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Summary and conclusions

There still seems to be further scope for geometrical frameworks as a means and basis of textile design construction today. As stated by Kappraff,[1] there are infinite possibilities offered by the application of geometrical symmetry:

> Symmetry is a concept that has inspired the creative work of artists and scientists; it is the common root of artistic and scientific endeavour. To an artist or architect symmetry conjures up feelings of order, balance, harmony and an organic relation between the whole and its parts. On the other hand, making these notions useful to a mathematician or scientist requires a precise definition. Although such a definition may make the idea of symmetry less flexible than the artists' intuitive feeling for it, that precision can actually help designers unravel the complexities of design and see greater possibilities for symmetry in their own work. It can also lead to practical techniques for generating patterns.

In sympathy with these considerations, in this book I have attempted to unlock the complexities of patterns and tilings and associated concepts in order greatly to enhance the creative scope of the designer.

Throughout Chapter 2 a comprehensive explanation has been given of fundamental concepts involved in the classification of finite, monotranslational and ditranslational symmetry groups. Group diagrams have been introduced as a means of representing a design's symmetry group and these act as a basis for understanding further geometrical concepts and classification systems in the ensuing chapters. The commonly accepted international notation has been used. However, because the allocation of letters and numbers to the '*pxyz*' notation, for both monotranslational and ditranslational designs, can appear quite complicated, a simplified version has been adopted. The '*pyxn*' and '*pnxy*' notations have been derived to denote monotranslational and ditranslational symmetry groups, respectively. The letter '*n*' has been used to represent a number and '*x*' and '*y*' to represent symmetrical characteristics in relation to *x* and *y* axes. For monotranslational designs the *x* axis has been taken to coincide with the longitudinal axis. Since monotranslational designs (or borders) are usually positioned as horizontal strips, this also seems a logical step forward from the school mathematics with which most textile designers are acquainted. (Adopting the convention of placing *x* and *y* axes horizontally and vertically (respectively) would appear therefore to be a useful step towards avoiding unnecessary confusion in the context of design.)

At the end of Chapter 2 a wide range of construction techniques has been discussed for finite, monotranslational and ditranslational designs. Simple methods of construction have been derived to construct intricate *dn* finite designs from *cn* and *dn*/2 designs. Six different types of design have been described and constructed for each of the seven monotranslational symmetry groups. These include one tiling design, three patterned tiling designs (with parallelogram-shaped tiles, asymmetric tiles and symmetric non-parallelogram-shaped tiles) and one simple and one interlocking pattern design. These monotranslational designs form the basis for the construction of the 17 symmetry groups of ditranslational design. Again analogously, patterns, patterned tilings and tiling designs have been discussed for each of the symmetry groups. Although construction techniques could be used for a variety of applications, particular emphasis was placed on their application to the construction of flat screen-printed textiles. This approach seems highly favourable for the application of symmetry groups to a specific branch of textiles. This simple method of symmetry group construction could be used as a technique for the production of screen printed textiles in both the teaching environment and commercially.

Chapter 3 was also based on the symmetry group classification system. It was recognised that, contrary to much popular thought, the motif contained within the fundamental region of a symmetry group need not necessarily be asymmetric. In fact, in certain cases, the motif within the fundamental region could be symmetrical although its orientation may be critical. This hypothesis gave some interesting and intriguing results. A new notation was developed to represent each of the finite, monotranslational and ditranslational design symmetry group subgroups. A range of schematic illustrations was given and a selection of further examples. Construction techniques were described and tabulated for all forms of these symmetry group subgroups for finite, monotranslational and ditranslational designs. It was particularly interesting to note that the projection of the crystal structure $C_6(CH_3)_4$ exhibited the $p2(d1)$ symmetry subgroup characteristics. Obviously it would be intriguing to find out if other crystallographic projections could also be categorised under this classification system.

Chapter 4 built on a classification system described by Grünbaum and Shephard in their monumental work *Tilings and Patterns*.[2] This work, which contains a vast array of information relating to the geometry of tilings and patterns, and is more penetrable than is conventionally the case with publications dealing with abstract algebra and group theory, is still regarded as being unapproachable to the average textile designer. In the interests of clarity, I have provided extensive illustrative material and, where appropriate, presented explanations of the characteristics of discrete patterns within the context of textile design. The notation used to represent these types of design has been adapted from that given by Grünbaum and Shephard. Finite pattern types have been denoted by $F(P)1_n$, $F(P)2_n$ and $F(P)3_n$ instead of $PF1_n$, $PF2_n$ and $PF3_n$, respectively. This is to account for the notation which had to be derived for finite tiling designs discussed in Chapter 5. Grünbaum and Shephard do not discuss finite or monotranslational tiling designs because, in a mathematical context, tiling designs cover the plane without gaps or overlaps of tiles rather than a finite portion of the plane. They represent monotranslational pattern types by PS1 to PS15 (pattern of the 'strip' variety). Since the 'border' designs in this book are described as 'monotranslational designs' rather than 'strips' it seems logical, in this context, to denote these types of pattern as Mt(P)1 to Mt(P)15. This notation was then easily adapted in Chapter 5 to represent monotranslational tiling designs which again are not considered in a mathematical sense, for the same reasons as described above. Grünbaum and Shephard denote the ditranslational discrete patterns by PP1 to PP51, but to avoid any confusion which may arise due to 'periodic' being associated with regular repetition by translational symmetry in one direction only (e.g. a sine wave) these pattern types, in this book, have been denoted by Dt(P)1 to Dt(P)51.

I hope I have developed an awareness of the different patterning effects that can occur within each symmetry group due to the symmetrical characteristics of the motif. This possibility does not appear to have been discussed in any detail in the context of textile design. Schematic illustrations of the translational symmetry groups are frequently represented in the literature by arrangements of asymmetric motifs, that is the primitive pattern types (although finite *dn* designs are quite often represented by a mixture of asymmetric and symmetric motifs). Explanation of the possibilities and potential patterning characteristics within each group is rarely presented. Further consideration of such possibilities may well offer a useful basis for design construction.

The construction techniques at the end of Chapter 4 have been derived from those described in Chapter 2. Illustrations of design construction have been given for each finite pattern type together with a selection of examples for each induced motif group for monotranslational and ditranslational pattern types. They illustrate the conditions held by discrete patterns although, in the context of textile design, these forms of pattern could be enhanced by the addition of detail particularly in the area of texture and background decoration. Patterned tiling designs, as described in Chapter 2, could also be derived from the construction techniques illustrated for discrete patterns, thus giving another method of surface decoration.

Chapter 5 built on the concepts discussed in Chapter 4, because isohedral tilings may be derived from discrete patterns by the 'Dirichlet relationship'. Again, these concepts are not new in the field of mathematics; however, they are yet to be exploited in the context of surface decoration. In Chapter 5 it was necessary to develop awareness of non-linear transformations. This was achieved through providing examples of 'Escher-like' metamorphoses including reference to one by M C Escher himself (Ernst).[3] Several examples were given to illustrate concepts relating to graph theory (which are areas normally inaccessible to the surface-pattern designer). With these extensive illustrations and explanations it is hoped that the designer will become aware of the vast range of possible tiling structures which may be used in imaginative design and decoration. It would certainly extend the application of mathematical concepts of isohedral tilings if they were used as a basis for design construction in textile, wallpaper or wrapping paper design, for example.

As stated previously, in a mathematical context, finite and monotranslational tiling designs do not exist owing to the fact that formally (and intuitively) tiling designs extend infinitely across the plane. However, because in the context of surface design these types of decoration can be used, it seemed appropriate to extend and associate the theory of finite and monotranslational pattern types with some forms of tiling design. Thus, a one-to-one relationship was used to develop the three finite and 15 monotranslational tiling types. These do not satisfy the Dirichlet relationship with their associated discrete patterns because this would have resulted in finite and monotranslational tiling designs extending to infinity which hardly seems appropriate in the context of surface design even if it is strictly correct in the mathematical sense. Construction techniques have been described and illustrated for all these types of finite and monotranslational tiling designs. Those with induced groups other than $c1$ have been derived from the associated primitive pattern types.

Construction techniques for ditranslational isohedral tilings have been developed which are based on the 11 topological structures. Processes have been described to evaluate the properties of a tiling by its incidence symbol and then consequently derive its method of construction. Illustrations and descriptions have been given for one example of each of the 11 topological structures. Any of the 93 isohedral tiling types may be constructed by these methods although the initial structures have been limited to ones with vertex positions corresponding to those of the associated Laves tiling. Thus, these construction methods could obviously be developed further to include all possible homeomorphic transformations of the initial underlying structures and hence a wider variety of forms of isohedral tiling within one type.

Finite, monotranslational and ditranslational tilings also extend the basis from which patterned tiling designs may be produced. (A motif may be added to a tile and mapped to all equivalent positions as described in Chapter 2.) Ditranslational isohedral tilings also increase the variety of topological structures and consequently the choice of shape of the fundamental region. This would then give further choice in the interlocking relationship between fundamental regions and hence adjacent motifs when constructing patterned tilings or patterns. This area of design, in the patterning of isohedral tilings (rather than just the marked variety described in Chapter 5), could lead to some effective and interesting results particularly in marking tilings composed of symmetric tiles.

Further design characteristics could also lead on from the monotranslational and particularly finite tilings developed in Chapter 5. Consecutive unit translations of monotranslational tilings in one direction (not parallel to the longitudinal axis) and finite tilings in two non-parallel directions may produce ditranslational non-monohedral tiling designs. Again with the addition of a pattern these could form patterned tilings or, with the removal of the boundaries of the tiles, an interesting form of interlocking pattern. Consequently, there is obviously scope for discovering new tiling and patterning effects by these methods.

The types of pattern and tiling designs discussed in this book only amount to a small portion of all possible forms of regularly repeating surface decoration. A pattern of motifs may form a design in itself or be a component of a patterned tiling design. The tiling in which a pattern is incorporated may not only be monohedral or isohedral but may be composed of two or three or more different shaped tiles. Geometric tilings comprising different shaped tiles were widely used by the Moors, for example, but rarely seem to be exploited as a basis of pattern design today.

An avenue of research which may prove useful in the area of surface design is the study of 'non-periodic' or 'aperiodic' tilings. These types of design are not regularly repeating but exhibit an intriguing mixture of a structured but disordered appearance. Although these types of design could not be translated, owing to their irregularity, some of their characteristics could be adapted and incorporated within a regularly repeating design. Elements of five-fold rotational symmetry, as exhibited in Penrose tilings, may be a worthwhile example.

Theories involved in the mathematics of design are fairly well developed. However, topics such as 'non-periodic tilings', 'isogonal tilings', 'Archimedian colourings', 'n-omino tilings', 'fractal patterns' and 'chaotic symmetry' are yet to be fully developed in contexts of art or surface decoration.

Another aspect of design technology which is yet to be fully exploited is the use of computer-aided design. Computer technology is developing rapidly but, as yet, its application to the construction of surface-pattern design is limited despite the time-saving value, efficiency and accuracy which it presents. Although software packages have been developed which enable the immediate production of the 17 symmetry groups of ditranslational designs (e.g. through 'Photoshop') they do not always result in an aesthetically pleasing, continuous flowing design. This is due to the fixed shape of the fundamental region area, often in its most rigid form, which does not allow blending or interlocking of adjacent motifs and/or design elements. Thus, the designs, although appealing, appear more rigidly geometric unless subtly modified after construction.

Advances in designer-friendly software packages will transform the methods of design production in academic institutions, colleges and industrial contexts by increasing the design scope, time-saving value, efficiency and accuracy. However, to appreciate their full implication it is beneficial to have a comprehensive understanding of the fundamental principles involved in the structure of design, and potential avenues through which creative ideas may be explored. Furthermore, to explore beyond symmetry group classification with respect to design construction can only enhance and extend the interests and creative limitations of surface-pattern designers. Consequently, an appreciation of the mathematical concepts may undoubtedly prove enriching. By using design, complex and intriguing aspects of geometry and crystallography can be displayed by means of eye catching, artistic interpretations. Thus, I hope that through this book I have brought an awareness to surface-pattern designers of the potential reward that may be gained from learning to appreciate principles of design geometry. Conversely in a mathematical context, because this book contains hundreds of original illustrations, I would like to think that they may prove useful in demonstrating geometric theories and principles and present an appealing method for representing their interpretation and application.

References

1 Kappraff J, *Connections: The Geometric Bridge Between Art and Science*, New York, McGraw-Hill, 1991.
2 Grünbaum B and Shephard G C, *Tilings and Patterns*, New York, Freeman, 1987.
3 Ernst B, *The Magic Mirror of Escher*, Norfolk, Tarquin Publications, 1985.