# 8 Dimensional Analysis

## 8.0 INTRODUCTION

Fluid flow is influenced by several parameters like, the geometry, fluid properties and fluid velocity. In the previous chapters analytical methods used in fluid flow studies were discussed. In the study of flow of real fluids analytical methods alone are found insufficient. Experimental methods and results have contributed heavily for the development of fluid mechanics. The solution of realistic problems usually involves both anlytical and experimental studies. Experiments are used to validate analytical results as well as generalize and extend their applications. Depending either solely on analytical methods or experiments for the design of systems is found to lead to inadequate performance and high cost.

Experimental work is rather costly and time consuming, particularly when more than three parameters are involved. Hence it is necessary to plan the experiments so that most information is obtained from fewest experiments. Dimensional analysis is found to be a very useful tool in achieving this objective. The mathematical method of dimensional analysis comes to our help in this situation. The number of parameters can be reduced generally to three by grouping relevant variables to form dimensionless parameters. In addition these groups facilitate the presentation of the results of the experiments effectively and also to generalize the results so that these can be applied to similar situations.

Flow through pipes can be considered as an example. Viscosity, density, flow velocity and diameter are found to influence the flow. If the effect of each of these parameters on flow is separately studied the number of experiments will be large. Also these results cannot be generalized and its usefulness will be limited. When the number of these variables are combined to form a dimensionless group like ($u\ D\ \rho/\mu$) few experiments will be sufficient to obtain useful information. This parameter can be varied by varying one of the variables which will be the easier one to vary, for example velocity $u$. The results will be applicable for various combinations of these parameters and so the results can be generalized and extended to new situations. The results will be applicable also for different fluids and different diameters provided the value of the group remains the same. Example 8.1 illustrates the advantage dimensional analysis in experiment planning. The use of the results of dimensional analysis is the basis for similitude and modal studies. The topic is discussed in the next chapter.

**Example 8.1.** *The drag force F on a stationary sphere in flow is found to depend on diameter D, velocity u, fluid density ρ and viscosity μ. Assuming that to study the influence of a parameter 10 experimental points are necessary, estimate the* ***total experimental points needed*** *to obtain complete information. Indicate how the number of experiments can be reduced.*

To obtain a curve $F$ *vs* $u$, for fixed values of ρ, μ and $D$, experiments needed = 10.

To study the effect of ρ these 10 experiments should be repeated 10 times with 10 values of ρ the total now being $10^2$.

The $10^2$ experiments have to repeated 10 times each for different values of μ.

Total experiments for $u$, ρ and μ = $10^3$.

To study the effect of variation of diameter all the experiments have to be repeated 10 times each. Hence total **experiments required = $10^4$.**

These parameters can be combined to obtain two dimensionless parameters,

$$\frac{F}{\rho u^2 D^2} = f\left(\frac{\rho\, uD}{\mu}\right)$$

(The method to obtain such grouping is the main aim of this chapter)

Now only 10 experiments are needed to obtain a comprehensive information about the effect of these five parameters.

Experiments can be conducted for obtaining this information by varying the parameter ($uD\rho/\mu$) and determining the values for $F/\rho u^2 D^2$. Note : It will be almost impossible to find fluids with 10 different densities and 10 different viscosities.

## 8.1 METHODS OF DETERMINATION OF DIMENSIONLESS GROUPS

**1. Intuitive method:** This method relies on basic understanding of the phenomenon and then identifying competing quantities like types of forces or lengths etc. and obtaining ratios of similar quantities.

Some examples are: Viscous force *vs* inertia force, viscous force *vs* gravity force or roughness dimension *vs* diameter. This is a difficult exercise and considerable experience is required in this case.

**2. Rayleigh method:** A functional power relation is assumed between the parameters and then the values of indices are solved for to obtain the grouping. For example in the problem in example 1 one can write

$$(\pi_1, \pi_2) = F^a\, \rho^b D^c \mu^d U^e$$

The values of *a, b, c, d*, and *e* are obtained by comparing the dimensions on both sides the dimensions on the L.H.S. being zero as π terms are dimensionless. This is also tedious and considerable expertise is needed to form these groups as the number of unknowns will be more than the number of available equations. This method is also called “indicial” method.

**3. Buckingham Pi theorem method:** The application of this theorem provides a fairly easy method to identify dimensionless parameters (numbers). However identification of the influencing parameters is the job of an expert rather than that of a novice. This method is illustrated extensively throughout this chapter.

## 8.2 THE PRINCIPLE OF DIMENSIONAL HOMOGENEITY

The principle is basic for the correctness of any equation. It states "If an equation truly expresses a proper relationship between variables in a physical phenomenon, then each of the additive terms will have the same dimensions or these should be dimensionally homogeneous."

For example, if an equation of the following form expresses a relationship between variables in a process, then each of the additive term should have the same dimensions. In the expression, $A + B = \boldsymbol{C}/D$, $A$, $B$ and $(C/D)$ each should have the same dimension. This principle is used in dimensional analysis to form dimensionless groups. Equations which are dimensionally homogeneous can be used without restrictions about the units adopted. Another application of this principle is the checking of the equations derived.

**Note :** Some empirical equations used in fluid mechanics may appear to be non homogeneous. In such cases, the numeric constants are dimensional. The value of the constants in such equations will vary with the system of units used.

## 8.3 BUCKINGHAM PI THEOREM

The statement of the theorem is as follows : If a relation among $n$ parameters exists in the form

$$f(q_1, q_2, \text{........} \ q_n) = 0$$

then the $n$ parameters can be grouped into $n - m$ independent dimensionless ratios or $\pi$ parameters, expressed in the form

$$g(\pi_1, \pi_2 \text{ ........ } \pi_{n-m}) = 0 \tag{8.3.1}$$

or $$\pi_1 = g_1 (\pi_2, \pi_3 \text{ ...... } \pi_{n-m})$$

where $m$ is the number of dimensions required to specify the dimensions of all the parameters, $q_1, q_2, \text{....} \ q_n$. It is also possible to form new dimensionless $\pi$ parameters as a discrete function of the $(n - m)$ parameters. For example if there are four dimensionless parameters $\pi_1$, $\pi_2$, $\pi_3$ and $\pi_4$ it is possible to obtain $\pi_5$, $\pi_6$ etc. as

$$\pi_5 = \frac{\pi_1}{\pi_3 \pi_4} \quad \text{or} \quad \pi_6 = \frac{\pi_1^{\,0.5}}{\pi_2^{\,2/3}}$$

The limitation of this exercise is that the exact functional relationship in equation 8.3.1 cannot be obtained from the analysis. The functional relationship is generally arrived at through the use of experimental results.

### 8.3.1 Determination of $\pi$ Groups

Irrespective of the method used the following steps will systematise the procedure.

**Step 1. List all the parameters that influence the phenomenon concerned.** This has to be very carefully done. If some parameters are left out, $\pi$ terms may be formed but experiments then will indicate these as inadequate to describe the phenomenon. If unsure the parameter can be added. Later experiments will show that the $\pi$ term with the doubtful parameters as useful or otherwise. Hence a careful choice of the parameters will help in solving the problem with least effort. Usually three type of parameters may be identified in fluid flow namely fluid properties, geometry and flow parameters like velocity and pressure.

**Step 2. Select a set of primary dimensions,** (mass, length and time), (force, length and time), (mass, length, time and temperature) are some of the sets used popularly.

**Step 3. List the dimensions of all parameters in terms of the chosen set** of primary dimensions. Table 8.3.1. Lists the dimensions of various parameters involved.

**Table 8.3.1. Units and Dimensions of Variables**

| Variable | Unit (SI) | Dimension | |
|---|---|---|---|
| | | MLT θ system | FLT θ system |
| Mass | kg | M | $FT^2/L$ |
| Length | m | L | L |
| Time | s | T | T |
| Force | N | $ML/T^2$ | F |
| Temperature | deg C or K | θ | θ |
| Area | $m^2$ | $L^2$ | $L^2$ |
| Volume | $m^3$ | $L^3$ | $L^3$ |
| Volume flow rate | $m^3/s$ | $L^3/T$ | $L^3/T$ |
| Mass flow rate | kg/s | M/T | FT/L |
| Velocity | m/s | L/T | L/T |
| Angular velocity | Rad/s | 1/T | 1/T |
| Force | N | $ML/T^2$ | F |
| Pressure, stress, | $N/m^2$ | $M/LT^2$ | $F/L^2$ |
| Bulk modulus | | | |
| Moment | Nm | $ML^2/T^2$ | FL |
| Work, Energy | J, Nm | $ML^2/T^2$ | FL |
| Power | W, J/s | $ML^2/T^3$ | FL/T |
| Density | $kg/m^3$ | $M/L^3$ | $FT^2/L^4$ |
| Dynamic viscosity | kg/ms, $Ns/m^2$ | M/LT | $FT/L^2$ |
| Kinematic viscosity | $m^2/s$ | $L^2/T$ | $L^2/T$ |
| Surface tension | N/m | $M/T^2$ | F/L |
| Specific heat | J/kg K | $L^2/T^2\ \theta$ | $L^2/T^2\theta$ |
| Thermal conductivity | W/mK | $ML/T^3\ \theta$ | F/Tθ |
| Convective heat | | | |
| transfer coefficient | $W/m^2$ K | $M/T^3\ \theta$ | F/LTθ |
| Expansion coefficient | (m/m)/K | 1/T | 1/T |

**Step 4. Select from the list of parameters a set of repeating parameters** equal to the number of primary dimensions. Some guidelines are necessary for the choice. (*i*) the chosen set should contain all the dimensions (*ii*) two parameters with same dimensions should not be chosen. say $L$, $L^2$, $L^3$, (*iii*) the dependent parameter to be determined should not be chosen.

**Step 5. Set up a dimensional equation with the repeating set and one of the remaining parameters,** in turn to obtain $n - m$ such equations, to determine $\pi$ terms numbering $n - m$. The form of the equation is,

$$\pi_1 = q_{m+1} \cdot q_1^{\,a} \cdot q_2^{\,b} \cdot q_3^{\,c} \,.....\, q_m^{\,d}$$

As the LHS term is dimensionless, an equation for each dimension in terms of $a, b, c, d$ can be obtained. The solution of these set of equations will give the values of $a, b, c$ and $d$. Thus the $\pi$ term will be defined.

**Step 6. Check whether $\pi$ terms obtained are dimensionless.** This step is essential before proceeding with experiments to determine the functional relationship between the $\pi$ terms.

**Example 8.2.** *The pressure drop $\Delta P$ per unit length in flow through a smooth circular pipe is found to depend on (i) the flow velocity, u (ii) diameter of the pipe, D (iii) density of the fluid* $\rho$, *and (iv) the dynamic viscosity* $\mu$.

(*a*) *Using $\pi$ theorem method, evaluate the dimensionless parameters for the flow.*

(*b*) *Using Rayleigh method (power index) evaluate the dimensionless parameters.*

Choosing the set mass, time and length as primary dimensions, the dimensions of the parameters are tabulated.

| S.No. | Parameter | Unit used | Dimension |
|---|---|---|---|
| 1 | Pressure drop/m, $\Delta P$ | ($N/m^2$/m ($N = kgm/s^2$) | $M/L^2T^2$ |
| 2 | Diameter, $D$ | m | $L$ |
| 3 | Velocity, $u$ | m/s | $L/T$ |
| 4 | Density, $\rho$ | kg/m$^3$ | $M/L^3$ |
| 5 | Dynamic viscosity, $\mu$ | kg/ms | $M/LT$ |

**There are five parameters and three dimensions. Hence two $\pi$ terms can be obtained**. As $\Delta P$ is the dependent variable $D$, $\rho$ and $\mu$ are chosen as repeating variables.

Let $\pi_1 = \Delta P\, D^a \rho^b u^c$, Substituting dimensions,

$$M^0L^0T^0 = \frac{M}{L^2T^2}\, L^a\, \frac{M^b}{L^{3b}}\, \frac{L^c}{T^c}$$

Using the principle of dimensional homogeneity, and in turn comparing indices of mass, length and time.

$$1 + b = 0 \quad \therefore \quad b = -1, \quad -2 + a - 3b + c = 0 \ \ \therefore \quad a + c = -1$$

$$-2 - c = 0 \quad \therefore \quad c = -2, \quad \text{Hence } a = 1.$$

Substituting the value of indices we obtain

Chapter 8

$\therefore \qquad \pi_1 = \Delta PD/\rho u^2;$

This represents the ratio of pressure force and inertia force.

Check the dimension :

$$\frac{M}{L^2T^2}\, L\, \frac{L^3}{M}\, \frac{T^2}{L^2} = M^0L^0T^0$$

Let $\pi_2 = \mu\, D^a\, \rho^b u^c$, substituting dimensions and considering the indices of $M$, $L$ and $T$,

$$M^0L^0T^0 = \frac{M}{LT}\, L^a\, \frac{M^b}{L^{3b}}\, \frac{L^c}{T^c}$$

$$1 + b = 0 \quad \text{or} \quad b = -1, \quad -1 + a - 3b + c = 0, \quad a + c = -2, -1 - c = 0, \quad c = -1 \quad a = -1$$

Substituting the value of indices,

$\therefore \qquad \pi_2 = \mu/u\rho D$

check, $$\frac{M}{LT}\, \frac{T}{L}\, \frac{L^3}{M}\, \frac{1}{L} = M^0L^0T^0$$

This term may be recognised as inverse of Reynolds number. So $\pi_2$ can be modified as $\pi_2 = \rho uD/\mu$ also $\pi_2 = (uD/v)$. The significance of this $\pi$ term is that it is the ratio of inertia force to viscous force. In case $D$, $u$ and $\mu$ had been choosen as the repeating, variables, $\pi_1 = \Delta PD^2/u\, \mu$ and $\pi_2 = \rho Du/\mu$. The parameter $\pi_1/\pi_2$ will give the dimensionless term. $\Delta P\, D/\rho u^2$. In this case $\pi_1$ represents the ratio pressure force/viscous force. This flow phenomenon is influenced by the three forces namely pressure force, viscous force and inertia force.

**Rayleigh method:** (Also called method of Indices). The following functional relationship is formed first. There can be two p terms as there are five variables and three dimensions.

$$\mathrm{D}P^aD^b\mathrm{r}^c\mathrm{m}^du^e = (\mathrm{p}_1\ \mathrm{p}_2), \text{ Substituting dimensions,}$$

$$\frac{M^a}{L^{2a}\, T^{2a}}\, L^b\, \frac{M^c}{L^{3c}}\, \frac{M^d}{L^dT^d}\, \frac{L^e}{T^e} = L^0\, M^0\, T^0$$

Considering indices of $M$, $L$ and $T$, three equations are obtained as below

$$a + c + d = 0, -2a + b - 3c - d + e = 0, -2a + d - e = 0$$

There are five unknowns and three equations. Hence some assumptions are necessary based on the nature of the phenomenon. As D$P$, the dependent variable can be considered to appear only once. We can assume $a = 1$. Similarly, studying the forces, m appears only in the viscous force. So we can assume $d = 1$. Solving $a = 1$, $d = 1$, $b = 0$, $c = -2$, $e = -3$, $(\mathrm{p}_1\ \mathrm{p}_2) = \mathrm{D}P\mathrm{m}/\mathrm{r}^2\, u^3$. Multiply and divide by $D$, then $\mathrm{p}_1 = \mathrm{D}PD/\mathrm{r}u^2$ and $\mathrm{p}_2 = \mathrm{m}/\mathrm{r}uD$. Same as was obtained by p theorem method. This method requires more expertise and understanding of the basics of the phenomenon.

**Example 8.3.** *The pressure drop $\Delta P$ in flow of incompressible fluid through rough pipes is found to depend on the length $l$, average velocity $u$, fluid density,* $\rho$, dynamic viscosity $\mu$, *diameter $D$ and average roughness height $e$.* ***Determine the dimensionless groups*** *to correlate the flow parameters.*

The variables with units and dimensions are listed below.

| S.No. | Variable | Unit | Dimension |
|---|---|---|---|
| 1 | $\Delta P$ | N/m$^2$ | $M/LT^2$ |
| 2 | $l$ | $L$ | $L$ |
| 3 | $u$ | m/s | $L/T$ |
| 4 | $\rho$ | kg/m$^3$ | $M/L^3$ |
| 5 | $\mu$ | kg/ms | $M/LT$ |
| 6 | $D$ | $L$ | $L$ |
| 7 | $e$ | $L$ | $L$ |

There are seven parameters and three dimensions. So four $\pi$ terms can be identified. Selecting $u$, $D$ and $\rho$ as repeating variables, (as these sets are separate equations, no problem will arise in using indices $a$, $b$ and $c$ in all cases).

Let $\pi_1 = \Delta P\, u^a D^b \rho^c,\ \pi_2 = L\, u^a D^b\, \rho^c,\ \pi_3 = \mu\, u^a\, D^b\, \rho^c,\ \pi_4 = e\, u^a D^b \rho^c$

Consider $\pi_1$, $M^0L^0T^0 = \dfrac{M}{LT^2}\dfrac{L^a}{T^a}L^b\dfrac{M^c}{L^{3c}}$

Equating the indices of $M$, $L$ and $T$,

$$1 + c = 0,\ c = -1,\ -1 + a + b - 3c = 0,\ -2 - a = 0, \quad a = -2,\ b = 0.$$

Substituting the value of indices we get

$\therefore$ $\boldsymbol{\pi_1 = \Delta P/\rho u^2}$

Consider $\pi_2$, $M^0L^0T^0 = L\dfrac{L^a}{T^a}L^b\dfrac{M^c}{L^{3c}}$

Equating indices of $M$, $L$ and $T$, $c = 0$, $1 + a + b - 3c = 0$, $a = 0$, $\therefore$ $b = -1$, $\therefore$ $\boldsymbol{\pi_2 = L/D}$

Consider $\pi_3$ $M^0L^0T^0 = \dfrac{M}{LT}\dfrac{L^a}{T^a}L^b\dfrac{M^c}{L^{3c}}$

Comparing the indices of $\mu$m, $L$ and $T$,

gives $1 + c = 0$ or $c = -1$, $-1 + a + b - 3c = 0$, $-1 - a = 0$ or $a = -1$, $\therefore$ $b = -1$

$\therefore$ $\boldsymbol{\pi_3 = \mu/\rho Du}$ **or** $\boldsymbol{\rho u D/\mu}$

Consider $\pi_4$, $M^0L^0T^0 = L\dfrac{L^a}{T^a}L^b\dfrac{M^c}{L^{3c}}$

This gives, $c = 0$, $1 + a + b - 3c = 0$, $-a = 0$, $b = -1$ $\therefore$ $\boldsymbol{\pi_4 = e/D}$

These $\pi$ terms may be checked for dimensionless nature.

The relationship can be expressed as $\dfrac{\Delta P}{\rho u^2} = f\left[\dfrac{L}{D}, \dfrac{e}{D}, \dfrac{\rho u D}{\mu}\right]$

## 8.4 IMPORTANT DIMENSIONLESS PARAMETERS

Some of the important dimensionless groups used in fluid mechanics are listed in Table 8.4.1. indicating significance and area of application of each.

**Table 8.4.1 Important Dimensionless Parameters**

| Name | Description | Significance | Applications |
|---|---|---|---|
| Reynolds Number, Re | $\rho uD/\mu$ or $uD/v$ | Inertia force/ Viscous force | All types of fluid dynamics problems |
| Froude Number Fr | $u/(gl)^{0.5}$ or $u^2/gl$ | Inertia force/ Gravity force | Flow with free surface (open channel and ships) |
| Euler Number Eu | $P/\rho u^2$ | Pressure force/ Inertia force | Flow driven by pressure |
| Cauchy Number Ca | $\rho u^2/E_v$ ($E_v$– bulk modulous) | Inertia force/ Compressibility force | compressible flow |
| Mach Number M | $u/c$, $c$–Velocity of sound | Inertia force/ Compressibility force | Compressible flow |
| Strouhal Number St | $\omega l/u$, $\omega$–Frequency of oscillation | Local inertia Force/ Convective inertia force | Unsteady flow with frequency of oscillation |
| Weber Number We | $\rho u^2 l/\sigma$, $\sigma$ = Surface tension | Inertia force/ Surface tension force | Problems influenced by surface tension free surface flow |
| Lift coefficient $C_L$ | $L/(1/2\ \rho Au^2)$ $L$ = lift force | Lift force/ Dynamic force | Aerodynamics |

## 8.5 CORRELATION OF EXPERIMENTAL DATA

Dimensional analysis can only lead to the identification of relevant dimensionless groups. The exact functional relations between them can be established only by experiments. The degree of difficulty involved in experimentation will depend on the number of $\pi$ terms.

## 8.5.1 Problems with One Pi Term

In this case a direct functional relationship will be obtained but a constant $c$ has to be determined by experiments. The relationship will be of the form $\pi_1 = c$. This is illustrated in example 8.4.

**Example 8.4.** *The drag force acting on a spherical particle of diameter D falling slowly through a viscous fluid at velocity u is found to be influenced by the diameter D, velocity of fall u, and the viscosity μ. Using the method of dimensional analysis obtain a relationship between the variables.*

The parameters are listed below using $M, L, T$ dimension set.

| S.No. | Parameter | Symbol | Unit | Dimension |
|---|---|---|---|---|
| 1 | Drag Force | $F$ | $N$ or kgm/s$^2$ | $ML/T^2$ |
| 2 | Diameter | $D$ | $m$ | $L$ |
| 3 | Velocity | $u$ | m/s | $L/T$ |
| 4 | Viscosity | $\mu$ | kg/ms | $M/LT$ |

There are four parameters and three dimensions. Hence only one π term will result.

$$\pi_1 = F\, D^a\, u^b\, \mu^c, \text{ Substituting dimensions,}$$

$$M^0L^0T^0 = \frac{ML}{T^2}\, L^a\, \frac{L^b}{T^b}\, \frac{M^c}{L^cT^c}\,, \qquad \text{Equating indices of } M, L \text{ and } T$$

$$0 = 1 + c,\ c = -1,\ 1 + a + b - c = 0,\ 2 + b + c = 0,\ b = -1,\ c = -1$$

$$\boldsymbol{\pi_1 = F/uD\mu \qquad \therefore\ \ F/uD\mu = \text{constant} = c}$$

or $F = cuD\mu$ or drag force varies directly with velocity, diameter and viscosity. A single test will provide the value of the constant. However, to obtain a reliable value for $c$, the experiments may have to be repeated changing the values of the parameters.

In this case an approximate solution was obtained theoretically for $c$ as $3\pi$. Hence drag force $F$ in free fall is given by $F = 3\pi\mu uD$. This can be established by experiments.

This relation is known as Stokes law valid for small values of Reynolds Number (Re << 1). This can be used to study the settling of dust in still air. Inclusion of additional variable, namely density will lead to another π term.

## 8.5.2 Problems with Two Pi Terms

In example 8.2 two π terms were identified. If the dimensional analysis is valid then a single universal relationship can be obtained. Experiments should be conducted by varying one of the group say $\pi_1$ and from the measurement the values of the other group $\pi_2$ is calculated. A suitable graph (or a computer program) can lead to the functional relationship between the π terms. Linear semilog or log/log plots may have to be used to obtain such a relationship. The valid range should be between the two extreme values used in the experiment. Extrapolation may lead to erroneous conclusions. This is illusration by example 8.5.

**Example 8.5.** *In order to determine the pressure drop in pipe flow per m length an experiment was conducted using flow of water at 20°C through a 20 mm smooth pipe of length 5 m. The variation of pressure drop observed with variation of velocity is tabulated below.The density of water = 1000 kg/m$^3$. Viscosity = $1.006 \times 10^{-3}$ kg/ms.*

| Velocity, m/s | 0.3 | 0.6 | 0.9 | 1.5 | 2.0 | 3 | 5 |
|---|---|---|---|---|---|---|---|
| Pressure drop, *N* | 404 | 1361 | 2766 | 6763 | 11189 | 22748 | 55614 |

Determine the functional relationship between the dimensionless parameters ($D\,\Delta P/\rho u^2$) and ($\rho uD/\mu$).

Using the data the two π parameters together with log values are calculated and tabulated below.

| $u$ | 0.3 | 0.6 | 0.9 | 1.5 | 2.0 | 3 | 5 |
|---|---|---|---|---|---|---|---|
| $D\Delta P/\rho u^2$ | 0.01798 | 0.01512 | 0.01366 | 0.01202 | 0.01119 | 0.01011 | 0.00890 |
| $\rho uD/\mu$ | 5964 | 11928 | 17894 | 29821 | 39761 | 59642 | 99400 |
| logRe | 3.78 | 4.08 | 4.25 | 4.48 | 4.6 | 4.78 | 4.997 |
| log($D\Delta P/\rho u$) | – 1.745 | – 1.821 | – 1.865 | – 1.92 | – 1.951 | – 1.995 | – 2.051 |

A plot of the data is shown in Fig. Ex. 8.5 (*a*). The correlation appears to be good. Scatter may indicate either experimental error or omission of an influencing parameter. As the direct plot is a curve., fitting an equation can not be done from the graph. A log log plot results in a straight line, as shown in the Fig. 8.5 (*b*). To fit an equation the following procedure is used.

The slope is obtained by taking the last values:

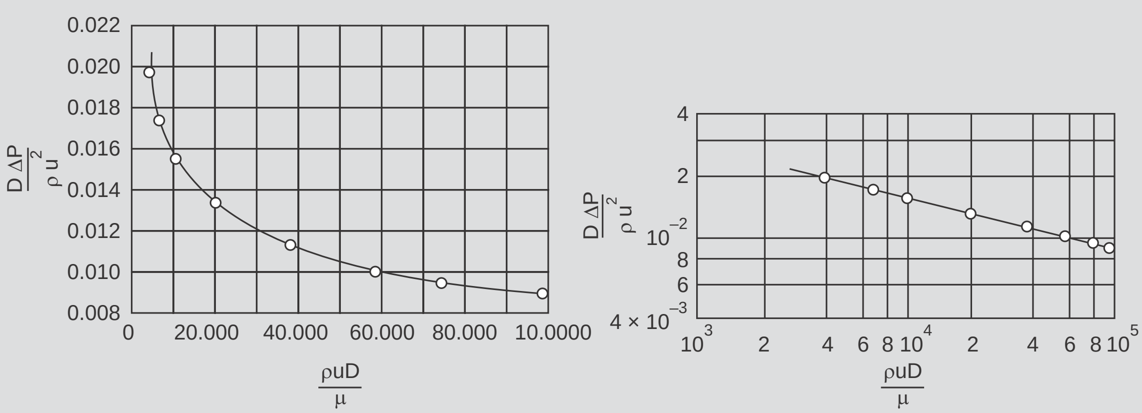
$$= \{-2.051 - (-1.745)\}/(4.997 - 3.78) = -0.2508$$

When extrapolating we can write, the slope using the same $-2.051 - (x)/(5 - 0) = -0.2508$

This gives $\quad x = -0.797.$

This corresponds to the value of 0.16. Hence we can write,

$$\frac{D\Delta P}{\rho u^2} = 0.16\left(\frac{PuD}{\mu}\right)^{-0.2508} = 0.16 \times \text{Re}^{-0.2508}$$




(a) (b)

**Figure Ex. 8.5**

### 8.5.3 Problems with Three Dimensionless Parameters

In this case experiments should be conducted for different constant values of $\pi_3$, varying $\pi_1$ and calculating the corresponding values of $\pi_2$. Such a set of experiments will result in curves of the form shown in Fig. 8.5.3.

These curves can also be converted to show the variation of $\pi_1$ with $\pi_3$ at constant values of $\pi_2$ by taking sections at various values of $\pi_2$. By suitable mathematical techniques correlation of the form below can be obtained.

$$\pi_2 = c\pi_1^{n_1}\,\pi_2^{n_2}$$

**When there are more than three π terms, two of these should be combined and the numbers reduced to three**. The procedure as described above can then be used to obtain the functional relationship.

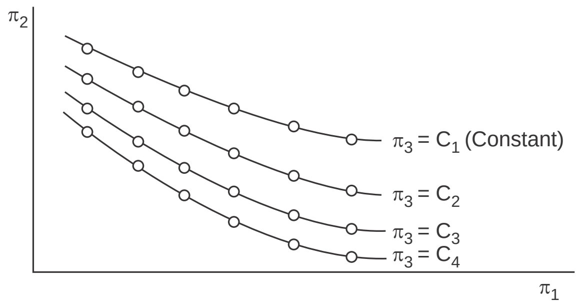



**Figure 8.5.1**

## SOLVED PROBLEMS

**Problem 8.1.** *The pressure drop ΔP in flow through pipes per unit length is found to depend on the average velocity μ, diameter D, density of the fluid ρ, and viscosity μ. Using FLT set of dimensions evaluate the dimensionless parameters correlating this phenomenon.*

The dimensions of the influencing parameters are tabulated below choosing FLT set.

| S.No. | Variables | Unit | Dimensions |
|---|---|---|---|
| 1 | Pressure drop per unit length, $\Delta P/l$ | $(N/m^2)/m$ | $F/L^3$ |
| 2 | Diameter, $D$ | m | $L$ |
| 3 | Velocity, $u$ | m/s | $L/T$ |
| 4 | Density, $\rho$ | $kg/m^3$ | $FT^2/L^4$ |
| 5 | Viscosity, $\mu$ | $Ns/m^2$ | $FT/L^2$ |

As there are five variables and three dimensions, two π terms can be obtained.

Using *D, u* and ρ as repeating parameters,

Let $$\pi_1 = \Delta P d^a u^b \rho^c \quad \text{or} \quad F^0 L^0 T^0 = \frac{F}{L^3} L^a \frac{L^b}{T^b} \frac{F^c\ T^{2c}}{L^{4c}}$$

Comparing the indices of *M, L* and *T* solving for *a, b* and *c*,

$$1 + c = 0, -3 + a + b - 4c = 0, -b + 2c = 0$$

∴ $$c = -1, b = -2, a = 1$$

Substituting the value of indices

∴ $$\boldsymbol{\pi_1 = D\Delta P/\rho u^2}$$

Let, $$\pi_2 = \mu D^a\ u^b\ \rho^c, \quad \text{or} \quad F^0\ L^0\ T^0 = \frac{F}{L^2} L^a \frac{L^b}{T^b} \frac{F^c\ T^{2c}}{L^{4c}}$$

Comparing the value of indices for *M, L* and *T*

∴ $$1 + c = 0, -2 + a + b - 4c = 0, 1 - b + 2c = 0$$

Solving, $a = -1$, $b = -1$, $c = -1$ substituting the values of $a$, $b$, $c$, $d$

∴ $$\boldsymbol{\pi_2 = \mu/\rho u D} \quad \text{or} \quad \boldsymbol{\rho u D/\mu}$$

∴ $$\frac{D\Delta P}{\rho u^2} = f\left[\frac{\rho u D}{\mu}\right]$$

The result is the same as in example 8.2. The dimension set choosen should not affect the final correlation.

**Problem 8.2.** *The drag force on a smooth sphere is found to be affected by the velocity of flow, u, the dimaeter D of the sphere and the fluid properties density* ρ *and viscosity* μ. *Using dimensional analysis obtain the dimensionless groups to correlate the parameters.*

The dimensions of the influencing variables are listed below, using M, L, T set.

| S.No. | Variables | Unit | Dimensions |
|---|---|---|---|
| 1 | Drag force, $F$ | $N$, (kgm/s$^2$) | $ML/T^2$ |
| 2 | Diameter, $D$ | m | $L$ |
| 3 | Velocity, $u$ | m/s | $L/T$ |
| 4 | Density, ρ | kg/m$^3$ | M/L$^3$ |
| 5 | Viscosity, μ | kg/ms | $M/LT$ |

**There are five variables and three dimensions. So two π terms can be obtained**. Choosing *D, u* and ρ as repeating variables,

Let $$\pi_1 = F\ D^a\ u^b\ \rho^c, \quad \text{or} \quad M^0 L^0 T^0 = \frac{ML}{T^2} L^a \frac{L^b}{T^b} \frac{M^c}{L^{3c}}$$

Comparing the values of indices for $M$, $L$ and $T$

$$1 + c = 1, \quad \therefore \quad c = -1, 1 + a + b - 3c = 0, -2 - b = 0$$

$\therefore$ $$b = -2, a = -2$$

Substituting the values of $a$, $b$, $c$

$\therefore$ $$\boldsymbol{\pi_1 = F/\rho u^2 D^2}$$

Let $$\pi_2 = \mu D^a u^b \rho^c \quad \text{or} \quad M^0L^0T^0 = \frac{M}{LT} L^a \frac{L^b}{T^b} \frac{M^c}{L^{3c}}$$

Comparing the values of indices of $M$, $L$ and $T$

$\therefore$ $$1 + c = 0, -1 + a + b - 3c = 0, -1 - b = 0 \quad \therefore \quad c = -1, b = -1, a = -1$$

Susbtituting the values of $a$, $b$, $c$.

$\therefore$ $$\boldsymbol{\pi_2 = \mu/\rho u D \quad \text{or} \quad \rho u D/\mu}$$

$\therefore$ $$\frac{F}{\rho u^2 D^2} = f\left[\frac{\rho\, uD}{\mu}\right];$$ Check for dimensions of $\pi_1$ and $\pi_2$.

$$\pi_1 = \frac{ML}{T^2} \frac{L^3}{M} \frac{T^2}{L^2} \frac{1}{L^2} = M^0L^0T^0 \quad \text{or} \quad \pi_2 = \frac{M}{L^3} \frac{L}{T} L \frac{LT}{M} = M^0L^0T^0$$

**Note:** the significance of the $\pi$ term. $F/\rho u^2D^2 \rightarrow F/\rho u\, Du \rightarrow F/mu \rightarrow$ Drag force/inertia force.

**Problem 8.3.** *The thrust force, F generated by a propeller is found to depend on the folllowing parameters: diameter D, forward velocity u, density ρ, viscosity μ and rotational speed N. Determine the dimensionless parameters to correlate the phenomenon.*

The influencing parameters with dimensions are listed below using $MLT$ set.

| S.No. | Parameters | Unit | Dimensions |
|---|---|---|---|
| 1 | Thrust force, $F$ | N | $ML/T^2$ |
| 2 | Diameter, $D$ | m | $L$ |
| 3 | Forward velocity, $u$ | m/s | $L/T$ |
| 4 | Density, $\rho$ | kg/m$^3$ | $M/L^3$ |
| 5 | Viscosity, $\mu$ | kg/ms | $M/LT$ |
| 6 | Rotational speed, $N$ | 1/s | $1/T$ |

**There are 6 variables and three dimensions. So three $\pi$ terms can be obtained.** Choosing $D$, $u$ and $\rho$ as repeating variables,

Let $$\pi_1 = F\, u^a D^b \rho^c, \quad \text{or} \quad M^0L^0T^0 = \frac{ML}{T^2} \frac{L^a}{T^a} L^b \frac{M^c}{L^{3c}}$$

Comparing indices of $M$, $L$ and $T$

$\therefore \quad 1 + c = 0,\ 1 + a + b - 3c = 0,\ -2 - a = 0$

$\therefore \quad a = -2,\ b = -2,\ c = -1$

Substituting the values of $a$, $b$, and $c$

$\therefore \quad \boldsymbol{\pi_1 = F/u^2D^2\,\rho}$, (Thrust force/Inertia force)

Let $\quad \pi_2 = \mu\, u^a D^b \rho^c \quad \text{or} \quad M^0L^0T^0 = \dfrac{M}{LT}\,\dfrac{L^a}{T^a}\,L^b\,\dfrac{M^c}{L^{3c}}$

Comparing the indices $M$, $L$ and $T$

$\therefore \quad 1 + c = 0,\ -1 + a + b - 3c = 0,\ -1 - a = 0$

$\therefore \quad a = -1,\ b = -1,\ c = -1$

Substituting the values of $a$, $b$ and $c$

$\therefore \quad \boldsymbol{\pi_2 = \mu/\rho u D \quad \text{or} \quad \rho u D/\mu}$ (Inertia force/Viscous force)

Let $\quad \pi_3 = N\, u^a\, D^b\, \rho^c, \quad \text{or} \quad M^0L^0T^0 = \dfrac{1}{T}\,\dfrac{L^a}{T^a}\,L^b\,\dfrac{M^c}{L^{3c}}$

Comparing the indices of $M$, $L$ and $T$

$c = 0,\ a + b - 3c = 0,\ -1 - a = 0, \quad \therefore \quad a = -1,\ b = 1$

Susbtituting the values of $a$, $b$ and $c$

$\therefore \quad \boldsymbol{\pi_3 = ND/u}$ (Rotational speed/Forward speed)

$$\therefore \quad F/u^2D^2\,\rho = f\left[\frac{uD\rho}{\mu}, \frac{ND}{u}\right]$$

**Problem 8.4.** *At higher speeds where compressibility effects are to be taken into account the performance of a propeller in terms of force exerted is influenced by the diameter, forward speed, rotational speed, density, viscosity and bulk modulus of the fluid. Evaluate the dimensionless parameters for the system.*

The influencing parameters and dimensions are tabulated below, using $M$, $L$, $T$ set.

| S.No. | Parameters | Unit | Dimensions |
|---|---|---|---|
| 1 | Force, $F$ | N/m$^2$ | M/LT$^2$ |
| 2 | Diameter, $D$ | m | $L$ |
| 3 | Forward velocity, $u$ | m/s | $L/T$ |
| 4 | Rotational speed, N | l/s | $1/T$ |
| 5 | Density, $\rho$ | kg/m$^3$ | $M/L^3$ |
| 6 | Viscosity, $\mu$ | kg/ms | $M/LT$ |
| 7 | Bulk Modulus, $E$ | (m$^3$/m$^3$)N/m$^2$ | $M/LT^2$ |

**There are seven variables and three dimensions, So four π terms are possible. Selecting D, u and ρ as repeating parameters,**

Let $\pi_1 = F\,\rho^a u^b d^c$, or $M^0L^0T^0 = \dfrac{M}{LT^2}\dfrac{M^a}{L^{3a}}\dfrac{L^b}{T^b}L^c$

The general procedure is to compare the indices of $M$, $L$ and T on both sides and from equations.

$$1 + a = 0, -1 - 3a + b + c = 0, -2 - b = 0$$

∴ $c = 0, b = -2, a = -1$

∴ $\pi_1 = \mathbf{F/\rho u^2}$ → (force exerted/inertia force)/m²

Let $\pi_2 = N\rho^a u^b D^c$, or $M^0L^0T^0 = \dfrac{1}{T}\dfrac{M^a}{L^{3a}}\dfrac{L^b}{T^b}L^c$

∴ $a = 0, -3a + b + c = 0, -1 - b = 0$

∴ $a = 0, b = -1, c = 1.$

$\pi_2 = \mathbf{ND/u}$ (or rotational speed/forward speed)

Let $\pi_3 = \mu\rho^a u^b D^c$, or $M^0L^0T^0 = \dfrac{M}{LT}\dfrac{M^a}{L^{3a}}\dfrac{L^b}{T^b}L^c$

∴ $1 + a = 0, -1 - 3a + b + c = 0, -1 - b = 0$

∴ $a = -1, b = -2, c = -1,$

∴ $\pi_3 = \mathbf{\mu/\rho uD}$ **or** $\mathbf{\rho uD/\mu}$ (Reynolds number)

Let $\pi_4 = E\rho^a u^b D^c$, or $M^0L^0T^0 = \dfrac{M}{LT^2}\dfrac{M^a}{L^{3a}}\dfrac{L^b}{T^b}L^c$

∴ $1 + a = 0, -1 - 3a + b + c = 0, -2 - b = 0$

∴ $a = -1, b = -2, c = 0$

$\pi_4 = \mathbf{E/\rho u^2}$ (Compressibility force/inertia force)

$$\therefore \quad \frac{F}{\rho u^2} = f\left[\frac{ND}{u}, \frac{\rho uD}{\mu}, \frac{E}{\rho u^2}\right]$$

**Problem 8.5.** *Using dimensional analysis, obtain a correlation for the frictional torque due to rotation of a disc in a viscous fluid. The parameters influencing the torque can be identified as the diameter, rotational speed, viscosity and density of the fluid.*

The influencing parameters with dimensions are listed below, using $M, L, T$ set.

| S.No. | Parameters | Unit | Dimensions |
|---|---|---|---|
| 1 | Torque, τ | Nm | $ML^2/T^2$ |
| 2 | Diameter, $D$ | m | $L$ |
| 3 | Rotational speed, $N$ | $l$/s | $1/T$ |
| 4 | Density, ρ | kg/m³ | $M/L^3$ |
| 5 | Viscosity, μ | kg/ms | $M/LT$ |

**There are five variables and three dimensions. So two $\pi$ parameters can be identified.**

Considering $D$, $N$ and $\rho$ as repeating variables.

Let $\pi_1 = \tau\, D^a N^b \rho^c$ or $M^0L^0T^0 = \dfrac{ML^2}{T^2} L^a \dfrac{1}{T^b} \dfrac{M^c}{L^{3c}}$

$\therefore$ $1 + c = 0,\ 2 + a - 3c = 0,\ -2 - b = 0$ $\therefore$ $c = -1,\ b = -2,\ a = -5,$

$\therefore$ $\boldsymbol{\pi_1 = \tau/\rho N^2 D^5}$

Let $\pi_2 = \mu\, D^a N^b \rho^c$ or $M^0L^0T^0 = \dfrac{M}{LT} L^a \dfrac{1}{T^b} \dfrac{M^c}{L^{3c}}$

$\therefore$ $1 + c = 0,\ -1 + a - 3c = 0,\ -1 - b = 0$ $\therefore$ $c = -1,\ b = -1,\ a = -2$

$\therefore$ $\boldsymbol{\pi_2 = \mu/\rho D^2 N}$**,** (Another form of Reynolds number, as DN $\rightarrow u$)

$\therefore$ $\dfrac{\tau}{\rho N^2 D^5} = f\left[\dfrac{\mu}{\rho D^2 N}\right]$ Check for the dimensions of $\pi_1$ and $\pi_2$

**Note:** Rotational speed can also be expressed as angular velocity, $\omega$. In that case $N$ will be replaced by $\omega$ as the dimension of both these variables is $1/T$.

**Problem 8.6.** *A rectangular plate of height, a and width, b is held perpendicular to the flow of a fluid. The drag force on the plate is influenced by the dimensions a and b, the velocity u, and the fluid properties, density* $\rho$ *and viscosity* $\mu$. *Obtain a correlation for the drag force in terms of dimensionless parameters.*

The parameters with dimensions are listed adopting $M$, $L$, $T$ set of dimensions.

| S.No. | Parameters | Unit | Dimensions |
|---|---|---|---|
| 1 | Drag force, $F$ | N | $ML/T^2$ |
| 2 | Width, $b$ | m | $L$ |
| 3 | Height, $a$ | m | $L$ |
| 4 | Velocity, $u$ | m/s | $L/T$ |
| 5 | density, $\rho$ | kg/m$^3$ | $M/L^3$ |
| 6 | Viscosity, $\mu$ | kg/ms | $M/LT$ |

**There are 6 parameters and three dimensions. Hence three $\pi$ terms can be obtained.** Selecting $b$, $u$ and $\rho$ as repeating variables.

Let $\pi_1 = F\, b^a u^b \rho^c$ or $M^0L^0T^0 = \dfrac{ML}{T^2} L^a \dfrac{L^b}{T^b} \dfrac{M^c}{L^{3c}}$

$\therefore$ $1 + c = 0,\ 1 + a + b - 3c = 0,\ -2 - b = 0$

$\therefore$ $c = -1,\ b = -2,\ a = -2$

$\therefore$ $\boldsymbol{\pi_1 = F/\rho u^2 b^2}$

Let $\quad \pi_2 = a\, b^a u^b \rho^c \quad \text{or} \quad M^0L^0T^0 = LL^a \dfrac{L^b}{T^b}\dfrac{M^c}{L^{3c}}$

$c = 0,\ 1 + a + b - 3c = 0,\ -b = 0,$

$\therefore \quad a = -1 \quad \therefore \quad \boldsymbol{\pi_2 = a/b}$

Let $\quad \boldsymbol{\pi_3 = \mu b^a u^b \rho^c} \quad \text{or} \quad M^0L^0T^0 = \dfrac{M}{LT} L^a \dfrac{L^b}{T^b}\dfrac{M^c}{L^{3c}}$

$\therefore \quad 1 + c = 0,\ -1 + a + b - 3c = 0,\ -1 - b = 0.$

$\therefore \quad b = -1,\ c = -1,\ b = -1$

$\therefore \quad \boldsymbol{\pi_3 = \mu/\rho ub \quad \text{or} \quad \pi_3 = \rho ub/\mu}$

$$\therefore \quad \frac{F}{\rho u^2 b^2} = f\left[\frac{a}{b}, \frac{\rho\, ub}{\mu}\right]$$

$\pi_3$ is Reynolds number based on length $b$. $\pi_1$ is (drag force/unit area)/inertia force.

**Problem 8.7.** *In film lubricated journal bearings, the frictional torque is found to depend on the speed of rotation, viscosity of the oil, the load on the projected area and the diameter. Evaluate dimensionless parameters for application to such bearings in general.*

The variables with dimensions are listed below, adopting $MLT$ set.

| S.No. | Variable | Unit | Dimensions |
|---|---|---|---|
| 1 | Frictional Torque, $\tau$ | Nm | $ML^2/T^2$ |
| 2 | Speed, $N$ | 1/s | $1/L$ |
| 3 | Load per unit area, $P$ | N/m$^2$ | $M/LT^2$ |
| 4 | Diameter, $D$ | m | $L$ |
| 5 | Viscosity, $\mu$ | kg/ms | $M/LT$ |

**There are five parameters and three dimensions. Hence two $\pi$ parameters can be found.** Considering $N$, $D$ and $\mu$ as repeating variables,

Let $\quad \pi_1 = \tau N^a D^b \mu^c \quad \text{or} \quad M^0L^0T^0 = \dfrac{ML^2}{T^2}\dfrac{1}{T^a} L^b \dfrac{M^c}{L^cT^c}$

$\therefore \quad 1 + c = 0,\ 2 + b - c = 0,\ -2 - a - c = 0 \quad \therefore \quad c = -1,\ a = -1,\ b = -3$

$\therefore \quad \boldsymbol{\pi_1 = \tau/N\mu D^3} \quad$ Also $\pi = \tau/\mu uD \quad$ ($\tau$–Torque)

Let $\quad \pi_2 = PN^aD^b\mu^c \quad \text{or} \quad M^0L^0T^0 = \dfrac{M}{LT^2}\dfrac{1}{T^a} L^b \dfrac{M^c}{L^cT^c}$

$\therefore \quad 1 + c = 0,\ -1 + b - c = 0,\ -2,\ -a - c = 0$

$\therefore \quad c = -1,\ a = -1,\ b = 0$

$$\therefore \quad \boldsymbol{\pi_2 = P/N\mu,} \quad \therefore \quad \frac{\tau}{N\mu D^3} = f\left[\frac{P}{N\mu}\right]$$

**Note :** $P/N\mu$ is also Reynolds number, try to verify.

**Problem 8.8.** *Obtain a relation using dimensional analysis, for the resistance to uniform motion of a partially submerged body (like a ship) in a viscous compressible fluid.*

The resistance can be considered to be influenced by skin friction forces, buoyant forces and compressibility of the fluid.

The variables identified as affecting the situation are listed below using *MLT* set.

| S.No. | Variable | Unit | Dimensions |
|---|---|---|---|
| 1 | Resistance to motion, $R$ | N/m$^2$ | $M/LT^2$ |
| 2 | Forward velocity, $u$ | m/s | $L/T$ |
| 3 | Length of the body, $l$ | m | $L$ |
| 4 | Density of the fluid, $\rho$ | kg/ms | $M/L^3$ |
| 5 | Viscosity, of the fluid, $\mu$ | kg/ms | $M/LT$ |
| 6 | Gravity, $g$ | m/s$^2$ | $L/T^2$ |
| 7 | Bulk modulus, $E_v$ | N/m$^2$ | $M/LT^2$ |

**There are seven parameters and three dimensions. So four $\pi$, terms are possible**. Considering velocity, density and length as repeating variables.

Let $\quad \pi_1 = Ru^a\rho^b l^c \quad \text{or} \quad M^0L^0T^0 = \dfrac{M}{LT^2}\dfrac{L^a}{T^a}\dfrac{M^b}{L^{3b}}L^c$

$\therefore \quad 1 + b = 0, -1 + a - 3b + c = 0, \quad -2 - a = 0$

$\therefore \quad a = -2, b = -1 \text{ and } c = 0$

$\therefore \quad \boldsymbol{\pi_1 = R/\rho u^2}$, Euler number.

Let $\quad \pi_2 = \mu\, u^a\, \rho^b\, l^c \quad \text{or} \quad M^0L^0T^0 = \dfrac{M}{LT}\dfrac{L^a}{T^a}\dfrac{M^b}{L^{3b}}L^c$

$\therefore \quad 1 + b = 0, -1 + a - 3b + c = 0, -2 - a = 0$

$\therefore \quad a = -1, b = -1 \quad \text{and} \quad c = -1$

$\therefore \quad \boldsymbol{\pi_2 = \mu/u\rho l.}$

Let $\quad \pi_3 = gu^a\rho^b l^c \quad \text{or} \quad M^0L^0T^0 = \dfrac{L}{T^2}\dfrac{L^a}{T^a}\dfrac{M^b}{L^{3b}}L^c$

$\therefore \quad b = 0, 1 + a - 3b + c = 0, -2 - a = 0$

$\therefore \quad a = -2, b = 0 \quad \text{and} \quad c = 1$

$\therefore \quad \boldsymbol{\pi_3 = gl/u^2} \rightarrow$ can also be expressed as $u/(\text{gL})^{0.5}$ (Froude number.)

Let $\quad \pi_4 = E_v u^a\rho^b l^c \quad \text{or} \quad M^0L^0T^0 = \dfrac{M}{LT^2}\dfrac{L^a}{T^a}\dfrac{M^b}{L^{3b}}L^c$

$\therefore \quad 1 + b = 0, -1 + a - 3b + c = 0, -2 - a = 0$

$\therefore \quad a = -2, b = -1 \text{ and } c = 0$

$\therefore \quad \boldsymbol{\pi_4 = E_v/\rho u^2}$

$$\frac{R}{\rho u^2} = f\left[\frac{\mu}{ul\rho}, \frac{gl}{u^2}, \frac{E_v}{\rho u^2}\right] \quad \text{or}$$

Euler number = $f$ (Reynolds number, Froude number and Mach number)

In the case of incompressible flow, this will reduce to

$$\frac{R}{\rho u^2} = f\left[\frac{ul\rho}{\mu}, \frac{u}{(gl)^{0.5}}\right] = f(\text{Re, Fr})$$

**Problem 8.9.** *The velocity of propagation of pressure wave, c through a fluid is assumed to depend on the fluid density ρ and bulk modulus of the fluid $E_v$. Using dimensional analysis obtain an expression for c in terms of ρ and $E_v$.*

**This is a case were there will be a direct relationship between the variables or one π term.**

**Note:** The definition of the bulk modulus is $dp/(dv/v)$, the dimension being that of pressure, $M/LT^2$, Writing $c = f(\rho, E_v)$

Let, $\quad \pi_1 = c\,\rho^a E_v^{\,b} \quad$ or $\quad M^0L^0T^0 = \dfrac{L}{T}\dfrac{M^a}{L^a}\dfrac{M^b}{L^b\,T^{2b}}$

$\therefore \quad a + b = 0,\ 1 - 3a - b = 0,\ -1 - 2b = 0, \quad \therefore \quad b = -0.5,\ a = 0.5$

$\therefore \quad \boldsymbol{\pi_1 = c(\rho/E_v)^{0.5}}, \quad$ or $\quad c = \text{const} \times (E_v/\rho)^{0.5}$

**Problem 8.10.** *Obtain a correlation for the coefficient of discharge through a small orifice, using the method of dimensional analysis.*

The following list of parameters can be identified as affecting the coefficient of discharge

| S.No. | Parameters | Unit | Dimensions |
|---|---|---|---|
| 1 | Diameter, $D$ | m | $L$ |
| 2 | Head, $H$ | m | $L$ |
| 3 | Gravity, $g$ | m/s$^2$ | $L/T^2$ |
| 4 | Density of the fluid, ρ | kg/m$^3$ | $M/L^3$ |
| 5 | Roughness height, $k$ | m | $L$ |
| 6 | Surface tension, σ | N/m | $M/T^2$ |
| 7 | Viscosity, μ | kg/ms$^2$ | $M/LT$ |

**There are seven variables and three dimensions. So four π terms can be identified.** Considering ρ, $g$ and $H$ as repeating variables

Let $\quad \pi_1 = D\,\rho^a g^b H^c \quad$ or $\quad M^0L^0T^0 = L\,\dfrac{M^a}{L^{3a}}\dfrac{L^b}{T^{2b}}\,L^c$

$\therefore \quad a = 0,\ 1 - 3a + b + c = 0,\ -2b = 0,$

$\therefore \quad c = -1 \quad \therefore \quad \boldsymbol{\pi_1 = D/H \quad \text{or} \quad H/D}$

Let $\pi_2 = k\rho^a g^b H^c$ or $M^0L^0T^0 = L\,\dfrac{M^a}{L^{3a}}\,\dfrac{L^b}{T^{2b}}\,L^c$

$\therefore$ $a = 0,\ 1 - 3a + b + c = 0,\ -2b = 0,$

$\therefore$ $a = 0,\ b = 0,\ c = -1 \quad \therefore \quad \boldsymbol{\pi_2 = k/H}$

Let $\pi_3 = \sigma\rho^a g^b H^c$ or $M^0L^0T^0 = \dfrac{M}{T^2}\,\dfrac{M^a}{L^{3a}}\,\dfrac{L^b}{T^{2b}}\,L^c$

$\therefore$ $a + 1 = 0,\ -3a + b + c = 0,\ -2 - 2b = 0,$

$\therefore$ $a = -1,\ b = -1,\ c = -2$

$\therefore$ $\boldsymbol{\pi_3 = \sigma/\rho g H^2}$

Let $\pi_4 = \mu\rho^a g^b H^c$ or $M^0L^0T^0 = \dfrac{M}{LT}\,\dfrac{M^a}{L^{3a}}\,\dfrac{L^b}{T^{2b}}\,L_c$

$\therefore$ $a + 1 = 0,\ -1 - 3a + b + c = 0,\ -1 - 2b = 0,$

$\therefore$ $a = -1,\ b = -1/2,\ c = -1.5.$

$\therefore$ $\boldsymbol{\pi_4 = \mu/(\rho H\sqrt{gH}\,).}$ As $C_d$ is dimensionless

$$C_d = f\left[\frac{D}{H}, \frac{k}{H}, (\sigma/\rho\, gH^2), \frac{\mu}{(\rho H\sqrt{gH}\,)}\right]$$

Check the dimensions of these $\pi$ terms.

**Problem 8.11.** *The volume flow rate of a gas through a sharp edged orifice is found to be influenced by the pressure drop, orifice diameter and density and kinematic viscosity of the gas. Using the method of dimensional analysis obtain an expression for the flow rate.*

The variables and dimensions are listed below, adopting MLT system

| S.No. | Variable | Unit | Dimensions |
|---|---|---|---|
| 1 | Volume flow rate, $Q$ | $m^3/s$ | $L^3/T$ |
| 2 | Pressure drop, $\Delta P$ | $N/m^2$ | $M/LT^2$ |
| 3 | Diameter, $D$ | m | $L$ |
| 4 | Density, $\rho$ | $kg/m^3$ | $M/L^3$ |
| 5 | Kinematic viscosity, $v$ | $m^2/s$ | $L^2/T$ |

**There are five parameters and three dimensions. So two $\pi$ terms can be obtained.** Choosing $\Delta P$, $D$ and $\rho$ as repeating variables,

Let $\pi_1 = Q\,\Delta P^a D^b \rho^c$ or $M^0L^0T^0 = \dfrac{L^3}{T}\,\dfrac{M^a}{L^aT^{2a}}\,L^b\,\dfrac{M^c}{L^{3c}}$

$\therefore$ $a + c = 0,\ 3 - a + b - 3c = 0,\ -1 - 2a = 0, \quad \therefore \quad a = -(1/2),\ c = 1/2,\ b = -2$

$\therefore$ $\boldsymbol{\pi_1 = (Q/D^2)\ (\rho/\Delta P)^{1/2}}$

Let $\pi_2 = v\Delta P^a D^b \rho^c$ or $M^0L^0T^0 = \dfrac{L^2}{T}\dfrac{M^a}{L^aT^{2a}}L^b\dfrac{M^c}{L^{3c}}$

$\therefore$ $a + c = 0,\ 2 - a + b - 3c = 0,\ -1 - 2a = 0,$

$\therefore$ $a = -(1/2),\ c = (1/2),\ b = -1$

$\therefore$ $\boldsymbol{\pi_2 = (v/D)\ (\rho/\Delta P)^{1/2}}$ or $\dfrac{Q}{D^2}\left(\dfrac{\rho}{\Delta P}\right)^{1/2} = f\left[\dfrac{v}{D}\left(\dfrac{\rho}{\Delta P}\right)^{1/2}\right]$

**Note :** $\pi_2$ can be also identified as Reynolds number. Try to verify.

**Problem 8.12.** *In flow through a sudden contraction in a circular duct the head loss h is found to depend on the inlet velocity u, diameters D and d and the fluid properties density* $\rho$ *and viscosity* $\mu$ *and gravitational acceleration, g. Determine dimensionless parameters to correlate experimental results.*

The influencing variables with dimensions are tabulated below with *MLT* set.

| S.No. | Variable | Unit | Dimensions |
|---|---|---|---|
| 1 | Loss of head, $h$ | m | $L$ |
| 2 | Inlet diameter, $D$ | m | $L$ |
| 3 | Outlet diameter, $d$ | m | $L$ |
| 4 | Velocity, $u$ | m/s | $L/T$ |
| 5 | Density, $\rho$ | kg/m$^3$ | $M/L^3$ |
| 6 | Viscosity, $\mu$ | kg/ms | $M/LT$ |
| 7 | Gravitational acceleration, $g$ | m/s$^2$ | $L/T^2$ |

**There are seven variables and three dimensions. Hence four $\pi$ parameters can be found.** Considering $D$, $\rho$ and $u$ as repeating variables,

Let $\pi_1 = h\,D^a\,\rho^b\,u^c$ or $M^0L^0T^0 = LL^a\dfrac{M^b}{L^{3b}}\dfrac{L^c}{T^c}$

$\therefore$ $b = 0,\ 1 + a - 3b + c = 0,\ c = 0 \quad \therefore \quad a = -1 \quad \therefore \quad \boldsymbol{\pi_1 = h/D}$

Let $\pi_2 = d\,D^a\,\rho^b\,u^c$ or $M^0L^0T^0 = LL^a\dfrac{M^b}{L^{3b}}\dfrac{L^c}{T^c}$

$\therefore$ $b = 0,\ c = 0,\ 1 + a - 3b + c = 0,\ a = -1 \quad \therefore \quad \pi_2 = \mathbf{d/D}$

Let $\pi_3 = \mu\,D^a\,\rho^b\,u^c$ or $M^0L^0T^0 = \dfrac{M}{LT}L^a\dfrac{M^b}{L^{3b}}\dfrac{L^c}{T^c}$

$\therefore$ $b + 1 = 0,\ -1 + a - 3b + c = 0,\ -1 - c = 0,$

$\therefore$ $b = -1,\ c = -1,\ a = -1$

$\therefore$ $\boldsymbol{\pi_3 = \mu/D\rho u}$ **or** $\boldsymbol{\rho Du/\mu}$

Let $\pi_4 = g\,D^a\,\rho^b\,u^c$ or $M^0L^0T^0 = \dfrac{L}{T^2}L^a\dfrac{M^b}{L^{3b}}\dfrac{L^c}{T^c}$

$\therefore$ $b = 0,\ 1 + a - 3b + c = 0,\ -2 - c = 0$

$\therefore$ $c = -2,\ a = 1 \quad \therefore \quad \boldsymbol{\pi_4 = gD/u^2}$

$$\therefore \quad \frac{h}{D} = f\left[\frac{d}{D}, \frac{\rho D u}{\mu}, \frac{gD}{u^2}\right]$$

**Note :** $gD/u^2$ is the ratio of Potential energy to Kinetic energy.

**Problem 8.13.** *The volume flow rate, Q over a V-notch depends on fluid properties namely density ρ, kinematic viscosity v, and surface tension σ. It is also influenced by the angle of the notch, head of fluid over the vertex, and acceleration due to gravity. Determine the dimensionless parameters which can correlate the variables.*

As θ, the notch angle is a dimensionless parameter, the other parameters are listed below with dimensions, adopting *MLT* set.

| S.No. | Variable | Unit | Dimension |
|---|---|---|---|
| 1 | Density, ρ | kg/m$^3$ | $M/L^3$ |
| 2 | kinematic vicosity, $v$ | m$^2$/s | $L^2/T$ |
| 3 | Surface tension, σ | N/m | $M/T^2$ |
| 4 | Head of fluid, $h$ | m | $L$ |
| 5 | Gravitational acceleration, $g$ | m/s$^2$ | $L/T^2$ |
| 6 | Flow rate, $Q$ | m$^3$/s | $L^3/T$ |

**There are six parameters and three dimensions. So three π terms can be identified.** Considering ρ, $g$ and $h$ as repeating variables.

Let $\pi_1 = Q\,\rho^a\, g^b\, h^c$ or $M^0L^0T^0 = \dfrac{L^3}{T}\,\dfrac{M^a}{L^{3a}}\,\dfrac{L^b}{T^{2b}}\,L^c$

$\therefore$ $a = 0,\ 3 - 3a + b + c = 0,\ -1 - 2b = 0$

$\therefore$ $b = -0.5,\ c = 2.5 \quad \therefore \quad \boldsymbol{\pi_1 = Q/g^{1/2}\,h^{5/2}}$

Let $\pi_2 = v\rho^a\, g^b\, h^c$ or $M^0L^0T^0 = \dfrac{L^2}{T}\,\dfrac{M^a}{L^{3a}}\,\dfrac{L^b}{T^{2b}}\,L^c$

$\therefore$ $a = 0,\ 2 - 3a + b + c = 0,\ -1 - 2b = 0$

$\therefore$ $b = -0.5,\ c = (-1.5) \quad \therefore \quad \boldsymbol{\pi_2 = v/g^{1/2}\,h^{3/2}}$

Let $\pi_3 = \sigma\,\rho^a\, g^b\, h^c$ or $M^0L^0T^0 = \dfrac{M}{T^2}\,\dfrac{M^a}{L^{3a}}\,\dfrac{L^b}{T^{2b}}\,L^c$

$\therefore$ $1 + a = 0,\ -3a + b + c = 0,\ -2 - 2b = 0 \quad \therefore \quad a = -1,\ b = -1,\ c = -2$

$$\therefore \quad \boldsymbol{\pi_3 = \sigma/\rho g h^2} \quad \therefore \quad Q = g^{1/2}\, h^{5/2}\, f\left[\frac{v}{g^{1/2}\,h^{3/2}}, \frac{\sigma}{\rho\, g h^2}, \theta\right]$$

**Note :** In case surface tension is not considered, $\pi_3$ will not exist. $\pi_2$ can be identified as Reynolds number.

**Problem 8.14.** *The capillary rise h is found to be influenced by the tube diameter D, density ρ, gravitational acceleration g and surface tension σ. Determine the dimensionless parameters for the correlation of experimental results.*

The variables are listed below adopting *MLT* set of dimensions.

| S.No. | Variable | Unit | Dimension |
|---|---|---|---|
| 1 | Diameter, $D$ | m | $L$ |
| 2 | Density, $\rho$ | kg/m$^3$ | $M/L^3$ |
| 3 | Gravitational acceleration, $g$ | m/s$^2$ | $L/T^2$ |
| 4 | Surface tension, $\sigma$ | N/m | $M/T^2$ |
| 5 | Capillary rise, $h$ | m | $L$ |

**There are five parameters and three dimensions and so two π parameters can be identified.** Considering $D$, ρ and $g$ as repeating variables,

Let $\pi_1 = h\, D^a\, \rho^b\, g^c$ or $M^0L^0T^0 = LL^a \dfrac{M^b}{L^{3b}} \dfrac{L^c}{T^{2c}}$

$\therefore$ $b = 0,\ 1 + a - 3b + c = 0,\ -2c = 0$

$\therefore$ $a = -1,\ b = 0,\ c = 0 \quad \therefore \quad \boldsymbol{\pi_1 = h/D}$

Let $\pi_2 = \sigma\, D^a \rho^b\, g^c$ or $M^0L^0T^0 = \dfrac{M}{T^2} L^a \dfrac{M^b}{L^{3b}} \dfrac{L^c}{T^{2c}}$

$\therefore$ $1 + b = 0,\ a - 3b + c = 0,\ -2 - 2c = 0 \quad \therefore \quad b = -1,\ c = -1$, and $a = -2$

$\therefore$ $\boldsymbol{\pi_2 = \sigma/D^2\, g\rho}$, $g\rho$ can also be considered as specific weight γ

$$\frac{h}{D} = f\left[\frac{\sigma}{D^2\gamma}\right],$$

**Note :** $\pi_2$ can be identified as 1/Weber number.

**Problems 8.15.** *Show that the power P, developed by a hydraulic turbine can be correlated by the dimensionless parameters* $P/\rho N^3 D^5$ *and* $N^2D^2/gh$*, where ρ is the density of water and N is the rotational speed, D is the runner diameter, h is the head and g is acceleration due to gravity.*

The parameters with dimensions are tabulated below using *MLT* set.

| S.No. | Variable | Unit | Dimension |
|---|---|---|---|
| 1 | Power, $P$ | W | $ML^2/T^3$ |
| 2 | Density, $\rho$ | kg/m$^3$ | $M/L^3$ |
| 3 | Speed, $N$ | 1/s | $1/T$ |
| 4 | Diameter, $D$ | m | $L$ |
| 5 | Head, $h$ | m | $L$ |
| 6 | Gravitational acceleration, $g$ | m/s$^2$ | $L/T^2$ |

**There are six parameters and three dimensions. So three $\pi$ terms can be found.** Choosing $\rho$, $D$ and $N$ as repeating variables,

Let $\pi_1 = P\,\rho^a\,D^b\,N^c$ or $M^0L^0T^0 = \dfrac{ML^2}{T^3}\,\dfrac{M^a}{L^{3a}}\,L^b\,\dfrac{1}{T^c}$

$\therefore$ $1 + a = 0,\ 2 - 3a + b = 0,\ -3 - c = 0 \quad \therefore \quad a = -1,\ c = -3,\ b = -5$

$\therefore$ $\boldsymbol{\pi_1 = P/\rho N^3 D^5}$ **(Power coefficient)**

Let $\pi_2 = h\,\rho^a\,D^b\,N^c$ or $M^0L^0T^0 = L\,\dfrac{M^a}{L^{3a}}\,L^b\,\dfrac{1}{T^c}$

$\therefore$ $a = 0,\ 1 - 3a + b = 0,\ c = 0, \quad \therefore \quad b = -1 \quad \therefore \quad \boldsymbol{\pi_2 = h/D.}$

Let $\pi_3 = g\,\rho^a\,D^b\,N^c$ or $M^0L^0T^0 = \dfrac{L}{T^2}\,\dfrac{M^a}{L^{3a}}\,L^b\,\dfrac{1}{T^c}$

$\therefore$ $a = 0,\ 1 - 3a + b = 0,\ -2 - c = 0$

$\therefore$ $c = -2,\ b = -1 \quad \therefore \quad \boldsymbol{\pi_3 = g/DN^2}$

$\boldsymbol{\pi_2 \times \pi_3 = gh/D^2N^2}$ **(Head coefficient)**

$$\therefore \quad \frac{P}{\rho N^3\,D^5} = f\left[\frac{gh}{D^2N^2}\right]$$

In this expression the first term is called power coefficient and the second one is called head coefficient. These are used in model testing of turbo machines.

**Problem 8.16.** *The power developed by hydraulic machines is found to depend on the head h, flow rate Q, density ρ, speed N, runner diameter D, and acceleration due to gravity, g. Obtain suitable dimensionless parameters to correlate experimental results.*

The parameters with dimensions are listed below, adopting *MLT* set of dimensions.

| S.No. | Variable | Unit | Dimension |
|---|---|---|---|
| 1 | Power, $P$ | W | $ML^2/T^3$ |
| 2 | Head, $h$ | m | $L$ |
| 3 | Flow rate, $Q$ | $m^3/s$ | $L^3/T$ |
| 4 | Density, $\rho$ | $kg/m^3$ | $M/L^3$ |
| 5 | Speed, $N$ | 1/s | $1/T$ |
| 6 | Diameter, $D$ | m | $L$ |
| 7 | Acceleration due to gravity, $g$ | $m/s^2$ | $L/T^2$ |

**There are seven variables and three dimensions and hence four $\pi$ terms can be formed.** Taking $\rho$, $D$ and $g$ as repeating variables.

Let $\pi_1 = P\,\rho^a\,D^b\,g^c$ or $M^0L^0T^0 = \dfrac{ML^2}{T^3}\,\dfrac{M^a}{L^{3a}}\,L^b\,\dfrac{L^c}{T^{2c}}$

$\therefore$ $1 + a = 0,\ 2 - 3a + b + c = 0,\ -3 - 2c = 0$

$\therefore$ $a = -1,\ c = -3/2 \quad \therefore \quad b = -7/2$

$\therefore$ $\boldsymbol{\pi_1 = P/\rho D^{7/2}\, g^{3/2}}$

Let $\pi_2 = h\, \rho^a\, D^b\, g^c$ or $M^0L^0T^0 = L\, \dfrac{M^a}{L^{3a}}\, L^b\, \dfrac{L^c}{T^{2c}}$

$\therefore$ $a = 0,\ 1 - 3a + b + c = 0,\ -2c = 0$

$\therefore$ $a = 0,\ b = -1,\ c = 0 \quad \therefore \quad \boldsymbol{\pi_2 = h/D}$

Let $\pi_3 = Q\, \rho^a\, D^b\, g^c$ or $M^0L^0T^0 = \dfrac{L^3}{T}\, \dfrac{M^a}{L^{3a}}\, L^b\, \dfrac{L^c}{T^{2c}}$

$\therefore$ $a = 0,\ 3 - 3a + b + c = 0,\ -1 - 2c = 0$

$\therefore$ $a = 0,\ c = -1/2,\ b = -5/2$

$\therefore$ $\boldsymbol{\pi_3 = Q/g^{1/2}\, D^{5/2}}$

Let $\pi_4 = N\, \rho^a\, D^b\, g^c$ or $M^0L^0T^0 = \dfrac{1}{T}\, \dfrac{M^a}{L^{3a}}\, L^b\, \dfrac{L^c}{T^{2c}}$

$\therefore$ $a = 0,\ -3a + b + c = 0,\ -1 - 2c = 0 \quad \therefore \quad a = 0,\ c = -1/2,\ b = 1/2$

$\therefore$ $\boldsymbol{\pi_4 = ND^{1/2}/g^{1/2}}$

The coefficients popularly used in model testing are given below. These can be obtained from the above four $\pi$ terms.

**1. Head coefficient** $\dfrac{gh}{N^2D^2} = \dfrac{\pi_2}{\pi_4{}^2} = \dfrac{hg}{DN^2\, D} = \dfrac{gh}{N^2D^2}$

**2. Flow coefficient** $\dfrac{Q}{ND^3} = \dfrac{\pi_3}{\pi_4} = \dfrac{Qg^{1/2}}{g^{1/2}\, D^{5/2}\, ND^{1/2}} = \dfrac{Q}{N\, D^3}$

**3. Power coefficient** $\dfrac{P}{\rho N^3\, D^5} = \dfrac{\pi_1}{\pi_4{}^3} = \dfrac{Pg^{3/2}}{\rho D^{7/2}\, g^{3/2}\, N^3\, D^{3/2}} = \dfrac{P}{\rho\, N^3 D^5}$

**4. Specific speed based on Q, for pumps, $\mathbf{N_{sp}}$**

$$\frac{N\sqrt{Q}}{(gh)^{3/4}} = \frac{(\text{flow coeff})^{1/2}}{(\text{head coeff})^{3/4}} = \frac{Q^{1/2}}{N^{1/2}D^{3/2}}\,\frac{N^{3/2}D^{3/2}}{(gh)^{3/4}} = \frac{N\sqrt{Q}}{(gh)^{3/4}}$$

(dimensional specific speed $N\sqrt{Q}/h^{3/4}$ is commonly used as mostly water is the fluid used)

**5. Specific speed based on power, for Turbines**

$$N_{st} = \frac{N\sqrt{P}}{\rho^{1/2}\,(gh)^{5/4}} = \frac{(\text{power coefficient})^{1/2}}{(\text{head coefficient})^{5/4}} = \frac{P^{1/2}(ND)^{5/2}}{\rho^{1/2}\, N^{3/2}\, D^{5/2}\,(gh)^{5/4}} = \frac{N\sqrt{P}}{\rho^{1/2}\,(gh)^{5/4}}$$

$\left(\text{Dimensional Specific speed } \dfrac{N\sqrt{P}}{h^{5/4}} \text{ is commonly used as water is used in most cases}\right)$

These are the popularly used dimensionally numbers in hydraulic turbo machinery.

**Problem 8.17.** *In forced convection in pipes heat transfer coefficient h is found to depend on thermal conductivity, viscosity, density, specific heat, flow velocity and the diameter. Obtain dimensionless parameters to correlate experimental results.*

The variables with dimensions are listed below using $MLT\,\theta$ set of dimensions, where $\theta$ is temperature.

| S.No. | Variable | Unit | Dimension |
|---|---|---|---|
| 1 | Convection coefficient, $h$ | W/m²K | $M/T^3\,\theta$ |
| 2 | Diameter $D$ | m | $L$ |
| 3 | Thermal conductivity, $k$ | W/mK | $ML/T^3\,\theta$ |
| 4 | Density, $\rho$ | kg/m³ | $M/L^3$ |
| 5 | Viscosity, $\mu$ | kg/ms | $M/LT$ |
| 6 | Specific heat, $c$ | Nm/kgK | $L^2/T^2\,\theta$ |
| 7 | Flow velocity, $u$ | m/s | $L/T$ |

**Three π terms are possible as there are seven variables and four dimensions.** Choosing $k$, $\mu$, $\rho$ and $D$ as repeating variables.

Let $\pi_1 = h\,k^a\,\mu^b\,\rho^c\,D^d$ or $M^0L^0T^0\,\theta^0 = \dfrac{M}{T^3\theta}\,\dfrac{M^a}{T^{3a}}\,\dfrac{L^a}{\theta^a}\,\dfrac{M^b}{L^bT^b}\,\dfrac{M^c}{L^{3c}}\,L^d$

$\therefore$ $1 + a + b + c = 0,\ a - b - 3c + d = 0,\ -3 - 3a - b = 0,\ -1 - a = 0$

$\therefore$ $a = -1,\ b = 0,\ c = 0,\ d = 1$

$\therefore$ $\boldsymbol{\pi_1 = hD/k}$ **(Nusselt number)**

Let $\pi_2 = u\,k^a\,\mu^b\,\rho^c\,D^d$ or $M^0L^0T^0\,\theta^0 = \dfrac{L}{T}\,\dfrac{M^a}{T^{3a}}\,\dfrac{L^a}{\theta^a}\,\dfrac{M^b}{L^bT^b}\,\dfrac{M^c}{L^{3c}}\,L^d$

$\therefore$ $a + b + c = 0,\ 1 + a - b - 3c + d = 0,\ -1 - 3a - b = 0,\ -a = 0,$

$\therefore$ $a = 0,\ b = -1,\ c = 1,\ d = 1$

$\therefore$ $\boldsymbol{\pi_2 = u\rho D/\mu}$ **(Reynolds number)**

Let $\pi_3 = c k^a\,\mu^b\,\rho^c\,D^d$ or $M^0L^0T^0\,\theta^0 = \dfrac{L^2}{T^2\theta}\,\dfrac{M^a}{T^{3a}}\,\dfrac{L^a}{\theta^a}\,\dfrac{M^b}{L^bT^b}\,\dfrac{M^c}{L^{3c}}\,L^d$

$\therefore$ $a + b + c = 0,\ 2 + a - b - 3c + d = 0,\ -2 - 3a - b = 0,\ -1 - a = 0,$

$\therefore$ $a = -1,\ b = 1,\ c = 0,\ d = 0$

$\therefore$ $\boldsymbol{\pi_3 = c\mu/k}$ **(Prandtl number),** $\dfrac{hD}{k} = f\left[\dfrac{uD\rho}{\mu}, \dfrac{c\mu}{k}\right]$

These are popular dimensionless numbers in convective heat transfer.

**Problem 8.18.** *The temperature difference $\theta$ at a location x at time $\tau$ in a slab of thickness L originally at a temperature difference $\theta_0$ with outside is found to depend on the thermal diffusivity $\alpha$, thermal conductivity k and convection coefficient h. Using dimensional analysis determine the dimensionless parameters to correlate the situation.*

The influencing parameters with dimensions are listed below, choosing $MLT\,\theta$ set.

| S.No. | Parameter | Unit | Dimension |
|---|---|---|---|
| 1 | Slab thickness, $L$ | m | $L$ |
| 2 | Location distance, $x$ | m | $L$ |
| 3 | Initial temperature difference, $\theta_0$ | deg K | $\theta_0$ |
| 4 | Temperature difference at time $\tau$, $\theta$ | deg K | $\theta$ |
| 5 | Time, $\tau$ | s | $T$ |
| 6 | Thermal diffusivity, $\alpha$ | $m^2/s$ | $L^2/T$ |
| 7 | Thermal conductivity, $k$ | W/mK | $ML/T^3\theta$ |
| 8 | Convection coefficient, $h$ | $W/m^2$ K | $M/T^3\theta$ |

**There are eight variables and four dimensions. Hence four $\pi$ terms can be identified.** Choosing $\theta_0$, $L$, $\alpha$ and $k$ as repeating variables,

Let $\pi_1 = \theta\, \theta_0^{\,a}\, L^b\, \alpha^c\, k^d$ or $M^0L^0T^0\theta^0 = \theta\, \theta^a\, L^b\, \dfrac{L^{2c}}{T^c}\, \dfrac{M^d\, L^d}{T^{3d}\, \theta^d}$

$\therefore$ $d = 0,\ b + 2c + d = 0,\ -c - 3d = 0,\quad 1 + a - d = 0,$

$\therefore$ $d = 0,\ c = 0,\ b = 0,\ a = -1$

$\therefore$ $\boldsymbol{\pi_1 = \theta/\theta_0}$

Let $\pi_2 = x\, \theta_0^{\,a}\, L^b\, \alpha^c\, k^d$ or $M^0L^0T^0\theta^0 = L\theta^a L^b\, \dfrac{L^{2c}}{T^c}\, \dfrac{M^d\, L^d}{T^{3d}\, \theta^d}$

$\therefore$ $a - d = 0,\ d = 0,\ 1 + b + 2c + d = 0,\ -c - 3d = 0,$

$\therefore$ $a = 0,\ b = -1,\ c = 0,\ d = 0$

$\therefore$ $\boldsymbol{\pi_2 = x/L}$

Let $\pi_3 = h\, \theta_0^{\,a}\, L^b\, \alpha^c\, k^d$ or $M^0L^0T^0\theta^0 = \dfrac{M}{T^3\theta}\, \theta^a L^b\, \dfrac{L^{2c}}{T^c}\, \dfrac{M^d\, L^d}{T^{3d}\, \theta^d}$

$\therefore$ $1 + d = 0,\ b + 2c + d = 0,\quad -3 - c - 3d = 0,\ -1 + a - d = 0,$

$\therefore$ $a = 0,\ b = 1,\ c = 0,\ d = -1$

$\therefore$ $\boldsymbol{\pi_3 = hL/k}$ **(Biot number)**

Let $\pi_4 = \tau\, \theta_0^{\,a}\, L^b\, \alpha^c\, k^d$ or $M^0L^0T^0\theta^0 = T\, \theta^a\, L^b\, \dfrac{L^{2c}}{T^c}\, \dfrac{M^d\, L^d}{T^{3d}\, \theta^d}$

$\therefore$ $d = 0,\ b + 2c + d = 0,\ 1 - c - 3d = 0,\ a - d = 0,$

$\therefore$ $a = 0,\ b = -2,\ c = 1,\ d = 0$

$\therefore$ $\boldsymbol{\pi_4 = \alpha\,\tau/L^2}$ **(Fourier number)**

$$\therefore \qquad \frac{\theta}{\theta_0} = f\left[\frac{x}{L}, \frac{hL}{k}, \frac{\alpha\tau}{L^2}\right]$$

There are the popular dimensionless numbers is conduction heat transfer.

**This problem shows that the method is not limited to fluid flow or convection only.**

**Problem 8.19.** *Convective heat transfer coefficient in free convection over a surface is found to be influenced by the density, viscosity, thermal conductivity, coefficient of cubical expansion, temperature difference, gravitational acceleration, specific heat, the height of surface and the flow velocity. Using dimensional analysis, determine the dimensionless parameters that will correlate the phenomenon.*

The variables with dimensions in the $MLT\,\theta$ set is tabulated below.

| S.No. | Variable | Unit | Dimension |
|---|---|---|---|
| 1 | Height, $x$ | m | $L$ |
| 2 | Temperature difference, $\Delta T$ | deg K | $\theta$ |
| 3 | Coefficient of cubical expansion, $\beta$ | $(m^3/m^3)$/deg K | $1/\theta$ |
| 4 | Acceleration due to gravity, $g$ | $m/s^2$ | $L/T^2$ |
| 5 | Density, $\rho$ | $kg/m^3$ | $M/L^3$ |
| 6 | Viscosity, $\mu$ | kg/ms | $M/LT$ |
| 7 | Specific heat, $c$ | J/kgK | $L^2/T^2\theta$ |
| 8 | Thermal conductivity, $k$ | W/mK | $ML/T^3\theta$ |
| 9 | Convective heat transfer coefficient, $h$ | $W/m^2K$ | $M/T^3\theta$ |

**There are nine variables and four dimensions. Hence five $\pi$ terms can be identified.** $\rho$, $\mu$, $x$ and $k$ are chosen as repeating variables.

$$\text{Let} \qquad \pi_1 = \Delta T\, \rho^a\, \mu^b x^c k^d \quad \text{or} \quad M^0L^0T^0\theta^0 = \theta\, \frac{M^a}{L^{3a}}\, \frac{M^b}{L^bT^b}\, L^c\, \frac{M^dL^d}{T^{3d}\theta^d}$$

$$\therefore \qquad a + b + d = 0, -3a - b + c + d = 0, b - 3d = 0, 1 - d = 0,$$

$$\therefore \qquad a = 2, b = -3, c = 2, d = 1$$

$$\therefore \qquad \boldsymbol{\pi_1 = \Delta T \rho^2\, x^2\, k/\mu^3}$$

$$\text{Let} \qquad \pi_2 = \beta\, \rho^a\, \mu^b x^c k^d \quad \text{or} \quad M^0L^0T^0\theta^0 = \frac{1}{\theta}\, \frac{M^a}{L^{3a}}\, \frac{M^b}{L^bT^b}\, L^c\, \frac{M^dL^d}{T^{3d}\theta^d}$$

$$\therefore \qquad a + b + d = 0, -3a - b + c + d = 0, \quad -b - 3d = 0, -1 - d = 0,$$

$$\therefore \qquad a = -2, b = 3, c = -2, d = -1$$

$$\therefore \qquad \boldsymbol{\pi_2 = \beta\, \mu^3/\rho^2 x^2 k}$$

$$\text{Let} \qquad \pi_3 = g\, \rho^a\, \mu^b x^c k^d \quad \text{or} \quad M^0L^0T^0\theta^0 = \frac{L}{T^2}\, \frac{M^a}{L^{3a}}\, \frac{M^b}{L^bT^b}\, L^c\, \frac{M^dL^d}{T^{3d}\theta^d}$$

$\therefore \quad a + b + d = 0,\ 1 - 3a - b + c + d = 0,\ -2 - b - 3d = 0,\ d = 0,$

$\therefore \quad a = 2,\ b = -2,\ c = 3,\ d = 0$

$\therefore \quad \boldsymbol{\pi_3 = g\, \rho^2\, x^3/\mu^2}$

Let $\quad \pi_4 = c\, \rho^a\, \mu^b x^c k^d \quad \text{or} \quad M^0L^0T^0\theta^0 = \dfrac{L^2}{T^2\theta}\,\dfrac{M^a}{L^{3a}}\,\dfrac{M^b}{L^bT^b}\,L^c\,\dfrac{M^dL^d}{T^{3d}\theta^d}$

$\therefore \quad a + b + d = 0,\ 2 - 3a - b + c + d = 0,\ -2 - b - 3d = 0,\ -1 - d = 0,$

$\therefore \quad a = 0,\ b = 1,\ c = 0,\ d = -1$

$\therefore \quad \boldsymbol{\pi_4 = c\, \mu/k}$ **(Prandtl number)**

Let $\quad \pi_5 = h\, \rho^a\, \mu^b x^c k^d \quad \text{or} \quad M^0L^0T^0\theta^0 = \dfrac{M}{T^3\theta}\,\dfrac{M^a}{L^{3a}}\,\dfrac{M^b}{L^bT^b}\,L^c\,\dfrac{M^dL^d}{T^{3d}\theta^d}$

$\therefore \quad 1 + a + b + d = 0,\ -3a - b + c + d = 0,\ -3 - b - 3d = 0,\ -1 - d = 0,$

$\therefore \quad a = 0,\ b = 0,\ c = 1,\ d = -1$

$\therefore \quad \boldsymbol{\pi_5 = hx/k}$ **(Nusselt number)**

As the $\pi$ terms are too many $\pi_1$, $\pi_2$ and $\pi_3$ are combined as $\pi_1 \times \pi_2 \times \pi_3$ to form the group known as Grashof number.

$$\boldsymbol{\pi_6} = \frac{\Delta T \rho^2 x^2 k}{\mu^3} \times \frac{\beta\mu^3}{\rho^2 x^2 k} \times \frac{g\rho^2 x^3}{\mu^2} = \frac{\Delta T \beta g x^3 \rho^2}{\mu^2} = \frac{\Delta T g \beta x^3}{v^2}$$

$$\therefore \quad \frac{hx}{k} = f\left[\frac{c\mu}{k}, \frac{\Delta T g \beta \rho^2 x^3}{\mu^2}\right]$$

**Note :** When there are more than three $\pi$ parameters the set should be reduced to three by judicial combination.

## OBJECTIVE QUESTIONS

**O Q. 8.1. Fill in the blanks:**

1. The dimension for force in the *MLT* set is ______________.
2. The dimension for mass in the *FLT* set is ______________.
3. If there are $n$ variables and $m$ dimensions, $\pi$-theorem states that ______________ dimensionless parameters can be obtained.
4. The dimension for thermal conductivity in the $MLT\,\theta$ system is ______________ and in $FLT\,\theta$ system is ______________.
5. For an expression to be dimensionally homogeneous, each additive term in the equations should have ______________.
6. One of the methods to check the correctness of an equation is to check for ______________ for each of the additive terms.
7. The limitation of dimensional analysis is that the ______________ has to be determined by experiments.
8. The approximate number of experiments to evaluate the influence of 5 parameters separately is ______________ assuming that 10 experiments are needed for each variable.

9. The choice of dimension set has ______________ on the final dimensionless numbers determined for a phenomenon.
10. When there are more than 3 Pi terms determined for a phenomenon, for experimentation the set must be reduced to 3 by ______________.

### Answers

(1) $ML/T^2$ (2) $FT^2/L$ (3) $n - m$ (4) $ML/T^3\ \theta$, $F/T\theta$ (5) the same dimensions (6) the sameness of dimensions (homogeneous) (7) the exact functional relation (8) $10^5$ (9) no effect (10) Combining terms in excess of two into a single $\pi$ term.

**O Q. 8.2. Fill in the blanks:**

1. Reynolds number is used in the study of ______________ flow.
2. Froude number is used in the study of ______________ flow.
3. Euler number is used in the study of ______________ flow.
4. Weber number is used in the study of ______________ flow.
5. Cauchy number is used when ______________ is important.
6. The number used in the study of oscillating flows is ______________.
7. Lift coefficient is the ratio of lift force to ______________.
8. Drag coefficient is the ratio of drag force to ______________.
9. The ratio of inertia to viscous force is called ______________ number.
10. The ratio of pressure force to inertia force is called ______________ number.

### Answers

(1) All types of flow (2) Free surface flow (3) Pressure driven flow (4) Free surface flow, Surface tension effects (5) Compressibility (6) Strouhal number (7) dynamic force (8) dynamic force (9) Reynolds numbers (10) Euler number.

**O Q. 8.3. State correct or incorrect:**

1. Dimensional analysis can be used to reduce the number of variables for investigation of a phenomenon.
2. Dimensional analysis can provide an exact functional relationship between variables affecting a phenomenon.
3. Dimensional analysis by clubbing variables into groups facilitates presentation of results of experiments in a compact form.
4. It is easier to investigate a problem varying the value of a group as a whole rather than individual variables.
5. Grouping of variables into dimensionless parameters reduces number of experiments.
6. Fluid dynamics problems can be completely solved by dimensional analysis.
7. Reynolds number is the ratio between gravitational force and the viscous force.
8. Froude number is the ratio between inertia force and gravitational force.
9. Euler number is the ratio between Inertia force and viscous force.
10. Weber number is used to study oscillating flow.

### Answers

(1) Correct : 1, 3, 4, 5, 8 Incorrect : 2, 6, 7, 9, 10

## EXERCISE PROBLEMS

**E 8.1.** Check whether the equation is dimensionally homogeneous. Find the unit conversion factor to make it dimensionally homogeneous.

$P = P_0 - (1/2)\ \rho u^2 - \rho g z$ ($P$-pressure, ρ-density, $u$-velocity, $z$-height above datum)

**($g_0$ = kgm/Ns$^2$)**

**E 8.2.** The centripetal acceleration of a particle in circular motion is dependent on velocity $u$ and radius $r$. Using dimensional analysis determine the functional relation. **[$a_r = f(r\omega^2)$]**

**E 8.3.** The velocity of sound, $c$ in a gaseous medium, depends on the pressure $P$ and density ρ of the gas. Find using dimensional analysis a functional relation. **($c\sqrt{P/\rho}$ = constant)**

**E 8.4.** In flow over a smooth flat plate, the boundary layer thickness δ is found to depend on the free stream velocity $u$, fluid density and viscosity and the distance $x$ from the leading edge. Express the correlation in the form of dimensionless groups. **($(\delta/x) = f(ux\ \rho/\mu)$)**

**E 8.5.** In flow over a smooth flat plate, the wall shear $\tau_w$ in the boundary layer depends on the free stream velocity, density and viscosity of the fluid and the distance from the leading edge. Determine the dimensionless parameters to express the relation between the variables.

**($(\tau_w/\rho u^2) = f(\mu/\rho u x)$)**

**E 8.6.** The volume flow $Q$ over a weir depends on the upstream height $h$, the width of the weir $b$, and acceleration due to gravity. Obtain a relationship between the variables in terms of dimensionless parameters. **($(Q/h^2\ (gh)^{1/2}) = f(b/h)$)**

**E 8.7.** The speed $u$ of free surface gravity wave in deep water depends on the depth $D$, wave length λ, density ρ and acceleration due to gravity. Determine, using dimensional analysis, a functional relationship between the varaibles. **($(u/(gD)^{1/2} = f(\lambda/D)$)**

**E 8.8.** Obtain a relationship for the torque τ to rotate a disk of diameter $D$ in a fluid of viscosity μ at an angular speed ω over a plate, with clearance $h$. **($(\tau/D^3\ \mu\omega) = f(h/D)$)**

**E 8.9.** The power $P$ to drive a fan is found to depend on the diameter $D$, density of the gas ρ, volume flow rate $Q$, and the speed $N$. Using the method of dimensional analysis obtain a correlation in terms of dimensionless numbers. **($(P/(\rho\ D^5N^3)) = f(Q/D^3\ N)$)**

**E 8.10.** Oil is moved up in a lubricating system by a rope dipping in the sump containing oil and moving up. The quantity of oil pumped $Q$, depends on the speed $u$ of the rope, the layer thickness δ, the density and viscosity of the oil and acceleration due to gravity. Obtain the dimensionless parameters to correlate the flow. $$(Q/u\delta^2) = f\left(\frac{\mu}{\rho u\delta}, \frac{u^2 g}{\delta}\right)$$

**E 8.11.** The time τ to drain a circular tank of diameter $D$ by an orifice of diameter $d$ is found to depend on the initial head $h$, the density and viscosity of the liquid and gravitational acceleration. Determine dimensionless parameters to correlate experimental results. (**Note** : Used in the determination of viscosity) $$(\tau\mu/h^2\ \rho) = f\left(\frac{D}{h}, \frac{d}{h}, \frac{gh^3\ \rho^2}{\mu^2}\right)$$

**E 8.12.** The instantaneous volume $Q$ drained by an orifice of diameter $d$ from a circular tank of diameter $D$, when the head is $h$ depends on the density and viscosity of the fluid and acceleration due to gravity. Determine the π terms to correlate the flow. $$(Q\rho/d\ \mu) = f\left(\frac{D}{d}, \frac{h}{d}, \frac{gh^3\ \rho^2}{\mu^2}\right)$$

**E 8.13.** In the atomization of a fluid by passing it through an orifice under pressure, the droplet size $d$ is believed to depend on the jet diameter $D$, jet velocity $u$, and the density, viscosity and surface tension of the lqiuid. Determine dimensionless parameters for the phenomenon.

$$(d/D) = f\left(\frac{\rho u D}{\mu}, \frac{\sigma}{\rho D u^2}\right)$$

**E 8.14.** The pressure $P$, developed in a jet pump is found to depend on the jet diameter $d$, diffuser diameter $D$, the velocity $u$ of the jet, the volume flow $Q$ and the density and viscosity of the fluid. Determine the dimensionless parameters to organize experimental results.

$$\left[\frac{Q}{ud^2}, \frac{\mu}{\rho u D}, \frac{d}{D}, \frac{P}{\rho u^2}\right]$$

**E 8.15.** A spherical ball of diameter $D$ and weight $w$ is balanced at the tip of a jet of diameter $d$ at a height $h$. The velocity of the jet is $u$. If the other parameters are the liquid density and viscosity, find the $\pi$ terms that can be used to characterize the phenomenon.

$$\left[\frac{h}{d}, \frac{D}{d}, \frac{\rho u D}{\mu}\right]$$

**E 8.16.** Players use spin in ball plays like tennis, golf etc. As the ball moves the spin rate will decrease. If the aerodynamic torque $\tau$ on the ball in flight depends on the forward speed $u$, density and viscosity of air, the ball diameter $D$, angular velocity of spin, $\omega$ and the roughness height $e$ on the ball surface, determine the dimensionless parameters to correlate situation.

$$\left[\frac{\tau}{\rho u^2 D^3}, \frac{\mu}{\rho u D}, \frac{\omega D}{u}, \frac{e}{D}\right]$$

**E 8.17.** The high pressure generated due to sudden closing of a valve in a pipeline (known as water hammer) is found to depend on the velocity of flow, the density of fluid and the bulk modulous $E_v$. Determine the functional relationship among the variables in terms of dimensionless parameters.

$$\left[\frac{P}{\rho u^2} = f(E_v/\rho u^2)\right]$$

**E 8.18.** The power required to drive a propeller in a gas medium depends upon the forward speed $u$, the rotational speed $N$, diameter $D$, density and viscosity of the gas and the speed of sound $c$ in the medium. Obtain dimensionless parameters to correlate experimental results.

$$\frac{P}{\rho D^2 u^2} = f\left[\frac{ND}{u}, \frac{\mu}{\rho u D}, \frac{c}{u}\right]$$

**E 8.19.** In an oven where materials are heated by convection, the heat transfer rate $Q$ ($W$) is believed to depend on the specific heat of air, temperature difference $\Delta\theta$ (between gas and heated body) a length parameter $L$, density, viscosity and flow velocity $u$ of the fluid. Determine the $\pi$ parameters for the situation.

$$\left[\frac{Q}{\rho u^3 L^2}, \frac{c\Delta\theta}{u^2}, \frac{\mu}{\rho u L}\right]$$

**E 8.20.** The deflection $\delta$ at the center due to fluid flow at velocity $u$, over a thin wire held between rigid supports is found to depend on length of the wire $L$, diameter of the wire, $d$ elastic modulus of the wire, $E$ and fluid properties density and viscosity and the velocity $u$.

Determine the $\pi$ terms for the problem. (use $\rho$, $d$ and $\mu$ as repeating variables)

$$\left[\frac{\delta}{d}, \frac{L}{d}, \frac{\rho u d}{\mu}, \frac{E\rho d^2}{\mu^2}\right] \text{ Also } E/\rho u^2$$

**E 8.21.** The volume of flue gas $Q$ flowing through a chimney of height $h$ and diameter $d$ is influenced by the density of the gas inside $\rho_g$, the density of air outside $\rho_a$ and acceleration due to gravity.

Determine $\pi$ terms to correlate the variables.

$$\left[\frac{Q}{d^2\sqrt{gD}}, \frac{h}{d}, \frac{\rho_g}{\rho_a}\right]$$