2 Pressure Distribution in Fluids

2.0 INTRODUCTION

Fluids are generally found in contact with surfaces. Water in the sea and in reservoirs are in contact with the ground and supporting walls. Atmospheric air is in contact with the ground. Fluids filling vessels are in contact with the walls of the vessels. **Fluids in contact with surfaces exert a force on the surfaces**. The force is mainly due to the specific weight of the fluid in the case of liquids. In the case of gases molecular activity is the main cause of force exerted on the surfaces of the containers. Gas column will also exert a force on the base, but this is usually small in magnitude. **When the whole mass of a fluid held in a container is accelerated or decelerated without relative motion between layers inertia forces also exert a force on the container walls**. This alters the force distribution at stationary or atatic conditions. Surfaces may also be immersed in fluids. A ship floating in sea is an example. In this case the force exerted by the fluid is called buoyant force. This is dealt with in a subsequent chapter. The force exerted by fluids vary with location. The variation of force under static or dynamic condition is discussed in this chapter.

This chapter also deals with pressure exerted by fluids due to the weight and due to the acceleration/deceleration of the whole mass of the fluid without relative motion within the fluid.

Liquids held in containers may or may not fill the container completely. When liquids partially fill a container a free surface will be formed. Gases and vapours always expand and fill the container completely.

2.1 PRESSURE

Pressure is a measure of force distribution over any surface associated with the force. **Pressure is a surface phenomenon and it can be physically visualised or calculated only if the surface over which it acts is specified**. Pressure may be defined as the force acting along the normal direction on unit area of the surface. However a more precise definition of pressure, P is as below:

$$\mathbf{P} = \lim_{A \to a} (\Delta F/\Delta A) = dF/dA \tag{2.1.1}$$

F is the resultant force acting normal to the surface area **A**. **'a'** is the limiting area which will give results independent of the area. This explicitly means that **pressure is the ratio of the elemental force to the elemental area normal to it.**

The force *dF* in the normal direction on the elemental area *dA* due to the pressure *P* is

$$\mathbf{dF = P\ dA} \tag{2.1.2}$$

The unit of pressure in the SI system is N/m^2 also called Pascal (Pa). As the magnitude is small kN/m^2 (kPa) and MN/m^2 (Mpa) are more popularly used. **The atmospheric pressure is approximately 10^5 N/m^2 and is designated as "bar"**. This is also a popular unit of pressure. In the metric system the popular unit of pressure is kgf/cm^2. This is approximately equal to the atmospheric pressure or 1 bar.

2.2 PRESSURE MEASUREMENT

Pressure is generally measured using a sensing element which is exposed on one side to the pressure to be measured and on the other side to the surrounding atmospheric pressure or other reference pressure. The details of some of the pressure measuring instruments are as shown in Fig. 2.2.1.

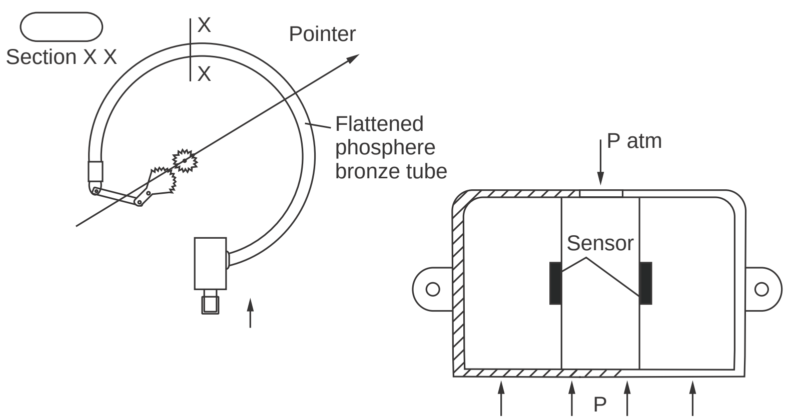


Figure 2.2.1 Pressure gauges

In the Borden gauge a tube of elliptical section bent into circular shape is exposed on the inside to the pressure to be measured and on the outside to atmospheric pressure. The tube will tend to straighten under pressure. The end of the tube will move due to this action and will actuate through linkages the indicating pointer in proportion to the pressure. Vacuum also can be measured by such a gauge. Under vacuum the tube will tend to bend further inwards and as in the case of pressure, will actuate the pointer to indicate the vacuum pressure. The scale is obtained by calibration with known pressure source.

The pressure measured by the gauge is called gauge pressure. The sum of the gauge pressure and the outside pressure gives the absolute pressure which actually is the pressure measured.

The outside pressure is measured using a mercury barometer (Fortins) or a bellows type meter called Aneroid barometer shown in Fig. 2.2.2. The mercury barometer and bellow type meter have zero as the reference pressure. The other side of the measuring surface in these cases is exposed to vacuum. Hence these meters provide the absolute pressure value.

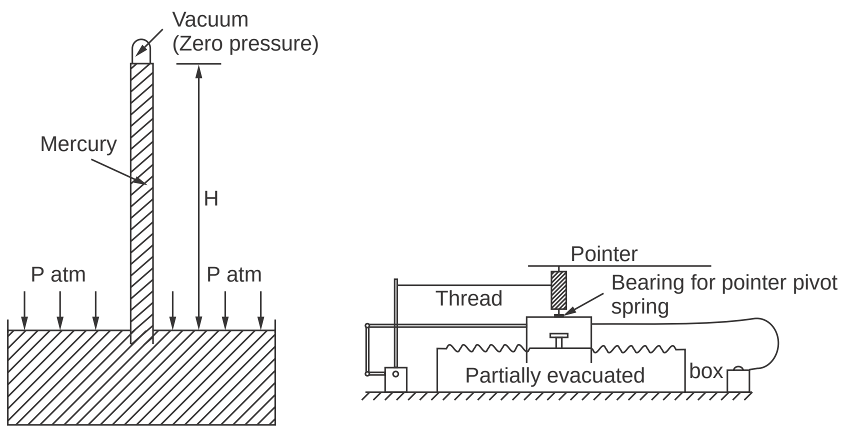


Figure 2.2.2 Barometer

When the pressure measured is above surroundings, then

Absolute pressure = gauge pressure + surrounding pressure

The surrounding pressure is usually the atmospheric pressure.

If the pressure measured is lower than that of surrounding pressure then

Absolute pressure = surrounding pressure – gauge reading

This will be less than the surrounding pressure. This is called Vacuum.

Electrical pressure transducers use the deformation of a flexible diaphragm exposed on one side to the pressure to be measured and to the surrounding pressure or reference pressure on the other side. The deformation provides a signal either as a change in electrical resistance or by a change in the capacitance value. An amplifier is used to amplify the value of the signal. The amplified signal is generally calibrated to indicate the pressure to be measured.

In this text the mension pressure means absolute pressure. Gauge pressure will be specifically indicated.

Example 2.1. *A gauge indicates 12 kPa as the fluid pressure while, the outside pressure is 150 kPa. Determine the absolute pressure of the fluid. Convert this pressure into kgf/cm²*

Absolute pressure = Gauge pressure + Outside pressure

= 150 + 12 = **162 kPa** or 1.62 bar.

$1.62 \text{ bar} = 1.62 \times 10^5 \text{ N/m}^2$

As $1\ kgf/cm^2 = 9.81\ N/cm^2 = 9.81 \times 10^4\ N/m^2 = 98100\ N/m^2$

$1.62 \times 10^5\ N/m^2 = 1.62 \times 10^5/98100 =$ **1.651 kgf/cm²**

Example 2.2. *A vacuum gauge fixed on a steam condenser indicates 80 kPa vacuum. The barometer indicates 1.013 bar. Determine the absolute pressure inside the condenser. Convert this pressure into head of mercury.*

Barometer reading = 1.013 bar = 101.3 kPa.

Absolute pressure = atmospheric pressure – vacuum gauge reading

Absolute pressure in the condenser = 101.3 – 80 = **21.3 kPa**

101.3 kPa = 760 mm of Hg. (standard atmosphere)

∴ 21.3 kPa = (21.3/101.3) × 760 = **159.8 mm of Hg**

2.3 PASCAL'S LAW

In fluids under static conditions pressure is found to be independent of the orientation of the area. This concept is explained by **Pascal's law** which **states that the pressure at a point in a fluid at rest is equal in magnitude in all directions**. Tangential stress cannot exist if a fluid is to be at rest. This is possible only if the pressure at a point in a fluid at rest is the same in all directions so that the resultant force at that point will be zero.

The proof for the statement is given below.

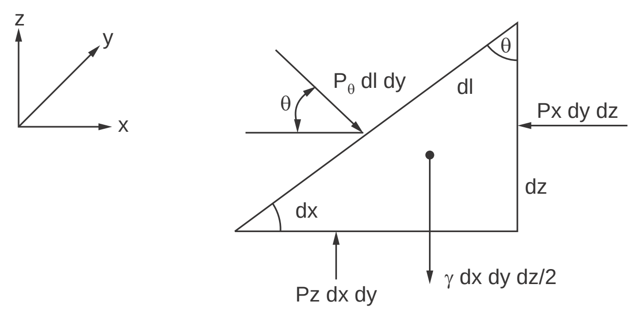


Figure 2.3.1 Pascals law demonstration

Consider a wedge shaped element in a volume of fluid as shown in Fig. 2.3.1. Let the thickness perpendicular to the paper be **dy**. Let the pressure on the surface inclined at an angle θ **to vertical be** P_θ and its length be **dl**. Let the pressure in the x, y and z directions be P_x, P_y, P_z.

First considering the x direction. For the element to be in equilibrium,

$$P_\theta \times dl \times dy \times \cos\theta = P_x \times dy \times dz$$

But, $dl \times \cos\theta = dz$ So, $P_\theta = P_x$

When considering the vertical components, the force due to specific weight should be considered.

$$P_z \times dx \times dy = P_\theta \times dl \times dy \times \sin\theta + 0.5 \times \gamma \times dx \times dy \times dz$$

The second term on RHS of the above equation is negligible, its magnitude is one order less compared to the other terms.

Also, $dl \times \sin \theta = dx$, So, $P_z = P_\theta$

Hence, $\mathbf{P_x = P_z = P_\theta}$

Note that the angle has been chosen arbitrarily and so this relationship should hold for all angles. By using an element in the other direction, it can be shown that

$$\mathbf{P_y = P_\theta} \quad \textbf{and} \quad \textbf{so } \mathbf{P_x = P_y = P_z}$$

Hence, the pressure at any point in a fluid at rest is the same in all directions. The pressure at a point has only one value regardless of the orientation of the area on which it is measured. This can be extended to conditions where fluid as a whole (like a rotating container) is accelerated like in forced vortex or a tank of water getting accelerated without relative motion between layers of fluid. Surfaces generally experience compressive forces due to the action of fluid pressure.

2.4 PRESSURE VARIATION IN STATIC FLUID (HYDROSTATIC LAW)

It is necessary to determine the pressure at various locations in a stationary fluid to solve engineering problems involving these situations. **Pressure forces are called surface forces. Gravitational force is called body force as it acts on the whole body of the fluid.**

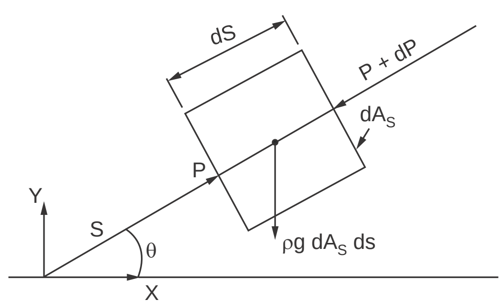


Figure 2.4.1 Free body diagram to obtain hydrostatic law

Consider an element in the shape of a small cylinder of constant area dA_s along the s direction inclined at angle θ to the horizontal, as shown in Fig. 2.4.1. The surface forces are P at section s and $P + dp$ at section $s + ds$. The surface forces on the curved area are balanced. The body force due to gravity acts vertically and its value is $\gamma \times ds \times dA_s$. A force balance in the s direction (for the element to be in equilibrium) gives

$$P \times dA_s - (P + dp) \times dA_s - \gamma \times dA_s \times ds \times \sin \theta = 0$$

Simplifying,

$$dp/ds = -\gamma \times \sin \theta \text{ or, } dp = -\gamma \times ds \times \sin \theta \tag{2.4.1}$$

This is the fundamental equation in fluid statics. The variation of specific weight γ with location or pressure can also be taken into account, if these relations are specified as (see also section 2.4.2).

$$\gamma = \gamma\,(P, s)$$

For x axis, $\theta = 0$ and $\sin\theta = 0$.

$$\therefore \quad dP/dx = 0 \tag{2.4.2}$$

In a static fluid with no acceleration, the pressure gradient is zero along any horizontal line i.e., planes normal to the gravity direction.

In y direction, $\theta = 90$ and $\sin\theta = 1$,

$$dP/dy = -\,\gamma = -\,\rho g/g_o \tag{2.4.4}$$

Rearranging and integrating between limits y_1 and y

$$\int_{p_1}^{p} dp = -\,\gamma \int_{y_1}^{y} dy \tag{2.4.5}$$

If γ is constant as in the case of liquids, these being incompressible,

$$P - P_1 = -\,\gamma \times (y - y_1) = -\,\rho\, g\,(y - y_1)/g_o \tag{2.4.6}$$

As P_1, y_1 and γ are specified for any given situation, P will be constant if y is constant. This leads to the statement,

The pressure will be the same at the same level in any connected static fluid whose density is constant or a function of pressure only.

A consequence is that the free surface of a liquid will seek a common level in any container, where the free surface is everywhere exposed to the same pressure.

In equation 2.4.6, if $y = y_1$ then $P = P_1$ and $dp = 0$. This result is used very extensively in solving problems on manometers.

2.4.1 Pressure Variation in Fluid with Constant Density

Consider the equation 2.4.6,

$$P - P_1 = -\,\gamma \times (y - y_1) = \gamma \times (y_1 - y) = \rho\, g\,(y_1 - y)/g_o \tag{2.4.7}$$

As y increases, the pressure decreases and vice versa **(y is generally measured in the upward direction)**. In a static fluid, the pressure increases along the depth. If the fluid is incompressible, then **the pressure at any y location is the product of head and specific weight, where head is the y distance** of the point from the reference location.

Example 2.3. *An open cylindrical vertical container is filled with water to a height of 30 cm above the bottom and over that an oil of specific gravity 0.82 for another 40 cm. The oil does not mix with water. If the atmospheric pressure at that location is 1 bar,* ***determine the absolute and gauge pressures*** *at the oil water interface and at the bottom of the cylinder.*

This has to be calculated in two steps, first for oil and then for water.

Density of the oil	$= 1000 \times 0.82 = 820$ kg/m^3
Gauge pressure at interface	$= (\rho \times g \times h)^*_{\text{oil}}$
	$= 820 \times 9.81 \times 0.4 =$ **3217.68 N/m^2**
Absolute pressure at interface	$= 3217.68 + 1 \times 10^5$ N/m^2
	$= 103217.68$ N/m^2 = **1.0322 bar**
Pressure due to water column	$= \rho \times g \times h = 1000 \times 9.81 \times 0.3 = 2943$ N/m^2

Gauge pressure at the bottom = gauge pressure at the interface + $(\rho \times g \times h)_{water}$

= 3217.68 + 1000 × 9.81 × 0.3 = **6160.68 N/m²**

Absolute pressure at bottom

= $6160.68 + 1 \times 10^5$

= **106160.68 N/m² or 1.0616 bar**

This value also equals the sum of absolute pressure at interface and the pressure due to water column.

***Note:** g_o is left out as $g_o = 1$ in SI units

Example 2.4. *The gauge pressure at the surface of a liquid of density 900 kg/m³ is 0.4 bar. If the atmospheric pressure is 1×10^5 Pa, calulate the absolute pressure at a depth of 50 m.*

$\mathbf{P_{50}}$ = atmospheric pressure + pressure at top surface + $\rho g h$

$= 1 \times 10^5 + 0.4 \times 10^5 + 900 \times 9.81 \times 50$ N/m²

$= 5.8145 \times 10^5$ Pa = **5.8145 bar (absolute)**

2.4.2 Pressure Variation in Fluid with Varying Density

Consider equation 2.4.4,

$$dP/dy = -\gamma$$

Gamma can be a function of either P or y or both, If $\gamma = \gamma(y)$ then

$$\int dP = -\gamma(y)\, dy, \quad \text{If } \gamma = \gamma(P) \text{ then } \int \gamma(P)\, dP = \int dy,$$

If $\gamma = \gamma(P, y)$ then the variables should be separated and integrated.

Example 2.5. *The local atmospheric pressure at a place at 30° C is 1 bar. Determine the pressure at an altitude of 5 km if (i) the air density is assumed to be constant (ii) if the temperature is assumed to be constant and (iii) if with altitude the temperature decreases linearly at a rate of 0.005°C per metre. Gas constant R = 287 J / kg K*

(*i*) constant air density, using equation 2.4.2, 2.4.3 and 2.4.5

$$\int_{p_1}^{p} dp = -\int_{y_1}^{y} \gamma dy$$

$$\gamma = (P/R\text{T}) \times g = \{1 \times 10^5/[287 \times (273 + 30)]\} \times 9.81 = 11.28 \text{ N/m}^3$$

Integrating between 0 and 5000 m

$$P - 1 \times 10^5 = -11.28 \times (5000 - 0), \text{ Solving, } P = 43{,}600 \text{ N/m}^2 = \mathbf{0.436\ bar}$$

(*ii*) isothermal

$P \times v$ = constant or (P/ρ) = constant or $(P/\rho\, g)$ = constant or (P/γ) = constant, at any location

i.e., $(P/\gamma) = (P_o/\gamma_o)$; $\gamma = (P \times \gamma_o)/P_o$

As $(dP/dy) = -\gamma = -(P \times \gamma_o)/P_o$, separating variables

$(d\text{P}/P) = -(\gamma_o/P_o)\, dy$

Integrating from zero altitude to y m

$$\int_{p_1}^{p} (dp/p) = -(\gamma_o/p_o) \int_0^y dy$$

In $(P/P_o) = -(\gamma_o/P_o) \times y$ (2.4.7)

$$P = P_o \exp[-(\gamma_o \times y/P_o)], \text{ Now } y = 5000, \qquad (2.4.8)$$

$$P = 1 \times 10^5 \exp[-(11.28 \times 5000)/(1 \times 10^5)] = 56{,}893 \text{ N/m}^2$$

$$= \mathbf{0.56893\ bar}$$

(*iii*) The condition reduces to the form, $T = T_o - cy$

$$Pv = RT; (P/\rho) = RT; \rho = (P/RT); \rho \times g = (P \times g/RT);$$

$$\gamma = Pg/RT = (g/R) \times [P/(T_o - c \times y)]$$

$$(dP/dy) = -\gamma = -(g/R) \times [P/(T_o - c \times y)]$$

or, $$(dP/P) = -(g/R) \times [dy/(T_o - c \times y)], \text{ Integrating,}$$

$$\int_{p_1}^{p} (dp/p) = -(g/R) \int_0^y dy/(T_1 - cy)$$

In $$(P/P_o) = -(g/R)\{1/(-c)\} \text{ In } \{(T_o - cy)/(T_o - c \times 0) \qquad (2.4.9)$$

$$= (g/Rc) \ln \{(T_o - cy)/T_o\}$$

or $$P/P_o = [(T_o - cy)/T_o]^{-g/RC} \qquad (2.4.10)$$

Substituting for $T_o = 303$ and $c = 0.005$, $y = 5000$ m and solving,

$$P = 55{,}506 \text{ N/m}^2 = \mathbf{0.55506\ bar}$$

2.5 MANOMETERS

Manometer is a device to measure pressure or mostly difference in pressure using a column of liquid to balance the pressure. It is a basic instrument and is used extensively in flow measurement. It needs no calibration. Very low pressures can be measured using micromanometers. The basic principle of operation of manometers is that at the same level in contigues fluid at rest, the pressure is the same. The pressure due to a constant density liquid (ρ) column if height h is equal to $\rho\, gh/g_o$. g_o in SI system of units has a numerical value of unity. Hence it is often left out in the equations. For dimensional homogenity g_o should be used. The principle of operation is shown in Fig. 2.5.1 (*a*) and some types of manometers are shown in Fig. 2.5.1 (*b*). In Fig. 2.5.1 (*a*), the pressure inside the conduit is higher than atmospheric pressure. The column of liquid marked *AB* balances the pressure existing inside the conduit. The pressure at point *C* above the atmospheric pressure (acting on the open limb) is given by $h \times (\gamma_1 - \gamma_2)$ where γ_1 and γ_2 are the specific weights of fluids 1 and 2, and h is the height of the column of liquid (*AB*). The pressure at the centre point *D* can be calculated as

$$P_d = P_c - \gamma_2 \times h'$$

Generally the pressure at various points can be calculated using the basic hydrostatic equation $dP/dy = -\gamma$ and continuing the summation from the starting point at which pressure is known, to the end point, where the pressure is to be determined.

Another method of solving is to start from a point of known pressure as datum and adding $\gamma \times \Delta y$ when going downwards and subtracting of $\gamma \times \Delta y$ while going upwards. The pressure at the end point will be the result of this series of operations.

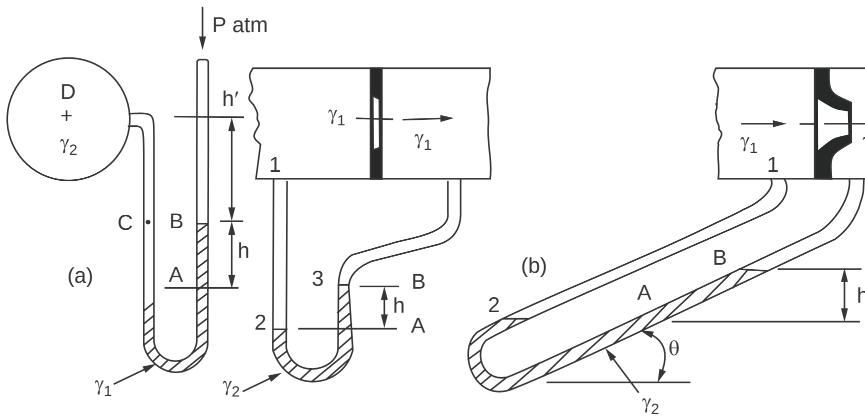


Figure 2.5.1 Types of manometers

$$\Delta P_{1-5} = \gamma_1\, \Delta y_1 + \gamma_2\, \Delta y_2 + \gamma_3\, \Delta y_3 + \gamma_4\, \Delta y_4$$

with proper sign for Δy values.

The advantages of using manometers are (*i*) their simplicity (*ii*) reliability and (*iii*) ease of operation and maintenance and freedom from frequent calibration needed with other types of gauges. As only gravity is involved, horizontal distances need not be considered in the calculation.

The sensitivity of simple manometers can be improved by using inclined tubes (at known angle) where the length of the column will be increased by (1/sin θ) where θ is the angle of inclination with the horizontal (Fig. 2.5.1 (*b*)).

Example 2.6. *A manometer is fitted as shown in Fig. Ex. 2.6. Determine the pressure at point A.*

With respect to datum at *B*, pressure at left hand side = pressure at right hand side

$P_C = P_B$ Consider the left limb

$$P_C = P_a + 0.125 \times 900 \times 9.81 + 0.9 \times 13600 \times 9.81$$

$$= P_a + 121178 \text{ N/m}^2$$

Consider the right limb $P_A = P_B - 0.9 \times 1000 \times 9.81 = P_a + 121178 - 0.9 \times 1000 \times 9.81$

$= P_a + 112349$ N/m^2 Expressed as gauge pressure

$$\mathbf{P_A} = 112349 \text{ N/m}^2$$

= **112.35 kPa** gauge

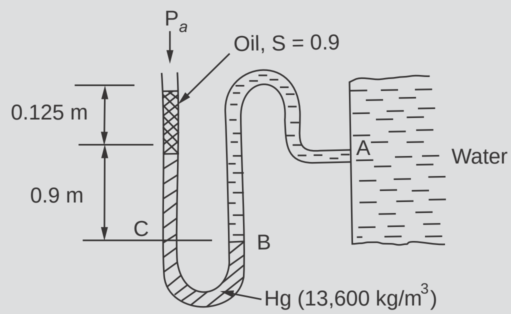


Figure Ex. 2.6

Example 2.7. *An inverted U-tube manometer is fitted between two pipes as shown in Fig.Ex.2.7. Determine the pressure at E if P_A = 0.4 bar (gauge)*

$$P_B = P_A - [(0.9 \times 1000) \times 9.81 \times 1.2]$$
$$= 40000 - [(0.9 \times 1000) \times 9.81 \times 1.2] = 29{,}405.2 \text{ N/m}^2$$
$$P_C = P_B - [(0.9 \times 1000) \times 9.81 \times 0.8] = 22342 \text{ N/m}^2$$
$$P_C = P_D = 22342 \text{ N/m}^2$$
$$\mathbf{P_E} = P_D + [1000 \times 9.81 \times 0.8] = 30190 \text{ N/m}^2 = \mathbf{30.19 \text{ kPa (gauge)}}$$

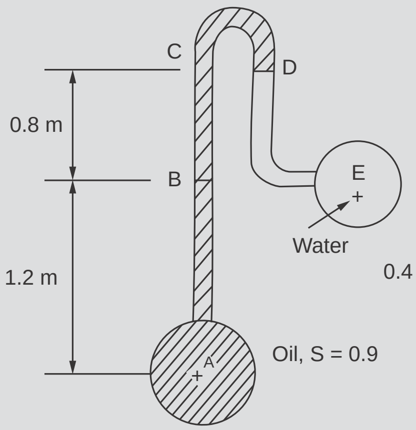


Figure Ex. 2.7

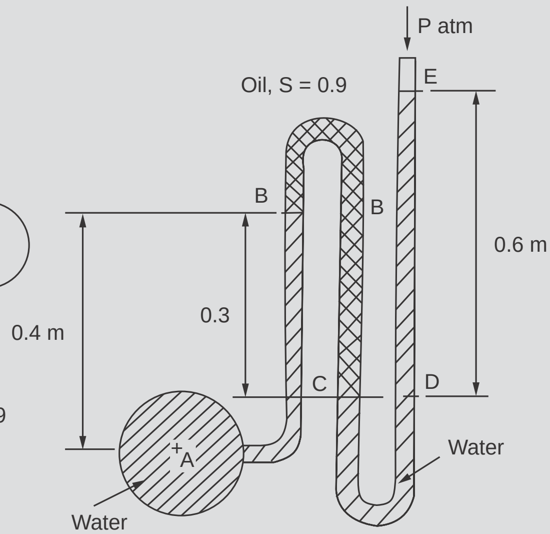


Figure Ex. 2.8

Example 2.8. *A multiple U-tube manometer is fitted to a pipe with centre at A as shown in Fig. Ex.2.8. Determine the pressure at A.*

Pressure at E = atmospheric pressure, P_{atm}

$$P_D = P_{atm} + (1000 \times 9.81 \times 0.6) = P_{atm} + 5886 \; P_a \text{ As } P_C = P_D$$
$$P_B = P_C - [0.9 \times 1000 \times 9.81 \times 0.3]$$
$$= P_{atm} + 5886 - 2648.7 = P_{atm} + 3237.3 \text{ Pa}$$
$$\mathbf{P_A} = P_B + [1000 \times 9.81 \times 0.4] = P_{atm} + 7161.3$$
$$= P_{atm} + 7161.3 \text{ N/m}^2 \text{ or } \mathbf{7161.3 \text{ kPa (gauge)}}$$

2.5.1 Micromanometer

Small differences in liquid levels are difficult to measure and may lead to significant errors in reading. Using an arrangement as shown in Fig. 2.5.1, the reading may be amplified. For improved accuracy the manometer fluid density should be close to that of the fluid used for measurement.

Chambers A and B are exposed to the fluid pressures to be measured. $P_A - P_B$ is the required value. These chambers are connected by a U tube having a much smaller area compared to the chambers A and B. The area ratio is the significant parameter. The volumes above this manometric fluid is filled with a fluid of slightly lower density.

Let pressure $P_A > P_B$ and let it cause a depression of Δy in chamber A. The fluid displaced goes into the U tube limb of area a. The displacement in the limb will therefore by $(y \times A/a)$ which becomes better readable.

Let the original level of manometric fluid in the U tube be at 2-2 and let the fluid levels originally in the chambers be 1-1. After connecting to the pressure sides let the level of manometric fluid be 3-3 on the high pressure side. Let the displacement in the chamber A be Δy. Let the specific weight of the pressure side fluid be γ_1 and that of the other fluid be γ_2 and that of the manometric fluid be γ_3. The fall in level of the manometric fluid from 2–3 on the left limb will equal the rise of the level from 3 to 4 in the right limb.

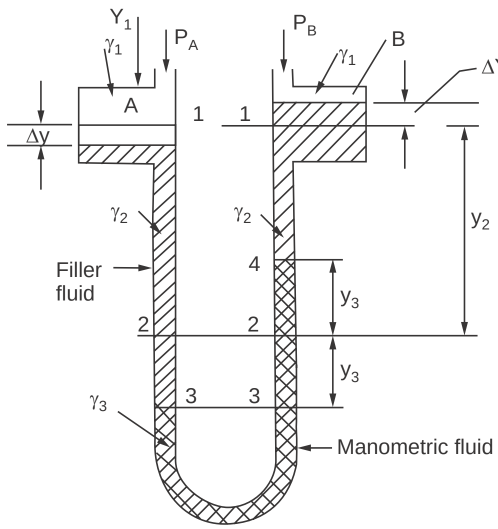


Figure 2.5.2 Micromanometer

Starting from level in chamber A and level 3 as datum

$$P_B = P_A + \{(y_1 + \Delta y) \times \gamma_1\} + \{(y_2 + y_3 - \Delta y) \times \gamma_2\} - \{2y_3 \times \gamma_3\}$$
$$- \{(y_2 - y_3 + \Delta y) \times \gamma_2\} - \{(y_1 - \Delta y) \times \gamma_1\}$$
$$= P_A - [2 \times y_3 \times (\gamma_3 - \gamma_2) + 2 \times \Delta y \times (\gamma_2 - \gamma_1)]$$

As $\Delta y = (a/A) \times y_3$

$$P_A - P_B = 2 \times y_3 \times [\gamma_3 - \gamma_2 \times \{1 - (a/A)\}] - [2 \times y_3 \times (a/A) \times \gamma_1] \quad (2.5.1)$$

Very often γ_1 is small (because gas is generally the medium) and the last term is negligible. So

$$P_A - P_B = 2 \times y_3 \times [\gamma_3 - \gamma_2 \times \{1 - (a/A)\}] \quad (2.5.2)$$

For a given instrument y_3 is a direct measure of $\Delta P \to (P_A - P_B)$.

To facilitate improved reading accuracy or increased value of y_3, it is necessary that $(\gamma_3 - \gamma_2)$ is small.

Example 2.9. *A micromanometer is to be used to find the pressure difference of air flowing in a pipeline between two points A and B. The air density is 1.2 kg/m^3. The micromanometer fluid is having a specific gravity of 1.1 and the filler fluid is water. Under measuring conditions, the manometric fluid movement on the pressure side is 5 cm. Determine the pressure difference between the two points A and B, if the area of the well chamber is 10 times that of the tube.*

Refer Fig. 2.5.1

$$y_3 = 5 \text{ cm} = 0.05 \text{ m};$$
$$\gamma_1 = 1.2 \times 9.81 \text{ N/m}^3,\ \gamma_2 = 1000 \times 9.81 \text{ N/m}^3;$$
$$\gamma_3 = 1.1 \times 1000 \times 9.81 \text{ N/m}^3; \text{ and } (a/A) = 1/10,$$

Using equation 2.5.1

$$\mathbf{P_A - P_B} = 2 \times y_3 \times [\gamma_3 - \gamma_2 \times \{1 - (a/A)\}] - \{2 \times y_3 \times (a/A) \times \gamma_1\}$$
$$= 2 \times 0.05 \times [1.1 \times 1000 \times 9.81) - 1000 \times 9.81)$$
$$\times (1 - 1/10)] - \{2 \times 0.05 \times (1/10) \times 1.2 \times 9.81\}$$
$$= 196.2 - 0.11772 = \mathbf{196.08\ N/m^2}$$

The second term due to air is negligible as it does not contribute even 0.1%. The advantage of this micromanometer is that the deflection is as high as 5 cm even for a pressure difference of 196.08 Pa. This helps to measure very low pressure differences with sufficient accuracy. In case ordinary manometer is used the deflection will be 5 mm only.

Example 2.10. *Determine the fluid pressure at a tapping connected with an inclined manometer if the rise in fluid level is 10 cm along the inclined tube above the reservoir level. The tube is inclined at 20° to horizontal as shown in figure. The density of manometric fluid is 800 kg/m³.*

The actual head, $y = 0.1 \times \sin 20 = 0.0342$ m

Pressure at the tapping point $= \gamma \times y = 800 \times 9.81 \times 0.0342$

$= 268.42$ N/m² (gauge)

Reading accuracy is improved as 3.42 cm is amplified to 10 cm.

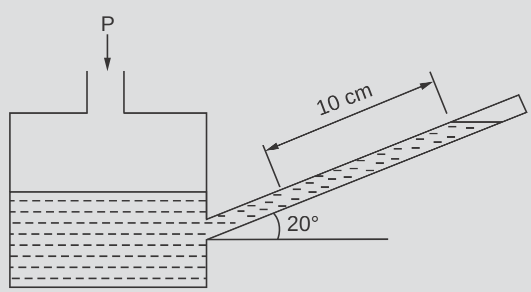


Figure Ex. 2.10

2.6 DISTRIBUTION OF PRESSURE IN STATIC FLUIDS SUBJECTED TO ACCELERATION, a_s

Consider the small cylindrical element of sectional area dA_s and length s inside the fluid, which is accelerated at a_s along the s direction. For equilibrium along s direction,

Surface forces + Body forces = Inertia forces

The net force in the s direction = rate of change of momentum is s direction.

Pressure force + Body force along s direction

$$= \{P \times dA_s - (P + dP) \times dA_s\} - \gamma \times dA_s \times ds \times \sin\theta$$

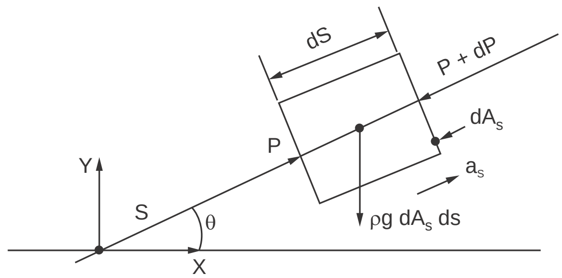


Figure 2.6.1 Free body diagram for accelerating fluid element

Inertial force = The rate of change of momentum = $\rho \times dA_s \times ds \times a_s$

Equating and simplifying,

$$dP/ds = -(\gamma \times \sin\theta + \rho \times a_s) \tag{2.6.1}$$

For the y direction, $\theta = 90°$

$$dP/dy = -(\gamma + \rho \times a_y) \tag{2.6.2}$$

dP/dy will be zero when, $\gamma = -\rho \times a_y$

For the x direction, $\theta = 0°$

$$dP/dx = -\rho \times a_x \tag{2.6.3}$$

This shows that when there is acceleration, a pressure gradient in x direction (horizontal direction) is also possible. The above three equations are to be used to determine the pressure distribution in cases where the fluid as a whole is accelerated without flow or relative motion in the fluid.

These equations can be integrated if a_s, γ, ρ are specified as functions of P or s. However, variable density problems are more involved in this situation and solutions become more complex.

2.6.1 Free Surface of Accelerating Fluid

The pressure gradient along any free surface is zero, as this surface is exposed to the same pressure all over. If the direction of free surface is s then $dP/ds = 0$. Using equation 2.6.1

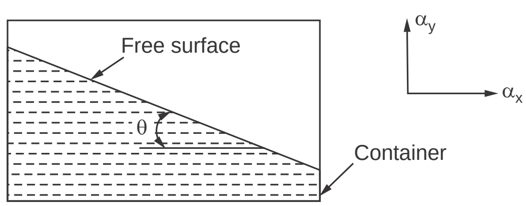


Figure 2.6.2 Free surface of accelerating fluid

$$\gamma \times \sin\theta = -\rho \times a_s \tag{2.6.5}$$

or
$$\theta = \sin^{-1}(-\rho \times a_s/\gamma) = \sin^{-1}(-a_s/g) \tag{2.6.6}$$

In general, for acceleration, in direction s inclined at θ to x direction, (two dimensional)

$$a_s = a_y \times \sin\theta + a_x \times \cos\theta$$

Substituting in equation 2.6.5 and rearranging

$$\tan\theta = -[a_x/(g + a_y)] \quad (2.6.7)$$

The consequence of these equations are

(*i*) If $a_x = 0$, the free surface will be horizontal

(*ii*) If $g = 0$, $\tan\theta = -a_x/a_y$

The constant pressure surface (free surface) will be normal to the resultant acceleration.

(*iii*) In general, the free surface angle will depend on a_x, a_y and g.

(*iv*) The free surfaces of liquids are constant pressure surfaces and hence follow equations 2.6.5–2.6.7.

When an open container filled with liquid accelerates, a free surface will be formed as specified by the above equations. **When gravity is not present, liquids may not assume a free surface but will be influenced only by surface tension. In space liquid spilling poses problems because of this condition**. When a closed container completely filled with liquid is accelerated a free surface cannot form. But the pressure at the various locations will be governed by these equations.

Chapter 2

2.6.2 Pressure Distribution in Accelerating Fluids along Horizontal Direction

Using the general expression for the model (fluid under acceleration) and the equation 2.6.1

$$(dP/ds) = -(\gamma \times \sin\theta + \rho \times a_s)$$

$\theta = 0$ for x direction, $ds = dx$, $a_s = a_x$ (x directional acceleration)

$$(dP/dx) = -(\rho \times a_x) \quad (2.6.8)$$

(*i*) For constant density conditions:

$$\int_{p_1}^{p_2} dp = -(\rho\, a_s) \int_{x_1}^{x_2} dx$$

$$(P_2 - P_1) = -(\rho \times a_x)(x_2 - x_1)$$

$$P_2 = P_1 - (\rho \times a_x)(x_2 - x_1) \quad (2.6.9)$$

a_x is positive in x direction (towards right) and negative in the $-x$ direction (left).

(*ii*) If density varies with pressure as, ρ = **AP + B** (A, B are constants):

Using equation 2.6.8, $[dP/(AP + B)] = -a_x \times dx$

Integrating, between the locations x_1 and x_2

$$(1/A) \times [\ln(AP + B)]_{p_1}^{p_2} = -a_x(x_2 - x_1) \text{ or}$$

$$\ln[(AP_2 + B)/(AP_1 + B) = -A \times a_x \times (x_2 - x_1)$$

$$(AP_2 + B) = (AP_1 + B)\exp[-A \times a_x \times (x_2 - x_1)]$$

or

$$P_2 = (1/A)\,[(AP_1 + B)\exp\{-A \times a_x \times (x_2 - x_1)\} - B] \quad (2.6.10)$$

This equation provides solution for pressure variation in the x direction when density varies linearly with pressure.

Example 2.11. *A cylinder (Figure Ex. 2.11) containing oil of specific gravity 0.8 is accelerated at 5 m/s^2 towards (i) right and (ii) left. Under this condition the pressure gauge fitted at the right end shows a reading of 150 kPa.* ***Determine the pressure at the left end*** *if the tube is 2 m long.*

Since the specific gravity of the oil is constant, equation 2.6.9 can be used to solve this problem.

$$P_2 = P_1 - \rho \times a_x \times (x_2 - x_1)$$

Case (i) a_x is towards right and so +ve and $(x_2 - x_1) = 2$ m

$$1{,}50{,}000 = P_1 - 800 \times 5 \times 2,$$

$$\therefore \quad \mathbf{P_1} = 1{,}58{,}000 \text{ N/m}^2 = \mathbf{158 \text{ kPa.}}$$

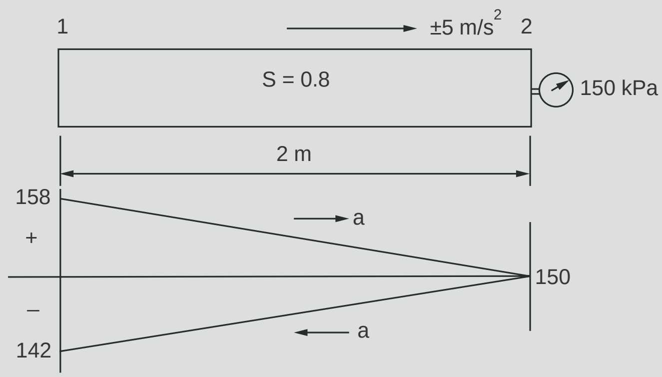


Figure Ex. 2.11

Case (ii) a_x is towards left and so –ve.

$$1{,}50{,}000 = P_1 + 800 \times 5 \times 2$$

$$\mathbf{P_1} = 1{,}42{,}000 \text{ Pa} = \mathbf{142 \text{ kPa}}. \text{ (note the unit of pressure used is N/m}^2\text{)}$$

Example 2.12. *A horizontal long cylinder containing fluid whose density varies as = 1.2 × 10^{-5} × P is accelerated towards right at 15 m/s^2.* ***Determine the pressure at a point which is 5 m to the left*** *of a point where the pressure gauge shows a reading of 250 kPa.*

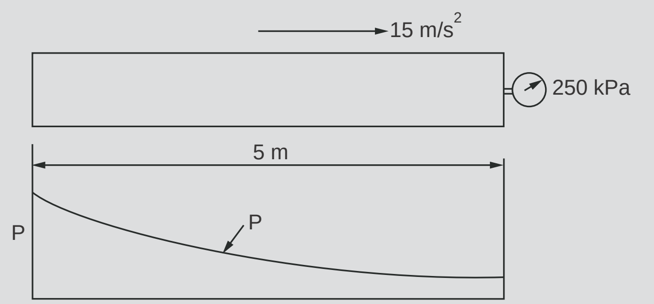


Figure Ex. 2.12

Equation 2.6.10 has to be used as density varies with pressure

$$P_2 = \{1/A\} \{AP_1 + B) \times \exp [- A \times a_x(x_2 - x_1)] - B\}$$

Here, $\quad B = 0, A = 1.2 \times 10^{-5}, a_x = 15 \text{ m/s}^2, x_2 - x_1 = 5 \text{ m}$

$$2{,}50{,}000 = \{1/1.2 \times 10^{-5}\} \times \{(1/1.2 \times 10^{-5} \times P_1) \times \exp [(-1.2 \times 10^{-5} \times 15 \times 5)]\}$$

$$\mathbf{P_1 = 250225 \text{ Pa} = 250.225 \text{ kPa.}}$$

Example 2.13. *A fluid of specific gravity 0.8 is filled fully in a rectangular open tank of size 0.5 m high, 0.5 m wide and 0.8 m long. The tank is uniformly accelerated to the right at 10 m/s²*. ***Determine the volume of fluid spilled from the tank***.

Since the fluid tank is accelerated in the horizontal direction $a_y = 0$.

Using equation 2.6.7, $\tan\theta = -a_x/(g + a_y) = -10/(9.81 + 0)$

With reference to the figure,

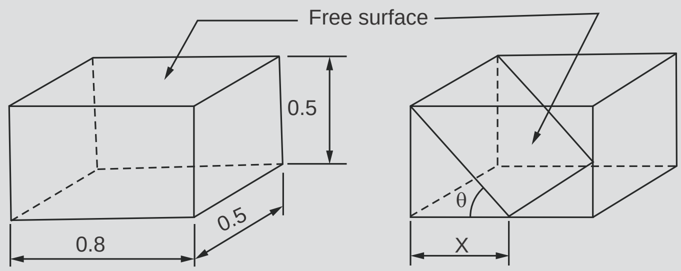


Figure Ex. 2.13

$$\tan\theta = -0.5/x = -10/9.81, \text{ So } x = 0.4905 \text{ m}$$

Remaining volume of fluid $= (1/2) \times 0.4905 \times 0.5 \times 0.5 = 0.0613125 \text{ m}^3$

Fluid tank volume or initial volume of fluid

$$= 0.5 \times 0.5 \times 0.8 = 0.2 \text{ m}^3$$

Fluid spilled $= 0.2 - 0.0613125 = \mathbf{0.1386875\ m^3}$

Example 2.14. *A U-tube as shown in figure filled with water to mid level is used to measure the acceleration when fixed on moving equipment. Determine the acceleration a_x as a function of the angle θ and the distance A between legs.*

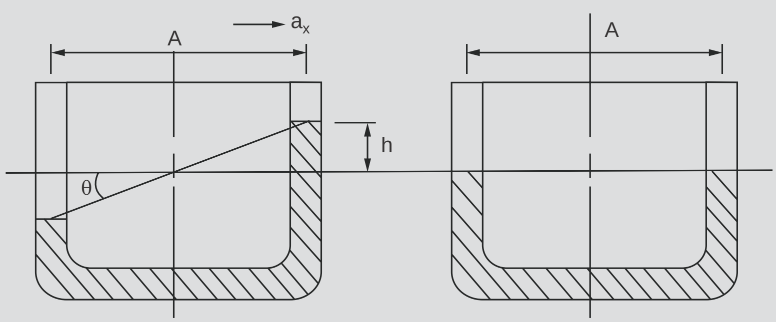


Figure Ex. 2.14

This is similar to the formation of free surface with angle θ, using eqn. 2.6.7

$$\tan\theta = -a_x/(g + a_y). \text{ As } a_y = 0, \tan\theta = -a_x/g$$

The acute angle θ will be given by, $\theta = \tan^{-1}(a_x/g)$

$$\mathbf{a_x} = g \times \tan\theta, \text{ As } \tan\theta = 2h/A, \mathbf{h = A\, a_x/2g}$$

Example 2.15. *Water is filled in a rectangular tank of 0.5 m high, 0.5 m wide and 0.8 m long to a depth of 0.25 m.* ***Determine the acceleration*** *which will cause water to just start to spill and also when half the water has spilled.*

Since the tank is half full, at the time of spill the free surface will be along the left top and right bottom. The angle of the free surface with horizontal at the time of starting of spill is $a_x/g = \tan\theta = 0.5/0.8 = 0.625$

$\therefore$ $\mathbf{a_x} = 0.625 \times 9.81 = \mathbf{6.13\ m/s^2}$

When half the water has spilled, the water will be at 0.4 m at bottom

$$\tan\theta = 0.5/0.4 = 1.25$$

$\therefore$ $a_x = 1.25 \times 9.81 = \mathbf{12.26\ m/s^2}$

Example 2.16. *A tank containing 1.5 m height of water in it is accelerating downwards at 3.5 m/s*^{2}*.* ***Determine the pressure*** *at the base of the tank above the atmospheric pressure.* ***What should be the acceleration*** *if the pressure on the base to be atmospheric?*

Using equation 2.6.2,

$$(dP/dy) = -(\gamma + \rho \times a_y);\ dP = -(\gamma + \rho \times a_y)\,dy$$

$$(P_2 - P_1) = (y_1 - y_2)(\gamma + \rho \times a_y)$$

$(y_1 - y_2) = 1.5$ m, as a_y is downwards and hence negative

$$(P_2 - P_1) = 1.5\,(\gamma - \rho \times a_y)$$

$$(P_2 - P_1) = 1.5\,(9810 - 1000 \times 3.5)/1000 = \mathbf{9.465\ kN/m^2}\ \text{(above atmospheric)}$$

At static conditions, the pressure would have been

$1.5 \times 1000 \times 9.81/1000 = 14.715\ kN/m^2$ (above atmospheric)

for the pressure at base to be atmospheric,

$P_2 - P_1 = 0 = 1.5\,[9810 - 1000\,a_y]$. i.e., $\mathbf{a_y = 9.81\ m/s^2}$

This is the situation of weightlessness, where $\mathbf{a_y = -g}$, the weight of water is zero

2.7 FORCED VORTEX

When a cylindrical container filled with a liquid is rotated about its axis, the liquid as a whole rotates. The angular velocity is the same at all points, but the linear velocity varies along the radius. The variation of the linear speed with radius causes a concave free surface to form, with fluid moving away from the centre.

The fluid rotates as a rigid body with velocity of $\omega \times r$ at a radius r (ω being the angular velocity). Fluid particles rotating in concentric circle with velocities of $r \times \omega$ along the tangent to the circles form a forced vortex. It is assumed that there is no relative sliding between layers.

The pressure variations and gradients caused by the rotation can be determined using equations 2.6.1 – 2.6.3. An element of fluid as shown in Fig. 2.7.1 is considered. The radius r is taken as positive along the outward direction.

Equation 2.6.1 gives

$$dP/dr = -\rho \times a_s = \rho \times r \times \omega^2 \text{ as } \theta = 0, \text{ and } a_r = -r \times \omega^2$$

$$dP/dy = -\gamma \text{ as } a_y = 0$$

Using the first equation, the pressure change along r_1 and r_2 is obtained as

$$(P_{r_2} - P_{r_1}) = \rho \times (\omega^2/2) \times (r_2^{\,2} - r_1^{\,2}) \qquad (2.7.1)$$

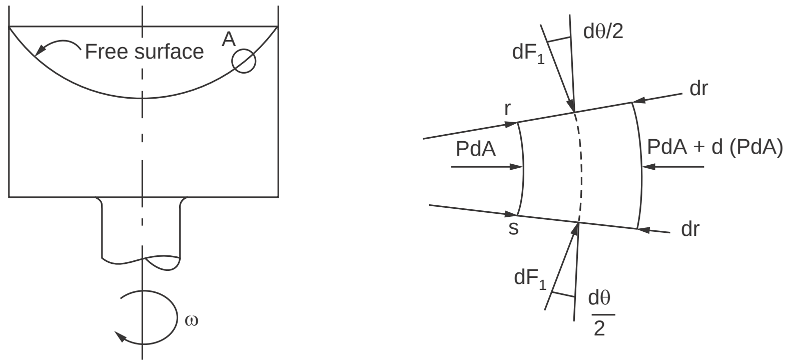


Figure 2.7.1 Free body diagram of rotating fluid element

From centre to any radius r,

$$(P_r - P_o) = \rho \times (\omega^2/2) \times r^2 = \rho \times (\omega r^2)/2 \quad (2.7.2)$$

If the pressure at the centre of the base or any radius is known, the pressure at all other points on the base can be calculated.

$$(P - P_b) = -\gamma \times (y - y_o) = -\gamma \times y, \text{ taking } y_o \text{ as the datum}$$

Here P is the pressure at the surface at any radius and P_b is the pressure at the base at the same radius and y is the height of liquid at that location. This gives

$$P_b = P + \gamma \times y \quad (2.7.3)$$

In order to determine the value of slope at any radius equation 2.6.7 is used. The surface profile is shown in Fig. 2.7.2.

$$\tan\theta = -a_x / (g + a_y)$$

For a rotating cylinder,

$$a_y = 0,\ a_x = -r \times \omega^2 \text{ and } \tan\theta = dy/dr = r\omega^2/g$$

Hence, $\quad dy/dr = r \times \omega^2/g \quad \therefore \quad dy = r\,dr \times \omega^2/g$

Integrating from centre to radius r and rearranging,

$$y_2 - y_1 = \frac{\omega^2}{2g}[r_2^2 - r_1^2] \quad (2.7.4)$$

$$y = y_o + [(\omega \times r)^2/(2 \times g)] \quad (2.7.4\ (a))$$

where y_o is the height of liquid at the centre. This shows that the free surface is a paraboloid. The height y at any radius depends on the angular velocity, radius and g.

In forced vortex, v/r = constant as $v = \omega.r$ and ω is constant

If $g = 0$, (space application) then $y \to \infty$ and the free surface becomes cylindrical or the liquid adheres to the surface in a layer.

A free vortex forms when the container is stationary and the fluid drains at the centre as in the case of draining a filled sink. Here the fluid velocity is inversely proportional to the radius (volume flow depends on area), the velocity near the centre being the highest ($v \times r$ = constant).

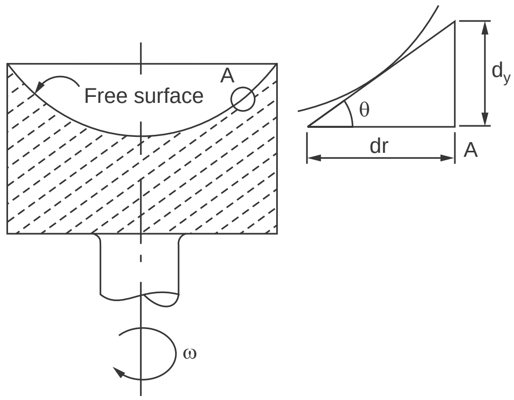
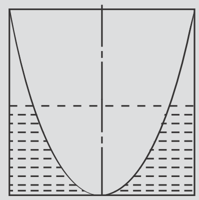


Figure 2.7.2 Forced Vortex—Free surface

Example 2.17. *A tall cylinder of 1 m dia is filled with a fluid to a depth of 0.5 m and rotated at a speed such that the height at the centre is zero. Determine the speed of rotation.*



It is to be noted here that the volume of a paraboloid of height h is equal to the volume of cylinder of half its height and the same radius. Hence the height at the outer radius is 1 m. Using equation 2.7.4

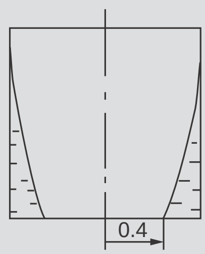
$y = y_o + [(\omega^2 \times r^2)/(2 \times g)]$, substituting the values,

$1 = 0 + (\omega^2 \times 0.5^2)/(2 \times 9.81)$,

$\therefore \quad \omega = 8.86$ rad/s

$\omega = 2\pi N/60$; $N = (8.86 \times 60)/(2 \times \pi) =$ **84.6 rpm**

Example 2.18. *Water is filled partially in a cylinder of 1 m dia and rotated at 150 rpm. The cylinder is empty at the bottom surface up to a radius of 0.4 m.* ***Determine the pressure at the extreme bottom edge****. Also calculate the height of liquid at the edge.*



Equation 2.7.1 is applicable.

$$(P_{r_2} - P_{r_1}) = \rho \times (\omega^2/2) \times (r_2^{\,2} - r_1^{\,2})$$

$$P_{r_1} = 0 \text{ (gauge) at } r_1 = 0.4,\ r_2 = 0.5 \text{ m}$$

$$\omega = 2\,\pi\,N/60 = 2 \times \pi \times 150/60 = \mathbf{15.71\ rad/s.}$$

$$P_{r_2} - 0 = 1000 \times (15.71^2/2) \times (0.5^2 - 0.4^2) \text{ N/m}^2$$

$$P_{r_2} = 11106.184 \text{ N/m}^2 \text{ (gauge)},$$

Using equation 2.7.4,

$$\mathbf{y_2 - y_1} = \frac{\omega^2}{2g}\,[r_2^{\,2} - r_1^{\,2}]$$

$$= 15.71^2\,(0.5^2 - 0.4^2)/2 \times 9.81 = \mathbf{1.132\ m}$$

SOLVED PROBLEMS

Problem 2.1. *Four pressure gauges A, B, C and D are installed as shown in figure in chambers 1 and 2. The outside pressure is 1.01 bar. The gauge A reads 0.2 bar, while the gauge*

B reads – 0.1 bar. Determine the pressures in chamber 1 and chamber 2 and the reading of gauge C and D.

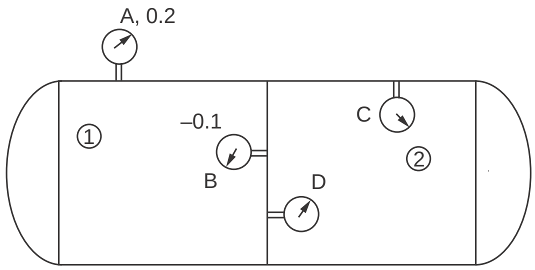


Figure P. 2.1

pressure in chamber 1 = atmospheric *pr* + reading of gauge A

= 1.01 + 0.2 = 1.21 bar

pressure in chamber 2 = pressure in chamber 1 + reading of gauge B

= 1.21 – 0.1 = 1.11 bar

atmospheric pressure = pressure in chamber 2 + reading of gauge C

reading of gauge C = 1.01 – 1.11 = – 0.1 bar

Gauge D reads the pressure in chamber 1 as compared to chamber 2

gauge reading D = pressure in chamber 1 – pressure in chamber 2

= 1.21 – 1.11 = 0.1 bar (opposite of gauge B)

Problem 2.2. *The pressures in chambers A, B, C and D as shown in Fig. P.2.2 are 3.4, 2.6, 1.8 and 2.1 bar respectively. Determine the readings of gauges 1 to 6.*

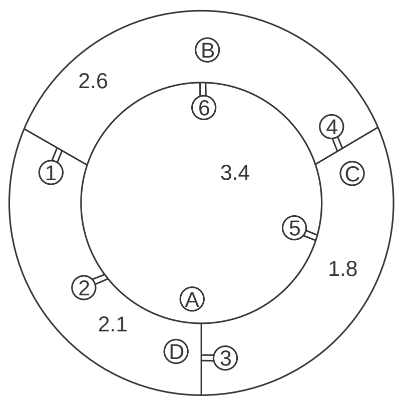


Figure P. 2.2

Gauge 1. This gauge measures the pressure in chamber B and the gauge is situated in chamber D, denoting the gauge reading by the corresponding suffix,

$$P_B = P_1 + P_D, \quad 2.6 = P_1 + 2.1$$

$\therefore$ $P_1 = 0.5$ bar

gauge 1 should show 0.5 bar

Gauge 2. This gauge measures the pressure in chamber A. The gauge is in chamber D.

$$P_A = P_2 + P_D, \qquad 3.4 = P_2 + 2.1$$

$\therefore$ $P_2 = 1.3$ bar,

By similar procedure the reading of gaug 3, 4, 5, 6 are obtained as below:

P_3: $P_D = P_3 + P_C$, $2.1 = 1.8 + P_3$ $\therefore$ $P_3 = 0.3$ bar,

P_4: $P_C = P_4 + P_B$, $1.8 = 2.6 + P_4$ $\therefore$ $P_4 = -0.8$ bar,

P_5: $P_C = P_5 + P_A$, $1.8 = 3.4 + P_5$ $\therefore$ $P_5 = -1.6$ bar,

P_6: $P_B = P_6 + P_A$, $2.6 = 3.4 + P_6$ $\therefore$ $P_6 = -0.8$ bar

The gauge readings show the pressure difference between the chambers connected and not absolute pressures. For example reading of $P_5 = -1.6$ bar. Such a vacuum is not possible.

Problem 2.3. *A container has h_w cm of water over which h_k cm of kerosene of specific gravity 0.9 floats. The gauge pressure at the base was 4 kN/m^2. If the ratio of $h_w/h_k = 1.25$,* ***determine the heights of the columns.***

Summing the pressures due to the two columns, (As $h_k = h_w/1.25$)

$$h_w \times 1000 \times 9.81 + [(h_w/1.25) \times 900 \times 9.81] = 4000$$

$$\therefore \qquad \mathbf{h_w} = 0.2371 \text{ m, or } \mathbf{23.71 \text{ cm}}$$

$$\therefore \qquad \mathbf{h_k} = 0.1896 \text{ m, or } \mathbf{18.96 \text{ cm}}$$

Problem 2.4. *A U-tube open to atmosphere is first filled to a sufficient height with mercury. On one side water of volume equal to 200 mm column over which kerosene of density 830 m^3/kg of volume equal to 250 mm column are added.* ***Determine the rise in the mercury column*** *in the other limb.*

Let the rise in mercury column be h, Then

$$h \times 13600 \times 9.81 = (0.2 \times 9.81 \times 1000) + (0.25 \times 9.81 \times 830)$$

Solving, $\qquad$ **h = 0.02996 m** or about **3 cm**.

Problem 2.5. *The pressure due to the atmosphere at the earths surface is 101.3 kPa.* ***Determine pressure at 10,000 m altitude****, assuming that the condition of air can be represented by the law $Pv^{1.4}$ = constant. Temperature at ground level is 27°C.*

The law can be written as $[P/(\rho g)^{1.4}]$ = const. or $P/\gamma^{1.4}$ = const.

Denoting the index as k, $P_1\,\gamma_1^{-k} = P_2\gamma_2^{-k} = P\,\gamma^{-k}$

Let the specific weight at altitude y be γ. Then $\gamma = (p/p_1)^{1/k}\gamma_1$

The hydrostatic equation is $dP/dy = -\gamma$ or $dP = -\gamma\, dy$ $\qquad$ ... (A)

The equation Pv^k = constant can be rewritten as,

$$P/\gamma^k = P_o/\gamma_o^{\,k}$$

$$\therefore \qquad \gamma^k = (P/P_o)\gamma_o^{\,k}$$

or

$$\gamma = P^{1/k}\, P_o^{-1/k}\, \gamma_o = \gamma_o P_o^{-1/k}\, P^{1/k},$$

substituting in A and separating variables

$$\therefore \qquad dP/P^{1/k} = -P_o^{-1/k}\,\gamma_o\, dy, \text{ integrating between limits } y_o \text{ and } y$$

$$[k/(k-1)]\;[P^{(k-1)/k} - P_o^{(k-1)/k}] = P_o^{-1/k}\,(gP_o/RT_o)\,(y - y_o)$$

$$= -P_o^{-1/k}\,(gP_o/RT_o)\,y$$

$$P^{(k-1)/k} = \{P_o^{(k-1)/k} - [((k-1)/k)]\;P_o^{(k-1)/k}\,(g/RT_o)\,y\}$$

$$P = P_o\,\{1 - [(k-1)/k]\,(g/RT_o)\,y\} \qquad \textbf{(P. 2.5.1)}$$

The values at various altitudes are calculated using the equation and compared with air table values. $T_o = 300$ K

Altitude, m	**1000**	**2000**	**4000**	**6000**	**8000**	**10000**	**20000**
Calculated P/P_o	**0.89**	**0.79**	**0.61**	**0.47**	**0.35**	**0.25**	**0.03**
From air tables	**0.8874**	**0.7848**	**0.6086**	**0.4657**	**0.3524**	**0.2622**	**0.0561**

Problem 2.6. *In a fresh water lake the specific weight of water γ is found to vary with depth y as $\gamma = K\,\gamma_o/(K + \gamma_o y)$ where K is the bulk modulus. At the surface $\gamma_o = 9810$ N/m³ and P_o = 101.3 kPa. If the pressure measured at 1500 m was 14860 kPa,* ***determine the value of K.***

$$dP/dy = -\gamma = -K\,\gamma_o/(K + \gamma_o\,y)$$

$$\therefore \quad dP = \{-K\,\gamma_o/(K + \gamma_o\,y)\}\,dy = -K\,\gamma_o dy/(K + \gamma_o\,y)$$

Integrating between the surface and the depth

$$P - P_o = -(K.\gamma_o/\gamma_o)\ln[(K + \gamma_o\,y)/(K + \gamma_o \times 0)] \qquad \textbf{- (P .2.6)}$$

$$= -K\ln[(K + \gamma_o\,y)/K]$$

$$\therefore \quad P = P_o - K\ln[(K + \gamma_o\,y)/K]$$

Note : y is –ve as measured downwards, substituting the given values

$$14860 \times 10^3 = 101.3 \times 10^3 - K\ln\{[K - (9810 \times 1500)]/K\}$$

$$14758.5 \times 10^3 = -K\ln[K - (14.715 \times 10^6)/K]$$

Solving by trial (generally K is of the order of 10^9)

Assumed value of K	2×10^9	2.5×10^9	3×10^9
RHS	14769×10^3	14758.5×10^3	14751×10^3

2.5×10^9 gives the value nearest to LHS

$$\therefore \quad \mathbf{K = 2.5 \times 10^9\ N/m^2}$$

The specific weight at this location is

$$\gamma = (2.5 \times 10^9 \times 9810)/\{2.5 \times 10^9 + [9810 \times (-1500)]\}$$

$$= 9868.08\ \text{N/m}^2$$

The pressure at various depths are tabulated,

Depth, m	1000	2000	4000	6000
P, kPa	9930	19798	39652	59665

Problem 2.7 *A chemical reaction vessel of the shape given in figure is full of water with the top of the longer limb sealed and the top of the smaller limb open to atmosphere.* ***Determine the pressure at B, the top of the longer limb.*** *The density of water is 992 kg / m³ at this condition. Using steam table indicate whether water will boil at this point if temperature is 30° C.*

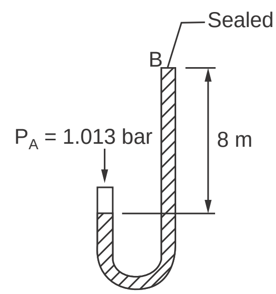


Figure P. 2.7

$$\mathbf{P_B} = 101.3 \times 10^3 - (8 \times 992 \times 9.81) = 23447.8 \text{ N/m}^2 \text{ or } \mathbf{23.45 \text{ kPa}}$$

At 30°C, the saturation pressure as read from steam table is 4.24 kPa, hence there will be no boiling at *B*. If the water just begins to boil, what should be the length of the limb?

$$4.246 \times 10^3 = 101.3 \times 10^5 - h \times 992 \times 9.81. \text{ Solving, } h = 9.973 \text{ m}$$

Problem 2.8. *A manometer of the shape shown in figure has limb A filled with water of specific gravity 1 and the other limb with oil of specific gravity 0.95. The area of the enlarged mouth portion is 50 times the area of the tube portion. If the pressure difference is 22 N/m², **calculate the height h**.*

Let the water level when $P_1 = P_2$ be at *x-x*. Then the pressures at these points are equal as the same liquid fills the volumes below *x-x*. Let the height of oil on the left limb above *x-x* be *H*. The height of water in the other side will be (γ_o/γ_w) H or 0.95 H (in this case). Now let pressure P_2 act on the oil side limb and let the level of water below move down by distance *h* to the level *yy*. The pressure on both limbs at the level *yy* are equal. Now the liquid heights in each limb can be calculated.

The rise of level in the water side will be $(a/A).h$ (As the filled volumes remain the same).

The fall of oil level in the other limit will be also $(a/A).h$

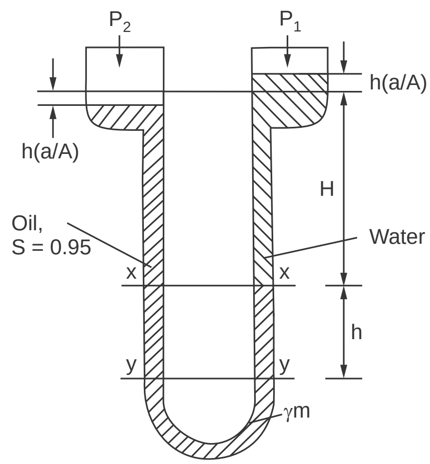


Figure P. 2.8

P_y is now calculated. Consider water side

$$P_y = P_1 + 9.81 \times 1000[0.95H + h + h(a/A)]$$

On the oil side, $P_y = P_2 + 9.81 \times 950[H + h - h(a/A)]$, As $P_2 - P_1 = 22$

$$22 = 9.81\,[-950H - 950h + (950 \times h/50) + 950H + 1000h + (1000 \times h/50)]$$

$$= 9.81\,[50h + 19h + 20h] = 9.81 \times 89h$$

$\therefore$ $22 = 9.81 \times 89h$, $\therefore$ **h = 0.0252 m or 25.2 mm**

A manometer with constant limb area will give a reading of only 2.24 mm of water. Thus the sensitivity is improved appreciably by this arrangement.

Problem 2.9. *A U-tube manometer has both its limbs enlarged to 25 times the tube area. Initially the tube is filled to some level with oil of specific weight γ_m. Then both limbs are filled with fluid of specific weight γ_s to the same level, both limbs being exposed to the same pressure. When a pressure is applied to one of the limbs the manometric fluid rises by h m. Derive an expression for the pressure difference in the limbs. In both cases assume that the liquid level remains in the enlarged section.*

Consider stationary condition, when both pressures are equal. Let the fluid with specific weight γ_s be having a height *H*.

After pressures are applied, consider pressures at y as the reference.

Consider the left limb:

$$P_y = P_2 + \left(H + \frac{h}{2} - \frac{h}{2}\frac{a}{A}\right)\gamma_s$$

Consider the right limb:

$$P_y = P_1 + \frac{h}{2}\,\gamma_m + \left(H - \frac{h}{2} + \frac{h}{2}\frac{a}{A}\right)\gamma_s$$

Equating and solving

$$\therefore \quad P_2 - P_1 = \frac{h}{2}\,\gamma_m + \left(H - \frac{h}{2} + \frac{h}{2}\frac{a}{A}\right)\gamma_s - \left(H + \frac{h}{2} - \frac{h}{2}\frac{a}{A}\right)\gamma_s$$

$$= \frac{h}{2}\,\gamma_m + \gamma_s\left(h\frac{a}{A} - h\right)$$

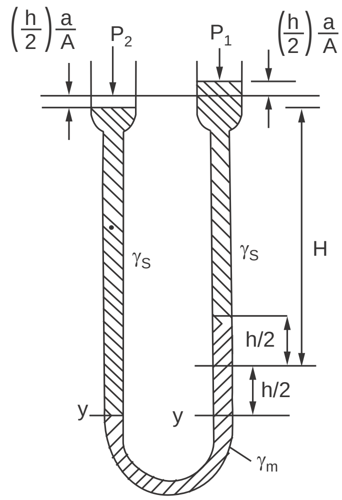


Figure P. 2.9

Let $P_2 - P_1 = 40$ N/m^2, $\gamma_m = 1000 \times 9.81$, $\gamma_s = 0.9 \times 1000 \times 9.81$

$$\therefore \qquad 40 = \frac{h}{2} \times 9810 + 0.9 \times 9810\left(\frac{h}{25} - h\right) = 1334.2\,h.$$

$$\therefore \qquad h = 0.02998 \text{ m or } 30 \text{ mm}.$$

If a U-tube with water was used the deflection will be of the order of 4 mm.

Problem 2.10. *A U-tube is filled first with a fluid of unknown density. Over this water is filled to depths as in figure. Lubricating oil of specific gravity 0.891 is filled over the water column on both limbs. The top of both limbs are open to atmosphere. Determine the density of the unknown fluid (dimensions in mm).*

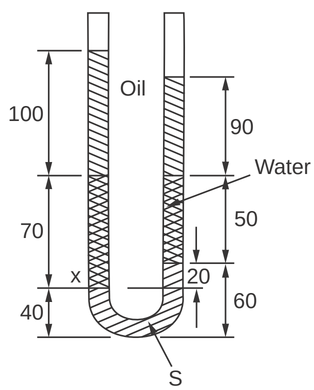


Figure P. 2.10

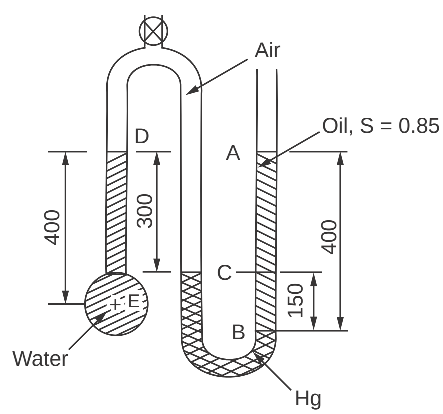


Figure P. 2.11

Consider level *x-x* in figure, on the left limb the pressure at this level is

$$P_{XL} = (70 \times 9.81 \times 1000/1000) + (100 \times 9.81 \times 891/1000)$$

On the right limb at this level,

$$P_{XR} = [(20/1000) \times 9.81 \times \rho] + [(50/1000) \times 9.81 \times 1000] + [(90/1000) \times 9.81 \times 891]$$

equating and solving, $\mathbf{\rho = 1445.5\ kg/m^3}$

Note: Division by 1000 is to obtain specific gravity.

Problem 2.11. *A compound manometer is used to measure the pressure in a pipe E carrying water. The dimensions are shown in Figure P.2.11. Determine the pressure in the pipe.*

Calculations can be started from the open limb where the pressure is known

$P_A = P_{atm} = 1.013$ bar = 101300 Pa

$P_B = P_A + \rho_o g h_o = 1.013 \times 10^5 + (850 \times 9.81 \times 0.4) = 104635.4$ Pa

$P_C = P_B - \rho_{Hg} g\, h_{Hg} = 104635.4 - (13600 \times 9.81 \times 0.15) = 84623$ Pa

$P_D = P_C = 84623$ Pa (300 mm air column does not contribute much)

$\mathbf{P_E} = P_D + \rho_w g h_w = 84623 + (1000 \times 9.81 \times 0.4) = 88547$ *Pa* or **88547 kPa**

Problem 2.12 *A U-tube with a distance of 120 mm between the limbs is filled with a liquid to mid level for use as a crude accelerometer fixed on a moving vehicle. When the vehicle is accelerated the difference in level between the limbs was measured as 32 mm.* ***Determine the acceleration.***

Let the angle connecting the liquid surfaces in the limbs be θ,

Then $\tan\theta = (h/2)/(L/2) = h/L$

using equation 2.6.7, $\tan\theta = a_x/(g + a_y)$

as $a_y = 0$, $\tan\theta = a_x/g$ or $\mathbf{a_x} = g(h/L) = 9.81 \times (0.032/0.12) = \mathbf{2.616\ m/s^2}$

Problem 2.13. *A container in the shape of a cube of 1 m side is filled to half its depth with water and placed on a plane inclined at 30° to the horizontal. The mass of the container is 50.97 kg. The coefficient of friction between the container and the plane is 0.30. Determine the angle made by the free surface with the horizontal when the container slides down. What will be the angle of the free surface if the container is hauled up with an acceleration of 3 m/s² along the plane.*

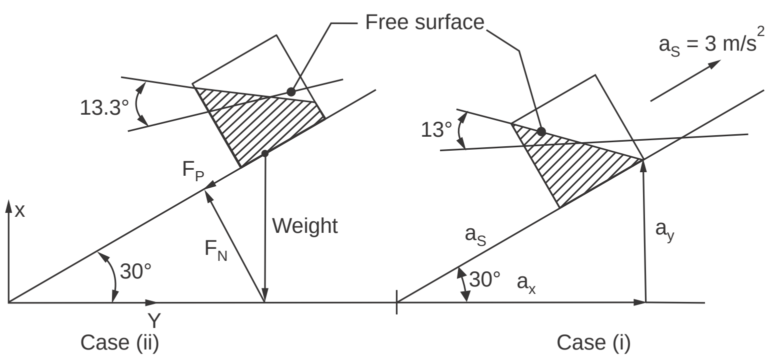


Figure P. 2.13

Irrespective of the inclination if the acceleration along and perpendicular to the horizontal are calculated, then the angle made by the free surface can be obtained using equation 2.6.7

$\tan\theta = -[a_x/(g + a_y)]$

The total mass $= (1 \times 1 \times 0.5 \times 1000) + 50.97 = 550.97$ kg

Case (*i*) Force along the plane

$$F_x = 550.97 \times 9.81 \times \cos 60 = 2702.5 \text{ N}$$

Force normal to plane $F_y = 550.97 \times 9.81 \times \sin 60 = 4680.9$ N

The friction force acting against F_x is $F_y\ \mu = 4680.9 \times 0.3 = 1404.26$ N

Net downward force along the plane $= F_x - F_y.\mu = (2701.5 - 1404.26) = 1298.24$ N

Acceleration along the plane, $a_s = F/m = 1298.24/550.97 = 2.356$ m/s^2

The component along horizontal, $a_x = 2.356 \times \cos 30 = -2.041$ m/s^2

The component along vertical, $a_y = -1.178\ m/s^2$ (downwards)

$$\mathbf{\tan\theta} = (-2.041)/(9.81 - 1.178) = +0.2364$$

$\therefore$ $\theta = +\mathbf{13.3°}$

Case (ii) $a_x = 3\cos 30 = 2.598$ m/s^2, $a_y = 3\sin 30 = 1.5$ m/s^2

$$\tan\theta = -[2.598/(9.81 + 1.5)]$$

$\therefore$ $\boldsymbol{\theta = -12.94°}$ **with horizontal**

Problem 2.14, *A tank 0.4 m × 0.2 m size and of height 0.4 m is filled with water upto a depth of 0.2 m. The mass of the container is 10 kg. The container slides without friction downwards on a surface making 30° with the horizontal. Determine the angle the free surface makes with the horizontal. If the tank is moved up with the same acceleration* ***determine the slope of the free surface.***

Refer Fig. P.2.13

Total mass $= 1000\ (0.4 \times 0.2 \times 0.2) + 10 = 26$ kg

Force along the surface $= 26 \times 9.81 \times \cos 60 = 127.53$ N

Acceleration $a_s = 127.5/26 = 4.905$ m/s^2

Acceleration along x direction $= 4.905 \times \cos 30 = -4.2478$ m/s^2

Acceleration along y direction $= 4.905 \times \sin 30 = -2.4525$ m/s^2

$\tan\theta = -[-4.2478/(9.81 - 2.4525)]$ $\therefore \theta = 30°$, same as the slope of the plane.

This is an interesting result. Try to generalise assuming other angles of inclination.

When moving up, with the same acceleration,

$$a_x = 4.2478,\ a_y = 2.4525,$$

$$\tan\theta = -[4.2478/(9.81 + 2.4525)]$$

$\therefore$ $\boldsymbol{\theta = -19.1°}$**, slope = 0.3464**

Problem 2.15. *An aircraft hydraulic line pressure is indicated by a gauge in the cockpit which is 3 m from the line. When the aircraft was accelerating at 10 m/s² at level flight, the gauge indicated 980 kPa.* ***Determine the pressure at the oil line using equation 2.6.9.*** *Specific gravity of oil is 0.9.*

$$P_2 = P_1 - (\rho.a_x)\ (x_2 - x_1)$$

$$\mathbf{P_2} = 980 \times 10^3 - [(900 \times 10) \times (-3)] = 1007 \times 10^3 \text{ Pa or } \mathbf{1007\ kPa}$$

Chapter 2

Problem 2.16. *A tank as in Fig.P.2.16 is filled with water. The left side is vented to atmosphere.* ***Determine the acceleration*** *along the right which will cause the pressure at A to be atmospheric.*

$$P_2 = P_1 - \rho.a_x (x_2 - x_1)$$

For the pressure at A to be atmospheric, there should be a reduction of 4 m of water column due to the acceleration.

Initial pressure all over the surface = P_{atm} + 4 m of water head

$$\tan \theta = 4/4 = a_x/g$$

$$\therefore \quad a_x = 9.81 \text{ m/s}^2$$

alternately, the general equation can be used, choosing B as origin,

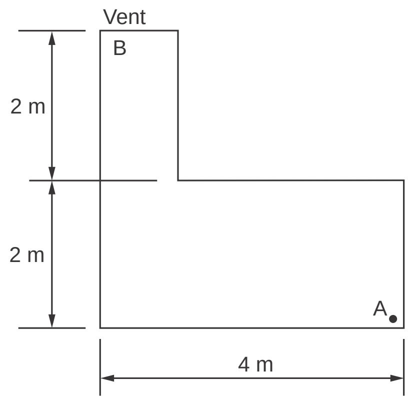


Figure P. 2.16

$$P = -[\gamma (a_x/g)x] - \gamma y$$

$$= -9810 (a_x/g) x - 9810y$$

$$P = -1000 a_x x - 9810 y$$

In this case, $P = 0, x = 4, y = -4$

$\therefore \quad 0 = -4000a_x + 4 \times 9810, \quad \therefore$ $\mathbf{a_x = 9.81 \text{ m/s}^2}$

Problem 2.17. *A fully air conditioned car takes a curve of radius 250 m at 90 kmph. The air within the car can be taken to move as a solid. A child holds a balloon with a string and it is vertical along straight road.* ***Determine the direction of the string*** *measured from the vertical during the turn.*

The balloon will move opposite to the pressure gradient at the location,

$$\tan \theta = a_x / (g + a_y),$$

During the travel along the curve, $a_x = r.\omega^2$, $a_y = 0$

speed, $v = 90 \times 1000/3600 = 25$ m/s

$$\omega = (v/\pi D).2\pi = (25 \times 2\pi)/(\pi/500) = 1/10 \text{ rad/s}$$

$\therefore \quad \tan \theta = 250 \times (1/10^2) \times (1/9.81) = 0.255, \quad \therefore$ $\mathbf{\theta = 14.3°}$

As pressure increases outwards, the balloon will turn inwards by 14.30 to the vertical.

Problem 2.18. *A U-tube shown in figure is filled with water at 30°C and is sealed at A and is open to atmosphere at D.* ***Determine the rotational speed along AB*** *in rad/s is the pressure at the closed end A should not fall below the saturation pressure of water at this temperature.*

From steam tables at 30°C, saturation pressure is read as 0.04241 bar

using equation 2.7.1 as the situation is similar to forced vortex

$$(P_{r_2} - P_{r_1}) = \rho(\omega^2/2) (r_2^2 - r_1^2)$$

$$r_1 = 0, \; P_{r_2} = 1.013 \text{ bar},$$

$$P_{r_1} = 0.04241 \text{ bar}, \rho = 1000 \text{ kg/m}^3, r_2 = 0.1 \text{ m}$$

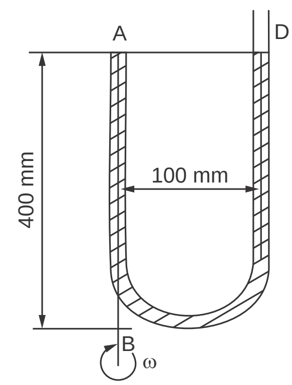


Figure P. 2.18

substituting, $(1.013 - 0.042421) \times 10^5 = (1000/2).\omega^2 \times 0.1^2$

$\therefore$ $\boldsymbol{\omega = 139.33}$ **rad/s**

Problem 2.19. *Gas centrifuges are used to produce enriched uranium. The maximum peripheral speed is limited to 300 m/s. Assuming gaseous uranium hexafluride at 325°C is used,* ***determine the ratio of pressures at the outer radius to the centre****. The molecular mass of the gas is 352. Universal gas constant = 8314 J/kgK.*

Equation 2.6.1 namely $dP/ds = -r \sin\theta + \rho\, a_s$

reduces when s is horizontal and in the case of rotation to

$dp/dr = \rho\, a_s = \rho r\omega^2$, For a gas $\rho = P/RT$, substituting

$dP/P = (\omega^2/RT)\, dr$, integrating between limits

$$\ln (P_2/P_1) = (\omega^2/2RT) r^2 = V^2/2RT \qquad \textbf{(P. 2.19)}$$

$V = 300$ m/s, $R = 8314/352$, $T = 325 + 273$ substituting

$$\ln (P_2/P_o) = 300^2 \times 352/[2 \times 8314 \times (325 + 273)] = 3.186$$

$\therefore$ $\mathbf{P_2/P_o = 24.19}$

Problem 2.20. *A container filled with oil of density 800 kg/m³ is shown in figure. The small opening at A is exposed to atmosphere.* ***Determine the gauge pressures at B, C, D and E*** *when (i) $a_x = 3.9$ m/s² and $a_y = 0$ (ii) $a_x = 2.45$ m/s² and $a_y = 4.902$ m/s².*

Determine also the values of a_x and a_y if $P_A = P_B = P_C$

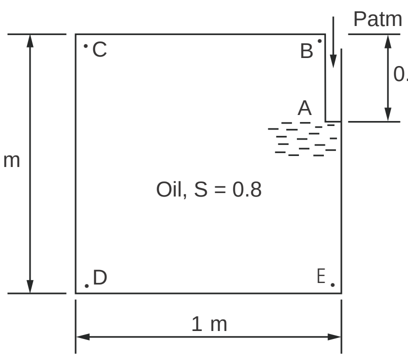


Figure P. 2.20

Case (i). Pressure at A is atmospheric in all cases. $a_x = 3.9$ m/s², $a_y = 0$, when accelerated along the x direction, the imaginary free surface angle θ is given by (as $a_y = 0$)

$$\tan\theta = -a_x/g = 3.9/9.81$$

$\therefore$ The slope is 3.9/9.81 = 0.39755 as the length is 1 m, C' will be above C by 0.0976 m head of fluid. As compared to A, B is at 0.3 liquid head above *i.e.*, P_B is lower

$\therefore$ $\mathbf{P_B} = -0.3 \times 9.81 \times 800 = \mathbf{-2354.4\ Pa}$

$\mathbf{P_C} = P_{C'} - P_C = 9.81 \times 800\,[0.39755 - 0.3] = \mathbf{765\ Pa}$

$\mathbf{P_D} = P_C + (1 \times 9.81 \times 800) = \mathbf{8613\ Pa}$

$\mathbf{P_E} = 0.7 \times 0.8 \times 9.81 = \mathbf{5493\ Pa}$

All the pressures are gauge pressures with atmospheric pressure as reference pressure

Case (ii). $a_x = 2.45\ m/s^2$, $a_y = 4.902\ m/s^2$

In this case $\tan\theta = a_x/(a_y + g)$ or the slope is 2.45/(9.81 + 4.902) = 0.16653

At B, the pressure is less than at A by a column of 0.3 m of liquid, but the weight is increased by the upward acceleration.

$\therefore$ $\mathbf{P_B} = 0.3 \times 800(9.81 + 4.902) = \mathbf{3531\ Pa}$

Now C' is at 0.16653 m above A. C is above C' by (0.3 – 0.1665) m

$$\mathbf{P_C} = -0.3 \times 800(9.81 + 4.902) \times (0.3 - 0.16653) = \mathbf{-1570\ Pa}$$

$$\mathbf{P_D} = P_C - [1 \times 0.3 \times 800(9.81 + 4.902)] = \mathbf{10200\ Pa}$$

$$\mathbf{P_E} = 0.7 \times 800(9.81 + 4.902) = \mathbf{8239\ Pa}$$

Case (iii). If $P_A = P_B$ then the weight of the liquid column should be zero due to the acceleration a_y,

$\therefore$ $\mathbf{a_y} = -g$ or $\mathbf{9.81\ m/s^2}$ upwards,

If $P_C = P_B$, automatically $B\ C$ should be constant pressure surface. So $\mathbf{a_x = 0}$

Problem 2.21. *At an instant an aircraft travelling along 40° to the horizontal at 180 m/s, decelerates at 4 m/s². Its path is along a concave upward circular curve of radius 2600 m.* ***Determine the position of the free surface of the fuel in the tank.***

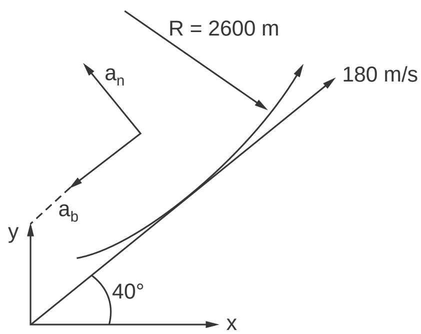


Figure P. 2.21

The path of the aircraft is shown in figure. The accelerations are indicated. The acceleration towards the centre of the curve is given by

$$a_x = V^2/R = 180^2/2600$$

$$= 12.5\ \text{m/s}^2\text{, towards centre}$$

The acceleration along the tangent $a_t = -4$ m/s²

The components along x and y directions are

$$a_x = -4 \cos 40 - 12.5 \sin 40 = -11.09\ \text{m/s}^2$$

$$a_y = -4 \sin 40 + 12.5 \cos 40 = 7.01\ \text{m/s}^2$$

$$\tan\theta = a_x / (a_y + \text{g}) = 11.09/(7.01 + 9.81) = 0.659,$$

$\therefore$ $\boldsymbol{\theta} = \mathbf{33.4°}$,

Slope of the free surface of fuel = 0.659

Problem 2.22. *A tanker lorry of cylindrical shape 6 m in length is filled completely with oil of density 830 kg/m³. The lorry accelerates towards the right. If the pressure difference between the front and back at the centre line should not exceed 40 kPa, (gauge) what should be the maximum acceleration. Neglect the weight component.*

using equation 2.6.9, $\Delta p = \rho a_x(x_2 - x_1)$,

$$40000 = 830 \times a_x \times 6 \quad \therefore\ \mathbf{a_x = 8.03\ m/s^2}$$

Problem 2.23. *Air fills the gap between two circular plates held horizontal. The plates rotate without any air flowing out. If the radius is 60 mm and if the speed is 60 rpm, determine the pressure difference between the centre and the circumference.*

The air in the gap can be considered to rotate as a single body. As the level is the same the head difference between the centre and the outer radius is given by

$$\mathbf{h} = (\omega r)^2/2g_o = [(2\pi 60/60) \times 0.06]^2/(2 \times 9.81) = \mathbf{7.24 \times 10^{-3}\ m\ of\ air}$$

Considering density of air to be about 1.2,

$$\textbf{head of water} = (7.24 \times 10^{-3} \times 1.2)/1000 = \mathbf{8.69 \times 10^{-6}\ m}$$

REVIEW QUESTIONS

1. Define and explain the concept "pressure".
2. State and prove Pascal's law. Explain the consequences of the law.
3. Distinguish between gauge pressure, absolute pressure and vacuum pressure.
4. Derive the expression for the pressure variation in a static fluid under gravitational forces. Indicate the modifications where pressure varies along vertical and horizontal directions.
5. Derive an expression for the distribution of force in static fluid subjected to whole body acceleration in a general direction – s.
6. Derive the expression for the angle made by the free surface in a liquid that is subjected to both acceleration and gravitation.
7. Derive an expression for the pressure distribution in an incompressible fluid accelerated horizontally.
8. Explain what is meant by forced vortex and derive the expression for the radial pressure distribution in forced vortex.
9. Explain the basic principle involved in measuring pressure and pressure difference using manometers. Indicate when the use of manometers is advantageous.
10. Explain how small pressure difference reading can be amplified by using a micro manometer or inclined tube manometer.

OBJECTIVE QUESTIONS

O Q.2.1 Fill in the blanks:

1. Pressure is defined as ________
2. Pascals law states ________
3. On a free surface of a liquid the pressure is ________
4. When gravitational forces are zero, the pressure exterted by a column of fluid is ________
5. The pressure exerted by a column of fluid of height y m and specific weight γ is ________
6. At zero horizontal accelerating conditions on earths surface, the free surface will be ________
7. Manometers use the principle of ________
8. Manometers are suitable for ________ pressure measurement.
9. In a forced vortex the height of liquid at the periphery of a cylinder of Radius R above that at the centre will be ________
10. The shape of free surface in a forced vortex is ________

Answers

1. As a measure of force distribution over any surface associated with a fluid (dF/dA) 2. that the pressure at a point in a fluid at rest is equal in magnitude in all directions 3. is the same at all points 4. is zero 5. $y\,\gamma$. 6. horizontal 7. basic hydrostatic equation, ($\Delta P = \Delta\, y\gamma$) 8. low 9. ($R^2\,\omega^2/2\,g_o$) 10. Paraboloidal.

O Q.2.2 Fill in the blanks:

1. If the density varies linearly with height the pressure will vary ________ with height.
2. When a fluid is decelerated at a rate equal to g in the vertical direction the pressure on the base will be ________
3. When a fluid in a container is accelerated along the x direction at a m/s^2, the angle the free surface will occupy is given by ________
4. In micromanometer, the density difference between the filler fluid and the manometer fluid should be ________
5. The capillary effect can be ________ when both limbs of a manometer have equal areas.
6. The shape of a forced vortex in the absence of gravity will be ________
7. The pressure at a point in fluid at rest is ________ of direction.
8. The pressure exerted by a liquid column on the base depends on the ________ of the liquid.
9. The level rise in the forced vortex is ________ of the fluid.
10. Due to horizontal acceleration, the free surface of the fluid will be 45° when the acceleration equals ________

Answers

1. exponentially 2. zero 3. tan θ = – a/g 4. small 5. neglected/equal on both sides 6. cylindrical 7. independent 8. specific weight 9. independent 10. $a_x = g$

O Q.2.3 Fill in the blanks with increases, decreases, or remains constant.

1. The pressure in a fluid at rest ________ with depth.
2. Along the free surface in a liquid, the pressure ________
3. In a fluid at rest the pressure at a point ________
4. As specific weight increases, the head of liquid for a given pressure ________
5. As the density of manometric fluid decreases, the manometric deflection for the same pressure difference ________
6. As a container with liquid is accelerated the pressure on the base along the direction of acceleration ________
7. The forced vortex rise ________ as density of the liquid increases.
8. The forced vortex rise ________ with rotational speed.
9. In a micromanometer, the gauge deflection will increase if the area ratio _______
10. In inclined tube manometer, the gauge reading ________ when the angle is reduced.

Answers

Increases : 1, 5, 8, 9, 10 **Decreases** : 4, 6 **Remains constant** : 2, 3, 7

O Q.2.4 Indicate whether the statements are correct or incorrect.

1. In a fluid at rest, the pressure at a point varies with direction.
2. In a fluid at rest the pressure at a constant level will be equal at all locations.
3. The pressure on the base of a liquid column will depend upon the shape of the column.
4. The pressure over a free surface of a fluid at rest will vary with location.
5. For low pressure measurement a manometric fluid with low density will be better.
6. In a manometer, the fluid column will rise if the pressure measured is above the atmosphere.
7. In a manometer, the fluid column will fall if the pressure inside is less than atmospheric.
8. The vacuum gauge reading will increase as the absolute pressure decreases.

9. The absolute pressure is equal to the vacuum gauge reading.
10. Absolute pressure = atmospheric pressure – vacuum gauge reading.

Answers

Correct : 2, 5, 6, 7, 8, 10 Incorrect : 1, 3, 4, 9

O Q.2.5 Choose the correct answer:

1. The gravity at a location is 5 m/s^2. The density of fluid was 2000 kg/m^3. The pressure exerted by a column of 1 m of the fluid will be
   (*a*) 400 N/m^2 (*b*) 10,000 N/m^2
   (*c*) 2000 N/m^2 (*d*) 5 N/m^2
2. In a circular cylinder of 0.2 m dia and 0.4 m height a fluid of specific weight 1200 × 9.81 N/m^3 is filled to the brim and rotated about its axis at a speed when half the liquid spills out. The pressure at the centre is
   (*a*) 0.2 × 1200 × 9.81 N/m^2 (*b*) Zero
   (*c*) 0.4 × 1200 × 9.81 N/m^2 (*d*) 0.1 × 1200 × 9.81 N/m^2
3. In a forced vortex
   (*a*) the fluid velocity is inversely proportional to the radius
   (*b*) the fluid rotates without any relative velocity
   (*c*) the rise depends on the specific weight
   (*d*) the rise is proportional to the cube of angular velocity
4. In a forced vortex, the level at a radius of 0.6 m is 0.6 m above the centre. The angular velocity in radians is
   (*a*) 11.44 (*b*) 5.72
   (*c*) 32.7 (*d*) 130.8
5. The shape of forced vortex under gravitational conditions is
   (*a*) hyperboloid (*b*) spherical
   (*c*) paraboloid (*d*) cylindrical
6. In a manometer using mercury as manometric fluid and measuring the pressure of water in a conduit, the manometric rise is 0.2 m. The specific gravity of mercury is 13.55. The water pressure in m of water is
   (*a*) 14.55 × 0.2 (*b*) 13.55 × 0.2
   (*c*) 12.55 × 0.2 (*d*) none of the above
7. A horizontal cylinder half filled with fuel is having an acceleration of 10 m/s^2. The gravitational forces are negligible. The free surface of the liquid will be
   (*a*) horizontal (*b*) slopes in the direction of acceleration
   (*c*) vertical (*d*) slopes in the direction opposite of acceleration
8. In a static fluid, with *y* as the vertical direction, the pressure variation is given by
   (*a*) $\frac{dp}{dy} = \rho$ (*b*) $\frac{dp}{dy} = -\rho$
   (*c*) $\frac{dp}{dy} = \gamma$ (*d*) $\frac{dp}{dy} = -\gamma$

9. The specific weight of a fluid is 20,000 N/m^3. The pressure (above atmosphere) in a tank bottom containing the fluid to a height of 0.2 m is
   (*a*) 40,000 N/m^2 (*b*) 2000 N/m^2
   (*c*) 4000 N/m^2 (*d*) 20,000 N/m^2
10. In a differential manometer a head of 0.6 m of fluid *A* in limb 1 is found to balance a head of 0.3 m of fluid *B* in limb 2. The ratio of specific gravities of *A* to *B* is
   (*a*) 2 (*b*) 0.5
   (*c*) cannot be determined (*d*) 0.18

Answers

(1) *b* (2) *b* (3) *b* (4) *b* (5) *c* (6) *c* (7) *c* (8) *d* (9) *c* (10) *b*

O Q.2.6 Match the pairs:

(*a*)	Free surface in forced vortex	(1) Vertical
(*b*)	Free surface in static fluid	(2) Paraboloid
(*c*)	Free surface in forced vortex without gravity	(3) Negative slope
(*d*)	Free surface in a horizontally accelerating fluid	(4) Horizontal

Answers

a - 2, *b* - 4, *c* - 1, *d* - 3

EXERCISE PROBLEMS

E.2.1. A chamber is at a pressure of 100 kN/m^2. A gauge fixed into this chamber Fig. E.2.1 to read the outside pressure shows 1.2 kN/m^2. Determine the outside pressure.

[101.2 kN/m^2 absolute]

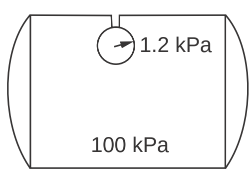


Figure E. 2.1

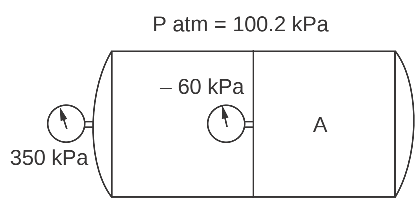


Figure E. 2.2

E.2.2. Determine the absolute and gauge pressures in chamber *A* as shown in Fig. E.2.2, the gauge pressure being referred to atmospheric pressure of 1.02 × 10^5 N/m^2. **[390.2, 290 kN/m^2]**

E.2.3. In an artificial atmosphere, the specific weight of air varies with the altitude y as $\gamma = c.y$, where γ is in N/m^3 and y is in m. The pressure at $y = 0$ is 5000 N/m^2. c is a dimensional constant having a unit of N/m^4. In this case c has a value of 1. Determine the expression for pressure variation with altitude. **[$P = 5000 - (y^2/2)$]**

E.2.4. Determine the pressure below 1000 m in the sea if the specific weight changes as $\gamma = K.\ \gamma_1/(K + \gamma_1.y)$ where K is the bulk modulus having a value of 2 × 10^9 N/m^2 and y is the depth in m. The surface pressure is 101.3 kN/m^2 and γ_1 = 9810 N/m^3. **[9935 kN/m^2]**

E.2.5. A vessel of the shape shown in Fig. E.2.5 is filled with a liquid of specific gravity 0.92. The pressure gauge at A reads 400 kN/m^2. Determine the pressure read by a gauge (Bourdon type) fixed at B. Neglect gauge height. **[454.15 kN/m^2]**

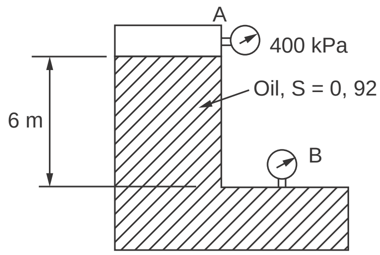


Figure E. 2.5

E.2.6. Determine the pressure above the atmosphere at point 3 for the manometer and dimensions shown in Fig. E.2.6. **[65 kN/m^2]**

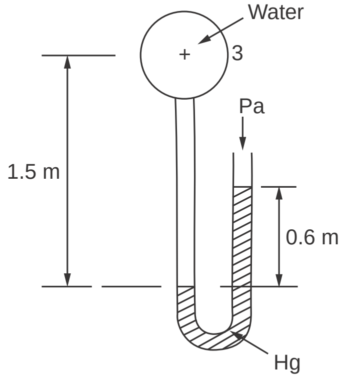


Figure E. 2.6

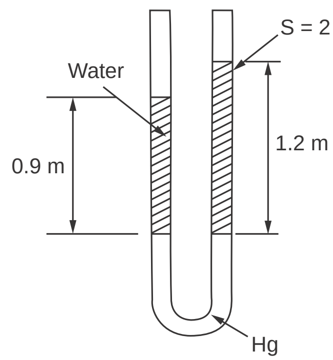


Figure E. 2.7

E.2.7. In a U-tube shown in Fig. E.2.7, open to atmosphere at both ends, a column of 0.9 m of water balances a column of 1.2 m of an unknown liquid. Determine the specific gravity of the unknown liquid. **[0.75]**

E.2.8. Determine the pressure at point X for the situation shown in Fig. E.2.8 **[8.04 kPa]**

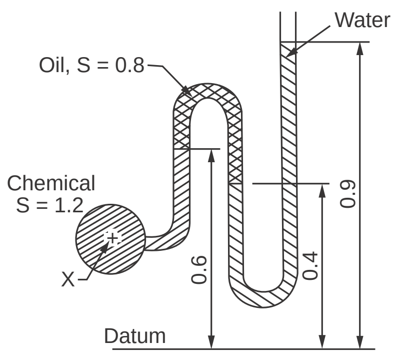


Figure E. 2.8

E.2.9. For the manometer shown in Figure E.2.9, determine the length *AB*. The pressure at point 1 and point 4 are 30 kPa and 120 kPa. **[68.3 cm]**

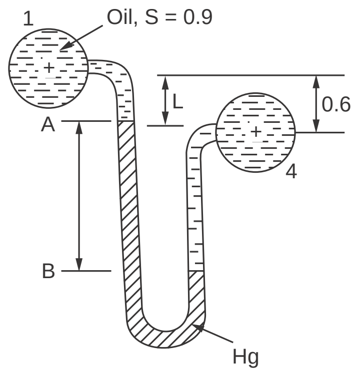


Figure E. 2.9

E.2.10. Determine the pressure at *A* above the atmosphere for the manometer set up shown in Fig. E.2.10. **[111.91 kPa]**

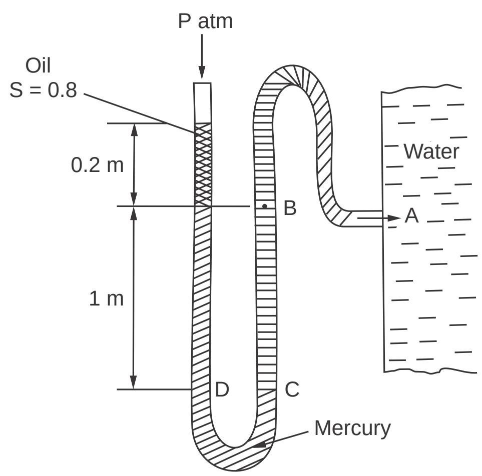


Figure E. 2.10

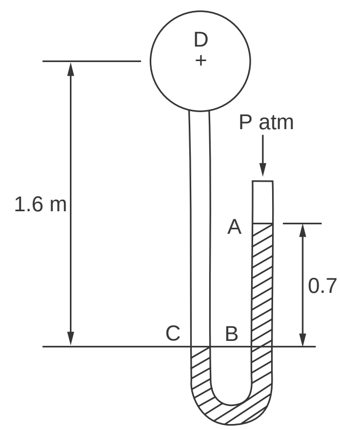


Figure E. 2.11

E.2.11. For the situation shown in Fig. E.2.11, determine the pressure at point *D*. The specific gravity of the oil is 0.9 and that of the manometer fluid is 0.7. **[94.04 kN/m^2]**

E.2.12. In a micromanometer the area of the well chamber is 12 times the area of the U tube section. The manometric fluid is having a specific gravity of 1.03 and the filler fluid is water. The flowing fluid in which the pressure is to be determined is air with a density of 1.2 kg/m^3 at the measuring condition. When pressures are equal, the level from the top to the filler fluid is

8 cm. The manometric fluid is 18 cm from top at filling. Under measuring condition the manometric fluid movement in one limb is 4 cm. Determine the pressure difference indicated. **[88.866 N/m²]**

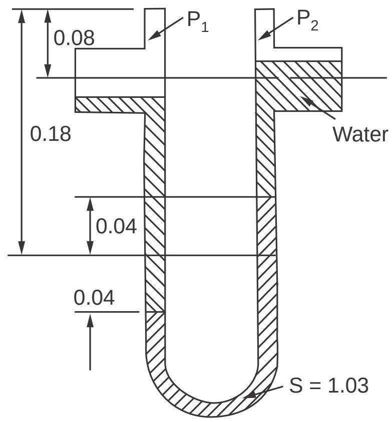


Figure E. 2.12

E.2.13. An inclined tube manometer with limb at 10° to horizontal shows a column length of 8 cm above the reservoir level. The specific weight of the fluid is 900 × 9.81 N/m³. Determine pressure above atmospheric level. **[122.65 N/m²]**

E.2.14. Determine the pressure difference between *A* and *B* shown in Fig. E.2.14.

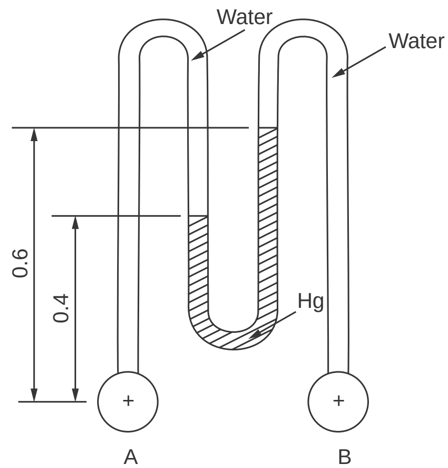


Figure E. 2.14

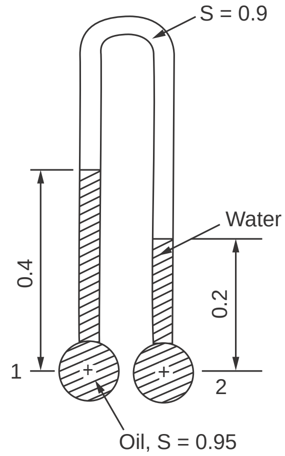


Figure E. 2.15

E.2.15. Determine the pressures at location 1 and 2 in Fig. E.2.15.

E.2.16. The atmospheric pressure at an elevation of 300 m was 100 kPa. when the temperature was 20°C. If the temperature varies at the rate of – 0.006° C/m, determine the pressure at height of 1500 m.

E.2.17. The pressure at sea level was 102 kPa and the temperature is constant with height at 5°C. Determine the pressure at 3000 m.

E.2.18. The bourden type pressure gauge in the oxygen cylinder of a deep sea diver when he is at a depth of 50 m reads 500 kPa. Determine the pressure of oxygen above atmospheric pressure. Assume sea water density is constant and is 1006 kg/m^3.

E.2.19. The density of a fluid at rest increases with depth as 1000 + 0.05h kg/m^3 where h is the depth in m from the surface. Determine the hydrostatic pressure at depth of 100 m.

E.2.20. A cylinder containing oil of specific gravity 0.92 as shown in Fig. E.2.20 is accelerated at 10 m/s^2 towards (*i*) right and (*ii*) left. The reading under accelerating conditions at the right end was 200 kN/m^2. The tube is 3 m long. Determine the pressure at the left end.

[227.6, 172.4 kN/m^2]

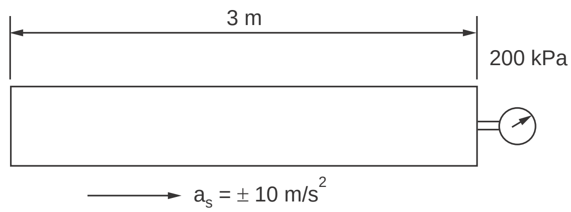


Figure E. 2.20

E.2.21. Using figure in Example 2.20, if the fluid density varies as $\rho = 0.3 + 8 \times 10^{-6} P$, where density is in kg/m^3 and P is in N/m^2 and if the pressure gauge at the right end reads 120 kN/m^2, determine the pressure at the left end, if the acceleration is to the right at 10 m/s^2.

[120.038 kN/m^2]

E.2.22. A rocket is accelerating horizontally to the right at 10 g. The pressure gauge is connected by a 0.6 m length tube to the left end of the fuel tank. If the pressure in the tank is 35 bar, and if fuel specific gravity is 0.8, determine the pressure gauge reading. **[35.471 × 10^5 N/m^2]**

E.2.23. A rectangular pan 0.3 m high, 0.6 m long and 0.3 m wide contains water to a depth of 0.15 m. Determine the acceleration which will cause water to spill. **[4.905 m/s^2]**

E.2.24. Determine the liquid level at the centre when a tall cylinder of 1.2 m dia filled upto a depth of 0.6 m is rotated at 77 rpm. **[0]**

E.2.25. A cylinder of radius 0.6 m filled partially with a fluid and axially rotated at 15 rad/s is empty upto 0.3 m radius. The pressure at the extreme edge at the bottom was 0.3 bar gauge. Determine the density of the fluid. **[987.65 kg/m^3]**

E.2.26. A tank containing liquid of specific gravity of 0.8 is accelerated uniformly along the horizontal direction at 20 m/s^2. Determine the decrease in pressure within the liquid per metre distance along the direction of motion.

E.2.27. The liquid in a tank when accelerated in the horizontal direction, assumes a free surface making 25° with the horizontal. Determine the acceleration.

E.2.28. A closed tank of cubical shape of 1 m side is accelerated at 3 m/s^2 along the horizontal direction and 6 m/s^2 in the vertical direction. Determine the pressure distribution on the base. Assume the base to be horizontal.

E.2.29. A closed cubical tank of 1.5 m side is filled to 2/3 of its height with water, the bottom face being horizontal. If the acceleration in the horizontal (along the right) and vertical directions are 5 m/s^2 and 7 m/s^2. Determine the pressures at the top and bottom corners.

E.2.30. A tube with closed ends filled with water is accelerated towards the right at 5 m/s^2. Determine the pressure at points 1, 2, 3 and 4. Calculate the acceleration that water will boil at point 4 at 40°C.

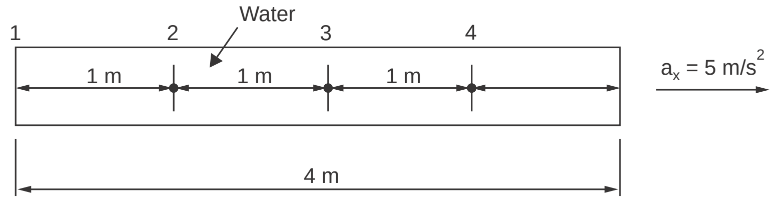


Figure E. 2.30

E.2.31. A cubical box of 1 m side is half filled with water and is placed in an inclined plane making 30° with the horizontal. If it is accelerated along the plane at 2 m/s^2 (*i*) upwards, (*ii*) downwards, determine the angle attained by the free surface.

E.2.32. A cylindrical vessel containing water is rotated as a whole. The pressure difference between radii 0.3 m and 0.6 m is 0.3 m of water. Calculate the rotational speed.

E.2.33. A small bore pipe 3 m long and one end closed is filled with water is inclined at 20° with the vertical and is rotated at 20 rpm with respect to a vertical axis passing through its mid point. The free surface is at the top of the pipe. Determine the pressure at the closed end.

E.2.34. The U-tube shown in Fig. E.2.34 is rotated at 120 rpm about the vertical axis along *A-A*. Determine the pressure at 1 and 2.

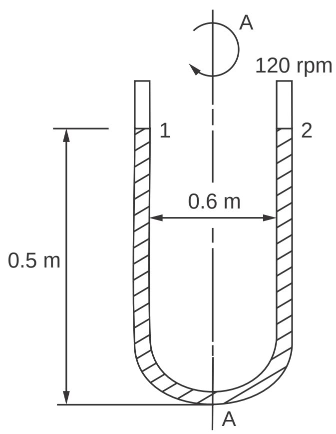


Figure E. 2.34

E.2.35. A hollow sphere of inside radius *r* is filled with water and is rotated about a vertical axis passing through the centre. Determine the circular line of maximum pressure.

E.2.36. A cylindrical vessel containing water is rotated about its axis at an angular speed ω (vertical). At the same time, the container is accelerated downwards with a value of *v* m/s^2. Derive an expression for the surface of constant pressure.

E.2.37. A box of cubical shape of 1.5 m side with base horizontal filled with water is accelerated upwards at 3 m/s^2. Determine the force on one of the faces.