A1
APPENDIX

Summation Notation

A1.1 SUMMATION AND ITS PROPERTIES

The addition of numbers is basic to our study of statistics. To avoid a detailed and repeated writing of this operation, the symbol Σ (the Greek capital letter *sigma*) is used as mathematical shorthand for the operation of addition.

Summation Notation Σ

The notation $\sum_{i=1}^{n} x_i$ represents the sum of n numbers $x_1, x_2, \ldots, x_n$ and is read as **the sum of all x_i with i ranging from 1 to n.**

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n$$

The term following the sign Σ indicates the quantities that are being summed, and the notations on the bottom and the top of the Σ specify the range of the terms being added. For instance,

$$\sum_{i=1}^{3} x_i = x_1 + x_2 + x_3$$

$$\sum_{i=1}^{4} (x_i - 3) = (x_1 - 3) + (x_2 - 3) + (x_3 - 3) + (x_4 - 3)$$

Example Suppose that the four measurements in a data set are given as $x_1 = 2$, $x_2 = 5$, $x_3 = 3$, $x_4 = 4$. Compute the numerical values of

(a) $\sum_{i=1}^{4} x_i$ (b) $\sum_{i=1}^{4} 6$ (c) $\sum_{i=1}^{4} 2x_i$

(d) $\sum_{i=1}^{4} (x_i - 3)$ (e) $\sum_{i=1}^{4} x_i^2$ (f) $\sum_{i=1}^{4} (x_i - 3)^2$

SOLUTION

(a) $\sum_{i=1}^{4} x_i = x_1 + x_2 + x_3 + x_4 = 2 + 5 + 3 + 4 = 14$

(b) $\sum_{i=1}^{4} 6 = 6 + 6 + 6 + 6 = 4(6) = 24$

(c) $$\sum_{i=1}^{4} 2x_i = 2x_1 + 2x_2 + 2x_3 + 2x_4 = 2\left(\sum_{i=1}^{4} x_i\right)$$
$$= 2 \times 14 = 28$$

(d) $$\sum_{i=1}^{4} (x_i - 3) = (x_1 - 3) + (x_2 - 3)$$
$$+ (x_3 - 3) + (x_4 - 3)$$
$$= \sum_{i=1}^{4} x_i - 4(3) = 14 - 12 = 2$$

(e) $\sum_{i=1}^{4} x_i^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 = 2^2 + 5^2 + 3^2 + 4^2 = 54$

(f) $$\sum_{i=1}^{4} (x_i - 3)^2 = (x_1 - 3)^2 + (x_2 - 3)^2$$
$$+ (x_3 - 3)^2 + (x_4 - 3)^2$$
$$= (2 - 3)^2 + (5 - 3)^2$$
$$+ (3 - 3)^2 + (4 - 3)^2$$
$$= 1 + 4 + 0 + 1 = 6$$

Alternatively, noting that $(x_i - 3)^2 = x_i^2 - 6x_i + 9$, we can write

$$\sum_{i=1}^{4} (x_i - 3)^2 = \sum_{i=1}^{4} (x_i^2 - 6x_i + 9)$$
$$= (x_1^2 - 6x_1 + 9) + (x_2^2 - 6x_2 + 9)$$
$$+ (x_3^2 - 6x_3 + 9) + (x_4^2 - 6x_4 + 9)$$
$$= \sum_{i=1}^{4} x_i^2 - 6\left(\sum_{i=1}^{4} x_i\right) + 4(9)$$
$$= 54 - 6(14) + 36 = 6$$

A few basic properties of the summation operation are apparent from the numerical demonstration in the example.

Some Basic Properties of Summation

If a and b are fixed numbers,

$$\sum_{i=1}^{n} bx_i = b \sum_{i=1}^{n} x_i$$

$$\sum_{i=1}^{n} (bx_i + a) = b \sum_{i=1}^{n} x_i + na$$

$$\sum_{i=1}^{n} (x_i - a)^2 = \sum_{i=1}^{n} x_i^2 - 2a \sum_{i=1}^{n} x_i + na^2$$

Exercises

1. Demonstrate your familiarity with the summation notation by evaluating the following expressions when $x_1 = 4, x_2 = -2, x_3 = 1$.

(a) $\sum_{i=1}^{3} x_i$ (b) $\sum_{i=1}^{3} 7$ (c) $\sum_{i=1}^{3} 5x_i$

(d) $\sum_{i=1}^{3} (x_i - 2)$ (e) $\sum_{i=1}^{3} (x_i - 3)$ (f) $\sum_{i=1}^{3} (x_i - 2)^2$

(g) $\sum_{i=1}^{3} x_i^2$ (h) $\sum_{i=1}^{3} (x_i - 3)^2$ (i) $\sum_{i=1}^{3} (x_i^2 - 6x_i + 9)$

2. Five measurements in a data set are $x_1 = 4, x_2 = 3, x_3 = 6, x_4 = 5, x_5 = 7$. Determine

(a) $\sum_{i=1}^{5} x_i$ (b) $\sum_{i=2}^{3} x_i$ (c) $\sum_{i=1}^{5} 2$

(d) $\sum_{i=1}^{5} (x_i - 6)$ (e) $\sum_{i=1}^{5} (x_i - 6)^2$ (f) $\sum_{i=1}^{4} (x_i - 5)^2$

A1.2 SOME BASIC USES OF Σ IN STATISTICS

Let us use the summation notation and its properties to verify some computational facts about the sample mean and variance.

$\Sigma(x_i - \bar{x}) = 0$

The total of the deviations about the sample mean is always zero. Since $\bar{x} = (x_1 + x_2 + \cdots + x_n)/n$, we can write

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n = n\bar{x}$$

Consequently, whatever the observations,

$$\begin{aligned}\sum_{i=1}^{n}(x_i - \bar{x}) &= (x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + (x_n - \bar{x}) \\ &= x_1 + x_2 + \cdots + x_n - n\bar{x} \\ &= n\bar{x} - n\bar{x} = 0\end{aligned}$$

We could also verify this directly with the second property for summation in A1.1, when $b = 1$ and $a = -\bar{x}$.

ALTERNATIVE FORMULA FOR s^2

By the quadratic rule of algebra,

$$(x_i - \bar{x})^2 = x_i^2 - 2\bar{x}x_i + \bar{x}^2$$

Therefore,

$$\begin{aligned}\Sigma(x_i - \bar{x})^2 &= \Sigma x_i^2 - \Sigma 2\bar{x}x_i + \Sigma \bar{x}^2 \\ &= \Sigma x_i^2 - 2\bar{x}\Sigma x_i + n\bar{x}^2\end{aligned}$$

Using $(\Sigma x_i)/n$ in place of $\bar{x}$, we get

$$\begin{aligned}\Sigma(x_i - \bar{x})^2 &= \Sigma x_i^2 - \frac{2(\Sigma x_i)^2}{n} + \frac{n(\Sigma x_i)^2}{n^2} \\ &= \Sigma x_i^2 - \frac{2(\Sigma x_i)^2}{n} + \frac{(\Sigma x_i)^2}{n} \\ &= \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\end{aligned}$$

We could also verify this directly from the third property for summation, in A1.1, with $a = \bar{x}$.

This result establishes that

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n - 1} = \frac{\Sigma x_i^2 - (\Sigma x_i)^2/n}{n - 1}$$

so the two forms of s^2 are equivalent.

SAMPLE CORRELATION COEFFICIENT

The sample correlation coefficient and slope of the fitted regression line contain a term

$$S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

which is a sum of the products of the deviations. To obtain the alternative form, first note that

$$(x_i - \bar{x})(y_i - \bar{y}) = x_i y_i - x_i \bar{y} - \bar{x} y_i + \bar{x}\bar{y}$$

We treat $x_i y_i$ as a single number, with index i, and conclude that

$$\begin{aligned} \Sigma (x_i - \bar{x})(y_i - \bar{y}) &= \Sigma x_i y_i - \Sigma x_i \bar{y} - \Sigma \bar{x} y_i + \Sigma \bar{x}\bar{y} \\ &= \Sigma x_i y_i - \bar{y} \Sigma x_i - \bar{x} \Sigma y_i + n \bar{x}\bar{y} \end{aligned}$$

Since $\bar{x} = (\Sigma x_i)/n$ and $\bar{y} = (\Sigma y_i)/n$,

$$\begin{aligned} \Sigma (x_i - \bar{x})(y_i - \bar{y}) &= \Sigma x_i y_i - \frac{(\Sigma y_i)}{n} \Sigma x_i - \frac{(\Sigma x_i)}{n} \Sigma y_i \\ &\quad + \frac{n(\Sigma x_i)}{n} \frac{(\Sigma y_i)}{n} \\ &= \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n} \end{aligned}$$

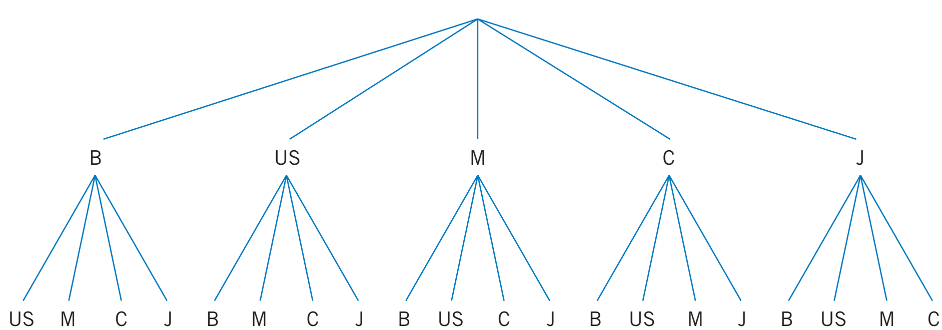
Consequently, either $\Sigma (x_i - \bar{x})(y_i - \bar{y})$ or $\Sigma x_i y_i - (\Sigma x_i)(\Sigma y_i)/n$ can be used for the calculation of S_{xy} with similar choices for S_{xx} and S_{yy}.

A2
APPENDIX

Rules for Counting

Some basic rules for counting can help us calculate probabilities. We begin with an example. Brazil (B), United States (U.S.), Mexico (M), Canada (C), and Japan (J) are in the final round of a volleyball tournament. How many ways can a winner and second place finisher be selected?

To systematically address this problem, we create a **tree diagram** where the possibilities for first place are represented by the initial 5 branches. For each choice of a first place finisher, there are four choices for second place. These are represented by the sets of 4 branches in the second stage. For instance, the left-most branch of the tree diagram depicts the case where Brazil is first and the United States second and so on.



There are $5 \times 4 = 20$ terminal points or ways for these teams to finish first and second. Generalizing the tree diagram to k choices followed by m choices, we obtain the product rule for counting.

Product Rule

An experiment consists of two parts where the first part has k distinct possible outcomes and, for each outcome of the first part, there are m distinct possible outcomes for the second part. Then, there are $k \times m$ distinct possible outcomes to the experiment.

The product rule is readily extended to experiments with more than two parts. Suppose we are interested in the winner, second, and third place finishers in the volleyball tournament. As above, by the product rule, there are $5 \times 4 = 20$ distinct possibilities for selecting a first and second place volleyball team. Again, by the product rule, each of these 20 outcomes must be paired with one of the 3 outcomes for a third place finisher so there are $20 \times 3 = 5 \times 4 \times 3 = 60$ possibilities for the first, second, and third place finishers.

The term **permutation** means an ordering or arrangement of objects. The number of different orderings or arrangements that can be formed with r objects selected from a group of n distinct objects is called the **number of permutations of r out of n** and is denoted by P_r^n.

In the volleyball example, $P_2^5 = 5 \times 4 = 20$ for the first two places and $P_3^5 = 5 \times 4 \times 3 = 60$ for the first three places. When r objects are selected, the first has n possibilities and the last $n - r + 1$ possibilities.

Rule of Permutations or Arrangements

The number of different arrangements of r objects selected from a group of n distinct objects is denoted by P_r^n,

$$P_r^n = n(n-1)\cdots(n-r+1)$$

If all n distinct objects are arranged in order, the number of possible arrangements is $P_n^n = n(n-1) \times \cdots \times 2 \times 1$, which is the product n times $n - 1$ on down through 2×1. This special product, of decreasing integers starting with n, is called **n factorial** and is denoted by $n!$.

When order is not important, we speak about collections of objects. The number of different collections of r objects selected from a group of n distinct objects is called the **number of combinations of r out of n.**

Rule of Combinations

The number of **combinations,** or different collections, of r objects selected from a group of n distinct objects is denoted by $\binom{n}{r}$,

$$\binom{n}{r} = \frac{P_r^n}{r!} = \frac{n(n-1)\cdots(n-r+1)}{r!} = \frac{n!}{r!(n-r)!}$$

The general formula for combinations is obtained by the following correspondence and an application of the product rule.

$$\begin{bmatrix}\text{Arrange } r \text{ objects}\\ \text{selected from } n\end{bmatrix} \quad \begin{matrix}\text{is the}\\ \text{same as}\end{matrix} \quad \begin{bmatrix}\text{first select } r\\ \text{objects from } n\end{bmatrix} \quad \begin{matrix}\text{and}\\ \text{then}\end{matrix} \quad \begin{bmatrix}\text{arrange the } r\\ \text{selected objects}\end{bmatrix}$$

$$P_r^n \qquad = \qquad \binom{n}{r} \qquad \times \qquad r!$$

According to the last form, $\binom{n}{r}$ is symmetric in r and $n - r$. Selecting r objects to be in the collection is the same as choosing which $n - r$ to exclude.

If 2 of the 5 volleyball teams must be selected to play another match, so the order of selection is not important, there are 10 ways to select 2 to play or the 3 teams which will not play.

$$\binom{5}{2} = \frac{5 \times 4}{2 \times 1} = 10 = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times (3 \times 2 \times 1)} = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} \text{ which equals } \binom{5}{3}$$

A3
APPENDIX

Expectation and Standard Deviation — Properties

The expected value of a discrete random variable is a summation of the products (value × probability). The key properties of expectations are then all inherited from the properties of summation. In this appendix, we indicate this development for some of the most useful properties of expectation and variance. The interested reader can consult Bhattacharyya and Johnson[1] for more details.

A3.1 EXPECTED VALUE AND STANDARD DEVIATION OF $cX + b$

The units of the random variable X may be changed by multiplying by a constant, for example,

$$X = \text{height in feet}, \qquad 12X = \text{height in inches}$$

or by adding a constant, for example,

$$\begin{aligned} X &= \text{temperature } (^\circ\text{F}) \\ X - 32 &= \text{degrees above freezing } (^\circ\text{F}) \end{aligned}$$

[1]G. K. Bhattacharyya and Richard A. Johnson, *Statistical Concepts and Methods*, John Wiley & Sons, New York, 1978.

The mean and standard deviation of the new random variables are related to $\mu = E(X)$ and $\sigma = \text{sd}(X)$.

If X is multiplied by a constant c,

Random Variable	Mean	sd
cX	$c\mu$	$\lvert c\rvert\sigma$

If a constant b is added to X,

Random Variable	Mean	sd
$X + b$	$\mu + b$	σ (unchanged)

Notice that adding a constant to a random variable leaves the standard deviation unchanged.

Example Let X have mean $= 3 = \mu$ and standard deviation $= 5 = \sigma$. Find the mean and sd of (a) $X + 4$, (b) $2X$, (c) $-X$, and (d) $\frac{1}{5}(X - 3)$.

SOLUTION By the foregoing properties,

Random Variable	Mean	sd
$X + 4$	$3 + 4 = 7$	5
$2X$	$2(3) = 6$	$2(5) = 10$
$-X = (-1)X$	$(-1)3 = -3$	$\lvert -1\rvert 5 = 5$

Finally, $(X - 3)$ has mean $3 - 3 = 0$ and sd $= 5$, so $\frac{1}{5}(X - 3)$ has mean $= \frac{1}{5}(0) = 0$ and sd $= \frac{1}{5}(5) = 1$.

Any random variable having $E(X) = \mu$ and $\text{Var}(X) = \sigma^2$ can be converted to a

Standardized variable

$$Z = \frac{X - \mu}{\sigma}$$

The standardized variable Z has mean $= 0$ and variance $= 1$. This was checked for $Z = \dfrac{X - 3}{5}$ in the example above.

*VERIFICATION OF THE MEAN AND sd EXPRESSIONS FOR $cX + b$

Consider the random variable $cX + b$, which includes the two cases above. The choice $b = 0$ gives cX and the choice $c = 1$ gives $X + b$. We restrict our verification to discrete random variables, where probability $f(x_i)$ is attached to x_i. Because $cX + b$ takes value $cx_i + b$ with probability $f(x_i)$,

$$(\text{value} \times \text{probability}) = (cx_i + b)f(x_i) = cx_i f(x_i) + bf(x_i)$$

and

$$\begin{aligned}\text{mean} &= \Sigma\,(\text{value} \times \text{probability})\\ &= \Sigma\, cx_i f(x_i) + \Sigma\, bf(x_i)\\ &= c\,\Sigma\, x_i f(x_i) + b\,\Sigma\, f(x_i) = c\mu + b \cdot 1\end{aligned}$$

Next,

$$\begin{aligned}\text{deviation} &= (\text{random variable}) - (\text{mean})\\ &= (cX + b) - (c\mu + b) = cX - c\mu = c(X - \mu)\end{aligned}$$

so

$$\begin{aligned}\text{variance} &= \Sigma\,(\text{deviation})^2 \times \text{probability}\\ &= \Sigma\, c^2(x_i - \mu)^2 f(x_i)\\ &= c^2\,\Sigma\,(x_i - \mu)^2 f(x_i) = c^2\sigma^2\end{aligned}$$

Taking the positive square root yields $\text{sd}\,(cX + b) = |c|\sigma$.

Finally, taking $c = 1/\sigma$ and $b = -\mu/\sigma$, we obtain

$$cX + b = \frac{1}{\sigma}X - \frac{\mu}{\sigma} = \frac{X - \mu}{\sigma} = Z$$

so the standardized variable Z has

$$\text{mean} \qquad c\mu + b = \frac{1}{\sigma}\mu - \frac{\mu}{\sigma} = 0$$

$$\text{sd} \qquad c\sigma = \frac{1}{\sigma}\sigma = 1$$

A3.2 ALTERNATIVE FORMULA FOR σ^2

An alternative formula for σ^2 often simplifies the numerical calculations. By definition,

$$\sigma^2 = \Sigma\,(\text{deviation})^2(\text{probability}) = \Sigma\,(x_i - \mu)^2 f(x_i)$$

but σ^2 can also be expressed as

$$\sigma^2 = \Sigma x_i^2 f(x_i) - \mu^2$$

To deduce the second form, we first expand the square of the deviation:

$$(x_i - \mu)^2 = x_i^2 - 2\mu x_i + \mu^2$$

Then, multiply each term on the right-hand side by $f(x_i)$ and sum:

First term		$= \Sigma x_i^2 f(x_i)$	
Second term	$-2\mu \Sigma x_i f(x_i)$	$= -2\mu^2$	since $\Sigma x_i f(x_i) = \mu$
Third term	$+ \mu^2 \Sigma f(x_i)$	$= \mu^2$	since $\Sigma f(x_i) = 1$
Result:	σ^2	$= \Sigma x_i^2 f(x_i) - \mu^2$	

Example Calculate σ^2 by both formulas.

SOLUTION

Calculation $\Sigma(x - \mu)^2 f(x)$				
x	$f(x)$	$xf(x)$	$(x - 2)^2$	$(x - 2)^2 f(x)$
1	.4	.4	1	.4
2	.3	.6	0	0
3	.2	.6	1	.2
4	.1	.4	4	.4
		$2.0 = \mu$		$1.0 = \sigma^2$

Calculation $\Sigma x^2 f(x) - \mu^2$			
x	$f(x)$	$xf(x)$	$x^2 f(x)$
1	.4	.4	.4
2	.3	.6	1.2
3	.2	.6	1.8
4	.1	.4	1.6
		$2.0 = \mu$	5.0
	$\sigma^2 = 5 - 2^2 = 1$		

A3.3 PROPERTIES OF EXPECTED VALUE FOR TWO RANDOM VARIABLES

The concept of expectation extends to two or more variables. With two random variables:

1. $E(X + Y) = E(X) + E(Y)$ (additivity or sum law of expectation).
2. If X and Y are independent, then $E(XY) = E(X)E(Y)$.

Remark: Property 1 holds quite generally, independence is not required.

*DEMONSTRATION

We verify both (1) and (2) assuming independence. Independence implies that $P[X = x, Y = y] = P[X = x]\,P[Y = y]$ for all outcomes (x, y). That is, the distribution of probability over the pairs of possible values (x, y) is specified by the product $f_X(x) f_Y(y) = P[X = x, Y = y]$.

The expected value, $E(X + Y)$, is obtained by multiplying each possible value $(x + y)$ by the probability $f_X(x) f_Y(y)$ and summing

$$\begin{aligned} E(X + Y) &= \sum_x \sum_y (x + y) f_X(x) f_Y(y) \\ &= \sum_x \sum_y x f_X(x) f_Y(y) + \sum_x \sum_y y f_X(x) f_Y(y) \\ &= \left(\sum_x x f_X(x)\right) \underbrace{\left(\sum_y f_Y(y)\right)}_{=1} + \underbrace{\left(\sum_x f_X(x)\right)}_{=1} \left(\sum_y y f_Y(y)\right) \\ &= E(X) + E(Y) \end{aligned}$$

Next,

$$\begin{aligned} E(XY) &= \sum_x \sum_y xy f_X(x) f_Y(y) \\ &= \sum_x x f_X(x) \left(\sum_y y f_Y(y)\right) = E(X) E(Y) \qquad \square \end{aligned}$$

Under the proviso that the random variables are **independent,** variances also add.

3. If X and Y are independent,

$$\operatorname{Var}(X + Y) = \operatorname{Var}(X) + \operatorname{Var}(Y)$$

*DEMONSTRATION

We set $\mu_1 = E(X)$ and $\mu_2 = E(Y)$, so by property 1,

$$E(X + Y) = \mu_1 + \mu_2$$

Then, since variance is the expected value of $(\text{variable} - \text{mean})^2$,

$$\begin{aligned} \operatorname{Var}(X + Y) &= E(X + Y - \mu_1 - \mu_2)^2 \\ &= E[(X - \mu_1)^2 + (Y - \mu_2)^2 + 2(X - \mu_1)(Y - \mu_2)] \\ &= E(X - \mu_1)^2 + E(Y - \mu_2)^2 + 2E(X - \mu_1)(Y - \mu_2) \\ &\qquad \text{(by the sum law of expectation, property 1)} \\ &= \operatorname{Var}(X) + \operatorname{Var}(Y) \end{aligned}$$

This last step follows since

$$\begin{aligned} E(X - \mu_1)(Y - \mu_2) &= E(XY - \mu_1 Y - X\mu_2 + \mu_1\mu_2) \\ &= E(XY) - \mu_1 E(Y) - \mu_2 E(X) + \mu_1\mu_2 \\ &= E(XY) - \mu_1\mu_2 \\ &= E(X)E(Y) - \mu_1\mu_2 = \mu_1\mu_2 - \mu_1\mu_2 = 0 \quad \text{by property 2} \qquad \square \end{aligned}$$

A4
APPENDIX

The Expected Value and Standard Deviation of $\overline{X}$

Some basic properties of the sampling distribution of $\overline{X}$ can be expressed in terms of the population mean and variance when the observations form a random sample. Let

$$\begin{aligned} \mu &= \text{population mean} \\ \sigma^2 &= \text{population variance} \end{aligned}$$

In a random sample, the random variables $X_1, \ldots, X_n$ are independent, and each has the distribution of the population. Consequently, each observation has mean μ and variance σ^2, or

$$\begin{aligned} E(X_1) &= \cdots = E(X_n) = \mu \\ \text{Var}(X_1) &= \cdots = \text{Var}(X_n) = \sigma^2 \end{aligned}$$

Next,

$$\overline{X} = \frac{1}{n}(X_1 + \cdots + X_n)$$

and n is a constant. Using the additivity properties of expectation and variance discussed in Appendix A3, we obtain

$$
\begin{aligned}
E(\overline{X}) &= \frac{1}{n} E(X_1 + \cdots + X_n) \\
&= \frac{1}{n} [E(X_1) + \cdots + E(X_n)] \quad \text{(mean of sum = sum of means)} \\
&= \frac{1}{n} [\mu + \cdots + \mu] = \frac{n\mu}{n} = \mu \\
\mathrm{Var}(\overline{X}) &= \frac{1}{n^2} \mathrm{Var}(X_1 + \cdots + X_n) \\
&= \frac{1}{n^2} [\mathrm{Var}(X_1) + \cdots + \mathrm{Var}(X_n)] \quad \text{(variances add due to independence)} \\
&= \frac{1}{n^2} [\sigma^2 + \cdots + \sigma^2] = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}
\end{aligned}
$$

Furthermore, taking the square root yields

$$
\mathrm{sd}(\overline{X}) = \sqrt{\mathrm{Var}(\overline{X})} = \frac{\sigma}{\sqrt{n}}
$$

B

APPENDIX

Tables

TABLE 1 Random Digits

Row										
1	0695	7741	8254	4297	0000	5277	6563	9265	1023	5925
2	0437	5434	8503	3928	6979	9393	8936	9088	5744	4790
3	6242	2998	0205	5469	3365	7950	7256	3716	8385	0253
4	7090	4074	1257	7175	3310	0712	4748	4226	0604	3804
5	0683	6999	4828	7888	0087	9288	7855	2678	3315	6718
6	7013	4300	3768	2572	6473	2411	6285	0069	5422	6175
7	8808	2786	5369	9571	3412	2465	6419	3990	0294	0896
8	9876	3602	5812	0124	1997	6445	3176	2682	1259	1728
9	1873	1065	8976	1295	9434	3178	0602	0732	6616	7972
10	2581	3075	4622	2974	7069	5605	0420	2949	4387	7679
11	3785	6401	0540	5077	7132	4135	4646	3834	6753	1593
12	8626	4017	1544	4202	8986	1432	2810	2418	8052	2710
13	6253	0726	9483	6753	4732	2284	0421	3010	7885	8436
14	0113	4546	2212	9829	2351	1370	2707	3329	6574	7002
15	4646	6474	9983	8738	1603	8671	0489	9588	3309	5860
16	7873	7343	4432	2866	7973	3765	2888	5154	2250	4339
17	3756	9204	2590	6577	2409	8234	8656	2336	7948	7478
18	2673	7115	5526	0747	3952	6804	3671	7486	3024	9858
19	0187	7045	2711	0349	7734	4396	0988	4887	7682	8990
20	7976	3862	8323	5997	6904	4977	1056	6638	6398	4552

TABLE 1 *(Continued)*

Row										
21	5605	1819	8926	9557	2905	0802	7749	0845	1710	4125
22	2225	5556	2545	7480	8804	4161	0084	0787	2561	5113
23	2549	4166	1609	7570	4223	0032	4236	0169	4673	8034
24	6113	1312	5777	7058	2413	3932	5144	5998	7183	5210
25	2028	2537	9819	9215	9327	6640	5986	7935	2750	2981
26	7818	3655	5771	4026	5757	3171	6435	2990	1860	1796
27	9629	3383	1931	2631	5903	9372	1307	4061	5443	8663
28	6657	5967	3277	7141	3628	2588	9320	1972	7683	7544
29	4344	7388	2978	3945	0471	4882	1619	0093	2282	7024
30	3145	8720	2131	1614	1575	5239	0766	0404	4873	7986
31	1848	4094	9168	0903	6451	2823	7566	6644	1157	8889
32	0915	5578	0822	5887	5354	3632	4617	6016	8989	9482
33	1430	4755	7551	9019	8233	9625	6361	2589	2496	7268
34	3473	7966	7249	0555	6307	9524	4888	4939	1641	1573
35	3312	0773	6296	1348	5483	5824	3353	4587	1019	9677
36	6255	4204	5890	9273	0634	9992	3834	2283	1202	4849
37	0562	2546	8559	0480	9379	9282	8257	3054	4272	9311
38	1957	6783	4105	8976	8035	0883	8971	0017	6476	2895
39	7333	1083	0398	8841	0017	4135	4043	8157	4672	2424
40	4601	8908	1781	4287	2681	6223	0814	4477	3798	4437
41	2628	2233	0708	0900	1698	2818	3931	6930	9273	6749
42	5318	8865	6057	8422	6992	9697	0508	3370	5522	9250
43	6335	0852	8657	8374	0311	6012	9477	0112	8976	3312
44	0301	8333	0327	0467	6186	1770	4099	9588	5382	8958
45	1719	9775	1566	7020	4535	2850	0207	4792	6405	1472
46	8907	8226	4249	6340	9062	3572	7655	6707	3685	1282
47	6129	5927	3731	1125	0081	1241	2772	6458	9157	4543
48	7376	3150	8985	8318	8003	6106	4952	8492	2804	3867
49	9093	3407	4127	9258	3687	5631	5102	1546	2659	0831
50	1133	3086	9380	5431	8647	0910	6948	2257	0946	1245
51	4567	0910	8495	2410	1088	7067	8505	9083	4339	2440
52	6141	8380	2302	4608	7209	5738	9765	3435	9657	6061
53	1514	8309	8743	3096	0682	7902	8204	7508	8330	1681
54	7277	1634	7866	9883	0916	6363	5391	6184	8040	3135
55	4568	4758	0166	1509	2105	0976	0269	0278	7443	2431
56	9200	7599	7754	4534	4532	3102	6831	2387	4147	2455
57	3971	8149	4431	2345	6436	0627	0410	1348	6599	1296
58	2672	9661	2359	8477	3425	8150	6918	8883	1518	4708
59	1524	3268	3798	3360	2255	0371	7610	9114	9466	0901
60	6817	9007	5959	0767	1166	7317	7502	0274	6340	0427

TABLE 1 *(Continued)*

Row										
61	6762	3502	9559	4279	9271	9595	3053	4918	7503	5169
62	5264	0075	6655	4563	7112	7264	3240	2150	8180	1361
63	5070	8428	5149	2137	8728	9110	2334	9709	8134	3925
64	1664	3379	5273	9367	6950	6828	1711	7082	4783	0147
65	6962	7141	1904	6648	7328	2901	6396	9949	6274	1672
66	7541	4289	4970	2922	6670	8540	9053	3219	8881	1897
67	5244	4651	2934	6700	8869	0926	4191	1364	0926	2874
68	2939	3890	0745	2577	7931	3913	7877	2837	2500	8774
69	4266	6207	8083	6564	5336	5303	7503	6627	6055	3606
70	7848	5477	5588	3490	0294	3609	1632	5684	1719	6162
71	3009	1879	0440	7916	6643	9723	5933	0574	2480	6893
72	9865	7813	7468	8493	3293	1071	7183	9462	2363	6529
73	1196	1251	2368	1262	5769	9450	7485	4039	4985	6612
74	1067	3716	8897	1970	8799	5718	4792	7292	4589	4554
75	5160	5563	6527	7861	3477	6735	7748	4913	6370	2258
76	4560	0094	8284	7604	1667	9286	2228	9507	1838	4646
77	7697	2151	4860	0739	4370	3992	8121	2502	7670	4470
78	8675	2997	9783	7306	4116	6432	7233	4611	7121	9412
79	3597	3520	5995	0892	3470	4581	1068	8801	1254	8607
80	4281	8802	5880	6212	6818	8162	0052	1755	7107	5197
81	0101	0907	9057	2263	0059	8553	7855	7758	1020	1264
82	8179	0109	4412	6044	7167	4209	5250	4570	1984	8276
83	8980	9662	9333	6598	2990	8173	1753	1135	1409	2042
84	3050	2450	9252	6724	2697	7933	9540	3700	6561	2790
85	4465	1307	8782	6763	9202	5594	7166	7050	4462	0426
86	1925	5402	1379	3556	5109	4846	9827	2881	5574	9027
87	8753	4602	1838	4624	4632	2512	2652	4804	1624	5116
88	2645	9197	4541	4822	7883	3352	3202	0906	3676	8141
89	4287	5473	4493	7086	4271	9140	3315	7073	4533	0653
90	5280	5426	7240	2154	7952	3804	8097	9328	8069	6894
91	9553	3136	2112	1369	5562	7360	5530	8074	6488	3682
92	2975	7924	0253	3503	9383	9454	3320	3234	9255	3527
93	2596	7274	8967	8138	6868	0385	4467	3792	3844	8700
94	4192	7440	6410	6064	4561	0411	9187	9940	2866	3345
95	3980	8594	9935	8560	0229	8778	2386	7852	4031	0627
96	1822	1177	6846	3997	5822	9188	2479	7951	3051	0110
97	8415	2623	2358	8895	5125	0173	3182	4151	4419	9049
98	2123	5798	5444	3282	8022	3931	4429	6028	5385	6845
99	1754	4076	3507	3705	7459	7544	6127	4820	3760	6476
100	3967	9997	0695	3562	9997	2934	8469	9706	4763	7132

TABLE 1 *(Continued)*

Row										
101	7604	6645	6633	6288	5488	8355	9295	9637	5410	0452
102	6357	0216	1685	4308	0391	1517	1952	0108	1258	5498
103	5241	0554	6072	2412	1915	4451	0633	0449	9059	6873
104	9683	0618	2433	0154	0816	9885	3562	7392	4406	2994
105	8073	7718	9374	0965	8861	0018	2152	1736	5187	9347
106	3685	5901	6296	7748	6815	8033	5646	8691	3885	1550
107	9354	1854	1914	2592	9939	2468	0190	5882	3964	6938
108	2604	3040	9664	3962	4600	1314	8163	7869	2059	8203
109	9371	8390	6971	4931	1142	8588	2240	9256	7805	0153
110	5463	5569	1657	2797	9026	7754	8501	1953	1364	7787
111	5832	6510	1728	0531	9770	5790	8294	2702	4318	2494
112	6977	1478	4053	5836	5773	5706	8840	6575	6984	0196
113	6653	3177	7173	1053	8117	5818	2177	7524	3839	2438
114	2043	3329	3149	8591	8213	7941	0324	0275	2808	5787
115	1892	6495	7363	8840	6126	5749	5841	5564	3296	8176
116	4279	6686	2795	2572	6915	5770	0723	5003	6124	0041
117	9018	3226	1024	4455	4743	8634	7086	9462	5603	4961
118	6588	0445	5301	0442	7270	4287	9827	7666	4020	6061
119	3258	2829	5949	6280	9178	3614	8680	6705	1311	2408
120	9213	0161	4449	9084	8199	7330	4284	5061	1971	1008

TABLE 2 Cumulative Binomial Probabilities

$$P[X \le c] = \sum_{x=0}^{c} \binom{n}{x} p^x (1 - p)^{n-x}$$

		p										
	c	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95
$n = 1$	0	.950	.900	.800	.700	.600	.500	.400	.300	.200	.100	.050
	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 2$	0	.902	.810	.640	.490	.360	.250	.160	.090	.040	.010	.002
	1	.997	.990	.960	.910	.840	.750	.640	.510	.360	.190	.097
	2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 3$	0	.857	.729	.512	.343	.216	.125	.064	.027	.008	.001	.000
	1	.993	.972	.896	.784	.648	.500	.352	.216	.104	.028	.007
	2	1.000	.999	.992	.973	.936	.875	.784	.657	.488	.271	.143
	3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 2 *(Continued)*

	c	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95
$n = 4$	0	.815	.656	.410	.240	.130	.063	.026	.008	.002	.000	.000
	1	.986	.948	.819	.652	.475	.313	.179	.084	.027	.004	.000
	2	1.000	.996	.973	.916	.821	.688	.525	.348	.181	.052	.014
	3	1.000	1.000	.998	.992	.974	.938	.870	.760	.590	.344	.185
	4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 5$	0	.774	.590	.328	.168	.078	.031	.010	.002	.000	.000	.000
	1	.977	.919	.737	.528	.337	.188	.087	.031	.007	.000	.000
	2	.999	.991	.942	.837	.683	.500	.317	.163	.058	.009	.001
	3	1.000	1.000	.993	.969	.913	.813	.663	.472	.263	.081	.023
	4	1.000	1.000	1.000	.998	.990	.969	.922	.832	.672	.410	.226
	5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 6$	0	.735	.531	.262	.118	.047	.016	.004	.001	.000	.000	.000
	1	.967	.886	.655	.420	.233	.109	.041	.011	.002	.000	.000
	2	.998	.984	.901	.744	.544	.344	.179	.070	.017	.001	.000
	3	1.000	.999	.983	.930	.821	.656	.456	.256	.099	.016	.002
	4	1.000	1.000	.998	.989	.959	.891	.767	.580	.345	.114	.033
	5	1.000	1.000	1.000	.999	.996	.984	.953	.882	.738	.469	.265
	6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 7$	0	.698	.478	.210	.082	.028	.008	.002	.000	.000	.000	.000
	1	.956	.850	.577	.329	.159	.063	.019	.004	.000	.000	.000
	2	.996	.974	.852	.647	.420	.227	.096	.029	.005	.000	.000
	3	1.000	.997	.967	.874	.710	.500	.290	.126	.033	.003	.000
	4	1.000	1.000	.995	.971	.904	.773	.580	.353	.148	.026	.004
	5	1.000	1.000	1.000	.996	.981	.938	.841	.671	.423	.150	.044
	6	1.000	1.000	1.000	1.000	.998	.992	.972	.918	.790	.522	.302
	7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 8$	0	.663	.430	.168	.058	.017	.004	.001	.000	.000	.000.	.000
	1	.943	.813	.503	.255	.106	.035	.009	.001	.000	.000	.000
	2	.994	.962	.797	.552	.315	.145	.050	.011	.001	.000	.000
	3	1.000	.995	.944	.806	.594	.363	.174	.058	.010	.000	.000
	4	1.000	1.000	.990	.942	.826	.637	.406	.194	.056	.005	.000
	5	1.000	1.000	.999	.989	.950	.855	.685	.448	.203	.038	.006
	6	1.000	1.000	1.000	.999	.991	.965	.894	.745	.497	.187	.057
	7	1.000	1.000	1.000	1.000	.999	.996	.983	.942	.832	.570	.337
	8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 9$	0	.630	.387	.134	.040	.010	.002	.000	.000	.000	.000	.000
	1	.929	.775	.436	.196	.071	.020	.004	.000	.000	.000	.000
	2	.992	.947	.738	.463	.232	.090	.025	.004	.000	.000	.000
	3	.999	.992	.914	.730	.483	.254	.099	.025	.003	.000	.000

(Column heads .05 through .95 are values of p.)

TABLE 2 *(Continued)*

		p										
	c	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95
	4	1.000	.999	.980	.901	.733	.500	.267	.099	.020	.001	.000
	5	1.000	1.000	.997	.975	.901	.746	.517	.270	.086	.008	.001
	6	1.000	1.000	1.000	.996	.975	.910	.768	.537	.262	.053	.008
	7	1.000	1.000	1.000	1.000	.996	.980	.929	.804	.564	.225	.071
	8	1.000	1.000	1.000	1.000	1.000	.998	.990	.960	.866	.613	.370
	9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 10$	0	.599	.349	.107	.028	.006	.001	.000	.000	.000	.000	.000
	1	.914	.736	.376	.149	.046	.011	.002	.000	.000	.000	.000
	2	.988	.930	.678	.383	.167	.055	.012	.002	.000	.000	.000
	3	.999	.987	.879	.650	.382	.172	.055	.011	.001	.000	.000
	4	1.000	.998	.967	.850	.633	.377	.166	.047	.006	.000	.000
	5	1.000	1.000	.994	.953	.834	.623	.367	.150	.033	.002	.000
	6	1.000	1.000	.999	.989	.945	.828	.618	.350	.121	.013	.001
	7	1.000	1.000	1.000	.998	.988	.945	.833	.617	.322	.070	.012
	8	1.000	1.000	1.000	1.000	.998	.989	.954	.851	.624	.264	.086
	9	1.000	1.000	1.000	1.000	1.000	.999	.994	.972	.893	.651	.401
	10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 11$	0	.569	.314	.086	.020	.004	.000	.000	.000	.000	.000	.000
	1	.898	.697	.322	.113	.030	.006	.001	.000	.000	.000	.000
	2	.985	.910	.617	.313	.119	.033	.006	.001	.000	.000	.000
	3	.998	.981	.839	.570	.296	.113	.029	.004	.000	.000	.000
	4	1.000	.997	.950	.790	.533	.274	.099	.022	.002	.000	.000
	5	1.000	1.000	.988	.922	.753	.500	.247	.078	.012	.000	.000
	6	1.000	1.000	.998	.978	.901	.726	.467	.210	.050	.003	.000
	7	1.000	1.000	1.000	.996	.971	.887	.704	.430	.161	.019	.002
	8	1.000	1.000	1.000	.999	.994	.967	.881	.687	.383	.090	.015
	9	1.000	1.000	1.000	1.000	.999	.994	.970	.887	.678	.303	.102
	10	1.000	1.000	1.000	1.000	1.000	1.000	.996	.980	.914	.686	.431
	11	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 12$	0	.540	.282	.069	.014	.002	.000	.000	.000	.000	.000	.000
	1	.882	.659	.275	.085	.020	.003	.000	.000	.000	.000	.000
	2	.980	.889	.558	.253	.083	.019	.003	.000	.000	.000	.000
	3	.998	.974	.795	.493	.225	.073	.015	.002	.000	.000	.000
	4	1.000	.996	.927	.724	.438	.194	.057	.009	.001	.000	.000
	5	1.000	.999	.981	.882	.665	.387	.158	.039	.004	.000	.000
	6	1.000	1.000	.996	.961	.842	.613	.335	.118	.019	.001	.000
	7	1.000	1.000	.999	.991	.943	.806	.562	.276	.073	.004	.000
	8	1.000	1.000	1.000	.998	.985	.927	.775	.507	.205	.026	.002
	9	1.000	1.000	1.000	1.000	.997	.981	.917	.747	.442	.111	.020
	10	1.000	1.000	1.000	1.000	1.000	.977	.980	.915	.725	.341	.118
	11	1.000	1.000	1.000	1.000	1.000	1.000	.998	.986	.931	.718	.460
	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 2 *(Continued)*

	c	p = .05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95
$n = 13$	0	.513	.254	.055	.010	.001	.000	.000	.000	.000	.000	.000
	1	.865	.621	.234	.064	.013	.002	.000	.000	.000	.000	.000
	2	.975	.866	.502	.202	.058	.011	.001	.000	.000	.000	.000
	3	.997	.966	.747	.421	.169	.046	.008	.001	.000	.000	.000
	4	1.000	.994	.901	.654	.353	.133	.032	.004	.000	.000	.000
	5	1.000	.999	.970	.835	.574	.291	.098	.018	.001	.000	.000
	6	1.000	1.000	.993	.938	.771	.500	.229	.062	.007	.000	.000
	7	1.000	1.000	.999	.982	.902	.709	.426	.165	.030	.001	.000
	8	1.000	1.000	1.000	.996	.968	.867	.647	.346	.099	.006	.000
	9	1.000	1.000	1.000	.999	.992	.954	.831	.579	.253	.034	.003
	10	1.000	1.000	1.000	1.000	.999	.989	.942	.798	.498	.134	.025
	11	1.000	1.000	1.000	1.000	1.000	.998	.987	.936	.766	.379	.135
	12	1.000	1.000	1.000	1.000	1.000	1.000	.999	.990	.945	.746	.487
	13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 14$	0	.488	.229	.044	.007	.001	.000	.000	.000	.000	.000	.000
	1	.847	.585	.198	.047	.008	.001	.000	.000	.000	.000	.000
	2	.970	.842	.448	.161	.040	.006	.001	.000	.000	.000	.000
	3	.996	.956	.698	.355	.124	.029	.004	.000	.000	.000	.000
	4	1.000	.991	.870	.584	.279	.090	.018	.002	.000	.000	.000
	5	1.000	.999	.956	.781	.486	.212	.058	.008	.000	.000	.000
	6	1.000	1.000	.988	.907	.692	.395	.150	.031	.002	.000	.000
	7	1.000	1.000	.998	.969	.850	.605	.308	.093	.012	.000	.000
	8	1.000	1.000	1.000	.992	.942	.788	.514	.219	.044	.001	.000
	9	1.000	1.000	1.000	.998	.982	.910	.721	.416	.130	.009	.000
	10	1.000	1.000	1.000	1.000	.996	.971	.876	.645	.302	.044	.004
	11	1.000	1.000	1.000	1.000	.999	.994	.960	.839	.552	.158	.030
	12	1.000	1.000	1.000	1.000	1.000	.999	.992	.953	.802	.415	.153
	13	1.000	1.000	1.000	1.000	1.000	1.000	.999	.993	.956	.771	.512
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 15$	0	.463	.206	.035	.005	.000	.000	.000	.000	.000	.000	.000
	1	.829	.549	.167	.035	.005	.000	.000	.000	.000	.000	.000
	2	.964	.816	.398	.127	.027	.004	.000	.000	.000	.000	.000
	3	.995	.944	.648	.297	.091	.018	.002	.000	.000	.000	.000
	4	.999	.987	.836	.515	.217	.059	.009	.001	.000	.000	.000
	5	1.000	.998	.939	.722	.403	.151	.034	.004	.000	.000	.000
	6	1.000	1.000	.982	.869	.610	.304	.095	.015	.001	.000	.000
	7	1.000	1.000	.996	.950	.787	.500	.213	.050	.004	.000	.000
	8	1.000	1.000	.999	.985	.905	.696	.390	.131	.018	.000	.000
	9	1.000	1.000	1.000	.996	.966	.849	.597	.278	.061	.002	.000
	10	1.000	1.000	1.000	.999	.991	.941	.783	.485	.164	.013	.001
	11	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.352	.056	.005
	12	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.602	.184	.036

TABLE 2 *(Continued)*

	c	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95
	13	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.833	.451	.171
	14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.794	.537
	15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 16$	0	.440	.185	.028	.003	.000	.000	.000	.000	.000	.000	.000
	1	.811	.515	.141	.026	.003	.000	.000	.000	.000	.000	.000
	2	.957	.789	.352	.099	.018	.002	.000	.000	.000	.000	.000
	3	.993	.932	.598	.246	.065	.011	.001	.000	.000	.000	.000
	4	.999	.983	.798	.450	.167	.038	.005	.000	.000	.000	.000
	5	1.000	.997	.918	.660	.329	.105	.019	.002	.000	.000	.000
	6	1.000	.999	.973	.825	.527	.227	.058	.007	.000	.000	.000
	7	1.000	1.000	.993	.926	.716	.402	.142	.026	.001	.000	.000
	8	1.000	1.000	.999	.974	.858	.598	.284	.074	.007	.000	.000
	9	1.000	1.000	1.000	.993	.942	.773	.473	.175	.027	.001	.000
	10	1.000	1.000	1.000	.998	.981	.895	.671	.340	.082	.003	.000
	11	1.000	1.000	1.000	1.000	.995	.962	.833	.550	.202	.017	.001
	12	1.000	1.000	1.000	1.000	.999	.989	.935	.754	.402	.068	.007
	13	1.000	1.000	1.000	1.000	1.000	.998	.982	.901	.648	.211	.043
	14	1.000	1.000	1.000	1.000	1.000	1.000	.997	.974	.859	.485	.189
	15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.997	.972	.815	.560
	16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 17$	0	.418	.167	.023	.002	.000	.000	.000	.000	.000	.000	.000
	1	.792	.482	.118	.019	.002	.000	.000	.000	.000	.000	.000
	2	.950	.762	.310	.077	.012	.001	.000	.000	.000	.000	.000
	3	.991	.917	.549	.202	.046	.006	.000	.000	.000	.000	.000
	4	.999	.978	.758	.389	.126	.025	.003	.000	.000	.000	.000
	5	1.000	.995	.894	.597	.264	.072	.011	.001	.000	.000	.000
	6	1.000	.999	.962	.775	.448	.166	.035	.003	.000	.000	.000
	7	1.000	1.000	.989	.895	.641	.315	.092	.013	.000	.000	.000
	8	1.000	1.000	.997	.960	.801	.500	.199	.040	.003	.000	.000
	9	1.000	1.000	1.000	.987	.908	.685	.359	.105	.011	.000	.000
	10	1.000	1.000	1.000	.997	.965	.834	.552	.225	.038	.001	.000
	11	1.000	1.000	1.000	.999	.989	.928	.736	.403	.106	.005	.000
	12	1.000	1.000	1.000	1.000	.997	.975	.874	.611	.242	.022	.001
	13	1.000	1.000	1.000	1.000	1.000	.994	.954	.798	.451	.083	.009
	14	1.000	1.000	1.000	1.000	1.000	.999	.988	.923	.690	.238	.050
	15	1.000	1.000	1.000	1.000	1.000	1.000	.998	.981	.882	.518	.208
	16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.977	.833	.582
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 18$	0	.397	.150	.018	.002	.000	.000	.000	.000	.000	.000	.000
	1	.774	.450	.099	.014	.001	.000	.000	.000	.000	.000	.000
	2	.942	.734	.271	.060	.008	.001	.000	.000	.000	.000	.000

(Column headings give p.)

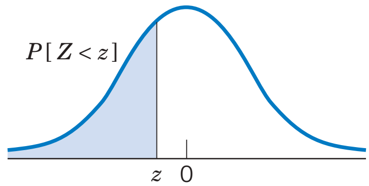
TABLE 2 *(Continued)*

		p										
		.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95
	c											
	3	.989	.902	.501	.165	.033	.004	.000	.000	.000	.000	.000
	4	.998	.972	.716	.333	.094	.015	.001	.000	.000	.000	.000
	5	1.000	.994	.867	.534	.209	.048	.006	.000	.000	.000	.000
	6	1.000	.999	.949	.722	.374	.119	.020	.001	.000	.000	.000
	7	1.000	1.000	.984	.859	.563	.240	.058	.006	.000	.000	.000
	8	1.000	1.000	.996	.940	.737	.407	.135	.021	.001	.000	.000
	9	1.000	1.000	.999	.979	.865	.593	.263	.060	.004	.000	.000
	10	1.000	1.000	1.000	.994	.942	.760	.437	.141	.016	.000	.000
	11	1.000	1.000	1.000	.999	.980	.881	.626	.278	.051	.001	.000
	12	1.000	1.000	1.000	1.000	.994	.952	.791	.466	.133	.006	.000
	13	1.000	1.000	1.000	1.000	.999	.985	.906	.667	.284	.028	.002
	14	1.000	1.000	1.000	1.000	1.000	.996	.967	.835	.499	.098	.011
	15	1.000	1.000	1.000	1.000	1.000	.999	.992	.940	.729	.266	.058
	16	1.000	1.000	1.000	1.000	1.000	1.000	.999	.986	.901	.550	.226
	17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.982	.850	.603
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 19$	0	.377	.135	.014	.001	.000	.000	.000	.000	.000	.000	.000
	1	.755	.420	.083	.010	.001	.000	.000	.000	.000	.000	.000
	2	.933	.705	.237	.046	.005	.000	.000	.000	.000	.000	.000
	3	.987	.885	.455	.133	.023	.002	.000	.000	.000	.000	.000
	4	.998	.965	.673	.282	.070	.010	.001	.000	.000	.000	.000
	5	1.000	.991	.837	.474	.163	.032	.003	.000	.000	.000	.000
	6	1.000	.998	.932	.666	.308	.084	.012	.001	.000	.000	.000
	7	1.000	1.000	.977	.818	.488	.180	.035	.003	.000	.000	.000
	8	1.000	1.000	.993	.916	.667	.324	.088	.011	.000	.000	.000
	9	1.000	1.000	.998	.967	.814	.500	.186	.033	.002	.000	.000
	10	1.000	1.000	1.000	.989	.912	.676	.333	.084	.007	.000	.000
	11	1.000	1.000	1.000	.997	.965	.820	.512	.182	.023	.000	.000
	12	1.000	1.000	1.000	.999	.988	.916	.692	.334	.068	.002	.000
	13	1.000	1.000	1.000	1.000	.997	.968	.837	.526	.163	.009	.000
	14	1.000	1.000	1.000	1.000	.999	.990	.930	.718	.327	.035	.002
	15	1.000	1.000	1.000	1.000	1.000	.998	.977	.867	.545	.115	.013
	16	1.000	1.000	1.000	1.000	1.000	1.000	.995	.954	.763	.295	.067
	17	1.000	1.000	1.000	1.000	1.000	1.000	.999	.990	.917	.580	.245
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.986	.865	.623
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 20$	0	.358	.122	.012	.001	.000	.000	.000	.000	.000	.000	.000
	1	.736	.392	.069	.008	.001	.000	.000	.000	.000	.000	.000
	2	.925	.677	.206	.035	.004	.000	.000	.000	.000	.000	.000
	3	.984	.867	.411	.107	.016	.001	.000	.000	.000	.000	.000
	4	.997	.957	.630	.238	.051	.006	.000	.000	.000	.000	.000
	5	1.000	.989	.804	.416	.126	.021	.002	.000	.000	.000	.000

TABLE 2 *(Continued)*

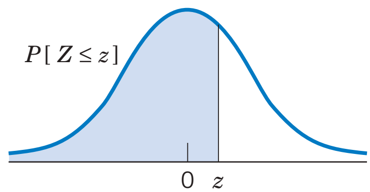
		p										
	c	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95
	6	1.000	.998	.913	.608	.250	.058	.006	.000	.000	.000	.000
	7	1.000	1.000	.968	.772	.416	.132	.021	.001	.000	.000	.000
	8	1.000	1.000	.990	.887	.596	.252	.057	.005	.000	.000	.000
	9	1.000	1.000	.997	.952	.755	.412	.128	.017	.001	.000	.000
	10	1.000	1.000	.999	.983	.872	.588	.245	.048	.003	.000	.000
	11	1.000	1.000	1.000	.995	.943	.748	.404	.113	.010	.000	.000
	12	1.000	1.000	1.000	.999	.979	.868	.584	.228	.032	.000	.000
	13	1.000	1.000	1.000	1.000	.994	.942	.750	.392	.087	.002	.000
	14	1.000	1.000	1.000	1.000	.998	.979	.874	.584	.196	.011	.000
	15	1.000	1.000	1.000	1.000	1.000	.994	.949	.762	.370	.043	.003
	16	1.000	1.000	1.000	1.000	1.000	.999	.984	.893	.589	.133	.016
	17	1.000	1.000	1.000	1.000	1.000	1.000	.996	.965	.794	.323	.075
	18	1.000	1.000	1.000	1.000	1.000	1.000	.999	.992	.931	.608	.264
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.988	.878	.642
	20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 25$	0	.277	.072	.004	.000	.000	.000	.000	.000	.000	.000	.000
	1	.642	.271	.027	.002	.000	.000	.000	.000	.000	.000	.000
	2	.873	.537	.098	.009	.000	.000	.000	.000	.000	.000	.000
	3	.966	.764	.234	.033	.002	.000	.000	.000	.000	.000	.000
	4	.993	.902	.421	.090	.009	.000	.000	.000	.000	.000	.000
	5	.999	.967	.617	.193	.029	.002	.000	.000	.000	.000	.000
	6	1.000	.991	.780	.341	.074	.007	.000	.000	.000	.000	.000
	7	1.000	.998	.891	.512	.154	.022	.001	.000	.000	.000	.000
	8	1.000	1.000	.953	.677	.274	.054	.004	.000	.000	.000	.000
	9	1.000	1.000	.983	.811	.425	.115	.013	.000	.000	.000	.000
	10	1.000	1.000	.994	.902	.586	.212	.034	.002	.000	.000	.000
	11	1.000	1.000	.998	.956	.732	.345	.078	.006	.000	.000	.000
	12	1.000	1.000	1.000	.983	.846	.500	.154	.017	.000	.000	.000
	13	1.000	1.000	1.000	.994	.922	.655	.268	.044	.002	.000	.000
	14	1.000	1.000	1.000	.998	.966	.788	.414	.098	.006	.000	.000
	15	1.000	1.000	1.000	1.000	.987	.885	.575	.189	.017	.000	.000
	16	1.000	1.000	1.000	1.000	.996	.946	.726	.323	.047	.000	.000
	17	1.000	1.000	1.000	1.000	.999	.978	.846	.488	.109	.002	.000
	18	1.000	1.000	1.000	1.000	1.000	.993	.926	.659	.220	.009	.000
	19	1.000	1.000	1.000	1.000	1.000	.998	.971	.807	.383	.033	.001
	20	1.000	1.000	1.000	1.000	1.000	1.000	.991	.910	.579	.098	.007
	21	1.000	1.000	1.000	1.000	1.000	1.000	.998	.967	.766	.236	.034
	22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.902	.463	.127
	23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.973	.729	.358
	24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.928	.723
	25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 3 Standard Normal Probabilities



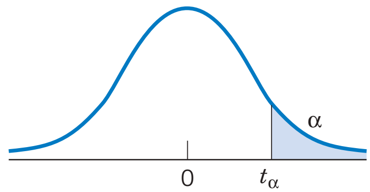
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
−.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

TABLE 3 *(Continued)*



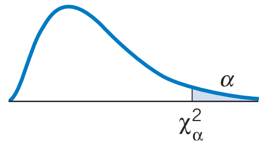
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998

TABLE 4 Percentage Points of *t* Distributions



d.f. \ α	.25	.10	.05	.025	.01	.00833	.00625	.005
1	1.000	3.078	6.314	12.706	31.821	38.204	50.923	63.657
2	.816	1.886	2.920	4.303	6.965	7.649	8.860	9.925
3	.765	1.638	2.353	3.182	4.541	4.857	5.392	5.841
4	.741	1.533	2.132	2.776	3.747	3.961	4.315	4.604
5	.727	1.476	2.015	2.571	3.365	3.534	3.810	4.032
6	.718	1.440	1.943	2.447	3.143	3.287	3.521	3.707
7	.711	1.415	1.895	2.365	2.998	3.128	3.335	3.499
8	.706	1.397	1.860	2.306	2.896	3.016	3.206	3.355
9	.703	1.383	1.833	2.262	2.821	2.933	3.111	3.250
10	.700	1.372	1.812	2.228	2.764	2.870	3.038	3.169
11	.697	1.363	1.796	2.201	2.718	2.820	2.981	3.106
12	.695	1.356	1.782	2.179	2.681	2.779	2.934	3.055
13	.694	1.350	1.771	2.160	2.650	2.746	2.896	3.012
14	.692	1.345	1.761	2.145	2.624	2.718	2.864	2.977
15	.691	1.341	1.753	2.131	2.602	2.694	2.837	2.947
16	.690	1.337	1.746	2.120	2.583	2.673	2.813	2.921
17	.689	1.333	1.740	2.110	2.567	2.655	2.793	2.898
18	.688	1.330	1.734	2.101	2.552	2.639	2.775	2.878
19	.688	1.328	1.729	2.093	2.539	2.625	2.759	2.861
20	.687	1.325	1.725	2.086	2.528	2.613	2.744	2.845
21	.686	1.323	1.721	2.080	2.518	2.601	2.732	2.831
22	.686	1.321	1.717	2.074	2.508	2.591	2.720	2.819
23	.685	1.319	1.714	2.069	2.500	2.582	2.710	2.807
24	.685	1.318	1.711	2.064	2.492	2.574	2.700	2.797
25	.684	1.316	1.708	2.060	2.485	2.566	2.692	2.787
26	.684	1.315	1.706	2.056	2.479	2.559	2.684	2.779
27	.684	1.314	1.703	2.052	2.473	2.552	2.676	2.771
28	.683	1.313	1.701	2.048	2.467	2.546	2.669	2.763
29	.683	1.311	1.699	2.045	2.462	2.541	2.663	2.756
30	.683	1.310	1.697	2.042	2.457	2.536	2.657	2.750
40	.681	1.303	1.684	2.021	2.423	2.499	2.616	2.704
60	.679	1.296	1.671	2.000	2.390	2.463	2.575	2.660
120	.677	1.289	1.658	1.980	2.358	2.428	2.536	2.617
∞	.674	1.282	1.645	1.960	2.326	2.394	2.498	2.576

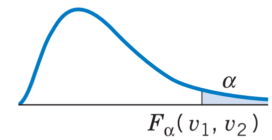
TABLE 5 Percentage Points of χ^2 Distributions



d.f. \ α	.99	.975	.95	.90	.50	.10	.05	.025	.01
1	.0002	.001	.004	.02	.45	2.71	3.84	5.02	6.63
2	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21
3	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34
4	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28
5	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09
6	.87	1.24	1.64	2.20	5.35	10.64	12.59	14.45	16.81
7	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48
8	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09
9	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67
10	2.56	3.24	3.94	4.87	9.34	15.99	18.31	20.48	23.21
11	3.05	3.81	4.57	5.58	10.34	17.28	19.68	21.92	24.72
12	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22
13	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69
14	4.66	5.62	6.57	7.79	13.34	21.06	23.68	26.12	29.14
15	5.23	6.26	7.26	8.55	14.34	22.31	25.00	27.49	30.58
16	5.81	6.90	7.96	9.31	15.34	23.54	26.30	28.85	32.00
17	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41
18	7.01	8.23	9.39	10.86	17.34	25.99	28.87	31.53	34.81
19	7.63	8.90	10.12	11.65	18.34	27.20	30.14	32.85	36.19
20	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57
21	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93
22	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29
23	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64
24	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98
25	11.52	13.11	14.61	16.47	24.34	34.38	37.65	40.65	44.31
26	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64
27	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96
28	13.56	15.30	16.93	18.94	27.34	37.92	41.34	44.46	48.28
29	14.26	16.04	17.71	19.77	28.34	39.09	42.56	45.72	49.59
30	14.95	16.78	18.49	20.60	29.34	40.26	43.77	46.98	50.89
40	22.16	24.42	26.51	29.05	39.34	51.81	55.76	59.34	63.69
50	29.71	32.35	34.76	37.69	49.33	63.17	67.50	71.42	76.15
60	37.48	40.47	43.19	46.46	59.33	74.40	79.08	83.30	88.38
70	45.44	48.75	51.74	55.33	69.33	85.53	90.53	95.02	100.43
80	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33
90	61.75	65.64	69.13	73.29	89.33	107.57	113.15	118.14	124.12
100	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81

TABLE 6 Percentage Points of $F(v_1, v_2)$ Distributions

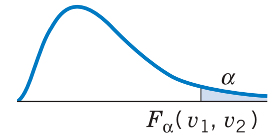
$\alpha = .10$



v_2 \ v_1	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.79
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.76
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.51
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.21
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.11
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.96
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.86
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.82
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.78
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.75
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.72
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.70
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.68
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.66
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.62
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.68	1.66	1.63	1.59
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.67	1.65	1.61	1.58
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.66	1.64	1.60	1.57
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.65	1.63	1.59	1.56
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.64	1.62	1.58	1.55
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.63	1.61	1.57	1.54
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.50	1.48	1.44	1.40
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24

TABLE 6 *(Continued)*

$\alpha = .05$



v_2 \ v_1	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60
1	161.5	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	246.0	248.0	249.3	250.1	251.1	252.2
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.57
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.43
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.74
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.30
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.01
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.79
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.62
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.49
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.38
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.30
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.22
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.16
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.11
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.06
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.02
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.95
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.89
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.86
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.84
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.94	1.90	1.85	1.80
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.92	1.88	1.84	1.79
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.89	1.85	1.81	1.75
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.88	1.84	1.79	1.74
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.78	1.74	1.69	1.64
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.69	1.65	1.59	1.53
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.60	1.55	1.50	1.43
∞	3.84	3.00	2.61	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.51	1.46	1.39	1.32

TABLE 7 Selected Tail Probabilities for the Null Distribution of Wilcoxon's Rank-Sum Statistic

$$P = P[W_s \geq x] = P[W_s \leq x^*]$$

Smaller Sample Size = 2

Larger Sample Size

3			4			5			6		
x	P	x^*	x	P	x^*	x	P	x^*	x	P	x^*
8	.200	4	10	.133	4	11	.190	5	13	.143	5
9	.100	3	11	.067	3	12	.095	4	14	.071	4
10	0	2	12	0	2	13	.048	3	15	.036	3
						14	0	2	16	0	2

7			8			9			10		
x	P	x^*	x	P	x^*	x	P	x^*	x	P	x^*
15	.111	5	16	.133	6	18	.109	6	19	.136	7
16	.056	4	17	.089	5	19	.073	5	20	.091	6
17	.028	3	18	.044	4	20	.036	4	21	.061	5
18	0	2	19	.022	3	21	.018	3	22	.030	4
			20	0	2	22	0	2	23	.015	3

Smaller Sample Size = 3

Larger Sample Size

3			4			5			6		
x	P	x^*	x	P	x^*	x	P	x^*	x	P	x^*
13	.200	8	16	.114	8	18	.125	9	20	.131	10
14	.100	7	17	.057	7	19	.071	8	21	.083	9
15	.050	6	18	.029	6	20	.036	7	22	.048	8
16	0	5	19	0	5	21	.018	6	23	.024	7
						22	0	5	24	.012	6
									25	0	5

7			8			9			10		
x	P	x^*	x	P	x^*	x	P	x^*	x	P	x^*
22	.133	11	24	.139	12	27	.105	12	29	.108	13
23	.092	10	25	.097	11	28	.073	11	30	.080	12
24	.058	9	26	.067	10	29	.050	10	31	.056	11
25	.033	8	27	.042	9	30	.032	9	32	.038	10
26	.017	7	28	.024	8	31	.018	8	33	.024	9
27	.008	6	29	.012	7	32	.009	7	34	.014	8
28	0	5	30	.006	6				35	.007	7
			31	0	5						

TABLE 7 *(Continued)*

Smaller Sample Size = 4

Larger Sample Size

4			5			6			7		
x	P	x^*	x	P	x^*	x	P	x^*	x	P	x^*
22	.171	14	25	.143	15	28	.129	16	31	.115	17
23	.100	13	26	.095	14	29	.086	15	32	.082	16
24	.057	12	27	.056	13	30	.057	14	33	.055	15
25	.029	11	28	.032	12	31	.033	13	34	.036	14
26	.014	10	29	.016	11	32	.019	12	35	.021	13
27	0	9	30	.008	10	33	.010	11	36	.012	12
			31	0	9				37	.006	11

8			9			10		
x	P	x^*	x	P	x^*	x	P	x^*
34	.107	18	36	.130	20	39	.120	21
35	.077	17	37	.099	19	40	.094	20
36	.055	16	38	.074	18	41	.071	19
37	.036	15	39	.053	17	42	.053	18
38	.024	14	40	.038	16	43	.038	17
39	.014	13	41	.025	15	44	.027	16
40	.008	12	42	.017	14	45	.018	15
			43	.010	13	46	.012	14
						47	.007	13

Smaller Sample Size = 5

Larger Sample Size

5			6			7			8		
x	P	x^*	x	P	x^*	x	P	x^*	x	P	x^*
34	.111	21	37	.123	23	41	.101	24	44	.111	26
35	.075	20	38	.089	22	42	.074	23	45	.085	25
36	.048	19	39	.063	21	43	.053	22	46	.064	24
37	.028	18	40	.041	20	44	.037	21	47	.047	23
38	.016	17	41	.026	19	45	.024	20	48	.033	22
39	.008	16	42	.015	18	46	.015	19	49	.023	21
			43	.009	17	47	.009	18	50	.015	20
									51	.009	19

TABLE 7 *(Continued)*

Larger Sample Size					
9			10		
x	P	x^*	x	P	x^*
47	.120	28	51	.103	29
48	.095	27	52	.082	28
49	.073	26	53	.065	27
50	.056	25	54	.050	26
51	.041	24	55	.038	25
52	.030	23	56	.028	24
53	.021	22	57	.020	23
54	.014	21	58	.014	22
55	.009	20	59	.010	21

Smaller Sample Size = 6

Larger Sample Size											
6			7			8			9		
x	P	x^*	x	P	x^*	x	P	x^*	x	P	x^*
47	.120	31	51	.117	33	55	.114	35	59	.112	37
48	.090	30	52	.090	32	56	.091	34	60	.091	36
49	.066	29	53	.069	31	57	.071	33	61	.072	35
50	.047	28	54	.051	30	58	.054	32	62	.057	34
51	.032	27	55	.037	29	59	.041	31	63	.044	33
52	.021	26	56	.026	28	60	.030	30	64	.033	32
53	.013	25	57	.017	27	61	.021	29	65	.025	31
54	.008	24	58	.011	26	62	.015	28	66	.018	30
			59	.007	25	63	.010	27	67	.013	29
									68	.009	28

10		
x	P	x^*
63	.110	39
64	.090	38
65	.074	37
66	.059	36
67	.047	35
68	.036	34
69	.028	33
70	.021	32
71	.016	31
72	.011	30
73	.008	29

TABLE 7 *(Continued)*

Smaller Sample Size = 7

Larger Sample Size

7			8			9			10		
x	P	x^*	x	P	x^*	x	P	x^*	x	P	x^*
63	.104	42	67	.116	45	72	.105	47	76	.115	50
64	.082	41	68	.095	44	73	.087	46	77	.097	49
65	.064	40	69	.076	43	74	.071	45	78	.081	48
66	.049	39	70	.060	42	75	.057	44	79	.067	47
67	.036	38	71	.047	41	76	.045	43	80	.054	46
68	.027	37	72	.036	40	77	.036	42	81	.044	45
69	.019	36	73	.027	39	78	.027	41	82	.035	44
70	.013	35	74	.020	38	79	.021	40	83	.028	43
71	.009	34	75	.014	37	80	.016	39	84	.022	42
			76	.010	36	81	.011	38	85	.017	41
						82	.008	37	86	.012	40
									87	.009	39

Smaller Sample Size = 8

Larger Sample Size

8			9			10		
x	P	x^*	x	P	x^*	x	P	x^*
80	.117	56	86	.100	58	91	.102	61
81	.097	55	87	.084	57	92	.086	60
82	.080	54	88	.069	56	93	.073	59
83	.065	53	89	.057	55	94	.061	58
84	.052	52	90	.046	54	95	.051	57
85	.041	51	91	.037	53	96	.042	56
86	.032	50	92	.030	52	97	.034	55
87	.025	49	93	.023	51	98	.027	54
88	.019	48	94	.018	50	99	.022	53
89	.014	47	95	.014	49	100	.017	52
90	.010	46	96	.010	48	101	.013	51
						102	.010	50

TABLE 7 *(Continued)*

Smaller Sample Size = 9						Smaller Sample Size = 10		
Larger Sample Size						Larger Sample Size		
9			10			10		
x	P	x^*	x	P	x^*	x	P	x^*
100	.111	71	106	.106	74	122	.109	88
101	.095	70	107	.091	73	123	.095	87
102	.081	69	108	.078	72	124	.083	86
103	.068	68	109	.067	71	125	.072	85
104	.057	67	110	.056	70	126	.062	84
105	.047	66	111	.047	69	127	.053	83
106	.039	65	112	.039	68	128	.045	82
107	.031	64	113	.033	67	129	.038	81
108	.025	63	114	.027	66	130	.032	80
109	.020	62	115	.022	65	131	.026	79
110	.016	61	116	.017	64	132	.022	78
111	.012	60	117	.014	63	133	.018	77
112	.009	59	118	.011	62	134	.014	76
			119	.009	61	135	.012	75
						136	.009	74

Source: Adapted from C. Kraft and C. van Eeden, *A Nonparametric Introduction to Statistics*, Macmillan, New York, 1968.

TABLE 8 Selected Tail Probabilities for the Null Distribution of Wilcoxon's Signed-Rank Statistic

$$P = P[T^{+} \geq x] = P[T^{+} \leq x^{*}]$$

n = 3

x	P	x^*
5	.250	1
6	.125	0
7	0	

n = 4

x	P	x^*
8	.188	2
9	.125	1
10	.062	0
11	0	

n = 5

x	P	x^*
12	.156	3
13	.094	2
14	.062	1
15	.031	0
16	0	

n = 6

x	P	x^*
17	.109	4
18	.078	3
19	.047	2
20	.031	1
21	.016	0
22	0	

n = 7

x	P	x^*
22	.109	6
23	.078	5
24	.055	4
25	.039	3
26	.023	2
27	.016	1
28	.008	0

n = 8

x	P	x^*
27	.125	9
28	.098	8
29	.074	7
30	.055	6
31	.039	5
32	.027	4
33	.020	3
34	.012	2
35	.008	1

n = 9

x	P	x^*
34	.102	11
35	.082	10
36	.064	9
37	.049	8
38	.037	7
39	.027	6
40	.020	5
41	.014	4
42	.010	3

n = 10

x	P	x^*
40	.116	15
41	.097	14
42	.080	13
43	.065	12
44	.053	11
45	.042	10
46	.032	9
47	.024	8
48	.019	7
49	.014	6
50	.010	5

n = 11

x	P	x^*
48	.103	18
49	.087	17
50	.074	16
51	.062	15
52	.051	14
53	.042	13
54	.034	12
55	.027	11
56	.021	10
57	.016	9
58	.012	8
59	.009	7

n = 12

x	P	x^*
56	.102	22
57	.088	21
58	.076	20
59	.065	19
60	.055	18
61	.046	17
62	.039	16
63	.032	15
64	.026	14
65	.021	13
66	.017	12
67	.013	11
68	.010	10

n = 13

x	P	x^*
64	.108	27
65	.095	26
66	.084	25
67	.073	24
68	.064	23
69	.055	22
70	.047	21
71	.040	20
72	.034	19
73	.029	18
74	.024	17
75	.020	16
76	.016	15
77	.013	14
78	.011	13
79	.009	12

n = 14

x	P	x^*
73	.108	32
74	.097	31
75	.086	30
76	.077	29
77	.068	28
78	.059	27
79	.052	26
80	.045	25
81	.039	24
82	.034	23
83	.029	22
84	.025	21
85	.021	20
86	.018	19
87	.015	18
88	.012	17
89	.010	16

TABLE 8 *(Continued)*

$n = 15$		
x	P	x^*
83	.104	37
84	.094	36
85	.084	35
86	.076	34
87	.068	33
88	.060	32
89	.053	31
90	.047	30
91	.042	29
92	.036	28
93	.032	27
94	.028	26
95	.024	25
96	.021	24
97	.018	23
98	.015	22
99	.013	21
100	.011	20
101	.009	19

Source: Adapted from C. Kraft and C. van Eeden, *A Nonparametric Introduction to Statistics*, Macmillan, New York, 1968.

Data Bank

The Jump River Electric Company serves several counties in northern Wisconsin. Much of the area is forest and lakes. The data on power outages from a recent summer include date, time, duration of outage (hours), and cause.

TABLE D.1 Power Outages

Date	Time	Duration	Cause
6/11	4:00 PM	5.50	Trees and limbs
6/12	8:00 PM	1.50	Trees and limbs
6/16	8:30 AM	2.00	Trees and limbs
6/17	5:30 AM	2.00	Trees and limbs
6/17	5:00 PM	8.00	Windstorm
6/21	4:30 PM	2.00	Trees and limbs
6/26	3:00 AM	3.00	Trees and limbs
6/26	2:00 PM	1.75	Unknown
6/26	12:00 AM	2.00	Lightning blew up transformer
7/03	6:00 PM	1.50	Trees and limbs
7/04	9:00 AM	2.50	Unknown
7/04	5:00 PM	1.50	Trees and limbs
7/05	5:00 AM	1.50	Trees and limbs
7/08	7:00 AM	3.50	Lightning
7/20	12:00 AM	1.50	Unknown
7/21	9:00 AM	1.50	Animal
7/28	12:00 PM	1.00	Animal

TABLE D.1 *(Continued)*

Date	Time	Duration	Cause
7/30	7:00 PM	1.00	Squirrel on transformer
7/31	7:30 AM	0.50	Squirrel on cutout
8/04	6:00 AM	2.50	Trees and limbs
8/06	8:00 PM	2.00	Beaver
8/09	5:30 AM	1.50	Fuse-flying squirrel
8/11	12:00 AM	3.00	Beaver-cut trees
8/13	1:00 AM	1.00	Unknown
8/15	12:30 AM	1.50	Animal
8/18	8:00 AM	1.50	Trees and limbs
8/20	4:00 AM	2.00	Transformer fuse
8/25	9:00 AM	1.00	Animal
8/26	2:00 AM	10.00	Lightning
8/27	3:00 AM	1.00	Trees and limbs

Madison recruits for the fire department need to complete a timed test that simulates working conditions. It includes placing a ladder against a building, pulling out a section of fire hose, dragging a weighted object, and crawling in a simulated attic environment. The times, in seconds, for recruits to complete the test for a Madison firefighter are shown in Table D.2.

TABLE D.2 Time to Complete Firefighters Physical Test (seconds)

425	389	380	421	438	331	368	417	403	416	385	315
427	417	386	386	378	300	321	286	269	225	268	317
287	256	334	342	269	226	291	280	221	283	302	308
296	266	238	286	317	276	254	278	247	336	296	259
270	302	281	228	317	312	327	288	395	240	264	246
294	254	222	285	254	264	277	266	228	347	322	232
365	356	261	293	354	236	285	303	275	403	268	250
279	400	370	399	438	287	363	350	278	278	234	266
319	276	291	352	313	262	289	273	317	328	292	279
289	312	334	294	297	304	240	303	255	305	252	286
297	353	350	276	333	285	317	296	276	247	339	328
267	305	291	269	386	264	299	261	284	302	342	304
336	291	294	323	320	289	339	292	373	410	257	406
374	268										

Natural resource managers have attempted to use the Satellite Landsat Multispectral Scanner data for improved land-cover classification. The intensities of reflected light recorded on the near-infrared band of a thermatic mapper are given in Table D.3. Table D.3a gives readings from areas known to consist of forest and Table D.3b readings from urban areas.

TABLE D.3a Near Infrared Light Reflected from Forest Areas

77	77	78	78	81	81	82	82	82	82	82	83	83	84	84	84	84	85	
86	86	86	86	86	87	87	87	87	87	87	87	89	89	89	89	89	89	89
90	90	90	91	91	91	91	91	91	91	91	91	91	93	93	93	93	93	93
94	94	94	94	94	94	94	94	94	94	94	94	95	95	95	95	95	96	96
96	96	96	96	97	97	97	97	97	97	97	97	97	98	99	100	100	100	100
100	100	100	100	100	101	101	101	101	101	101	102	102	102	102	102			
102	103	103	104	104	104	105	107											

TABLE D.3b Near Infrared Light Reflected from Urban Areas

71	72	73	74	75	77	78	79	79	79	79	80	80	80	81	81	81	82	82	82	82
84	84	84	84	84	84	85	85	85	85	85	85	86	86	87	88	90	91	94		

Beginning accounting students need to learn to audit in a computerized environment. A sample of beginning accounting students took a test that is summarized by two scores shown in Table D.4: the Computer Attitude Scale (CAS), based on 20 questions, and the Computer Anxiety Rating Scale (CARS), based on 19 questions. (Courtesy of Douglas Stein.) Males are coded as 1 and females as 0.

TABLE D.4 Computer Attitude and Anxiety Scores

Gender	CAS	CARS	Gender	CAS	CARS
0	2.85	2.90	1	3.30	3.47
1	2.60	2.32	1	2.90	3.05
0	2.20	1.00	1	2.60	2.68
1	2.65	2.58	0	2.25	1.90
1	2.60	2.58	0	1.90	1.84
1	3.20	3.05	1	2.20	1.74
1	3.65	3.74	0	2.30	2.58
0	2.55	1.90	0	1.80	1.58
1	3.15	3.32	1	3.05	2.47
1	2.80	2.74	1	3.15	3.32
0	2.40	2.37	0	2.80	2.90
1	3.20	3.11	0	2.35	2.42
0	3.05	3.32	1	3.70	3.47
1	2.60	2.79	1	2.60	4.00
1	3.35	2.95	0	3.50	3.42
0	3.75	3.79	0	2.95	2.53
0	3.00	3.26	1	2.80	2.68
1	2.80	3.21			

Data were collected on students taking the course Conditioning 1, designed to introduce students to a variety of training techniques to improve cardiorespiratory fitness, muscular strength, and flexibility (Table D.5). (Courtesy of K. Baldridge.)

c1 Gender (1 = male, 2 = female)
c2 Pretest percent body fat
c3 Posttest percent body fat
c4 Pretest time to run 1.5 miles (seconds)
c5 Posttest time to run 1.5 miles (seconds)
c6 Pretest time to row 2.5 kilometers (seconds)
c7 Posttest time to row 2.5 kilometers (seconds)
c8 Pretest number of situps completed in 1 minute
c9 Posttest number of situps completed in 1 minute

TABLE D.5 Physical Fitness Improvement

Gender	Pretest % Fat	Posttest % Fat	Pretest Run	Posttest Run	Pretest Row	Posttest Row	Pretest Situps	Posttest Situps
1	15.1	12.4	575	480	621	559	60	67
1	17.1	18.5	766	672	698	595	42	45
2	25.5	15.0	900	750	840	725	32	36
2	19.5	17.0	715	610	855	753	28	38
2	21.7	20.6	705	585	846	738	46	54
1	17.7	18.5	820	670	630	648	18	41
2	22.7	17.2	880	745	860	788	22	33
2	26.6	22.4	840	725	785	745	29	39
2	36.4	32.5	1065	960	780	749	27	40
1	9.7	8.0	630	565	673	588	32	49
2	31.0	25.0	870	780	746	689	39	54
2	6.0	6.0	580	494	756	714	37	49
2	25.1	22.8	1080	806	852	838	43	38
1	15.1	13.4	720	596	674	576	48	62
2	23.0	21.1	780	718	846	783	30	38
2	9.7	9.7	945	700	890	823	38	44
1	7.0	6.1	706	657	652	521	52	63
1	16.6	15.1	650	567	740	615	38	47
2	21.7	19.5	686	662	762	732	29	47
2	23.7	21.7	758	718	830	719	35	42
2	24.7	21.7	870	705	754	734	24	37
1	11.6	8.9	480	460	640	587	55	60
2	19.5	16.0	715	655	703	731	36	44
1	4.8	4.2	545	530	625	571	40	45
2	31.1	21.6	840	790	745	728	30	36

TABLE D.5 *(Continued)*

Gender	Pretest % Fat	Posttest % Fat	Pretest Run	Posttest Run	Pretest Row	Posttest Row	Pretest Situps	Posttest Situps
1	23.2	19.9	617	622	637	620	35	32
1	12.5	12.2	635	600	805	736	37	52
2	29.3	21.7	790	715	821	704	41	42
2	19.5	16.0	750	702	1043	989	42	45
1	10.7	9.8	622	567	706	645	40	46
2	26.6	21.7	722	725	741	734	49	61
2	27.6	23.7	641	598	694	682	40	47
1	25.9	18.8	708	609	685	593	20	21
1	27.5	16.0	675	637	694	682	35	37
1	9.4	7.6	618	566	610	579	36	50
1	12.5	9.8	613	552	610	575	46	50
2	27.5	30.9	705	660	746	691	31	36
2	18.3	15.0	853	720	748	694	31	34
1	6.6	4.7	496	476	623	569	45	45
2	23.7	21.7	860	750	758	711	42	55
2	23.2	22.1	905	636	759	726	26	30
2	18.0	14.8	900	805	823	759	31	29
2	22.7	18.3	767	741	808	753	28	32
1	19.9	14.8	830	620	632	586	39	45
1	6.6	5.7	559	513	647	602	44	49
1	14.3	9.8	699	652	638	602	34	35
1	21.7	18.5	765	735	674	615	41	48
1	13.9	9.4	590	570	599	571	40	42
1	20.1	19.3	770	672	675	611	37	48
1	15.1	8.9	602	560	656	578	47	58
2	14.8	11.0	741	610	768	687	34	41
2	23.4	19.1	723	641	711	695	36	41
2	23.7	16.3	648	601	802	740	44	51
1	28.0	17.7	842	702	790	765	26	29
1	4.7	4.7	558	540	660	600	40	46
2	26.6	23.7	750	565	720	670	36	52
2	21.7	18.3	608	592	707	697	42	46
1	11.6	8.9	537	495	610	572	55	60
2	22.7	18.3	855	694	800	712	41	50
2	21.7	18.3	630	614	785	743	50	55
2	25.9	21.9	902	820	771	717	34	38
1	28.4	21.7	780	664	756	703	57	64
2	21.9	20.8	665	670	673	667	41	44
2	27.5	22.7	675	646	689	674	49	50
1	5.7	3.8	473	472	551	546	53	53
2	22.7	22.0	715	682	678	672	40	43
2	33.2	25.7	795	740	817	721	30	31
2	16.0	13.6	688	615	811	705	45	49
1	14.3	11.6	530	497	589	570	39	50

TABLE D.5 *(Continued)*

Gender	Pretest % Fat	Posttest % Fat	Pretest Run	Posttest Run	Pretest Row	Posttest Row	Pretest Situps	Posttest Situps
1	22.7	19.5	840	705	788	780	29	32
2	25.0	18.3	690	618	816	701	42	50
1	4.2	3.2	545	527	577	543	55	59
2	25.7	21.7	760	727	849	724	39	44
1	13.9	9.4	620	515	689	580	32	41
1	12.2	7.6	605	564	661	614	35	38
1	21.7	20.6	688	625	750	686	39	32
1	7.0	5.1	590	529	631	619	60	65
2	34.6	27.5	720	694	690	698	34	45
1	3.2	3.2	500	459	644	599	35	37
1	6.1	5.1	540	492	579	546	56	60
2	28.4	21.7	885	825	804	733	33	36

Grizzly bears are magnificent animals that weigh up to several hundred pounds and can run over 30 mph in short distances. Males range over several miles but the females tend to stay closer to home. Grizzly bears usually keep to themselves. However, several grizzly bear attacks on humans are reported each year. Because of their large size, they can menace hikers and campers in some circumstances. Even so, it is important that a healthy grizzly bear population be maintained for future generations.

Harry Reynolds, of the Alaska Game and Fish Department, has studied grizzly bears for over 20 years in an effort to protect them and learn what they need to survive. He and his colleagues typically spot bears from an airplane. Once a bear is located, they try to shoot it with a dart so that it falls asleep for a few minutes. During this brief period, they check its overall health and take measurements of the size of the bear. They also roll the bear onto a tarp, which is then lifted by a scale attached to a tripod. Age is given in years, weight in pounds, and the other size measurements in centimeters.

Occasionally, the researchers have to run to a vehicle for protection when the sedated bear awakes too quickly. These measurements are not easy to make!

TABLE D.6 Grizzly Bear Data

Bear	Sex	Age	Weight	Length	Neck	Girth	Head Length	Head Width
1	M	9	245	199	84	135	24.5	39.0
2	M	9	200	198	83	129	22.5	39.3
3	F	4	102	174	65	104	20.3	33.4
4	F	5	180	183	57	101	17.9	32.7
5	F	5	225	170	57	100	18.6	33.3
6	F	5	230	188	62	103	19.3	32.9
7	F	13	315	185	62	118	22.1	36.9
8	F	19	280	193	63	114	22.2	35.5
9	F	5	235	177	56	93	19.4	33.4
10	F	2	70	152	46	80	16.8	31.3
11	M	2	91	157	58	102	17.3	30.6
12	F	6	280	181	61	100	19.4	34.5
13	M	11	465	204	84	132	23.7	38.8
14	M	4	125	191	60	97	20.1	33.4
15	F	6	265	189	59	100	20.4	33.3
16	F	3	107	156	45	81	16.3	28.1
17	F	2	68	158	43	77	16.0	29.9
18	M	2	75	163	45	77	17.8	29.7
19	M	2	82	164	48	80	16.7	32.3
20	F	6	245	176	66	104	20.5	34.5
21	F	4	215	176	55	103	19.1	32.0
22	M	2	84	169	53	89	19.9	31.5
23	F	17	109	176	62	112	19.0	32.3
24	F	5	240	185	61	103	20.5	33.6
25	F	5	247	175	57	102	18.5	33.3

TABLE D.6 *(Continued)*

Bear	Sex	Age	Weight	Length	Neck	Girth	Head Length	Head Width
26	F	3	130	148	45	77	13.6	27.4
27	F	3	130	166	53	86	17.5	30.0
28	F	7	280	177	57	106	18.7	33.1
29	M	11	620	224	95	164	24.2	42.0
30	M	3	150	151	49	85	18.0	32.4
31	F	1	115	134	45	78	14.4	26.5
32	F	1	115	137	43	77	14.0	26.8
33	F	4	285	188	64	110	19.6	34.0
34	M	4	415	199	77	137	23.0	38.5
35	F	1	120	135	49	77	14.0	26.3
36	M	6	578	219	90	134	24.8	38.6
37	M	2	140	148	56	88	17.0	29.9
38	F	2	125	131	48	80	16.3	27.2
39	M	4	265	178	63	102	18.8	32.2
40	F	2	135	153	49	83	16.2	28.7
41	M	2	212	171	51	88	17.8	31.4
42	F	2	190	168	49	81	17.4	30.3
43	M	5	359	198	76	114	21.4	36.6
44	F	7	187	166	57	95	19.4	32.4
45	M	1	62	114	37	66	12.4	26.9
46	F	17	283	188	67	113	20.5	37.0
47	M	1	126	145	46	84	15.2	27.4
48	F	4	196	168	56	90	18.2	31.5
49	M	4	345	192	69	112	20.3	35.2
50	M	2	195	175	53	92	18.2	30.2
51	M	1	100	125	41	73	14.1	24.5
52	F	1	50	107	33	58	12.1	21.1
53	M	1	100	131	45	74	15.1	26.7
54	F	1	65	119	39	53	13.2	22.5
55	F	1	105	136	42	71	15.0	25.5
56	M	1	110	144	43	70	15.4	26.5
57	F	1	100	133	45	66	16.4	25.2
58	M	1	63	116	38	62	13.3	23.5
59	F	1	70	123	41	63	13.5	24.2
60	F	1	96	139	46	76	14.5	25.6
61	M	1	95	142	48	76	14.4	25.6

Salmon fisheries support a primary industry in Alaska and their management is of high priority. Salmon are born in freshwater rivers and streams, but then swim out into the ocean for a few years before returning to spawn and die. In order to identify the origins of mature fish and equably divide the catch of returning salmon between Alaska and the Canadian provinces, researchers have studied the growth of their scales. The growth the first year in freshwater is measured by the width of the growth rings for that period of life, and marine growth is measured by the width of the growth rings for the first year in the ocean environment. The scales are first magnified 100 times and then the measurements are made in hundredths of an inch. A set of these measurements, collected by the Alaska Department of Fish and Game, are given in Table D.7. (Courtesy of K. Jensen and B. Van Alen.)

TABLE D.7 Radius of Growth Zones for Freshwater and First Marine Year

Males		Females	
Freshwater Growth	First Year Marine Growth	Freshwater Growth	First Year Marine Growth
147	444	131	405
139	446	113	422
160	438	137	428
99	437	121	469
120	405	139	424
151	435	144	402
115	394	161	440
121	406	107	410
109	440	129	366
119	414	123	422
130	444	148	410
110	465	129	352
127	457	119	414
100	498	134	396
115	452	139	473
117	418	140	398
112	502	126	434
116	478	116	395
98	500	112	334
98	589	117	455
83	480	97	439
85	424	134	511
88	455	88	432
98	439	99	381
74	423	105	418
58	411	112	475
114	484	98	436
88	447	80	431

TABLE D.7 *(Continued)*

Males		Females	
Freshwater Growth	First Year Marine Growth	Freshwater Growth	First Year Marine Growth
77	448	139	515
86	450	97	508
86	493	103	429
65	495	93	420
127	470	85	424
91	454	60	456
76	430	115	491
44	448	113	474
42	512	91	421
50	417	109	451
57	466	122	442
42	496	68	363

The U. S. Department of Agriculture and State Agriculture Experiment Stations cooperate on the investigation of barleys for possible use in brewing processes. One year, the malt extract (%) was obtained for 40 different varieties from one experiment station.

TABLE D.8 Malt Extract

75.3	77.9	77.6	76.6	78.3	77.9	77.5	77.6	77.1	78.0
77.9	76.3	75.7	77.4	77.4	76.9	77.9	77.4	78.1	77.4
76.4	79.1	80.0	76.9	78.5	78.4	77.8	80.4	75.9	77.0
79.2	76.2	77.0	75.9	77.9	78.4	76.7	76.4	76.6	77.4

An ongoing study of wolves is being conducted at the Yukon-Charley Rivers National Preserve. Here are some of the physical characteristics of wolves that were observed. (Courtesy of John Burch National Park Service, Fairbanks, Alaska.)

TABLE D.9 Physical Characteristics of Wolves

Sex	Age	Weight (lb)	Body Length (cm)	Tail Length (cm)	Canine Length (mm)
M	4	71	134	44	28.7
F	2	57	123	46	27.0
F	4	84	129	49	27.2
M	4	93	143	46	30.5
M	4	101	148	48	32.3
M	1	84	127	42	25.8
M	2	88	136	47	26.6
M	3	117	146	46	29.1
F	2	90	143	43	27.1
M	4	86	142	51	29.2
F	6	71	124	42	28.2
F	8	71	125	42	27.8
M	0	86	139	54	24.0
M	2	93	140	45	29.0
M	2	86	133	44	29.3
F	3	77	122	45	27.4
F	2	68	125	51	27.3
M	3	106	123	53	29.0
F	0	73	122	43	24.5

Researchers studying sleep disorders needed to obtain data from the general population to serve as a reference set. They randomly selected state employees who worked in certain divisions, and many agreed to spend a night in the sleep lab and be measured. Female = 0 and male = 1. The body weight index (BMI) is a person's weight (kg) divided by the square of his or her height (m). The percent of rapid eye movement (PREM) sleep is the percent of total sleep time that is spent in rapid eye movement sleep. The number of breathing pauses per hour (BPH) is the total number of breathing pauses divided by the total hours of sleep. The snoring frequency (SNORF) is a response to the survey question, "According to what others have told you, please estimate how often you snore?" The possible responses were (1) rarely, (2) sometimes, (3) irregular pattern but at least once a week, (4) three to five nights a week, or (5) every or almost every night. (Courtesy of T. Young.)

TABLE D.10 Sleep Data

Gender	Age	BMI	PREM	BPH	SNORF	Gender	Age	BMI	PREM	BPH	SNORF
0	41	24.65	0.200	0.00	5	0	55	25.10	0.326	0.19	0
0	49	23.14	0.162	0.00	4	0	31	28.17	0.124	0.21	1
0	39	27.61	0.308	0.00	5	0	58	32.68	0.216	5.76	1
0	51	33.50	0.083	0.00	4	0	43	20.07	0.191	0.00	0
0	32	43.36	0.205	2.79	4	0	53	24.92	0.189	7.46	0
0	37	31.04	0.109	3.87	5	0	52	29.10	0.194	0.69	2
0	36	32.42	0.154	0.00	5	0	43	26.40	0.077	0.00	2
0	30	31.06	0.328	3.42	5	0	41	20.13	0.138	0.12	1
0	44	44.15	0.227	4.39	5	0	34	23.77	0.260	0.00	0
0	43	38.29	0.104	5.14	5	0	41	29.06	0.163	0.31	2
0	34	21.63	0.184	0.00	5	0	49	24.45	0.164	0.00	0
0	47	34.78	0.215	1.13	5	0	41	21.15	0.142	0.41	0
0	40	25.16	0.206	0.00	2	0	47	19.47	0.224	0.00	1
0	49	37.02	0.123	2.58	5	0	59	28.76	0.176	0.99	0
0	54	31.25	0.313	0.00	0	0	34	26.77	0.109	0.00	0
0	38	32.58	0.227	0.13	5	0	42	40.06	0.191	0.14	0
0	36	34.65	0.172	0.00	4	0	44	24.99	0.316	1.01	2
0	39	29.55	0.223	2.84	5	0	32	17.86	0.174	0.00	0
0	45	31.96	0.146	0.00	4	0	31	25.99	0.109	0.00	0
0	49	21.64	0.250	0.00	5	0	35	20.42	0.227	0.25	2
0	47	30.67	0.244	3.98	5	0	58	23.03	0.202	0.00	1
0	33	23.43	0.228	0.00	2	0	38	41.14	0.220	1.24	2
0	43	30.24	0.193	0.00	1	0	40	22.49	0.149	0.00	0
0	43	37.83	0.230	7.11	5	0	43	23.71	0.149	0.00	0
0	39	22.46	0.141	0.17	5	1	44	30.22	0.162	2.33	5
0	50	33.53	0.094	3.66	5	1	59	27.39	0.103	0.00	3
0	57	24.35	0.174	0.33	4	1	51	24.34	0.203	1.30	5
0	36	37.64	0.000	25.36	2	1	49	27.17	0.179	0.00	5
0	34	28.03	0.205	0.00	1	1	29	28.09	0.228	0.00	4
0	43	25.24	0.223	0.00	1	1	40	30.06	0.180	3.18	5
0	59	33.71	0.175	4.71	0	1	45	40.82	0.082	2.34	5

TABLE D.10 *(Continued)*

Gender	Age	BMI	PREM	BPH	SNORF	Gender	Age	BMI	PREM	BPH	SNORF
1	55	32.46	0.082	38.00	5	1	47	24.00	0.220	0.53	5
1	51	22.95	0.175	3.29	4	1	36	31.76	0.196	6.73	5
1	38	21.97	0.123	0.00	5	1	59	27.12	0.255	6.18	5
1	36	25.57	0.179	0.00	4	1	42	24.34	0.137	1.46	5
1	39	26.67	0.216	0.24	5	1	59	35.14	0.160	3.56	5
1	38	28.95	0.231	0.55	5	1	37	26.53	0.122	3.80	4
1	42	35.65	0.139	0.20	5	1	55	31.28	0.170	0.58	4
1	34	43.65	0.131	47.52	4	1	31	35.61	0.066	19.60	4
1	55	42.70	0.191	5.41	5	1	53	27.55	0.083	8.01	4
1	42	25.04	0.242	0.00	5	1	41	34.92	0.207	15.28	4
1	47	31.28	0.147	7.26	5	1	45	33.20	0.228	4.37	5
1	46	31.02	0.124	22.55	5	1	41	29.94	0.134	2.42	5
1	39	25.43	0.169	10.35	5	1	50	26.26	0.179	0.16	5
1	37	26.29	0.178	7.05	5	1	40	34.94	0.174	8.76	5
1	41	29.59	0.259	1.37	5	1	54	28.60	0.197	0.00	4
1	58	29.24	0.170	3.89	5	1	39	31.55	0.135	0.81	5
1	57	28.72	0.152	2.33	5	1	49	22.84	0.109	1.53	2
1	33	28.56	0.126	1.09	5	1	46	18.89	0.114	0.00	0
1	58	30.04	0.160	0.18	4	1	39	25.70	0.233	0.00	0
1	54	35.43	0.117	1.24	5	1	41	26.89	0.247	1.94	1
1	46	34.78	0.151	19.47	5	1	46	31.29	0.230	0.00	2
1	46	25.66	0.223	0.00	3	1	40	29.76	0.000	0.71	0
1	55	31.80	0.087	4.58	5	1	44	26.71	0.163	0.39	2
1	56	29.74	0.162	18.77	5	1	41	24.69	0.209	0.16	0
1	35	24.43	0.423	0.16	5	1	37	26.58	0.074	0.00	0
1	49	23.33	0.196	21.58	4	1	44	23.61	0.234	0.00	0
1	50	26.45	0.199	0.00	4	1	44	29.86	0.147	5.13	2
1	42	27.38	0.155	2.63	5	1	41	28.41	0.097	0.19	1
1	46	27.06	0.123	0.00	5	1	46	23.12	0.156	0.19	0
1	37	24.82	0.175	0.18	4	1	34	29.88	0.195	0.00	0
1	38	30.64	0.190	70.82	5	1	41	18.02	0.149	0.00	0
1	44	37.61	0.254	7.61	5	1	44	20.90	0.181	0.21	0
1	48	26.85	0.219	0.42	4	1	39	22.39	0.223	0.00	1
1	53	28.23	0.103	0.33	4	1	35	29.23	0.152	0.00	2
1	55	29.48	0.150	4.57	5	1	38	23.83	0.163	0.00	2
1	56	27.76	0.185	7.82	5	1	47	22.23	0.129	0.76	1
1	57	32.08	0.205	28.26	5	1	47	28.04	0.146	2.53	0
1	58	27.55	0.128	26.37	5	1	50	32.82	0.183	12.86	0
1	37	31.98	0.241	5.38	1	1	48	20.09	0.083	1.59	2
1	30	24.09	0.188	0.68	5	1	38	26.63	0.229	1.81	2
1	51	33.80	0.169	0.90	5	1	37	30.72	0.203	2.13	2
1	44	28.08	0.130	0.53	5	1	50	24.73	0.188	0.00	2
1	45	30.83	0.105	6.86	4	1	37	33.35	0.274	17.78	2
						1	44	24.70	0.170	0.42	2
						1	49	18.24	0.223	0.00	1
						1	41	24.86	0.216	2.31	0

Insecticides, including the long banned DDT, which imitate the human reproductive hormone estrogen, may cause serious health problems in humans and animals. Researchers examined the reproductive development of young alligators hatched from eggs taken from (1) Lake Apopka, which is adjacent to an EPA Superfund site, and (2) Lake Woodruff, which acted as a control. The contaminants at the first lake, including DDT, were thought to have caused reproductive disorders in animals living in and around the lake. The concentrations of the sex steroids estradiol and testosterone in the blood of alligators were determined by radioimmunoassay both at about six months of age and then again after the alligators were stimulated with LH, a pituitary hormone. (Courtesy of L. Guillette.)

The data are coded as (* indicates missing):

x_1 = 1, Lake Apopka, and = 0, Lake Woodruff

x_2 = 1 male and = 0 female

x_3 = E2 = estradiol concentration (pg/ml)

x_4 = T = testosterone concentration (pg/ml)

x_5 = LHE2 = estradiol concentration after receiving LH (pg/ml)

x_6 = LHT = testosterone concentration after receiving LH (pg/ml)

TABLE D.11 Alligator Data

Lake Apopka						Lake Woodruff					
Lake x_1	Sex x_2	E2 x_3	T x_4	LHE2 x_5	LHT x_6	Lake x_1	Sex x_2	E2 x_3	T x_4	LHE2 x_5	LHT x_6
1	1	38	22	134	15	0	1	29	47	46	10
1	1	23	24	109	28	0	1	64	20	82	76
1	1	53	8	16	12	0	1	19	60	*	*
1	1	37	6	220	13	0	1	36	75	19	72
1	1	30	7	114	11	0	1	27	12	118	95
1	0	60	19	184	7	0	1	16	54	33	64
1	0	73	23	143	13	0	1	15	33	99	19
1	0	64	16	228	13	0	1	72	53	29	20
1	0	101	8	163	10	0	1	85	100	72	0
1	0	137	9	83	7	0	0	139	20	82	2
1	0	88	7	200	12	0	0	74	4	170	75
1	0	73	19	220	21	0	0	83	18	125	45
1	0	257	8	194	37	0	0	35	43	19	76
1	0	138	10	221	3	0	0	106	9	142	5
1	0	220	15	101	5	0	0	47	52	24	62
1	0	88	10	141	7	0	0	38	8	68	20
						0	0	65	15	32	50
						0	0	21	7	140	4
						0	0	68	16	110	3
						0	0	70	16	58	18
						0	0	112	14	78	5

Market researchers track attitudes and preferences of potential purchasers. An increasing number of brands are being linked to social causes and this study[2] was conducted to help determine if adding a social cause to products would cause a difference in consumers' preferences regarding credit cards. Questions about credit card use and preferences, as well as attitudes toward sharing wealth with others, were included.

Female = 0 and male = 1. The three age groups 19–24, 25–34, and 35–44 are coded as 1, 2, and 3, respectively. Charge is the average monthly amount charged on credit cards. The last three variables are measured on seven point rating scales. The frequency of credit card use, FreqCC, ranges from never (1) to several times a day (7), with weekly (4) in the middle. "Money given to charities goes for good causes" and "I consider myself a compassionate person" scored from strongly disagree (1) to strongly agree (7). (Courtesy of Neeraj Arora)

TABLE D.12 Credit Card Data

Row	Gender	Age	Charge	FreqCC	Charity	Compassion
1	0	3	100	1	6	6
2	1	3	150	3	5	7
3	0	3	500	3	7	7
4	1	3	0	1	5	7
5	1	3	0	2	6	6
6	0	1	100	4	5	5
7	1	3	1100	6	6	6
8	1	3	1000	5	5	7
9	0	3	200	6	7	7
10	1	3	1000	5	7	7
11	1	2	200	5	7	7
12	1	1	500	4	5	5
13	1	3	100	4	5	6
14	1	3	20	3	6	7
15	0	3	0	1	7	7
16	0	2	400	6	6	5
17	1	1	0	1	5	7
18	1	3	0	1	6	6
19	0	1	800	7	6	5
20	1	3	1000	7	7	7
21	1	1	100	4	5	4
22	1	3	500	4	5	5
23	1	3	0	4	4	5
24	1	2	300	4	5	5
25	1	2	400	5	5	5
26	1	1	50	4	6	7

[2] N. Arora and T. Henderson. "Embedded Premium Promotion: Why it Works and How to Make it More Effective," *Marketing Science*, **26**, (2007), pp. 514–531.

TABLE D.12 *(Continued)*

Row	Gender	Age	Charge	FreqCC	Charity	Compassion
27	1	2	100	4	6	6
28	1	2	200	5	4	7
29	0	2	1	6	5	5
30	1	2	2000	7	1	4
31	1	1	450	3	7	7
32	1	2	100	2	3	6
33	1	3	0	1	5	6
34	1	2	600	4	6	6
35	1	3	100	3	6	7
36	1	3	700	5	7	7
37	1	3	500	4	6	7
38	1	2	300	7	6	6
39	1	3	100	4	7	7
40	1	3	1500	7	7	5
41	0	3	600	6	6	6
42	1	1	40	2	6	7
43	1	2	1000	7	4	5
44	1	3	100	3	6	7
45	1	2	300	3	6	6
46	1	1	0	1	7	7
47	1	3	150	5	6	6
48	1	2	10	2	6	7
49	0	2	1000	5	7	7
50	1	2	500	5	7	7
51	0	3	0	2	4	6
52	0	1	500	4	5	4
53	1	2	450	4	5	5
54	1	2	200	5	6	7
55	1	2	25	3	2	6
56	1	2	500	5	5	6
57	1	3	70	4	2	4
58	0	3	300	4	5	1
59	1	3	300	5	4	6
60	1	1	0	2	4	6
61	1	1	0	1	5	6
62	1	2	200	5	5	5
63	0	2	2000	4	4	4
64	0	3	0	1	7	7
65	1	3	1500	7	4	5
66	1	2	0	1	3	3
67	1	1	60	3	6	6
68	1	3	500	5	7	7
69	0	3	0	1	5	7
70	0	3	1200	5	5	6
71	0	3	2000	6	5	5

TABLE D.12 *(Continued)*

Row	Gender	Age	Charge	FreqCC	Charity	Compassion
72	0	2	700	6	2	7
73	1	3	0	2	5	4
74	0	1	200	3	5	6
75	0	3	3000	6	6	6
76	1	3	50	5	5	3
77	0	3	300	4	6	6
78	1	3	3000	6	3	7
79	0	3	0	1	6	7
80	0	1	0	1	7	7
81	1	2	0	1	6	7
82	1	3	50	6	4	4
83	1	3	500	6	5	6
84	0	1	50	2	2	2
85	1	1	40	3	4	5
86	0	3	150	2	4	7
87	1	3	25	3	5	7
88	0	3	800	5	3	6
89	0	3	3500	7	5	6
90	1	2	100	5	3	7
91	0	2	200	3	1	7
92	0	3	400	5	5	6
93	1	1	300	5	3	3
94	1	2	350	5	6	6

Answers to Selected Odd-Numbered Exercises

CHAPTER 1

1.1 Population: Entire set of responses from all U.S. teens 13 to 17. Sample: responses from the 1055 teens contacted.

1.3 (a) person living in Chicago, (b) whether a person is an illegal alien(yes) or not (no), (c) entire set of yes/no responses from all persons living in Chicago.

1.5 (a) Anyone who golfs (b) The statistical population is the collection of estimated hole sizes, one for each golfer. The sample is the collection of 46 hole sizes the psychologists recorded.

1.7 It is likely that any person visiting the site does so because they have a problem with procrastination. The scores recorded would not be representative of the general population.

1.9 Population: The entire collection of responses, from all residents, of their favorite establishment. Sample: the responses from the persons who filled out the form. This is a self-selected sample, and not representative, as only those with strong opinions would fill out form.

1.11 (a) anecdotal, (b) sample based, (c) sample based

1.13 "Too long" is not well defined. Could, for instance, say half the time it takes longer than 5 minutes.

1.21 (b) Including votes only from those with strong enough opinions to call in, like big dogs jumping above the fence, would produce unrepresentative results.

CHAPTER 2

2.1 (a) 21.7% (c) 32.7%, 45.5%, 78.3%

2.3 Relative frequencies are 7, 10, 13, 5, 2, 1, 1, 1

2.5 (a) The relative frequencies are Drive alone 0.625, Car pool 0.075, Ride bus 0.175, and Other 0.125.

2.7 3 belongs to two classes.

2.9 5 doesn't belong to any class.

2.11 (a) Yes (d) No (e) No

2.13 (c) 9/50 = .18

2.17 (c) 1/15 = .067

2.19 (a) The frequency table

Class	Frequency
0.45–0.90	2
0.90–1.35	6
1.35–1.80	11
1.80–2.25	5
2.25–2.70	6

2.21 The frequencies are 12, 15, 10, 10 and 8. The classes are not of equal length.

2.23 The stem-and-leaf display:

0	6
1	2234455567777889
2	000000222445567799
3	022444566
4	1167
5	12

2.25 The double-stem display:

0	6	3	022444
1	22344	3	566
1	55567777889	4	11
2	00000022244	4	67
2	5567799	5	12

2.27 The five-stem display Leaf Unit = 1.0

18	1
18	2
18	
18	66
18	8
19	
19	2
19	44555
19	
19	888
20	
20	3
20	
20	
20	
21	0001
21	
21	
21	6
21	89
22	1
22	
22	
22	7

2.29 (a) Median = 3 and $\bar{x}$ = 3
(c) Median = 1 and $\bar{x}$ = 1

2.31 (a) $\bar{x}$ = 254 (b) Median since one large observation heavily influences the mean.

2.33 Claim ignores variability. July with 105 is hot.

2.35 (a) $\bar{x}$ = 8.48 (b) Either

2.37 Mean = 118.05 is center where sum of positive deviations balance the sum of negative deviations. Median = 117.00 has at least half of weights the same or smaller and at least half the same or larger.

2.39 (a) 2.60 (b) 2.00

2.43 181.5

2.45 (b) $\bar{x}$ = 26.62; sample median = 24
(c) Q_1 = 22, Q_2 = 24, and Q_3 = 30

2.47 (a) Sample median = 153
(b) Q_1 = 135.5, Q_2 = 153, and Q_3 = 166.5

2.49 Sample median = 94, Q_1 = 73 and Q_3 = 105

2.51 (a) Median = 110; Q_1 = 60, Q_3 = 340

(b) 90th percentile = 400
2.53 (b) Mean = 24.33°C; median = 24.44°C
2.57 (b) $s^2 = 7$ and $s = 2.646$
2.59 (b) $s^2 = 18.667$ and $s = 4.32$
2.63 (a) $s^2 = 1.30$ (c) $s^2 = .286$
2.67 (a) $s^2 = 155{,}226$ (b) $s = 393.99$
2.71 (a) Median = 68.4
(b) $\bar{x} = 68.343$ (c) $s = 2.419$
2.73 (a) $s^2 = 239.32$ (b) 25.00
(c) 8 is nearly half as variable.
2.75 Interquartile range = 31.0
2.77 No. Extremes more variable than middle.
2.81 (a) $\bar{x} = 2.600$ and $s = 1.303$ (b) proportions .5, 1.0 and 1.0, respectively.
2.83 (a) −1.037 (b) 1.677
2.87 (a) Mean = 24.83, standard deviation = 7.68 (b) 23/24 = .958
2.89 Mean = 1.925, standard deviation = 1.607
2.91 (a) 14.0 (b) maximum 55 Truman and minimum −8 (gain) Bush (c) 63
2.95 Only the value 215 from the second period is outside of limits.
2.97 First an upward trend and then a downward trend. Control limits not appropriate.
2.99 The relative frequencies of the occupation groups are:

Occupation Group	Relative Frequency 2007	2000
Goods producing	0.139	0.161
Service (private)	0.722	0.702
Government	0.139	0.136
Total	1.000	0.999

2.103 (a) Yes (b) Yes (c) No
2.105 (a) Mean = 227.4 pounds is center where sum of positive deviations balance the sum of negative deviations. Median = 232.5 has at least half of weights the same or smaller and at least half the same or larger. (b) 82.7 (c) 1.120
2.107 (a) Median = 9 (b) $\bar{x} = 9.033$
(c) $s^2 = 3.895$
2.109 (a) $\bar{x} = 7, s = 2$ (c) Mean = −21, standard deviation = 6.
2.113 (a) $\bar{x} = 5.375$ and $s = 3.424$ (b) Median = 5.0 (c) Range = 13
2.115 (a) Median = 4.505, $Q_1 = 4.30$, $Q_3 = 4.70$ (b) 4.935
(c) $\bar{x} = 4.5074, s = .368$
2.117 (a) Median = 6.7, $Q_1 = 6.4$, $Q_3 = 7.1$
(b) $\bar{x} = 6.73$ and $s = .466$
2.119 (b) Not reasonable with time trend
2.121 Mean = 83.34 and standard deviation = 38.37
2.129 $\bar{x} = 109.1, s = 65.8$

CHAPTER 3

3.1 (c) The pill seems to reduce the proportions of severe and moderate nausea.
3.3 The proportion who study more than 10 hours is highest for the physical sciences. It is .70 while social sciences have the lowest proportion .48.
3.7 (b)

	Major B	H	P	S	Total
Male	.245	.082	.102	.286	.715
Female	.122	0	.082	.082	.286
Total	.367	.082	.184	.368	1.001

3.11 (b) Research hospital best for either condition.
3.15 (b) Positive—better players are paid more.
3.17 No. There is not a straight line relationship.
3.19 (b) $r = -.415$
3.21 (a) Figure (c) (b) Figure (b)
(c) Figure (a)
3.23 $r = -.578$
3.25 $r = .891$
3.27 (b) $r = .279$
3.33 (b) $r = .996$
3.37 (b) 16 or more

3.39 (a) $\hat{\beta}_0 = .52 \quad \hat{\beta}_1 = .70$
3.41 (a) $\hat{\beta}_0 = 2.143 \quad \hat{\beta}_1 = 1.085$
3.43 (b) $\hat{y} = -157.6 + 1.385x$
3.47 (c)

2-Wheel	4-Wheel	Total
.343	.657	1.000
.444	.556	1.000

3.49 (a) negative. Only so much time
(c) no relation
3.51 (b) $r = .988$
3.53 (b) $r = -.158$
3.55 (c) $\hat{y} = 1.15 + .37x$
3.57 (a) x = road roughness and y = gas consumption
3.59 (b) $r = .649$
3.61 (b) $r = .437$

CHAPTER 4

4.1 (a) (iv) (d) (vi) (f) (i)
4.3 (a) (ii) (b) (iii)
4.5 (a) $\{0, 1\}$
(b) $\{0, 1, \ldots, 344\}$
(c) $\{t: 90 < t < 425.4\}$
4.7 (a) $S = \{BJ, BL, JB, JL, LB, LJ, BS, JS, LS, SB, SJ, SL\}$
(b) $A = \{LB, LJ, LS\}$
$B = \{JL, LJ, JS, SJ, LS, SL\}$
4.9 .1
4.11 (a) Yes
(b) No, because the total is less than 1
(c) Yes
4.13 .2
4.15 .799
4.17 (c) $\frac{3}{8}$
4.19 (a) $S = \{e_1, e_2, e_3\}$ where e_i = [the ticket has number i]; $P(e_1) = \frac{2}{8}$, $P(e_2) = \frac{3}{8}$, $P(e_3) = \frac{3}{8}$
(b) $\frac{5}{8}$
4.21 (c) $P(A) = \frac{5}{36}$, $P(B) = \frac{1}{6}$, $P(C) = \frac{1}{2}$, $P(D) = \frac{1}{6}$
4.23 (b) $\frac{4}{9}$
4.25 (a) $\frac{1}{15}$ (b) $\frac{11}{15}$ (c) $\frac{12}{15}$
4.27 $S = \{N, YN, YYN, YYYN, YYYYN, YYYYY\}$
4.29 (b) $\frac{1}{7}$
4.33 (b) $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$
4.35 (b) (i) $AB = \{e_6, e_7\}$
(iii) $A\overline{B} = \{e_4, e_5\}$
4.37 (b) $AB = \{e_2, e_6, e_7\}$, $P(AB) = .40$
(d) $\overline{A}\,\overline{C} = \{e_3, e_4\}$, $P(\overline{A}\,\overline{C}) = .24$
4.39 (b) $A \cup B = \{e_1, e_3, e_4\}$, $AB = \{e_3\}$
4.41 (a) $P(A) = .30$ and $P(B) = .36$
(c) $P(A \cup B) = .40$
4.43 (b) $P(A\overline{B}) = .13$, $P(\overline{A}B) = .38$, $P(\overline{A}\,\overline{B}) = .37$
4.45 (b) $P(\overline{A}B) = .36$
(c) $P(\overline{A} \cup B) = .73$ (d) .59
4.47 (a) $P(\overline{A}) = .65$ (b) $P(A\overline{B}) = .15$
4.49 $\frac{7}{18}$
4.51 (a) $P(B) = .35$
4.55 Higher, they are dependent.
4.57 (a) .48 (b) .52 (c) .507
4.59 $P(B \mid A) = .0099$, not independent
4.63 (a) $P(A) = .4$, $P(B) = .4$
(b) Not independent because $P(AB) = .1 \neq P(A)P(B)$
4.65 (b) 4.8% (c) .360
4.67 (a) .688 (c) .769
4.69 (a) .25 (c) $\frac{1}{22}$ (e) $\frac{9}{22}$
4.71 (a) $\frac{2}{9}$ (b) $\frac{7}{12}$
4.73 (a) BC, $P(BC) = 0$
(c) $\overline{B}$, $P(\overline{B}) = .8$
4.75 .99991
4.77 (b) .896
4.79 (a) .55 (b) Yes
4.81 $P(B \mid A) = .577$
4.83 (a) 20 (c) 231 (e) 4060
4.85 (a) 210 (b) 90
4.87 (a) 462 (b) 210
4.91 (a) .491 (b) .084
4.93 No
4.97 (a) .018 (b) .393
4.99 (a) 330 (b) .182
4.105 (a) $A = \{23, 24\}$

(d) $A = \{t : 0 \leq t < 500.5\}$

4.107 $A = \{p : .10 < p < 100\}$, p is percent

4.109 (a) .5 (b) .75 (c) .167

4.111 .5

4.113 (a) $\frac{4}{9}$ (b) $\frac{2}{9}$

4.115 (b) 2/3

4.117 (a) Either a faulty transmission or faulty brakes or both
(b) Transmission, brakes, and exhaust system all faulty

4.119 $P(A) = .3$, $P(AB) = .10$, $P(A \cup B) = .5$

4.123 (a) .25 (b) .73

4.125 (a) .25 (c) .34

4.127 $P(A|B) = .8$, they are independent.

4.129 (a) $P(A)P(C) = .15$, $P(AC) = .15$, independent
(b) $P(A\overline{B})P(C) = .1$, $P(A\overline{B}C) = .15$, not independent

4.131 (a) .44 (b) .50
(c) Not independent

4.135 b(ii) .64 b(iii) .49

4.137 $\frac{1}{165} = .0061$

4.139 (a) .108 (b) .515

4.141 (a) .008

CHAPTER 5

5.1 (a) Discrete (b) Continuous
(c) Continuous

5.3 (a)

Possible choices	x
{1, 3}	2
{1, 5}	4
{1, 6}	5
{1, 7}	6
{3, 5}	2
{3, 6}	3
{3, 7}	4
{5, 6}	1
{5, 7}	2
{6, 7}	1

(b)

x	1	2	3	4	5	6
$f(x)$	.2	.3	.1	.2	.1	.1

5.9 (c)

x	2	3	4	5	6
$f(x)$	.1	.4	.1	.2	.2

5.13 (a)

x	1	2	3	4	Total
$f(x)$	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	1

Yes, a probability distribution

(b)

x	1	2	3	4	Total
$f(x)$	$-\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{2}{3}$	1

Not a probability distribution; $f(1)$ is negative.

5.15

x	0	1	2
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

5.17

x	0	1	2	3	4
$f(x)$	.4096	.4096	.1536	.0256	.0016

5.19 (b) .5

5.21 $f(2) = .2$, $f(4) = .2$, $f(6) = .2$

5.25 (a) .94 (b) .47 (c) .41

5.27 (a) .45 (b) .25
(c) The capacity must be increased by 2 to a total of 4.

5.29 (b) $E(X) = 3$, $\sigma = 1$

5.31 \$931.20

5.33 $\mu = 2.4$, $\sigma = .980$

5.35 (c) \$113,500

5.37 (a) $E(X) = 1.381$ (b) $\sigma = 1.318$

5.41 (b)

x	0	1	2	3	4	Total
$f(x)$	.2401	.4116	.2646	.0756	.0081	1.0000

(c) $E(X) = 1.2$

5.43 Median = 3

5.45 (a) Trials are dependent
(c) Bernoulli model plausible

5.47 Identify S = yellow, F = other colors
(a) Yes, $p = .48$ (b) No (c) No

5.49 (a) Yes, $P(S) = .5$

(b) Not independent

5.51 (a) Bernoulli trials model is not appropriate.
(b) Appropriate

5.53 (a) .2401 (b) .0081

5.55 (a) .00729 (b) .0729

5.57 (c)

x	0	1	2
$f(x)$	$\frac{81}{256}$	$\frac{126}{256}$	$\frac{49}{256}$

5.59 (a) Yes, $n = 10$, $p = \frac{1}{6}$
(b) No, because the number of trials is not fixed.

5.61 (b) .132

5.63 (a) .9850 (b) .1265 (c) .3255

5.65 (a) .7383 (b) .6836 (c) 3

5.69 (a) .233 (b) .014 (c) .014

5.71 (a) .837 (b) .901

5.73 (a) Mean = 15 and sd = 1.936 (c) Mean = 12 and sd = 2.898

5.75 .411

5.77 Mean = 49.05, sd = 6.681

5.79 (b) Mean = 12.308, sd = 2.919

5.81 (a) $P[X \le 4 \text{ or } X \ge 13] = .072$
(b) $P[X \le 3 \text{ or } X \ge 12] = .073$

5.83 (c) The proportions for all restaurants are "in control."

5.85 The point .917 for this restaurant is "out of control."

5.87 (b)

x	0	1	2
$f(x)$	.400	.467	.133

5.89

w	0	1	2
$f(w)$	.405	.470	.125

5.91 (a) The probability distribution is given in the first two columns.

x	$f(x)$	$x f(x)$	$x^2 f(x)$
0	$\frac{1}{12}$	0	0
1	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{5}{12}$
2	$\frac{5}{12}$	$\frac{10}{12}$	$\frac{20}{12}$
3	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{9}{12}$
Total	1.00	1.50	$\frac{34}{12}$

(b) $\mu = 1.5$ and $\sigma^2 = 7/12 = .583$

5.93 (a) $\mu = 1.0$ (b) $\sigma^2 = .6$

5.95 (a) .002 (b) −$0.60

5.97 (a) .30 (b) $\sigma = 1.342$

5.99 (a)

x	0	1	3
$f(x)$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{1}{6}$

(b) 1.0

5.101 (a)

x	−15	−5	5	15
$f(x)$	.1458	.3936	.3543	.1063

(b) −$0.79 (c) No

5.103 (a) $F(3) = .44$, $F(4) = .72$, $F(5) = .90$, $F(6) = 1.00$

5.105 $\mu = 3.667$, $\sigma = .943$

5.107 (a) Not plausible, because the assumption of independence is questionable.
(b) Plausible

5.109 .125

5.111 .146

5.113 (a) Binomial distribution with $n = 6$, $p = .4$
(b) .821, .047, 2.4

5.115 (b) 1.759 (c) .22

5.117 $P[X \le 10] = .048$. This small probability casts doubt on the claim that $p = .7$

5.119 (c) $E(X) = 2.0$ and $\text{Var}(X) = 1.2$

5.121 $P[X \ge 10] = .048$, $P[X = 10] = .031$

5.123 (a) .598 (c) $\mu = 3.2$ and $\sigma = 1.6$

CHAPTER 6

6.1 (c) Probability density function

6.3 The interval 1.5 to 2 has higher probability.

6.5 Median = 1.414

6.9 (c) $Z = \dfrac{X - 161}{5}$

6.11 (b) $Z = \dfrac{X - 250}{6}$

6.13 (a) .7967 (c) .1515

6.15 (c) .8907 (d) .0510

6.17 (b) .8164 (d) .0695

6.19 (a) −.842 (c) $z = .97$

6.21 (a) $z = -.93$ (d) $z = .753$

6.23 (a) .6293 (d) $z = .524$

6.25 (a) .3446 (c) .2119
(e) .9093
6.27 (a) $b = 67.64$
6.29 (a) .0668 (c) .9198
6.31 .8468
6.33 .2024
6.35 (a) .0128
6.37 (a) (iii) .3830 (b) 16.5 to 17.5
6.41 (a) Normal .0236
(c) With continuity correction .7025
6.47 (a) .0923 (b) .0728
6.49 .1562 using continuity correction
6.51 .6004 no continuity correction
6.55 (a) 0.5
(b) First quartile = .25, second quartile = .5, and third quartile = .75
6.59 (a) $z = -1.38$ (c) $z = .64$
6.61 (a) .2676 (c) .0026
6.63 (a) .8371 (c) .8057
(e) .6096
6.65 (a) .2000 (c) .2047
6.67 .1057
6.69 (a) 0.5793
6.71 (b) 305.3 ounces
6.73 (b) (i) .1537 using continuity correction
6.75 (a) .0281 using continuity correction
6.77 (a) .0401 using continuity correction
6.79 (b) .0418 with continuity correction

CHAPTER 7

7.1 (a) Statistic
(c) Parameter
7.5 (b)

$\bar{x}$	Probability $f(\bar{x})$
3	$\frac{1}{9}$
4	$\frac{2}{9}$
5	$\frac{3}{9}$
6	$\frac{2}{9}$
7	$\frac{1}{9}$
Total	1

7.7 No, very best pictures become sample.
7.9 (a) $X = 4$ for 6 dots
7.11 (a) $E(\bar{X}) = \mu = 83$, s.d.$(\bar{X}) = 19$
7.13 (a) .3224 (c) .0806
7.15 $E(\bar{X}) = 5$, s.d.$(\bar{X}) = \sqrt{\frac{4}{3}}$
7.17 (b) $E(\bar{X}) = 1.0$ (c) mean = 1.0, variance = $\frac{.6}{36}$ = .0167
7.19 (a) $E(\bar{X}) = 115$ (b) sd $(\bar{X}) = 8.98$
(c) $N(115, 8.98)$
7.21 (a) .1587 (c) .0786
7.23 (a) $\bar{X}$ is approximately $N(41{,}000, 500)$. (b) .1587
7.27 .681
7.31 (b) $\sigma = 1.136$
7.33 (b) more variability in two months
7.35 (b) Sampling distribution of R:

Value of R	Probability
0	4/16
2	6/16
4	4/16
6	2/16
Total	1

7.37 (c) mean = 2 variance = $\frac{.4}{81}$ = .0049
7.39 (b) Exactly a normal distribution
(c) .7066
7.41 (a) $N(12.1, (3.2)/3)$ (c) About 26% smaller than 10
7.43 (a) mean = 1.9, sd = .20 hour
(b) Approximately $N(1.9, .20)$
7.45 (a) $E(\bar{X}) = 4.660$ and s.d.$(\bar{X}) = .1289$
(b) Approximately $N(4.660, .1289)$
7.47 (b) $\sigma = .781$ (d) .095
7.49 (b) 305.29

CHAPTER 8

8.1 (a) S.E. = 5.814, 98% error margin = 13.55
(c) S.E. = 8.165, 90% error margin = 13.43
8.3 (a) $\bar{x} = 6.167$, estimated S.E. = .365
(c) $\bar{x} = 6.6$, estimated S.E. = .313

8.5 $\bar{x} = 95.74$, 90% error margin $= 5.73$
8.7 (a) 2.143 (b) 3.525
8.9 (a) $n = 246$ (b) $n = 171$
8.13 $n = 68$
8.15 (2.03, 3.14)
8.19 (28.94, 31.46) grams
8.21 (17.1, 19.5) days
8.23 (a) (3.604, 3.996)
8.25 (8.00, 9.20) miles
8.27 (88.91, 102.57)
8.29 (−.00497, .00157)
8.31 (1.521, 1.909) centimeters
8.33 (b) In long run, covers with proportion .95
8.35 (b) Yes, in the middle (d) No
8.37 (a) $H_0: \mu = 14, H_1: \mu < 14$ days
(b) $H_0: \mu = 4.50, H_1: \mu > 4.50$ dollars
8.39 (a) Correct decision if $\mu = 14$. Wrong decision if $\mu < 14$, Type II error.
(b) Correct decision if $\mu = 4.50$ dollars. Wrong if $\mu > 4.50$, Type II error.
8.43 (a) a $= .025$ (c) $c = 41.1$
8.45 $Z = 2.26.$, fail to reject H_0 (b) Fail to reject H_0 when $\mu > 2.0$.
8.47 (a) Reject H_0 (c) .01 (e) .005
8.49 (a) $Z = 1.58$, reject H_0 (b) Falsely reject $H_0: \mu = 3.5$
8.51 (a) $H_0: \mu = 183, H_1: \mu > 183$ days
(b) $Z = 1.88$, reject H_0 (c) Reject H_0 when $\mu = 183$
8.53 $H_0: \mu = 3.0, H_1: \mu > 3.0$,
$R: Z \geq 1.645$
Observed $z = 1.44$; H_0 is not rejected.
8.55 $H_0: \mu = 3000, H_1: \mu \neq 3000$,
$R: |Z| \geq 1.96$
Observed $z = 2.13$; H_0 is rejected at $\alpha = .05$.
8.57 (a) (i) p = probability that reading is a favorite leisure time activity of a randomly selected adult. (ii) $\hat{p} = .299$ (iii) .029
(b) (i) p = probability that a randomly selected pet owner buys their pet holiday presents. (ii) $\hat{p} = .666$ (iii) .044
8.59 (a) $n = 632$ (c) $n = 752$
8.61 (b) (.49, .53).
8.63 (a) $\hat{p} = .287$ (b) S. E. $= .0467$
(c) (.178, .396)
8.65 (a) $H_0: p = .26$ and $H_1: p > .26$
8.67 (c) (i) $H_0: p = .60$ versus $H_1: p \neq .60$
(ii) $\dfrac{\hat{p} - .6}{.0558}$ (iii) $R: |Z| \geq 2.33$
8.69 (a) $H_0: p = .30$ versus $H_1: p > .30$
(b) $R: Z = \dfrac{\hat{p} - .3}{\sqrt{.3 \times .7/60}} \geq 1.645$
(c) Fail to conclude that more than 30% have posters. (d) reject H_0
8.71 (b) P−value $= .0244$
8.73 $\hat{p} = .6225$ so $Z = 2.915$ and P−value .002. Strong evidence that p greater than .55
8.75 (a) (.47, .55) (b) (3889, 4490)
8.77 (.58, .85)
8.79 (a) $\bar{x} = 85.022$ (b) S.E. $= 1.201$
(c) 2.354
8.81 (b) A factor of 16
8.83 (a) 2.8 (b) (124.6, 129.2)
8.85 (a) Correct
8.87 The alternative hypothesis H_1 is the assertion that is to be established; its opposite is the null hypothesis H_0.
(a) $H_0: \mu = 50, H_1: \mu < 50$
(d) $H_0: \mu = 16, H_1: \mu \neq 16$
8.89 (a) μ = mean number of days toothpaste lasts.
(c) $R: Z = \dfrac{\sqrt{n}(\bar{X} - 30.5)}{s} \geq c$
(d) $Z = 2.51$ reject H_0 (e) .006
8.91 (b) P−value $.014/2 = .007$ reject H_0.
8.93 (b) $\sigma = .781$
(c) $H_0: \mu = .7$ versus $H_1: \mu > .7$,
$Z = \dfrac{\bar{X} - .7}{S/\sqrt{64}}$, $R: Z \geq 1.645$.
Observed $z = 2.80$; H_0 is rejected at $\alpha = .05$.
8.95 (a) $\hat{p} = .0875$ (b) .0124
8.97 (a) (.190, .255)
8.99 (c) $R: Z = \dfrac{\sqrt{137}(\hat{p} - .25)}{\sqrt{(.25 \times .75)}} \geq c$
(d) $Z = 1.92$, reject H_0 (e) .027

8.101 (a) $H_0: p = .8 \quad H_1: p \neq .8$

$Z = \dfrac{\hat{p} - .8}{\sqrt{(.8 \times .2)/200}}$, $R: |Z| \geq 1.96$,

Observed $z = -4.24$, H_0 is rejected at $\alpha = .05$.

P–value $P[|Z| \geq 4.24] < .0001$ is extremely small. The genetic model is strongly contradicted.

(b) (.62, .74)

8.103 (a) larger must cover more often, (.260, .303) (b) reject H_0 at level $\alpha = .05$.

8.105 (a) (1.55, 1.88) (b) Observed $z = -2.464$, reject H_0: $\mu = 1.9$.

8.107 (90.54, 106.16)

CHAPTER 9

9.1 (a) 2.015 (b) −2.160

9.3 (b) 3.747 (c) −1.717

9.5 (a) 2.015 (b) 2.120

9.7 (b) Between .025 and .05

(c) Between .05 and .10

9.9 (a) (72.75, 79.45)

(b) Center = 76.1, Length = 6.7

(c) Usually different since S is random

9.11 (128.17, 147.03)

9.13 (3.2, 4.0) kg

9.15 (a) $\bar{x} = 22.4$, $s = 5.98$

(b) (17.7, 27.1)

9.17 (5.372, 5.594)

9.19 H_0: $\mu = 800$, $H_1: \mu > 800$, $T = \dfrac{\bar{X} - 800}{S/\sqrt{10}}$, d.f. = 9, $R: T \geq 1.833$

Observed $t = 2.02$; H_0 is rejected at $\alpha = .05$.

9.21 (a) Cannot tell, mean unknown

(b) In the long run, 95% of the intervals obtained using this procedure will cover.

9.23 (b) (23.38, 29.86) feet

9.25 $H_0: \mu = 128$, H_1: $\mu > 128$

$T = \dfrac{\bar{X} - 128}{S/\sqrt{20}}$, d.f. = 19,

$t_{.05} = 1.729$,

$R: T \geq 1.729$

Observed $t = 2.13$, H_0 is rejected at $\alpha = .05$.

9.27 $H_0: \mu = 83$, $H_1: \mu \neq 83$ pounds.

$T = \dfrac{\bar{X} - 83}{S/\sqrt{8}}$, d.f. = 7,

$t_{.025} = 2.365$, $R: T \leq -2.365$ or $T \geq 2.365$

Observed $t = -2.566$; H_0 is rejected at $\alpha = .05$.

9.29 (b) (83, 89) (c) normal population

9.31 (b) Reject H_0 when it is true

(d) When H_0 prevails, H_0 will be rejected in about proportion .05 of the times.

9.33 (a) H_0 is rejected at $\alpha = .05$.

9.35 (a) (1.28, 12.28)

(c) Observed $t = -3.53$; H_0 is rejected at $\alpha = .05$.

9.39 (a) $\chi^2_{.10} = 23.54$

(b) $\chi^2_{.95} = 12.34$

(c) $\chi^2_{.10} = 51.81$

(d) $\chi^2_{.95} = 2.73$

9.41 (a) .773 (b) (.566, 1.220)

9.43 H_0: $\sigma = .6$, $H_1: \sigma < .6$

$\chi^2 = \dfrac{(n - 1)S^2}{(.6)^2}$, d.f. = 39,

$R: \chi^2 \leq \chi^2_{.95} = 25.7$

Observed $\chi^2 = 24.4$; H_0 is rejected at $\alpha = .05$.

9.45 (1.56, 4.15)

9.47 (a) (15.99, 27.60)

9.49 (4.70, 10.83)

9.51 (a) 2.015 (c) −2.015

9.55 (39.4, 54.6)

9.57 (a) $\bar{x} = 3.007$, 95% error margin = .298

(b) (2.763, 3.251)

9.59 (a) H_0: $\mu = 42$, H_1: $\mu < 42$

(b) $T = \dfrac{\bar{X} - 42}{S/\sqrt{21}}$, d.f. = 20

(c) $R: T \leq -t_{.01} = -2.528$

(d) Observed $t = -3.23$, H_0 is rejected at $\alpha = .01$.

9.63 (a) $H_0: \mu = 65, H_1: \mu > 65$

(b) $T = \dfrac{\overline{X} - 65}{S/\sqrt{9}}$, d.f. $= 8$,

$t_{.05} = 1.860, R: T \geq 1.860$.

(c) Observed $t = .18$; fail to reject H_0. The claim is not demonstrated.

9.65 $H_0: \mu = 1500, H_1: \mu > 1500$

$T = \dfrac{\overline{X} - 1500}{S/\sqrt{5}}$, d.f. $= 4$, $t_{.05} = 2.132$, $R: T \geq 2.132$. Observed $t = \dfrac{1620 - 1500}{90/\sqrt{5}} = 2.98$;

H_0 is rejected at $\alpha = .05$.

9.67 (a) (423.09, 456.05)

(b) $t = -1.99$, reject H_0

9.69 (a) $\chi^2_{.05} = 14.07$ (c) $\chi^2_{.95} = 2.17$

9.71 (a) $H_0: \mu = 55.0$ versus $H_1: \mu \neq 55.0$

(b) $T = \dfrac{\overline{X} - 55.0}{S/\sqrt{n}}$

(c) $R: T \geq c$ or $T \leq -c$

(d) $t = 2.75$ and $c = 2.228$ so reject H_0 at level $\alpha = .05$.

9.73 (a) $\chi^2 = \dfrac{(n - 1)S^2}{324}$, d.f. $= 9$,

$R: \chi^2 \leq 3.33$

Observed $\chi^2 = 3.25$; H_0 is rejected.

9.75 (a) Cannot tell, mean unknown

(b) In the long run, 95% of the intervals obtained using this procedure will cover.

9.77 $t = 2.25$, reject H_0

CHAPTER 10

10.1 First group, using first letter of subject,
{B, C}, {B, E}, {B, H}, {B, P}
{C, E}, {C, H}, {C, P}
{E, H}, {E, P}
{H, P}

10.3 (a) Using first letter of name,
{(S, G), (T, E)} {(S, G), (T, R)}
{(S, G), (E, R)} {(S, J), (T, E)}
{(S, J), (T, R)} {(S, J), (E, R)}
{(J, G), (T, E)} {(J, G), (T, R)}
{(J, G), (E, R)}

(b) There are three sets each consisting of three pairs.

10.5 (a) $\bar{x} - \bar{y} = 7$, S.E. $= 2.48$

(b) (2.1, 11.9)

10.7 $H_0: \mu_1 - \mu_2 = 0$,
$H_1: \mu_1 - \mu_2 \neq 0, R: |Z| \geq 1.96$
Observed $z = -4.17$; null hypothesis is rejected at $\alpha = .05$.

10.9 μ_1 = population mean sentence length for magazine 1..

(a) $H_0: \mu_1 - \mu_2 = .2, H_1: \mu_1 - \mu_2 > 2$

(b) $Z = \dfrac{\overline{X} - \overline{Y}}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$, $R: Z \geq -1.645$

(c) Observed $z = 1.69$, H_0 is rejected, $P-$value $= .0455$

10.11 H_0 is rejected at $\alpha = .05$.

10.13 (a) $s^2_{pooled} = 3.20$ (b) 1.789

(c) $t = -1.46$

10.15 (a) $s^2_{pooled} = 2.400$ (b) $t = 3.35$ and $P-$value $= .001$, reject $H_0: \mu_1 - \mu_2 = 0$ (c) (.77, 3.23)

10.17 $H_0: \mu_1 - \mu_2 = 0$,
$H_1: \mu_1 - \mu_2 > 0$,

$Z = \dfrac{\overline{X} - \overline{Y}}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$ Observed $z = 2.91$,

$P-$value $= P[Z \geq 2.91] = .0018$.
Strong evidence in support H_1.

10.19 (a) $t = -1.81$, d.f $= 18$, reject $H_0: \mu_1 - \mu_2 = 0$ (c) $(-9.09, .69)$

10.21 Cannot reject $H_0: \mu_1 - \mu_2 = 4.1$

10.23 (a) Diet and other exercise.

10.25 (a) $t = 3.34$ with d.f $= 11$, reject $H_0: \mu_1 - \mu_2 = .80$ at $\alpha = .05$

10.27 (a) Reject H_0. Evidence is strong since $P-$value $= .004$.

(b) $t = 2.43$, reject H_0

10.29 We drew slips with α, β, τ so group 1 is {alpha, beta, tau}.

10.33 (a) $t = 2.07$ (b) d.f. $= 5$

10.35 (a) $t = 1.46$, the claim is not established at $\alpha = .05$.

10.37 (a) Observed $t = 2.87$; H_0 is rejected at $\alpha = .05$.
(b) $(1.52, 7.90)$.

10.39 $H_0\colon \mu_D = 0$, $H_1\colon \mu_D \neq 0$, $T = \dfrac{\overline{D}}{S_D/\sqrt{n}}$, d.f. $= 8$, $R\colon |T| \geq 2.306$
Observed $t = 1.49$; H_0 is not rejected. The difference is not significant.

10.43 (a) $t = 8.25$, d.f $= 9$, P– value less than .001 (b) $(60.27, 60.47)$

10.45 (a) $(.17, .36)$
(b) $z = 4.56$ reject H_0 at level $\alpha = .05$, P– value less than .0001

10.49 $H_0 : p_1 = p_2$, $H_1 : p_1 > p_2$ (suffix 1 refers to abused group), $R\colon Z \geq 2.33$. Observed $z = 3.01$, H_0 is rejected at $\alpha = .01$. There is strong evidence in support of the conjecture.

10.51 $H_0 : p_1 = p_2$, $H_1 : p_1 \neq p_2$ (suffix 1 refers to "with carbolic acid"), $R\colon |Z| \geq 1.96$.
Observed $z = 2.91$; H_0 is rejected at $\alpha = .05$. P–value $= .0036$.

10.55 (a) $H_0 : p_1 = p_2$, $H_1 : p_1 > p_2$ (suffix 1 refers to "smokers").
Observed $z = 4.29$, P–value less than .00001. The conjecture is strongly supported.

10.57 (b) $(-.14, -.08)$

10.59 (a) $(.17, .38)$
(b) $(-.045, .103)$

10.61 $(3.89, 6.11)$

10.63 (a) $H_0\colon \mu_1 - \mu_2 = -150$ versus $H_1 : \mu_1 = \mu_2 < -150$
(c) $z = -1.79$, so reject H_0 at $\alpha = .05$, P–value $= .0367$.

10.65 (a) $(.13, 1.09)$
(b) $(4.31, 4.97)$

10.67 (a) 4.857 (b) $t = 2.03$, d.f. $= 7$

10.69 $(-10.0, 42.0)$, d.f. $= 11$

10.71 (a) $H_0 : \mu_1 - \mu_2 = 0$, $H_1 : \mu_1 - \mu_2 \neq 0$ (suffix 1 refers to "city A"), $R\colon |Z| \geq 2.33$.
Observed $z = -4.97$; reject H_0.
(b) $(-7.93, -2.87)$

10.73 (a) Do not reject H_0 (b) $(-.026, .630)$

10.75 (a) $t = 2.00$, fail to reject H_0
(b) $(.68, 18.65)$

10.77 Denote growth with additive (x).
$$t = \frac{d}{s_d/\sqrt{n}} = 2.93 \quad \text{d.f.} = 9$$
greater growth established at $\alpha = .05$.

10.79 (a) $t = -1.95$, reject H_0.

10.81 $H_0 : p_1 = p_2$, $H_1 : p_1 < p_2$ (suffix 1 refers to "seeded"), $R\colon Z \leq c$.
$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$
Observed $z = -1.77$, P–value $= P[Z \leq -1.77] = .0384$. Fairly strong evidence in support of the conjecture.

10.83 (a) $H_0 : p_1 = p_2$, $H_1 : p_1 > p_2$ (suffix 1 refers to "uremic"), with $\alpha = .01$, the rejection region is $R\colon Z \geq 2.33$. Observed $z = 2.64$, reject H_0 at $\alpha = .01$. Strong evidence of higher incidence.
(b) $(.05, .29)$.

10.85 (a) Fail to reject H_0
(b) $(-500.2, -12.2)$

10.87 (a) Do not pool (b) $(-58.0, -16.0)$

10.89 $z = 2.961$, reject H_0

10.91 $z = 13.13$, reject H_0

CHAPTER 11

11.1 Intercept $= 3$, slope $= 2$

11.3 (a) $x =$ duration of training, $y =$ measure of performance
(c) $x =$ level of humidity, $y =$ growth rate of fungus

11.5 $\beta_0 = 6$, $\beta_1 = 3$ and $\sigma = 3$

11.7 (a) $E(Y) = -1$, s.d.$(Y) = 4$

11.11 (b) No, only the mean is higher.

11.13 (d) The fitted line is $\hat{y} = 8.5 - 1.5x$
11.15 (b) SSE $= 5.50$
(c) $s^2 = 1.375$
11.17 (b) $\hat{\beta}_1 = .962$, $\hat{\beta}_0 = 2.154$
11.19 (a) $\hat{y} = 2.751 + .809\,x$
(c) $s^2 = 21.30$
11.21 (a) $\hat{\beta}_1 = .5033$, $\hat{\beta}_0 = 89.61$
(c) $s^2 = 31.37$
11.25 (a) $\hat{\beta}_0 = 3.50$, $\hat{\beta}_1 = 2.50$, $s^2 = 3.833$
(b) $H_0: \beta_1 = 0$, $H_1: \beta_1 \neq 0$, $R: |T| \geq t_{.025} = 3.182$, d.f. $= 3$ Observed $t = 4.04$, reject H_0 at the 5% level.
(c) (8.94, 13.06).
11.27 (.530, 4.470)
11.29 (b) $\hat{y} = 41.58 + .8694x$
(c) (.72, 1.02)
11.31 (a) (.813, .942) (b) (.716, 1.038)
11.33 (a) $Y = \beta_0 + \beta_1 x + e$ and fit $\hat{y} = 994 + .10373x$ with $\widehat{\text{s.d.}(e)} = s = 299.4$.
(b) Observed t-ratio $= 3.48$ with P–value .002. Reject H_0: $\beta_1 = 0$ at the 5% level.
11.35 (a) $Y = \beta_0 + \beta_1 x + e$ and fit $\hat{y} = .3381 + .83099x$ with $\widehat{\text{s.d.}(e)} = s = .1208$.
(b) Observed t-ratio $= 9.55$ with P–value approximately zero. Reject H_0: $\beta_1 = 0$ at 5% level.
11.37 (a) $\hat{y} = 24.78 + 1.413x$
(c) (29.4, 31.4)
11.39 $r^2 = .380$
11.41 (a) $r^2 = .855$ (b) same
11.43 (a) $r^2 = .956$ (b) $r = .978$
11.45 Proportion explained $= r^2 = .799$.
11.49 $x =$ (leaf length) $\times$ (leaf width) is the area of a rectangle containing leaf. Slope should be less than one.
11.51 (b) SSE $= 15.66$ (c) $s^2 = 2.24$
11.53 (a) $\hat{y} = 73.18 - 2.214x$
(b) $H_0: \beta_1 = -2$, $H_1: \beta < -2$, $R: T \leq t_{.05} = -1.771$, d.f. $= 13$.
Observed $t = -.54$, fail to reject H_0.
11.55 (b) $H_0: \beta_1 = 0$, $H_1: \beta_1 > 0$, $R: T \geq t_{.05} = 1.943$, d.f. $= 6$.
Observed $t = 7.41$, reject H_0.
(d) (822.4, 985.8) dollars
11.57 (b) $\hat{y} = 17.64 - 1.416x$
(c) $(-1.803, -1.029)$
11.59 $r = -.925$, .856 explained
11.61 (a) $\hat{\beta}_0 = -1.071$, $\hat{\beta}_1 = 2.7408$
(c) Estimated S.E. $(\hat{\beta}_0) = 2.751$
(e) $r^2 = .828$
11.63 (a) $\hat{y} = -87.17 + 1.2765x$
(b) Observed $t = 5.92$, d.f. $= 16$; reject H_0: $\beta_1 = 0$.
11.65 (a) $\hat{y} = 24.96 + 3.306x$
(c) 74.55 degrees
11.67 (b) MARINGRW $= 478 - 0.236$ FRESHGRW

CHAPTER 12

12.1 (b) $\hat{y} = 3.92 - .53x$
(c) $r^2 = .842$, but relation is not a straight line.
12.3 (a) $y' = \dfrac{1}{y^{1/3}}$, $x' = x$
12.5 (b) No. beetles $= 6.14 - 0.899 \log_e$(distance)
(c) $(-1.52, -.28)$
12.7 $E(Y) = -11$
12.9 (a) (.81, 1.20)
(b) $R: T > t_{.05} = 1.68$ d.f $= 48$
$t = (.0166 - .0140)\ /\ .00107 = 2.43$, reject H_0
12.11 (a) $\hat{\beta}_1 = -3.22$, $\hat{\beta}_2 = -.0207$, and $\hat{\beta}_0 = 45.3$
(c) .795 (d) 11.58
12.13 (a) $R: |T| > t_{.025} = 2.086$ d.f $= 20$
$t = -3.2243/.4562 = -7.07$ reject H_0 (b) $t = -2.41$, reject H_0
12.15 (b) $\hat{y} = 46.55 - 11.77 \log_{10}(x)$, $r^2 = .749$

(c) $(-17.66, -5.88)$

12.17 (a) $\widehat{\log_{10} y} = -1.16 + .0305x$, $r^2 = .988$

12.19 (a) $(.151, .529)$
(b) $R: T > t_{.05} = 1.740$ d.f. $= 17$
$t = (.1826 - .10)/.0451 = 1.83$, reject H_0

12.21 (b) The residuals tend to increase over time. Errors dependent.

12.25 (a) $\hat{y} = 81.2 + 0.998x + 0.00993x^2$
(b) .998 (c) $R: T > t_{.05} = 1.860$ d.f. $= 8$ $t = .9983/.1389 = 7.19$, reject H_0

12.27 (a) $C3 = -.167 + .237C1$
$r^2 = .925$ (c) $(9.67, 11.32)$

12.29 (a) $\hat{y} = 50.4 + .1907x_2$ and $r^2 = .03$
(c) Even three variables do not predict well.

12.33 (b) $\hat{y} = 9.999 + .155(\text{gender}) + .9015(\text{initial no.})$ but P-value for gender large

CHAPTER 13

13.1 Observed $\chi^2 = 13.90$, d.f. $= 5$, $\chi^2_{.05} = 11.07$. The model of a fair die is contradicted.

13.3 Observed $\chi^2 = 2.00$, d.f. $= 3$, $\chi^2_{.05} = 7.81$. The model is not contradicted.

13.5 Observed $\chi^2 = 17.336$, d.f. $= 3$, $\chi^2_{.025} = 9.35$ Strong evidence of mislabeling.

13.7 Observed $\chi^2 = 12.44$, d.f. $= 5$, $\chi^2_{.05} = 11.07$. Reject H_0 at $\alpha = .05$.

13.9 (a)

x	0	1	2	3
Probability	.216	.432	.288	.064

(b) Observed $\chi^2 = 1.098$,. d. f. $= 2$, $\chi^2_{.05} = 5.99$. The binomial model is not contradicted.

13.11 (c) With $\alpha = .05$ and d.f. $= 1$, $\chi^2_{.05} = 3.84$
Observed $\chi^2 = 9.298$, reject H_0.

13.13 Observed $\chi^2 = 5.580$, d.f. $= 3$, $\chi^2_{.05} = 7.81$. The two brands are not shown to be significantly different in quality.

13.15 (b) Observed $\chi^2 = 9.780$, d.f. $= 3$, $\chi^2_{.05} = 7.81$. Reject H_0
(c) p_1: $(.53, .79)$

13.17 (a) Observed $z = 3.00$, reject H_0. P-value $= .0013$

13.21 Observed $\chi^2 = 15.734$, d.f. $= 2$, $\chi^2_{.05} = 5.99$. Reject H_0. The appeals decision and type of representation are dependent.

13.23 Observed $\chi^2 = 27.847$ d.f. $= 2$, $\chi^2_{.01} = 9.21$. Reject the null hypothesis of independence at $\alpha = .01$.

13.25 Observed $\chi^2 = 4.134$, d.f. $= 2$, $\chi^2_{.05} = 5.99$. Fail to reject the null hypothesis of independence.

13.27 (b) Observed $\chi^2 = 16.238$ d.f. $= 1$, $\chi^2_{.01} = 6.25$. Reject H_0.

13.29 Observed $\chi^2 = 16.44$, d.f. $= 9$, $\chi^2_{.05} = 16.92$. Fail to reject H_0.

13.31 Observed $\chi^2 = 4.977$, d.f. $= 3$, $\chi^2_{.10} = 6.25$. Fail to reject H_0. The stated conjecture is not contradicted.

13.33 Observed $\chi^2 = 48.242$, d.f. $= 1$, $\chi^2_{.01} = 6.63$. The null hypothesis of homogeneity is rejected even at $\alpha = .01$.

13.35 (a) Observed $\chi^2 = 12.053$ statistic, d.f. $= 3$, $\chi^2_{.05} = 7.81$. Reject H_0. Drugs are different.
(b) Drug 1: $(.32, .55)$
Drug 3: $(.26, .48)$

13.37 (a) Observed $z = 2.78$ and P-value $= .0027$. Strong evidence for H_1.
(b) $(.09, .48)$

13.39 Observed $\chi^2 = 0.153$, d.f. $= 2$, $\chi^2_{.05} = 5.99$. Not significant.

13.41 Observed $\chi^2 = 2.764$, d.f. $= 1$, $\chi^2_{.05} = 3.84$. Fail to reject H_0.

13.43 Observed $\chi^2 = 18.338$, d.f. $= 1$, $\chi^2_{.01} = 6.63$. Reject H_0.

CHAPTER 14

14.1 (b)–(d)

ANOVA Table

Source	Sum of Squares	d.f.
Treatment	28	3
Error	20	4
Total	48	7

14.3 (d)

ANOVA Table

Source	Sum of Squares	d.f.
Treatment	32	2
Error	22	8
Total	54	10

14.5

ANOVA Table

Source	Sum of Squares	d.f.
Treatment	30	3
Error	26	8
Total	56	11

14.7

ANOVA Table

Source	Sum of Squares	d.f.
Magazines	4414.80	2
Error	148196.39	57
Total	152611.19	59

14.9 (a) $F_{.10}(3, 5) = 3.62$
(b) $F_{.10}(3, 10) = 2.73$

14.11 $F_{.05}(5, 30) = 2.53$ and $F = 2.46$, fail to reject H_0.

14.13 $F_{.05}(2, 9) = 4.26$ and $F = 8.26$, reject H_0.

14.15 $F_{.05}(2, 8) = 4.46$ and $F = 5.82$, reject H_0.

14.17 (a) With 26 d.f., $t_{.0083} = 2.559$
(b) With df $= 26$, $t_{.005} = 2.779$

14.19 $t_{.00833} = 2.45$ with 75 d.f.
For $\mu_1 - \mu_2$: t interval $.90 \pm .90$, multiple-t interval $.90 \pm 1.33$

14.21 $\dfrac{t_{.05}}{t_{.005}} = .595$ for 15 d.f.

14.23 (b) Treatment SS $= 56$, Block SS $= 78$, Residual SS $= 64$
(c) Treatment d.f. $= 2$, Block d.f. $= 3$, and Residual d.f. $= 2 \cdot 3 = 6$

14.25

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F
Treatment	2	56	28	2.62
Blocks	3	78	26	2.44
Error	6	64	10.667	
Total	11	198		

Fail to reject equal treatment means. Blocks not significant.

14.27 (a) Randomize the position of the loaves in the oven.
(b) $F_{.05}(2, 8) = 4.46 > 3.92$, no significant treatment difference $F_{.05}(4, 8) = 3.84 < 5.31$, block effects are significant.

14.29 (a) $F_{.05}(5, 25) = 2.603 < 106.79$, treatments are highly significant. $F_{.05}(5, 25) < 6.11$, block effects are significant.

14.33 (a) $F_{.05}(7, 13) = 2.83$
(c) $F_{.10}(7, 12) = 2.28$

14.35 $F = 20.85$ with P–value .000

14.37 (b) Treatment SS $= 56$, Block SS $= 138$, Residual SS $= 32$
(c) Treatment d.f. $= 2$, Block d.f. $= 3$, Residual d.f. $= 2 \cdot 3 = 6$

CHAPTER 15

15.1 (a) Probability $= \frac{1}{15}$ for each rank collection.
(b) $P[W_B = 5] = \frac{2}{15}$

15.3 (a) $P[W_S \geq 39] = .063$
(c) $c = 66$

15.5 (a) $W_A = 2 + 4 + 5 = 11$
(b) $W_S = 1 + 3 = 4$
15.7 $W_S = 47$, reject H_0.
15.9 $W_S = 62$, fail to reject H_0.
15.11 (a) *BBBBBBBBBA*
(b) $P[W_A = 10] = .1$
15.13 .048
15.15 $S = 11$; with 7 ties, $n = 18$; do not reject H_0.
15.17 (a) .021 (c) $c = 31$
15.19 (b) $P[T^+ = t] = .125$ for $t = 0$, 1, 2 and 4, 5, 6
15.21 (a) $S = 11$; with $n = 13$, reject H_0.
(b) $T^+ = 83$, reject H_0.
15.23 $r_{Sp} = .80$
15.25 $z = -.60$, do not reject H_0 ($n = 10$ is not large).
15.27 $W_A = 12$
15.29 (a) Each triple has probability $\frac{1}{10}$
(b) $P[W_A = 8] = .2$
15.31 (a) .098 (c) $c = 21$
15.33 $W_S = 51 \leq 69$, reject H_0.
15.35 (a) $r_{Sp} = -0.524$
(b) Do not reject independence.
(c) $\alpha = .05$ ($n = 8$ may be too small)
15.37 (a) $S = 4$ (b) $T^+ = 14$

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