16 Reciprocating Pumps

16.0 INTRODUCTION

There are two main types of pumps namely the dynamic and positive displacement pumps. Dynamic pumps consist of centrifugal, axial and mixed flow pumps. In these cases pressure is developed by the dynamic action of the impeller on the fluid. Momentum is imparted to the fluid by dynamic action. This type was discussed in the previous chapter. Positive displacement pumps consist of reciprocating and rotary types. These types of pumps are discussed in this chapter. In these types a certain volume of fluid is taken in an enclosed volume and then it is forced out against pressure to the required application.

16.1 COMPARISON

Dynamic pumps	Positive displacement pumps
1. Simple in construction.	More complex, consists of several moving parts.
2. Can operate at high speed and hence compact.	Speed is limited by the higher inertia of the moving parts and the fluid.
3. Suitable for large volumes of discharge at moderate pressures in a single stage.	Suitable for fairly low volumes of flow at high pressures.
4. Lower maintenance requirements.	Higher maintenance cost.
5. Delivery is smooth and continuous.	Fluctuating flow.

16.2 DESCRIPTION AND WORKING

The main components are:

1. Cylinder with suitable valves at inlet and delivery.
2. Plunger or piston with piston rings.
3. Connecting rod and crank mechanism.

4. Suction pipe with one way valve.
5. Delivery pipe.
6. Supporting frame.
7. Air vessels to reduce flow fluctuation and reduction of acceleration head and friction head.

A diagramatic sketch is shown in Fig. 16.2.1.

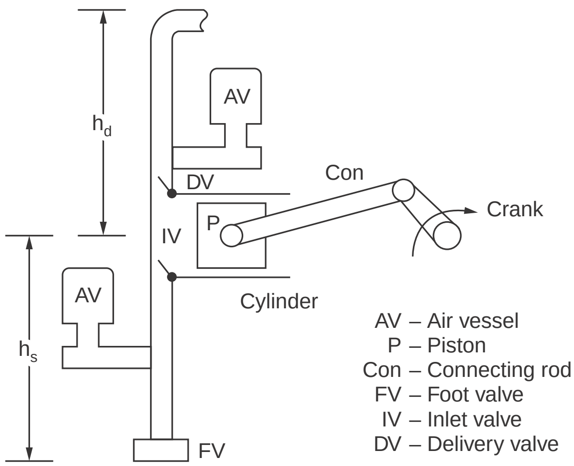

Figure 16.2.1 Diagramatic view of single acting reciprocating pump

The action is similar to that of reciprocating engines. As the crank moves outwards, the piston moves out creating suction in the cylinder. Due to the suction water/fluid is drawn into the cylinder through the inlet valve. The delivery valve will be closed during this outward stroke. During the return stroke as the fluid is incompressible pressure will developed immediately which opens the delivery valve and closes the inlet valve. During the return stroke fluid will be pushed out of the cylinder against the delivery side pressure. The functions of the air vessels will be discussed in a later section. The volume delivered per stroke will be the product of the piston area and the stroke length. In a single acting type of pump there will be only one delivery stroke per revolution. Suction takes place during half revolution and delivery takes place during the other half. As the piston speed is not uniform (crank speed is uniform) the discharge will vary with the position of the crank. The discharge variation is shown in figure 16.2.2.

In a single acting pump the flow will be fluctuating because of this operation.

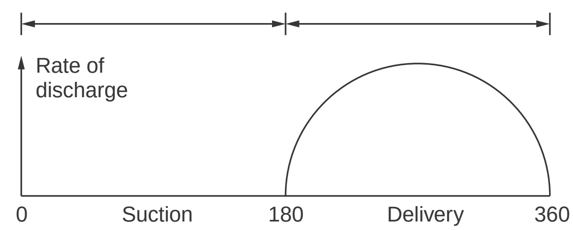

Figure 16.2.2 Flow variation during crank movement of single acting pump

Fluctuation can be reduced to some extent by double acting pump or multicylinder pump.

The diagramatic sketch of a double acting pump is shown in figure 16.2.3.

In this case the piston cannot be connected directly with the connecting rod. A gland and packing and piston rod and cross-head and guide are additional components. There will be nearly double the discharge per revolution as compared to single acting pump. When one side of the piston is under suction the other side will be delivering the fluid under pressure. As can be noted, the construction is more complex.

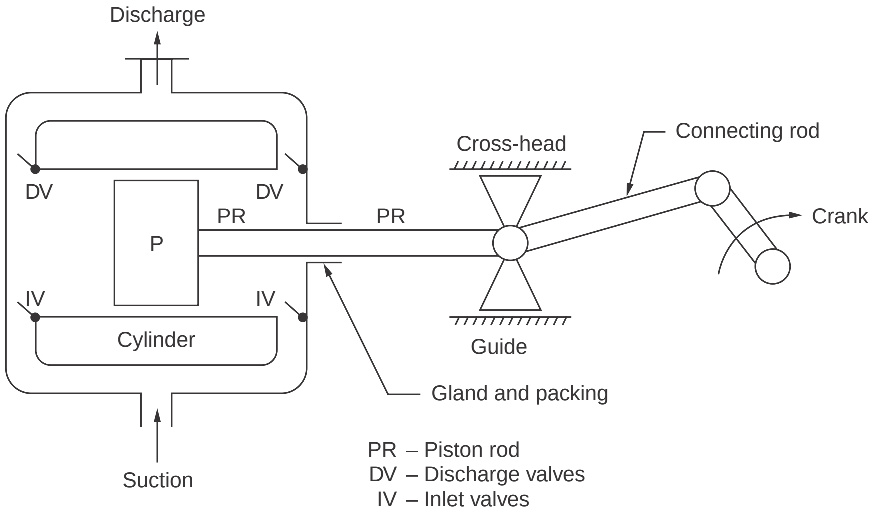

Figure 16.2.3 Diagramatic view of a double action pump

16.3 FLOW RATE AND POWER

Theoretical flow rate per second for single acting pump is given by, $Q_{SA} = \dfrac{L\,A\,N}{60}$ m³/s

(16.3.1)

Where L is the length of stroke, A is the cylinder or piston area and N is the revolution per minute. It is desirable to express the same in terms of crank radius and the angular velocity as simple harmonic motion is assumed.

$$\omega = \frac{2\pi N}{60},\ N = \frac{60\,\omega}{2\pi},\ r = \frac{L}{2}$$

$$Q_{SA} = \frac{2r\,.\,A \times 60\,\omega}{2\pi \times 60} = \frac{A\omega r}{\pi}\ \text{m}^3/\text{s} \tag{16.3.1a}$$

In double acting pumps, the flow will be nearly twice this value. If the piston rod area is taken into account, then

$$Q_{DA} = \frac{A\,L\,N}{60} + (A - A_{pr})\,\frac{L\,N}{60}\ \text{m}^3/\text{s} \tag{16.3.2}$$

Compared to the piston area, the piston rod area is very small and neglecting this will lead to an error less than 1%.

$$\therefore \qquad Q_{DA} = \frac{2ALN}{60} = \frac{2Awr}{\pi} \text{ m}^3/\text{s}. \qquad (16.3.2a)$$

For multicylinder pumps, these expressions, (16.3.1), (16.3.1*a*), (16.3.2), and (16.3.2*a*) are to be multiplied by the number of cylinders.

16.3.1 Slip

There can be leakage along the valves, piston rings, gland and packing which will reduce the discharge to some extent. This is accounted for by the term slip.

$$\text{Percentage of slip} = \frac{Q_{th} - Q_{ac}}{Q_{th}} \times 100 \qquad (16.3.3)$$

Where Q_{th} is the theoretical discharge given by equation (16.3.1) and 2 and Q_{ac} is the measured discharge.

$$\text{Coefficient of discharge, } C_d = \frac{Q_{ac}}{Q_{th}} \qquad (16.3.4)$$

It has been found in some cases that $\mathbf{Q_{ac} > Q_{th}}$, due to operating conditions. In this case the slip is called **negative slip**. When the delivery pipe is short or the delivery head is small and the accelerating head in the suction side is high, the delivery valve is found to open before the end of suction stroke and the water passes directly into the delivery pipe. Such a situation leads to negative slip.

Theoretical power $= mg(h_s + h_d)$ W

where m is given by $Q \times \delta$. (16.3.5)

Example 16.1 *A single acting reciprocating pump has a bore of 200 mm and a stroke of 350 mm and runs at 45 rpm. The suction head is 8 m and the delivery head is 20 m. Determine the theoretical discharge of water and power required. If slip is 10%, what is the actual flow rate ?*

Theoretical flow volume $Q = \frac{LAN}{60} = \frac{0.35 \times \pi \times 0.2^2}{4} \times \frac{45}{60}$

$= 8.247 \times 10^{-3}$ m³/s or 8.247 l/s or 8.247 kg/s

Theoretical power = (mass flow/s) × head in $m \times g$ Nm/s or W

$= 0.9 \times 8.247 \times (20 + 8) \times 9.81$

= 2039 W or 2.039 kW

$$\text{Slip} = \frac{Q_{th} - Q_{ac}}{Q_{th}},\ 0.1 = \frac{8.247 - Q_{ac}}{8.247}$$

$\therefore$ $\mathbf{Q_{actual} = 7.422}$ **l/s**

The actual power will be higher than this value due to both solid and fluid friction.

Example 16.2 *A double acting reciprocating pump has a bore of 150 mm and stroke of 250 mm and runs at 35 rpm. The piston rod diameter is 20 mm. The suction head is 6.5 m and the delivery head is 14.5 m. The discharge of water was 4.7 l/s. Determine the slip and the power required.*

$$\mathbf{Q} = \frac{L\,A_1\,N}{60} + \frac{L\,A_2\,N}{60} = \frac{L\,N}{60}[A_1 + A_2]$$

$$= \frac{0.25 \times 35}{60}\left[\frac{\pi \times 0.15^2}{4} + \frac{\pi}{4}(0.15^2 - 0.02^2)\right]$$

$$= \frac{0.25 \times 35 \times \pi}{60 \times 4}[2 \times 0.15^2 - 0.02^2]$$

$$= \mathbf{5.108 \times 10^{-3}\ m^3/s\ or\ 5.108\ l/s\ or\ 5.108\ kg/s}$$

It piston rod area is not taken into account

$$Q = 5.154\ \text{l/s.}$$

An error of 0.9% rather negligible.

$$\mathbf{Slip} = \frac{5.108 - 4.7}{5.108} \times 100 = \mathbf{7.99\%}$$

Theoretical power $= m\,g\,h = 4.7 \times 9.81 \times (14.5 + 6.5)\ \text{W} = \mathbf{968\ W}$

The actual power will be higher than this value due to mechanical and fluid friction.

16.4 INDICATOR DIAGRAM

The pressure variation in the cylinder during a cycle consisting of one revolution of the crank. When represented in a diagram is termed as indicator diagram. The same is shown in figure 16.4.1.

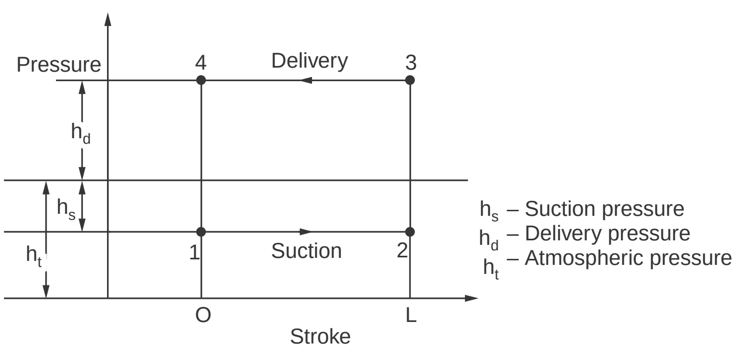

Figure 16.4.1 Indicator diagram for a crank revolution

Figure represents an ideal diagram, assuming no other effects are involved except the suction and delivery pressures. Modifications due to other effects will be discussed later in the section.

Point 1 represents the condition as the piston has just started moving during the suction stroke. 1-2 represents the suction stroke and the pressure in the cylinder is the suction pressure below the atmospheric pressure. The point 3 represents the condition just as the piston has started moving when the pressure increases to the delivery pressure. Along 3-4 representing the delivery stroke the pressure remains constant. The area enclosed represents the work done during a crank revolution to some scale

$$\text{Power} = Q\,\rho\, g(h_s + h_d) = \rho\, g\, L\, A\, N\, (h_s + h_d)/60 \qquad (16.4.1)$$

16.4.1 Acceleration Head

The piston in the reciprocating pump has to move from rest when it starts the suction stroke. Hence it has to accelerate. The water in the suction pipe which is also not flowing at this point has to be accelerated. Such acceleration results in a force which when divided by area results as pressure. When the piston passes the mid point, the velocity gets reduced and so there is retardation of the piston together with the water in the cylinder and the pipe. This again results in a pressure. These pressures are called acceleration pressure and is denoted as head of fluid ($h = P/\rho g$) for convenience. Referring to the figure 16.4.2 shown below the following equations are written.

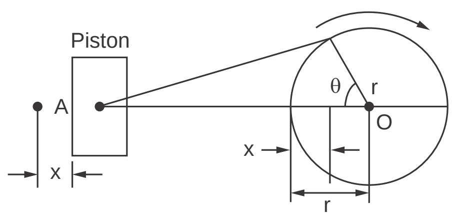

Figure 16.4.2 Piston Crank Configuration

Let ω be the angular velocity.

Then at time t, the angle travelled $\theta = \omega t$

Distance $\quad x = r - r\cos\theta = r - r\cos\omega t$

Velocity at this point,

$$v = \frac{dx}{dt} = \omega r \sin wt \qquad (16.4.2)$$

The acceleration at this condition

$$\ddot{x} = \frac{dv}{dt} = \omega^2 r \cos wt \qquad (16.4.3)$$

This is the acceleration in the cylinder of area A. The acceleration in the pipe of area a is

$$= \frac{A}{a}\omega^2\, r\cos\omega t. \qquad (16.4.4)$$

Accelerating force = mass × acceleration

$$\text{mass in the pipe} = \rho\, al \text{ kg} = \frac{\gamma al}{g}$$

$$\therefore \quad \text{Acceleration force} = \frac{\gamma al}{g} \times \frac{A}{a} \omega^2 r \cos \omega t \qquad (16.4.5)$$

$$\text{Pressure} = \text{force/area}$$

$$= \frac{ral}{g} \cdot \frac{1}{a} \cdot \frac{A}{a} \omega^2 r \cos \omega t$$

$$= \frac{rl}{g} \cdot \frac{A}{a} \omega^2 r \cos \theta$$

$$\text{Head} = \text{Pressure}/\gamma$$

$$h_a = \frac{l}{g} \cdot \frac{A}{a} \omega^2 r \cos \theta \qquad (16.4.6)$$

This head is imposed on the piston in addition to the static head at that condition. This results in the modification of the indicator diagram as shown in figure 16.4.3.

(*i*) Beginning of suction stroke: θ = 0, cos θ = 1

$$\therefore \qquad h_{as} = \frac{l_s}{g} \cdot \frac{A}{a_s} \cdot \omega^2 r$$

This is over and above the static suction head. Hence the pressure is indicated by 1′ in the diagram.

(*ii*) Middle of stroke: θ = 90 ∴ $h_{as} = 0$. There is no additional acceleration head.

(*iii*) End of stroke: θ = 180. cos θ = – 1

$$\therefore \qquad h_{as} = -\frac{l_s}{g} \cdot \frac{A}{a_s} \cdot \omega^2 r$$

This reduces the suction head. Hence the pressure is indicated at 2′ in the diagram.

Similarly during the beginning of the delivery stroke

$$\theta = 0, \cos \theta = 1$$

$$h_{ad} = \frac{l_d}{g} \cdot \frac{A}{a_d} \cdot \omega^2 r$$

This head is over and above the static delivery pressure. The pressure is indicated by point 3′ in the diagram. At the middle stroke $h_{ad} = 0$. At the end of the stroke $h_{ad} = -\frac{l_a}{g} \cdot \frac{A}{a_d} \cdot \omega^2 r$.

This reduces the pressure at this condition and the same is indicated by 4′, in the diagram.

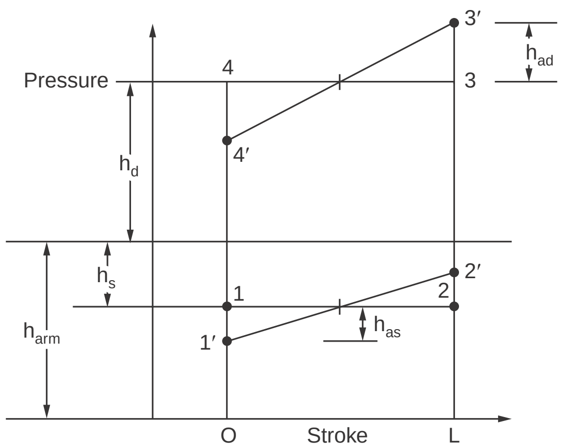

Figure 16.4.3 Modified indicator diagram due to acceleration head

The effect of acceleration head are:

1. No change in the work done.

2. Suction head is reduced. This leads to the problem of separation in suction pipe in case the pressure at 1′ is around 2.5 m of head of water (absolute). As the value depends on ω which is directly related to speed, the speed of operation of reciprocating pumps is limited. Later it will be shown than the installation of an air vessel alleviates this problem to some extent.

Example 16.3 *Calculate and tabulate the distance travelled as a fraction of stroke length as the crank rotates from 0 to 180°.*

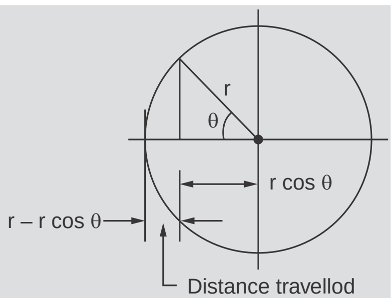

Figure Ex. 16.3 (a)

Distance travelled = $r - r\cos\theta$

Stroke = $2r$.

∴ Distance travelled as a fraction of, L

$$= \frac{r - r\cos\theta}{2r} = \frac{1-\cos\theta}{2}$$

Values are calculated for 0, 30, 60, 90, 120, 150, 180° and tabulated below.

Angle	0	30	60	90	120	150	180
Distance moved	0	.0677	0.250	0.5	0.75	0.933	1

Note: The distances of piston movement is not uniform with crank angle.

For the data, speed = 40 rpm and r = 0.15 m. calculate the velocity and acceleration as the crank moves from one dead centre to the next.

$$\text{Velocity} = \omega r \sin\theta = \frac{2\pi N}{60} \,.\, r \sin\theta$$

$$\text{Acceleration} = \omega^2 r \cos\theta = \left(\frac{2\pi N}{60}\right)^2 \,.\, r \,.\, \cos\ \theta$$

The values are calculated using the specified data and tabulated below.

Angle	0	30	60	90	120	150	180
Velocity m/s	0	0.314	0.544	0.628	0.544	0.314	0
Acceleration m/s^2	2.632	2.279	1.316	0	– 1.316	– 2.279	– 2.632

Note: The velocity follows sine curve and acceleration the cosine curve.

The acceleration is highest at start of stroke and decreases up to the middle of stroke and becomes zero and then decelerates at an increasing rate.

This can be illustrated as below.

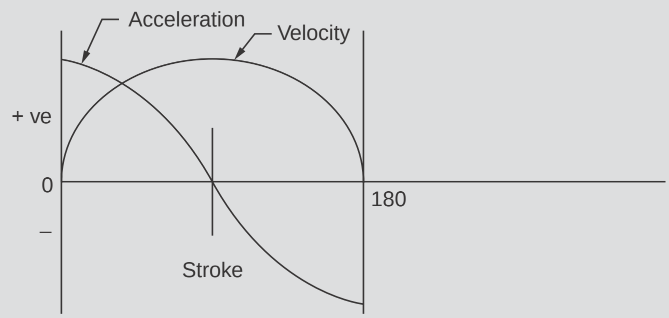

Figure Ex. 16.3 (b) Velocity and acceleration plot during a stroke

Example 16.4 *A single acting reciprocating pump of 200 mm bore and 300 mm stroke runs at 30 rpm. The suction head is 4 m and the delivery head is 15 m. Considering acceleration determine the pressure in the cylinder at the beginning and end of suction and delivery strokes. Take the value of atmospheric pressure as 10.3 m of water head. The length of suction pipe is 8 m and that of delivery pipe is 20 m. The pipe diameters are 120 mm each.*

Acceleration head on the suction side, $h_{as} = \dfrac{l_s}{g}\dfrac{A}{a_s} \cdot \omega^2 r$

A – piston diameter, a_s – pipe diameter, $r = L/2$

$$\frac{A}{a_s} = \frac{\pi \times 0.2^2}{4} \times \frac{4}{\pi \times 0.12^2} = 2.78,\ \omega = \frac{2\pi N}{60}$$

$$\therefore \qquad \mathbf{h_{as}} = \frac{8}{9.81} \times 2.78 \times \left(\frac{2 \times \pi \times 30}{30}\right)^2 \times 0.15 = \mathbf{3.35\ m}$$

At start of suction, $H_{BS} = h_{atm} - h_s - h_{as}$ = 10.3 – 4 – 3.35 = 2.95 m absolute or 7.05 m vacuum.

At end of suction H_{es} = 10.3 – 4 + 3.35 = **9.65 m absolute or** 0.65 m vacuum.

$$h_{ad} = \frac{20}{9.81} \times 2.78 \times \left(\frac{2\pi \times 30}{60}\right)^2 \times 0.15 = \mathbf{8.38\ m\ of\ water\ column}$$

At starting of delivery, $\mathbf{H_{Bd}}$ = 10.3 + 15 +8.38 = **33.68 m absolute or 23.38 m gauge**

At end of delivery, $\mathbf{H_{ed}}$ = 10.3 + 15 – 8.38 = **16.92 m absolute or 6.62 m gauge**

This applies for both single acting and double acting pumps.

16.4.2 Minimum Speed of Rotation of Crank

During the suction stroke, the head at the suction side is given by

$$h = h_{atm} - h_s - h_{as}$$

In case this head is below 2.5 m of head of water, water may vaporise at this point and the flow will be disrupted causing separation in the liquid column. Pumping will be discontinuous.

In order to avoid this, the acceleration head which can be changed should be limited. As this depends on the speed there is a limitation to the operating speed.

During delivery stroke also, there is a possibility of separation which may be caused by the layout of the delivery pipe. Two alternatives are shown in figure 16.4.4. The first method is to have a horizontal bend at the pump level and then to have the vertical line. In this case separation is avoided as at the bend the column of water above it exerts a pressure above 2.5 m (absolute). In the second arrangement the pressure at the bend is given by $(h_{atm} - h_{ad})$ and this may be below 2.5 m of water hence the preferred arrangement is to have a horizontal bend immediately after the pump.

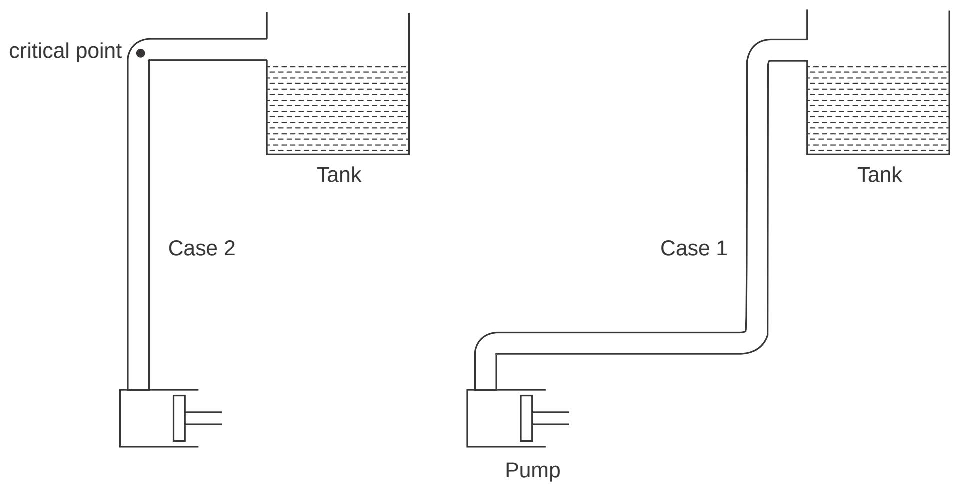

Figure 16.4.4 Delivery pipe arrangement

Example 16.5 *A single acting reciprocating pump of 200 mm plunger diameter and 300 mm stroke length has a suction head of 4 m. The suction pipe diameter is 110 mm and is 9 m long. The pressure at the beginning of suction should be above 2.5 m water column (absolute) to avoid separation. Determine the highest speed at which the pump can operate.*

At the beginning the pressure required is 2.5 m. This equals the difference between the absolute pressure and the sum of suction and accelerating head.

$$\therefore \quad 2.5 = 10.3 - 4 - h_{as} \quad \therefore \quad h_{as} = 3.8 \text{ m}$$

$$h_{as} = \frac{l}{g} \cdot \frac{A}{a} \cdot \omega^2 r \quad (\cos\theta = 1)$$

$$3.8 = \frac{9}{9.81} \times \frac{\pi \times 0.2^2}{4} \times \frac{4}{\pi \times .011^2} . \omega^2 \times 0.15$$

Solving $\omega^2 = 8.353$, $\omega = 2.89$ radians/second.

$$\omega = \frac{2\pi N}{60}, \quad \mathbf{N} = \frac{\omega \times 60}{2\pi} = \frac{2.89 \times 60}{2 \times \pi} = \mathbf{27.6\ rpm}$$

This will be the same for double acting pump also.

Example 16.6 *The delivery pipe of a reciprocating pump is taken vertically up and then given a horizontal bend. The pump diameter is 180 mm and the stroke is 300 mm. The pipe diameter is 100 mm and the length is 18 m. The speed is 30 rpm. Check whether separation will occur at the bend. Separation is expected to take place if the absolute pressure is 2.5 m head. Atmospheric pressure may be taken as 10.3 m head of water.*

At the top of the pipe, the static head is zero. The only pressure is the accelerating head. To avoid separation $(P_{atm} - P_a) > 2.5$ m.

$$\mathbf{P_a} = \frac{l}{g} . \frac{A}{a} . \omega^2 r = \frac{18}{9.81} \times \left(\frac{\pi \times 0.18^2}{4} \times \frac{4}{\pi \times 0.1^2}\right) \times \left(\frac{2\pi \times 30}{60}\right)^2 \times 0.15$$

$$= \mathbf{8.8\ m}$$

Pressure P at the bend = 10.3 – 8.8 = 1.5 m

Hence separation will occur at the bend.

In the above problem if the pipe is taken first horizontally and then vertically, what will be the pressure at the bend. The delivery pressure is 15 m. The pressure at the bend will be the sum of the atmospheric pressure and the static pressure minus the acceleration head. The acceleration head itself is 8.8 m head.

$$P = 10.3 + 15 - 8.8 = 16.5 \text{ m head.}$$

Hence the arrangement is safe against separation.

16.4.3 Friction Head

When air vessels are not fixed in a pump, the velocity variation of water in the cylinder is given by equation (16.4.2)

$$v = \omega r \sin \omega t = \omega r \sin \theta$$

In the pipe the velocity variation will be in the ratio of areas.

$$\therefore \qquad v_p = \frac{A}{a} . \omega r \sin\theta$$

Friction head $\qquad h_f = f\, l\, v^2 / 2gd$

$$= \frac{fl}{2gd} . \left(\frac{A}{a} . \omega r \sin\theta\right)^2 \tag{16.4.7}$$

The maximum value of friction heed

$$h_{f\,max} = \frac{fl}{2gd} . \left(\frac{A}{a} . \omega r\right)^2 \tag{16.4.8}$$

At the beginning of the stroke $\theta = 0°$ $\therefore$ $h_f = 0$

At middle of stroke, $\theta = 90°$ and $h_f = h_{f\,max}$

At end of stroke, $\theta = 180°$ and $h_f = 0$.

The friction head leads to another modification of the indicator diagram shown in figure 16.4.5. With h_s and h_d as static suction and delivery heads, the pressure at the various locations are indicated below.

Suction stroke: At the start, head = $h_s + h_{as}$

At middle position, head = $h_s + h_{fs}$

At the end, $h = h_s - h_{as}$

Delivery stroke: At the start, head = $h_d + h_{ad}$

At middle position, head = $h_d + h_f$

At the end, head = $h_d - h_{ad.}$

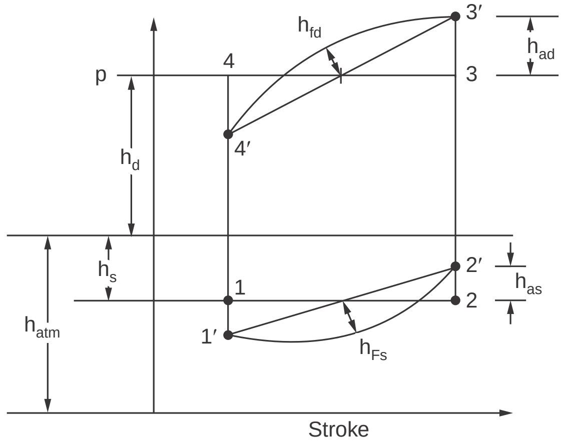

Figure 16.4.5 Variation of indicator diagram taking pipe friction into account

It can be observed that friction head increases the work done as seen by the increased area enclosed in the indicator diagram. The introduction of air vessels will reduce the friction work considerably. In order to calculate the work done, it will be desirable to calculate the average friction head. As the variation is parabolic, the average is found to be 2/3 of the maximum.

$$h_{fav} = 2/3\ h_{f\,max} \tag{16.4.9}$$

The total head against which work is done equals

$$h_{to} = h_s + h_d + 2/3\ h_{f\,max\,s} + 2/3\ h_{f\,max\,d} \tag{16.4.10}$$

The addition work due to friction is given by

$$2/3\ Q\,\rho\,g\,(h_{fs} + h_{fd}) \tag{16.4.11}$$

Later it will be seen that the use of air vessels causes the velocity in the pipe to be constant without fluctuations and this reduces the work to be overcome by friction.

Example 16.7 *A single acting pump with 200 mm bore diameter and 320 mm stroke runs at 30 rpm. The suction pipe diameter is 110 mm. The delivery pipe diameter is 100 mm. The suction and delivery pipes are 10 m and 22 m long. The friction factor is 0.01. Determine the frictional head at the suction and delivery.*

Assume no air vessels are fitted.

$$h_f = \frac{flV^2}{2gd} = \frac{4fl}{2gd}\left(\frac{A}{a}\,.\,\omega r \sin\theta\right)^2$$

Maximum occurs at the middle of stroke.

$$\mathbf{h_{fs\ max}} = \frac{4 \times 0.01 \times 10}{2 \times 9.81 \times 0.11}\left[\frac{200^2}{110^2}\,.\,\frac{2\pi \times 30}{60} \times 0.15\,.\,\sin 90\right]^2$$

$$= \mathbf{0.45\ m\ \ head}$$

$$\mathbf{h_{fd\ max}} = \frac{4 \times 0.01 \times 22}{2 \times 9.81 \times 0.1}\left[\frac{200^2}{100^2} \times \frac{2\pi \times 30}{60} \times 0.15\,.\,\sin 90\right]^2$$

$$= \mathbf{1.59\ m\ head.}$$

Note: Two types of equations for frictional head are used:

$$\frac{4flv^2}{2gd} \text{ and } \frac{flv^2}{2gd}$$

When $4f$ is used, f is the coefficient of friction commonly denoted as c . When f alone is used it is called Darcy friction factor and

Darcy friction factor = 4 c_f

Now the popular use is the second equation using f from Moody diagram.

16.5 AIR VESSELS

Air vessel is a strong closed vessel as shown in figure 16.5.1. The top half contains compressed air and the lower portion contains water or the fluid being pumped. Air and water are separated by a flexible diaphragm which can move up or down depending on the difference in pressure between the fluids. The air charged at near total delivery pressure/suction pressure from the top and sealed. The air vessel is connected to the pipe lines very near the pump, at nearly the pump level. On the delivery side, when at the beginning and up to the middle of the delivery stroke the head equals $h_s + h_f + h_a$, higher than the static and friction heads. At this time part of the water from pump will flow into the air vessel and the remaining will flow through the delivery pipe. This will increase the compressed air pressure. At the middle stroke position the head will be sufficient to just cause flow. The whole of the flow from pump will flow to the delivery pipe. At the second half of the stroke the head will be equal to $h_s + h_f - h_a$. At the position the head will be not sufficient to cause flow. The compressed air pressure will act on the water and water charged earlier into the air vessel will now flow out. Similar situation prevails on the suction side. At the start and up to the middle of the suction stroke the head at the pump is higher than static suction head by the amount of acceleration head. The flow will be more and part will flow into the air vessel. The second half of the stroke water will flow out of the air vessel. In this process the velocity of water in the delivery pipe beyond the air vessel is uniform, and lower than the maximum velocity if air vessel is not fitted. Similar situation prevails in the suction side also. The effect is not only to give uniform flow but reduce the friction head to a considerable extent saving work. Without air vessel the friction head increases, reaches a maximum value at the mid stroke and then decreases to zero. With air vessel the friction head is lower and is constant throughout the stroke. This is due to the constant velocity in the pipe.

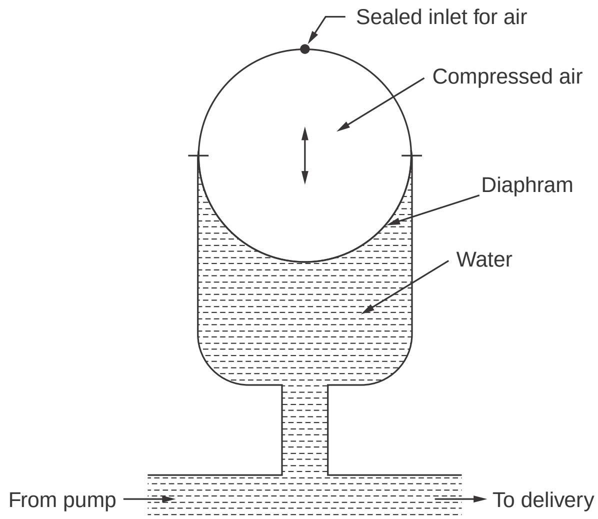

Figure 16.5.1 Air vessel

The advantages of installing air vessels are:

(*i*) The flow fluctuation is reduced and a uniform flow is obtained.

(*ii*) The friction work is reduced.

(*iii*) The acceleration head is reduced considerably.

(*iv*) Enables the use of higher speeds.

The maximum friction head of water without air vessel, refer eqn. (16.4.8),

$$h_{f\,max} = \frac{4fl}{2gd}\left(\frac{A}{a}\,.\,\omega r\right)^2$$

The average friction head = 2/3 $h_{f\,max}$. (refer eqn. 16.4.9). When the air vessel is placed near the pump, the uniform velocity,

$$v = \frac{A}{a}\cdot\frac{LN}{60} = \frac{A}{a}\,.\,2r\,.\,\frac{60\omega}{2\pi} = \frac{A}{a}\cdot\frac{\omega r}{\pi} \qquad (16.5.1)$$

$$h_f = \frac{4fl}{2gd}\cdot\left(\frac{A}{a}\cdot\frac{\omega r}{\pi}\right)^2 \qquad (16.5.2)$$

$$\frac{h_f}{h_f a_v} = \frac{3}{2} \cdot \frac{1}{\pi^2} = 0.152 \tag{16.5.3}$$

∴ Reduction is 84.8%

Naturally the work done due to friction will reduce by this percentage.

Example 16.8 *In a single acting reciprocating pump with plunger diameter of 120 mm and stroke of 180 mm running at 60 rpm, an air vessel is fixed at the same level as the pump at a distance of 3 m. The diameter of the delivery pipe is 90 mm and the length is 25 m. Friction factor is 0.02. Determine the reduction in accelerating head and the friction head due to the fitting of air vessel.*

Without air vessel :

$$\mathbf{h_{ad}} = \frac{l}{g} \cdot \frac{A}{a} \cdot \omega^2 r = \frac{25}{9.81} \cdot \frac{0.12^2}{0.09^2} \cdot \left(\frac{2\pi \times 60}{60}\right)^2 \times 0.09$$

$$= \mathbf{16.097\ m}$$

With air vessel :

$$\mathbf{h'_{ad}} = \frac{3}{9.81} \cdot \frac{0.12^2}{0.09^2} \cdot \left(\frac{2\pi \times 60}{60}\right)^2 \times 0.09 = \mathbf{1.932\ m}$$

Reduction = 16.097 – 1.932 = **14.165 m**

Fitting air vessel reduces the acceleration head.

Without air vessel :

$$\text{Friction head} \quad h_f = \frac{4\,f\,l.V^2}{2gd} = \frac{4fl}{2gd}\left[\frac{A}{a} \cdot \omega r \sin\theta\right]^2$$

At θ = 90°

$$\mathbf{h_{f\,max}} = \frac{4 \times 0.02 \times 25}{2 \times 9.81 \times 0.09}\left[\frac{0.12^2}{0.09^2} \cdot \frac{2\pi \times 60}{60} \times 0.09 \times 1\right]^2 = \mathbf{1.145\ m}$$

With air vessel, the velocity is constant in the pipe.

$$\text{Velocity} = \frac{A\,L\,N}{60} \times \frac{4}{\pi\,.\,d^2} = \frac{\pi \times 0.12^2}{4} \times \frac{0.18 \times 60 \times 4}{60 \times \pi \times 0.09^2}$$

$$= 0.102 \text{ m/s}$$

$$\text{Friction head} = \frac{4 \times 0.02 \times 25}{2 \times 9.81 \times 0.09} \times 0.102^2 = \mathbf{0.012\ m}$$

Percentage saving over maximum

$$= \frac{1.145 - 0.012}{1.145} \times 100 = 99\%$$

Air vessel reduces the frictional loss.

16.5.1 Flow into and out of Air vessel

Single acting pump: The flow into the delivery side is only during half a revolution. This amount has to flow during the full revolution:

The average velocity in the pipe = $\left(\dfrac{L\,N}{60} \times \dfrac{A}{a}\right)$.

Double acting pump: There are two discharger per revolution. The average velocity in this case = $\dfrac{2L\,N}{60} \times \dfrac{A}{a}$.

Hence the frictional head will be different in single acting and double acting pumps. This is illustrated in figure 16.5.2.

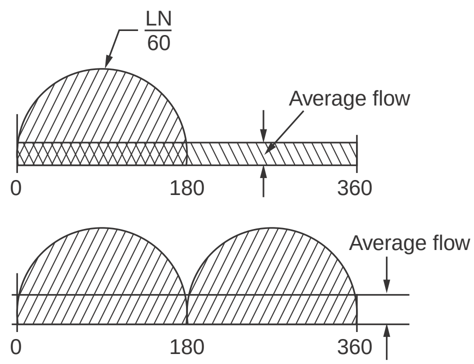

Figure 16.5.2 Pipe flow in single acting and double acting pumps

Single acting pump :

The flow from cylinder, $Q = A\,\omega r \sin\theta$

With air vessel the average velocity (refer 16.5.1) = $\dfrac{A}{a} \cdot \dfrac{\omega r}{\pi}$

Flow through pipe $= a \cdot \dfrac{A}{a} \cdot \dfrac{\omega r}{\pi} = \dfrac{A\,\omega r}{\pi}$

Flow into or out of the air vessel with θ = 0 to 360 = $A\,\omega r\,\sin\theta - \dfrac{A\,\omega r}{\pi}$

$$= A\,\omega r\left(\sin\theta - \frac{1}{\pi}\right) \qquad (16.5.4)$$

Double acting pump: Flow through the pump = $\dfrac{2A\,\omega r}{\pi}$

At any point of time flow from cylinder = $A\,\omega r \sin\theta$

Flow into air vessel = $A\,\omega r\left(\sin\theta - \dfrac{2}{\pi}\right)$ (16.5.5)

Example 16.9 *Determine the rate of flow in and out of the air vessel on the delivery side in a single acting centrifugal pump of 200 mm bore and 300 mm stroke running at 60 rpm. Also find the angle of crank rotation at which there is no flow into or out of the air vessel.*

At any instant of time the flow rate from the pump cylinder is

$$Q = A\,\omega r \sin\theta$$

Beyond the air vessel the velocity in the pipe is constant

$$v = \frac{A}{a} \cdot \frac{\omega r}{\pi}$$

The flow rate $= a \times \frac{A}{a} \cdot \frac{\omega r}{\pi} = \frac{A\,\omega r}{\pi}$

Volume flow rate into the air vessel,

$$q = \text{Volume flow rate from cylinder} - \text{Volume flow rate beyond the air vessel}$$

$$q = A\,\omega r \sin\theta - \frac{A\,\omega r}{\pi} = A\,\omega r\left[\sin\theta - \frac{1}{\pi}\right]$$

In this case

$$\mathbf{q} = \frac{\pi \times 0.2^2}{4} \times \frac{2\pi \times 60}{60} \times \frac{0.3}{2}\left[\sin\theta - \frac{1}{\pi}\right]$$

$$= \mathbf{0.0296}\left[\mathbf{\sin\theta - \frac{1}{\pi}}\right].$$

Delivery											Suction		
θ	0	30	60	90	120	150	180	210	240	270	300	330	360
q	– 0.318	0.182	0.548	.0682	0.548	0.182	– 0.318	– 0.818	– 1.184	– 1.318	– 1.118	– 0.818	– 0.318

At no flow condition, the quantity within the bracket showed be zero.

or $$\boldsymbol{\theta} = \sin^{-1}\left(\frac{1}{\pi}\right) = \mathbf{18.56^\circ}$$

Also $\theta = 161.44°$.

At two locations there is no flow into or out of the air vessel. Similar situation prevails on the suction side also.

16.6 ROTARY POSITIVE DISPLACEMENT PUMPS

In order to avoid the complexity of construction and restriction on speed of the reciprocating pumps, rotary positive displacement pumps have been developed. These can run at higher speeds and produce moderately high pressures. These are very compact and can be made for very low delivery volumes also. These are extensively used for pumping lubricant to the engine parts and oil hydraulic control systems. These are not suited for water pumping. Some types described in this section are: (*i*) Gear pump, (*ii*) Lobe pump and Vane pump.

16.6.1 Gear Pump

These are used more often for oil pumping. Gear pumps consist of two identical mating gears in a casing as shown in figure 16.6.1. The gears rotate as indicated in the sketch. Oil is trapped in the space between the gear teeth and the casing. The oil is then carried from the lower pressure or atmospheric pressure and is delivered at the pressure side. The two sides are sealed by the meshing teeth in the middle. The maximum pressure that can be developed depends on the clearance and viscosity of the oil. The operation is fairly simple. One of the gear is the driving gear directly coupled to an electric motor or other type of drives.

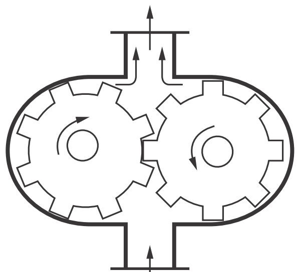

Figure 16.6.1 Gear pump

These pumps should be filled with oil before starting. The sketch shows an external gear pump. There is also another type of gear pump called internal gear pump. This is a little more compact but the construction is more complex and involved and hence used in special cases where space is a premium.

16.6.2 Lobe Pump

This type is also popularly used with oil. The diagramatic sketch of a lobe pump is shown in figure 16.6.2. This is a three lobed pump. Two lobe pump is also possible. The gear teeth are replaced by lobes. Two lobes are arranged in a casing. As the rotor rotates, oil is trapped in the space between the lobe and the casing and is carried to the pressure side. Helical lobes along the axis are used for smooth operation. Oil has to be filled before starting the pump. Lobe type of compressors are also in use. The constant contact between the lobes makes a leak tight joint preventing oil leakage from the pressure side.

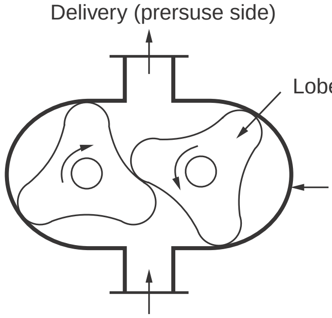

Figure 16.6.2 Lobe pump

The maximum pressure of operation is controlled by the back leakage through the clearance. This type of pump has a higher capacity compared to the gear pump.

16.6.3 Vane Pump

This is another popular type not only for oil but also for gases. A rotor is eccentrically placed in the casing as shown in figure 16.6.3. The rotor carries sliding vanes in slots along the length. Springs control the movement of the vanes and keep them pressed on the casing. Oil is trapped between the vanes and the casing. As the rotor rotates the trapped oil is carried to the pressure side. The maximum operating pressure is controlled by the back leakage.

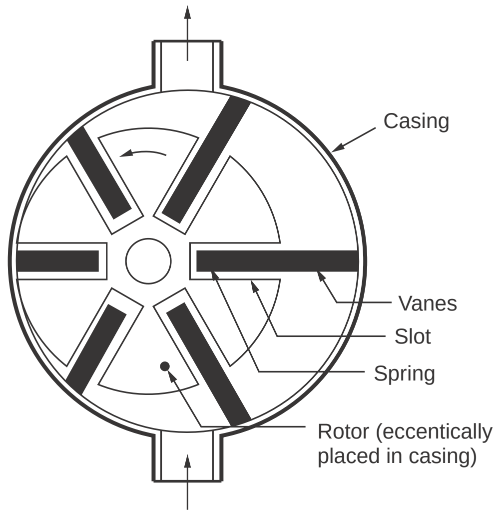

Figure 16.6.3 Vane pump

SOLVED PROBLEMS

Problem 16.1 *A single acting reciprocating water pump of 180 mm bore and 240 mm stroke operates at 40 rpm. Determine the discharge if the slip is 8%. What is the value of coefficient of discharge. If the suction and delivery heads are 6 m and 20 m respectively determine the theoretical power. If the overall efficiency was 80%, what is the power requirement.*

$$\text{Theoretical discharge} = \frac{A\,L\,N}{60}$$

$$= \frac{\pi \times 0.18^2}{4} \times 0.24 \times \frac{40}{60} = 4.0775 \times 10^{-3}\ \text{m}^3/\text{s}$$

$$= 4.0715\ \text{l/s} = 4.0715\ \text{kg/s}$$

$$\text{Slip} = 8\%$$

$$\therefore \quad \text{Actual flow} = 4.0715 \times \frac{92}{100} = 3.746\ \text{l/s or kg/s}$$

$$\text{Coefficient of discharge} = \frac{3.746}{4.0715} = 0.92$$

$$\text{Theoretical power} = mgH = 3.746 \times 9.81 \times 26$$

$$= 955.45\ \text{W}$$

$$\text{Actual Power} = 955.45/0.8$$

$$= \mathbf{1194.3\ W\ or\ 1.1943\ kW}$$

If it is a double acting pump, in case the piston rod diameter is neglected, the flow and power will be double this value. The slip and coefficient discharge and efficiency remaining the same.

Problem 16.2 *It is desired to have a discharge of water of 10 l/min using a reciprocating pump running at 42 rpm. The bore to stroke ratio is to be 1 : 1.5. It is expected that the slip will be 12%. Determine the bore and stroke for (a) single acting pump, and (b) double acting pump. If the total head is 30 m and the overall efficiency is 82%, determine the power required in both cases.*

Single acting pump :

Theoretical discharge $= \dfrac{A\,L\,N}{60}$ m^3/s

Actual discharge $= (1 - \text{slip})\,\dfrac{A\,L\,N}{60}$ m^3/s

Actual per minute $= (1 - \text{slip}) \times A\,L\,N$

$$A = \frac{\pi\,D^2}{4}, \qquad L = 1.5\,D,\ (1 - 0.12)\,\frac{\pi\,D^2}{4} \times 1.5\,D \times 42 = 0.01$$

Solving $\quad D^3 = \dfrac{0.01 \times 4}{0.88 \times \pi \times 1.5 \times 42}$

Solving, $\quad$ **D = 62.7 mm** and **Stroke = 94 mm**

$$\textbf{Power} = \frac{mgh}{\eta} = \frac{10 \times 9.81 \times 30}{0.82 \times 60} = \mathbf{60\ W}$$

Double acting : (Neglecting piston rod diameter)

$$0.01 = (1 - 0.12)\left(\frac{2\pi\,D^2}{4} \times 1.5\,D \times 42\right)$$

Solving: $\quad$ **D = 48.6 mm, L = 97.2 mm**

The advantage of double acting pump is compactness and lower weight as can be seen form the values.

The power required will be double that of the single acting pump

P = 120 W.

Problem 16.3 *A reciprocating pump with plunger diameter of 120 mm and 200 mm stroke has both delivery and suction pipes of 90 mm diameter. The suction length is 9 m and the delivery length is 18 m. The atmospheric head is 10.3 m of water head. Determine the suction head and the delivery head due to acceleration at speeds 30, 40, 50, 60 rpm operating speeds.*

Delivery side: $\mathbf{h}_{\mathbf{a\,max\,d}} = \dfrac{l}{g} \cdot \dfrac{A}{a} \cdot \omega^2 r = \dfrac{18}{9.81} \times \dfrac{0.12^2}{0.09^2}\left(\dfrac{2\pi \times N}{60}\right)^2 \times 0.1$

$= \mathbf{3.577 \times 10^{-3}\,N^2}$

The values are tabulated below

N rpm	30	40	50	60
$h_{a\ max\ d}$	3.22	5.72	8.94	12.88

Suction side: $\mathbf{h_{a\ max\ s}} = \frac{l}{g} \cdot \frac{A}{a} \cdot \omega^2 r = \frac{9}{9.81} \times \frac{0.12^2}{0.09^2} \left(\frac{2\pi \times N}{60}\right)^2 \times 0.1$

$= \mathbf{1.7886 \times 10^{-3}\ N^2}$.

The value are tabulated below for various speeds.

N rpm	30	40	50	60
$h_{a\ max\ d}$	1.61	2.86	4.47	6.44

As the separation limit is 2.5 m absolute, the maximum suction head at 60 rpm (for example) will be equal to (10.3 = 2.5 – 6.44) = 1.36 m only. It can be seen that this speed itself is on the higher side.

Problem 16.4 *In a reciprocating pump the bore is 180 mm and stroke is 280 mm. Water level is 5 m from the pump level. The suction pipe is 110 m diameter and 9 m long. The atmospheric pressure head is 10.3 m water. Determine the maximum speed if the head at pipe suction should not be less than 2.5 m head of water. If the suction pipe diameter is increased 125 mm and length reduced to 6 m, what will be the maximum speed ?*

Suction head = 5 m

$r = L/2 = 0.14$ m

Acceleration head available

$= 10.3 - 5 - 2.5 = 2.8$ m

$$h_{as} = \frac{l_s}{g} \cdot \frac{A}{a} \cdot \omega^2 r$$

$$2.8 = \frac{9}{9.81} \times \frac{0.18^2}{0.11^2}\, \omega^2 \times 0.14$$

$\therefore$ $\omega^2 = (2.8 \times 9.81 \times 0.11^2)/(9 \times 0.18^2 \times 0.14) = 8.413$

$$\omega = 2.8533 = \frac{2\pi N}{60}$$

$$\mathbf{N} = \frac{2.8533 \times 60}{2\pi} = \mathbf{27.25\ rpm}$$

Fairly low speed.

At the changed condition,

$$2.8 = \frac{6}{9.81} \times \frac{0.18^2}{0.125^2} \cdot \omega^2 \times 0.14$$

Solving, $\omega = 3.9711$

$\therefore$ $\mathbf{N = 37.92\ rpm.}$

Problem 16.5 *In a single acting reciprocating pump the bore and stroke are 90 and 160 mm. The static head requirements are 4 m suction and 15 m delivery. If the pressure at the end of delivery is atmospheric determine operating speed. The diameter of the delivery pipe is 90 mm and the length of the delivery pipe is 22 m. Determine the acceleration head at θ = 30 from the start of delivery.*

In this case, the acceleration head equals the static delivery head.

$$\therefore \quad 15 = \frac{22}{9.81} \times \frac{0.09^2}{0.09^2} \cdot \omega^2 \times 0.08$$

Solving, $\omega = 9.1437$

$$\mathbf{N} = \frac{\omega \times 60}{2\pi} = \frac{9.1437 \times 60}{2 \times \pi}$$

$$= \mathbf{87.32\ rpm}$$

At the position 30° from start of delivery,

$$\mathbf{h_a} = \frac{l}{g} \cdot \frac{A}{a} \cdot \omega^2 r \cos\theta$$

$$= \frac{22}{9.81} \times \frac{0.09^2}{0.09^2} \cdot 9.1437^2 \times 0.08 \times \cos 30$$

$$= \mathbf{12.99\ m.}$$

Problem 16.6 *A reciprocating pump handling water with a bore of 115 mm and stroke of 210 mm runs at 35 rpm. The delivery pipe is of 90 mm diameter and 25 m long. An air vessel of sufficient volume is added at a distance of 2 m from the pump. Determine the acceleration head with and without the air vessel.*

Without air vessel:

$$\mathbf{h_a} = \frac{l}{g} \cdot \frac{A}{a} \cdot \omega^2 r$$

$$= \frac{25}{9.81} \times \frac{0.115^2}{0.09^2} \times \left(\frac{2\pi \times 35}{60}\right)^2 \times 0.105 = \mathbf{5.869\ m}$$

With air vessel l reduces to 2 m.

$$\therefore \quad \mathbf{h'} = \frac{5.869 \times 2}{25} = \mathbf{0.47\ m}$$

A considerable reduction.

Problem 16.7 *The bore and stroke of a reciprocating pump are 10 cm and 15 cm. The pump runs at 40 rpm. The suction pipe is 9 cm diameter and 12 m long. Determine the absolute pressure at suction if static suction is 3.5 m. Take h_{atm} = 10.3 m. If an air vessel is fitted at 1.5 m from the pump determine the absolute pressure at suction.*

Chapter 16

Without air vessel:

$$h_{as} = \frac{l_s}{g} \cdot \frac{A}{a_s} \cdot \omega^2 r = \frac{12}{9.81} \times \frac{0.1^2}{0.09^2} \cdot \left(\frac{2\pi \times 40}{60}\right)^2 \times 0.075$$

$$= 1.987 \text{ m}$$

Absolute pressure = 10.3 – 3.5 – 1.987 = **4.812.** Safe against separation

With air vessel :

$$h_a' = 1.987 \times 1.5/12 = 0.248 \text{ m}$$

Absolute pressure = 10.3 – 35 – 0.248 = **6.55 m**

The pump can be run at a higher speed.

Problem 16.8 *In a reciprocating pump delivering water the bore is 14 cm and the stroke is 21 cm. The suction lift is 4 m and delivery head is 12 m. The suction and delivery pipe are both 10 cm diameter, length of pipes are 9 m suction and 24 m delivery. Friction factor is 0.015. Determine the theoretical power required. Slip is 8 percent. The pump speed is 36 rpm.*

Volume delivered assuming single acting,

$$= A\ L\ N/60 = \frac{\pi \times 0.14^2}{4} \times 0.21 \times \frac{36}{60}$$

$$= 1.9396 \times 10^{-3} \text{ m}^3\text{/s or } 1.9396 \text{ kg/s}$$

Slip is 8%

∴ Actual mass delivered = 1.9396 × 0.92 = 1.784 kg/s

Total static head = 4 + 12 = 16 m head

Friction head in the delivery pipe:

Maximum velocity, $v = \frac{A}{a}\omega r = \frac{0.14^2}{0.1^2} \times \frac{2\pi \times 36}{60} \times 0.105 = 0.7758$ m/s

$$h_{fd} = \frac{flv^2}{2gd} = \frac{0.015 \times 24}{2 \times 9.81 \times 0.1} \cdot [0.7758]^2 = 0.11 \text{ m}$$

Average is, 2/3 $\mathbf{h_{fd}}$ = **0.07363 m**

Friction head in the suction pipe ;

Velocity is the same as diameters are equal

$$h_{fs} = \frac{0.015 \times 9}{2 \times 9.81 \times 0.1} \times [0.7758]^2 = 0.0414 \text{ m}$$

Average = 2/3 h_{fs} = 0.0414 × 2/3 = 0.02761 m

Total head = 16 + 0.07363 + 0.02761 = 16.10124 m

Theoretical Power = 1.784 × 9.81 × 16.1024 = **282 W.**

Problem 16.9 *The bore and stroke of a single acting reciprocating water pump are 20 cm and 30 cm. The suction pipe is of 15 cm diameter and 10 m long. The delivery pipe is 12 cm diameter and 28 m long. The pump is driven at 32 rpm. Determine the acceleration heads and the friction head, f = 0.02. Sketch the indicator diagram. The suction and delivery heads from atmosphere are 4 m and 16 m respectively.*

$$\mathbf{h_{as\,max}} = \frac{l_s}{g} \cdot \frac{A}{a_s} \cdot \omega^2 r = \frac{10}{9.81} \times \frac{0.2^2}{0.15^2} \left(\frac{2\pi \times 32}{60}\right)^2 \times 0.15 = \mathbf{3.05\ m}$$

$$\mathbf{h_{ad\,max}} = \frac{l_d}{g} \cdot \frac{A}{a_d} \cdot \omega^2 r = \frac{28}{9.81} \times \frac{0.2^2}{0.12^2} \left(\frac{2\pi \times 32}{60}\right)^2 \times 0.15 = \mathbf{13.35\ m}$$

$$V_{s\,max} = \frac{A}{a} \omega r = \frac{0.2^2}{0.15^2} \times \frac{2\pi \times 32}{60} \times 0.15 = 0.8936$$

$$\mathbf{h_{fs}} = \frac{fl_s}{2gd_s} \cdot v_s^2 = \frac{0.2 \times 10 \times 0.8436^2}{2 \times 9.81 \times 0.15} = \mathbf{0.5427\ m}$$

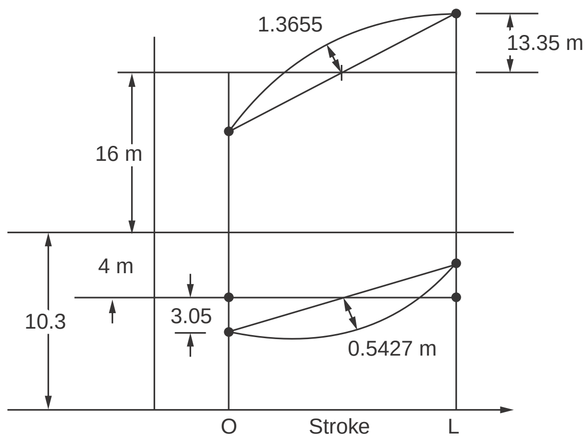

Figure P.16.9 Problem model

$$\mathbf{h_{fd}} = \frac{fl_d}{2gd_d} \cdot V_d^2,\ V_{d\,max} = \frac{A}{a} \cdot \omega r = \frac{0.2^2}{0.12^2} \times \frac{2\pi \times 32}{60} \times 0.15$$

$$= \mathbf{1.396\ m/s}$$

$$\mathbf{h_{fd\,max}} = \frac{0.02 \times 28}{2 \times 9.81 \times 0.12} \times 2.396^2 = \mathbf{1.3655\ m.}$$

Problem 16.10 *Using the data from problem 16.9 determine the theoretical power required.*

Flow rate $= \dfrac{A\,L\,N}{60} = 0.2^2 \times 0.3 \times 32/60$

$= 6.4 \times 10^{-3}$ m^3/s or 6.4 kg/s

Total head $= 16 + 4 + 2/3(1.3655 + 0.5427) = 21.272$ m

Power $= 21.272 \times 9.81 \times 6.4 =$ **1336 W.**

Problem 16.11 *A single acting reciprocating of pump handles water. The bore and stroke of the unit are 20 cm and 30 cm. The suction pipe diameter is 12 cm and length is 8 m. The delivery pipe diameter is 12 cm and length is 24 m. f = 0.02. The speed of operation is 32 rpm. Determine the friction power with and without air vessels.*

Without air vessels $V = \frac{A}{a}\omega r = \frac{0.2^2}{0.12^2} \times \frac{2\pi \times 32}{60} \times 0.15$

$= 1.3963$ m/s

$$\mathbf{h_{fs\,max}} = \frac{fl_s}{2gd_s} \times v^2 = \frac{0.02 \times 8}{2 \times 9.81 \times 0.12} \times (1.3963)^2 = \mathbf{0.1325\ m}$$

$$\mathbf{h_{fd\,max}} = \frac{0.02 \times 24}{2 \times 9.81 \times 0.12} \times \left(\frac{0.2^2}{0.12^2} \cdot \frac{2\pi \times 32}{60} \times 0.15\right)^2 = \mathbf{0.3975\ m}$$

Total average friction head

$$= \frac{2}{3}[0.3975 + 0.1325] = 0.3533 \text{ m}$$

Flow rate $= \frac{A\,L\,N}{60} = \frac{\pi \times 0.2^2}{4} \times 0.3 \times \frac{32}{60} = 5.0265 \times 10^{-3}$ m^3/s

or $= 5.0265$ kg/s

Friction power $= 5.0265 \times 9.81 \times 0.3533 =$ **17.42 W**

When air vessels are installed,

Average velocity in suction pipe

$$= \frac{A}{a}\frac{L\,N}{60} = \frac{0.02^2}{0.12^2} \times 0.3 \times \frac{32}{60} = 0.4444 \text{ m/s}$$

$$h_{fs} = \frac{fl_s\,v^2}{2g\,d_s} = \frac{0.02 \times 8 \times 0.4444^2}{2 \times 9.81 \times 0.12} = 0.013424 \text{ m}$$

As diameters are equal velocity are equal

$$h_{fd} = \frac{0.02 \times 24 \times 0.4444^2}{2 \times 9.81 \times 0.12} = 0.040271 \text{ m}$$

Friction power $= 5.0265 \times 9.81 \times (0.013424 + 0.040271) =$ **2.65 W**

The percentage reduction is $\frac{17.42 - 2.65}{17.42} \times 100 =$ **84.8%**

By use of air vessels there is a saving of 84.8% in friction power.

Problem 16.12 *Show that in a double acting pump the work saved by fitting air vessels is about 39.2%.*

In a double acting pump during a revolution, the discharge

$$Q = \frac{2A\,L\,N}{60}$$

Velocity in the pipe with air vessel

$$= \frac{2\;A\;L\;N}{a \times 60} = \frac{2A \times 2r}{a \times 60} \times \frac{60 \times \omega}{2\pi} = \frac{2A}{a} \times \frac{\omega r}{\pi}$$

Friction head $\quad = \dfrac{fl}{2gd} \,.\, v^2 = \dfrac{fl}{2gd} \,.\, \left(\dfrac{2A}{a} \times \dfrac{\omega r}{\pi}\right)^2$

Without air vessel,

Maximum friction head $= \dfrac{fl}{2gd}\left(\dfrac{A}{a} \,.\, \omega r\right)^2$

Average value is $\quad = \dfrac{2}{3} \times \dfrac{fl}{2gd}\left(\dfrac{A}{a}\,\omega r\right)^2$

The ratio of effective friction head is also the ratio of power as power = mgH, and mg are constant for a pump.

$$\frac{h_f \text{ with air vessel}}{h_f \text{ without air vessel}} = \frac{\dfrac{f\,l}{2gd}\left(\dfrac{2A}{a} \times \dfrac{\omega r}{\pi}\right)^2}{\dfrac{2}{3} \,.\left(\dfrac{f\,l}{2gd}\right)\left(\dfrac{A}{a}\,\omega r\right)^2} = \frac{6}{\pi^2} = 0.608$$

$\therefore\quad$ Reduction is **39.2%.**

Problem 16.13 *In a double acting pump, determine the angle at which there will be no flow in or out of the air vessel.*

Refer equation (16.5.5). Flow to or from air vessel

$$Q = A\omega r\left(\sin\theta - \frac{2}{\pi}\right)$$

When there is no flow in or out of the air vessel,

$$Q = 0,$$

$\therefore \qquad \sin\theta = \dfrac{2}{\pi} \quad \therefore \quad \theta = \sin^{-1}\left(\dfrac{2}{\pi}\right) = \mathbf{39.54°}$

and $\qquad (180 - 39.54) = \mathbf{140.46°}$

This is as against about 18° in the case of single acting pump.

Chapter 16

Problem 16.14 *Tabulate the flow rate as a product of Aωr for various angles of θ in the case of double acting pump.* $\left(\sin\theta - \frac{2}{\pi}\right)$ *is calculated and tabulated.*

Crank Angle θ	0	30	60	90	120	150	180
Flow rate Aωr times	– 0.637	– 0.137	0.2294	0.3634	0.2294	– 0.137	– 0.637

Note that this is different from example 16.9

For the following data speed is 30 rpm, and $r = 0.15$ m, bore = 20 cm.

at 30°, flow is $\left(\sin\theta - \frac{2}{\pi}\right) - 0.137$

$$\therefore \quad \mathbf{q} = -0.137 \times \frac{\pi \times 0.2^2}{4} \times \frac{2 \times \pi \times 30}{60} \times 0.15$$

$$= -2.028 \times 10^{-3} \text{ m}^3\text{/s or } 2.028 \text{ l/s out of the air vessel.}$$

At the starting of delivery stroke, flow is zero, but there is flow in the pipe. This should come out of the air vessel.

Problem 16.15 *In a single acting pump of 16 cm bore and 24 cm stroke, the delivery pipe is 20 m long. f = 0.02. The speed 45 rpm. Determine the friction head on the delivery side, with and without air vessel for pipe diameters of 8 cm and 12 cm.*

Without air vessel : Effective friction head

$$\mathbf{h_f} = \frac{2}{3} \cdot \frac{fl}{2gd}\left(\frac{A}{a}\omega r\right)^2$$

for 8 cm pipe diameter,

$$= \frac{2}{3} \times \frac{0.02 \times 20}{2 \times 9.81 \times 0.08}\left(\frac{0.16^2}{0.08^2} \times 2\pi \times \frac{45}{60} \times 0.12\right)^2$$

$$= \mathbf{0.8692\ m}$$

For 12 cm dia, h_f = 0.1145 m (obviously, larger the pipe diameter lower the friction head)

With air vessel :

$$\mathbf{h_f} = \frac{fl}{2gd}\left(\frac{A}{a} \cdot \frac{\omega r}{\pi}\right)^2$$

$$= \frac{0.02 \times 20}{2 \times 9.81 \times 0.08}\left(\frac{0.16^2}{0.08^2} \times \frac{2\pi \times 45}{60} \times \frac{0.12}{\pi}\right)^2 = \mathbf{0.1321\ m}$$

For **d = 0.12 m, $\mathbf{h_f}$ = 0.0174 m.**

REVIEW QUESTIONS

1. List some types of positive displacement pumps.
2. List the advantages and limitations of reciprocating pumps over dynamic pumps.
3. Compare single acting and double acting pumps.
4. Explain the reason for the limitation of speed of operation of reciprocating pumps.
5. Compare reciprocating and rotary type of positive displacement pumps.
6. Describe working of vane pump.
7. Describe the working of gear pump.
8. Describe the working of lobe pump.
9. Define "slip" in reciprocating pump. Can it be negative ?
10. What is the cause of negative slip ?
11. Explain the function of air vessel.
12. What are the advantages of installing air vessels in reciprocating pumps.
13. Define separation. Explain how this controls the speed of operation.
14. Explain what causes additional head than static head in reciprocating pumps.

OBJECTIVE QUESTIONS

I. Fill in the blanks :

1. Air vessels _________ the flow.
2. Reciprocating pumps are suitable for _________ pressures.
3. Reciprocating pumps are suitable for _________ volumes.
4. Air vessels _________ friction head.
5. Acceleration heads _________ affect the power required.
6. The delivery pipe _________ be bent after the vertical run to avoid separation.
7. The friction head is _________ after the installation of air vessels.
8. The friction head _________ and then _________ in reciprocating pumps without air vessel.
9. Acceleration head is affected by _________ and _________.
10. Double acting pump will give a _________ flow.

Answers

(1) Smoothens (2) High (3) Low (4) Reduce (5) Does not (6) Should not (7) Reduced (8) Increases, decreases (9) Length of pipe, speed (10) Smoother.

EXERCISE PROBLEMS

16.E.1 A single acting pump running at 30 rpm delivers 6.5 l/s of water. The bore and stroke are 20 cm and 30 cm respectively. Determine the percentage slip and coefficient of discharge.

(8.5%, 0.92)

16.E.2 A double acting water pump with bore of 30 cm and stroke of 40 cm has piston rod of 5 cm diameter. The pump runs at 60 rpm and delivers 50 l/s. Determine the slip. Consider area occupied by the piston rod. Also find the coefficient of discharge. **(10.4%, 0.896)**

16.E.3 A single acting pump has a bore and stroke of 300 mm and 400 mm respectively. The discharge is 50 l/s. If the slip is 2%, determine the speed of operation. **(54 rpm)**

16.E.4 The bore and stroke of a double acting pump running at 60 rpm are 0.15 m and 0.3 m respectively. The total head against which works 30 m. If the efficiency is 80% determine the power required. **(4050 W)**

16.E.5 The bore and stroke of a single acting reciprocating pump are 300 mm and 450 mm respectively. The static suction head is 4 m. The suction pipe is 125 mm in diameter and 8 m long. If the separation head is 2.5 m determine the maximum speed of operation of the pump. Atmospheric head is 10.3 m of water. Also calculate the discharge at this speed and the maximum friction head on the suction side. $f = 0.02$. What will be pressure at starting, middle and end of stroke ?

(34 rpm, 18 l/s, 1.4 m, 2.5 m, 4.9 m, 10.1 m)

16.E.6 Determine the maximum speed of operation of a single acting reciprocating pump to avoid separation, give the bore and stroke as 10 cm and 15 cm and static suction head as 4 m and suction pipe length as 6 m and diameter as 25 mm. **(20.9 rpm)**

16.E.7 A single acting reciprocating pump running at 24 rpm has a bore and stroke of 12.5 cm and 30 cm. The static suction 4 m. Determine the pressure at start, middle and end of suction stroke. The suction pipe of 75 mm diameter is 9 m long. Atmospheric pressure is 10.3 m of water

(3.9, 4, 8.7)

16.E.8 A single acting pump takes water from well at 4 m from pump level. The bore and stroke are 20 cm and 30 cm. The suction pipe of 10 cm diameter is 8 m long. If the pump runs at 30 rpm check whether separation will occur. **(Separation will occur as head is 1.5 m at start)**

16.E.9 A single acting reciprocating pump running at 20 rpm has a bore and stroke of 40 cm each. The suction and delivery heads are 4 and 20 m. The suction and delivery pipes of 20 cm diameter are 4 m and 20 m long. $f = 0.02$. Determine the power required if the efficiency is 80%. **(4.99 kW)**

16.E.10 A reciprocating pump running at 60 rpm has a bore of 300 mm and stroke of 450 mm. The delivery pipe of 150 mm diameter is 50 m long. Determine the saving in friction power due to fitting of an air vessel on the delivery side. $f = 0.02$. **(890 W)**

16.E.11 Determine the maximum suction head for a reciprocating pump running at 90 rpm, if bore and stroke are 10 cm and 25 cm respectively. The suction pipe of 100 mm diameter is 5 m long. Separation occurs when the pressure in the cylinder during suction reaches 2 m.

(2.65 m)

16.E.12 The bore and stroke of a reciprocating pump are 25 cm and 50 cm. The delivery pipe is of 100 mm diameter. The delivery is to a tank 15 m above the pump. Determine the speed if separation should not occur. The separation pressure is 2.3 m. The tank is at a distance of 30 m horizontally from the pump. There is no air vessel.

Case (*i*) Pipe is vertical up to 15 m and then horizontal.

Case (*ii*) Pipe is horizental for 30 m and then vertical.

[Case (*i*) 10.1 rpm, Case (*ii*) 17 rpm)

16.E.13 The bore and stroke of a double acting pump running at 40 rpm are 0.2 m and 0.4 m. The delivery pipe of 15 cm diameter is 36 m long. The static delivery head is 10 m. All air vessel is fitted at the level of the pump at a distance of 3 m from the pump. $f = 0.032$. Determine the pressure in the pump at the beginning of delivery stroke. **(12.2 m)**

16.E.14 Determine the change in the maximum speed of operation due to fitting an air vessel on the suction side of a pump of 300 mm bore and 500 mm stroke. The suction pipe of 200 mm diameter is 10 m long. The suction lift is 3.5 m. **(26.1 rpm and 48.6 rpm)**

16.E.15 Determine the friction power on the delivery side without and with fitting air vessel on a pump running at 60 rpm if the bore and stroke are 250 mm and 500 mm. The delivery pipe of 100 mm diameter is 50 m long. $f = 0.04$. **(2.53 kW)**

16.E.16 A double acting pump of 175 mm bore and 350 mm stroke runs at 150 rpm. The suction pipe is of 150 mm diameter. Determine the crank angle at which there will be no flow from or to the air vessel. **(39.5° or 40.5°).**