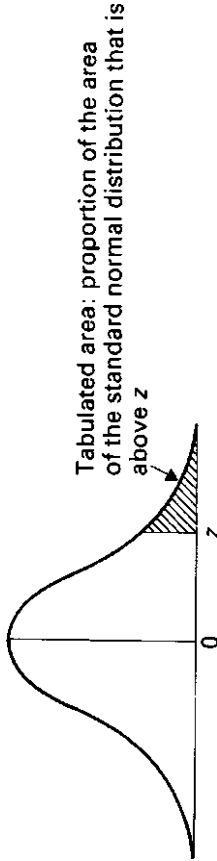


Table A1 Areas in tail of the standard normal distribution.Adapted from Table 3 of White *et al.* (1979) with permission of the authors and publishers.

z	Second decimal place of z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681

1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0275	0.0222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003

Table A2 Percentage points of the standard normal distribution.

<i>P</i> -value	Percentage points	
	One-sided	Two-sided
0.5	0.00	0.67
0.4	0.25	0.84
0.3	0.52	1.04
0.2	0.84	1.28
0.1	1.28	1.64
0.05	1.64	1.96
0.02	2.05	2.33
0.01	2.33	2.58
0.005	2.58	2.81
0.002	2.88	3.09
0.001	3.09	3.29
0.0001	3.72	3.89

Table A3 Percentage points of the t distribution.Adapted from Table 7 of White *et al.* (1979) with permission of the authors and publishers.

d.f.	One-sided P -value								
	0.25	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
	Two-sided P -value								
	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
1	1.00	3.08	6.31	12.71	31.82	63.66	127.32	318.31	636.62
2	0.82	1.89	2.92	4.30	6.96	9.92	14.09	22.33	31.60
3	0.76	1.64	2.35	3.18	4.54	5.84	7.45	10.21	12.92
4	0.74	1.53	2.13	2.78	3.75	4.60	5.60	7.17	8.61
5	0.73	1.48	2.02	2.57	3.36	4.03	4.77	5.89	6.87
6	0.72	1.44	1.94	2.45	3.14	3.71	4.32	5.21	5.96
7	0.71	1.42	1.90	2.36	3.00	3.50	4.03	4.78	5.41
8	0.71	1.40	1.86	2.31	2.90	3.36	3.83	4.50	5.04
9	0.70	1.38	1.83	2.26	2.82	3.25	3.69	4.30	4.78
10	0.70	1.37	1.81	2.23	2.76	3.17	3.58	4.14	4.59
11	0.70	1.36	1.80	2.20	2.72	3.11	3.50	4.02	4.44
12	0.70	1.36	1.78	2.18	2.68	3.06	3.43	3.93	4.32
13	0.69	1.35	1.77	2.16	2.65	3.01	3.37	3.85	4.22
14	0.69	1.34	1.76	2.14	2.62	2.98	3.33	3.79	4.14
15	0.69	1.34	1.75	2.13	2.60	2.95	3.29	3.73	4.07
16	0.69	1.34	1.75	2.12	2.58	2.92	3.25	3.69	4.02
17	0.69	1.33	1.74	2.11	2.57	2.90	3.22	3.65	3.96
18	0.69	1.33	1.73	2.10	2.55	2.88	3.20	3.61	3.92
19	0.69	1.33	1.73	2.09	2.54	2.86	3.17	3.58	3.88
20	0.69	1.32	1.72	2.09	2.53	2.84	3.15	3.55	3.85
21	0.69	1.32	1.72	2.08	2.52	2.83	3.14	3.53	3.82
22	0.69	1.32	1.72	2.07	2.51	2.82	3.12	3.50	3.79
23	0.68	1.32	1.71	2.07	2.50	2.81	3.10	3.48	3.77
24	0.68	1.32	1.71	2.06	2.49	2.80	3.09	3.47	3.74
25	0.68	1.32	1.71	2.06	2.48	2.79	3.08	3.45	3.72
26	0.68	1.32	1.71	2.06	2.48	2.78	3.07	3.44	3.71
27	0.68	1.31	1.70	2.05	2.47	2.77	3.06	3.42	3.69
28	0.68	1.31	1.70	2.05	2.47	2.76	3.05	3.41	3.67
29	0.68	1.31	1.70	2.04	2.46	2.76	3.04	3.40	3.66
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03	3.38	3.65
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97	3.31	3.55
60	0.68	1.30	1.67	2.00	2.39	2.66	2.92	3.23	3.46
120	0.68	1.29	1.66	1.98	2.36	2.62	2.86	3.16	3.37
∞	0.67	1.28	1.65	1.96	2.33	2.58	2.81	3.09	3.29

Table A4 Two-sided P -values for the t distribution, according to the value of the test statistic.

The final column shows P -values for infinite degrees of freedom, equivalent to P -values from the normal distribution.

Value of test statistic (t)	Degrees of freedom for t							
	5	6	7	8	9	10	12	14
1.5	0.194	0.184	0.177	0.172	0.168	0.165	0.159	0.156
1.6	0.170	0.161	0.154	0.148	0.144	0.141	0.136	0.132
1.7	0.150	0.140	0.133	0.128	0.123	0.120	0.115	0.111
1.8	0.132	0.122	0.115	0.110	0.105	0.102	0.097	0.093
1.9	0.116	0.106	0.099	0.094	0.090	0.087	0.082	0.078
2.0	0.102	0.092	0.086	0.081	0.077	0.073	0.069	0.065
2.1	0.090	0.080	0.074	0.069	0.065	0.062	0.058	0.054
2.2	0.079	0.070	0.064	0.059	0.055	0.052	0.048	0.045
2.3	0.070	0.061	0.055	0.050	0.047	0.044	0.040	0.037
2.4	0.062	0.053	0.047	0.043	0.040	0.037	0.034	0.031
2.5	0.054	0.047	0.041	0.037	0.034	0.031	0.028	0.025
2.6	0.048	0.041	0.035	0.032	0.029	0.026	0.023	0.021
2.7	0.043	0.036	0.031	0.027	0.024	0.022	0.019	0.017
2.8	0.038	0.031	0.027	0.023	0.021	0.019	0.016	0.014
2.9	0.034	0.027	0.023	0.020	0.018	0.016	0.013	0.012
3.0	0.030	0.024	0.020	0.017	0.015	0.013	0.011	0.010
3.1	0.027	0.021	0.017	0.015	0.013	0.011	0.009	0.008
3.2	0.024	0.019	0.015	0.013	0.011	0.009	0.008	0.006
3.3	0.021	0.016	0.013	0.011	0.009	0.008	0.006	0.005
3.4	0.019	0.014	0.011	0.009	0.008	0.007	0.005	0.004
3.5	0.017	0.013	0.010	0.008	0.007	0.006	0.004	0.004
3.6	0.016	0.011	0.009	0.007	0.006	0.005	0.004	0.003
3.7	0.014	0.010	0.008	0.006	0.005	0.004	0.003	0.002
3.8	0.013	0.009	0.007	0.005	0.004	0.003	0.003	0.002
3.9	0.011	0.008	0.006	0.005	0.004	0.003	0.002	0.002
4.0	0.010	0.007	0.005	0.004	0.003	0.003	0.002	0.001
4.1	0.009	0.006	0.005	0.003	0.003	0.002	0.001	0.001
4.2	0.008	0.006	0.004	0.003	0.002	0.002	0.001	0.001
4.3	0.008	0.005	0.004	0.003	0.002	0.002	0.001	0.001
4.4	0.007	0.005	0.003	0.002	0.002	0.001	0.001	0.001
4.5	0.006	0.004	0.003	0.002	0.001	0.001	0.001	<0.001
4.6	0.006	0.004	0.002	0.002	0.001	0.001	0.001	<0.001
4.7	0.005	0.003	0.002	0.002	0.001	0.001	0.001	<0.001
4.8	0.005	0.003	0.002	0.001	0.001	0.001	<0.001	<0.001
4.9	0.004	0.003	0.002	0.001	0.001	0.001	<0.001	<0.001
5.0	0.004	0.002	0.002	0.001	0.001	0.001	<0.001	<0.001

Table A5 Percentage points of the χ^2 distribution.

Adapted from Table 8 of White *et al.* (1979) with permission of the authors and publishers.

d.f. = 1. In the comparison of two proportions (2×2 χ^2 or Mantel–Haenszel χ^2 test) or in the assessment of a trend, the percentage points give a two-sided test. A one-sided test may be obtained by halving the *P*-values. (Concepts of one- and two-sidedness do not apply to larger degrees of freedom, as these relate to tests of multiple comparisons.)

d.f.	P-value							
	0.5	0.25	0.1	0.05	0.025	0.01	0.005	0.001
1	0.45	1.32	2.71	3.84	5.02	6.63	7.88	10.83
2	1.39	2.77	4.61	5.99	7.38	9.21	10.60	13.82
3	2.37	4.11	6.25	7.81	9.35	11.34	12.84	16.27
4	3.36	5.39	7.78	9.49	11.14	13.28	14.86	18.47
5	4.35	6.63	9.24	11.07	12.83	15.09	16.75	20.52
6	5.35	7.84	10.64	12.59	14.45	16.81	18.55	22.46
7	6.35	9.04	12.02	14.07	16.01	18.48	20.28	24.32
8	7.34	10.22	13.36	15.51	17.53	20.09	21.96	26.13
9	8.34	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	9.34	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	10.34	13.70	17.28	19.68	21.92	24.73	26.76	31.26
12	11.34	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	12.34	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	13.34	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	14.34	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	15.34	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	16.34	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	17.34	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	18.34	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	19.34	23.83	28.41	31.41	34.17	37.57	40.00	45.32
21	20.34	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	21.34	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	22.34	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	23.34	28.24	33.20	36.42	39.36	42.98	45.56	51.18
25	24.34	29.34	34.38	37.65	40.65	44.31	46.93	52.62
26	25.34	30.43	35.56	38.89	41.92	45.64	48.29	54.05
27	26.34	31.53	36.74	40.11	43.19	46.96	49.64	55.48
28	27.34	32.62	37.92	41.34	44.46	48.28	50.99	56.89
29	28.34	33.71	39.09	42.56	45.72	49.59	52.34	58.30
30	29.34	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	39.34	45.62	51.81	55.76	59.34	63.69	66.77	73.40
50	49.33	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	59.33	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	69.33	77.58	85.53	90.53	95.02	100.43	104.22	112.32
80	79.33	88.13	96.58	101.88	106.63	112.33	116.32	124.84
90	89.33	98.65	107.57	113.15	118.14	124.12	128.30	137.21
100	99.33	109.14	118.50	124.34	129.56	135.81	140.17	149.45

Table A6 Probits.

Adapted from Table 4 of Pearson & Hartley (1966) with permission of the Biometrika Trustees.

%	Decimal place of %									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	-∞	-3.09	-2.88	-2.75	-2.65	-2.58	-2.51	-2.46	-2.41	-2.37
1	-2.33	-2.29	-2.26	-2.23	-2.20	-2.17	-2.14	-2.12	-2.10	-2.07
2	-2.05	-2.03	-2.01	-2.00	-1.98	-1.96	-1.94	-1.93	-1.91	-1.90
3	-1.88	-1.87	-1.85	-1.84	-1.83	-1.81	-1.80	-1.79	-1.77	-1.76
4	-1.75	-1.74	-1.73	-1.72	-1.71	-1.70	-1.68	-1.67	-1.66	-1.65
5	-1.64	-1.64	-1.63	-1.62	-1.61	-1.60	-1.59	-1.58	-1.57	-1.56
6	-1.55	-1.55	-1.54	-1.53	-1.52	-1.51	-1.51	-1.50	-1.49	-1.48
7	-1.48	-1.47	-1.46	-1.45	-1.45	-1.44	-1.43	-1.43	-1.42	-1.41
8	-1.41	-1.40	-1.39	-1.39	-1.38	-1.37	-1.37	-1.36	-1.35	-1.35
9	-1.34	-1.33	-1.33	-1.32	-1.32	-1.31	-1.30	-1.30	-1.29	-1.29
10	-1.28	-1.28	-1.27	-1.26	-1.26	-1.25	-1.25	-1.24	-1.24	-1.23
11	-1.23	-1.22	-1.22	-1.21	-1.21	-1.20	-1.20	-1.19	-1.19	-1.18
12	-1.18	-1.17	-1.17	-1.16	-1.16	-1.15	-1.15	-1.14	-1.14	-1.13
13	-1.13	-1.12	-1.12	-1.11	-1.11	-1.10	-1.10	-1.09	-1.09	-1.08
14	-1.08	-1.08	-1.07	-1.07	-1.06	-1.06	-1.05	-1.05	-1.05	-1.04
15	-1.04	-1.03	-1.03	-1.02	-1.02	-1.02	-1.01	-1.01	-1.00	-1.00
16	-0.99	-0.99	-0.99	-0.98	-0.98	-0.97	-0.97	-0.97	-0.96	-0.96
17	-0.95	-0.95	-0.95	-0.94	-0.94	-0.93	-0.93	-0.93	-0.92	-0.92
18	-0.92	-0.91	-0.91	-0.90	-0.90	-0.90	-0.89	-0.89	-0.89	-0.88
19	-0.88	-0.87	-0.87	-0.87	-0.86	-0.86	-0.86	-0.85	-0.85	-0.85
20	-0.84	-0.84	-0.83	-0.83	-0.83	-0.82	-0.82	-0.82	-0.81	-0.81
21	-0.81	-0.80	-0.80	-0.80	-0.79	-0.79	-0.79	-0.78	-0.78	-0.78
22	-0.77	-0.77	-0.77	-0.76	-0.76	-0.76	-0.75	-0.75	-0.75	-0.74
23	-0.74	-0.74	-0.73	-0.73	-0.73	-0.72	-0.72	-0.72	-0.71	-0.71
24	-0.71	-0.70	-0.70	-0.70	-0.69	-0.69	-0.69	-0.68	-0.68	-0.68
25	-0.67	-0.67	-0.67	-0.67	-0.66	-0.66	-0.66	-0.65	-0.65	-0.65
26	-0.64	-0.64	-0.64	-0.63	-0.63	-0.63	-0.63	-0.62	-0.62	-0.62
27	-0.61	-0.61	-0.61	-0.60	-0.60	-0.60	-0.59	-0.59	-0.59	-0.59
28	-0.58	-0.58	-0.58	-0.57	-0.57	-0.57	-0.57	-0.56	-0.56	-0.56
29	-0.55	-0.55	-0.55	-0.54	-0.54	-0.54	-0.54	-0.53	-0.53	-0.53
30	-0.52	-0.52	-0.52	-0.52	-0.51	-0.51	-0.51	-0.50	-0.50	-0.50
31	-0.50	-0.49	-0.49	-0.49	-0.48	-0.48	-0.48	-0.48	-0.47	-0.47
32	-0.47	-0.46	-0.46	-0.46	-0.46	-0.45	-0.45	-0.45	-0.45	-0.44
33	-0.44	-0.44	-0.43	-0.43	-0.43	-0.43	-0.42	-0.42	-0.42	-0.42
34	-0.41	-0.41	-0.41	-0.40	-0.40	-0.40	-0.40	-0.39	-0.39	-0.39
35	-0.39	-0.38	-0.38	-0.38	-0.37	-0.37	-0.37	-0.37	-0.36	-0.36
36	-0.36	-0.36	-0.35	-0.35	-0.35	-0.35	-0.34	-0.34	-0.34	-0.33
37	-0.33	-0.33	-0.33	-0.32	-0.32	-0.32	-0.32	-0.31	-0.31	-0.31
38	-0.31	-0.30	-0.30	-0.30	-0.30	-0.29	-0.29	-0.29	-0.28	-0.28
39	-0.28	-0.28	-0.27	-0.27	-0.27	-0.27	-0.26	-0.26	-0.26	-0.26
40	-0.25	-0.25	-0.25	-0.25	-0.24	-0.24	-0.24	-0.24	-0.23	-0.23
41	-0.23	-0.23	-0.22	-0.22	-0.22	-0.21	-0.21	-0.21	-0.21	-0.20
42	-0.20	-0.20	-0.20	-0.19	-0.19	-0.19	-0.19	-0.18	-0.18	-0.18
43	-0.18	-0.17	-0.17	-0.17	-0.17	-0.16	-0.16	-0.16	-0.16	-0.15
44	-0.15	-0.15	-0.15	-0.14	-0.14	-0.14	-0.14	-0.13	-0.13	-0.13
45	-0.13	-0.12	-0.12	-0.12	-0.12	-0.11	-0.11	-0.11	-0.11	-0.10
46	-0.10	-0.10	-0.10	-0.09	-0.09	-0.09	-0.09	-0.08	-0.08	-0.08
47	-0.08	-0.07	-0.07	-0.07	-0.07	-0.06	-0.06	-0.06	-0.06	-0.05
48	-0.05	-0.05	-0.05	-0.04	-0.04	-0.04	-0.04	-0.03	-0.03	-0.03
49	-0.03	-0.02	-0.02	-0.02	-0.02	-0.01	-0.01	-0.01	-0.01	0.00

(continued)

Table A6 Probits (continued)

%	Decimal place of %									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50	0.00	0.00	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02
51	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.05	0.05
52	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07	0.07	0.07
53	0.08	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.10	0.10
54	0.10	0.10	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.12
55	0.13	0.13	0.13	0.13	0.14	0.14	0.14	0.14	0.15	0.15
56	0.15	0.15	0.16	0.16	0.16	0.16	0.17	0.17	0.17	0.17
57	0.18	0.18	0.18	0.18	0.19	0.19	0.19	0.19	0.20	0.20
58	0.20	0.20	0.21	0.21	0.21	0.21	0.22	0.22	0.22	0.23
59	0.23	0.23	0.23	0.24	0.24	0.24	0.24	0.25	0.25	0.25
60	0.25	0.26	0.26	0.26	0.26	0.27	0.27	0.27	0.27	0.28
61	0.28	0.28	0.28	0.29	0.29	0.29	0.30	0.30	0.30	0.30
62	0.31	0.31	0.31	0.31	0.32	0.32	0.32	0.32	0.33	0.33
63	0.33	0.33	0.34	0.34	0.34	0.35	0.35	0.35	0.35	0.36
64	0.36	0.36	0.36	0.37	0.37	0.37	0.37	0.38	0.38	0.38
65	0.39	0.39	0.39	0.39	0.40	0.40	0.40	0.40	0.41	0.41
66	0.41	0.42	0.42	0.42	0.42	0.43	0.43	0.43	0.43	0.44
67	0.44	0.44	0.45	0.45	0.45	0.45	0.46	0.46	0.46	0.46
68	0.47	0.47	0.47	0.48	0.48	0.48	0.48	0.49	0.49	0.49
69	0.50	0.50	0.50	0.50	0.51	0.51	0.51	0.52	0.52	0.52
70	0.52	0.53	0.53	0.53	0.54	0.54	0.54	0.54	0.55	0.55
71	0.55	0.56	0.56	0.56	0.57	0.57	0.57	0.57	0.58	0.58
72	0.58	0.59	0.59	0.59	0.59	0.60	0.60	0.60	0.61	0.61
73	0.61	0.62	0.62	0.62	0.63	0.63	0.63	0.63	0.64	0.64
74	0.64	0.65	0.65	0.65	0.66	0.66	0.66	0.67	0.67	0.67
75	0.67	0.68	0.68	0.68	0.69	0.69	0.69	0.70	0.70	0.70
76	0.71	0.71	0.71	0.72	0.72	0.72	0.73	0.73	0.73	0.74
77	0.74	0.74	0.75	0.75	0.75	0.76	0.76	0.76	0.77	0.77
78	0.77	0.78	0.78	0.78	0.79	0.79	0.79	0.80	0.80	0.80
79	0.81	0.81	0.81	0.82	0.82	0.82	0.83	0.83	0.83	0.84
80	0.84	0.85	0.85	0.85	0.86	0.86	0.86	0.87	0.87	0.87
81	0.88	0.88	0.89	0.89	0.89	0.90	0.90	0.90	0.91	0.91
82	0.92	0.92	0.92	0.93	0.93	0.93	0.94	0.94	0.95	0.95
83	0.95	0.96	0.96	0.97	0.97	0.97	0.98	0.98	0.99	0.99
84	0.99	1.00	1.00	1.01	1.01	1.02	1.02	1.02	1.03	1.03
85	1.04	1.04	1.05	1.05	1.05	1.06	1.06	1.07	1.07	1.08
86	1.08	1.08	1.09	1.09	1.10	1.10	1.11	1.11	1.12	1.12
87	1.13	1.13	1.14	1.14	1.15	1.15	1.16	1.16	1.17	1.17
88	1.18	1.18	1.19	1.19	1.20	1.20	1.21	1.21	1.22	1.22
89	1.23	1.23	1.24	1.24	1.25	1.25	1.26	1.26	1.27	1.28
90	1.28	1.29	1.29	1.30	1.30	1.31	1.32	1.32	1.33	1.33
91	1.34	1.35	1.35	1.36	1.37	1.37	1.38	1.39	1.39	1.40
92	1.41	1.41	1.42	1.43	1.43	1.44	1.45	1.45	1.46	1.47
93	1.48	1.48	1.49	1.50	1.51	1.51	1.52	1.53	1.54	1.55
94	1.55	1.56	1.57	1.58	1.59	1.60	1.61	1.62	1.63	1.64
95	1.64	1.65	1.66	1.67	1.68	1.70	1.71	1.72	1.73	1.74
96	1.75	1.76	1.77	1.79	1.80	1.81	1.83	1.84	1.85	1.87
97	1.88	1.90	1.91	1.93	1.94	1.96	1.98	2.00	2.01	2.03
98	2.05	2.07	2.10	2.12	2.14	2.17	2.20	2.23	2.26	2.29
99	2.33	2.37	2.41	2.46	2.51	2.58	2.65	2.75	2.88	3.09

Table A7 Critical values for the Wilcoxon matched pairs signed rank test.Reproduced from Table 21 of White *et al.* (1979) with permission of the authors and publishers. N = number of non-zero differences; T = smaller of T_+ and T_- ; Significant if $T <$ critical value.

N	One-sided P -value				N	One-sided P -value			
	0.05	0.025	0.01	0.005		0.05	0.025	0.01	0.005
	Two-sided P -value					Two-sided P -value			
	0.1	0.05	0.02	0.01		0.1	0.05	0.02	0.01
5	1				30	152	137	120	109
6	2	1			31	163	148	130	118
7	4	2	0		32	175	159	141	128
8	6	4	2	0	33	188	171	151	138
9	8	6	3	2	34	201	183	162	149
10	11	8	5	3	35	214	195	174	160
11	14	11	7	5	36	228	208	186	171
12	17	14	10	7	37	242	222	198	183
13	21	17	13	10	38	256	235	211	195
14	26	21	16	13	39	271	250	224	208
15	30	25	20	16	40	287	264	238	221
16	36	30	24	19	41	303	279	252	234
17	41	35	28	23	42	319	295	267	248
18	47	40	33	28	43	336	311	281	262
19	54	46	38	32	44	353	327	297	277
20	60	52	43	37	45	371	344	313	292
21	68	59	49	43	46	389	361	329	307
22	75	66	56	49	47	408	397	345	323
23	83	73	62	55	48	427	397	362	339
24	92	81	69	61	49	446	415	380	356
25	101	90	77	68	50	466	434	398	373
26	110	98	85	76					
27	120	107	93	84					
28	130	117	102	92					
29	141	127	111	100					

Table A8 Critical ranges for the Wilcoxon rank sum test.

Reproduced from Table A7 of Cotton (1974) with permission of the author and publishers.

n_1, n_2 = sample sizes of two groups; T = sum of ranks in group with smaller sample size; significant if T on boundaries or outside critical range.

n_1, n_2	One-sided P -value			n_1, n_2	One-sided P -value		
	0.025	0.005	0.0005		0.025	0.005	0.0005
	Two-sided P -value				Two-sided P -value		
	0.05	0.01	0.001		0.05	0.01	0.001
2, 8	3, 19			4, 13	18, 54	14, 58	10, 62
2, 9	3, 21			4, 14	19, 57	14, 62	10, 66
2, 10	3, 23			4, 15	20, 60	15, 65	10, 70
2, 11	4, 24			4, 16	21, 63	15, 69	11, 73
2, 12	4, 26			4, 17	21, 67	16, 72	11, 77
2, 13	4, 28			4, 18	22, 70	16, 76	11, 81
2, 14	4, 30			4, 19	23, 73	17, 79	12, 84
2, 15	4, 32			4, 20	24, 76	18, 82	12, 88
2, 16	4, 34			4, 21	25, 79	18, 86	12, 92
2, 17	5, 35			4, 22	26, 82	19, 89	13, 95
2, 18	5, 37			4, 23	27, 85	19, 93	13, 99
2, 19	5, 39	3, 41		4, 24	28, 88	20, 96	13, 103
2, 20	5, 41	3, 43		4, 25	28, 92	20, 100	14, 106
2, 21	6, 42	3, 45					
2, 22	6, 44	3, 47		5, 5	17, 38	15, 40	
2, 23	6, 46	3, 49		5, 6	18, 42	16, 44	
2, 24	6, 48	3, 51		5, 7	20, 45	17, 48	
2, 25	6, 50	3, 53		5, 8	21, 49	17, 53	
				5, 9	22, 53	18, 57	15, 60
3, 5	6, 21			5, 10	23, 57	19, 61	15, 65
3, 6	7, 23			5, 11	24, 61	20, 65	16, 69
3, 7	7, 26			5, 12	26, 64	21, 69	16, 74
3, 8	8, 28			5, 13	27, 68	22, 73	17, 78
3, 9	8, 31	6, 33		5, 14	28, 72	22, 78	17, 83
3, 10	9, 33	6, 36		5, 15	29, 76	23, 82	18, 87
3, 11	9, 36	6, 39		5, 16	31, 79	24, 86	18, 92
3, 12	10, 38	7, 41		5, 17	32, 83	25, 90	19, 96
3, 13	10, 41	7, 44		5, 18	33, 87	26, 94	19, 101
3, 14	11, 43	7, 47		5, 19	34, 91	27, 98	20, 105
3, 15	11, 46	8, 49		5, 20	35, 95	28, 102	20, 110
3, 16	12, 48	8, 52		5, 21	37, 98	29, 106	21, 114
3, 17	12, 51	8, 55		5, 22	38, 102	29, 111	21, 119
3, 18	13, 53	8, 58		5, 23	39, 106	30, 115	22, 123
3, 19	13, 56	9, 60		5, 24	40, 110	31, 119	23, 127
3, 20	14, 58	9, 63		5, 25	42, 113	32, 123	23, 132
3, 21	14, 61	9, 66	6, 69				
3, 22	15, 63	10, 68	6, 72	6, 6	26, 52	23, 55	
3, 23	15, 66	10, 71	6, 75	6, 7	27, 57	24, 60	
3, 24	16, 68	10, 74	6, 78	6, 8	29, 61	25, 65	21, 69
3, 25	19, 71	11, 76	6, 81	6, 9	31, 65	26, 70	22, 74
				6, 10	32, 70	27, 75	23, 79
4, 4	10, 26			6, 11	34, 74	28, 80	23, 85
4, 5	11, 29			6, 12	35, 79	30, 84	24, 90
4, 6	12, 32	10, 34		6, 13	37, 83	31, 89	25, 95
4, 7	13, 35	10, 38		6, 14	38, 88	32, 94	26, 100
4, 8	14, 38	11, 41		6, 15	40, 92	33, 99	26, 106
4, 9	15, 41	11, 45		6, 16	42, 96	34, 104	27, 111
4, 10	15, 45	12, 48		6, 17	43, 101	36, 108	28, 116
4, 11	16, 48	12, 52		6, 18	45, 105	37, 113	29, 121
4, 12	17, 51	13, 55					

Table A8 Critical ranges for the Wilcoxon rank sum test (continued)

n_1, n_2	One-sided P -value			n_1, n_2	One-sided P -value		
	0.025	0.005	0.0005		0.025	0.005	0.0005
	Two-sided P -value				Two-sided P -value		
	0.05	0.01	0.001		0.05	0.01	0.001
6, 19	46, 110	38, 118	29, 127	9, 15	79, 146	70, 155	60, 165
6, 20	48, 114	39, 123	30, 132	9, 16	82, 152	72, 162	61, 173
6, 21	50, 118	40, 128	31, 137	9, 17	84, 159	74, 169	63, 180
6, 22	51, 123	42, 132	32, 142	9, 18	87, 165	76, 176	65, 187
6, 23	53, 127	43, 137	33, 147	9, 19	90, 171	78, 183	66, 195
6, 24	55, 131	44, 142	34, 152	9, 20	93, 177	81, 189	68, 202
				9, 21	95, 184	83, 196	70, 209
7, 7	36, 69	32, 73	28, 77	10, 10	78, 132	71, 139	63, 147
7, 8	38, 74	34, 78	29, 83	10, 11	81, 139	74, 146	65, 155
7, 9	40, 79	35, 84	30, 89	10, 12	85, 145	76, 154	67, 163
7, 10	42, 84	37, 89	31, 95	10, 13	88, 152	79, 161	69, 171
7, 11	44, 89	38, 95	32, 101	10, 14	91, 159	81, 169	71, 179
7, 12	46, 94	40, 100	33, 107	10, 15	94, 166	84, 176	73, 187
7, 13	48, 99	41, 106	34, 113	10, 16	97, 173	86, 184	75, 195
7, 14	50, 104	43, 111	35, 119	10, 17	100, 180	89, 191	77, 203
7, 15	52, 109	44, 117	36, 125	10, 18	103, 187	92, 198	79, 211
7, 16	54, 114	46, 122	37, 131	10, 19	107, 193	94, 206	81, 219
7, 17	56, 119	47, 128	38, 137	10, 20	110, 200	97, 213	83, 227
7, 18	58, 124	49, 133	39, 143	11, 11	96, 157	87, 166	78, 175
7, 19	60, 129	50, 139	41, 148	11, 12	99, 165	90, 174	81, 183
7, 20	62, 134	52, 144	42, 154	11, 13	103, 172	93, 182	83, 192
7, 21	64, 139	53, 150	43, 160	11, 14	106, 180	96, 190	85, 201
7, 22	66, 144	55, 155	44, 166	11, 15	110, 187	99, 198	87, 210
7, 23	68, 149	57, 160	45, 172	11, 16	114, 194	102, 206	90, 218
8, 8	49, 87	43, 93	38, 98	11, 17	117, 202	105, 214	92, 227
8, 9	51, 93	45, 99	40, 104	11, 18	121, 209	108, 222	94, 236
8, 10	53, 99	47, 105	41, 111	11, 19	124, 217	111, 230	97, 244
8, 11	55, 105	49, 111	42, 118	12, 12	115, 185	106, 194	95, 205
8, 12	58, 110	51, 117	43, 125	12, 13	119, 193	109, 203	98, 214
8, 13	60, 116	53, 123	45, 131	12, 14	123, 201	112, 212	100, 224
8, 14	63, 121	54, 130	46, 138	12, 15	127, 209	115, 221	103, 233
8, 15	65, 127	56, 136	47, 145	12, 16	131, 217	119, 229	105, 243
8, 16	67, 133	58, 142	49, 151	12, 17	135, 225	122, 238	108, 252
8, 17	70, 138	60, 148	50, 158	12, 18	139, 233	125, 247	111, 261
8, 18	72, 144	62, 154	51, 165	13, 13	137, 214	125, 226	114, 237
8, 19	74, 150	64, 160	53, 171	13, 14	141, 223	129, 235	116, 248
8, 20	77, 155	66, 166	54, 178	13, 15	145, 232	133, 244	119, 258
8, 21	79, 161	68, 172	56, 184	13, 16	150, 240	137, 253	122, 268
8, 22	82, 166	70, 178	57, 191	13, 17	154, 249	140, 263	125, 278
9, 9	63, 108	56, 115	50, 121	14, 14	160, 246	147, 259	134, 272
9, 10	65, 115	58, 122	52, 128	14, 15	164, 256	151, 269	137, 283
9, 11	68, 121	61, 128	53, 136	14, 16	169, 265	155, 279	140, 294
9, 12	71, 127	63, 135	55, 143	15, 15	185, 280	171, 294	156, 309
9, 13	73, 134	65, 142	56, 151				
9, 14	76, 140	67, 149	58, 158				

Table A9 Random numbers.

Reproduced from Table XXXIII of Fisher and Yates (1963) following Armitage (1971) by permission of the authors and publishers.

03	47	43	73	86	36	96	47	36	61	46	98	63	71	62	33	26	16	80	45	60	11	14	10	95
97	74	24	67	62	42	81	14	57	20	42	53	32	37	32	27	07	36	07	51	24	51	79	89	73
16	76	62	27	66	56	50	26	71	07	32	90	79	78	53	13	55	38	58	59	88	97	54	14	10
12	56	85	99	26	96	96	68	27	31	05	03	72	93	15	57	12	10	14	21	88	26	49	81	76
55	59	56	35	64	38	54	82	46	22	31	62	43	09	90	06	18	44	32	53	23	83	01	30	30
16	22	77	94	39	49	54	43	54	82	17	37	93	23	78	87	35	20	96	43	84	26	34	91	64
84	42	17	53	31	57	24	55	06	88	77	04	74	47	67	21	76	33	50	25	83	92	12	06	76
63	01	63	78	59	16	95	55	67	19	98	10	50	71	75	12	86	73	58	07	44	39	52	38	79
33	21	12	34	29	78	64	56	07	82	52	42	07	44	38	15	51	00	13	42	99	66	02	79	54
57	60	86	32	44	09	47	27	96	54	49	17	46	09	62	90	52	84	77	27	08	02	73	43	28
18	18	07	92	46	44	17	16	58	09	79	83	86	19	62	06	76	50	03	10	55	23	64	05	05
26	62	38	97	75	84	16	07	44	99	83	11	46	32	24	20	14	85	88	45	10	93	72	88	71
23	42	40	64	74	82	97	77	77	81	07	45	32	14	08	32	98	94	07	72	93	85	79	10	75
52	36	28	19	95	50	92	26	11	97	00	56	76	31	38	80	22	02	53	53	86	60	42	04	53
37	85	94	35	12	83	39	50	08	30	42	34	07	96	88	54	42	06	87	98	35	85	29	48	39
70	29	17	12	13	40	33	20	38	26	13	89	51	03	74	17	76	37	13	04	07	74	21	19	30
56	62	18	37	35	96	83	50	87	75	97	12	25	93	47	70	33	24	03	54	97	77	46	44	80
99	49	57	22	77	88	42	95	45	72	16	64	36	16	00	04	43	18	66	79	94	77	24	21	90
16	08	15	04	72	33	27	14	34	09	45	59	34	68	49	12	72	07	34	45	99	27	72	95	14
31	16	93	32	43	50	27	89	87	19	20	15	37	00	49	52	85	66	60	44	38	68	88	11	80
68	34	30	13	70	55	74	30	77	40	44	22	78	84	26	04	33	46	09	52	68	07	97	06	57
74	57	25	65	76	59	29	97	68	60	71	91	38	67	54	13	58	18	24	76	15	54	55	95	52
27	42	37	86	53	48	55	90	65	72	96	57	69	36	10	96	46	92	42	45	97	60	49	04	91
00	39	68	29	61	66	37	32	20	30	77	84	57	03	29	10	45	65	04	26	11	04	96	67	24
29	94	98	94	24	68	49	69	10	82	53	75	91	93	30	34	25	20	57	27	40	48	73	51	92

16	90	82	66	59	83	62	64	11	12	67	19	00	71	74	60	47	21	29	68	02	02	37	03	31
11	27	94	75	06	06	09	19	74	66	02	94	37	34	02	76	70	90	30	86	38	45	94	30	38
35	24	10	16	20	33	32	51	26	38	79	78	45	04	91	16	92	53	56	16	02	75	50	95	98
38	23	16	86	38	42	38	97	01	50	87	75	66	81	41	40	01	74	91	62	48	51	84	08	32
31	96	25	91	47	96	44	33	49	13	34	86	82	53	91	00	52	43	48	85	27	55	26	89	62
66	67	40	67	14	64	05	71	95	86	11	05	65	09	68	76	83	20	37	90	57	16	00	11	66
14	90	84	45	11	75	73	88	05	90	52	27	41	14	86	22	98	12	22	08	07	52	74	95	80
68	05	51	18	00	33	96	02	75	19	07	60	62	93	55	59	33	82	43	90	49	37	38	44	59
20	46	78	73	90	97	51	40	14	02	04	02	33	31	08	39	54	16	49	36	47	95	93	13	30
64	19	58	97	79	15	06	15	93	20	01	90	10	75	06	40	78	78	89	62	02	67	74	17	33
05	26	93	70	60	22	35	85	15	13	92	03	51	59	77	59	56	78	06	83	52	91	05	70	74
07	97	10	88	23	09	98	42	99	64	61	71	62	99	15	06	51	29	16	93	58	05	77	09	51
68	71	86	85	85	54	87	66	47	54	73	32	08	11	12	44	95	92	63	16	29	56	24	29	48
26	99	61	65	53	58	37	78	80	70	42	10	50	67	42	32	17	55	85	74	94	44	67	16	94
14	65	52	68	75	87	59	36	22	41	26	78	63	06	55	13	08	27	01	50	15	29	39	39	43
17	53	77	58	71	71	41	61	50	72	12	41	94	96	26	44	95	27	36	99	02	96	74	30	83
90	26	59	21	19	23	52	23	33	12	96	93	02	18	39	07	02	18	36	07	25	99	32	70	23
41	23	52	55	99	31	04	49	69	96	10	47	48	45	88	13	41	43	89	20	97	17	14	49	17
60	20	50	81	69	31	99	73	68	68	35	81	33	03	76	24	30	12	48	60	18	99	10	72	34
91	25	38	05	90	94	58	28	41	36	45	37	59	03	09	90	35	57	29	12	82	62	54	65	60
34	50	57	74	37	98	80	33	00	91	09	77	93	19	82	74	94	80	04	04	45	07	31	66	49
85	22	04	39	43	73	81	53	94	79	33	62	46	86	28	08	31	54	46	31	53	94	13	38	47
09	79	13	77	48	73	82	97	22	21	05	03	27	24	83	72	89	44	05	60	35	80	39	94	88
88	75	80	18	14	22	95	75	42	49	39	32	82	22	49	02	48	07	70	37	16	04	61	67	87
90	96	23	70	00	39	00	03	06	90	55	85	78	38	36	94	37	30	69	32	90	89	00	76	33

(continued)

Table A9 Random numbers (continued).

53	74	23	99	67	61	32	28	69	84	94	62	67	86	24	98	33	41	19	95	47	53	53	38	09
63	38	06	86	54	99	00	65	26	94	02	82	90	23	07	79	62	67	80	60	75	91	12	81	19
35	30	58	21	46	06	72	17	10	94	25	21	31	75	96	49	28	24	00	49	55	65	79	78	07
63	43	36	82	69	65	51	18	37	88	61	38	44	12	45	32	92	85	88	65	54	34	81	85	35
98	25	37	55	26	01	91	82	81	46	74	71	12	94	97	24	02	71	37	07	03	92	18	66	75
02	63	21	17	69	71	50	80	89	56	38	15	70	11	48	43	40	45	86	98	00	83	26	91	03
64	55	22	21	82	48	22	28	06	00	61	54	13	43	91	82	78	12	23	29	06	66	24	12	27
85	07	26	13	89	01	10	07	82	04	59	63	69	36	03	69	11	15	83	80	13	29	54	19	28
58	54	16	24	15	51	54	44	82	00	62	61	65	04	69	38	18	65	18	97	85	72	13	49	21
34	85	27	84	87	61	48	64	56	26	90	18	48	13	26	37	70	15	42	57	65	65	80	39	07
03	92	18	27	46	57	99	16	96	56	30	33	72	85	22	84	64	38	56	98	99	01	30	98	64
62	95	30	27	59	37	75	41	66	48	86	97	80	61	45	23	53	04	01	63	45	76	08	64	27
08	45	93	15	22	60	21	75	46	91	98	77	27	85	42	28	88	61	08	84	69	62	03	42	73
07	08	55	18	40	45	44	75	13	90	24	94	96	61	02	57	55	66	83	15	73	42	37	11	61
01	85	89	95	66	51	10	19	34	88	15	84	97	19	75	12	76	39	43	78	64	63	91	08	25
72	84	71	14	35	19	11	58	49	26	50	11	17	17	76	86	31	57	20	18	95	60	78	46	75
88	78	28	16	84	13	52	53	94	53	75	45	69	30	96	73	89	65	70	31	99	17	43	48	76
45	17	75	65	57	28	40	19	72	12	25	12	74	75	67	60	40	60	81	19	24	62	01	61	16
96	76	28	12	54	22	01	11	94	25	71	96	16	16	88	68	64	36	74	45	19	59	50	88	92
43	31	67	72	30	24	02	94	08	63	38	32	36	66	02	69	36	38	25	39	48	03	45	15	22
50	44	66	44	21	66	06	58	05	62	68	15	54	35	02	42	35	48	96	32	14	52	41	52	48
22	66	22	15	86	26	63	75	41	99	58	42	36	72	24	58	37	52	18	51	03	37	18	39	11
96	24	40	14	51	23	22	30	88	57	95	67	47	29	83	94	69	40	06	07	18	16	36	78	86
31	73	91	61	19	60	20	72	93	48	98	57	07	23	69	65	95	39	69	58	56	80	30	19	44
78	60	73	99	84	43	89	94	36	45	56	69	47	07	41	90	22	91	07	12	78	35	34	08	72

84	37	90	61	56	70	10	23	98	05	85	11	34	76	60	76	48	45	34	60	01	64	18	39	96
36	67	10	08	23	98	93	35	08	86	99	29	76	29	81	33	34	91	58	93	63	14	52	32	52
07	28	59	07	48	89	64	58	89	75	83	85	62	27	89	30	14	78	56	27	86	63	59	80	02
10	15	83	87	60	79	24	31	66	56	21	48	24	06	93	91	98	94	05	49	01	47	59	38	00
55	19	68	97	65	03	73	52	16	56	00	53	55	90	27	33	42	29	38	87	22	13	88	83	34
53	81	29	13	39	35	01	20	71	34	62	33	74	82	14	53	73	19	09	03	56	54	29	56	93
51	86	32	68	92	33	98	74	66	99	40	14	71	94	58	45	94	19	38	81	14	44	99	81	07
35	91	70	29	13	80	03	54	07	27	96	94	78	32	66	50	95	52	74	33	13	80	55	62	54
37	71	67	95	13	20	02	44	95	94	64	85	04	05	72	01	32	90	76	14	53	89	74	60	41
93	66	13	83	27	92	79	64	64	72	28	54	96	53	84	48	14	52	98	94	56	07	93	89	30
02	96	08	45	65	13	05	00	41	84	93	07	54	72	59	21	45	57	09	77	19	48	56	27	44
49	83	43	48	35	82	88	33	69	96	72	36	04	19	76	47	45	15	18	60	82	11	08	95	97
84	60	71	62	46	40	80	81	30	37	34	39	23	05	38	25	15	35	71	30	88	12	57	21	77
18	17	30	88	71	44	91	14	88	47	89	23	30	63	15	56	34	20	47	89	99	82	93	24	98
79	69	10	61	78	71	32	76	95	62	87	00	22	58	40	92	54	01	75	25	43	11	71	99	31
75	93	36	57	83	56	20	14	82	11	74	21	97	90	65	96	42	68	63	86	74	54	13	26	94
38	30	92	29	03	06	28	81	39	38	62	25	06	84	63	61	29	08	93	67	04	32	92	08	09
51	29	50	10	34	31	57	75	95	80	51	97	02	74	77	76	15	48	49	44	18	55	63	77	09
21	31	38	86	24	37	79	81	53	74	73	24	16	10	33	52	83	90	94	76	70	47	14	54	36
29	01	23	87	88	58	02	39	37	67	42	10	14	20	92	16	55	23	42	45	54	96	09	11	06
95	33	95	22	00	18	74	72	00	18	38	79	58	69	32	81	76	80	26	92	82	80	84	25	39
90	84	60	79	80	24	36	59	87	38	82	07	53	89	35	96	35	23	79	18	05	98	90	07	35
46	40	62	98	82	54	97	20	56	95	15	74	80	08	32	16	46	70	50	80	67	72	16	42	79
20	31	89	03	43	38	46	82	68	72	32	14	82	99	70	80	60	47	18	97	63	49	30	21	30
71	59	73	05	50	08	22	23	71	77	91	01	93	20	49	82	96	59	26	94	66	39	67	98	60

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