6 Bernoulli Equation and Applications

6.0 INTRODUCTION

In chapter five flow of ideal fluids was discussed. The main idea was the study of flow pattern. The determination of equal flow paths and equal potential lines was discussed. No attempt was made to determine the numerical value of these quantities.

In this chapter the method of determination of the various energy levels at different locations in the flow is discussed. In this process first the various forms of energy in the fluid are identified. Applying the law of conservation of energy the velocity, pressure and potential at various locations in the flow are calculated. Initially the study is limited to ideal flow. However the modifications required to apply the analysis to real fluid flows are identified.

The material discussed in this chapter are applicable to many real life fluid flow problems. The laws presented are the basis for the design of fluid flow systems.

Energy consideration in fluid flow:

Consider a small element of fluid in flow field. The energy in the element as it moves in the flow field is conserved. This principle of conservation of energy is used in the determination of flow parameters like pressure, velocity and potential energy at various locations in a flow. The concept is used in the analysis of flow of ideal as well as real fluids.

Energy can neither be created nor destroyed. It is possible that one form of energy is converted to another form. The total energy of a fluid element is thus conserved under usual flow conditions.

If a stream line is considered, it can be stated that the total energy of a fluid element at any location on the stream line has the same magnitude.

6.1 FORMS OF ENERGY ENCOUNTERED IN FLUID FLOW

Energy associated with a fluid element may exist in several forms. These are listed here and the method of calculation of their numerical values is also indicated.

6.1.1 Kinetic Energy

This is the energy due to the motion of the element as a whole. If the velocity is V, then the kinetic energy for m kg is given by

$$KE = \frac{mV^2}{2g_o} \text{ Nm} \tag{6.1.1}$$

The unit in the SI system will be Nm also called Joule (J)

{(kg m^2/s^2)/(kg m/N s^2)}

The same referred to one kg (specific kinetic energy) can be obtained by dividing 6.1.1 by the mass m and then the unit will be Nm/kg.

$$KE = \frac{V^2}{2g_o}\text{, Nm/kg} \tag{6.1.1b}$$

In fluid flow studies, it is found desirable to express the energy as the head of fluid in m. This unit can be obtained by multiplying equation (6.1.1) by g_o/g.

$$\text{Kinetic head} = \frac{V^2}{2g_o}\frac{g_o}{g} = \frac{V^2}{2g} \tag{6.1.2}$$

The unit for this expression will be $\frac{m^2 s^2}{s^2 m}$ = m

Apparently the unit appears as metre, but in reality it is Nm/N, where the denominator is weight of the fluid in N.

The equation in this form is used at several places particularly in flow of liquids. But the energy associated physically is given directly only be equation 6.1.1.

The learner should be familiar with both forms of the equation and should be able to choose and use the proper equation as the situation demands. **When different forms of the energy of a fluid element is summed up to obtain the total energy, all forms should be in the same unit.**

6.1.2 Potential Energy

This energy is due to the position of the element in the gravitational field. While a zero value for *KE* is possible, the value of potential energy is relative to a chosen datum. The value of potential energy is given by

$$PE = mZ\, g/g_o \text{ Nm} \tag{6.1.3}$$

Where m is the mass of the element in kg, Z is the distance from the datum along the gravitational direction, in m. The unit will be (kg m m/s^2) × (Ns^2/kgm) *i.e.,* Nm. The specific potential energy (per kg) is obtained by dividing equation 6.1.3 by the mass of the element.

$$PE = Z\, g/g_0 \text{ Nm/kg} \tag{6.1.3. b}$$

This gives the physical quantity of energy associated with 1 kg due to the position of the fluid element in the gravitational field above the datum. As in the case of the kinetic energy, the value of *PE* also is expressed as head of fluid, *Z*.

$$PE = Z\,(g/g_o)\,(g_o/g) = Z\ m. \tag{6.1.4}$$

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This form will be used in equations, but as in the case of KE, one should be familiar with both the forms and choose the suitable form as the situation demands.

6.1.3 Pressure Energy (Also Equals Flow Energy)

The element when entering the control volume has to flow against the pressure at that location. The work done can be calculated referring Fig. 6.1.1.

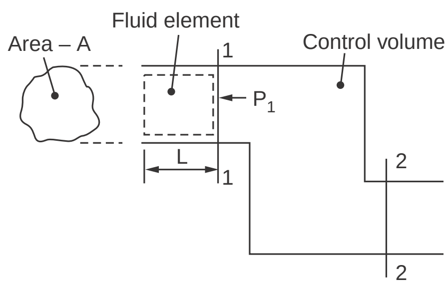

Figure 6.1.1 Flow work calculation

The boundary of the element of fluid considered is shown by the dotted line, Force = $P_1 A$, distance to be moved = L, work done = $P_1 AL = P_1\ mv$ as AL = volume = mass × specific volume, v. $\therefore$ flow work = $P\ mv$.

The pressure energy per kg can be calculated using $m = 1$. The flow energy is given by

$$FE = P.v = P/\rho, \text{ Nm/kg} \quad (6.1.5)$$

Note: $$\frac{N}{m^2}\frac{m^3}{kg} \rightarrow \frac{Nm}{kg}$$

As in the other cases, the flow energy can also expressed as head of fluid.

$$FE = \frac{P}{\rho}\frac{g_o}{g}, \text{ m} \quad (6.1.5a)$$

As specific weight $\gamma = \rho\ g/g_o$, the equation is written as,

$$FE = P/\gamma, \text{ m} \quad (6.1.5b)$$

It is important that in any equation, when energy quantities are summed up consistent forms of these set of equations should be used, that is, all the terms should be expressed either as head of fluid or as energy (J) per kg. These are the three forms of energy encountered more often in flow of incompressible fluids.

6.1.4 Internal Energy

This is due to the thermal condition of the fluid. This form is encountered in compressible fluid flow. For gases (above a datum temperature) $IE = c_v\ T$ where T is the temperature above the datum temperature and c_v is the specific heat of the gas at constant volume. The unit for internal energy is J/kg (Nm/kg). When friction is significant other forms of energy is converted to internal energy both in the case of compressible and incompressible flow.

6.1.5 Electrical and Magnetic Energy

These are not generally met with in the study of flow of fluids. However in magnetic pumps and in magneto hydrodynamic generators where plasma flow in encountered, electrical and magnetic energy should also be taken into account.

6.2 VARIATION IN THE RELATIVE VALUES OF VARIOUS FORMS OF ENERGY DURING FLOW

Under ideal conditions of flow, if one observes the movement of a fluid element along a stream line, the sum of these forms of energy will be found to remain constant. However, there may be an increase or decrease of one form of energy while the energy in the other forms will decrease or increase by the same amount. For example when the level of the fluid decreases, it is possible that the kinetic energy increases. When a liquid from a tank flows through a tap this is what happens. In a diffuser, the velocity of fluid will decrease but the pressure will increase. In a venturimeter, the pressure at the minimum area of cross section (throat) will be the lowest while the velocity at this section will be the highest.

The total energy of the element will however remain constant. In case friction is present, a part of the energy will be converted to internal energy which should cause an increase in temperature. But the fraction is usually small and the resulting temperature change will be so small that it will be difficult for measurement. From the measurement of the other forms, it will be possible to estimate the frictional loss by difference.

6.3 EULER'S EQUATION OF MOTION FOR FLOW ALONG A STREAM LINE

Consider a small element along the stream line, the direction being designated as s.

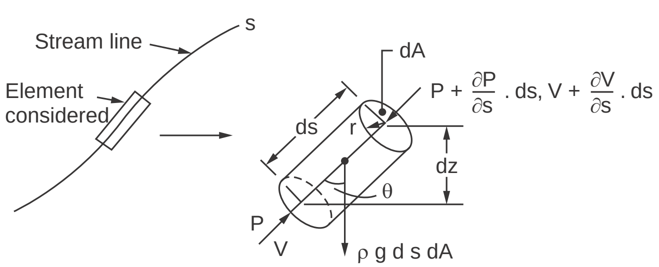

Figure 6.3.1 Euler's equation of Motion – Derivation

The net force on the element are the body forces and surface forces (pressure). These are indicated in the figure. Summing this up, and equating to the change in momentum.

$$PdA - \{P + (\partial P/\partial s\} dA - \rho g \, dA \, ds \cos\theta = \rho \, dA \, ds \, a_s \qquad (6.3.1)$$

where a_s is the acceleration along the s direction. This reduces to,

$$\frac{1}{\rho}\frac{\partial P}{\partial s} + g\cos\theta + a_s = 0 \qquad (6.3.2)$$

(**Note**: It will be desirable to add g_o to the first term for dimensional homogeneity. As it is, the first term will have a unit of N/kg while the other two terms will have a unit of m/s^2. Multiplying by g_o, it will also have a unit of m/s^2).

$$a_s = dV/dt, \text{ as velocity, } V = f(s, t), (t = \text{time}).$$

$$dV = \frac{\partial V}{\partial s} ds + \frac{\partial V}{\partial t} dt \quad \text{dividing by } dt,$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial t} \qquad \text{As } \frac{ds}{dt} = V,$$

and as $\cos\theta = dz/ds$, equation 6.3.2 reduces to,

$$\frac{1}{\rho}\frac{\partial P}{\partial s} + g\frac{\partial z}{\partial s} + V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} = 0 \qquad (6.3.2.\ a)$$

For steady flow $\partial V/\partial t = 0$. Cancelling ∂s and using total derivatives in place of partials as these are independent quantities.

$$\frac{dp}{\rho} + gdz + VdV = 0 \qquad (6.3.3)$$

(**Note**: in equation 6.3.3 also it is better to write the first term as $g_o.dp/\rho$ for dimensional homogeneity).

This equation after dividing by g, is also written as,

$$\frac{dp}{\gamma} + d\left(\frac{V^2}{2g}\right) + dz = 0 \quad \text{or} \quad d\left[\frac{P}{\gamma} + \frac{V^2}{2g} + z\right] = 0 \qquad (6.3.4)$$

which means that the quantity within the bracket remains constant along the flow.

This equation is known as Euler's equation of motion. The assumptions involved are:

1. Steady flow
2. Motion along a stream line and
3. Ideal fluid (frictionless)

In the case on incompressible flow, this equation can be integrated to obtain Bernoulli equation.

6.4 BERNOULLI EQUATION FOR FLUID FLOW

Euler's equation as given in 6.3.3 can be integrated directly if the flow is assumed to be incompressible.

$$\frac{dP}{\rho} + gdz + VdV = 0, \quad \text{as } \rho = \text{constant}$$

$$\frac{P}{\rho} + gz + \frac{V^2}{2} = \text{const. or } \frac{P}{\rho} + z\left(\frac{g}{g_0}\right) + \frac{V^2}{2g_0} = \text{Constant} \qquad (6.4.1)$$

The constant is to be evaluated by using specified boundary conditions. The unit of the terms will be energy unit (Nm/kg).

In SI units the numerical value of $g_o = 1$, kg m/N s^2. Equation 6.4.1 can also be written as to express energy as head of fluid column.

$$\frac{P}{\gamma} + z + \frac{V^2}{2g} = \text{constant} \qquad (6.4.2)$$

(γ is the specific weight N/m^3). In this equation all the terms are in the unit of head of the fluid.

The constant has the same value along a stream line or a stream tube. The first term represents (flow work) pressure energy, the second term the potential energy and the third term the kinetic energy.

This equation is extensively used in practical design to estimate pressure/velocity in flow through ducts, venturimeter, nozzle meter, orifice meter etc. In case energy is added or taken out at any point in the flow, or loss of head due to friction occurs, the equations will read as,

$$\frac{P_1}{\rho} + \frac{V_1{}^2}{2g_o} + \frac{z_1 g}{g_o} + W - \frac{h_f g}{g_o} = \frac{P_2}{\rho} + \frac{V_2{}^2}{2g_o} + \frac{z_2 g}{g_o} \qquad (6.4.3)$$

where W is the energy added and h_f is the loss of head due to friction.

In calculations using SI system of units g_o may be omitting as its value is unity.

Example 6.1 *A liquid of specific gravity 1.3 flows in a pipe at a rate of 800 l/s, from point 1 to point 2 which is 1 m above point 1. The diameters at section 1 and 2 are 0.6 m and 0.3 m respectively. If the pressure at section 1 is 10 bar,* ***determine the pressure at section 2.***

Using Bernoulli equation in the following form (6.4.2)

$$\frac{P}{\gamma} + z + \frac{V^2}{2g} = \text{constant},$$

Taking the datum as section 1, the pressure P_2 can be calculated.

$V_1 = 0.8 \times 4/\pi \times 0.6^2 = 2.83$ *m/s*, $V_2 = 0.8 \times 4/\pi \times 0.3^3 = 11.32$ *m/s*

$P_1 = 10 \times 10^5$ N/m^2, γ = sp. gravity × 9810. Substituting.

$$\frac{10 \times 10^5}{9810 \times 1.3} + 0 + \frac{2.83^2}{2 \times 9.81} = \frac{\text{P}_2}{9810 \times 1.3} + 1 + \frac{11.32^2}{2 \times 9.81}$$

Solving, $\mathbf{P_2 = 9.092}$ **bar** (9.092×10^5 N/m^2).

As P/γ is involved directly on both sides, gauge pressure or absolute pressure can be used without error. However, it is desirable to use absolute pressure to avoid negative pressure values (or use of the term vacuum pressure).

Example 6.2 *Water flows through a horizontal venturimeter with diameters of 0.6 m and 0.2 m. The guage pressure at the entry is 1 bar.* ***Determine the flow rate*** *when the throat pressure is 0.5 bar (vacuum). Barometric pressure is 1 bar.*

Using Bernoulli's equation in the form,

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

and noting $Z_1 = Z_2$, $P_1 = 2 \times 10^5$ N/m^2 (absolute)

$P_2 = 0.5 \times 10^5$ N/m^2 (absolute), $\gamma = 9810$ N/m^3

$V_1 = Q \times 4/(\pi \times 0.60^2) = 3.54\,Q$, $V_2 = Q \times 4/(\pi \times 0.20^2) = 31.83Q$

$$\frac{2 \times 10^5}{9810} + 0 + \frac{3.54^2}{2 \times 9.81} Q^2 = \frac{0.5 \times 10^5}{9810} + 0 + \frac{31.83^2\, Q^2}{2 \times 9.81}$$

Solving, $\mathbf{Q = 0.548}$ **m³/s**, $V_1 = 1.94$ m/s, $V_2 = 17.43$ m/s.

Example 6.3 *A tap discharges water evenly in a jet at a velocity of 2.6 m/s at the tap outlet, the diameter of the jet at this point being 15 mm. The jet flows down vertically in a smooth stream.* ***Determine the velocity and the diameter*** *of the jet at 0.6 m below the tap outlet.*

The pressure around the jet is atmospheric throughout. Taking the tap outlet as point 1 and also taking it as the datum using Bernoulli equation.

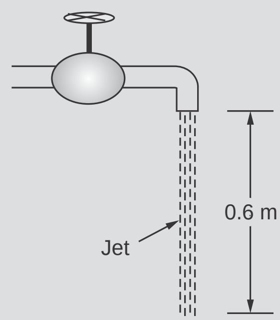

Figure Ex. 6.3 Problem model

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g},$$

$P_1 = P_2$, $Z_2 = 0$,

$Z_2 = -\,0.6$ m, $V_1 = 2.6$ m/s

$$\therefore \quad \frac{2.6^2}{2 \times 9.81} = -\,0.6 + \frac{V_2^2}{2 \times 9.81}$$

$\therefore$ $\mathbf{V_2 = 4.3}$ **m/s.**

using continuity equation (one dimensional flow) and noting that density is constant.

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi \times 015^2}{4} \times 2.6 = \frac{\pi \times D^2}{4} \times 4.3, \quad \therefore \quad \mathbf{D = 0.01166\ m\ or\ 11.66\ mm}$$

As the potential energy decreases, kinetic energy increases. As the velocity is higher the flow area is smaller.

Entrainment of air may increase the diameter somewhat.

Example 6.4 *Water flows in a tapering pipe vertically as shown in Fig. Ex.6.4.* ***Determine the manometer reading "h"****. The manometer fluid has a specific gravity of 13.6. The flow rate is 100 l/s*

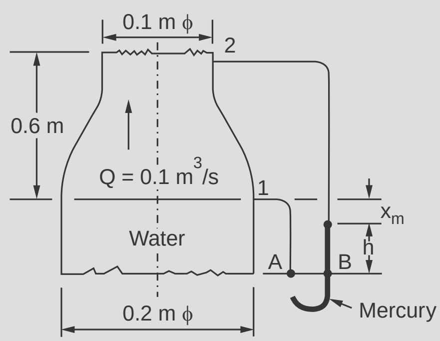

Figure Ex. 6.4 Problem model

The velocities at sections 1 and 2 are first calculated.

$$V_1 = 4 \times 0.1/(\pi \times 0.2^2) = 3.183 \text{ m/s},$$

$$V_2 = 4 \times 0.1/(\pi \times 0.1^2) = 12.732 \text{ m/s}$$

It is desired to determine $P_1 - P_2$. Rearranging Bernoulli equation for this flow,

$$\frac{P_1 - P_2}{\gamma} = 0.6 + (12.732^2 - 3.183^2)/(2 \times 9.81) = 8.346 \text{ m of water}$$

For water γ = 9810 N/m^3. For the manometer configuration, considering the level AB and equating the pressures at A and B

$$\frac{P_1}{\gamma} + x + h = \frac{P_2}{\gamma} + 0.6 + x + sh$$

(where x, h are shown on the diagram and s is specific gravity)

$$\therefore \quad \frac{P_1 - P_2}{\gamma} = 0.6 + h(s - 1), \text{ substituting the values,}$$

$$8.346 = 0.6 + h(13.6 - 1)$$

$\therefore$ **h = 0.6148 m or 61.48 cm**

6.5 ENERGY LINE AND HYDRAULIC GRADIENT LINE

The total energy plotted along the flow to some specified scale gives the energy line. When losses (frictional) are negligible, the energy line will be horizontal or parallel to the flow direction. For calculating the total energy kinetic, potential and flow (pressure) energy are considered.

Energy line is the plot of $\frac{P}{\gamma} + Z + \frac{V^2}{2g}$ along the flow. It is constant along the flow when losses are negligible.

The plot of $\frac{P}{\gamma} + Z$ along the flow is called the hydraulic gradient line. When velocity increases this will dip and when velocity decreases this will rise. An example of plot of these lines for flow from a tank through a venturimeter is shown in Fig. 6.5.1.

The hydraulic gradient line provides useful information about pressure variations (static head) in a flow. The difference between the energy line and hydraulic gradient line gives the value of dynamic head (velocity head).

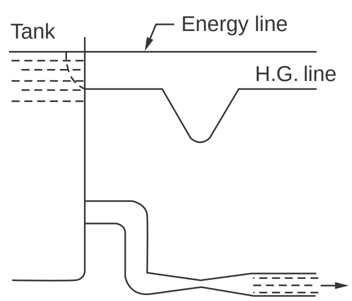

Figure 6.5.1 Energy and hydraulic gradient lines

6.6 VOLUME FLOW THROUGH A VENTURIMETER

Example 6.5 *Under ideal conditions show that the volume flow through a venturimeter is given by*

$$Q = \frac{A_2}{\{1-(A_2/A_1)^2\}^{0.5}}\left[2g\left(\frac{P_1-P_2}{\gamma}+(Z_1-Z_2)\right)\right]^{0.5}$$

where suffix 1 and 2 refer to the inlet and the throat.

Refer to Fig. Ex. 6.5

Volume flow $= A_1 V_1 = A_2 V_2$

$$\therefore \quad V_1 = \frac{A_2}{A_1} V_2,\ V_1^2 = \left(\frac{A_2}{A_1}\right)^2 . V_2^2,$$

$$\therefore \quad (V_2^2 - V_1^2) = V_2^2\left[1-\left(\frac{A_2}{A_1}\right)^2\right]$$

Applying Bernoulli equation to the flow and considering section 1 and 2,

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

Rearranging,

$$\left[2g\left\{\frac{P_1-P_2}{\gamma}+(Z_1-Z_2)\right\}\right]^{0.5} = V_2\left[1-\left(\frac{A_2}{A_1}\right)^2\right]^{0.5}$$

$$V_2 = \frac{1}{[1-(A_2/A_1)^2]^{0.5}}\left[2g\left\{\frac{P_1-P_2}{\gamma}+(Z_1-Z_2)\right\}\right]^{0.5}$$

$\therefore$ Volume flow is

$$A_2V_2 = \frac{A_2}{[1-(A_2/A_1)^2]^{0.5}}\left[2g\left\{\frac{P_1-P_2}{\gamma}+(Z_1-Z_2)\right\}\right]^{0.5} \qquad (6.6.1)$$

This is a general expression and can be used irrespective of the flow direction, inclination from horizontal or vertical position. This equation is applicable for orifice meters and nozzle flow meters also.

In numerical work consistent units should be used.

Pressure should be in N/m^2, Z in m, A in m^2 and then volume flow will be m^3/s.

A coeficient is involved in actual meters due to friction.

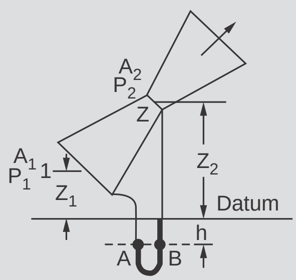

Figure Ex. 6.5 Venturimeter-flow

Example 6.6 *Show that when a manometric fluid of specific gravity S_2 is used to measure the head in a venturimeter with flow of fluid of specific gravity S_1, if the manometer shows a reading of hm, the volume flow is given by*

$$Q = \frac{A_2}{\left[1 - (A_2/A_1)^2\right]^{0.5}} \left[2gh\left(\frac{S_2}{S_1} - 1\right)\right]^{0.5}$$

Comparing the equation (6.6.1) with the problem at hand, it is seen that it is sufficient to prove,

$$h\left(\frac{S_2}{S_1} - 1\right) = \frac{P_1 - P_2}{\gamma_1} + (Z_2 - Z_1)$$

Considering the plane A–B in the manometer and equating the pressures at A and B Fig. Ex. 6.5 : The manometer connection at the wall measures the static pressure only)

$$P_1 + Z_1\,\gamma_1 + h\gamma_1 = P_2 + Z_2\gamma_1 + h\gamma_2$$

$$(P_1 - P_2) + (Z_1 - Z_1)\,\gamma_1 = h(\gamma_2 - \gamma_1), \text{ dividing by } \gamma_1,$$

$$\frac{P_1 - P_2}{\gamma_1} + (Z_1 - Z_2) = h\left(\frac{\gamma_2}{\gamma_1} - 1\right) = h\left(\frac{S_2}{S_1} - 1\right)$$

Hence volume flow,

$$Q = \frac{A_2}{[1 - (A_2/A_1)^2]^{0.5}} \left[2gh\left(\frac{S_2}{S_1} - 1\right)\right]^{0.5} \qquad (6.6.2)$$

This equation leads to another conclusion. The fluid head, H, causing the flow is equal to the manometer reading $h[(S_2/S_1) - 1]$ and flow is independent of the inclination if the reading of the manometer and the fluids are specified.

i.e., As the manometer reading converted to head of flowing fluid, $H = h[(S_2/S_1) - 1]$

$$Q = \frac{A_2}{[1 - (A_2/A_1)^2]^{0.5}} [2gH]^{0.5} \qquad (6.6.3)$$

If the pressure at various locations are specified, these equations are applicable for orifice and nozzle meters also.

Example. 6.7 ***Determine the flow rate*** *through the siphon Fig. Ex. 6.7 when flow is established. Also determine the pressure at A.*

The pressure at C and B are atmospheric. Considering locations C and B and taking the datum at B, applying Bernoulli equation, noting that the velocity at water surface at $C = 0$.

$$0 + 0 + V_B^2/2g = 3 + 0 + 0$$

$$\therefore \quad V_B = 7.672 \text{ m/s.}$$

$$\therefore \quad \textbf{Flow rate} = (\pi D^2/4) \times V$$

$$= (\pi \times 0.1^2/4) \times 7.672$$

$$= \mathbf{0.06\ m^3/s}$$

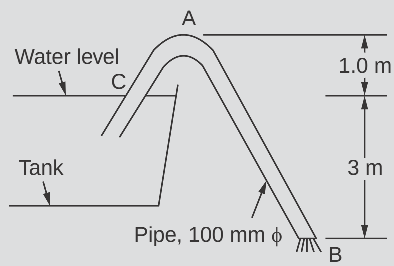

Figure Ex. 6.7 Problem model

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The velocity at A is the same as velocity at B. Now considering locations C and A,

$$3 + 0 + 0 = 4 + (P_A/\gamma) + 7.672^2/ (2 \times 9.81)$$

$\therefore$ $P_A/\gamma = -4$m or -4m of water head or 4m water-head below atmospheric pressure.

Check: Consider points A and B

$$4 + \frac{P_A}{\gamma} + \frac{V_A^2}{2g} = \frac{V_B^2}{2g} + 0 + 0 \quad \text{as } V_A = V_B, \quad \frac{\mathbf{P_A}}{\gamma} = \mathbf{-4\ m} \text{ checks.}$$

Example. 6.8 *Water flows in at a rate of 80 l/s from the pipe as shown in Fig. Ex. 6.8 and flows outwards through the space between the top and bottom plates. The top plate is fixed.* ***Determine the net force acting on the bottom plate****. Assume the pressure at radius r = 0.05 m is atmospheric.*

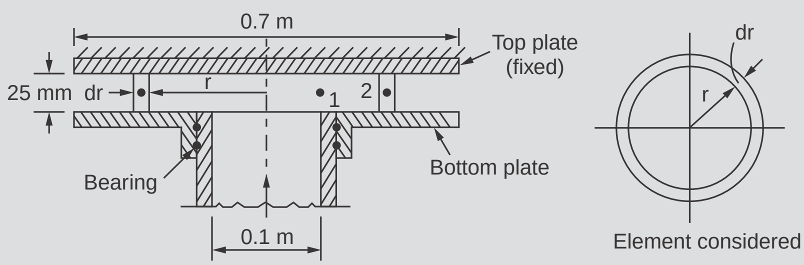

Figure Ex. 6.8 Problem model

Consider an element area of width dr (annular) in the flow region at a distance r as shown in figure. The pressure at this location as compared to point 1 can be determined using Bernoulli equation.

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}, \; P_1 \text{ is atmospheric}$$

As $Z_1 = Z_2$, $$P_2 - P_1 = \frac{\gamma}{2g}(V_1^2 - V_2^2)$$

$$V_1^2 = (0.08/2\pi \times 0.05 \times 0.025)^2 = 103.75$$

$$V_2^2 = (0.08/2\pi \times 0.025 \times r)^2 = 0.2594/r^2$$

$(P_2 - P_1)$ is the pressure difference which causes a force at the area $2\pi r dr$ at r.

The force on the element area of the bottom plate = $2\pi r dr\,(P_2 - P_1)$

Substituting and nothing $\gamma = \rho g/g_0$, the elemental force dF is given by,

$$dF = \rho\, \pi r dr \left[103.75 - \frac{0.2594}{r^2}\right],$$

Integrating between the limits $r = 0.05$ to 0.35,

$$\textbf{Net force} = 1000 \times \pi \left[(103.75\,(0.35^2 - 0.05^2)/2) - \left(0.2594 \ln \frac{0.35}{0.05}\right)\right] = \mathbf{17970\ \ N}$$

6.7 EULER AND BERNOULLI EQUATION FOR FLOW WITH FRICTION

Compared to ideal flow the additional force that will be involved will be the shear force acting on the surface of the element. Let the shear stress be τ, the force will equal $\tau\, 2\pi r\, ds$ (where r is the radius of the element, and $A = \pi r^2$)

Refer **Para 6.3** and **Fig. 6.3.1.** The Euler equation 6.3.3 will now read as

$$\frac{dP}{\rho} + VdV + gdZ - \frac{2\tau ds}{\rho r} = 0$$

$$\frac{dP}{\gamma} + d\left(\frac{V^2}{2g}\right) + dZ - \frac{2\tau ds}{\gamma\, r} = 0$$

ds can also be substituted in terms of Z and θ

Bernoulli equation will now read as (taking s as the length)

$$\frac{P_1}{\gamma} + \frac{V_1^{\,2}}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^{\,2}}{2g} + Z_2 + \frac{2\tau s}{\gamma r}$$

The last term is the loss of head due to friction and is denoted often as h_L,h_f in head of fluid in metre height (check for the unit of the last term).

Example 6.9 *The delivery line of a pump is 100 mm ID and it delivers water at a height of 12 m above entry. The pipe ends in a nozzle of diameter 60 mm. The total head at the entry to the pipe is 20 m.* ***Determine the flow rate if losses in the pipe is given by 10 $V_2^2/2g$****. where V_2 is the velocity at nozzle outlet. There is no loss in the nozzle.*

Equating the total energy at inlet and outlet,

$$20 = 12 + \frac{V_2^2}{2g} + 10\,\frac{V_2^2}{2g},$$

$$\therefore \quad V_2^{\,2} = \frac{8 \times 2 \times 9.81}{11}, \; V_2 = 3.777 \text{ m/s}$$

$$\textbf{Flow} = A_2\, V_2 = \frac{\pi \times 0.06^2}{4} \times 3.777 = 0.01068 \text{ m}^3\text{/s} = \mathbf{0.64\ m^3/min.}$$

(If losses do not occur then, V_2 = 12.53 m/s and flow will be 2.13 m^3/min)

Example 6.10 *A tank with water level of 12 m has a pipe of 200 mm dia connected from its bottom which extends over a length to a level of 2 m below the tank bottom.* ***Calculate the pressure at this point*** *if the flow rate is 0.178 m^3/s. The losses due to friction in the pipe length is accounted for by 4.5 $V_2^2/2g$.*

Taking location of the outlet of the pipe as the datum, using Bernoulli equation and accounting for frictional drop in head (leaving out the atmospheric pressure which is the same at the water level and at outlet).

$$14 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + 4.5\,\frac{V_2^2}{2g}$$

$$\therefore \qquad V_2 = 0.178/\pi \times 0.1 \times 0.1 = 5.67 \text{ m/s}$$

$$14 - 5.5 \times \frac{5.67^2}{2 \times 9.81} = \frac{P_2}{\gamma} = 5 \text{ m of water head.}$$

$$\therefore \qquad P_2 = 9810 \times 5 \text{ N/m}^2 = \mathbf{0.49\ bar} \text{ (above atmospheric pressure)}$$

Example 6.11 *A vertical pipe of diameter of 30 cm carrying water is reduced to a diameter of 15 cm. The transition piece length is 6 m. The pressure at the bottom is 200 kPa and at the top it is 80 kPa. If frictional drop is 2 m of water head,* ***determine the rate*** *of flow.*

Considering the bottom as the datum,

$$\frac{200 \times 10^3}{9810} + 0 + \frac{V_1^2}{2g} = \frac{80 \times 10^3}{9810} + 6 + \frac{V_2^2}{2g} + 2$$

$$V_2^2 = V_1^2\,(0.3/0.15)^4 = 16V_1^2$$

$$\therefore \qquad \frac{120 \times 10^3}{9810} - 8 = 15\frac{V_1^2}{2g}, \text{ Solving, } V_1 = 2.353 \text{ and } V_2 = 9.411 \text{ m/s}$$

$$\therefore \qquad \textbf{Flow rate} = A_1\,V_1 = A_2 V_2 = \mathbf{0.166\ m^3/s}$$

6.8 CONCEPT AND MEASUREMENT OF DYNAMIC, STATIC AND TOTAL HEAD

In the Bernoulli equation, the pressure term is known as static head. It is to be measured by a probe which will be perpendicular to the velocity direction. Such a probe is called static probe. The head measured is also called Piezometric head. (Figure 6.8.1 (*a*))

The velocity term in the Bernoulli equation is known as dynamic head. It is measured by a probe, one end of which should face the velocity direction and connected to one limb of a manometer with other end perpendicular to the velocity and connected to the other limb of the manometer. (Figure 6.8.1 (*b*))

The total head is the sum of the static and dynamic head and is measured by a single probe facing the flow direction. (Figure 6.8.1 (*c*))

The location of probes and values of pressures for the above measurements are shown in Fig. 6.8.1.

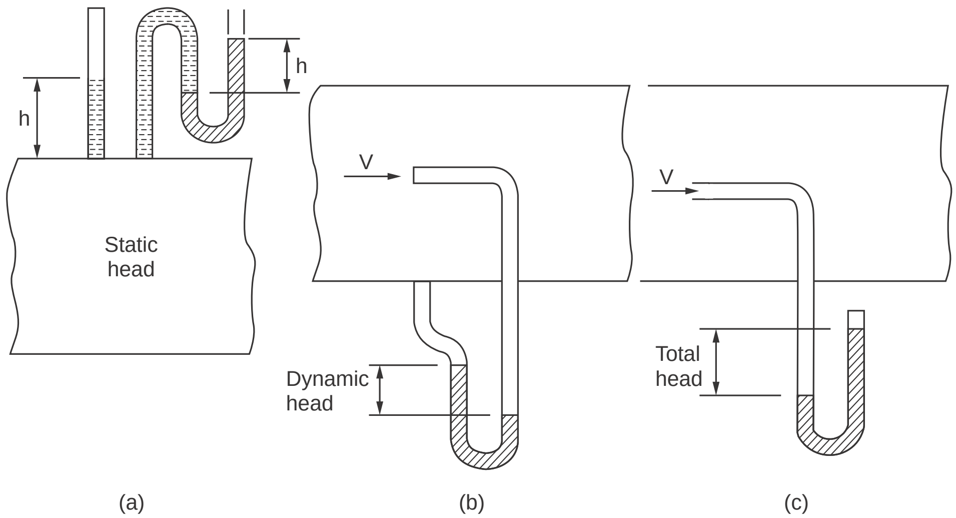

Figure 6.8.1 Pressure measurement

6.8.1 Pitot Tube

The flow velocity can be determined by measuring the dynamic head using a device known as pitot static tube as shown in Fig. 6.8.2. The holes on the outer wall of the probe provides the static pressure (perpendicular to flow) and hole in the tube tip facing the stream direction of flow measures the total pressure. The difference gives the dynamic pressure as indicated by the manometer. The head will be h $(s - 1)$ of water when a differential manometer is used $(s > 1)$.

The velocity variation along the radius in a duct can be conveniently measured by this arrangement by traversing the probe across the section. This instrument is also called pitot–static tube.

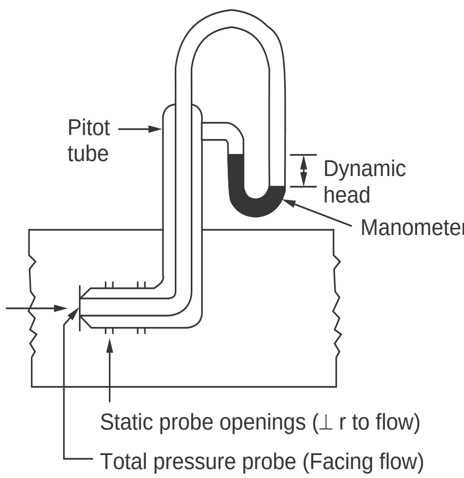

Figure 6.8.2 Pitot-Static tube

Example 6.12 *The dynamic head of a water jet stream is measured as 0.9 m of mercury column.* ***Determine the height to which the jet will rise*** *when it is directed vertically upwards.*

Considering the location at which the dynamic head is measured as the datum and converting the column of mercury into head of water, and noting that at the maximum point the velocity is zero,

$$0.9 \times 13.6 + 0 + 0 = 0 + 0 + Z \quad \therefore \mathbf{Z = 12.24\ m}$$

Note. If the head measured is given as the reading of a differential manometer, then the head should be calculated as 0.9 (13.6 – 1) m.

Example 6.13 *A diverging tube connected to the outlet of a reaction turbine (fully flowing) is called "Draft tube". The diverging section is immersed in the tail race water and this provides additional head for the turbine by providing a pressure lower than the atmospheric pressure at the turbine exit. If the turbine outlet is open the exit pressure will be atmospheric as in Pelton wheel. In a draft tube as shown in Fig. Ex. 6.13,* ***calculate the additional head provided by the draft tube.*** *The inlet diameter is 0.5 m and the flow velocity is 8 m/s. The outlet diameter is 1.2 m. The height of the inlet above the water level is 3 m. Also calculate the pressure at the inlet section.*

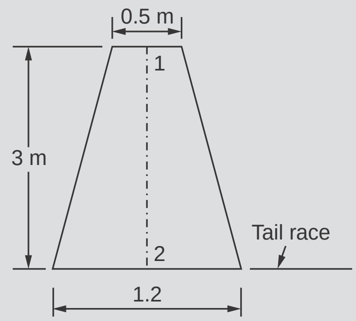

Figure Ex. 6.13 Draft tube

Considering sections 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} Z_1 = \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_2$$

Considering tail race level, 2 as the datum, and calulating the velocities

$$V_1 = 8 \text{ m/s}, V_2 = 8 \times \frac{0.5^2}{1.2^2} = 1.39 \text{ m/s.}$$

$$P_2 = \text{atmospheric pressure}, Z_2 = 0, Z_1 = 3$$

$$\frac{P_1}{\gamma} + \frac{8^2}{2 \times 9.81} + 3 = \frac{1.39^2}{2 \times 9.81}$$

$\therefore$ $\frac{P_1}{\gamma}$ = **– 6.16 m of water.** (Below atmospheric pressure)

Additional head provided due to the use of draft tube will equal 6.16 m of water

Note: This may cause cavitation if the pressure is below the vapour pressure at the temperature condition. Though theoretically the pressure at turbine exit can be reduced to a low level, cavitation problem limits the design pressure.

SOLVED PROBLEMS

Problem 6.1 *A venturimeter is used to measure the volume flow. The pressure head is recorded by a manometer. When connected to a horizontal pipe the manometer reading was h cm. If the reading of the manometer is the same when it is connected to a vertical pipe with flow upwards and (ii) vertical pipe with flow downwards, discuss in which case the flow is highest.*

Consider equation 6.6.2

$$Q = \frac{A_2}{\left[1 - (A_2/A_1)^2\right]^{0.5}} \left[2gh\left(\frac{S_2}{S_1} - 1\right)\right]^{0.5}$$

As long as '*h*' remains the same, the volume flow is the same for a given venturimeter as this expression is a general one derived without taking any particular inclination.

This is because of the fact that the manometer automatically takes the inclination into account in indicating the value of $(Z_1 - Z_2)$.

Problem 6.2 *Water flows at the rate of 600 l/s through a horizontal venturi with diameter 0.5 m and 0.245 m. The pressure gauge fitted at the entry to the venturi reads 2 bar.* ***Determine the throat pressure****. Barometric pressure is 1 bar.*

Using Bernoulli equation and neglecting losses

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2,\ P_1 = 2 \text{ bar (gauge)} = 3 \text{ bar (absolute) } 3 \times 10^5 \text{ N/m}^2$$

$$V_1 = \frac{Q}{(\pi \times d^2/4)} = \frac{0.6}{(\pi \times 0.5^2/4)} = 3.056 \text{ m/s} \quad \text{can also use}$$

$$V_2 = V_1 \left(\frac{D_2}{D_1}\right)^2$$

$$V_2 = \frac{0.6}{(\pi \times 0.245^2/4)} = 12.732 \text{ m/s, Substituting}$$

$$\frac{3 \times 10^5}{9810} + \frac{3.056^2}{2 \times 9.81} + 0 = \frac{P_2}{9810} + \frac{12.732^2}{2 \times 9.81} + 0$$

$\therefore$ $\mathbf{P_2}$ = 223617 N/m^2 = **2.236 bar (absolute) = 1.136 bar (gauge)**

Problem 6.3 *A venturimeter as shown in Fig P. 6.3 is used measure flow of petrol with a specific gravity of 0.8. The manometer reads 10 cm of mercury of specific gravity 13.6.* ***Determine the flow rate.***

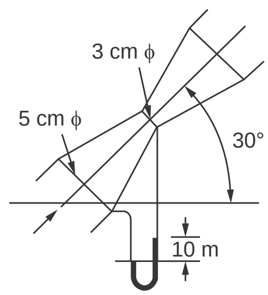

Figure P. 6.3 Problem model

Using equation 6.6.2

$$Q = \frac{A_2}{\left[1 - (A_2/A_1)^2\right]^{0.5}} \left[2gh\left(\frac{S_2}{S_1} - 1\right)\right]^{0.5}$$

$A_2 = (\pi/4)\ 0.03^2$ as D_2 = 3 cm

$\therefore$ $(A_2/A_1)^2 = (D_2/D_1)^4 = (0.03/0.05)^4$,

h = 0.10 m S_2 = 13.6, S_1 = 0.8, Substituting,

$$\boldsymbol{Q} = \frac{(\pi \times 0.03^2/4)}{\left[1 - (0.03/0.05)^4\right]^{0.5}} \left[2 \times 9.81 \times 0.1\left(\frac{13.6}{0.8} - 1\right)\right]^{0.5}$$

= **4.245 × 10^{-3} m^3/s or 15.282 m^3/hr** or 4.245 l/s or 15282 l/hr or 3.396 kg/s

Problem 6.4 *A liquid with specific gravity 0.8 flows at the rate of 3 l/s through a venturimeter of diameters 6 cm and 4 cm. If the manometer fluid is mercury (sp. gr = 13.b)* ***determine the value of manometer reading, h.***

Using equation (6.6.2)

$$Q = \frac{A_2}{\left[1-(A_2/A_1)^2\right]^{0.5}}\left[2gh\left(\frac{S_2}{S_1}-1\right)\right]^{0.5}$$

$$A_1 = \frac{\pi \times 0.06^2}{4} = 2.83 \times 10^{-3}\ \text{m}^2\ ;$$

$$A_2 = \frac{\pi \times 0.04^2}{4} = 1.26 \times 10^{-3}\ \text{m}^2$$

$$3 \times 10^{-3} = \frac{1.26 \times 10^{-3}}{\left[1-\left(\dfrac{1.26\times 10^{-3}}{2.83\times 10^{-3}}\right)^2\right]^{0.5}}\left[2 \times 9.81 \times h\left(\frac{13.6}{0.8}-1\right)\right]^{0.5}$$

Solving, **h = 0.0146 m = 14.6 mm.** of mercury column.

Problem 6.5 *Water flows upwards in a vertical pipe line of gradually varying section from point 1 to point 2, which is 1.5m above point 1, at the rate of 0.9m³/s. At section 1 the pipe dia is 0.5m and pressure is 800 kPa. If pressure at section 2 is 600 kPa,* ***determine the pipe diameter at that location.*** *Neglect losses.*

Using Bernoulli equation,

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{800 \times 10^3}{9810} + \frac{\left(0.9 \times 4/\pi \times 0.5^2\right)}{2 \times 9.81} + 0 = \frac{600 \times 10^3}{9810} + \frac{V_2^2}{2 \times 9.81} + 1.5$$

Solving, $V_2 = 19.37$ m/s.

Flow = area × velocity, $\dfrac{\pi \times d_2^2}{4} \times 19.37 = 0.9\ \text{m}^3/\text{s}$

Solving for d_2, **Diameter of pipe at section 2 = 0.243 m**

As (p/γ) is involved directly on both sides, gauge pressure or absolute pressure can be used without error. However it is desirable to use absolute pressure to aviod nagative pressure values.

Problem 6.6 ***Calculate the exit diameter, if*** *at the inlet section of the draft tube the diameter is 1 m and the pressure is 0.405 bar absolute. The flow rate of water is 1600 l/s. The vertical distance between inlet and outlet is 6 m.*

Applying Bernoulli equation between points 1 and 2, neglecting losses

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$V_1 = \frac{Q \times 4}{\pi \times D_1^{\;2}} = \frac{1600 \times 10^{-3} \times 4}{\pi \times 1^2} = 2.04 \text{ m/s}$$

P_2 = atmospheric pressure; Z_2 = 0 (datum); Z_1 = 6 m

$$\frac{0.405 \times 10^5}{9810} + \frac{2.04^2}{2 \times 9.81} + 6 = \frac{1.013 \times 10^5}{9810} + \frac{V_2^2}{2 \times 9.81} + 0 \quad \therefore V_2 = 0.531 \text{ m/s}$$

$$\frac{A_2}{A_1} = \frac{V_1}{V_2} = \frac{D_2^2}{1^2} = \frac{2.04}{0.531} \quad \therefore \quad \mathbf{D_2 = 1.96\ m}$$

0.405 bar absolute means vacuum at the inlet section of the draft tube. This may cause "cavitation" if this pressure is below the vapour pressure at that temperature. Though theoretically the pressure at turbine exit, where the draft tube is attached, can be reduced to a vary low level, cavitation problem limits the pressure level.

Problem 6.7 *Water flows at the rate of 200 l/s upwards through a tapered vertical pipe. The diameter at the bottom is 240 mm and at the top 200 mm and the length is 5m. The pressure at the bottom is 8 bar, and the pressure at the topside is 7.3 bar.* ***Determine the head loss through the pipe****. Express it as a function of exit velocity head.*

Applying Bernoulli equation between points 1 (bottom) and 2 (top) and considering the bottom level as datum.

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + \text{losses}$$

$$\frac{8 \times 10^2}{9810} + \frac{(200 \times 10^{-3} \times 4)/\pi \times 0.24^2)^2}{2 \times 9.81} + 0$$

$$= \frac{7.3 \times 10^5}{9810} + \frac{(200 \times 10^{-3} \times 4)/(\pi \times 0.2^2)^2}{2 \times 9.81} + 5 + \text{losses}$$

$\therefore$ **Losses = 1.07 m**

$$1.07 = X\frac{V_2^2}{2g} = X\left[\frac{200 \times 10^{-3} \times 4}{\pi \times 0.22}\right]^2 /2 \times 9.81 \quad \therefore X = 0.516,$$

$$\textbf{Loss of head} = \mathbf{0.516}\ \frac{\mathbf{V_2^2}}{\mathbf{2g}}$$

Problem 6.8 ***Calculate the flow rate of oil*** *(sp. gravity, 0.8) in the pipe line shown in Fig. P. 6.8. Also calculate* ***the reading "h"*** *shown by the differential manometer* ***fitted to the pipe*** *line which is filled with mercury of specific gravity 13.6.*

Applying Bernoulli equation (neglecting losses) between points 1 and 2

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$P_1 = 2 \times 10^5$ N/m^2; $P_2 = 0.8 \times 10^5$ N/m^2 ;

$Z_1 = 0$, $Z_2 = 2$ m

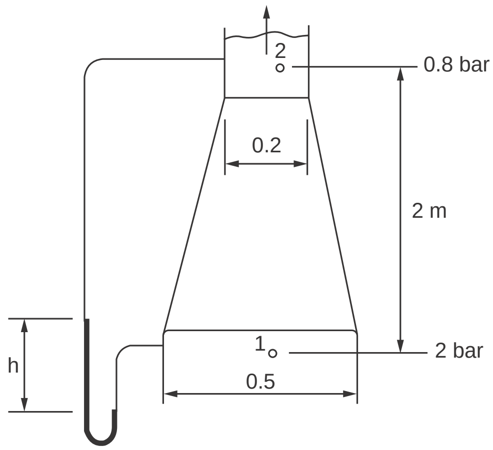

Figure P. 6.8

Applying continuity equation between points 1 and 2

$$A_1V_1 = A_2V_2,\ V_2 = V_1\frac{A_1}{A_2} = V_1\left(\frac{\pi \times 0.5^2/4}{\pi \times 0.2^2/4}\right) = 6.25\ V_1$$

$$\frac{2 \times 10^5}{9810 \times 0.8} + \frac{V_1^2}{2 \times 9.81} + 0 = \frac{0.8 \times 10^5}{9810 \times 0.8} + \frac{(6.25V_1)^2}{2 \times 9.81} + 2 \quad \therefore V_1 = 2.62 \text{ m/s}$$

Flow rate, $\mathbf{Q} = A_1\ V_1 = \frac{\pi \times 0.5^2}{4} \times 2.62 = 0.514$ m^3/s $=$ **514 l/s**

Using equation (6.6.2) (with $A_2 = 0.031$ m^2, $A_1 = 0.196$ m^2)

Flow rate, $$Q = \frac{A_2}{\left[1-(A_2/A_1)^2\right]^{0.5}}\left[2gh\left(\frac{S_2}{S_1}-1\right)\right]^{0.5}$$

$$0.514 = \frac{0.031}{\left[1-\left(\frac{0.031}{0.196}\right)^2\right]^{0.5}}\left[2 \times 9.81 \times h\left(\frac{13.6}{0.8}-1\right)\right]^{0.5}$$

Solving, **h = 0.854 m**

Problem 6.9 *Water flows at the rate of 400 l/s through the pipe with inlet (1) diameter of 35 cm and (2) outlet diameter of 30 cm with 4m level difference with point 1 above point 2. If $P_1 = P_2$ = 2 bar absolute,* ***determine the direction of flow.***

Consider datum as plane 2

$$\text{Total head 1,}\quad \frac{2\times10^5}{9810}+\frac{\left(0.4\times4/\pi\times0.35^2\right)^2}{2\times9.81}+4 = 25.27 \text{ m water column}$$

$$\text{Total head at 2,}\quad \frac{2\times10^5}{9810}+\frac{\left(0.4\times4/\pi\times0.3^2\right)^2}{2\times9.81}+0 = 22.02 \text{ m of water column}$$

The total energy at all points should be equal if there are no losses. This result shows that there are losses between 1 and 2 as the total energy at 2 is lower. **Hence the flow will take place from points 1 to 2**.

Problem 6.10 *Petrol of relative density 0.82 flows in a pipe shown Fig. P.6.10. The pressure value at locations 1 and 2 are given as 138 kPa and 69 kPa respectively and point 2 is 1.2m vertically above point 1.* ***Determine the flow rate****. Also calculate* ***the reading of the differential manometer*** *connected as shown. Mercury with S = 13.6 is used as the manometer fluid.*

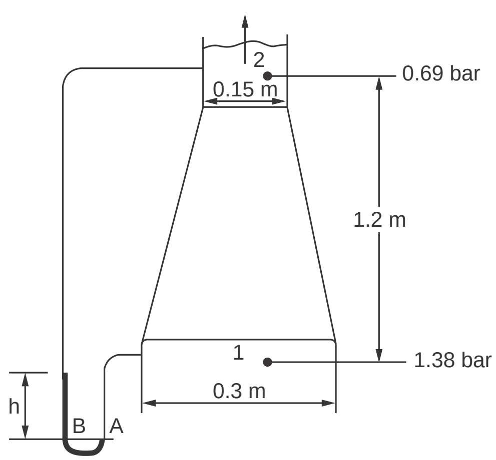

Figure P. 6.10 Problem Model

Considering point 1 as a datum and using Bernoulli equation.

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2,\ Z_1=0,\ Z_2=1.2 \text{ m},\ V_2=V_1\frac{A_1}{A_2}=V_1\left(\frac{D_1^2}{D_2^2}\right)$$

$$\therefore \qquad V_2^{\,2}=V_1^{\,2}\left(\frac{D_1^4}{D_2^4}\right)=16\,V_1^{\,2} \text{ as } D_1/D_2=2$$

$$\frac{138\times10^3}{0.82\times9810}+\frac{V_1^2}{2g}+0=\frac{69\times10^3}{0.82\times9810}+16\left(\frac{V_1^2}{2g}\right)+1.2$$

$$\frac{(138-69)10^3}{0.82\times9810}-1.2=15\frac{V_1^2}{2g}.\quad \text{Solving, } V_1 = 3.106 \text{ m/s}$$

$$\therefore \qquad \textbf{Volume flow} = \frac{\pi\times0.3^2}{4}\times3.106 = \mathbf{0.22\ m^3/s\ or\ 180\ kg/s}$$

The flow rate is given by equation 6.6.2

$$Q=\frac{A_2}{\left[1-\left(\frac{A_2}{A_1}\right)^2\right]^{0.5}}\left[2gh\left(\frac{S_2}{S_1}-1\right)\right]^{0.5},\ \frac{S_2}{S_1}=\frac{13.6}{0.82}$$

$$0.22=\frac{\pi\times0.15^2/4}{\left[1-\left(\frac{0.15}{0.3}\right)^4\right]^{0.5}}\left[2\times9.81\times h\left(\frac{13.6}{0.82}-1\right)\right]^{0.5}$$

Solving, $\mathbf{h = 0.475\ m}$ of mercury column

Problem 6.11 *Water flows downwards in a pipe as shown in Fig. P.6.11. If pressures at points 1 and 2 are to be equal,* ***determine the diameter of the pipe at point 2****. The velocity at point 1 is 6 m/s.*

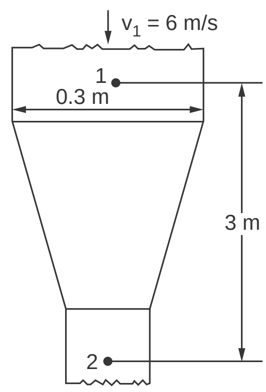

Figure P. 6.11 Problem model

Applying Bernoulli equation between points 1 and 2 (taking level 2 as datum)

$$\frac{P_1}{\gamma}+\frac{6^2}{2\times9.81}+3=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+0$$

as $P_1 = P_2$, $\mathbf{V_2 = 9.74\ m/s}$

Using the relation $A_1 V_1 = A_2 V_2$,

$$\frac{\pi\times0.3^2\times6}{4}=\frac{\pi\times d^2\times9.74}{4}$$

$\therefore$ $\mathbf{d = 0.2355\ m.}$

Problem 6.12 *A siphon is shown in Fig P. 6.12. Point A is 1m above the water level, indicated by point 1. The bottom of the siphon is 8m below level A. Assuming friction to be negligible,* ***determine the speed of the jet at outlet and also the pressure at A.***

Using Bernoulli equation, between 1 and 2.

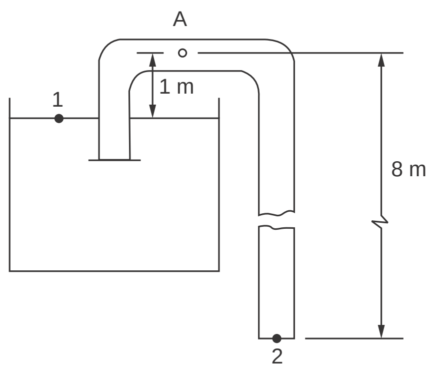

Figure P. 6.12 Problem model

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 \,,$$

$$P_1 = P_2 = \text{atmospheric pressure.}$$

Consider level 1 as datum. The velocity of water at the surface is zero.

$$\therefore \qquad 0 + 0 = \frac{V_2^2}{2g} - 7$$

$$\therefore \qquad \mathbf{V_2} = \sqrt{7 \times 2 \times 9.81} = \mathbf{11.72\ m/s = V_A}$$

Considering surface 1 and level *A*. As flow is the same,

$$\frac{P_1}{\gamma} + 0 + 0 = \frac{P_A}{\gamma} + 1 + \frac{V_A^2}{2g}$$

Considering $P_1/\gamma = 10.3$ m of water,

$$\frac{\mathbf{P_A}}{\gamma} = \frac{P_1}{\gamma} - 1 - \frac{V_2^2}{2g} = 10.3 - 1 - 7$$

$$= \mathbf{2.3\ m\ of\ water\ column}\ \text{(absolute)}$$

Problem 6.13 *A pipe line is set up to draw water from a reservoir. The pipe line has to go over a barrier which is above the water level. The outlet is 8 m below water level.* ***Determine the maximum height of the barrier if the pressure*** *at this point should not fall below 1.0 m of water to avoid cavitation. Atmospheric pressure is 10.3 m.*

Considering outlet level 3 as datum and water level as 1 and appyling Bernoulli equation,

$$Z_3 = 0,\ Z_1 = 8,\ V_1 = 0,\ P_1 = P_3$$

$$\therefore \qquad 8 = \frac{V_3^2}{2g} \quad \therefore \quad V_3 = \sqrt{8 \times 9.81 \times 2} = 12.53 \text{ m/s}$$

Chapter 6

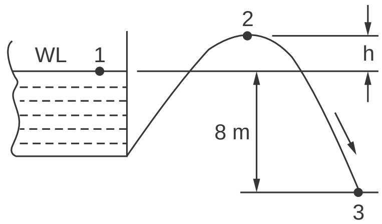

Figure P. 6.13 Problem model

Considering the barrier top as level 2

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3, \quad \text{As } V_2 = V_3, Z_3 = 0, P_2/\gamma = 1$$

$$1 + Z_2 = 10.3$$

∴ $\mathbf{Z_2 = 9.3}$ **m.** Therefore the barrier can **be 1.3 m above water level.**

Problem 6.14 ***Determine the flow rate of water*** *across the shutter in an open canal if the water level upstream of shutter is 5m and downstream is 2m. The width of the canal is 1m and flow is steady.*

Applying Bernoulli equation between point 1 in the upstream and point 2 in the downstream on both sides of the shutter, both surface pressures being atmospheric.

$$\frac{V_1^2}{2g} + 5 = \frac{V_2^2}{2g} + 2 \tag{1}$$

Applying continuity equation, flow rate, $Q = A_1V_1 = A_2V_2$

$(1 \times 5)\ V_1 = (1 \times 2)\ V_2, \quad \because \quad V_2 = 2.5\ V_1,$ Substituting in equation (1),

$$\frac{V_1^2}{2 \times 9.81} + 5 = \frac{(2.5V_1)^2}{2 \times 9.81} + 2,$$

∴ $V_1 = 3.35$ m/s, $V_2 = 8.37$ m/s. $\mathbf{Q = 16.742\ m^3/s}$,

Problem 6.15 *Uniform flow rate is maintained at a shutter in a wide channel. The water level in the channel upstream of shutter is 2m. Assuming uniform velocity at any section if the flow rate per m length is 3m³/s/m,* ***determine the level downstream****.*

Assume velocities V_1 and V_2 upstream and downstream of shutter and the datum as the bed level. Using Bernoulli equation

$$2 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} \tag{A}$$

Considering unit width from continuity $1 \times 2 \times V_1 = 1 \times h_2 \times V_2$ (B)

∴ $V_2 = (2/h_2)\ V_1$, from flow rate $V_1 = 3/2 = 1.5$ m/s ∴ $V_2 = \dfrac{3}{h_2}$

Substituting

$$2 + \frac{1.5^2}{2 \times 9.81} = h_2 + \frac{3^2}{h^2 \times 2 \times 9.81}$$

Simplifying, this reduces to $h_2^3 - 2.1147\, h_2^2 + 0.4587 = 0$

Solving, h_2 can be **2 m, – 0.425 m, 0.54 m**

h_2 = 0.54 m is the acceptable answer. 2m being trivial.

Using *B*, $0.54 \times V_2 \times 1 = 2 \times 1.5 = 3.$ $\therefore V_2 = 5.56$ m/s.

check using *A*, $2 + 0.1147 = 0.54 + 1.57$ checks.

The difference between the dynamic head values will equal the difference between the datum heads. This may be checked using the calculated velocity values.

Problem 6.16 *A pump with centre line 2m above the sump water level develops 50m head of water. The suction pipe is of 150 mm ID. The loss of head in the suction line is given by 5* $V_s^2/2g$*. The delivery line is of 100 mm dia and the loss in the line is 12* $V_d^2/2g$*. The water is delivered through a nozzle of 75 mm dia. The delivery is at 30m above the pump centre line.* ***Determine the velocity at the nozzle outlet and the pressure at the pump inlet***.

Let the velocity at the nozzle be V_n

Velocity in the delivery pipe $= V_d = V_n \times \frac{75^2}{100^2} = \frac{9}{16} V_n$

Velocity in suction pipe $V_s = V_n \left(\frac{75}{150}\right)^2 = \frac{V_n}{4}$

Kinetic head at outlet $= \frac{V_n^2}{2g}$

Loss in delivery pipe $= \frac{V_d^2}{2g} = 12 \times \left(\frac{9}{16}\right)^2 \frac{V_n^2}{2g} = 3.797 \frac{V_n^2}{2g}$

Loss in suction pipe $= \frac{V_s^2}{2g} = \frac{5}{16} \frac{V_n^2}{2g} = 0.3125 \frac{V_n^2}{2g}$

Equating the head developed to the static head, losses and kinetic head,

$$50 = 30 + 2 + \frac{V_n^2}{2g} [1 + 3.797 + 0.3125]$$

$$18 \times 2 \times 9.81 = V_n^2 \,[5.109]$$

$\therefore$ **Velocity at the nozzle V_n = 8.314 m/s**

Pressure at suction : Taking datum as the water surface and also the velocity of the water to be zero at the surface,

P_1 as atmospheric, 10.3 m of water column, Kinetic head $V^2/2g$, loss $5V^2/2g$

$$10.3 = \frac{P_2}{\gamma} + 2 + \left(\frac{(8.314/4)^2}{2 \times 9.81}\right) \times (5 + 1) \quad (\text{as } V_s = V_n/4)$$

$$\therefore \qquad \frac{\mathbf{P_2}}{\boldsymbol{\gamma}} = 10.3 - 3.321 \text{ m} = \mathbf{6.979 \text{ m absolute}}$$

or 3.321 m below atmospheric pressure.

Problem 6.17 *A liquid jet at a velocity V_0 is projected at angle θ. Describe the path of the free jet. Also calculate the* ***maximum height and the horizontal distance travelled****.*

The horizontal component of the velocity of jet is $V_{xo} = V_o \cos\theta$. The vertical component $V_{zo} = V_o \sin\theta$.

In the vertical direction, distance travelled, Z, during time t, (using the second law of Newton)

$$Z = V_{zo}\, t - (1/2)\, gt^2 \tag{A}$$

The distance travelled along x direction

$$X = V_{xo}\, t \text{ or } t = X/V_{xo} \tag{B}$$

Solving for t from B and substituting in A,

$$Z = \frac{V_{zo}}{V_{xo}}\, X - \frac{1}{2}\,\frac{g}{V_{xo}^2}\, X^2 \tag{C}$$

Z value can be maximised by taking dz/dx and equating to zero

$$\frac{dz}{dx} = \frac{V_{zo}}{V_{xo}} - \frac{1}{2}\,\frac{g}{V_{xo}^2}\, 2X, \quad \frac{V_{zo}}{V_{xo}} = \frac{gX}{V_{xo}^2} \quad \therefore \quad X = V_{zo}V_{xo}/g$$

Substituting in C,

$$Z_{max} = \frac{V_{zo}}{V_{xo}} \cdot \frac{V_{zo}V_{xo}}{g} - \frac{1}{2}\,\frac{g}{V_{xo}^2} \cdot \frac{V_{zo}^2 V_{xo}^2}{g^2}$$

$$= \frac{1}{2}\,\frac{V_{zo}^2}{g} = \frac{V_0^{\,2}\sin^2\theta}{2g}, \quad Z_{max} = V_o^{\,2}\sin^2\theta/2g \tag{D}$$

The maximum height is achieved when θ = 90°.

$\therefore$ $\mathbf{X_{mas} = 2 \text{ times x as } Z_{max.}}$

$$X_{max} = 2V_o^{\,2}\sin\theta\cos\theta\,/g = V_o^{\,2}\sin 2\theta/g \tag{E}$$

Maximum horizontal reach is at θ = 45° or 2θ = 90° and for this angle it will reach half the vertical height.

This describes an inverted parabola as shown in Fig. P.6.17

Bernoulli equation shows that $Z_t + V_t^{\,2}/2\text{g}$ = constant along the rejectory. V_t is the velocity at that location when air drag is neglected. Pressure is assumed to be uniform all over the trejectory as it is exposed to atmosphere all along its travel. Hence

$$Z_t + V_t^2/2g = \text{constant for the jet.}$$

(**Note:** Velocity at time $t = V_{zo}\,t = V_0 \sin\theta + a \times t$, where $a = -g$, so the velocity decreases, becomes zero and then turns – ve)

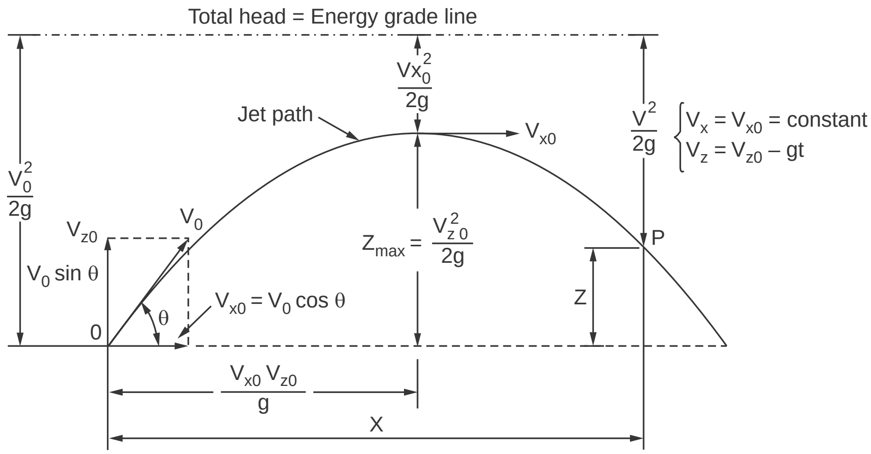

Figure P. 6.17 Jet trejectory

Problem 6.18 *A jet issuing at a velocity of 20 m/s is directed at 30° to the horizontal.* ***Calculate the height cleared*** *by the jet at 25m from the discharge location? Also* ***determine the maximum height the jet will clear*** *and the corresponding horizontal location.*

Ref Fig. P. 6.17

$$V_{xo} = V_o \cos 30 = 20 \cos 30 = 17.32 \text{ m/s;}$$

$$V_{zo} = V_o \sin 30 = 20 \sin 30 = 10 \text{ m/s;}$$

at time t, $X = V_{xo}t$; $Z = V_{zo}\,t - (1/2)\,gt^2$, Substituting for t as X/V_{xo} with $X = 25$ m

$$Z = \frac{V_{zo}}{V_{xo}} X - \frac{1}{2}\frac{g}{V_{xo}^2} X^2 \tag{A}$$

Height cleared, $\quad \mathbf{Z_{25}} = \dfrac{10}{17.32} \times 25 - \dfrac{1}{2}\,\dfrac{9.81}{17.32^2} \times 25^2 = \mathbf{4.215\ m}$

Maximum height of the jet trajectory $= \dfrac{V_{zo}^2}{2g} = \dfrac{10^2}{2 \times 9.81} = \mathbf{5.097\ m}$

Corresponding horizontal distance $= \dfrac{V_{xo}V_{zo}}{g} = \dfrac{17.32 \times 10}{9.81} = \mathbf{17.66\ m}$

Total horizontal distance is twice the distance travelled in reaching

$$\mathbf{Z_{max} = 35.32\ m}$$

It would have crossed this height also at 10.43 m from the starting point (check using equations derived in Problem 6.17).

Problem 6.19 ***Determine the velocity of a jet*** *directed at 40° to the horizontal to clear 6 m height at a distance of 20m. Also determine the maximum height this jet will clear and the total horizontal travel. What will be the horizontal distance at which the jet will be again at 6m height*.

From basics, referring to Fig. P. 6.17,

$$V_{xo} = V_o \cos 40, \quad V_{zo} = V_o \sin 40,$$

$$X = V_{xo}\, t,\ t = \frac{X}{V_{xo}}, \quad Z = V_{zo}\, t - (1/2)\, gt^2$$

Substituting for t as X/V_{xo}

$$Z = \frac{V_{zo}}{V_{xo}} X - \frac{1}{2} \frac{g}{V_{xo}^{\ 2}} X^2 \qquad (A)$$

Substituting the values,

$$6 = \frac{V_o \sin 40}{V_o \cos 40} \times 20 - \frac{1}{2} \times \frac{9.81 \times 20^2}{V_o^{\ 2} \cos^2 40}$$

$$6 = 20 \tan 40 - \frac{1}{2} \frac{9.81 \times 20^2}{V_o^{\ 2} \cos^2 40} \qquad (B)$$

$$\therefore \quad V_o^{\ 2} = \frac{9.81 \times 20^2}{2 \cos^2 40 (20 \tan 40 - 6)} = 310 \quad \therefore \quad \mathbf{V_o = 17.61\ m/s.}$$

Maximum height reached

$$= V_{zo}^{\ 2}/2g = (V_o \sin 40)^2/2g$$

$$= (17.61 \times \sin 40)^2 /2 \times 9.81 = \mathbf{6.53\ m}$$

The X value corresponding to this is, (**half total horizontal travel**)

$$X = V_{xo} V_{zo}/g = 17.61^2 \sin 40 \cos 40/9.81 = \mathbf{15.56\ m.}$$

This shows that the jet clears 6m height at a distance of 20 m as it comes down. The jet would have cleared this height at a distance less than 15.56 m also. By symmetry, this can be calculated as – (20 – 15.56) + 15.56 = 11.12 m

check by substituting in equation B.

$$11.12 \tan 40 - \frac{1}{2} \times \frac{9.81 \times 11.12^2}{17.61^2 \cos^2 40} = 6$$

When both Z and X are specified unique solution is obtained. Given V_o and Z, two values of X is obtained from equation A.

Problem 6.20 ***Determine the angle at which a*** *jet with a given velocity is to be projected for obtaining maximum horizontal reach*.

Refer Problem 6.17. $\quad X = V_{xo}\, t,\ Z = V_{zo}\, t - (1/2)\, gt^2$

The vertical velocity at any location/time is given by,

$$V_{zt} = \frac{dz}{dt} = V_{zo} - gt$$

The horizontal distance travelled will be half the total distance travelled when

$$V_{zt} = 0 \text{ or } t = V_{zo}/g$$

Total X distance travelled during time $2t$.

$$X = 2\,V_{xo}\,V_{zo}/g = 2\,V_o^2 \cos\theta \sin\theta/g = V_o^2 \sin 2\theta/g$$

For X to be maximum sin 2θ should be maximum or 2θ = 90° or

θ = 45°. For maximum horizontal reach, the projected angle should be 45°.

The maximum reach, $X = V_o^2/g$ as $\sin 2\theta = 1$.

Problem 6.21 ***Determined the angle at which*** *a jet with an initial velocity of 20 m/s is to be projected to clear 4m height at a distance of 10* m.

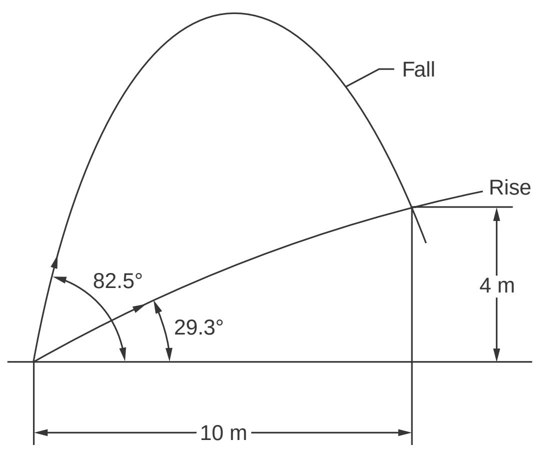

Figure P. 6.21 Jet trajectory

Refer Problem 6.17, eqn. C

$$Z = \frac{V_{zo}}{V_{xo}} x - \frac{1}{2}\frac{gx^2}{V_{xo}^2}$$

Substituting in terms of V_o and θ.

$$Z = \frac{V_o \sin\theta}{V_o \cos\theta} x - \frac{1}{2}\frac{gx^2}{V_o^2 \cos^2\theta} = x\tan\theta - \frac{1}{2}\frac{gx^2}{V_o^2}(\sec^2\theta)$$

$$Z = x\tan\theta - \frac{1}{2}\frac{gx^2}{V_o^2}(1 + \tan^2\theta)$$

Substituting the given values, $4 = 10\tan\theta - \frac{1}{2}\frac{9.81\times 10^2}{20^2}(1+\tan^2\theta)$

Hence, $\tan^2\theta - 8.155\tan\theta + 4.262 = 0$, solving **tan θ = 7.594 or 0.5613**

This corresponds to θ = 82.5° or 29.3°. In the first case it clears the height during the fall. In the second case it clears the height during the rise. See Fig. P.6.21.

Problem 6.22 *From a water tank two identical jets issue at distances H_1 and H_2 from the water level at the top. Both reach the same point at the ground level of the tank. If the* ***distance*** *from the ground level to the jet levels are y_1 and y_2.* ***Show that $H_1y_1 = H_2y_2$.***

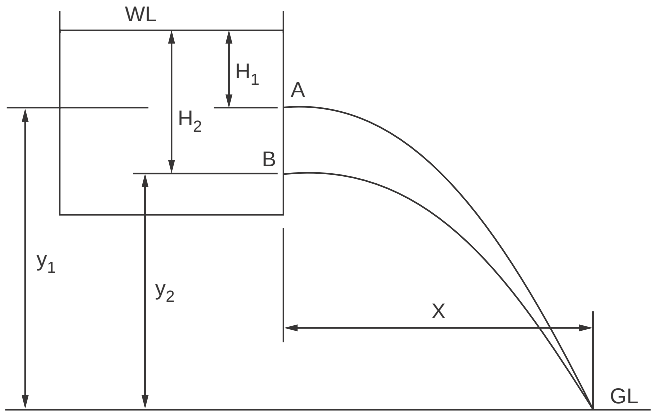

Figure P. 6.22 Problem model

In this case the jets issue out at *A* and *B* horizontally and so the position can be taken as the Z_{max} position.

Referring to Problem 6.17, eqn. (*D*)

$$Z_{max} = \frac{V_{zo}^2}{2g}, \quad y_1 = \frac{V_{zo1}^2}{2g} \quad \text{or} \quad V_{zo1} = \sqrt{2gy_1}$$

Similarly, $$y_2 = \frac{V^2_{zo2}}{2g} \quad \text{or} \quad V_{zo2} = \sqrt{2gy_2} \qquad (A)$$

(V_{zo1} and V_{zo2} are the *Z* components at point where the jet touches the ground)

$$X_{max} = \frac{V_{zo}V_{xo}}{g} \text{ and so } \frac{V_{zo1}V_{xo1}}{g} = \frac{V_{zo2}V_{xo2}}{g} \qquad (B)$$

$$V_{xo1} = \sqrt{2gH_1}\,,\ V_{xo2} = \sqrt{2gH_2} \qquad (C)$$

Substituting results (*A*) and (*C*) in equation (*B*), and simplifying,

$$\frac{\sqrt{2gH_1}\,\sqrt{2gy_1}}{g} = \frac{\sqrt{2gH_2}\,\sqrt{2gy_2}}{g} \quad \therefore \quad \mathbf{H_1\,y_1 = H_2\,y_2}$$

Problem 6.23 *A jet of water initially 12 cm dia when directed vertically upwards, reaches a maximum height of 20 m. Assuming the jet remains circular* ***determine the flow rate and area of jet at 10 m height***.

As $V = 0$ at a height of 20 m, Bernoulli equation reduces to

$$\frac{V^2}{2g} = 20,$$

$$\therefore \qquad V = (20 \times 9.81 \times 2)^{0.5} = 19.809 \text{ m/s}$$

$$\textbf{Flow rate} = \text{area} \times \text{velocity} = \frac{\pi \times 0.12^2}{4} \times 19.809 = \mathbf{0.224\ m^3/s}$$

When the jet reaches 10 m height, the loss in kinetic energy is equal to the increase in potential energy. Consider this as level 2 and the maximum height as level 1 and ground as datum,

$$P_1 = P_2,\ V_1 = 0,\ Z_2 = Z_1 - 10 = (20 - 10) = 10$$

$$20 = 10 + \frac{V^2{}_{20}}{2g} \quad \therefore \quad \frac{V_2{}^2}{2g} = 10,$$

$$\therefore \qquad \mathbf{V_2} = (10 \times 2 \times 9.81)^{0.5} = \mathbf{14\ m/s}$$

$$\textbf{Flow rate} = \text{area} \times \text{velocity},\ 0.224 = \frac{\pi \times D^2}{4} \times 14 \quad \therefore \quad \mathbf{D = 0.1427\ m}$$

Problem 6.24 *Water is discharged through a 150 mm dia pipe fitted to the bottom of a tank. A pressure gauge fitted at the bottom of the pipe which is 10 m below the water level shows 0.5 bar. Determine the flow rate. Assume the frictional loss as* $4.5V_2^2/2g$.

Applying Bernoulli equation between the water level, 1 and the bottom of the pipe, 2 and this level as datum

$$\frac{P_1}{\gamma} + \frac{V_1{}^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2{}^2}{2g} + Z_2 + \text{losses}$$

$$0 + 0 + 10 = \frac{0.5 \times 10^5}{9810} + \frac{V_2{}^2}{2 \times 9.81} + 0 + 4.5\frac{V_2{}^2}{2 \times 9.81}$$

Solving, $\mathbf{V_2 = 4.18\ m/s}$

$$\textbf{Flow rate} = \frac{\pi \times 0.15^2}{4} \times 4.18 = \mathbf{0.0739\ \mathit{m^3/s} = 73.9\ l/s.}$$

Problem 6.25 *An open tank of diameter D containing water to depth* h_o *is emptied by a smooth orifice at the bottom. Derive an expression for the time taken to reduce the height to h. Also find the time* t_{max} *for emptying the tank.*

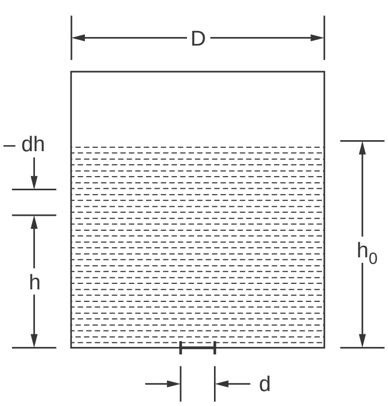

Figure P. 6.25 Problem model

Considering point 1 at the top of the tank and point 2 at the orifice entrance, and point 2 as datum

$$P_{atm} + \frac{V_1{}^2}{2g} + h = \frac{V_2{}^2}{2g} + p_{atm}$$

$$\therefore \qquad \frac{V_1{}^2}{2g} + h = \frac{V_2{}^2}{2g}$$

Also $\qquad V_1{}^2 = V_2{}^2\left[\frac{d}{D}\right]^4$

$$\therefore \qquad V_2 = \sqrt{\frac{2gh}{1-(d/D)^4}}$$

Let the level at the time considered be h.

The drop in level dh during time dt is given by (as dh is negative with reference to datum)

$$\frac{dh}{dt} = -\frac{V_2 A_2}{A_1} = -\left(\frac{d}{D}\right)^2 \sqrt{\frac{2gh}{1-\left(\frac{d}{D}\right)^4}}$$

Taking $\left(\frac{d}{D}\right)^2$ inside and rearranging

$$\frac{dh}{dt} = -\sqrt{\frac{2gh}{\left(\frac{D}{d}\right)^4 - 1}}$$

Separating variables and integrating

$$\int_{h_o}^{h} \frac{dh}{\sqrt{h}} = -\sqrt{\frac{2g}{\left(\frac{D}{d}\right)^4 - 1}} \cdot \int_0^t dt$$

$$2\left[\sqrt{h_o} - \sqrt{h}\right] = \sqrt{\frac{2g}{\left(\frac{D}{d}\right)^4 - 1}} \cdot t \qquad (A)$$

$$t = 2(\sqrt{h_o} - \sqrt{h}) \Big/ \sqrt{\frac{2g}{\left(\frac{D}{d}\right)^4 - 1}} = \sqrt{h_o} - \sqrt{h} \Big/ \sqrt{\frac{g/2}{\left(\frac{D}{d}\right)^4 - 1}} \qquad (B)$$

Equation (A) can be rearranged to give

$$\frac{h}{h_0} = \left[1 - \frac{t\sqrt{g/2\,h_o}}{\sqrt{\left(\frac{D}{d}\right)^4 - 1}}\right]^2 \qquad (C)$$

Equation (B) will be useful to find the drop in head during a given time interval.

Consider a numerical problem.

Let $\quad$ **D = 0.5 m, d = 0.025 m, h_o = 0.5 m**,

Time for emptying is calculated as h = 0,

$$t = \sqrt{h_o} \Big/ \sqrt{\frac{g/2}{\left(\frac{D}{d}\right)^4 - 1}}$$

$$= \sqrt{0.5} / \sqrt{\frac{9.81/2}{\left(\frac{0.5}{0.025}\right)^4 - 1}} = 127.7 \text{ seconds.}$$

To find the drop in level in say 100 seconds.

$$\frac{h}{h_o} = \left[1 - \frac{100\sqrt{9.81/2 \times 0.5}}{\sqrt{\left(\frac{0.5}{0.25}\right)^4 - 1}}\right]^2 = 0.0471$$

$\therefore$ Drop in head = 0.5 (1 – 0.0471) = 0.4764 m

In case $d << D$, then $V_2 = \sqrt{2gh}$ when head is h m

$$\frac{dh}{dt} = -\frac{A_2 V_2}{A_1} = -V_2\left(\frac{d}{D}\right)^2 = -\left(\frac{d}{D}\right)^2 . \sqrt{2gh}$$

Separating variables and integrating

$$\int_{h_o}^{h} \frac{dh}{\sqrt{h}} = -\left(\frac{d}{D}\right)^2 \sqrt{2g} . \int_0^t dt$$

$$2\left[\sqrt{h_o} - \sqrt{h}\right] = \left(\frac{d}{D}\right)^2 \sqrt{2g} . t$$

In this case to empty the tank,

$$2\sqrt{0.5} = \left(\frac{0.025}{0.5}\right)^2 . \sqrt{2 \times 9.81} . t.$$

Solving $\qquad$ **t = 127.71 s**.

The same answer because the same diameter of the orifice is used. Say d = 0.01 m, then time for employing is 1130 sec.

Problem 6.26 *Two identical jets issuing from a touch as shown in figure reach the ground at a distance of 10 m.* ***Determine the distances indicated as h and H***.

Consider top jet:

x distance travelled in time t is 10 m.

$$\therefore \quad V_{xo1} t = 10 \qquad (A)$$

$$t = 10/V_{xo1}$$

The height drop is as V_{zo} as start is zero,

$$\therefore \quad V_{zo1} t = H = \frac{1}{2} g t^2 \qquad (B)$$

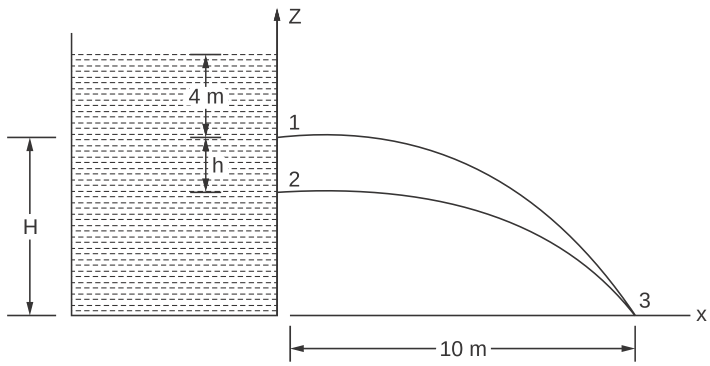

Figure P. 6.26 Problem model

Substituting for t

$$H = \frac{1}{2} g \frac{100}{V_{xo2}} \quad \therefore \quad V^2_{xo} = \frac{50g}{H}$$

As jet issues from the nozzle it has any x directional velocity V_{xo1}, is present.

$$V^2_{xo1} = 2\,g4 = 8g \qquad (C) \text{ (as head available in 4 m)}$$

Substituting, $8g = \frac{50g}{H}$ or $\mathbf{H = 6.25\ m.}$

Considering the second jet.

$$V_{xo2}\, t = 10,\ t = \frac{10}{V_{xo2}},$$

The head drop in $(H - h)$ m. As in the previous case $V_{zoc} = 0$ at start

$$H - h = \frac{1}{2} gt^2. \text{ Substituting}$$

$$H - h = \frac{1}{2} g \frac{1V_o}{V_{xo2}^{\ 2}} = \frac{50g}{V_{xo2}^{\ 2}} \qquad (D)$$

As at start only V_{xo2} is present,

$$V_{xo2}^{\ 2} = (4 + h)\, g \times 2$$

Substituting in (D)

$$H - h = \frac{50g}{(4+h)\, g \times 2} = \frac{25}{4+h}, \text{ as } \mathbf{H = 6.25\ m.}$$

$$6.25 - h = \frac{25}{4+h}. \text{ This leads to}$$

$$h^2 - 2.25h = 0, \quad \text{or} \quad \mathbf{h = 2.25\ m.}$$

It may be also noted from problem 6.22.

$$H \times 4 = (H - h)\,(4 + h).$$

$$6.25 \times 4 = 4 \times 6.25$$

Hence this condition is also satisfied.

OBJECTIVE QUESTIONS

O Q. 6.1. Fill in the blanks:

1. Kinetic energy of fluid element is due to its ——————————.
2. The amount of kinetic energy per kg is given by the expression —————————— the unit used being head of fluid.
3. The kinetic energy in the unit Nm/kg is given by the expression ——————————.
4. Potential energy of a fluid element is due to its ——————————.
5. Potential energy of a fluid element in head of fluid is given by ——————————.
6. Potential energy of a fluid element in Nm/kg is given by ——————————.
7. Pressure energy or flow energy of a fluid element is given in head of fluid by the expression ——————————.
8. Pressure energy or flow energy of a fluid element in the unit Nm/kg is given by the expression ——————————.
9. Internal energy is due to ——————————.
10. In the analysis of incompressible fluid flow, internal energy is rarely considered because ——————————.
11. Electrical and magnetic energy become important in the flow of ——————————.

Answers

(1) motion (2) $V^2/2g$, where V is the velocity (3) $V^2/2g_o$ where g_o is the force conversion constant having a unit of m kg/Ns^2 (4) location in the gravitational field. (5) Z, the elevation from datum (6) Zg/g_o, Z being elevation (7) $(P/\rho)\ (g_o/g) = p/\gamma.g_o$ where γ is specific weight. (8) P/ρ (9) the microscopic activity of atoms/molecules of the matter, exhibited by the temperature (10) temperature change is generally negligible (11) plasma.

O Q. 6.2. Fill in the blanks:

1. Eulers equation is applicable for flow along a ——————————.
2. Bernoulli equation is applicable for flows which are ——————————.
3. Bernoulli equation states that the total head ——————————.
4. Total head in a steady incompressible irrotational flow is the sum of ——————————.
5. In steady flow along a horizontal level as the velocity increases the pressure ——————————.
6. The pressure along the diverging section of a venturi ——————————.
7. Cavitation will occur when the pressure at a point ——————————.
8. Draft tube —————————— the available head in the case of reaction turbines.
9. Energy line along the flow —————————— if there are no losses.
10. Hydraulic grade line represents the sum of —————————— along the flow.
11. If a pump supplies energy to the flow the energy line —————————— .
12. If there are frictional losses the energy grade line will —————————— .

Answers

(1) stream line (2) incompressible, steady and irrotational (3) remains constant if there are no irreversibilities (4) dynamic head, pressure head and potential head (5) decreases (6) increases (7) goes below the vapour pressure of the fluid at that temperature. (8) increases (9) will be horizontal parallel to the flow (10) pressure and potential head (11) will increase by a step (12) dip.

O Q. 6.3. Indicate whether the statement is correct or incorrect.

1. Energy line along the direction of flow will dip if there are losses.
2. When a pump supplies energy to a flow stream, the energy line will decreases by a step.
3. For ideal flows the energy line will slope upward along the flow.
4. If velocity increases, the hydraulic grade line will dip along the flow direction.
5. If the differential manometer reading connected to a venturimeter is the same, the flow will be independent of the position or flow direction.
6. For the same reading of the differential manometer connected to a vertical venturimeter, the flow rate will be larger if flow is downwards.
7. A pitot probe connected perpendicular to flow will indicate the total head.
8. A pitot probe facing the flow will indicate the dynamic head.
9. A pitot probe facing the flow will indicate the total head.
10. A pitot-static tube has probes both facing the flow and perpendicular to flow.
11. Flow will take place along hydraulic gradient.
12. Flow will take place along energy gradient.

Answers

Correct – 1, 4, 5, 9, 10, 11 Incorrect – 2, 3, 6, 7, 8, 12

O Q. 6.4. Choose the correct answer:

1. For a free jet the maximum horizontal reach will depend on
   (*a*) the angle of projection only (*b*) the initial velocity only
   (*c*) the fluid flowing in the jet (*d*) the angle of projection and initial velocity.
2. Bernoulli equation is applicable for
   (*a*) steady rotational flow (*b*) steady rotational compressible flow
   (*c*) steady irrotational incompressible flow (*d*) unsteady irrotational incompressible flow
   (*e*) all flows.
3. In a steady flow along a stream line at a location in the flow, the velocity head is 6 m, the pressure head is 3 m, the potential head is 4 m. The height of hydraulic gradient line at this location will be
   (*a*) 13 m (*b*) 9 m (*c*) 10 m (*d*) 7 m
4. In a flow along a varying flow cross section, as the area decreases
   (*a*) the energy line will slope up
   (*b*) the hydraulic gradient line will slope up
   (*c*) the hydraulic gradient line will slope down
   (*d*) the energy line will slope down.
5. In a steady flow of incompressible fluid, as the diameter is doubled, the velocity will
   (*a*) be halved (*b*) be doubled
   (*c*) increase four fold (*d*) decrease four fold.
6. In steady flow in a varying section pipe if the diameter is doubled the kinetic energy will
   (*a*) be doubled (*b*) increase 4 times
   (*c*) increase 8 times (*d*) decrease to one sixteenth.

7. In a source type of flow, the kinetic energy along the radius will vary (constant thickness of fluid along radius)
   (*a*) proportional to radius
   (*b*) directly proportional to the square root of radius
   (*c*) inversely proportional to the square of radius
   (*d*) proportional to the fourth power of radius
8. In a vertical flow of incompressible fluid along a constant pipe section under steady conditions, the pressure along flow direction will
   (*a*) remain constant (*b*) decrease
   (*c*) increase (*d*) increase or decrease depending on the fluid.
9. The differential manometer connected to two points along a pipe line gives a reading of h m. The flow will be
   (*a*) highest if the pipe is horizontal
   (*b*) independent of the slope of pipe and direction of flow
   (*c*) highest if flow is downwards
   (*d*) will depend on the fluid.

Answers

(1) *d* (2) *c* (3) *d* (4) *c* (5) *d* (6) *d* (7) *c* (8) *b* (9) *b*.

O Q. 6.5. Match the sets

1.

Set A	Set B
1. Bernoulli equation	(*a*) potential function
2. Continuity equation	(*b*) stream line
3. Eulers equation	(*c*) total head
4. Laplace equation	(*d*) conservation of mass.

2.

Set A	Set B
1. potential energy	(*a*) plasma flow
2. kinetic energy	(*b*) temperature
3. internal energy	(*c*) position
4. electrical energy	(*d*) velocity.

Answers

(1) 1*c*, 2*d*, 3*b*, 4*a* (2) 1*c*, 2*d*, 3*b*, 4*a*

EXERCISE PROBLEMS

E 6.1. A pipe inclined at 45° to the horizontal converges from 0.2 m dia to 0.1 m at the top over a length of 2 m. At the lower end the average velocity is $2m/s$. Oil of specific gravity 0.84 flows through the pipe. Determine the pressure difference between the ends, neglecting losses. If a mercury manometer (specific gravity 13.6) is used to measure the pressure, determine the reading of the manometer difference in m of mercury. Oil fills the limbs over mercury in the manometer. **(36.854 N/m^2, 0.201m)**

E 6.2. Oil of specific gravity of 0.9 flows through a venturimeter of diameters 0.4 and 0.2 m. A U-tube mercury manometer shows a head 0.63 m. Calculate the flow rate. **(0.105 m^3/s)**

E 6.3. Water flows from a reservoir 240 m above the tip of a nozzle. The velocity at the nozzle outlet is 66 m/s. The flow rate is 0.13 m^3/s. Calculate (1) the power of the jet. (2) the loss in head due to friction. **(283.14 kW, 17.98 m)**

E 6.4. Water flows in the middle floor tap at 3 m/s. Determine the velocities at the taps in the other two floors shown in Fig. E. 6.4.

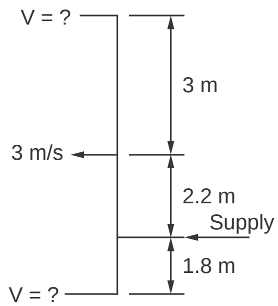

Figure E. 6.4

E 6.5. Oil flows through a horizontal pipe will line which has a diameter of 0.45 m at the start. After some distance the diameter of reduces to 0.3 m at which point the flow divides into pipes of 0.15 m and 0.225 m diameter. The velocity at the beginning is 1.8 m/s. The velocity in the pipe line of 0.225 m dia is 3.6 m/s. If the pressure at the start is 20 m head of oil and the specific gravity of the oil is 0.91 determine the pressure at the fork and also at the end of the two branch pipes. Neglect losses.

(V at fork = 4.05 m/s, $V_{0.15}$ = 8.1 m/s, P_{fork} = 13.33 m, $P_{0.15}$ = 16.821 m, $P_{0.225}$ = 19.5 m)

E. 6.6. A nozzle of 25 mm dia. directs a water jet vertically with a velocity of 12 m/s. Determine the diameter of the jet and the velocity at a height of 6 m. **(38.25 mm, 5.13 m/s)**

E 6.7. A pipe line is 36 m above datum. The pressure and velocity at a section are 410 kN/m^2 and 4.8 m/s. Determine the total energy per kg with reference to the datum. **(774.7 Nm.kg)**

E 6.8. The supply head to a water nozzle is 30 m gauge. The velocity of water leaving the nozzle is 22.5 m/s. Determine the efficiency and power that can be developed if the nozzle diameter is 75mm. **(84.3%, 25.2 kW)**

E 6.9. The suction pipe of a pump slopes at 1 m vertical for 5 m length. If the flow velocity in the pipe is 1.8 m/s and if the pressure in the pipe should not fall by more than 7 m of water, determine the maximum length. **(35.8 m)**

E 6.10. The pressure at the entry to the pipe line of 0.15 m dia. is 8.2 bar and the flow rate at this section is 7.5 m^3/min. The pipe diameter gradually increases to 0.3 m and the levels rises by 3 m above the entrance. Determine the pressure at the location. Neglect losses. **(8.14 bar)**

E 6.11. A tapering pipe is laid at a gradient of 1 in 100 downwards. The length is 300 m. The diameter reduces from 1.2 m to 0.6 m. The flow rate of water is 5500 l/min. The pressure at the upper location is 0.8 bar. Determine the pressure at the lower location. **(0.73 bar)**

E 6.12. A horizontal pipe carrying water is gradually tapering. At one section the diameter is 150 mm and flow velocity is 1.5 m/s. (*i*) If the drop in pressure is 1.104 bar at a reduced section determine the diameter at the section. (*ii*) If the drop in pressure is 5 kN/m^2, what will be the diameter? Neglect losses. **(47.6 mm, 100 mm)**

E 6.13. The diameter of a water jet at nozzle exist is 75 mm. If the diameter at a height of 12 m is 98.7mm, when the jet is directed vertically, determine the height to which the jet will rise. **(18 m)**

E 6.14. Calculate the height to which the jet, issuing at 18.8 m/s will rise when (*i*) The jet is directed vertically (*ii*) when it is directed at 45°. Also find the horizontal distance travelled in this case. **(18 m, 9 m, 9 m)**

E 6.15. A jet directed at 30° reaches a maximum height of 3 m at a horizontal distance of 18 m. Determine the issuing velocity of the jet. **(16.9 m/s)**

E 6.16. Determine the flow rate of a fluid of specific gravity 0.83 upward in the set up as shown in Fig. E. 6.16.

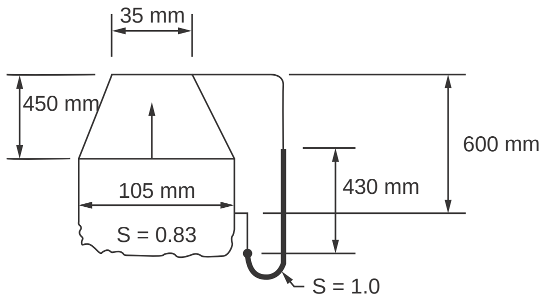

Figure E. 6.16

E 6.17. Determine the flow rate and also the pressure at point 2 in the siphon shown in Fig. E. 6.17. Diameter of the pipe is 2.5 cm. **(4.2 l/s, 45.1 kPa ab)**

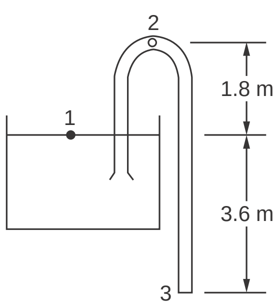

Figure E. 6.17

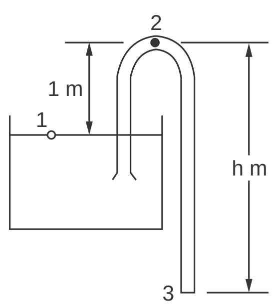

Figure E. 6.18

E 6.18. In the setup of siphon for water flow, shown in Fig. E. 6.18, determine the value of '*h*' and also the pressure at point 2, if velocity at 3 was 11.7 m/s. **(8 m, 22.8 kPa ab)**

E 6.19. A siphon is used to draw water from a tank. The arrangement is shown in Fig. E. 6.19. Calculate the flow rate and pressure at point 2. The frictional loss equals 40 $V^2/2g$. Atmospheric pressure is 10.2 m of water. **(43 l/min, 1.13 m of water)**

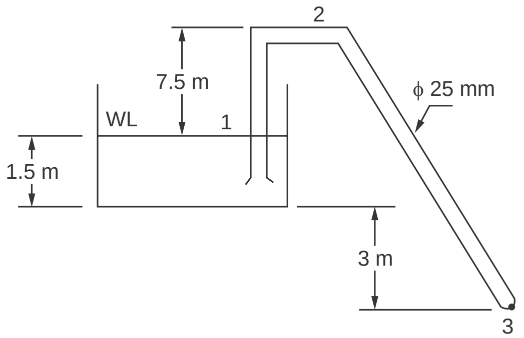

Figure E. 6.19

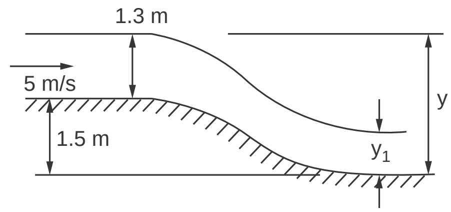

Figure E. 6.20

E 6.20. Water flows through a channel as shown in Fig. E. 6.20. Determine the possible values of depth of water down stream. Neglect losses. Assume uniform velocity of 5 m/s upstream.

E 6.21. Water flows up an inclined duct as shown in Fig. E. 6.21 Determine the possible depth of water upstream.

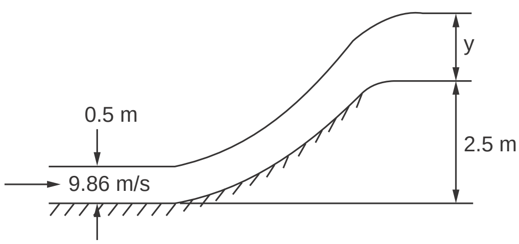

Figure E. 6.21

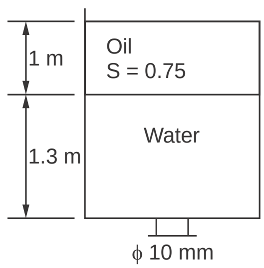

Figure E. 6.22

E 6.22. Neglecting losses, determine the flow rate in the setup shown in Fig. E. 6.22.

E 6.23. Derive an expression for the variation of jet radius *r* with distance *y* downwards for a jet directed downwards. The initial radius is *R* and the head of fluid is *H*.

E 6.24. For the venturimeter shown in Fig. E. 6.24, determine the flow rate of water.

E 6.25. A horizontal pipe divides into two pipes at angles as shown in Fig. E. 6.25. Determine the necessary forces along and perpendicular to the pipe to hold it in place. Assume that these are no losses.

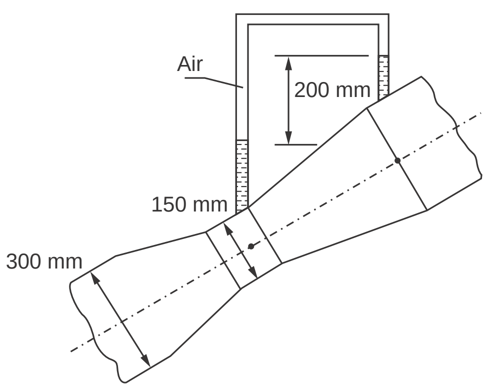

Figure E. 6.24

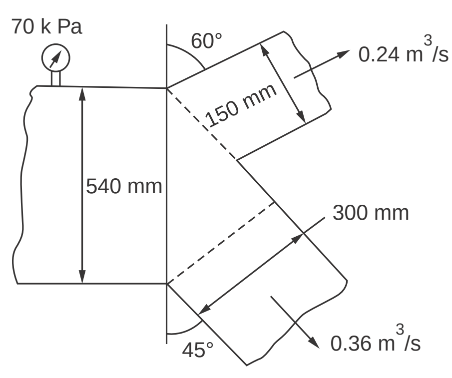

Figure E. 6.25