5 Fluid Flow—Basic Concepts—Hydrodynamics

5.0 INTRODUCTION

In the previous three chapters the pressure distribution in static fluids and its effect on surfaces exposed to the fluid was discussed. In this chapter the flow of ideal fluids will be discussed. The main attempt in this chapter is to visualise flow fields. A flow field is a region in which the flow is defined at all points at any instant of time. The means to that is to define the velocities at all the points at different times. It should be noted that the velocity at a point is the velocity of the fluid particle that occupies that point. In order to obtain a complete picture of the flow the fluid motion should be described mathematically. Just like the topography of a region is visualised using the contour map, the flow can be visualised using the velocity at all points at a given time or the velocity of a given particle at different times. It is then possible to also define the potential causing the flow.

Application of a shear force on an element or particle of a fluid will cause continuous deformation of the element. Such continuing deformation will lead to the displacement of the fluid element from its location and this results in fluid flow. The fluid element acted on by the force may move along a steady regular path or randomly changing path depending on the factors controlling the flow. The velocity may also remain constant with time or may vary randomly. In some cases the velocity may vary randomly with time but the variation will be about a mean value . It may also vary completely randomly as in the atmosphere. The study of the velocity of various particles in the flow and the instantaneous flow pattern of the flow field is called flow kinematics or hydrodynamics. Such a study is generally limited to ideal fluids, fluids which are incompressible and inviscid. In real fluid shows, beyond a certain distance from the surfaces, the flow behaves very much like ideal fluid. Hence these studies are applicable in real fluid flow also with some limitations.

5.1 LAGRANGIAN AND EULARIAN METHODS OF STUDY OF FLUID FLOW

In the Lagrangian method a single particle is followed over the flow field, the co-ordinate system following the particle. The flow description is particle based and not space based. A moving coordinate system has to be used. This is equivalent to the observer moving with the particle to study the flow of the particle. This method is more involved mathematically and is used mainly in special cases.

In the Eularian method, the description of flow is on fixed coordinate system based and the description of the velocity etc. are with reference to location and time *i.e.*, $V = V\ (x, y, z, t)$ and not with reference to a particular particle. Such an analysis provides a picture of various parameters at all locations in the flow field at different instants of time. This method provides an easier visualisation of the flow field and is popularly used in fluid flow studies. However the final description of a given flow will be the same by both the methods.

5.2 BASIC SCIENTIFIC LAWS USED IN THE ANALYSIS OF FLUID FLOW

(i) Law of conservation of mass: This law when applied to a control volume states that the net mass flow through the volume will equal the mass stored or removed from the volume. Under conditions of steady flow this will mean that the mass leaving the control volume should be equal to the mass entering the volume. The determination of flow velocity for a specified mass flow rate and flow area is based on the continuity equation derived on the basis of this law.

(ii) Newton's laws of motion: These are basic to any force analysis under various conditions of flow. The resultant force is calculated using the condition that it equals the rate of change of momentum. The reaction on surfaces are calculated on the basis of these laws. Momentum equation for flow is derived based on these laws.

(iii) Law of conservation of energy: Considering a control volume the law can be stated as "the energy flow into the volume will equal the energy flow out of the volume under steady conditions". This also leads to the situation that the total energy of a fluid element in a steady flow field is conserved. This is the basis for the derivation of Euler and Bernoulli equations for fluid flow.

(iv) Thermodynamic laws: are applied in the study of flow of compressible fluids.

5.3 FLOW OF IDEAL / INVISCID AND REAL FLUIDS

Ideal fluid is nonviscous and incompressible. Shear force between the boundary surface and fluid or between the fluid layers is absent and only pressure forces and body forces are controlling.

Real fluids have viscosity and surface shear forces are involved during flow. However the flow after a short distance from the surface is not affected by the viscous effects and approximates to ideal fluid flow. The results of ideal fluid flow analysis are found applicable in the study of flow of real fluids when viscosity values are small.

5.4 STEADY AND UNSTEADY FLOW

In order to study the flow pattern it is necessary to classify the various types of flow. The classification will depend upon the constancy or variability of the velocity with time. In the next three sections, these are described. In steady flow the property values at a location in the flow are constant and the values do not vary with time. The velocity or pressure at a point remains constant with time. These can be expressed as $V = V(x, y, z)$, $P = P(x, y, z)$ etc. In steady flow a picture of the flow field recorded at different times will be identical. In the case of unsteady flow, the properties vary with time or $V = V(x, y, z, t)$, $P = P(x, y, z, t)$ where t is time.

In unsteady flow the appearance of the flow field will vary with time and will be constantly changing. In turbulent flow the velocity at any point fluctuates around a mean value, but the mean value at a point over a period of time is constant. For practical purposes turbulent flow is considered as steady flow as long as the mean value of properties do not vary with time.

5.5 COMPRESSIBLE AND INCOMPRESSIBLE FLOW

If the density of the flowing fluid is the same all over the flow field at all times, then such flow is called incompressible flow. Flow of liquids can be considered as incompressible even if the density varies a little due to temperature difference between locations. Low velocity flow of gases with small changes in pressure and temperature can also be considered as incompressible flow. Flow through fans and blowers is considered incompressible as long as the density variation is below 5%. If the density varies with location, the flow is called compressible flow. In this chapter the study is mainly on incompressible flow.

5.6 LAMINAR AND TURBULENT FLOW

If the flow is smooth and if the layers in the flow do not mix macroscopically then the flow is called laminar flow. For example a dye injected at a point in laminar flow will travel along a continuous smooth line without generally mixing with the main body of the fluid. Momentum, heat and mass transfer between layers will be at molecular level of pure diffusion. In laminar flow layers will glide over each other without mixing.

In turbulent flow fluid layers mix macroscopically and the velocity/temperature/mass concentration at any point is found to vary with reference to a mean value over a time period. For example $u = \overline{u} + u'$ where u is the velocity at an instant at a location and $\overline{u}$ is the average velocity over a period of time at that location and u' is the fluctuating component. This causes higher rate of momentum/heat/mass transfer. A dye injected into such a flow will not flow along a smooth line but will mix with the main stream within a short distance.

The difference between the flows can be distinguished by observing the smoke coming out of an incense stick. The smoke in still air will be found to rise along a vertical line without mixing. This is the laminar region. At a distance which will depend on flow conditions the smoke will be found to mix with the air as the flow becomes turbulent. Laminar flow will prevail when viscous forces are larger than inertia forces. Turbulence will begin where inertia forces begin to increase and become higher than viscous forces.

5.7 CONCEPTS OF UNIFORM FLOW, REVERSIBLE FLOW AND THREE DIMENSIONAL FLOW

If the velocity value at all points in a flow field is the same, then the flow is defined as uniform flow. The velocity in the flow is independent of location. Certain flows may be approximated as uniform flow for the purpose of analysis, though ideally the flow may not be uniform.

If there are no pressure or head losses in the fluid due to frictional forces to be overcome by loss of kinetic energy (being converted to heat), the flow becomes reversible. The fluid can be restored to its original condition without additional work input. For a flow to be reversible, no surface or fluid friction should exist. The flow in a venturi (at low velocities) can be considered as reversible and the pressures upstream and downstream of the venturi will be the same in such a case. The flow becomes irreversible if there are pressure or head losses.

If the components of the velocity in a flow field exist only in one direction it is called one dimensional flow and $V = V(x)$. Denoting the velocity components in x, y and z directions as u, v and w, in one dimensional flow two of the components of velocity will be zero. In two dimensional flow one of the components will be zero or $V = V(x, y)$. In three dimensional flow all the three components will exist and $V = V(x, y, z)$. This describes the general steady flow situation. Depending on the relative values of u, v and w approximations can be made in the analysis. In unsteady flow $V = V(x, y, z, t)$.

5.8 VELOCITY AND ACCELERATION COMPONENTS

The components of velocity can be designated as

$$u = \frac{dx}{dt},\ v = \frac{dy}{dt} \quad \text{and} \quad w = \frac{dz}{dt}$$

where t is the time and dx, dy, dz are the displacements in the directions x, y, z.

In general as $\quad u = u(x, y, z, t),\ v = v(x, y, z, t) \quad$ and $\quad w = w(x, y, z, t)$

Defining acceleration components as

$$a_x = \frac{du}{dt},\ a_y = \frac{dv}{dt} \text{ and } a_z = \frac{dw}{dt}, \quad \text{as } u = u\,(x, y, z, t)$$

$$a_x = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t} + \frac{\partial u}{\partial t}$$

$$= u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

Similarly,

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

and

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

The first three terms in each case is known as convective acceleration terms, because these represent the convective act of moving from one position to another. The last term is known as local accleration term, because the flow at a point is changing with time. Under steady flow conditions, only the convective acceleration terms will exist.

5.9 CONTINUITY EQUATION FOR FLOW—CARTESIAN CO-ORDINATES

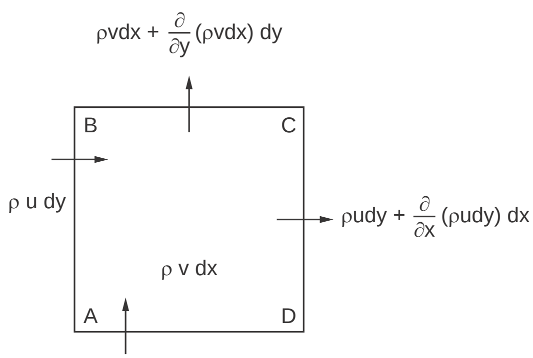

Figure. 5.9.1 Derivation of continuity equation

Consider an element of size dx, dy, dz in the flow as shown in Fig 5.9.1.

Applying the law of conservation of mass, for a given time interval,

The net mass flow into the element through all the surfaces

= The change in mass in the element.

First considering the $y - z$ face, perpendicular to the x direction and located at x, the flow through face during time dt is given by

$$\rho u\, dy\, dz\, dt \tag{5.9.1}$$

The flow through the $y - z$ face at $x + dx$ is given by

$$\rho u\, dy\, dz\, dt + \frac{\partial}{\partial x}(\rho\, u\, dy\, dz\, dt)\, dx \tag{5.9.2}$$

The net mass flow in the x direction is the difference between the quantities given by (5.9.1) and (5.9.2) and is equal to

$$\frac{\partial}{\partial x}(\rho u)\, dx\, dy\, dz\, dt \tag{5.9.3}$$

Similarly the net mass through the faces $z - x$ and $x - y$ in y and z directions respectively are given by

$$\frac{\partial}{\partial x}(\rho v)\, dx\, dy\, dz\, dt \tag{5.9.4}$$

$$\frac{\partial}{\partial x}(\rho w)\, dx\, dy\, dz\, dt \tag{5.9.5 }$$

The change in the mass in the control volume equals the rate of change of density × volume × time or

$$\frac{\partial \rho}{\partial t}\, dx\, dy\, dz\, dt \tag{5.9.6}$$

The sum of these quantities should equal zero, cancelling common terms $dx\, dy\, dz\, dt$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = \frac{\partial \rho}{\partial t} \tag{5.9.7}$$

This is the general equation. For steady flow this reduces to

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \tag{5.9.8}$$

For incompressible flow this becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{ 5.9.9}$$

Whether a flow is steady can be checked using this equation when the velocity components are specified. For two dimensional steady incompressible flow, the equation reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5.9.10}$$

For one dimensional flow with varying area, the first term of the general equation alone need be considered. For steady flow

$$\frac{\partial(\rho u\, dy\, dz)}{\partial x} = 0 \text{ as } dy\, dz = dA. \text{ Integrating } \rho u A = \text{constant. or}$$

$$\rho_1 u_1 A_1 = \rho_2\, u_2 A_2 \tag{5.9.11}$$

This equation is used to calculate the area, or velocity in one dimensional varying area flow, like flow in a nozzle or venturi.

5.10 IRROTATIONAL FLOW AND CONDITION FOR SUCH FLOWS

Irrotational flow may be described as flow in which each element of the moving fluid suffers no net rotation from one instant to the next with respect to a given frame of reference. In flow along a curved path fluid elements will deform. **If the axes of the element rotate equally towards or away from** each **other, then the flow will be irrotational**. This means that as long as the algebraic average rotation is zero, the flow is irrotational. The idea is illustrated in Fig. 5.10.1

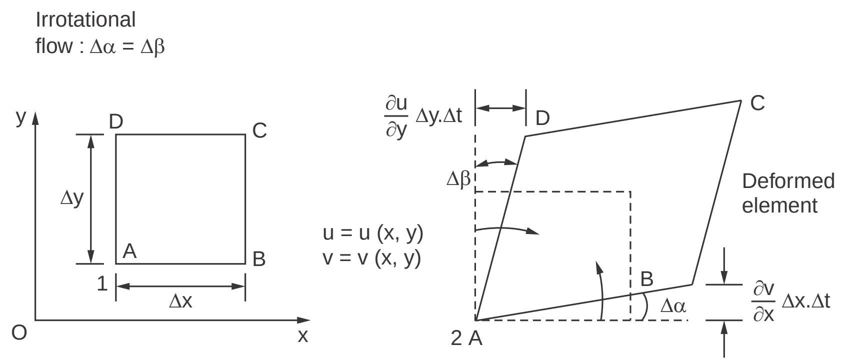

Figure 5.10.1 Rotation in Flow

An element is shown moving from point 1 to point 2 along a curved path in the flow field. At 1 the undeformed element is shown. As it moves to location 2 the element is deformed. The angle of rotation of x axis is given by $(\partial v/\partial y)$. $\Delta y.\Delta t$. The angle of rotation of y axis is given by $(\partial u/\partial y)$. $\Delta y.\Delta t$. (It is assumed that $\Delta x = \Delta y$. For irrotational flow, the angle of rotation of the axes towards each other or away from each other should be equal *i.e.,* the condition to be satisfied for irrotational flow is,

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \quad \text{or} \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \qquad (5.10.1)$$

Another significance of irrotational flow is that it is defined by a potential function ϕ for the flow described in para 5.15.

In case there is rotation, then the rotation is given by (with respect to the Z axis in the case of two dimensional flow along x and y)

$$\omega_z = (1/2)\,(\partial v/\partial x - \partial u/\partial y) \qquad (5.10.2)$$

and $\omega_z = 0$ for irrotational flow.

5.11 CONCEPTS OF CIRCULATION AND VORTICITY

Considering a closed path in a flow field as shown in Fig. 5.11.1, circulation is defined as the line integral of velocity about this closed path. The symbol used is Γ.

$$\Gamma = \oint_L u \; ds = \oint_L u \cos \beta \; dL$$

where dL is the length on the closed curve, u is the velocity at the location and β is the angle between the velocity vector and the length dL.

The closed path may cut across several stream lines and at each point the direction of the velocity is obtained from the stream line, as its tangent at that point.

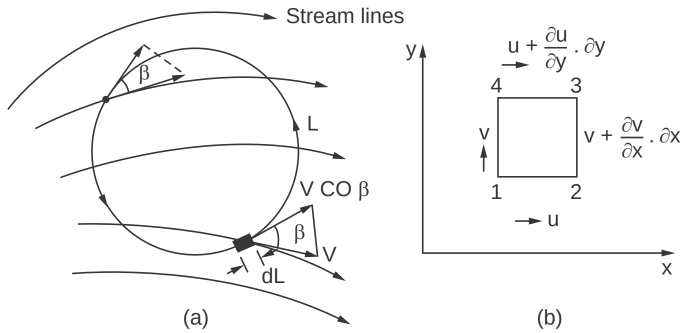

Figure. 5.11.1 Circulation in flow

The integration can be performed over an element as shown in Fig. 5.11.1 (*b*).

In the cartesian co-ordinate if an element $dx. \, dy$ is considered, then the circulation can be calculated as detailed below:

Consider the element 1234 in Fig. 5.3*b*. Starting at 1 and proceeding counter clockwise,

$$d\Gamma = u\,dx + [v+(\partial v/\partial x)dx]dy - [u+(\partial u/\partial y).dy]\,dx - vdy$$
$$= [\partial v/\partial x - \partial u/\partial y]dxdy \qquad (5.11.1)$$

Vorticity is defined as circulation per unit area. i.e.,

Vorticity = circulation per unit area, here area is $dx\,dy$, so

$$\text{Vorticity} = \frac{d\Gamma}{dxdy} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \qquad (5.11.2)$$

For irrotational flow, vorticity and circulation are both zero. In polar coordinates

$$\text{Vorticity} = \frac{\partial v_\theta}{\partial r} - \frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\theta}{r}$$

5.12 STREAM LINES, STREAM TUBE, PATH LINES, STREAK LINES AND TIME LINES

The analytical description of flow velocity is geometrically depicted through the concept of stream lines. The velocity vector is a function of both position and time. If at a fixed instant of time a curve is drawn so that it is tangent everywhere to the velocity vectors at these locations then the curve is called a stream line. Thus stream line shows the mean direction of a number of particles in the flow at the same instant of time. **Stream lines are a series of curves drawn tangent to the mean velocity vectors of a number of particles in the flow. Since stream lines are tangent to the velocity vector at every point in the flow field, there can be no flow across a stream line.**

A bundle of neighbouring stream lines may be imagined to form a passage through which the fluid flows. Such a passage is called a stream tube. Since the stream tube is bounded on all sides by stream lines, there can be no flow across the surface. Flow can be only through the ends. A stream tube is shown diagrammatically in Figure 5.12.1.

Under steady flow condition, the flow through a stream tube will be constant along the length.

Path line is the trace of the path of a single particle over a period of time. Path line shows the direction of the velocity of a particle at successive instants of time. In steady flow path lines and stream lines will be identical.

Streak lines provide an instantaneous picture of the particles, which have passed through a given point like the injection point of a dye in a flow. In steady flow these lines will also coincide with stream lines.

Path lines and streak lines are shown in Figure 5.12.1.

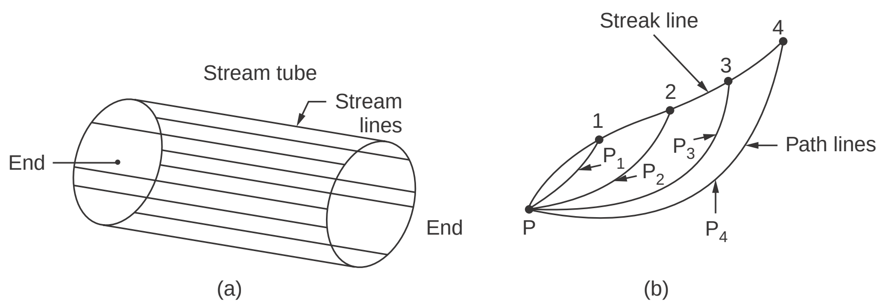

Figure 5.12.1 Stream tube, Path lines and Streak lines

Particles P_1, P_2, P_3, P_4, starting from point P at successive times pass along path lines shown. At the instant of time considered the positions of the particles are at 1, 2, 3 and 4. A line joining these points is the streak line.

If a number of adjacent fluid particles in a flow field are marked at a given instant, they form a line at that instant. This line is called time line. Subsequent observations of the line may provide information about the flow field. For example the deformation of a fluid under shear force can be studied using time lines.

5.13 CONCEPT OF STREAM LINE

In a flow field if a continuous line can be drawn such that the tangent at every point on the line gives the direction of the velocity of flow at that point, such a line is defined as a stream line. In steady flow any particle entering the flow on the line will travel only along this line. This leads to visualisation of a stream line in laminar flow as the path of a dye injected into the flow.

There can be no flow across the stream line, as the velocity perpendicular to the stream line is zero at all points. The flow along the stream line can be considered as one dimensional flow, though the stream line may be curved as there is no component of velocity in the other directions. Stream lines define the flow paths of streams in the flow. The flow entering between two stream lines will always flow between the lines. The lines serve as boundaries for the stream.

In the cartesian co-ordinate system, along the stream line in two dimensional flow it can be shown that

$$\frac{dx}{u} = \frac{dy}{v} \quad \text{or} \quad v\,dx - u\,dy = 0 \tag{5.13.1}$$

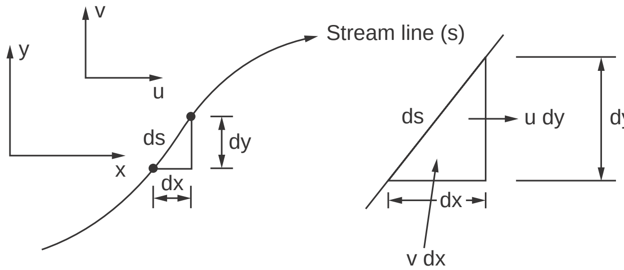

Figure 5.13.1 Velocity components along a stream line

Referring to Fig. 5.5 considering the velocity at a point and taking the distance ds and considering its x and y components as dx and dy, and noting that the net flow across ds is zero,

the flow along y direction $= dx\,v$

the flow along x direction $= dy\,u$

These two quantities should be equal for the condition that the flow across ds is zero, thus proving the equation (5.13.1).

In the next para, it is shown that stream lines in a flow can be described by a stream function having distinct values along each stream line.

5.14 CONCEPT OF STREAM FUNCTION

Refer to Fig. 5.14.1 showing the flow field, co-ordinate system and two stream lines.

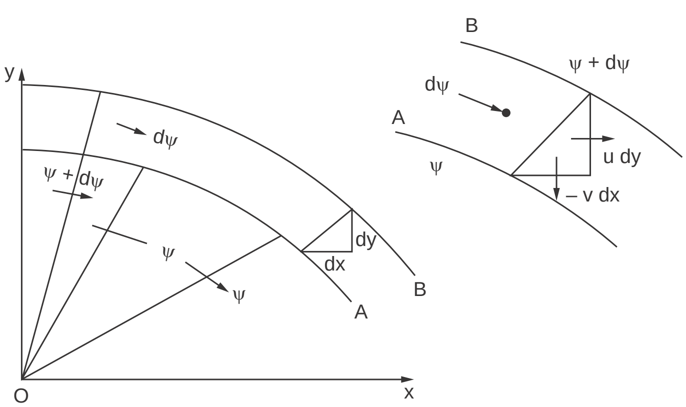

Figure. 5.14.1 Stream function—Definition

Stream function is a mathematical expression that describes a flow field. The definition is based on the continuity principle. It provides a means of plotting and interpreting flow fields. Considering the stream line *A* in figure, the flow rate across any line joining 0 and any point on *A* should be the same as no flow can cross the stream line *A*. Let the slow rate be denoted as ψ. Then ψ is a constant of the streamline *A*. If ψ can be described by an equation in *x* and *y* then stream line *A* can be plotted on the flow field. Consider another stream line *B* close to *A*. Let the flow between stream lines *A* and *B* be *d*ψ. The flow across any line between *A* and *B* will be dψ. Now taking components in the *x* and *y* directions,

$$d\psi = u\,dy - v\,dx \tag{5.14.1}$$

If the stream function ψ can be expressed as $\psi = \psi(x, y)$ (as it has a value at every point) then

$$d\psi = \frac{\partial \psi}{\partial x}\,dx + \frac{\partial \psi}{\partial y}\,dy \tag{5.14.2}$$

and comparing the above two equations, it is seen that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \tag{5.14.3}$$

In the practical point of view equation 5.14.3 can be considered as the definition of stream function.

As a result of the definition, if the stream function for a stream line is known, then the velocity at each point can be determined and vice versa.

If the velocity is expressed for a flow field in terms of *x* and *y* then the stream function value can be obtained by integrating equation 5.14.1.

$$\psi = \int \frac{\partial \psi}{\partial x}\,dx + \int \frac{\partial \psi}{\partial y}\,dy + c \tag{5.14.4}$$

The constant provides the difference in flow between various stream lines. By substituting for the values of *u* and *v* in the continuity equation 5.9.10 in terms of ψ,

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0 \tag{5.14.5}$$

As the value of the derivative is the same irrespective of the order in which it is taken the continuity equation is automatically satisfied by the stream function.

If the value of stream function is expressed in terms of *x* and *y*, stream lines can be plotted and the flow values can also be obtained between the stream lines. There are only a limited number of flows which are simple enough that stream function can be easily obtained. Many real flows can be obtained by the combination of the simple flows. It is also possible to combine two flows and then obtain the stream lines for the combined flow. This technique of superposition is found very useful in the analysis of more complex flows, with complex boundary conditions.

5.15 POTENTIAL FUNCTION

Flow is caused by a driving potential. It will be useful to have an idea of the potential at various locations.

If a fluid flow is irrotational, then equation 5.10.1 is satisfied

i.e.,
$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

Fluid flows which approximate to this condition are found to be large in number. Converging flows, and flows outside the boundary layer are essentially irrotational.

If this condition is satisfied everywhere in a flow except at a few singular points, it is mathematically possible to define a velocity potential function ϕ as

$$u = -\frac{\partial \phi}{\partial x}, \quad v = -\frac{\partial \phi}{\partial y} \tag{5.15.1}$$

The negative sign indicates that ϕ decreases in the direction of velocity increase.

These partial derivatives are known as potential gradients and give the flow velocity in the direction of the gradient. **Potential functions exist only in irrotational flow whereas stream functions can be written for all flows**. Substituting these in the continuity equation, an equation known as Laplace's equation results. Considering the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = -\frac{\partial^2 \phi}{\partial x^2}, \quad \frac{\partial v}{\partial y} = -\frac{\partial^2 \phi}{\partial y^2}$$

Substituting,
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \tag{5.15.2}$$

This is similar to heat conduction equation with temperature T replacing ϕ as potential.

Substituting this in equation 5.10.1 (irrotational flow)

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial v}{\partial x} = -\frac{\partial^2 \phi}{\partial x \partial y} \cdot \frac{\partial u}{\partial y} = -\frac{\partial^2 \phi}{\partial y \partial x} \tag{5.15.3}$$

as these two quantities are equal the irrotationality condition is satisfied.

Potential function and stream functions are orthogonal to each other. The proof is given in solved problem 5.1. The method of determination of potential function given the velocities or the stream function is described under solved problems 5.11, 5.12 and 5.13.

5.16 STREAM FUNCTION FOR RECTILINEAR FLOW FIELD (POSITIVE X DIRECTION)

It is often found necessary to analyse flow fields around immersed bodies. The extent of the approaching flow is often large and possesses straight and parallel stream lines, and the velocity distribution is uniform at a distance from the object. Such a flow is termed as **rectilinear flow** and is of practical importance. The flow can be described by the condition, **u = constant and v = 0.**

$$\therefore \qquad \psi = \int \text{const. } dy + \int (0)\, dx = c_1 y + c_2$$

where c_1 and c_2 are constants. By applying the boundary at $y = 0$, *i.e.,* coincident with x axis, $c_2 = 0$.

So $\qquad y = Cy = uy \qquad$ (5.16.1. *a*)

and $\quad \phi = cx = ux \qquad$ (5.16.1. *b*)

In polar coordinates

$\psi = ur \sin\theta, \quad \phi = ur \cos\theta \qquad$ (5.16.1. *c*)

Considering uniform flow at an angle α with x-axis

$\psi = u(y \cos\alpha - x \sin\alpha), \quad \phi = u(x \cos\alpha + y \sin\alpha) \qquad$ (5.16.1. *d*)

The stream lines are shown in Fig. 5.16.1.

The circulation Γ around any closed curve will be zero in this flow (check) Potential function is orthogonal to stream function.

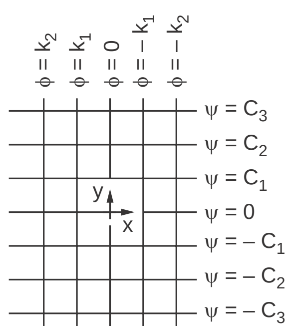

Figure. 5.16.1 Rectilinear flow stream and potential lines

It y distances are equally spaced, with distance 'a' then $C_0 = 0$, $C_1 = ua$, $C_3 = 2ua$ etc.

5.17 TWO DIMENSIONAL FLOWS—TYPES OF FLOW

There are only a few types of flow for which stream and potential functions can be determined directly. For other flows can be generally approximated as combinations of these flows. In this section, the simple flows are described.

5.17.1 Source Flow

A source flow consists of a symmetrical flow field with radial stream lines directed outwards from a common point, the origin from where fluid is supplied at a constant rate q. As the area increases along the outward direction, the velocity will decrease and the stream lines will spread out as the fluid moves outwards. The velocity at all points at a given radial distance will be the same.

The equations describing the flow are:

Velocity at radius r for flow rate of strength q is given by

$$u_r = q/2\pi r \tag{5.17.1}$$

The velocity in the tangential direction is zero

$$u_\theta = 0 \tag{5.17.2}$$

Stream function is represented by

$$\psi = (q/2\pi)\theta \tag{5.17.3}$$

The potential function is represented by

$$\phi = -(q/2\pi)\ln r \tag{5.17.4}$$

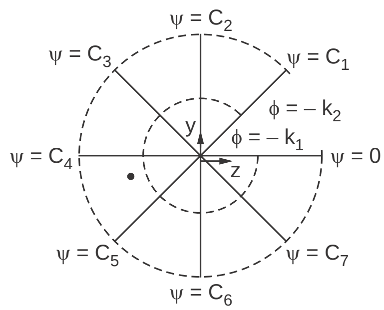

Figure 5.17.1 Potential and stream lines for source flow

The origin is a singular point. The circulation Γ around any closed curve is zero.

The stream lines are shown in Fig. 5.17.1.

Here C_1, C_2 etc are simply $\left(\frac{q}{2\pi}\right)\theta$, where θ is the angle of the stream line.

5.17.2 Sink Flow

Sink is the opposite of source and the radial streamlines are directed inwards to a common point, origin, where the fluid is absorbed at a constant rate. The velocity increases as the fluid moves inwards or as the radius decreases, the velocity will increase. In this case also the velocity at all points at a given radial distance from the origin will be the same. The origin is a singular point. The circulation around any closed curve is zero. The equations describing the flow are

$$u_r = -(q/2\pi r)\,,\; u_\theta = 0, \tag{5.17.5}$$

$$\psi = -(q/2\pi)\theta\,,\; \phi = (q/2\pi)\ln r \tag{5.17.6}$$

The stream lines are shown in Fig. 5.17.2.

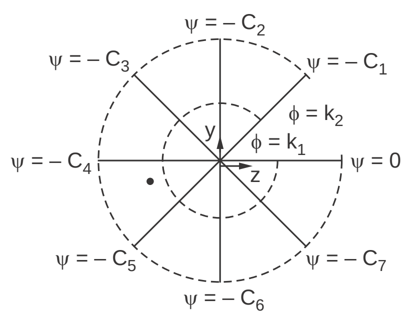

Figure 5.17.2 Stream and potential lines for sink flow

5.17.3 Irrotational Vortex of Strength *K*

(Free vortex, counter clockwise is taken as +ve. The origin is at the centre and is a singular point).

The equations describing the flow are

$$u_r = 0,\; u_\theta = (K/2\pi r) \tag{5.17.7}$$

$$\psi = -(K/2\pi)\ln r,\; \phi = -(K/2\pi)\theta \tag{5.17.8}$$

Circulation $\Gamma = K$ for closed curve enclosing origin and $\Gamma = 0$ for any other closed curves.

In this case the velocity varies inversely with radius. At $r = o$, velocity will tend to be ∞ and that is why the centre is a singular point.

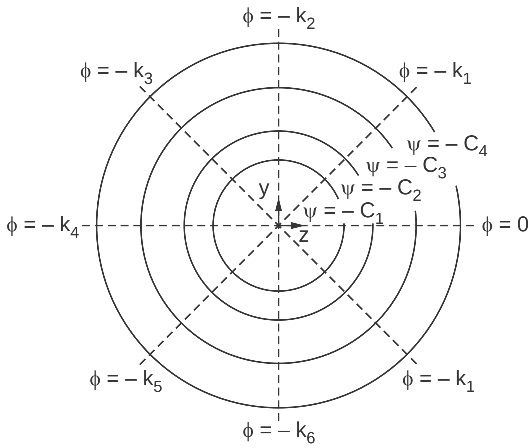

Figure 5.17.3 Irrotational vortex

Forced vortex is discussed in solved problem 5.3.

5.17.4 Doublet of Strength Λ

The centre is at the origin and is a singular point. Such a flow is obtained by allowing a source and sink of equal strengths merge and

$\Lambda = q\ ds/2\pi$, where ds is the distance between them.

The equations describing the flow are

$$u_r = -(\Lambda/r^2)\cos\theta\ ,\ u_\theta = -(\Lambda/r^2)\sin\theta \tag{5.17.9}$$

$$\psi = -(\Lambda\sin\theta/r),\ \phi = -(\Lambda\cos\theta/r) \tag{5.17.10}$$

The equation and the plot are for the limiting condition, $ds \to 0$. In this case Λ takes a definite value.

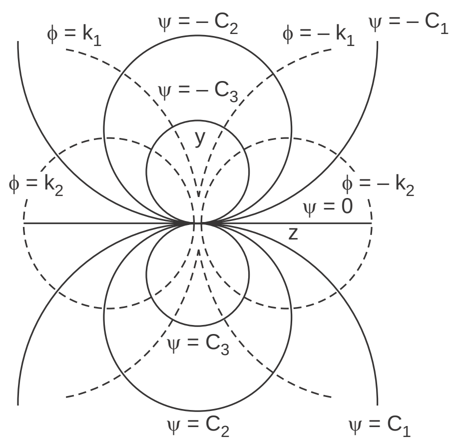

Figure 5.17.4 Potential and stream line for doublet

5.18 PRINCIPLE OF SUPERPOSING OF FLOWS (OR COMBINING OF FLOWS)

Some of the practical flow problems can be more easily described by combination of the simple flows discussed in previous article. For example, if in uniform flow a cylinder like body is interposed, the flow area reduces. The stream lines nearer the body move closer to each other and the flow far removed from the body is still uniform. This flow can be visualised by the combination of uniform flow and a source. The wake flow (behind the body) can be visualised by means of a sink and uniform flow. As equations for stream lines are available for flows like uniform flow, source, sink etc, it is found useful to study such combination of flows.

The simple rule for such a combination of two flows A and B is

$$\psi = \psi_A + \psi_B$$

where ψ describes the combined flow and ψ_A and ψ_B describe the component flows. Similarly $\phi = \phi_A + \phi_B$

Some of the examples follow.

5.18.1 Source and Uniform Flow (Flow Past a Half Body)

The combined stream lines are shown in Fig. 5.18.1.

The velocity in uniform flow along the x direction is u and along y direction is zero. The flow rate of the source is q.

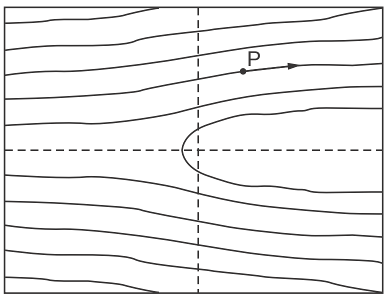

Figure 5.18.1 Source and uniform flow

The equations describing the flows are,

For source flow $\psi_1 = (q/2\pi)\,\theta$, For uniform flow $\psi_2 = cy = uy$

$\therefore\ \psi = \psi_1 + \psi_2 = (q/2\pi)\,\theta + uy$, In polar coordinates

$\psi = (q/2\pi)\theta + ur \sin\theta$

For uniform flow $\phi_2 = -\,ux$, For source flow $\phi_1 = -\,(q/2\pi) \ln r$, Combining

$$\phi = \phi_1 + \phi_2 = -\,(q/\,2\pi) \ln r - ux, \text{ in polar coordinates.}$$

$\phi = -\,(q/2\pi) \ln r - ur \cos\ \theta$

5.18.2 Source and Sink of Equal Strength with Separation of 2a Along x-axis

For source flow $\psi_1 = (q/2\pi)\,\theta_1$, for sink flow $\psi_2 = -\,(q/2\pi)\,\theta_2$,

Combining $\psi = \psi_1 + \psi_2 = (q/2\pi)\theta_1 - (q/2\pi)\theta_2 = (q/2\pi)(\theta_1 - \theta_2)$

Similarly using

$$\phi_1 = -\,(q/2\pi) \ln r_1 \text{ and } \phi_2 = (q/2\pi) \ln r_2$$

$$\phi = \phi_1 + \phi_2 = (q/2\pi) \ln (r_2/r_1)$$

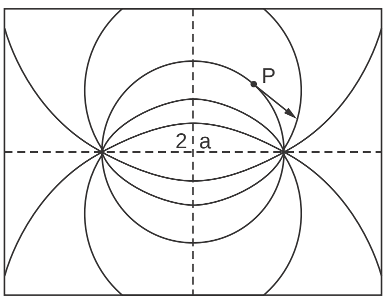

Figure 5.18.2 Source and sink of equal strength

5.18.3 Source and Sink Displaced at 2a and Uniform Flow (Flow Past a Rankine Body)

In this case refer para 5.18.1 and 2,

$\psi_1 = (q/2\pi)\,(\theta_1 - \theta_2),\ \psi_2 = uy$

$\therefore\quad \psi = (q/2\pi)(\theta_1 - \theta_2) + uy$

$= (q/2\pi)(\theta_1 - \theta_2) + ur \sin\theta$

$\phi_1 = (q/2\pi)\ln(r_2/r_1)$ and $\phi_2 = -\,ux$

$\therefore\quad \phi = (q/2\pi)\ln(r_2/r_1) - ur\cos\theta$

here r is the distance from the origin to the point and θ is the angle made by this line with x axis.

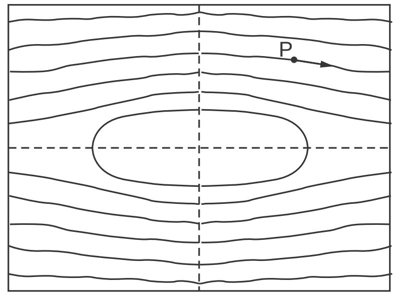

Figure 5.18.3 Source, sink and uniform flow

5.18.4 Vortex (Clockwise) and Uniform Flow

Refer results of section 5.17.3 for the vortex $\psi = (K/2\pi)\ln r$ (clockwise)

For uniform flow

$\psi = uy$

$\therefore\quad \psi = (K/2\pi)\ln r + uy,$

In polar coordinates,

$\psi = (K/2\pi)\ln r + ur\sin\theta$

For vortex $\phi_1 = (K/2\pi)\theta$, For uniform flow

$\phi_2 = -\,ux$

$\therefore\quad \phi = (K/2\pi)\theta - ux$

In polar coordinates,

$\phi = (K/2\pi)\theta - ur\cos\theta$

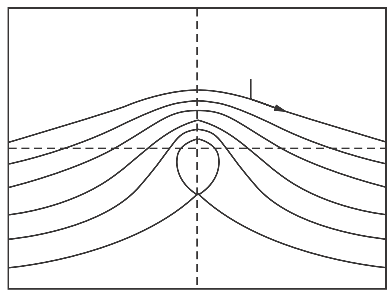

Figure 5.18.4 Vortex and uniform flow

5.18.5 Doublet and Uniform Flow (Flow Past a Cylinder)

Refer results of para 5.17.4. For doublet $\psi_1 = \Lambda \sin\theta/r$,

For uniform flow $\quad \psi_2 = uy = ur\sin\theta$

$\therefore\quad \psi = (\Lambda\sin\theta/r) + ur\sin\theta$

defining $\quad a^2 = \Lambda/u,\ \psi = ur\,[1 - (a^2/r^2)]\sin\theta$

$\phi_1 = -\,(\Lambda\cos\theta/r),$

$\phi_2 = -\,ux = -\,ur\cos\theta$

$\therefore\quad \phi = -\,ur\,[1 + (a^2/r^2)]\cos\theta$

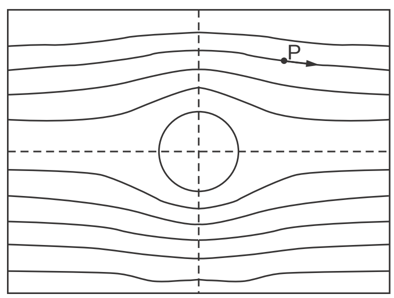

Figure 5.18.5 Flow past a cylinder

5.18.6 Doublet, Vortex (Clockwise) and Uniform Flow

Refer results of para 15.18.5 and 15.17.3

$\psi = ur\,[1 - (a^2/r^2)]\sin\theta + (K/2\pi)\ln r$

$\phi = -\,ur\,[1 + (a^2/r^2)]\cos\theta + (K/2\pi)\theta$

where $\quad a^2 = \Lambda/u$, and for $K < 4\pi au$

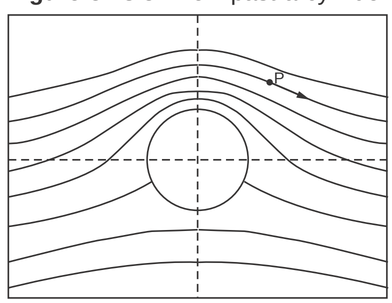

Figure 5.18.6 Doublet, vortex and uniform flow

5.18.7 Source and Vortex (Spiral Vortex Counterclockwise)

Refer results of para 5.17.1 and 3

$\psi = (q/2\pi)\,\theta - (K/2\pi)\ln r$

$\phi = -(q/2\pi)\ln r - (K/2\pi)\,\theta$

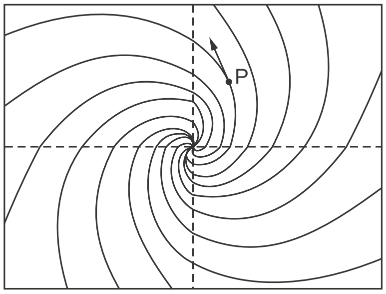

Figure 5.18.7 Source and vortex

5.18.8 Sink and Vortex (Spiral Vortex Counterclockwise)

Refer results of para 15.17.2 and 3

$\psi = -(q/2\pi)\theta - (K/2\pi)\ln r$

$\phi = (q/2\pi)\ln r - (K/2\pi)\theta$

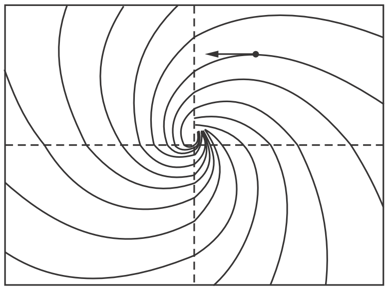

Figure 5.18.9 Sink and vortex

5.18.9 Vortex Pair (Equal Strength, Opposite Rotation, Separation by 2a)

Refer results of para 15.17.3

$$\psi = (K/2\pi)\ln(r_2/r_1),$$

$$\phi = (K/2\pi)(\theta_2 - \theta_1)$$

Many more actual problems can be modelled by the use of this basic principle.

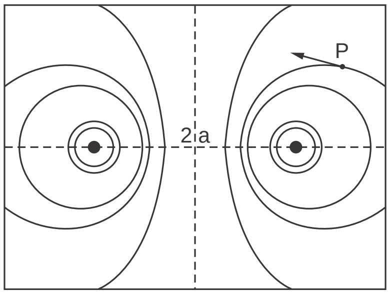

Figure 5.18.8 Vortex pair

5.19 CONCEPT OF FLOW NET

The plot of stream lines and potential flow lines for a flow in such a way that these form curvilinear squares is known as flow net. The idea that stream lines and potential lines are orthogonal is used in arriving at the plot.

Such a plot is useful for flow visualisation as well as calculation of flow rates at various locations and the pressure along the flow. The lines can be drawn by trial or electrical or magnetic analogue can also be used.

An example is shown in Fig. 5.19.1 for flow through a well rounded orifice in a large tank. The flow rate along each channel formed by the stream lines will be equal. The pressure drop between adjacent potential lines will also be equal.

With the advent of computer softwares for flow analysis, the mechanical labour in the plotting of such flow net has been removed. However the basic idea of flow net is useful.

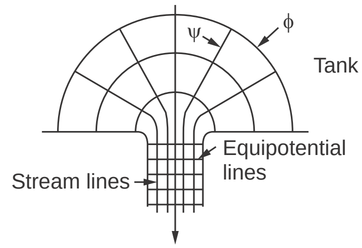

Figure 5.19.1 Flow Net

SOLVED PROBLEMS

Problem 5.1. *Prove that the stream function and potential function lead to orthogonality of stream lines and equipotential flow lines.*

$$\psi = \psi\,(x, y)$$

$$\therefore \qquad d\psi = \frac{\partial \psi}{\partial x}\,dx + \frac{\partial \psi}{\partial y}\,dy$$

Substituting from definition of $\frac{\partial \psi}{\partial x}$ and $\frac{\partial \psi}{\partial y}$ as $-v$ and u

$$\therefore \qquad \partial\psi = -vdx + udy$$

as ψ is constant along a stream line $d\psi = 0$,

$$\therefore \qquad vdx = udy$$

The slope of the stream line at this point is thus given by

$$\frac{dy}{dx} = \frac{v}{u} \qquad (1)$$

Similarly, $\phi = \phi\,(x, y)$

$$\therefore \qquad d\phi = \frac{\partial \phi}{\partial x}\,dx + \frac{\partial \phi}{\partial y}\,dy$$

Substituting for $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ as $-u$ and $-v$

$$d\phi = -udx - vdy$$

as $d\phi = 0$ along an equipotential line. $udx = -vdy$

$$\therefore \qquad \frac{\partial y}{\partial x} = -u/v \qquad (2)$$

These values of slopes show that the two sets of lines are perpendicular to each other. Hence stream lines and equipotential lines are orthogonal.

Problem 5.2. *Determine the stream function in the case of free vortex.*

In free vortex flow the stream lines are concentric circles around the singular point. The flow is irrotational except at the singularity. This condition can be used to show that $v_t\, r$ = constant where v_t is the tangential velocity. This is done by considering the circulation around the element going along 1234.

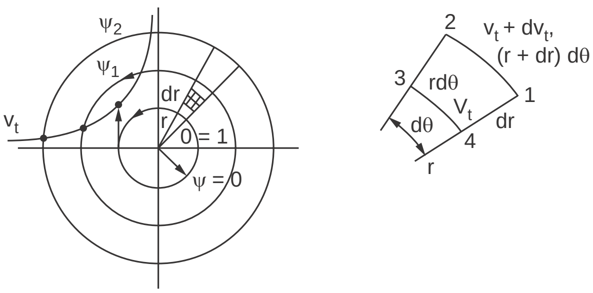

Figure P. 5.2

$$d\Gamma = (v_t + dv_t)\,(r + dr)\, d\,\theta + 0 - v_t\, rd\,\theta + 0 = 0$$

neglecting second order terms and simplifying

$$v_t .\, dr + r\, dv_t = d\,(rv_t) = 0 \quad \text{or} \quad v_t\, r = \text{constant for the flow}$$

This relationship holds at all locations except the centre (singular point)

The circulation along any streamline can be calculated by the usual procedure of

$$\oint_L v\, dL \quad \text{and} \quad \Gamma = 2\pi r v_t$$

As $v_t\, r$ = constant for the flow, circulation is constant for the vortex and Γ is known as vortex strength. The stream function can be determined by integration

$$v_t = \frac{\Gamma}{2\pi r} = \frac{-\,\partial\psi}{\partial r},\; v_r = 0 \qquad \text{(radial velocity)}$$

$$\therefore \qquad \psi = \int -\frac{\Gamma}{2\pi r}\, dr + \int (0) r d\theta + C$$

taking that $\psi = 0$ for the stream line at r =1

$$\psi = \frac{\Gamma}{2\pi} \ln r \qquad (1)$$

For a clockwise vortex

$$\psi = \frac{\Gamma}{2\pi} \ln r$$

Problem 5.3. *Determine the stream function for a forced vortex.*

A forced vortex is obtained by rotating the fluid as a whole. The flow is characterised by the equation $v_t = -\,\omega r$ and $v_r = 0$ (– sign for clockwise vortex)

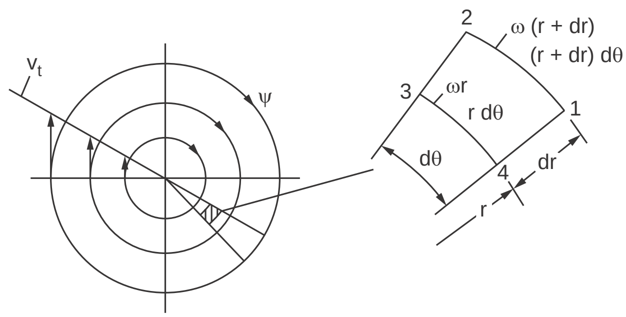

Figure P. 5.3

∴ $$\psi = \int (\omega r)\, dr + \int (0)\, rd\, \varphi + C = \omega r^2/2 + C$$

taking $\psi = 0$ for the stream line at $r = 0$, C will vanish and so $\psi = \omega r^2/2$

An important aspect of the flow is that the flow is rotational

This can be shown by considering an element in the flow as shown in Fig. P.5.3 and calculating the circulation and then vorticity

$$d\,\Gamma = -\,\omega\,(r + dr)\,(r + dr)\,d\,\theta + 0 + \omega\, r\, rd\,\theta + 0 = -\,2\,\omega\, rd\, rd\,\theta$$

as vorticity = $d\,\Gamma$/area and as area = $r\,d\theta\,dr$

Vorticity = $-\,2\,\omega$. Hence flow is rotational. The vorticity is directly related to the angular velocity of the mass.

Problem 5.4. *In a two dimensional flow the x and y directional velocities u and v are given by*

$$u = -\frac{x}{x^2 + y^2}, \qquad v = -\frac{y}{x^2 + y^2}$$

1. *Show that the flow is steady and* 2. *Check whether the flow is irrotational*

To check for steady flow, the continuity equation should be used.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = -\left[\frac{(x^2 + y^2) - 2x^2}{(x^2 + y^2)^2}\right] = \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

$$\frac{\partial v}{\partial y} = -\left[\frac{(x^2 + y^2) - 2y^2}{(x^2 + y^2)^2}\right] = \frac{y^2 - x^2}{(x^2 + y^2)^2}$$

The sum is zero and this satisfies the continuity equation and so the flow is steady. To check for irrotationality, the condition to be satisfied is,

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \quad \frac{\partial v}{\partial x} = \frac{2xy}{(x^2 + y^2)^2}, \quad \frac{\partial u}{\partial y} = \frac{2xy}{(x^2 + y^2)^2},$$

The difference is zero and hence the flow is irrotational.

Problem 5.5. *Check whether the following velocity relations satisfy the requirements for steady irrotational flow.*

(i) $u = x + y,\ v = x - y$ (ii) $u = xt^2 + 2y,\ v = x^2 - y\,t^2$ (iii) $u = xt^2,\ v = xyt + y^2$

To check for steady flow use continuity equation:

(i) $\frac{\partial u}{\partial x} = 1, \frac{\partial v}{\partial y} = -1$ $\therefore$ $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ So the flow is steady

(ii) $\frac{\partial u}{\partial x} = t^2, \frac{\partial v}{\partial y} = -t^2$ $\therefore$ satisfies the continuity equation and flow is steady

(iii) $\frac{\partial u}{\partial x} = t^2, \frac{\partial v}{\partial y} = xt + 2y$

This does not satisfy the requirements for steady flow

To Check for irrotational flow: $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$

(i) $\frac{\partial u}{\partial y} = 1, \frac{\partial v}{\partial x} = 1$ $\therefore$ flow is irrotational

(ii) $\frac{\partial u}{\partial y} = 2, \frac{\partial v}{\partial x} = 2x$ $\therefore$ flow is not irrotational

(iii) $\frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial x} = \text{yt}$ $\therefore$ flow is not irrotational

Problem 5.6. *Check whether the following flows are (i) steady and (ii) irrotational*

(i) $u = 2y,\ v = -3x,$ *(ii)* $u = 3xy,\ v = 0,$ *(iii)* $u = -2x,\ v = 2y$

(a) *Check for steady flow:* $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

(i) $\frac{\partial u}{\partial x} = 0, \frac{\partial v}{\partial y} = 0$ So steady flow prevails

(ii) $\frac{\partial u}{\partial x} = 3y, \frac{\partial v}{\partial y} = 0$ This is not a steady flow

(iii) $\frac{\partial u}{\partial x} = -2, \frac{\partial v}{\partial y} = 2$ So steady flow prevails

(b) *Check for irrotational flow:* $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$

(i) $\frac{\partial u}{\partial y} = 2, \frac{\partial v}{\partial x} = -3$ Hence not irrotational

Chapter 5

(*ii*) $\frac{\partial u}{\partial y} = 3x, \frac{\partial v}{\partial x} = 0$ Hence not irrotational

(*iii*) $\frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial x} = 0$ Hence irrotational

Problem 5.7. *The stream function for a flow is given by $\psi = xy$. Is the flow irrotational? Determine (i) u, v (ii) the vorticity and (iii) circulation.*

(*i*) From the definition stream function, $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, as $\psi = xy$,

$$v = -\frac{\partial \psi}{\partial x} = -y,\ u = \frac{\partial \psi}{\partial y} = x$$

$\therefore$ $u = x, \quad v = -y$

For irrotational flow

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}, \quad \frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial x} = 0, \ \therefore \text{Flow is irrotational}$$

(*ii*) vorticity and (*iii*) circulation will be zero for irrotational flow.

Problem 5.8. *Describe the method of determination of the stream function given the velocity relationship and also determine the stream function given*

$$u = 4xy \quad \text{and} \quad v = c - 2y^2$$

The method used for the determination of stream function is described below

(1) First check for continuity

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0,$$

(2) Let $u = f_1 (x, y)$

$\therefore$ $u = \frac{\partial \psi}{\partial y} = f_1 (x, y)$

$\therefore$ $\psi = \int f_1 (x, y)\, dy + f(x)$

where the second terms is a function of x only

(3) Let $-v = \frac{\partial \psi}{\partial x} = f_2 (x, y)$, using equation 1 determine the derivative.

$$\frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x} \left[\int f_1 (x, y)\, dy\right] + f'(x) = f_2 (x, y) = -v$$

comparing the terms with $f_2 (x, y)$, $f'(x)$ can be obtained

(4) $\psi = \int f(x)dx + \int f_1(x, y)dy + \text{constant}$

(1) $u = 4xy$, $v = c - 2y^2$

$$\frac{\partial u}{\partial x} = 4y,\ \frac{\partial v}{\partial y} = -4y,\ \therefore \text{ continuity is satisfied}$$

(2) $$u = \frac{\partial \psi}{\partial y} = 4xy$$

$\therefore$ $$\psi = \int 4xy\, dy = 2xy^2 + f(x) \quad \text{(A)}$$

where $f(x)$ is a function of x only

(3) $$\frac{\partial \psi}{\partial x} = -v = 2y^2 - c$$

$$= 2y^2 + f'(x) \qquad \text{(Using equation } A\text{)}$$

Differentiating equation (A) *w.r.t.* x and comparing $f'(x) = -c$, $f(x) = -cx$

(4) Now substitute for $f(x)$ in A

$$\psi = 2xy^2 - cx + \text{constant} \quad \text{(B)}$$

Check (use equation B)

$$u = \frac{\partial \psi}{\partial y} = 4xy,\ v = -\frac{\partial \psi}{\partial x} = -2y^2 + c$$

Problem 5.9. *Determine the stream function given, $u = 2x + y$, $v = x - 2y$*

(1) Check for continuity

$$\frac{\partial u}{\partial x} = 2,\ \frac{\partial v}{\partial y} = -2 \quad \therefore \quad \text{Satisfies continuity}$$

(2) $$\frac{\partial \psi}{\partial y} = u = 2x + y$$

$$\psi = \int (2x + y)\, dy + f(x)$$

$$= 2xy + (y^2/2) + f(x) \quad \text{(A)}$$

(3) Using A, $$\frac{\partial \psi}{\partial x} = 2y + f'(x) = -v = 2y - x$$

$$f'(x) = -x,\ f(x) = -x^2/2$$

$\therefore$ $$\psi = 2xy + (y^2/2) - (x^2/2) + \text{Constant}$$

Check $$u = \frac{\partial \psi}{\partial y} = 2x + y,\ v = -\frac{\partial \psi}{\partial x} = -2y + x$$

Problem 5.10. *Explain how the validity of a given potential function ϕ is established. Validate the potential function given as (i) $\phi = y^2 - x^2$ (ii) $\phi = xy$*

A potential function should satisfy the laplace equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

It should also satisfy the condition for irrotational flow

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

Case (i) $$\frac{\partial \phi}{\partial x} = -2x, \frac{\partial^2 \phi}{\partial x^2} = -2, \frac{\partial \phi}{\partial y} = 2y, \frac{\partial^2 \phi}{\partial x^2} = 2$$

Hence Laplace equation is satisfied. To check for irrotational flow

$$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial (y^2 - x^2)}{\partial x} = 2x, \qquad \therefore \quad \frac{\partial u}{\partial y} = 0$$

$$v = -\frac{\partial \phi}{\partial y} = -\frac{\partial (y^2 + x^2)}{\partial y} = -2y, \qquad \therefore \quad \frac{\partial v}{\partial x} = 0$$

So the flow is irrotational and hence the function is valid.

Case (ii) $$\phi = xy, \frac{\partial \phi}{\partial x} = y \quad \therefore \quad \frac{\partial^2 \phi}{\partial x^2} = 0, \frac{\partial \phi}{\partial y} = x, \quad \therefore \quad \frac{\partial^2 \phi}{\partial y^2} = 0$$

Hence valid. $$u = -\frac{\partial \phi}{\partial x} = -y \quad \therefore \quad \frac{\partial u}{\partial y} = -1, v = -\frac{\partial \psi}{\partial y} = -x \quad \therefore \quad \frac{\partial v}{\partial x} = -1$$

Hence irrotationality is satisfied. The function is a valid potential function.

Problem 5.11. *Explain how the potential function can be obtained if the stream function for the flow is specified.*

(1) Irrotational nature of the flow should be checked first. Stream function may exist, but if the flow is rotational potential function will not be valid.

(2) The values of u and v are obtained from the stream function as

$$\frac{\partial \psi}{\partial y} = u \text{ and } \frac{\partial \psi}{\partial x} = -v$$

(3) From the knowledge of u and v, ϕ can be determined using the same procedure as per the determination of stream function

$$u = -\frac{\partial \phi}{\partial x}$$

$$\therefore \qquad \phi = -\int u dx - f(y) \qquad \text{(A)}$$

where $f(y)$ is a function of y only

$\frac{\partial \phi}{\partial y}$ is determined and equated to $-v$

Comparing $f'(y)$ is found and then $f(y)$ is determined and substituted in equation A

$$\phi = -\int u dx - f(y) + C$$

Problem 5.12. *For the following stream functions, determine the potential function*

(i) $\psi = (3/2)(x^2 - y^2)$ *(ii)* $\psi = -8xy$ *(iii)* $\psi = x - y$

(*i*) $$u = \frac{\partial \psi}{\partial y} = -3y, \quad u = -3y, \quad -v = \frac{\partial \psi}{\partial x} = 3x, \quad v = -3x$$

To check for irrotationality $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$, here both are – 3, so checks

(2) $$u = -3y, \quad \text{also} \quad u = -\frac{\partial \phi}{\partial x} \quad \therefore \quad \phi = \int 3y dx + f(y)$$

$$\phi = 3xy + f(y) \qquad \text{(A)}$$

Differentiating equation A with respect to y and equating to v,

$$\frac{\partial \phi}{\partial y} = 3x + f'y = -v = 3x$$

$\therefore$ $f'(y) = 0$ and so $f(y) =$ constant

Substituting in A, $\phi = 3xy$ + constant

check $$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0, \quad \frac{\partial \phi}{\partial y} = 3y, \quad \frac{\partial^2 \phi}{\partial x^2} = 0, \text{ So also } \frac{\partial^2 \phi}{\partial y^2} = 0$$

So checks, $$u = -\frac{\partial \phi}{\partial x} = -3y, \quad v = -\frac{\partial \phi}{\partial y} = -3x \text{ also checks,}$$

(*ii*) $$\psi = -8xy$$

$$u = \frac{\partial \psi}{\partial y} = -8x \quad \therefore \quad u = -8x, \quad -v = \frac{\partial \psi}{\partial x} = -8y$$

$$\therefore \quad v = 8y$$

Check for irrotationality : $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$, in this case both are zero. Hence flow is irrotational.

$$\frac{\partial \phi}{\partial x} = -u = 8x \quad \phi = \int 8x dx + f(y) = 4x^2 + f(y)$$

differentiating this expression with respect to y, $\frac{\partial \phi}{\partial y} = f'(y) = -v = -8y$

$\therefore$ $f(y) = -4y^2$ $\therefore$ $\phi = 4x^2 - 4y^2$

Check for countinuity, Laplace equation etc. as an exercise. Also calculate u and v from this expression and check

(*iii*) $\psi = x - y,$

$$u = \frac{\partial \psi}{\partial y} = -1, \quad v = -\frac{\partial \psi}{\partial x} = -1 \quad \therefore \quad \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = 0,$$

Hence flow is irrotational.

$$\frac{\partial \phi}{\partial x} = -u = 1, \quad \phi = \int 1\,dx + f(y) = x + f(y)$$

Differentiating *w.r.t.* y,

$$\frac{\partial \phi}{\partial y} = f'(y) = -v = 1 \qquad \therefore \quad f(y) = y$$

$\therefore$ $\phi = x + y$ (Check for other conditions)

Note: In all cases, a constant can be added to the function ψ as well as ϕ.

A similar procedure is to be adopted to obtain stream function when potential function is specified. In this case stream function will always exist. Only in the case of potential function, the condition for irrotationality is to be checked.

Problem 5.13. *Given that $u = x^2 - y^2$ and $v = -2xy$, determine the stream function and potential function for the flow.*

Check for continuity;

$$\frac{\partial u}{\partial x} = 2x, \ \frac{\partial v}{\partial y} = -2x.$$

Hence satisfies the condition $\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0$

Check for rotation:

$$\frac{\partial u}{\partial y} = -2y, \ \frac{\partial v}{\partial x} = -2y \qquad \therefore \quad \text{flow is irrotational}$$

(*i*) To determine the stream function

$$u = \frac{\partial \psi}{\partial y} = x^2 - y^2 \quad \psi = x^2 y - (y^3/3) + f(x)$$

Differentiating this expression w.r.t. x,

$$\frac{\partial \psi}{\partial x} = 2xy + f'(x) = -v = 2xy$$

$\therefore$ $f'(x) = 0$ and $f(x)$ = constant.

$\therefore$ $\psi = x^2 y - y^3/3$ + constant

Check: $\frac{\partial \psi}{\partial y} = x^2 - y^2 = u$, checks. $\frac{\partial \psi}{\partial x} = 2xy = -v$, checks

Also $$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=0 \text{ i.e., } 2y-2y=0$$

(*ii*) To determine the potential function

$$\frac{\partial\phi}{\partial x}=-u=-x^2+y^2\,,\ \phi=-x^3/3+y^2x+f(y)$$

Differentiating w.r.t. y,

$$\frac{\partial\psi}{\partial y}=2xy+f'(y)=-v=2xy$$

$\therefore$ $f'(y)=0$ and $f(y)=$ constant

$\therefore$ $\phi=-x^3/3+y^2x+c$

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=-2x+2x=0. \text{ Hence checks.}$$

(also check calculating u, v) and for orthogonality.

Problem 5.14. *Derive an expression for the stream function for (i) uniform flow of 10 m/s along the x direction (ii) uniform flow of 5 m/s parallel to the negative y direction (iii) the combination of the two.*

(*i*) When the flow is uniform along the x direction with velocity u

$\psi_1=uy$ and in polar coordinates $\psi_1=ur\cos\theta$ $\therefore$ $\psi_1=10y$

(*ii*) For uniform flow along negative y direction with velocity v

$\psi_2=-(-vx)=vx=5x$

(*iii*) Combining ψ_1 and ψ_2, $\psi=10y+5x$

The combined streamlines are shown in Fig. P. 5.14.

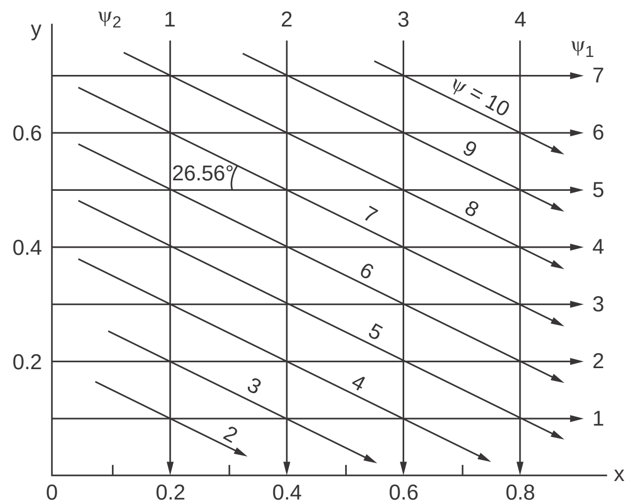

Figure P. 5.14

Chapter 5

At any point the resultant velocity is

$$(u^2 + v^2)^{0.5} = (10^2 + 5^2)^{0.5} = 11.18 \text{ m/s}$$

The direction is given by $\theta = \tan^{-1}(5/10) = 26.6^0$ to the x axis

Poblem 5.15. *Determine the stream function for a uniform flow in the negative x direction towards the origin at 5m/s combined with a sink flow of strength 12.*

Show the resulting stream lines.

For uniform flow $\psi_1 = -uy = -5y$,

For the sink $\psi_2 = -12\,\theta/2\pi$

For the combined stream lines

$$\psi = \psi_1 + \psi_2 = -5y - (12\theta/2\pi)$$

The combined flow is shown in Fig. P. 5.15

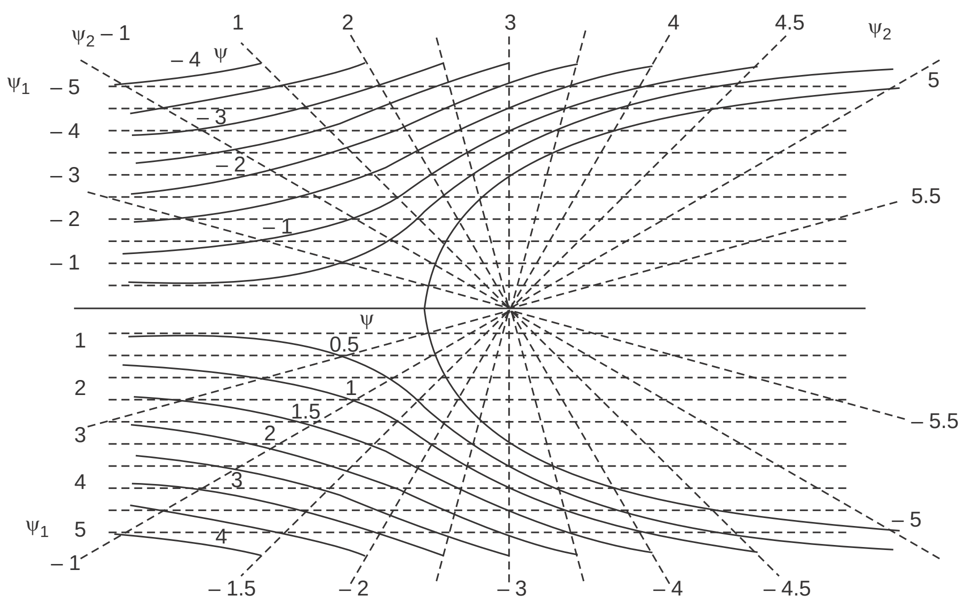

Figure P. 5.15

Problem 5.16. *Determine the stream function for a uniform x directional flow towards the origin from the positive x direction at 5m/s and a source of strength 12 m*

For uniform flow $\psi_1 = -uy = -5y$,

For the sink $\psi_2 = 12\,\theta/2\pi$

For the combined stream lines

$$\psi = \psi_1 + \psi_2 = -5y + (12\theta/2\pi)$$

The combined flow is shown in Fig. P. 5.16

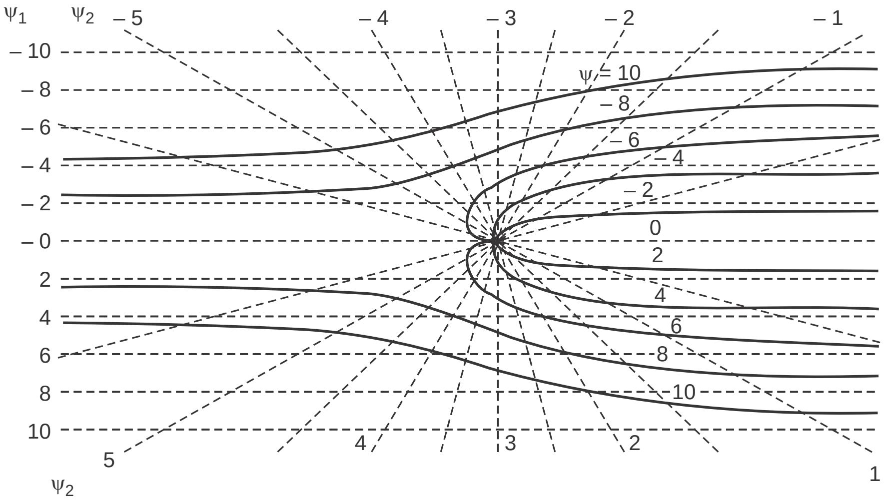

Figure P. 5.16

Problem 5.17. The velocity components at point (2, 2) is specified by the equation $u = x^2 + 3y$ and $v = -2xy$. Determine the accelerations and vorticity at this point.

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} = (x^2 + 3y)(2x) + (-2xy)\,3 + 0$$

$$= 2x^3 + 6xy - 6xy = 2x^3 = 16 \text{ units}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial v}{\partial t} = (x^2+3y)(-2y) + (-2xy)(-2x) + 0$$

$$= -2x^2\text{y} - 6y^2 + 4x^2y = 2x^2y - 6y^2$$

$$= 2 \times 8 - 6 \times 4 = -8 \text{ units}$$

$$w_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\,(-2y - 3) = -(y + 1.5)$$

$$= -3.5 \text{ units}$$

Problem 5.18. *In a two dimensional flow, determine a possible x component given* $v = 2y^2 + 2x - 2y$. *Assume steady incompressible flow.*

The continuity equation is $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$

$$\therefore \qquad \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} = -[4y - 2] = 2 - 4y.$$

$$\therefore \qquad u = \int (2 - 4y)\,dx = 2x - 4xy + f(y)$$

There are numerous possibilities for $f(y)$.

One possibility is $f(y) = 0$.

$\therefore \quad u = 2x - 4xy.$

Problem 5.19. *The velocity components in a flow are given by $u = 4x$, $v = -4y$. Determine the stream and potential functions. Plot these functions for $\psi = 60, 120, 180,$ and 240 and $\phi = 0, -60, -120, -180, +60, +120, +180$. Check for continuity*

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 4 - 4 = 0 \text{ checks}$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

flow is irrotational and so ϕ exists

$$\psi = \int 4x\,dy + f(x) = 4xy + f(x) \text{ , differentiating w.r.t. } y$$

$$\frac{\partial \psi}{\partial y} = -v = -(-4y) = 4y + f'(x) \quad \therefore \quad f'(x) = \text{constant}$$

$$\psi = 4xy + \text{constant or } \psi = 4xy$$

$$\phi = \int -4x\,dx + f(y) = -2y^2 + f(y),$$

$$\frac{\partial \psi}{\partial y} = -v = 4y = f'(y) \quad \therefore \quad f(y) = 2y^2$$

$$\phi = 2y^2 - 2x^2 + c \quad \text{or} \quad \phi = 2y^2 - 2x^2$$

To plot the stream function, the values of y are calculated for various values of x, using $\psi = 4xy$ or $y = \psi/4x$. The calculated values of y for $x = 1$ to 15 and $\psi = 60$ to 240 are tabulated below.

x / ψ	1	2	3	4	5	10	15
60.000	15.00	7.50	5.00	3.75	3.00	1.50	1.00
120.00	30.00	15.00	10.00	7.50	6.00	3.00	2.00
180.00	45.00	22.50	15.00	11.25	9.00	4.50	3.00
240.00	60.00	30.00	20.00	15.00	12.00	6.00	4.00

These values are shown plotted in Fig P. 5.19

To plot the potential function the values of x or y are calculated for given values y or x using

$$x = \pm\sqrt{y^2 - (\phi/2)} \quad \text{or} \quad y = \pm\sqrt{x^2 + (\phi/2)}$$

The values of y so calculated are tabulated below.

The range used is $x \to 1$ to 15, $\phi \to 0, -60, -120, -180$.

x / ϕ	1	2	3	4	5	10	15
– 60.0	5.57	5.83	6.25	6.75	7.42	11.4	15.97
– 120.0	7.81	8.00	8.31	8.72	9.22	12.65	16.88
– 180.0	9.54	9.70	9.95	10.30	10.72	13.78	17.75

y / ϕ	0	1	2	3	4	5	10	15
60.00	5.48	5.57	5.83	6.25	6.75	7.42	11.4	15.97
120.00	7.75	7.81	8.00	8.31	8.72	9.22	12.65	16.88
180.00	9.49	9.54	9.70	9.95	10.30	10.72	13.78	17.75

(when values of x or y exceeded 15, then the corresponding values of y or x for exact values of 15 is calculated and used)

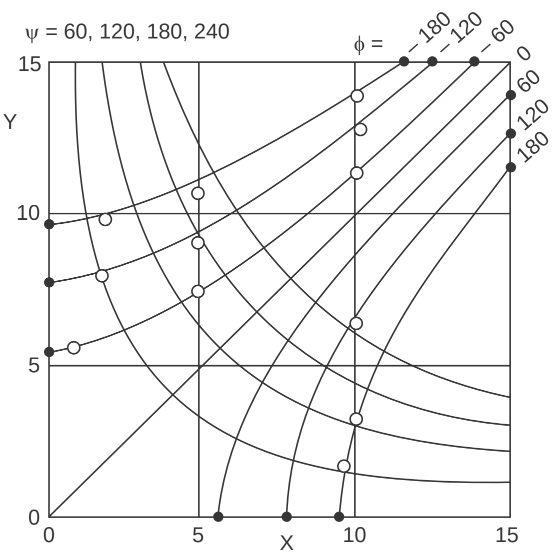

Figure P. 5.19 Flux plot for plot of potential and stream lines

OBJECTIVE QUESTIONS

O Q. 5.1. Fill in the blanks:

1. Ideal fluid is defined as ________________.
2. Real fluids exhibit ________________.
3. Steady flow is defined as flow where the flow parameters ________________.
4. Incompressible flow is defined as flow when ________________ does not vary ________________.
5. Under unsteady flow conditions the flow parameters vary with ________________.
6. Hydrodynamics deals with ________________.
7. Irreversibility in flow is due to ________________.
8. The laws of thermodynamics apply to ________________ flow.
9. The various laws applicable for steady incompressible flow are ________________.
10. In turbulent flow the velocity at a point ________________.
11. In laminar flow momentum transfer takes place at ________________ level.

Answers

1. fluid with zero viscosity 2. viscosity 3. does not vary with time 4. density, with location 5. time 6. ideal fluid flow providing mathematical model for such flow 7. Frictional effects 8. compressible 9. Law of conservation of mass, Newtons laws of motion, Law of conservation of energy 10. varies with time about a mean velocity 11. molecular/microscopic level.

O Q. 5.2. Fill in the blanks:

1. Stream line is defined as the line along which ________________ at any point is ________________ to the line.
2. Path line is defined as ________________.
3. Streak line is defined as ________________.
4. Irrotational flow is defined as ________________.
5. Circulation is defined as ________________.
6. Vorticity is defined as ________________.
7. Stream function is defined by ________________
8. Potential flow function is defined by ________________.
9. Potential flow function exists only if the flow is ________________.
10. Continuity equation is derived using the law of ________________.
11. A doublet is defined as a combination of ________________.
12. The equation for a free vortex is ________________.
13. The slope for stream line is ________________.
14. The slope for velocity potential line is ________________.

Answers

1. the velocity vector is, tangent. 2. line described over time by a particle which has passed through a given point. 3. the line showing the location of various particles that passed through a specified point. 4. there is no net rotation of the fluid particles along the flow – equal deformation along the axes as the flow proceeds. 5. the line integral over a closed path, the product of differential length on the path and the velocity component along the length. 6. circulation per unit area. 7. a function describing the flow field in terms of velocities at various locations – a

function describing the stream lines for the flow field ($\partial\psi/dy = u$, $\partial\psi/\partial x = -v$) 8. a function which describes the flow field potential – a function describing the equipotential lines ($\partial\phi/\partial x = -u$, $\partial\phi/\partial y = -v$). 9. irrotational. 10. law of conservation of mass. 11. a combination of a source and sink of equal strength. 12. $u_\theta r$ = constant 13. v/u 14. $-(u/v)$

O Q. 5.3. Fill in the blanks

1. Rectilinear flow is defined as ______________.
2. A source is defined as ______________.
3. A sink is defined as ______________.
4. The stream function for rectilinear flow is ______________.
5. The stream function for source/sink is ______________.
6. The stream lines and equipotential lines for a flow field are ______________.
7. The x and y directional velocities in a flow is specified by the stream function by ______________.
8. The x and y directional velocities in a flow field is given by the potential function as ______________.
9. The condition to satisfied by irrotational flow is ______________.
10. The stream function for a combination of flows with ψ_A and ψ_B is ______________.

Answers

1. a flow having stream lines parallel to one of the axes axis 2. flow with radial stream lines, directed outwards 3. flow with radial stream lines directed towards the centre 4. $\psi = cy$ where c is a constant equal to the velocity 5. $\psi = q\ \theta/2\pi$, q = total flow, θ = angle (in polar co-ordinate), $\psi = -q\ \theta/2\pi$ 6. perpendicular to each other. 7. $u = \partial\psi/\partial y$ $v = -\partial\psi/\partial y$ (8) $u = -\partial\phi/\partial x$, $= -\partial\phi/\partial y$ 9. $\partial v/\partial x = \partial u/\partial y$ 10. $\psi = \psi_A + \psi_B$.

O Q. 5.4. Indicate whether the statements are correct or incorrect.

1. An ideal fluid flow is a good approximation for real fluid flow if viscosity is small.
2. Compressible flow is flow of gases.
3. Turbulent flow is unsteady flow.
4. A stream line shows the path of a particle in any flow.
5. For every stream function a potential function should exist.
6. For every potential function a stream function should exist.
7. Stream function can exist only for irrotational flow.
8. Potential function can exist only for irrotational flow.
9. Circulation will be zero for irrotational flow.
10. Free vertex flow is irrotational.
11. Forced vertex flow is irrotational.

Answers

Correct: 1, 2, 6, 8, 9, 10 Incorrect : 3, 4, 5, 7, 11

O Q. 5.5. Choose the correct answer:

1. A flow is defined by $u = 2(1 + t)$, $v = 3(1 + t)$ where t is the time. The velocyity at $t = 2$ is
   (*a*) 6 (*b*) 9 (*c*) 10.82 (*d*) 6.7.
2. The value of local acceleration in the x direction for flow with $u = 2(1 + t)$ is given by
   (*a*) 0 (*b*) $2t$ (*c*) 2 (*d*) t.
3. The value of x directional convective acceleration in the case of flow with $u = 2(1 + t)$ and $v = 3(1 + t)$ equals
   (*a*) 0 (*b*) 5 (*c*) 1 (*d*) $5t$.

4. When $u = 3 + 2xy + 4t^2$, $v = xy^2 + 3t$. The x directional acceleration is given by
   (a) $2x^2y^2 + 4xy^2 + 6y + 8t^2y + 6tx$ (b) $4xy$
   (c) x/y (d) $2x^2y^2 + 6tx$.
5. The continuity equation is satisfied by
   (a) $u = A \sin xy$, $v = -A \sin xy$ (b) $u = x + y$. $v = x - y$
   (c) $u = 2x^2 + cy$, $v = 3y^2$ (d) $u = x + 2y$, $v = 2x + y$.
6. The following represent steady incompressible flow.
   (a) $u = 4xy + 2y^2$, $v = 6xy + 3x^2$ (b) $u = x^2 + y^2$, $v = -2xy + 7$
   (c) $u = x/y$, $v = y/x$ (d) $u = 2x + y$, $v = 4y + x$.
7. If $\psi = 3x^2y - y^3$. The values u and v are
   (a) $6xy$, $3x^2 - 3y^2$ (b) $3x^2 - 3y^2$, $6xy$
   (c) $(3x^2 - 3y^2)$, $-6xy$ (d) $3y^2 - 3x^2$, $6xy$.
8. This is a valid potential function
   (a) $\phi = c \ln x$ (b) $\phi = c \cos x$
   (c) $\phi = 3xy$ (d) $\phi = c\,(x^2 + y^2)$.
9. The continuity equation for incompressible two dimensional steady flow is
   (a) $\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} = 0$, (b) $\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial t} = 0$,
   (c) $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$, (d) $\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial t} = 0$.
10. The velocity components in the x and y directions in terms of stream function ψ is given by
   (a) $u = \dfrac{\partial \psi}{\partial x}$, $v = \dfrac{\partial \psi}{\partial y}$ (b) $u = \dfrac{\partial \psi}{\partial y}$, $v = -\dfrac{\partial \psi}{\partial x}$
   (c) $u = -\dfrac{\partial \psi}{\partial y}$, $v = \dfrac{\partial \psi}{\partial x}$ (d) $u = -\dfrac{\partial \psi}{\partial x}$, $v = \dfrac{\partial \psi}{\partial y}$.

Answers

1. c, 2. c, 3. a, 4. a, 5. b, 6. b, 7. c, 8. c, 9. c, 10. b.

O Q. 5.6. Choose the correct answer

1. The velocity components in the x and y directions in terms of potential function ϕ is given by
   (a) $u = -\dfrac{\partial \phi}{\partial y}$, $v = \dfrac{\partial \phi}{\partial x}$ (b) $u = \dfrac{\partial \phi}{\partial y}$, $v = -\dfrac{\partial \phi}{\partial x}$
   (c) $u = \dfrac{\partial \phi}{\partial x}$, $v = \dfrac{\partial \phi}{\partial y}$ (d) $u = -\dfrac{\partial \phi}{\partial x}$, $v = -\dfrac{\partial \phi}{\partial y}$.
2. The condition for irrotational flow is
   (a) $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ (b) $\dfrac{\partial u}{\partial x} = -\dfrac{\partial v}{\partial y}$
   (c) $\dfrac{\partial v}{\partial x} = \dfrac{\partial u}{\partial y}$ (d) $\dfrac{\partial v}{\partial x} = -\dfrac{\partial u}{\partial y}$.

3. The equation of a stream line in two dimensional steady flow can be expressed as

(*a*) $\frac{u}{dx} = \frac{dy}{v}$ (*b*) $\frac{dy}{u} = \frac{dx}{v}$

(*c*) $-\frac{dy}{u} = \frac{dx}{v}$ (*d*) $-\frac{dx}{u} = \frac{dy}{v}$.

4. The flow rate between stream lines with values ψ_1 and ψ_2 is given by

(*a*) $\psi_1 + \psi_2$ (*b*) $\psi_1 + C\psi_2$

(*c*) $\psi_2 - \psi_1$ (*d*) $C\,\psi_1 + \psi_2$.

5. The continuity equation is the result of application of the following law to the flow field

(*a*) First law of thermodynamics (*b*) Conservation of energy

(*c*) Newtons second law of motion (*d*) Conservation of mass.

6. A path line describes

(*a*) The velocity direction at all points on the line

(*b*) The path followed by particles in a flow

(*c*) The path over a period of times of a single particle that has passed out at a point

(*d*) The instantaneous position of all particles that have passed a point.

7. The relationship between stream and potential functions ψ and ϕ is

(*a*) $\frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}$ (*b*) $\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y}$

(*c*) $\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$ (*d*) $\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \psi}{\partial y^2}$.

8. The value that will satisfy potential function ϕ is

(*a*) $\phi = x^2 + y^2$ (*b*) $\phi = \sin x$

(*c*) $\phi = x + y$ (*d*) $\phi = \ln(x + y)$.

9. The stream function is

(*a*) constant along an equipotential line (*b*) along a stream line

(*c*) defined only in irrotational flow (*d*) defined only for incompressible flow.

10. A potential function

(*a*) is constant along a stream line

(*b*) is definable if a stream function is available for the flow

(*c*) describes the flow if it is rotational

(*d*) describes the flow if it is irrotational.

Answers

1. *d*, 2. *c*, 3. *d*, 4. *c*, 5. *d*, 6. *c*, 7. *c*, 8. *c*, 9. *b*, 10. *d*.

O Q. 5.7. Match the pairs

1.

Set *A*	Set *B*
A. Ideal fluid	1. Irreversible flow
B. Steady flow	2. Incompresible flow
C. Low velocity gas flow	3. Zero viscosity
D. Friction	4. Velocity at a point is constant.

Answers

$A - 3, B - 4, C - 2, D - 1.$

2.

Set A	Set B
A. Stream line	1. Path of particles that passed a point
B. Streak line	2. The path of a single particle that passed a point
C. Equipotential line	3. Shows velocity
D. Path line	4. Described by potential function.

Answers

$A - 3, B - 1, C - 4, D - 2.$

EXERCISE PROBLEMS

E 1. Given $u = kx$ in a two dimensional flow determine v. **[$-ky + f(x)$]**

E 2. Given velocity potential, determine the velocity components u and v.

(*a*) $\phi = \ln xy$ **[– (1/x), (1/y)]**

(*b*) $\phi = 3(x^2 + y^2)$ **[– 6x , – 6y)]**

(*c*) $\phi = a \cos xy$ **[aysinxy, axsinxy]**

E 3. Given stream function $\psi = 3x - 4y$, calculate the slope of the line and also the value of resultant velocity. Does it satisfy continuity equation? Is the flow irrotational? **[θ = 37°, v = 5 units]**

E 4. Given $\phi = x(2y - 1)$, determine ψ. **[$-y^2 + y + f(y)$]**

E 5. Given $\psi = 4x^2 - 4y^2$, find ϕ. **[$\phi = 8xy + c$]**

E 6. Find the relationship between a and b if in steady flow $u = bx$ and $v = ay$, **[b = – a]**

E 7. Show that for two dimensional steady flows with velocity components u and v,

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \quad \text{and} \quad a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}.$$

E 8. Given $u = 2y$, $v = x$, sketch the flow. Also find a_x, a_y and a.

E 9. If $u = 3 + 2xy$ and $v = xy^2$ determine a_x, a_y and a.

E 10. If $u = 3y$ and $v = 2$, determine a_x and a_y.

E 11. If $u = 0$, $v = 3xy$, determine a_x and a_y.

E 12. If $u = -2y$, $v = 3xy$, determine a_x and a_y.

E 13. Determine the normal and tangential components of acceleration for a circular stream line.

E 14. Find ϕ and ψ given $u = 2x$ and $v = -2y$. [$\psi = 2xy + c_1$, $\phi = -(x^2 - y^2) + c^2$]

E 15. If $u = 2$, $v = 8x$ determine ψ.

E 16. Tabulate the values of x and y for $\psi = 0, 1, 2, 3$ given (*i*) $\psi = 10y$.

(*ii*) $\psi = -20x$, (*iii*) $\psi = 10y - 20x$.

E 17. Determine the stream function if it exists. Also check for irrotationality.

(*i*) $u = 5$, $v = 6$ (*ii*) $u = 3 + x$, $v = 4$

(*iii*) $u = 3xy$, $v = 1.5x^2$ (*iv*) $u = 3x$, $v = 3y$

(*v*) $u = 4 + 2x$, $v = -y$.

E 18. Calculate the values of x and y for stream lines. $\psi = 0, 1, 2, 3$ and 4, given $\psi = 1.2xy$ (for one quadrant).

E 19. Tabulate and plot $\psi = 1.5x^2 + y^2$ for positive values of x, y.

E 20. If $\psi = x^2 - y$, find u and v and also the vorticity.

E 21. A source discharging 1 m^3/sm is at (– 1, 0) and a sink taking in $1m^3/sm$ is at (+1, 0). If this is combined with uniform flow of $u = 1.5$ m/s, left to right, calculate the length of the resolution of closed body contour.

E 22. Two sources one of strength 8π and the other of 16π are located at (2, 0) and (– 3, 0) respectively. For the combined flow field, calculate the location of the stagnation point. Also plot $\psi = 4\pi$, $\psi = 8\pi$ and $\psi = 0$.

E 23. A source of $20m^3/sm$ at (0, 0) is combined with a uniform flow with $u = 3m/s$ from left to right. Determine ψ for the flow.

E 24. Given $u = x^2 + 2x - y$ and $v = -2xy - 2y$, determine ψ. Also compute vorticity.