3 Forces on Surfaces Immersed in Fluids

3.0 INTRODUCTION

In the previous chapter the pressure distribution in fluids in static and dynamic condition was discussed. **When a fluid is in contact with a surface is exerts a normal force on the surface.** The walls of reservoirs, sluice gates, flood gates, oil and water tanks and the hulls of ships are exposed to the forces exerted by fluids in contact with them. The fluids are generally under static condition. For the design of such structures **it is necessary to determine the total force** on them. It is **also necessary to determine the point of action of this force.** The point of action of the total force is known as **centre of pressure or pressure centre**. From the basic hydrodynamic equation it is known that the force depends on the pressure at the depth considered.

i.e., $P = \gamma h$. Force on an elemental area dA as a depth, h, will be

$$dF = \gamma h dA \tag{3.0.1}$$

The **total force is obtained by integrating** the basic equation over the area

$$F = \gamma \int_A h dA \tag{3.0.2}$$

From the definition of centre of gravity or centroid

$$\int_A h dA = \bar{h} A \tag{3.0.3}$$

where $\bar{h}$ is the depth of the centre of gravity of the area.

To determine the point of action of the total force, moment is taken of the elemental forces with reference to an axis and equated to the product of the total force and the distance of the centre of pressure from the axis namely h_{cp}

$$F.\, h_{cp} = \int_A h dF = \gamma \int_A h^2 dA \tag{3.0.4}$$

The integral over the area is nothing but the second moment or the moment of inertia of the area about the axis considered.

Thus there is a need to know the centre of gravity and the moment of inertia of areas.

3.1 CENTROID AND MOMENT OF INERTIA OF AREAS

In the process of obtaining the resultant force and centre of pressure, the determination of first and second moment of areas is found necessary and hence this discussion. The moment of the area with respect to the y axis can be obtained by summing up the moments of elementary areas all over the surface with respect to this axis as shown in Fig. 3.1.1.

$$\text{Moment about } y \text{ axis} = \int_A x\, dA \qquad (3.1.1)$$

$$\text{Moment about } x \text{ axis} = \int_A y\, dA \qquad (3.1.2)$$

The integral has to be taken over the area. If moments are taken with respect to a parallel axis at a distance of k from the y axis equation 3.1.1. can be written as

$$\int_A (x - k)\, dA = \int_A x\, dA - k \int_A dA = \int_A x\, dA - k\, A \qquad (3.1.3)$$

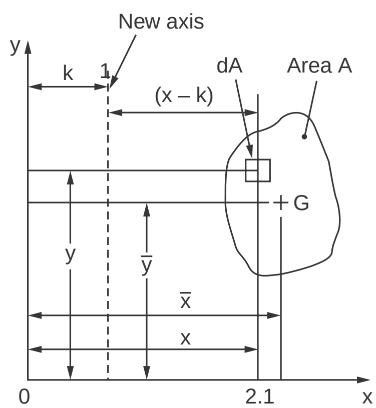

Figure. 3.1.1 First moment and second moment of an area

As k is a constant, it is possible to choose a value of $\bar{x} = k$, such that the moment about the axis is zero. The moment about the axis through the centre of gravity is always zero.

$$\int_A x\, dA - \bar{x}\, A = 0$$

Such an axis is called **centroidal** y axis. The value of $\bar{x}$ can be determined using

$$\bar{x} = (1/A) \int_A x\, dA \qquad (3.1.4)$$

Similarly the centroidal x axis passing at $\bar{y}$ can be located using

$$\bar{y} = (1/A) \int_A y\, dA \qquad (3.1.5)$$

The point of intersection of these centroidal axes is known as the **centroid** of the area.

It can be shown that the moment of the area about any line passing through the centroid to be zero.

With reference to the Fig. 3.1.1, the second moment of an area about the y axis. I_y is defined as

$$I_y = \int_A x^2\, dA \qquad (3.1.6)$$

Considering an axis parallel to y axis through the centroid and taking the second moment of the area about the axis and calling it as I_G, where $\bar{x}$ is the distance from the axis and the centroid.

$$I_G = \int_A (x - \bar{x})^2 \, dA \tag{3.1.7}$$

$$I_G = \int_A x^2 \, dA - 2\bar{x} \int_A x \, dA + \int_A \bar{x}^2 \, dA$$

By definition $\int_A x^2 \, dA = I_y, \int_A x^2 \, dA = \bar{x} A$

As $\bar{x}^2$ is constant, $\int_A \bar{x}^2 \, dA = \bar{x}^2 A$. Therefore

$$I_G = I_y - 2\bar{x}^2 A + \bar{x}^2 A = I_y - \bar{x}^2 A \tag{3.1.8}$$

or $$I_y = I_G + \bar{x}^2 A \tag{3.1.9}$$

Similarly $$I_y = I_G + \bar{y}^2 A \tag{3.1.10}$$

The moment of inertia of an area about any axis is equal to the sum of the moment of inertia about a parallel axis through the centroid and the product of the area and the square of the distance between this axis and centroidal axis. These two equations are used in all the subsequent problems.

The second moment is used in the determination of the centre of pressure for plane areas immersed in fluids.

The product of inertia is defined as

$$I_{xy} = \int_A xy \, dA = I_{Gxy} + \bar{x}\,\bar{y}\, A \tag{3.1.11}$$

It can be shown that whenever any one of the axes is an axis of symmetry for the area, $I_{xy} = 0$.

The location of the centre of gravity, moment of inertia through the centroid I_G and moment of inertia about edge I_{edge} (specified) for some basic shapes are given in Table 3.1.

Table 3.1 Centre of Gravity and Moment of Inertia for some typical shapes

Shape	CG	I_G	I_{base}
1. Triangle, side b height h and base zero of x axis	$h/3$	$bh^3/36$	$bh^3/12$
2. Triangle, side b height h and vertex zero of x axis	$2h/3$	$bh^3/36$	$bh^3/12$
3. Rectangle of width b and depth D	$D/2$	$bD^3/12$	$bD^3/3$
4. Circle	$D/2$	$\pi D^4/64$	–
5. Semicircle with diameter horizontal and zero of x axis	$2D/3\pi$	–	$\pi D^4/128$
6. Quadrant of a circle, one radius horizontal	$4R/3\pi$	–	$\pi R^4/16$
7. Ellipse : area $\pi bh/4$ Major axis is b, horizontal and minor axis is h	$h/2$	$\pi bh^3/64$	–

8. Semi ellipse with major axis as horizontal and $x = 0$	$2h/3\pi$	–	$\pi bh^3/128$
9. Parabola (half) area $2bh/3$ (from vertex as zero)	$y_g = 3h/5$ $x_g = 3b/8$	–	$2bh^3/7$

3.2 FORCE ON AN ARBITRARILY SHAPED PLATE IMMERSED IN A LIQUID

Case 1 : Surface exposed to gas pressure : For plane surface, force = area × pressure The contribution due to the weight of the gas column is negligible. The resultant acts at the centroid of the area as the pressure at all depths are the same.

Case 2 : Horizontal surface at a depth y.

$P = -y \times \gamma$ and as y is – ve, force = $Ay\gamma$ in which y may also be expressed as head of the fluid. The resultant force acts vertically through the centroid of the area, Here also the pressure at all locations are the same.

Case : Plane inclined at angle θ with horizontal. Refer Fig. 3.2.1

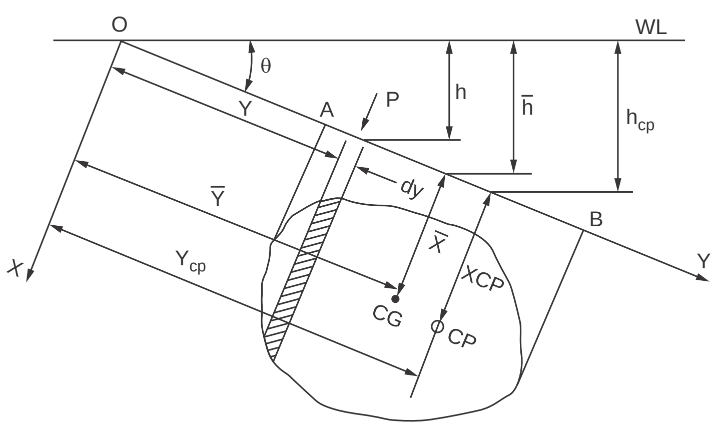

Figure. 3.2.1 Plane surface immersed in liquid at an angle

Consider the plane AB of the given shape immersed in the liquid at an angle θ to the horizontal (free surface). Let the trace of the plane (the end view of the line where it meets the horizontal plane) be "O". Consider this line as reference and set up the axes as shown in figure. Consider the elemental area dA. The force dF on the elemental area is given by

$$dF = P\,dA = \gamma h dA = \gamma\, y \sin\theta\, dA$$

The total force over the whole area is obtained by integration of this expression over the whole area.

$$F = \int_A \gamma\, y \sin\theta\, dA = \gamma \sin\theta \int_A y\, dA$$

From the definition of centroidal axis at $\bar{y} = \int_A y\, dA = \bar{y}\, A$. So

$$F = \gamma A \bar{y} \sin \theta \qquad (3.2.1)$$

Calling the depth at $\bar{y}$ (distance of centroid from the surface) as $\bar{h}$,

As $$\bar{h} = \bar{y} \sin \theta, F = \gamma A \bar{h} \qquad (3.2.2)$$

This equation is extensively used in the calculation of total force on a surface. Equations (3.1.4), (3.1.5) and those given in Table 3.1 are used to obtain the location of the centroid.

The following important conclusions can be drawn from this equation.

1. $\gamma\bar{h}$ equals the pressure at the centroid. The total force thus equals the product of area and the pressure at the centroid.

2. A special case of the situation is a vertical surface where θ = 90° and sin θ = 1. and so $\bar{h} = \bar{y}$ in this case.

3.3 CENTRE OF PRESSURE FOR AN IMMERSED INCLINED PLANE

The centre of pressure is determined by taking moments of the force on elementary areas with respect to an axis (say O in Fig. 3.2.1) and equating it to the product of the distance of the centre of pressure from this axis and the total force on the area (as calculated in section 3.2). For surfaces with an axis of symmetry, the centre of pressure will lie on that axis. In other cases x_{cp} and y_{cp} are calculated by the use of moments.

With reference to the Fig. 3.2.1, let CP (x_{cp}, y_{cp}) be the centre of pressure.

$$x_{cp} F = \int_A x \; P \, dA \quad \text{and} \quad y_{cp} F = \int_A y \; P \, dA$$

The area element considered here being *dxdy*. Referring to the Fig. 3.1.1

$$x_{cp} \gamma \bar{y} A \sin \theta = \int_A \gamma \, x \, y \sin \theta \, dA \qquad (P = \gamma y \sin \theta, F = \gamma \bar{y} A \sin \theta)$$

$$x_{cp} = (1/\bar{y} A) \int_A x \; y \, dA, \qquad \text{As} \int_A x \; y \, dA = I_{xy}$$

From equations 3.1.9 and 3.1.10

$$x_{cp} = I_{xy} / \bar{y} A = (I_{Gxy} + \bar{x} A \bar{y})$$
$$= (I_{Gxy} / A \bar{y}) + \bar{x} \qquad (3.3.1)$$

In case $x = \bar{x}$ or $y = \bar{y}$ is an axis of symmetry for the area, $I_{Gxy} = 0$ and the centre of pressure will lie on the axis of symmetry.

Along the y direction (more often the depth of centre of pressure is required)

$$y_{cp} = I_x / \bar{y} A, \text{As } I_X = I_G + \bar{y}^2 A$$

$$y_{cp} = (I_G / \bar{y} A) + \bar{y} \qquad (3.3.2)$$

I_G is the moment of inertia along the centroidal axis and $\bar{y}$ is the location of centroid along the y direction.

If the plane is vertical, then, $\bar{y} = \bar{h}$ the depth to the centroid.

In case the height h_{cp} is required instead of y_{cp}, (along the plane) then substituting

$$y_{cp} = h_{cp}/\sin\theta \quad \text{and} \quad \bar{y} = \bar{h} / \sin\theta \tag{3.3.2a}$$

Substituting in equation (3.3.2)

$$h_{cp} / \sin\theta = [I_G/\{(\bar{h} / \sin\theta) \times A\}] + \bar{h}/\sin\theta$$

$$\therefore \quad h_{cp} = [(I_G \sin^2\theta)/ \bar{h} A)] + \bar{h} \tag{3.3.3}$$

This is the general equation when the depth of the centre of pressure is required in the case of inclined planes. If θ = 90° (vertical surface), then $\sin^2\theta = 1$.

These equations are fairly simple, the main problem being the calculation of the moment of inertia for odd shapes.

Example 3.1. *The wall of a reservoir is inclined at 30° to the vertical. A sluice 1m long along the slope and 0.8 m wide is closed by a plate. The top of the opening is 8 m below the water level.* ***Determine the location of the centre of pressure*** *and the total force on the plate.*

The angle with the horizontal is 60°. The depth of centre of gravity,

$$\bar{h} = 8 + (0.5 \times \sin 60) / 2 = 8.433 \text{ m}$$

$$\textbf{Total force} = \gamma \bar{h} A = 1000 \times 9.81 \times 8.433 \times 1 \times 0.8 = \textbf{66182 N}$$

$$\boldsymbol{h_{cp}} = (I_G \sin^2\theta / \bar{h} A) + \bar{h},\ I_G = (1/12)\ bd^3$$

$$[(1/12)\ 0.8 \times 1^3 \times \sin^2 60/8.433 \times 0.8] + 8.433 = \textbf{8.44 m}$$

Distance along the wall surface, 8.44/ cos 30 = 9.746 m

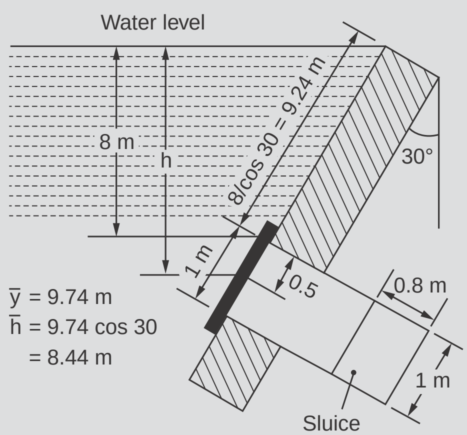

Figure Ex. 3.1 Problem model

Example 3.2. ***Determine the total force and its point of action*** *on an annular lamina of 1m ID and 3 m OD placed at an inclination of 30 degrees to the horizontal under water. The depth of centre of lamina from water surface is 8 m.*

Total force = $\gamma A \bar{h}$ = 1000 × 9.81 × π (3² – 1¹) × 8/4

= **493104.38 N** (depth is directly specified)

Depth of centre of pressure = $(I_G \sin^2 \theta / \bar{h} A) + \bar{h}$, $I_G = (\pi / 64)(D^4 - d^4)$

= $[(\pi/64)(3^4 - 1^4) \sin^2 30 - \{8 \times ((3^2 - 1^2)\pi/4)\}] + 8$

= **8.0195 m**

3.3.1 Centre of Pressure for Immersed Vertical Planes

Case 1: A rectangle of width b and depth d, the side of length b being horizontal.

Case 2: A circle of diameter d.

Case 3: A triangle of height h with base b, horizontal and nearer the free surface.

Assuming the depth of CG to be P m in all the cases. $\bar{h} = \bar{y}$ in the case

$$h_{cp} = (I_G/A\,\bar{y}) + \bar{y}$$

Case 1: $I_G = bd^2/12$, $\bar{y} = P$, $A = bd$

$$\mathbf{h_{cp}} = (bd^3/12\; bd\; P) + P = \mathbf{(d^2/12\,P) + P} \qquad (3.3.4)$$

Case 2: $I_G = \pi\, d^4/64$, $\bar{y} = P$, $A = \pi\, d^2/4$

$$\mathbf{h_{cp}} = (\pi\, d^4 \times 4/64\; \pi d^2\, P) + P = \mathbf{(d^2/16\,P) + P} \qquad (3.3.5)$$

Case 3: $I_G = bh^3/36$, $\bar{y} = P$, $A = bh/2$

$$\mathbf{h_{cp}} = (2\; b\; h^3/\,36\; b\; h\; P) + P = \mathbf{(h^2/18\,P) + P} \qquad (3.3.6)$$

These equations can be used as a short cut under suitable situations.

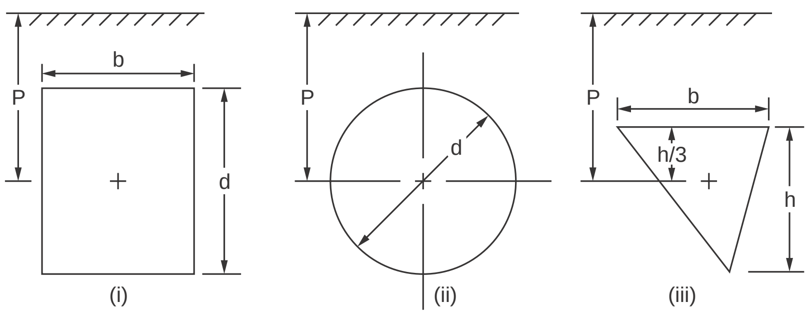

Figure 3.3.1 Vertical Surfaces

Example 3.3. *An oil tank is filled to a height of 7.5 m with an oil of specific gravity 0.9. It has a rectangular gate 1m wide and 1.5 m high provided at the bottom of a side face.* ***Determine the resultant force on the gate and also its point of action.***

Force on the gate from oil side = $\gamma A \bar{h}$ = (0.9 × 1000 × 9.81) (1 × 1.5) (6 + 0.75) = **89394 N**

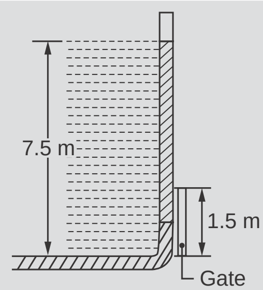

Figure Ex. 3.3

$$\mathbf{h_{cp}} = (I_G / \bar{y}\, A) + \bar{y};\ I_G = bd^3/12;\ \bar{y} = \mathbf{6.75\ m}$$

$$= ((1 \times 1.5^3/12)/6.75 \times 1 \times 1.5) + 6.75 = \mathbf{6.78\ m}$$

Check using eqn. 3.3.4, which is simpler,

$$\mathbf{h_{cp}} = (d^2/12P) + P = (1.5^2/12 \times 6.75) + 6.75 = \mathbf{6.78\ m}$$

The resultant force will act at a distance of 6.78 m from the surface of oil at the centre line of the gate.

Example 3.4. ***Determine the net force and its point of action*** *over an L shaped plate submerged vertically under water as shown in Fig. Ex. 3.4. The top surface of the plate is 1.5 m below water surface.*

The plate can be considered as two rectangles (*i*) ABCD 1m wide and 2 m deep and (*ii*) CEFG 2.5 m wide and 1 m deep.

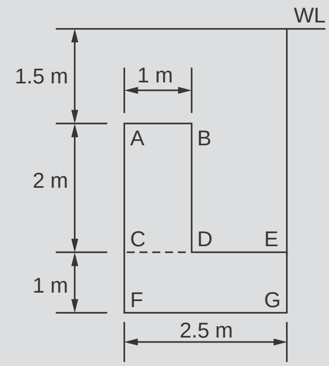

Figure Ex. 3.4

$$F_i = \gamma\, h_i A_i = (1000 \times 9.81)$$
$$(1.5 + 2.0/2)\ (1 \times 2) = 49050 \text{ N}$$

$$F_{ii} = \gamma\, h_{ii} A_{ii} = (1000 \times 9.81)$$
$$(1.5 + 2 + 1/2)\ (1 \times 2.5) = 98100 \text{ N}$$

Total force on the plate $= \mathbf{147150\ N}$

Considering *ABCD* $\quad h_{cpi} = (I_{gi} / \bar{h}_i\, A_i) + \bar{h}_i$

$$= [(1 \times 2^3/12)/(1 \times 2.5 \times 2)] + 2.5 = 2.633 \text{ m}$$

Also by equation (3.5.1), $\quad h_{cpi} = (d^2/12P) + P = (2^2/12 \times 2.5) = 2.633$ m

$$h_{cpii} = (I_{gii}/h_{ii}\, A_{ii}) + h_{ii} = ((2.5 \times 1^3/12)\ (1/4 \times 2.5)) + 4 = 4.021 \text{ m}$$

Also by equation 3.3.4, $h_{cpii} = (1^2/12 \times 4) + 4 = 4.021$ m

In order to locate the point of action of the resultant force, moment is taken with reference to the surface to determine the depth.

$$\mathbf{h_{cpy}} = (F_i\, h_{cpi} + F_{ii}\, h_{cpii})/(F_i + F_{ii})$$
$$= (49050 \times 2.633 + 98100 \times 4.021)/(49050 + 98100) = \mathbf{3.5583\ m}$$

Moment is taken about *AF* to determine the lateral location

$$\mathbf{h_{cpx}} = [(49050 \times 0.5) + (98100 \times 1.25)]/(49050 + 98100) = \mathbf{1.0\ m}$$

The resultant force acts at a depth of 3.5583 m and at a distance of 1.0 m from the edge *AF*.

3.4 COMPONENT OF FORCES ON IMMERSED INCLINED RECTANGLES

Consider a case of a rectangle of $a \times d$, with side d inclined at θ to the horizontal, immersed in a fluid with its centroid at a depth of $\bar{h}$ m. For this case it can be shown that (*i*) **The horizontal component of the resultant force equals the force on the vertical projection of the area and** (*ii*) **The vertical component equals the weight of the fluid column above this area.**

The net force acting perpendicular to the area is given by $= \gamma A \bar{h}$

The horizontal component equals $= \gamma A\, \bar{h}\, \sin\theta$

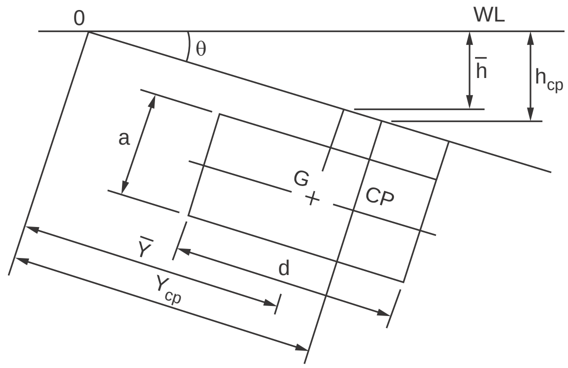

Figure 3.4.1

The vertical projection of the area = $A \sin \theta$

The centroid of this area will also be at $\bar{h}$

The force on the projected area = $\gamma A \bar{h} \sin \theta$.

Hence the horizontal component of the force equals the force on the vertical projection of the area.

The vertical component = $\gamma A \bar{h} \cos \theta$

The horizontal projection of the area = $A \cos \theta$

The volume of the fluid column above this surface = $A \cos \theta \, \bar{h}$

The weight of the fluid column = $\gamma A \bar{h} \cos \theta$

Hence the vertical component of the force equals the weight of the fluid column above the area.

It can also be shown that the location of the action of the horizontal component will be at the centre of pressure of the projected area and the line **of action of the vertical component will be along the centroid of the column of the liquid above the plane.**

Using equation (3.3.2), denoting the distance along the plane as y,

$y_{cp} = (I_G / \bar{y} \,.\, A) + \bar{y}$, and in case **the edge is at the free surface,** $\bar{y} = d/2$

The equation reduces to $y_{cp} = (2/3)\, d$ or $(2/3)\, \bar{y} \sin \theta$ from free surface (3.4.1)

Example 3.5. *An automatic gate which will open beyond a certain head h is shown in Fig. Ex. 3.5.* ***Determine the ratio of h/L.*** *Neglect the weight of gate, friction etc.*

Consider 1 m width of the gate

The vertical force = $L \gamma h$ and acts at a distance $L/2$ from O.

The horizontal force on the gate = $\gamma h \, h/2$ and acts at $h/3$ distance from O.

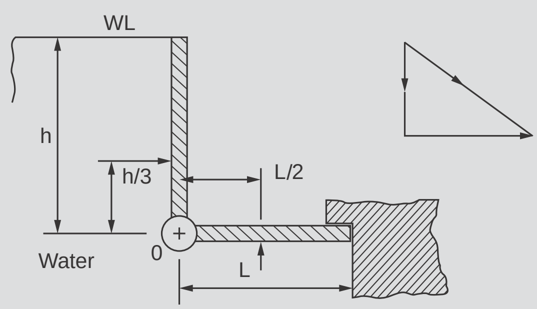

Figure Ex. 3.5

Note : When the water level is at the top of vertical plate, the centre of pressure will be at $(2/3)h$ from top. See 3.4.1.

Taking moments about O, $L\ \gamma\ h\ (L/2) = \gamma\ h\ (h/2)\ (h/3)$

$$L^2 = h^2/3 \text{ (or) } \mathbf{L = h/(3)^{0.5} = 0.5774\ h}$$

For example if $L = 2$ m, then $h = 3.4641$ m.

Example 3.6. *A modified form of automatic gate is shown in Fig. Ex. 3.6. Determine the value of h in terms of L, W, and* L_w *where W is the weight for unit width of gate and* L_w *is the distance from O to the line of action of W.*

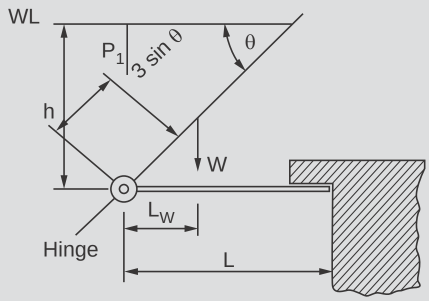

Figure Ex. 3.6

Considering unit width,

The vertical force on the horizontal side = $\gamma\ h\ L$ and acts upwards at $L/2$ from O.

Weight of the gate = W and acts downwards at L_w from O.

Total force normal to the plate $F = \gamma\ .\ A.\ \bar{h}$

For unit width, $A = h/\sin\theta$ and $\bar{h} = h/2$

Using equation (3.4.1.), the line of action along the inclined side can be obtained as $h/3 \sin\theta$ from bottom edge, or $\left(\dfrac{2}{3}\dfrac{h}{\sin\theta} \text{ from top}\right)$

Pressure force normal to plate = $\gamma\,(h/2)\,(h/\sin\theta)$ and acts at $(h/3\sin\theta)$ from O.

Taking moments about O,

$$W.L_w + \gamma\,(h/2)\,(h/\sin\theta)\,(h/\ 3\sin\theta) = \gamma\ h\ L\ L/2$$

$$\mathbf{(\gamma / 6 \sin^2\theta)\ h^3 - (\gamma L^2 / 2)\ h + W\ L_w = 0}$$

This is a cubic equation in h and can be solved by trial. With the calculators available presently cubic equations can be solved directly.

Example 3.7. *An automatic gate which opens beyond a particular head is as shown in Fig. Ex. 3.6. For the following data, determine the value of water head h to open the gate.* $\theta = 50°$, $L = 1m$, $W = 8000\ N$, $L_w = 0.265\ m$.

Using the equation derived in example 3.6,

$$(\gamma / 6 \sin^2\theta)\ h^3 - \gamma\,(L^2/2)\ h + W\ L_w = 0$$

$$(9810/\ 6 \sin^2 50)\ h^3 - 9810(1^2 / 2)\ h + 8000 \times 0.265 = 0$$

or $$2786\ h^3 - 4905\ h + 2120 = 0$$

Solving by trial, $h = 1$ m., **Hence the gate will open when water rises to 1m above *O***, (The other solution is 0.506 m).

3.5 FORCES ON CURVED SURFACES

(*i*) Vertical forces : The vertical force on a curved surface is given by the weight of the liquid enclosed by the surface and the horizontal free surface of the liquid. The force acts along the centre of gravity of the volume. In case there is gas pressure above the surface, the force due to gas pressure equals the product of horizontal projected area and the gas pressure and acts at the centroid of the projected area.

If the other side of the surface is exposed to the same gas pressure, force due to the gas pressure cancels out. This applies to doubly curved surfaces and inclined plane surfaces.

***(ii)* Horizontal forces: The horizontal force equals the force on the projected area of the curved surface and acts at the centre of pressure of the projected area.** The value can be calculated using the general equation.

$$F = \gamma A \bar{h},$$

where A is the projected area and $\bar{h}$ is the depth of the centroid of the area.

These two statements can be proved as indicated below. Refer to Fig. 3.5.1. The volume above the surface can be divided into smaller elements. At the base of each element, the vertical force equals the weight of the small element. Thus the total vertical force equals the sum of the weights of all the elements or the weight of the liquid enclosed between the area and the horizontal surface. Consider an imaginary vertical surface $A'B'$. The element between $A'B'$ and the surface AB is in equilibrium. $A'B'$ gives the projected vertical area. The horizontal force on this area due to liquid pressure should equal the horizontal force on the curved surface for the volume $A'B'$ AB to be in equilibrium. Hence the horizontal force equals the force on the projected area due to liquid pressure.

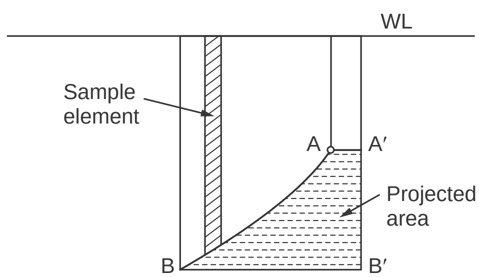

Figure 3.5.1

Example 3.8. *Determine the force exerted by sea water (sp. gravity = 1.025) on the curved portion AB of an oil tanker as shown in Fig. Ex. 3.8. Also determine the direction of action of the force.*

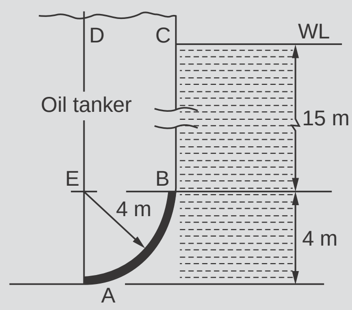

Figure Ex. 3.8

Consider 1m width perpendicular to paper,

The horizontal component of the **force** acting on the curved portion AB

$$= \gamma A \bar{h} = (1025 \times 9.81)(4 \times 1)(15 + 4/2)$$

$$= \mathbf{683757\ N}$$

Line of action of this horizontal force $= \bar{h} + (I_G/\bar{h} A)$

$$= 17 + [1 \times 4^3/12]\ [1/(17 \times 4 \times 1)] = 17.0784\ \mathbf{m}$$

from top and towards left

The vertical force is due to the volume of sea water displaced.

Vertical force = [volume *BCDE* + volume *ABE*] γ

$= [(15 \times 4 \times 1) + (4^2 \times \pi \times 1/4)]\ [1025 \times 9.81]$

= **729673 N** (acts upwards).

To find the location of this force : Centre of gravity of the column *BCDE* is in the vertical plane 2 m from the edge.

Centre of gravity of the area *ABE* = $(4 - 4R/3\pi)$ from the edge

$= (4 - 4 \times 4/3\pi) = 2.302$ m from the edge.

Taking moments of the area about the edge, the line of action of vertical force is

$= [(2.302 \times 4^2\ \pi/4) + 2 \times (15 \times 4)]\ /\ [(4^2\ \pi/4) + (15 \times 4)]$ = **2.0523 m from the edge.**

The resultant force = $(683757^2 + 729673^2)^{0.5}$ = **999973.16 N**

The direction of action to the vertical is,

$\tan\theta = 683757 / 729673 = 0.937 \quad \therefore \quad \theta = 43.14°$

The answer can be checked by checking whether the resultant passes through the centre of the circle (as it should) by taking moments about the centre and equating them.

$729673(4 - 2.0523) - 683757 \times 2.078 = 337$

Compared to the large values, the difference is small and so the moments are equal and the resultant can be taken as zero. Hence the resultant can be taken to pass through the centre of the cricle.

Example 3.9. *Determine the magnitude and direction of the resultant force due to water on a quadrant shaped cylindrical gate as shown in Fig. Ex. 3.9. Check whether the resultant passes through the centre.*

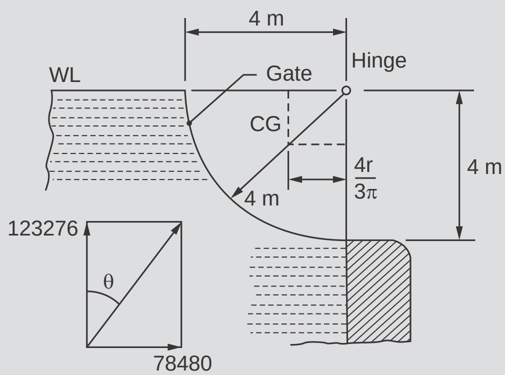

Figure Ex. 3.9

The horizontal force = $\gamma A \bar{h}$ where *A* is the projected area.

Considering unit width,

Horizontal force = 9810 × 2 × 4 × 1 = 78480 N, to the right

It acts at the centre of pressure of the projected area

i.e., at = 1.333 m from the bottom (*i.e.,* (1/3) × 4)

Vertical force = the weight to the liquid displaced

$= \pi \times 4^2 \times 1 \times 9810/4$ = 123276 N, upwards.

It acts at $4r/3\pi$ = 1.698 m, from the hinge.

Resultant force = $(123276^2 + 78480^2)^{0.5}$ = 146137 N

Angle is determined by $\tan\theta$ = 78480/123276 = 0.6366,

∴ θ = 32.48° where θ is the angle with vertical.

To check for the resultant to pass through the centre the sum of moment about *O* should be zero.

$$78480 \times (4 - 1.3333) - 123276 \times 1.698 = 42.$$

Compared to the values the difference is small and these can be assumed to the equal. Hence the resultant passes through the centre of the cricle.

3.6 HYDROSTATIC FORCES IN LAYERED FLUIDS

Two fluids may sometimes be held in a container one layer over the other. In such cases the total force will equal the some of the forces due to each fluid. The centre of pressure has to determined for each layer separately with reference to the centroid of each area. The location of the point of action of the total force can be determined taking moments about some convenient references.

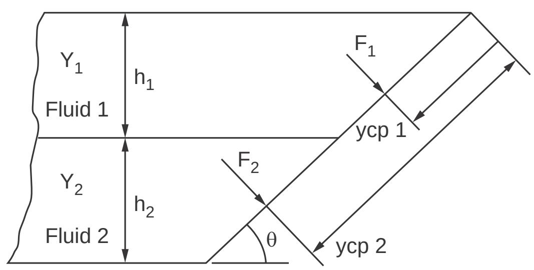

Figure 3.6.1

Total force, $F = F_1 + F_2 + ... = P_1A_1 + P_2A_2 + ...$

The depth of centre of pressure of fluid 1 is determined using the eqn. (4.3.2)

$$y_{cp1} = -\frac{\rho_1 g \sin\theta\, I_{xx}}{P_1 A_1} + \bar{y}_1 \tag{3.8.1}$$

This distance is with respect to the centroid of the area.

Note : (P_1 / ρ_1) gives the head of the fluid as ρ is different for different fluids, this form is prferable.

Example 3.10. *A tank 20 m deep and 7 m wide is layered with 8 m of oil, 6 m of water and 4 m of mercury. Determine the total hydrostatic force and resultant centre of pressure on the side. Specific gravity of oil is 0.881 and that of mercury is 13.6.*

Pressures at the centroid of each layer is

$$P_{cg1} = 881 \times 4 \times 9.81 = 34570.44 \text{ N/m}^2$$

$$P_{cg2} = (881 \times 4 \times 9.81) + (1000 \times 3 \times 9.81) = 64000 \text{ N/m}^2$$

$$P_{cg3} = (881 \times 4 \times 9.81) + (1000 \times 3 \times 9.81) + (13600 \times 2 \times 9.81)$$

$$= 330832 \text{ N/m}^2$$

$F_1 = P_{cg1} \times A_1 = 34570 \times 8 \times 7 = 1935944$ N

$F_2 = P_{cg2} \times A_2 = 64000 \times 6 \times 7 = 2688018$ N

$F_3 = P_{cg3} \times A_3 = 330832 \times 4 \times 7 = 9263308$ N

Total force = 13.887×10^6 N/m^2

$$y_{cp1} = -\frac{\rho_1 g \sin\theta\, I_{xx}}{P_1 A_1} = \frac{881 \times 9.81 \times 8^3 \times 7}{12 \times 34570} = 1.333 \text{ m as } \theta = 90°$$

$$y_{cp2} = \frac{1000 \times 9.81 \times 6^3 \times 7}{12 \times 2688018} = 0.46 \text{ m},$$

$$y_{cp3} = \frac{13600 \times 9.81 \times 4^3 \times 7}{12 \times 9263308} = 0.54 \text{ m}$$

The line of action of the total force is determined by taking moment about the surface.

$$y \times 13.887 \times 10^6 = (5.333 \times 1935944) + (11.46 \times 268808) + (16.54 \times 9263308)$$

Solving y = 13.944 m.

Example 3.11. *A tank contains water upto 3 m height over which oil of specific gravity 0.8 is filled to 2 m depth. Calculate the pressure at 1.5 m, 2 m and 2.5 m. Also calculate the total force on a 6 m wide wall.*

(*i*) At 1.5 m depth, $P_{1.5} = (0.8 \times 1000) \times 9.81 \times 1.5 = 11770$ N/m^2

$P_{2.0} = (0.8 \times 1000) \times 9.81 \times 2 = 15700$ N/m^2

$P_{2.5} = 15700 + (1000 \times 9.81 \times 0.5) = 20600$ N/m^2

At the base, $P = 15700 + (1000 \times 3 \times 9.81) = 45130$ N/m^2

Total force, $F = (15700 \times 2 \times 6/2) + \{(45130 + 15700) \times 3 \times 6/2\} = 641670$ N

SOLVED PROBLEMS

Problem 3.1. *The force due to water on a circular gate of 2m dia provided on the vertical surface of a water tank is 12376 N. Determine the level of water above the gate. Also* ***determine the depth of the centre of pressure from*** *the centre of the gate.*

Total force on the gate = $\gamma A \bar{h} = (1000 \times 9.81)(\pi \times 2^2/4) \times \bar{h} = 123276.09$ N

Solving $\bar{h}$ = 4 m. So the depth of water above the centre = 4 m

Centre of action of this force

$= (I_G / \bar{h} A) + \bar{h} = (\pi \times 2^4/64)/(4 \times \pi \times 2^2/4) + 4$

= **4.0625 m, from the water level**

or 0.0625 m from the centre of the gate

Check using eqn. (3.4.2), $h_{cp} = (D^2/16\bar{h}) + \bar{h} = (2^2/(16 \times 4) + 4 = 4.0625$ m

Chapter 3

Problem 3.2. *A circular plate of 3 m dia is vertically placed in water with its centre 5 m from the free surface. (i) Determine the force due to the fluid pressure on one side of the plate and also its point of application. (ii) Also find the diameter of a concentric circle dividing this area into two so that force on the inner circular area will equal the force on the annular area. (iii) Determine also the centre of pressures for these two areas separately. (iv) Show that the centres of pressure of the full area lies midway between these two centres of pressure.*

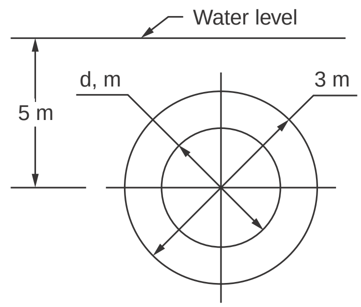

Figure P. 3.2

(*i*) The total force on the circular plate

$$\gamma A \bar{h} = 9810 \times (\pi \times 3^2/4) \times 5 = 346714 \text{ N}$$

Centre of pressure $= \bar{h} + (I_G/\bar{h} A) = 5 + (\pi \times 3^4 \times 4/64 \times 5 \times \pi \times 3^2)$

$=$ **5.1125 m.**

(*ii*) The given condition is, force on the circular area = force on annular area, with outside diameter being 3 m. For the areas the centroid depth $\bar{h}$ is the same.

$$\gamma A_1 \bar{h} = \gamma A_2 \bar{h}$$

assuming that the diameter of the smaller circle to be d,

$$A_1 = \pi (3^2 - d^2)/4, A_2 = \pi d^2/4 \text{ equating}$$

$\therefore$ $3^2 - d^2 = d^2$ or $2d^2 = 3^2$ or $d = 2.12132$ m.

The force on the inner circular area = $9810 (\pi \times 3^2/2 \times 4) \times 5 =$ **173357 N**

(Checks as it is half of 346714 N)

Centre of pressure for the circle

$= 5 + [\pi \times 2.12132^4 \times 4/64 \times \pi \times 2.12132^2 \times 5] =$ **5.05625 m**

Centre of pressure for the annulus

$= 5 + [\{\pi (3^4 - 2.12132^4)/64\}/\{5 \times \pi (3^2 - 2.12132^2)/4\}]$

$=$ **5.16875 m**

The mid point of these two is = (5.16875 + 5.05625)/2 = 5.1125 m

(same as the centre of pressure of full area)

Problem 3.3. *A plane 3 × 4 m is vertically placed in water with the shorter side horizontal with the centroid at a depth of 5m. (i) Determine the total force on one side and the point of action of the force. (ii) Also determine the size of an inner rectangle with equal spacing on all sides, the total force on which will equal the force on the remaining area. (iii) Determine the centre of pressures for these areas and compare the moments of these forces about a horizontal axis passing through the centre of pressure for the whole area.*

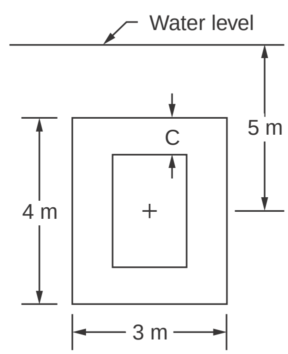

Figure P. 3.3

(i) The total force = $\gamma A \bar{h}$ = 1000 × 9.81 × 5 × 4 × 3 = 588600 N

Centre of pressure = 5 + (1/2) (3 × 4^3/5 × 12) = **5.2667 m**

(ii) Assume a spacing of C m on all sides to form the inner rectangle. The depth CG is the same for both areas

As $\gamma A_1 \bar{h} = \gamma A_2 \bar{h}$,

$A_1 = A_2$; (4 × 3) = 2(4 – 2C) (3 – 2C)

This reduces to $C^2 - 3.5\,C + 1.5 = 0$;

Solving C = 0.5 or 3.0. As C = 3 is not possible,

C = 0.5. The smaller rectangle is of 2 m × 3 m size,

(Check : 2 × 3 = 6, 4 × 3 = 12, half the area)

Pressure on the smaller rectangle = 1000 × 9.81 × 5 × 3 × 2 = **294300 N**

(iii) Centre of pressure for the inner area = 5 + (1/2) (2 × 3^3/2 × 3 × 5) = **5.15 m**

Centre of pressure for the outer area

= 5 + (1/2) (3 × 4^3 – 2 × 3^3) / [5 × (4 × 3 – 3 × 2)] = **5.3833 m**

(iv) Moment of the force on the inner area about the CP of the whole area

= 294300 (5.26667 – 5.15) = 34335 Nm (clockwise, acts above)

Moment of the force on the outer area

= 294300 (5.3833 – 5.26667)

= 34335 Nm (anti clockwise, acts below)

The moments are equal but are opposite in sign and the total is zero.

Problem 3.4. *A water tank has an opening gate in one of its vertical side of 10 m × 5m size with 5m side in the horizontal direction. If the water level is upto the top edge of the gate, locate three horizontal positions so that equal forces will act at these locations due to the water pressure.*

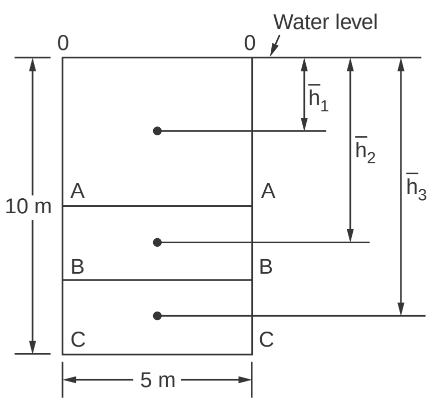

Figure P. 3.4

To solve this problem the area should be divided into three parts in each of which the force will be equal to 1/3 of the total force on the surface. Then the centre of pressure for each of the areas should be located to obtain the points of application of the forces.

Let $\bar{h}_1$ be the centroid of the top portion of the surface on which the force acting is equal to 1/3 of the total. Then

(*i*) $$\gamma \bar{h}_1 A_1 = (1/3)\, \gamma A\, \bar{h}$$

$$\gamma \bar{h}_1 (2\, \bar{h}_1 \times 5) = (1/3) \times \gamma \times (10 \times 5) \times 5$$

$$\therefore \quad \bar{h}_1^{\,2} = 25/3$$

$$\therefore \quad \bar{h}_1 = 2.887 \text{ m},$$

Therefore the depth of the top strip is 2 × 2.887 = 5.774 m

Centre of pressure for first strip $= [I_{G1}/\bar{h}_1 A_1] + \bar{h}_1$

$= [5 \times 5.774^3/12 \times 2.887\ (5 \times 5.774)] + 2.887 =$ **3.8493 m**

(*ii*) For the second strip, let the centroid be $\bar{h}_2$.

The depth of this second strip $= 2(\bar{h}_2 - 2\, \bar{h}_1)$

Force on the second strip, $\gamma\, \bar{h}_2 A_2 = (1/3)\, \gamma\, A\, \bar{h}$

$$\gamma\, \bar{h}_2 \times 5 \times 2\, (\bar{h}_2 - 2\, \bar{h}_1) = (1/3) \times \gamma \times 5 \times 10 \times 5$$

$$\bar{h}_2^{\,2} - 2\, \bar{h}_1\, \bar{h}_2 - (25/3) = 0 \quad \text{or} \quad \bar{h}_2^{\,2} - 2 \times 2.887\, \bar{h}_2 - (25/3) = 0$$

$$\bar{h}_2^{\,2} - 5.774\, \bar{h}_2 - 8.33 = 0, \text{ solving } \bar{h}_2 = 6.97 \text{ m}$$

The depth for second strip is $= 2(\bar{h}_2 - 2\, \bar{h}_1)$

$= 2\,(6.97 - 2 \times 2.887) = 2.392$ m

Centre of pressure for this second strip $= [I_{G2}/\bar{h}_2 A_2] + \bar{h}_2$

$= [5 \times 2.392^3/12 \times 6.97 \times (5 \times 2.392)] + 6.97 =$ **7.038 m**

(*iii*) The depth of the third and bottom strip is

$= 10 - 5.774 - 2.392 = 1.834$ m

$$\bar{h}_3 = 10 - (1.834/2) = 9.083 \text{ m}$$

Centre of pressure for third strip is $= [I_{G3}/\bar{h}_3 A_3] + \bar{h}_3$

$= 5 \times 1.834^3/(12 \times 9.083 \times 5 \times 1.834) + 9.083 =$ **9.114 m**

To keep the gate closed supports at these locations will be optimum *i.e.*, at depths of 3.849, 7.038 and 9.114 m. The average of these values will equal the depth of centre of pressure for the whole gate *i.e.,* 6.667 m (check).

Problem 3.5. *A triangular surface is kept vertical in water with one of its edges horizontal and at the free surface. If the triangle is divided by a line drawn from one of the vertices at the free surface such that the total force is equally divided between the parts, determine the ratio by which the opposite side is divided.*

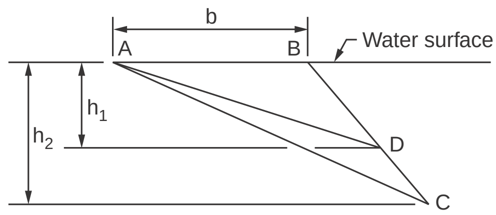

Figure P. 3.5

Consider the triangle *ABC*. Let the *AD* divided the triangle such that the total force on *ABD* equals the force on *ACD*.

Let h_1 be the height of triangle *ABD* and let h_2 be the height of triangle *ABC* along the depth.

The force on *ABC* = 2 × the force on *ABD*

The centroids will be at $h_1/3$ and $h_2/3$ for these triangles.

$$\gamma\,(bh_2/2)(h_2/3) = 2\,(\gamma\,bh_1/2)(h_1/3),\ \text{rearranging}$$

$$h_2^2 = 2\,h_1^2 \quad \text{or} \quad (h_2/h_1) = (2)^{0.5}$$

But $BC/BD = (h_2/h_1) = (2)^{0.5}$

The opposite side is divided the ratio of $(2)^{0.5}$ *i.e.*, $BC/BD = 1.4142$

∴ **CD/BD = 0.4142**

Problem 3.6. *In a water reservoir, the vertical gate provided for opening is a semicircular plate of dia 3 m with diameter horizontal and at the water level.* ***Determine the total pressure and its point of action if water level*** *is up to the top edge of the gate.*

Total force on the gate = $\gamma A\bar{h}$

$\bar{h}$ = centre of gravity of the semicircular surface $2D/3\pi$

$= 2 \times 3/3 \times \pi = 0.6366$ m

Total force $= 1000 \times 9.81 \times (\pi \times 3^2/4 \times 2) \times 0.6366 = 22072$ N = 22.072 kN

$I_{base} = \pi D^4/128$ (about the diameter)

Depth centre of pressure $= I_{base}/A\bar{h}$

$= (\pi \times 3^4/128)(2 \times 4/\pi \times 3^2)(1/0.6366)$

$=$ **0.8836 m**

Problem 3.7. *A water tank is provided with a gate which has a shape of a quadrant of a circle of 3 m radius. The gate is positioned in such a way that one straight edge of it is horizontal. Determine the force acting on the gate due to water and its point of action if the tank is filled with water upto 2 m above the edge.*

Distance of centre of gravity of the gate from the top edge $= 2D/3\pi = 4r/3\pi$

Total pressure on the gate $= \gamma\,\bar{h}\,A$

$= 1000 \times 9.81\,[2 + 4 \times 3/3\,\pi]\,[\pi \times 3^2/4] =$ **226976 N**

Moment of inertia for the gate with reference to the diameter = $\pi D^4/ 2 \times 128$

$$I_G = I - \bar{y}^2 A$$

Moment of inertia with reference to the centroid

$= [(1/2) \times \pi D^4/128] - (\pi D^2/4 \times 4) (2D / 3 \pi)^2$

$= [(1/2) \times \pi D^4/128] - \pi D^4/36 \pi^2$

$= \pi D^4 [(1/256) - (1/36 \pi^2)]$, Simplyfying

$I_G = \pi D^4/ 916$

Depth of centre of pressure = $[I_G/\bar{h} A] + \bar{h}$

$\bar{h} = 2 + 4r/3\pi = 2 + 4/\pi = 3.2732$ m as $r = 3$,

Depth of centre of pressure

$= [(\pi \times 6^4/916)/(3.2732 \times \pi 3^2/4)] + 3.2732$

$= 0.1921 + 3.2732 =$ **3.4653 m**

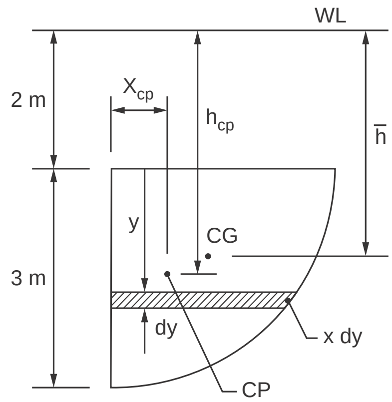

Figure P. 3.7

To determine the location of centre of pressure (as there is no line of symmetry with reference to the axes), moment of elementary forces of the elementary strips is taken with reference to the y axis and equated to the product of total force and the distance to the centre of pressure from x axis.

Circle equation is, $x^2 + y^2 = 9$ (taking centre as 0, 0)

Area of strip = $x.dy$, Force on the strip = $\gamma.h.x.dy$

Moment of force with respect to y axis $dM = \gamma\, h\, x\, dy\, x/2 = \frac{\gamma}{2}\, hx^2 dy$,

[force acts at a distance of $x/2$ from y axis]

As $h = y + 2$ and $x^2 = (9 - y^2)$, $dM = (\gamma/2)\,(y + 2)\,(9 - y^2)\, dy$

Integrating the above expression from $y = 0$ to $y = 3$

$$M = (\gamma/2) \int_0^3 [- y^3 - 2y^2 + 9y + 18]\, dy$$

$$= \frac{\gamma}{2}\left[-\frac{y^4}{4} - \frac{2}{3}y^3 + \frac{9}{2}y^2 + 18y\right]_0^3$$

$= (\gamma/2)\,[56.25] = (9810/2) \times 56.25 = 275,906$ Nm

Equating the moment for the total force

$226,976 \times x_p = 275,906 \quad \therefore \quad x_p = 1.2156$ m from the left edge

The centre of pressure is located at 3.4655 m below the free surface and 1.2156 m from the vertical edge.

Problem 3.8. *A right angle triangle of 2m × 2m sides lies vertically in oil of specific gravity 0.9 with one edge horizontal and at a depth of 2m. Determine the net force on one side and its point of action.*

The centre of gravity lies at 1/3 height from base. The moment of inertia about the *CG* is $bh^3/36$ where b is the base and h is the height.

Total force = $\gamma A \bar{h}$ = 1000 × 9.81 × 0.9 × (2 × 2/2)(2 + 2/3) = **47088 N**

(if water F = 47088/0.9 N)

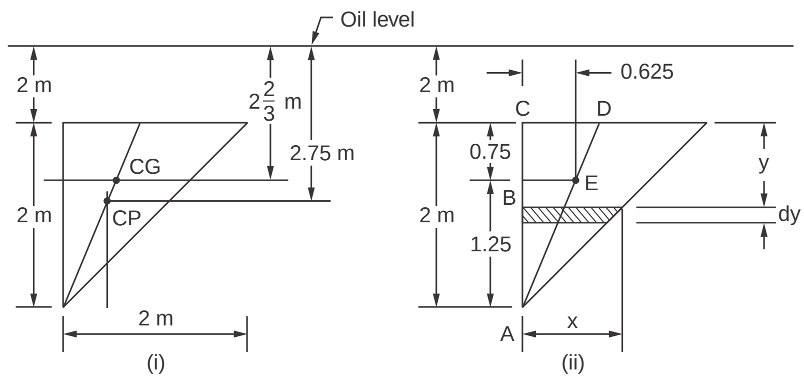

Figure P. 3.8

The depth of the centre of pressure = $h_{cp} = \bar{h} + (I_G/\bar{h}\,A)$

$$= (2 + 2/3) + [(2 \times 3^3/36)/(2 + 2/3)\,(2 \times 2/2)] = 2.75 \text{ m}$$

In the x direction, the centre of pressure will be on the median line, which is the line of symmetry. Referring to the figure,

$$(BE/CD) = (AB/AC) = (1.25/2),\ CD = 1 \text{ so, BE} = 0.625 \text{ m}$$

The centre of pressure is **2.75 m from top and 0.625 m from the vertical side.**

Check:

A strip of width dy is considered. Force on the strip = $\gamma\, h\, A = \gamma\,(y + 2)\,(x\,dy)$

In this case, by similar triangle ($AF = FG = 2 - y$) $\therefore$ $x = (2 - y)$

Force $dF = \gamma\,(y + 2)\,(2 - y)\,dy$ and Moment about the vertical edge = $dF\,(x/2)$

$dM = \gamma\,((2 - y)/\,2)\,(y + 2)\,(2 - y)\,dy$, Integrating from 0 to 2 for value of y,

$$M = (\gamma/2)\int_0^2 (y^3 - 2y^2 - 4y + 8)\,dy$$

$$= (9810 \times 0.9 / 2)\left[\frac{y^4}{4} - \frac{2}{3}y^3 - \frac{4}{2}y^2 + 8y\right]_0^2 = 29430 \text{ Nm}$$

Taking moments of the total force,

$$x_{cp} = 29430/47088 = 0.625 \text{ m (checks)}$$

Whenever there is a line of symmetry for the axis, the centre or pressure will be on it.

Problem 3.9. *Determine the centre of pressure and the total force for the combined area as shown in Fig. P.3.9. Assume water is the liquid.*

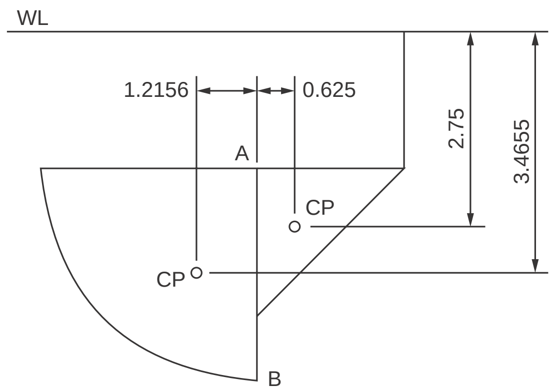

Figure P. 3.9

The shape is a combination of the shapes of problems P.3.7 and P.3.8.

The available values from these problems are

(*i*) For the Quadrant of circle the centre of pressure is at depth = 3.4655 m and distance from side = 1.2156 m

(*ii*) For the triangle, the centre of pressure is at depth = 2.75 m and distance from the side = 0.625 m

(note *CP* is independent of density as long as density is constant)

The forces are : C from the problems P.3.7 and P.3.8)

For triangle = 47088 × (1/0.9) = 52320 N

For quadrant = 226976 N

To locate the depth moment is taken about the surface.

Taking moments about *AB*, depth

$$y = [(226976 \times 3.4655) + (47088 \times 2.75/0.9)]/(226976 + 52320)$$

$$= \mathbf{3.3313\ m}$$

Taking moments about the common edge horizontal location is,

$$x = [(1.2156 \times 226976) - (0.625 \times 52320)]/(226976 + 52320)$$

$$= \mathbf{0.8708\ m} \text{ to the left of the common edge}$$

The combined centre of pressure lies at a depth of 3.3313 m and 0.8708 m to the left from the vertical common edge.

Problem 3.10. *An oil tank has an opening of 2 m square with diagonal horizontal in one of its vertical wall as shown in Fig. P. 3.10. Determine the total force and torque required to close the opening by a hinged gate exactly if the oil (sp. gravity 0.90) level is 5m above the centreline of the gate.*

The centre of gravity for the plate is on its diagonal.

Moment of inertia = Moment of inertia of the top triangle + Moment of inertia of bottom triangle = $bh^3/12 + bh^3/12 = 2\,bh^3/12$

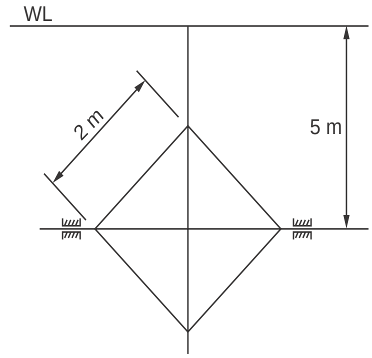

Figure P. 3.10

$$h = \sqrt{2^2 + 2^2}/2 = \sqrt{8}/2,\ b = \sqrt{8}$$

Moment of Inertia $= 2(\sqrt{8})\,(\sqrt{8}/2)^3/12 = 1.3333\ \text{m}^4,\ h = 5,\ A = 4$

Depth of centre of pressure = (1.3333/5 × 4) + 5 = **5.0667 m**

The centre of pressure lies on the vertical diagonal at a depth of 5.0667 m

Total force on the gate = $\gamma A\bar{h}$ = (1000 × 9.81 × 0.9) × 4 × 5 = **176.580 kN**

Torque required to close the gate = (5.0667 – 5) 176580 = **11,778 Nm**

Problem 3.11. *A hinged gate is held in position by a counter weight W as shown in Fig. P. 3.11. The gate is L m long along the slope and b m wide. The counter weight, W acts perpendicular to the gate which is inclined at angle θ. Determine the height of water for the movement of the gate outwards. Neglect the weight of the gate.*

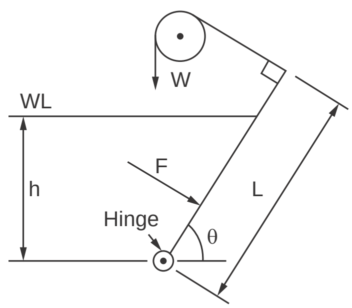

Figure P. 3.11

Let h m of water cause the gate to just start to move out,

Force on the gate = $\gamma A\,\bar{h} = \gamma\,(hb/\sin\theta)\,h/2$. The force acts at ($h$/3.sin θ) from the hinge (position of centre of pressure). The tension of the rope will equal W. Taking moments about the hinge,

$$W.L = \gamma\,(hb/\sin\theta)\,(h/2)\,(h/3\sin\theta)$$

∴ $$\mathbf{h^3 = 6\,W\,L\,\sin^2\theta\gamma b}$$

Problem 3.12. *A square shaped vertical closing for an opening in a water tank is pivotted along the middle, and held in place by a torque. Show that the torque required remains constant irrespective of the height of the fluid above the opening as long as it submerges the opening completely.*

The torque required = Net force × distance from centroid to centre of pressure

$$= \gamma A\,\bar{h}\,(h_{cp} - \bar{h}) = \gamma A\,\bar{h}\,(I_G / A\,\bar{h}) = \gamma I_G$$

As I_G depends only on the plate dimension, the torque is independent of the height of fluid subject to the condition that it submerges the plate fully. As force increases the distance of centre of pressure from *CG* decreases and hence this result.

Problem 3.13. *A square gate of side, a closing an opening is to be hinged along a horizontal axis so that the gate will open automatically when the water level reaches a certain height above the centroid of the gate. Determine the distance of this axis from the centroid if the height h is specified.*

The gate will begin to open if the hinge is at the level of the centre of pressure and the water level just begins to rises, in which case the centre of pressure will move upwards causing the opening.

$(h_{cp} - \bar{h})$ gives the distance of the hinge from the centroid

$$(h_{cp} - \bar{h}) = (I_G / A\bar{h}),\ \bar{h} = h/2,\ I_G = a^4/12 \text{ (square plate of side } a)$$

$$\therefore \quad (h_{cp} - \bar{h}) = (a^4\ 2/12\ a^2\ h) = a^2/6h$$

The location of the hinge should be at ($a^2/6h$) below the centroid.

If for example h = 6 m and a = 2 m, then

$$(h_{cp} - \bar{h}) = 4/(6 \times 6) = 1/9\text{m, below the centroidal axis.}$$

Problem 3.14. *A gate of rectangular shape hinged at A divides the upstream and downstream sides of a canal 5 m wide as shown in figure. Determine the angle θ in terms of the head h_1 and h_2 for equilibrium. Neglect the weight of the gate. The angle between the plates is 90°.*

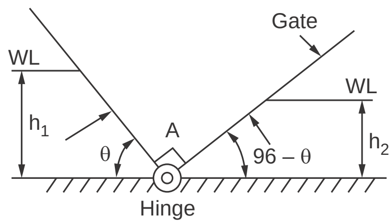

Figure P. 3.14

On the left side, the force is given by (per unit width) $\gamma (h_1/2)(h_1/\sin\theta)$ and it acts at $(1/3)(h_1/\sin\theta)$, from hinge. On the right side (as $\sin(90° - \theta) = \cos\theta$) the force is $\gamma(h_2/2)(h_2/\cos\theta)$ and acts at $(1/3)(h_2/\cos\theta)$, from hinge in both cases perpendicular to the plate.

Taking moments about the hinge,

$$\gamma (h_1/2)(h_1/\sin\theta)(h_1/2)(h_1/3\sin\theta) = \gamma (h_2/2)(h_2/\cos\theta)(h_2/3\cos\theta)$$

$$\therefore \quad \mathbf{(h_1/h_2)^3 = tan^2}\ \theta;\ \text{If } h_1 = h_2,\ \theta = 45° \text{ as } \tan^2\theta = 1\ \tan\theta = 1$$

This checks the expression. If h_1 = 2 m and h_2 = 3 m, then

$$(2/3)^3 = \tan^2\theta,\ \theta = 28.56° \text{ or the gate is tilted towards left.}$$

Problem 3.15. *Determine the total force and location of the centre of pressure on a rectangular plate 11 m long and 6 m wide with a triangular opening immersed in water at an angle of 40° to the horizontal as shown in figure. The top edge of the inclined plate is 6 m from the free surface.*

(*i*) Considering the whole plate without the opening

$$\bar{h} = 6 + (11/2)\sin 40 = 9.54 \text{ m}$$

$$\text{Total force} = \gamma\,\bar{h}\,A = 9810 \times 9.54 \times 11 \times 6/1000 = 6176.77 \text{ kN}$$

Depth of centre of pressure = $\bar{h} + (I_G \sin^2 \theta / A\,\bar{h})$

$$= 9.54 + [6 \times 11^3/12]\ [\sin^2 40/(11 \times 6)\ 9.54] = 9.977 \text{ m}$$

(*ii*) Considering the triangular hole portion only

$$\bar{h} = 6 + (4 + 5/3) \sin 40 = 9.6425 \text{ m}$$

$$\text{Total force} = \gamma\, \bar{h}\, A = 9810 \times 9.6425 \times (4 \times 5/2)/1000 = 945.33 \text{ kN}$$

Depth of centre of pressure

$$= 9.6425 + [4 \times 5^3/36]\ [\sin^2 40/9.6425\ (4 \times 5/2)] = 9.702 \text{ m}$$

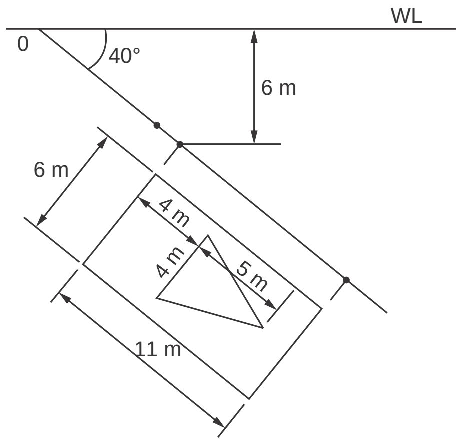

Figure P. 3.15

To determine the line of action of the resultant, moment is taken about *O*, at the surface

$$[9.977 \times 6176770/\sin 40] - [9.702 \times 945330/\sin 40]$$

$$= [h/\sin 40]\ [6176770 - 945330]$$

where h is the centre of pressure of the composite area.

Solving, depth of centre of pressure of the composite area of the gate = **10.027 m**

Calculate the depth of the *CG* of the area and check whether it is lower than 10.027 m.

CG is at 9.52 m depth. Net force on the composite area of the gate

$$= (6176.77 - 945.33) = 5231.44 \text{ kN}$$

Problem 3.16. *Determine the resultant force and the direction of its action on the segmental gate shown in Fig. P. 3.16.*

Height, $h = 4 \times 2 \sin 30 = 4$ m. Considering 1 m width, the horizontal force = 9810 × 2 × 4 = 78480 N. The line of action (centre of pressure) is 1.333 m from the bottom.

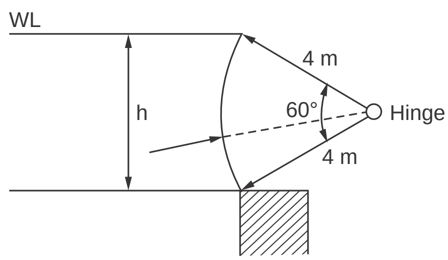

Figure P. 3.16

Vertical force (upwards) equals the weight of displaced. Volume of segment of circle

$$= [(\pi R^2 \times 60/360) - (2 R \sin 30 \times R \cos 30/2)] \times 1 = 1.4494\ m^3$$

Weight = 1.449 × 9810 = **14218 N** upwards direction.

tan θ = 78480/14218, θ = 79.73° from vertical

The net force is $(78480^2 + 14218^2)^{0.5}$ = **79757.5 N**

The centre of gravity of a segment of a circle from centre is given by

(2/3) $R \sin^3 \theta/(\text{Rad } \theta - \sin \theta \cos \theta)$ where θ is half of the segment angle.

Substituting the values

$$h_{CG} = (2/3) \times 4 \times \sin^3 30/[(\pi / 6) - \sin 30 \cos 30] = 3.67974 \text{ m}$$

taking moments about the centre,

$$(14218 \times 3.67974) - (2 - 1.33) \times 78477 = 0.$$

The quantities are equal. So the resultant passes through the centre and as the resulting moment about the centre is zero. **The line of action will pass through *O* and its direction will be (90 – 79.73)° with horizontal, as shown in figure.**

Problem 3.17. *A roller gate as shown in Fig. P. 3.17 has a span of 5 m. Determine the magnitude and direction of the resultant force on the cylinder when water just begins to overflow. Neglect weight of the gate.*

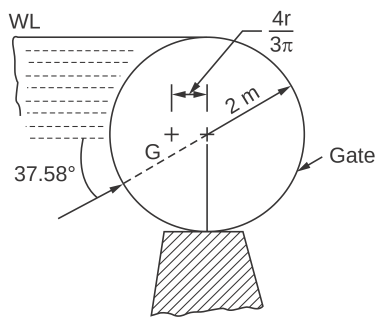

Figure P. 3.17

The horizontal force equals the force on the projected area,

$$\gamma A\ \bar{h} = 9810 \times 4 \times 5 \times 2 = 392400 \text{ N}$$

This acts at a distance of 1.3333 m from bottom twoards the right.

The vertical upward force is equal to the weight of the water displaced

$$= 5 \times 9810 \times \pi \times 2^2/2 = 308190 \text{ N}$$

It acts at a distance of $(4r/3\pi)$ = 0.84883 m left of centre and upwards

$$\text{Resultant} = (392400^2 + 308190^2)^{0.5} = 498958 \text{ N}$$

Taking moments about the centre,

$$392400 \times (2 - 1.3333) - 0.84883 \times 308189 = 0.$$

As the net moment about the centre is zero the resultant passes through the centre. The direction with horizontal is given by tan θ = 308190 / 392400, θ = 37.58°.

Problem 3.18. *Determine the magnitude and direction of the force on the elliptical tank portion AB as shown in Fig P. 3.18.*

The horizontal force on the elliptical portion equals the force on the projected area, considering 1 m width, $\bar{h}$ = 5m, $A = 4 \times 1$

$$\gamma A\ \bar{h} = 9810 \times 4 \times 5 = 196200 \text{ N acts at 1.333 m from bottom}$$

Vertical force on the elliptical portion equals the weight of water above this area. Area of ellipse = $\pi\, bh/4$ where b and h are minor and major axis. $b = 6$, $h = 8$. $A = 12\pi$. The area here is 1/4 th of the ellipse. Hence, $A = 3\,\pi\, m^2$ (ellipse portion)

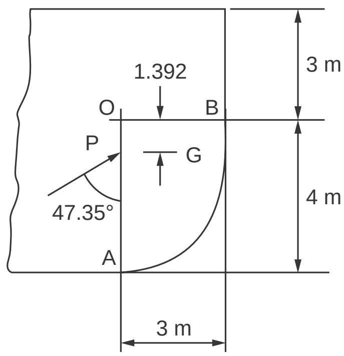

Figure P. 3.18

Rectangular portion above

$= 3 \times 3 = 9$ m^2. Volume $= 1 \times (3\,\pi + 9)$ m^3

Weight = 9810 $(3\,\pi + 9)$

= 180747 N = Total vertical force

The centre of gravity of the quarter of elliptical portion

$= (4b/3\pi)$,

$= 4 \times 3/3\,\pi = 4/\pi\, m$ from the major axis (as $b = 3$ m)

Centre of gravity of the rectangle = 1.5 m from the wall

Taking moments and solving, the location x of vertical force

$= [9 \times 1.5 + 3\,\pi\,(3 - 4/\pi)] / [9 + 3\,\pi]$ = **1.616 m** from wall

Resultant force = $(196200^2 + 180747^2)^{0.5}$ = 266766 N

To determine the line of action, let this line cut OA at a distance of h below O at P.

Then, taking moments about P, $(2.666 - h)$ 192600 = (3 – 1.616) 180747

∴ **h = 1.392 m, the line of action passes through a point P, 1.392 m below O at an angle of 47.35° from vertical.**

Problem 3.19. *Determine the resultant force on the wall of a tank ABC as shown in Fig. P 3.19.*

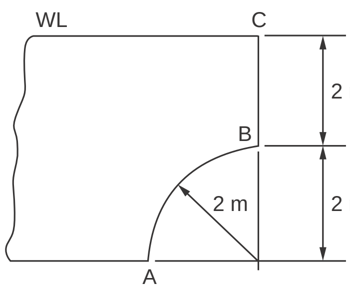

Figure P. 3.19

Considering unit width,

Horizontal force equals the force on the projected area = $\gamma A\, \overline{h}$

$= 9810 \times 4 \times 2$ = **78480 N**

This force acts at 2.6667 m from the top

The vertical force equals the weight of the volume above the surface (unit width)

Vertical force $= 9810 \times (4 \times 2 - \pi\, 2^2/4) \times 1$ = **47661 N** (downward)

Resultant $= (78480^2 + 47661^2)^{0.5}$ = **91818 N**

Angle with vertical θ : $\tan^{-1}$ (78480/47661) = **58.73°**

To fix the line of action, the line of action of the vertical force should be determined.

Problem 3.20. *The shape described as $x = 0.2y^2$ forms the wall of a gate. Derive expressions for the horizontal force, vertical force and the moment on the gate with respect to O, as shown in Fig. P. 3.20. Calculate the values for y = 3m.*

Consider unit width,

Horizontal force $= \gamma A\, \overline{h} = \gamma \times y \times (y/2) = \gamma\, y^2/2$

This force acts at $y/3$ from the bottom. Vertical force: weight of volume above the surface. Assuming unit width, the volume = area × width

$$A = \int_0^y x.dy = 0.2 \int_0^y y^2\, dy$$

$$= 0.2\, y^3/3 = xy/3$$

Vertical force = $(xy/3) \times \gamma \times$ width

$$= (0.2\, y^3/3) \times \gamma \times \text{width}$$

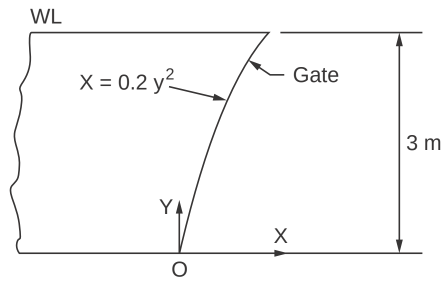

Figure P. 3.20

The position of line of action can be determined taking moment about the y axis. Let it be $\bar{x}$ from y axis.

$$\bar{x}\ .\ xy/3 = \int_0^y x\ .\ dy\ .\ x/2 = \frac{0.04}{2}\int_0^y y^4\ dy = \frac{0.04\ y^5}{10} = \frac{x^2\ y}{10},$$

$\therefore$ $\bar{x} = (3x/10) = 0.06\, y^2$

Clockwise moment about

$$O = (\gamma\, y^2/2) \times (y/3) + (0.2\, y^3/3)\, \gamma \times 0.06\, y^2$$

For $y = 3$ m and unit width

Horizontal force = 9810 (3 × 3/2) = 44145 N

Vertical force = $0.2\, y^3\, \gamma/3 = 0.2 \times 27 \times 9810/3 = 17658$ N

Clockwise moment = $(\gamma \times 27/6)\,(1 + 0.024 \times 9) = 53680$ N m

The direction of the force with the vertical can be found using

$$\tan^{-1}(44145/17658) = 68.2°$$

Resultant = $(44145^2 + 17658^2)^{0.5}$ = **47546 N**

To locate the actual line of action of the force, perpendicular distance from O × force = moment

$\therefore$ Distance = 53680/47456 = 1.129 m

It cuts the vertical from O at 1.129/sin 68.2 = **1.216 m**

Problem 3.21. *A hemispherical bulge of 3m diameter inwards is as shown in Fig. P. 3.21. Determine for the given dimensions, the magnitude and direction of the resultant force on the wall of the bulge (i) when water is full (ii) water level comes to the top of the bulge and (iii) water level upto the centre of the bulge.*

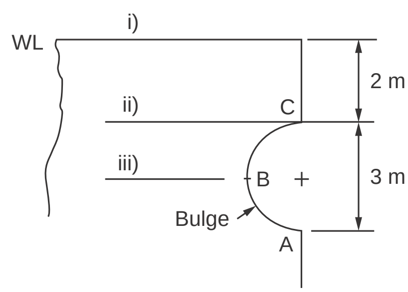

Figure P. 3.21

Force on the surface ABC is required. The projected area = $\pi\, D^2/4$

(*i*) Horizontal force = $9810 \times 3.5 \times \pi \times 3^2/4 = 242700$ N

Depth of centre of pressure

$$= 3.5 + (\pi/64)\,(3^4 \times 4/3.5 \times \pi \times 3^2) = 3.66071 \text{ m}$$

Vertical force = weight of the liquid displaced

$\gamma\,(1/2)\,(4\,\pi\,R^3/3) = 69343$ N and acts upwards

For the hemisphere, centre of action from surface = $3\,R/8 = 0.5625$ m from wall.

Note : The vertical force on the surface *AB* is due to the liquid column above it and acts upwards. The vertical force on the surface *BC* is due to the liquid column above it and acts downwards. **So the net force is due to the weight of the volume of liquid displaced and acts upwards.**

The resultant is given by $[242700^2 + 69343^2]^{0.5} = 252412$ N

The direction is given by (angle with vertical) θ,

$$\theta = \tan^{-1}\,(24700/69343) = 74.05°.$$

The angle with the horizontal will be 15.95°

Check whether the resultant passes through the centre by taking moment. It does.

$\{0.5625 \times 69343 - (0.16071 \times 242700)\} \cong 0$

(*ii*) When water level comes up to the edge, horizontal force

$$= \gamma\,h\,A = 9810 \times 1.5 \times (\pi \times 3^2/4) = 104014 \text{ N}$$

Horizontal force acts at $1.5 + (\pi\,3^4/64)\,(1/1.5)\,(4/\pi \times 3^2) = 1.875$ m

The vertical force remains the same.

Resultant = $[104014^2 + 69343^2]^{0.5} = 125009$ N

Does the resultant pass through the centre? Check.

Line of action, angle with vertical = $\tan^{-1}\,(104014/69343) = 56.31°$

(*iii*) When water comes to the centre, horizontal force

$$= 9810 \times 0.75 \times (\pi \times 3^2/8) = 26004 \text{ N}$$

$$I_G = I_b - A\,(\bar{h}^2),\ \bar{h} = 2D/3\,\pi,\ I_b = \pi\,D^4/128$$

$$\therefore \qquad I_G = 0.55565$$

Centre of pressure = $0.75 + [0.55565/(0.6366 \times \pi \times 1.5^2/2)]$

= **0.9973 m, down from B.**

Vertical force = $\gamma\,(1/4)\,(4\,\pi\,R^3/3) = 34672$ N (upwards)

Resultant = 43340N, θ = 36.87° with vertical.

Problem 3.22. *An oil tank of elliptical section of major axis 3 m and minor axis 2 m is completely filled with oil of specific gravity 0.9. The tank is 6 m long and has flat vertical ends. Determine the forces and their direction of action on the two sides and the ends.*

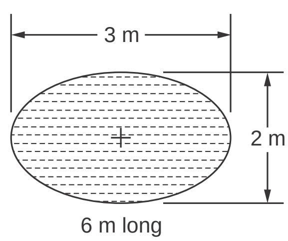

Figure P. 3.22

Considering the surface of the left half of the tank, horizontal force = $\gamma\,\bar{h}\,A$

$$= 9810 \times 0.9 \times 1 \times 2 \times 6 = 105948 \text{ N to the left}$$

Line of action = $1 + (6 \times 2^3/12)\,(1/1)\,(1/6 \times 2) = 1.333$ m from top. Similar force acts on the right half of the tank to the right, at the same level.

Vertical force on the left half = Weight of displaced liquid

= 9810 × 0.9 × (π × 3 × 2/4 × 2) = 20803 N

downward and the location is $4h/3\pi = 4 \times 1.5/3\pi = 0.63662$ m, from centre line

Resultant = $(105948^2 + 20803^2)^{0.5}$ = **107971 N**

Direction (with vertical) = $\tan^{-1}$ (105948/20803) = **78.89°**. Similar force acts on the other half.

Ends: Elliptical surfaces : $F = \gamma \bar{h} A$ = 9810 × 0.9 × 1 × π × 3 × 2/4 = **41606 N**

Line of action = 1 + (π × 3 × 2^3/64) (1/1) (4/π × 3 × 2)

= **1.25 m from top.**

Problem 3.23. *A square section tank of 3 m side and 2 m length as shown in Fig. P. 3.23 has the top of one side wall in the shape of a cylinder as indicated. The tank is filled with water as indicated. Determine the horizontal and vertical forces on the curved surface. Also locate the line of action of the resultant force. The water is under a gauge pressure of 20,000 N/m².*

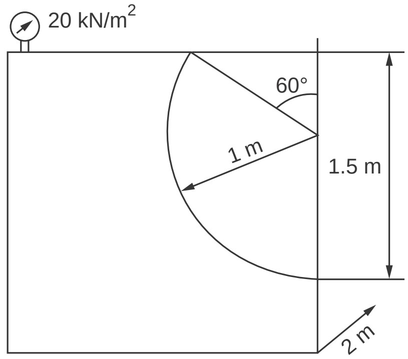

Figure P. 3.23

The horizontal force is the force due to water pressure on the projected area. It can be split up into two components (*i*) due to the water column and (*ii*) due to the pressure on the fluid

The horizontal force = $\gamma \bar{h} A + PA$

= 9810 × 0.75 × (1.5 × 2) + 20,000 × (1.5 × 2) = 22072.5 + 60,000

= 82072.5 N (to the right)

The first component acts at the centre of pressure and the second at the centre of gravity.

Centre of pressure due to fluid pressure

= 0.75 + (1/12) (2 × 1.5^3/ 0.75 × 2 × 1.5) = 1.0 m (from top).

Location of the net force is determined by taking moments about the top.

= {(22072.5 × 1) + (60000 × 0.75)} / (22072.5 + 60,000)

= 0.8172 m from top.

The vertical force can also be considered as the result of two action (*i*) the weight of displaced volume and (*ii*) the pressure on the projected area.

= 9810 [(120 × π × 1^2 × 2/360) + (cos 60 × sin 60 × 2/2)]

+ sin 60 × 2 × 20000

= 24794 + 34641 = 59435 N (upwards)

The resultant = $(82072.5^2 + 59435^2)^{0.5}$ = 101333 N

The resultant acts at an angle (with vertical), $\tan^{-1}$ (82072.5 / 59434.9) = 54.09°

Note : The problem can also be solved by considering an additional head of fluid equal to the gauge pressure.

Problem 3.24. *Determine the vertical and horizontal forces on the cylinder shown in Fig. P. 3.24. The cylinder is in equilibrium.*

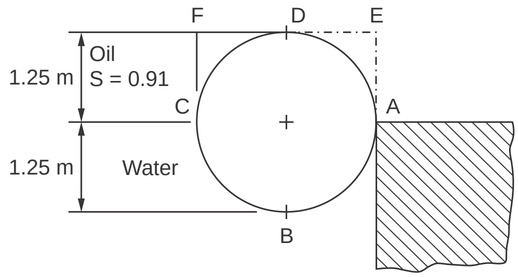

Figure P. 3.24

The horizontal force can be calculated as the sum of forces due to the oil and due to the water on projected area. Consider 1 m length. The horizontal force on *AB* and *BC* are equal and opposite.

The other horizontal force due to oil on *CD* is

$$= 9810 \times 0.91 \times (1.25/ 2) \times 1.25 \times 1$$

= **6974.3 N** and this force acts at

$[(1.25/2) + 1 \times 1 \times 1.25^3 \times 2/12 \times 1.25 \times 1.25 \times 1) = 0.833$ m from the top surface

The vertical upward force equals the weight of water displaced + weight of oil displaced (*AEDG* + *CDG*)

$$= [9810 \times (\pi \times 1.25^2/2)] + [(1.25 \times 1.25) + \pi \times 1.25^2/4) \times 0.91 \times 9810]$$

$$= (24077 + 24904) \text{ N}$$

Total upward force = **48981 N**

Workout the resultant as an exercise.

Problem 3.25. *A bridge is in the form of an elliptical arch and wter flows just touching the bottom. The major axis is 8 m and the minor axis is 3 m. Determine the upward force due to water pressure. The bridge is 4m wide.*

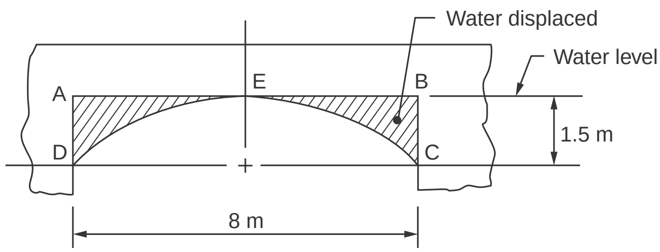

Figure P. 3.25

The upward force is due to the weight of water displaced as shown in figure.

Area of half ellipse = $\pi bh/8 = \pi \times 8 \times 3/8 = 3\pi \text{m}^2$

Area of rectangle = $8 \times 1.5 = 12 \text{ m}^2$

Weight of water displaced = $(12 - 3\pi) \times 4 \times 9810 = 101052$ N

Problem 3.26. *A channel is closed by two swinging lock gates each of 4 m wide and 6 m height and when closed the angle between them is 120°. On the upstream side the water level is 5.5 m and in the downstream it is 2m. Determine (i) normal force on each gate and (ii) the reaction between the gates. If the gates are hinged at 0.5 m and 5.5 m from the base, determine the reaction at each hinge.*

The normal force on each gate on the upstream side

$$= 9810 \times (5.5 /2) \times 5.5 \times 4 = 593505 \, N$$

This force acts at 5.5 / 3 = 1.8333 m from the bottom.

The normal force on the downstream side on each gate

$$= 9810 \times (2/2) \times 2 \times 4 = 78480 \text{ N}.$$

This force acts at 2/3 = 0.667 m from bottom.

Net normal force = 593505 – 78480 = 515025 N

To determine the reaction R considering the equilibrium and taking moments about the point A.

$$515025 \times 2 = R \times 4 \sin 30$$

R = 515025 N. This acts perpendicular to the contact as shown. The total reaction at the hinges should also equal this value. To determine the reaction at each hinge, moments can be taken with reference to the other hinge.

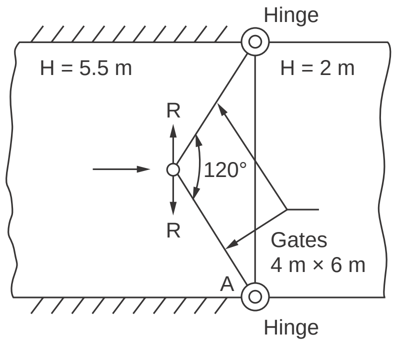

Figure P. 3.26

Taking moments from the top hinge at 5.5 m from base, (as half the force only is causing the reaction at the hinge, and as the reaction is at 30° to the plane of gate)

$$(593505/2)\,(5.5 - 1.83333) - (78480/2)\,(5.5 - 0.6667)$$

$$= R_1\,(5.5 - 0.5) \sin 30$$

R_1 = 359373 N and by similar calculation

R_2 = 155652 N, (check total as 515025)

Problem 3.27. *A dam section is 6 m wide and 20 m high. The average specific gravity of the material is 2.8. Determine the height of water which may just cause overturning of the dam wall.*

When the moment with downstream corner A of the structures weight equals the moment of the pressure force on the structure, the structure will tilt. *i.e.* Weight × AB = Pressure force × AC

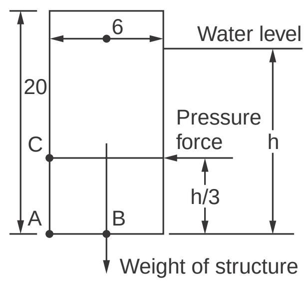

Figure P. 3.27

Considering 1m length, (CP is h/3 from bottom), taking moments

$$20 \times 6 \times 2.8 \times 9810 \times 3 = 9810 \times (h/2) \times h \times (h/3)$$

Solving, **h = 18.22 m.**

Problem 3.28. *Determine the resultant vertical force on the curved structure AB and also the line of action. At any section the height, x = 0.5 Z² where z is the width. Consider a width of 1m. Water stands upto x = 2m. (note the constant 0.5 should be dimensional, 1/m).*

The area and centre of gravity are to be determined by integration. Considering a strip at x and width dx,

$$A = \int_0^x Z\,dx \text{ where } Z = (x/0.5)^{0.5}$$

$$\therefore \quad A = 0.9428\,x^{1.5}$$

To obtain the line of centre of gravity from $z = 0$ line

$$A\,\overline{Z} = \int_0^x Z\,dx\,Z/2 = \int_0^x x\,dx = x^2/2$$

$$\therefore \qquad \overline{Z} = (x^2/2)\,(1/0.9428\,x^{1.5}) = x^{0.5}/1.8856$$

The vertical force due to gas pressure = vertical projected area × pressure

Top width, $Z = (3/0.5)^{0.5} = 2.45$ m. Considering 1 m width.

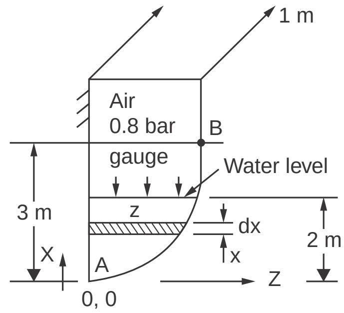

Figure P. 3.28

$$P = 0.8 \text{ bar} = 0.8 \times 10^5\ N/m^2$$

Pressure force = $2.45 \times 1 \times 0.8 \times 10^5$ N = 195959 N. This acts along 1.225 m from $Z = 0$ position. The vertical force due to the weight of water which stands upto $x = 2$ m over the surface.

Weight = volume × sp. weight = area × depth × sp. weight

$$A = 0.9428\,x^{1.5},\ x = 2,\ A = 2.6666 \text{ m}^2$$

Force due to water = 2.6666 × 9810 = 26160 N. This force acts at

$$\overline{Z} = x^{0.5}/1.8856 = 0.75 \text{ m from the vertical at } Z = 0$$

Total force = 195959 + 26160 = 222119 N, To determine the line of action:

$$195959 \times 1.225 + 26160 \times 0.75 = Z\,(222119),$$

$$z = 1.169 \text{ m.}$$

REVIEW QUESTIONS

1. Explain the importance of the study of fluid forces on surfaces and submerged bodies.
2. Explain the concept of centroid of an area or centre of gravity. What will be the value of first moment of area about the centroid.
3. Explain the concept of Moment of Inertia of a surface and the application of the same in the study of forces due to fluid pressure on surfaces.
4. Derive an expression for the force on a thin plate of given arbitrary shape immersed in a liquid at an angle θ to the free surface.
5. Obtain simplified expressions for the centre of pressure of vertical planes. (*i*) plate (*ii*) circle (*iii*) triangle.
6. Show that in the case of a rectangle inclined to the horizontal, immersed in a fluid with its centroid at a depth, h (*i*) the horizontal component of the resultant force equals the force on the vertical projection of the area and (*ii*) the vertical component equals the weight of the fluid column above this area.
7. Explain how force on curved surfaces due to fluid pressure is determined.

OBJECTIVE QUESTIONS

O Q.3.1. Fill in the blanks

1. The point of action of resultant fluid forces is called _________.
2. The moment of area about any axis through the centre of gravity will be _________.
3. The second moment of area about an axis through the centre of gravity will _________ compared to any other axis.
4. The centre of pressure will generally be _________ the centroid.
5. The hydrostatic force on a submerged plane surface depends on the _________ of the centroid.
6. The force due to liquid pressure acts _________ to the surface.
7. The law for calculating hydrostatic pressure is _________.
8. The second moment about any axis differs from the second moment through a parallel axis through the centroid by _________.
9. The distance of centre of pressure from its centroid for a vertical area immersed in liquid is given by _________.
10. The vertical distance between the centriod and centre of pressure over a plane area immersed at an angle θ to the free surface is given by _________.
11. The pressure at the same horizontal level in a static liquid is _________.

Answers

(1) Centre of pressure, (2) zero, (3) lower, (4) below, (5) depth, (6) normal, (7) $(dp/dy) = -\gamma$, (8) $A\,x^2$, x-distance between the axes, 9. $I_G / A\,\bar{h}$, 10. $I_G \sin^2\theta / A\,\bar{h}$ 11. the centre.

O Q.3.2 Fill in the blanks:

1. The horizontal force on a curved surface immersed in a liquid is equal to the force on _________.
2. The vertical force on a curved surface equals the _________.
3. The line of action of horizontal force on a curved surface immersed in a liquid is_________.
4. The line of action of vertical force on a curved surface immersed in a liquid is _________.
5. The force due to gas pressure on curved surface in any direction _________.
6. The resultant force on cylindrical or spherical surfaces immersed in a fluid passes through _______.

Answers

(1) the vertical projected area, (2) the weight of column of liquid above the surface, (3) the centre of pressure of the vertical projected area, (4) the centriod of the liquid column above the surface, (5) equals the product of gas pressure and projected area in that direction, (6) the centre.

O Q.3.3 Fill in the blanks using increases, decreases or remains constant :

1. The force due to liquid pressure _________ with depth of immersion.
2. The distance between the centriod and the center of pressure _________ with depth of immersion.
3. When a plane is tilted with respect to any centriodal axis the normal force on the plane due to liquid pressure _________.
4. The location of centre of pressure of a plane immersed in a liquid _________ with change in density of the liquid.

Answers

Increases : 1, Decreases : 2, Remains Constant : 3, 4.

O Q.3.4 Indicate whether the statements are correct or incorrect :

1. The centre of pressure on a plane will be at a lower level with respect to the centroid.
2. In a plane immersed in a liquid the centre of perssure will be above the centroid.
3. The resultant force due to gas pressure will act at the centroid.
4. The vertical force on an immersed curved surface will be equal to the column of liquid above the surface.
5. The normal force on an immersed plane will not change as long as the depth of the centroid is not altered.
6. When a plane is tilted along its centroidal axis so that its angle with horizontal increases, the normal force on the plane will increase.

Answers

(1) Correct : 1, 3, 5 (2) Incorrect : 2, 4, 6.

O Q.3.5 Choose the correct answer :

1. The pressure at a depth 'd' in a liquid, (above the surface pressure) is given by

   (*a*) $\rho\, g$ (*b*) $\gamma\, d$

   (*c*) $-\gamma\, d$ (*d*) $(\rho/g)d$ (usual notations)

2. The density of a liquid is 1000 kg/m^3. At location where g = 5 m/s^2, the specific weight of the liquid will be

   (*a*) 200 N/m^3 (*b*) 5000 N/m^3

   (*c*) 5000 × 9.81 / 5 N/m^3 (*d*) 5000 × 5 / 9.81 N/m^3

3. The centre of pressure of a rectangular plane with height of liquid h m from base

   (*a*) h/2 m from bottom (*b*) h/3 m from top

   (*c*) h/3 m from bottom

   (*d*) can be determined only if liquid specific weight is known.

4. The horizontal force on a curved surface immersed in a liquid equals

   (*a*) the weight of the column of liquid above the surface

   (*b*) the pressure at the centroid multiplied by the area

   (*c*) the force on the vertical projection of the surface

   (*d*) the pressure multiplied by the average height of the area.

5. The location of the centre of pressure over a surface immersed in a liquid is

   (*a*) always above the centroid

   (*b*) will be at the centroid

   (*c*) will be below the centroid

   (*d*) for higher densities it will be above the centroid and for lower densities it will be below the centroid.

6. The pressure at a point y m below a surface in a liquid of specific weight γ as compared to the surface pressure, P will be equal to

   (*a*) $P + (y/\gamma)$ (*b*) $P + y\gamma$

   (*c*) $P - (y\, g/\gamma)$ (*d*) $P + (y\, .\, g/\gamma)$.

7. When the depth of immersion of a plane surface is increased, the centre of pressure will
   (*a*) come closer to the centroid
   (*b*) move farther away from centroid
   (*c*) will be at the same distance from centroid
   (*d*) depend on the specific weight of the liquid.
8. A sphere of R m radius is immersed in a fluid with its centre at a depth h m The vertical force on the sphere will be
   (*a*) $\gamma\,(4/3)\pi R^3$ (*b*) $\gamma\pi R^2 h$
   (*c*) $\gamma\,(\pi R^2 h + 8\pi R^2/3)$ (*d*) $\gamma\,(\pi R^2 h - 8\pi R^2/3)$.

Answers

(1) *b* (2) *b* (3) *c* (4) *c* (5) *c* (6) *b* (7) *a* (8) *a*.

O Q.3.6 Match the sets A and B :

(I)

A	B
1. Specific weight	(*a*) m^3
2. Density	(*b*) m^4
3. Second moment of area	(*c*) N/m^2
4. First moment of area	(*d*) kg/m^3
5. Pressure	(*e*) N/m^3

Answers

1 – *e*, 2 – *d*, 3 – *b*, 4 – *a*, 5 – *c*.

(II)

A	B
1. Centroid	(*a*) always positive
2. Centre of pressure	(*b*) area moment zero
3. Free surface	(*c*) resultant force
4. Second moment of area	(*d*) constant pressure

Answers

1 – *b*, 2 – *c*, 3 – *d*, 4 – *a*.

(III) Moment of inertia of various shapes :

1. Circle about centroidal axis	(*a*) $B\,h^3/36$
2. Rectangle about centroidal axis	(*b*) $D^4/64$
3. Triangle about centroidal axis	(*c*) $D^4/128$
4. Semicircle about base	(*d*) $B\,h^3/12$

Answers

1 – *b*, 2 – *d*, 3 – *a*, 4 – *c*.

EXERCISE PROBLEMS

E.3.1. Determine the centroid of the following shapes shown in Fig E. 3.1 from the given reference lines.

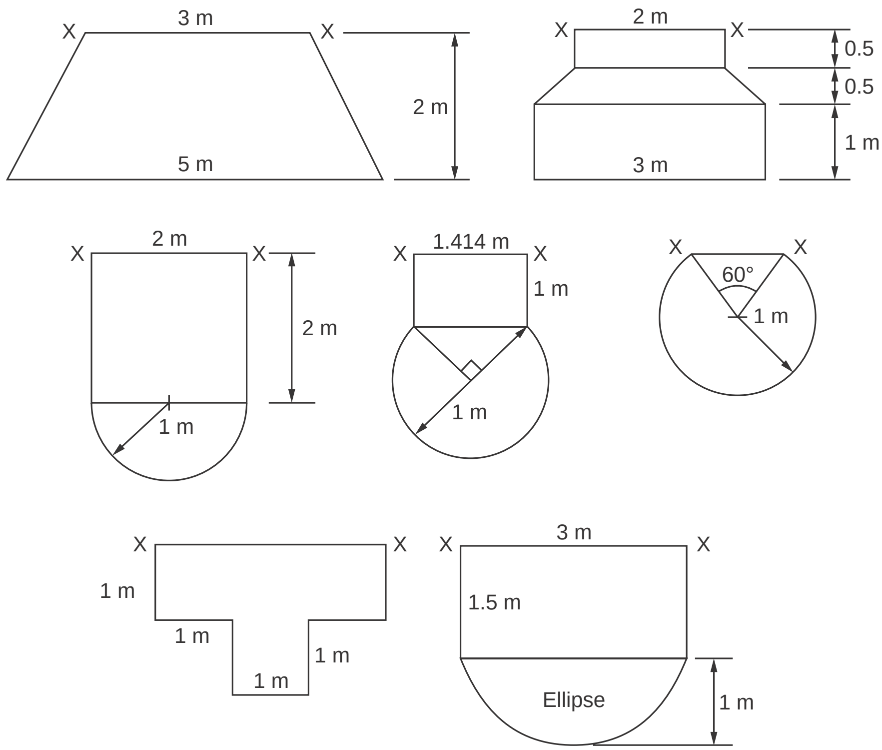

Figure E. 3.1

E.3.2. For the shapes in Fig E. 3.1, determine the moment of inertia of the surfaces about the axis *xx* and also about the centroid.

E.3.3. From basics (by integration) determine the forces acting on one side of a surface kept vertical in water as shown in Fig. E. 3.3.

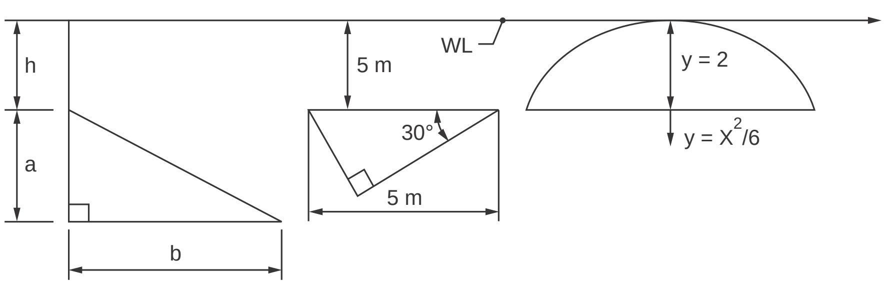

Figure E. 3.3

E.3.4. Determine the magnitude and location of the hydrostatic force on one side of annular surface of 2 m *ID* and 4 m *OD* kept vertical in water.

E.3.5. Determine the moment required to hold a circular gate of 4 m dia, in the vertical wall of a reservoir, if the gate is hinged at (*i*) the mid diameter (*ii*) at the top. The top of the gate is 8 m from the water surface.

E.3.6. Determine the compressive force on each of the two struts supporting the gate, 4 m wide, shown in Fig. E. 3.6.

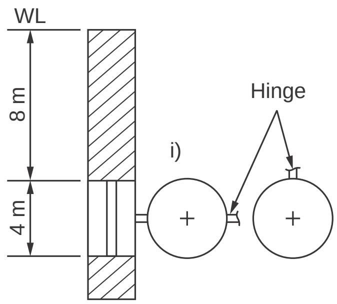

Figure E. 3.5

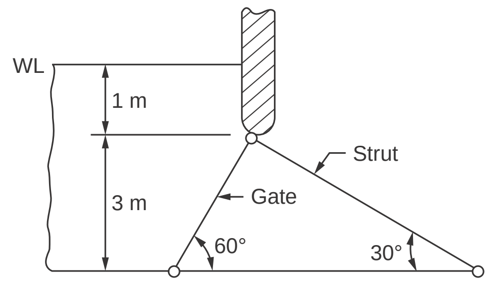

Figure E. 3.6

E.3.7. An annular plate of 4 m *OD* and 2 m *ID* is kept in water at an angle of 30° with the horizontal, the centre being at 4 m depth. Determine the hydrostatic force on one side of the plane. Also locate the centre of pressure.

E.3.8. A tank contains mercury upto a height of 0.3 m over which water stands to a depth of 1 m and oil of specific gravity 0.8 stands to a depth of 0.5 m over water. For a width of 1 m determine the total pressure and also the point of action of the same.

E.3.9. A trapezoidal gate of parallel sides 8 m and 4 m with a width of 3 m is at an angle of 60° to the horizontal as shown in Fig. E. 3.9 with 8 m length on the base level. Determine the net force on the gate due to the water. Also find the height above the base at which the resultant force acts.

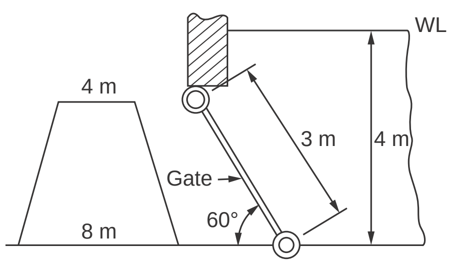

Figure E. 3.9

E.3.10. Show that the resultant force on a submerged plane remains unchanged if the area is rotated about an axis through the centroid.

E.3.11. A gate as shown in Fig. E. 3.11 weighing 9000 N with the centre of gravity 0.5 m to the right of the vertical face holds 3 m of water. What should be the value of counter weight W to hold the gate in the position shown.

E.3.12. A rectangular gate of 2 m height and 1 m width is to be supported on hinges such that it will tilt open when the water level is 5 m above the top. Determine the location of hinge from the base.

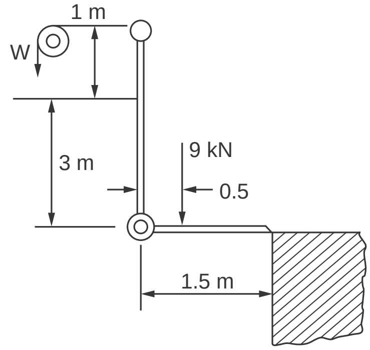

Figure E. 3.11

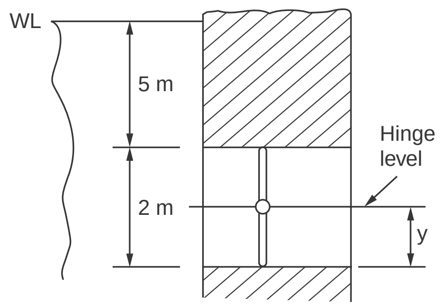

Figure E. 3.12

E.3.13. Show that as the depth of immersion increases, the centre of pressure approaches the centroid.

E.3.14. Determine the magnitude and line of action of the hydrostatic force on the gate shown in Fig. E. 3.14. Also determine the force at the edge required to lift the gate. The mass of the gate is 2500 kg and its section is uniform. The gate is 1 m wide.

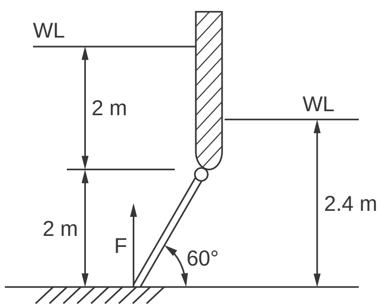

Figure E. 3.14

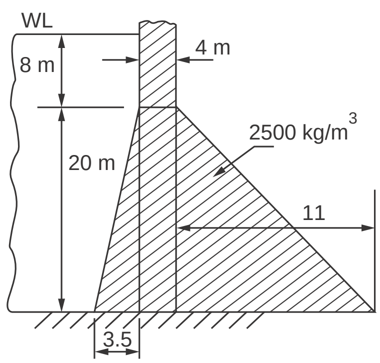

Figure E. 3.15

E.3.15. A dam section is shown in figure. Determine the location where the resultant hydrostatic force crosses the base. Also calculate the maximum and minimum compressive stress on the base.

E.3.16. An automatic flood gate 1.5 m high and 1 m wide is installed in a drainage channel as shown in Fig. E. 3.16. The gate weighs 6 kN. Determine height of water backing up which can lift the gate.

E.3.17. Compressed air is used to keep the gate shown in Fig. E. 3.17 closed. Determine the air pressure required.

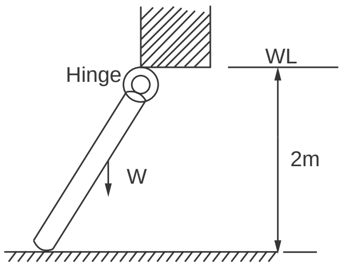

Figure E. 3.16

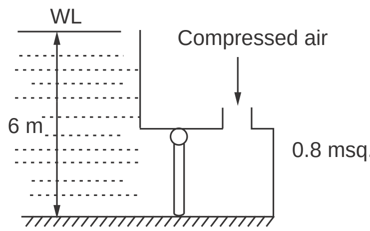

Figure E. 3.17

E.3.18. A spherical container of 6 m diameter is filled with oil of specific gravity 0.73. Determine the resultant force on one half of the sphere divided along the vertical plane. Also determine the direction of action of the force.

E.3.19. An inverted frustum of a cone of base dia 1 m and top dia 6 m and height 5 m is filled with water. Determine the force on one half of the wall. Also determine the line of action.

E.3.20. A conical stopper is used in a tank as shown in Fig. E. 3.20. Determine the force required to open the stopper.

E.3.21. Determine the total weight/m length of a gate made of a cylindrical drum and a plate as shown in Fig. E. 3.21, if it is in equilibrium when water level is at the top of the cylinder.

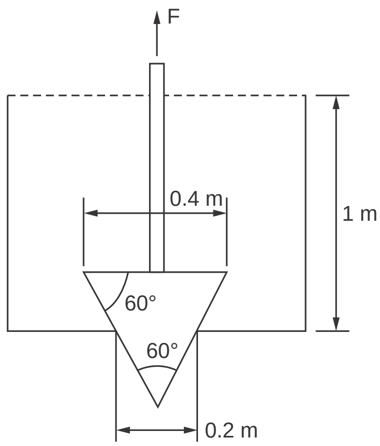

Figure E. 3.20

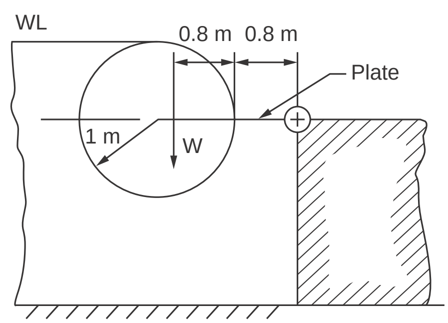

Figure E. 3.21

E.3.22. A gate 12 m long by 3 m wide is vertical and closes an opening in a water tank. The 3 m side is along horizontal. The water level is up to the top of the gate. Locate three horizontal positions so that equal forces acting at these locations will balance the water pressure.

[*3.4641 m, 8.4452 m 10.9362 m*]