## Chapter 19

## Measurement of Centeral Tendency

If we take the achievement scores of the students of a class and arrange them in a frequency distribution, We can easily find that there are few students who either score very high or very low. The marks of most of the students lie somewhere between the highest and the lowest scores of the whole class. This tendency of a group of a distribution is named as central tendency and the typical score lying between the extremes and shared by most of the students is referred to as a measure of central tendency. In this way, a measure of central tendency as Tate defines, "is a sort of average or typical value of the items in the series and its function is to summarize the series in terms of this average value" (1955, p. 78). The most common measures of central tendency are-
(i) Arithmetic mean or mean
(ii) median and
(iii) mode

Each of them, in its own way can be called a: representative of the characteristics of the whole group and thus the performance of the group as a whole can be described by the single value which each of these measures gives. The values of mean, median or mode also' help us in comparing two or more groups or frequency distributions in terms of typical or characteristics performance. In the following pages we will study these measures of central tendency.

## ARITHMETIC MEAN (M)

It is the simplest but most useful measure of central tendency. It is nothing but' the 'average' which we compute in our High School Arithmetic and therefore can be easily defined as the sum of all the values of the items in a series divided by the number of items. It is designated by the symbol M .

## Computation of Mean in the Case of Ungrouped Data

Let $X_{1}, X_{2}, X_{3}, X_{4}, X_{5}, X_{6}, X_{7}, X_{8}, X_{9}, X_{10}$ be the scores obtained by 10 students in an Achievement Test. Then the Arithmetic mean or Mean scores of the group of these students can be calculated as-

$$
M=\frac{X_{1}+X_{2}+X_{3}+\ldots+X_{10}}{10}
$$

In this way, the formula for calculating mean of an un grouped data is $M=\frac{\Sigma X}{N}$, where $\Sigma X$ stands for the sum of the scores or values of the items and $N$ for the total numbers of items is a series of group.

## Computation of Mean in the Case of Grouped Data

(Data in the Form of Frequency Distribution)
(i) In frequency distribution where frequencies are greater than 1 , the mean is calculated by the formula $M=\frac{\Sigma F X}{N}$, where $X$ represents the mid-point of class interval, $F$ its respective frequency and, the total of all the frequencies.
We can illustrate the use of this formula by taking the frequency distribution previously given

| Scores | $f$ | Mid-Point | $(F X)$ |
| :---: | :---: | :---: | :---: |
| $65-69$ | 1 | 67 | 67 |
| $60-64$ | 3 | 62 | 186 |
| $55-59$ | 4 | 57 | 228 |
| $50-54$ | 7 | 52 | 64 |
| $45-49$ | 9 | 47 | 423 |
| $40-44$ | 11 | 42 | 462 |
| $35-39$ | 4 | 37 | 296 |
| $30-34$ | 2 | 32 | 128 |
| $25-29$ | 1 | 27 | 54 |
| $20-24$ |  |  | 22 |
|  |  |  | Sfx $=230$ |

$$
M=\frac{\Sigma F X}{N}=\frac{2230}{50}=446
$$

Arithmetic Mean $=44.6$
(iii) Short cut Method of computing the mean of grouped data

Mean for the grouped data can be computed easily with the help of the following formula:

$$
M=A+\frac{\Sigma X}{N} \times i
$$

Where A stands for assumed mean, $i$ for class interval, $f$ for the respective frequency of the midvalue, $N$ for the total frequency and $x^{\prime}$ for $\frac{x-a}{i}$ (the quotient obtained after division of the difference between the mid-value of the class and assumed mean by $i$ ).

The use of this formula can be easily understood through the-following illustration

Let Assumed Mean $(A)=42$

| Scores | $f$ | $x$ | $x^{\prime}=\frac{X-A}{i}$ | $f x^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: |
| $65-69$ | 1 | 67 | 5 | 5 |
| $60-64$ | 3 | 62 | 4 | 12 |
| $55-59$ | 4 | 57 | 3 | 12 |
| $50-54$ | 7 | 52 | 2 | 14 |
| $45-49$ | 9 | 47 | 1 | 9 |
| $40-44$ | 11 | 44 | 0 | 0 |
| $35-9$ | 8 | 37 | -1 | -8 |
| $30-34$ | 4 | 32 | -3 | -8 |
| $25-29$ | 2 | 27 | -4 | -4 |
| $20-24$ | 1 |  | $\Sigma F X^{\prime}=26$ |  |

$$
x^{\prime}=\frac{\Sigma x^{\prime}}{N} \times i=42+\frac{26}{50} \times 5=42+2.6=44.6
$$

## MEDIAN ( $\mathrm{M}_{\mathrm{d}}$ )

If the items of a series are arranged in ascending or descending order of magnitude, the measure or value of the central item in the series is termed the median. In this way, as Bloomers are Lindquist define:
"The Median of a distribution is the point on the score scales below which one-half or 50 percent of the scores fall."

Thus, median is the score or value 'of the central item which divides the series into two equal parts. In this connection it should be clearly understood that central item itself is not the median. It is only the measure or value of the central item that is known as median. For example, if we arrange in ascending or descending order the marks of 5 students, then the marks obtained by 3rd student from either side will be termed as the median of the scores of the group of students under consideration.

## Computation in the Case of Ungrouped Data

There may arise two situations-
(i) When $N$ (the number of items in series) is odd - In the case where $N$ i.e. number of students in the above example, is odd (not divisible by 2 ) then the median can be computed by the formula.

$$
M_{d}=\text { The measure of value of the }(N+1) / 2 \text { th item. }
$$

Example-Let the scores obtained by 7 students in an Achievement Test be 17, 47, 15, 35, 25, 29, 39,44 . Then first of all, for calculating median we have to arrange the scores in ascending or descending order like $15,17,25,39,44,47$. Here $N(=7)$ is odd and therefore the score of the $(N+1) / 2$ th or 4th student, i.e. 35 is the median of given scores.
(ii) Where $N$ (the number of items in a series) is even-In the case where $N$ is even (divisible by 2 ), then the median is determined by this following formula-

$$
M_{d}=\frac{\text { Total value of }(N / 2) \text { th and }(N / 2+1) \text { th item }}{2}
$$

Example-Let there be a group of 8 students, whose scores are 17, 47, 15, 35, 39, 50, 44.
For calculating the median of these scores we will proceed as under:
Arrangement of scores in proper order 15, 17, 25, 35, 39, 44, 47, 50.
The score of the $(N / 2)$ th $=35$ i.e. 4 th students
The score of the $(N / 2+1)$ th $=39$ i.e. 5 th student
Then Median $=\frac{35+39}{2}=37$
Computation of the median for grouped data (data in the form of a frequency distribution)
If the data is available in the form of a frequency distribution like the following:

| Scores | $f$ |
| :---: | :---: |
| $65-69$ | 1 |
| $60-64$ | 3 |
| $55-59$ | 4 |
| $50-54$ | 7 |
| $45-49$ | 9 |
| $40-44$ | 11 |
| $35-39$ | 8 |
| $30-34$ | 4 |
| $25-29$ | 2 |
| $20-24$ | 1 |
|  | $N=50$ |

Then calculating of median first requires the location of median class. Actually as defined earlier, median is the measure or score of the central.

Therefore, it is needed which is the central item whose measure we aim to determine. It is done through the formula given above in the case of ungrouped data depending upon the odd and even. nature of total frequencies $(N)$. Here in the present distribution $N(=50)$ is even, therefore, median will fall somewhere between the scores of 25 th and 26 th items in the given distribution. In the present example if we add frequencies from the above or below we can know that the class interval designated as $40-44$ can be labelled as the class where the score representing median lies.

After estimating the median class the median of the distribution can be interpolated from the following formula.

$$
M_{d}=L+\frac{N / 2-F}{f} \times i
$$

Where $L=$ Exact lower limit of the median class.
$F=$ Total of all the frequencies before the median class.
$f=$ Frequency of the median class.
$i=$ Class interval.
$N=$ Total of all the frequencies.
By applying the above formula we can compute the median of the given distribution in the following way:

$$
\begin{aligned}
M_{d} & =39.5+\left(\frac{50 / 2}{11}\right) \times 5=39.5+\frac{10}{11} \times 5 \\
& =39.5+\frac{50}{11}=39.5+4.55=44.05
\end{aligned}
$$

Some Special situations in the computation of Median

| Scores <br> $(a)$ | $f$ | Scores <br> $(b)$ | $f$ | Scores <br> $(c)$ | $f$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $55-59$ | 5 | $45-49$ | 2 | $20-21$ | 2 |
| $50-54$ | 3 | $40-44$ | 5 | $18-19$ | 1 |
| $45-49$ | 8 | $35-39$ | 6 | $16-17$ | 0 |
| $40-44$ | 18 | $30-34$ | 0 | $14-15$ | 0 |
| $35-39$ | 15 | $25-29$ | 8 | $12-13$ | 2 |
| $30-34$ | 10 | $20-24$ | 3 | $10-11$ | 0 |
| $25-29$ | 7 | $15-19$ | 2 | $8-9$ | 0 |
| $20-24$ | 2 |  |  | $6-7$ | 2 |
|  |  |  |  | $4-5$ | 1 |
|  |  |  |  | $2-3$ | 1 |
|  |  |  |  | $0-1$ | 1 |

Let us think about the medians of the above distribution.
(a) We know by definition that median is the point on the score scale below and above which $50 \%$ cases lie. Observing through this definition the score representing median should be a common score falling between the class 35-39 and 40-44. This score is nothing but the upper limit of the class 35-39 which is also the lower limit of the class 40-44. Therefore, in this case median is $39-5$.
(b) In the 2 nd distribution, if we try to add the frequencies from below we see that up to class interval $25-29,13$ cases lie and by adding frequencies from above we also find that up to the class interval 35-39, 13 cases lie. In this way, the class interval 30-34 divides the distribution into two equal parts below and above which $50 \%$ cases lie. It leads us to conclude that median should be the mid-point of the class interval 30-34 and therefore 32 is the median of this distribution.
(c) In the 3 rd case if we add the frequencies from below we find that upto the class interval $6-7,5$ cases lie and by adding the frequencies from above.
We also find that upto the class $12-13,5$ cases lie. The median should fall in the mid-way between the two classes $8-9$, and $10-11$. It should be the common score represented by both these classes. This score is noting but the upper limit of the class $8-9$ and lower limit of the class $10-11$ and therefore, it should be 9.5.

## MODE ( $\mathrm{M}_{\mathrm{O}}$ )

Mode is defined to be the size of the variable (say a score) which occurs most frequently. It is the point on the score scale that corresponds to the maximum frequency of the distribution. In any series it is the value of the item, which is most characteristic or common and is usually repeated maximum number of items.

## Computation of Mode

## (a) In case of ungrouped data

In the case of ungrouped data mode can be easily computed merely by looking at it. All that one has to do is to find out the score which is repeated maximum number of times.

Example- Suppose we have to find out the value of the mode from the following scores of the students:

25, 29, 24, 27, 28, 25, 29
Here the score 2.5 is repeated maximum number of times and thus value of the mode in this case is 25 .

## (b) In Case of grouped data

In the case where data, is available in the form of a frequency distribution, the mode $\left(M_{o}\right)=3 M_{d}-2 M$ where $M_{d}$ is the median and $M$, the mean of the given distribution of all mean as well as median of the distribution are computed and then with the help of the above formula Mode is calculated. For illustration we can take the distribution previously given in the Table 16.2. We-know the mean and median of this distribution. Now we can use these results for the computation of the mode.

$$
\begin{aligned}
M_{d}=44.05, M & =44.6 \\
\text { Therefore, } M_{o} & =3 \times 44.05-2 \times 44.6 \\
& =132.15-89.2 \\
& =42.95
\end{aligned}
$$

## When to use the Mean, Median and Mode

Computation of any of the three-mean, median and mode-provides a measure of central tendency. Now which of them should be computed for a particular distribution is a question that can be raised quite
often. Below we pay attention over this aspect in light of the characteristics and nature of all these measures.

## When to Use the Mean

(i) Mean is the most reliable accurate measure of the central tendency of a distribution in comparison to median and mode. It has the greatest stability as there are less fluctuations in the means of the samples drawn from the same population. Therefore, in the case where a reliable and accurate measure of central tendency is needed, we compute mean for the given data.
(ii) Mean can be given an algebraic treatment and is better suited to further arithmetical computation. Therefore, it can be easily employed for the computation of various statistics like Standard Deviation, Coefficient of correlation etc. Hence, when we need to know such statistics, mean is computed for the given data.
(iii) In computation of the mean we give equal weightage to every item in the series. Therefore, it is affected by the value of each item in that series. Sometimes, there are extreme items which seriously affect the position of the mean. Therefore, it is not proper to compute mean for the series that have extreme items. It should be calculated only when the series has no extreme items and each score carries equal weight in determining the central tendency.

## When to Use the Median

(i) Median is the exact mid-point of a series as $50 \%$ cases lie below and above it. Therefore, when the exact mid-point of the distribution is desired, median is to be computed.
(ii) Median is not affected by the extreme scores in the series. Therefore, when a series contains extreme scores, the median is perhaps the most representative central measure.
(iii) In case of an open end distribution (incomplete distribution " 80 and above" or " 20 and below" etc.) mean is impossible to be calculated.
(iv) Mean cannot be calculated graphically. But in case of median we can compute it graphically. Therefore, when we have suitable graphs like Frequency curve, Polygon etc. we should try to compute median.
(v) The median is specifically useful for the data the items of which cannot precisely be measured quantitatively e.g. qualities like health, culture, honesty, intelligence etc.

## When to Use the Mode

(i) In many crude mode can be computed by just having a look at the date. It gives the quickest, although approximate, measure of central tendency. Therefore, in cases where a quick and approximate measure of central tendency is all that is desired, we compute mode.
(ii) Mode is that value' of the item which occurs most frequently or is repeated maximum number of times in a given series. Therefore, when we need to know; the most often recurring score or value of the items in a series, we compute mode. On account of the characteristic mode has unique importance in the large scale manufacturing of consumption goods. In finding the sizes of the shoes and ready-made clothes which will fit most men, the manufacturer makes use of the average indicated by mode.
(iii) Mode can be computed from the histogram and other frequency curves. Therefore, when we already have a graphically representation of the distribution in the form of such figures, it is appropriate to compute mode instead of mean.

## EXERCISES

## Essay Type Questions

1. Compute Median for the following Ungrouped data:
(i) $16,2,10,9,4,7,12,4,150$.
(ii) $8,3,10,5,2,11,14,12$.

Ans. (i) 10 (ii) 9
2. Find the crude Mode for the following data:
$15,14,8,14,14,11,9,9,11$.
Ans. 14
3. Compute the Mean, Median and Mode for the following distribution:

| Scores | $f$ | Scores | $f$ | Scores | $f$ | Scores | $f$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(a)$ |  | $(b)$ |  | $(c)$ |  | $(d)$ |  |
| $70-71$ | 2 | $120-122$ | 2 | $45-49$ | 2 | $135-144$ | 1 |
| $68-69$ | 2 | $117-119$ | 2 | $40-44$ | 3 | $125-134$ | 2 |
| $66-67$ | 3 | $114-116$ | 2 | $35-39$ | 2 | $115-124$ | 8 |
| $64-65$ | 4 | $111-113$ | 4 | $30-34$ | 17 | $105-114$ | 22 |
| $62-63$ | 6 | $108-110$ | 5 | $25-29$ | 30 | $95-104$ | 33 |
| $60-61$ | 7 | $105-107$ | 9 | $20-24$ | 25 | $85-94$ | 22 |
| $58-59$ | 5 | $102-104$ | 6 | $15-19$ | 15 | $75-84$ | 9 |
| $56-57$ | 1 | $99-101$ | 3 | $10-14$ | 3 | $65-74$ | 2 |
| $54-55$ | 2 | $96-98$ | 4 | $5-9$ | 2 | $55-64$ | 1 |
| $52-53$ | 3 | $93-95$ | 2 | $0-4$ | 1 |  |  |
| $50-51$ | 1 | $90-92$ | 1 |  |  |  |  |
|  | $N-36$ |  | $N=40$ |  | $N=100$ |  | $N=100$ |

Ans.
(a)
$M=6111$
$M_{d}=6121$
$M_{o}=6141$
(b)
$M=106.00$
$M=105.83$
$M_{o}=105.49$
(c)
$M=25.05$
$M_{d}=25.17$
$M_{o}=25.41$
(d)
$M=993$
$M_{d}=993$
$M_{o}=993$
4. What do you understand by mean and median? Explain by computing these for the scores in a test given below. (Taking 2 as class interval)

| 72 | 75 | 77 | 67 | 72 |
| :--- | :--- | :--- | :--- | :--- |
| 91 | 78 | 65 | 86 | 83 |
| 67 | 82 | 76 | 76 | 70 |
| 83 | 71 | 63 | 72 | 72 |
| 61 | 67 | 84 | 69 | 64 |

5. What do you mean by measures of central tendency? Name different measures of central tendency and discuss them in brief.
6. What is Arithmetic Mean $(M)$ ? How is it computed in the cases of ungrouped and grouped data? Discuss with hypothetical examples.
7. What is median $\left(M_{d}\right)$ ? How is it computed in the cases of ungrouped and grouped data? Discuss with. the help of a hypothetical example.
8. What is mode $\left(M_{o}\right)$ ? How is it computed 'in the cases of ungrouped and grouped data? Discuss with the help of a hypothetical example.
9. Explain which of the three mean, median and mode should be computed for a particular distribution in a specified situation.
10. Calculate the Mean, Median and Mode of the following frequency distribution.

| Class Interval | Frequency |
| :---: | :---: |
| $195-199$ | 1 |
| $190-194$ | 2 |
| $185-189$ | 4 |
| $180-184$ | 5 |
| $175-179$ | 8 |
| $170-174$ | 10 |
| $65-169$ | 6 |
| $160-164$ | 4 |
| $155-159$ | 4 |
| $150-154$ | 2 |
| $145-149$ | 3 |
| $140-144$ | 1 |

Ans. $M=170.8, M_{d}=172, M_{o}=174.4$
5. Calculate the mean, Median and Mode of the following distribution.

| Class Interval | $35-39$ | $30-34$ | $25-29$ | $20-24$ | $15-19$ | $10-14$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 4 | 8 | 15 | 10 | 8 | 5 |

Ans. $M=24.5, M_{d}=28.83, M_{o}=37.49$

## Short Answer Type Questions (Answer in 100-120 words)

1. What are the measures of central tendency? Name their various types.
2. What is mean? How is it computed?
3. What is median? How is it computed?
4. What is mode? How is it computed?
5. When and where is the need of the computation of mean for a given data?
6. When and where is the need of the computation of median for a given data?
7. When and where is it useful to compute mode in the case of a given data?

## Chapter 20

## Measures of Variability

Measure of central tendency-mean, median and mode-provide central value or typical representative of a set of scores as a whole. Through these measures we can represent a characteristic or quality of the whole group by a single number. By comparing such typical representative of the different sets of scores we can compare the achievement of the two groups. But these representative numbers give us merely an idea of the general achievement of the group as a whole, and does not show how the individual scores are spread out. Therefore, through measures of central tendency we are unable to know much about the distribution of scores in a series or characteristics on items in a group. Hence, measures of central tendency provide insufficient base for the comparison of two or more frequency distribution or sets of scores. It can be made more clear from the following example.

Let there be two small groups of boys and girls whose scores in an achievement test are such as the following:

Test Scores of Group A (boys) - 40, 38, 36, 17, 20, 19, 18, 3, 5, 4
Test Scores of Group B (girls) - 19, 20, 22, 18, 21, 23, 17, 20, 22, 18.
Now the value of the Mean in both the cases is 20 and thus, so far as the mean goes, there is no difference in the performance of the two groups. Now the question arises, can we take both sets of scores as identical? Definitely there is a lot of difference between the performance of two groups. Whereas the test scores of group A are found to range from 30 to 40 , the scores in group B range from 18 to 23. First group in composed of individuals who have wide individual difference. It contains either very capable or very individuals. The second group, on the other hand is composed of average individuals. Individuals in this latter group are less variable than those of the former. Looking in this way, there is a great need of paying consideration to the variability or dispersion of the scores in the sets of scores or series if we want to describe and compare them.

## DIFFERENT MEASURES OF VARIABILITY OR DISPERSION

These are chiefly, four measures of indicating variability or dispersion within the set of scores. They are:
(a) The Range (R)
(b) The Quartile Deviation (Q)
(c) The Average Deviation (AD)
(d) The Standard Deviation (SD)

Each of the above measures of variability gives us the degree of variability or dispersion by the use of a single number and tells us how the individual scores are scattered or spread over throughout the distribution or gives data.

In the following pages we will discuss these measures in brief.

## Range (R)

Range is the simplest measure of variability or dispersion. It is calculated by subtracting the lowest scores in the series from the highest. But it is very rough measure of the variability of a series. It takes only extreme scares into consideration and tells nothing about the variation of the individual items.

## Quartile Deviation (Q)

It is computed by the formula $Q=\left(Q_{3}-Q_{1}\right) / 2$, where $Q_{1}$ and $Q_{3}$ represent the 1 st and 3rd quartiles of the distribution under consideration. The amount $Q_{3}$ and $Q_{1}$ is nothing but the difference of range between 3 rd and 1 st quartile. It is designated as the inter quartile range. For computing Quartile Deviation, this interquartile range is divided by 2 and therefore, $Q$ uartile Deviation is also named as semi-interquartile range. In this way, for computing $Q$, the values of $Q_{1}$ and $Q_{3}$ are first determined and then by applying the above formula we try to get the value of Quartile Deviation.

## Average Deviation (AD)

"Average Deviation or AD" as Garrett defines it, "is the mean of the deviation of all the separate scores in the series taken from their mean (occasionally from the median or mode)." (1971, p. 481)

It is the simplest measure of variability that takes into account the fluctuation or variation of all the items in a series.
(i) Computation of Average Deviation ( $A D$ ) from ungrouped data.

In the case of ungrouped data $A D$ is calculated by the formula-

$$
A D=\frac{\Sigma|x|}{N}
$$

Where $x=X-M=$ Deviation of the score from the mean of the series and x signifies that in the deviation values we ignore the algebraic signs +ve and -ve .

The use of this formula can be explained through the following example.
Example-Find out the Average Deviation of the scores 15, 10, 6, 8, 11 of a series.
Solution-The mean of given series $=15+10+6+8+11=10$

| Scores | Deviation from the mean <br> $(X-M)=x$ | $\|x\|$ |
| :---: | :---: | :---: |
| 15 | 5 | 5 |
| 10 | 0 | 0 |
| 6 | -4 | 4 |
| 8 | -2 | 2 |
| 11 | 1 | 1 |
| $N=5$ |  | $\|x\|=12$ |

By applying the formula- $A D=\frac{\Sigma|x|}{N}=12-2.4$
(ii) Computation of Average Deviation from grouped data

From grouped data AD can be computed by formula- $A D=\frac{\Sigma|x|}{N}$
Use of this formula can be understood through the following Illustration.

| Score | $f$ | Mid-point $X$ | $f x$ | $x=(X-100.06)$ | $f x$ | $\|f x\|$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $110-114$ | 4 | 112 | 448 | 11.94 | 44.76 | 47.76 |
| $105-109$ | 4 | 107 | 428 | 6.94 | 27.76 | 27.76 |
| $100-104$ | 3 | 102 | 306 | 1.94 | 5.82 | 5.82 |
| $95-99$ | 0 | 97 | 0 | -3.06 | 0 | 0 |
| $90-94$ | 3 | 92 | 276 | -8.08 | -24.18 | 24.18 |
| $85-89$ | 3.87 | 261 | -13.36 | -39.18 | 39.18 |  |
| $80-84$ | 1 | 82 | 82 | -18.06 | -18.06 | 18.06 |
|  | $N=18$ |  | 1801 |  |  | 162.67 |

First of all Mean is computed

$$
\text { Here Mean }=\frac{\Sigma f x}{N}=\frac{1801}{18}=100.06
$$

Then we calculate the values of $x$ by subtracting. Mean from the respective values of $x$ and enter them into V column. By multiplying these values by the respective class frequencies and ignoring the algebraic sign we get the values of $\Sigma|x|$ Afterward we apply the formula below

$$
A D=\frac{\Sigma|f x|}{N}=\frac{162.76}{18}=9.04
$$

## STANDARD DEVIATION (SD)

Standard Deviation of a set of a scores is defined as the square root of the average of the squares of the deviation of each from the mean.

Symbolically we can say that $S D=\sqrt{\frac{\Sigma(X-M)^{2} N}{}}=\sqrt{\frac{\Sigma x^{2}}{N}}$, where $X$ stands for individual score, M for mean of the given set of scores. $N$ for total number of the scores and x for the deviation of each score from the mean.

Standard Deviation is regarded as a most stable and reliable measure of variability as it employs mean for its computation. It is often called as root-mean square deviation and is denoted by the Greek letter sigma.

## (a) Computation of Standard Deviation (SD) from Ungrouped Data

SD can be, computed from the ungrouped scores by the following formula $\sigma=\sqrt{\frac{\Sigma x^{2}}{N}}$.
Below we illustrate the use of this formula by taking a particular example.

Example-Calculate SD for the following set of test scores:
52, 50, 56, 68, 65, 57, 70
Solution-Mean of the given scores $=480 / 8=60$

| Scores $X$ | Deviation from the mean <br> $(X-M)$ or $x$ | Squares of Deviations |
| :---: | :---: | :---: |
| 52 | -8 | 64 |
| 50 | -10 | 100 |
| 56 | -4 | 16 |
| 68 | 8 | 64 |
| 65 | 5 | 25 |
| 62 | 2 | 4 |
| 57 | -3 | 9 |
| 70 | 10 | 100 |
|  |  | $S x^{2}=382$ |

Now $\sigma=\sqrt{\frac{\Sigma x^{2}}{N}}=\sqrt{\frac{382}{8}}=\sqrt{47.75}=6.91$

## (b) Computation of SO from the Grouped Data"

SD in case of grouped data can be computed by formula $\sigma=\sqrt{\frac{\Sigma x^{2}}{N}}$.
The use of the formula can be understood through the solution of the following example.
Problem - Compute SD for the frequency distribution given below on the extreme left. The mean of this distribution is 115 .

| I.Q. Scores | $f$ | $X$ | $M$ | $X$ | $x^{2}$ | $f x^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $127-129$ | 1 | 128 | 115 | 13 | 169 | 169 |
| $124-128$ | 2 | 125 | 115 | 10 | 100 | 200 |
| $121-123$ | 3 | 122 | 115 | 7 | 49 | 147 |
| $118-120$ | 1 | 119 | 115 | 4 | 16 | 16 |
| $115-117$ | 6 | 116 | 115 | 1 | 1 | 6 |
| $112-114$ | 4 | 113 | 115 | -2 | 4 | 16 |
| $109-111$ | 3 | 110 | 115 | -5 | 25 | 75 |
| $106-108$ | 2 | 107 | 115 | -8 | 64 | 128 |
| $103-105$ | 1 | 114 | 115 | -11 | 121 | 121 |
| $100-102$ | J | 101 | 115 | -14 | 196 | 196 |
| $N=24$ |  |  |  |  |  | Sfx $x^{2}=1074$ |

Now $\sigma=\sqrt{\frac{\Sigma x^{2}}{N}}=\sqrt{\frac{1074}{24}}=\sqrt{44.75}=6.69$

In the above computation work we have made use of M , the mean of the distribution. If not given in the example, it can be computed in the following way:

Calculation of Mean-Let Assumed Mean be 116.

| Scores | $f$ | $x$ (Mid-value) | $x^{\prime}=\frac{X-A}{i}$ | $f x^{\prime}$ |
| :---: | :---: | :---: | :---: | :---: |
| $127-129$ | 1 | 128 | 4 | 4 |
| $124-126$ | 2 | 125 | 3 | 6 |
| $121-123$ | 3 | 122 | 2 | 6 |
| $118-120$ | 1 | 119 | 1 | 1 |
| $115-117$ | 6 | 116 | 0 | 0 |
| $112-114$ | 4 | 113 | -1 | -4 |
| $109-111$ | 3 | 110 | -2 | -6 |
| $106-108$ | 2 | 107 | -3 | -6 |
| $103-105$ | 1 | 104 | -4 | -4 |
| $100-102$ | 1 | 101 | -5 | -5 |
|  | $N=24$ |  |  | $f x^{\prime}=-8$ |

Formula: $\quad M=A+\frac{\Sigma x^{\prime}}{N} \times i=116-\frac{8}{24} \times 3=116-1=115$

$$
\text { Mean }=115
$$

## (c) Computation of SD from Grouped Data by Short-cut Method

SD from grouped data can also be computed by the following formula:

$$
\sigma=\sqrt{\frac{\Sigma f x^{\prime 2}}{N}-\left(\frac{\Sigma f x^{\prime 2}}{N}\right)}
$$

Where the notation have the same meaning as desired earlier. The use of this formula can be explained by solving the example given under the case (b).

| I.Q. Scores | $f$ | $X$ | $x^{\prime}=\frac{X-A}{i}$ | $f x^{\prime}$ | $f x^{\prime 2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $127-129$ | 1 | 128 | 4 | 4 | 16 |
| $124-126$ | 2 | 125 | 3 | 6 | 18 |
| $121-123$ | 3 | 122 | 2 | 6 | 12 |
| $11-120$ | 1 | 119 | 1 | 1 | 1 |
| $115-117$ | 6 | 116 | 0 | 0 | 0 |
| $112-114$ | 4 | 113 | -1 | -4 | 4 |
| $109-111$ | 3 | 110 | -2 | -6 | 12 |
| $106-108$ | 2 | 107 | -3 | -6 | 18 |
| $103-105$ | 1 | 104 | -4 | -4 | 16 |
| $100-102$ | 1 | 101 | -5 | -5 | 25 |
|  | $N=24$ |  |  | $S f x^{\prime} 1=-8$ | $S f x^{\prime 2}=122$ |

$$
\begin{aligned}
\text { Formula- } \sigma & =i \sqrt{\frac{\Sigma f x^{\prime 2}}{N}-\left(\frac{\Sigma f x^{\prime}}{N}\right)^{2}}=3 \sqrt{\frac{122}{4}-\left(\frac{-8}{24}\right)^{2}} \\
& =3 \sqrt{\frac{122}{4-24 \times 24}}=\frac{3}{24} \sqrt{122 \times 24-34}=\frac{1}{8} \sqrt{2864}=\frac{53.51}{8}
\end{aligned}
$$

Standard Deviation $=6.69$

## EXERCISE

## Essay Type Questions

1. What do you understand by dispersion or variability of the scores in a given series? Discuss in brief the different measures of variability.
2. Calculate average Deviation from the following Data:
(a) Scores 30, 35, 36, 42, 46, 38, 34, 35
(b)

| Scores | $80-84$ | $85-89$ | $90-94$ | $95-991$ | $00-104$ | $105-109$ | $110-114$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | 4 | 4 | 3 | 0 | 3 | 3 | 1 |

$\mathrm{N}=18$
Ans. (a) 3.9, (b) 2.04
3. Compute Standard Deviation for each of the four frequency distributions a, b, c, d, given in problem 3 of the chapter 18.
Ans. (a) 4.99, (b) 7.73, (c) 7.7, (d) 13.4
4. What are the measures of variability or dispersion? Discuss the need of their computation.
5. What is average deviation? Discuss the procedure of its computing from the ungrouped as well as the grouped data with the help of an hypothetical examples.
6. What is Standard Deviation? Discuss the procedure of its computation from the ungrouped as well as grouped data with the help of some hypothetical data. Calculate Mean and Standard Deviation for the following data.
7. Calculate mean and standard deviation for the following data:

| (a) Scores | $f$ | (b) Score | $f$ |
| :---: | :---: | :---: | :---: |
| $60-69$ | 4 | $40-44$ | 1 |
| $50-59$ | 4 | $35-39$ | 2 |
| $40-49$ | 4 | $30-34$ | 3 |
| $30-39$ | 10 | $25-29$ | 4 |
| $20-29$ | 8 | $15-19$ | 15 |
| $10-19$ | 5 | $10-14$ | 5 |
| $0-9$ | 5 | $5-9$ | 8 |

Ans. (a) $M=36.75, S D=17.815$, (b) $M=18.98, S D=8.52$
8. Calculate Mean and Standard Deviation for the following data:

| (a) Scores | $f$ | (b) Scores | $f$ |
| :--- | :--- | :---: | :---: |
| $45-49$ | 2 | $55-59$ | 1 |
| $40-44$ | 3 | $50-54$ | 1 |
| $35-39$ | 5 | $45-49$ | 3 |
| $30-34$ | 9 | $40-4$ | 4 |
| $25-29$ | 6 | $35-39$ | 6 |
| $20-24$ | 4 | $30-34$ | 7 |
| $15-19$ | 1 | $25-29$ | 12 |
|  |  | $20-24$ | 6 |
|  |  | $15-19$ | 8 |
|  |  | $10-14$ | 2 |

Ans. (a) $M=32, \quad$ (b) $M=29.6$
$S D=7.415, S D=10.45$
9. Compute Mean and Standard Deviation for the following data:

| (a) Scores | $f$ | (b) Scores | $f$ | (c) Scores | $f$ |
| :---: | :--- | :---: | :---: | :---: | :---: |
| $45-49$ | 2 | $90-93$ | 1 | $85-87$ | 1 |
| $40-44$ | 3 | $86-89$ | 3. | $82-84$ | 3 |
| $35-39$ | 2 | $82-85$ | 8 | $79-81$ | 2 |
| $30-34$ | 6 | $78-81$ | 5 | $76-78$ | .3 |
| $25-29$ | 8 | $74-77$ | 7 | $7-75$ | 3 |
| $20-24$ | 8 | $70-73$ | 6 | $70-72$ | 2 |
| $15-19$ | 7 | $66-69$ | 4 | $67-69$ | 2 |
| $10-14$ | 5 | $62-65$ | 2 | $64-66$ | 3 |
| $5-9$ | 9 |  |  | $61-63$ | 1 |

Ans. (a) $M=22.4, \quad$ (b) $M=77.06, \quad$ (c) $M=14.15$
$S D=113, S D=7.13, S D=6.936$
10. Compute Quartile Deviation from the following data:

| (a) Scores | $f$ | (b) Scores | $f$ |
| :---: | :---: | :---: | :---: |
| $45-49$ | 2 | $135-144$ | 1 |
| $40-44$ | 3 | $125-134$ | 2 |
| $35-39$ | 2 | $115-124$ | 8 |
| $30-34$ | 17 | $105-114$ | 22 |
| $25-29$ | 30 | $95-104$ | 33 |
| $20-24$ | 25 | $85-94$ | 22 |
| $15-19$ | 15 | $75-84$ | 9 |
| $10-14$ | 3 | $65-74$ | 2 |
| $5-9$ | 2 | $55-64$ | 1 |
| $0-4$ | 1 |  |  |

Ans. (a) $4.5 \quad$ (b) 8.85
11. Compute Standard Deviation for the data presented in the problems 10 and 11 of the chapter 18. Ans. Problem 10, $S D=12.62$, Problem 11, $S D=7.017$

## Short Answer Type Questions (Answer in 100-120 words)

1. What is a range as a measure of variability? Illustrate its computation through an example.
2. What are the measures of variability or dispersion? Name the various measures of variability.
3. What is quartile deviation? How is it computed? Explain with the help of an example.
4. Tell about the average deviation as a measure of variability.
5. Tell about the standard deviation as a measure of variability.
6. Write short note on the need of computing measure of variability or dispersion.
