =15.66 \mathrm{~m} / \mathrm{s} \\
\therefore & Q & =\pi D_{2} b_{2} V_{f 2} \\
& & 0.3 & =\pi \times D_{2} \times 0.1 D_{2} \times 3 \\
& D_{2}{ }^{2} & =\frac{0.3}{\pi \times 0.1 \times 3},
\end{array}
$$



Figure P. 15.11

Solving,

$$
\therefore \quad \mathbf{b}_{2}=0.05642 \mathrm{~m}
$$

$$
\begin{aligned}
\mathbf{D}_{2} & =0.5642 \mathrm{~m} \\
\mathbf{b}_{2} & =0.05642 \mathrm{~m} \\
\mathbf{D}_{1} & =0.2821 \mathrm{~m} \\
\mathbf{b}_{1} & =0.1128 \mathrm{~m} \\
u_{2} & =\frac{\pi D N}{60}, 15.66=\frac{\pi \times 0.5642}{60} \times N
\end{aligned}
$$

Solving

$$
\text { N = } 530 \text { rpm. }
$$

Problem 15.12 A centrifugal pump running at 900 rpm and delivering $0.3 \mathrm{~m}^{3} / \mathrm{s}$ of water against a head of 25 m , the flow velocity being $3 \mathrm{~m} / \mathrm{s}$. If the manometric efficiency is $82 \%$ determine the diameter and width of the impeller. The blade angle at outlet is $25^{\circ}$.

The velocity diagram at outlet is as shown. The inlet whirl is generally assumed as zero unless mentioned.

$$
\eta_{m}=\frac{g H}{u_{2} V_{u 2}}
$$



Figure P. 15.12

$$
\begin{array}{ll}
\therefore & u_{2} V_{u 2}=\frac{9.81 \times 25}{0.82}=299.09 \\
& \tan 25=\frac{V_{f}}{u_{2}-V_{u 2}} \quad \therefore \quad u_{2}-V_{u 2}=\frac{3}{\tan 25}=6.43352  \tag{B}\\
\therefore & V_{u 2}=u_{2}-6.43352 \\
\text { Solving } \quad u_{2} \times\left(u_{2}-6.43352\right)=299.09 \\
& u_{2}{ }^{2}-6.43352 u_{2}-299.09=0 \\
u_{2}=20.808 \mathrm{~m} / \mathrm{s} \text { (the other solution being negative). }
\end{array}
$$

or

$$
\begin{array}{ll} 
& u_{2}=\frac{\pi D N}{60}, \\
\therefore \quad & \mathbf{D}=\frac{u_{2} \times 60}{\pi N}=\frac{20.808 \times 60}{\pi \times 900}=\mathbf{0 . 4 4 1 6} \mathbf{~ m} \text { or } \mathbf{4 4 . 1 6} \mathbf{~ c m} \\
& Q=\pi D_{2} b_{2} V_{f 2} \\
\therefore \quad & \mathbf{b}_{2}=\frac{Q}{\pi D_{2} V_{f 2}}=\frac{0.3}{\pi \times 0.4416 \times 3}=\mathbf{0 . 0 7 2 1} \mathbf{~ m} \text { or } 7.21 \mathbf{~ c m} .
\end{array}
$$

Problem 15.13. An axial flow pump running at 600 rpm deliver $1.4 \mathrm{~m}^{3} / \mathrm{s}$ against a head of 5 m . The speed ratio is 2.5 . The flow ratio is 0.5 . The overall efficiency is 0.83 . Determine the power required and the blade angles at the root and tip and the diffuser blade inlet angle. Inlet whirl is zero.
$\beta_{1}$ - Blade angle at inlet
$\beta_{2}$ - Blade angle at outlet
$\alpha_{2}$ - Diffuser blade inlet angle
Power $=\frac{1.4 \times 10^{3} \times 5 \times 9.81}{0.83 \times 10^{3}}=82.73 \mathrm{~kW}$

$$
\begin{aligned}
u_{t} & =2.5 \sqrt{2 g H} \\
& =2.5 \sqrt{2 \times 9.81 \times 5} \\
& =24.76 \mathrm{~m} / \mathrm{s}, \\
V_{f} & =0.5 \sqrt{2 g H}=4.95 \mathrm{~m} / \mathrm{s} \\
D_{0} & =\frac{24.76 \times 60}{\pi \times 600}=0.788 \mathrm{~m}
\end{aligned}
$$

$$
Q=\frac{\pi\left(D_{0}{ }^{2}-D_{1}{ }^{2}\right) V_{f}}{4}
$$



Figure P. 15.13

$$
\therefore \quad 1.4=\pi\left(0.788^{2}-D_{1}^{2}\right) 4.95
$$

Solving, $\quad D_{1}=0.51 \mathrm{~m}$
At Tip : From inlet triangle,

$$
\beta_{1 t}=\tan ^{-1}\left(\frac{4.95}{24.76}\right)=11.3^{\circ}
$$

From outlet triangle, $\boldsymbol{\beta}_{2 \mathrm{t}}=V_{f} /\left(u-V_{u 2}\right)$

$$
\begin{aligned}
u_{2} V_{u 2} & =g H, 24.76 \times V_{u 2}=9.81 \times 5, V_{u 2}=1.981 \mathrm{~m} / \mathrm{s} \\
\therefore \quad \beta_{2 \mathrm{t}} & =\tan ^{-1}\left(\frac{4.95}{24.76-1.981}\right)=\mathbf{1 2 . 2 6}{ }^{\circ} \\
& \alpha_{2 t}=\tan ^{-1}\left(\frac{4.95}{1.981}\right)=68.2^{\circ}, \text { can also be given as }\left(180-68.2^{\circ}\right)
\end{aligned}
$$

At Root :

$$
u_{2} V_{u 2}=g H, \quad u_{2}=\pi \times 0.51 \times 600 / 60=16.02 \mathrm{~m} / \mathrm{s}
$$

$$
V_{u 2}=5 \times 9.81 / 16.02=3.06 \mathrm{~m} / \mathrm{s}, V_{f}=\text { constant }
$$

$$
\beta_{1 \mathrm{R}}=\tan ^{-1}\left(\frac{4.95}{16.02}\right)=\mathbf{1 7 . 1 7}^{\circ}
$$

$$
\beta_{2 \mathbf{R}}=\tan ^{-1}\left(4.95 /(16.02-3.06)=\mathbf{2 0 . 9}{ }^{\circ}\right.
$$

$$
\alpha_{2 R}=\tan ^{-1}\left(\frac{4.95}{3.06}\right)=58.3^{\circ}
$$

$\alpha$ values can also be given as ( $\mathbf{1 8 0} \mathbf{- 5 8 . 3 ) ^ { \circ } \text { . }}$
Problem 15.14 A centrifugal pump of impeller diameter 0.4 m runs at 1450 rpm. The blades are curved back at $30^{\circ}$ to the tangent at the outlet. The velocity of flow is 3 m per second. Determine the theoretical maximum lift if the outlet velocity is reduced by the diffuser by $50 \%$.

Inlet whirl is assumed to be zero

$$
\begin{aligned}
u_{2} & =\frac{\pi \times 0.4 \times 1450}{60}=30.37 \mathrm{~m} / \mathrm{s} \\
V_{u 2} & =30.37-\frac{3}{\tan 30}=25.17 \mathrm{~m} / \mathrm{s} \\
V_{2} & =\left(25.17^{2}+3^{2}\right)^{0.5}=25.35 \mathrm{~m}
\end{aligned}
$$

Head imparted $=\frac{30.37 \times 25.17}{9.81}=77.92 \mathrm{~m}$

Static head

$$
=77.92-\frac{25.35^{2}}{2 \times 9.81}=45.17 \mathrm{~m}
$$

Without diffuser the pump can pump to a head of 45.17 m theoretically.
If velocity is reduced to $50 \%$ of the value
New velocity $\quad=12.675 \mathrm{~m} / \mathrm{s}$
$\therefore$ Head recovered $\quad=\frac{25.35^{2}-12.675^{2}}{2 \times 9.81}=24.57 \mathrm{~m}$

## $\therefore$ Theoretical maximum lift

$$
=45.17+24.57=69.74 \mathbf{~ m}
$$

Problem 15.15 A centrifugal pump running at 900 rpm has an impeller diameter of 500 mm and eye diameter of 200 mm . The blade angle at outlet is $35^{\circ}$ with the tangent. Determine assuming zero whirl at inlet, the inlet blade angle. Also calculate the absolute velocity at outlet and its angle with the tangent. The flow velocity is constant at $3 \mathrm{~m} / \mathrm{s}$. Also calculate the manometric head.

The velocity diagrams are as shown.

## Consider inlet

$$
\begin{aligned}
u_{1} & =\frac{\pi \times 0.2 \times 900}{60}=9.42 \mathrm{~m} / \mathrm{s} \\
V_{f 1} & =3 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Blade angle at inlet

$$
\begin{aligned}
& \tan \beta_{1} & =\frac{V_{f 1}}{u_{1}}=\frac{3}{9.42} \\
\therefore & \boldsymbol{\beta}_{1} & =\mathbf{1 7 . 6 6}^{\circ}
\end{aligned}
$$

Considering outlet

$$
u_{2}=\frac{\pi \times 0.5 \times 900}{60}=23.56 \mathrm{~m} / \mathrm{s}
$$

$$
V_{u 2}=u_{2}-\frac{V_{f 2}}{\tan 35}=23.56-\frac{3}{\tan 35}=19.28 \mathrm{~m} / \mathrm{s}
$$

$$
\tan \alpha_{2}=\frac{3}{19.28}
$$

$$
\therefore \quad \alpha_{2}=8.85^{\circ}
$$

$$
V_{2}=\sqrt{3^{2}+19.28^{2}}=19.51 \mathrm{~m} / \mathrm{s}
$$

The outlet velocity is $19.51 \mathrm{~m} / \mathrm{s}$ at an angle of $8.85^{\circ}$ to the tangent. (taken in the opposite direction of $u$ ).

$$
\text { Manometric head } \quad=\frac{23.56 \times 19.28}{9.81}=46.3 \mathrm{~m}
$$

Problem 15.16 A centrifugal pump running at 900 rpm delivers $800 \mathrm{l} / \mathrm{s}$ against a head of 70 m . The outer diameter of the impeller is 0.7 m and the width at outlet is 7 cm . There is recirculation of $3 \%$ of volume delivered. There is a mechanical loss of 14 kW . If the manometric efficiency is $82 \%$ determine the blade angle at outlet, the motor power and the overall efficiency.

The velocity diagram at outlet is as shown. The inlet whirl is assumed as zero. Backward curved vane is assumed (This is the general case).

$$
u_{2}=\frac{\pi D_{2} N}{60}=\frac{\pi \times 0.7 \times 900}{60}=32.99 \mathrm{~m} / \mathrm{s}
$$

From manometric efficiency, $V_{u 2}$ is determined

$$
\begin{aligned}
\eta_{m} & =\frac{g H}{u_{2} V_{u 2}} \\
0.82 & =\frac{9.81 \times 70}{32.99 \times V_{u 2}} \\
\therefore \quad V_{u 2} & =25.385 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The flow through the impeller is increased by $3 \%$


Figure P. 15.16

$$
\therefore \quad V_{f 2}=\frac{Q \times 1.03}{\pi D_{2} b_{2}}=\frac{0.8 \times 1.03}{\pi \times 0.7 \times 0.07}=5.353 \mathrm{~m} / \mathrm{s}
$$

From velocity triangle,

$$
\tan \beta_{2}=\frac{V_{f 2}}{u_{2}-V_{u 2}}=\frac{5.353}{32.99-25.385}
$$

Solving

$$
\beta_{2}=35.14^{\circ}
$$

Motor Power $\quad=Q \times 1.03 \times u_{2} V_{u 2}+$ Mechanical losses

$$
=(0.8 \times 1.03 \times 32.99 \times 25.385)+14
$$

$$
=690.06+14=704.06 \mathbf{k W}
$$

$$
\begin{aligned}
\text { Overall efficiency } & =1.03 \times 0.8 \times 9.81 \times 70 \times \frac{1}{704.06} \\
& =\mathbf{0 . 8 0 3 6} \text { or } \mathbf{8 0 . 3 6 \%}
\end{aligned}
$$

Problem 15.17 The pressure difference between the suction and delivery sides of a pump is 25 m . The impeller diameter is 0.3 m and the speed is 1450 rpm . The vane angle at outlet $30^{\circ}$ with the tangent. The velocity of flow is 2.5 m . Determine the manometric efficiency. If frictional losses in the impeller is 2 m calculate the fraction of total energy converted to pressure energy in the impeller. Also calculate the pressure rise in the pump casing.

$$
u_{2}=\frac{\pi \times 0.3 \times 1450}{60}=22.78 \mathrm{~m} / \mathrm{s}
$$

$$
\begin{aligned}
V_{u 2} & =22.78-\frac{2.5}{\tan 30}=18.45 \mathrm{~m} / \mathrm{s} \\
V_{2} & =\left(18.45^{2}+2.5^{2}\right)^{0.5}=18.62 \mathrm{~m} / \mathrm{s} \\
& =\frac{22.78 \times 18.45}{9.81}=42.84 \mathrm{~m} \\
\eta_{\mathrm{m}} & =\frac{25}{42.84}=\mathbf{0 . 5 8 3 5} \text { or } 58.35 \%
\end{aligned}
$$

Kinetic energy at exit of impeller

$$
=18.62^{2} / 2 \times 9.81=17.67 \mathrm{~m}
$$

$\therefore$ Energy conversion in the impeller

$$
=42.84-17.67=25.17 \mathrm{~m}
$$

$$
\text { Frictional loss } \quad=2 \mathrm{~m}
$$

## Energy fraction for pressure

$$
=\frac{25.17-2}{42.84}=\mathbf{0 . 5 4 0 9} \text { or } \mathbf{5 4 . 0 9 \%}
$$

Total pressure rise $\quad=25 \mathrm{~m}$
Pressure rise in the impeller $=(25.17-2)=23.17 \mathrm{~m}$
$\therefore \quad$ Pressure rise in the pump casing $=\mathbf{1 . 8 3} \mathbf{~ m}$.
Problem 15.18 A centrifugal pump running at 1450 rpm has impeller of 350 mm OD and 150 mm ID. The blade angles as measured with the radial direction are $60^{\circ}$ and $65^{\circ}$ at inlet and outlet. The hydraulic efficiency is $85 \%$. The width of the impeller at inlet is 50 mm . Flow velocity is constant.

Determine the static and stagnation pressure risc across the impeller and the power input to the


Figure P. 15.18

$$
V_{u 2}=u_{2}-\frac{V_{f 2}}{\tan \beta_{2}}=26.572-\frac{6.575}{\tan 25^{\circ}}=12.472 \mathrm{~m} / \mathrm{s}
$$

Energy input to the pump $\quad=\frac{u_{2} V_{u 2}}{g}=33.78 \mathrm{~m}$
Energy to the fluid $\quad=0.85 \times 33.78=28.72 \mathrm{~m}$

Stagnation pressure rise through the impeller

$$
=\left[\frac{P_{2}-P_{1}}{\gamma}+\frac{V_{2}^{2}-V_{1}^{2}}{2 g}\right]=28.72 \mathrm{~m}
$$

Static pressure rise $=28.72-\left[\frac{V_{2}{ }^{2}-V_{1}{ }^{2}}{2 g}\right]$

$$
V_{2}=\left(V_{f 2}{ }^{2}+V_{u 2}{ }^{2}\right)^{1 / 2}=\left(6.575^{2}+12.472^{2}\right)^{1 / 2}=16.1 \mathrm{~m} / \mathrm{s}
$$

## $\therefore$ Static pressure rise

$$
=28.72-\frac{16.1^{2}-6.575^{2}}{2 \times 9.81}=\mathbf{1 7 . 7 1} \mathbf{~ m}
$$

(Heads can be converted to pressure using $H \gamma=P$ )
Flow rate

$$
=\pi \times 0.15 \times 0.05 \times 6.575=0.1549 \mathrm{~m}^{3} / \mathrm{s}
$$

$\therefore$ Power input $\quad=\frac{33.78 \times 01549 \times 10^{3} \times 9.81}{10^{3}}=\mathbf{5 1 . 3} \mathbf{k W}$.
Problem 15.19 A centrifugal pump with $O D=0.6 \mathrm{~m}$ and $I D=0.3 \mathrm{~m}$ runs at 900 rpm and discharges $0.2 \mathrm{~m}^{3} / \mathrm{s}$ of water against a head of 55 m . The flow velocity remains constant along the flow. The peripheral area for flow is $0.0666 \mathrm{~m}^{2}$. The vane angle at outlet is $25^{\circ}$. The entry is radial. Determine the manometric efficiency and the inlet vane angle.

Radial entry means zero whirl at inlet. The velocity triangles are as shown.
$u_{2}=\frac{\pi D_{2} N}{60}=\frac{\pi \times 0.6 \times 900}{60}=28.27 \mathrm{~m} / \mathrm{s}$
From outlet triangle,

$$
\begin{array}{cc} 
& V_{u 2}=u_{2}-\frac{V_{f 2}}{\tan 25} \\
& V_{f 2}=0.2 / 0.0666=3 \mathrm{~m} / \mathrm{s} \\
\therefore & V_{u 2}=28.27-\frac{3}{\tan 25}=21.84 \mathrm{~m} / \mathrm{s} \\
\therefore & \eta_{\mathrm{m}}=\frac{g H}{u_{2} V_{u 2}}=\frac{55 \times 9.81}{28.27 \times 21.84}=0.8739 \\
& \quad \text { or } 87.39 \%
\end{array}
$$



Figure P. 15.19

From inlet triangle,

$$
\begin{aligned}
& \tan \beta_{1}=\frac{V_{f 1}}{u_{1}}, u_{1}=\frac{\pi \times 0.3 \times 900}{60}=14.14 \mathrm{~m} / \mathrm{s} \\
\therefore \quad & \tan \beta_{1}=\frac{3}{14.14} \quad \therefore \quad \beta_{1}=12^{\circ}
\end{aligned}
$$

Problem 15.20 A five stage centrifugal pump with blades radial at outlet runs at 500 rpm delivering $0.25 \mathrm{~m}^{3} / \mathrm{s}$ against a total head of 100 m . The diameter of the impellers is 0.6 m and the flow velocity is $5 \mathrm{~m} / \mathrm{s}$. Determine the manometric efficiency and the width of the impellers.

Inlet whirl is assumed zero. The velocity diagram is shown.

Here $\mathbf{u}_{2}=\mathbf{V}_{\mathbf{u} 2}$

$$
\begin{aligned}
u_{2} & =\frac{\pi D N}{60}=\frac{\pi \times 0.6 \times 500}{60} \\
& =15.708 \mathrm{~m} / \mathrm{s} \\
& =\frac{u_{2} V_{u 2}}{g}=\frac{u_{2}{ }^{2}}{g}=\frac{15.708}{9.81} \\
& =25.15 \mathrm{~m}
\end{aligned}
$$

$$
\text { Manometric head } \quad=\frac{u_{2} V_{u 2}}{g}=\frac{u_{2}{ }^{2}}{g}=\frac{15.708^{2}}{9.81}
$$



Figure P. 15.20

Head delivered by each impeller is $100 / 5=20 \mathrm{~m}$
$\therefore \quad$ Manometric efficiency $=20 / 25.15=0.7952=\mathbf{7 9 . 5 2 \%}$
Flow rate $=\pi D b V_{f}=0.25=\pi \times 0.6 \times 5 . b_{2}$
Solving $\mathbf{b}_{2}=0.0265 \mathrm{~m}$ or $\mathbf{2 6 . 5} \mathbf{~ m m}$.
Problem 15.21 A centrifugal pump with an impeller diameter of 0.4 m runs at 1450 rpm. The angle at outlet of the backward curved vane is $25^{\circ}$ with tangent. The flow velocity remains constant at $3 \mathrm{~m} / \mathrm{s}$. If the manometric efficiency is $84 \%$ determine the fraction of the kinetic energy at outlet recovered as static head.

The whirl at inlet is zero. The velocity triangle is as shown.

$$
\begin{aligned}
u_{2} & =\frac{\pi D N}{60}=\frac{\pi \times 0.4 \times 1450}{60} \\
& =30.37 \mathrm{~m} / \mathrm{s} \\
V_{u 2} & =u_{2}-\frac{V_{f 2}}{\tan \beta_{2}} \\
& =30.37-(3 / \tan 25)=23.94
\end{aligned}
$$



Figure 15. P. 21
$\mathrm{m} / \mathrm{s}$

$$
V_{2}=\sqrt{{V_{u 2}}^{2}+{V_{f 2}}^{2}}=\sqrt{23.94^{2}+3^{2}}=24.12 \mathrm{~m} / \mathrm{s}
$$

Total head developed $=\frac{30.37 \times 23.94}{9.81}=74.11 \mathrm{~m}$

Kinetic head $\quad=\frac{24.12^{2}}{2 \times 9.81}=29.65 \mathrm{~m}$
Kinetic head at inlet $\quad=\frac{3^{2}}{2 \times 9.81}=0.46 \mathrm{~m}$
$\therefore \quad$ Static head at impeller exit (using Bernoulli equation between inlet and outlet)

$$
=74.11+0.46-29.65=44.92 \mathrm{~m}
$$

Actual static head $\quad=\eta_{m} \times \frac{u_{2} V_{u 2}}{9.81}=0.84 \times 74.11=62.25 \mathrm{~m}$
Static head recovered $=62.25-44.92=17.33 \mathrm{~m}$
Let the fraction be $\phi$

$$
\frac{\phi \times 24.12^{2}}{2 \times 9.81}=17.33,
$$

Solving $\quad \phi=0.5823$
Problem 15.22 A centrifugal pump running at 900 rpm delivers $1 \mathrm{~m}^{3} / \mathrm{s}$ against a head of 12 m . The impeller diameters are 0.5 m and 0.3 m respectively. The blade angle at outlet is $20^{\circ}$ to the tangent. Determine the manometric efficiency and the power required. Mechanical efficiency $=98 \%$. Also estimate the minimum speed at which the pump will start delivery. The impeller width at outlet is $\mathbf{1 0} \mathbf{~ c m}$.

Whirl at inlet is zero is assumed

$$
\begin{aligned}
u_{2} & =\frac{\pi \times 0.5 \times 900}{60}=23.56 \mathrm{~m} / \mathrm{s} \\
V_{f 2} & =1 / \pi \times 0.5 \times 0.1=6.366 \mathrm{~m} / \mathrm{s} \\
V_{u 2} & =u_{2}-\frac{V_{f 2}}{\tan 20}=23.56-\frac{6.366}{\tan 20}=6.07 \mathrm{~m} / \mathrm{s} \\
\frac{u_{2} V_{u 2}}{g} & =\frac{23.56 \times 6.07}{9.81}=14.58 \mathrm{~m}
\end{aligned}
$$

$\therefore \quad$ Manometric efficiency

$$
\begin{aligned}
& =\frac{12}{14.58}=0.8232=83.32 \% \\
& =\frac{1 \times 10^{3} \times 14.58 \times 9.81}{0.98 \times 10^{3}}=145.9 \mathrm{~kW}
\end{aligned}
$$

Starting speed is given by the expression

$$
\frac{u_{2}^{2}-u_{1}^{2}}{2 g} \geq \frac{u_{2} V_{u_{2}}}{g}
$$

$$
\frac{\pi^{2} N^{2}\left(0.5^{2}-0.3^{2}\right)}{60^{2} \times 2 \times 9.81}=14.58
$$

Solving $\quad N=801.6 \mathbf{r p m}$
The pump will start delivering at 801.6 rpm .
Problem 15.23 If the backward curved bladed impeller of $40^{\circ}$ outlet angle, running an 1440 rpm is operated in the opposite direction, find the ratio of power and exit velocities. The diameter of the impeller is 0.3 m and the flow velocity is 0.20 of blade velocity.

The respective velocity diagrams are shown.
Backward curved

$$
\begin{aligned}
u_{2} & =\frac{\pi \times 0.3 \times 1440}{60}=22.62 \mathrm{~m} / \mathrm{s} \\
V_{f 2} & =22.62 \times 0.2=4.52 \mathrm{~m} / \mathrm{s} \\
V_{u 2} & =22.62-\frac{4.52}{\tan 40}=17.23 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Input head $=\frac{u_{2} V_{u 2}}{g}=\frac{22.62 \times 17.23}{9.81}$

$$
\begin{aligned}
& =\mathbf{3 9 . 7 2} \mathbf{~ m} \\
\mathbf{V}_{2} & =\left[17.23^{2}+4.52^{2}\right]^{0.5}=\mathbf{1 7 . 8 1} \mathbf{~ m} / \mathbf{s}
\end{aligned}
$$

Static pressure rise in the impeller

$$
=39.72-\frac{V_{2}{ }^{2}}{2 g}=39.72-\frac{17.81^{2}}{2 \times 9.81}=23.55 \mathrm{~m}
$$




Forward curved
Figure P. 15.23

Forward curved

$$
\begin{gathered}
u_{2}=22.62 \mathrm{~m} / \mathrm{s}, V_{f 2}=4.52 \mathrm{~m} / \mathrm{s} \\
V_{u 2}=u_{2}+\frac{V_{f 2}}{\tan 40}=28 \mathrm{~m} / \mathrm{s}
\end{gathered}
$$

Input head

$$
=\frac{u_{2} V_{u 2}}{g}=\frac{22.62 \times 28}{9.81}=64.58 \mathrm{~m}
$$

$$
\mathbf{V}_{2}=\left[28^{2}+4.52^{2}\right]^{0.5}=\mathbf{2 8 . 3 6} \mathbf{~ m} / \mathbf{s}
$$

Static pressure rise in the impeller

$$
=64.58-\frac{28.36^{2}}{2 \times 9.81}=64.58-41=\mathbf{2 5 . 5 8} \mathbf{~ m}
$$

Ratio of power :

$$
F B / B B=\frac{64.58}{39.72}=1.63
$$

Ratio of velocities at outlet : $\quad F B / B B=\frac{28.36}{17.81}=\mathbf{1 . 5 9}$
Static pressure rise is found to be nearly equal.

Problem 15.24 A centrifugal pump when tested with Brine of density $1190 \mathrm{~kg} / \mathrm{m}^{3}$ discharged $60 \mathrm{l} / \mathrm{s}$ against a pressure of 300 kPa . It is desired to investigate the change in power when a similar pump is used to pump petrol of density $700 \mathrm{~kg} / \mathrm{m}^{3}$ against the same pressure. It is desired to keep the speed the same. Check whether any change in the drive motor is required.

Assume an overall efficiency of $70 \%$ in both cases.
With Brine :
Head developed $\quad=\frac{300 \times 10^{3}}{1190 \times 9.81}=25.7 \mathrm{~m}$

$$
\text { Power }=\frac{60 \times 1.19 \times 9.81 \times 25.7}{0.7 \times 10^{3}}=25.71 \mathbf{k W}
$$

## With petrol :

$$
\begin{aligned}
\text { Head developed } & =\frac{300 \times 10^{3}}{700 \times 9.81}=43.69 \mathrm{~m} \\
\text { Power } & =\frac{60 \times 0.7 \times 9.81 \times 43.69}{0.7 \times 10^{3}}=\mathbf{2 5 . 7 1} \mathbf{~ k W}
\end{aligned}
$$

## There is no need to change the motor.

As long as $\gamma H$ is the same, other conditions remaining constant, the power will be the same.

Problem 15.25 A centrifugal pump was tested for cavitation initiation. Total head was 40 m and flow rate was $0.06 \mathrm{~m}^{3} / \mathrm{s}$. Cavitation started when the total head at the suction side was 3 m . The atmospheric pressure was $760 \mathrm{~mm} H g$ and the vapour pressure at this temperature was 2 kPa . It was proposed to install the pump where the atmospheric pressure is 700 mm Hg and the vapour pressure at the location temperature is 1 kPa . If the pump develops the same total head and flow, can the pump be fixed as the same height as the lab setup? What should be the new height.

It is necessary to consider the suction point
Total head = Vapour pressure + velocity head .
$\therefore \quad$ Velocity head $=$ Total head - Vapour pressure in head of water

$$
\therefore \quad \frac{V_{s}^{2}}{2 g}=3-\frac{2 \times 10^{3}}{10^{3} \times 9.81}=2.769 \mathrm{~m}
$$

Cavitation parameter $\sigma$ is defined by

$$
\sigma=\frac{V_{s}^{2}}{2 g H}=2.796 / 40=0.0699
$$

Considering the point at the sump level and the suction point

$$
\frac{P_{1}}{\gamma}+\frac{V_{1}^{2}}{2 g}+Z_{1}=\frac{\mathrm{P}_{\mathrm{atm}}}{\gamma}-h_{f}
$$

$$
\begin{aligned}
\therefore \quad\left(Z_{1}+h_{f}\right) & =\frac{\mathrm{P}_{\mathrm{atm}}}{\gamma}-\sigma H-\frac{P_{v}}{\gamma} \\
& =\left(\frac{760 \times 13.6}{1000}\right)-2.796-\frac{2 \times 10^{3}}{10^{3} \times 9.81}=7.336 \mathrm{~m}
\end{aligned}
$$

At the new location (head and flow being the same, friction loss will be the same)

$$
\left(Z^{\prime}+h_{f}^{\prime}\right)=\frac{700 \times 13.6}{1000}-2.796-\frac{1 \times 10^{3}}{10^{3} \times 9.81}=6.622 \mathrm{~m}
$$

As

$$
\begin{aligned}
h_{f} & =h_{f}^{\prime} \\
Z-Z^{\prime} & =0.716 \mathrm{~m}
\end{aligned}
$$

The pump cannot be set at the same height.
It should be lowered by $\mathbf{0 . 7 1 6} \mathbf{~ m}$
The new height is 6.622 m .

## REVIERW QUESTIONS

1. What are the types of casings used in centrifugal pumps?
2. What are the advantages of centrifugal pumps over reciprocating pumps?
3. What are the advantages of double suction pumps?
4. Define manometric head and manometric efficiency of a centrifugal pump.
5. Explain why priming is necessary to start pumping by centrifugal pump.
6. Define shut off head.
7. List the types of impellers and indicate where each of them are used.
8. Explain the functions of a foot value. Indicate how it works.
9. Explain what is meant by slip. What are the effects of slip?
10. What is cavitation? Where does it occur in centrifugal pumps?
11. Define critical cavitation parameter and write down the expression for the same.
12. Explain why backward curved blades are more popularly used.

## OBJECTIVE QUESTIONS

## I. Choose the correct answer

1. Manometric head of a centrifugal pump is given by
(a) Static head + losses
(b) Static head
(c) $u_{2} V_{u 2} / g$
(d) Static head + losses + exit kinetic head.
2. The dimensionless specific speed of a centrifugal pump
(a) $\frac{N \sqrt{P}}{H^{3 / 4}}$
(b) $\frac{N \sqrt{Q}}{H^{5 / 4}}$
(c) $\frac{N \sqrt{Q}}{(g H)^{3 / 4}}$
(d) $\frac{N \sqrt{Q}}{H^{3 / 4}}$.
3. The manometric efficiency of a centrifugal pump is given by
(a) $u_{2} V_{u 2} / g$
(b) $2 H / u_{2} V_{u 2}$
(c) $g H / u_{2} V_{u 2}$
(d) $u_{2} V_{u 2} / g H$.
4. The shut off head of a centrifugal pump is
(a) $\frac{u_{2}^{2}-u_{1}^{2}}{2 g}$
(b) $\frac{u_{2}{ }^{2}-u_{1}{ }^{2}}{g}$
(c) $\frac{u_{2}{ }^{2}}{2 g}$
(d) $\frac{u_{2}-V_{u 2}}{g}$.
5. Slip in the case of a centrifugal pump.
(a) Reduces the flow rate.
(b) Reduces the energy transfer.
(c) Reduces the speed.
(d) Increases cavitation.

## Answers

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\begin{array}{llll}
\text { (1) } d & \text { (2) } c & \text { (3) } c & \text { (4) } a
\end{array} \text { (5) } b \text {. }
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## EXERCISE PROBLEMS

E 15.1 A centrifugal pump with 30 cm impeller delivers $30 \mathrm{l} / \mathrm{s}$ against a head of 24 m when running at 1750 rpm . What will be the delivery of a homologous pump of 15 cm impeller diameter assuming same efficiencies and speed.
(3.75 l/s)

E 15.2 Determine the discharge from centrifugal pump running at 1000 rpm , the head being 14.5 m . The vane angle at outlet is $30^{\circ}$ to the periphery. The impeller diameter is 0.3 m and width is 0.05 m . The manometric efficiency of the pump is $85 \%$.
(Q $1.14 \mathrm{~m}^{3} / \mathrm{s}$ )
E 15.3 A centrifugal pump impeller is 0.5 m in diameter and delivers $2 \mathrm{~m}^{3} / \mathrm{min}$ of water. The peripheral velocity is $10 \mathrm{~m} / \mathrm{s}$ and the flow velocity is $2 \mathrm{~m} / \mathrm{s}$. The blade outlet angle is $35^{\circ}$. Whirl at inlet is zero. Determine the power and torque delivered by the impeller.
( $2.18 \mathrm{~kW}, 54.5 \mathrm{mN}$ )
E 15.4 A centrifugal pump running at 750 rpm delivers $60 \mathrm{l} / \mathrm{s}$ against a head of 20 m . If the pump is speeded up so that it runs at 1200 rpm , Determine the head and discharge. Compare the power. $\quad\left(H=32 \mathrm{~m}, \mathrm{Q}=96 \mathrm{l} / \mathrm{s}, \mathrm{P}_{1}=\mathbf{1 1 . 7 7 2 \mathrm { kW } , \mathrm { P } _ { 2 } = 3 0 . 1 3 \mathrm { kW } )}\right.$
E 15.5 A centrifugal pump with 2.3 diameter impeller running at 327 rpm delivers $7.9 \mathrm{~m}^{3} / \mathrm{s}$ of water. The head developed is 72.8 m . The width of the impeller at outlet is 0.22 m . If the overall efficiency is $91.7 \%$ determine the power to drive the pump. Also determine the blade angle at exit.
( $61.52 \mathrm{~kW}, 13^{\circ}$ )
E15.6 A centrifugal pump running at 1000 rpm works against a head of 80 m delivering $1 \mathrm{~m} / \mathrm{s}$. The impeller diameter and width are 80 cm and 8 cm respectively. Leakage loss is 3 percent of discharge. Hydraulic efficiency is $80 \%$. External mechanical loss is 10 kW . Calculate the blade angle at outlet, the power required and the overall efficiency.
( $\left.15.5^{\circ}, 1020 \mathrm{~kW}, 76.9 \%\right)$
E 15.7 The diameters of a centrifugal pump impeller is 600 mm and that of the eye is 300 mm . The vane angle at inlet is $30^{\circ}$ and that at outlet is $45^{\circ}$. If the absolute velocity of water at inlet is $2.5 \mathrm{~m} / \mathrm{s}$ determine the speed and manometric head. The whirl at inlet is zero.
( $\mathbf{2 7 5 . 8} \mathbf{~ r p m , ~} 5.44 \mathrm{~m}$ )
E 15.8 The speed of a centrifugal pump was 240 rpm and it is required to develop 22.5 m head when discharging $2 \mathrm{~m}^{3} / \mathrm{s}$ of water. The flow velocity at outlet is $2.5 \mathrm{~m} / \mathrm{s}$. The vanes at outlet are set back at $30^{\circ}$ to the tangential direction. Determine the manometric efficiency and the power required to drive the pump. Impeller diameter is 1.5 m .
( $\eta_{\mathrm{m}}=81 \%, 545 \mathrm{~kW}$ )

E 15.9 In a three stage pump the diameter and width at outlet of each impeller is 37.5 cm and 2 cm respectively. The discharge required is $3 \mathrm{~m}^{3} / \mathrm{min}$ at 900 rpm . The vanes are set back at $45^{\circ}$ to the tangent at the outlet. If the manometric efficiency a $84 \%$ determine the total head developed by the pump. ( 68.7 m )
E 15.10 The diameter of a centrifugal pump impeller is 0.8 m . The width at outlet is 0.12 m . It delivers $1.8 \mathrm{~m}^{3} / \mathrm{s}$ through a height of 77 m , when running at 900 rpm . The blades are curved back at an angle of $25^{\circ}$ to the tangent at outlet. Calculate the manometric efficiency and power required to run the pump.
( $0.8047,1690 \mathrm{~kW}$ )
E 15.11 A backward curved bladed impeller of diameter 24 cm rotates at 2400 rpm . The blade angle is $45^{\circ}$. The flow velocity is $20 \%$ of the peripheral velocity and is constant. If the direction is reversed it will act as a forward bladed impeller. Determine the static pressure rise in the impeller in both cases. Also find the ratio of exit velocities and power.
(42.66 in both, 1.48, 1.5)

E 15.12 The impeller of a centrifugal pump of diameters 32 cm and 16 cm rotates at 90 radians/ second. The width at inlet is 5 cm . The vane angles at inlet and outlet are $25.7^{\circ}$ and $14.3^{\circ}$ respectively. Assuming zero whirl at inlet determine the head developed.
( 11.17 m )
E 15.13 A centrifugal pump delivers $50 \mathrm{l} / \mathrm{s}$ when running at 1500 rpm . The inner and outer diameters are 0.15 m and 0.25 m respectively. The blades are curved at $30^{\circ}$ to the tangent at the outlet. The flow velocity is $2.5 \mathrm{~m} / \mathrm{s}$ and is constant. The suction and delivery pipe diameters are 15 cm and 10 cm , respectively. The pressure head at suction is 4 m below atmosphere. The pressure at the delivery is 18 m above atmosphere. The power required was 18 kW . Determine the vane angle at inlet for zero whirl at inlet. Also find the manometric efficiency and overall efficiency.
( $\left.12^{\circ}, \mathbf{7 7 . 2 \%}, 64.6 \%\right)$
E 15.14 The impeller diameter of a pump is 400 mm . The speed is 1450 rpm . The working head is 60 m . Velocity of flow is $3 \mathrm{~m} / \mathrm{s}$. Determine the manometric efficiency if the vane angle at outlet is $30^{\circ}$.
(77\%)
E 15.15 The inlet and outlet diameters of a centrifugal impeller are 0.2 m and 0.4 m respectively. The vane angle at outlet is $45^{\circ}$. The pump speed is 1000 rpm . The flow velocity is constant at 3 $\mathrm{m} / \mathrm{s}$. The entry of the water is at radial direction. Determine the vane angle at inlet, the work done for 1 kg , the absolute velocity at outlet and its direction $\alpha_{2}$.

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\left(\beta_{1}=15.9^{\circ}, 374 \mathrm{Nm} / \mathrm{kg}, 18.1 \mathrm{~m} / \mathrm{s}, 9.5^{\circ}\right)
$$

E 15.16 A centrifugal pump discharges $125 l$ of water per second against a head of 35 m while running at 11 revolutions per second. The inner and outer diameters of the impeller are 0.6 m and 0.3 m . The flow area remains constant at $0.06 \mathrm{~m}^{2}$. The vane angle at outlet is $45^{\circ}$. Water enters the impeller radially. Determine the vane angle at inlet and the manometric efficiency.

E 15.17 A centrifugal pump running at 1450 rpm delivers $0.11 \mathrm{~m}^{3} / \mathrm{s}$ of water against a head of 23 m . The impeller diameter is 250 mm and the width is 50 mm . The manometric efficiency is $75 \%$ determine vane angle at outlet.
E 15.18 The diameters of a centrifugal pump impeller are 750 mm and 400 mm respectively. The vane is backward curved and the outlet angle is $35^{\circ}$. If the speed is 1000 rpm determine the angle at inlet. Also calculate the work done per kg velocity of flow is $6 \mathrm{~m} / \mathrm{s}$.
( $16^{\circ}, \mathbf{1 2 5 0} \mathrm{Nm} / \mathrm{kg} / \mathrm{s}$ )
E 15.19 A multi stage pump is required to deliver of $2 \mathrm{l} / \mathrm{s}$ water against a maximum discharge head of 240 m . The diameter of radial bladed impeller should not be more than 15 cm . Assume a speed of 2800 rpm . Determine the impeller diameter, number of stages and power. Overall efficiency is 0.7 .
( 5 stages, $148 \mathrm{~mm}, 6.7 \mathrm{~kW}$ )

E 15.20 A radial bladed centrifugal pump running at 1440 rpm is to deliver $30 \mathrm{l} / \mathrm{min}$ of water against a head of 20 m . Assuming flow velocity as $3 \mathrm{~m} / \mathrm{s}$. Determine the diameter and width of the impeller at the outlet.
( $\mathbf{1 8 6} \mathbf{~ m m}, \mathbf{2 8 . 5} \mathbf{~ m m}$ )
E 15.21 A centrifugal pump running at 1000 rpm delivers $250 \mathrm{l} / \mathrm{s}$. The flow velocity at outlet is $3 \mathrm{~m} / \mathrm{s}$. The blades are swept back at $30^{\circ}$ to the tangent at outlet. The hydraulic efficiency is $80 \%$. Determine the diameter and width of the impeller at outlet. ( $41.9 \mathbf{~ c m}, 6.33 \mathrm{~cm}$ )
E 15.22 The impeller diameters of a centrifugal pump are 30 cm and 15 cm . The width at outlet is 6 mm . Vanes at outlet are curved back by $45^{\circ}$ to the tangent. Determine the increase in pressure as the water flows through the impeller.
( 28.1 m )
E 15.23 Calculate the least diameter of a centrifugal pump impeller to just start delivering against a head of 40 m when running at 1450 rpm if the inner diameter is 0.4 times the outer diameter. Manometric efficiency is 0.9 .
( 0.4244 m )
E 15.24 A centrifugal pump started cavitating when the total pressure at suction was 3.26 m . The barometric pressure was 750 mm Hg and the vapour pressure at that condition was 1.8 kPa . Determine the value of critical cavitation parameter for the pump.
The total head across the pump was 36.5 m and the discharge $0.048 \mathrm{~m}^{3} / \mathrm{s}$. The pump is to be relocated where the barometric pressure is 622 mm Hg and the vapour pressure was 830 Pa , how much must the hight above the sump be reduced to avoid cavitation. Assume the same total head and discharge.
$\left(\sigma_{\mathrm{c}}=0.084,1.65 \mathrm{~m}\right)$

