9 Similitude and Model Testing

9.0 INTRODUCTION

Fluid flow analysis is involved in the design of aircrafts, ships, submarines, turbines, pumps, harbours and tall buildings and structures. Fluid flow is influenced by several factors and because of this the analysis is more complex. For many practical situations exact soluations are not available. The estimates may vary by as much as ± 20%. Because of this it is not possible to rely solely on design calculations and performance predictions. **Experimental validation of the design is thus found necessary.** Consider the case of a hydraulic turbine of 50 MW size. It will be a very costly failure if the design performance and the actual performance differ. If we can predict its performance before manufacturing the unit it will be very useful. Model testing comes to our aid in this situation. **Constructing and testing small versions of the unit is called model testing. Similarity of features enable the prediction of the performance** of the full size unit from the test results of the smaller unit. The application of dimensional analysis is helpful in planning of the experiments as well as prediction of the performance of the larger unit from the test results of the model.

9.1 MODEL AND PROTOTYPE

In the engineering point of view model can be defined as the representation of physical system that may be used to predict the behavior of the system in the desired aspect. **The system whose behavior is to be predicted by the model is called the prototype**. The discussion in this chapter is about physical models that resemble the prototype but are generally smaller in size. These may also operate with different fluids, at different pressures, velocities etc. As models are generally smaller than the prototype, these are cheaper to build and test. Model testing is also used for evaluating proposed modifications to existing systems. The effect of the changes on the performance of the system can be predicted by model testing before attempting the modifications. Models should be carefully designed for reliable prediction of the prototype performance.

9.2 CONDITIONS FOR SIMILARITY BETWEEN MODELS AND PROTOTYPE

Dimensional analysis provides a good basis for laying down the conditions for similarity. The *PI* theorem shows that the performance of any system (prototype) can be described by a functional relationship of the form given in equation 9.2.1.

$$\pi_{1p} = f(\pi_{2p}, \pi_{3p} \ldots\ldots \pi_{np}) \quad \ldots(9.2.1)$$

The *PI* terms include all the parameters influencing the system and are generally ratios of forces, lengths, energy etc. If a model is to be similar to the prototype and also function similarly as the prototype, then the *PI* terms for the model should also have the same value as that of the prototype or the same functional relationship as the prototype. (eqn. 9.2.1)

$$\pi_{1m} = f(\pi_{2m}, \pi_{3m} \ldots\ldots \pi_{nm}) \quad \ldots(9.2.2)$$

For such a condition to be satisfied, the model should be constructed and operated such that simultaneously

$$\pi_{1m} = \pi_{1p}, \pi_{2m} = \pi_{2p}, \ldots\ldots \pi_{nm} = \pi_{np} \quad \ldots(9.2.3)$$

Equation 9.2.3 provides the model design conditions. It is also called similarity requirements or modelling laws.

9.2.1 Geometric Similarity

Some of the *PI* terms involve the ratio of length parameters. All the similar linear dimension of the model and prototype should have the same ratio. This is called geometric similarity. The ratio is generally denoted by the scale or scale factor. One tenth scale model means that the similar linear dimensions of the model is 1/10 th of that of the prototype. For complete similarity all the linear dimensions of the model should bear the same ratio to those of the prototype. There are some situations where it is difficult to obtain such similarity. Roughness is one such case. In cases like ship, harbour or dams distorted models only are possible. In these cases the depth scale is different from length scale. Interpretation of the results of the tests on distorted models should be very carefully done. Geometric scale cannot be chosen without reference to other parameters. For example the choice of the scale when applied to the Reynolds number may dictate a very high velocity which may be difficult to achieve at a reasonable cost.

9.2.2 Dynamic Similarity

Similitude requires that π terms like Reynolds number, Froude number, Weber number etc. be equal for the model and prototype. These numbers are ratios of inertia, viscous gravity and surface tension forces. This condition implies that the ratio of forces on fluid elements at corresponding points (homologous) in the model and prototype should be the same. This requirement is called dynamic similarity. This is a basic requirement in model design. If model and prototype are dynamically similar then the performance of the prototype can be predicted from the measurements on the model. In some cases it may be difficult to hold simultaneously equality of two dimensionless numbers. In such situations, the parameter having a larger influence on the performance may have to be chosen. This happens for example in the case of model tasting of ships. Both Reynolds number and Froude number should be simultaneously held equal between the model and prototype. This is not possible as this would require either fluids with a very large difference in their viscosities or the use of very large velocities with the model. This is illustrated in problem 9.14.

9.2.3 Kinematic Similarity

When both geometric and dynamic similarities exist, then velocity ratios and acceleration ratios will be the same throughout the flow field. This will mean that the streamline patterns will be the same in both cases of model and prototype. This is called kinematic similarly. **To achieve complete similarity between model and prototype all the three similarities - geometric, dynamic and kinematic should be maintained.**

9.3 TYPES OF MODEL STUDIES

Model testing can be broadly classified on the basis of the general nature of flow into four types. These are

(1) Flow through closed conduits

(2) Flow around immersed bodies

(3) Flow with free surface and

(4) Flow through turbomachinery

9.3.1 Flow through Closed Conduits

Flow through pipes, valves, fittings and measuring devices are dealt under this category. The conduits are generally circular, but there may be changes along the flow direction. As the wall shear is an important force, Reynolds number is the most important parameter. The pressure drop along the flow is more often the required parameter to be evaluated. Compressibility effect is negligible at low mach numbers. (M<0.3).

From dimensional analysis the pressure drop can be established as

$$\Delta P/\rho u^2 = f\left(\frac{\rho u L}{\mu}, \frac{\varepsilon}{L}, \frac{D}{L}\right) \qquad \text{...(9.3.1)}$$

The geometric scale is given by the ratio, scale = L_m/L_p.

This requires $\dfrac{D_m}{D_p} = \dfrac{\varepsilon_m}{\varepsilon_p} = \dfrac{L_m}{L_p} = \lambda.$

Reynolds number similarity leads to the condition for velocity ratio as

$$\frac{u_m \rho_m L_m}{\mu_m} = \frac{u_p \rho_p L_p}{\mu_p} \quad \therefore \quad \frac{u_m}{u_p} = \frac{\mu_m}{\mu_p} \frac{\rho_p}{\rho_m} \frac{L_p}{L_m} \qquad \text{...(9.3.2)}$$

If the fluid used for the model and prototype are the same, then $\dfrac{u_m}{u_p} = \dfrac{L_p}{L_m}$ or $u_m = u_p/\lambda$.

As λ is less than one, the velocity to be used with the model has to be higher compared to the prototype. Otherwise a different fluid with higher viscosity should be chosen to satisfy the requirements.

The pressure drop in the prototype is calculated as in equation (9.3.3)

From equality of, $\Delta P / \rho u^2$, $$\Delta P_P = \frac{\rho_p}{\rho_m}\left(\frac{u_p}{u_m}\right)^2 \Delta P_m \tag{9.3.3}$$

As ΔP_m is measured, using the model, the pressure drop in the prototype can be predicted.

When Reynolds numbers are large the inertia forces are predominant and viscous forces will be small in comparison. In such cases, the Reynolds number similarity becomes unimportant. However, the model should be tested at various Reynolds numbers to determine the range at which its effect on pressure drop becomes negligible. After this is established the model test results can be applied without regard to Reynolds number similarity, in this range.

Another condition is the onset of cavitation at some locations in the flow, particularly in testing components where at some points the local velocity may become high and pressure may drop to a level where cavitation may set in. Unless cavitation effects are the aim of the study, such condition should be avoided. In case cavitation effects are to be studied, then similarity of cavitation number should be established. *i.e.* $(p_r - p_v)/(\rho u^2/2)$. Where p_r is the reference pressure and p_v is the vapour pressure at that temperature.

9.3.2 Flow Around Immersed Bodies

Aircraft, Submarine, cars and trucks and recently buildings are examples for this type of study. In the sports area golf and tennis balls are examples for this type of study. Models are usually tested in wind tunnels. As viscous forces over the surface and inertia forces on fluid elements are involved in this case also, Reynolds number of the model and prototype should be equal. Gravity and surface tension forces are not involved in this case and hence Froude and Weber numbers need not be considered. Drag coefficient, defined by [Drag force /(1/2) $\rho u^2 l^2$)] is the desired quantity to be predicted. Generally the following relationship holds in this case.

$$C_D = \frac{D}{(1/2)\rho u^2 l^2} = f\left[\frac{l_1}{l}, \frac{\varepsilon}{l}, \frac{\rho u l}{\mu}\right] \tag{9.3.4}$$

where l is a characteristic length of the system and l_1 represents the other length parameter affecting the flow and ε is the roughness of the surface.

When the flow speed increases beyond Mach number 0.3 compressibility effect on similarity should be considered. Using the similitude, measured values of drag on model is used to estimate the drag on the prototype.

$$D_p = D_m \frac{\rho_p}{\rho_m}\left(\frac{u_p}{u_m} \times \frac{l_p}{l_m}\right)^2 \tag{9.3.5}$$

From Reynolds number similitude

$$u_m = \frac{\mu_m}{\mu_p}\frac{\rho_p}{\rho_m}\frac{l_p}{l_m}u_p = \frac{v_m}{v_p}\frac{l_p}{l_m}u_p \tag{9.3.6}$$

When same fluid is used for both prototype and model

$$u_m = (l_p/l_m)\, u_p \qquad ...(9.3.7)$$

The model velocity should be higher by the geometric scale.

If the prototype is to operate at 100 kmph and if the scale is 1:10, then the model should operate at 1000 kmph, which will mean a high Mach number. The model will be influenced by compressibility effect due to the operation at high Mach numbers. The prototype however will be operating at low Mach numbers where compressibility effect is negligible. Hence the performance prediction will be in error.

This may be overcome by using different fluids say water in place of air. Using equation 9.3.6, as kinematic viscosity of air is about 10 times that of water, the velocity will now be at a reasonable level. Another method is to pressurise the air in the wind tunnel, thus increasing the density, and reducing the required velocity of the model.

Where expense is of no consideration due to the requirement of utmost reliability as in space applications and development of new aircraft, full scale models are also used.

In some cases at higher ranges, the Reynolds number is found to have little influence on drag. Strict Reynolds similarity need not be used in such situations. The variation of drag due to variation in Reynolds number for cylinder and sphere is shown as plotted in Fig. 9.3.1. It may be seen that above Re = 10^4 the curve is flat. If the operation of the prototype will be at such a range, then Reynolds number equality will not be insisted for model testing.

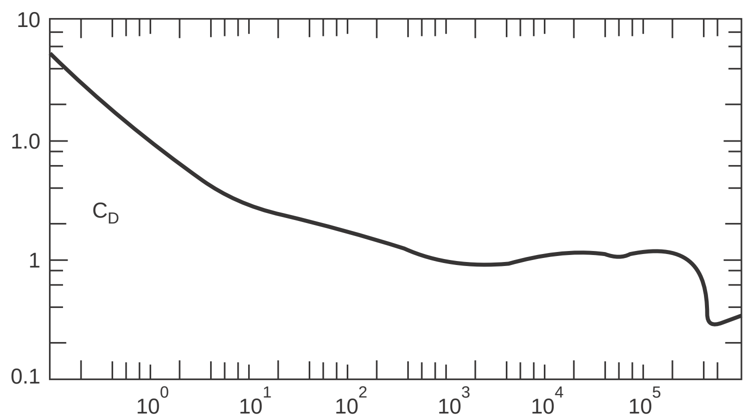

Figure 9.3.1 Variation of drag with Reynolds number for flow over cylinder

Another situation arises in testing of models of high speed aircraft. In this case the use of Mach number similitude requires equal velocities while the Reynolds number similarity requires increased velocity for the model as per geometric scale. In such cases distorted model is used to predict prototype performance.

9.3.3 Flow with Free Surface

Flow in canals, rivers as well as flow around ships come under this category. In these cases gravity and inertia forces are found to be governing the situation and hence Froude number becomes the main similarity parameter.

In some cases Weber number as well as Reynolds number may also influence the design of the model.

Considering Froude number, the velocity of the model is calculated as below.

$$\frac{u_m}{\sqrt{g l_m}} = \frac{u_p}{\sqrt{g l_p}} \tag{9.3.8}$$

$$\therefore \qquad u_m = u_p \sqrt{\frac{l_m}{l_p}} = u_p \sqrt{\text{scale}} \tag{9.3.9}$$

In case Reynolds number similarity has to be also considered, substituting this value of velocity ratio, the ratio of kinematic viscosities is given as

$$\frac{v_m}{v_p} = (\text{scale})^{3/2} \tag{9.3.10}$$

As these situations involve use of water in both model and prototype, it is impossible to satisfy the condition of equations 9.3.9 and 9.3.10 simultaneously. In such a case distorted model may have to be selected.

If surface tension also influences the flow, it is still more difficult to choose a fully similar model.

In many practical applications in this type of situation the influence of Weber and Reynolds number is rather small. Hence generally models are designed on the basis of Froude number similarity.

A special situation arises in the case of ships. The total drag on the ship as it moves is made up of two components: (1) The viscous shearing stress along the hull, (2) Pressure induced drag due to wave motion and influenced by the shape of the hull.

As it is not possible to build and operate a model satisfying simultaneously the Reynolds number similarity and Froude number similarity ingenious methods have to be adopted to calculate the total drag. The total drag on the model is first measured by experiment. The shear drag is analytically determined and the pressure drag on the model is calculated by subtracting this value. The drag on the prototype is determined using Froude number similarity. The calculated value of viscous drag is then added to obtain the total drag.

In case of design of river model, if the same vertical and horizontal scales are used, the depth will be low for the model and surface tension effects should be considered. But the use of distorted model, (vertical scaling smaller than horizontal scaling) overcomes this problem.

9.3.4 Models for Turbomachinery

Pumps as well as turbines are included in the general term turbomachines. Pumps are power absorbing machines which increase the head of the fluid passing though them. Turbines are power generating machines which reduce the head of the fluid passing through them.

The operating variables of the machines are the flow rate Q, the power P and the speed N. The fluid properties are the density and viscosity. The machine parameters are the diameter and a characteristic length and the roughness of the flow surface. Power, head and efficiency can be expressed as functions of π terms as in equation 9.3.11 (refer problem 9.21).

$$\text{Power} = f_1\left(\frac{l}{D}, \frac{\varepsilon}{D}, \frac{Q}{N\,D^3}, \frac{\rho N D^2}{\mu}\right) \quad (9.3.11)$$

The term ε / D is not important due to the various sharp corners in the machine. The dimensionless term involving power is defined as power coefficient, defined as $C_p = P/\rho\, N^3\, D^3$. The head coefficient is defined as $C_h = gh/N^2\, D^2$. The term $Q/N\, D^3$ is called flow coefficient. If two similar machines are operated with the same flow coefficient, the power and head coefficients will also be equal for the machines. This will then lead to the same efficiency. Combining flow and head coefficients in the case of pumps will give the dimensionless specific speed of the pump.

$$N_{sp} = \frac{N\sqrt{Q}}{(gh)^{3/4}} \quad (9.3.12)$$

Popularly used dimensional specific speed for pumps is defined as

$$N_{sp} = \frac{N\sqrt{Q}}{h^{3/4}} \quad (9.3.12a)$$

In the case of turbines, combining power and flow coefficients, the specific speed is obtained as

$$N_{st} = \frac{N\sqrt{P}}{\rho^{1/2}(gh)^{5/4}} \quad (9.3.13)$$

Popularly used dimensional speed for turbines is

$$N_{st} = \frac{N\sqrt{P}}{h^{5/4}} \quad ...(9.3.13a)$$

In model testing at a particular speed, the flow rate at various delivery heads can be measured. This can be used to predict the performance of the pump at other speeds using the various coefficients defined. The procedure for turbines will also be similar. The model can be run at a constant speed when the head is varied, the power and flow rate can be measured. The performance of the prototype can be predicted from the results of the tests on the geometrically similar model.

9.4 NONDIMENSIONALISING GOVERNING DIFFERENTIAL EQUATIONS

When differential equations describing the phenomenon is not available, the method of dimensional analysis is used to obtain similarity conditions. When differential equations describing the system are available, similarity parameters can be deduced by non dimensionalising the equations.

Consider the continuity and x directional momentum equations for two dimensional flow,

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

The various quantities can be made dimensionless by dividing by reference quantities, as given below

$$u^* = \frac{u}{U},\ v^* = \frac{v}{V},\ P^* = \frac{p}{P_0},\ x^* = \frac{x}{L},\ y^* = \frac{y}{L},\ t^* = \frac{t}{\tau}$$

Then $$\frac{\partial u}{\partial x} = \frac{U}{L}\frac{\partial u^*}{\partial x^*},\qquad \frac{\partial^2 u}{\partial x^2} = \frac{U}{L^2}\frac{\partial^2 u^*}{\partial x^{*2}}$$

Similar method is used in the case of other terms.

Substituting, the momentum equation reduces to the form

$$\left[\frac{L}{\tau U}\right]\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial t^*}=-\left[\frac{P_0}{\rho U^2}\right]\frac{\partial P^*}{\partial x^*}+\left[\frac{\mu}{\rho UL}\right]\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

It may be noted that the non dimensionalised equation is similar to the general equation except for the terms in square brackets. These are the similarity parameters thus identified.

$$\frac{L}{\tau U},\ \frac{P}{\rho U^2},\ \frac{\mu}{\rho UL}$$

In case gravity force is added, gL/U^2 will be identified. These are forms of Strouhal, Euler, Reynolds and Froude numbers. As the equation describes the general unsteady flow all the numbers are involved. If other forms of forces like surface tension is added. Weber number can be identified. If equations for compressible flow is used, Mach number can be obtained by a similar method.

9.5 CONCLUSION

In all the problems in this chapter on model testing the π terms identified in chapter 8 are used. Reference may be made to the problems in chapter 8. The discussions in this chapter is limited to basics. In actual model making and testing as well as interpretation of results many other finer details have to be considered for obtaining accurate predictions about the performance of the prototype.

SOLVED PROBLEMS

Problem 9.1 *To study the pressure drop in flow of water through a pipe, a model of scale 1/10 is used.* ***Determine the ratio of pressure drops*** *between model and prototype if water is used in the model. In case air is used determine the ratio of pressure drops.*

Case (i) Water flow in both model and prototype.

Reynolds number similarity is to be maintained.

$$\frac{u_m d_m \rho_m}{\mu_m} \frac{u_p d_p \rho_p}{\mu_p} \quad \therefore \quad \frac{u_m}{u_p} = \frac{\mu_m}{\mu_p} \times \frac{d_p}{d_m} \times \frac{\rho_p}{\rho_m}$$

As viscosity and density values are the same,

$$\frac{u_m}{u_p} = \frac{d_p}{d_m} = 10,$$

The pressure drop is obtained using pressure coefficient

$$[\Delta P/(1/2)\ \rho u^2]_m = [\Delta P/(1/2)\ \rho u^2]_p$$

$\therefore$ $$\frac{\Delta \mathbf{p_m}}{\Delta \mathbf{p_p}} = \frac{\rho_\mathbf{m} \mathbf{u_m}^2}{\rho_\mathbf{p} \mathbf{u_p}^2}$$, As $\rho_m = \rho_p$ and $u_m/u_p = 10$, $\Delta P_m/\Delta P_P = 10^2 =$ **100.**

Case (ii) If air is used in the model, then

$$\frac{u_m}{u_p} = \frac{\mu_m}{\mu_p} \times \frac{d_p}{d_m} \times \frac{\rho_p}{\rho_m}, \quad \frac{\Delta p_m}{\Delta p_p} = \frac{\rho_m}{\rho_p}\left(\frac{\mu_m}{\mu_p} \times \frac{d_p}{d_m} \times \frac{\rho_p}{\rho_m}\right)^2$$

$$= 100\ \frac{\rho_p}{\rho_m}\left(\frac{\mu_m}{\mu_p}\right)^2$$

From data tables at 20^0 C, $\rho_{air} = 1.205$ kg/m^3, $\mu_{air} = 18.14 \times 10^{-6}$ kg/ms,

$\rho_w = 1000$ kg/m^3, $\mu_w = 1.006 \times 10^{-3}$ kg/ms

$\therefore$ $$\frac{\Delta \mathbf{P_m}}{\Delta \mathbf{P_p}} = 100 \times \frac{1000}{1.205} \times \left(\frac{18.14 \times 10^{-6}}{1.006 \times 10^{-3}}\right)^2 = \mathbf{26.98}$$

This illustrates that it may be necessary to use a different fluid in the model as compared to the prototype.

Problem 9.2 *To determine the pressure drop in a square pipe of 1 m side for air flow, a square pipe of 50 mm side was used with water flowing at 3.6 m/s. The pressure drop over a length of 3 m was measured as 940 mm water column. Determine the corresponding* ***flow velocity*** *of air in the larger duct and also the* ***pressure drop*** *over 90 m length. Kinematic viscosity of air = 14.58 $\times 10^{-6}$ m^2/s. Density = 1.23 kg/m^3.* Kinematic viscosity of water = 1.18 × 10^{-6} m^2/s

For pipe flow, Reynolds number analogy should be used. Also the drag coefficients will be equal.

For square section hydraulic mean diameter = 4 $A/P = 4a^2/4a = a$ (side itself)

$$Re = uD/v = 3.6 \times 0.05/1.18 \times 10^{-6} = 152542$$

For air $$152542 = \frac{1 \times u}{14.58 \times 10^{-6}} \quad \therefore \quad \mathbf{u = 2.224\ m/s}$$

Drag coefficient $F/\rho u^2$ should be the same for both pipes.

$$\frac{F_{air}}{F_w} \frac{\rho_{air} u_{air^2}}{\rho_w u_w^2}$$

The pressure drop equals the shear force over the area. For square section, area = a^2, perimeter = $4a$

$$\therefore \qquad \Delta P = \frac{4FL}{a},\ \Delta P_{air} = \frac{4F_{air}L_{air}}{a_{air}},\ \Delta P_{w} = \frac{4F_w L_w}{a_w}$$

Dividing and substituting for F_{air}/F_w

$$\frac{\Delta P_{air}}{\Delta P_w} = \frac{L_{air}}{L_w} \times \frac{a_w}{a_{air}} \times \frac{F_{air}}{F_w} = \frac{L_{air}}{L_w} \times \frac{a_w}{a_{air}} \times \frac{\rho_{air}}{\rho_w}\left(\frac{u_{air}}{u_w}\right)^2$$

$$= \frac{90 \times 0.05^2 \times 1.23}{1 \times 3 \times 1000}\left(\frac{2.224}{3.6}\right)^2 = 3.521 \times 10^{-5}$$

$$\mathbf{\Delta P_{air}} = 940 \times 3.521 \times 10^{-5} = \textbf{0.033 mm of water column}$$

Problem 9.3 *Water at 15°C flowing in a 20 mm pipe becomes turbulent at a velocity of 0.114 m/s. What will be the* ***critical velocity of air at 10°C*** *in a similar pipe of 40 mm diameter. Density of air = 1.23kg/m³. Dynamic viscosity of air = 17.7 × 10⁻⁶ kg/ms.*

Density of water = 1000 kg/m³. Dynamic viscosity of water = 1.12 × 10⁻³ kg/ms.

As roughness etc are similar, for pipe flow, reynolds number similarity is to be used.

$$\frac{114 \times 0.02 \times 1000}{112 \times 10^{-3}} = \frac{u_{air} \times 0.04 \times 1.23}{127 \times 10^{-6}}, \quad \therefore \quad \mathbf{u_{air} = 0.732\ m/s}$$

Problem 9.4 *A model of 1/8 geometric scale of a valve is to be designed. The diameter of the prototype is 64 cm and it should control flow rates upto 1m³/s.* ***Determine the flow required for model*** *testing. The valve is to be used with brine in a cooling system at –10°C. The kinematic viscosity of brine at the saturated condition is 6.956 × 10⁻⁶ m²/s. For model testing water at 30°C is used. Kinematic viscosity is 0.8315 × 10⁻⁶ m²/s*

This is a situation of flow through closed conduits. Reynolds number similarity is required.

$$\frac{u_m d_m}{v_m} = \frac{u_p d_p}{v_p}, \qquad \therefore \quad \frac{u_m}{u_p} = \frac{d_p}{d_m} \times \frac{v_m}{v_p}, \qquad Q_p = \frac{\pi D_p^{\,2}}{4} u_p$$

$$1 = \pi \times \frac{0.64^2}{4} u_p \qquad \therefore \quad \mathbf{u_p = 3.1085\ m/s}$$

$$\therefore \qquad \mathbf{u_m} = 3.1085 \times 8 \times 0.8315 \times 10^{-6}/6.956 \times 10^{-6} = \mathbf{2.9726\ m/s,}$$

$$d_m = d_p/8 = 0.64/8 = 0.08 \text{ m}$$

$$\mathbf{Q_m} = \frac{\pi d_m^{\,2}}{4} u_m = \frac{\pi}{4} \times 0.08^2 \times 2.9726 = \mathbf{0.0149\ m^3/s}$$

If the valve is to be used with water, then the model velocity has to be 8 × 3.1085 m/s. *i.e.* 24.87 m/s, which is rather high.

The pressure drop can also be predicted from the model measurements using

$$\left(\frac{\Delta p}{\rho u^2}\right)_p = \left(\frac{\Delta p}{\rho u^2}\right)_m$$

Problem 9.5 *To predict the drag on an aircraft at a flight speed of 150 m/s, where the condition of air is such that the local speed of sound is 310 m/s, a pressurised low temperature tunnel is used. Density, viscosity and local sonic velocity at tunnel condition are 7.5 kg/m^3, 1.22 × 10^{-5} Ns/m^2 and 290 m/s.* ***Determine the flow velocity and the scale*** *of the model. Assume full dynamic similarity should be maintained. Density and viscosity at the operating conditions are 1.2 kg/m^3 and 1.8 × 10^{-5} Ns/m^2.*

In addition to Reynolds number similarity compressibility effect should be considered. For Mach number similarity,

$$M_p = M_m,\ \frac{u_p}{c_p} = \frac{u_m}{c_m} \quad \therefore \quad u_m = 150 \times 290/310 = \mathbf{140.32\ m/s}$$

For Reynolds number similarity

$$\frac{u_m \rho_m L_m}{\mu_m} = \frac{u_p \rho_p L_p}{\mu_p}$$

$$\frac{\mathbf{L_m}}{\mathbf{L_p}} = \frac{u_p}{u_m} \times \frac{\rho_p}{\rho_m} \times \frac{\mu_m}{\mu_p} = \frac{150}{140.32} \times \frac{1.2}{7.5} \times \frac{1.8 \times 10^{-5}}{1.22 \times 10^{-5}} = \mathbf{0.252}$$

or about 1/4th scale. When both Match number similarity and Reynolds number similarity should be maintained, generally the size of the model has to be on the higher side Drag force similarity is given by $(F/\rho u^2 L^2)_m = (F/\rho u^2 L^2)_p$

$$\frac{F_m}{F_p} = \frac{\rho_m u_m^{\,2} L_m^{\,2}}{\rho_p u_p^{\,2} L_p^{\,2}} = \frac{7.5}{1.2} \times \frac{140.32^2}{150^2} \times (0.252)^2 = 0.347$$

As the model size is larger, the force ratio is high.

Problem 9.6 *An aircraft fuselage has been designed for speeds of 380 kmph. To estimate power requirements the drag is to be determined. A model of 1/10 size is decided on. In order to reduce the effect of compressibility, the model is proposed to be tested at the same speed in a pressurized tunnel.* ***Estimate the pressure required.*** *If the drag on the model was measured as 100 N, predict the drag on the prototype.*

This is fully immersed flow. Hence Reynolds number similarity is required.

$$\frac{u_m L_m \rho_m}{\mu_m} = \frac{u_p L_p \rho_p}{\mu_p}$$

A viscosity is not affected by pressure and as velocities are equal,

$$L_m \rho_m = L_p \rho_p \quad \therefore \quad \rho_m/\rho_m = L_p/L_m = 10$$

At constant temperature, pressure ratio will be the same as density ratio.

$$\therefore \qquad \mathbf{P_m} = \frac{L_p}{L_m} P_p = \mathbf{10 \times P_p}$$

or 10 times the operating pressure of the aircraft.

The π parameter for drag force, D, gives

$$\frac{D_m}{(1/2)\rho_m u_m^{\,2} L_m^{\,2}} = \frac{D_p}{(1/2)\rho_p u_p^{\,2} L_p^{\,2}} \text{ as } u_m = u_p$$

$\therefore$ $\mathbf{D_p} = D_m\,(\rho_p L_p^{\,2}/\rho_m L_m^{\,2}) = 100 \times (1/10)\, 10^2 =$ **1000 or 1 kN**

Problem 9.7 *The performance of an aeroplane to fly at 2400 m height at a speed of 290 kmph is to be evaluated by a 1/8 scale model tested in a pressurised wind tunnel maintaining similarity. The conditions at the flight altitude are temperature = – 1⁰C, pressure = 75 kN/m².*

$\mu = 17.1 \times 10^{-6}$ kg/ms. The test conditions are 2150 kN/m², and 15⁰C.

$\mu = 18.1 \times 10^{-6}$ kg/ms. The drag resistance on the model measured at 18 m/s and 27 m/s. are 4.7N and 9.6N. ***Determine the drag on the prototype.***

At the given flight conditions, Velocity of sound is

$$C = \sqrt{g_0 kRT} = \sqrt{1 \times 1.4 \times 287 \times 272} = 330 \; m/s = 1190 \text{ kmph}$$

Mach number = 290/1190 = 0.24 < 0.3

Hence Reynolds number similarity only need be considered.

Density at test conditions = $2150 \times 10^3/(287 \times 288) = 26.01$ kg/m³

Density at flight conditions = $75 \times 10^3/(287 \times 272) = 0.961$ kg/m³

Equating Reynolds numbers, assuming length L,

Velocity at flight condition = 290000/3600 = 80.56 m/s

$$\frac{80.56 \times L \times 0.961}{17.1 \times 10^{-6}} = u \times \frac{L}{8} \times \frac{26.01}{18.1 \times 10^{-6}} \qquad \therefore \quad \mathbf{u = 25.195 \; m/s}$$

This is also low subsonic. Drag can be obtained using drag coefficient $F/\rho A u^2$

$$\frac{F_m}{\rho_m A_m u_m^{\,2}} = \frac{F_p}{\rho_p A_p u_p^{\,2}} \qquad \therefore \quad \frac{F_p}{F_m} = \frac{\rho_p}{\rho_m} \times \left(\frac{u_p}{u_m}\right)^2 \times \frac{A_p}{A_m}$$

$$= \frac{26.01}{0.961} \times \left(\frac{80.56}{25.195}\right)^2 \times 8^2 = 24.165$$

By interpolation using equality of F/u^2, **drag at 25.195 m/s** model speed is obtained as **8.78 N.** $\therefore$ **Drag on prototype =** 8.78 × 24.165 = **212 N**

Problem 9.8 *In a test in a wind tunnel on 1:16 scale model of a bus, at an air speed of 35m/s, the drag on the model was measured as 10.7N, If the width and frontal area of the prototype was 2.44 m and 7.8 m²,* ***estimate the aerodynamic drag*** *force on the bus at 100 kmph. Conditions of air in the wind tunnel are the same as at the operating conditions of the bus. Assume that coefficient of drag remains constant above Reynolds number 10^5.*

$v = 1.006 \times 10^{-6}$ m²/s. Also ***determine the power required.***

The width of the model = 2.44/16 = 0.1525 m.

$$Re = \frac{0.1525 \times 35}{15.06 \times 10^{-6}} = 3.5 \times 10^5, \quad \text{This condition is above } 10^5.$$

Area of the model = $7.8/16^2$

$$C_D = \frac{F}{(1/2)\rho u^2 A} = \frac{10.7 \times 2 \times 16^2}{1.205 \times 35^2 \times 7.8} = 0.4758$$

Drag force on the prototype at 100 kmph. (27.78 m/s)

$$0.4758 = \frac{F}{(1/2)1.205 \times 7.8 \times (27.78)^2} \quad \therefore \quad \mathbf{F = 1725\ N \quad or \quad 1.725\ kN}$$

Power required = 1725 × 27.78 W = 47927 W or **47.927 kW.**

Problem 9.9 *A water tunnel operates with a velocity of 3m/s at the test section and power required was 3.75 kW. If the tunnel is to operate with air, **determine for similitude the flow velocity and the power required.***

$$\rho_a = 1.25 \text{ kg/m}^3,\ v_a = 14.8 \times 10^{-6} \text{ m}^2/\text{s}, \quad v_w = 1.14 \times 10^{-6} \text{ m}^2/\text{s}$$

In this case Reynolds number similarity is to be maintained. The length dimension is the same.

$$\frac{u_a}{v_a} = \frac{u_w}{v_w}$$

$$\therefore \text{ Velocity of air, } \quad \mathbf{u_a} = \frac{u_w}{v_w} v_a = \frac{3 \times 14.8 \times 10^{-6}}{1.14 \times 10^{-6}} = \mathbf{38.95\ m/s}$$

Power can be determined from drag coefficient, by multiplying and dividing by u as

$F \times u$ power $\quad \dfrac{F \times u}{\rho A u^2 u} = \dfrac{P}{\rho A u^3}$ As A is the same,

$$P_{air} = P_w \frac{\rho_{air}}{\rho_w} \frac{u_{air}{}^3}{u_w{}^3} = 3.75 \times \frac{1.28}{1000} \times \left(\frac{38.95}{3}\right)^3 = \mathbf{10.5\ kW}$$

Problem 9.10 *The performance of a torpedo, 1 m diameter and 4 m long is to be predicted for speeds of 10 m/s. If a scale model of 1/25 size is used to predict the performance using a water tunnel, **determine the flow velocity** required. The ratio of density between sea water and fresh water is 1.02 and the viscosity ratio is 1.05. Also **determine the value of Reynolds number,** if the density of water was 1000 kg/m^3 and kinematic viscosity was 0.832 × 10^{-6} m^2/s.*

This is a fully submerged flow. Hence Reynolds number similarity should be maintained in the test. *i.e.* $\dfrac{D_p u_p \rho_p}{\mu_p} = \dfrac{D_m u_m \rho_m}{\mu_m}$,

$$u_m = u_p \times \frac{D_p}{D_m} \times \frac{\rho_p}{\rho_m} \times \frac{\mu_m}{\mu_p} = 10 \times 25 \times 1.02/1.05 = 242.85 \text{ m/s}$$

This is a very high speed generally not achievable in water tunnel.

$$Re = D_m\, u_m\, \rho_m/\mu_m = \frac{1}{25} \times \frac{242.85 \times 1000}{1000 \times 0.832 \times 10^{-6}} = 11.67 \times 10^6$$

For values of $Re > 10^5$ the coefficient of drag remains constant. Hence strict Reynolds number similarity need not be insisted on beyond such value.

In this case for example, velocity around 2.5 m/s may be used for the test which corresponds to $Re = 1.2 \times 10^5$.

Problem 9.11 *A 1/6 scale model of a submarine is tested in a wind tunnel using air of density 28.5 kg/m³ and viscosity 18.39 × 10⁻⁶ kg/ms at a speed of 36.6 m/s.* ***Calculate the corresponding speed and drag*** *of the prototype when submerged in sea water with density 1025 kg/m³ and viscosity 1.637 × 10⁻³ kg/ms if the model resistance was 67 N.*

Reynolds number similarity should be considered in this case. Let L be the length of the prototype.

$$\frac{36.6 \times L \times 28.5}{6 \times 18.39 \times 10^{-6}} = \frac{u_p \times L \times 1025}{1.637 \times 10^{-3}} \quad \therefore \mathbf{u_p = 15.2 \text{ m/s}}$$

Using drag coefficient

$$\frac{F_m}{\rho_m u_m^2 L_m^2} = \frac{F_p}{\rho_p u_p^2 L_p^2}$$

$$\therefore \qquad \mathbf{F_p} = F_m \times \frac{\rho_p}{\rho_m} \left(\frac{u_p}{u_m}\right)^2 \left(\frac{L_p}{L_m}\right)^2 = 67 \times \frac{1025}{28.5} \left(\frac{15.2}{36.6}\right)^2 (6)^2 = \mathbf{14961 \text{ N}}$$

Problem 9.12 *A sonar transducer in the shape of a sphere of 200 mm diameter is used in a boat to be towed at 2.6 m/s in water at 20°C. To determine the drag on the transducer a model of 100 mm diameter is tested in a wind tunnel, the air being at 20° C. The drag force is measured as 15 N. Determine the* ***speed of air for the test. Estimate the drag*** *on the prototype.*

As it is fully immersed type of flow, Reynolds number similarity should be maintained.

The density and kinematic viscosity values are :

$$\rho_{air} = 1.205 \text{ kg/m}^3, \quad v_{air} = 15.06 \times 10^{-6} \text{ m}^2/\text{s}$$
$$\rho_w = 1000 \text{ kg/m}^3, \quad v_w = 1.006 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\frac{u_m D_m}{v_m} = \frac{u_p D_p}{v_p}$$

$$\mathbf{u}_m = u_p \frac{D_p}{D_m} \frac{v_m}{v_p} = 2.6 \times \frac{200}{100} \times \frac{15.06 \times 10^{-6}}{1.006 \times 10^{-6}} = \mathbf{77.85 \text{ m/s}}$$

Mach number will be about 0.25. Hence compressibility effect will be negligible.

The coefficient of drag should be same for this condition. As $A \propto D^2$

$$\frac{F_m}{\rho_m u_m^2 D_m^2} = \frac{F_p}{\rho_p u_p^2 D_p^2},$$

$$\therefore \qquad \mathbf{F_p} = 15 \times \frac{1000}{1.205} \times \left[\frac{2.6 \times 200}{77.85 \times 100}\right]^2 = \mathbf{55.54 \text{ N}}$$

Problem 9.13 *In order to predict the flow conditions after the turbine outlet (tail race) of a hydroelectric plant delivering 2400 m³/s, a model of 1/75 scale is proposed.* ***Determine the flow rate required.***

This is a free surface flow. Hence Froude number similarity is to be maintained.

$$Fr_m = Fr_p \quad \text{or} \quad \frac{u_m}{\sqrt{l_m}} = \frac{u_p}{\sqrt{l_p}} \quad \text{or} \quad \frac{u_m}{u_p} = \sqrt{\frac{l_m}{l_p}}$$

As flow ($Q = Au$) depends on area which varies as L^2

$$\therefore \qquad \frac{Q_m}{Q_p} = \frac{A_m u_m}{A_p u_p} = \frac{{L_m}^2}{{L_p}^2}\sqrt{\frac{L_m}{L_p}} = \left(\frac{L_m}{L_p}\right)^{2.5}$$

$$\therefore \qquad \mathbf{Q_m} = 2400\left(\frac{1}{75}\right)^{2.5} = \mathbf{0.04927\ m^3/s}$$

Problem 9.14 *The total drag on a ship having a wetted hull area of 2500 m² is to be estimated. The ship is to travel at a speed of 12 m/s. A model 1/40 scale when tested at corresponding speed gave a total resistance of 32 N. From other tests the frictional resistance to the model was found to follow the law $F_{sm} = 3.7\, u^{1.95}$ N/m² of wetted area. For the prototype the law is estimated to follow $F_{sp} = 2.9\, u^{1.8}$ N/m² of wetted area* ***Determine the expected total resistance.***

The total resistance to ships movement is made up of (*i*) wave resistance and (*ii*) frictional drag. For wave resistance study Froude number similarity should be maintained. For frictional resistance Reynolds number similarity should be maintained. But it is not possible to maintain these similarities simultaneously. In the case of ships the wave resistance is more difficult to predict. Hence Froude number similarity is used to estimate wave resistance. Frictional drag is estimated by separate tests. From the Froude number similarity,

$$\mathbf{u_m} = u_p\sqrt{\frac{l_m}{l_p}} = 12/40^{0.5} = \mathbf{1.897\ m/s}$$

The **skin friction drag** for the model is calculated using this velocity.

$$F_{sm} = 3.7 \times 1.897^{1.95} \times A_m \quad \text{as} \quad A_m = 2500/40^2$$

$$= 3.7 \times 1.897^{1.95} \times 2500/40^2 = \mathbf{20.16\ N}$$

Wave drag on the model = 32 – 20.16 = **11.84 N**

The wave drag is calculated using $(F/\rho\, u^2L^2)_m = (F/\rho\, u^2L^2)_p$

Noting that sea water is denser with $\rho = 1025$ kg/m³

$$F_{wp} = F_{wm}\,\frac{\rho_p}{\rho_m}\left(\frac{L_p}{L_m}\right)^2\left(\frac{u_p}{u_m}\right)^2 = 11.84 \times \frac{1025}{1000}(40)^2\left(\frac{12}{1.897}\right)^2 = 774.38 \times 10^3\ \text{N}$$

Skin friction drag for the prototype

$\mathbf{F_{sp}} = 2.9\, {u_p}^{1.8} \times A_p = 2.9 \times 12^{1.8} \times 2500 = 635.13 \times 10^3$ N

$\therefore$ Total resistance = 1.41×10^6 N or **1.41 MN**

Problem 9.15 *A scale model of a ship of 1/30 size is to be towed through water. The ship is 135 m long. For similarity* ***determine the speed with which the model should be towed.*** *The ship is to travel at 30 kmph.*

Froude number similarity is to be maintained.

$$\frac{u_m}{\sqrt{gL_m}} = \frac{u_p}{\sqrt{gL_p}} \qquad \therefore \quad \mathbf{u_m} = u_p\sqrt{\frac{L_m}{L_p}} = \frac{30 \times 1000}{3600} \times \frac{1}{\sqrt{30}} = \mathbf{1.52\ m/s}$$

Problem 9.16 *The wave resistance of a ship when travelling at 12.5 m/s is estimated by test on 1/40 scale model. The resistance measured in fresh water was 16 N.* ***Determine the speed of the model and the wave resistance of the prototype*** *in sea water. The density of sea water = 1025 kg/m³.*

Froude number similarity is to be maintained.

$$\therefore \qquad \mathbf{u_m} = u_p \sqrt{\frac{L_m}{L_p}} = 12.5 \sqrt{\frac{1}{40}} = \mathbf{1.976\ m/s}$$

The wave resistance is found to vary as given below.

$$\frac{F_m}{\rho_m u_m^2 L_m^2} = \frac{F_p}{\rho_p u_p^2 L_p^2}$$

$$\therefore \qquad \mathbf{F_p} = F_m \times \frac{\rho_p}{\rho_m}\left(\frac{u_p}{u_m}\right)^2\left(\frac{L_p}{L_m}\right)^2 = 16 \times \frac{1025}{1000}\left(\frac{12.5}{1.976}\right)^2 (40)^2$$

$$= \mathbf{1049.6 \times 10^3\ N} \quad \text{or} \quad \mathbf{1050\ kN}$$

Problem 9.17 *Vortex shedding at the rear of a structure of a given section can create harmful periodic vibration. To predict the shedding frequency, a smaller model is to be tested in a water tunnel. The air speed is expected to be about 65 kmph. If the geometric scale is 1:6 and if the water temperature is 20° C determine the speed to be used in the tunnel. Consider air temperature as 40° C. If the shedding frequency of the model was 60 Hz* ***determine the shedding frequency*** *of the prototype. The dimension of the structure are diameter = 0.12 m, height = 0.36 m.*

The frequency of vortex shedding can be related by the equation

$$\omega = F\,(d, h, u, \rho, \mu)$$

Dimensional analysis leads to the π terms relation, (refer Chapter 8)

$$\frac{\omega D}{u} = f\left(\frac{D}{H}, \frac{\rho u D}{\mu}\right)$$

The model dimension can be determined as

$$D_m = 1/6\ D_p = 0.12/0.6 = 0.02\ \text{m},\ H_m = 1/6\ H_p = 0.36/0.6 = 0.06\ m$$

$$\therefore \qquad \frac{D}{H} = \frac{0.02}{0.06} = \frac{1}{3}, \qquad \text{Reynolds similarity requires}$$

$$\frac{\rho_m u_m D_m}{\mu_m} = \frac{\rho_p u_p D_p}{\mu_p} \qquad \therefore \quad u_m = u_p \frac{\rho_p}{\rho_m}\frac{\mu_m}{\mu_p}\frac{D_p}{D_m}$$

The property values of air and water at the given temperatures are,

$$\rho_p = 1.128\ \text{kg/m}^3, \quad \mu_p = 19.12 \times 10^{-6}\ \text{kg/ms}$$

$$\rho_m = 1000\ \text{kg/m}^3, \quad \mu_p = 1.006 \times 10^{-6}\ \text{kg/ms}$$

$$u_p = 65 \times 1000/3600 = 18.056\ \text{m/s}$$

$$\therefore \qquad \mathbf{u_m} = 18.056 \times \frac{1.128}{1000} \times \frac{1.006 \times 10^{-3}}{19.12 \times 10^{-6}}\,\frac{6}{1} = \mathbf{6.43\ m/s}$$

Chapter 9

Vortex shedding frequency is determined. Using the third π parameter,

$$\frac{\omega_m D_m}{u_m} = \frac{\omega_p D_p}{u_p}$$

$$\therefore \qquad \omega_p = \frac{u_p}{u_m}\frac{D_p}{D_m}\ \omega_m = \frac{18.06}{6.43} \times \frac{1}{6} \times 60 = \mathbf{28.08\ Hz.}$$

The drag also can be predicted from the model. The drag for unit length can be expressed in the dimensionless from as $D/d\rho u^2$ where D is the drag and d is the diameter. Thus

$$\frac{D_p}{d_p \rho_p {u_p}^2} = \frac{D_m}{d_m \rho_m {u_m}^2} \quad \therefore \quad D_p = D_m \cdot \frac{d_p}{d_m} \cdot \frac{\rho_p}{\rho_m} \cdot \frac{{u_p}^2}{{u_m}^2}$$

Problem 9.18 *In order to determine the drag on supporting columns (of a bridge) of* 0.3 *m diameter, due to water flowing at a speed of 14.5 km/hr, a column of 0.25 m diameter was tested with air flow. The resistance was measured as 227 N/m, under similar conditions of flow.* ***Determine the force on the bridge*** *column per m length.* $v_{air} = 1.48 \times 10^{-5}\ m^2/s$. $v_w = 1.31 \times 10^{-6}\ m^2/s, \rho_a = 1.23\ kg/m^3$

Similarity requires equal Reynolds numbers

Velocity of flow of water = 14.5 × 1000/3600 = 4.028 m/s

$$\frac{4.028 \times 0.3}{1.31 \times 10^{-6}} = \frac{u_a \times 0.25}{1.48 \times 10^{-5}}$$

∴ Velocity of air, u_a = 54.61 m/s

The force can be obtained by the dimensional parameter (drag coefficient)

$F/\rho A u^2$, here $A = 1 \times D$

∴ The parameter in this case for force is

$$\frac{F}{\rho D u^2} \quad \frac{F_w}{\rho_w D_w {u_w}^2} = \frac{F_a}{\rho_a D_a {u_a}^2} \quad \text{or}$$

$$\mathbf{F_w} = F_a\left(\frac{\rho_w}{\rho_a}\right)\left(\frac{D_w}{D_a}\right)\left(\frac{u_w}{u_a}\right)^2 = 227 \times \frac{1000}{1.23} \times \frac{0.3}{0.25} \times \left(\frac{4.028}{54.61}\right)^2 = \mathbf{1205\ N}$$

Problem 9.19 *To ascertain the flow characteristics of the spillway of a dam, 1/20 geometric scale model is to be used. The spillway is 40 m long and carries 300 m³/s at flood condition.* ***Determine the flow rate required*** *to test the model. Also* ***determine the time scale*** *for the model. Viscous and surface tension effects may be neglected.*

This situation is open surface flow. Froude number similarity is required.

$$\frac{u_m}{(gL_m)^{0.5}} = \frac{u_p}{(gL_p)^{0.5}} \quad \text{or} \quad \therefore \ \frac{u_m}{u_p} = \left(\frac{L_m}{L_p}\right)^{0.5}$$

As $\qquad Q = uA = uL^2,\ Q_m = u_m L_m^2,\ Q_p = u_p L_p^2,\ \dfrac{Q_m}{Q_p} = \dfrac{u_m L_m^2}{u_p L_p^2}$

$$\therefore \qquad \mathbf{Q_m} = Q_p \frac{u_m}{u_p}\frac{L_m^{\;2}}{L_p^{\;2}} = Q_p \left(\frac{L_m}{L_p}\right)^{0.5}\left(\frac{L_m}{L_p}\right)^{2} = Q_p \left(\frac{L_m}{L_p}\right)^{2.5}$$

$$= 300\left(\frac{1}{20}\right)^{2.5} = \mathbf{0.168\ m^3/s}$$

Time scale can be determined from velocities, as velocity = length/time.

$$\frac{u_m}{u_p} = \frac{L_m}{L_p}\frac{t_p}{t_m} \quad \therefore \quad \frac{\mathbf{t_m}}{\mathbf{t_p}} = \frac{L_m}{L_p}\frac{u_p}{u_m} = \frac{L_m}{L_p}\left(\frac{L_p}{L_m}\right)^{0.5} = \left(\frac{L_m}{L_p}\right)^{0.5} = \left(\frac{1}{20}\right)^{0.5} = \mathbf{0.2236}$$

Problem 9.20 *A fan when tested at ground level with air density of 1.3 kg/m*^{3}*, running at 990 rpm was found to deliver 1.41 m*^{3}*/s at a pressure of 141 N/m*^{2}*. This is to work at a place where the air density is 0.92 kg/m*^{3}*, the speed being 1400 rpm.*

Determine the volume delivered and the pressure rise.

For similarity condition the flow coefficient Q/ND^3 should be equal.

As D is the same,

$$\frac{Q_1}{N_1} = \frac{Q_2}{N_2} \qquad \text{or} \qquad \mathbf{Q_2} = Q_1 \frac{N_2}{N_1} = 1.41 \times \frac{1400}{990} = \mathbf{2\ m^3/s}$$

The head coefficient $H/\rho N^2D^2$ is used to determine the pressure rise.

$$\mathbf{\Delta P_2} = \Delta P_1 \frac{\rho_2 N_2^{\;2}}{\rho_1 N_1^{\;2}} = 141 \times \frac{0.92}{1.3} \times \left(\frac{1400}{990}\right)^2 = \mathbf{199.55\ N/m^2}$$

Problem 9.21 *A centrifugal pump with dimensional specific speed (SI) of 2300 running at 1170 rpm delivers 70 m*^{3}*/hr. The impeller diameter is 0.2 m. Determine* ***the flow, head and power if*** *the pump runs at 1750 rpm . Also* ***calculate the specific speed*** *at this condition.*

The head developed and the power at test conditions are determined first. (At 1170 rpm).

$$N_s = N\sqrt{Q}/H^{3/4} = 1170\sqrt{70}/H^{3/4} = 2300 \quad \therefore\ \mathbf{H = 6.9\ m}$$

$$\textbf{Power} = \text{mg H} = 9.81 \times 70000 \times 6.9/3600 = \mathbf{1316\ W}$$

When operating at 1750 rpm, using flow coefficient Q/ND^3, as D is the same

$$\mathbf{Q_2} = 70\left(\frac{1750}{1170}\right) = \mathbf{104.7\ m^3/hr}$$

Using head coefficient, H/N^2D^2, $\mathbf{H_2} = H_1\,(N_1/N_2)^2 = 6.9 \times (1750/1170)^2 = \mathbf{15.44\ m}$

Using power coefficient : P/rN^3D^5,

$$\mathbf{P_2} = P_1 \times \left[\frac{N_2}{N_1}\right]^3 = 1316 \times \left[\frac{1750}{1170}\right]^3 = \mathbf{4404\ W}$$

Specific speed for the model

$$N_s = N\sqrt{Q}/H^{3/4} = 1750\sqrt{104.7}/(15.44)^{3/4} = 2300$$

Note: Specfic speeds are the same.

Problem 9.22 *A pump running at 1450 rpm with impeller diameter of 20 cm is geometrically similar to a pump with 30 cm impeller diameter running at 950 rpm. The discharge of the larger pump at the maximum efficiency was 200 litres/s at a total head of 25m.* ***Determine the discharge*** *and head of the smaller pump at the maximum efficiency conditions. Also determine the* ***ratio of power*** *required.*

The *PI* terms of interest are the head coefficient, power coefficient scale and $Q/\omega D^3$ called flow coefficient $(\omega \propto N)$ (Refer chapter 8, Problem 8.16).

Considering flow coefficient, denoting the larger machines as 1 and the smaller as 2,

$$\frac{Q_1}{\omega_1 {D_1}^3} = \frac{Q_2}{\omega_2 {D_2}^3} \quad \therefore \quad \mathbf{Q_2} = Q_1 \frac{\omega_2}{\omega_1} \frac{{D_2}^3}{{D_1}^3} = 200 \times \frac{1450}{950} \left(\frac{20}{30}\right)^3 = \mathbf{90.45\ l/s}$$

Considering head coefficient, (g being common)

$$\frac{gh_1}{{\omega_1}^2 {D_1}^2} = \frac{gh_2}{{\omega_2}^2 {D_2}^2} \quad \therefore \; h_2 = h_1 \left(\frac{\omega_2}{\omega_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2$$

$$\therefore \qquad \mathbf{h_2 =} 25 \times \left[\frac{1450}{950}\right]^2 \left(\frac{20}{30}\right)^2 = \mathbf{25.885\ m}$$

Consider power coefficient $\dfrac{P_1}{\rho_1 {\omega_1}^3 {D_1}^5} = \dfrac{P_1}{\rho_1 {\omega_1}^3 {D_1}^5}$, as $\rho_1 = \rho_2$,

$$\frac{\mathbf{P_2}}{\mathbf{P_1}} = \left(\frac{\omega_2}{\omega_1}\right)^3 \left(\frac{D_2}{D_1}\right)^5 = \left(\frac{1450}{950}\right)^3 \left(\frac{20}{30}\right)^5 = \mathbf{0.468}$$

As efficiencies should be the same, $Q_1\, \rho_1\, h_1 = Q_2\, \rho_2\, h_2$, with $\rho_1 = \rho_2$

$$0.200 \times 25 = 0.09045 \times 25.885/0.468, \; 5.00 = 5.00 \text{ (checks)}$$

Specific speed $= N\sqrt{Q}/H^{3/4} = 1450\sqrt{0.09045}\,/\,25.885^{3/4} = 38$ (dimensional)

For larger pump, specific speed = $950\sqrt{0.2}/25^{3/4}$ = 38, checks.

Problem 9.23 *A V notch is to be used with utectic calcium chloride solution at 30°C. Density = 1282 kg/m³, v_e = 2.267 × 10⁻⁶ m²/s. The flow rate has to be found for various heads. Water was used for the test at 20°C. Density = 1000 kg/m³, v_w = 1.006 × 10⁻⁶ m²/s. Neglecting the effect of surface tension,* ***determine the ratio of corresponding heads*** *and* ***mass flow rates of water and*** *the solution at the corresponding heads.*

Dimensional analysis shows (Neglecting surface tension effects), Q being volume flow rate that for similarity the following parameters should be equal. (suffix c refers to the solution properties) (Refer chapter 8, Problem 8.13).

$$\frac{Q}{g^{1/2} h^{5/2}} = f\left[\frac{g^{1/2} h^{3/2}}{v}, \theta\right]$$

$$\therefore \qquad \frac{{h_w}^{3/2}}{v_w} = \frac{{h_c}^{3/2}}{v_c}$$

$$\therefore \qquad \frac{\mathbf{h_c}}{\mathbf{h_w}} = \left(\frac{v_c}{v_w}\right)^{2/3} = \left[\frac{2.267 \times 10^{-6}}{1.006 \times 10^{-6}}\right]^{2/3} = \mathbf{1.71884}$$

$$\frac{Q_c}{h_c^{\,5/2}} = \frac{Q_w}{h_w^{\,5/2}}$$

$$\therefore \qquad \frac{\mathbf{Q_c}}{\mathbf{Q_w}} = \left(\frac{h_c}{h_w}\right)^{5/2} = (1.71884)^{5/2} = \mathbf{3.873}$$

Ratio of mass flow rates = 3.873 × (1282/1000) = **4.97**

Problem 9.24 *The discharge Q through an orifice is found to depend on the parameter* $\rho D\sqrt{gH}/\mu$, *when surface tension effect is neglected. Determine the ratio of flow rates of water and refrigerant 12 at 20°C under the same head.* ***What should be the ratio of heads for the*** *same flow rate.* $\mu_R = 2.7 \times 10^{-4}$ *kg/ms.,* $\mu_w = 1.006 \times 10^{-3}$ *kg/ms. Density of refrigerent = 923 kg/m*^{3}.

$$Q_w \propto \rho_w D\sqrt{gH}/\mu_w \quad \text{and} \quad Q_R \propto \rho_R D\sqrt{gH}/\mu_r, \text{ Dividing}$$

$$\frac{\mathbf{Q_R}}{\mathbf{Q_w}} = \frac{\rho_R}{\rho_w} \times \frac{\mu_R}{\mu_w} = \frac{923}{1000} \times \frac{1.006 \times 10^{-3}}{2.7 \times 10^{-4}} = \mathbf{3.44}$$

For the same flow rate

$$\frac{\rho_w D\sqrt{gH_w}}{\mu_w} = \frac{\rho_R D\sqrt{gH_R}}{\mu_R}$$

$$\therefore \qquad \frac{\mathbf{H_w}}{\mathbf{H_R}} = \left(\frac{\mu_w}{\mu_R} \times \frac{\rho_R}{\rho_w}\right)^2 = \left(\frac{1.006 \times 10^{-3}}{2.7 \times 10^{-4}} \times \frac{923}{1000}\right)^2 = \mathbf{11.827}$$

OBJECTIVE QUESTIONS

O Q. 9.1 Fill in the blanks.

1. The representation of a physical system used to predict the behaviour of the system is called __________.
2. The system whose behaviour is predicted by the model is called __________.
3. Models are generally __________ in size compared to prototype.
4. When the prototype is very small __________ model is used.
5. Models may also be used to predict the effect of __________ to an existing system.
6. Dimensionless parameters provide __________ conditions for model testing.
7. For geometric similarity ratio of __________ should be equal.
8. For dynamic similarity ratio of __________ should be equal.
9. If stream lines are similar between model and prototype it is called __________ similarity.
10. When geometric and dynamic similarities exist then automatically __________ will exist.

Answers

1. Model 2. Prototype 3. Smaller 4. Enlarged/larger 5. Modifications 6. Similarity 7. Linear dimensions of model and prototype 8. Forces at corresponding locations of model and prototype 9. kinematic 10. kinematic similarity.

O Q. 9.2 Fill in the blanks.

1. For rivers, harbours etc. ________ models are used.
2. Reason for distorted models for rivers etc. is because of ________.
3. For complete similarity in general the ________ number for the model and prototype should be the same.
4. When viscous and inertia forces are important ________ number similarity should be used.
5. When gravity and inertia forces are important ________ number similarity is used.
6. When the ratio of velocities and accelerations are equal at corresponding locations in the model and prototype it is called ________ similarity.
7. When gravity and surface tension are important, then ________ number similarity is used.
8. When wave resistance is important ________ similarity is used.
9. For flow with free surface ________ number similarity is used.
10. To consider compressible flow effects ________ number similarity is used.
11. When periodic motion is to be considered ________ number similarity is used.

Answers

1. Distorted 2. Very small flow height for model 3. Reynolds, Froude, Weber, Mach 4. Reynolds 5. Froude 6. Dynamic 7. Weber 8. Froude 9. Froude 10. Mach 11. Strouhal

O Q. 9.3 Fill in the blanks.

1. For incompressible flow through closed ducts ________ similarity is used.
2. For compressible flow through closed ducts ________ and ________ similarity are used.
3. For flow around immersed bodies ________ similarity is used in case of incompressible flow.
4. For flow around immersed bodies ________ and ________ similarities are used in the case of compressible flow.
5. In case of same fluid properties it is ________ to have Reynolds and Mach analogy simultaneously.
6. ________ Wind tunnels are used to have simultaneously Reynlods and Mach analogy.
7. In flow with free surface ________ similarity is used.
8. For studying wave drag ________ similarity is used.
9. At high Reynolds numbers the coefficient of drag does not significantly ________.
10. Froude and Reynolds similarities ________ be maintained simultaneously.

Answers

1. Reynolds 2. Reynolds and Mach 3. Reynolds 4. Reynolds, Mach 5. Impossible 6. Pressurised 7. Froude 8. Froude 9. vary 10. cannot

O Q. 9.4 State correct or incorrect.

1. Geometric similarity will automatically lead to kinematic similarity.
2. Under kinematic similarity conditions dynamic similarity will result automatically.
3. Geometric similarity will lead to dynamic similarity.
4. Webr number similarity is used to study wave drag.

5. Froude number similarity is used to study wave drag.
6. Mach number similarity need not be considered for low velocities.
7. For obtaining simultaneously Reynolds number and Mach number similarity pressurised tunnels are used.
8. When surface tension forces prevail Froude number similarity should be used.
9. When gravity forces prevail Froude number similarity should be used.
10. For fluctuating flow Strouhal number similarity should be used.
11. Distorted models are used to study river flow.
12. At high Reynolds numbers viscous drag coefficient remains constant.

Answers

Correct : 2, 5, 6, 7, 9, 10, 11, 12
Incorrect : 1, 3, 4, 8

EXERCISE PROBLEMS

Note: Property values are not specified. These should be obtained from tables of properties.

E 9.1. An airship is to operate in air at 20ºC and 1 bar at 20 m/s speed. A model of scale 1/20 is used for tests in a wind tunnel, the test speed being 75 m/s. Determine the pressure of the tunnel for dynamic similarity. The air temperatures are equal. If the drag force on the model was 250 N. Determine the drag on the prototype **(539 kPa, 1340 N)**

E 9.2. One fifth scale model of an automobile is tested in a towing water tank. Determine the ratio of speeds of the model and prototype. Assume 20ºC in both cases. It is found that the coefficient of drag remains constant for the model after speeds of 4 m/s and the drag at this speed was 182 N. Estimate the drag on the prototype when operating at 90 kmph.

((u_m/u_p) = 0.345, 219 N)

E 9.3. A torpedo 533 mm dia and 6700 mm long is to travel in water at 28 m/s. A model of 1/5 scale is to be tested in a wind tunnel. Air speed in the tunnel should not exceed 110 m/s to avoid compressibility effect. If air can be pressurised with temperature remaining at 20ºC determine the minimum pressure required. Dynamic viscosity is not affected by pressure. If the drag force at this condition on the model was 618N, Determine the drag on the prototype.

(1.93 Mpa, 43. 7 kN)

E 9.4. A dynamically similar model of an airfoil of 1/10 scale was tested in a wind tunnel at zero angle of attack at a Reynoles number of 5.5×10^6 (based on chord length). The temperature and pressure in the wind tunnel are 15ºC and 10 atm absolute. The prototype has a chord length of 2 m and it is to fly at 15ºC and 1 atm. Determine the speed in the wind tunnel and the prototype speed. **(both 39.2 m/s)**

E 9.5. A 3 m dia weather balloon of spherical shape is to travel at 1.5 m/s. To determine the drag a model of dia 50 mm is to be tested in a water tunnel. Under dynamically similar conditions the drag on the model was measured as 3.78 N. Calculate the test speed and estimate the drag on the balloon. **(6.21 m/s, 0.978 N)**

E 9.6. A model of an automobile of scale 1/5 is tested in water tunnel for obtaining the performance of the prototype. If the prototype is to travel at 27.78 m/s in air at 15ºC, determine the speed in the tunnel. Also determine the ratio of drag forces. If at a point the pressure coefficient was –1.4, determine the static pressure at the point.

E 9.7. The power requirement of a tractor tailor with a frontal area of 0.625 m^2 when travelling at 22.4 m/s is to be estimated by a model to be tested in a wind tunnel. The scale for the model is 1/4. When tested at 1 atm and 15º*C* at a speed of 89.6 m/s, the drag on the model was measured as 2.46 kN. Determine the drag on the prototype and the power required.

(2.46 kN, 55.1 kW)

E 9.8. Water flows at the rate of 40 m^3/s through a spillway of a dam. The width of the spillway is 65 m. A model of 1.5 m width is proposed for laboratory test. Determine the flow rate of the model.

(3.24×10^{-3} m^3/s)

E 9.9. The pressure drop through an elbow of 150 mm diameter is to be determined by test on a model. The flow through the elbow is water at 20ºC. The flow velocity is 5 m/s. The model is to be tested with water at 20ºC and the velocity is limited to 10 m/s. Determine the dimeter of the model. If the pressure drop in the model was measured as 15 kpa. Determine the pressure drop in the prototype.

E 9.10. In a flow the geometric scale is 1/4. The density scale is 1. If inertial gravitational, surface tension and viscous effects are important determine the viscosity and surface tension scales.

E 9.11. The drag on a solid body having a characteristic length of 2.5 mm, moving with a velocity of 15 m/s through water is to be estimated by a study on a model having 50 times this characteristic length.

(*i*) In case a water tunnel is used determine the velocity. Also determine the ratio between model and prototype drag.

(*ii*) Repeat the above in case a wind tunnel is used.

Assume a temperature 20ºC in all cases.

E 9.12. The drag characteristics of a new design of an automobile at 32 kmph and 144 kmph. speeds are to be estimated. The characteristic length of the unit is 7 m. If a model of 1.2 m is to be used in an atmospheric pressure wind tunnel, determine the air velocity required. In case a pressurised tunnel with a pressure of 8 atm is used, determine the velocity required. In this case the model size is to be 0.6 m. Assume same temperature in all cases.

E 9.13. An open channel of rectangular section of width 7 m carries water to a depth of 1 m and a flow rate of 2 m^3/s. A model to have Froude number similarity is to be designed. The discharge scale is 1/1000. Determine the depth of flow in the model.

E 9.14. A 1/50 scale model of a ship is tested in a towing tank to determine the wave drag on the ships hull. The ship is to designed to cruise at 18 knots (knot = 1852 m). Determine the velocity with which the model is to be towed. Also determine the ratio of drag values on model and prototype. Neglect viscous drag. **(1.31m/s, 1.25×10^5)**

E 9.15. It is proposed to design a centrifugal pump to deliver 4.1 m^3/s of water at 200 *m* head when running at 1200 rpm. The diameter of the impeller is to be 1 m. A laboratory model of 1/5 th scale is proposed for testing. The model is to run at the same speed. Determine the operating head and discharge of the model. Assume both model and prototype operate at the same efficiency. **(8 m, 0.0328 m^3/s)**

E 9.16. Oil flows over a submerged body horizontally at a velocity 15 m/s. The property values for oil are: kinematic viscosity = 3.45×10^{-5} m^2/s, density = 833 kg/m^3. An enlarged model is used with 8:1 scale in a water towing tank. Determine the velocity of the model to achieve dynamic similarity. If drag force on the model is 3.5 N, predict the drag force on the prototype. Kinematic viscosity of water = 1.14×10^{-6} m^2/s. **(0.062 m/s, F = 2666 N)**

E 9.17. A model 1/50 scale of a boat when tested at 1 m/s in water gave a wave resistance of 0.02 N. Determine the velocity of operation of the boat for similarity. Also determine the drag force and the power required for cruising the boat. **(7.1 m/s, 2500 N, 17.75 kW)**

E 9.18. A model of an aeroplane of 1/20 size is to be tested in a pressurised wind tunnel at the same speed as that of the prototype to get over compressibility effects. If the temperatures are the same, determine the pressure in the wind tunnel in atm. The aeroplane is to be operated at 0.8 atm.

E 9.19. The drag force on a sphere submerged in water at 20ºC, when moved at 1.5 m/s was measured as 10 N. An enlarged model of 3:1 scale was tested in a pressurised wind tunnel at a pressure of 1.5 MN/m^2 and temperature of 20ºC. Determine the velocity for dynamic similarity. Also determine the drag force on the model. Kinematic viscosity of water = 1.006×10^{-6} m^2/s. Viscosity of air = 18.14×10^{-6} kg/ms. Density of air = 17.83 kg/m^3.

E 9.20. Determine the flow rate of air at 80º*C* in a 50 mm diameter pipe that will give dynamic similarity for flow of 50 l/s of water at 60ºC in a 400 mm diameter pipe if the pressure of air is 4 bar. Kinematic viscosity of water = 0.478×10^{-6} m^2/s. Dynamic viscosity of air = 20.1×10^{-6} kg/ms. Density = 4.24 kg/m^3.

E 9.21. The flow rate over a spillway of a dam was 150 m^3/s. If the flow rate over the model was 1.35 m^3/s. Determine the linear scale. If the force at a certain point on the model was measured as 5 N, determine the force at the corresponding point on the prototype.

E 9.22. A ship 180 m long is to cruise at a speed of 40 kmph in sea water whose viscosity is 1.2 cp and specfic weight is 10 kN/m^3. If a model 3 m length is to satisfy both Reynolds number and Froude number similarity calculate the kinematic viscosity of the fluid to be used with the model. Comment on the results.

E 9.23. A small insect of about 1 mm dia moves slowly in sea water. To determine the drag an enlarged model of 100:1 scale, tested in glycerin at a velocity of 30 cm/s measured a drag of 1.3 N, Determine the speed of travel of the insect and also the drag force on it. μ_w = 0.001 kg/ms, ρ_w = 998 kg/m^3, μ_w = 1.5 kg/ms, ρ_g =1263 kg/m^3 **(2.53 cm/s, 7.31×10^{-7} N)**

E 9.24. The slope of the free surface of a steady wave in one dimensional flow in a shallow liquid layer is described by the equation below. Non dimensionless the equation and obtain dimensionless groups to characterise the flow. Determine the condition for dimensional similarity.

$$\frac{\partial h}{\partial x} = -\frac{u}{g} \cdot \frac{\partial u}{\partial x}.$$

E 9.25. One dimensional unsteady flow in a thin liquid layer is described by the equation below. Non dimensionlise the equation and obtain dimensionless groups to characetrise the flow.

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} = -g\frac{\partial h}{\partial x}.$$

E 9.26. Steady incompressible two dimensional flow, neglecting gravity is described by the equations below. Non dimensioalise the equation and obtain the diminsionless groups that characterise the flow.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,\ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\frac{\partial^2 u}{\partial y^2}.$$

E 9.27. A large veturimeter is calibrated using 1/10 scale model. If the same fluid conditions are used for the model and prototype determine the discharge ratio. Assume dynamic similarity conditions. **(1/10)**

E 9.28. The scale ratio between model and prototype of a spillway is 1/25. Determine the ratio of velocities and discharges. If the prototype discharges 3000 m^3/s. Calculate the model discharges. **(1/5, 1/3125, 0.96 m^3/s)**

E 9.29. The performance of a spherical balloon to be used in air at 20ºC is to be obtained by a test in a water tank using 1/3 scale model. The diameter of the model is 1 m and when dragged at 1.2 m/s measured a drag of 200 N. Determine the expected drag on the prototype if the water temperature was 15ºC.

$v_a = 15.06 \times 10^{-6}$ m²/s, $\rho_a = 1.205$ kg/m³ $v_w = 1.2015 \times 10^{-6}$ m²/s **(42.2 N)**

E 9.30. In model testing if both Reynolds number similarity and Froude number similarity should be simultaneously maintained, determine the ratio of kinematic viscosity of fluids to be used with the model and prototype. **[v_m/v_p = (geometric scale)$^{1.5}$]**

E 9.31. Determine the ratio of drag coefficients of prototype to model when tested at 1/3 rd density of air, but at the same Mach number, the geometric scale being 1/5. **(25:1)**

E 9.32. A ship model of scale 1/50 showed a wave resistance of 30 N at its design speed. Determine the prototype wave resistance.

E 9.33. A turbine model of 1/5 scale uses 2 m³/s of water. The prototype turbine has to work with a flow rate of 15 m³/s. Determine the speed ratio and power ratio. **(16.67, 1.48)**

E 9.34. A centrifugal fan in operation when tested gave the following data. Volume delivered : 2.75 m³/s. Total pressure 63.5 mm of wate column. Power absorbed : 1.75 kW. A geometrically similar fan of 1/4 size is to be used running at twice the speed of the operating fan. Assuming same conditions for the air determine the volume delivered, total pressure and power absorbed.

(2.32 m³/s, 142.8 mm w.c, 3.2 kW)

E 9.35. The drag on a ship 122 m long and with 2135 m² wetted area is to be estimated. A model towed at 1.3 m/s through fresh water had a total drag resistance at 15.3 N. The skin resistance was separately analyzed and found to follow the law $F = c\, u_m^{1.9}$. When tested at 3 m/s, the skin resistance was 14.33 Nm² and the ships skin resistance is estimated to follow the law $F = c\, u_s^{1.85}$ and has a value of 43 N/m². Determine the corresponding speed of the ship and the power needed to propel it. Density of the sea water is 1025 kg/m³ **(7.51 m/s, 6530 kW)**

E 9.36. The prototype weir is to discharge under a head of 1.2 m. A scale model of 1/10 size is proposed and the available flow is 42.5 l/s. Determine the heads necessary and the corresponding discharge of the prototype weir. **(0.12 m, 13.44 m³/s)**

E 9.37. The skin resistance of a ship model is given by 5.22 $u_m^{1.95}$ N/m². For the prototype the skin resistance is given by 4.95 $u_p^{1.9}$ N/m². The model of 1/20 scale with wetted area of 4 m² when towed in fresh water at 1.2 m/s. measured a total resistance of 46.2 N. Determine the total resistance of the ship in sea water at speeds corresponding to that of the model. Density of sea water = 1025 kg/m³.