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Introduction

The presence of symmetry in our surroundings may be perceived, on the one hand, as a source of delight and intrigue or, on the other hand, as unattractively, constrained rigid order. Taking a psychological perspective of patterns, Gombrich commented that it is our search for meaning, our effort to find order, which determines the appearance of patterns, rather than the structure described by mathematicians.[1]

Primitive art, decorating the surfaces of archaeological treasure dating from before Christ, displays material evidence that people from different times and cultures had a natural perception of the balance and configurations derived from geometric shapes. Owen Jones,[2] in his classic work *The Grammar of Ornament* comments on this as follows:

> . . . the eye of the savage, accustomed only to look upon Nature's harmonies, would readily enter into the perception of the true balance both of form and colour; in point of fact, we find that it is so, that in savage ornament the true balance of both is always maintained.

Design and ornament throughout the ages appear to have been influenced by the aesthetic effect of due proportion present in the striking features of natural forms. The attraction of balance, harmony and complexity in nature, from microscopic to immense structures, has appealed to and affected both scientist and artist. For example, the biologist and philosopher, Ernst Heinrich Haeckel, was particularly interested in, and made detailed studies of, microscopic life, some of which displayed unusual and fascinating symmetrical characteristics.[3] D'Arcy Thompson, a mathematically minded biologist, observed that the beauty of a snow crystal depends on its mathematical regularity and symmetry and thought that the number of variants of a single type, all related but no two the same, vastly increased our pleasure and admiration of their form.[4]

In this context, the naturally formed attraction of the snowflake is dependent on a specific, invariant regular hexagonal structure whose intricacy of design elements is unpredictable and infinitely variable. Such a relationship between a basic formal structure and the individuality of stylistic approaches to its decoration forms a framework for the construction of regularly repeating designs.

A designer may be presented with a suitable structure, in the form of a lattice, along with a set of geometric rules, and from this, he or she may derive numerous symmetric decorative effects. Although each set of rules is geometrically predetermined (owing to the laws of crystallography), like the form of a snowflake, there is infinite design variation within each set of geometric constraints which, in the context of design construction, is dependent on the nature and artistic inclination of the designer. As the designer, Day, discovered, the art of the pattern designer is not merely to devise pretty combinations of form, but to work within these rules to produce beautiful results however unpromising the conditions of origin.[5]

An intuitive awareness of order may contribute to the way in which a designer fills out and completes the details of his or her design to achieve a satisfying sense of balance and harmony. However, as is recognised by all practising designers, their initial art work must be adapted to fit together with regular repetition, in other words the framework of their design must be contained within the mathematical constraints of geometry. With reference to printed textiles, Flower[6] stated that even the most sensitive and personal piece of work must eventually rely on geometry if it is to be printed in repeat. Some designers may not feel that it is advantageous to explore the avenues of design geometry owing to the restriction it imposes on the 'free' style of their artwork.[6] However, as illustrated by William Morris, a high order of symmetry in a design need not necessarily restrain its free-flowing nature. For example, his wallpaper designs 'Net Ceiling', 'Spring Thicket', 'Triple Net', 'Borage', 'Sunflower', 'Ceiling' and 'Autumn Flowers' all display reflectional properties, but their floral arrangements still drift freely and retain balance continuously throughout the designs.[7]

Another factor adding to the reluctance of the textile designer to penetrate the theory of geometry is 'mathematics' itself. The term 'mathematics' is often perceived in an unfavourable light by designers owing to its association with impenetrable theories and incomprehensible language and terminology. (This is not surprising as, generally, mathematicians are reluctant to use any more words than necessary and the substitution of letters from the Greek alphabet is infinitely preferable). With this in mind, Oleg Grabar observed that while it is legitimate enough at professional mathematical levels to see arbitrary signs and numbers as language, that language is hardly accessible to most mortals.[8] Thus the complicated terminology used in mathematical theories immediately hinders any progress in its application in areas other than its own or those which are very closely related. However, the two-dimensional theory relating to three-dimensional crystallographic groups is becoming increasingly utilised by archaeologists and cultural anthropologists in ascertaining intercultural influences manifested in the geometry of patterns on textiles, ceramics and other decorated objects (e.g. Washburn and Crowe).[9] There is now a range of comprehensive literature which provides a full understanding of the symmetry group classification system in the area of textile design (initiated by H J Woods in the 1930s) and I hope that this book will add to this understanding.

In general, surface-pattern designers have been aware of the importance of geometry in the construction of regularly repeating designs. However, J Kappraff, in his fascinating book *Connections: The Geometric Bridge Between Art and Science* states that, more often than not, the designer is not conscious of the geometric constraints of space, and that the success of a design depends to a large degree on how well the artist is attuned to the problems and possibilities presented by these constraints.[10] He goes on to say that 'nowhere is this tension between artists and their art more evident than with regard to the issue of symmetry'.

This book, therefore, develops and applies mathematical thinking from areas such as geometry, graph theory and topology, to the context of regularly repeating surface-pattern design. The classification of designs is investigated and explained in depth, and differences are demonstrated between the symmetrical characteristics of individual elements within a design and the overall design structure.

The constraints imposed on the pattern designer by geometric theory relate to the different ways in which motifs may be organised in a pattern to produce regularly repeating designs. Examples of such patterns, in the context of which these geometric theories evolved, would be the arrangements of atoms within crystalline structures.

The theories relating to crystalline structures were well developed by the late nineteenth century. The discovery of X-ray diffraction by Max von Laue in 1912 was applied to the analysis of crystal structures by William Henry and William Laurence Bragg in 1915. As described in Senechal's book[11] *Crystalline Symmetries: An Informal Mathematical Introduction*, Bragg showed that the diffraction of X-rays by crystals could be interpreted as reflections by the lattice planes of the crystal. When a beam of parallel monochromatic X-rays of wavelength λ is passed through a crystal, the reflected rays will emerge from the crystal in phase if the wavelength λ, the interplanar spacing d, and the angle of reflection θ satisfy Bragg's condition: $n\lambda = 2d \sin\theta$, where n is an integer. If this condition is satisfied, and the emerging waves strike a photographic plate, they will create a pattern of bright spots. The X-ray crystallographer begins with these spots and works backwards to deduce the geometry of the structure that gave rise to them.[11]

In the mid-1930s H J Woods, a physicist working in the University of Leeds, published a remarkable series of papers in which he attempted to demystify the mathematical rules pertaining to the geometrical structures of two-dimensional patterns relating to three-dimensional crystal structures. His primary objective was to encourage an understanding among textile designers of the principles of geometric symmetry. He said that every designer should be familiar with the outlines, at least of the 'science' of design which was, in fact, only a simplified and specialised part of that branch of physics devoted to the study of crystalline forms. Crystallography, in turn, to the mathematician, was nothing but an application of group theory.[12]

Woods' papers presented the concepts associated with two-dimensional pattern structures in a simplified form suitable for textile designers. They included the interpretation and explanation of the geometrical principles of finite designs (referred to as 'point groups'), monotranslational designs (referred to as 'borders') and ditranslational designs (referred to as 'plane groups'). This theory formed a foundation of symmetry group classification for the textile designer and gave insight into the rules of symmetry and thus, further access to design analysis.

The symmetry group classification has been extensively explained and utilised in archaeological and anthropological investigations, the results of which have established pattern preference and/or change over specific periods of time, thus suggesting intercultural influences in design creation (see Bier,[13] Grabar[8] and Hann[14]). However, there seems to be further scope available for the application of this classification system to the construction of surface-pattern designs today.

From the geometrical viewpoint there are several different methods for dividing designs into separate classes. For example, a design may be regarded as a pattern comprised of motifs or as a tiling composed of tiles. (In general, the term 'pattern' is used to describe any type of surface design (including a tiling) which contains, what Christie refers to as, a 'device' which is regularly repeated at unit intervals by translational symmetry.[15] However, throughout this book, pattern and tiling designs are treated separately as described above.) Again, pattern and tiling designs may be subdivided into, for example, patterns/tilings comprised of one-shaped motifs/tiles and those comprised of motifs/tiles of two, three, four or five and so on different shapes.

Chapter 2 discusses the broadest of these geometrical classification systems, which may be applied to any regularly repeating design: the classification by symmetry group. It begins by establishing the fundamental principles relating to regularly repeating designs and then applies these principles to the construction of design symmetry groups with particular emphasis, where appropriate, being placed on the construction of designs by the flat screen-printing method. (In this context the construction techniques may be applied to paper and textile printing, for example.) The construction processes relate to a selection of design types such as simple tiling designs, patterned tilings and patterns. The differentiation between design types by their symmetry groups then produces a basis from which additional design categorisations may follow.

Designs with only translational symmetry in their structures are often assumed to be constructed from or represented by patterns with asymmetric motifs. However, this need not necessarily be the case. In Chapter 3 an idea is explored which suggests that each symmetry group may be built up from symmetric motifs without inducing further symmetries into its structure. This is followed by the development of a classification system and construction methods for finite, monotranslational and ditranslational designs which account for symmetric motifs within fundamental regions.

Chapter 4 discusses the features of 'discrete patterns' and their classification and construction by a method which may easily be adapted for screen printing. Although in the field of mathematics the concept of a discrete pattern is not a new development, in the context of surface-pattern design it illustrates an important aspect of a design's structure. Patterns which are classed in the same symmetry group may either have asymmetric motifs, or symmetric motifs which may or may not be positioned on axes or points of symmetry in the design structure. In each case, the positioning and symmetries of the motifs will significantly alter the appearance and geometric characteristics of the design. Construction techniques are discussed for finite, monotranslational and ditranslational discrete pattern types and the patterned tiling designs which may be derived from them.

Chapter 5 involves the description, classification and construction of isohedral tilings. These are special forms of tilings which relate to the discrete patterns discussed in Chapter 4. The concepts, terminology and properties used to categorise these types of design are comprehensively explained. Following this, finite and monotranslational tiling types are derived and constructed from their associated discrete pattern types. (In general, tiling designs are perceived as covering an entire surface with translational symmetry occurring in two non-parallel directions in their structures. However, when finite and monotranslational designs are included in the tiling category, further options become available in the area of surface design). The construction of the ditranslational isohedral tilings relates to the 'Laves' tilings which possess the 11 different topological structures of the 93 ditranslational isohedral tiling types.

This book discusses theoretical concepts behind the geometry of regularly repeating designs associated with and built upon the foundation of symmetry group classification of crystalline structures. The aim throughout the following chapters is to begin from elementary geometrical concepts involved in different design structures and then to derive comprehensive construction techniques. Consequently it is hoped to broaden the scope of the surface-pattern designer by increasing their knowledge in the otherwise impenetrable theory of geometry with the view of increasing their creativity and design potential. As Christie commented in his book *Pattern Design*, geometric formulation has always resulted in a permanent enlargement of the apparatus used by pattern designers, by introducing new ideas and fresh aspects of design. Furthermore, he added that conscious recognition and deliberate exploration of rhythmic expansion as the basic principle in ornament designing is a fundamental event, not only in the history of ornament, but also in the education of every designer.[15]

Throughout this book, the momentous work *Tilings and Patterns* by Grünbaum and Shephard has been of great inspiration and significant value in presenting and suggesting avenues of research.[16] Some of these avenues have provided a foundation from which to extend the application of the mathematical theory to a design context suitable for textiles and other forms of surface decoration. The vast majority of the illustrative material used to represent and explain these mathematical concepts is original and has been constructed using the 'Harvard Graphics' software package.

References

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