4 Buoyancy Forces and Stability of Floating Bodies

4.0 ARCHIMEDES PRINCIPLE

In the previous chapter the forces due to fluid on surfaces was discussed. In this chapter the forces due to fluid on floating and submerged bodies is discussed. It is applicable in the design of boats, ships, balloons and submersibles and also hydrometers. In addition to the discussion **of forces** the **stability of floating bodies** due to small disturbances is also discussed.

If an object is immersed in or floated on the surface of fluid under static conditions a force acts on it due to the fluid pressure. This force is called **buoyant force**. The calculation of this force is based on Archimedes principle.

Archimedes principle can be stated as (*i*) **a body immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced and (*ii*) a floating body displaces its own weight of the liquid in which it floats.**

Other possible statements are: The resultant pressure force acting on the surface of a volume partially or completely surrounded by one or more fluids under non flow conditions is defined as **buoyant force and acts vertically on the volume.** The buoyant force is equal to the weight of the displaced fluid and **acts upwards through the centre of gravity of the displaced fluid.** This point is **called the centre of buoyancy** for the body.

This principle directly follows from the general hydrostatic equation, $F = \gamma Ah$ and is applied in the design of ships, boats, balloons and other such similar systems. The stability of such bodies against tilting over due to small disturbance can be also checked using this principle.

4.1 BUOYANCY FORCE

Consider the immersed or floating body shown in Fig. 4.1.1. The total force on the body can be calculated by considering the body to consist of a large number of cylindrical or prismatic elements and calculating the sum of forces on the top and bottom area of each element.

(*i*) **Immersed body.** Consider a prismatic element :

Let the sectional area be dA, Force on the top $dF_1 = dA\,\gamma\,h_1$ and

Force on the base $dF_2 = dA\,\gamma\,h_2$ (cancelling P_{atm}, common for both terms)

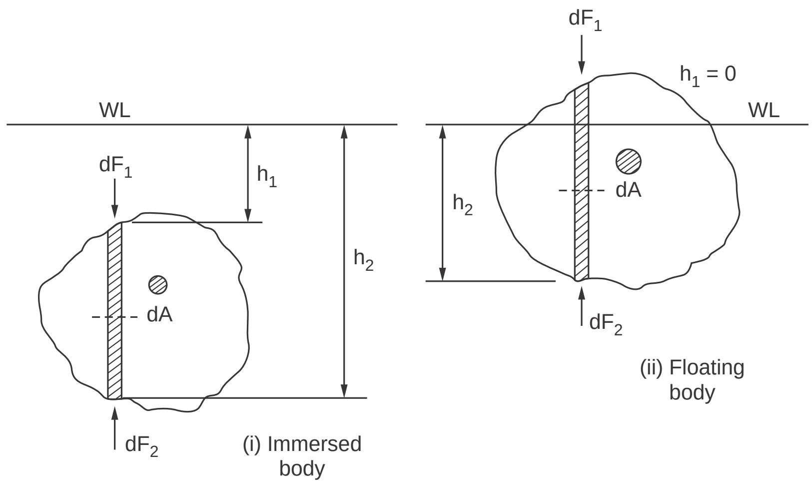

Figure 4.1.1 Proof for Archimedes principle

Net force on the element $(dF_2 - dF_1) = \gamma\, dA\, (h_2 - h_1) = \gamma\, dV$.

where dV is the volume of the element. This force acts upwards. as $h_2 > h_1$

Summing up over the volume, $F = \gamma V$ (or) the weight of the volume of liquid displaced.

(*ii*) **Floating body.** Considering an element of volume dV, Force on the top of the element $dF_1 = dA.\, P_a$ and Force on the base of the element $dF_2 = dA\, (\gamma\, h_2 + P_a)$

$$dF_2 - dF_1 = \gamma\, dA\, h_2 = \gamma\, dV$$

where dV is the volume of the fluid element displaced. Summing up over the area,

$$F = \gamma V, \text{ the weight of volume displaced.}$$

It is seen that the equation holds good in both cases – immersed or floating.

Example. 4.1 *A cylinder of diameter 0.3 m and height 0.6 m stays afloat vertically in water at a depth of 1 m from the free surface to the top surface of the cylinder.*

Determine the buoyant force on the cylinder. *Check the value from basics*

Buoyant force = Weight of water displaced

$= (\pi \times 0.3^2/4)\ 0.6 \times 1000 \times 9.81 = 416.06$ N

This acts upward at the centre of gravity G

Check: Bottom is at 1.6 m depth. Top is at 0.6 m depth

Buoyant force = Force on the bottom face – Force on top face

$= (\pi \times 0.3^2/4)\ (1000 \times 9.81 \times 1.6 - 1000 \times 9.81 \times 1.0)$

= 416.16 N

Example. 4.2 ***Determine the maximum weight that may be supported*** *by a hot air balloon of 10 m diameter at a location where the air temperature is 20° C while the hot air temperature is 80° C. The pressure at the location is 0.8 bar. R = 287 J/kg. K.*

The forces that act on the balloon are its weight downward and the buoyant force upwards. The buoyant force equals the weight of the cold surrounding air displaced. The difference between these two gives the maximum weight that may be carried by the balloon. The volume of cold air displaced equals the volume of the balloon. The pressure is assumed to be the same both inside and outside of the balloon.

Volume of balloon = $(4 \times \pi \times 5^3/3) = 523.6\ m^3$

Mass of hot air = $(PV/RT) = 0.8 \times 10^5 \times 523.6/287 \times (273 + 80)$

= 413.46 kg.

Weight of hot air = m.g = 413.46 × 9.81 = 4056 N

Weight of cold air = $[0.8 \times 10^5 \times 523.6 \times 9.81] /[287 \times (273 + 20)]$

= 4886.6 N

Weight that can be carried by the balloon = 4886.6 – 4056

= 830.6 N (*i.e.*, about 84.67 kg mass under earths gravity)

This should include the weight of the balloon material and fittings.

4.2 STABILITY OF SUBMERGED AND FLOATING BODIES

There are three possible situations for a body when immersed in a fluid.

(*i*) If the **weight of the body is greater** than the weight of the liquid of equal volume then the **body will sink into** the liquid (To keep it floating additional upward force is required).

(*ii*) If the **weight of the body equals** the weight of equal volume of liquid, then the body will submerge and may **stay at any location** below the surface.

(*iii*) If the **weight of the body is less** than the weight of equal volume of liquid, then the body will be partly submerged and **will float in** the liquid.

Comparison of densities cannot be used directly to determine whether the body will float or sink unless the body is solid over the full volume like a lump of iron. However the apparent density calculated by the ratio of weight to total volume can be used to check whether a body will float or sink. If apparent density is higher than that of the liquid, the body will sink. If these are equal, the body will stay afloat at any location. If it is less, the body will float with part above the surface.

A submarine or ship though made of denser material floats because, the weight/volume of the ship will be less than the density of water. In the case of submarine its weight should equal the weight of water displaced for it to lay submerged.

Stability of a body: A ship or a boat should not overturn due to small disturbances but should be stable and return, to its original position. Equilibrium of a body exists when there is no resultant force or moment on the body. A body can stay in three states of equilibrium.

(*i*) **Stable equilibrium: Small disturbances will create a correcting couple** and the body will go back to its original position prior to the disturbance.

(*ii*) **Neutral equilibrium Small disturbances do not create any additional force** and so the body remains in the disturbed position. No further change in position occurs in this case.

(*iii*) **Unstable equilibrium:** A small **disturbance creates a couple which acts to increase the disturbance** and the body may tilt over completely.

Under equilibrium conditions, two forces of equal magnitude acting along the same line of action, but in the opposite directions exist on a floating/submerged body. These are the gravitational force on the body (weight) acting downward along the centroid of the body and buoyant force acting upward along the centroid of the displaced liquid. Whether floating or submerged, under equilibrium conditions these two forces are equal and opposite and act along the same line.

When the position of the body is disturbed or rocked by external forces (like wind on a ship), the position of the centre of gravity of the body **(with respect to the body)** remains at the same position. But the shape of the displaced volume of liquid changes and so its centre of gravity shifts to a new location. Now these two forces constitute a couple which may correct the original tilt or add to the original tilt. If the couple opposes the movement, then the body will regain or go back to the original position. If the couple acts to increase the tilt then the body becomes unstable. These conditions are illustrated in Fig 4.2.

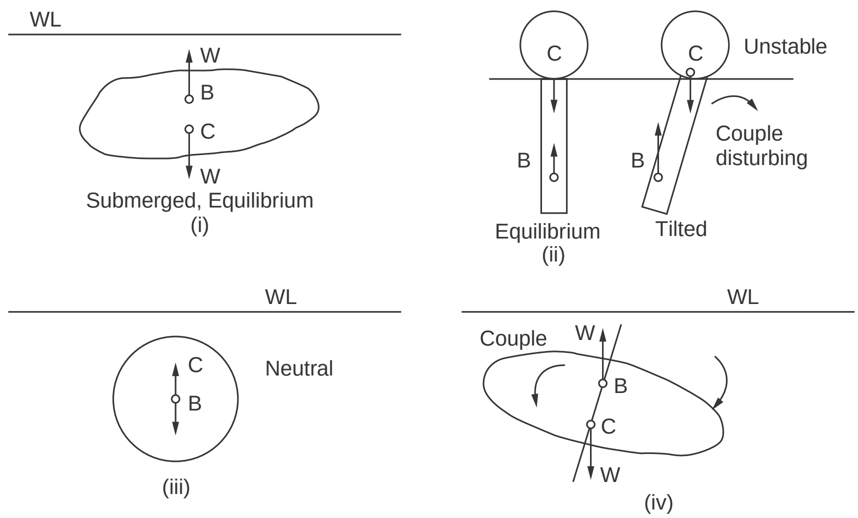

Figure 4.2.1 Stability of floating and submerged bodies

Figure 4.2.1 (*i*) and 4.2.1 (*ii*) shows bodies under equilibrium condition. Point C is the centre of gravity. Point B is the centre of buoyancy. It can be seen that the gravity and buoyant forces are equal and act along the same line but in the opposite directions.

Figure 4.2 (*iii*) shows the body under neutral equilibrium. The centre of gravity and the centre of buoyancy conicide.

Figures 4.2.1 (*iv*) and 4.2.1 (*v*) shows the objects in Figures 4.2.1 (*i*) and 4.2.1 (*ii*) in a slightly disturbed condition. Under such a condition a couple is found to form by the two forces, because the point of application of these forces are moved to new positions. In the case of Figure 4.2.1 (*iv*) the couple formed is opposed to the direction of disturbance and tends to return the body to the original position. This body is in a state of stable equilibrium. The couple is called righting couple. In the case of Figure 4.2.1 (*ii*) the couple formed is in the same direction as the disturbance and hence tends to increase the disturbance. This body is in unstable equilibrium. In the case of figure 4.2.1 (*iii*) no couple is formed due to disturbance as both forces act at the same point. Hence the body will remain in the disturbed position.

In the case of top heavy body (Figure 4.2 (*ii*)) the couple created by a small disturbance tends to further increase the tilt and so the body is unstable.

It is essential that the stability of ships and boats are well established. The equations and calculations are more involved for the actual shapes. Equations will be derived for simple shapes and for small disturbances. (**Note:** For practical cases, the calculations will be elaborate and cannot be attempted at this level.)

4.3 CONDITIONS FOR THE STABILITY OF FLOATING BODIES

(*i*) **When the centre of buoyancy is above the centre of gravity of the floating body, the body is always stable** under all conditions of disturbance. A righting couple is always created to bring the body back to the stable condition.

(*ii*) **When the centre of buoyancy coincides with the centre of gravity,** the two forces act at the same point. A disturbance does not create any couple and so **the body just remains in the disturbed position.** There is no tendency to tilt further or to correct the tilt.

(*iii*) **When the centre of buoyancy is below the centre of gravity** as in the case of ships, **additional analysis is required to establish stable conditions** of floating.

This involves the **concept of metacentre and metacentric height.** When the body is disturbed the centre of gravity still remains on the centroidal line of the body. The shape of the displaced volume changes and the centre of buoyancy moves from its previous position.

The location M at which the line of action of buoyant force meets the centroidal axis of the body, when disturbed, is defined as **metacentre.** The distance of this point from the centroid of the body is called metacentric height. This is illustrated in Figure 4.3.1.

If the metacentre is above the centroid of the body, the floating body will be stable. If it is at the centroid, the floating body will be in neutral equilibrium. If it is below the centroid, the floating body will be unstable.

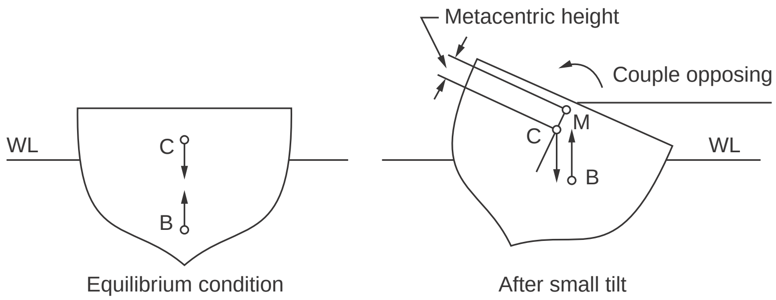

Figure 4.3.1 Metacentric height, stable condition

When a small disturbance occurs, say clockwise, then the centre of gravity moves to the right of the original centre line. The shape of the liquid displaced also changes and the centre of buoyancy also generally moves to the right. If the distance moved by the centre of buoyancy is larger than the distance moved by the centre of gravity, the resulting couple will act anticlockwise, correcting the disturbance. If the distance moved by the centre of gravity is larger, the couple will be clockwise and it will tend to increase the disturbance or tilting.

The distance between the metacentre and the centre of gravity is known is **metacentric height**. The magnitude of the righting couple is directly proportional to the metacentric height. Larger the metacentric height, better will be the stability. Referring to Fig 4.4.1, the centre of gravity G is above the centre of buoyancy B. After a small clockwise tilt, the centre of buoyancy has moved to B'. The line of action of this force is upward and it meets the body centre line at the metacentre M which is above G. In this case metacentric height is positive and the body is stable. It may also be noted that the couple is anticlockwise. If M falls below G, then the couple will be clockwise and the body will be unstable.

4.4 METACENTRIC HEIGHT

A floating object is shown in Figure 4.4.1 in section and plan view (part). In the tilted position, the submerged section is *FGHE*. Originally the submerged portion is *AFGHD*. Uniform section is assumed at the water line, as the angle of tilt is small. The original centre of buoyancy *B* was along the centre line. The new location *B′* can be determined by a moment balance. Let it move through a distance *R*. Let the weight of the wedge portion be *P*.

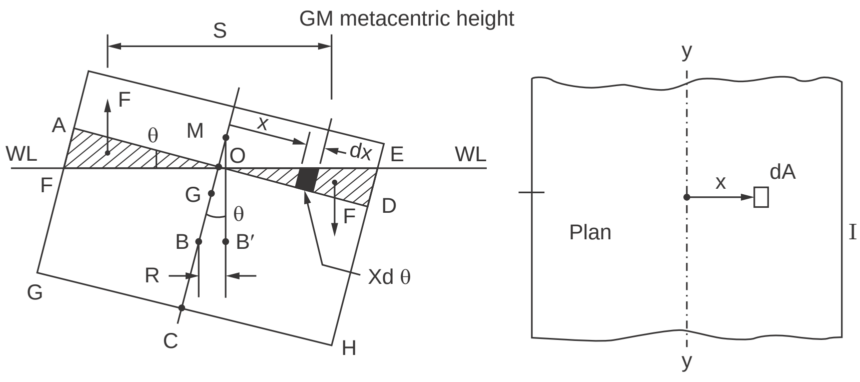

Figure 4.4.1 Metacentric height – derivation

The force system consists of the original buoyant force acting at *B* and the forces due to the wedges and the resultant is at *B′* due to the new location of the buoyant force.

Taking moments about *B*, $P \times S = W \times R$

The moment $P \times S$ can be determined by taking moments of elements displaced about *O*, the intersection of water surface and centre line.

Consider a small element at x with area dA

The height of the element = $\theta \times x$ (as θ is small, expressed in radians)

The mass of the element $\gamma \times \theta\, dA$ (γ – specific weight). The moment distance is x.

$$\therefore \quad P \times S = \gamma\,\theta \int_A x^2\, dA = \gamma\, I\, \theta \tag{4.4.1}$$

where $I = \int_A x^2\, dA$, moment of inertia about the axis $y - y$

$W \times R = \gamma\,\theta\, I$, but $W = V\gamma$ where V is the displaced volume

$$\therefore \quad \gamma\,\theta\, I = V\gamma R \tag{4.4.2}$$

From the triangle MBB′, $R = MB \sin\theta$ or $R = MB\,\theta$

$$\therefore \quad MB = R/\theta = I/V \tag{4.4.3}$$

Both I and V are known. As $V = W/\gamma$, the metacentric height is given by,

$$MG = MB \pm GB \tag{4.4.4}$$

GB is originally specified. So the metacentric height can be determined. If *G* is above *B* –ve sign is used. If *G* is below *B* +ve sign is to be used.

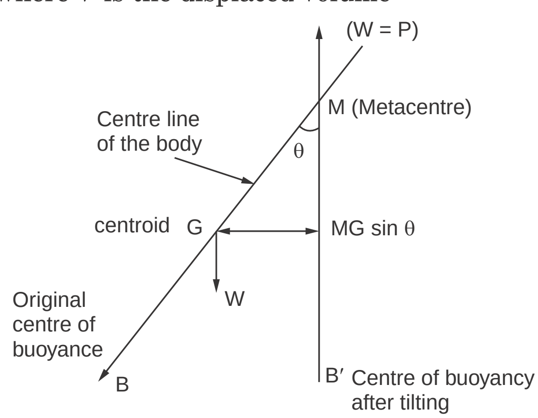

Figure 4.4.2

B = W acts vertically along $B'M$. in the upward direction W acts vertically downwards at G. The distance between the couple formed is $MG \sin\theta$. Hence the righting couple

$$= \gamma V \overline{MG}\ \theta = W\ \overline{MG}\ \sin\theta.$$

Example. 4.3 *A ship's plan view is in the form of an ellipse with a major axis of 36 m and minor axis of 12 m. The mass of the ship is 1000 tons. The centre of buoyancy is 1.8 m below the water level and the centre of gravity is 0.3 m below the water level.* ***Determine the metacentric height for rolling*** *(y – y axis) and* ***pitching*** *(x – x* axis).

$$MG = (I/V) \pm GB,\ GB = 1.8 - 0.3 = 1.5 \text{ m}$$

For rolling:

$$\mathbf{I} = x\ (bh^3/64) = \boldsymbol{\pi \times 36 \times 12^3/64} = 3053.63 \text{ m}^4,\ V = W/\gamma = \text{m}/\rho$$

Considering sea water of density 1030 kg/m^3 and V as the liquid volume displaced.

$$V = 1000{,}000/1030 = 970.87 \text{ m}^3,\ MB = I/V = 3053.63/970.87$$

$$MG = MB - GB = (3053.62/970.87) - 1.5 = 3.15 - 1.5 = \mathbf{1.65\ m}$$

(–ve) sign as B is below G). This is positive and so the ship is stable about rolling by small angles. For **pitching**

$$\mathbf{MG} = (\pi \times 12 \times 36^3/64 \times 970.87) - 1.5 = \mathbf{28.3\ m}$$

Highly stable in this direction. This situation is for small angles and uniform section at the water line.

4.4.1 Experimental Method for the Determination of Metacentric Height

The weight of the ship should be specified, say W. A known weight W_1 is located at a distance of X from the centre line. A plumb bob or pendulum is used to mark the vartical. The weight is now moved by $2X$ m so that it is at a distance of X m on the otherside of the centre line. The angle of tilt of the pendulum or plumb bob is measured. Then the disturbing moment is $W_1\ 2X$. This equals the restoring couple $W\ MG \sin\theta$. For small angles $\sin\theta \simeq \theta$

$$W_1\, 2X = W\ \overline{MG}\ \sin\theta = W\ \overline{MG}\ \theta \ (\theta \text{ in radians})$$

Metacentric height $MG = 2\ W_1\ X/W\ \theta$ (X is half the distance or distance of weight from centre and θ is the angle in radians). The angle can be measured by noting the length of the pendulum and the distance moved by the plumb bob weight.

Example. 4.4 *A ship displacing 4000 tons has an angle of tilt of 5.5° caused by the movement of a weight of 200 tons through 2m from one side of centre line to the other.* ***Determine the value of metacentric height.***

$$\overline{\mathbf{MG}} = (2W_1\ X/W\ \theta)$$

$$= 2 \times 200 \times 1000 \times 9.81 \times 1/4000 \times 1000 \times 9.81\ (.5.5 \times \pi/180)$$

$$= \mathbf{1.042\ m}$$

(here X is half the distance moved) check using the degree of tilt and $MG = 2MX/W \sin\theta$.

SOLVED PROBLEMS

Problem 4.1 *Determine* ***the diameter of a hydrogen filled*** *balloon to support a total of 1 kg at a location where the density of air is 0.8 kg/m^3 and that of the hydrogen in the balloon is 0.08 kg/m^3.*

The weight that can be supported equals the difference in weights of air and hydrogen.

$$9.81 \times 1 = (4/3) \times \pi \times r^3 \times (0.8 - 0.08) \times 9.81, \text{ Solving } r = 0.692 \text{ m}$$

$$\mathbf{D = 2r = 1.384\ m}$$

Problem 4.2 *Ship weighing 4000 tons and having an area of 465 m^2 at water line submerging to depth of 4.5 m in sea water with a density of 1024 kg/m^3 moves to fresh water.* ***Determine the depth of submergence in fresh water.*** *Assume that sides are vertical at the water line.*

Originally the weight of the ship equals the weight of sea water displaced. (omitting 9.81 in both numerator and denominator)

Volume of sea water displaced = 4000 × 1000/1024 = 3906.25 m^3

To support the same weight, the volume of fresh water displaced

= 4000 × 1000/1000 = 4000 m^3

Extra volume = 4000 – 3906.25 = 93.75 m^3

Area at this level = 465 m^2, Equivalent Depth = 93.75/465 = 0.2 m

∴ The depth of submergence in fresh water = 4.5 + 0.2 = **4.7 m**

Problem 4.3 *A bathy sphere of mass 6800 kg (empty) and having a diameter of 1.8 m is to be used in an ocean exploration. It is supported by a cylindrical tank of 3 m dia and 6 m lenght of mass 4500 kg when empty and filled with oil of specific gravity 0.7.* ***Determine the maximum mass of equipment*** *that can be supported in the bathy sphere. Assume density of sea water as 1024 kg/m^3. Neglect metal thickness. The important thing to note is that the limiting condition is when the supporting cylinder just submerges.*

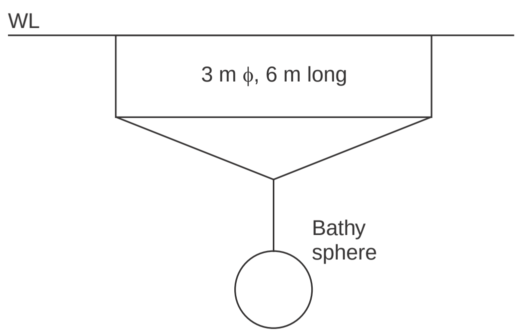

Figure P. 4.3

∴ The total volume displaced = volume of cylinder + volume of sphere

$$= (\pi \times 3^3 \times 6/4) + (4 \times \pi \times 0.9^3/3) = 45.465 \text{ m}^3$$

Weight of water displaced = 45.465 × 1024 × 9.81 = **456717 N**

The weight of cylinder and oil = 9.81 [4500 + ($\pi \times 3^2 \times 6 \times 700/4$)] = **335384.8 N**

The weight of empty sphere = 6800 × 9.81 = 66708 N

Total weight = 335384.8 + 66708 = **402092.8 N**

The additional weight that can be supported = 456717 – 402092.8

= 54624.5 N (about 5568 kg of mass)

Problem 4.4 *A cubical block with a density of 2500 kg/m^3 fully submerged in water is used to hold down a box 0.3 × 0.6 × 0.9 m size just submerged in water. The box has a mass of 110 kg.* ***Determine the weight of the block.***

The upward forces are due to buoyancy on the block and buoyancy on the box. The downward forces are due to the weight of the box and the block.

Let the side of the block be h m.

Total upward force $= (0.3 \times 0.6 \times 0.9 \times 9810) + (h^3 \times 9810)$

Downward force $= 9.81\ [110 + h^3 \times 2500]$

Equating, $1589.22 + 9810\ h^3 = 1079.1 + 24525\ h^3$

Solving $h = 0.326065$ m

Weight of the block $= 2500 \times 9.81 \times 0.326065^3 =$ **850.2 N (86.7 kg)**

Problem 4.5 ***Determine the volume and specific weight*** *of an object which weighs 22 N in water and 30 N in oil of specific gravity 0.80.*

Obviously this object is immersed in the fluid completely during weighment. If it is just floating, its weight is balanced by the buoyant force and so the apparent weight will be zero. Let its volume be V m^3. When in water, it displaces V m^3 and so also when in oil. Let buoyant force when in water be W_w and when in oil W_o

Let the real weight of the object be W kg

$$W - W_w = 22\ N\ ;\ W - W_O = 30\ N$$

Subtracting $W_w - W_O = 8$ N

$V(9810 - 0.80 \times 9810) = 8$; $\therefore$ $\mathbf{V = 4.0775 \times 10^{-3}\ m^3}$ or 4.0775 litre

$\therefore$ $W_w = 4.0775 \times 9.81 = 40$ N

Substituting for W_w in equaion 1

$$W = 22 + W_w = 62 \text{ N} \text{ ; } \textbf{specific weight} = W/V = \mathbf{15205.5\ N/m^3}$$

Problem 4.6 *A hydrometer (to measure specific gravity of a liquid) is in the form of a sphere of 25 mm dia attached to a cylindrical stem of 8 mm dia and 250 mm length. The total mass of the unit is 14 grams.* ***Determine the depth of immersion*** *of the stem in liquids of specific gravity of 0.75, 0.85, 0.95. 1.05 and 1.15. Cheak whether the intervals are uniform.*

The volume of liquid displaced × sepecific weight = Weight of the hydrometer

The volume is made up of the sphere and the cylindrical of length h.

$$[(4/3)\ \pi\ (0.025/2)^3 + \pi\ (0.004)^2\ h] \times \text{sp. gravity} \times 9810 = 14 \times 9.81 \times 1/1000$$

$$[0.8181 \times 10^{-6} + 5.026 \times 10^{-5}\ h] \times \text{sp. gravity} = 1.4 \times 10^{-5}$$

This reduces to $h = (0.2786/\text{sp. gravity}) - 0.1628$

The only unknown is h and is tabulated below

Sp. gravity	0.75	0.85	1.00	1.05	1.15
h, mm	208.6	164.9	115.8	102.5	79.4

The intervals in mm are : 43.7, 34.5, 27.9, 23.1 and hence not uniform. This is due to the combined spherical and cylindrical shape.

Only the sphere will be immersed when the sp. gravity of liquid equals 1.711. Usually the major portion of the weight is placed in the spherical portion. So the buoyant force creates a righting couple and the instrument is stable till the spherical portion alone is immersed.

Problem 4.7 *The specific weight of a liquid varies as $\gamma = 9810\ (1 + y)$ where y is measured in m, downward from the surface. A block 1 m × 2 m area and 2 m deep weighing 19620 N floats in the liquid with the 2 m side vertical.* ***Determine the depth of immersion.***

The weight of liquid displaced = weight of the body

To determine the weight of the liquid displaced, consider a small thickness by at distance y. (as $\gamma = 9810\ (1 + y)$)

The weight of the element $dW = 1 \times 2dy \times \gamma = 2\ (1 + y)\ 9810\ dy$

Let the depth of immersion be D. Then integrating the expression and equating it to the weight of the solid.

$$2 \times 9810 \int_0^D (1 + y)\, dy = 19620 \ ; \quad \therefore \quad \left[y + y^2/2\right]_0^D = 1$$

or $D^2 + 2D - 2 = 0$, Solving **D = 0.732 m** (the other root is negative). This is less than the depth of the body. So the assumption that the body floats is valid. Check whether it will be stable.

Problem 4.8 *An iceberg floats in sea water with 1/7 of the volume outside water.* ***Determine the density of ice.*** *The density of sea water is 1025 kg/m*^{3}.

The weight of any floating body equals the weight of liquid displaced by it. The iceberg displaces 6/7 of its volume of sea water. Let the vulume of iceberg be V m^3.

Then $\quad 1 \times \gamma \times V = (6/7)V \times 1025 \times 9.81; \quad \therefore \quad \gamma = 8618.79$ N/m^3

$\therefore$ $\quad$ density of **ice = γ/g = 878.57 kg/m**^{3}

Problem 4.9 *Two spheres, one heavier and weighing 12000 N and of diameter 1.2 m and the other lighter and weighing 4000 N, are tied with a rope and placed in water. It was found that the spheres floated vertically with the lighter sphere just submerging.*

Determine the diameter of the lighter sphere and the tension in the rope.

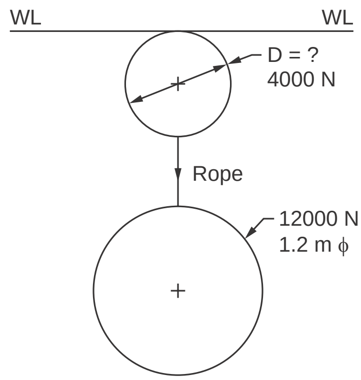

Figure P. 4.9

The buoyant force on the heavier sphere equals the weight of water displaced. Buoyant force on the heavier sphere = $(4/3) \times \pi \times 0.6^3 \times 9810 = 8875.9$ N

The weight of the sphere = 12000 N.

The difference between these two is the tension in the rope.

The **tension in the rope** = 12000 – 8875.9 = **3124.1 N**

The weight of the lighter sphere and the rope tension together should balance the buoyant force on the smaller sphere of diameter D.

$$(4/3) \times \pi \times R^3 \times 9810 = 4000 + 3124.1 \ ; \quad \therefore \quad \mathbf{D = 1.1152\ m}$$

Problem 4.10 *A mass of volume 0.1 m*^{3} *attached to a balloon of 0.3 m*^{3} *at a pressure of 1.4 bar (abs) weigh totally 2000 N. The unit is released in the sea.* ***Determine the level to which the unit will sink****. Assume the specific weight of sea water as 10000 N/m*^{3} *and the air temperature in the balloon remains constant.*

When the unit sinks to a level such that the weight equals the buoyant force, it will stop sinking further. The buoyant force equals the weight of water displaced. As it goes down in the water the volume of the balloon shrinks due to the increase in surrounding pressure.

Let the volume of the balloon at this level be V m^3.

$(0.1 + V)\ 10000 = 2000; \quad \therefore \quad V = 0.1$ m^3. The original volume was 0.3 m^3.

As $(P_2/P_1) = (V_1/V_2)$; $P_2 = 1.4 \times (0.3/0.1) = 4.2$ bar (abs)

The depth at which this pressure reached is given by

$$10000 \times y = (4.2 - 1.013) \times 10^5; \text{ Solving } y = 31.87 \text{ m.}$$

Hence the unit will **sink to a depth of 31.87 m**

Problem 4.11 *A helium balloon is floating (tied to a rope) at a location where the specific weight of air is 11.2 N/m³ and that of helium is 1.5 N/m³. The empty balloon weighs 1000 N.* ***Determine the diameter of the balloon*** *if the tension in the rope was 3500 N.*

The buoyant force on the balloon = Rope tension + weight of balloon

Volume × (sq weight of surrounding fluid air – sp. weight of helium)

= Rope tension + weight of balloon

$V \times (11.2 - 1.5) = 3500 + 1000;$

$\therefore \quad V = 463.92 \text{ m}^3; (4\pi R^3/3) = 463.92 \text{ m}^3;$

$\therefore$ **D = 9.6 m**

Problem 4.12 ***Determine the diameter of the sphere to open a cylindrical gate*** *hinged at the top and connected to the sphere as shown in Fig. P.4.12 when the water level reaches 6 m above the centre of the gate. The gate weighs 4500 N and its centroid coincides with the centroid of the semicircle. The sphere weighs 1500 N/m³. The width of the gate is 1.5 m.*

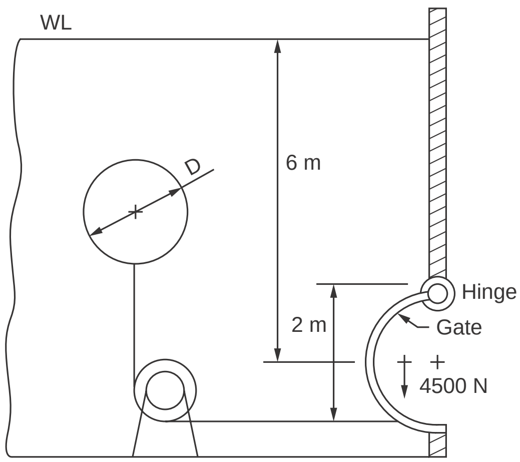

Figure P. 4.12

The forces on the gate are (*i*) tension in the rope equal to the buoyant force on the sphere minus the weight of the sphere. (*ii*) horizontal force due to pressure and (*iii*) vertical force due to pressure and weight.

Buoyant force = $(4/3)\, \pi R^3$ [9810 – 1500] N acts at 2 m from hinge.

as $h = 6$ m, horizontal force due to water = 9810 × 6 × 2 × 1.5 N = 176580 N

Centre of pressure = 6 + (1/12) (1.5 × 2^3/6 × 2 × 1.5) = 6.0556 m, acts from the hinge at: 1.0556 m.

Vertical force = Weight of gate + Pressure force (Pressure force equals the weight of the volume displaced, acts upwards), $R = 1$, $L = 1.5$ m,

Vertical upward force = – 4500 + π (R^2/2) L × 1000 × 9.81 = 18614 N.

It acts at the centroid of the semicircle, $(4R/3\pi) = 0.4244$ m from hinge.

Taking moments about the hinge,

$$176580 \times 1.0555 = 18614 \times 0.4244 + (4/3)\,\pi R^3\,(9810 - 1500)$$

Solving, **R = 1.7244 m or D = 3.4488 m**

Problem 4.13 *An empty storage tank of square section 6 m side and 1.2 m high of mass 2250 kg is buried under loose soil at a depth of 1 m. The density of the soil is 480 kg/m³. A spring causes water to seep below the tank.* ***Determine the height of water*** *that may cause the tank to break free and start to rise.*

At the point when the tank begins to break free due to water seeping all around, it can be considered that the tank floats with a weight equal to its own and the weight of soil above it. The total weight should equal the weight of water displaced.

If h is the height upto which the water rises, then

$$(2250 + 6 \times 6 \times 1 \times 480)\ 9.81 = 9.81 \times 1000 \times 6 \times 6 \times h \quad \therefore \quad \mathbf{h = 0.5425\ m}$$

When water rises to about 0.5425 m from battom the tank will begin to break free.

Problem 4.14 *A wooden pole of 45 cm² section and 3 m length is hinged at 1.2 m above the water surface and floats in water at an angle of θ with vertical* ***Determine the value of the angle.*** *The pole weighs 90 N.*

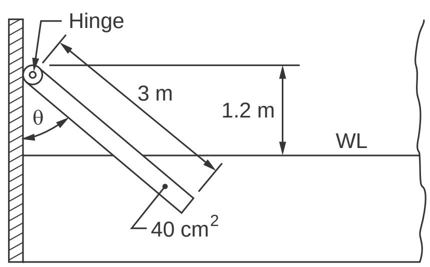

Figure P. 4.14

The problem is solved by taking moment of the weight at the hinge and equating it to the moment of the buoyancy force at the hinge. Taking moment **along the pole.**

Moment of the weight = (3/2) 90 = 135 Nm.

Length of the submerged portion = $3 - (1.2/\cos\theta)$,

Weight of the displaced water or buoyancy force

$$= (45/10^4) \times 9810\,[3 - (1.2/\cos\theta)].$$

The distance along the pole it acts = $(1.2/\cos\theta) + (1/2)\,[3 - (1.2/\cos\theta)]$

$$= (1/2)\,[3 + (1.2/\cos\theta)]$$

Moment of buoyant force = $(45/10^4)\,(9810/2)\,[3^2 - (1.2^2/\cos^2\theta)]$

Equating to 135 Nm and solving, $\boldsymbol{\theta = 45°}$.

Check = $(45/10^4)\,(9810/2)\,[3^2 - (1.2^2/\cos^2 45)] = 135.08$ Nm. *checks*

This can be extended to analyse the water level control valve in tanks.

Problem 4.15 *A wooden pole of 0.16 m square section of length 3m and weighing 425 N and of dimensions as shown in Fig. P. 4.15 floats in oil of specific gravity 0.815. The depth of oil above the hinge (friction negligible) is 2 m.* ***Determine the angle of inclination of the pole*** *with horizontal. Also determine the oil level for the pole to float vertically.*

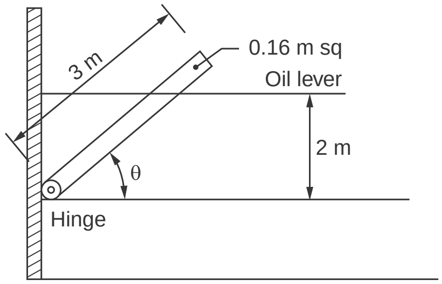

Figure P. 4.15

The problem is solved by taking moment about the hinge for the weight and the buoyant force and equating them. Let the angle of inclination be θ with horizontal. The moment for the weight about the hinge (along × direction) = 425 × (3/2) cos θ. The weight of oil displaced (buoyant force)

$$= (2/\sin\theta) \times 0.16^2 \times 9810 \times 0.815 \text{ N}$$

Moment arm = (2/sin θ) (1/2) (cos θ). Equating the moments,

$$425\,(3/2)\cos\theta = (2/\sin\theta) \times 0.16^2 \times 9810 \times 0.815\,(2/\sin\theta)(1/2)\cos\theta$$

Solving, **θ = 53.26˚.**

When the pole begins to float vertically, the weight equals the buoyant force.

$h \times 0.16^2 \times 9810 \times 0.815 = 425$; **h = 2.0765 m.**

If the level rises above this value, a vertical force will act on the hinge.

Problem 4.16 *In order to keep a weight of 160 N just submerged in a liquid of specific gravity of 0.8, a force of 100 N acting upward is required. The same mass requires a downward force of 100 N to keep it submerged in another liquid.* ***Determine the specific weight of the second liquid.***

The volume of fluid displaced in both cases are equal as the weight is submerged.

Case (i) The buoyant force in this case = (160 – 100) N = 60 N

The volume of the fluid displaced = $60/(0.8 \times 1000 \times 9.81)$ m^3 = 7.65×10^{-3} m^3

Case (ii) The buoyant force = 160 + 100 = 260 N

Specific weight of the other liquid = $260/7.65 \times 10^{-3}$

= 34008 N/m^3

(**Note** : Buoyant force = volume displaced × sp. weight of liquid)

density will be = (34008/9.81) = **3466.7 kg/m^{3c}**

Problem 4.17 *A cylindrical container of 0.4 m dia. and 0.9 m height weighing 60 N contains oil of specific weight 8600 N/m^3 to a depth of 0.3 m.* ***Determine the depth upto which it will float in water.*** *Also calculate the depth oil in the container so that the depth of oil and depth of immersion are equal.*

Case (*i*) Total weight of the container = $60 + \pi \times 0.2^2 \times 0.3 \times 8600$ N = 384.21 N

Volume of water displaced = $384.21/(1000 \times 9.81)$ m^3 = 0.0392 m^3

∴ Depth of immersion = $0.0392/(\pi \times 0.2^2)$ = **0.3117 m**

Case (ii) Let h be the depth of immersion. The depth of oil will also be h,

$$60 + (\pi \times 0.2^2 \times h \times 8600) = \pi \times 0.02^2 \times h \times 9810, \text{ Solving } \mathbf{h = 0.3946\ m}$$

Chapter 4

Problem 4.18 *A cylindrical hydrometer weighing 0.04 N has a stem diameter of 6 mm.* ***Determine the distance between the markings*** *for 0.8, 1.0 and 1.1 specific gravity values. Indicate the direction of the markings as up or down also.*

The volume displaced in each case equals the weight of the hydrometer.

(*i*) Relative density = 0.8. Let the volume displaced be V_1

$$V_1 \times 9810 \times 0.8 = 0.04; \quad \therefore \quad V_1 = 5.097 \times 10^{-6}\ \text{m}^3$$

(*ii*) Relative density = 1. Let the volume displaced be V_2

$$V_2 \times 9810 = 0.04\ ; \quad \therefore \quad V_2 = 4.0775 \times 10^{-6}\ \text{m}^3.$$

$$(V_1 - V_2) = \pi\ (D^2/4)\ l_1;$$

Solving $\boldsymbol{l_1}$ **= 0.036052 m or 3.6 cm.** This is downwards as V_2 is less than V_1.

(*iii*) Relative density = 1.1. Let the volume displaced be V_3.

$$V_3 = 0.04/(9810 \times 1.1) = 3.707 \times 10^{-6}\ \text{m}^3$$

$(V_3 - V_3)\ \pi\ (D^2/4) \times l_1$; $\boldsymbol{l_2}$ **= 0.01311 m or 1.31 cm**

This is also downwards as V_3 is less than V_2. As density increases, the depth of immersion decreases and is non linear.

Problem 4.19 *A wooden cylinder having a specific gravity of 0.6 has a concrete cylinder of the same diameter and 0.2 m length attached to it at one end. The specific gravity of the concrete is 2.5.* ***Determine the length of the wooden cylinder*** *for the composite block to float vertically.*

The limiting condition is for the composite block to float with top surface at water level. Let "h" be the length of the wooden cylinder. The weight of the composite block

$$= (\pi \times R^2 \times 0.2 \times 9810 \times 2.5) + (\pi \times R^2 \times h \times 9810 \times 0.6)$$

This equals the weight of water displaced when the block just floats.

The weight of water displaced = $\pi \times R^2 \times 9810\ (0.2 + h)$. Equating and solving,

$$0.5 + 0.6\ h = 0.2 + h \quad \therefore \quad h = 0.75\ \text{m}$$

The wooden cylinder **should be atleast 0.75 m long for the composite cylinder to float vertically.**

Work the problem for 1 m long cylinder and find the length above the water line.

Problem 4.20 *A right circular cylinder of diameter D m and height h m with a relative density of (S < 1) is to float in water in a stable vertical condition.* ***Determine the limit of the ratio D/h*** *for the required situation.*

For stability, the limiting condition is that the metacentre approach the centre of gravity. Using equation (4.4.3) and (4.4.4), (V – volume displaced), $MB = I/V$

$$MG = (I/V) \pm GB. \text{ Here } MG = 0 \text{ for the limiting condition.}$$

$(I/V) = GB \quad I = \pi\ D^4/64,\ V = \pi\ D^2\ h\ \text{S}/4, \quad \therefore \quad (I/V) = D^{2/}/16hS$

Also from basics

$$GB = (h/2) - (h\ S/2) = h\ (1 - S)/2; \text{ equating, } (D^2/16hs) = [h\ (1 - S)]/2$$

$$\therefore \qquad (D/h) = 2\ [2\ S\ (1 - S)]^{0.5} \qquad \text{... (1)}$$

For example if $\quad S = 0.8,\ \dfrac{D}{h} = 1.1314$

$\therefore \quad D > h.$

The diameter should be larger than the length. This is the reason why long rods float with length along horizontal. The same expression can be solved for limiting density for a given D/h ratio. Using equation 1

$$(D/h)^2 = 8\,S\,(1 - S) \quad \text{or} \quad 8\,S^2 - 8\,S + (D/h)^2 = 0$$

$$S = \{1 \pm [1 - (4/8)\,(D/h)^2]^{0.5}\}/2,$$

say **if (D/h) = 1.2,** then **S = 0.7646 or 0.2354**

Problem 4.21 *A right circular cylinder of 0.3 m dia and 0.6 m length with a specific weight of 7500 N/m³ is to float vertically in kerosene of specific weight of 8900 N/m³.* ***Determine the stability of the cylinder.***

$$MG = (I/V) - GB;\ I = \pi \times 0.3^4/64$$

Volume displaced, $V = (\pi \times 0.3^2 \times 0.6 \times 7500)/(4 \times 8900)$

Location of G = 0.3 m from bottom

Location of B = 0.3 × (7500/8900) m from the bottom.

$\therefore \quad GB = 0.3 - 0 - 3 \times (7500/8900)$

$$MG = [(\pi \times 0.3^4 \times 4 \times 8900)/\ 64 \times \pi \times 0.3^2 \times 0.6 \times 7500)] - [0.3 - (0.3 \times 7500/8900)] = -\,0.036 \text{ m.}$$

Hence, the cylinder is unstable.

Check : (use the eqn. 1 in problem 4.20)

$$D = 2h\ [2S\ (1 - S)]^{0.5}, \text{ here } S = 7500/8900$$

Substituting, $D = 2 \times 0.6\ [2 \times (7500/8900) \times (1 - 7500/8900)]^{0.5}$

$= 0.61788$ m. $D > h$.

This is the value of D which is required for stability. The given cylinder is of lower diameter and hence unstable.

Problem 4.22 ***Derive the expression for (D/h) for a hollow right circular cylinder*** *of outer diameter D and inner diameter kD and height h, to float vertically in a liquid with relative density S.*

The limiting condition for stability is $MG = 0$ or $(I/V) = GB$

With usual notations (refer P 4.20)

$$I = D^4\,(1 - k^4)/64;\ V = \pi\,D^2\,(1 - k^3)\,h\,S/4$$

where V is the volume of the liquid displaced. G is located at $h/2$ from base and B is located at $h\,S/2$ from base. $GB = h\,(1 - S)/2$;

Equating, $[D^4\,(1 - k^4)/64]\ [4/(D^2\,(1 - k^2)\,hS] = h\,(1 - S)/2$. Solving

$$\mathbf{(D/h) = 2[2S\ (1 - S)/(1 + k^2)]^{0.5}} \qquad ...(1)$$

For example if $k = 0$ this becomes a solid cylinder and the expression reduces to $(D/h) = 2\,[2S\,(1 - S)]^{0.5}$ as in problem Problem 4.20.

Consider a thin cylinder, where $k = 0.9$ and $S = 0.8$ then, $(D/h) = 0.841$ (compare with Problem 4.20).

Problem 4.23 Check the stability *of a hollow cylinder with D = 1.2 m and h = 1.8 m with a specific gravity 0f 0.33333 to float in water. The ID is 0.5D.*

Refer Problem 4.20, equation 1.

Here, $k = 0.5$, $h = 1.8$ m. For stability, the minimum

Value of D is given by $D = 2h\ [2S\ (1 - S)/(1 + k^2)]^{0.5}$

Substituting the values, for stability

$$\mathbf{D} = 2 \times 1.8\ [2 \times 0.33333\ (1 - 0.33333)/(1 + 0.5^2)]^{0.5}$$
$$= \mathbf{2.147\ m > 1.8}$$

The specified diameter is only 1.2 m. **So it is not stable.** Considering the calculated value,

$$D = 2.147 \text{ m}, I = \pi \times 2.147^4\ (1 - 0.5^4)/64.$$
$$V = \pi \times 2.147^2\ (1 - 0.5^2)\ 1.8 \times 0.333333/4;\ (I/V) = 0.60002 \text{ and}$$
$$GB = 1.8\ (1 - 0.33333)/2 = 0.6. \text{ Hence, } MG = 0. \text{ So, } \textit{checks.}$$

Problem 4.24 *Determine the* ***metacentric height of a torus of mean diameter D*** *with a section diameter d and specific gravity 0.5 when it floats in water with its axis vertical.*

The specific gravity is 0.5. So it floats such that half its volume will be displaced.

$$MG = (I/V) - GB\ ;\ I = \pi\ [(D + d)^4 - (D - d)^4]/64$$

as the section along the free surface is annular with $OD = D + d$ and $ID = D - d$.

$$V = (1/2)\ (\pi d^2/4)\pi D.$$

Centre of gravity is on the water surface. Centre of buoyancy will be at the CG of displaced volume equals $2d/3\pi$.

$$GB = (2d/3\pi)$$

$\therefore$ $$MG = \{\pi\ [(D + d)^4 - (D - d)^4] \times 8/[64 \times \pi^2 \times D \times d^2]\ \} - (2d/3\pi)$$

Simplifying $$MG = \{[(D + d)^4 - (D - d)^4]/[8 \times \pi \times d \times d^2]\} - (2d/3\pi)$$

Problem 4.25 Determine the metacentric height of a torus of D = *1.8 m and d = 0.6 m with specific gravity 0.5 when floating in water with axis vertical.*

Refer Problem 4.24 $\mathbf{MG} = \{[(D + d)^4 - (D - d)^4]/[8 \times \pi \times D \times d^2]\} - \{2d/3\pi\}$

$$= ([2.4^4 - 1.2^4]/[8 \times \pi \times 1.8 \times 0.6^2]\} - \{2 \times 0.6/3\pi\} = \mathbf{1.7826\ m}$$

[**Note :** If the relative density is different from 0.5, the determination of the value of GB is more involved as the determination of the position of CG is difficult]

Problem 4.26 *Derive on expression for the ratio of length, h to side, a of a square log to float stably in a vertical direction. The relative density of the log is S.*

The limiting condition for floating in a stable position is that metacentre and centre of gravity coincide. or $(I/V) = GB$.

$I = (a^4/12)$ where a is the side of square. The volume displaced V = a^2hS where h is the immersion height. Position of $G = h/2$ and position of $B = h\ S/2$, $GB = h\ (1 - S)]/2$

$$I/V = [(a^4/12)(1/a^2hS)] = [h\ (1 - S)/2];$$

$\therefore$ $$(a/h) = [6S\ (1 - S)]^{0.5}$$

Consider $S = 0.8$, then $(a/h) = 0.98$, $S = 0.5$, then, $(a/h) = 1.23$

as S decreases (a/h) increases.

The sides should be longer than the height. The is the reason why long logs float with length along horizontal. The expression can be generalised for a rectangular section with sides a and k.a (where k is a fraction). Then the stability is poorer along the shorter length ka.

$$I = ak^3\ a^3/12;\ V = h\ Sa^2\ k\ ;\ GB = h(1 - S)/2$$

$$(k^3a^4/12)\ (1/a^2khS) = h(1 - S)/2;\ (a/h) = [65\ (1 - S)]^{0.5}/k \qquad \text{...A}$$

Here the side has to be still larger or the height shorter. This expression can be used also to determine the limiting density for a given (side/height) ratio to float stably in a vertical position.

Consider the general eqn. A, reordering,

$$(k^2 \times a^2/h^2) = 6S - 6S^2 \text{ or } S^2 - S + (k^2a^2/6h^2) = 0$$

$$\therefore \quad S = \{ 1 \pm [1 - (4k^2a^2)/(6h^2)]^{0.5} \}/2$$

if $k = 1$, then $S = [1 \pm \{1 - (4a^2/6h^2)\}^{0.5}]/2$

Problem 4.27 *Derive the expression for the ratio of base diameter to the height of a cone to float in a fluid in a stable condition given the relative density between the solid and the fluid as S.*

This case is different from the cylinder due to variation of area along the height (Refer Problem 4.20). The situation is shown in the Fig. P. 4.27. The cone displaces liquid upto a depth h where the diameter is d.

The limiting conditions is that $MG = 0$ or $(I/V) = GB$. In this case the volume displaced and the relationship between D and d and H and h are to be established.

$$(1/3)(D^2/4)\, HS = (1/3)\, (d^2/4)\, h\ ;\ D^2\, HS = d^2h;\ S = \frac{d^2}{D^2} \cdot \frac{h}{H}$$

Also as $\quad (h/H) = (d/D)$

$$h/H = (d/D) = S^{1/3}$$

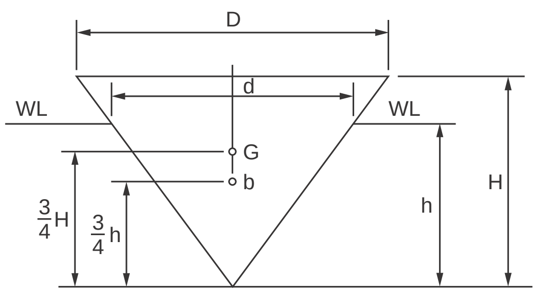

Figure P. 4.27

Volume displaced $\quad = (1/3)(\pi d^2/4)\, h = (1/3)\, (\pi D^2/4)\, HS$

$$I = \pi d^4/64 = \pi D^4\, S^{4/3}/64\ (d^4 = D^4\, .\, S^{4/3})$$

$$\therefore \quad I/V = (D^4\, S^{(4/3)}/64)/(D^2\, HS/12) = (3/16)\, (D^3/H)\, S^{1/3} \quad \text{...A}$$

G = 3H/4 (from vertex) and $B = (3h/4) = (3/4)\, HS^{1/3}$ (from vertex)

$$\therefore \quad GB = (3/4)\, H\, (1 - S^{1/3}) \quad \text{...B}$$

Equating A and B

$$(3D^2S^{1/3}/16H) = (3/4)\, H\, (1 - S^{1/3})$$

$(D^2/H^2) = 4\, (1 - S^{1/3})/S^{1/3}$ **or** $\mathbf{H^2 = D^2S^{1/3}/4\, (1 - S^{1/3})}$

In actual case $\mathbf{H^2}$ should be less than this value for stability.

Problem 4.28 *A conical wooden block of 0.4 m dia and 0.6 m high has a relative density of 0.8 for a fluid in which it floats.* ***Determine whether it can float in a stable condition.***
For stability, the limiting value of H is given by

$$H^2 = D^2\, S^{1/3}/4\,(1 - S^{1/3}) = 0.4^2 \times 0.8^{1/3}/4\,(1 - 0.8^{1/3}) = 0.518 \text{ m}^2$$

$\therefore$ $H = 0.7197$ m. The actual value of is $0.6 < 0.7197$ and so the cone will float in a stable position.

Check: $H = 0.6$ m, $h = H\,S^{1/3} = 0.557$ m; $D = 0.4$ m, $d = DS^{1/3} = 0.371$ m

$$I = \pi d^4/64 = 9.332 \times 10^{-4} \text{ m}^4;\; V = (1/3)\,(\pi D^2/4)\,HS$$

$$V = (1/3)\,(\pi \times 0.4^2/4)\,0.6 \times 0.8 = 0.0201 \text{ m}^3;$$

$$\therefore \quad I/V = 0.04642 \text{ m}$$

$$GB = (3/4)\,(0.6 - 0.557) = 0.03226 \text{ m};$$

$$MG = (I/V) - GB = 0.04642 - 0.03226 = 0.01416 \text{ m}.$$

This is positive and hence stable.

Problem 4.29 *A rectangular pontoon 10 m long, 8m wide and 3m deep weighs 6×10^5 N and carries a boiler of 4 m dia on its deck which weighs 4×10^5 N. The centre of gravity of each may be taken to be at the geometric centre.* ***Determine the value of the meta centric height of the combined unit,*** *when it floats in river water. Calculate also the restoring torque for a tilt of 5° from vertical.*

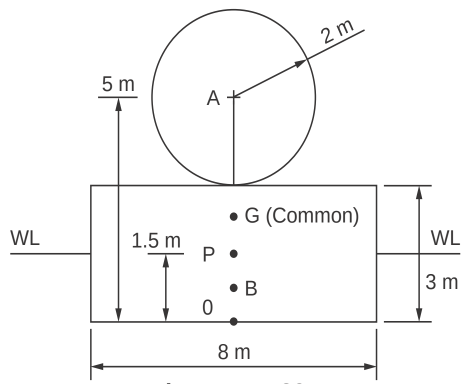

Figure P. 4.29

Assuming the centres to be on the vertical line for the combined unit, the position of the centre of gravity from base can be determined by taking moments about O.

$$OP \times 6 \times 10^5 + OA \times 4 \times 10^5$$

$$= OG \times 10^6;\; OP = 1.5 \text{ m};\; OA = 5 \text{ m};$$

Solving, $OG = 2.9$ m, Total weight $= 10^6$ N

Depth of immersion: $10^6 = 10 \times 8 \times h \times 9810$; $\therefore$ $h = 1.2742$ m

$$OB = 1.2742/2 = 0.6371 \text{ m};\; GB = OG - OB = 2.2629$$

$$\mathbf{MG} = (I/V) - GB = [(1/12) \times 10 \times 8^3]/[10 \times 8 \times 1.2742] - 2.2629$$

$$\mathbf{= 1.9227 \text{ m}}$$

This is positive and hence the unit is stable.

Restoring torque $= W\,MG\,\theta$ (θ in radian)

$= 10^6 \times 1.9227 \times (\pi \times 5/180) =$ **1.68×10^5 Nm.**

REVIEW QUESTIONS

1. Prove Archimedes principle from basics.
2. State the conditions for the stability of floating bodies.
3. Define centre of buoyancy.
4. Define metacentre and metacentric height.
5. Describe an experimental method to determine the metacentric height of a boat.
6. Derive an expression for the height to diameter ratio of a cylinder of specific gravity S to float with its axis vertical in a stable condition.
7. Describe how the density of liquid can be estimated using a cylindrical hydrometer.

OBJECTIVE QUESTIONS

O Q. 4.1 Fill in the blanks

1. When a body floats in water the buoyancy force equals _____.
2. The weight of volume of liquid displaced by a floating body equals _____.
3. The centre of buoyancy is defined as ______.
4. The statement of Archimedes Principle is ______.
5. The three states of equilibrium of a floating body are______.
6. When a small tilt is given to a body floating in stable equilibrium it will ____.
7. When a small tilt is given to a body floating in neutral equilibrium it will ____.
8. When a small tilt is given to a body floating in unstable equilibrium it will ___.
9. If the centre of gravity coincides with the centre of buoyancy, the floating body will be in _______ equilibrium.
10. If the centre of gravity is below the centre of buoyancy the body will always be in _____ equilibrium.
11. If the centre of gravity is above the centre of buoyancy the metacentric height should be ______ stable equilibrium.
12. Metacentric height is equal to _____.
13. The righting moment due to a tilt of a floating body equals_____.
14. The condition for a cylinder of given diameter to length ratio to float vertically in stable equilibrium is _____.
15. The condition for a square prism of given side to length ratio to float vertically in stable equilibrium is _____.
16. The height to diameter ratio for stable floating condition of a cone is ______.
17. Metacenter is the point at which ________ cuts the body centre line.
18. The body displaces $1 m^3$ of water when it floats. It's weight is ______.
19. As fluid density increases the hydrometer will sink by a ______ distance.

Answers

1. The weight of volume of water displaced 2. the weight of the body 3. the centre of gravity of the displaced volume 4. the buoyant force on a floating body equals the weight of the displaced volume and a floating body displaces it's own weight of liquid in which it floats 5. satble, neutral and unstable 6. return to the original position 7. remain in the new position 8. overturn 9. neutral 10. stable 11. positive 12. distance between the metacentre and centre of gravity 13. $W\,MG \sin\theta$, $W\,MG\,\theta$ 14. $(d/h) = 2[2s(1 - s)]^{0.5}$ 15. $(a/h) = [6s(1 - s)]^{-0.5}$ 16. $(H/D)^2 = s^{1/3}/[4(1 - s^{1/3})]$ 17. The line of action of the buoyant force in the displaced position 18. 9810 N 19. shorter

O Q. 4.2 Fill in the blanks with increases, decreases or remains constant

1. Stability of a floating body improves as the metacentric height ____
2. The position of a floating body will ______ when a small tilt is given if the metacentric height is positive.
3. As the density of the floating body increases the distance between the centre of gravity and centre of buoyancy _______
4. The stability of a floating body deteriotes as the metacentric height ________.
5. The volume of liquid displaced by a floating body of wieght W will _______ irrespective of the shape of the body.
6. When a given body floats in different liquids the volume displaced will ________ with increase in the specific gravity of the fluid.

7. For a given floating body in stable equilibrium the righting couple will ______ with increasing metacentric height.
8. For a given shape of a floating body the stability will improve when the density of the body_____
9. The metacentric height of a given floating body will ________ if the density of the liquid decreases.
10. For a body immersed in a fluid the buoyant force _______ with increase in density of the body.

Answers

Increases 1, 7, 8 Decreases 3, 4, 6, 9, 10 Remains constant 2, 5

O Q. 4.3 Indicate whether the following statements are correct or incorrect

1. A floating body will displace the same volume of liquid irrespective of the liquid in which it floats.
2. The buoyant force on a given body immersed in a liquid will be the same irrespective of the liquid.
3. A floating body will displace a volume of liquid whose weight will equal the weight of the body.
4. As the metacentric height increases the stability of a floating body will improve.
5. When the metacentric height is zero the floating body will be in stable equilibrium.
6. When the centre of buyoancy is below the metacenter the floating body will be in stable equilibrium.
7. When the centre of gravity is below the centre of buoyancy the floating body will be unstable.
8. When the metacentre is between the centre of gravity and centre of buoyancy the body will be unstable.
9. When the length of a square log is larger than the side of section the log will float horizontally.
10. A given cubic piece will float more stably in mercury than in water.

Answers

Correct : 3, 4, 6, 8, 9, 10 Incorrect 1, 2, 5, 7

O Q. 4.4 Choose the correct answer

1. If a body is in stable equilibrium the metacentric height should be

   (*a*) zero (*b*) positive

   (*c*) negative (*d*) depends on the fluid.
2. When a heavy object is immersed in a liquid completely the centre of byoyancy will be at

   (*a*) The centre of gravity of the object.

   (*b*) The centre of gravity of the volume of the liquid displaced.

   (*c*) Above the centre of gravity of the object.

   (*d*) Below the centre of gravity of the displaced volume.
3. An object with specific gravity 4 weighs 100N in air. When it is fully immersed in water its weight will be

   (*a*) 25 N (*b*) 75 N

   (*c*) 50 N (*d*) None of the above.
4. A solid with a specific weight 9020 N/m^3 floats in a fluid with a specific weight 10250 N/m^3. The percentage of volume submerged will be

   (*a*) 90% (*b*) 92%

   (*c*) 88% (*d*) 78%.
5. An object weighs 50 N in water. Its volume is 15.3 *l*. Its weight when fully immersed in oil of specific gravity 0.8 will be

(*a*) 40 N (*b*) 62.5 N
(*c*) 80 N (*d*) 65 N.

6. When a ship leaves a river and enters the sea
(*a*) It will rise a little (*b*) It will sink a little
(*c*) There will be no change in the draft. (*d*) It will depend on the type of the ship.
7. When a block of ice floating in water in a container begins to melt the water level in the container
(*a*) will rise (*b*) will fall
(*c*) will remains constant (*d*) will depend on the shape of the ice block.
8. Two cubes of equal volume but of specific weights of 0.8 and 1.2 are connected by a weightless string and placed in water.
(*a*) one cube will completely submerged and the other will be completely outside the surface.
(*b*) heavier cube will go down completely and the lighter one to 0.25 times its volume.
(*c*) will float in neutral equilibrium.
(*d*) heavier cube will submerge completely and the lighter one will submerge to 0.8 times its volume.
9. For *a* floating body to be in stable equilibrium (with usual notations)
(*a*) $I/V = GB$ (*b*) $I/V < GB$
(*c*) $I/V > GB$ (*d*) $I/V = MG$.
10. A cube of side, *a* floats in a mercury/water layers with half its height in mercury. Considering the relative density of mercury as 13.6, the relative density of the cube will be
(*a*) 6.3 (*b*) 7.3
(*c*) 6.8 (*d*) $a \times 13.6/2$

Chapter 4

Answers

(1) *b*, (2) *b*, (3) *b*, (4) *c*, (5) *c*, (6) *a*, (7) *b*, (8) *c*, (9) *c*, (10) *b*

O Q. 4.5 Match the sets A and B

A	B
1. Metacentric height	(*a*) weight of displaced volume
2. *G* below *B*	(*b*) *CG* of displaced volume
3. Centre of buoyancy	(*c*) Stability
4. Buoyant force	(*d*) Always stabe.

Answers

(1) *c*, (2) *d*, (3) *b*, (4) *a*

EXERCISE PROBLEMS

E 4.1 Determine the buoyant force on a cube of 2 m side which stays afloat in water with its top face horizontal and 0.2 m above the free surface. **[70632 N]**

E 4.2 A hot air filled balloon of 8m diameter is used to support a platform. The surrounding air is at 20° C and 1 bar while the hot air inside the balloon is at a temperature of 70° C. Determine the buoyant force and the weight that may be supported by the balloon.
[3074.93 N, 403.37 N]

E 4.3 A closed cylindrical drum of 3 m dia and 2 m height is filled fully with oil of specific gravity 0.9 is placed inside an empty tank vertically. If water is filled in the tank, at what height of water level, the drum will start floating. Neglect the self weight of the drum.
[1.80 m from bottom]

E 4.4 A balloon is filled with hydrogen of density 0.08 kg/m^3. To support 50 N of weight in an atmospheric condition where the sir/density is 0.9 kg/m^3. Calculate the diameter of the balloon? **[2.28 m]**

E 4.5 A box of size 1m × 2m × 3m and weight 1000 N to lie just submerged in water is held down with a cubic block placed on it. If the density of the cubic block material is 2000 kg/m^3, find the dimension of the cubic block. The cubic block is also submerged in water. **[1.81 m]**

E 4.6 Determine the depth of immersion of a cubic block of 2 m side weighing 20 kN which floats in a liquid whose specific weight varies as 9810 (1 + depth in m). **[1 m]**

E 4.7 An object weighs 20 N when fully submerged in water. The same object weighs 35 N when fully submerged in an oil of specific gravity 0.8. Determine its volume and density.
[7.65 × 10^{-3} m^{-3}, 1265.88 kg/m^3]

E 4.8 Determine the specific gravity of a liquid when a hydrometer which is in the form of a sphere of 20 mm dia attached with a cylindrical stem of 5 mm dia and 200 mm length showed a depth of immersion of the stem of 100 mm. The total mass of hydrometer is 15 grams. **[0.8]**

E 4.9 Determine the metacentric height of a ship for rolling ($Y - Y$ aixs) and pitching ($X - X$ axis) whose plan view is in the form of an ellipse with major axis of 40 m and minor axis of 15 m. The weight of the ship is 9000 kN and the centre of buoyancy is 2 m below the water level and the centre of gravity is 0.5 m below the water level. Assume density of sea water as 1025 kg/m^3.
[5.9 m, 51.15 m]

E 4.10 Determine the metacentric height of a ship which displaces 5000 kN of water when it tilts by 6° due to the movement of 300 kN weight through 3 m from one side of center line to the other.
[1.72 m]

E 4.11 A cylinder with diameter 0.25 m and length 0.5 m floats in water. Determine its stability if its specific weight is 8000 N/m^3. **[unstable]**

E 4.12 A hollow cylinder with *ID* 0.8 m, *OD* 1.6 m and height 2 m floats in water. Check the stability of the cylinder if its specific gravity is 0.4. For stability of the cylinder, what is the required outer diameter? **[unstable, 2.479 m]**

E 4.13 A torus of D = 2 m and d = 0.5 with specific gravity 0.5 floats in water. Determine its metacentric height. **[2.6 m]**

E 4.14 Determine the D/h ratio for a stable floating log of circular cross section with density 800 kg/m^3. **[1.13]**

E 4.15 Determine the maximum density of a conical wooden block of 0.5 m dia and 0.8 m height to float stably in water. **[756 kg/m^3]**

E 4.16 Determine the metacentric height of the combined unit of a rectangular pontoon, 9 m long, 7 wide and 2 m deep weighing 500 kN carrying on its deck a boiler of 3 m dia weighing 300 kN. The centre of gravity of each unit may be taken to be at the geometric centre and along the same line. Also calculate the restoring torque for a tilt of 4° from vertical. Assume the centre to be on the vertical line. **[1.865 m, 1.0416 × 10^5 Nm]**

E 4.17 The stem of a hydrometer is of cylindrical shape of 2.8 mm dia and it weighs 0.0216 N. It floats 22.5 mm deeper in an oil than in alcohol of specific gravity 0.821. Determine the specific gravity of the oil. **[0.78]**

E 4.18 A metal piece floats in mercury of specific gravity 13.56. If the fraction of volume above the surface was 0.45, determine the specific gravity of the metal. **[7.458]**

E 4.19 A piece of material weighs 100 N in air and when immersed in water completely it weighs 60 N. Calculate the volume and specific gravity of the material. **[0.00408 m^3, 1.67]**

E 4.20 A wooden block when floating in glycerin projects 76 mm above the surface of the liquid. If the specific gravity of the wood was 0.667, how much of the block wil project above the surface in water. Specific gravity of glycerin is 1.6. **[50 mm]**

E 4.21 A long log of 2.5 m dia and 4.5 m length and of specific gravity of 0.45 floats in water. Calculate the depth of floatation.

E 4.22 A tank of 1.5 m dia and 2 m length open at one end is immersed in water with the open end in water. Water rises by 0.6 m inside. The water level is 1 m from the top. Determine the weight of the tank.

E 4.23 A ship with vertical sides near the water line weighs 4000 tons and the depth of immersion is 6.7056 m in sea water of specific gravity 1.026. When 200 tons or water ballast is discharged, the depth of immersion is 6.4 m. Calculate the depth of immersion in fresh water.

E 4.24 Determine whether a cylinder of 0.67 m dia and 1.3 m length will float vertically in stable condition in oil of specific gravity 0.83.

E 4.25 A sphere of 1.25 m dia floats half submerged in water. If a chain is used to tie it at the bottom so that it is submerged completely, determine the tension in the chain.

E 4.26 The distance between the markings of specific gravity of 1 and 1.1 is 10 mm, for a hydrometer of 10 mm dia. Determine the weight of the unit. **[0.08475 N]**

E 4.27 The difference in specific gravities of 1.1 and 1.4 is to be shown by 40 mm by a hyrometer of mass 25 gram. What should be the diameter of the stem. **[4.95 mm]**

E 4.28 A sphere of specific gravity 1.2 is immersed in a fluid whose sepcific gravity increases with depth y as $1 + 20 \times 10^{-6}\, y$, ($y$ in mm). Determine the location of the centre of the sphere when it will float in nuteral equilibrium.

E 4.29 A cube side 60 cm is made of two equal horizontal layers of specific gravity 1.4 and 0.6 and floats in a bath made of two layers of sepecific gravity 1.2 and 0.9, the top layer being 60 cm thick. Determine the location of the base from the liquid surface. **[0.2 m]**

E 4.30 A barge is of rectangular section of 26.7 m × 10 m and is 3 m in height. The mass is 453.62 tons with load. The centre of gravity is at 4 m from bottom. Determine the metacentric height for rotation along the 26.7 m centreline. Investigate the stability. Also determine the restoring torque if it is rotated by 5° about the axis. **[1.753 m, stable, 1904 kN/m]**

E 4.31 A wedge of wood of specific gravity 0.65 and base width 0.5 m and height 0.5 m is forced into water by 666 N. If the wedge is 50 cm wide, determine the depth of immersion.

E 4.32 A cube of side 40 cm weighing 1050 N is lowered into a tank containing water over a layer of mercury. Determine the position of the block under equilibrium.

E 4.33 An iceberg of specific gravity 0.92 floats in ocean water of specific gravity 1.02. If 3000 m^3 protrudes above the water level calculate the total volume of the iceberg.

E 4.34 Show that in the case of cylindrical hydrometer, the difference in the height of immersion in liquid of specific gravity S, over the height h of immersion in water is given by $h = V(S - 1) \times A \times S$ where V is the submerged volume in water, and A is the sectional area of the stem.