

# **Geometric symmetry in patterns and tilings**

**Clare E Horne**



The Textile Institute



WOODHEAD PUBLISHING LIMITED

## **Geometric symmetry in patterns and tilings**



# Geometric symmetry in patterns and tilings

---

Clare E Horne



The Textile Institute



CRC Press

Boca Raton Boston New York Washington, DC

WOODHEAD PUBLISHING LIMITED

Cambridge England

Published by Woodhead Publishing Limited in association with The Textile Institute  
Abington Hall, Abington  
Cambridge CB1 6AH, England  
www.woodhead-publishing.com

Published in North and South America by CRC Press LLC  
2000 Corporate Blvd, NW  
Boca Raton FL 33431, USA

First published 2000, Woodhead Publishing Ltd and CRC Press LLC  
© 2000, Woodhead Publishing Ltd  
The author has asserted her moral rights.

This book contains information obtained from authentic and highly regarded sources. Reprinted material is quoted with permission, and sources are indicated. Reasonable efforts have been made to publish reliable data and information, but the author and the publishers cannot assume responsibility for the validity of all materials. Neither the author nor the publishers, nor anyone else associated with this publication, shall be liable for any loss, damage or liability directly or indirectly caused or alleged to be caused by this book.

Neither this book nor any part may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, microfilming and recording, or by any information storage or retrieval system, without permission in writing from the publishers.

The consent of Woodhead Publishing and CRC Press does not extend to copying for general distribution, for promotion, for creating new works, or for resale. Specific permission must be obtained in writing from Woodhead Publishing or CRC Press for such copying.

Trademark notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation, without intent to infringe.

British Library Cataloguing in Publication Data  
A catalogue record for this book is available from the British Library.

Library of Congress Cataloging in Publication Data  
A catalog record for this book is available from the Library of Congress.

Woodhead Publishing ISBN 1 85573 492 3  
CRC Press ISBN 0-8493-1048-2  
CRC Press order number: WP1048

Cover design by The ColourStudio  
Typeset by Best-set Typesetter Ltd, Hong Kong  
Printed by T J International, Cornwall, England

This book is dedicated to my family.



---

# Contents

Foreword	ix
Acknowledgements	x
<b>1 Introduction</b>	<b>1</b>
References	4
<b>2 Classification of designs by symmetry group</b>	<b>7</b>
2.1 Introduction	7
2.2 Symmetry and its relevance to designs	7
2.3 Symmetry operations	7
2.4 Symmetry group	10
2.5 Figures and designs	10
2.6 Classification of finite designs	11
2.7 Structure of translational designs	14
2.8 Generating functions	25
2.9 Classification of monotranslational designs	28
2.10 Classification of ditranslational designs	29
2.11 Construction of finite designs	33
2.12 Construction of monotranslational designs	39
2.13 Construction of ditranslational designs	47
2.14 Summary	76
References	77
<b>3 Classification of designs by symmetry group and design unit</b>	<b>79</b>
3.1 Introduction	79
3.2 Notation	82
3.3 Finite designs	82
3.4 Monotranslational designs	82
3.5 Ditranslational designs	82
3.6 Construction of finite designs	94
3.7 Construction of monotranslational designs	98
3.8 Construction of ditranslational designs	110
3.9 Summary	126
References	129
<b>4 Classification of discrete patterns</b>	<b>131</b>
4.1 Introduction	131
4.2 Monomotif pattern	131
4.3 Discrete pattern	133
4.4 Primitive pattern	135
4.5 Induced motif groups	137
4.6 Motif-transitive subgroups	142
4.7 Classification of finite discrete pattern types	145
4.8 Classification of monotranslational discrete pattern types	149
4.9 Classification of ditranslational discrete pattern types	149
4.10 Construction of finite pattern types	151



4.11 Construction of monotranslational discrete pattern types	151
4.12 Construction of ditranslational discrete pattern types	158
4.13 Summary	170
References	170
<b>5 Classification of isohedral tilings</b>	<b>171</b>
5.1 Introduction	171
5.2 Isohedral tiling	172
5.3 Dirichlet tiling	178
5.4 Topology of tilings	180
5.5 Incidence symbols	189
5.6 Marked isohedral tilings	196
5.7 Classification of finite isohedral tiling types	197
5.8 Classification of monotranslational isohedral tiling types	197
5.9 Classification of ditranslational isohedral tiling types	203
5.10 Construction of finite isohedral tiling types	203
5.11 Construction of monotranslational isohedral tiling types	205
5.12 Construction of ditranslational isohedral tiling types	211
5.13 Summary	230
References	232
<b>6 Summary and conclusions</b>	<b>233</b>
References	236
Index	237

---

# Foreword

*Geometric Symmetry in Patterns and Tilings* results from one of a series of exciting and innovative research projects emanating from the School of Textile Industries at University of Leeds.

This particular project was conducted under my supervision, and was aided by scholarship funding from the Worshipful Company of Clothworkers of the City of London. It extends the Leeds tradition of research into pattern symmetry initiated in the 1930s by H J Woods, a physicist (and mathematician), whose contribution in laying the foundations for current thinking on the geometrical characteristics of patterns is, today, widely acknowledged by scholars in the field.

Whilst many symmetry concepts have their origin in the area of crystallography, an appreciation of their usefulness has, in recent years, extended to many disciplines and realms of study. Washburn and Crowe made a major contribution in the area of anthropology in their largely pioneering work *Symmetries of Culture*. The mathematical treatise *Tilings and Patterns* by Grünbaum and Shephard stands as a major contribution to the conceptual development of the subject. *Visions of Symmetry*, Schattschneider's monumental study of the work of M C Escher, has not only stimulated an insight into the periodic drawings and patterns of the artist but has also encouraged an understanding of symmetry concepts beyond a mathematically aware audience to inspire the creation of original decorative patterns.

Recent research projects at Leeds have employed symmetry concepts in the investigation of patterns produced in a range of historical and/or cultural contexts and as a systematic means of generating printed-textile designs. Layer symmetry principles have been employed in the analysis of woven-fabric structures, and as a basis for developing a systematic means of designing woven fabrics.

The present book focuses principally on characteristics of surface-pattern design, and presents a comprehensive means of classifying patterns and tilings. A wide range of original illustrative material is included.

M A Hann,  
*Reader in International Textile Design*  
*University of Leeds*

---

# Acknowledgements

This book has been developed from research activities undertaken whilst studying in the School of Textile Industries at the University of Leeds. Consequently, first I would like to express my gratitude to the School of Textile Industries, the Worshipful Company of Clothworkers, and in particular to my supervisor, Dr. M A Hann, and the Head of Department at the time, Professor D Johnson, for supporting my research.

I am also sincerely grateful to all my family and friends for their support, encouragement and understanding whilst I have been compiling this work. I would especially like to thank my family, Brenda, Tony, Christopher, Alison and Jenny. I am also greatly indebted to many friends who have shown their continual care and consideration, in particular Rachel Segal, Mark Colpus, Graham Gifford, Marion Small and Wendy Cawthray.

Finally, I would like to thank Woodhead Publishing for maintaining their interest in my work and particularly Patricia Morrison for showing such patience and support over a long period of time.